Some Notes on the Performance of Small High Speed Wind Tunnels

-by-

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SUMMARY

There is a need for high speed wind tunnels of modest dimensions and costs in establishments teaching the fundamentals of compressible flow. This paper discusses briefly the layouts and performances of various types of tunnel which might be suitable for colleges and universities. The subject is still in its infancy and this report should not be regarded as more than an attempt at reviewing the present position.
### 1. Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>velocity of sound</td>
</tr>
<tr>
<td>$C_p$</td>
<td>specific heat of dry air at constant pressure</td>
</tr>
<tr>
<td>$C_v$</td>
<td>specific heat of dry air at constant volume</td>
</tr>
<tr>
<td>$J$</td>
<td>mechanical equivalent of heat</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number</td>
</tr>
<tr>
<td>$m$</td>
<td>mass flow ratio</td>
</tr>
<tr>
<td>$N$</td>
<td>r.p.m. of vacuum pump</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure</td>
</tr>
<tr>
<td>$R$</td>
<td>gas constant in equation of state</td>
</tr>
<tr>
<td>$S$</td>
<td>working section area</td>
</tr>
<tr>
<td>$s$</td>
<td>number of stages in air compressor</td>
</tr>
<tr>
<td>$t$</td>
<td>time</td>
</tr>
<tr>
<td>$T$</td>
<td>absolute temperature</td>
</tr>
<tr>
<td>$U$</td>
<td>velocity</td>
</tr>
<tr>
<td>$V$</td>
<td>volume of storage tank</td>
</tr>
<tr>
<td>$V_s$</td>
<td>swept volume per stroke in vacuum pump</td>
</tr>
<tr>
<td>$\eta_d$</td>
<td>diffuser efficiency</td>
</tr>
<tr>
<td>$\eta$</td>
<td>efficiency of pump</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
</tr>
<tr>
<td>$\mu$</td>
<td>mass flow in unit time</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>ratio of specific heats</td>
</tr>
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</table>

### Suffices

<table>
<thead>
<tr>
<th>Suffices</th>
<th>Definition</th>
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<tbody>
<tr>
<td>a</td>
<td>refers to atmospheric conditions</td>
</tr>
<tr>
<td>b</td>
<td>refers to stagnation conditions in the settling chamber of a straight-through tunnel</td>
</tr>
<tr>
<td>o</td>
<td>refers to stagnation conditions upstream of working-section</td>
</tr>
<tr>
<td>R</td>
<td>refers to stagnation conditions in reservoir</td>
</tr>
</tbody>
</table>

### Introduction

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2. "Introduction"
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The various types of high speed wind tunnel can be classified as follows:

(a) Continuous flow
   (i) Direct drive return flow
   (ii) Direct flow 'straight-through' tunnel
   (iii) Induction tunnel return flow
   (iv) Induction 'straight-through' tunnel

(b) Intermittent flow
   (i) 'Straight-through' with high pressure reservoir at entry
   (ii) 'Straight-through' with vacuum tank at exit
   (iii) Induction 'straight-through' tunnel

(c) As (a) and (b) but using gases other than air (e.g., superheated steam).

Since we are considering tunnels likely to be within the resources of colleges and universities the schemes considered will be limited to those using less than 100 b.h.p. The calculations have been confined to tunnels having working sections of dimensions 2in. x 2in.; the value of smaller tunnels for research and demonstration purposes is likely to be severely limited. The calculations can, however, be readily generalised to tunnels of other sizes.

In estimating the performance of the different designs it will be assumed that

(i) the shape of the nozzle (effuser) has been correctly designed, due allowance having been made for boundary layer thickness;

(ii) the subsonic diffuser has an efficiency above 75 per cent.

(iii) the kinetic energy at the diffuser exit is small enough to be neglected when compared with that at the working-section.

3. Continuous tunnels

3.1 Direct-drive return flow

A sketch of a typical tunnel layout is shown in Fig.1. The tunnel is driven by an electric motor through a speed-up gear box to an axial or centrifugal compressor. The former is preferable owing to the simplified geometry of the ducting.

A cooler is required to remove the heat of compression but may at the same time serve as an efficient honeycomb. The working section has glass sides for flow observation (schlieren or shadowgraph).
These sides are hinged or made easily detachable, in order to facilitate changing the supersonic liners.

Table I gives the pressure ratio and approximate horse-power required by a tunnel with 2 in. sq. working section over a range of Mach numbers. The results are plotted in Figs. 8 and 9. The difficulty involved in matching a compressor to drive the tunnel efficiently over a wide range of Mach number is apparent.

3.2 Direct flow 'straight-through' tunnel

A diagrammatic layout of a typical form of this tunnel is shown in Fig.2. The tunnel is supplied with air from a reciprocating compressor via an after cooler, oil separator and reservoir.

The main advantage of this scheme over the return flow tunnel is that the performance of the compressor need not necessarily be closely matched to that of the tunnel. The compressor discharge pressure must, however, be greater than the minimum operating pressure \(p_{\text{min}}\), given in Table 2, and the air flow \(W \text{ lb/sec}\) must also equal at least the values quoted. The electric motor b.h.p. will be greater, for a corresponding Mach number, than the value quoted in Table I as the compressor efficiency will be lower.

3.3 Induction, continuous, return flow tunnel

A typical layout is shown in Fig.3. The main advantage over scheme 3.1 is that the compressor and tunnel characteristics need not be closely matched, but this is gained at the expense of extra power required by the compressor. In general the induction principle is applied when relatively small supplies of high pressure air or gas are available.

A disadvantage is that Mach numbers above 2.0 have as yet not been attained when the induction principle is used, although further development and research may help to raise the limit. The remarks in the following paragraph relating to ejector slots apply here.

3.4 Induction, continuous, straight-through tunnel

The scheme is shown in Fig.4. The important design feature of this tunnel is that of the ejector slot. To obtain maximum operating efficiency at any given Mach number the ratio of ejector slot area to working section area must be matched to the required blowing pressure. The ejector slot is usually designed to give at the slot a Mach number of unity. This condition is dependent only on the blowing pressure and is independent of tunnel operating conditions.

Typical design curves can be obtained from Reference 1, while Table 3 gives the quantity of high pressure air required by a 2 in. sq. tunnel when the blowing pressure is 100 lb./sq.in. above atmospheric.

Recently this type of tunnel has been driven by a jet engine of 5,000 lb. static thrust placed downstream of the working section. With this arrangement, Mach numbers up to 0.9 can be obtained with working section areas of about 10 sq.ft.

4. Intermittent Tunnels

4.1 'Straight-through' intermittent with high pressure reservoir

A diagrammatic layout of the tunnel is shown in Fig.5.

Table 4 gives the size of reservoir required for a 2 in. sq. working section when \(p_R\) is 100 lb./sq.in. absolute and \(p_b\) is equal
to $p_{\text{b min}}$ (see Table 2). The time of running is 30 secs.

4.2 'Straight-through' intermittent with vacuum tank at exit

A diagrammatic layout of the tunnel is shown in Fig. 6.

In order to run the tunnel at constant subsonic speeds, a throat at the end of the working-section must be fitted. It is essential for efficient tunnel operation to use a quick acting valve with a minimum time lag. The main advantage of this scheme is that Mach numbers up to 4.5 can easily be obtained provided that efficient vacuum pumps are used.

Table 5 gives the volume $V$ of the vacuum tank for a 2in. sq. working-section when the time of running is 30 secs. The initial pressure in the vacuum tank is assumed to be 0.44 lb./sq.in. absolute.

4.3 'Straight-through' intermittent induction tunnel

The layout of this tunnel is similar to that shown in Fig. 4 except that a storage cylinder must be provided between the compressor and ejector box.

Table 6 gives the necessary storage volume $V$ for a 2in. sq. tunnel running for 30 secs. The initial reservoir pressure is 100 lb./sq.in. and the constant blowing pressure 55 lb./sq.in.

The advantage of using the induction type intermittent tunnel will be seen to lie in the relatively small volume of reservoir required.

5 Continuous and intermittent tunnels using gases other than air

5.1 Tunnels in which the gas in the working-section is not air

A considerable saving in power could be obtained by using a gas lighter than air, but having the same value of $\gamma$. However, this advantage is offset by the added complications of the operating plant. Most schemes considered have been concerned with gases such as freon that have a much smaller speed of sound than air, and thus a given Mach number is obtained using less power. For freon, however, the value of $\gamma$ is smaller than that for air.

5.2 Induction tunnels using steam in place of high pressure compressed air as the ejecting fluid

This type of tunnel is similar to that shown in Fig. 4 except that superheated steam is supplied to the ejector box in place of the compressed air. This scheme offers distinct advantages in establishments where a steam boiler plant is already in existence.

Typical operating characteristics can be obtained from Ref. 2.

6. Conclusions

The performance of a variety of small high speed wind tunnels have been discussed with brief references to the advantages and disadvantages of each.

A number of simple calculations, needed for the preliminary design of such tunnels, are described briefly in the Appendix.

/REFERENCES...
### REFERENCES

<table>
<thead>
<tr>
<th>Number</th>
<th>Author</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Lilley, G.M. and Holder, D.W.</td>
<td>Experiments on an induction type high speed tunnel driven by low pressure steam. October, 1948. (To be published shortly in the College of Aeronautics series).</td>
</tr>
</tbody>
</table>
TABLE I

Performance of a 2in sq. direct-drive return flow tunnel

<table>
<thead>
<tr>
<th>Mach number (working section)</th>
<th>$P_4$</th>
<th>Mass flow W lb/sec.</th>
<th>Adiabatic h.p.</th>
<th>Actual h.p. required (approx.)</th>
</tr>
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<tr>
<td>M.</td>
<td>$P_3$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>1.03</td>
<td>1.03</td>
<td>1.52</td>
<td>10.0</td>
</tr>
<tr>
<td>0.7</td>
<td>1.07</td>
<td>1.25</td>
<td>3.72</td>
<td>18.5</td>
</tr>
<tr>
<td>0.9</td>
<td>1.27</td>
<td>1.36</td>
<td>8.65</td>
<td>38.5</td>
</tr>
<tr>
<td>1.0</td>
<td>1.28</td>
<td>1.375</td>
<td>19.60</td>
<td>78.5</td>
</tr>
<tr>
<td>1.25</td>
<td>1.29</td>
<td>1.315</td>
<td>19.0</td>
<td>78.5</td>
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<tr>
<td>1.50</td>
<td>1.58</td>
<td>1.17</td>
<td>23.50</td>
<td>78.5</td>
</tr>
<tr>
<td>2.00</td>
<td>1.99</td>
<td>0.81</td>
<td>30.8</td>
<td>47.5</td>
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<tr>
<td>2.50</td>
<td>2.69</td>
<td>0.52</td>
<td>32.4</td>
<td>47.5</td>
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<tr>
<td>3.00</td>
<td>4.46</td>
<td>0.325</td>
<td>30.4</td>
<td>43.5</td>
</tr>
<tr>
<td>4.00</td>
<td>10.95</td>
<td>0.13</td>
<td>22.5</td>
<td>43.5</td>
</tr>
</tbody>
</table>

$P_3$ is the static pressure upstream of the compressor.

$P_4$ is the static pressure downstream of the compressor.

Notes: Mass flow calculated for atmospheric stagnation pressure and temperature upstream of working-section.
The actual horse-power required is based on average compressor efficiencies for a compressor rated at a pressure ratio of 4:1.
TABLE 2
Continuous direct flow, 2in. sq, 'straight-through' tunnel

<table>
<thead>
<tr>
<th>Working-section Mach number M.</th>
<th>Blowing pressure $P_b$ min. lb/sq.in. absolute</th>
<th>Mass Flow $M$ lb/sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>15.15</td>
<td>1.06</td>
</tr>
<tr>
<td>0.7</td>
<td>15.75</td>
<td>1.35</td>
</tr>
<tr>
<td>0.9</td>
<td>18.65</td>
<td>1.725</td>
</tr>
<tr>
<td>1.0</td>
<td>18.8</td>
<td>1.76</td>
</tr>
<tr>
<td>1.25</td>
<td>18.95</td>
<td>1.695</td>
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<tr>
<td>1.50</td>
<td>23.2</td>
<td>1.845</td>
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<td>2.0</td>
<td>29.25</td>
<td>1.61</td>
</tr>
<tr>
<td>2.5</td>
<td>42.5</td>
<td>1.505</td>
</tr>
<tr>
<td>3.0</td>
<td>65.6</td>
<td>1.45</td>
</tr>
<tr>
<td>4.0</td>
<td>161.0</td>
<td>1.425</td>
</tr>
</tbody>
</table>

TABLE 3
Continuous induction 'straight-through' tunnel

Working-section 2.0 in. sq. Blowing pressure 100 lb./sq.in. above atmospheric.

<table>
<thead>
<tr>
<th>Working-section Mach number M.</th>
<th>Area Ejector Slot Area Working-section</th>
<th>Mass flow ratio m</th>
<th>High pressure air required lb./sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.006</td>
<td>15.5</td>
<td>0.07</td>
</tr>
<tr>
<td>0.7</td>
<td>0.009</td>
<td>12.5</td>
<td>0.10</td>
</tr>
<tr>
<td>0.9</td>
<td>0.012</td>
<td>9.5</td>
<td>0.14</td>
</tr>
<tr>
<td>1.4</td>
<td>0.037</td>
<td>3.0</td>
<td>0.41</td>
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</table>
**TABLE 4**

'Straight-through' intermittent tunnel with high pressure reservoir at entry. Working-section 2.0 in.sq. Initial reservoir pressure 100 lb./sq.in. absolute, Time of running 30.0 secs.

<table>
<thead>
<tr>
<th>Working-section Mach number M.</th>
<th>Blowing pressure $P_b$ min lb/sq.in. absolute</th>
<th>Volume of Receiver V cu.ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>15.15</td>
<td>89.0</td>
</tr>
<tr>
<td>0.7</td>
<td>15.75</td>
<td>114.0</td>
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<tr>
<td>0.9</td>
<td>18.65</td>
<td>151.0</td>
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<tr>
<td>1.0</td>
<td>18.8</td>
<td>154.5</td>
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<tr>
<td>1.25</td>
<td>18.95</td>
<td>149.0</td>
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<td>1.50</td>
<td>23.2</td>
<td>171.0</td>
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<td>2.00</td>
<td>29.25</td>
<td>161.0</td>
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<td>2.50</td>
<td>42.5</td>
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<td>264.0</td>
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<tr>
<td>4.00</td>
<td>161.0</td>
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</table>

**TABLE 5**

'Straight-through' intermittent tunnel with vacuum tank at exit

Working-section 2in.sq. Time of running 30 secs. Initial pressure in tank is 0.44 lb./sq.in. absolute.

<table>
<thead>
<tr>
<th>Working-section Mach number M.</th>
<th>Volume of tank V cu.ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>600.0</td>
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<tr>
<td>0.7</td>
<td>765.0</td>
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<td>0.9</td>
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<td>1.50</td>
<td>1065.0</td>
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<tr>
<td>2.0</td>
<td>940.0</td>
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<tr>
<td>2.5</td>
<td>903.0</td>
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<tr>
<td>3.0</td>
<td>920.0</td>
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<tr>
<td>4.0</td>
<td>808.0</td>
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</table>
TABLE 6

'Straight-through' intermittent induction tunnel

Working-section 2in. sq. Time of running 30 secs. Initial reservoir pressure 100 lb./sq.in. above atmospheric. Constant blowing pressure \( P_b = 55 \text{ lb./sq.in. absolute.} \)

<table>
<thead>
<tr>
<th>Working section Mach number M</th>
<th>Area Ejector Slot</th>
<th>Mass Flow ratio m.</th>
<th>Volume of reservoir V. cu.ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.013</td>
<td>14.0</td>
<td>9.6</td>
</tr>
<tr>
<td>0.7</td>
<td>0.022</td>
<td>11.0</td>
<td>14.8</td>
</tr>
<tr>
<td>0.9</td>
<td>0.030</td>
<td>8.5</td>
<td>20.9</td>
</tr>
<tr>
<td>1.4</td>
<td>0.080</td>
<td>3.0</td>
<td>52.5</td>
</tr>
</tbody>
</table>
APPENDIX

1. The mass flow through the working-section

The mass flow in unit time through the working-section is,

$$ \mu = pSU $$  \hspace{1cm} \text{A 1.1} $$

The isentropic change of pressure from the settling chamber to the working-section is given by,

$$ \frac{P_0}{p} = \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\frac{\gamma}{\gamma-1}} $$  \hspace{1cm} \text{A 1.2} $$

If we combine equations A 1.1 and A 1.2 together with the relations,

$$ a^2 = \frac{\gamma p}{\rho} $$  \hspace{1cm} \text{A 1.3} $$

$$ \frac{P}{\rho \gamma} = \text{constant} $$  \hspace{1cm} \text{A 1.4} $$

then,

$$ \mu = \rho_0 a_0 SM \left(1 + \frac{\gamma-1}{2} M^2 \right)^{-\frac{\gamma+1}{2(\gamma-1)}} $$  \hspace{1cm} \text{A 1.5} $$

2. The pressure ratio required by the compressor of a direct-drive return flow wind tunnel

The pressure rise through the compressor (refer to Fig.1),

$$ \frac{P_4}{P_3} = \frac{P_4}{P_0} \cdot \frac{P_0}{p} \cdot \frac{p}{P_3} $$  \hspace{1cm} \text{A 2.1} $$

where, $P_4 - P_0$ is the pressure drop across the cooler.

$$ \frac{p_0}{p} = \left(1 + \frac{\gamma-1}{2} M^2 \right)^{\gamma/\gamma-1} $$ \hspace{1cm} \text{see equation A 1.2} $$

$$ \frac{P_3}{p} = \left(1 + \frac{\gamma-1}{2} \eta_0 M^2 \right)^{\gamma/\gamma-1} $$

and $\eta_0$ is the overall efficiency of the return circuit diffuser.
If we make the necessary substitutions,

\[
\frac{P_b}{P_3} = \frac{P_b}{P_o} \left( \frac{1 + \frac{\gamma-1}{2} \frac{M^2}{\eta_\sigma M^2}}{1 + \frac{\gamma-1}{2} \eta_\sigma M^2} \right)^{\gamma/\gamma-1}
\]

.............. A 2.2

The value of \( P_b/P_o \) is approximately unity and we then write,

\[
\frac{P_b}{P_3} = \left( \frac{1 + \frac{\gamma-1}{2} M^2}{1 + \frac{\gamma-1}{2} \eta_\sigma M^2} \right)^{\gamma/\gamma-1}
\]

.............. A 2.3

3. The adiabatic horse-power required by the compressor of a direct-drive return flow tunnel

The adiabatic h.p. = \[ \frac{\mu C_P J T_3}{550} \left[ \frac{\gamma-1}{\gamma} \left( \frac{P_b}{P_3} \right)^{\gamma-1} - 1 \right] \] .... A 3.1

If we substitute the value of \( P_b/P_3 \) found from equation A 2.3,

the adiabatic h.p. = \[ \frac{\mu C_P J T_3}{550} \left[ \frac{1 + \frac{\gamma-1}{2} M^2}{1 + \frac{\gamma-1}{2} \eta_\sigma M^2} \right] \] .... A 3.2

4. Limiting values for the diffuser efficiency \( \eta_\sigma \)

When \( M \geq 1.0 \) the limiting conditions for \( \eta_\sigma \) are,

(a) A normal shock exists at the end of the working-section and is followed by isentropic compression in the diffuser. This is equivalent to \( \eta_\sigma \) sub. = 1.0 where

\[ \eta_\sigma \] sub is the efficiency of a subsonic diffuser.

(b) A normal shock exists at the end of the working-section and is followed by constant pressure in the diffuser. This is equivalent to \( \eta_\sigma \) sub. = 0.

For (a) it can be shown that,

\[
\eta_\sigma = \left( \frac{\gamma+1}{\gamma-1} \right)^{1/\gamma} \left( \frac{\gamma+1}{2\gamma M^2 - \gamma+1} \right) - \frac{2}{(\gamma-1)M^2}
\]

.............. A 4.1

(\( \eta_\sigma \) sub = 1)
For (b) it can be shown that,

\[ \eta_\sigma = \left[ \frac{2 \gamma M^2 - \gamma + 1}{\gamma + 1} \right]^{\frac{\gamma-1}{\gamma}} - 1 \]

\[ \frac{\gamma-1}{2} \frac{M^2}{Y} \]

\[ \eta_\sigma \]

\[ A 4.2 \]

5. The energy ratio of a return flow tunnel

Energy ratio = \[ \frac{\text{rate of flow of kinetic energy at working-section}}{\text{power input}} \]

If we use the adiabatic horse-power then,

\[ E.R. = \frac{1}{2} \mu \frac{U^2}{\mu C_p J T_3} \left[ \left( \frac{P_4}{P_3} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right] \]

\[ A 5.1 \]

If we substitute the value of \( \frac{P_4}{P_3} \) from equation A 2.3 and assume that \( T_3 = T_\infty \) then,

\[ E.R. = \frac{\left( 1 + \eta_\sigma \frac{\gamma-1}{2} \frac{M^2}{Y} \right)}{\left( 1 + \frac{\gamma-1}{2} \frac{M^2}{Y} \right)} \left( 1 - \eta_\sigma \right) \]

\[ A 5.2 \]

The energy ratio for small values of \( M \), i.e., \( M \to 0 \) is,

\[ E.R. = \frac{1}{1 - \eta_\sigma} \]

\[ A 5.3 \]

and for large values of \( M \),

\[ E.R. = \frac{\eta_\sigma}{1 - \eta_\sigma} \]

\[ A 5.4 \]

But from equations A 4.1 and A 4.2 it can be shown that as \( M \to \infty \), \( \eta_\sigma \to 0 \). Therefore equation A 5.4 becomes,

\[ E.R. = 0 \]

\[ A 5.5 \]
6. To calculate the time of running of an intermittent 'straight-through' tunnel operated from compressed air storage

If the high pressure reservoir is pumped up to a pressure $P_R$, then the following analysis gives the time of running until the pressure in the reservoir falls to the constant blowing pressure $P_b$. (It is not essential to maintain constant blowing pressure but if this is not done the pressure will fall continuously in the working-section and the temperature and density will also vary during the run).

If $V$ is the volume of the reservoir then in time $dt$ the mass of air in the reservoir falls by $Vdp$. The air flowing through the tunnel in time $dt$ is $\mu dt$, where $\mu$ is the mass flow through the tunnel in unit time. Hence,

$$\frac{d\mu}{dt} = -\frac{\mu}{V} \quad \text{............... A 6.1.}$$

If we assume that no heat interchange exists between the reservoir, the walls of the tunnel and the atmosphere, then the heat energy given in time $dt$ to the atmosphere by the air leaving the tunnel is,

$$\mu C_p T dt \quad \text{where} \quad T \quad \text{is the temperature of air in the reservoir at time} \quad t.$$

The change in time $dt$ of the internal energy of the air inside the reservoir is,

$$V C_v \, d (pT)$$

hence,

$$\mu C_p T dt = -V C_v \, d (pT) \quad \text{............... A 6.2}$$

If we combine equations A 6.2 and A 6.1 then we obtain for pressure changes in the reservoir the isentropic law,

$$\frac{P}{P_b} = \text{constant} \quad \text{............... A 6.3}$$

It can be shown that provided no losses occur in throttling between pressures $P_R$ and $P_b$, the mass flow through the tunnel is given by,

$$\mu = P_b \, \sqrt{\frac{V}{RT}} \, S \, M \left[ 1 + \frac{y-1}{2(y-1)} M^2 \right] - \frac{y+1}{2(y-1)}$$

$$\quad \text{............... A 6.4}$$

where

$$P_b \, S = P_b \sqrt{\frac{V}{RT}} \quad \text{............... A 6.5}$$

But $T$ the temperature of air in the reservoir varies with time, therefore $\mu$ must vary with time.
If we neglect frictional losses in the expansion from the settling chamber, at pressure \( p_b \), to the working-section, it can be shown that,

\[
\frac{p_b}{p_a} = \left( \frac{1 + \frac{\gamma - 1}{2} \frac{m^2}{\gamma}}{1 + \frac{\gamma - 1}{2} \eta_0 \frac{m^2}{\gamma}} \right)^{\gamma/(\gamma - 1)} \quad ................. \text{A 6.6}
\]

where \( p_a \) is the atmospheric pressure at the exit from the diffuser, whose efficiency is \( \eta_0 \).

From equations A 6.1, A 6.3, A 6.4, and A 6.5 it follows that,

\[
\Delta t = \left( \frac{2}{\gamma + 1} \right) \frac{p_b \sqrt{V}}{\mu_i R T_R} \left[ 1 - \left( \frac{p_b}{p_R} \right)^{\frac{\gamma + 1}{2\gamma}} \right] \quad ................. \text{A 6.7}
\]

where, \( \Delta t \) = time of running in secs.

\( T_R \) = initial temperature of air in reservoir

\( \mu_i \), the initial mass flow \( p_b \sqrt{V \over R_T} \) \( \mu \frac{m}{\gamma} \left[ 1 + \frac{\gamma - 1}{2} \frac{m^2}{\gamma} \right]^{\gamma/(\gamma - 1)} \)

7. To calculate the time of running of an intermittent 'straight-through' tunnel operated from a vacuum tank at exit

If \( V \) is the volume of the vacuum tank and \( \mu \) the constant mass flow through the tunnel then,

\[
\frac{d\mu}{dt} = \frac{\mu}{V} = \text{constant} \quad ................. \text{A 7.1}
\]

The change of internal energy in the tank in time \( dt \) is

\[ V C_V d (pT) \]

and the heat energy extracted from the atmosphere at temperature \( T_a \), in time \( dt \) is,

\[ \mu C_p T_a dt \]

Hence,

\[ \mu C_p T_a dt = V C_V d (pT) \quad ................. \text{A 7.2} \]

If we integrate equation A 7.2 we find that,

\[
\Delta t = \frac{V \Delta p}{\mu vRT_a} \quad ................. \text{A 7.3}
\]

(where, \( ... \))
where, \( \Delta t \) is the time of running
\( \Delta p \) is the pressure rise in the vacuum tank.

If we substitute the value of \( \mu \) found from equation A 1.5,
\[
\Delta t = \frac{\Delta p}{p_o} \cdot \frac{V}{S} \left[ \frac{1 + \frac{Y-1}{2} \frac{M^2}{Y-1}}{M \gamma a_o} \right] \quad \text{A 7.4}
\]

The pressure rise
\[
\Delta p = P_R - P_i \quad \text{A 7.5}
\]
where \( P_i \) is the initial pressure in the vacuum tank

and
\[
\frac{P_R}{P_o} = \left( \frac{1 + \frac{Y-1}{2} \frac{M^2}{Y-1}}{1 + \frac{Y-1}{2} \frac{M^2}{Y-1}} \right) \quad \text{A 7.6}
\]

8. To calculate the time of running of an intermittent 'straight-through' induction tunnel

It can be shown that,
\[
\frac{dp}{dt} = -\frac{\mu}{V m} \quad \text{A 8.1}
\]
where \( m = \frac{\text{mass of air flowing through the tunnel working-section}}{\text{mass of air from the high pressure cylinder}} \)

Let \( P_R \) and \( T_R \) be the initial reservoir pressure and temperature respectively and \( P_b \) the constant blowing pressure. If similar assumptions to those stated in Appendix 6 are used, then
\[
\Delta t = \left( \frac{2}{Y+1} \right) \frac{P_R V m_i}{\mu R T_R} \left[ 1 - \left( \frac{P_b}{P_R} \right)^{\frac{Y+1}{2}} \right] \quad \text{A 8.2}
\]

where \( m_i = \text{initial mass flow ratio.} \)
9. The adiabatic horse-power required by the compressor in an induction type tunnel

If the compressor has $s$ stages the adiabatic h.p. =

\[
\frac{\mu RT_1}{550} \left( \frac{\gamma}{\gamma - 1} \right)^{\frac{\gamma - 1}{\gamma}} \left( \frac{P_2}{P_1} \right)^{\frac{\gamma - 1}{\gamma}} - 1\]

\[\text{A 9.1}\]

where $\mu$ mass of air discharged per sec.

$P_2$ exit pressure

$P_1$ inlet pressure

$T_1$ inlet temperature.

10. The time required to pump up a high pressure reservoir

Let a reservoir of volume $V$ at pressure $p_a$ be supplied with a mass flow $\mu$ at temperature $T_a$ then the change of density $d\rho$ inside the reservoir in time $dt$ is given by

\[
d\rho = \frac{\mu}{V} dt \]

\[\text{A 10.1}\]

hence,

\[
\frac{\Delta t}{V} = \frac{\rho a}{\gamma R T_a} \left[ \frac{P_R}{P_a} - 1 \right] \]

\[\text{A 10.2}\]

where $P_R$ is the final reservoir pressure.

11. The time required to evacuate a tank

The volumetric efficiency of a vacuum pump

\[
\eta = \frac{\text{Effective suction volume per stroke}}{\text{swept volume per stroke}}
\]

and $\eta = f(p)$.

If the pump runs at $N$ r.p.m. and the swept volume per stroke is $v_s$ then the volume of air removed in unit time from a tank, volume $V$, connected to its inlet is

\[
\eta \frac{v_s N}{60} \]

\[\text{A 11.1}\]

But the change of mass of air in the tank in time $dt$ is $V d\rho$, hence

\[
\frac{d\rho}{dt} = - \frac{\rho \eta v_s N}{60 V} \]

\[\text{A 11.2}\]
If the operation takes place at constant temperature,

\[ \Delta t = \frac{V}{v_s N} \int_{p_0}^{p} \frac{dp}{p} \]

where \( p_0 \) is the initial pressure in the vacuum tank, \( p \) is the final pressure and \( \Delta t \) is the time taken.

In an efficient vacuum pump \( \eta \) approximately equals unity. In this case,

\[ \Delta t = \frac{60 V}{N v_s} \log_\frac{p_0}{P} \]

where, \( P = \frac{p_0}{p} \).
DIRECT-DRIVE RETURN FLOW WIND TUNNEL

FIG. 1.

CONTINUOUS DIRECT FLOW 'STRAIGHT-THROUGH' TUNNEL.

FIG. 2
STRAIGHT - THROUGH INTERMITTENT 'PRESSURE' TUNNEL

FIG. 5.

STRAIGHT - THROUGH INTERMITTENT 'VACUUM' TUNNEL

FIG. 6.
Diffuser Efficiency for Sub- and Supersonic Entry Velocities (for Air $\gamma = 1.4$)
DIRECT-DRIVE RETURN FLOW WIND TUNNEL

PRESSURE RATIO FOR COMPRESSOR

PRESERPE RATIO ACROSS COMPRESSOR

\[ \frac{P_4}{P_3} \]

PRESSURE CONSTANT IN RETURN CIRCUIT \( \theta_{\text{sub}} = 0 \)

DIFFUSER EFFY. FOUND FROM PRACTICAL CURVE (SEE FIG. 7)

MACH. NUMBER IN WORKING SECTION

FIG. 8
NOMENCLATURE

- M = WORKING-SECTION MACH NUMBER
- $P_4/P_3$ = COMPRESSOR PRESSURE RATIO
- $W$ = FLOW OF AIR, LB/SEC.
- N = COMPRESSOR R.P.M.
- $T_3$ AND $P_3$ TEMPERATURE AND PRESSURE AT INTAKE TO COMPRESSOR.

TUNNEL REQUIREMENT

TYPICAL COMPRESSOR PERFORMANCE CURVES OF CONSTANT

$N/\sqrt{T_3}$

COMPRESSOR CHARACTERISTICS FOR A DIRECT DRIVE RETURN FLOW WIND TUNNEL.