DESIGN AND CALIBRATION OF AN AIR EJECTOR TO OPERATE AGAINST VARIOUS BACK PRESSURES

by

Ronald G. A. Chisholm

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UTIA TECH. NOTE NO. 39
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SUMMARY

As part of a wind tunnel investigation of GETOL aerodynamics, an air ejector was designed and built to act as a "step-down transformer" between a high pressure air supply and the models. It consisted of a central primary jet discharging into a constant area mixing tube. The thrust and mass augmentation of this ejector were determined for various mixing tube back pressures. These experiments were carried out for a convergent primary nozzle and a supersonic one and for two diameters of the mixing tube with the latter nozzle. The effect of the primary mass flow on the mass and thrust augmentation was obtained for a sonic primary.

A comparison between the experimental results and those predicted by a theory developed in this paper was made. The agreement between theory and experiment was generally within ten percent except when the mixed velocity profile was very non-uniform.
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LIST OF SYMBOLS, SUBSCRIPTS AND SUPERSCRIPTS

Symbols

\( P \) pressure, p.s.f.a.
\( \varrho \) density, slubs/ft\(^3\)
\( T \) temperature, \(^{\circ}\)F absolute
\( A \) area, ft\(^2\)
\( V \) velocity, ft/sec
\( M \) Mach no.
\( a \) speed of sound, ft/sec.
\( \gamma \) adiabatic exponent
\( R \) gas constant; 1715 \( \frac{ft^2}{sec^2 \degree R} \)
\( j \) mechanical equivalent of heat; 778 ft.lb/B. t. u.
\( h \) enthalpy or specific heat content of gas, \( C_p T \), B. t. u. /lb.
\( C_q \) discharge coefficient of jet
\( G \) mass flow, slugs/sec.
\( W \) weight flow, lbs/hr.
\( F \) impulse function, lbs.
\( J \) momentum flux, lbs.
\( T_h \) thrust, lbs.
\( \mu \) mass augmentation \(- \frac{G_3}{G_1}\)
\( \psi \) thrust augmentation
\( C_f \) local skin friction coefficient
\( \Theta \) angle
\( \lambda \) flow parameter \(- \frac{V^2}{V^1}\)
\( \frac{L}{D} \) ratio of the mixing tube length to its diameter

\( \rho \) dynamic pressure, p.s.f.a.

\( r \) radius

**Subscripts**

1 exit plane of primary jet in shroud

j exit plane of primary jet without shroud

2 exit plane of secondary flow

3 exit plane of mixing tube

4 exit plane of nozzle or diffuser

"oi" stagnation conditions at point"i"

a ambient or atmospheric conditions

p conditions at plane P - primary measuring station

S inner surface of mixing tube

m maximum flow quantity

M conditions at the inlet plane of mixing

**Superscript**

* sonic conditions
1. INTRODUCTION

As some experimental work in the UTIA subsonic wind tunnel on ground effect was anticipated using a wing with a peripheral jet, a study of the laboratory compressed air supply was made and revealed that the flow from the reservoir would have too large a stagnation pressure. This meant that if ducted directly to the model wing, the flow when exhausted to the atmosphere through the slots would be sonic and badly overchoked. Also, if the flow were brought into the model at this high stagnation pressure, sealing problems would arise in addition to the heavy losses of total head that would occur in dumping the flow into the plenum chamber of the wing. Moreover, it was desired, for reasons of similitude, to have the jet leave the slots at the leading and trailing edges of the model over a range of subsonic speeds. It was therefore decided to utilize a device which would reduce the velocity of the air jets and at the same time not lose any of the thrust available from the flow's large total head. Hence, the choice of an air ejector.

However, the design of an air ejector to give a maximum thrust augmentation is a difficult one in view of the number of variables involved. If one examines the applications of the ejector principle of using a primary jet with a large kinetic energy to induce a secondary stream to flow through a constant area mixing tube, one finds they fall into two main categories. The first of these has a low ratio of the secondary area to the primary and consequently a low mass augmentation. This is compensated by their ability to pump a secondary stream at a low stagnation pressure to a reservoir at a much higher pressure. Hence the name 'jet pump' may be applied to these.

The second type of ejector is one which has a large ratio of the secondary area to the primary and consequently has a large mass augmentation. Furthermore, it is not able to pump the secondary stream against as large a pressure rise as the first kind. It has many fan-type applications and might be classified under the heading 'mass augmentors'.

However, one might introduce a third class of ejectors, which on the basis of the ratio of secondary area to the primary, and their pumping abilities, might fall somewhere between these other two. This third group would include those ejectors so designed as to give a maximum thrust augmentation. In general, thrust augmentation depends on the geometry of the ejector, the flow properties of the primary and secondary streams and the exit conditions at the end of the mixing tube. But experimental studies have been concerned more with mass augmentation and pumping qualities than with thrust augmentation. In almost all of the analytical solutions for the flow through an ejector the continuity, momentum and energy equation are written before and after mixing. Very few attempts have been made to determine the effect of changing the ejector geometry and the secondary velocity profile on thrust augmentation. Von Karman has considered the problem only
for incompressible flow and has shown the effect of the secondary inlet velocity profile. However, further experimental evidence is required.

Although no attempt is made to indicate the optimum design for thrust augmentation the author wished to bring some attention to this problem. As our ejector was expected to operate under a load due to the losses in the ducting and a slight contraction of the flow on entering the model a number of tests were made with various nozzles attached to the end of the mixing tube. In this manner the performance of the ejector was deduced from a knowledge of the primary mass flow, the static pressure of the secondary flow measured at the plane of mixing, and the mixed-flow velocity profile. These tests showed how the mass and thrust augmentation decreased with an increasing load on the ejector.

However, as the load imposed by the model wing's ducting greatly reduced the mass and thrust augmentation, the mixing tube diameter had later to be reduced from 4 in. to 2.75 in. to allow for a diffuser section to be fitted to the end of the mixing tube. The pressure rise across this diffuser was approximately equal to the pressure drop through the model wing ducting so that the mixing tube operated nearly as if it were exhausting to the atmosphere.

2. COMPRESSIBLE THEORY FOR EJECTORS

2.1 Introductory Comments

Before proceeding to write the equations of fluid mechanics and thermodynamics that govern the flow through an ejector some mention of the way that flow properties vary in an ejector is necessary in order to understand its function as a pumping device.

For simplicity, consider a duct of constant area. Into this duct a jet having a high velocity is injected and as it expands it entrains the surrounding air by a viscous process, causing a local region of low pressure. In order to appreciate the pumping action of an air ejector more fully, it will be assumed that there is no chemical interaction between the primary and the secondary fluids, and the effects of the skin friction at the wall will be ignored.

The conservation of momentum states, assuming the static pressure and the density are constant across any section, that

\[ \frac{P}{s} A + \int V^2 dA = \text{constant} \]

Let \( V = \overline{V} + V_1 \) where \( \overline{V} \) = mean velocity and \( V_1 \) = the deviation from the mean. Substituting this into \( \int V^2 dA \) gives

\[ \frac{P}{s} A + \overline{V}^2 A + \int V_1^2 dA = \text{constant} \]
This equation reveals one of the underlying principles observed in an ejector. Initially the velocity profile is very peaked and the integral \( \int V^2 dA \), which depends on the non-uniformity of the profile, is a maximum. Since the middle term is a constant because of the continuity, the pressure is at its lowest value near the inlet of the duct. As the flow mixes along the duct \( \int V^2 dA \) diminishes and there is a conversion of momentum to pressure until at the point where the mixing is complete the flow is uniform and as \( \int V^2 dA = 0 \) the pressure is a maximum. This rising pressure as the flow travels along the mixing tube is exemplified by some static pressure measurements along the tube (see Fig. 18). The effect of adding a diffuser to the end of the constant area duct is to reduce the pressure at any position along the duct but increase the mean velocity. The effect of a nozzle or a load acting at the end of the duct is just the opposite.

In order to determine the magnitude of the change in pressure between the inlet and the outlet of the duct, one must specify a relationship for the velocity over the cross-section. Physically, this distribution will be determined by the ratio of the secondary area to the primary, by the primary and secondary stagnation conditions and the mixing tube exit conditions.

The momentum equation may also be written

\[
\frac{p}{\rho} + \lambda V^2 = \text{constant}
\]

where

\[
\lambda = \frac{V^2}{\bar{V}^2} = \frac{A \int V^2 dA}{[\int V dA]^2}
\]

The flow parameter \( \lambda \) is a measure of the degree of non-uniformity of the flow and is unity when the velocity profile is uniform.

The pressure rise across the duct assuming the flow is completely mixed at the duct outlet (i.e. \( \lambda_3 = 1 \)) is

\[
P_3 - P_M = \rho \bar{V}^2 (\lambda_M - 1)
\]

From this expression we can see that a very non-uniform velocity profile at station M or \( \lambda_M \) very large produces a large pressure rise in the mixing tube.

In order to simplify the analysis a stepwise velocity distribution is assumed at the inlet. That is, the secondary and primary velocity profiles are assumed uniform over their respective areas but a discontinuity exists at the initial plane of contact between the two flows. For large primary velocities this distribution is usually obtained physically.
Hence the flow parameter \( \lambda \) may be written

\[
\lambda_M = \frac{\sqrt{V_M^2}}{\sqrt{V_M^2}} = \frac{A_1 V_1^2 + A_2 V_2^2}{A_3} \left( \frac{A_1 V_1 + A_2 V_2}{A_3} \right)^2
\]

or

\[
\lambda_M = \frac{(A+1)(1 + AV^2)}{(1 + AV)^2}
\]

where \( \frac{A_2}{A_1} = A \) and \( \frac{V_2}{V_1} = V \).

This has its largest value when \( V = 0 \) or \( V_1 \to \infty \) then

\[
\lambda_M = A+1 = \frac{A_2}{A_1} + 1
\]

Using the stepwise velocity distribution we get for the pressure rise across the duct

\[
P_3 - P_M = \frac{\rho}{(A+1)^2} \left( V_1^2 - V^2 \right)
\]

\[
= \frac{\rho}{(A+1)^2} \left[ \frac{A_1 V_1^2 + A_2 V_2^2}{A_3} - \left( \frac{A_1 V_1 + A_2 V_2}{A_3} \right)^2 \right]
\]

\[
= \frac{\rho A_1^2 V_1^2}{A_3^2} \left[ \frac{A_3}{A_1} + \frac{A_3}{A_1} \frac{A_2}{A_1} (\frac{V_2}{V_1})^2 - (1 + 2 \frac{A_2}{A_1} \frac{V_2}{V_1} + (\frac{A_2}{A_1})^2 (\frac{V_2}{V_1})^2) \right]
\]

or

\[
P_3 - P_M = \frac{\rho}{(A+1)^2} \frac{V_2^2 A}{(1 - V)^2}
\]

The maximum pressure rise will exist when \( V = 0 \) or \( V_2 \to 0 \) then

\[
(P_3 - P_M)_m = \frac{\rho V_1^2 A}{(A+1)^2}
\]

This is a maximum when \( A = 1 \) and hence one wants a small value of \( \frac{A_2}{A_1} \) to obtain a maximum pressure rise.

For incompressible flow \( \rho_0 \to -1 \),

\[
\frac{1}{2} \rho V_1^2 = \rho_1, \quad V = \frac{V_2}{V_1} = \sqrt{\frac{\rho_2}{\rho_1}}
\]
substituting, in the previous expression for the pressure rise, with
the condition \( P_0 = P_3 \)

\[
P_0 - P_M = \frac{2z^2}{(A+1)^2} A \left( 1 - \sqrt{\frac{q_z}{q_1}} \right)^2
\]

or

\[
(A+1) \frac{q_z}{q_1} = 2A \left( 1 - 2 \sqrt{\frac{q_z}{q_1}} + \frac{q_z}{q_1} \right)
\]

or

\[
(A^2+1) \frac{q_z}{q_1} + 4A \sqrt{\frac{q_z}{q_1}} - 2A = 0
\]

The roots to this quadratic are

\[
\sqrt{\frac{q_z}{q_1}} = \frac{-4A \pm \sqrt{16A^2 - 4(-2A)(A^2+1)}}{2(A^2+1)}
\]

\[
= \frac{-2A + \sqrt{2A(A^2+1-2A)}}{A^2+1}
\]

for \( A > 10 \)

\[
\sqrt{\frac{q_z}{q_1}} = \frac{-2A + \sqrt{2A \left[ 1 + \left( \frac{2}{A} + \frac{1}{A} \right)^2 \right]}}{A^2+1}
\]

The mass augmentation is given by

\[
\mu = \frac{G_3}{G_1} = 1 + \frac{G_z}{G_1} = 1 + \sqrt{\frac{q_z}{q_1}} \frac{A_z}{A_1}
\]

substituting in the expression for \( \sqrt{\frac{q_z}{q_1}} \) from above

\[
\mu = 1 + \frac{-2 + \sqrt{2} \sqrt{A + \frac{1}{A} + 2}}{1 + \frac{1}{A^2}}
\]

or for \( A > 10 \)

\[
\mu = \sqrt{2A} - 1
\]

As can be seen from these expressions a large area ratio is desirable for mass augmentation.
2.2 General Theory for Ejectors

Consider a simple ejector as illustrated in Fig. 1. The fluid from the high pressure reservoir, indicated by (1) in the sketch, enters the mixing tube at station M together with the secondary fluid indicated by (2). The two fluids then mix with one another in passing through the mixing tube M = 3.

In considering the flow bounded by the control surface (shown dotted) the following assumptions are made:

1) The flow is considered one-dimensional and steady,
2) There is no heat exchange through the walls of the mixing tube,
3) The primary and secondary fluids are of the same molecular structure and thus their equations of state are the same,
4) There is no chemical reaction between the primary and the secondary flows.

Now, considering the flows at the inlet and the outlet of the mixing tube, the equations of continuity, momentum and energy may be written. The continuity equation states that

\[ G_1 + G_2 = G_3 \]  \hspace{1cm} (1)

where \( G = \int S V dA \)

The momentum equation states,

\[ \int S_2 V_2^2 dA_3 - \int S_2 V_2^2 dA_2 - \int S_1 V_1^2 dA_1 \]

\[ = P A_1 + P_2 A_2 - P_3 A_3 - \int C_f \frac{S V^2}{2} dA_5 - \int p \sin \Theta dA_5 \]  \hspace{1cm} (2)

the energy equation states that for an adiabatic process

\[ G_1 h_{01} + G_2 h_{02} = G_3 h_{03} \]  \hspace{1cm} (3)

These three equations are sufficient to describe completely the flow process between stations M and 3 and provided that suitable assumptions are made regarding the velocity distributions, the mixing tube exit pressure, the local skin friction coefficient and the mixing tube geometry.

In order to facilitate the analytical solution, the following assumptions are generally made:

5) The primary velocity front is uniform and hence

\[ \frac{V_1^2}{2} = \frac{V_f^2}{2} \]
6) The mixing tube is circular in cross-section and the flow is symmetrical with respect to the centre of the jet.
7) $A_1 + A_2 = A_3$.
8) The skin friction term is neglected.

The momentum equation then becomes

$$J_3 - J_1 - J_2 = P_1 A_1 + P_2 A_2 - P_3 A_3$$

where

$$J = \int \rho V^2 \, dA$$

In order to evaluate the first two terms of Eq. (4) we must have some knowledge of the dependence of $\rho$ and $V$ on the radius, e.g. for isentropic flow.

$$\rho = \rho_0 \left(1 - \frac{\gamma - 1}{2} \frac{V^2}{a_0^2}\right)$$

But, as this complicates the integration the density will be assumed constant across a particular section. Now, introducing the flow parameter $\lambda$ defined previously, Eq. (4) becomes

$$\lambda_3 \frac{G_3^2}{g_3 A_3} - \lambda_2 \frac{G_2^2}{g_2 A_2} - J_1 = P_1 A_1 + P_2 A_2 - P_3 A_3$$

If one considers an annular primary the velocity profile may be expressed by the power law $\frac{V}{V_m} = \left(\frac{R}{R_0}\right)^n$. For this case, one obtains

$$\lambda = \frac{(n+2)^2}{4(n+1)}$$

If one considers a central primary the secondary and mixed-flow velocity profiles may be approximated by $\frac{V}{V_m} = 1 - \left(\frac{R}{R_0}\right)^n$. For this case,

$$\lambda = \frac{n+2}{n+1}$$

In order to simplify the right hand side of Eq. (5) the mixing process must be specified.

For the case of constant pressure mixing $P_1 = P_2 = P_3$ and so Eq. (5) becomes

$$\lambda_3 \frac{G_3^2}{g_3 A_3} - \lambda_2 \frac{G_2^2}{g_2 A_2} - J_1 = 0$$

In this case, $P_3$ must be less than the secondary stagnation pressure or the ejection would not pump at all, i.e., the constant-pressure condition cannot exist when $P_3 = P_{02} = P_a$.

For the case of constant-area mixing and a subsonic or balanced supersonic primary $P_1 = P_2 = P_M$ and so Eq. (5) becomes
2.3 Theory for Constant Area Mixing

In order to simplify the analytical solution still further, the velocity profiles of the secondary and the mixed flows are assumed to be uniform. Hence, \( \lambda_1 = \lambda_2 = \lambda_3 = 1 \) and the momentum equation, Eq. (7) becomes

\[
\sum q_i A_i \dot{V}_i^2 + P_i A_i + \sum q_j A_j \dot{V}_j^2 + P_j A_j = \sum q_k A_k \dot{V}_k^2 + P_k A_k
\]

and using the equation of state

\[
J = g A V^2 = \gamma P A \frac{V^2}{\gamma R T} = \gamma P A M^2
\]

or

\[
\dot{\mathcal{J}}_1 + \dot{\mathcal{J}}_2 = \dot{\mathcal{J}}_3
\]

where

\[
\dot{\mathcal{J}} = P A (1 + \gamma M^2)
\]

The equation of state for both flows is

\[
\rho = \frac{g R T}{V^2}
\]

Now using the equation of state

\[
G = \frac{g A V}{\sqrt{R T}} = \frac{\gamma P_0}{\sqrt{R T_0}} \frac{A V}{\sqrt{R T}} = \frac{P}{P_0} \sqrt{\frac{T_0}{T}}
\]

Introducing Bernoulli's equations for compressible flow, Eqs. 4.14a, and 4.14b, from Ref. 36 gives

\[
G = \sqrt{\frac{\gamma}{R T_0}} \frac{P_0 A}{\gamma R T} \frac{M}{(1 + \frac{\gamma - 1}{2} M^2) \frac{\gamma + 1}{2(\gamma - 1)}} = \sqrt{\frac{\gamma}{R T_0}} P A M \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{2}}
\]

Now Eq. (1) may be written

\[
\frac{C_G \phi_1}{\sqrt{\phi_1}} \frac{P_0}{P_a} \frac{A_1}{A_3} M_1 \left[1 + \frac{\gamma - 1}{2} M_1^2\right]^{-\frac{\gamma + 1}{2(\gamma - 1)}} + \frac{P_0}{P_a} \frac{A_2}{A_3} \frac{M_2}{\sqrt{T_0}} \left[1 + \frac{\gamma - 1}{2} M_2^2\right]^{-\frac{\gamma + 1}{2(\gamma - 1)}}
\]

\[
= \frac{P_a}{P_0} \frac{M_3}{\sqrt{T_0}} \left[1 + \frac{\gamma - 1}{2} M_3^2\right]^{\frac{1}{2}}
\]

where \( C_G \) is the discharge coefficient of the primary nozzle. Eq. (8) may be written since \( \dot{\mathcal{J}} = P_a A (1 + \gamma M^2) \frac{P}{P_0} \) and using Eq. 4.14b from Ref. 36,

\[
\frac{P_0}{P_a} \frac{A_1}{A_3} \phi_1 + \frac{P_0}{P_a} \frac{A_2}{A_3} \phi_2 = \frac{P_a}{P_0} \left(1 + \gamma M_3^2\right)
\]
where
\[ \phi = \frac{1 + \gamma M^2}{\left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}}} \]

Substituting Eq. (1) in (3) gives
\[ T_{o3} = \frac{G_1 T_{o1} + G_2 T_{o2}}{G_1 + G_2} \quad (12) \]

Equations (10), (11) and (12) are three non-linear simultaneous equations containing four unknowns \( M_2 \), \( M_3 \), \( \frac{P_3}{P_a} \) and \( T_{o3} \). Given a fixed geometry, the primary mass flow and stagnation conditions in addition to the secondary stagnation conditions, these equations can only be solved if the pressure ratio \( \frac{P_3}{P_a} \) is specified.


In order to facilitate an analytical solution to Eqs. (10), (11), and (12), it is assumed that \( M_2 < 0.3 \) and \( M_3 < 0.3 \) and terms involving the Mach no. to higher than the second power are ignored.

The continuity equation (Eq. 10) then becomes
\[ \frac{C_a}{\sqrt{T_{o1}}} \frac{P_{o1}}{P_a} \frac{A_1}{A_3} M_1 \left[1 + \frac{\gamma-1}{2} M_1^2\right]^{-\frac{\gamma+1}{2(\gamma-1)}} + \frac{P_{o2}}{P_a} \frac{A_2}{A_3} \frac{M_2}{\sqrt{T_{o2}}} = \frac{P_3}{P_a} \frac{M_3}{\sqrt{T_{o3}}} \quad (13) \]

Upon expanding \( \phi \), and dropping the higher powers of \( M_2 \), the momentum equation (Eq. 11) becomes,
\[ \frac{P_{o1}}{P_a} \frac{A_1}{A_3} \phi_1 + \frac{A_2}{A_3} \frac{P_{o2}}{P_a} \left(1 + \frac{\gamma}{2} M_2^2\right) = \frac{P_3}{P_a} \left(1 + \gamma M_3^2\right) \quad (14) \]

If we further assume that \( T_{o1} = T_{o2} \), the energy equation degenerates into the continuity equation and equations (13) and (14) govern the flow through the ejector.

Convergent Nozzle - Primary Choked

For this case \( M_1 = 1 \) and so Eq. (13) becomes
\[ \frac{C_a}{\sqrt{T_{o1}}} \frac{P_{o1}}{P_a} \frac{A_1}{A_3} \left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}} + \frac{P_{o2}}{P_a} \frac{A_2}{A_3} M_2 = \frac{P_3}{P_a} M_3 \quad (15) \]

and Eq. (14) becomes
\[ \frac{P_{o1}}{P_a} \frac{A_1}{A_3} \left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}} + \frac{A_2}{A_3} \frac{P_{o2}}{P_a} \left(1 + \frac{\gamma}{2} M_2^2\right) = \frac{P_3}{P_a} \left(1 + \gamma M_3^2\right) \quad (16) \]
Combining Eqs. (15) and (16) gives a quadratic in \( M_2 \) or \( M_3' \). If \( M_2 \) is eliminated the result is

\[
RM_3^2 + BM_3 + C = 0
\]  

where

\[
A = \frac{A_1}{A_2} \frac{P_i}{P_{o2}} \left[ 2 - \frac{P_i}{P_{o2}} \frac{A_3}{A_2} \right]
\]

\[
B = 2 \frac{A_1}{A_2} \frac{A_3}{A_2} \frac{P_i}{P_{o2}} \frac{P_{o1}}{P_{o2}} \left( \frac{\gamma + 1}{2} \right) \frac{\gamma^2 - 2}{(\gamma - 1)} C Q_1
\]

\[
C = \frac{2}{\gamma} \left[ \frac{A_3}{A_2} \frac{P_i}{P_{o2}} - 1 - \frac{A_1}{A_2} \frac{P_{o1}}{P_{o2}} \left( \frac{\gamma + 1}{2} \right) - C Q_1 \right]^{\gamma - 1} \left[ \frac{A_3^2}{A_2^2} \frac{P_{o1}}{P_{o2}} \left( \frac{\gamma + 1}{2} \right) \frac{\gamma^2 - 2}{(\gamma - 1)} \right]^{-\gamma - 1}
\]

As can be seen from the above expressions for the coefficients \( A, B, \) and \( C \), \( A \) is always positive provided \( \frac{P_i}{P_{o2}} \frac{A_3}{A_2} < 2 \) or for the case where \( \frac{P_i}{P_{o2}} = 1 \), provided \( \frac{P_i}{P_{o2}} < 2 \) or \( \frac{A_1}{A_2} < 1 \), which would generally be the case of interest. Furthermore, as \( B \) is always positive only the positive square root of \( B^2 - 4AC \) applies.

However, it may be easier to plot Eq. (15) and (16) for specific values of \( \frac{P_i}{P_{o2}} \). The intersection of the two curves then gives the appropriate value of \( M_2 \) and \( M_3 \) for a given primary mass flow.

2.4(a) Equations Governing the Flow Through a Nozzle at the Exit of the Mixing Tube

In deriving the equations for the flow through a nozzle the following assumptions are made:

1) The flow is subsonic at the exit of the nozzle; so \( P_4 = P_a \).
2) The flow process between planes 3 and 4 is isentropic (see Fig. 2).

The equations of continuity, momentum and energy may be written

\[
G_3 = G_4
\]  

\[
P_3 A_3 (1 + \gamma M_3^2) + \sum_{walls} P dA = P_0 A_4 (1 + \gamma M_4^2)
\]

\[
G_3 \omega_3 = G_4 \omega_4
\]
Now Eq. (18) may be written
\[
\frac{P_3}{P_a} = \frac{A_4}{A_3} \frac{M_4}{M_3} \left[ 1 + \frac{\gamma - 1}{2} \frac{M_4^2}{M_3^2} \right]^{1/2} \tag{21}
\]
Since the flow is isentropic the energy equation gives
\[
\left( \frac{P_3}{P_a} \right)^{\frac{\gamma - 1}{2}} = \frac{1 + \frac{\gamma - 1}{2} M_4^2}{1 + \frac{\gamma - 1}{2} M_3^2} \tag{22}
\]
Combining these two equations gives
\[
\left( \frac{P_3}{P_a} \right)^{\frac{\gamma + 1}{2}} = \left( \frac{A_4}{A_3} \right)^2 \left( \frac{M_4}{M_3} \right)^2 \tag{23}
\]
Eliminating $M_4$ from Eq. (21) by using Eqs. (22) and (23) gives
\[
M_3^2 = \frac{1}{\frac{\gamma - 1}{2} \left( \frac{P_3}{P_a} \right)^{\frac{\gamma - 1}{2}} \left[ 1 - \left( \frac{A_4}{A_3} \right)^2 \left( \frac{P_3}{P_a} \right)^{\frac{\gamma - 1}{2}} \right]} \tag{24}
\]
This is the fourth equation we require along with Eqs. (10), (11), and (12) to determine the performance of an ejector with a nozzle or a load acting at the end of the mixing tube. For the special case where $M_2$ and $M_3$ are small, $T_01$ and $T_02$ and $M_1 = 1$, values of $M_3$ and $\frac{P_3}{P_a}$ can be obtained from a plot of Eq. (24) and substituted into Eq. (15) and (16). The values of $M_2$ obtained from these two equations can again be plotted. The intersection of the two curves would give the values of $M_3$, $\frac{P_3}{P_a}$ and $M_2$ corresponding to a certain primary mass flow and certain geometry.

2.4(b) Equations Governing the Flow Through a Diffuser

The efficiency of the diffuser may be defined as
\[
\eta = \frac{P_3 - P_3^1}{P_4 - P_3^1}
\]
where $P_3$ = actual pressure at plane 3.

$P_3^1$ = pressure at plane 3 if there were no diffuser losses, and if $P_4$ were the same.

The efficiency $\eta$ in the case of conical diffusers is dependent upon the expansion angle, the Reynolds number and the Mach number.

The definition of $\eta$ can be rewritten
\[
\frac{P_3}{P_a} = 1 - \eta \left( 1 - \frac{P_3^1}{P_a} \right) \tag{25}
\]
as
\[
P_4 = P_a
\]
and substituted into Eq. (24) to obtain the required fourth equation to complete the set.
However, it is more convenient to introduce a polytropic exponent \( \eta \) which is determined by (as shown in Ref. 3)

\[
\eta = \frac{\gamma \eta}{1 - \gamma(1 - \eta)}
\]  
(26)

This enables Eq. (23) to be written

\[
\left(\frac{P_3}{P_2}\right)^{\eta+1} = \left(\frac{A_4}{A_3}\right)^{2} \left(\frac{M_4}{M_3}\right)^{2}
\]

and similarly the exponent involving \( \gamma \) is replaced by \( \eta \) in Eq. (24).

An alternative solution is obtained from a consideration of the efficiency of transforming kinetic energy into pressure (Ref. 8). The value of the efficiency for a uniform velocity front at the inlet and outlet of the diffuser is given by

\[
\eta = \frac{P_4 - P_3}{\frac{1}{2} \rho V_3^3 \left(1 - \left(\frac{A_3}{A_4}\right)^2\right)}
\]

or as

\[
Q_{3}^2 = \frac{\gamma P_3}{S_3}
\]

Then for \( M_3 \leq 0.4 \)

\[
M_3^2 = \frac{P_3/P_4 - 1}{\frac{1}{2} \gamma \left[1 - \left(\frac{A_3}{A_4}\right)^2\right]}
\]

This equation can be used, in a similar manner as Eq. (24), along with Equations (10), (11) and (12) to give the performance of the ejector operating under a decreased back pressure.

2.5 Parameters Affecting Thrust Augmentation

In defining the term 'thrust augmentation' one is confronted with the problem as to what one really means by this expression. It would seem natural to define it as the ratio of the thrust obtained by some system of which the primary jet is a component to the thrust obtained from the primary jet alone. Consider, say, the system shown in Fig. 3 where the control volume is indicated by the dotted line. The approaching stream has the speed \( V_0 \).

Writing the momentum equation for the flow passing through the control volume, we get

\[
G_3 V_3 - G_1 V_0 - G_2 V_0 = -P_3 A_3 + P_2 A_4 - P_2 A_{\text{shroud + primary system}}
\]

as

\[
G_1 + G_2 = G_3 \quad , \quad A_1 + A_2 = A_3 \quad , \quad A_{\text{shroud + primary system}}
\]

we have

\[
G_3 (V_3 - V_0) = -(P_3 - P_2) A_3 - P_2 (A_4 - A_{\text{shroud + primary system}}) - \int P \, ds + P_2 A_{\text{shroud + primary system}}
\]
now \[ -\int p dS_x + P_a A_N - P_a (A_d - A_o) = TH \]
\[ \begin{align*}
0 &= G_3 (V_3 - V_0) + (P_3 - P_a) A_3
\end{align*} \tag{28} \]

or in the special case where \( V_0 = 0 \) and \( P_3 = P_a \)
\[ T = G_3 V_3 \tag{29} \]

As can be seen from Eq. (29) the thrust acting on the system is just the momentum flux of the flow leaving the ejector mixing tube. It is for this reason that the thrust augmentation \( \psi \) is defined as

\[ \psi = \text{mixed-flow momentum flux} \div \text{thrust from primary alone} \]

i.e.
\[ \psi = \frac{J_3}{TH} \tag{30} \]

where in the case of a subsonic or balanced supersonic jet
\[ TH_i = J_i = S_i A_i V_i^2 \]

or in the case of a choked jet exhausting to the atmosphere.
\[ TH_i = J_i + (P_i - P_a) A_i \]

Due to the non-linearity of the compressible equations it is extremely difficult to arrive at an analytical expression for a maximum thrust augmentation expressed as some function of the primary and secondary stagnation conditions and the ratio of the secondary to the primary areas. Moreover, as the thrust augmentation is influenced by the ejector geometry and the mixing process, it may well be that neither constant pressure nor constant area mixing give the maximum thrust augmentation. However, in order to understand how thrust augmentation arises these two types of mixing processes are considered for both incompressible and compressible flow theory with the primary jet always unchoked.

CONSTANT PRESSURE MIXING

From Eq. (30) we have
\[ \psi = \frac{J_3}{TH} \frac{TH_i}{TH_i} \]

Now consider a mixing tube so designed that the pressure along the wall of the mixing tube between stations \( M \) and \( 3 \) is constant. We then have

\[ P_1 = P_2 = P_3 \]

The momentum equation may be written
\[ P A_1 + \rho_1 A_1 V_1^2 + P_2 A_2 + \rho_2 A_2 V_2^2 + \int_{\text{wall}} P dS_x
\]
\[ = P_3 A_3 + \rho_3 A_3 V_3^2 \tag{31} \]
now as the mixing process is at constant pressure.

\[ \int p dS_x = p_3 \int dS_x = 0 \quad \text{if} \quad A_1 + A_2 = A_3 \]

The momentum equation may then be written

\[ J_1 + J_2 = J_3 \quad (32) \]

and since

\[ T_{H1} = J_1 \]

so

\[ \frac{J_3}{T_{H1}} = 1 + \frac{J_2}{J_1} \]

We also have

\[ \frac{T_{H1}}{T_{Hj}} = \frac{J_1}{J_j} = \frac{\delta_1 V_1^2}{\delta_j V_j^2} = \frac{P_1 M_1^2}{P_j M_j^2} \quad (33) \]

**Incompressible Theory**

As \( \delta = \text{constant} \) and since for incompressible flow we have from Bernoulli’s equation

\[ \frac{P_1}{P_3} = 1 + \frac{1}{2} \delta V_1^2 \quad (34) \]

\[ \frac{P_2}{P_3} = 1 + \frac{1}{2} \delta V_2^2 \quad (35) \]

so

\[ \frac{T_{H1}}{T_{Hj}} = \frac{V_1^2}{V_j^2} = \frac{P_1 - P_3}{P_2 - P_3} \quad (36) \]

\[ \frac{J_3}{J_j} = \frac{(P_2 - P_3) A_2}{(P_3 - P_1) A_1} \quad (37) \]

Substituting equations (36) and (37) in Eq. (30), we get

\[ \psi = \frac{P_1 - P_3}{P_2 - P_3} \left[ 1 + \left( \frac{V_3}{V_1} \right)^2 \frac{A_2}{A_1} \right] \quad (38) \]

**Compressible Theory**

Since

\[ P_3 = P_1 \]

therefore

\[ \frac{J_3}{J_j} = \frac{P_3 M_3^2}{P_1 M_1^2} \quad (39) \]

and since

\[ P_1 = P_2 \]

therefore

\[ \frac{J_3}{J_1} = \frac{A_2}{A_1} \left( \frac{M_2}{M_1} \right)^2 \quad (40) \]

so that

\[ \psi = \frac{P_3 M_1^2}{P_2 M_2^2} \left[ 1 + \left( \frac{M_2}{M_1} \right)^2 \frac{A_2}{A_1} \right] \quad (41) \]

As can be seen from the above results the maximum thrust augmentation will occur when \( \frac{J_2}{J_1} \) is made a maximum. This will correspond to some area ratio for a particular primary mass flow. Moreover, as the thrust augmentation depends on the square of the
secondary velocity or Mach number one should perhaps choose a value of the area ratio $A_2 / A_1$ smaller than that for the case of mass augmentors.

CONSTANT AREA MIXING (see Fig. 1)

If the primary is unchoked then $P_1 = P_2 = P_M$ and as $A_1 + A_2 = A_3$ then the momentum equation may be written

$$\frac{J_3}{J_1} = 1 + \frac{J_2}{J_1} - \frac{(P_3 - P_M)}{J_1} A_3 \quad (42)$$

**Incompressible Theory**

$$\frac{J_1}{J_1} = \frac{V_1^2}{V_2^2} = \frac{P_{01} - P_1}{P_{01} - P_2} \quad (43)$$

and since

$$P_{01} = P_M + \frac{1}{2} \rho V_1^2 \quad (44)$$

$$P_{02} = P_M + \frac{1}{2} \rho V_2^2 \quad (45)$$

then

$$\frac{J_2}{J_1} = \frac{P_{02} - P_M}{P_{01} - P_M} \frac{A_2}{A_1} \quad (46)$$

Also,

$$\frac{P_3 - P_M}{J_1} A_3 = \frac{P_3 - P_M}{2 \left( P_{01} - P_M \right)} \frac{A_2}{A_1} \left( 1 + \frac{A_1}{A_2} \right) \quad (47)$$

Now substituting Eq. (46), (47) in (42) and then into Eq. (30) along with Eq. (44) gives

$$\psi = \frac{P_{01} - P_2}{P_{01} - P_M} \left[ 1 + \frac{P_{02} - P_M}{P_{01} - P_M} \frac{A_2}{A_1} \left( 1 - \frac{P_3 - P_M}{P_{02} - P_M} \left( 1 + \frac{A_1}{A_2} \right) \right)^2 \right] \quad (48)$$

As can be seen from Eq. (48) the thrust augmentation is influenced greatly by the two pressure ratios $\frac{P_{02} - P_M}{P_{01} - P_M}$ and $\frac{P_3 - P_M}{P_{02} - P_M}$. As in the case of constant pressure mixing one wants $P_{02} - P_M$ or the secondary velocity as high as possible.

For the special case where $P_3 = P_{02} = P_a$ the thrust augmentation becomes

$$\psi = \left( \frac{V_1}{V_2} \right)^2 \left[ 1 + \frac{1}{2} \left( \frac{V_2}{V_1} \right)^2 \left( \frac{A_2}{A_1} - 1 \right) \right] \quad (49)$$

**Compressible Theory**

In this case

$$\frac{J_1}{J_1} = \frac{R M_1^2}{P_2 M_2^2} = \frac{P_1}{P_2} \left[ \left( \frac{P_1}{P_M} \right)^{\frac{2+1}{2}} - 1 \right] \quad (50)$$

and

$$\frac{P_3 - P_M}{J_1} A_3 = \frac{1}{\gamma M_1^2} \left( \frac{P_3}{P_M} - 1 \right) \left( \frac{A_2}{A_1} + 1 \right) \quad (51)$$
and

\[
\frac{J_2}{J_1} = \frac{A_2}{A_1} \left( \frac{M_2}{M_1} \right)^2.
\]  

(52)

Substituting Eq. (51) and (52) in Eq. (30) gives

\[
\psi = \frac{J_1}{J_1} \left[ 1 + \left( \frac{M_2}{M_1} \right)^2 \frac{A_2}{A_1} - \frac{1}{\gamma M_1^2} \left( \frac{P_3}{P_M} - 1 \right) \left( \frac{A_2}{A_1} + 1 \right) \right]
\]

(53)

if we assume \( P_3 = P_a = P_{o2} \)

and \( M_2 < 0.3 \) then

\[
\frac{P_2}{P_2} = 1 + \frac{\gamma}{2} M_2^2
\]

Eq. (53) reduces to

\[
\psi = \frac{J_1}{J_1} \left[ 1 + \frac{1}{2} \left( \frac{M_2}{M_1} \right)^2 \left( \frac{A_2}{A_1} - 1 \right) \right]
\]

(54)

This result is similar to that obtained for constant pressure mixing. In both constant pressure and constant area mixing, it would seem that a large ratio of the secondary Mach number to the primary is necessary to achieve a good thrust augmentation. As indicated by Eq. (41) for constant pressure mixing, the thrust augmentation is always greater than unity. In the case of constant area mixing, however, this is only true provided \( M_2 > \sqrt{\frac{1}{\gamma} \left( \frac{P_3}{P_M} - 1 \right)} \) when the ratio \( A_2 \rightarrow A_1 \).

3. DESIGN OF EJECTOR

3.1 Air Supply

Before designing the ejector, a brief study was made of the compressed air supply. The compressor which supplies the compressed air to the laboratories and the shop is an Ingersoll Rand model 50-B two-stage, air cooled, single acting type 40 stationary air compressor and is driven by a fifty horsepower induction motor. The capacity of the compressor is 230 cfm at N.T.P and it is capable of compressing this flow to 100 psig. The storage tank has a capacity of 52 cubic feet and the intercooler is capable of reducing the temperature of the air from the compressor down to about 85°F. Due to the long length of the pipe between the storage tank and the subsonic tunnel, where the ejector was located, a drop in total head of about 20 - 30 psi occurred.

3.2 Design of Primary Nozzle and Flow Control

In order to obtain maximum thrust conditions over a wide range of primary stagnation conditions, a convergent nozzle with a tapered, concentric plug was used to produce a variable exit area (see Fig. 7). The nozzle can slide forward, by adjusting a micrometer screw, towards the plug which is fixed to the walls of the mixing tube by three struts. The primary nozzle when closed against the plug is
located about three inches beyond the entrance of the mixing tube (see Fig. 4) to obtain maximum duct thrust (see Ref. 3).

As it was anticipated that a supersonic primary nozzle might improve the mass augmentation the original plug and convergent nozzle were later replaced by a de Laval nozzle giving a Mach number $M_1 = 1.85$ and an area ratio of $\frac{A_2}{A_1} = 67$.

3.3 Design of Secondary Intake and Mixing Tube

As the secondary flow was to be taken from the wind tunnel room a horn shaped fairing to provide a smooth inlet flow was attached to a cone section (Fig. 4). This, in turn, was attached to the beginning of the mixing section. The angle of the cone was chosen to be five degrees to provide maximum duct thrust.

In designing the mixing tube, a cylindrical tube of constant diameter was chosen. The ratio of the secondary area to the primary varied depending upon the primary nozzle position with respect to the conical plug. When the first mixing tube of diameter 4 inches proved to be unsatisfactory a new mixing tube of 2.75 inches diameter was designed. This new mixing tube was perforated with holes for the transmission of sound and installed inside the former mixing tube. Between the two cylindrical tubes a half-inch layer of steel wool was packed to absorb the sound produced by the turbulent mixing.

3.4 Mixing Tube Exit Nozzles and Diffuser

Five conical nozzles varying in diameter from 2.5 inches to 3.875 inches were fitted on the end of the mixing tube to simulate the effect of a load on the ejector. The angle of each cone was chosen to be ten degrees.

When the first mixing tube of diameter 4 inches was found to be unsatisfactory to cope with the loads produced by the model wing, a diffuser was designed to fit onto the end of the 2.75 inch diameter mixing tube to counterbalance these loads. The diffuser exit diameter was 3.913 inches and the included cone angle was twelve degrees.

4. INSTRUMENTATION AND MEASUREMENTS

4.1 Measurement of Primary Flow Quantities

In order to ensure a regular velocity profile along a diameter of the primary, a special piece of smooth steel pipe was chosen with a length corresponding to about 75 pipe diameters. The diameter of this calibration pipe was chosen to be slightly larger than the one inch supply line in order to ensure that the flow velocity in the pipe was low enough to permit the use of incompressible flow theory. A total
pressure probe was installed about 45 pipe diameters downstream of the calibration pipe inlet and a traverse was made along one diameter. About four diameters upstream a static pressure tap was installed and connected to a test gage with a range of 0-100 psi and a sensitivity of ± 1/2 psi. A second connection to this pressure tap was made and went to one side of a U-tube manometer (see Fig. 6). The other side was connected to the total pressure probe. In this manner, the dynamic pressure of the stream was determined.

In order to determine the temperature of the flow a vapour actuated thermometer with a remote indicating dial was employed. The long bulb containing the vapour was strapped to the outside of the pipe and covered with a two inch layer of insulating material. As the pipe was insulated reasonably well the temperature measured was nearly the recovery temperature. As the velocity of the flow was very small the dynamic contribution to the recovery temperature was also small and so it was assumed that \( T_d = T_r \). From measurements of the static pressure and the recovery temperature, the density was calculated and hence the velocity was deducable from readings of the dynamic head as measured on the U-tube manometer.

As the ratio of the mean velocity in the pipe to that on the axis was required, the data from the pitot traverse of the primary was made non-dimensional and a curve of velocity versus probe position was plotted (see Fig. 8).

From this curve, the mean velocity was found to be \( \bar{V} = 0.835 \) \( V_{axial} \) for a Reynold's number of \( Re = 5.6 \times 10^5 \). This result is slightly larger than the one obtained by extrapolating Stanton and Pannell's results as given in Ref. 35. Their value for the same Reynold's number was \( \bar{V} = 0.82 \) \( V_{axial} \). As the Reynold's number for the other runs was roughly the same only the velocity at the centre of the pipe was measured and the mean velocity calculated from the primary calibration result. Hence, the primary mass flow was found from three measurements, the static pressure, the recovery temperature, and the dynamic head.

An expression for the primary mass flow in terms of these three quantities is given below

\[
G_i = \xi \frac{R}{\gamma} \frac{g_1}{\gamma} A_i V_i
\]

where \( g_1 = \frac{P}{R T_u} \) and \( P_{o1} - P = \Delta P = \frac{1}{2} g_1 V_i^2 \)

so

\[
G_i = \xi \sqrt{2 g_1} A_i \sqrt{\Delta P_{ax}}
\]

where

\[
\xi = \frac{\bar{V}}{V_{axial}} = 0.835.
\]
In estimating the thrust of the primary an estimate of the exit area of the nozzle was required. This was achieved in the case of the choked primary, knowing the nozzle and plug geometry and the relative position between the two. In determining the pressure thrust, the static pressure as measured on the test gage was assumed equal to the total pressure. This is a good approximation since the flow velocity in the calibration pipe was very small. The very small loss in total head due to pipe friction between the measuring station on the primary calibration pipe and the nozzle exit was ignored. The static pressure at the (choked) nozzle exit was then calculated from the relation

\[ P_i = \frac{P_{01}}{\left(\frac{y+1}{2}\right)^{\frac{y-1}{2}}} = 0.528 \, P_{01} \]

The pressure component of the thrust was then determined from the expression

\[ (P_i - P_a) \, A_i \]

To obtain the momentum thrust at the nozzle exit only the primary mass flow and the stagnation temperature were required since the jet was choked. The momentum flux for a choked jet is given by

\[ J_i = G_i \, V_i = G_i \sqrt{\frac{2 \gamma R T_{01}}{\gamma + 1}} = 44.6 \, G_i \sqrt{\frac{T_{01}}{T_{01}}} \]

The total thrust is given by the sum of these two components. It is

\[ T_H = 44.6 \, G_i \sqrt{\frac{T_{01}}{T_{01}}} + (P_i - P_a) \, A_i \]

4.2 Measurement of Secondary Flow Quantities

As the ejector inlet was open to the atmosphere the stagnation temperature and pressure were determined by the ambient conditions. As the flow velocity of the secondary was small, being of the order \( M_2 \approx 0.25 - 0.35 \), incompressible flow theory could be employed. In computing the density of the flow the static temperature was assumed equal to the total or the ambient pressure since the velocity was small. The static pressure of the secondary was measured at three taps on the wall of the ejector arranged at 120° intervals, in the same plane. The plane of these three taps was located slightly upstream from the end of the primary nozzle. The three pressure readings were averaged and the result was used to calculate the mean velocity of the secondary stream. The density and pressure were assumed not to vary across the inlet. The secondary mass flow was calculated from the relation based on incompressible flow

\[ G_2 = \frac{P_{2e}}{RT_{2e}} \]

where

\[ G_2 = \frac{P_2}{RT_2} A_2 V_2 = A_2 \sqrt{2(P_2 - P_a) G_2} \]
The velocity at the nozzle exit was calculated from the relationship \( \mathbf{v}_4 = \mathbf{v}_1 + \mathbf{v}_2 \) and the exit momentum flux was obtained from the relation \( \mathbf{j}_4 = q_0 A_4 v_4^2 \).

These measurements of the secondary flow quantities provided a check on the measurement of the primary and the mixed flow.

4.3 Measurement of Mixed Flow Quantities

Initially, it was hoped that two total pressure rakes extending across two mutually perpendicular diameters would give a good estimation of the mixed mass flow and the mixed momentum flux. However, due to the centre line of the nozzle not coinciding with the centre line of the plug, the mixed flow was found to be unsymmetrical along a diameter. Moreover, as the total pressure rake was placed at a position downstream from the primary nozzle of about five mixing tube diameters, the mixing process was far from complete and consequently the small number of probes gave an inaccurate estimation of the velocity profile.

It was for this reason that a traversing gear having a single pitot probe (see Fig. 5) was fitted to the end of the ejector at a distance of about seven times the diameter of the mixing tube, measured from the exit of the primary nozzle. Measurements of the velocity along any diameter and at any radius were possible.

If the velocity profiles along two mutually perpendicular diameters were not similar, two additional traverses were made inclined at an angle of 45° with respect to the former. From the measurements of the velocity at various radii the mixed mass flow was deduced from the relation \( G_3 = G_4 = q V_m \int_0^{2\pi} \mathbf{r} d\theta \int_0^R \frac{v}{V_m} r dr \)

where \( V_m \) is the average maximum velocity over all traverses. The integral

\( A(\theta) = \int_0^R \frac{v}{V_m} r dr \)

was approximated using the trapezoidal rule. The second portion of this double integral was also approximated using the trapezoidal rule.

Instead of using a balance to determine the mixed thrust or momentum flux, the latter was estimated using the relation

\[ \mathbf{j}_4 = q V_m^2 \int_0^{2\pi} d\theta \int_0^R (\frac{v}{V_m})^2 r dr \]

It should be noted that the momentum flux depends upon the velocity profile of the mixed flow. If one assumes a velocity profile of the shape \( \frac{v}{V_m} = 1 - \left(\frac{r}{R}\right)^n \), then the mixed momentum flux is

\[ \mathbf{j}_4 = q A_4 \lambda V_4^2 = q A_4 V_4^2 \frac{m + 2}{m + 1} \]
for a uniform flow $m \to \infty \quad \lambda \to 1$
while for a very peaked flow $m \to 0 \quad \lambda \to 2$

4.4 Experiments

Experimental Procedure for Determining the Effect of an Increased Back Pressure at the Exit of the Mixing Tube

Before attaching any nozzles to the end of the mixing tube, some measurements were made at constant primary mass flow of the primary and secondary flow properties for various primary exit areas or alternatively for area ratios $34 < \frac{A_2}{A_1} < 112$. As it was found that the primary exit area had some effect on the ejector performance, it was decided to operate at the largest possible value of the primary pressure $P_{01}$.

In order to determine the effect of an increased back pressure five nozzles, varying in diameter from 2.5 in. to 3.812 in., giving an area ratio $0.40 < \frac{A_4}{A_3} < 0.90$, were fitted onto the end of the mixing tube. The primary and secondary flow quantities were measured and two or four pitot traverses were made, depending upon the velocity profile, at the exit of each nozzle. For the nozzles with a diameter $\leq 3$ inches the velocity was almost constant along a diameter (see Fig. 11) and in estimating the value of $V_4$ the velocity corresponding to $r = 0.707R$ was chosen. The mass and thrust augmentation were computed using the results of the secondary measurements and also using the mixed measurements. The two results were compared by plotting the mass and thrust augmentation versus mixing tube back pressure.

As it was also important to determine the mass and thrust augmentation at reduced primary mass flows, some of the compressed air was bled off and measurements of the primary and secondary flow quantities were made for $250 \text{ lbs/hr.} < G_1 < 900 \text{ lbs/hr.}$ These experiments were done for $\frac{A_2}{A_1} = 112$ and for a mixing tube nozzle of 3 in. diameter.

In determining the performance of the ejector with the supersonic primary, the secondary and primary flow quantities were used to deduce the mass and thrust augmentation for nozzles attached to the exit of the mixing tube having the diameters 2 1/2 inches, 3 inches, 3 1/4 inches and 3 1/2 inches. The mixed flow measurements were also used to determine the performance with no exit nozzle attached. These results were then compared on a graph (Fig. 12) with those for a choked primary nozzle.

Static Pressure Drop in the Ducting to the Model Wing Slots and Due to the Reduction of the Final Exit Area

In order to estimate the load or increase in back pressure acting at the end of the mixing tube due to the model and ducting, the system was
coupled to the ejector and run at the maximum primary mass flow and with the wing slots fully open. A length of 12 1/2 feet of flexible steel tubing was first connected to the ejector and although the mass augmentation was found to decrease slightly the drop in static pressure across the length of the pipe was found to be zero. This zero loss is attributed to the fact that the flow at the end of the mixing tube is not completely mixed. Hence, the mixing process is accompanied by an increase in static pressure for some distance along the pipe before the pressure drops to atmospheric at the exit of the pipe due to wall friction.

On adding to the 4 inch diameter pipe a contracting section, an elbow and a diffuser the back pressure at the exit of the mixing tube was found to be greater than atmospheric by about eleven inches of water. This was mainly due to the exit area of the diffuser being about one half the area of the mixing tube. When the model was connected onto the end of the diffuser, the back pressure was increased to about 15 inches of water above atmospheric. This large load on the ejector seriously reduced the mass augmentation to about \( \mu = 3 \) or a drop of about 70% compared with no load. The thrust augmentation suffered a more severe decrease falling well below unity.

As the load due to the model and ducting crippled the pumping capabilities of the ejector it was decided to reduce the area of the mixing tube by about 50% and to add a diffuser at its downstream end so as to maintain the pressure there reasonably near atmospheric. The addition of this liner inside the previous mixing tube meant that a convergent section to reduce the inlet area from 5 inches down to 2.7 inches had to be added at the entrance of the mixing tube (see Fig. 4).

Experimental Tests with the Mixing Tube Liner Installed and a Supersonic Nozzle

With the smaller diameter mixing tube in place and a de Laval nozzle connected to the primary, the area ratio was \( \frac{A_2}{A_1} = 31 \), and the ratio of \( \frac{L}{D} = 7.75 \). The primary and secondary flow properties were measured with and without the diffuser. With the diffuser in place at the end of the mixing tube four nozzles varying in diameter from 2 1/2 inches to 3 1/2 inches were fitted onto it and the performance of the ejector and diffuser operating against various pressures was again determined. The mixed mass flow and momentum flux were again determined by doing two pitot traverses along mutually perpendicular diameters.

As the performance of the ejector for various values of back pressure higher than atmospheric was desired, a very fine mesh screen and two coarse mesh screens were fitted inside the 2 1/2 inch and 3 inch nozzles. The static pressure drop across the fine screen
was roughly one dynamic head while that across the coarse was about two-tenths of a dynamic head.

The efficiency of the diffuser was obtained from a measurement of the static pressure rise across the diffuser and the theoretical pressure rise obtained by knowing the mixed exit velocity and the geometry of the diffuser.

5. RESULTS AND DISCUSSION

5.1 Effect of a Back Pressure Greater than Atmospheric at the End of the Mixing Tube

These experiments were done primarily with a convergent nozzle and the mass and thrust augmentation for an increased back pressure were found to be as shown in Fig. 9.

In the case of mass augmentation Fig. 9 indicates that the induced mass flow decreases very slowly until a back pressure of about 35 psf (6.7 inches H$_2$O) is attained. Here, the induced mass flow begins to fall off much more rapidly until, say, for example at a back pressure of 90 psf or 17 inches H$_2$O, the mass augmentation has dropped to one-half its value for no load. In comparing the results obtained from the secondary pressure measurements with those from the mixed velocity traverse, they were found to be in good agreement.

In contrast to the mass augmentation, the thrust augmentation decreased more rapidly with an increase in back pressure. However, for very small increments in back pressure above atmospheric the thrust augmentation decreased very slowly. At a back pressure of about 78 psf or 15 inches H$_2$O, the thrust augmentation had been reduced to about one half its value for no load. For the larger values of the back pressure the secondary pressure measurements gave a good indication of the mixed momentum flux. However, for very small loads the secondary results gave a lower value of the thrust augmentation than that from the mixed flow results. This is attributed to the fact that for small loads the mixed velocity profile is far from uniform and $\bar{V}_4^2 < \bar{V}_4'$. As the secondary results can only give $\bar{V}_4$, they are insufficient to determine accurately the thrust augmentation when the mixed velocity profile is non-uniform. For a ratio of $\frac{L}{D} = 7$, the velocity profiles for various mixing tube nozzles are as shown in Fig. 11. It should be pointed out that the mixed flow was unsymmetrical about the axis of the jet. Furthermore, when traverses were made to give the mixed exit velocity, fluctuations in stagnation pressures were observed which ranged from 2 to 5 percent of the mean.
5.2 Effect of the Primary Exit Area

With the primary mass flow held constant and the area of the primary varied by withdrawing the nozzle from the plug, the effect of the ratio $\frac{A_2}{A_1}$ was found to be as shown in Fig. (10). Although the primary momentum thrust remained almost constant during the test, the enlarging of the primary area caused the pressure thrust to drop and hence also the total primary thrust. Moreover, as the thrust augmentation increased slightly with increasing the ratio $\frac{A_2}{A_1}$, the mixed momentum flux was by far largest at the larger values of $\frac{A_2}{A_1}$. While the thrust augmentation seemed to depend on the ratio of $\frac{A_2}{A_1}$ the mass augmentation was only slightly dependent on it decreasing slightly as $\frac{A_2}{A_1}$ decreased.

5.3 Effect of the Type of Primary Nozzle

Before installing the smaller diameter mixing tube to reduce the area ratio $\frac{A_2}{A_1}$, the convergent nozzle and the plug used to control the primary mass flow were removed. In its place a de Laval nozzle was used to determine the performance of the ejector with a supersonic primary. As shown in Fig. (12), the mass and thrust augmentation were substantially increased by the supersonic nozzle. However, it is difficult to decide how much of the improvement is due to the conversion from a choked jet to a supersonic jet for the following reason. The plug or cone used to control the primary mass flow and its supporting strut had considerable drag due to the high velocity air surrounding it. This would extract some momentum from the primary stream thereby reducing slightly the induced mass flow and hence the mass and thrust augmentation. However, as the mass augmentation was increased by about 20% and the thrust augmentation by about 10% over that for the convergent nozzle, it would seem that despite the elimination of the plug, the supersonic nozzle might be superior.

The results shown in Fig. (12) are based solely on the measurements of the primary and secondary flows. They are felt to be reliable for the mass augmentation and the thrust augmentation for nearly uniform mixed velocity profiles. Furthermore, the mixed velocity profiles are similar for a convergent primary nozzle and a supersonic one (see Fig. 14) and hence the last statement is justifiable since it has been proven for the case of a convergent nozzle.

Also shown in Fig. (12) are some theoretical results for a supersonic primary, calculated from Eqs. (13) and (14). The agreement between theory and experiment is fairly good for the case of mass augmentation, the errors falling in the range of 8 - 12 percent. This variation in the difference between the results for theory and experiment is perhaps due to assumptions (5) and (8) on Pages 6 and 7.
Unfortunately, the difference between theory and experiment became even more pronounced for the case of thrust augmentation. As the graphical solution of Eqs. (13) and (14) gave the appropriate values of $V_2$ and $V_3$, any deviation from a non-uniform velocity profile would produce a very large error in the momentum flux which depends on the mean of the velocity squared $V^2$. In comparing theory and experiment the percentage error in thrust augmentation varied from about 10 - 20 percent.

5.4 Effect of the Primary Mass Flow

When the primary mass flow was varied between 250 lbs/hr $< G_1 < 900$ lbs/hr the mixed and the secondary weight flow were found to increase almost linearly with the primary as indicated in Fig. (13). However, the mass augmentation decreased from seven down to four in the range of primary weight flows tested. It should be noted that this was only for the case where the mixing tube back pressure was about 13 inches H$_2$O above atmospheric. The thrust augmentation was found to remain almost constant as the primary weight flow was varied.

5.5 Effect of the Ratio $\frac{A_2}{A_1}$ and an Exit Diffuser on the Performance When Operating Against Various Back Pressures

As mentioned before, the mass and thrust augmentation decreased very rapidly with an increasing load when the area ratio $\frac{A_2}{A_1} = 67$. However, when the area ratio was reduced to about one half its original value and a diffuser was fitted to the end of the liner, the mass and thrust augmentation were found to vary much less rapidly with changes in back pressure, as shown in Fig. 15 and Fig. 16. The mass and thrust augmentation for $\frac{A_2}{A_1} = 31$ were less than for the case of $\frac{A_2}{A_1} = 67$ only when the mixing tube back pressure was less than about 8 inches of water above atmospheric. However, at larger loads the combination of a smaller diameter mixing tube and a diffuser was much superior to the larger diameter mixing tube. For instance, at a back pressure of 18 inches of water, the mass augmentation was increased by about 50%, while the thrust augmentation was increased about 75% using a smaller diameter mixing tube and a diffuser. Moreover, the thrust augmentation remains almost constant over a substantial range of back pressures, decreasing quickly as very large absolute values of back pressure are attained. These two curves for different area ratios of the secondary to the primary indicate the importance of choosing a design appropriate to the back pressures to be experienced. This efficiency of the diffuser at the end of the mixing tube (see Fig. 4) was calculated from the relationship for incompressible flow.

From Bernoulli's equation the theoretical pressure rise across the diffuser is $\Delta P_{th} = P_4 - P_3 = g_3 - g_4$. 
now from continuity \[ V_3 A_3 = V_4 A_4 \]

or \[ q_3 = q_4 \left( \frac{A_4}{A_3} \right)^2 \]

so \[ \Delta P_{th} = q_4 \left[ \left( \frac{A_4}{A_3} \right)^2 - 1 \right] \]

\[ \frac{A_4}{A_3} = \left( \frac{3.913}{2.75} \right)^2 = 2.025 \]

The average measured value of \( V_4 = 225 \) ft/sec.

and of \( \Delta P = 27 \) in. H\(_2\)O

Hence \( \Delta P_{th} = \frac{0.00238}{2} \left( 225 \right)^2 \left[ \left( 2.025 \right)^2 - 1 \right] = 188 \) p.s.f.

and the diffuser efficiency

\[ \eta = \frac{\Delta P}{\Delta P_{th}} = \frac{138}{188} = 0.738 \text{ or } 73.8\% \]

The velocity profiles at the end of the various nozzles attached to the end of the diffuser are shown in Fig. 17. Although the flow is well mixed at the end of the mixing tube, the effect of a diverging section at the mixing tube exit produces a radial velocity component and hence the velocity profile becomes very non-uniform as the ratio \( A_4/A_3 \) is increased.

6. **CONCLUSIONS**

In considering the performance curves for two different designs of an air ejector operating against various loads, one realizes the necessity of choosing a design that is only slightly affected by changes in back pressure. If a load (back-pressure) is applied to the end of a mixing tube with a large ratio of the secondary area to the primary, the mass and thrust augmentation decrease very rapidly as the load is increased. If the area ratio is reduced the rate of decrease might be lessened, however, at the expense of a substantial loss in mass aug-
mentation. Part or all of this loss in mass augmentation can be offset by adding a diffuser at the end of the mixing tube. Moreover, with a suitable choice of $\frac{A_p}{A_i}$ and a diffuser the thrust augmentation can be made relatively constant over a certain range of back pressures.

From the experiments performed with various primary areas, the effect of changing the primary area while holding the secondary area and the primary entrainment area constant has only a small effect on the ejector's performance. If an annular primary nozzle is used it is the primary entrainment area which determines the performance.

If an accurate theoretical estimation of thrust augmentation is desired, some consideration must be given to the mixed flow velocity profile at the end of the mixing tube. Without this consideration the theory, as the experimental results based on the secondary pressure measurements show, tend to be a considerable amount less than the true value.

In reviewing the literature on ejector theory there seemed to be nearly a different set of parameters chosen by each author in solving the ejector's flow equations. In choosing the flow stagnation conditions and the Mach number as the variables herein it is hoped that they will be more in keeping with the variables usually used to describe compressible fluid flow.
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APPENDIX I

Jet Pump Efficiency

In determining the pumping ability of the air ejector operating under various conditions two definitions were considered. The first definition is similar to the one used to evaluate the efficiency of other types of hydraulic pumping devices. Here, the efficiency is defined as the ratio of the increase in the energy of the secondary stream to the loss in energy of the primary.

\[ \eta = \frac{G_2 (P_{03} - P_{o2})}{G_1 (P_{01} - P_{03})} \]

where

- \( P_{03} - P_{o2} \) \( \equiv \) net jet pump head
- \( P_{01} - P_{03} \) \( \equiv \) net driving head.

If the secondary and mixed stream Mach numbers are small, Bernoulli's equation for incompressible flow may be applied.

Hence

\[ P_{03} - P_{o2} = P_3 - P_2 + q_3 - q_2 \]

If

\[ P_3 = P_a = P_{o2} \]

and so

\[ P_{03} - P_{o2} = q_3 \]

Similarly

\[ P_{01} - P_{03} = P_1 - P_3 + \mathcal{Z} q_1 - q_3 \]

where

\[ \mathcal{Z}_1 = 1 + \frac{M_i^2}{4} + \frac{M_i^4}{40} + \frac{M_i^6}{1600} + \cdots \]

If the primary nozzle is supersonic and if it is operating at its design conditions \( P_1 = P_2 \) and as \( P_3 = P_a = P_{o2}, P_1 - P_3 = -q_2 \)

so

\[ P_{01} - P_{03} = \mathcal{Z} q_1 - q_2 - q_3 \]

Substituting this in our equation for the jet pump efficiency gives:

\[ \eta = \frac{G_2}{G_1} \frac{q_1}{q_3} \frac{G_2}{G_1} - 1 \]

For the case of a sonic primary

\[ \eta = \frac{G_2}{G_1} \left[ \frac{1}{P - P_a} + \mathcal{Z}_1 \frac{q_1}{q_3} - 1 \right] \]
Experimental Results for the Convergent Primary Nozzle

\[ \mu = \frac{G_3}{G_1} = 6.70 \quad \text{or} \quad \frac{G_2}{G_1} = 5.70 \]

\[ \bar{V}_3 = 269 \text{ ft/sec.} \quad q_3 = \frac{0.00238}{2} (269)^2 = 86 \text{ psf.} \]

so \[ p_{01} = 91.7 \text{ psia.} \quad p_1 = 0.528 (91.7) = 484 \text{ psia} \]

so \[ P_1 - P_a = (48.4 - 14.7) 144 = 4860 \text{ psf.} \]

\[ Z_1 = 1.275 \quad \frac{\rho_1}{\rho_a} = \frac{P_1}{P_a} \left( \frac{M_1}{M_3} \right)^2 = \frac{48.4}{14.7} \left( \frac{1}{0.261} \right)^2 = 48.3 \]

so \[ \gamma = 5.70 \left[ \frac{1}{4860 + 1.275(48.3) - 1} \right] = 0.0486 \text{ or } 4.86\% \]

Experimental Results for the Supersonic Nozzle

\[ \mu = 7.96 \quad \frac{G_2}{G_1} = 6.96 \]

\[ \frac{\rho_1}{\rho_a} = \frac{P_1}{P_a} \left( \frac{M_1}{M_3} \right)^2 = \frac{278}{299} = 0.865 \]

\[ Z_1 = 1.275 \quad \frac{\rho_1}{\rho_a} = \frac{\rho_1}{\rho_a} \left( \frac{V_2}{V_3} \right)^2 = 2.239 \left( \frac{V_2}{V_3} \right)^2 \left( \frac{M_1}{M_3} \right)^2 \]

\[ = 2.239 \left( 0.865 \right) \left( \frac{1.90}{0.253} \right)^2 = 109 \]

\[ \gamma = \frac{6.96}{109 - 0.865 - 1} = 0.065 \text{ or } 6.5\% \]

These efficiencies are very low compared to the value of \( \gamma = 35\% \)
mentioned in Ref. (31) for a liquid jet pump. Moreover, the efficiency is extremely low compared with that obtainable from other pumping devices.

The second definition used to estimate the pumping ability of the ejector is taken from Ref. (29). The efficiency \( \gamma \) is defined by Hembold as

\[ \gamma = \frac{G_3}{G_1} \frac{\bar{V}_3^2 - \bar{V}_2^2}{\bar{V}_1^2 - \bar{V}_2^2} = \frac{\Delta K.E.}{\Delta K.E.} \]

where \( \bar{V}_3^2 - \bar{V}_2^2 \) = increase in specific kinetic energy of the mixed stream due to the primary driving stream

\[ \bar{V}_1^2 - \bar{V}_2^2 \] = loss in specific kinetic energy of the primary stream.
APPENDIX II

Calculation of Ejector Performance

In order to clarify the technique used to solve the ejector equations, a sample calculation is given. As the secondary and mixed Mach numbers were small Eqs. (13) and (14) were used to find \( M_3 \left( \frac{P_3}{P_a} \right) \) and \( M_2 \left( \frac{P_3}{P_a} \right) \). When there was a nozzle at the end of the mixing tube Eq. (24) was used in addition to give a relationship between \( M_3 \) and \( \frac{P_3}{P_a} \). In the case of a diffuser at the end of the mixing tube, Eq. (27) was used.

In this example, the primary is supersonic and the diameter of the mixing tube is four inches. To illustrate the technique of solving the equations when a nozzle or diffuser is applied at the end of the mixing tube, the case where \( \frac{R_3}{R_4} = 1.51 \), or a nozzle having an exit diameter \( D_4 = 3 \, \frac{1}{4} \) inches is considered.

It is assumed that \( T_{01} \approx T_{02} \) and thus \( T_{01} = T_{03} \). This was nearly so experimentally. It is also assumed that the primary discharge coefficient \( CQ_1 = 1 \).

Using the above assumptions, Eq. (13) may be written

\[
\frac{P_{01}}{P_a} \frac{A_1}{A_3} M_1 \left[ 1 + \frac{\gamma - 1}{2} M_1^2 \right] - \frac{\gamma + 1}{2(\gamma - 1)} \frac{P_{02}}{P_a} M_2 \frac{A_2}{A_3} = \frac{P_3}{P_a} M_3
\]

The measured quantities for the case of the supersonic primary were \( P_{01} = 98.6 \) psia. and \( \frac{A_2}{A_1} = \frac{12.38}{0.1768} = 70 \).

It was assumed that \( P_1 \approx P_a = 14.7 \) psia and so

\[
\frac{P_{01}}{P_a} = 6.71.
\]

Now from the isentropic flow relationship

\[
M_1 = \sqrt{\frac{2}{\gamma - 1} \left[ 1 - \left( \frac{P_{01}}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} \right]}
\]

\[
= \sqrt{\frac{2}{1.4 - 1} \left[ 1 - \left( \frac{6.71}{14.7} \right)^{0.285} \right] = 1.90
\]

now \( A_1 = 0.1765 \) in\(^2\) \( A_3 = 12.56 \) in\(^2\)

and thus \( \frac{A_1}{A_3} = 0.01405; \frac{A_2}{A_3} = 1 - \frac{A_1}{A_3} = 0.986 \)
So the continuity equation may be written

\[ \frac{P_3}{P_a} M_3 = 6.71 (0.01405) + 1.90 \left[ 1 + 0.2 (1.90)^2 \right]^{-3} + 0.986 M_2 \]

or

\[ \frac{P_3}{P_a} M_3 = 0.0345 + 0.986 M_2 \]  \hspace{1cm} (13)

Using the previous assumptions, Eq. (14) may be written

\[ M_2^2 = \frac{2}{\gamma} \left[ \frac{P_3}{P_a} (1 + \gamma M_3^2) \frac{A_3}{A_2} - \frac{P_3}{P_a} \frac{A_3}{A_2} \phi_1 - 1 \right] \]

Now

\[ \phi_1 = \frac{1 + \gamma M_2^2}{(1 + \gamma - 1 M_2^2)^{\gamma - 1}} = \frac{1 + 1.4 (1.90)^2}{(1 + 0.2 (1.90)^2)^{3.5}} = 0.904 \]

Substituting in the previous values of \( \frac{P_3}{P_a}, \frac{A_3}{A_2}, \) and \( \frac{A_3}{A_2} \) in Eq. (14) gives

\[ M_2^2 = \frac{2}{1.4} \left[ \frac{P_3}{P_a} (1 + 1.4 M_3^2) \frac{1}{0.986} - \frac{6.71}{70} (0.904) - 1 \right] \]

\[ = 1.43 \left[ 1.014 \frac{P_3}{P_a} (1 + 1.4 M_3^2) - 1.0866 \right] \]  \hspace{1cm} (14)

Equations (13) and (14) may now be used to solve for \( M_3 \) and \( M_2 \) provided the back pressure \( P_3 \) is known as a function of \( M_3 \). This relationship between \( M_3 \) and \( \frac{P_3}{P_a} \) is given in Eq. (24) and is shown in Fig. 19 for various values of \( \frac{A_3}{A_4} \). The curve of interest in this example is \( \frac{A_3}{A_4} = 1.51 \).

From this curve values of \( \frac{P_3}{P_a} \) are given for specific values of \( M_3 \), e.g. for

\[ M_3 = 0.15, \quad \frac{P_3}{P_a} = 1.020. \]

These values of \( M_3 \) and \( \frac{P_3}{P_a} \) are now substituted into Eqs. (13) and (14) and a value of \( M_2 \) is obtained for each \( M_3 \) or \( \frac{P_3}{P_a} \). The values of \( M_3 \) and \( M_2 \) for each \( \frac{P_3}{P_a} \) are then plotted on a graph (see Fig. 20). The intersection of Eq. (13) and (14) gives the solution to the ejector equations for this case. As can be seen from the graph the solution is \( M_3 = 0.1575 \) and \( M_2 = 0.1375 \). The corresponding value of \( \frac{P_3}{P_a} \) to this \( M_3 \) is obtained from Fig. 19. It is \( \frac{P_3}{P_a} = 1.023 \).

The velocity of sound

\[ a = \sqrt{\gamma RT} = \sqrt{\gamma R T_0} \sqrt{\frac{T}{T_0}} \]
$T_0 = T_a$ or room temperature = $80^\circ F = 540^\circ R$. Consider, first, the secondary flow, from isentropic flow tables for $M_2 = 0.1375$.

$$\frac{T_2}{T_0} = 0.996$$

so

$$a_2 = \sqrt{1.4 (1715) 540 \sqrt{0.996}} = 1136 (0.995) = 1130 \text{ ft/sec.}$$

$$\overline{V}_2 = M_2a_2 = 0.1375 (1130) = 155 \text{ ft/sec.}$$

The secondary mass flow is

$$G_2 = \rho_2 R_2 \overline{V}_2 = \rho_2 \frac{P_2}{P_a} \frac{T_a}{\overline{T}_2} \overline{V}_2$$

now

$$\frac{P_2}{P_0} = \frac{P_2}{P_a} \frac{14.7 (14.4)}{1715 (540)} = 0.00228 \text{ slugs/ft}^3$$

$$\frac{\rho_2}{\rho_0} = 0.990 \text{ from isentropic flow tables}$$

so

$$G_2 = 0.00228 (0.990) \frac{12.4}{144} 155 = 0.0304 \text{ slugs/sec.}$$

The mixed mass flow

$$G_3 = G_1 + G_2$$

$$G_3 = 0.0079 + 0.0304 = 0.0383 \text{ slugs/sec.}$$

As a check on this latter calculation the mixed mass flow may be calculated directly using the value of $\overline{V}_3$, i.e.

$$G_3 = \rho_3 \overline{A}_3 \overline{V}_3$$

where

$$\rho_3 = \frac{P_3}{RT_3} = \frac{P_3}{P_a} \frac{P_a}{RT_a} \frac{T_a}{T_3}$$

and as

$$T_0 = \frac{T_0}{T_3}$$

from isentropic flow tables

$$\frac{T_0}{T_3} = 1.007 \text{ for } M_3 = 0.16$$

$$\rho_3 = 1.023 (0.00228) 1.007 = 0.00235 \text{ slugs/ft}^3.$$

also

$$\overline{V}_3 = M_3a_3 = 0.1575 \sqrt{1.4 (1715) 540 \frac{1}{1.007}} = 179 \text{ ft/sec.}$$

so

$$G_3 = 0.00235 \left( \frac{12.56}{144} \right) 179 = 0.0368 \text{ slugs/sec.}$$

Previously, using the value of the secondary velocity $G_3 = 0.0383 \text{ slugs/sec.}$ The % difference is

$$\Delta G_{\text{error}} = \frac{0.0015}{0.0368} = 0.041 \text{ or } 4.1\%$$
This difference is perhaps due to numerical errors. In estimating the mass augmentation an average value of $G_3$ is taken.

$$G_3 = \frac{0.0383 + 0.0368}{2} = 0.0375 \text{ slugs/sec.}$$

The primary mass flow is

$$G_1 = \sqrt{\frac{\gamma}{R T_{01}}} \frac{P_{01} \Delta A_1}{(1 + \frac{\gamma-1}{2} M_1^2)} \left(\frac{\gamma+1}{\gamma-1}\right) = \sqrt{\frac{1.4}{1715(540)}} \left[\frac{986(0.1765)(1.90)}{1 + 0.2(1.90)^2}\right] = 0.0079 \text{ slugs/sec.}$$

The mass augmentation $\mu = \frac{G_3}{G_1} = \frac{0.0375}{0.0079} = 4.74$

The next quantity of interest is the thrust augmentation.

The mixed momentum flux

$$J_4 = \frac{G_3 \Delta A_4 V_4^2}{G_4 A_4} = \frac{G_3^2}{G_4 A_4} = \frac{(0.0375)^2}{0.00235(0.144)} = 10.3 \text{ lbs.}$$

The primary momentum flux

$$J_1 = \gamma P_1 \Delta A_1 M_1^2$$

now

$$P_1 = P_2 = P_{a2} - q_2$$

where

$$q_2 = \frac{1}{2} \frac{S_2}{V_2} = \frac{1}{2} \frac{S_{o2}}{V_o} \frac{V_2}{V_o}^2 = 1/2 \ 0.00228 \ (0.990) \ (155)^2 = 27.2 \text{ psf} = 0.188 \text{ psi.}$$

so

$$P_1 = 14.7 - 0.2 = 14.5 \text{ psia}$$

$$J_1 = 1.4 \ (14.5) \ 0.1765 \ (1.90)^2 = 12.9 \text{ lbs.}$$

Hence the thrust augmentation

$$\psi = \frac{J_4}{J_1} = \frac{10.3}{12.9} = .798.$$
FIG. 1 - THE SIMPLE EJECTOR.

FIG. 2 MIXING TUBE EXIT NOZZLE AND DIFFUSER.
FIG. 3 - A PARTICULAR THRUST AUGMENTING SYSTEM.
FIG. 4 - A SCHEMATIC LONGITUDINAL X-SECTION OF THE EJECTOR.
FIG. 5: PHOTOGRAPH OF EJECTOR ASSEMBLY; WITH LOAD NOZZLE AND TRAVERSING PITOT
FIG. 6  PRIMARY INSTRUMENTATION
FIG. 7 PRIMARY NOZZLE ASSEMBLY
FIG. 8 VELOCITY PROFILE - PRIMARY CALIBRATION PIPE
FIG. 9 - PERFORMANCE CURVES FOR A 4 in. DIAMETER MIXING TUBE AT VARIOUS BACK PRESSURES.
FIG. 10 - PERFORMANCE CURVES FOR VARIOUS VALUES OF THE PRIMARY AREA.
FIG. 11 VELOCITY PROFILES AT THE EXIT OF THE LOAD NOZZLE - DIA. MIXING TUBE - 4 INCHES

Diameter of Load Nozzle

- 4 inches
- 3 1/4 inches
- 3 1/2 inches
- 2 1/2 inches

$W_1 = 900 \text{ lbs/hr}$

$\frac{A_2}{A_1} = 112$

$\frac{L}{D} = 7 - 7.5$
FIG. 12 PERFORMANCE CURVES FOR A CHOKED PRIMARY AND A SUPERSONIC PRIMARY
FIG. 13 - PERFORMANCE CURVES FOR VARIOUS VALUES OF THE PRIMARY MASS FLOW.
FIG. 14 - A COMPARISON OF THE VELOCITY PROFILES AT THE EXIT OF THE MIXING TUBE NOZZLES BETWEEN THE CHOKED PRIMARY AND THE SUPERSONIC PRIMARY.
FIG. 15 - PERFORMANCE CURVES FOR VARIOUS BACK Pressures AND FOR TWO MIXING TUBE DIAMETERS.
FIG. 16 - PERFORMANCE CURVES FOR VARIOUS LOADS AND FOR TWO DIAMETERS OF THE MIXING TUBE.
FIG. 17 - VELOCITY PROFILES FOR THE 2.75 IN. MIXING TUBE EXIT NOZZLES.
FIG. 18 - STATIC PRESSURE DISTRIBUTION ALONG THE MIXING TUBE LENGTH. (AS OBTAINED FROM REF. 3)
FIG. 19 THE MIXED MACH NO. AS A FUNCTION OF THE MIXING TUBE BACK PRESSURE FOR VARIOUS VALUES OF $A_3/A_4$ AS GIVEN BY EQUATION 24
A TYPICAL GRAPHICAL SOLUTION TO EQUATIONS (13) AND (14)