THE ANALYSIS OF THE DYNAMIC RESPONSE OF LINEAR VISCOELASTIC MATERIALS

by

R. C. Tennyson, D. Zimcik and J. D. Tulk


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SUMMARY

A combined analytical and experimental programme was undertaken to demonstrate a direct method of determining the complex modulus of a linear viscoelastic material as a function of frequency. Fourier analysis of a dynamic strain pulse (produced by the detonation of an explosive charge) propagating down a long circular cylindrical rod was employed to evaluate the effects of attenuation and dispersion. In addition, experiments were performed with the split Hopkinson bar apparatus to provide data on the range of linear behaviour. Results are presented for two materials tested at room temperature (75 °F), polymethylmethacrylate (PMMA) and a clear epoxy system (Hysol) cured at room temperature. It was found that the results obtained on PMMA were in agreement with the data presented by other investigators at lower frequency levels.
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NOTATION

cross-sectional area

\[ \frac{2}{T} \int_0^T \varepsilon(\xi) \cos nw\xi d\xi, \text{ Fourier coefficient} \]

\[ \frac{2}{T} \int_0^T \varepsilon(\xi) \sin nw\xi d\xi, \text{ Fourier coefficient} \]

\[(E^*/\rho)^{1/2} \sec5/2, \text{ one-dimensional phase velocity} \]

\[(E/\rho)^{1/2}, \text{ elastic one-dimensional wave velocity} \]

diameter

modulus of elasticity

components of complex modulus

\[(E_1^2 + E_2^2)^{1/2} \]

force

strain gauge calibration constant

specimen length

differential operators

time

temperature

axial displacement

axial velocity

axial co-ordinate

\[(\omega/c) \tan5/2, \text{ attenuation coefficient} \]

phase lag

axial strain

\[d\varepsilon/dt \]

viscous coefficient

dummy variable in time

mass density
\( \sigma \) \hspace{1cm} \text{axial stress}

\( \tau \) \hspace{1cm} \text{pulse period}

\( \omega \) \hspace{1cm} \text{frequency}

\textbf{Subscripts}

\( g \) \hspace{1cm} \text{refers to strain gauge}

\( I \) \hspace{1cm} \text{incident}

\( o \) \hspace{1cm} \text{output}

\( R \) \hspace{1cm} \text{reflected}
1. **INTRODUCTION**

The current widespread use of plastics to fabricate load carrying structures (including composite systems) requires the design engineer to know the mechanical properties of these materials. In general, it can be said that plastics (both thermoset and thermoplastic types) demonstrate viscoelastic behaviour under load, with an equation of state of the form,

$$F(\sigma, \varepsilon, t, T) = 0$$

Most viscoelastic solids exhibit mechanical properties which are 'rate dependent', accompanied by significant internal friction loss especially when subjected to high frequency dynamic loading. Extensive testing of polymers has been directed towards determining their mechanical properties due to creep, stress relaxation and dynamic loading. Considerable theoretical and experimental work can be found in the literature describing the viscoelastic behaviour of a wide range of materials.

Although actual viscoelastic solids are generally nonlinear, it is possible to characterize their response in many instances as linear for a prescribed frequency or time domain. Consequently, only two functions corresponding to the bulk and shear moduli are required to describe the constitutive relation for an isotropic material. The four principal methods of measuring these functions are:

1) the response to a step-function strain (the stress-relaxation function)
2) the response to a step-function stress (the creep compliance)
3) the steady-state response to a sinusoidal strain (the dynamic or complex modulus)
4) the steady-state response to a sinusoidal stress (the complex compliance).

Cases (1) and (2) have been found to yield results for \( t > 1 \sim 10 \text{ sec} \) whereas (3) and (4) provide data in the frequency range \( \omega \geq 1 \sim 10 \text{ sec}^{-1} \). The work contained in this report is concerned only with the determination of the dynamic response of a linear viscoelastic material. Current methods for studying dynamic behaviour have employed steady-state sinusoidal vibration tests resulting in complex modulus estimates for a limited range of frequencies (\( \sim 10^4 \text{ Hz} \)), although ultrasonic investigations have also been performed on 'rubberlike' polymers at much higher frequencies (\( \sim 10^8 \text{ Hz} \)). However, most dynamic data have been obtained from impact tests, such as the split Hopkinson bar. (An excellent review of the literature on this subject can be found in Ref. 8). The major deficiency in the use of this data arises because of the lack of an appropriate mechanical model. On the other hand, as noted by Kolsky (6), if the values of the complex modulus are known over a given frequency range, the viscoelastic response of the material to any arbitrary loading (having a spectrum defined within the frequency range) can be calculated in terms of Fourier integrals (see Ref. 9 for example). Clearly, if a mechanical model were available which adequately described dynamic viscoelastic behaviour, the complex modulus could be determined in terms of the model parameters and thus the data from such tests as the split Hopkinson bar could be used (10,11).

This report describes a direct method for evaluating the complex modulus of a linear viscoelastic material over a wide range of frequencies. The basic
approach employed a Fourier analysis of a one-dimensional dynamic strain pulse of arbitrary shape propagating down a finite length circular cylindrical rod. After several reflections, the effects of dispersion and attenuation were estimated in terms of the change in Fourier components. A general analysis based on Kolsky's work \(6,7\) is presented for two materials, (polymethylmethacrylate and an epoxy).

2. BASIC THEORY

For a linear viscoelastic material, the generalized form of the stress-strain equation characterizing a one-dimensional system (ex; a cylindrical rod) is given by

\[
P \sigma = Q \varepsilon
\]  

(2-1)

where \(P\) and \(Q\) are linear differential operators

\[
P = p_0 + p_1 D + p_2 D^2 + p_3 D^3 + \ldots
\]  

\[
Q = q_0 + q_1 D + q_2 D^2 + q_3 D^3 + \ldots
\]  

(2-2)

and

\[
D = \frac{d}{dt}. \quad \text{The standard linear solid is usually described by}
\]

\[
P_0 \sigma + p_1 D \sigma = q_0 \varepsilon + q_1 D \varepsilon
\]  

(2-3)

In terms of a mechanical model representation composed of linear springs and dashpots, the coefficients of Eq. (2-3) are,

\[
p_0 = E_a + E_v
\]  

\[
p_1 = \eta_v
\]  

\[
q_0 = E_a E_v
\]  

\[
q_1 = \eta_v E_v
\]  

(2-4)

where \(E_a\), \(E_v\) and \(\eta_v\) define the spring stiffness and dashpot viscosity as shown in Fig. 1. The individual element stress-strain relations are given by

\[
\sigma_i = E_i \varepsilon_i
\]

\[
\sigma_v = \eta_v D \varepsilon_v
\]  

(2-5)

However, since it is very difficult to formulate an appropriate mechanical model in terms of Eq. (2-1) (without recourse to higher order operators than Eq.(2-3)) which is in agreement with experimental data, an alternative form of the constitutive relation can be employed.

Consider a simple harmonic variation in strain imposed on a linear viscoelastic material \(6,7\). The corresponding stress, as given by Eq. (2-1), will also vary periodically at the same frequency, but out of phase by some amount \(\delta\). Thus,

\[
\varepsilon = \varepsilon_0 \cos wt
\]

\[
\sigma = \sigma_0 \cos (\omega t + \delta)
\]  

(2-6)
In complex form, Eq. (2-6) can be written as,

\[ \varepsilon = \varepsilon_0 e^{i\omega t} \]
\[ \sigma = (E_1 + i E_2) \varepsilon_0 e^{i\omega t} \]  

(2-7)

where \((E_1 + i E_2)\) is defined as the complex modulus of the material and is a function of frequency. Comparing Eqs. (2-6) and (2-7) yields,

\[ E^* = (E_1^2 + E_2^2)^{1/2} = \frac{\sigma_0}{\varepsilon_0} \]

(2-8)

\[ \tan \delta = \frac{E_2}{E_1} \]

Hence the mechanical properties of a linear viscoelastic material can be described by the variation in the complex modulus with frequency.

2.1 Fourier Analysis of One-Dimensional Dynamic Strain Pulse

The equation of motion describing the propagation of a one-dimensional stress wave in a uniform homogeneous isotropic medium is

\[ \frac{\partial \sigma}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2} \]  

(2-9)

Substituting Eq. (2-7) into Eq. (2-9) yields

\[ (E_1 + i E_2) \frac{\partial^2 u}{\partial x^2} = \rho \frac{\partial^2 u}{\partial t^2} \]  

(2-10)

for sufficiently small displacements that \( \varepsilon = \partial u/\partial x \).

The solution to Eq. (2-10) in terms of the strain, assuming a simple harmonic input

\[ \varepsilon(0, t) = \varepsilon_0 e^{i\omega t} \]

i.e., \( \varepsilon(x, t) = \varepsilon_0 e^{i\omega t} \)

is given as

\[ \varepsilon(x, t) = \varepsilon_0 e^{\alpha x + i\omega(t - \frac{x}{c})} \]  

(2-11)

where

\[ \alpha = \frac{\omega}{c} \tan \frac{\delta}{2} \]
\[ c = (E^*/\rho)^{1/2} \sec \frac{\delta}{2} \]  

(2-12)

\( \alpha \) is defined as the attenuation coefficient and \( c \) is the velocity of propagation of a sinusoidal strain pulse at a particular frequency \( \omega \). If the coordinate system is not stationary but moves in the direction of the strain pulse with a velocity \( V \), then Eq. (2-11) can be re-written in the form,
\[ \epsilon(x, t) = \epsilon_0 e^{-\alpha x} \left[ \cos\left(t - \frac{x}{c} + \frac{X}{V}\right) + i \sin\left(t - \frac{x}{c} + \frac{X}{V}\right) \right] \]  

(2-13)

Since a linear viscoelastic solid is described by a linear differential equation, an arbitrary strain pulse can be treated as the superposition of its Fourier components. Thus the appropriate expression for the strain can be written as the sum of \( m \) terms in a Fourier series

\[ \epsilon(x, t) = \sum_{n=1}^{m} \left[ \epsilon_n \cos n\omega \left(t - \frac{x}{c_n} + \frac{X}{V}\right) + i \epsilon'_n \sin n\omega \left(t - \frac{x}{c_n} + \frac{X}{V}\right) \right] \]  

(2-14)

where \( \epsilon_n, \epsilon'_n, \alpha_n \) and \( c_n \) are functions of frequency and

\[ A_n = e^{-\alpha_n x} \left[ \epsilon_n \cos \omega \left( \frac{x}{c_n} - \frac{X}{V} \right) - i \epsilon'_n \sin \omega \left( \frac{x}{c_n} - \frac{X}{V} \right) \right] \]  

(2-15)

\[ B_n = e^{-\alpha_n x} \left[ \epsilon_n \sin \omega \left( \frac{x}{c_n} - \frac{X}{V} \right) + i \epsilon'_n \cos \omega \left( \frac{x}{c_n} - \frac{X}{V} \right) \right] \]

If \( A_n \) and \( B_n \) are evaluated at two spatial locations \( x_1 \) and \( x_2 \), then Eqs. (2-15) can be solved to yield the following values for \( \alpha_n \) and \( c_n \) for each harmonic;

\[ \alpha_n = \frac{1}{2(x_2-x_1)} \ln \frac{A_n^2(x_1) + B_n^2(x_1)}{A_n^2(x_2) + B_n^2(x_2)} \]  

\[ c_n = \left\{ \frac{1}{n\omega(x_2-x_1)} \arctan \left[ \frac{A_n(x_2)B_n(x_1) - A_n(x_1)B_n(x_2)}{A_n(x_1)A_n(x_2) + B_n(x_1)B_n(x_2)} \right] + \frac{1}{V} \right\}^{-1} \]  

(2-16)

Therefore, from Eqs. (2-8), (2-12) and (2-16), the complex modulus can be determined as a function of frequency. The Fourier coefficients \( A_n(x), B_n(x) \) can be readily computed for an arbitrary strain pulse \( \epsilon(x, t) \) from the relations

\[ A_n(x) = \frac{2}{\tau} \int_0^\tau \epsilon(\xi) \cos n\omega \xi \, d\xi \]  

(2-17)

\[ B_n(x) = \frac{2}{\tau} \int_0^\tau \epsilon(\xi) \sin n\omega \xi \, d\xi \]

where \( \tau \) is the period of the base harmonic of the pulse.
2.2 Split Hopkinson Bar Analysis

The split Hopkinson bar test has been used extensively to determine the dynamic response characteristics of materials under uniaxial compressive loading\(^{(12\text{ to }15)}\). Although Bell\(^{(16)}\) has shown major deficiencies in the interpretation of results based on the assumptions used with this method, his criticisms are confined mainly to large nonlinear plastic deformations in metals. The primary theoretical assumption requires the stress and strain to be uniformly distributed in the specimen throughout the impact test period.

The apparatus (Fig. 2), which is based on the configuration used in Ref. 17, is composed of elastic input-output steel cylindrical rods containing a specimen sandwiched between them. Using a high pressure gas reservoir, a cylindrical projectile is fired down the barrel and impacted on the input bar. A one-dimensional elastic strain pulse of magnitude \(\varepsilon_I\) is then propagated along the input bar and recorded by a surface-bonded foil electrical resistance strain gauge mounted a short distance (~1.0 in) from the specimen interface. Subsequently, the incident strain pulse \((\varepsilon_I)\) is partially reflected \((\varepsilon_R)\) and transmitted through the specimen with an intensity dependent upon the relative acoustic impedances of the materials. Again, at the second interface, the strain pulse is partially reflected and transmitted through the output bar \((\varepsilon_O)\) and recorded by another strain gauge. A typical set of traces obtained with and without a specimen is shown in Fig. 3. Interpretation of the signal traces is given in Fig. 4. For sufficiently short specimens, multiple reflections of the strain pulse occur within the sample prior to the original wave front reaching the free ends of the input-output bars. In the simplified analysis used to determine dynamic stress-strain behaviour, the effects of longitudinal inertia and radial motion (resulting in contact surface frictional forces) are neglected. It has been demonstrated that these effects are indeed negligible \(^{(13)}\) providing that the short specimens have a (length/diameter) ratio of at least unity and the contact surfaces are coated with a lubricant. Of particular significance are the tests conducted in this investigation to determine under what conditions an epoxy specimen containing a capsulated foil strain gauge (1/8 in. in length) would exhibit the same strain-time response as the split Hopkinson bar data. A comparison of the results is shown in Figs. 5 and 6. A specimen ratio of \(L/D = 1\) provides the best correlation. In order to evaluate the specimen response, the following equations are used.

Assuming the input-output bars behave in a linear elastic manner, a one-dimensional elastic wave theory can be used. At any time \(\tau\), the displacements at the input-output interfaces of the specimen are given by

\[
\begin{align*}
U_1 &= C \int_0^\tau (\varepsilon_I - \varepsilon_R) \, dt \\
U_2 &= C \int_0^\tau \varepsilon_O \, dt
\end{align*}
\]

respectively, where \(C\) is the elastic one-dimensional wave velocity in the steel bars. Hence, the interface velocities are given by

\[
\begin{align*}
V_1 &= C (\varepsilon_I - \varepsilon_R) \\
V_2 &= C \varepsilon_O
\end{align*}
\]
Thus, if the specimen's initial length is \( L \), the average strain and strain rate existing in the material at any time \( \tau \) can be estimated by the relations

\[
\begin{align*}
\varepsilon_s &= \frac{U_l - U_2}{L} = \frac{C}{L} \int_0^\tau (\varepsilon_I - \varepsilon_R - \varepsilon_o) \, dt \\
\dot{\varepsilon}_s &= \frac{V_l - V_2}{L} = \frac{C}{L} (\varepsilon_I - \varepsilon_R - \varepsilon_o)
\end{align*}
\] (2-20)

The average stress acting on the specimen can be calculated by using the simple one-dimensional elastic equations to determine the interface forces,

\[
\begin{align*}
F_1 &= EA (\varepsilon_I + \varepsilon_R) \\
F_2 &= EA \varepsilon_o
\end{align*}
\] (2-21)

where \( A \) denotes the input-output bars' cross-sectional area, and \( E \) is the elastic modulus of the steel bars. Hence the average stress in the specimen at any time \( \tau \) can be estimated by

\[
\sigma_s = \frac{F_1 + F_2}{2A_s} = \frac{E A}{2A_s} (\varepsilon_I + \varepsilon_R + \varepsilon_o)
\] (2-22)

3. EXPLOSIVE ROD EXPERIMENT

The experimental programme was divided into two phases. Tests were conducted on long circular cylindrical rods fabricated from a clear casting epoxy system (Hysol XC9 - C419 resin and 3561 hardener) cured at room temperature and on commercial extruded 'plexiglas' (polymethylmethacrylate). The split Hopkinson bar tests were performed only on the epoxy since considerable data was already available in the literature on PMMA. All tests were done at room temperature (~ 75°F). The primary purpose of the split Hopkinson bar experiments was to provide additional data on the epoxy to correlate with the explosive rod tests.

3.1 Construction of Specimens

One aspect of the programme was concerned with the comparison of a surface strain gauge signal with that measured internally in the specimen. Considerable work has been done with capsulated strain gauges (18 to 22) indicating that for short time measurements (in which local heating does not affect the material response), accurate strain data can be obtained. Based on the rise time characteristics of the strain gauges (23), it was estimated that their response was accurate up to frequencies of the order of \( 10^6 \) Hz. In order to employ the capsulated strain gauge technique, it was necessary to cast the specimen in layers. For this reason, a liquid epoxy plastic was chosen as one of the experimental materials. This was a commercially available plastic in the form of a resin and hardener. When combined in the ratio of 100 parts resin to 29 parts hardener, the resulting mixture had a curing time of approximately 3 hours at room temperature and could be cast in any shape. In addition, McCullough (24) has shown that if this epoxy is cast in layers, the bond at the interface exhibits the same properties as the material due to cross-linking of the molecules, providing the two layers are cast within 3 to 4 days.
The cast rod was formed in a brass mold which bolted together to form a cylinder 0.375 inches in diameter and 20 inches long. Brass plates were fitted on both ends to form an enclosed cavity. One-half of the cylindrical rod was poured first and allowed to cure for 24 hours. An epoxy-backed strain gauge (BLH SR-4 FAE-03N) was bonded at midlength to the flat surface of the plastic ensuring that the gauge was aligned longitudinally and centered laterally. The epoxy bond was allowed to cure for 24 hours. The mold was then bolted together and the top half of the cylinder was formed by pouring plastic into the upright mold, from the top end. During the curing process, air bubbles were formed and since the liquid plastic was very viscous and the mold was enclosed (except for one end), it was found necessary to pour this last section in two stages, about 6 hours apart, to allow these bubbles to escape. Removal of the completed cylindrical rod was facilitated by the use of a liquid releasing agent applied to the mold before casting. The ends of the completed specimen were then cut to position the imbedded gauge exactly in the centre of the rod, and machined flat. A second strain gauge (BLH SR-4 FAE-03N) was subsequently bonded to the surface of the specimen at precisely the same location as the first. Low resistance wire was attached to the leads of the strain gauges, thus completing the specimen.

A specimen of PMMA was made from 0.375 inch diameter commercial extruded plexiglas. The surface was smoothed of any scratches or irregularities by polishing. A strain gauge (BLH SR-4 FAE-03N) was bonded to the surface in a manner similar to above and the ends of the rod were machined flat. The same low resistance wire was attached to the leads. For both specimens, care was taken not to locally strengthen the surface of the material by rigidly fastening the leads to the specimen.

3.2 Test Apparatus

Testing of the specimens was performed using the apparatus shown in Fig. 7. Two linear bearings held the specimen vertically and allowed it to move freely in the vertical direction yet prohibited horizontal displacement or azimuthal rotation. Flexural motion of the rod was also highly restricted. The lower end of the rod rested in a cavity, 0.375 inches in diameter, in an explosive cap machined from nylon rod 1 inch in diameter. The nylon cap was restrained by a brass ring, both of which were constrained from below by a steel plate. Alignment of the specimen was achieved by the use of six screws on each support.

The explosive (silver acetylide - silver nitrate) was placed in the nylon cap cavity just below the rod. Two wires running longitudinally through the cap between the cavity and the outer circumference were connected to a 6 volt D.C. source by means of a toggle switch. A single strand from No.36 wire was soldered to the ends of these two wires and imbedded in the explosive. Ignition of the explosive was achieved by closing the circuit and heating the wire. Upon ignition, the specimen was free to travel 3 inches vertically before being restrained. The design of the apparatus permitted unrestricted one-dimensional motion for a period long enough to observe many traverses of the length of the rod by the strain pulse due to the explosive alone without a reaction from the supports, as shown in Fig. 8.

3.3 Test Procedure

With the specimen aligned in the testing apparatus as described above, a D.C. power supply was connected to the strain gauges by means of the circuit
shown in Fig. 9. A constant voltage (10 V) was supplied from the source. The voltage drop across the gauges was connected to the vertical input of a dual beam Tektronix type 565 oscilloscope through a 2A7 differential comparator. When two gauges were used, the signal from each was displayed on a different channel but both were triggered from one signal. Since the changes in voltage due to strain were small compared to the zero-strain voltage, the differential comparator enabled only the voltage changes to be recorded on an expanded scale. The oscilloscope was set to display only a single sweep showing the initial nine or ten traverses of the pulse and was triggered internally by the unfiltered input signal. A polaroid camera attached to the oscilloscope recorded the response using high speed film.

A strain pulse was generated in the specimen using the explosive silver acetylide-silver nitrate. This particular material was selected because it could readily be manufactured in the laboratory under controlled conditions, it was easily ignited and it had a high Brisance number. The explosive was ground into a fine power and packed uniformly in the detonating cap to deliver a uniform strain over the cross-section of the rod. Since it is well known that viscoelastic properties are a function of temperature, the strain gauge current was held to a minimum value (15 ~ 20 milliamps) for only a short time prior to testing. The warm-up period leading to zero drift for the gauges was approximately one minute, thus, reducing the effects of localized heating, particularly for the imbedded gauge.

3.4 Analysis of Results

As described previously, the photographic record of the oscilloscope display contained the variation in voltage across the strain gauge as a function of time. It can be shown for the circuitry used (Fig.9) that the change in voltage is related to the strain by the following equation,

\[ \Delta \varepsilon = \Delta E_g \left\{ \frac{R_g + R}{KR E_g} \right\} \]  

where \( \Delta \varepsilon \) is the change in strain associated with a voltage change \( \Delta E_g \),

- \( R_g \) is the gauge resistance
- \( R \) is the circuit resistance
- \( K \) is the gauge factor

and \( E_g \) is the applied voltage across the gauge and is equal to the differential comparator voltage.

For the strain gauges (BLH SR-\( \frac{1}{4} \) FAE-03N) used in this experiment, \( R_g \) was 120 ± 2 ohms and \( K \) was 1.94 ± 2%. The changes in voltage were read from the photographic records by means of a travelling microscope capable of traversing two directions with verniers calibrated to 0.001 cm.

When the strain pulse was generated in the rod by the explosion, it travelled through the rod until it reached the end. Here it was reflected without

* Appendix A summarizes the manufacturing process used to produce the explosive.
distortion by the flat surface and reversed its direction of travel and sign. That is, compression pulses were reflected as tensile strains and vice versa. Each time the pulse passed the strain gauge, a change in voltage was recorded. Since the pulse was not distorted upon reflection from the ends, it was assumed that measuring the pulse after several reflections was equivalent to measuring a pulse down a rod many times the length of the specimen. This method was used to compare the initial strain profile with a later pulse after it had travelled a distance sufficiently large enough to produce changes in its shape due to the effects of attenuation and dispersion. The sweep rate of the oscilloscope was set to allow many traverses of the strain pulse to be recorded to facilitate this procedure. Ten traverses could easily be recorded before the specimen was arrested by the end constraint (refer to Fig. 6).

Since both compressive and tensile strains were alternately recorded, the period of the base frequency was taken to be the time required for both a compressive and a tensile pulse to pass the gauge. Approximately 45 voltage readings were taken on the travelling microscope for each period corresponding to time intervals of $10^{-5}$ sec. between each data point. A numerical integration using the trapezoidal rule was carried out to evaluate Eq. (2-17) to obtain the Fourier coefficients for each harmonic. In order to increase the accuracy of the results, each time interval was subdivided into 10 parts and the voltage co-ordinate interpolated by a linear approximation for these generated data points. The argument for the sine or cosine function used in the integration was taken to correspond to the midpoint of each time sub-interval. This was done for two periods of each photographic trace to obtain approximations for the Fourier coefficients $A_n(x)$ and $B_n(x)$ for two positions. The co-ordinate velocity ($V$) was obtained directly from the trace knowing the time of travel and the distance between the two periods. A comparison between the predicted strain pulse for a typical test run based on the Fourier series representation with the measured data from the photograph is shown in Fig. 10. Equations (2-16) were then used to calculate the attenuation coefficient $\alpha_n$ and phase velocity $c_n$ respectively for each harmonic. All calculations were performed on an IBM 1130 computer. The computer programme is listed in Appendix B.

One objective of the investigation was to determine if the longitudinal strain pulse travelling in a rod could be measured at the surface without the interference of Rayleigh surface waves or distortional waves. For this reason, the epoxy plastic specimen had a strain gauge imbedded on a mid-surface plane along the longitudinal axis as well as on the surface immediately above. Figure 11 shows a photographic record of the oscilloscope display for a test using this specimen. The upper beam recorded the voltage changes across the gauge on the surface of the specimen while the lower beam recorded the signal from the imbedded gauge. Both beams were set to trigger from the lower beam when the change in voltage reached 2 mv. With the exception of the first passage of the strain pulse, both curves are smooth and changes in voltage from the zero-strain voltage are identical at any given time. The initial traverse of the pulse for the external gauge shows some distortion from the other signal but this is very minor and decays completely with this pulse. Consequently, it is possible to measure the longitudinal strain pulse at the surface of the specimen with no distortion.

This fact adds a great deal of flexibility to the present analysis. Since the epoxy plastic was obtained in liquid form and cured in the laboratory,
it was possible to imbed a gauge on the midsurface without much difficulty. However, for the great majority of plastics which are available as solids only, this could pose a problem. However, as shown here, these materials may also be subjected to the same method of analysis since the longitudinal strain pulse can be measured accurately by use of a surface strain gauge.

An equally important fact can be seen from Figs. 12 and 13. These photographic records show that very little time elapsed from the end of one pulse to the start of the next reflected pulse. Therefore, the strain pulse generated in the specimen was almost equal to the specimen length (15 in). Since the length of the strain gauge was only 0.0625 inches, there was minimal averaging of the strain over the length of the gauge. This is particularly evident from the sharp peaks obtained at the maximum strain for the first few traverses of the pulse. Consequently, the strain gauge output closely approximated the actual strain at that point and time.

Using the above results, the attenuation coefficient ($\alpha_n$) and phase velocity ($c_n$) for the frequency range 2 KHz to 50 KHz were calculated for PMMA by the methods outlined previously. The values obtained from three test runs are listed in Tables I, II and III. Table IV contains a summary of the average properties as a function of frequency. Plots of the average values for $\alpha$ and $c$ are contained in Figs. 14 and 15 respectively. Over the entire frequency range considered, the phase velocity was found to be approximately constant at $8.9 \times 10^4$ in/sec. This value agrees well with that obtained by Kolsky(7), $9.0 \times 10^4$ in/sec, which also corresponds closely to the value predicted by linear elastic theory ($E_1/\rho)^{1/2}$ if $\delta$ is small and $E^* \approx E_1$.

For the lower end of the frequency range considered in this analysis, the calculated values of attenuation coefficient follow a linear relationship with frequency (refer to Fig.16) as had been assumed by Kolsky(7) and shown to be true for PMMA for frequencies less than 1 KHz by Lethersich(25). The constant of proportionality assumed by Kolsky(7) was $c^{-1}$ tan $\delta/2$. As mentioned above, the value of 'c' agreed quantitatively with that obtained by Kolsky and the value of tan$\delta/2$ obtained experimentally was 0.019, as compared to a value of 0.020 used by Kolsky. The value of tan$\delta/2$ obtained by Lethersich was 0.019 for a frequency of 800 Hz. Consequently, the values obtained for attenuation coefficient in the lower frequency range are in excellent agreement with other researchers' findings. However, at the higher frequencies considered, the calculated values of attenuation coefficient deviate from this linear relationship. At a frequency of approximately 25 KHz, a local maximum occurs and the attenuation coefficient begins to decrease with increasing frequency. Because it has been shown that the dynamic pulse wavelength must be larger than the lateral dimension of the specimen in order to neglect the effects of lateral inertia and preserve the one-dimensional assumption, there was an upper limit placed on the frequency range. Consequently, only 20 harmonics of the Fourier series were used. However, even at the maximum frequency considered, the wavelength of a sinusoidal strain pulse was many times the lateral dimension of the rod. Therefore the change in behaviour of attenuation coefficient was not due to effects such as distortional waves. Peaking of the attenuation coefficient has also been observed for other materials.

The complex modulus components were also calculated, the results of which are plotted in Figs. 17 and 18. It is readily seen that for the range of frequencies studied, $E^* \approx E_1$, thus indicating that PMMA behaves much
like a linear elastic material with a modulus (~ 9 x 10^5 PSI) which is approximately twice its static value.

The calculated values of attenuation coefficient and phase velocity for the epoxy are contained in Tables V, VI and VII for three test runs, with a summary of the average properties listed in Table VIII. Plots of these parameters are given in Figs. 19 and 20. Similar to the behaviour of PMMA, the phase velocity was found to be approximately constant over the frequency range at a value of 6.61 x 10^4 in/sec, and a local peak in the distribution of \( \alpha \) with frequency was also observed. The maximum value of tan\( \delta \) occurred at approximately 10 KHz, where tan\( \delta \) \( \approx \) 0.068. Plots of the complex modulus and its components are given in Figs. 21 and 22. Again, it was found that the epoxy behaved like a quasi-linear elastic material with \( E^* \approx E_1 = 4.65 \times 10^5 \) psi over the frequency range studied. Of particular significance is the fact that this value is only slightly higher than the static modulus (~ 4 \times 10^5 psi) thus indicating that the epoxy is not as 'rate-sensitive' as PMMA. This behaviour difference can probably be attributed to the cross-linked, three-dimensional molecular network of the epoxy which results in a rigid structure. On the other hand, PMMA, which is a thermoplastic, has no primary chemical bonding between molecules which form long chain (linear branching) aggregates.

3.5 Error

Error from all sources was small. Since the length of the strain pulse was of the order of 15 inches and the length of the strain gauge was 0.0625 inches, there was very little averaging of the strain over the length of the gauge. Measurements of voltage changes and time co-ordinates on the travelling microscope were accurate to within 0.5%. The numerical integration was accurate to within 1%. Deviation of local temperature due to strain gauge heating from room temperature was kept minimal, and the effect of distortional waves was also negligible. Consequently, the estimated error in the calculated values of attenuation coefficient and phase velocity was less than 1% for each test.

However, the error from test to test was larger than this value. This was very likely the result of slightly eccentric loading of the specimen. It was necessary to pack the explosive extremely uniformly (which was difficult to do because of the small amount required) in the cavity in order to achieve a uniform stress over the total cross-sectional area. Any slight variations would produce flexural vibrations in the specimen which were not considered in the mathematical derivations based on an assumed uniform, one-dimensional wave system. As a result, the values obtained from any test were probably affected by the presence of these flexural vibrations, thus accounting for the scatter observed from test to test. However, the linear bearings supporting the specimen restricted flexural vibrations to small amplitudes. Therefore the flexural vibrations caused only small perturbations about the true values and thus did not seriously invalidate the assumption of a one-dimensional system.

4. SPLIT HOPKINSON BAR EXPERIMENT

In order to obtain additional data on the dynamic response of viscoelastic materials as a function of strain rate, a split Hopkinson bar was designed and constructed. The following report describes the apparatus in detail, including results obtained on the epoxy (Hysol XC9-C419 resin with 3561 hardener).
4.1 Construction of Specimens

All test samples used in this phase of the programme were fabricated from Hysol epoxy, as noted above. The specimens were machined into 3/8 inch diameter cylindrical rods from larger blocks of cured epoxy, some of which contained capsulated strain gauges. The same casting procedures as described previously were employed. In order to ensure complete stress transmission at the interfaces, each end of the test specimens was machined flat and parallel and then polished.

4.2 Split Hopkinson Bar Apparatus

The design of the split Hopkinson bar and gun apparatus was based on a similar device used by Maiden and Green(17). The gun consists of a barrel connected to a reservoir chamber with a 'quick-release' poppet valve to seal off the reservoir and a piston arrangement to operate the valve (refer to Fig. 23). The barrel was manufactured from a stainless steel tube, 100 inches in length, having an internal diameter of 7/8 inch. The bore of the tube was accurately honed and polished to a uniform diameter so that the projectile (machined to include a small clearance) could move freely*. The reservoir was of cylindrical shape and mounted directly to the barrel. The poppet valve was aligned such that in the closed position, the face of the valve head sealed off the reservoir from the barrel. The stem of the valve passed through the rear wall of the reservoir and was fastened to a piston within the valve actuating cylinder. All seals in the system were standard rubber 'O' rings. Each of the three chambers of the gun was connected to a pressure control console and hence to a source of compressed gas (usually nitrogen). Thus each chamber could be pressurized and vented independently.

To fire the gun, the projectile was first retracted to the breech end of the barrel using a small vacuum pump connected to the reservoir sections. The poppet valve was then closed by pressurizing the back end of the cylinder. Having sealed the reservoir, the desired test run pressure was introduced. Subsequent pressurization of the front end of the cylinder and venting the back end resulted in the sudden withdrawal of the piston, thus opening the poppet valve. The high pressure in the main reservoir then accelerated the projectile down the barrel which, upon impacting the 'striker' (or input) bar, initiated the dynamic stress wave propagating into the specimen. An end cap fastened to the muzzle of the barrel constrained the projectile to remain in the barrel while, at the same time, it permitted free axial movement of the bar apparatus containing the specimen.

The striker and anvil bars (also known as the input and output bars, respectively) were constructed from straight, ground steel, drill rod, 15 inches in length and 3/8 inch in diameter. These were mounted in tandem in bushings, (Fig.25) which permitted free axial movement, with the specimen sandwiched between them. The interfaces of the bars were machined flat and parallel, and then polished to a mirror finish. The final assembly was then placed in an adjustable frame (as shown in Fig. 26) which allowed the bar apparatus to be accurately aligned with respect to the axis of the gun barrel. A check on the

* Calibration tests indicated (Fig.24) that a minimum reservoir pressure of approximately 13 psi was required to overcome the barrel friction for the given projectile.
alignment was provided by testing with no specimen. Complete stress wave transmission was found, as shown in Fig. 3.

Pairs of bonded foil resistance strain gauges (BLH SR-4 FAE-03N 125 6L) were mounted diametrically opposed on the input-output bars in order to evaluate the dynamic response of the specimen based on the theory outlined in section 2.2. Using conventional Wheatstone bridge circuits (Fig. 27), the signal outputs were monitored on a dual beam oscilloscope.

In order to obtain quasi-static, low strain-rate data on the epoxy material, a simple arrangement composed of an aluminum load cell in combination with a specimen was tested in a standard compression machine. A schematic of the facility set-up is contained in Fig. 28.

4.3 Analysis of Results

Quasi-Static and Low Strain-Rate Tests

Prior to the high strain-rate experiments, a series of tests were made with a conventional compression machine. The samples were machined to the shape shown in Fig. 28, with foil strain gauges bonded to the narrow section. The output signals were recorded on polaroid film and the results analyzed using a travelling microscope. A plot summarizing the response of the epoxy is shown in Fig. 29. Except for the range of large strains, the linear portion of the curves all have the same slope of \(4 \times 10^5\) psi. This value is in complete agreement with extensive data obtained from other tests performed on the same epoxy.

Dynamic Test Results

The first set of experiments conducted in the split Hopkinson bar involved specimens of varying length with capsulated strain gauges. Figure 30 describes the circuitry used to obtain the strain-time output. Preliminary tests were done with the strain gauges in an equilibrium state (i.e., no long term drift of the signal). As shown in Fig. 31, the localized heating led to a dynamic response which was quite different than that obtained from analysis of the split Hopkinson bar data (based on the equations given in Section 2.2). Once the duration of the 'warm-up' period was reduced to minimize the thermal effects, it was found that an optimum (length/diameter) ratio of about one provided the best correlation of the internal strain-time response with analysis. (refer to Figs. 5 and 6).

Based on these results, a set of dynamic tests was performed on the epoxy at various strain-rates. A summary of the response characteristics is contained in Fig. 32. The maximum increase in the slope of the stress-strain curves was from \(4 \times 10^5\) psi (static value) to \(4.3 \times 10^5\) psi corresponding to a strain-rate \(\sim 1200\) sec\(^{-1}\). From the complex modulus data, \(E^* \approx E_1 \approx 4.6 \times 10^5\) psi at \(\omega \approx 2\) KHz. Thus it can be concluded that both sets of data are in close agreement.

5. CONCLUSIONS

The explosive rod test coupled with a Fourier analysis of the dynamic one-dimensional strain pulse has been shown to be an effective means of determining the attenuation coefficient and phase velocity (and thus the complex modulus) as a function of frequency for a linear viscoelastic material. In
particular, for the two materials studied (epoxy and PMMA), it was found that for the frequency range of 2 KHz to 50 KHz, the complex modulus was essentially equal to the elastic component ($E_1$). The results for PMMA were in close agreement with data obtained at lower frequencies by other researchers. The response of the epoxy system was also found to correspond to data obtained from split Hopkinson bar tests. In general, the attenuation coefficients for both materials varied (to a first approximation) linearly with frequency up to a maximum value, after which they decreased in magnitude as frequency increased.
REFERENCES


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"Compressive Strain-Rate Tests on Six Selected Materials at Strain Rates from $10^{-3}$ to $10^4$ In/In/Sec", Trans. of the ASME, J. Applied Mech., September, 1966.

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Skjelbreia, L.  

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Brasier, R. I.  
Baker, W. E.  

Dove, R. C.  

22. Lewis, J. L.  
Goldsmith, W.  
Cunningham, D. M.  

23. Oi, K.  

24. McCullough, J. R.  

25. Lethersich, W.  
APPENDIX A: PREPARATION OF SILVER ACETYLIDE-SILVER NITRATE

The silver acetylide-silver nitrate explosive was prepared in the laboratory in the following manner according to the equation:

\[ 3 \text{AgNO}_3 + \text{C}_2\text{H}_2 \rightarrow \text{Ag}_2\text{C}_2\text{AgNO}_3 + 2\text{HNO}_3 \]

1. Measure 10gm. AgNO₃ and pour crystals into a gas washing bottle.
2. Add 40 cc. warm distilled water.
3. Add 6 cc. concentrated HNO₃.
4. Stir until the AgNO₃ crystals are all dissolved.
5. Bubble pure acetylene through the above until precipitation is complete (about 15 minutes).
6. Allow the precipitate to settle and decant the excess liquid.
7. Wash with an equal volume of pure acetone. Mix well and allow to precipitate.
8. Wash with acetone 8 further times. Each time allow the precipitate to settle and decant the excess liquid before washing.
APPENDIX B

Computer Listing of Fourier Analysis to Evaluate Phase Velocity and Attenuation Coefficient as a Function of Frequency.
LIST SOURCE PROGRAM
*ARITHMETIC TRACE

SURROUTINE FORAN ( N, VSF, TSF, GV, GF, RG, R, DX, VEL, V, T, NPUL1, NPUL2, INTRAC, N)
DIMENSION Y(2, 60), T(60), ATOT(2, 20), BTOT(2, 20), C(20), ALPHA(20),
ISUM(20), ASUM(20), V(2, 60), ZERO(20),
1A(2, 20), B(2, 20), YY(2, 25), NPUL(2)
NPUL1 = NPUL(1)
NPUL2 = NPUL(2)
N = N + 1
DO 14 I = 1, 2
ZERO(I) = V(I, 1)
DO 14 J = 1, N
V(I, J) = ZERO(I) - V(I, J)
14 V(I, J) = V(I, J) * VSF
DO 801 J = 1, N
801 T(J) = T(J) * TSF
DT = T(2) - T(1)
WRITE (3, 501) NPIC, INTRAC, NPUL1, NPUL2
501 FORMAT (1H1, 12H RUN NUMBER, 13H 10X, 13H TRACE NUMBER, 13H 10X, 114H PULSE NUMBERS, 12H 4H AND, 12H)
DO 555 J = 1, N
555 V(I, J) = V(I, J)
DO 56 J = 2, N
56 T(J) = T(J - 1) + DT
DT = DT / 10.
PI = 3.1415
CONST = (R + RG) / (GV * R * GF)
DO 25 I = 1, 2
DO 25 J = 1, N
25 Y(I, J) = V(I, J) * CONST
N1 = N - 1
DO 15 L = 1, 2
DO 15 K = 1, M
P = 2. * PI * K / T(N)
ATOT(L, K) = 0.
BTOT(L, K) = 0.
DO 17 J = 1, N1
DY = (Y(L, J + 1) - Y(L, J)) / 10.
YY(L, 1) = Y(L, J)
DO 17 JJ = 2, 11
YY(L, JJ) = Y(L, J) + (JJ - 1) * DY
ARG = (P * T(J) + (JJ - 1.5) * DT))
ATOT(L, K) = ATOT(L, K) + ((YY(L, JJ) + YY(L, JJ - 1)) / 2.) * DT * COS(ARG))
17 BTOT(L, K) = BTOT(L, K) + ((YY(L, JJ) + YY(L, JJ - 1)) / 2.) * DT * SIN(ARG))
AL(L, K) = (2. * ATOT(L, K)) / T(N)
15 RL(L, K) = (2. * BTOT(L, K)) / T(N)
WRITE (3, 503)
503 FORMAT (/ 13H FOR COS COEF, 5X, 13H FOR SIN COEF//)
DO 200 I = 1, 2
DO 200 J=1,M
200 WRITE (3,55) A(I,J)*B(I,J)
55 FORMAT (2E16.7)
WRITE (9,502)
502 FORMAT (/17H ATTENUATION COEF,15H PHASE VELOCITY,10H FREQUENCY//)
DO 20 I=1,M
    SSUM(I)=(A(I,1)*A(I,1)+B(I,1)*B(I,1))/(A(2,1)*A(2,1)+B(2,1)*B(2,1))
    ASUM(I)=(A(2,I)*B(1,I)-A(I,1)*B(2,1))/(A(1,1)*A(2,1)+B(1,1)*B(2,1))
    ALPHA(I)=1./(2.*DX)*ALOG(SSUM(I))
    C(I)=1./(1./(I*P*DX))*ATAN(ASUM(I))+(1./VEL))
    P=I/T(N)
20 WRITE(3,22) ALPHA(I),C(I),P
22 FORMAT (1E15.7)
RETURN
END

FEATURES SUPPORTED
ARITHMETIC TRACE
CORE REQUIREMENTS FOR FORAN
COMMON O VARIABLES 904 PROGRAM 950
RELATIVE ENTRY POINT ADDRESS IS 03F9 (HEX)
END OF COMPILATION

// DUP
*DELETF FORAN
D 26 NAME NOT FOUND IN LET/FLET

*STORE WS UA FORAN
CART ID 0001 DB ADDR 60F3 DB CNT 0042
FOR

*I0CS*\(1132 \text{ PRINTER, CARD, TYPEWRITER, PLOTTER})
*LIST SOURCE PROGRAM
*ARITHMETIC TRACE

**DIMENSION** \(V(2,60), T(60), NPUL(2)\)

\(M = 20\)
\(DX = 108.\)
\(GV = 2.111\)
\(TSF = 0.0004\)
\(N = 43\)
\(VSF = 0.004\)
\(GF = 1.95\)
\(RG = 119.8\)
\(R = 455.8\)

DO 11 \(I = 1, 2\)
READ \((2,500)\) PIC,NTRAC,NPUL \((1)\)
500 FORMAT \((3110)\)
DO 11 \(J = 1, N\)
13 FORMAT \((2F10.6)\)
READ \((2,13)\) \(T(J), V(I,J)\)
\(V(I,J) = V(I,J)\)
11 \(T(J) = T(J)\)
PAUSE 7777
CALL FORAN(F,N,Vsf,TSF,GV,GF,RG,R,DX,VEL,V,T,NPUL,1NTRAC,M)
STOP
END

**FEATURES SUPPORTED**
ARITHMETIC TRACE
I0CS

**CORE REQUIREMENTS FOR**
COMMON 0 VARIABLES 394 PROGRAM 186

END OF COMPILATION

// XEQ
TABLE I

Run No. 23
Weight of Explosive used: 0.08774 gm
Maximum strain rate: 90 sec⁻¹
Specimen: 18" PMMA rod
Measurements taken from external gauge.

<table>
<thead>
<tr>
<th>Frequency (cps)</th>
<th>Attenuation Coefficient (in⁻¹)</th>
<th>Phase Velocity (in/sec)</th>
</tr>
</thead>
<tbody>
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<td>5.85 x 10⁻³</td>
<td>8.87 x 10⁴</td>
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<td>3.60</td>
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### TABLE II

Run No. 24  
Weight of Explosive used: 0.09440 gm  
Maximum strain rate: 100 sec⁻¹  
Specimen: 18” PMMA rod  
Measurements taken from external gauge.

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<th>Frequency (cps)</th>
<th>Attenuation Coefficient (in⁻¹)</th>
<th>Phase Velocity (in/sec)</th>
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</thead>
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<tr>
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</table>
TABLE III

Run No.26
Weight of Explosive used: 0.08496 gm
Maximum strain rate: 80 sec\(^{-1}\)
Specimen: 18" PMMA rod
Measurements taken from external gauge

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<th>Frequency (cps)</th>
<th>Attenuation Coefficient (in(^{-1}))</th>
<th>Phase Velocity (in/sec)</th>
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<tr>
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<td>19.9</td>
<td>8.84</td>
</tr>
</tbody>
</table>
TABLE IV

Summary of Average Values of Properties for Polymethylmethacrylate (density = 2.33 slugs/ft³)

<table>
<thead>
<tr>
<th>Frequency (cps)</th>
<th>Average Attenuation Coefficient (in⁻¹)</th>
<th>Average Phase Velocity (in/sec)</th>
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<tr>
<td>2.38 x 10³</td>
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TABLE V

Run No. 19
Weight of explosive used: 0.08102 gm
Maximum strain rate: 140 sec⁻¹
Specimen: 15" epoxy rod
Measurements taken from internal gauge.

<table>
<thead>
<tr>
<th>Frequency (cps)</th>
<th>Attenuation Coefficient (in⁻¹)</th>
<th>Phase Velocity (in/sec)</th>
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TABLE VI

Run No.20
Weight of Explosive used: 0.07796 gm
Maximum strain rate: 140 sec⁻¹
Specimen: 15" epoxy rod.
Measurements taken from internal gauge.

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<tr>
<th>Frequency (cps)</th>
<th>Attenuation Coefficients (in⁻¹)</th>
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TABLE VII

Run No. 22
Weight of explosive used: 0.08172 gm
Maximum strain rate: 140 sec^-1
Specimen 15" epoxy rod
Measurements taken from internal gauge.

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<th>Attenuation Coefficient (in^-1)</th>
<th>Phase Velocity (in/sec)</th>
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</tr>
<tr>
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<td>6.58</td>
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</tbody>
</table>
### Table VIII

Summary of Average Values of Properties for Epoxy (density = 2.21 slugs/ft³)

<table>
<thead>
<tr>
<th>Frequency (cps)</th>
<th>Average Attenuation Coefficient (in⁻¹)</th>
<th>Average Phase Velocity (in/sec)</th>
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<td>2.13 x 10³</td>
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</tr>
<tr>
<td>42.6</td>
<td>17.7</td>
<td>6.61</td>
</tr>
</tbody>
</table>
FIG. 1  STANDARD LINEAR MODEL
FIG. 2 SPLIT HOPKINSON BAR APPARATUS

BAR APPARATUS SCHEMATIC

PROJECTILE  STRIKER BAR  STRAIN GAUGES  ANVIL BAR

SAMPLE

BRIDGE CIRCUITRY

DUAL TRACE OSCILLOSCOPE

GUN AND BAR APPARATUS
FIG. 3  TYPICAL STRAIN GAUGE TRACES FROM SPLIT HOPKINSON BAR
FIG. 4 INTERPRETATION OF SIGNAL TRACES FROM THE SPLIT HOPKINSON BAR APPARATUS
FIG. 5 COMPARISON OF INTERNALLY & EXTERNALLY MEASURED STRAIN-TIME HISTORIES
Minimum Warm-up Period

\[ \frac{L}{D} \approx 2.7 \]

FIG. 6 COMPARISON OF INTERNALLY & EXTERNALLY MEASURED STRAIN-TIME HISTORIES
FIG. 7 EXPLOSIVE ROD APPARATUS

FIG. 8 PROPAGATION OF DYNAMIC STRAIN PULSES ALONG CYLINDRICAL ROD
DC Power Supply
\( V = 10 \text{ volts} \)

Strain Gauge
\( R_g = 120 \, \Omega \)

Oscilloscope
\( R = 470 \, \Omega \)

FIG. 9 STRAIN GAUGE CIRCUITRY
FIG. 10  COMPARISON OF FOURIER SERIES FIT WITH MEASURED DATA
FIG. II  TYPICAL PHOTOGRAPHIC RECORD OF OSCILLOSCOPE DISPLAY FOR EPOXY SPECIMEN. OUTPUT FROM EXTERNAL GAUGE RECORDED ON UPPER CHANNEL; IMBETDED GAUGE ON LOWER CHANNEL. VERTICAL SCALE (STRAIN): $3 \times 10^{-3}$/DIV. HORIZONTAL SCALE (TIME): 0.1 mSEC./DIV.
FIG. 12  TYPICAL PHOTOGRAPHIC RECORD OF OSCILLOSCOPE
DISPLAY FOR THE IMBEDDED GAUGE FOR EPOXY
SPECIMEN.
VERTICAL SCALE (STRAIN): $1.5 \times 10^{-3}$/DIV.
HORIZONTAL SCALE (TIME): 0.2 m SEC./DIV.
FIG. 13 TYPICAL PHOTOGRAPHIC RECORD OF OSCILLOSCOPE DISPLAY OF OUTPUT FROM EXTERNAL GAUGE FOR PMMA.

VERTICAL SCALE (STRAIN): $6 \times 10^{-4}$/DIV.
HORIZONTAL SCALE (TIME): 0.2 mSEC./DIV.
FIG. 15

PHASE VELOCITY FOR PMMA

Phase Velocity (in/sec)

Frequency (cps)
FIG. 16 ATTENUATION COEFFICIENT FOR PMMA
FIG. 17 PLOT OF COMPLEX MODULUS COMPONENTS ($E^*$, $E_1$) AS A FUNCTION OF FREQUENCY FOR PMMA
FIG. 18 PLOT OF COMPLEX MODULUS COMPONENT ($E_2$) AS A FUNCTION OF FREQUENCY FOR PMMA
FIG. 19 ATTENUATION COEFFICIENT FOR "HYSOL" EPOXY PLASTIC
FIG. 20  PHASE VELOCITY FOR "HYSOL" EPOXY PLASTIC
FIG. 21  PLOT OF COMPLEX MODULUS COMPONENTS \( (E^*, E_1) \) AS A FUNCTION OF FREQUENCY FOR EPOXY PLASTIC
FIG. 22 PLOT OF COMPLEX MODULUS COMPONENT \( (E_2) \) AS A FUNCTION OF FREQUENCY FOR EPOXY PLASTIC
FIG. 23 SCHEMATIC VIEW OF AIR GUN

POPPET VALVE

PROJECTILE

BARREL

VALVE ACTIVATING CYLINDER

MAIN RESERVOIR
FIG. 24  THE EFFECT OF FIRING PRESSURE ON MAXIMUM INCIDENT COMpressive STRESS FOR A 3 LB PROJECTILE
FIG. 25 DETAIL OF BARS

FIG. 26 VIEW OF BARS, BAR SUPPORTS AND INSTRUMENTATION
FIG. 27 SPLIT HOPKINSON BAR INSTRUMENTATION

Strain Gauges
(SR4 foil gauges, 0.030 inch gauge length)

Dual Beam Oscilloscope
(Tektronix 565)
FIG. 28 APPARATUS for LOW STRAIN-RATE TESTING

- Epoxy Sample
- Bridge Circuits
- Operational Amplifier Circuits
  - Amplifier
  - Differentiator
- Strain Signal
- Stress Signal
- Dual Beam Oscilloscope (Tektronix 565)
FIG. 29 STRESS-STRAIN RELATIONSHIP (LOADING IN COMPRESSION)

- **QUASI-STATIC**
- \( \varepsilon \approx 6 \times 10^{-3} \)
- \( \varepsilon \approx 6 \times 10^{-2} \)
- \( \varepsilon \approx 1.5 \times 10^{-1} \)

![Stress-Strain Relationship Graph](image-url)
FIG. 30  INTERNALLY GAUGED SAMPLE EXPERIMENT

-0
POWER
SUPPLY
TO
CRO.

470Ω

1/2

3/8

75

50

25

0

50 100 150 200
TIME μSEC.

STRAIN %
FIG. 31 COMPARISON OF INTERNALLY AND EXTERNALLY MEASURED STRAIN-TIME HISTORIES
CONFIGURATION 'A' 1/2 INCH SAMPLE
SAMPLE AT EQUILIBRIUM TEMPERATURE

- - - INTERNALLY MEASURED STRAIN (LOCAL)
--- EXTERNALLY MEASURED STRAIN (AVERAGE)
Fig. 32 Stress-strain curves for epoxy plastic at high strain rates.
A combined analytical and experimental programme was undertaken to demonstrate a direct method of determining the complex modulus of a linear viscoelastic material as a function of frequency. Fourier analysis of a dynamic strain pulse (produced by the detonation of an explosive charge) propagating down a long circular cylindrical rod was employed to evaluate the effects of attenuation and dispersion. In addition, experiments were performed with the split Hopkinson bar apparatus to provide data on the range of linear behaviour. Results are presented for two materials tested at room temperature (75°F), polymethylmethacrylate (PMMA) and a clear epoxy system (Hysol) cured at room temperature. It was found that the results obtained on PMMA were in agreement with the data presented by other investigators at lower frequency levels.

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