HYPER VELOCITY LAUNCHERS
PART I: SIMPLE LAUNCHERS

by

I. I. Glass
ACKNOWLEDGEMENTS

I wish to express my thanks to Dr. G. N. Patterson for his encouragement and interest in this project.

The help received from Mr. A. Benoit, Mr. G. F. Bremner, and Mr. D. E. Rothe with some of the drawings and calculations and the critical reading of the manuscript by Dr. A. H. Makomaski and Mr. J. J. L. Brennan are appreciated. The assistance given by Mr. Robert G. Dunn, ARL, and Mr. R. S. Penner, DRB, in obtaining reports and other technical data is acknowledged with thanks.

This work was supported by ARL under Contract No. USAF AF-33 (657) - 7874.
SUMMARY

A critical survey is made of hypervelocity launchers and their research applications to reentry physics, hypervelocity impact, gasdynamics and aerodynamics. The present portion of the survey (Part I) deals mainly with simple launchers (constant area, single stage, light-gas guns) in order to illustrate some of the basic concepts affecting their design and operation. The effects of counterpressure, boundary layer, bore friction, and gas imperfections are considered. It is noted that there are many outstanding problems associated with the possible acceleration of simple aerodynamic models to velocities of 50,000 ft/sec and beyond. It appears that microparticles might be accelerated to supervelocities of 100,000 ft/sec or more by explosive spray or electrodynamic techniques. Subsequent portions of the survey will deal with the topics outlined in the table of contents.
# TABLE OF CONTENTS

## HYPERVELOCITY LAUNCHERS

**Part I: Simple Launchers***

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>2.</td>
<td>THEORETICAL PERFORMANCE OF SIMPLE LAUNCHERS</td>
<td>3</td>
</tr>
<tr>
<td>2.1</td>
<td>Simple Launchers and Simple Shock Tubes</td>
<td>3</td>
</tr>
<tr>
<td>2.2</td>
<td>Some Properties of Nonstationary Waves for Launcher Analysis</td>
<td>13</td>
</tr>
<tr>
<td>2.3</td>
<td>Internal Ballistics of Simple Launchers</td>
<td>18</td>
</tr>
<tr>
<td>2.3.1</td>
<td>Case of an Evacuated Barrel</td>
<td>18</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Effect of Counterpressure</td>
<td>25</td>
</tr>
<tr>
<td>2.3.3</td>
<td>Boundary Layer Effects</td>
<td>30</td>
</tr>
<tr>
<td>2.3.4</td>
<td>Effect of Bore Friction</td>
<td>49</td>
</tr>
<tr>
<td>2.3.5</td>
<td>Effects of Gas Imperfections</td>
<td>52</td>
</tr>
<tr>
<td>2.4</td>
<td>Extension of Existing Methods of Analysis</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>REFERENCES</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>TABLES</td>
<td></td>
</tr>
<tr>
<td></td>
<td>FIGURES</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>DRIVERS FOR HYPERVELOCITY LAUNCHERS</td>
<td></td>
</tr>
<tr>
<td>3.1</td>
<td>Effects of Driver Pressure and Temperature</td>
<td></td>
</tr>
<tr>
<td>3.2</td>
<td>Chamberage and Chamber Length</td>
<td></td>
</tr>
<tr>
<td>3.3</td>
<td>Combustion-Heated Driver Gases</td>
<td></td>
</tr>
<tr>
<td>3.4</td>
<td>Piston-Type Compressors and Heaters</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>Electrical Discharges as Heaters and Compressors</td>
<td></td>
</tr>
<tr>
<td>3.6</td>
<td>Explosive Compression and Heating</td>
<td></td>
</tr>
<tr>
<td>3.7</td>
<td>Spherical Implosions and Explosions for High-Enthalpy, High-Pressure Drivers</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>LAUNCHER STAGING</td>
<td></td>
</tr>
<tr>
<td>4.1</td>
<td>Types of Single-Stage and Multi-Stage Launchers</td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td>Accelerated Reservoir Launchers</td>
<td></td>
</tr>
</tbody>
</table>

* Part I consists of Sections 1 and 2 of the proposed outline. The remaining parts will be issued when they are completed. Relevant references, tables, and figures are contained in each part.

iv
5. OBSERVED PERFORMANCE OF ACTUAL LAUNCHERS

5.1 Combustion and Piston-Driven Launchers
5.2 Spark-Discharge or "Pot-Shot" Launchers
5.3 Explosive-Compression Launchers
5.4 Implosion-Driven Launchers

6. POSSIBLE IMPROVEMENTS IN LAUNCHER PERFORMANCE

6.1 Energy Augmentation of the Driver Gas
6.2 Shot Acceleration through Energy Addition During Flight
6.3 Constant Base Pressure Launchers

7. EXPLOSIVE ACCELERATION TO SUPERVELOCITIES

8. ELECTRODYNAMIC ACCELERATORS

9. HYPERVELOCITY RESEARCH

9.1 Impact, Cratering, and Solid-State Physics
9.2 Aerophysical and Aerodynamic Research
9.3 Chemical Kinetics
9.4 Radiation and Wake Studies
9.5 Radiative Wave Interactions and Energy Addition to Plasma Sheaths
9.6 Surface Properties and Accommodation Coefficients

10. CONCLUSIONS

APPENDIX A - Hypervelocity Launcher Instrumentation

1. Measurement of Velocity
2. Pressure Measurements
3. Temperature and Heat Transfer Measurements
4. Spectroscopic Methods
5. Radiative Properties and Chemical Kinetics
6. Optical Methods
7. Telemetry
8. Aerodynamic Coefficients
# NOTATION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>area ($ft^2$, $cm^2$), nondimensional quantity (Eq. 100)</td>
</tr>
<tr>
<td>a</td>
<td>sound speed ($ft/sec$, $cm/sec$)</td>
</tr>
<tr>
<td>$C_f$</td>
<td>boundary layer skin friction coefficient (local)</td>
</tr>
<tr>
<td>$C_p$</td>
<td>specific heat at constant pressure ($ft lb/slug^0R$, $cal/gm^0K$)</td>
</tr>
<tr>
<td>$C_v$</td>
<td>specific heat at constant volume ($ft lb/slug^0R$, $cal/gm^0K$)</td>
</tr>
<tr>
<td>D</td>
<td>hydraulic diameter ($4 \times \text{area}/\text{perimeter}$) (ft, cm)</td>
</tr>
<tr>
<td>d</td>
<td>diameter (ft, cm)</td>
</tr>
<tr>
<td>F</td>
<td>frictional force (lb, dynes)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>nondimensional friction force</td>
</tr>
<tr>
<td>$f_s$</td>
<td>solid friction coefficient</td>
</tr>
<tr>
<td>g</td>
<td>gravitational acceleration ($981 \text{cm}/\text{sec}^2$, $32.2 \text{ft}/\text{sec}^2$)</td>
</tr>
<tr>
<td>k</td>
<td>heat conduction coefficient ($\text{slug ft}/\text{sec}^3 \text{degree}$, $\text{gm cm}/\text{sec}^3 \text{degree}$; see also Eq. (139))</td>
</tr>
<tr>
<td>L</td>
<td>length (ft, cm)</td>
</tr>
<tr>
<td>L/d</td>
<td>launcher length in calibres</td>
</tr>
<tr>
<td>M</td>
<td>Mach number ($u/a$)</td>
</tr>
<tr>
<td>$M_s$</td>
<td>shock Mach number</td>
</tr>
<tr>
<td>m</td>
<td>mass (slugs, gm); molecular weight (slugs, gm)</td>
</tr>
<tr>
<td>P</td>
<td>Riemann variable ($2a/((\gamma - 1) + u)$)</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number ($Pr = \mu C_p/k$)</td>
</tr>
<tr>
<td>p</td>
<td>pressure ($\text{lb}/\text{ft}^2$, $\text{dynes}/\text{cm}^2$)</td>
</tr>
<tr>
<td>Q</td>
<td>Riemann variable ($2a/((\gamma - 1) - u)$)</td>
</tr>
<tr>
<td>q</td>
<td>heat transfer coefficient ($\text{BTU}/\text{ft}^2 \text{sec}$, $\text{cal}/\text{cm}^2 \text{sec}$)</td>
</tr>
<tr>
<td>R</td>
<td>gas constant ($\text{ft lb}/\text{slug}^0R$, $\text{cal}/\text{gm}^0K$)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\mathcal{R}$</td>
<td>universal gas constant (ft·lb/slug mole°R, cal/gm mole°K)</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number ($Re = \frac{u}{\nu}$) (see Eqs. 105 and 106)</td>
</tr>
<tr>
<td>$r$</td>
<td>recovery factor (Eq. 127)</td>
</tr>
<tr>
<td>$S$</td>
<td>entropy (ft·lb/slug°R, cal/gm°K); stress (psi)</td>
</tr>
<tr>
<td>$St$</td>
<td>Stanton number $\left[ q = St \frac{C_p}{u} \frac{1}{\rho} (T_r - T_w) \right]$</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature (°R, °K)</td>
</tr>
<tr>
<td>$T_o$</td>
<td>stagnation temperature</td>
</tr>
<tr>
<td>$T_r$</td>
<td>recovery temperature</td>
</tr>
<tr>
<td>$T_w$</td>
<td>wall temperature</td>
</tr>
<tr>
<td>$t$</td>
<td>time (sec)</td>
</tr>
<tr>
<td>$\tilde{t}$</td>
<td>nondimensional time (Eq. 55c)</td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>nondimensional time (Eq. 57)</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity, component in x-direction (ft/sec, cm/sec)</td>
</tr>
<tr>
<td>$\hat{u}$</td>
<td>$(2/\gamma_4 - 1) a_4$ escape speed (ft/sec, cm/sec)</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td>$u_3/\hat{u}$ nondimensional velocity (Eq. 55c)</td>
</tr>
<tr>
<td>$\Delta u$</td>
<td>$u_3/a_4$ nondimensional velocity (Eqs. 57, 74, 142)</td>
</tr>
<tr>
<td>$V$</td>
<td>relative velocity (ft/sec, cm/sec)</td>
</tr>
<tr>
<td>$\nu'$</td>
<td>velocity component in y-direction (ft/sec, cm/sec)</td>
</tr>
<tr>
<td>$w$</td>
<td>shock wave velocity</td>
</tr>
<tr>
<td>$x$</td>
<td>distance (ft, cm); degree of ionization</td>
</tr>
<tr>
<td>$\bar{x}$</td>
<td>nondimensional distance (Eq. 53c)</td>
</tr>
<tr>
<td>$\Delta x$</td>
<td>nondimensional distance (Eq. 57)</td>
</tr>
<tr>
<td>$y$</td>
<td>distance (ft, cm); see also Eq. (76)</td>
</tr>
<tr>
<td>$Z$</td>
<td>compressibility factor ($p = Z \rho RT$); also Eq. 51</td>
</tr>
</tbody>
</table>
\( \lambda \) \((\gamma + 1)/(\gamma - 1)\); acceleration (ft/sec\(^2\), cm/sec\(^2\)); degree of dissociation; diffusivity (Eq. 94); nondimensional quantity (Eq. 129)

\( \beta \) \((\gamma - 1)/2\gamma\)

cpecific heat ratio for a frozen flow

\( \Gamma \) specific heat ratio (\(C_P/C_V\))

\( \gamma_e \) specific heat ratio for equilibrium flow

\( \delta \) boundary layer thickness (in, cm)

\( \delta^* \) boundary layer displacement thickness (in, cm)

\( \eta \) efficiency (Eq. 140); nondimensional length (Eq. 84)

\( \theta \) momentum thickness (in, cm, Eq. 112)

\( \mu \) viscosity coefficient (slug/ft sec, gm/cm sec); Poisson's ratio

\( \nu \) kinematic viscosity (\(\nu = \mu/\rho\)) (ft\(^2\)/sec, cm\(^2\)/sec)

\( \rho \) density (slugs/ft\(^3\), gm/cm\(^3\))

\( \zeta \) nondimensional velocity (Eq. 108)

\( \zeta \) boundary layer shear stress; (psi, dynes/cm\(^2\)); time interval

\( \phi \) frictional stress (psi, dynes/cm\(^2\)) (see Eq. 135), also see Eq. 117.
1. INTRODUCTION

In the years following the Second World War, which saw the advent of manned space vehicles, the interest in attainable velocities for atmospheric and space flights grew from about 1000 ft/sec to 50,000 ft/sec and for space-impact simulation experiments to well over 200,000 ft/sec (Ref. 1). To keep pace with the requirements of space-vehicle design various research facilities were developed based either on the concept of a hyper-velocity flow over a stationary model or a model moving at hypervelocities through a stationary gas. The shock tube, hypersonic shock tunnel, the hot-shot tunnel, and the plasma jet are examples of testing facilities of varying flow duration wherein the gas moves over the model. Whereas the light-gas gun, electro-dynamic accelerators and explosively accelerated projectiles are examples of facilities where the models move through a known atmosphere at hypervelocities or impact on selected targets.

In all of these devices the goal is to convert available mechanical, chemical or electrical potential energy into directed kinetic energy of relatively small masses moving at hypervelocities. No single facility can be considered as superior in all respects, but rather as a complementary device. For example, the light-gas gun is ideally suited for well-controlled aerodynamic and aerophysical studies whereas explosively or electrodynamically launched pellets or microparticles that attain velocities of 100,000 ft/sec would be very desirable for impact experiments at meteoric velocities. At the time of writing, masses of 2.0 to 7.0 gm have been accelerated explosively to velocities up to 70,000 ft/sec (Ref. 2) whereas projectiles launched from 0.22 calibre light gas guns have achieved a velocity of 32,800 ft/sec (Ref. 3). It appears possible that microparticles (0.1 to 10 micron radius) may be accelerated by explosive implosive sprays to velocities over 100,000 ft/sec and by electrodynamic means to a similar value.

Although hypervelocities are not easily obtained in a laboratory they are a common occurrence in space. For example the mean orbital velocities of Mercury, Venus, Earth and Mars are 30, 22, 18\(\frac{3}{2}\), and 15 mile/sec, respectively. Meteors are known to achieve velocities greater than 40 mile/sec, which is nearly an order better than what has been achieved in accelerating small projectiles by any device in a laboratory to date. Consequently, astronomers had the first opportunity (1803) to study hypervelocity flight in a natural laboratory (astroballistics) and the attendant problems of heat transfer, ablation, and impact (Ref. 4).

Hypervelocities, in a laboratory were probably first produced through the use of shaped charges (1937). The method started with the launching of a large number of particles of unknown characteristics and then a controlled experiment involving a single pellet was finally achieved (1950) (Ref. 4)

Conventional guns are not capable of launching a shot at speeds exceeding 10,000 ft/sec no matter how large the explosive charge that is used
to drive it or how light the shot may be, even in an evacuated barrel. The
difficulty arises from the fact that the driver gases generated by conventional
propellants or explosives are essentially too heavy (or have a large acoustic
impedance). Consequently, the pressures that would otherwise be available
to accelerate the shot are expended in accelerating the heavy driver gas. To
overcome this problem use has been made of driver gases that are light in
weight and possess high sound speeds (or a small acoustic impedance) such as
hydrogen and helium, especially at high temperatures. Since these are pure
gases and are usually not generated as the products of an explosive, the
hydrogen or helium must be compressed and heated by other means (chemical,
electrical, or mechanical). These facts led to the invention of the light-gas
gun.

The light-gas gun in an aeroballistic range forms a very
versatile and powerful research facility for the study of a great number of
physical problems associated with hypervelocities. The first light-gas gun
developed by Crozier and Hume (conceived in 1946) is described in some de­
tail in Ref. 5. Use was made of 0.32 to 0.41 in. dia. gun barrels, 39 to 54
in. long, propelling a 4.47 gm sphere at a maximum muzzle velocity of 12,230
ft/sec. By 1955 there were apparently several guns in operation including
one at Aberdeen Proving Grounds under the direction of Dr. A. Charters, and
at NOL, Silver Spring, under Dr. Z.I. Slawsky. Both groups pioneered later
improvements in light-gas gun technology (Ref. 6).

The first light-gas gun in Canada was developed at CARDE
(1957) under the direction of Dr. G.V. Bull. This was a 0.5 in. dia. gun
capable of firing 2 gm models up to 26,000 ft/sec. CARDE is unique in its
development of the largest (14 in. dia.) light-gas gun capable of firing 100 lb.
models at 5000 ft/sec (Ref. 7a).

The light-gas guns described previously made use of high­
pressure reservoirs of hydrogen or helium as the light driver gas in order to
accelerate the model to hypervelocities. Improvements were later made by
using chemical explosives and combustion coupled with area changes, pistons,
and multiple driving chambers or imploding shock waves to further increase
the temperature and pressure of the driver gas in order to attain velocities in
excess of 30,000 ft/sec. Another very useful method of raising the driver gas
temperature and pressure has been through the instantaneous addition of
electrical energy by means of a rapid discharge from a bank of condensers.

Many schemes have also been developed for using electro­
magnetic and electrostatic accelerators. However, to date these methods
have not proved to be very successful for the launching of aerodynamic models
owing to many practical difficulties that have as yet not been solved (Ref. 8).
However, as noted above electrodynamic accelerators and explosive spray
techniques have proved successful in accelerating microparticles to the highest
velocities achieved to date.
Eq. (1) it can be seen that under the above assumptions for a 1 cm. dia. barrel accelerations of $7 \times 10^6$ g would be produced. These are very high values indeed and model design for high-g loading becomes a very serious problem in the attainment of hypervelocities.

The previous remarks are well illustrated in Fig. 2, which shows graphically the effects of the three main parameters in Eq. 4, that is, the increase of muzzle velocity with launcher length, base pressure and low weight models for cylinders of single calibre. The example of the nylon cylinder considered above is indicated as a single point. Although the above analysis is based on very idealized conditions it does give an indication of the ultimate muzzle velocity that one might expect to attain under the best of conditions in a light-gas gun, and 50,000 ft/sec appears as a reasonable limit. In addition, it will be shown subsequently that the concept of a constant base pressure has led to the development of the so called "accelerated reservoir light-gas gun" (Ref. 11).

It was noted from Eq. (4) that a piston of vanishing mass would lead to very large muzzle velocities. This concept immediately suggests that it is worth comparing the performance of the idealized simple launcher considered previously with the idealized simple shock tube (Ref. 12). As seen from Fig. 3, the simple shock tube consists of a tube of uniform cross-section containing a diaphragm that separates a high-pressure (temperature) state (4) from a low-pressure (temperature) state (1). When the diaphragm is ruptured a wave system is produced consisting of a shock wave and contact surface moving into the channel and a rarefaction wave into the chamber. The pressure and velocity across the contact surface are equal but temperature, density, or entropy are generally not equal. The contact surface separates two thermodynamically different states produced by the isentropic expansion of the high-pressure gas in the chamber and the shock wave compression of the gas in the channel.

It is assumed that the contact surface undergoes infinite acceleration to a uniform velocity at the origin. The maximum velocity that the contact surface can attain is the so called "escape speed" of the gas into a vacuum, that is, when the channel pressure ($p_1$) is zero. Consequently, the contact surface, which drives the shock wave ahead of itself, may be considered as a massless piston and it is instructive to examine this type of flow in some detail in order to determine what velocities can be obtained by accelerating the gas itself at varying pressures. It is also worth noting that the maximum velocity of the shock wave produced in the channel depends directly on the magnitude of the escape speed and it will be shown later that this property is a function of the initial sound speed of the gas before expansion and its specific heat ratio.

The equations of motion of the nonstationary flow through a one-dimensional expansion wave are given by (Ref. 12).
mass: \[ \rho_t + \rho u_x + u \rho_x = 0 \]  

(momentum: \[ u_t + u u_x + \frac{1}{\rho} \rho_x = 0 \]  

energy or entropy: \[ \left( \frac{p}{\rho^\gamma} \right)_t + u \left( \frac{p}{\rho^\gamma} \right)_x = 0 \]  
or \[ S_t + u S_x = 0 \]  

(7)  

entropic equation of state: \[ \frac{p}{\rho^\gamma} = \exp \left( \frac{S - S_0}{C_v} \right) \rho_{\infty} \]  
or \[ p = A(S) \rho^\gamma \]  

The three nonlinear partial differential equations and the equation of state provide four equations for the four unknown dependent variables \( u, p, \rho, \) and \( S \) in terms of the independent variables \( x \) and \( t \).

If the flow is isentropic everywhere then from Eq. (8),

\[ p_x = \left( \frac{\partial p}{\partial \rho} \right)_S \rho_x + \left( \frac{\partial p}{\partial S} \right)_\rho S_x \]  

which reduces to

\[ p_x = a^2 \rho_x \]  
where \( a^2 = \left( \frac{\partial p}{\partial \rho} \right)_S \)

and

\[ p = A \rho^\gamma, \]  
where \( A \) is now a constant. (8b)

Therefore, Eq. (6) can be restated as,

\[ u_t + u u_x + \frac{a^2}{\rho} \rho_x = 0 \]  

(6a)

and Eq. (7) is no longer required.

Equations (5) and (6a) can be solved by using the "method of characteristics" (Ref. 13) which reduces the two partial differential equations, Eqs. (5) and (6), to the following four ordinary differential equations. Along the \( Q \)-characteristic lines or Mach lines (Fig. 3),

\[ \frac{dx}{dt} = u - a \]  

\[ \frac{du}{d\rho} = \frac{a}{\rho} \]  
or \[ \frac{du}{dp} = \frac{1}{\rho a} \]  

(11)
or making use of Eq. (8b) and integrating,

\[
\frac{2a}{\gamma - 1} - u = Q, \text{ a constant}
\]

Along the P-characteristic lines,

\[
\frac{dx}{dt} = u + a \tag{12}
\]
\[
\frac{du}{dp} = -\frac{a}{\rho} \quad \text{or} \quad \frac{du}{dp} = -\frac{1}{\rho a} \tag{13}
\]

or

\[
\frac{2a}{\gamma - 1} + u = P, \text{ a constant}
\]

It is of interest to note that from the differential form of Eqs. (11) and (13) that the velocity increment (\(\Delta u\)) attained by the gas as a result of a given pressure drop (\(\Delta p\)) is inversely proportional to the so called "acoustic impedance" of the gas (\(\rho a\)). Consequently, this quantity must be kept small in order to achieve efficient conversion of the random thermal energy of the gas into directed kinetic energy and in turn the acceleration of a model by the gas. In essence, this is the principle on which a shock tube or a light-gas gun operates. From Table 1, it is seen that the light gases hydrogen and helium possess the lowest value of the acoustic impedance and as expected they are widely used as driver gases in light-gas guns.

From Fig. 3, it can be seen that a P-characteristic line crosses all of the Q-characteristic lines. Consequently, for a Q-rarefaction wave the following relations apply everywhere,

\[
\frac{dx}{dt} = u - a \tag{14}
\]
\[
\frac{2a}{\gamma - 1} + u = \text{const} = \frac{2a_4}{\gamma_4 - 1}
\]

The complementary equations hold through a P-rarefaction wave.

The escape speed (\(\hat{u}\)), which is attained along a particle path after a very long time (Ref. 12), is obtained from Eq. (14) when a \(\to 0\), or,

\[
\hat{u} = \frac{2a_4}{\gamma_4 - 1} \tag{15}
\]

That is, the particle path and the slopes (Eqs. (10), (12)) of the P and Q-characteristic lines ultimately all coincide at the tail of the wave for a com-
plete expansion into a vacuum. It is of interest to compare the escape speed of hydrogen, helium, air, and of SF$_6$. From Table I, at $0^\circ$C,

\[
\begin{align*}
\hat{u}_{H_2} &= \frac{2 \times 4165}{1.40 - 1} = 20,830 \text{ ft/sec} \\
\hat{u}_{He} &= \frac{2 \times 3182}{1.67 - 1} = 9,500 \text{ ft/sec} \\
\hat{u}_{air} &= \frac{2 \times 1087}{1.4 - 1} = 5,440 \text{ ft/sec} \\
\hat{u}_{SF_6} &= \frac{2 \times 432}{1.1 - 1} = 8,640 \text{ ft/sec}
\end{align*}
\]

The low molecular weight of hydrogen and its smaller value of $(\gamma' - 1)$ gives it an exceedingly large escape speed even at low temperatures by comparison to helium. On the other hand, even though SF$_6$ has a very high molecular weight the very small value of $(\gamma' - 1)$ produces a much better value of $\hat{u}$ than for air. However, it was shown above that more pressure energy would go into accelerating the heavy gas to a fixed velocity (Eq. 11), compared to the light gases. Consequently, the available base pressures would be small and for practical launcher lengths SF$_6$ would not be an efficient driver gas for a light-gas gun although the escape speed is reasonably large. That is, the value of having $\gamma' \rightarrow 1$, in this case, is nullified by the large molecular weight of SF$_6$, which has a value of 146. What is required is a combination of low molecular weight and $\gamma'$ close to unity. This ideal requirement is approached by using partially dissociated or ionized hydrogen (see Subsection 2.3.5).

Using the thermal equation of state for a perfect gas,

\[ p = \rho RT \]  

and Eq. (8b) one obtains

\[
\frac{a}{a_4} = \left[ \frac{T}{T_4} \right]^\frac{1}{2} = \left[ \frac{\rho}{\rho_4} \right]^\frac{\gamma'}{2} = \left[ \frac{p}{p_4} \right]^\frac{\gamma' - 1}{2\gamma'}  
\]  

(17)

and using Eq. (14),

\[
\frac{a}{a_4} = \left[ 1 - \frac{\gamma' - 1}{2} \frac{u}{a_4} \right] = \left[ 1 + \frac{(\gamma' - 1)}{2} M \right]^{-1}  
\]  

(18)

where, $M = \frac{u}{a}$

\[
\frac{T}{T_4} = \left[ 1 - \frac{\gamma' - 1}{2} \frac{u}{a_4} \right]^2 = \left[ 1 + \frac{\gamma' - 1}{2} M \right]^{-2}  
\]  

(19)
From Eq. (22) for isentropic expansion of a perfect gas one obtains an alternate index of conversion of pressure energy into directed energy as,

$$\frac{du}{dp} = \frac{a_4}{\gamma_4 p_4} \left[ \frac{p_4}{p} \right]^{\gamma_4 + 1} = -\frac{1}{(\gamma_4 p_4 a_4)} \left[ \frac{p_4}{p} \right]^{\gamma_4 + 1} = -\frac{V}{p_4} \sqrt{\frac{T_4}{\gamma_4 m_4}} \left[ \frac{p_4}{p} \right]^{\gamma_4 + 1}$$

(23)

From Eq. (23) it is seen that for a given expansion \((p_4/p)\), the temperature, pressure, specific heat ratio and molecular weight of the driver gas must be optimized to obtain the largest velocity increment. In the range of \(1 \leq \gamma \leq 5/3\), for physically available gases, the power index varies only from 1.0 to 0.8, respectively. Consequently, for given values of \(p_4\) and \(p_4/p\), the dominating term is \(a_4\), and a large sound speed or a light gas is required for acceleration to hypervelocities. For example, consider an expansion to sonic velocity, then from Eqs. (21) and (22), when \(M = 1\) (at the diaphragm station, Fig. 3), \(u/a_4 = 2/(\gamma_4 + 1)\), and the values of \(u\) for H2, He, air and SF6 are 3470, 2390, 905 and 411 ft/sec respectively, for initial sound speeds at 0°C. The superiority of hydrogen is quite apparent.

Since \(\gamma_4\) appears in the denominator of Eq. (23) as well as in the power index, significant improvements in the quantity \(du/dp\) can come from both terms by using a low value of \(\gamma_4\). It is interesting to note that increasing the pressure \((p_4)\), while keeping the other driver gas quantities fixed reduces the value of \(du/dp\). This arises from the fact that the density \((\rho_4)\) of the gas or its acoustic impedance \((\rho_4 a_4)\) is increased.

If considerations of barrel strength fix the value of the driver pressure \((p_4)\), then the quantity \(a_4/\gamma_4 = \frac{V}{p_4} (T/\gamma m_4)\) can be maximized by making \((T/\gamma m_4)\) large or by using gases at high temperature that have low molecular weights \((m_4)\) and low values of the specific heat ratio \((\gamma_4)\), which automatically ensures a larger power index \((\gamma_4 + 1)/\gamma_4\). Dissociated hydrogen possesses the qualifications (essentially a low value of the acoustic impedance), and this topic will be considered in greater detail in Subsection 2.3.5.

It is worth noting that the parameter \((T/\gamma m)_4\) was in fact used in Ref. 7b, as an optimizing quantity for hypervelocity launcher driver gases.
evolved as explosive products. However, as Eq. (23) indicates the parameter may be used in this form only if $p_4$ is fixed, otherwise $(T/\gamma m p^2)_4$ should be used for performance analysis.

It is also worth noting from Eq. (15) that the maximum velocity attained by a massless piston (escape speed) only depends on the initial gas temperature, whereas the dynamic analysis (Eq. 4) showed the importance of large base pressures for the acceleration of a piston of finite mass. An examination of Eq. (21) shows that the idealized concept of the constant base pressure in the expansion of a gas can be attained only when $a_4 \to \infty$. Consequently, one might suspect that both of these parameters ($a_4, p_4$) are of great importance in the operation of an actual light-gas gun. In practice, the attainment of high temperature is invariably associated with high pressure so that these conditions are usually satisfied in a successful launcher. However, it is not possible to launch a projectile from a gun at hypervelocities on the basis of even enormous pressures alone, as Langweiler has shown (Refs. 5, 9). It is worthwhile noting from Eq. (22) that when the escape speed is achieved the base pressure $p$ is zero. That is, all the random energy has been converted into directed energy. This hypothetical case applied only to the acceleration of gas particles and the limit is not applicable when accelerating an actual mass since the base pressure drops and the accelerating force becomes vanishingly small requiring hypothetically infinite launcher lengths (see Subsection 2.8.1.).

From Eq. (15) it is seen that a 25-fold increase in temperature (6800°K) would produce a 5-fold increase in the escape speed (104,000 ft/sec in hydrogen) in a perfect gas. It is of some interest to examine the strength of the shock wave that would be produced in the channel of a simple shock tube containing air under these conditions. The shock relations in a perfect gas in stationary coordinates are given by (Ref. 12)

\[
V_1 = u_1 - w_1 = w_1, \quad V_2 = u_2 - w_1
\]  
\text{(24)}

where, $u_1 = 0, u_2$ = contact surface velocity, $w_1$ = shock velocity and $V_1, V_2$ are the relative velocities with respect to the shock wave.

mass: \[
\rho_1 V_1 = \rho_2 V_2
\]  
\text{(25)}

momentum: \[
p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2
\]  
\text{(26)}

energy: \[
C_p T_1 + \frac{1}{2} V_1^2 = C_p T_2 + \frac{1}{2} V_2^2
\]  
\text{(27)}

From the above equations the following relations can be derived,

\[
\frac{\rho_1}{\rho_2} = 1 - \frac{u_2}{w_1}
\]  
\text{(28)}
Equations (28) and (29) apply to a perfect or imperfect gas and are known sometimes as the mechanical shock conditions. Equations (30) apply only to a perfect gas since the energy equation for a perfect gas, Eqs. (27), was used in their derivation. In particular the relation between pressure and density is known as the Rankine-Hugoniot equation across a shock front. A plot of pressure ratio vs density ratio results in a Hugoniot curve (as distinct from an isentrope) for a perfect gas. However, for an imperfect gas each initial pressure gives a different Hugoniot wave.

For strong shock waves \((p_1/p_2) \to 0\), and from Eqs. (30) and (28)

\[
\frac{p_2}{p_1} \to \frac{\gamma + 1}{\gamma - 1}
\]

From Eq. (15), knowing that \(u_2 \to \hat{u}\), or in the operation of a shock tube the maximum value of the particle velocity behind a shock wave (contact surface velocity) cannot exceed the escape speed of the driver gas,

\[
u_2 \to \frac{2 a_4}{\gamma - 1} = \hat{u}
\]

\[
\frac{w_1}{a_1} = M_s \left| \frac{p_2}{p_1} \to \infty \right| = \frac{\gamma + 1}{2} \cdot \frac{\hat{u}}{a_1} = \frac{\gamma + 1}{\gamma - 1} \cdot \frac{a_4}{a_1}
\]

where, \(M_s = \text{shock Mach number}\)
The important role of the escape speed or the sound speed ratio $a_4/a_1$ is clearly illustrated. From Eq. (34) it can be seen that even with extremely large pressures in the chamber (diaphragm pressure ratio $p_4/p_1 \to \infty$) one cannot attain really large contact surface velocities or shock Mach numbers unless $a_4$ or $T_4$ is very large. This illustrates the importance of having high-pressure, high-temperature drivers for hypervelocity shock tube operation or for light-gas guns.

For example, from Eq. (34) it can be concluded that the limiting shock Mach number in an air-driven shock tube (containing air in the chamber and channel both initially at the same temperature) is given by

$$M_s \bigg|_{p_4/p_1 \to \infty} = \gamma_1 + 1/ \gamma_4 - 1 = 6.$$  

However, for a shock Mach number $M_s = 6$, the condition $p_1/p_2 \to 0$ is not precisely satisfied and

$$p_2/p_1 = (\gamma_1 + 1)/ (\gamma_1 - 1) - \xi,$$

(4) is a small quantity) and a more precise evaluation of the limiting value of $M_s$ from the shock tube equations then yields (Ref. 12),

$$M_s \bigg|_{p_4/p_1 \to \infty} = \gamma_1 + 1/ \gamma_4 - 1 \cdot \frac{a_4}{a_1} \frac{1}{M_s} \bigg|_{p_4/p_1 \to \infty}$$

For the case of hydrogen driving air, at room temperature Eq. (34) gives $M_s = 6 a_4/a_1$ or $M_s = 23$, and for the example noted before for hydrogen at 68000 K, and air at 2730 K, $M_s = 115$, a very large shock Mach number indeed. From Eq. (35) the results for the previous cases give values of $M_s = 6.16, 23.04$, and 115.01. That is, for large $M_s$ , $\xi \to 0$, and Eq. (34) gives the correct limit.

From the foregoing it can be concluded that it is possible to accelerate a gas originally in a stationary reservoir (chamber) by means of a nonstationary expansion wave to very high velocities provided the chamber gas pressure and temperature are high. If the channel pressure is high, then to satisfy the condition of infinitely large diaphragm pressure ($p_4/p_1 \to \infty$) the chamber pressure must be correspondingly large. The question now arises if the high-speed gas can impart this velocity by accelerating a projectile of finite mass under actual conditions when $a_4$ is not exceedingly large and $p_b/p_4 < 1$, that is, for realistic initial chamber temperature and pressure and subsequent falling base pressures with correspondingly decreasing accelerations. This topic will be considered in detail in Subsection 2.3.
2.2 Some Properties of Nonstationary Waves for Launcher Analysis

Although a few basic properties of one-dimensional expansion waves and shock waves in a perfect, inviscid, gas were considered in the previous section, it may be worthwhile as a reference for subsequent sections to examine the more general case where area changes, entropy changes, heat transfer, and skin friction produced by the viscous boundary layer may also exist in the flow.

Consider the simple launcher shown in Fig. 4. It is assumed that the diameter is a constant throughout and that the projectile is a cylinder driven by the high-pressure, high-temperature gas in state (4). The barrel would usually be evacuated to reduce the effects of counterpressure. Even under rarefied conditions a wave system will be produced in front of the projectile. However, the effect on the projectile motion will be unimportant since the number of gas particles involved is small. At higher density, energy is required to accelerate an ever growing volume of gas in front of the piston and also to increase its internal energy in the form of active degrees (translation and rotation) and inert degrees of freedom (vibration, electronic excitation, dissociation, and ionization). In this case the projectile retardation can be significant since its kinetic energy is being degraded to supply the aforementioned energy sinks. Alternately a smaller initial pressure ratio exists across the piston and consequently it is accelerated to a lower final velocity.

It can also be seen that a viscous boundary layer is produced on the tube walls, whose leading edges extend from the rarefaction wave head to the piston, and from the shock wave to the piston. The rate of growth of this boundary layer and its characteristics are quite different in both regions. The boundary layer thickness is shown schematically in Fig. 4, as greater behind the shock wave, as a result of the relatively rarefied conditions resulting from an evacuated barrel. The adverse effects of a boundary layer growth behind the piston in the expansion region can be significant since the pressure changes which are caused by skin friction and heat transfer are propagated along the P and Q-characteristic lines and they act to reduce the base pressure on the projectile. In turn, this decreases the muzzle velocity. These effects are characterized by the boundary layer shear stress (τ) and the heat transfer rate (q).

In addition, the projectile will experience a retardation as a result of its contact with the barrel walls. This effect can be characterized by the kinetic friction stress between two solids (ϕ) and incorporated in the equation of motion for the projectile (Subsection 2.3.4).

Although area changes have not been shown in Fig. 4, the following characteristic equations of nonstationary motion apply to a perfect gas in a channel of slowly varying cross-sections (dlnA/dx << 1) and with boundary layer effects (C_f, q) (Ref. 14, 12).
\[
\frac{\delta + P}{\delta t} = -au \frac{\partial \ln A}{\partial x} - a \frac{\partial \ln \delta}{\partial t} + \frac{a}{\gamma R} \frac{\delta + S}{\delta t} + (\gamma - 1) \frac{a}{\gamma R} \frac{DS}{Dt} - \frac{2u^2C_f}{D} \tag{36}
\]

\[
\frac{\delta - Q}{\delta t} = -au \frac{\partial \ln A}{\partial x} - a \frac{\partial \ln \delta}{\partial t} + \frac{a}{\gamma R} \frac{\delta - S}{\delta t} + (\gamma - 1) \frac{a}{\gamma R} \frac{DS}{Dt} + \frac{2u^2C_f}{D} \tag{37}
\]

\[
\frac{DS}{Dt} = \left[ \frac{4q}{\rho D} + \frac{2u^3C_f}{D} \right] / T \tag{38}
\]

where, \( \frac{D}{Dt} = \frac{\partial}{\partial t} + u \frac{\partial}{\partial x} \)

represents differentiation along a particle path whose characteristic slope is

\[
\frac{dx}{dt} = u,
\]

\[
\frac{\delta + \gamma}{\delta t} = \frac{\partial}{\partial t} + (u + a) \frac{\partial}{\partial x}
\]

represents differentiation along a \( \frac{P}{Q} \)-characteristic with slope \( \frac{dx}{dt} = (u + a) \),

\[
\frac{2u^2C_f}{D} = \text{frictional force/unit mass}
\]

\[
\frac{4q}{\rho D} = \text{heat transferred/unit mass}
\]

that is, the effects of skin friction and heat transfer are averaged over the section.

If the Reynolds analogy between skin friction and heat transfer is applied to the nonstationary boundary layer then (Ref. 15),

\[
q = C_p \int u \left( \frac{C_f}{2} Pr \right)^{-2/3} \th \tag{39}
\]

where, \( Pr = \frac{\mu C_p}{k} \), Prandtl number

\[
\theta = (T_r - T_w), \text{ temperature potential, (recovery temperature-wall temperature)}
\]

Using these relations Eq. (38) can be restated so that only the skin friction coefficient appears,
\[
\frac{D(S/R)}{Dt} = \left[ M^2 + \frac{1}{\gamma^2} \frac{\theta}{T} P_r - \frac{2}{3} \right] \frac{2Y}{D} u C_f \quad (40)
\]

Equations (36) to (38) show how the values of the Riemann quantities \( P \) and \( Q \) change along the two cross characteristics and how the entropy varies along the third characteristic line, the particle path. It is seen that changes in \( P \) and \( Q \) arise from area changes, skin friction, and heat transfer and in turn this will affect the base pressure of the projectile at any given instant in the barrel.

It is worth noting that external energy addition during the piston motion or mass addition would require a modification of the above equations to include these additional parameters. Even with the absence of the latter two effects, Eqs. (36) to (38) represent a formidable set of nonlinear partial differential equations. They can be solved by applying certain assumptions and using a step-by-step graphical integration in finite difference form. Many examples, including effects of boundaries and discontinuities, can be found in Ref. 14. A particular application that might be useful in the analysis of boundary layer effects on the projectile motion is given in Refs. 16 and 17.

Gas imperfections (vibration, electronic excitation, dissociation and ionization) are not taken into account in the above expressions of the characteristic equations. These effects may be quite important for a particular set of operating conditions and would have to be considered. The characteristic equations then become much more complex and Mollier diagrams (for equilibrium flow) and rate equations (for nonequilibrium flow) describing the chemical kinetics must be used. An analysis of a nonstationary nonequilibrium rarefaction wave is given in Ref. 18, and the same method can be applied to launcher analysis.

As an example of the flow through a nonstationary expansion wave, including effects of equilibrium gas imperfections, consider for convenience the case of air expanded through a rarefaction wave from 8000\(^0\)K and a density of 0.1 NTP. The major constituents of the gas are (Ref. 19)

\[
\begin{align*}
N_2 &= 3.08 \times 10^{-1}, \\
NO &= 5.64 \times 10^{-3}, \\
\text{NO}^+ &= 1.03 \times 10^{-3}, \\
\text{e} &= 2.22 \times 10^{-3}, \\
N &= 9.45 \times 10^{-1}, \\
N^+ &= 7.94 \times 10^{-4}, \\
O &= 4.14 \times 10^{-1}, \\
O^+ &= 2.72 \times 10^{-4}, \\
A &= 9.38 \times 10^{-3}.
\end{align*}
\]

From the above species obtain by summation, \( Z = 1.686 \), \( C_{Va} = 2.844 \), \( \Gamma = 1.593 \) (where \( Z \) = compressibility factor in the thermal equation of state \( p = Z \rho RT \), \( C_{Va} \) = active specific heat for translation and rotation, \( \Gamma \) = frozen flow isentropic index, \( \Gamma = \frac{C_{pa}}{C_{Va}} = 1 + \frac{ZR}{C_{Va}} \); \( \rho = \frac{A \rho^n}{} \) ). The results are shown in Figs. 5 to 10, respectively for the flow of a perfect gas, frozen flow and equilibrium flow. The perfect gas and frozen flow results are obtained by using the values of \( \gamma = 1.40 \) and \( \Gamma = 1.593 \) respectively, in the isentropic expansion. The equilibrium flow case is computed by using a Mollier diagram (Ref. 20) and graphical integration (Ref. 12). The sound speed in the frozen case is given by \( a^2 = \Gamma \rho / \rho = \Gamma ZRT \) and in the equilibrium case \( a^2 = \frac{\gamma}{\gamma - 1} \rho \frac{\partial p}{\partial \rho} = \frac{\gamma e}{\gamma - 1} ZRT \), where,

\[
\gamma = \left( \frac{\partial p}{\partial \rho} \right)_s.
\]
Figures 5 to 7 show the variation of temperature, sound speed and density with pressure for the three cases. Note that the variation of pressure with density in Fig. 7 gives a linear relation on a logarithmic plot, with a slope of $\gamma = 1.40$ and $\Gamma = 1.59$, for the perfect gas and frozen flow, respectively. Figure 8 shows the superior acoustic impedance (larger values of $1/\rho a$) for the equilibrium flow compared with a perfect gas flow (see Table 2 for details). A graphical integration of this curve yields the variation of particle velocity with pressure as shown in Fig. 9. The effect of the acoustic impedance shows up in the improved particle velocities for the equilibrium flow as the inert degrees of freedom are deexcited and add heat to the flow thereby increasing the kinetic and thermal energies of the gas.

A cross-plot of Fig. 6 and 9 yields the variation of particle velocity with sound speed. The constant slopes of the perfect and frozen flows are given explicitly by (Eq. 13), $du/da = -2/(\gamma - 1) = -5$ and $du/da = -2/(\Gamma - 1) = -3.37$ respectively. Such a relation does not exist for the equilibrium flow. From the above it can be concluded that launcher calculations can be refined by including the effects of gas imperfections for frozen or equilibrium flows. In the case of a nonequilibrium flow the computations would be considerably more difficult since they would involve the chemical kinetic rate equations and some of these may not even be known to a good degree of accuracy. Consequently, the relaxation times or distances that would define the zone of nonequilibrium flow between the frozen and equilibrium flows would not be too well defined.

In the case of normal shock waves with gas imperfections Eqs. 25 to 30, have to be modified since a unique relation (Rankine-Hugoniot) between pressure and density no longer exists. As a result the following equations must be solved simultaneously to obtain a solution,

\begin{align*}
\text{mass:} & \quad \rho_1 V_1 = \rho_2 V_2 \\
\text{momentum:} & \quad p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2 \\
\text{energy:} & \quad h_1 + \frac{1}{2} V_1^2 = h_2 + \frac{1}{2} V_2^2 \\
\text{state (thermally imperfect gas):} & \quad p = Z \rho RT \\
\text{compressibility factor:} & \quad Z = Z(p, T) \\
\text{enthalpy:} & \quad h = h(p, T)
\end{align*}

(25) (26) (41) (42) (43) (44)

The above set provides six equations for the six unknowns $(V, \rho, p, T, Z, h)$. Some details regarding the methods of solution can be found in Ref. 12. Results of such computations for gases in equilibrium behind the normal shock wave appear in Figs. 11 and 12. Figure 11 shows the properties of such shock waves in air. It is seen that gas imperfections are most prominent at high
shock Mach numbers ($M_s$) and at the lower densities or pressures ($p_1$) where fewer molecules can have a larger share of the thermal energy which has been converted from the directed energy to excite the inert degrees of freedom. The result is that the temperature drops rather markedly. The cooling increases the density since the gains in pressure are quite small. The perfect gas values ($\gamma = 1.4$) are also shown for comparison. The concentration of electrons/cm$^3$ as a function of shock Mach number has been included. It is seen that the number density increases rapidly with shock Mach number and initial air density (even though the degree of ionization may be lower). Similar results for helium and hydrogen appear in Figs. 12 and 13. Further details concerning both perfect and imperfect gas effects for these gases appear in Refs. 21 to 27.

The passage of gas molecules through a shock wave or an expansion wave represents a disturbance in the original thermodynamic and dynamic state. Adjustments to the new conditions take place through molecular or atomic collisions as energy is partitioned among the various degrees of freedom. The process of adjustment takes place relatively slowly through an expansion wave (except at the focal point of a centred wave) in a perfect gas and no entropy changes occur. However, in an imperfect gas the adjustment of the inert degrees takes place in a finite time (relaxation time) or distance, giving rise to entropy and other flow changes as the energy of deexcitation is added along a particle path and a nonequilibrium flow region therefore exists.

Similarly, for shock fronts nonequilibrium regions exist behind the sharp front where the active degrees are excited in a few collisions. The kinetic temperature falls exponentially as equilibrium is attained through the equipartition of energy that will take an ever increasing number of collisions depending on the magnitude of the excitation energy involved. This process is again accompanied by a rise in entropy for stronger shock waves. The relaxation time especially (or distance) will depend on the local mean free path and at rarefied conditions can be relatively large (Refs. 12, 21, 28a).

The other remaining transition front that occurs in nonstationary flows is the contact surface. Ideally, it is a sharp front that separates two different thermodynamic states across which the pressure and velocity are continuous but all the other thermodynamic quantities are generally discontinuous. Shock waves and rarefaction waves will undergo refraction at an interface (Ref. 12) and will give rise to transmitted and reflected waves, separated by a modified contact surface. In reality, diffusion of heat and chemical species will occur across the contact front. If this diffusion is laminar then the contact front thickness will grow as the square root of time (Ref. 28b). However, if the diffusion is modified by some initially turbulent process like the breaking of a diaphragm in a shock tube, then the extent of the contact region can be very large indeed (Ref. 12). This can have a very profound influence on the flow properties of quasi-steady states and their nonstationary waves.
2.3 Internal Ballistics of Simple Launchers

In Subsection 2.1, two idealized launchers were considered in order to obtain some limiting hypervelocities attainable by a projectile moving under a constant base pressure and a massless projectile (gas particles themselves) accelerated by an isentropic nonstationary rarefaction wave with a falling base pressure. In the present subsection the more realistic case of a projectile of finite mass accelerated by a rarefaction wave will be considered. Until the recent invention of a projectile driven by a more uniform base pressure (Ref. 11), the foregoing was the operating principle of all light-gas guns.

In practice, the acceleration of a projectile is also affected by the existence of a counterpressure in the barrel (this can readily be overcome by evacuating the barrel); boundary layer growth on the wall of the barrel which reduces the base pressure; deceleration effects generated by the bore friction between the barrel surface and the projectile; nonequilibrium effects and gas imperfections. All of these factors should be incorporated in an analysis in order to obtain the actual base pressures during the expansion of the high-pressure high-enthalpy gas. Some consideration will be given to the above effects in the following subsections.

2.3.1 Case of an Evacuated Barrel

Consider the simple launcher shown in Fig. 4, for the case when the driving chamber is long enough to prevent the reflected rarefaction wave from overtaking the projectile of mass \( m \) and assume that boundary layer and bore friction can be neglected. Let the pressure at the projectile base be \( P_3 \) and at the head \( P_2 \). Let the piston velocity at any instant be \( u_3 \). Note that all flow quantities vary between the head of the rarefaction wave and the piston base as well as between the piston head and the compression wave or shock wave. Apply Newton's second law,

\[
m \frac{d^2x}{dt^2} = A(p_3 - p_2) \tag{45}
\]

Assume that the barrel is evacuated so that \( p_2 \) may be neglected and Eq. (45) reduces to a simplified form of Lagrange's ballistic problem of finding the subsequent states of the gas and the motion of the piston (Ref. 29) (Lagrange appears to have dealt with this problem at the end of the 18th Century) From Eq. (45),

\[
m \frac{d^2x}{dt^2} = Ap_3 \tag{46}
\]

or

\[
p_3 = \frac{m}{A} u_3 \frac{du_3}{dx} \tag{47}
\]

(Note that the piston or projectile velocity can be expressed as a function of
t or x whereas all other physical quantities depend on both x and t and their variations are expressed by partial differentials). It was shown previously that for a Q-rarefaction wave (Eq. 21),

\[
\left( \frac{p_3}{p_4} \right)^{\frac{\gamma_4 - 1}{2 \gamma_4}} = 1 - \frac{\gamma_4 - 1}{2} \cdot \frac{u_3}{a_4}
\]

Substituting in Eq. (47),

\[
\frac{m \cdot u_3}{A p_4} \cdot \frac{d u_3}{dx} = \left( 1 - \frac{\gamma_4 - 1}{2} \cdot \frac{u_3}{a_4} \right)^2 \frac{\gamma_4}{\gamma_4 - 1}
\]

As noted above when \( a_4 \to \infty \), then \( p_3 \to p_4 \) and Eq. (49) reduces to Eq. (4), for the constant base pressure case.

Equation (49) can be integrated as follows,

\[
\frac{A p_4 x}{m} = \int_0^{u_3} \left[ \frac{u_3 du_3}{1 - \frac{\gamma_4 - 1}{2} \cdot \frac{u_3}{a_4}} \right] \frac{2 \gamma_4}{\gamma_4 - 1}
\]

Let, \( Z = 1 - \frac{\gamma_4 - 1}{2} \cdot \frac{u_3}{a_4} \)

and,

\[
\frac{A p_4 x}{m} = \left( \frac{2a_4}{\gamma_4 - 1} \right)^2 \left[ \int_1^Z \frac{dZ}{Z \gamma_4} - \int_1^Z \frac{dZ}{Z^{1/\beta_4}} \right]
\]

where, \( \alpha_4 = \frac{\gamma_4 + 1}{\gamma_4 - 1} \) and \( \beta_4 = \frac{\gamma_4 - 1}{2 \gamma_4} \)

or \( \frac{\gamma_4 - 1}{2} \cdot \frac{A p_4 x}{m a_4^2} = \frac{2}{\gamma_4 + 1} \left[ 1 - \frac{\gamma_4 - 1}{2} \cdot \frac{u_3}{a_4} \right]^{\frac{\gamma_4 + 1}{\gamma_4 - 1}} - \left[ 1 - \frac{\gamma_4 - 1}{2} \cdot \frac{u_3}{a_4} \right]^{\frac{2}{\gamma_4 - 1}} + \frac{\gamma_4 - 1}{\gamma_4 + 1}
\)
\[
\frac{\gamma_4 - 1}{2} \cdot \frac{Ap_4 x}{ma_4^2} = \left[ \frac{u_3}{\bar{u}} - \frac{\gamma_4 - 1}{\gamma_4 + 1} \right] \left[ 1 - \frac{u_3}{\bar{u}} \right] - \frac{\gamma_4 + 1}{\gamma_4 - 1} + \frac{\gamma_4 - 1}{\gamma_4 + 1} \tag{53b}
\]

or in nondimensional form,
\[
\bar{x} = \frac{1}{\bar{\alpha}_4 - 1} \left[ \frac{\bar{u} - \frac{1}{\bar{\alpha}_4}}{(1 - \bar{u})\bar{\alpha}_4} + \frac{1}{\bar{\alpha}_4} \right] \tag{53c}
\]

where, \( \bar{x} = \frac{Ap_4 x}{m \bar{u}^2} \), and \( \bar{u} = u_3/\bar{u} \)

Equation (53) gives an explicit relation for the projectile position in the barrel (\( x \)) or the launcher length (\( x = L \)) required to accelerate a model of a given mass (\( m \)) and cross-sectional area (\( A \)) to a prescribed muzzle velocity (\( u_3 \)), as a function of the initial conditions of the driver gas in state (4). When \( u_3 \to 0 \), then \( x \to 0 \), as expected. When \( u_3 \to \bar{u} = \frac{2a_4}{\gamma_4 - 1} \), then \( p_3 \to 0 \), and for finite driver pressures (\( p_4 \)), \( x \to \infty \), that is an infinitely long frictionless launcher would hypothetically be required to accelerate a finite mass to the gas escape speed. Alternately, \( u_3 \to \bar{u} \), for a massless piston (\( m \to 0 \)) for all lengths, \( x > 0 \), that is, the contact surface case in a shock-tube expansion into a vacuum (Eq. 15). Similarly, from Eq. (49), write,

\[
\frac{Ap_4 t}{m} = - \frac{2a_4}{\gamma_4 - 1} \int_{1}^{Z} \frac{dZ}{Z^{1/\gamma_4}} \tag{54}
\]

or \( \frac{Ap_4 t}{ma_4} = \frac{2}{\gamma_4 + 1} \left[ 1 - \frac{\gamma_4 - 1}{2} \right] \left[ \frac{u_3}{a_4} \right] \frac{\gamma_4 + 1}{\gamma_4 - 1} - \frac{2}{\gamma_4 + 1} \) \tag{55a}

or, \( \frac{\gamma_4 + 1}{2} \cdot \frac{Ap_4 t}{ma_4} = \left[ 1 - \frac{u_3}{\bar{u}} \right] \frac{\gamma_4 + 1}{\gamma_4 - 1} - 1 \) \tag{55b}

or in nondimensional form,
\[
\bar{\tau} = \frac{1}{\bar{\alpha}_4} \left[ \frac{1}{(1 - \bar{u})\bar{\alpha}_4} - 1 \right] \tag{55c}
\]

where, \( \bar{\tau} = \frac{Ap_4 t}{m \bar{u}} \) and \( \bar{u} = u_3/\bar{u} \)

Equation (55) gives an explicit relation for the time (\( t \)) required to achieve a given projectile velocity (\( u_3 \)), as a function of the initial conditions of the driver
gas and the shot characteristics. It is seen that when \( u_3 = 0, t = 0 \), and when \( u_3 \to \hat{u} \), then for finite driver pressures \( (P_4) \), \( t \to \infty \), that is, a hypothetically infinite time is required to accelerate a finite mass to the escape speed of the driver gas. Alternately, \( u_3 \to \hat{u} \), when \( m \to 0 \) for all \( t > 0 \), the case of a shock tube expansion into a vacuum noted previously.

It is worthwhile considering the example used before under the kinematic solution (Eq. 4). For a nylon cylinder of single calibre, Eq. (53) can be reduced to (where, \( \rho_m \) is the model density),

\[
\frac{x}{d} = \frac{2 \gamma_4}{\gamma_4 - 1} \frac{\rho_m}{P_4} \left( \left[ \frac{u_3}{\hat{u}} - \frac{\gamma_4 - 1}{\gamma_4 + 1} \right] \left[ 1 - \frac{u_3}{\hat{u}} \right] \frac{\gamma_4 + 1}{\gamma_4 - 1} + \frac{\gamma_4 - 1}{\gamma_4 + 1} \right)
\]

( for a single calibre model)

The launcher length in calibres \( (x/d) \), is a minimum for a fixed muzzle velocity when the ratio of the model density \( (\rho_m) \) to driver gas density \( (\rho_4) \) is small. Since \( \rho_4 \) is a function of the initial pressure and temperature and since the temperature also occurs in \( \hat{u} \), it is not immediately apparent that both \( p_4 \) and \( a_4 \) should be large to reduce the launcher length for a given velocity. However, a few simple examples will illustrate this point.

Consider the case of a high-pressure \( (100,000 \text{ psi}) \), cold hydrogen driver \( (a_4 = 4165 \text{ ft/sec}, \hat{u} = 20,830 \text{ ft/sec}, \rho_4 = 0.61 \text{ gm/cm}^3) \). Assume \( u_3/\hat{u} = 0.5 \) (i.e., \( u_3 = 10,400 \text{ ft/sec} \)), then a substitution in Eq. (56) gives \( x/d = 250 \), and \( p_3/P_4 = 7.8 \times 10^{-3} \). That is, unlike the dynamic solution (Eq. 4) for a constant base pressure of 100,000 psi, the present results show that a single calibre nylon projectile \( (137 \text{ m gm}^* \text{ for a 0.22 calibre bore}) \) driven by an isentropic expansion over a launcher length which is 25% greater than the constant base pressure case only achieves about 1/5 of the muzzle velocity and the base pressure has dropped to less than 1/100 of its initial value. To achieve \( u_3/\hat{u} = 0.75 \), or 15,650 ft/sec, an impractical launcher length of 27,400 calibres would be required and the pressure ratio \( (p_3/p_4) \) would have dropped to an insignificant value of \( 6 \times 10^{-5} \).

If on the other hand the hydrogen (ideal gas case) was heated 100 fold \( (27,300^\circ \text{K}) \) at a pressure of 100,000 psi, then \( \rho_4 \) is decreased by a factor of 100 \( (\rho_4 = 6.1 \times 10^{-3}) \) and \( \hat{u} \) is increased by a factor of 10 (i.e., \( 2.083 \times 10^5 \text{ ft/sec} \)). Consequently, for the previous muzzle velocities of \( u_3 = 10,400 \text{ ft/sec}, x/d = 9.2 \) and \( p_3/p_4 = 0.70 \); \( u_3 = 15,630, x/d = 23 \) and \( p_3/p_4 = 0.58 \). That is, the high sound speed acts to maintain a slowly falling base pressure, which is available to further drive the projectile to very large muzzle velocities. Consequently, the effect of high sound speeds is equivalent to approaching the constant base pressure solution, as noted previously (Eq. 21). For this particular case (high-pressure, high-temperature hydrogen) a muzzle velocity of 36,000 ft/sec is obtained at 200 calibres (the imperfect gas value would have been greater), compared with the 50,000 ft/sec obtained for the constant base pressure solution.

\[* \rho_m = 1 \text{ gm/cc} \]
Additional numerical results using helium as a driver gas at various temperatures and pressures in order to launch a 1 gm projectile from a 0.30 calibre barrel at various muzzle velocities appear in Figs. 14 and 15. From Fig. 14, assuming a launcher length of 200 calibres (5 ft) it is seen that an initial pressure of 400,000 psi and 5000°K or 100,000 psi and 20,000°K yields a muzzle velocity of 20,000 ft/sec. Alternately, from Fig. 15, 35,000 ft/sec can be obtained at 200 calibres when the initial pressure is 500,000 psi and the temperature is 30,000°K. The base pressures at the end of such ideal launchings are 10,800 psi, 24,300 psi and 56,500 psi respectively. If a base pressure of only a few hundred psi at the end of a launching is a practical requirement, then the high-temperature drivers are much superior since additional muzzle velocity can be obtained by increasing the length of the launch tube to 400 or 500 calibre lengths depending on the initial conditions.

In general, the graphs show that for a given mass (≈ 1 gm) large muzzle velocities (≈ 20,000 ft/sec) can only be achieved at reasonable calibre lengths (≈ 200) and pressures (≈ 100,000 psi) when the temperature is high (≈ 20,000°K). Otherwise, if the mass is increased or the temperature is lowered excessive pressures and launcher lengths are required.

Equations (53) and (55) can be written in an alternate dimensionless form as follows. By letting,

\[ x = \frac{Ap_4}{ma_4^2} x, \quad t = \frac{Ap_4}{ma_4} t, \quad u = \frac{u_3}{a_4} \quad (57) \]

then, Eqs. (53) and (56) become,

\[ \Delta x = \left( -\frac{4}{\gamma_4^2 - 1} \right) \left[ 1 - \frac{\gamma_4 - 1}{2} \Delta u \right] - \frac{\gamma_4 + 1}{\gamma_4 - 1} \left( \frac{2}{\gamma_4 - 1} \right) \left[ 1 - \frac{\gamma_4 - 1}{2} \Delta u \right]^{-2} \frac{1}{\gamma_4 - 1} + \frac{2}{\gamma_4 + 1} \quad (58) \]

\[ \Delta t = -\frac{2}{\gamma_4 + 1} \left[ 1 - \frac{\gamma_4 - 1}{2} \Delta u \right] \frac{\gamma_4 + 1}{\gamma_4 - 1} - \frac{2}{\gamma_4 + 1} \quad (59) \]

The last two relations may be compared with their equivalent forms, Eqs. (53c) and (55c). A plot and cross-plot of Eqs. (58) and (59) for two values of \( \gamma_4 \), corresponding to hydrogen and helium (1.40 and 1.66), are shown in Figs. 16 and 17. Very little can be said about these nondimensional curves, except what has already been mentioned before, that is, when \( \Delta u \to 3 \), for \( \gamma_4 = 1.66 \), and \( \Delta u \to 5 \), for \( \gamma_4 = 1.40 \), then \( x \to \infty \) and \( t \to \infty \), as shown in
Fig. 16. Figure 17 gives an improved plot of $\hat{x}$ vs $\hat{u}$ for small $\hat{x}$. The advantage of these nondimensional plots is that a single curve of $\hat{x}$ vs $\hat{u}$ suffices for a given specific heat ratio ($\gamma_4$). On the other hand, some of the physical properties of the curves which were discussed previously are not explicitly revealed in this type of plot owing to the fact that several subvariables are contained in each nondimensional quantity.

A very useful nondimensional carpet plot of Eq. (53c), which makes use of the gas escape speed ($\hat{u}$), as a nondimensionalising quantity, is shown in Fig. 18. This plot has the advantage of displaying the independent variable ($\hat{x}$) as a function of both dependent variables ($\hat{u}$, $\gamma_4$) over a very wide range of values. The carpet plot illustrates very well the two extreme conditions when $\hat{u} \to 0$, in which case $\hat{x} \to 0$, independent of $\gamma_4$, and when $\hat{u} \to 1$, $\hat{x} \to \infty$. As shown in the plot, the rate of approach is greatest for gases with $\gamma_4 \to 1$, as the term $(1 - \hat{u})^{\gamma_4}$ approaches zero very rapidly in this case since $\gamma_4 \to \infty$.

The values used in constructing Fig. 18 are given in Table 3*.

A much more revealing carpet plot is shown in Fig. 19 for a 200 calibre launcher and 1 calibre plastic projectile ($\rho = 1.2$ gm/cc) driven by hot helium at high pressure. From the definition of $\hat{x}$ it is seen that it now only depends on $p_4$ and $a_4^2$ (or $T_4$), consequently lines of constant $\hat{x}$ and $u_3$ can be plotted as functions of $p_4$ and $a_4$, as illustrated in Fig. 19. It is seen that the isovelocity lines are hyperbolic in shape and the lines of constant nondimensional distance ($\hat{x}$), appear as radial lines. Consequently, to attain a given muzzle velocity it is pointless to work near the asymptotes where if the temperature is too low then very little is gained by going to even enormous pressures in the driver, and if the pressures are too small then little is gained by going to extremely high temperatures. Large velocity gains can be obtained in the range $2 < \hat{x} < 10$, with reasonable increases in either pressure or temperature or both. Consequently, for an ideal launcher of fixed calibre and model density optimum design pressures and temperatures can probably be realised in order to achieve a given muzzle velocity.

It is perhaps worth noting (Ref. 33) that the unsteady flow ideal internal ballistics equation, Eq. (53), can be compared with the dynamic solution Eq. (4), by rewriting Eq. (53), as follows,

---

* The computations were done by the Numerical Analysis Group, CARDE, Quebec and supplied by Dr. H. McMahon. The author wishes to express his appreciation for this assistance.
By plotting $u_3/u_{\text{max}}$ vs $a_4/\gamma_4u_{\text{max}}$ for various specific heat ratios an almost single collapsed curve is obtained for all $\gamma_4$, as shown in Fig. 20. It is seen that $u_3/u_{\text{max}} \rightarrow 1$ when $a_4/\gamma_4u_{\text{max}} \rightarrow \infty$, regardless of $\gamma_4$. This result can be readily verified from Eq. (60). It simply restates the previous remark that a constant base pressure is hypothetically possible if $a_4 \rightarrow \infty$ (Eq. 21). The lines of constant $\gamma_4u_3/a_4$ (or $u_3/u_{\text{max}} = \text{const.}$) indicate the small gains in $u_3/u_{\text{max}} \geq 0.8$ regardless of the increase in driver sound speed for a fixed pressure (that is, for $u_3/u_{\text{max}} \geq 0.7$, then $\gamma_4u_{\text{max}}/a_4 \leq 1$). In the example following (Eq. (4), Subsection 2.1, for a one calibre nylon cylinder driven at a constant base pressure of 100,000 psi, over a launcher length of 200 calibres, $u_{\text{max}} \approx 55,000$ ft/sec. It is seen from Fig. 20 that the ideal unsteady flow launcher requires a sound speed in helium of 37,000 ft/sec at this pressure in order to realise a muzzle velocity of even 30,000 ft/sec. This illustrates again the important interplay between driver pressure and temperature in order to attain a required muzzle velocity. This problem will be reconsidered in Section 5 for actual operating launchers.

It is noted in Refs. 32 that the driver gas temperatures are limited by radiation losses and by erosion products from the launcher inlet, sabot or model base which increases the gas molecular weight and the heat capacity, thereby lowering the temperature and the sound speed. An empirical correction was obtained which gave the following results (Ref. 32c, Fig. 4). An optimum sound speed (temperature) in helium appears to be about 18,000 ft/sec over the driver gas pressure range $60 \times 10^3 < p_4 < 350 \times 10^3$ psi (corresponding muzzle velocities of $15 \times 10^3 < u_3 < 22 \times 10^3$ ft/sec, 200 calibre launch tube, 1 cal plastic projectile, no chamberage, infinite chamber length). For each velocity there is also a minimum pressure. Increasing the pressure, or temperature (sound speed) only causes a reduction in velocity - a surprising result. This empirical correction appears to correlate the data from AEDC as well as from a similar gun at the NASA Ames Laboratory.

It is not known at this time whether in fact erosion and heat losses will set an upper limit to the hypervelocities that can be obtained from a light-gas gun. However, there is little doubt about the importance of these two parameters (see Subsection 2.4).
2.3.2 Effect of Counterpressure

In Subsection 2.2 (Fig. 4) a qualitative description was given of the wave system formed in front of an accelerating projectile or piston when the barrel is not evacuated. It is seen from Fig. 4 that the pressures acting on the projectile base and projectile head are functions of time or position, not only because of the formation of expansion and compression waves, respectively, but also owing to unsteady wall boundary layer effects and the changing values of solid friction between the projectile and barrel. Consequently, the dynamic equation, (Eq. 45), for a projectile of given mass \(m\) and cross-sectional area \(A\),

\[
mu_3 \frac{du_3}{dx} = A (p_3 - p_2)
\]

is usually solved subject to the simplifying assumptions that the viscous and frictional effects do not affect the unsteady pressures \(p_3\) and \(p_2\). A graphical procedure using the method of characteristics can then be employed (Ref. 14), wherein the piston path (integrating Eq. 61) and the characteristic lines are plotted simultaneously. The presence of a counterpressure \(p_2\), also means that the projectile acceleration is reduced or that the forward motion of the projectile is resisted by the presence of the gas in the barrel.

It is seen from Fig. 4 that the compression waves overtake and coalesce to form a shock wave of monotonically increasing strength. It is of interest to determine the birth point of the shock wave, which occurs at the intersection of the closest \(P\)-characteristic with the head of the wave \((x_i, t_i)\), as shown in Fig. 21.

Assume that the piston starting from rest moves with a constant acceleration \(\alpha\) and traces a path as shown in Fig. 21. (It is seen from Fig. 26 that at very small times this is a reasonable assumption.) As the piston moves it generates \(P\)-compression pulses at o, k, n, and j. The pulse from o is the head of the \(P\)-compression wave and moves into the known gas at rest (1). Consider a general characteristic from point \(n\), at small times. The distance \(x_n\) is given by (Ref. 12),

\[
x_n = \frac{\alpha}{2} t_n^2
\]

The piston velocity and the gas particle velocity on the characteristic line are identical and given by,

\[
u_n = \alpha t_n
\]

From Eq. (11), for a \(Q\)-characteristic, \(Q_1 = Q_n\), or

\[
a_n = a_1 + \frac{\sqrt{1 - \frac{1}{2} u_n}}{2} u_n
\]
The velocity of the $P$-characteristic emanating from $(n)$ is given by,

$$\frac{dx}{dt} = u_n + a_n = a_1 + \frac{\gamma_1 + 1}{2} \varepsilon t_n$$

(65)

The equation of this line becomes,

$$(x - x_n) = (a_1 + \frac{\gamma_1 + 1}{2} \varepsilon t_n)(t - t_n)$$

(66)

If point $(n)$ approaches $(k)$ very close to the origin, then it intersects with the head of the wave $(x = a_1 t)$ at $(x_i, t_i)$ to give the point $(i)$ where the shock is first formed, since a unique isentropic solution can no longer exist there. Solving for $x_i$ and $t_i$ from the two simultaneous equations (Eq. 65 and $x = a_1 t$) yields,

$$x_i = \frac{2a_1^2}{(\gamma_1 + 1) \varepsilon} (1 + \frac{\gamma_1 \varepsilon}{2a_1} t_k)$$

(67)

$$t_i = \frac{2a_1}{(\gamma_1 + 1) \varepsilon} (1 + \frac{\gamma_1 \varepsilon}{2a_1} t_k)$$

The time $t_k$ was chosen arbitrarily small, but rigorously it must approach zero and the shock birth point is given by,

$$x_i = \frac{2a_1^2}{(\gamma_1 + 1) \varepsilon}$$

(68)

$$t_i = \frac{2a_1}{(\gamma_1 + 1) \varepsilon}$$

During this time the piston has moved a distance ($t_i = t_j$),

$$x_j = \frac{\varepsilon}{2} t_i^2 = \frac{2a_1^2}{(\gamma_1 + 1)^2}$$

(69)

The distance of the shock formation point to the projectile head is given by,

$$x_i - x_j = \frac{2\gamma_1 a_1^2}{(\gamma_1 + 1)^2}$$

(70)

Considering the example following Eq. (4), Subsection 2.1, for a 1 calibre nylon cylinder in a 1.0 cm. dia. launcher under a constant base pressure of 100,000 psi moving in a barrel filled with air at standard conditions (1 atm, 20°C, $a_1 = 1127$ ft/sec), then $(x_i - x_j) \sim 1$ mm. That is, the shock
wave is formed almost instantly. Under such conditions it is possible to express the instantaneous counterpressure at the head of the projectile in terms of the Rankine-Hugoniot shock wave relations and the projectile velocity. Therefore, from Eqs. (29) and (30),

\[
\frac{u_2}{a_1} = \frac{(p_2/p_1) - 1}{\sqrt{\frac{\gamma_1}{\beta_1}(1 + \alpha_1 p_2/p_1)}}
\]  

(71)

or

\[
\frac{p_2}{p_1} = 1 + \frac{\gamma_1}{2} \cdot \frac{u_2}{a_1} \left\{ \frac{\gamma_1 + 1}{2} \cdot \frac{u_2}{a_1} + \left( \frac{\gamma_1 + 1}{2} \cdot \frac{u_2}{a_1} \right)^2 + 4 \right\}
\]  

(72)

Also from Eq. (48),

\[
\frac{p_3}{p_4} = \left( 1 - \frac{\gamma_4 - 1}{2} \cdot \frac{u_3}{a_4} \right)^{1/2}
\]  

(73)

and since at the projectile \( u_3 = u_2 = u \), the dynamic equation, Eq. (45), \( m \frac{du}{dt} = A(p_3 - p_2) \), can be expressed as a first order differential equation of the form,

\[
\frac{m}{A} \frac{du}{dt} = f(u)
\]  

(74)

The term \( f(u) \) is a complex function of \( u, \gamma_1 \) and \( \gamma_4 \) and Eq. (74) has to be integrated numerically. Solutions of Eq. (74) in nondimensional form appear in Figs. 22 and 23 (Ref. 34a) for air and helium drivers at room temperature and varying pressures. Equations (72) and (73) are then identified by a bar(\( \hat{u} \)) over all thermodynamic quantities and Eq. (74) is written as \( \frac{du}{dt} = f(\hat{u}) \), where \( p_2/p_4 = \hat{p}_2, p_1/p_4 = \hat{p}_1, a_2/a_4 = \hat{a}_2, a_1/a_4 = \hat{a}_1, u/a_4 = \hat{u}, tp_4A/a_4m = \hat{t} \). It should be noted that instead of unity in the bracket of Eq. (73), the chamberage factor \( \sqrt{\frac{\gamma_4 + 1}{2}} \) was used in the equation. This improves the expansion velocity (Subsection 3.2), but should not differ from the constant cross-section case by more than a few percent. If a constant frictional force (\( F \)) is included as an approximation to account for the effects of viscous and solid friction, then Eq. (74) can be written as \( \frac{d\hat{u}}{dt} = f(\hat{u}) - \hat{F} \), where \( \hat{F} = F/Ap_4 \).

An examination of Figs. 22 and 23 shows that for a fixed barrel pressure (\( p_1 = 1 \text{ atm} \)) a high driver pressure (\( p_4 \)) means a higher asymptotic velocity and the longer it takes to achieve. The effect of a constant frictional retardation (200 lb) on initial acceleration and final muzzle velocity are greatest at the lower driver pressures where it becomes a sizable fraction.
of the driver force (base pressure) and can therefore significantly reduce the action of the driver gas. The location in the (x, t)-plane of 5, 10 and 15 gm pistons (projectiles) at the end of a 10 ft launcher is also shown. As expected, the lightest piston achieves the highest velocity and in the shortest actual time. (A second integration of the equations would yield the time-distance (x, t)-plane).

In the case of an air driver in the range $10 \leq p_4/p_1 \leq 500$ projectiles of up to 20 gm appear to achieve 90% of the asymptotic velocity in 10 ft and about 99% for a projectile below 5 gm. For a constant frictional force of 200 lb the final velocity is reduced by 6% at high driver pressures ($p_4/p_1 = 500$) but rises to 74% at low driver pressures ($p_4/p_1 = 10$) as noted above. The results for a helium driver are similar even in the larger range of $50 \leq p_4/p_1 \leq 5000$. The asymptotic values of $u$ are achieved in a shorter time at even the higher driver pressures. Since the sound speed in helium is three-fold as great as in air, the curves show that all other quantities remaining fixed a given projectile will achieve a greater muzzle velocity at the end of the 10 ft. launcher in shorter time when helium is the driver even with counter-pressure. The effects of friction are reduced for helium driver and would arise from the fact that the net driving force (base pressure) is greater for the gas with the higher sound speed.

Seigel (Ref. 33) gives a relation for the effect of counterpressure, which appears to agree very well with exact numerical solutions for an infinite length chambered gun. The results are also applicable with minor changes to a gun without chamberage. The ratio of projectile velocity with counter-pressure ($u_{p1}$) to that of an evacuated barrel ($u_{p1} = 0$) is given by

$$\frac{u_{p1}}{u_{p1} = 0} = \left( 1 - \frac{p_1}{p_4} \right) \left( \frac{1 - e^{-y}}{y} \right)$$

(75)

where, the nondimensional launcher distance ($y$) is expressed as

$$y = \frac{(\gamma_1 + 1) \gamma_1 p_1 A_1 x}{ma_1^2}$$

(76)

A plot of $u_{p1}/u_{p1} = 0$ vs $y$ appears in Fig. 24. It is seen that $u_{p1} = 0$ for $p_1 \to 0$ and then it becomes exponentially smaller than unity for all $p_1 > 0$ (until the meaningless hypothetical value (no motion) of zero is reached when $p_1 = p_4$). The equation is only applicable until the base pressure and head pressure become equal ($p_3 = p_2$), that is, for larger values of $y$. The projectile then achieves a limiting velocity ($u_1$), which can be obtained from Eqs. (72) and (73) by equating $p_3$ and $p_2$ and solving for $u_1$ as a function of $p_1$, $p_4$, $\gamma_1$ and $\gamma_4$. It is worth noting that the limiting piston velocity is precisely the massless contact surface velocity of an equivalent shock tube problem with
a diaphragm pressure ratio \( p_4/p_1 \) (Ref. 12). This velocity is achieved instantly under ideal conditions in a shock tube, but only after some time or distance in a launcher owing to the mass of the projectile which must be accelerated to the limiting velocity. If no counterpressure or friction exists then the escape speed is the limiting velocity and is achieved hypothetically for infinitely long launching barrels (Eq. 53c).

The launcher position \( x \) where this occurs is then obtained by choosing several values of \( u_3 > u_1 \) and solving for \( x \) from Eq. (53). By using Eqs. (75) and (76) it is now possible to plot a new curve of \( u_2/u_p1 = 0 \) vs \( y \). The intercept gives the position where \( p_3 = p_2 \), as shown on Fig. 24, for a particular case only. For example, using a cold hydrogen driver \( p_4 = 100,000 \) psi, \( a_4 = 4165 \) ft/sec) against a counterpressure of air \( p_1 = 14.7 \) psi, \( a_1 = 1087 \) ft/sec) yields from Eqs. (72) and (73) a limiting projectile velocity \( u_2 \) of nearly 9000 ft/sec. Alternately, from the shock tube relations (Ref. 12) find for \( p_4/p_1 = 6800, M_S = 10 \) and \( u_2/u_1 = 8.2 \) or \( u_2 \sim 9000 \) ft/sec. It should be restated that frictional effects were not taken into account in this development, and in practice the actual velocities would generally be lower (Ref. 34).

An actual case where a piston path in a gun tunnel (Ref. 35) was determined by a microwave technique is shown in Fig. 25. The 3 in. dia., 100 gm, piston was hollowed out for lightness and was made of dural. The piston quickly accelerates to achieve its maximum velocity of about 2000 ft/sec against a counterpressure of air at 1 atm when driven by an air driver at 200 atm, both at 20°C. A nondimensional plot of piston velocity versus time is shown in Fig. 26. It is seen that a maximum piston speed of about 2030 ft/sec is achieved after nearly 10 msec or 17.5 ft, whereas from Eqs. (72) and (73) one obtains 2100 ft/sec. This excellent agreement indicates that frictional effects cannot be very significant at low projectile velocities. A solution for the position \( x \) in the barrel when \( p_3 = p_2 \) for this particular case, as outlined, shows that \( x = 17.5 \) ft (or \( y = 0.65 \)) is a good value (see Fig. 24). At this distance the speed in an evacuated barrel would have been 2370 ft/sec, i.e. a loss of 370 ft/sec (or nearly 18%) occurs at that station owing to the counter-pressure of 1 atm.

It is also interesting to note that the initial piston acceleration is apparently fairly constant (as assumed in Eq. 62) and has a value of 42.5 \( x 10^3 \) g's. Consequently, it is found from Eq. (70) that the shock forms at 5.4 in. from the piston face. In practice the shock formation may be closer to the piston face since under the conditions given in Fig. 25, the instantaneous acceleration can be calculated as 93.7 \( x 10^3 \) g's. The actual decreased acceleration would arise mainly from starting friction.

It was already noted that the effects of counterpressure can be overcome quite simply by evacuating the barrel. Nevertheless, the foregoing analysis is of practical significance when designing multistage launchers. In this case the projectile (piston) now acts as a shock compressor for a light driver gas such as hydrogen or helium. Imperfect gas effects in the driver
chamber as well as in the barrel would have to be taken into account so that the simple analysis given above would have to be modified. The effects of heat transfer and skin friction would also affect the final conditions in the hot gas compressed by the piston. Additional details are given in Subsection 3.4.

2.3.3 Boundary Layer Effects

From the available literature on hypervelocity launchers it is not possible at this date to give a decision as to the importance of viscous effects in causing projectile velocities in actual launchers to be less than the calculated values. Like bore friction and nonequilibrium effects in the driver gas, the skin friction and heat transfer arising from boundary layer formation will undoubtably contribute to an experimentally determined velocity decrement of a projectile. However, there may be factors like erosion of the barrel inlet, sabot, or projectile material by the high-temperature, high-pressure driver gas, which may easily account for most of the observed decrement under extreme driving conditions. The individual contributions to this velocity decrement have not been separated as yet. Consequently, some information will be presented in all of these areas in the following. Whenever hypervelocity launcher information is lacking a comparison will be made with a similar facility, like the shock tube, especially when dealing with boundary layer effects, in order to illustrate the mathematical methods, in the hope that they will be extended to include hypervelocity launchers.

It was noted previously (Subsection 2.2 and Fig. 4) that when a projectile starts to move in a gun barrel it induces a nonstationary boundary layer from the head of the rarefaction wave to the base and from the head of the compression wave or shock wave to the front face of the projectile. Unlike shock-tube flows, owing to the actual monotonically increasing (and then decreasing) velocity of the projectile throughout its course in the barrel, no quasi-steady flow regions are ever produced. Consequently, the boundary layer growth takes place under conditions where the dynamic and thermodynamic quantities vary with position \((x)\) and time \((t)\). In the expansion wave zone as well as in the compression wave region (even in an evacuated barrel the compression and shock waves exist) the flow is accelerating but the pressure gradient is favourable in the former and adverse in the latter. Whereas heat is always transferred out of one boundary layer in the compression wave (for an evacuated barrel this would be negligibly small), it is transferred into the boundary at the initial portion of the rarefaction wave and out of the layer for the rest of the wave, for a cold driver gas. The solutions to such complex and unsteady flow boundary layer problems are extremely difficult unless some simplifying assumptions are made.

It appears that one of the first attempts to account for the effects of boundary layer heat transfer in conventional guns was made by Hicks and Thornhill in a classified paper on "The Heating of a Gun-Barrel", in 1942 (Ref. 36). The paper was declassified in 1951 and can be found as Appendix II (Ref. 10a), "The Heating of a Gun Barrel by Propellant Gases". Incompressible
steady flow, turbulent boundary layer theory, without radiation effects for a smooth surface was used in the analysis. The Reynolds analogy between skin friction and heat transfer was assumed to apply in order to determine the heat transfer coefficient. As a consequence of the assumptions this analysis shows that boundary layer closure or pipe flow would usually not occur in conventional guns. The heat loss from the gas is transferred to the gun barrel surface and is then dissipated throughout the metal by conduction. It is shown that the heat losses are nearly linear with the shot travel. The losses ranged from 5 to 8% of the total propellant energy over barrel lengths of 37 and 73 calibres. Simultaneously, the projectile had acquired a muzzle kinetic energy of 21 to 31% of the propellant energy for a two-pounder gun having the same mass ratio of explosive charge to shot. The absolute heat loss increased with this ratio but the percentage of the total energy loss decreased with increasing charge to shot ratio. Computed temperatures in the barrel were found to vary from a maximum of about 1100°C at the surface to 160°C at a depth of only 1 mm at 10 m sec (as will be shown subsequently, this maximum value is rather high).

The next attempt to account for boundary layer effects was made for shock-tube flows by Donaldson and Sullivan (Ref. 37) on the basis of viscous diffusion in an incompressible gas without heat transfer, using as a model an infinite plate which is impulsively set into motion at a prescribed velocity (Rayleigh's problem). It was shown by Emrich and Curtis (Ref. 38) and by Glass and Martin (Refs. 39, 40) that this theory did not describe the observed flow properties in a shock tube (except for very weak shock waves when it will be indicated subsequently that the analogy becomes exact). Significant contributions to the theory of nonstationary compressible boundary layers and pipe flows were then made in rapid succession by Trimpi and Cohen (Ref. 41), Williams (Ref. 42), Hollyer (Ref. 43), Bromberg (Ref. 44) and Mirels (Ref. 45). Further details can be found in Ref. 12.

Briefly, Trimpi and Cohen linearized the characteristic equations Eqs. (36) to (40) and transformed them to ordinary differential equations, involving the skin friction coefficient which could then be integrated along particle paths. The assumption is made that steady flow values for skin friction can be used in lieu of the unknown unsteady values. (Unsteady values supplied by Mirels were used by Boyer (Ref. 46) to account for shock wave attenuation but the agreement with experiment was not very good). The effect of heat transfer out of the gas can be thought of as equivalent to generating rarefaction waves whereas heat flow into the gas as equivalent to generating compression waves. Skin friction always generates compression waves. These waves move upstream and downstream at acoustic speed with respect to the moving gas and change its thermodynamic and dynamic properties as a function of time and position. In the analysis, the entire expansion wave was collapsed into a single line (artificial expansion shock) at the head of the wave in order to convert the rarefaction wave into a part of the uniform flow region (3) (Fig. 3). Such an assumption would be very unsatisfactory for treating boundary layers in launchers since no uniform state exists behind the base of the projectile.
Williams (Ref. 42) makes use of successive approximations to solve the characteristic equations. He also uses the steady flow values for the skin friction coefficient. However, he does consider the flow in an actual rarefaction wave. In a shock-tube flow this zone is isolated from any influence downstream of the tail of the expansion wave, since a disturbance pulse originating anywhere between the shock wave and the tail can only become parallel to the tail of the wave but cannot cross it (Fig. 3). In a launcher flow this would not be possible in any event, owing to the isolating effect by the projectile itself. Consequently, the treatment given by Williams might be usefully applied to launchers. He also assumes that a turbulent pipe flow exists everywhere for simplicity and uses steady pipe flow properties to evaluate the effects on the nonstationary flow. The assumption of a pipe flow is not unreasonable for launchers of small diameter. However, these results might be improved by using nonstationary turbulent boundary layer properties or pipe flow properties as they become available.

Williams calculations are in qualitative agreement with experiments for a rarefaction wave produced in a 3.5 cm. dia. shock tube 15 m sec after the diaphragm was ruptured (Fig. 27). It is seen that the particle velocity is reduced from its ideal ($\gamma = 1.4$) value, as expected. Similar data for the remaining physical quantities are not given and it is difficult to assess the net decrease in velocity that one might expect in an equivalent launcher problem. The analysis was only done for the expansion region that exists in the chamber, up to the diaphragm station ($M = 1$).

A different approach to the laminar boundary layer problem in the expansion region can be found in Refs. 47 and 48. However, it is expected that the turbulent boundary layer and pipe flow will be more significant in launcher applications. Such analyses still remain to be done since the problem is made very difficult by the existence of flow gradients and the completely nonuniform nature of the flow.

At this point it would be quite useful to consider in more detail some of the features of nonstationary boundary layers in shock-tube flows, in order to illustrate some of their essential characteristics compared with steady flows. This will also provide a background for the more difficult analyses that are required for the boundary layers or pipe flows that are developed behind a moving projectile in a launcher barrel. Consider Figs. 4 and 28, it is seen that it would be possible to put the two nonstationary boundary layers with their moving leading edges at the shock front and at the rarefaction wave head into a steady-flow frame of reference by fixing a set of coordinate axes to the wave front. This imposes no difficulty at the shock but requires an assumption that the rarefaction wave fan can be replaced by a single plane (expansion shock). This assumption becomes progressively worse as the pressure ratio across the diaphragm increases. In the limit, for a perfect gas, state (3) disappears and the rarefaction wave fan occupies a length which is sixfold greater (for Air/Air, $\gamma = 1.4$) than the region occupied by state (2), between the shock and the contact surface. The two boundary layers in steady and unsteady coordinates
are shown in Figs. 28 and 29. The gas thermal boundary layer and the wall thermal layer appear in Fig. 30. (The notation used in Subsection 2.1 above, has been retained; that is, for a shock wave $V_1 = w_1$, $V_2 = w_1 - u_2$, $V_1/V_2 = f_2/f_1$, and $1 \leq f_2/f_1 \leq 6$ for $1 \leq V_1/a_1 \leq \infty$, for $\gamma = 1.4$. Similarly, for an expansion wave $V_1 = a_4$ and $V_2 = a_4 + u_3$. For $\gamma = 1.4$, in the range $0 \leq u_3 \leq 5a_4$, $1 \geq V_1/V_2 \geq 1/6$. It is seen that, in steady co-ordinates, the wall now has a velocity equal to the wave velocity ($V_1 = V_w$). The expansion wave profile is reminiscent of a boundary layer with slip flow, while behind the shock wave the profile has the largest velocity at the wall and smallest in the free stream. That is, the boundary layer is similar in appearance to the boundary layer that would be produced on a flat plate in a wind tunnel if the surface of the plate were moving like a conveyor belt from the leading edge downstream at a velocity greater than the free stream velocity. It is seen that the displacement thickness of such a layer would be opposite (negative) to what is usually encountered and would give rise to an effectively diverging wall behind the shock wave.

a) **Laminar Boundary Layer**

Following the method used by Mirels in Refs. 49 and 50, the Prandtl boundary layer equations are applied (except at the leading edges of the two waves) to the compressible flow for $x > 0$ with the assumption $dp/dx = 0$.

**Continuity:**

$$\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0$$  \hspace{1cm} (77)

**Momentum:**

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right)$$  \hspace{1cm} (78)

(Note: $v$ is the velocity component in the $y$-direction).

**Energy:**

$$\int C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{1}{2} \left( \frac{\partial u}{\partial y} \right)^2$$  \hspace{1cm} (79)

**State:**

$$p = \int \rho RT$$  \hspace{1cm} (80)

**Boundary conditions for shock wave (Fig. 28):**

$$u(x, 0) = -w_1 = -V_1 \text{ (zero slip)}$$

$$v(x, 0) = 0$$  \hspace{1cm} (81)

$$T(x, 0) = T_w \text{ (constant wall temperature)}$$
(It was shown by Mirels that \( T_w = T_1 \) except for stronger shocks.)

\[
\begin{align*}
    u(x, \infty) &= (u_2 - w_1) = -V_1 \\
    T(x, \infty) &= T_2
\end{align*}
\]

Boundary conditions for the expansion wave (Fig. 28):

\[
\begin{align*}
    u(x, 0) &= a_4 = V_1 \\
    u(x, 0) &= 0 \\
    T(x, 0) &= T_w \quad \text{(constant wall temperature)}
\end{align*}
\]

(\( T_w \approx T_4 = T_1 \) for all expansions).

\[
\begin{align*}
    u(x, \infty) &= a_4 + u_3 = V_2 \\
    T(x, \infty) &= T_3
\end{align*}
\]

The fact that the flow velocity \( u_2 = V_1 - V_2 \), the wall velocity \( V_1 \) and the wall to free stream temperature ratio \( T_w/T_2 \) or \( T_w/T_3 \) changes for each diaphragm pressure ratio in the shock tube introduces an added complexity in the boundary layer analysis. In addition, the results cannot be compared directly with steady flow cases on flat plates or cones because of the different velocity and temperature profiles. Nevertheless, the unsteady boundary layers behind shock waves provide simple means of investigating the properties of high-temperature flows with large heat transfer rates and gas imperfections that cannot be obtained too readily in other facilities.

Numerical solutions to the above equations in the range \( 0 \leq V_1/V_2 \leq 6 \) are given in Refs. 49 and 50. The case of \( V_1/V_2 = 0 \) is the Blasius solution for the incompressible flow past a fixed, semi-infinite plate. When \( V_1/V_2 = 1 \pm \xi \) (where, \( \xi \) is a small quantity, \( \xi \ll 1 \); otherwise, \( \xi = 0 \) \( V_1/V_2 = 1 \), \( u_2 = 0 \), and a boundary layer does not exist, see Fig. 29) an analytical solution involving the first order in \( (V_1/V_2 - 1) \) is possible, which reduces to the Rayleigh relation for the nonstationary boundary layer:

\[
\frac{V_1 - V}{V_1 - V_2} = \frac{u}{u_2} = \text{erf} \left( \frac{\sqrt{\frac{V_2}{x}}}{\sqrt{V_2}} \cdot \frac{y}{2} \right) = \text{erf} \left( \frac{y}{2\sqrt{V_2 \xi}} \right)
\]

A nondimensional boundary layer parameter \( \eta \) is defined as

\[
\eta = \sqrt{\frac{V_2}{2}} \int_{0}^{y} \frac{T_1}{T} \, dy
\]
The boundary layer thickness $\delta$ is defined as that value of $y$ when $y = \delta$ or $\gamma = \gamma_\delta$, and at that point

$$\frac{V_1 - V}{V_1 - V_2} = 0.99 \quad \text{or} \quad \frac{V}{V_2} = 0.99 \pm 0.01 \frac{V_1}{V_2} \quad (85)$$

(The plus sign is used for $1 \leq V_1/V_2 \leq 6$ and the minus sign for $0 \leq V_1/V_2 \leq 1$.)

The variations of $V/V_2$ with $\gamma$ are given in Refs. 49 and 50. Consequently, when $V/V_2$ assumes the value of Eq. (85), $\gamma_\delta$ is known, and the reduced quantities $\gamma/\gamma_\delta$ versus $(V_1 - V)/(V_1 - V_2)$ can be plotted. For the Rayleigh case, we write

$$\frac{V_1 - V}{V_1 - V_2} = \text{erf} \left( K \frac{\gamma}{\gamma_\delta} \right) \quad (86)$$

where $K = \text{constant}$.

When

$$\gamma = \gamma_\delta, \quad \frac{V_1 - V}{V_1 - V_2} = 0.99 \quad \text{and} \quad K \approx 1.8$$

or

$$\frac{V_1 - V}{V_1 - V_2} = \text{erf} \left( 1.8 \frac{\gamma}{\gamma_\delta} \right) \quad (87)$$

Equation (87) along with the reduced values for the Blasius ($V_1/V_2 = 0$) and the strong shock cases ($V_1/V_2 = 6$) are shown on Fig. 31. It is seen that the velocity profiles encountered in a shock tube on the whole lie closer to the Rayleigh profile except for strong expansions which are closer to the Blasius profile (with the assumption of an expansion shock). On this plot, it is seen that the slope of the profile at the wall increases from the Blasius to the Rayleigh to the strong shock case, and the curves are also in this order.

From Eq. (84), when $y = \delta$, $\gamma = \gamma_\delta$ and $T = T_2$,

$$\frac{\delta}{\gamma_\delta} = \frac{T_2}{T_1} \sqrt{\frac{2 \times \nu}{V_2}} \quad (88)$$

For the Blasius case, when $V/V_2 = 0.99$, $\gamma_\delta = 3.5$. For $T_2/T_1 = 1$,

$$\delta = 5 \sqrt{\frac{x \nu}{V_2}}$$

(This is the incompressible result for the boundary layer growth). From Eq. (83) for the Rayleigh case.
\[
erf \sqrt{\frac{V_2}{x_s^2}} \cdot \frac{\delta}{2} = 0.99
\]

or
\[
\delta = 3.6 \sqrt{\frac{x_s V_2}{V_2}}
\]

For the strong shock case \((V_1/V_2 = 6)\), \(V/\sqrt{V_2} = 1.05\) and \(\gamma_{\delta} = 1.55\)

or
\[
\delta = 2.2 \frac{T_2}{T_w} \sqrt{\frac{x_s V_2}{V_2}}
\]

The shear stress \(\tau_w\) at the wall is given by
\[
\tau_w = \left(\frac{\mu \partial u}{\partial y}\right)_w = \mu w \sqrt{\frac{V_2}{2 x_s y_w}} \cdot f''(0)
\]

The friction \(f''(0)\) is tabulated in Refs. 49 and 50. If the local skin friction is defined as
\[
C_f = \frac{\tau_w}{\frac{1}{2} \int_w (V_1 - V_2)^2} = \frac{\tau_w}{\frac{1}{2} \int_w u_2^2}
\]

and the Reynolds number as
\[
Re = \frac{V_2 x_s}{y_w} \left(1 - \frac{V_1}{V_2}\right)^2
\]

(The reason for this definition of \(Re\) is given subsequently in Eq. 106.)

then
\[
C_f \sqrt{Re} = \frac{\sqrt{2} f''(0)}{\sqrt{\frac{V_1}{V_2} - 1}}
\]

From Ref. 50, when
\[
\frac{V_1}{V_2} = 0, \quad f''(0) = 0.4696
\]

or
\[
C_f \sqrt{Re} = 0.664
\]

36
the incompressible flow (Blasius) value.

When
\[ \frac{V_1}{V_2} = 1 , \quad f''(0) = \frac{1}{\pi} \left( \frac{V_1}{V_2} - 1 \right) \]

and \( C_f \sqrt{Re} = 1.128 \), the Rayleigh value.

Finally, when
\[ \frac{V_1}{V_2} = 6 , \quad f''(0) = 8.101 \text{ and } C_f \sqrt{Re} = 2.291 \]

Thus, the Rayleigh value of \( C_f \sqrt{Re} \) is greater than the corresponding Blasius value by a factor of 1.70, and the strong shock value is greater by a factor of 3.45 (as might have been expected from the slopes of the curves at the wall in Fig. 31).

The theory (independent formulations of Refs. 51 and 49) for the laminar compressible boundary layer (with heat transfer) developed on the wall behind a plane shock wave has been substantiated by the work of Ref. 51, where the density profiles were measured with an interferometer. By assuming that the Crocco relation between temperature and velocity applies (Ref. 15), the velocity profiles can be calculated, as shown in Fig. 32. The discrepancy at the larger values of \( n \) is attributed to the thin boundary layer (0.3 to 0.6 mm) and other uncertainties. However, the agreement is considered as "moderately good".

A much more detailed interferometric study of the laminar boundary layer appears in Ref. 52, where it is concluded that the available data from several sources indicates that Mirels' theory "can be considered correct in the range of conditions \( 0.16 < M_2 < 1.60 \), that is, over the subsonic and supersonic flow range behind the shock wave.

Expressions for the recovery factor and heat transfer functions and their numerical values are also given in the above references over the range \( 0 \leq V_1/V_2 \leq 6 \) for a Pr = 0.72. It is shown that the recovery factor \( r \), rises from \( r = 0.845 \) (Blasius) to \( r = 0.885 \) (Rayleigh) to \( r = 0.920 \) for \( V_1/V_2 = 6 \).

In addition, it is shown in Ref. 50 that the thermal boundary layer in the wall of the tube can be described in a manner similar to the Rayleigh gas velocity boundary layer. The problem is that of bringing two media (the gas and the wall) at different uniform temperatures into contact at a time \( t = 0 \), and is given by the one-dimensional heat conduction equation (Fig. 30):

\[ \frac{\partial T}{\partial x} = \frac{\chi_w}{V_1} \frac{\partial^2 T}{\partial y^2} \]  \( \text{(93)} \)

37
where
\[ \chi_w = \left( \frac{k}{\rho C_p} \right)_w \]  

is the diffusivity of the wall material. The boundary conditions for \( x > 0 \) are given by
\[ T(x, 0) = T_w \]
\[ T(x, -\infty) = T_1 \quad \text{and} \quad T(x, -\infty) = T_4 \]

for the shock wave and expansion wave, respectively.

The solution of Eqs. (93) to (95) is given by
\[ \frac{T_w - T}{T_w - T_1} = \text{erf} \left( \frac{-y}{2 \sqrt{\frac{x \chi_w}{V_1}}} \right) \]

For a Prandtl number = 1, the recovery temperature \( T_r \) and the total temperature \( T_0 \) are identical (for \( \text{Pr} < 1 \), \( 0.8 < T_r/T_0 < 1 \) for \( \infty \geq M \geq 1 \); that is, the ratio is less than unity). Consequently, from Refs. 50 and 12, for \( \gamma = 1.4 \), for \( V_1/V_2 > 1 \),
\[ \frac{T_r}{T_1} - 1 = \frac{1}{3} (M_s^2 - 1) \]  

and for \( V_1/V_2 < 1 \),
\[ \frac{T_r}{T_4} - 1 = -2 \left[ 3 \left( \frac{p_3}{p_4} \right)^{1/7} - 2 \right] \left[ 1 - \left( \frac{p_3}{p_4} \right)^{1/7} \right] \]

Since \( w_1/a_1 = M_s > 1 \), \( T_r/T_1 > 1 \) and heat transfer always occurs from the gas to the wall behind the shock wave. From Eq. (98) it is seen that in the range \( 1 \geq p_3/p_4 \geq (2/3)^7 \); \( 0 \leq M_3 \leq 2.5 \), \( T_r/T_4 < 1 \), and heat transfer occurs from the wall to the cold gas. For stronger expansion waves \( T_r/T_4 > 1 \), and heat is transferred from the cold gas to the tube walls. Therefore, it is possible for stronger expansion waves to have heat transfer into the gas for the initial portion of the fan \( (M_3 < 2.5) \) and into the tube wall for the latter portion of the fan \( (M_3 > 2.5) \) at the same time for cold drivers.

The temperature rise is given in Ref. 50 as
\[ \frac{T_w - T_1}{T_1} = A_1 \left( \frac{T_r}{T_1} - 1 \right) \]

for \( V_1/V_2 > 1 \).
where
\[ A_1 \sim \sqrt{\frac{\alpha_w}{\alpha_1}} \cdot \frac{k_1}{k_w} \]  
and
\[ \frac{T_w - T_4}{T_4} = A_4 \left( \frac{T_r}{T_4} - 1 \right) \]

for \( V_1/V_2 < 1 \)

where,
\[ A_4 \sim \sqrt{\frac{\alpha_w}{\alpha_4}} \cdot \frac{k_4}{k_w} \]

If \( k \propto T \) and \( C_p = \) constant, then \( k/\sqrt{\rho} \) is independent of \( T \), that is, it depends on \( \sqrt{p} \) for a given gas and unique curves of \( \Delta T/\sqrt{p} \) should be obtained for varying pressure levels behind the shock wave. (Typical values of \( \alpha \) cm\(^2\)/sec at 0 - 18°C are: gold = 1.209, silver = 1.700, mild steel = 0.173, air at 1 atm = 0.179, helium at 1 atm = 1.54, hydrogen at 1 atm = 1.36; for air \( k = 5.5 \times 10^{-5} \) cal cm/sec cm\(^2\)oC, helium \( k = 34 \times 10^{-5} \), hydrogen \( k = 42 \times 10^{-5} \), for mild steel (1 per cent carbon) \( k = 0.107 \) cal cm/sec cm\(^2\)oC -- Refs. 24 and 53.) Consequently for air - steel, \( A \sim 0.0005 \); H\(_2\) - steel, \( \sim 0.001 \); He - steel, 0.001; H\(_2\) - steel at 100 atm and 5000 K, \( k = 0.0125 \), \( C_p = 21.9 \), \( \Delta = 11.6 \). With the value of \( A \) for air - steel, Eq. (99) shows that the wall temperature will remain within 1 percent of its original value behind the shock up to shock Mach numbers \( M_s \sim 8 \). For the entire range of expansion waves it will remain up to 0.2 per cent of its initial wall temperature. The results for cold helium and hydrogen drivers would not be too different. Because the heat conductivity of metals is several orders greater than that of the gas, the metal acts as a very large sink and its temperature remains essentially constant. However for high-pressure, high-temperature drivers the values of \( A \) could be orders greater so that the wall surface temperature might go up considerably (hundreds of degrees) owing to convection and radiation heat transfer at very high temperatures. Mirels (Ref. 50) also shows that the thickness ratio of the metal thermal boundary layer (\( \Delta T \)) and the gas thermal boundary layer (\( \delta T \)) in the Rayleigh case are of order unity for a gas like air in a steel shock tube (i.e., \( \Delta T/\delta_{T} \sim \sqrt{\alpha_w/\alpha_1} \)).

The laminar boundary layer surface temperatures rise as a function of shock Mach number in air in a shock tube, using thin film gauges on a glass wall (Fig. 33, Ref. 54) shows very good agreement with theory (Mirels) and substantiates this type of analysis. It is seen that for all shocks the agreement with \( \rho \mu \) variable in the boundary layer rather than the wall value \( (\rho \mu)_{1} \) gives the best agreement. The Prandtl number used in the analysis was \( Pr = 0.72 \).

Further substantiation of Mirels' analysis comes from heat transfer data as shown in Fig. 34 where the Stanton number (St) is plotted.
against shock Mach number. The Reynolds number \( \text{Re} \) is defined in Eq. (104) and is arbitrarily based on the free stream value of \( \sqrt{2} \). Excellent agreement is obtained, which shows that \( \text{St} \sqrt{\text{Re}} \) is nearly constant and independent of pressure over the shock Mach number range. An extension of Mirels' work for stronger shock waves can be found in Ref. 55.

Heat transfer measurements in the rarefaction region for a laminar boundary layer in nitrogen can be found in Ref. 56a). A comparison with the theoretical predictions of Trimpi and Cohen (Refs. 47, 48) is also made. It is found that in general the agreement of wall temperature and heat transfer are not good except close to the head of the rarefaction wave where the theory is at its best. The surface temperature decreased linearly with time and the heat transfer from the shock tube walls varied directly as the square root of time. This result is unlike the flow behind a shock wave where the surface temperature remains constant and the heat transfer rate varies inversely as the root of time, for a laminar layer.

However, for very weak rarefaction waves the linear decrease in wall temperature is followed by a flat portion, corresponding to the uniform cold flow region (like the shock-wave flow), and then an additional (more rapid) decrease takes place after transition to turbulence. The experimental results indicate that the boundary layer in the rarefaction wave region is still far from being fully understood.

b) Transition

The problem of predicting transition from a laminar to a turbulent layer is more complex in the case of boundary layers developed behind travelling shock waves or expansion waves than in steady flows, owing to the nonstationary nature of the problem. The boundary layer itself in this case generates disturbances which are propagated by the characteristic lines throughout the flow. Where the boundary layer occupies a sizeable fraction of the tube area, the isentropic portion of the flow is no longer uniform even in a quasi-steady region. In a launcher or shock tube expansion region the nonuniformities of the flow and those induced by the boundary layer are coupled.

However, the transition from a laminar to a turbulent boundary layer can be studied experimentally using optical methods (shadowgraph, schlieren, and interferometer) and by means of the thin-film heat gauge (see Appendix A; a historical note of its development is given in Ref. 51). It was noted above that in shock-tube flows for the laminar boundary layer the wall temperature remains constant. This is indicated by the flat portion of the heat gauge trace (Fig. 35). When transition to turbulent flow occurs at the wall, then the curve suddenly rises (definite change in slope) and becomes somewhat wavy. The time between the passage of the shock over the gauge and transition to turbulence is given by \( \tau = (t_3 - t_2) \), and the phenomenon moves along with the shock speed \( w_1 \). (A similar argument applies to the
expansion wave assuming it is collapsed onto its head and moves with a speed $a_4$). The flow particle which first goes turbulent on hitting the gauge has been in motion for a total flow time $\Delta t = (t_3 - t_1)$. From the figure,

$$\frac{\Delta t}{\tau} = \frac{a + b}{a} = \frac{w_1 \tau + u_2 \Delta t}{w_1 \tau}$$

or

$$\Delta t = \frac{w_1 \tau}{w_1 - u_2} = \frac{V_1}{V_2} \cdot \tau = \frac{\rho_2}{\rho_1} \cdot \tau \tag{103}$$

That is, the particle transition time is equal to the gauge transition time multiplied by the stationary velocity factor $V_1/V_2 = \rho_2/\rho_1$. In the case of the Rayleigh boundary layer, when $V_1/V_2 = 1$, then $\Delta t = \tau$, i.e., the two transition times are equal, since all particles start instantly. For the strong shock case, in a perfect gas ($\gamma = 1.4$) it is sixfold greater and for an imperfect gas it can be more than an order of magnitude greater.

It is now possible to define a transition Reynolds number $Re_T$ based on this particle time. The characteristic distance is the particle time multiplied by the external flow velocity $u_2$, and is equivalent to the distance $b$ from the leading edge of a flat plate to the transition point in the steady case. That is, in both cases it is the distance that the particle has moved with respect to the wall before undergoing transition.

Therefore,

$$Re_T = \frac{u_2 b}{\nu_w} \tag{104}$$

or

$$Re_T = \frac{u_2^2 \tau}{\nu_w} \cdot \frac{V_1}{V_2} = \frac{(V_1 - V_2)^2 \tau}{\nu_w} \cdot \frac{V_1}{V_2} \tag{105}$$

It should be noted that this $Re_T$ is greater by a factor of $V_1/V_2$ than that based on the Rayleigh flow. (The relation $Re_T = u_2 b/\nu_2$ based on the free stream kinematic viscosity has also been used arbitrarily and is roughly $(T_2/T_1)^2$ greater than that based on the wall value of $\nu_w$).

From the point of view of the co-ordinates fixed in the shock wave, an appropriate Reynolds number $Re_x$ for any distance $x$ measured behind the shock wave can be defined because $x = a$ is the distance between the shock and particle path in question at that station.

Since

$$\frac{b}{a} = \frac{u_2 \Delta t}{w_1 \tau} = \frac{u_2}{w_1} \cdot \frac{V_1}{V_2}$$
Therefore

$$Re_x = \frac{u_2^2}{w_1} \cdot \frac{V_1}{V_2} \cdot \frac{x}{\sqrt{w}} = \frac{V_2x}{\sqrt{w}} \left(1 - \frac{V_1}{V_2}\right)^2$$  (106)

Equation (106) was derived by Mirels in Ref. 50.

An excellent summary of transition behind moving normal shock waves is given in Ref. 54. The experimental results are summarized in Fig. 36, where the transition Reynolds number (ReT) is plotted against the speed ratio (V1/V2) and temperature ratio (Tw/T2). It is seen that for weak shock waves (small cooling effects or values of Tw/T2 closer to unity) ReT increases with the unit Reynolds number (Re/1 = u2/\sqrt{2}) of the free stream. The apparent reason being that the boundary layer is thinner at the higher pressure levels. However, when the shock Mach number (M_S) is large and the cooling rate is very high (Tw/T2 < 0.1) then the unit Reynolds number is not a parameter. The boundary layer is very stable and large values of ReT are obtained. Destabilization of the boundary layer as a result of excessive cooling which has been observed in steady flows were not noticed in shock tube flows. A somewhat different transition plot (Fig. 37) based on a boundary-layer thickness Reynolds number (Re_\delta) shows approximately the same trend in the data.

A similar study of transition in the expansion wave region can be found in Ref. 56a). The results are illustrated schematically in Fig. 38. It is seen that whereas the transition phenomenon moves with the shock speed* in the hot gas (ReT is independent of the measuring station for a given set of initial conditions), the transition point in the cold gas depends on the measuring station, that is, the transition time increases with distance from the diaphragm station. However, for a fixed set of conditions the transition point is independent of the extent of the expansion wave (for all M_3 > 1, say), but the transition point does rise with decreasing chamber pressure (p_4). Because the transition point depends on the measuring station it is not possible to define a unique transition Reynolds number (the difficulty is compounded by the non-uniform nature of expansion wave flows which do not lend themselves to defining a Reynolds number as for shock wave flows: except for the artificial "expansion shock" case). It was also found that small steps (0.001 in.) in the tube wall were not prone to trip the boundary layer (to induce transition) in the case of the rarefaction wave but did so readily in the hot gas. Since the expansion wave flow has a favourable pressure gradient this may not be too surprising.

A recent theoretical study of boundary layer transition behind a moving wave based on classical amplification of Tollmien-Schlichting waves (Ref. 56b) shows no correlation between theory and experiment (see inset in Fig. 36). It is seen that the results differ by orders of magnitude. At wall temperature ratios T_2/T_1 ~ 0.5, the analysis shows that the boundary layer is nearly infinitely stable. The authors conclude that transition must be induced by other types of disturbances rather than the classical waves. It is seen that this problem is far from solved in the region behind the shock front and it is even more complex in the region behind the expansion wave.

* see (Ref. 57c) for comments
c) Turbulent Boundary Layer

Solutions for the turbulent boundary layer behind moving shock waves are given in Ref. 50. The same concept is used, i.e., that it is possible to reduce the problem to a steady-flow case by fixing the reference axes to the shock front. The usual empirical, semi-infinite flat-plate theory without pressure gradient is applied to the apparently moving wall in the shock tube.

The integral form of the momentum equation is given by

$$\frac{\zeta w}{\rho V^2} \frac{d}{dx} \int_0^\infty \frac{\rho V}{\rho_2 V_2} (1 - \frac{V}{V_2}) dy = \frac{d\theta}{dx}$$

(107)

where $\theta$, as defined by Eq. (107), is the momentum thickness of the boundary layer. (Note, as above, "V" is now the average relative velocity $(w_1 - u)$ whereas the written "v" is the average velocity component in the y-direction in the boundary layer). If the boundary layer thickness is given as $\delta$, then a turbulent similarity parameter can be defined as $\xi = y/\delta$. The velocity profile relative to the shock-tube wall is assumed to follow a power law expressed by

$$\frac{V - V_1}{V_2 - V_1} = \frac{1}{\xi^n} \text{ for } 0 \leq \xi \leq 1$$

and

$$\frac{V - V_1}{V_2 - V_1} = 1 \text{ for } \xi \geq 1$$

(108)

The above assumes that the actually existing boundary-layer profile in the shock tube in region (2) or (3) can be expressed by,

$$\frac{u}{u_2} = \xi \frac{1}{n} \text{ or } \frac{u}{u_3} = \xi \frac{1}{n}$$

(109)

In what follows, region (2) will be used for illustration, but the same procedure also applies to region (3). A value of $n = 7$ is usually accepted for work in steady flow. (However, the data of Ref. 56c) indicate that $n = 5$ gives a somewhat better fit for shock-tube velocity profiles, whereas the data of Ref. 52 confirm the value of $n = 7$, in many cases).

Similarly, the thermal boundary-layer profile is expressed as a function of $\zeta$ by using Crocco’s relation (usually used for laminar boundary layers) which assumes that $Pr = 1$, or $T_0 = T_r$, or that the thermal and velocity profiles have the same thickness (see the bibliography and remarks in Ref. 56c),

$$\frac{\rho_2}{\rho} = \frac{T}{T_2} = \frac{T_w}{T_2} (1 + b \frac{1}{n} \xi - c \frac{2}{n} \xi^2)$$
for 

\[ 0 \leq \zeta \leq 1 \]

and

\[ \frac{\rho_2}{\rho} = \frac{T}{T_2} = 1 \text{ for } \zeta \geq 1 \]

where

\[ b = \frac{T_r}{T_2} - 1, \quad c = \left( \frac{T_r}{T_2} - 1 \right) \frac{T_2}{T_w} \quad (110) \]

It is shown in Ref. 50 that

\[ \frac{T_w - T_1}{T_1} \sim 10\% \text{ for } M_s \gg 1 \]

when

\[ \left( \frac{x V_2}{V_2} \right)^{0.3} \quad M_s \text{ order of } 10^4 \]

Consequently, even for moderately strong shocks, \( T_w \sim T_1 \). Thus \( T_w \) may be replaced by \( T_1 \), and \( T_r \) by \( T_0 \) in the above equations. (Since for air \( Pr \sim 0.72 \), \( T_r \) is usually retained to give a "better" estimate of \( T/T_2 \)).

Since by definition

\[ \mathcal{S}^* = \int_{0}^{\infty} \left( 1 - \frac{\rho V}{\rho_2 V_2} \right) dy \quad (111) \]

and from Eq. (107)

\[ \theta = \int_{0}^{\infty} \frac{\rho V}{\rho_2 V_2} \left( 1 - \frac{V}{V_2} \right) dy \quad (112) \]

A substitution of Eqs. (108) and (109) into Eqs. (110) and (112) yields

\[ \frac{\mathcal{S}^*}{\zeta} = 1 - \left( \frac{\rho T_2}{T_w} \right) \left[ \frac{V_1}{V_2} I_{n-1} + \left( 1 - \frac{V_1}{V_2} \right) I_n \right] \quad (113) \]

\[ \frac{\theta}{\zeta} = n \frac{T_2}{T_w} \left( 1 - \frac{V_1}{V_2} \right) \left[ \frac{V_1}{V_2} I_{n-1} + \left( 1 - 2 \frac{V_1}{V_2} \right) I_n - \left( 1 - \frac{V_1}{V_2} \right) I_{n+1} \right] \quad (114) \]

where the \( I \)'s are functions of \( b \) and \( c \) and are defined by the integral,

\[ I_a = \int_{0}^{1} \frac{z^a \ dy}{1 + bz - cz^2} \quad (115) \]

44
where \( a = (n - 1), n, \) and \( (n + 1) \).

The following results have been developed in Refs. 50 and 56c),

\[
\frac{\tau_w}{\rho_2 V_2^2} = 0.0225 \phi \left(1 - \frac{V_1}{V_2}\right) \left|1 - \frac{V_1}{V_2}\right|^{3/4} \left(\frac{V_2}{V_2^*}\right)^{1/4} \tag{116}
\]

for

\[
n = 7, \quad \phi = \left(\frac{\mu_m}{\mu_2}\right)^{1/4} \left(\frac{T_2}{T_m}\right)^{3/4} \tag{117}
\]

\[
\delta = 0.0574 \left(\phi = \frac{1 - V_1/V_2}{\theta/\delta}\right)^{4/5} \left|1 - \frac{V_1}{V_2}\right|^{1/2} \left(\frac{V_2}{V_2^*}\right)^{1/5} . \tag{118}
\]

\[
C_f Re^{1/5} = 0.0920 \left[\frac{T_w}{T_2} \frac{\theta}{\delta} \left(\frac{1}{1 - V_1/V_2}\right) \frac{\nu_m}{\nu_w} \left(\frac{f_m}{\rho_m^{1/3}}\right)^{1/4} \frac{(1 - V_1/V_2)}{|1 - V_1/V_2|}\right] \tag{119}
\]

where

\[
T_m = 0.5 (T_w + T_2) + 0.22 (T_r - T_2) \tag{120}
\]

\[
\frac{\tau_w}{\rho_2 V_2^2} = 0.0488 \phi \left(1 - \frac{V_1}{V_2}\right) \left|1 - \frac{V_1}{V_2}\right|^{2/3} \left(\frac{V_2}{V_2^*}\right)^{1/3} \tag{121}
\]

for

\[
n = 5, \quad \phi = \left(\frac{\mu_m}{\mu_2}\right)^{1/3} \left(\frac{T_2}{T_m}\right)^{2/3} \tag{122}
\]

\[
\delta = 0.130 \left(\phi = \frac{1 - V_1/V_2}{\theta/\delta}\right)^{3/4} \left|1 - \frac{V_1}{V_2}\right|^{1/2} \left(\frac{V_2}{V_2^*}\right)^{1/4} . \tag{123}
\]

\[
C_f Re^{1/4} = 0.195 \left[\frac{T_w}{T_2} \frac{\theta}{\delta} \left(\frac{1}{1 - V_1/V_2}\right) \frac{\nu_m}{\nu_w} \left(\frac{f_m}{\rho_m^{3/4}}\right)^{1/4} \frac{(1 - V_1/V_2)}{|1 - V_1/V_2|}\right] \tag{124}
\]

\[
q_w = C_p, m \frac{T_w - T_r}{V_2 (1 - V_1/V_2)} P_{r, m}^{-2/3} \tau_w \tag{125}
\]

\[
\frac{T_r}{T_2} = 1 + \left(\frac{V_1}{V_2} - 1\right)^2 \frac{v_2^2 r(0)}{2 T_2 \epsilon_{p, m}} \tag{126}
\]
Typical experimental results from Ref. 56c) for a 2 x 7-in. shock tube are shown in Figs. 39 to 41. For these data, the boundary layer thickness was defined as the value of $y$ where $u/u_2 = 0.99$ or $\delta^*/\delta = 0.996$. Since it is difficult to determine $\delta$ accurately, the ratio $u/u_2$ is plotted against $y/\delta^*$ in Fig. 39 to avoid scatter, as $\delta^*$ is an integrated quantity, and variations in $\delta$ do not affect it strongly. The temperature profiles were obtained directly from the interferometric density measurements and the velocity profiles were obtained with the aid of Eq. (110). It is seen that the experimental values tend to follow the 1/5 rather than the 1/7 power law for the given initial conditions. This in itself is not too indicative since Ref. 52 did verify the 1/7 power law over a wide range. The important point is that Mirels' analysis is substantiated by the experimental results. The plots of $\theta$, $\delta^*$, and $\theta$ appear on Fig. 40. The experimental points are closer to the 1/5 power law profile for $\delta$, but there is only a small difference between the 1/5 and 1/7 power law for the theoretical values of $\delta^*$ and $\theta$. For the stronger shock wave ($p_2/p_1 = 8$), the scatter in the experimental results is about 25 to 30 per cent for $\theta$ and $\delta^*$ at the higher Re ($10^8$), and may be due to the fact that the measurements were close to the eddying contact region. It is seen that thick boundary layers are developed only after a short distance from the shock front at low pressures.

In Fig. 41 the local skin friction $C_f$ shows the typical decline with increasing Reynolds number $Re_x$, as the boundary layer thickens. For a given $Re_x$, the theoretical skin friction decreases somewhat with an increase in $V_1/V_2$ from 2 to 3.5. The agreement with theory shown in Fig. 41 is very good for $V_1/V_2 = 2$, $(u_2/a_1 = 0.79, M_2 = 0.67, T_2/T_w = 1.38)$. The experimental points for $V_1/V_2 = 3.5$, $(u_2/a_1 = 1.88, M_2 = 1.25, T_2/T_w = 2.3)$ are higher than that predicted by theory and are close to the values obtained for $V_1/V_2 = 2$. A later interferometric study of the turbulent boundary layer (Ref. 52) gives some excellent substantiating data for Mirels' analysis, especially when the flow behind the shock wave is supersonic. In addition, turbulent heat transfer data from Ref. 54, also shows very good agreement for $1.5 < M_8 < 5.5$ with the analysis developed by Mirels, as indicated in Fig. 42. In this case as well the Reynolds number is defined by Eq. (104) but is arbitrarily based on $\sqrt{2}$ of the free stream.

It can be concluded that the available experimental work where the initial conditions were satisfactory substantiates the analysis of Ref. 50 of the zero-pressure-gradient, compressible turbulent boundary layer developed on the wall of a shock tube behind a plane shock wave. Similar complete theoretical and experimental data for the turbulent flow behind a rarefaction wave are not available as yet. Such results would be very helpful in the performance analysis of hypervelocity launchers. Some relevant recent theoretical and experimental investigations of boundary layer flows behind moving shock waves in shock tubes can be found in Refs. 57a to c.
d) Boundary-Layer Closure

Of some interest is the distance behind the shock wave when the turbulent boundary layer has grown to such an extent that it closes and a turbulent pipe flow results. This may be illustrated by referring to Fig. 35 and by assuming that the boundary layer closes at distance \(a\), behind the shock wave. It is seen that the total test time that would be available with uniform flow is given by \(T\). Consequently, there is nothing gained by making the channel longer than \((a + b)\), that is, this particular particle path may be thought of as a new contact surface. As a result, the ratio of the total channel length to the boundary layer closing distance is given by

\[
\frac{a + b}{a} = \frac{V_1}{V_2} = \frac{w_1/u_2}{w_1/u_2 - 1} = \frac{1}{1 - u_2/w_1} = \frac{\rho_2}{\rho_1}
\]

(128)

This ratio has a value of unity for very weak shocks, and a value of six for strong shocks, in air, considered as a perfect gas. For an imperfect gas, the ratio \(\rho_2/\rho_1\) depends on the initial pressure and can exceed six by sizeable factors.

From Eqs. (118) and (123), it is seen that

\[
\delta = \frac{bx}{Re_x} = \frac{bx}{Re_x}
\]

(129)

where

\[
b = b\left(\frac{V_1}{V_2}, p_1, T_1\right)
\]

is a function of the shock velocity ratio and the initial conditions. The parameter depends on the power index \(n\). If the Reynolds number is now changed so that it is based on the hydraulic diameter, then

\[
\frac{x}{D} = \left(\frac{\delta}{D}\right)\frac{1}{1 - \alpha} \left(\frac{Re_D}{b}\right)\frac{1}{1 - \alpha} \quad \text{or} \quad \frac{\delta}{D} = c\left(\frac{x}{D}\right)^{1 - \alpha}
\]

(130)

where

\[
c = c\left(\frac{V_1}{V_2}, p_1, T_1\right)
\]

Under the assumption of a flat plate with zero pressure gradient the boundary layer would close at a distance behind the shock when \(\delta = 1/2\ D\), or from Eq. (130),

\[
\frac{a}{D} = \left(\frac{1}{2b}\right)\frac{1}{1 - \alpha} \left(\frac{Re_D}{b}\right)^{\alpha} \frac{\alpha}{1 - \alpha}
\]

(131)
It is noted in Ref. 12 that the closing distance lies in the range of $20 < x/D < 140$ for the shock strengths ($M_s$) in the range $1.5 < M_s < 12$ and pressures $p_1 < 0.1$ atm for tubes of hydraulic diameters $2 < D < 3$ in. (Similar results can also be found in Ref. 52, where it is also shown that the experimental pipe-flow profiles do not appear to differ from the turbulent boundary layer profiles). This wide variation in $x/D$ can hardly be taken as an index. It is an indication that the nonstationary boundary layer closure phenomenon is far from understood.

In summary, it can be stated that from the available theoretical and experimental researches that the laminar and turbulent boundary layers behind moving shock waves are reasonably well understood at the present time, with the exception of boundary layers where strong imperfect gas effects also exist. Transition to turbulent flow occurs at the appropriate Blasius-type Reynolds numbers $Re \sim 10^6$ (based on the free stream kinematic viscosity) for low heat transfer (cooling) and at Reynolds numbers of an order of magnitude greater or more for high cooling rates. Boundary layer closure, resulting in pipe flow, is not too well understood and this problem requires a systematic study in order to be able to predict its occurrence.

In the case of flows inside and behind rarefaction waves, the boundary layer flow problem has not been fully solved to date. There are several theories but these have not been substantiated by experiment. The difficulty arises from the fact that the boundary layer in this region grows in a flow that has completely nonuniform properties and makes a theoretical analysis rather difficult. The transition Reynolds number cannot be readily defined but appears to lie in the million range when defined by local flow properties and the time a particle has been in motion (Blasius flow). Boundary layer closure is even less understood under these circumstances.

In view of the above, it would appear reasonable to analyse launcher flows on the basis of pipe flow existing throughout the barrel for small calibre guns (dia $< 1/2$ in.), owing to the large Reynolds numbers, and assuming that one can apply turbulent skin friction and heat transfer coefficients corresponding to nonstationary flows noted previously. Some reasonable assumptions will have to be made as a result of the completely unsteady nature of the flow when using these coefficients.

Some work in this direction is already underway at NOL (Ref. 58). The method developed by Williams (Ref. 42) for the effects of boundary layers in the rarefaction wave is being utilized in the analysis. However, the steady flow turbulent skin friction coefficient ($C_f = 0.049 \ Re^{-1/5}$) is being used along with the Reynolds analogy $(1/2 \ C_f = q/\rho \ u * P_{*}^{2/3})$ between skin friction and heat transfer (Eq. 39). The results are to be published in the near future. It would be very useful to extend this work by using the appropriate nonstationary values of the turbulent boundary layer skin friction coefficient, as noted previously.

In lieu of any definitive study of the effects of boundary layer skin friction and heat transfer on the reduction in the muzzle velocities of pro-
jectiles accelerated in hypervelocity launchers, an empirical curve (Fig. 43) based on the experience at NOL (Ref. 33) is offered as a reasonable guide. Figure 43 depends on a qualitative dimensional analysis in which it was concluded that the dimensionless ratio of projectile velocity to driver sound speed is the most significant parameter. The graph indicates that the effects of skin friction are small at low velocities but are very significant at high muzzle velocities. On the other hand, a high sound speed (high temperature) driver gas would tend to reduce the effects of skin friction for a given hypervelocity. Since this plot is based on experience with relatively low temperature driver gases, this last conclusion still has to be verified.

It should be noted that when pistons (or diaphragms) are used in multi-stage guns to produce a reservoir of high-temperature high-pressure driver gas multiple shock reflections occur. The waves in turn interact with the nonstationary boundary layer on the wall and will further modify the state of the reservoir (Ref. 12). These effects may have to be considered for some performance analyses.

2.3.4 Effect of Bore Friction

The solid friction between the projectile and the launcher bore will reduce the muzzle velocity of the shot. This decrease is difficult to predict theoretically and to measure accurately experimentally. Continuous measurements of base pressure using piezo-pressure gauges located in the projectile proper and of the projectile trajectory using microwave techniques could result in the isolation of the effects of friction. Some experiments along these lines have actually been done (Ref. 9).

It appears that bore friction can be simulated reasonably well by an average constant force at least, for nylon models, (see Refs. 34 and 62 to 64). It was shown in Subsection 2.3.2, Figs. 22 and 23, that an arbitrarily fixed value of solid friction (200 lb) regardless of the driver gas pressure will artificially reduce the muzzle velocity by about 50% at low driver pressures and by approximately 5% at higher driver pressures. In lieu of exact data it is usual to reduce the driver gas pressure by about 5% in rifled and unrifled bores (Refs. 36 and 59), to account for the decrement in muzzle velocity resulting from bore friction.

It is apparent that like boundary layer friction this aspect of hypervelocity launcher performance needs further study in order to make it possible to predict realistic launcher hypervelocities. It is also difficult to apportion the decrement in velocity resulting from bore friction and that due to viscous effects for a given launcher and a given set of initial conditions in the driver gas. From available evidence it appears that the boundary layer causes a much larger deceleration of the projectile at hypervelocities than bore friction.

Rather than use a constant retardation force due to friction as noted above, the following simplified approach (since heat transfer, melting
or ablation of the model are neglected) may be used as a first approximation to this problem (Ref 34d). Consider the forces (neglecting acceleration loads) acting on a cylindrical projectile moving with uniform velocity \( u_3 \) in a rigid launcher barrel, as shown in the sketch below. Assume that the initial clearance between projectile and barrel is hypothetically zero, yet the piston is free to move. If the projectile

\[
\text{SKETCH 1. Some Effects of Bore Friction}
\]

base pressure is \( p_3 \), and the counterpressure \( p_2 \), then these pressures will try to deform the moving projectile so as to decrease its length (1) and thereby increase the diameter (d). Since the barrel is assumed to be rigid, such a deformation cannot occur and hence a normal wall stress \( p \) is generated and in turn produces the bore friction.

In an actual case this stress will decrease from the base to the head of the projectile as a result of the decreasing action of \( p_3 \) towards the head and that of \( p_2 \) towards the base. However, for simplicity it is assumed that a uniform axial average stress exists given by,

\[
S_{\text{axial}} = \frac{p_3 + p_2}{2} \tag{132}
\]

which generates a uniform normal pressure \( p \), between projectile and launcher. If the resulting radial and tangential stresses are introduced in a free-body diagram, a second order ordinary differential equation can be formed subject to the boundary conditions that the tangential and radial stresses at the centre of the cylinder must be finite and that the deformation at the periphery of the cylinder (maximum radius) must vanish. A solution of the differential equation shows that the radial and tangential stresses are constant and equal to the normal stress \( p \), that is,

\[
S_{\text{radial}} = p = S_{\text{tangential}} \tag{133}
\]
The value of $p$ is given explicitly by the relation

$$ p = \frac{\mu}{1 - \mu} \cdot \frac{p_3 + p_2}{2} \tag{134} $$

where, $\mu$ = Poisson's ratio

The frictional retardation force per unit area ($\phi$) is then expressed by,

$$ \phi = f_s p \tag{135} $$

where, $f_s$ = coefficient of solid friction, and the total retarding force is

$$ F = \int d l f_s p \tag{136} $$

Using the above relations and Eqs. (45) and (74), the equation of motion of the projectile can be rewritten as,

$$ m \frac{du}{dt} = A(p_3 - p_2) - 2A \frac{1}{d} f_s \frac{\mu}{1 - \mu} (p_3 + p_2) \tag{137} $$

As noted in the foregoing very little is known about the actual variation of $f_s$ under operating conditions in a launcher barrel. If it is assumed that $f_s$ is a constant for the contacting materials regardless of velocity, then for a nylon projectile in a steel barrel if $f_s \sim 1/3$, $\mu \sim 1/8$ and $p_3 \sim 10^5$ psi, then neglecting counterpressure ($p_2 \sim 0$), for a single calibre 1/4 in. dia. nylon cylinder, the total retarding force is $F \sim 500$ lb, and when $p_3 \sim 10^3$ psi, $F \sim 5$ lb, by using the equations developed previously. The assumed conditions (zero clearance, no deformation, uniform normal force) are rather conservative. Despite this the values for the total frictional retardation appear to have the correct magnitude (except perhaps at very high pressures) compared with those used in Refs. 34a) and b) to account for the observed piston path.

It is worth noting from Eq. (137) that the projectile achieves a maximum velocity when $du/dt = 0$, or

$$ \frac{p_3}{p_2} = \frac{1 + k}{1 - k} \tag{138} $$

where,

$$ k = 2 \frac{1}{d} f_s \frac{\mu}{1 - \mu} \tag{139} $$

* I wish to thank Dr. A. H. Makomaski for developing this relation. (See also theory of thick-walled cylinders, viz. J. P. Den Hartog, Strength of Materials, McGraw-Hill, 1949).
For the above example, \( k \sim \frac{1}{10} \), for constant \( f_s \) & \( \mu \), or \( \frac{p_3}{p_2} \sim 1.2 \), that is, the effect of friction increases \( p_3 \) over \( p_2 \) by about 20%, in order to overcome bore friction. It is of interest to note that when the counterpressure \( p_2 = 0 \) (evacuated barrel) then \( \frac{du}{dt} = 0 \) applies only when the base pressure \( p_3 = 0 \), since all the other quantities are constant. This result is consistent with the assumption that the frictional retardation depends on \( f_s p_3 \), and hence eventually becomes vanishingly small. In an actual case frictional forces will develop from causes other than axial stress such as surface roughness, thermal stresses, and other model deformations, which may be more severe than the above assumptions. On the other hand, if the model surface melts, bore friction will be reduced as a result of lubrication. Consequently, a more realistic equation than Eq. (137) is required to determine the optimum launcher length (\( \frac{du}{dt} = 0 \)) in a real case.

As noted above, considerable research is still required in order to provide realistic answers to questions regarding bore friction. For example, gas leakage and melting of the projectile material at the wall will reduce the frictional retardation. However, gas leakage will also cause a reduction in muzzle velocity which might be erroneously attributed to friction.

It is worth noting that Ref. 34 c) reports that nylon pistons, (3.2 in. dia., in the weight range of 250-90 gm driven by an initial pressure ratio \( \frac{p_4}{p_1} = 500/15 \) psi and maximum velocities of 1200 - 1550 ft/sec, extrapolated very well to 1700 ft/sec which is the theoretical zero mass or contact surface case. One may therefore conclude that in these experiments with well-fitting, stiff, nylon pistons, the effect of bore friction was reduced to a negligible value. The results of Refs. 34 a) and b) however, required fairly large constant friction forces in the equations of motion to account for the observed velocity decrement and it appears that in these cases the piston fit may not have been as carefully made as in Ref. 34 c). This conclusion is also reached in Ref. 34 d). Although the above results were obtained in gun-tunnels at relatively low piston velocities they are instructive but no definite conclusion can be carried over to hypervelocity launchers where speeds which can be twenty fold greater should be considered. Research at hypervelocities on various aspects of bore friction is presently being conducted at NRL, Washington and ARO, Tullahoma (Refs. 34 e, f, and 65). Undoubtedly, the effects of bore friction will be better understood when the research results become available. However, the results shown in Ref. 65 indicate that the effects of viscous and bore friction are small.

2.3.5 Effects of Gas Imperfections

Caloric and thermal gas imperfections can affect the performance of hypervelocity launchers in the following ways. The rapid expansion of the high-pressure, high-enthalpy driver gas through the nonstationary rarefaction wave may give rise to a frozen, nonequilibrium or equilibrium flow process (Refs. 18, 60). For example, a one calibre cylinder travelling at 30,000 ft/sec in a 1/4 in. dia x 4 ft long launcher barrel has a launch or dwell time of about 150 \( \mu \) sec. Since this time is very short (initially, com-
pared with some chemical equilibrium time) an analysis must be made of the particular driving conditions to determine the actual base pressures on the model. Initially the flow would be frozen. As the number of molecular collisions increased and equipartition of energy took place, the flow would pass to a non-equilibrium and finally to an equilibrium state. The latter is most desirable since the deexcitation energies of the inert modes (vibration, dissociation, electronic excitation and ionization, will enhance the acoustic impedance of the driver gas (see Fig. 8 and Subsection 2.2), which results in a higher muzzle velocity. Consequently, it is important to determine whether equilibrium conditions will prevail for most of the launch period. This can be computed for an inviscid expansion using the methods outlined in Ref. 18.

However, in a real flow the formation of the viscous boundary layer is coupled to the inviscid rarefaction region thereby changing its flow properties. In addition, the heat losses arising from convective heat transfer across the boundary layer into the metal launcher tube and that lost by the hot gas through radiation will ultimately limit the temperatures and pressures that can be obtained in practice in a driver chamber.

If a counterpressure exists then conditions in the gas between the shock wave and the head of the projectile will also be affected by gas imperfections and boundary layer growth and in turn this will influence the expansion process. Consequently, the perfect gas relations developed in Subsection 2.3.2 will no longer provide an accurate estimate of counterpressure. It is possible to develop a set of equations to describe most of these effects but they would be very difficult to solve without some simplifications.

The present subsection will follow the work of Bjork (Ref. 61), in estimating launcher performance based on a simplified flow of hydrogen or helium considered as an imperfect but inviscid gas. Although his calculations were done for a specific driver heated by an electrical discharge, the results can be applied generally to drivers using hot hydrogen or helium.

Bjork has shown that when hydrogen is expanded through a non-stationary rarefaction wave the equilibrium flow isentropic index, $\gamma_e = (\partial p/\partial \rho)_s$, has a constant value, along a given isentrope even for high-temperature hydrogen. This remarkable result is illustrated in Fig. 44. It is seen that in the expansion from 100,000 psi to 200 psi, in the temperature range (1800°K < T < 12,000°K) along lines of constant entropy (13 cal/gm°K < S < 27 cal/gm°K), the isentropic index ($\gamma_e$) has a constant value along each isentrope. (A minimum value, $\gamma_e \sim 1.23$, occurs at T ~ 6200°K.) This fact makes it possible to use the ideal internal ballistic equation (for example, Eq. 53 c or Fig. 18) for hydrogen even when gas imperfections occur by replacing the perfect gas index ($\gamma$) with the one for equilibrium conditions ($\gamma_e$).

It is also of interest to reproduce Bjork's derivation of the optimum type of driver gas as compared with the analysis contained in Subsection 2.1, as follows. If it were possible to maintain the driver pressure ($p_d$) as the base pressure throughout the motion of the projectile in a launcher
tube of length \( x \) (see Eq. 1), then the work done on the projectile would be \( p_4 A x \) and in turn it would appear as kinetic energy \( 1/2 \, m \, u_3^2 \). However, in an actual expansion process the base pressure is continually falling and therefore only a fraction of this work can be done on the projectile. This fact can be described in terms of a driver efficiency \( \eta \) which relates the actual conversion of work done by the base pressure on the projectile into kinetic energy to that which would be possible with a constant base pressure over a given launcher length \( x \), that is,

\[
\eta = 1/2 \, m \, u_3^2 / A p_4 x
\]  

Using the notation of Eq. (53 c),

\[
x = \frac{1}{\alpha_4 - 1} \left[ \frac{\bar{u} - \frac{\alpha_4}{(1 - \bar{u}) \alpha_4}}{\alpha_4} + \frac{1}{\alpha_4} \right]
\]

\[
\bar{u} = \frac{u_3}{\hat{u}}, \quad \bar{x} = \frac{A p_4 x}{m \hat{u}^2}, \quad \hat{u} = \frac{2}{\gamma_4 - 1} \, a_4
\]

Then Eq. (140) can be rewritten as,

\[
\eta = 1/2 \, \frac{\bar{u}^2}{\bar{x}}
\]  

by substituting \( \bar{u} = u_3 / \hat{u} = u_3 / (\alpha_4 - 1) \, a_4 \), or \( u_3 / a_4 = \bar{u} / (\alpha_4 - 1) \). Therefore, from Eq. (53 c) above, \( \bar{x} = f(\bar{u}) = F(u_3 / a_4) \), and Eq. (141) can be conveniently plotted against \( u_3 / a_4 = \bar{u} \), which might be called the muzzle Mach number (see Eq. 57). For example, consider the case when \( \alpha_4 = 4, 6, 9 \) and \( 12 \) and \( \gamma_4 = (\alpha_4 + 1)/(\alpha_4 - 1) = 5/3, 7/5, 5/4 \) and \( 13/11 \), respectively. From Table 3 or Fig. 18, for \( \gamma_4 = 7/5 \), when \( \bar{u} = 1 \) or \( \bar{u} = 1/5 \), then \( \bar{x} = 0.0587 \), or \( \eta = 0.341 \). Similar results can be obtained over the entire range of parameters as shown in Fig. 45. It is seen from this figure that the driver efficiency \( \eta \) approaches unity when the muzzle Mach number approaches zero. This simply says that initially, when \( x \rightarrow 0 \), the base pressure \( p_3 \) approaches the driver pressure \( p_4 \) and there is virtually no loss in kinetic energy compared with the constant base pressure case. However, when \( u_3 \rightarrow \hat{u} \) or \( u_3 / a_4 \rightarrow (\alpha_4 - 1) \), \( x \rightarrow \infty \) and the base pressure \( p_3 \rightarrow 0 \). Under these conditions \( A p_4 x \rightarrow \infty \), and \( 1/2 \, m \hat{u}^2 \), although a large quantity, is negligible by comparison with \( A p_4 x \) and therefore \( \eta \rightarrow 0 \). That is, the four curves approach zero asymptotically as \( u_3 / a_4 \) approaches values of \( 3, 5, 8 \) and \( 11 \), respectively. The same result can be obtained analytically by substituting Eq. (53c) for \( x \) in Eq. (141) and taking the limits for \( \eta \) when \( \bar{u} \rightarrow 0 \) and \( \bar{u} \rightarrow 1 \). It may be verified that for \( 1 \leq \gamma_4 \leq 5/3 \) or \( \infty \geq \alpha_4 \geq 4 \), \( \eta \rightarrow 1 \) for \( \bar{u} \rightarrow 0 \) and \( \eta \rightarrow 0 \) for \( \bar{u} \rightarrow 1 \). From Eqs. (140) or (141) the driver efficiency can be rewritten as,

\[
\eta = \frac{1/2 \, m a_4^2}{A p_4 x} \cdot \left( \frac{u_3}{a_4} \right)^2
\]  

54
For a gun of fixed geometry, projectile mass, and initial driver conditions 
\( \gamma_{i} = 1/2 \frac{ma}{Ap_{4}x} \) is a constant and \( \gamma \alpha (u_{3}/a_{4})^{2} \), or on a log log plot the 
lines of fixed \( \gamma_{i} \), are straight lines with a constant slope of 2, (600°), (also on 
the line \( u_{3}/a_{4} = 1 \), the intercepts are automatically given by \( \gamma_{i} \)], as shown on 
Fig. 45. It is readily seen that these lines cut the curves of \( \gamma \) vs \( u_{3}/a_{4} \), 
derived from Eq. (53c), in such a manner that for a given \( \gamma_{i} \), \( \gamma \) is a 
maximum for the gas with the highest \( \alpha_{4} \) or smallest \( \gamma_{4} \). This restates 
the discussion following Eq. (141) and the results derived from Eq. (15) that 
for a fixed \( a_{4} \), the escape speed \( \tilde{u} \to \infty \) as \( \gamma_{4} \to 1 \) and the efficiency im-
proves with decreasing \( \gamma_{4} \) for a given \( u_{3}/a_{4} \). In other words, a high sound 
speed coupled with a low value of \( \gamma_{4} \) is very desirable for efficient drivers.

It should be restated that a low value of \( \gamma_{4} \) for a perfect gas 
can only be obtained by using a heavy molecule (see Table 1) and a correspond-
ingly low sound speed, which defeats its use as a driver gas. However, as a 
result of gas imperfections a reasonably low value of \( \gamma_{4} \) can occur simulta-
nously with a large sound speed, as seen from Fig. 44. Consequently, 
partially dissociated hydrogen becomes an excellent driver gas (or partially 
ionized hydrogen or helium - see also Fig. 51b). Consequently, the acoustic 
impedance of dissociated hydrogen under these conditions is low and as the 
inert degrees become deexcited the energies of dissociation, electronic excita-
tion, and vibration become available to further increase the projectile velocity 
for a given pressure change as compared with the perfect gas value (see Eq. 11).

Consequently, to achieve high muzzle velocities the same re-
quirements apply to an imperfect gas as well as a perfect gas, that is, a 
large driver pressure (\( p_{4} \)), a high sound speed (\( a_{4} \)), and a low value of the 
isentropic index (\( \gamma_{4} \)). The driver pressure is limited by structural con-
siderations to about \( 10^{5} \) psi (perhaps up to \( 5 \times 10^{5} \) psi by using special tech-
niques in constructing the driver chamber). However, the combination of \( a_{4} \) 
and \( \gamma_{4} \) for an imperfect gas must be optimized. Bjork suggests that the 
combination,

\[
\xi = \frac{2 \gamma_{4}^{2} a_{4}}{\gamma_{4} - 1} = \gamma_{4}^{2} \alpha
\]

should be maximum.

The driver efficiency \( \eta \) (Eq. 140), can now be rewritten as

\[
\eta = \frac{1/2 m \tilde{u}_{3}^{2}}{p_{4}Ax} \cdot \left( \frac{u_{3}}{\xi} \right)^{2}
\]

If Eq. (144) is now replotted on a log log scale against \( u_{3}/\xi \), by using Eq. 
(141), then Fig. 46 results. It is seen that in the range \( 4 \leq \gamma_{4} \leq 9 \) or \( 5/3 \geq 
\gamma_{4} \geq 5/4 \), which is useful for launcher design, the curves tend to merge as 
a single curve, indicating that \( \xi \) is a very useful parameter for maximizing 
the combination of \( a_{4} \) and \( \gamma_{4} \) in this range.
However, it is worthwhile comparing the maximizing parameter derived by Bjork \[2 \gamma_4^2 a_4/(\gamma_4 - 1)\] with the simpler one derived analytically for a perfect gas for a constant driver pressure \(p_4\) (see Eq. 23), that is, \(a_4/\gamma_4 = \Phi\) (note that \(\Phi = \xi \beta_4/\gamma_4^2\) where \(\beta_4 = (\gamma_4 - 1)/(2 \gamma_4)\). The efficiency can now be rewritten as

\[
\eta = \frac{1/2 m \Phi^2}{P_4 A x} \left( \frac{u_3^2}{\Phi} \right)
\]

A plot of \(\eta\) vs \(u_3/\Phi\) is shown on Fig. 47. It is seen that the simpler optimizing parameter \(\Phi\) appears to be very useful over a good portion of the range shown and all curves tend to collapse into a single curve for \((u_3/\Phi) \leq 2\). Beyond this value they diverge. Consequently, it may be concluded that \(a_4/\gamma_4\) should be a maximum for a driver gas in order to attain the highest muzzle velocity. [For a perfect gas this means that the quantity \((T/\gamma_4 m)_4\) should be a maximum. However, for an imperfect gas like dissociated hydrogen \((ZT/\gamma_4 m)_4\) should be made a maximum (see the discussion following Eq. 23). It is worth repeating that the latter result is based on the fact noted by Bjork that the isentropic index \((\gamma_4)\) remains a constant in the nonstationary expansion of hydrogen even though it is an imperfect gas.]

Bjork has calculated 5 cases for hydrogen and 5 for helium as driver gases, for pressures \(p_4 = 10^5\) psi, and temperatures \(4700^0\text{K} \leq T_4 \leq 12,000^0\text{K}\), in order to compare their respective muzzle velocities in launching a 0.22 in. (0.559 cm) dia. projectile weighing 194 gm [aluminum cylinder of about half a calibre (0.293 cm) at 2.7 gm/cm\(^3\)]. The pressure can be considered as a practical value without going to sophisticated driver chamber design. The half calibre cylinder is also considered as a practical limit for aluminum without the model breaking up during the launching acceleration period.

The values are listed in Tables 4 and 5. In Table 4, helium appears as a perfect gas. Under these pressures and temperatures negligible gas imperfections occur, since the first ionization potential of helium is the highest of all the elements (24.58 electron volts). Consequently, the entropy is not required and \(\gamma_4 = 5/3\). The values of entropy and the equilibrium value of \(\gamma_4\) are listed for hydrogen. The parameters \(\xi\) and \(\Phi\) are also given and it is seen that \(\Phi\) has a smaller spread than \(\xi\) when \(\gamma_4\) is small, illustrating the influence in the denominator of the term \((\gamma_4 - 1)\) on \(\xi\) under these conditions.

The resulting muzzle velocities appear in Table 5. The parameter \(P_4 A x/m\) for the actual initial conditions considered previously has a value of \(1.15 \times 10^{12}\) cm\(^2\)/sec\(^2\). The other values give an indication of the range of muzzle velocities that can be expected by changing this quantity in a given experiment. The value 1.1 \(u_3\) also appears in order to take into account the increase in muzzle velocity resulting from chamberage (see Subsection
This figure is based on the experience at the NASA Ames Laboratory with their 0.22 in or 0.559 cm dia. x 134 cm long light-gas gun noted previously. A plot of 1.1 \( u_3 \) vs \( \xi \) and \( \Phi \) is shown in Fig. 48. It is seen that the influence of \((\gamma_4 - 1)\) in Bjork's curve \((\xi)\) results in a better continuous curve for helium and imperfect hydrogen when \(\gamma_4\) is small than for the \(\Phi\)-curve. The individual curves for helium and hydrogen on the \(\Phi\)-plot are rather good on their own, and no attempt has been made to form them into a reasonable single curve.

The importance of making \(p_4Ax/\\text{m} \) large for a given \(\\xi (a_4, \gamma_4)\) is clearly illustrated in Table 5. It also reflects the remark made previously that to take advantage of the increased heating of a driver gas it is also important to simultaneously raise the pressure \(p_4\) (as well as increase \(A/\\text{m}\) or \(x\) - see Fig. 19). Very little gain in \(u_3\) occurs when \(\xi\) increases from about 30 to 100 for \(p_4Ax/\\text{m} = 2 \times 10^{11} \text{cm}^2/\text{sec}^2\), but a very substantial gain in \(u_3\) takes place when this parameter is raised to \(5 \times 10^{12} \text{cm}^2/\text{sec}^2\). It was already noted that \(p_4\) may be increased fivefold in future designs and by using plastic models \(m\) may be decreased threefold. Consequently, hypervelocities of 40,000 ft/sec or more might be expected in the near future.

It can be seen from the foregoing tables and graphs that for a given launcher with the same initial conditions, hydrogen outperforms helium as a driver gas by as much as 30 to 40\% over the temperature range of 12,000 to 47000 K. Hydrogen at 47000 K and 100,000 psi gives a greater velocity than helium at 120000 K and 100,000 psi. This fact is not only important because of the added practical difficulties of obtaining high gas temperatures, but in practice the gun barrel inlet erosion problem and the resulting gas contamination will tend to limit the use of helium as a driver gas. It is possible, as Bjork has noted, that when the techniques of further increasing driver gas temperatures to very high values are mastered, that helium may well outperform hydrogen as a driver gas. An analysis as well as an experimental verification will be required to obtain a final decision. The heat loss resulting from hydrogen recombination at the wall, its higher thermal conductivity, and lower ionization potential may favour helium. The safety aspect of using helium would certainly be an additional consideration. A recent analysis performed at NRL substantiates Bjork's conclusions and helium outperformed hydrogen at temperatures in excess of 14,000 K (Ref. 62). However, this result is not decisive in the regions of partially ionized helium and hydrogen and additional analyses are required. Ultimately though, the question will probably be settled by practical considerations such as erosion as noted above.

It is worth noting that by using Eq. (22) it is possible to determine the base pressure on the projectile as it leaves the 240 calibre launcher listed in Tables 4 and 5. For a value of the parameter \(p_4Ax/\\text{m} = 5 \times 10^{12} \text{cm}^2/\text{sec}^2\), one obtains \(p_3 = 7100 \text{ psi}\) for Case 1 and \(p_2 = 660 \text{ psi}\) for Case 10. Similarly for the actual value of \(p_4Ax/\\text{m} = 1.15 \times 10^{12} \text{cm}^2/\text{sec}^2\) for a reasonable case such as Case 6, one obtains \(p_3 = 250 \text{ psi}\). Consequently, the
launcher is much too short for Case 1, above, since there is still a large base pressure available to increase the muzzle velocity by a significant amount. In view of the above, a launcher of 400 - 500 calibres might prove to be very practical for high-pressure, high-temperature drivers despite frictional effects.

Two additional figures from Ref. 61 are included since they are of importance in the performance analysis of electrically heated hydrogen drivers. Figure 49 shows the variation of internal energy with entropy on constant temperature and density lines (a modified Mollier diagram). Figure 50 replots the data shown in Fig. 49 except that pressure becomes the ordinate rather than internal energy. The usefulness of these two figures is shown schematically in Fig. 51.

Assume that an initial driver state (A) is given as defined by the temperature and density. If electrical energy is now added a line of constant density (say \( \rho / \rho_s = 100 \)) can now be followed (the volume and the mass of gas are fixed) until it intersects the maximum design pressure line (\( p = 10^5 \) psi). The point of intersection (state B) gives the final temperature (\( \sim 12,000^0\text{K} \)) and the internal energy of the gas (\( \sim 300,000 \text{ joules/gm} \)). If the gas was initially at room temperature then its internal energy is about 3,000 joules/gm. Consequently, 297,000 joules/gm must be added to attain this pressure (\( p_4 \sim 10^5 \) psi) and temperature (\( T_4 \sim 12 \times 10^3 \text{ OK} \)).

State B or state (4) can now be located on Fig. 50 and the expansion of the hot driver gas can be followed along the isentrope \( S = 27 \text{ cal/gm} \). As this isentrope intersects the density-pressure lines, points such as shown in Fig. 44 can be obtained which on a log log plot form a straight line whose slope (in this case \( \tan^{-1} = 1.57 \)) gives the value of \( \gamma_e \), as noted previously. Since \( \gamma_e \) for imperfect hydrogen in this range is fortuitously a constant throughout the expansion, the perfect-gas ballistics relation (Eq. 53c) can be used, as discussed. Otherwise, the graphical method treated in Subsection 2.2 (Figs. 5 to 10) would have to be utilized in conjunction with the dynamic equation, Eq. (47), by expressing \( p_3 \) in terms of \( u_3 \) using a polynomial fit to the graphical curve and integrating for each particular case. Such a procedure becomes much more involved, as expected.

In Fig. 53, some additional data on dissociated and ionized hydrogen at low pressures are included as a guide. It is seen that for hydrogen at 1.0 atmosphere a low value of \( \gamma_e = 1.15 \) occurs at a temperature \( T \sim 2800^0\text{K} \) when the molecular weight \( m \sim 1.8 \text{ gm/mole} \) or a degree of dissociation \( \sim 0.11 \). The second, and smaller minimum occurs at \( T \sim 15,000^0\text{K} \), \( m \sim 0.8 \text{ gm/mole} \) and a degree of ionization \( x \sim 0.25 \). Consequently, advantage can be taken of high escape speeds for hydrogen resulting from a low value of \( \gamma_e \) when partial dissociation or partial ionization occurs (see Ref. 12 for exact expressions for \( \gamma_e \)). For fully ionized hydrogen (with a molecular weight of one quarter of undissociated hydrogen), the sound speed is again \( a_c = \sqrt[4]{5/3} \sqrt{RT/m} \) and increases as the square root of \( T \). The extension of such calculations for higher pressures for hydrogen and helium would provide useful data for launcher analyses.
2.4 Extension of Existing Methods of Analysis

The foregoing subsections dealt with the simplest type of launcher consisting of a tube of constant cross section containing a projectile driven by the expansion of a high-pressure high-temperature driver gas. Using this simple model it was possible to develop an analytical expression that would predict the muzzle velocity for a given set of initial conditions. In practice, these ideal muzzle velocities are not achieved. Consequently, the first question that arises is, what are the causes for the observed velocity decrement? However, even if these causes were fully understood, it still does not remedy the inefficient method through which hypervelocities are achieved as a result of decreasing temperatures and pressures at the base of the projectile produced by the nonstationary expansion wave. Therefore, the second question that may be asked is, how can the ideally efficient constant base pressure acceleration be approached in order to obtain significant hypervelocities of 50,000 ft/sec or greater?

Figure 52 a) illustrates the first question by showing the significant reduction (about 25%) in the actual final projectile velocity compared with the predicted result for a 37 mm 0.60 calibre, 6.8 gm, aluminum cylinder and a 20 mm, 0.30 calibre, 1.0 gm aluminum cylinder (Ref. 62). Although the NRL light-gas gun from which the data are taken is not a simple launcher, the results do illustrate the point of a velocity decrement in an actual launcher (in a compound launcher there are additional factors that contribute to the observed decrement in muzzle velocity. Similar results for a 0.30 calibre AEDC launcher appear in Fig. 52 b), which utilizes piston-heated hydrogen to drive a 0.44 gm projectile. It is seen that the agreement with an empirically derived correlation formula is very good indeed. The correlation formula (Fig. 52 b) is seen to contain three velocity decrements \( c_1 \), \( c_2 \), and \( c_3 \) which depend on piston reversal, initial projectile motion, and loss in temperature, arising primarily from driver gas contamination by metal vapours owing to launcher inlet erosion. The corrections \( c_1 \) and \( c_2 \) are associated with two-stage launchers, while \( c_3 \) would be common to all high-temperature high-pressure drivers. (Further details will be given in Section 5). It should be noted that possible decrements in muzzle velocity due to nonequilibrium, bore friction, and boundary layer effects are entirely absent. Although the empirical correction correlates the AEDC results as well as similar NASA Ames results, they are based on a limited number of rounds which could have been interpreted in a somewhat different manner to yield a modified empirical correction formula. Such formulas are very useful from practical considerations, since analytical relations that account for observed velocity decrements are still lacking.

The loss in muzzle velocity in a simple launcher can be attributed to the following major causes,

a) Loss in stagnation enthalpy or temperature in the driver gas as a result of radiation, heat conduction, erosion of the launcher inlet, and vaporization of sabot and projectile materials, which produces a net decrease in projectile base pressure.
b) Possible nonequilibrium during the expansion which prevents the addition of the inert energies to the flow thereby reducing the base pressure.

c) Loss in projectile base pressure arising from the boundary layer induced behind the rapidly moving shot.

d) Direct retardation of the projectile by bore friction.

These losses must be accepted as inevitable for real launchers. However, what is essential is to formulate realistic mathematical descriptions of these effects and solve the resulting equations by using high-speed digital computers in order to supply the most realistic predictions of launcher performance. The equations should take into account the four possible major sources of velocity decrement in order to provide accurate predictions of muzzle velocities for a given light-gas gun. It should be emphasized that in all calculations it is desirable to use real equations of state and appropriate rate equations to ascertain whether the gas flow is in a frozen, nonequilibrium or equilibrium state. Undoubtedly, this makes the analysis very complex but some group which has a computing capability should do this in order to ascertain the magnitude of these effects. Studies of this type have now been initiated by several investigators for more complex launchers (Refs. 58, 63, 64).

A simpler and useful set of calculations that should be performed would consist of curves which showed the deviations produced by each one of the four effects noted previously from the ideal internal ballistics relation (Eq. 53), as well as their combined effect on a simple launcher. In order to make these calculations meaningful it will be necessary to verify them experimentally in specially instrumented launchers that can provide a history of the state of the gas in the launching chamber and of the projectile motion in the barrel of a simple launcher. More complex launchers will require additional information pertaining to the other components of the system. This may be a very difficult task and should be confined to one or two groups whose specific interest is launcher design and performance. Considerable progress in this direction has already been reported in Refs. 32 a) to c) and Ref. 65.

Assuming that such performance data will be readily available in the near future, the second question of reducing the limitations of the present-day simple, light-gas launchers, might be approached as follows. Considerable increases in hypervelocity can still be achieved through improved structural design and energy addition. That is, higher driver gas temperatures and pressures are required (provided that these quantities are not limited by contamination effects - Ref. 32). Even though the basic driving principle (nonstationary expansion) is unchanged, much larger hypervelocities could be attained by this head-on approach. However, this will require sophisticated driver techniques such as the use of explosively generated implosion waves (see Subsection 3.7). The attainment of high temperatures and pressures
result in very high accelerations on the projectile. It was shown (following Eq. 4) that a 1 calibre nylon projectile in a 10 mm dia (0.4 in) launcher barrel would be subjected to $7 \times 10^6$ g on application of a pressure of $10^5$ psi. For a 5 mm barrel this rises to $14 \times 10^6$ g. If the pressure is now increased to $3 \times 10^5$ or $4 \times 10^5$ psi, the accelerations would rise to $42 \times 10^6$ g and $56 \times 10^6$ g, respectively. It is not known if materials will be available that will enable the projectile to emerge without being shattered into fragments under such conditions.

Another approach will consist of unique designs such as the accelerated reservoir methods (Ref. 11), which provides the model with a relatively constant base pressure. The addition of electrical energy behind the base of the shot as it proceeds along the barrel may be another possibility. Some of the above approaches will be considered in greater detail in the subsequent subsections. However, it is doubtful if these methods will provide controlled hypervelocities of aerodynamic models in the 100,000 ft/sec range unless counterfire techniques or other entirely new methods of acceleration are developed.
REFERENCES

   Winslow, Jr., P.C.

   Ghering, J.W.

   Curtis, J.S.


   Hume, W.


7.a) Bull, G.V.  Hypervelocity Research in the CARDE Free Flight Ranges. Galbraith Building Opening Ceremonies, University of Toronto, 1961 (also private communication).

   McMahon, H.M.
   Letarte, M.

   Bergslien, R.M.


   Editor


   Hall, J.G.


<table>
<thead>
<tr>
<th>No.</th>
<th>Author(s)</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>24.</td>
<td>King C. R.</td>
<td>Compilation of Thermodynamic Properties, Transport Properties and Theoretical Rocket</td>
</tr>
</tbody>
</table>
32. a) Stephenson, W. B. Anderson, D. E.

32. b) Lukasiewicz, J. Stephenson, W. B. Clemens, P. L. Anderson, D. E.

32. c) Stephenson, W. B.

33. Seigel, A. E.

34. a) Bray, K. N. C. Pennelegion, L.

34. b) Cox, R. N. Winter, D. F. T.
   A Theoretical and Experimental Study of an Intermittent Hypersonic Wind Tunnel Using Free-Piston Compression. ARDE Report (B) 9161, Fort Halstead, 1961.

34. c) Stollery, J. L. Maull, D. J. Belcher, B. J.
   The Imperial College Hypersonic Gun Tunnel, J. R. Aero. S. 64, 589, pp. 24-32, 1960.

34. d) Jungowski, V.

34. e) Swift, H. F. Porter, C. D.

* see also, Seigel, A. E. An Experimental Solution to the Lagrange Ballistic Problem, Navord Report 2693, 1952.
<table>
<thead>
<tr>
<th></th>
<th>Author(s)</th>
<th>Reference/Description</th>
</tr>
</thead>
</table>
47. Trimpi, R. L.
Cohen, N. B.


48. Cohen, N. B.


49. Mirels, H.


50. Mirels, H.

Boundary Layer Behind Shock or Thin Expansion Wave Moving into Stationary Fluid, NACA TN 3712, 1956.

51. Bershader, D.
Allport, J.


52. Asbridge, J. R.


53. Gray, D. E., Editor


54. Hartunian, R. A.
Russo, A. L.
Marrone, P. V.


55. Mirels, H.


56.a) Chabai, A. J.


56.b) Ostrach, S.
Thornton, P. R.


TABLE 1

PHYSICAL PROPERTIES OF GASES

<table>
<thead>
<tr>
<th>Gas</th>
<th>Molecular Weight</th>
<th>γ</th>
<th>α = (γ+1) / γ - 1</th>
<th>β = (γ-1) / 2γ</th>
<th>C_v (Cal/gm/°C)</th>
<th>Viscosity (micro poise)</th>
<th>Refraction Point °C</th>
<th>Boiling Point °C</th>
<th>Sound Speed ft/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>28.96</td>
<td>5.95</td>
<td>0.144</td>
<td>0.172</td>
<td>185</td>
<td>1.00029</td>
<td>-185.7</td>
<td>1087</td>
<td></td>
</tr>
<tr>
<td>Argon</td>
<td>39.94</td>
<td>4.00</td>
<td>0.200</td>
<td>0.075</td>
<td>222</td>
<td>1.00028</td>
<td>-1009</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CO₂</td>
<td>44.01</td>
<td>1.300</td>
<td>0.115</td>
<td>0.151</td>
<td>150</td>
<td>1.00045</td>
<td>-78.5</td>
<td>846</td>
<td></td>
</tr>
<tr>
<td>Carbon Tetra-</td>
<td>153.84</td>
<td>16.38</td>
<td>0.058</td>
<td>0.1167</td>
<td>100</td>
<td>1.0018</td>
<td>-422</td>
<td></td>
<td></td>
</tr>
<tr>
<td>chloride</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C₄H₁₀</td>
<td>72.15</td>
<td>24.3</td>
<td>0.039</td>
<td>0.418</td>
<td>67</td>
<td>1.0017</td>
<td>28</td>
<td>606</td>
<td>+</td>
</tr>
<tr>
<td>Ethane</td>
<td>30.05</td>
<td>10.09</td>
<td>0.090</td>
<td>0.368</td>
<td>95</td>
<td>1.0008</td>
<td>-88.3</td>
<td>996</td>
<td>+</td>
</tr>
<tr>
<td>C₂H₄</td>
<td>120.9</td>
<td>1.255</td>
<td>0.102</td>
<td>0.2862</td>
<td>100</td>
<td>1.00072</td>
<td>-103.9</td>
<td>1046</td>
<td></td>
</tr>
<tr>
<td>Ethylene</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Freon 12</td>
<td>120.9</td>
<td>1.139</td>
<td>0.061</td>
<td>0.1297</td>
<td>123</td>
<td>1.00072</td>
<td>-28</td>
<td>480</td>
<td>+</td>
</tr>
<tr>
<td>He</td>
<td>4.003</td>
<td>4.00</td>
<td>0.200</td>
<td>0.746</td>
<td>195</td>
<td>1.000035</td>
<td>-268.9</td>
<td>3182</td>
<td></td>
</tr>
<tr>
<td>H₂</td>
<td>2.016</td>
<td>5.91</td>
<td>0.145</td>
<td>2.24</td>
<td>88</td>
<td>1.00014</td>
<td>-252.8</td>
<td>4165</td>
<td></td>
</tr>
<tr>
<td>Krypton</td>
<td>82.9</td>
<td>3.90</td>
<td>0.204</td>
<td>0.036*</td>
<td>246</td>
<td>1.00014</td>
<td>-152.9</td>
<td>701</td>
<td>+</td>
</tr>
<tr>
<td>CH₄</td>
<td>16.03</td>
<td>7.39</td>
<td>0.119</td>
<td>0.402</td>
<td>109</td>
<td>1.00045</td>
<td>-161.5</td>
<td>1417</td>
<td></td>
</tr>
<tr>
<td>Methane</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ne</td>
<td>20.18</td>
<td>4.12</td>
<td>0.196</td>
<td>0.148*</td>
<td>312</td>
<td>1.00067</td>
<td>-245.9</td>
<td>1428</td>
<td></td>
</tr>
<tr>
<td>Nitrogen</td>
<td>28.02</td>
<td>5.95</td>
<td>0.144</td>
<td>0.177</td>
<td>176</td>
<td>1.00030</td>
<td>-195.8</td>
<td>1098</td>
<td></td>
</tr>
<tr>
<td>SF₆</td>
<td>146.06</td>
<td>21.9</td>
<td>0.044</td>
<td>0.141</td>
<td>150</td>
<td>1.00078</td>
<td>-63.8</td>
<td>432</td>
<td></td>
</tr>
<tr>
<td>Xe</td>
<td>131.3</td>
<td>4.00</td>
<td>0.200</td>
<td>0.023*</td>
<td>226</td>
<td>1.00070</td>
<td>-107.1</td>
<td>557</td>
<td>+</td>
</tr>
<tr>
<td>Sulfur Hexafluoride</td>
<td>146.06</td>
<td>1.096</td>
<td>0.044</td>
<td>0.141</td>
<td>150</td>
<td>1.00078</td>
<td>-63.8</td>
<td>432</td>
<td></td>
</tr>
<tr>
<td>Xenon</td>
<td>131.3</td>
<td>4.00</td>
<td>0.200</td>
<td>0.023*</td>
<td>226</td>
<td>1.00070</td>
<td>-107.1</td>
<td>557</td>
<td>+</td>
</tr>
</tbody>
</table>

* Calculated ( ~ 2 cal/gm mole)
† Toxic
+ Calculated from, a² = γRT

Table entries are rounded to three significant digits.
## Table 2

**Equilibrium Flow of Air Through a Rarefaction Wave**

<table>
<thead>
<tr>
<th>T °K</th>
<th>P atm</th>
<th>p lb/ft²</th>
<th>( \dot{m}/\dot{m}_0 )</th>
<th>( \dot{m} ) slugs/ft²</th>
<th>( \alpha = \sqrt{\frac{\gamma}{\gamma - 1}} )</th>
<th>( \frac{1}{\rho \alpha} ) (ft/sec)(ft²/lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8000</td>
<td>4.98</td>
<td>10540</td>
<td>1.02 x 10⁻¹</td>
<td>2.56 x 10⁻⁴</td>
<td>6920</td>
<td>1.162</td>
</tr>
<tr>
<td>7500</td>
<td>2.51</td>
<td>5310</td>
<td>5.60 x 10⁻²</td>
<td>1.41 x 10⁻⁴</td>
<td>6580</td>
<td>1.153</td>
</tr>
<tr>
<td>7000</td>
<td>1.127</td>
<td>2380</td>
<td>2.82 x 10⁻²</td>
<td>7.08 x 10⁻⁵</td>
<td>6200</td>
<td>1.145</td>
</tr>
<tr>
<td>6500</td>
<td>4.68 x 10⁻¹</td>
<td>990</td>
<td>1.28 x 10⁻²</td>
<td>3.21 x 10⁻⁵</td>
<td>5925</td>
<td>1.136</td>
</tr>
<tr>
<td>6000</td>
<td>1.62 x 10⁻¹</td>
<td>343</td>
<td>5.03 x 10⁻³</td>
<td>1.26 x 10⁻⁵</td>
<td>5535</td>
<td>1.128</td>
</tr>
<tr>
<td>5500</td>
<td>4.99 x 10⁻²</td>
<td>105.4</td>
<td>1.76 x 10⁻³</td>
<td>4.42 x 10⁻⁶</td>
<td>5180</td>
<td>1.128</td>
</tr>
<tr>
<td>5000</td>
<td>1.41 x 10⁻²</td>
<td>29.8</td>
<td>5.33 x 10⁻⁴</td>
<td>1.34 x 10⁻⁶</td>
<td>4980</td>
<td>1.114</td>
</tr>
<tr>
<td>4800</td>
<td>7.48 x 10⁻³</td>
<td>15.8</td>
<td>3.16 x 10⁻⁴</td>
<td>7.93 x 10⁻⁷</td>
<td>4710</td>
<td>1.112</td>
</tr>
<tr>
<td>4600</td>
<td>4.17 x 10⁻³</td>
<td>8.81</td>
<td>1.86 x 10⁻⁴</td>
<td>4.67 x 10⁻⁷</td>
<td>4580</td>
<td>1.113</td>
</tr>
<tr>
<td>4400</td>
<td>2.29 x 10⁻³</td>
<td>4.84</td>
<td>1.11 x 10⁻⁴</td>
<td>2.78 x 10⁻⁷</td>
<td>4400</td>
<td>1.115</td>
</tr>
<tr>
<td>4200</td>
<td>1.26 x 10⁻³</td>
<td>2.66</td>
<td>6.60 x 10⁻⁵</td>
<td>1.66 x 10⁻⁷</td>
<td>4240</td>
<td>1.120</td>
</tr>
<tr>
<td>4000</td>
<td>7.24 x 10⁻⁴</td>
<td>1.53</td>
<td>3.90 x 10⁻⁵</td>
<td>9.99 x 10⁻⁸</td>
<td>4160</td>
<td>1.131</td>
</tr>
<tr>
<td>3800</td>
<td>4.47 x 10⁻⁴</td>
<td>0.944</td>
<td>2.62 x 10⁻⁵</td>
<td>6.58 x 10⁻⁸</td>
<td>4060</td>
<td>1.149</td>
</tr>
<tr>
<td>3600</td>
<td>2.82 x 10⁻⁴</td>
<td>0.596</td>
<td>1.78 x 10⁻⁵</td>
<td>4.47 x 10⁻⁸</td>
<td>3970</td>
<td>1.180</td>
</tr>
<tr>
<td>3400</td>
<td>2.00 x 10⁻⁴</td>
<td>0.423</td>
<td>1.32 x 10⁻⁵</td>
<td>3.31 x 10⁻⁸</td>
<td>3960</td>
<td>1.224</td>
</tr>
<tr>
<td>3200</td>
<td>1.41 x 10⁻⁴</td>
<td>0.298</td>
<td>1.00 x 10⁻⁵</td>
<td>2.51 x 10⁻⁸</td>
<td>3890</td>
<td>1.271</td>
</tr>
<tr>
<td>3000</td>
<td>1.10 x 10⁻⁴</td>
<td>0.232</td>
<td>7.98 x 10⁻⁶</td>
<td>2.00 x 10⁻⁸</td>
<td>3880</td>
<td>1.302</td>
</tr>
</tbody>
</table>
TABLE 3 - Sheet 1

IDEALIZED INTERNAL BALLISTICS EQUATION SHOWING THE VARIATION OF \( \bar{x} \) WITH \( u \) FOR VARIOUS VALUES OF \( \gamma \)

\[ u \quad \gamma = 1.10 \quad \bar{x} \quad \gamma = 1.15 \quad \bar{x} \quad \gamma = 1.20 \quad \bar{x} \]

\[ \begin{array}{ccc}
\bar{u} & \bar{x} & \bar{u} \\
0.499999 -01 & 0.2730509 -02 & 0.499999 -01 & 0.2140083 -02 & 0.499999 -01 & 0.1898738 -02 \\
0.999999 -01 & 0.2631691 -01 & 0.999999 -01 & 0.1549837 -01 & 0.999999 -01 & 0.1198784 -01 \\
1.499999 00 & 0.1577597 00 & 1.499999 00 & 0.1570465 00 & 1.499999 00 & 0.1440208 01 \\
1.999999 00 & 0.1523404 00 & 1.999999 00 & 0.2440670 00 & 1.999999 00 & 0.1360893 00 \\
2.499999 00 & 0.1425694 01 & 2.499999 00 & 0.1802259 00 & 2.499999 00 & 0.3857700 00 \\
2.999999 00 & 0.1275916 02 & 2.999999 00 & 0.2872939 01 & 2.999999 00 & 0.1066533 02 \\
3.499999 00 & 0.1283775 03 & 3.499999 00 & 0.1013413 02 & 3.499999 00 & 0.2969861 01 \\
3.999999 00 & 0.1031748 03 & 3.999999 00 & 0.3747773 02 & 3.999999 00 & 0.8528750 01 \\
4.499999 00 & 0.0701652 04 & 4.499999 00 & 0.1501733 03 & 4.499999 00 & 0.2578502 02 \\
4.999999 00 & 0.0743591 05 & 4.999999 00 & 0.6666555 03 & 4.999999 00 & 0.8379078 02 \\
5.499999 00 & 0.0481419 06 & 5.499999 00 & 0.3360145 04 & 5.499999 00 & 0.2996218 03 \\
5.999999 00 & 0.0627988 07 & 5.999999 00 & 0.2010629 05 & 5.999999 00 & 0.1213771 04 \\
6.499997 00 & 0.1130873 09 & 6.499997 00 & 0.1802709 06 & 6.499997 00 & 0.5790720 04 \\
6.999997 00 & 0.3118355 10 & 6.999997 00 & 0.1476222 07 & 6.999997 00 & 0.3438307 05 \\
7.499997 00 & 0.1541545 12 & 7.499997 00 & 0.2173887 08 & 7.499997 00 & 0.2764396 06 \\
7.999996 00 & 0.1793797 14 & 7.999996 00 & 0.5715863 09 & 7.999996 00 & 0.3462302 07 \\
8.499996 00 & 0.8043100 16 & 8.499996 00 & 0.3772486 11 & 8.499996 00 & 0.8775627 08 \\
8.999995 00 & 0.4261667 20 & 8.999995 00 & 0.1341427 14 & 8.999995 00 & 0.8090545 10 \\
\end{array} \]

Note: \(-02 \equiv x 10^{-2}, 02 \equiv x 10^{2}, \) etc.
## TABLE 3 - Sheet 2

<table>
<thead>
<tr>
<th>( \bar{u} )</th>
<th>( \bar{x} )</th>
<th>( \gamma = 1.25 )</th>
<th>( \gamma = 1.30 )</th>
<th>( \gamma = 1.35 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.4999999 -01</td>
<td>.1768459 -02</td>
<td>.4999999 -01</td>
<td>.1607130 -02</td>
<td>.4999999 -01</td>
</tr>
<tr>
<td>.9999999 -01</td>
<td>.1030390 -01</td>
<td>.9999999 -01</td>
<td>.9325929 -02</td>
<td>.9999999 -01</td>
</tr>
<tr>
<td>00</td>
<td>.3487695 -01</td>
<td>00</td>
<td>.3976746 -01</td>
<td>00</td>
</tr>
<tr>
<td>00</td>
<td>.9661297 -01</td>
<td>00</td>
<td>.7730891 -01</td>
<td>00</td>
</tr>
<tr>
<td>.2499999 00</td>
<td>.2451091 00</td>
<td>.2499999 00</td>
<td>.1823296 00</td>
<td>.2499999 00</td>
</tr>
<tr>
<td>.5989937 00</td>
<td>.2999999 00</td>
<td>.4113156 00</td>
<td>.2999999 00</td>
<td>.3159680 00</td>
</tr>
<tr>
<td>.2499999 00</td>
<td>.2455622 01</td>
<td>.2499999 00</td>
<td>.2050028 01</td>
<td>.3499999 00</td>
</tr>
<tr>
<td>00</td>
<td>.3597352 01</td>
<td>00</td>
<td>.2659999 00</td>
<td>00</td>
</tr>
<tr>
<td>.2499999 00</td>
<td>.2451204 01</td>
<td>.2499999 00</td>
<td>.4709890 01</td>
<td>.4499999 00</td>
</tr>
<tr>
<td>00</td>
<td>.2490270 02</td>
<td>00</td>
<td>.1126319 02</td>
<td>00</td>
</tr>
<tr>
<td>.5499996 00</td>
<td>.7251600 02</td>
<td>.5499998 00</td>
<td>.2870055 02</td>
<td>.5499996 00</td>
</tr>
<tr>
<td>.5999998 00</td>
<td>.2331332 03</td>
<td>.5999998 00</td>
<td>.7320758 02</td>
<td>.5999998 00</td>
</tr>
<tr>
<td>.2499997 00</td>
<td>.2446748 03</td>
<td>.2499997 00</td>
<td>.2439272 03</td>
<td>.6499997 00</td>
</tr>
<tr>
<td>.6999997 00</td>
<td>.3739811 04</td>
<td>.6999997 00</td>
<td>.8717234 03</td>
<td>.6999997 00</td>
</tr>
<tr>
<td>.7499997 00</td>
<td>.2093485 05</td>
<td>.7499997 00</td>
<td>.3838604 04</td>
<td>.7499997 00</td>
</tr>
<tr>
<td>.7999996 00</td>
<td>.1681829 06</td>
<td>.7999996 00</td>
<td>.2234259 05</td>
<td>.7999996 00</td>
</tr>
<tr>
<td>00</td>
<td>.2492486 07</td>
<td>00</td>
<td>.2237622 06</td>
<td>00</td>
</tr>
<tr>
<td>.8499996 00</td>
<td>.3490269 08</td>
<td>.8499996 00</td>
<td>.5357029 07</td>
<td>.8499995 00</td>
</tr>
<tr>
<td>$\bar{u}$</td>
<td>$\bar{x}$</td>
<td>$\gamma = 1.40$</td>
<td>$\gamma = 1.45$</td>
<td>$\gamma = 1.50$</td>
</tr>
<tr>
<td>--------</td>
<td>--------</td>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>0.499999 -01</td>
<td>0.1591285 -02</td>
<td>0.499999 -01</td>
<td>0.156083 -02</td>
<td>0.499999 -01</td>
</tr>
<tr>
<td>0.999999 -01</td>
<td>0.8244324 -02</td>
<td>0.999999 -01</td>
<td>0.7915149 -02</td>
<td>0.999999 -01</td>
</tr>
<tr>
<td>1.499999 00</td>
<td>0.2449511 -01</td>
<td>1.499999 00</td>
<td>0.2297192 -01</td>
<td>1.499999 00</td>
</tr>
<tr>
<td>1.999999 00</td>
<td>0.5876464 -01</td>
<td>1.999999 00</td>
<td>0.5370585 -01</td>
<td>1.999999 00</td>
</tr>
<tr>
<td>2.499999 00</td>
<td>0.1269775 00</td>
<td>2.499999 00</td>
<td>0.1127912 00</td>
<td>2.499999 00</td>
</tr>
<tr>
<td>2.999999 00</td>
<td>0.2599961 00</td>
<td>2.999999 00</td>
<td>0.2238069 00</td>
<td>2.999999 00</td>
</tr>
<tr>
<td>3.499999 00</td>
<td>0.5195063 00</td>
<td>3.499999 00</td>
<td>0.4319213 00</td>
<td>3.499999 00</td>
</tr>
<tr>
<td>3.999999 00</td>
<td>0.1033560 01</td>
<td>3.999999 00</td>
<td>0.8268065 00</td>
<td>3.999999 00</td>
</tr>
<tr>
<td>4.499999 00</td>
<td>0.2080489 01</td>
<td>4.499999 00</td>
<td>0.1594347 01</td>
<td>4.499999 00</td>
</tr>
<tr>
<td>4.999999 00</td>
<td>0.4399993 01</td>
<td>4.999999 00</td>
<td>0.3140590 01</td>
<td>4.999999 00</td>
</tr>
<tr>
<td>5.499999 00</td>
<td>0.9266075 01</td>
<td>5.499999 00</td>
<td>0.6410974 01</td>
<td>5.499999 00</td>
</tr>
<tr>
<td>5.999999 00</td>
<td>0.2119212 02</td>
<td>5.999999 00</td>
<td>0.1378736 02</td>
<td>5.999999 00</td>
</tr>
<tr>
<td>6.499999 00</td>
<td>0.5261889 02</td>
<td>6.499999 00</td>
<td>0.318557 02</td>
<td>6.499999 00</td>
</tr>
<tr>
<td>6.999999 00</td>
<td>1.463515 03</td>
<td>6.999999 00</td>
<td>0.8167866 02</td>
<td>6.999999 00</td>
</tr>
<tr>
<td>7.499999 00</td>
<td>4.778956 03</td>
<td>7.499999 00</td>
<td>0.2416589 03</td>
<td>7.499999 00</td>
</tr>
<tr>
<td>7.999999 00</td>
<td>1.9792176 04</td>
<td>7.999999 00</td>
<td>0.8861613 03</td>
<td>7.999999 00</td>
</tr>
<tr>
<td>8.499999 00</td>
<td>1.1997599 05</td>
<td>8.499999 00</td>
<td>0.4587673 04</td>
<td>8.499999 00</td>
</tr>
<tr>
<td>8.999999 00</td>
<td>1.466625 06</td>
<td>8.999999 00</td>
<td>0.4484641 05</td>
<td>8.999999 00</td>
</tr>
<tr>
<td>( \gamma = 1.55 )</td>
<td>( \gamma = 1.60 )</td>
<td>( \gamma = 1.667 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( u )</td>
<td>( x )</td>
<td>( u )</td>
<td>( x )</td>
<td>( u )</td>
</tr>
<tr>
<td>0.49999999</td>
<td>0.1517344</td>
<td>0.49999999</td>
<td>0.1501424</td>
<td>0.49999999</td>
</tr>
<tr>
<td>0.99999999</td>
<td>0.7462089</td>
<td>0.99999999</td>
<td>0.7299598</td>
<td>0.99999999</td>
</tr>
<tr>
<td>0.14999999</td>
<td>0.2093886</td>
<td>0.14999999</td>
<td>0.202824</td>
<td>0.14999999</td>
</tr>
<tr>
<td>0.19999999</td>
<td>0.4717528</td>
<td>0.19999999</td>
<td>0.4495452</td>
<td>0.19999999</td>
</tr>
<tr>
<td>0.24999999</td>
<td>0.9512846</td>
<td>0.24999999</td>
<td>0.8929444</td>
<td>0.24999999</td>
</tr>
<tr>
<td>0.29999999</td>
<td>0.1804885</td>
<td>0.29999999</td>
<td>0.166538</td>
<td>0.29999999</td>
</tr>
<tr>
<td>0.34999999</td>
<td>0.3314926</td>
<td>0.34999999</td>
<td>0.300532</td>
<td>0.34999999</td>
</tr>
<tr>
<td>0.39999999</td>
<td>0.6006429</td>
<td>0.39999999</td>
<td>0.5336868</td>
<td>0.39999999</td>
</tr>
<tr>
<td>0.44999999</td>
<td>0.1089469</td>
<td>0.44999999</td>
<td>0.9464632</td>
<td>0.44999999</td>
</tr>
<tr>
<td>0.49999999</td>
<td>0.2003842</td>
<td>0.49999999</td>
<td>0.1697434</td>
<td>0.49999999</td>
</tr>
<tr>
<td>0.54999998</td>
<td>0.3785968</td>
<td>0.54999998</td>
<td>0.3116222</td>
<td>0.54999998</td>
</tr>
<tr>
<td>0.59999998</td>
<td>0.7455573</td>
<td>0.59999998</td>
<td>0.5941753</td>
<td>0.59999998</td>
</tr>
<tr>
<td>0.64999997</td>
<td>0.1558322</td>
<td>0.64999997</td>
<td>0.1196186</td>
<td>0.64999997</td>
</tr>
<tr>
<td>0.69999997</td>
<td>0.3543569</td>
<td>0.69999997</td>
<td>0.2602975</td>
<td>0.69999997</td>
</tr>
<tr>
<td>0.74999997</td>
<td>0.9094905</td>
<td>0.74999997</td>
<td>0.6336948</td>
<td>0.74999997</td>
</tr>
<tr>
<td>0.79999996</td>
<td>0.2797346</td>
<td>0.79999996</td>
<td>0.1825746</td>
<td>0.79999996</td>
</tr>
<tr>
<td>0.84999996</td>
<td>0.1152375</td>
<td>0.84999996</td>
<td>0.6906896</td>
<td>0.84999996</td>
</tr>
<tr>
<td>0.89999995</td>
<td>0.8146018</td>
<td>0.89999995</td>
<td>0.4325421</td>
<td>0.89999995</td>
</tr>
</tbody>
</table>
TABLE 4
(Ref. 61)

DRIVER GAS PARAMETERS FOR HIGH-PRESSURE,
HIGH-TEMPERATURE, HYDROGEN AND HELIUM

<table>
<thead>
<tr>
<th>Case</th>
<th>Gas</th>
<th>T₄ deg K</th>
<th>S cal/gm deg</th>
<th>γ₄</th>
<th>a₄ km/sec</th>
<th>β km/sec</th>
<th>ρ km/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>H₂</td>
<td>12,000</td>
<td>27.0</td>
<td>1.57</td>
<td>11.49</td>
<td>99.4</td>
<td>7.32</td>
</tr>
<tr>
<td>2</td>
<td>H₂</td>
<td>10,000</td>
<td>25.2</td>
<td>1.34</td>
<td>9.09</td>
<td>96.0</td>
<td>6.78</td>
</tr>
<tr>
<td>3</td>
<td>H₂</td>
<td>8,000</td>
<td>22.5</td>
<td>1.25</td>
<td>7.30</td>
<td>91.3</td>
<td>5.84</td>
</tr>
<tr>
<td>4</td>
<td>H₂</td>
<td>6,200</td>
<td>19.9</td>
<td>1.23</td>
<td>5.88</td>
<td>77.4</td>
<td>4.78</td>
</tr>
<tr>
<td>5</td>
<td>H₂</td>
<td>4,700</td>
<td>17.9</td>
<td>1.24</td>
<td>5.06</td>
<td>64.8</td>
<td>4.07</td>
</tr>
<tr>
<td>6</td>
<td>He</td>
<td>12,000</td>
<td>----</td>
<td>1.67</td>
<td>6.44</td>
<td>53.7</td>
<td>3.86</td>
</tr>
<tr>
<td>7</td>
<td>He</td>
<td>10,000</td>
<td>----</td>
<td>1.67</td>
<td>3.88</td>
<td>49.0</td>
<td>3.52</td>
</tr>
<tr>
<td>8</td>
<td>He</td>
<td>8,000</td>
<td>----</td>
<td>1.67</td>
<td>5.26</td>
<td>43.8</td>
<td>3.15</td>
</tr>
<tr>
<td>9</td>
<td>He</td>
<td>6,200</td>
<td>----</td>
<td>1.67</td>
<td>4.63</td>
<td>38.6</td>
<td>2.77</td>
</tr>
<tr>
<td>10</td>
<td>He</td>
<td>4,700</td>
<td>----</td>
<td>1.67</td>
<td>4.03</td>
<td>33.6</td>
<td>2.41</td>
</tr>
</tbody>
</table>
TABLE 5
(Ref. 61)
MUZZLE VELOCITIES FOR DRIVERS SHOWN IN TABLE 4

<table>
<thead>
<tr>
<th>p_4 Ax/m</th>
<th>2 x 10^{11} cm^2/sec^2</th>
<th>1.15 x 10^{12} cm^2/sec^2</th>
<th>3 x 10^{12} cm^2/sec^2</th>
<th>5 x 10^{12} cm^2/sec^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>u_3 km/sec</td>
<td>1.1u_3 km/sec</td>
<td>u_3 km/sec</td>
<td>1.1u_3 km/sec</td>
</tr>
<tr>
<td>1</td>
<td>4.95</td>
<td>5.45</td>
<td>9.26</td>
<td>10.2</td>
</tr>
<tr>
<td>2</td>
<td>4.89</td>
<td>5.38</td>
<td>9.10</td>
<td>10.0</td>
</tr>
<tr>
<td>3</td>
<td>4.73</td>
<td>5.20</td>
<td>8.66</td>
<td>9.53</td>
</tr>
<tr>
<td>4</td>
<td>4.51</td>
<td>4.96</td>
<td>8.00</td>
<td>8.80</td>
</tr>
<tr>
<td>5</td>
<td>4.31</td>
<td>4.74</td>
<td>7.46</td>
<td>8.21</td>
</tr>
<tr>
<td>7</td>
<td>4.06</td>
<td>4.47</td>
<td>6.67</td>
<td>7.34</td>
</tr>
<tr>
<td>8</td>
<td>3.90</td>
<td>4.29</td>
<td>6.29</td>
<td>6.92</td>
</tr>
<tr>
<td>9</td>
<td>3.71</td>
<td>4.08</td>
<td>5.86</td>
<td>6.45</td>
</tr>
<tr>
<td>10</td>
<td>3.51</td>
<td>3.86</td>
<td>5.40</td>
<td>5.94</td>
</tr>
</tbody>
</table>

Note: $p_4 = 10^5$ psi, $A = 0.266 \text{ cm}^2$, $x = 134 \text{ cm}$, $m = 0.211 \text{ gm}$, $P_4 \frac{Ax}{m} = 1.15 \times 10^{12} \text{ cm}^2/\text{sec}^2$. 
FIG. 1  IDEALIZED LAUNCHER HAVING A CONSTANT BASE PRESSURE TO DRIVE THE MODEL

FIG. 2  VARIATION OF MUZZLE VELOCITY (u) WITH LAUNCHER LENGTH (L/d) FOR ONE CALIBRE CYLINDRICAL MODELS OF CONSTANT DENSITY AND AT A CONSTANT BASE PRESSURE
FIG. 3  THE WAVE SYSTEM IN A SIMPLE SHOCK TUBE
FIG. 4 A SIMPLE LAUNCHER
FIG. 5 TEMPERATURE VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN AIR
FIG. 6  SOUND SPEED VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN AIR

Equilibrium

Nitrogen Recombination

Frozen

Perfect

\( a (\text{ft.} / \text{sec.}) \times 10^{-3} \)

\( p (\text{lb.} / \text{ft.}^2) \)
FIG. 7 DENSITY VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN AIR

The graph shows the relationship between density (\(\rho\)) and pressure (\(p\)) in air, with logarithmic scales on both axes. The graph includes three lines representing different states:

- **Equilibrium**: The most gradual line, indicating a state of equilibrium where the density changes with pressure at a slower rate.
- **Perfect Gas**: A line showing a steeper increase in density with pressure, characteristic of a perfect gas.
- **Frozen**: A line that remains nearly constant, representing a state where the density remains relatively unchanged with pressure.

The pressure range is indicated as \(10^{-1} \text{ lb. / ft.}^2 \) to \(10^{4} \text{ lb. / ft.}^2\). The density values are given in slugs/ft.\(^3\).
FIG. 8 ACOUSTIC IMPEDANCE VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN AIR

\[ u = - \frac{1}{\rho_a} \int_{p_4}^P \frac{1}{p} \, dp \text{ (Eq. 13)} \]
FIG. 9 PARTICLE VELOCITY VERSUS PRESSURE THROUGH A NONSTATIONARY EXPANSION WAVE IN AIR

- Frozen
- Perfect
- One-Dimensional Steady Flow
- Expansion Wave - Equilibrium

Equilibrium

$u (\text{ft./sec.}) \times 10^{-3}$

$p (\text{lb.}/\text{ft.}^2)$

$p_4 = 10,540 \text{ lb.}/\text{ft.}^2$
FIG. 10 PARTICLE SPEED VERUS SOUND SPEED THROUGH A NONSTATIONARY EXPANSION WAVE IN AIR

Equilibrium

Frozen

Perfect
FIG. 11 VARIATION OF DENSITY ($\rho_2/\rho_1$), TEMPERATURE ($T_2/T_1$), PRESSURE ($p_2/p_1$) AND ELECTRON NUMBER DENSITY ($N_e$) WITH NORMAL SHOCK WAVE MACH NUMBER ($M_s$) IN AIR (CHEMICAL EQUILIBRIUM, $T_1 = 294^\circ$K, Ref. 21)
FIG. 12 VARIATION OF a) DEGREE OF IONIZATION ($x$), b) PRESSURE RATIO ($p_2/p_1$), c) TEMPERATURE RATIO ($T_2/T_1$), AND d) DENSITY RATIO ($\rho_2/\rho_1$) BEHIND NORMAL SHOCK WAVES ($M_s$) IN HELIUM (CHEMICAL EQUILIBRIUM, $T_1 = 300^\circ$K, REF. 22)
FIG. 12 b)

\[ p_1 \text{ mm Hg} \]

\[ \frac{p_2}{p_1} \]

\[ M_s \]

\[ \gamma = 5/3 \]
FIG. 12 c)

\[ \gamma = \frac{5}{3} \]

\[ p_1 \text{ mm Hg} \]

\[ T_2/T_1 \]

\[ M_s \]
FIG. 13 VARIATION OF a) COMPRESSIBILITY FACTOR ($Z$), b) PRESSURE RATIO ($p_2/p_1$), c) TEMPERATURE RATIO ($T_2/T_1$), AND d) DENSITY RATIO ($\rho_2/\rho_1$) BEHIND NORMAL SHOCK WAVES ($M_s$) IN HYDROGEN (CHEMICAL EQUILIBRIUM, $T_1 = 300^\circ$K, REF. 25)
FIG. 13 d

$p_1$ mm Hg

$\gamma = 7/5$
FIG. 14  IDEAL INTERNAL BALLISTICS EQUATION FOR A 1.0 GM PROJECTILE TO ATTAIN A MUZZLE VELOCITY OF 20,000 FT/SEC IN A 0.30 CALIBRE HELIUM GUN (Ref. 30)
FIG. 15  IDEAL INTERNAL BALLISTICS EQUATION FOR A 1.0 GM PROJECTILE IN A 0.30 CAL. HELIUM GAS GUN DRIVEN AT VARYING PRESSURES AND TEMPERATURES (Ref. 30)
FIG. 16  DIMENSIONLESS VELOCITY ($\hat{u}$) AND TIME ($\hat{t}$) VERSUS PROJECTILE POSITION ($\hat{x}$) (Ref. 31)
FIG. 17  DIMENSIONLESS VELOCITY ($\hat{u}$) VERSUS PROJECTILE POSITION ($\hat{x}$) (Ref. 27)
Enlargement of Fig. 18 (Sheet 1)

IDEALIZED INTERNAL BALLISTICS EQ’N

\[
\bar{x} = \frac{1}{\alpha - 1} \left[ \bar{u} - \frac{1}{\alpha} \right] + \frac{1}{\alpha} \\
\bar{x} = \frac{A_4 x}{m \hat{u}^2} \quad \bar{u} = \frac{u_3}{\hat{u}} \\
\hat{u} = \frac{2\alpha_4}{\chi_4 - 1}
\]
FIG. 19  CARPET PLOT OF MUZZLE VELOCITY \(u_3\) AND NONDIMENSIONAL LAUNCHER LENGTH \(\chi\) VERSUS DRIVER SOUND SPEED \(a_4\) AND PRESSURE \(p_4\)

(Ref. 32)
FIG. 20  NONSTATIONARY FLOW SOLUTION FOR LAUNCHER VELOCITY \( u_3 \) VERSUS DRIVER SOUND SPEED \( a_4 \) USING THE CONSTANT BASE PRESSURE (KINEMATIC SOLUTION) MAXIMUM VELOCITY \( u_{\text{max}} \) AS A NON-DIMENSIONALIZING PARAMETER (NO CHAMBERAGE, INFINITE CHAMBER LENGTH, \( 1 \leq \gamma \leq 5/3 \), Ref. 33)
FIG. 21 FORMATION OF A SHOCK WAVE AHEAD OF AN ACCELERATING PROJECTILE
FIG. 22  EFFECT OF A CONSTANT COUNTERPRESSURE (AIR, \( p_1 = 1 \text{ ATM} \)) AND FRICTIONAL FORCE (200 Lb) ON PROJECTILES DRIVEN BY AIR AT DIFFERENT PRESSURES (\( p_4 \)) (Ref. 34a)
FIG. 23  EFFECT OF A CONSTANT COUNTERPRESSURE (AIR, $p_1 = 1$ ATM) AND FRICTIONAL FORCE (200 LB) ON PROJECTILES DRIVEN BY HELIUM AT DIFFERENT PRESSURES ($p_4$) (Ref. 34a)
FIG. 24  APPROXIMATE SOLUTION FOR HYPERVELOCITY LAUNCHER MUZZLE VELOCITIES WITH COUNTER- PRESSURE (Ref. 33)

FOR ALL VALUES OF $D_D/D_i$, $\gamma_4$, $\gamma_1$, AND FOR $X_4 = 0$

FOR CASE WHEN $U_p$, APPROACHES $U_L$

SEE TEXT.
FIG. 25  PISTON PATH IN A GUN TUNNEL OBTAINED FROM MICROWAVE TRACKING (Ref. 35)

AIR - 20°C
\( \rho_4 = 200 \text{at.} \)
\( \rho_1 = 1 \text{at.} \)
PISTON WEIGHT = 100g., 3 IN. DIA.
FRINGE SEPARATION = 1.019"
FIG. 26 NONDIMENSIONAL PLOT OF PISTON VELOCITY AS A FUNCTION OF TIME (Ref. 35) (See Fig. 25)

\( \xi = \frac{x}{20}, \tau = \frac{1126 t}{20}, U_p = \frac{u}{1126}, \ddot{\xi} = \frac{g^2}{\alpha^2} \xi / dt^2 = 21.5 \)

OR \( \alpha / g = 21.5(1126)^2 / 32.2 \times 20 = 42.5 \times 10^3 \)
VELOCITY DISTRIBUTION THROUGH THE SUBSONIC REGION OF A RAREFACTION WAVE WITHOUT ($\gamma = 1.4$) AND WITH BOUNDARY LAYER EFFECTS ($N_2$, $\gamma = 1.4$, $t = 15$ msec, $p_1 = 0.5$ AND 0.1 atm., 3.5 cm. DIA. SHOCK TUBE, REF. 42)
FIG. 28  COORDINATE AXES FOR A MOVING AND STATIONARY SHOCK WAVE OR EXPANSION WAVE IN A SHOCK TUBE
FIG. 29 LIMITING BOUNDARY LAYER PROFILES IN A SHOCK TUBE IN AIR; \( \gamma = 1.4 \)
FIG. 30 VELOCITY AND TEMPERATURE PROFILES IN THE BOUNDARY LAYER BEHIND A MOVING SHOCK WAVE (REFERENCE AXES FIXED TO SHOCK)
FIG. 31 LAMINAR BOUNDARY LAYER VELOCITY PROFILES FOR A MOVING WALL IN THE RANGE OF WALL TO FREE STREAM VELOCITY RATIOS OF $0 \leq V_1/V_2 \leq 6$ (AFTER REF'S. 47 AND 48)
FIG. 32 \( P_{21} = 2.75 \) \( \frac{P_{2}}{P_{1}}\) \( P_{21} = 7 \) \( P_{21} = 3.05 \) \( P_{21} = 6.66 \) \( P_{21} = 7.0 \) \( P_{21} = 7.3 \)

\[ \eta = \frac{y}{x} \sqrt{Re} \quad Re = \frac{u_2 x}{y_2} \]

COMPARISON OF EXPERIMENTAL (INTERFEROMETRIC) AND THEORETICAL LAMINAR BOUNDARY LAYER VELOCITY PROFILES ON A SHOCK TUBE WALL (REF. 51)
FIG. 33  GLASS WALL SURFACE TEMPERATURE RISE FOR LAMINAR BOUNDARY LAYER BEHIND SHOCK WAVE  (REF. 54)
FIG. 34  LAMINAR HEAT TRANSFER PARAMETER ($St/\sqrt{Re}$)
BEHIND A MOVING SHOCK WAVE ($M_s$) (REF. 54)
ILLUSTRATION OF THE MEASUREMENT OF BOUNDARY LAYER TRANSITION BEHIND A MOVING SHOCK WAVE WITH A THIN FILM HEAT GAUGE
Fig. 36  Boundary layer transition correlation in terms of transition Reynolds number \( \text{Re}_T = \frac{u_2^2}{\nu} \frac{\nu}{\nu_2} \) (Ref. 54)
FIG. 37  BOUNDARY LAYER TRANSITION CORRELATION IN TERMS OF BOUNDARY LAYER THICKNESS (REF. 54)
FIG. 38 ILLUSTRATION OF BOUNDARY LAYER TRANSITION BEHIND SHOCK AND RAREFACTION WAVES IN A SHOCK-TUBE FLOW (Ref. 56a)

- x x x - transition in hot gas
- • • • - transition with stronger rarefaction waves
- Δ Δ Δ - transition in cold gas (weak rarefaction waves)
- x₁ - observation point

Wall Temperature Records

x₁ - observation point
FIG. 39  THEORETICAL AND EXPERIMENTAL VELOCITY PROFILES
(u/u₂) IN A TURBULENT BOUNDARY LAYER (y/δ*) BEHIND
A PLANE SHOCK WAVE IN A 2 x 7 in. SHOCK TUBE;
p₂/p₁ = 8.03, p₁ = 120 mmHg, T₁ = 533°C, w₁/a₁ = 2.65,
M₂ = 1.65, u₂/a₁ = 1.88 (Ref. 57)
FIG. 40 THEORETICAL AND EXPERIMENTAL VARIATION OF THE BOUNDARY LAYER THICKNESS ($\delta$), DISPLACEMENT THICKNESS ($\delta^M$), AND MOMENTUM THICKNESS ($\theta$) WITH THE DISTANCE BEHIND THE SHOCK WAVE ($X_s$) AND THE REYNOLDS NUMBER ($Re_x$) IN A 2 x 7 in. SHOCK TUBE: $p_2/p_1 = 8.03$, $p_1 = 120$ mmHg, $T_1 = 533^\circ$R, $w_1/a_1 = 2.65$, $M_2 = 1.25$, $u_2/a_1 = 1.88$ (Ref. 57)
FIG. 41 VARIATION OF THE LOCAL SKIN FRICTION COEFFICIENT \( (C_f) \) (FROM INTERFEROMETRIC MEASUREMENTS) WITH REYNOLDS NUMBER \( (Re_X) \) FOR TURBULENT BOUNDARY LAYER BEHIND A PLANE SHOCK WAVE (Ref. 57)
FIG. 42  TURBULENT BOUNDARY LAYER HEAT TRANSFER BEHIND MOVING NORMAL SHOCK WAVES ($M_s$) (Ref. 54)
FIG. 43  EMPIRICAL REDUCTION IN MUZZLE VELOCITY AS A RESULT OF BOUNDARY LAYER EFFECTS (Ref. 33)
FIG. 44  VALUES OF THE ISENTROPIC INDEX ($\gamma$) ALONG ISENTROPES IN THE EXPANSION OF HIGH-PRESSURE HIGH-TEMPERATURE HYDROGEN (Ref. 61)
FIG. 45 VARIATION OF DRIVER EFFICIENCY ($\eta$) WITH MUZZLE MACH NUMBER ($\tilde{u}$) (Ref. 61)
FIG. 46  VARIATION OF DRIVER EFFICIENCY ($\eta$) WITH THE NONDIMENSIONAL MUZZLE VELOCITY ($u_3/\xi$) (Ref. 61)
FIG. 47  VARIATION OF DRIVER EFFICIENCY ($\eta$) WITH THE NONDIMENSIONAL MUZZLE VELOCITY ($u_3/\varphi$)
FIG. 48 VARIATION OF THE THEORETICAL MUZZLE VELOCITY

$(1.1 u_3)$ WITH THE OPTIMIZING LAUNCHER PARAMETERS

- $(a)$ (Ref. 61) AND $\xi$
- $(b)$
FIG. 49  A MODIFIED MOLLIER DIAGRAM, INTERNAL ENERGY VS ENTROPY, FOR HYDROGEN IN CHEMICAL EQUILIBRIUM (Ref. 61)
FIG. 50  PRESSURE VS ENTROPY FOR HYDROGEN IN CHEMICAL EQUILIBRIUM (Ref. 61)
FIG. 51a) METHOD OF FINDING THE STATE OF THE DRIVER GAS AFTER ENERGY ADDITION AND AFTER THE NON-STATIONARY EXPANSION (Ref. 61)
MOLECULAR WEIGHT OF A HYDROGEN PLASMA

EQUILIBRIUM SOUND SPEED IN A HYDROGEN PLASMA

EQUILIBRIUM SPECIFIC HEAT RATIO OF A HYDROGEN PLASMA

FIG. 51 b) PROPERTIES OF HYDROGEN AT LOW PRESSURES AND HIGH TEMPERATURES (Ref. 26b)
FIG. 52 a) PREDICTED AND ACTUAL VELOCITY-DISPLACEMENT CURVES FOR TWO NRL LIGHT-GAS GUNS (Ref. 62)
CORRELATION FORMULA:

\[
\frac{u_3}{(u_3)_{\text{calc}}} = 1 - (c_1 + c_2 + c_3)
\]

FIG. 52b) CORRELATION OF EXPERIMENTAL RESULTS
FOR AN AEDC 0.5 in. dia., 230 CALIBRES,
TWO-STAGE LAUNCHER (ROUND MASSES 1 to 3.5 gm, VELOCITIES 11,800 to 26,700 ft/sec).
(Ref. 32b)