THE PLASMA CURRENT MULTIPLIER

by

W. T. Shmayda

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Finally I wish to thank my children, Paula, Christopher-Robert and Tamara, for their efforts to be quiet all those evenings I worked at home.
Abstract

The Plasma Current Multiplier (PCM) is a well-stabilized low pressure arc discharge with the capability of multiplying the initial electron current injected into the device. Experimentally the PCM gain per unit length \( g \) was found to decrease with increasing arc current at very low arc current densities, to remain constant at moderate arc current densities, and to decrease asymptotically towards unity at extremely high current densities. Theoretically, sheath thickening and neutral rarefaction due to local gas heating have been identified as the phenomena responsible for the PCM gain behaviour at the very low and high arc current densities respectively. Experimental confirmation of the gain dependence on mass and tube radius originally predicted by the Stangeby and Allen theory was extended to several atomic species. An emissionless source of electrons was constructed and tested, thus expanding the operating capacity of this device to reactive gases.
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**Notation**

**a**  
Ionization efficiency coefficient (Fig. 46), constant in energy balance equation (3.13)

**A**  
Argon

**A⁺**  
Singly charged argon ion

**b₀**  
Impact parameter for 90° scattering of an average particle (3.19)

**B²**  
Scaling parameter (A.9)

**c**  
Random velocity

**C**  
Peculiar velocity ($\vec{c} - \vec{u}$)

**e**  
Electron charge

**E**  
Longitudinal electric field

**f**  
Particle velocity distribution function

**f_M**  
Maxwellian velocity distribution function

**f_Mₑ**  
Maxwellian electron velocity distribution function

**F**  
Force vector

**g**  
Gain for a PCM of unit length

**H**  
Magnetic field intensity

**I_a**  
Arc current

**I₀**  
Initial arc current, emission current

**J_a**  
Arc current density

**Jₑ**  
Cross-sectionally averaged random electron current density

**Jₑw**  
Random electron current density at the wall

**J_w**  
Ion wall current density

**k**  
Boltzmann constant

**Kn**  
Ion Knudsen number ($\lambda_i/r$

**L**  
Length

**L**  
Inductance
Electron mass
Neutral mass
Local number density
Local number density at radius r
Cross-sectionally averaged electron concentration
Axial electron concentration
Number density at fill conditions
Neutral concentration at 133.3 Pascals (1 torr) and 293°K (Loschmidt number/760) (= 3.54 x 10^16 particles/cc)
Pressure
Speed dependent collision cross-section
Electron-neutral momentum transfer collision cross-section
Electron-ion momentum transfer collision cross-section
Total collision cross-section (= nq)
Average electron-atom momentum transfer cross-section
Average electron impact neutral ionization cross-section
Average ion-neutral momentum transfer collision cross-section
Total averaged excitation cross-sections for radiation processes
Radius
Wall radius
Reaction rate at r
Resistor attached to a segment
Dimensionless distance (A.7). Ratio of α/γ (3.9)
Value of α/γ for free fall case (= 0.546)
Dimensionless wall radius
Average rate coefficient for process 1 via electron impact (= <v_e Q_ei>)
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<td>$S_i$</td>
<td>Mean neutral ionization rate coefficient</td>
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<td>$T$</td>
<td>Temperature</td>
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<td>$u$</td>
<td>Axial drift velocity</td>
</tr>
<tr>
<td>$\vec{V}$</td>
<td>Instantaneous velocity vector</td>
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<td>$V_B$</td>
<td>Bohm velocity $= \sqrt{kT_e/M}$</td>
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<td>$V$</td>
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<td>$V(r)$</td>
<td>Potential at radius $r$</td>
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<td>$V_c$</td>
<td>Cathode voltage</td>
</tr>
<tr>
<td>$V_{float}$</td>
<td>Floating segment voltage</td>
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<tr>
<td>$V_i$</td>
<td>First neutral ionization level</td>
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<td>$z$</td>
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**Greek Symbols**

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<td>Average Bohm velocity to instantaneous Bohm velocity ratio $(1.2)$</td>
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<td>$\beta$</td>
<td>Arc current density to average random electron current density ratio $(j_a/j_e)$</td>
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<td>$\gamma$</td>
<td>Ratio of the cross-sectionally averaged electron concentration to the concentration at the sheath interface $(1.5)$</td>
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<td>$\Gamma$</td>
<td>Sum of neutral and ion temperatures $(^\circ K)$</td>
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<td>$\delta$</td>
<td>Neutral atomic diameter</td>
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<td>$\delta_o$</td>
<td>Debye length $(8.16)$</td>
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<td>$\varepsilon_g$</td>
<td>Mean gas energy</td>
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<td>$\varepsilon_i$</td>
<td>Mean ion energy</td>
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<td>$\varepsilon_0$</td>
<td>Permittivity of free space $(= 8.85 \times 10^{-12} \text{ Fd/m})$</td>
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<tr>
<td>$\eta$</td>
<td>Dimensionless energy $(= eV/kT_e)$</td>
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<td>$\eta_V$</td>
<td>Viscosity $(3.4)$</td>
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<td>$\theta$</td>
<td>Scattering angle in center of mass frame</td>
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<td>$\lambda$</td>
<td>Gain constant $(1.9)$, wavelength, mean free path $(3.5)$</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
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<td>$\lambda_{20}$</td>
<td>Electron mean free path at 293°K and 133.3 Pascals (1 torr)</td>
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<td>$\Lambda$</td>
<td>Debye length to impact parameter ($\delta_o$) ratio ($= \delta / \delta_o$)</td>
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<td>$\mu$</td>
<td>Reduced mass, mobility (B.7)</td>
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<td>$\mu_0$</td>
<td>Permeability of free space ($= 4\pi \times 10^{-7}$ henry/m)</td>
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<td>$v_{ea}$</td>
<td>Collision frequency</td>
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<td>$v$</td>
<td>Average electron-atom collision frequency for momentum transfer</td>
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<td>$\xi$</td>
<td>Total number of ions generated per electron per second</td>
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<td>$\rho$</td>
<td>Mass density</td>
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<td>$\sigma$</td>
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<td>Time constant ($= L/R$), collision interval</td>
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<td>$\phi$</td>
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<td>$\phi(\mathbf{r})$</td>
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**Subscripts**

- $a$: Arc, atomic
- $D$: Drift
- $e$: Electron
- $exp$: Experimental
- $f$: Fill condition
- $g$: Gas
- $i$: Ion, ionization
- $m$: Metastable
- $n$: Neutral
- $o$: Initial, along centreline
- $s$: Sheath
- $w$: Wall
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<td>GPC</td>
<td>Gain per centimeter</td>
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<td>T-L</td>
<td>Tonks-Langmuir</td>
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<td>( \dot{v}_B )</td>
<td>Cross-sectionally averaged Bohm velocity</td>
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<tr>
<td>( &lt;v_e \sigma_{ei}&gt; )</td>
<td>Velocity-averaged collision cross-section for process i to occur via electron impact</td>
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CHAPTER 1. BACKGROUND

1.1 Introduction

Electrical gas discharges have been the object of scientific study for more than a century. The description of the atomic processes involved however, has become less phenomenological and more fundamental within the last few decades. The motivation for such research has been diverse. Electrical discharges involve intrinsically interesting physical effects. Discharges provide a convenient medium for the examination of fundamental atomic, plasma and spectroscopic phenomena. Applications of electrical discharges are very extensive: fluorescent lighting, gas lasers, plasma chemistry, and model studies for atmospheric processes. Ion propulsion, thermonuclear fusion and magnetohydrodynamic power generation are promising future applications.

During the investigation of current limiting processes in low-pressure mercury discharges, Stangeby and Allen discovered the ability of the arc discharge to act as a current amplifier (Ref. 41). Consider a wall-stabilized arc discharge operating in a metal tube. If the metal walls are biased slightly more negatively than they would otherwise float, the arc current density could be induced to grow exponentially with discharge length. This phenomenon is a controlled, steady-state electron avalanche. It has been termed plasma current multiplication. The device is called a Plasma Current Multiplier (PCM).

Prior to this study, the properties of the PCM had not been examined in any detail. The earlier work was primarily confined to low current levels (a few ma/cm\(^2\) maximum) and to four gas species: Mercury, Argon, Helium and Hydrogen. The influence of gas pressure had not been examined. On the basis of the preliminary work however, extrapolation to higher current densities suggested the PCM could be useful in the power field:

(1) as a current control device (HVDC current breaker for instance), and

(2) as an emissionless power cathode: i.e., a device capable of generating a high density electron current without the inherent drawbacks of the thermionic emitters.

These applications however required the PCM to operate in the high current density regime. Consequently the task of the present study required the construction of a high current density PCM and the examination of its properties at these current density levels.

The present study reports extension of PCM theoretical and experimental understanding in several respects. The principal new finding is that PCM current gain saturates at high current densities. This unexpected effect will be shown to depend on gas heating and the concomitant local neutral rarefaction. A range of other potential effects on the current gain have also been examined both experimentally and theoretically. A major extension of the original simple PCM theory has been made to cover the effects of pressures and current levels not previously included. The new theory has drawn extensively on the gas laser literature. While two of the key

equations used for predicting ion laser operation are still phenomeno-
logical, they have been found adequate to explain high order effects in
ion lasers such as the excited-state population densities of the ionized
species. It is therefore not surprising that these equations have been
found to adequately explain the PCM operation, which is based on a first
order effect, namely the ion wall current.

PCM operation has been demonstrated for a wide variety of gases which
constitutes a further confirmation of theory. An especially simple and
direct verification of theory was made by comparison of the PCM current
gains for the closely related gas pair of hydrogen and deuterium.

PCM operation requires an initial source of electrons to be injected
into the device from a cathode. The simplest cathodes are thermionic.
However, such electron sources generally have short lifetimes in any gas
and are totally unusable in a reactive gas such as oxygen. A new cathode
was devised by incorporating an electrodeless microwave discharge into
the PCM. Emissionless electrons are drawn from a microwave plasma and
introduced into the PCM to initiate the electron avalanche. Continuous
operation in O2 was demonstrated to be feasible with this electron source.
Applications of such a high discharge device may be found where filament
replacement is inconvenient or undesirable, for example in the radioactive
environment of neutral beam injectors for fusion reactors.

Other applications of the PCM are conceivable; refer to Appendix M.
However the exploration of such possibilities was beyond the scope of the
present work. It is believed that the present study constitutes a com-
plete elucidation of the major effects influencing PCM operation and is
thus a basis for any future work on PCM applications.

1.2 PCM Fundamentals

Physically the PCM consists of three elements: an electron source,
a long segmented tube and an electron collector. Figure 53 schematically
illustrates a three-segment idling PCM. In the idling mode the electron
emission from the filament is exactly equal to the arc current received at
the PCM anode. The segmented tube is made up of short metallic cylinders
aligned end to end. Although in close proximity to each other, each segment
is electrically isolated from its neighbours. A number of considerations
dictate the use of a sequence of short, metallic, annular segments: short
to allow easy initiation of the arc through each segment, metallic to
facilitate the impressing of potentials, and finally segmented to permit
the wall potential to follow the axial centreline voltage gradient more
closely. Segmenting avoids discharge shorting, and minimizes excessive
sheath voltage drops. Metal sputtering and wall overheating are associated
with high ion wall-impact energies.

The voltage along the centreline of a cylindrical PCM is illustrated
in Fig. 54. As one moves from the filament end (i.e. from the ground poten-
tial side of Fig. 54) towards the anode, the centreline voltage is seen to
increase in a continuous fashion. The anode and cathode regions are typi-
cally contained within the first and last segments of the PCM. The remainder
of the segments contain the positive column. The voltage gradient within the
positive column is approximately constant and to first order fixed by the
choice of fill pressure and tube radius. The column length is arbitrary. Any number of segments can be inserted between the cathode and anode.

The wall potential of any typical segment is represented by the short horizontal dash. These potentials are seen to increase discontinuously as one moves towards the anode. The floating wall potential of any given segment will be lower than the centreline voltage contained within that segment. The Tonks-Langmuir (T-L) theory (Ref. 8) predicts the wall voltages for both long and short ion mean-free-path cases. Figure 55 illustrates the radial voltage profile and the floating wall potential for the long ion mean-free-path case. In this figure both axes have been nondimensionalized. The radial position is given in terms of the tube radius whereas the voltage is in terms of the local electron temperature.

During the initiation of the positive column, electrons accumulate on the walls more rapidly than the ions. A net negative charge, relative to the centreline potential, develops on the walls. This potential restricts the electron wall current while simultaneously enhancing the ion wall current. In steady state, the net rate of charge deposition on the walls will be zero. For an argon discharge, this condition is attained when the wall potential is 5.6 units below the centreline value.

The arc discharge within the segment may be artificially divided into two regions:

(A) the plasma: a region which has approximately equal electron and ion charge concentrations and is characterized by weak electric fields,

(B) the sheath: a region in which charge neutrality is not applicable and the electric fields are strong.

For a typical argon discharge the plasma-sheath interface occurs at the point at which the plasma potential is approximately 1.1 units below the centreline value (labelled in Fig. 55 as $\eta_{\text{sheath}}$).

If the radial voltage profile had been drawn to scale in this figure, the sheath region would be a fraction of a percent of the tube radius. The sheath thickness depends on

$$\frac{1}{\sqrt{\eta_e}}$$

Decreasing the electron concentration in the sheath by biasing, causes the sheath to increase. However this increase is obscured by the relative smallness of the sheath size. Macroscopically, therefore, any impressed voltage on the segment appears to be taken up by the original sheath which is vanishingly thin.

Ionization occurs in the bulk of the plasma. The ions which are formed roll down the potential hill towards the wall. Their arrival rate at the sheath edge is determined exclusively by the radial plasma potential. The factors which determine this ion current (Eq. A.12) are specified in the T-L theory (Ref. 8):
\[ J_w = \alpha n_s v_B \tag{1.1} \]

where \( \alpha \) is a parameter introduced by Allen and Thonemann (Ref. 40) to account for the ion energy distribution, \( n_s \) is the charge particle concentration at the plasma-sheath interface and \( v_B \) is the Bohm velocity.

\[ v_B = \sqrt{\frac{kT_e}{M_i}} \]

Here \( T_e \) is electron temperature and \( M_i \) is ion mass. Since ions are generated throughout the discharge volume, the ion energy at the sheath will depend on the radial location at which the ionization occurred. The resultant energy dispersion in falling from differing radial potentials to the sheath potential has been considered in the T-L theory (Ref. 8) and formalized in the Allen-Thonemann work (Ref. 40) by the introduction of the alpha (\( \alpha \)) parameter. If \( v_B \) is the average Bohm velocity at the sheath edge, \( \alpha \) is defined by

\[ \alpha = \frac{v_B}{v_B} \tag{1.2} \]

The electrons are confined within the plasma region by the strong repelling fields of the sheath. In the idling mode only the most energetic plasma electrons can overcome the potential barrier and reach the tube walls. The random electron current density at the sheath edge will be

\[ j_e = n_s \frac{e}{4} \tag{1.3} \]

In the idling mode, the electron current to the wall will be regulated by the wall potential to exactly balance the ion wall current.

Biasing the wall potential negatively with respect to its floating value will reduce random electron flux density in the sheath region immediately adjacent to the walls. The number of electrons capable of penetrating the imposed barrier to reach the walls will decrease. To convert the idling PCM into the multiplying mode, each segment must be coupled to earth via a resistor as shown in Fig. 56. By judicious choice of resistor, one can reduce the biasing voltage below the floating potential to repel virtually all plasma electrons. Since the majority of the radial voltage drop occurs in the sheath region during biasing, the original plasma potential will remain unaltered. As a result the original ion velocity distribution will not be affected and the ion wall current will continue to arrive at the original rate. To neutralize these ions, electrons must be drawn from earth through the biasing resistors (refer to Fig. 56). Upon recombination with these external electrons, the ions will return to the plasma as neutrals. As a consequence of the recombination with electrons from this new source, the effective number of electrons in the arc will be increased. These new electrons are released into the plasma to be added to the arc current as the returning neutrals are once again ionized.
The rate of arc current increase will be proportional to the ion wall current. Consider an element of plasma of cross-sectional area \( \pi R^2 \) and \( \Delta z \) long. Some of the \( \pi R^2 j_{arc} \) electrons entering the \( \pi R^2 \Delta z \) volume will be involved in ionizing collisions with the local neutral concentration. As a result an ion flux of magnitude \( 2\pi R j_{wall} \Delta z \) will flow to the walls. If the walls are strongly biased and all plasma electrons are prevented from reaching them, external electrons will have to be supplied at the rate: \( 2\pi R j_{wall} \Delta z \). Consequently the net increase in the arc current density per incremental length of plasma will be:

\[
\frac{d}{dz} j_{arc} = \frac{2j_{wall}}{R}
\]  

(1.4)

The ion wall current \( (j_w) \) can be related to the arc current. If \( \beta \) is defined as the ratio of the arc current density to the electron random current density (Ref. 8),

\[
\beta = \frac{j_a}{j_e}
\]  

(1.5)

and if \( \gamma \) represents the ratio of the cross-sectionally averaged electron density to the value at the sheath edge

\[
\gamma = \frac{n_e}{n_s}
\]  

(1.6)

it may be readily shown that (Ref. 41)

\[
j_w = \frac{\alpha}{\beta \gamma} \sqrt{\frac{2 m_m}{M}} j_a
\]  

(1.7)

Substitution of this relation into Eq. (1.4) and the subsequent integration yields

\[
I_a = I_o e^{\lambda z}
\]  

(1.8)

where \( z \) is the axial length of the discharge, \( I_o \) is the initial current at \( z = 0 \) and \( \lambda \) is the gain constant. Here \( \lambda \) is defined as

\[
\lambda = \frac{\alpha}{\beta \gamma} \sqrt{\frac{2 m_m}{M}} \frac{2}{R}
\]  

(1.9)

In this model, the constants \( \alpha, \beta, \gamma \) were assumed to be current independent. From the T-L theory for the long mean free path case

\[
\frac{\alpha}{\gamma} = \frac{0.7722}{2} = 0.546
\]
The drift to random current ratio ($\beta$), on the other hand, was left undetermined. It could only be deduced experimentally via Langmuir probes or from the slope of the PCM gain. However this model predicts the gain constant will depend on the species mass and discharge radius in a simple way. Experimental confirmation was undertaken in four different gases: Mercury, Helium, Hydrogen and Argon. For low arc current densities (ranging from 8.4 ma/mm$^2$ to 180 ma/mm$^2$), the authors (Ref. 41) found that both $\lambda R$ and $\lambda w/M$ were approximately constant. These observations are consistent with Eq. (1.9).

CHAPTER 2. THE EXPERIMENT

2.1 Introduction

The previous work on the PCM was restricted to low current densities and covered a rather narrow operating range. Extrapolation of that work to higher current densities suggested promising PCM current-handling capabilities and hence potential applications as a power control device. The final arc current achievable in the PCM, according to the low current PCM theory, seemed to depend simply on the choice of the active gas:

$$\text{PCM Gain } \propto \exp \left[ \frac{1}{\sqrt{M_i}} \right]$$

and on the structural dimensions:

$$\text{PCM Gain } \propto \exp \left[ \frac{Z}{r_w} \right]$$

where $M_i$ is the ion mass

$r$ the multiplying length

$r_w$ the tube radius

Although the phenomena normally associated with high arc current densities: gas pumping, self-magnetic field effects, cumulative ionization and neutral heating were expected to modulate the PCM characteristics, it was not apparent which factors were important or at what current densities they would play a role. Gas pressure was expected, from physical arguments, to influence the PCM characteristic through the $\alpha/\beta$ factor. However this area had not yet been explored.

Decoupling the electron source from the PCM could further enhance the PCM applicability and usefulness. If possible, both electron source and PCM could be operated in their respective optimum pressure regions, and subsequently coupled by a differentially pumped system. Decoupling also facilitated the use of non-thermionic electron sources and hence the potential use of the PCM in reactive gases or in situations which were hampered by short filament lifetimes.
Accordingly the present experimental study was undertaken to:

1. test the low current PCM theory for an extended number of gases,

2. select experimentally the best gas for high current work, given the voltage limitations of the existing research equipment,

3. extend the PCM operation to as wide a range of pressures and currents as possible,

4. investigate the PCM ability to operate without a thermionic source.

Experimental studies indicated the practical advantage of using Argon in the PCM. Gain was found to be high, presumably because of the unusually high value of $\alpha/\beta_0$ (see Table 4). Operating voltages, however, were lower than for the higher gain gases Hydrogen and Helium. Accordingly the bulk of the low current research and all of the high current work was undertaken in Argon. In Argon the PCM optimum operation was in the 10-30 range. For lower pressures, gains decreased dramatically. For higher pressures, gains exhibited small increases per incremental pressure increase while the tendency for intersegment arcing increased dramatically.

2.2 Experimental Apparatus

2.2.1 Introduction

In the Plasma Current Multiplier, PCM, a series of short electrically-insulated metal tubes separate an emission source from the anode. In the non-amplifying mode, a positive column is struck through the tubes between the emitter and the collector. Each segment develops a floating potential, which is negative with respect to the local centreline voltage. In the non-amplifying mode the electron current to the wall is regulated by the wall potential to balance the ion wall current. If each segment is biased approximately 20 V more negatively than the floating potential, the high energy electrons normally lost to the walls, will be reflected back into the positive column. The ions in the plasma, unaffected by the additional voltage drop which appears in the sheath only, arrive at the wall at the original rate (although with a higher impact velocity at the wall itself) and extract electrons from the metal surface. As a result a net electron current flows from earth to each segment and subsequently into the plasma. This property of adding electrons to the arc current gives the PCM its capability of amplifying approximately exponentially since ionization rates are proportional to electron density.

Steady-state operation of a PCM at current levels much greater than one amperes requires the increased expense and inconvenience of segment cooling. In order to avoid this, the present work employed a 95 kJ, 1 kV capacitor bank to operate the PCM in a pulsed mode. The actual PCM pulse duration, typically 1/4 second, is long enough to be steady-state so far as all plasma processes are concerned, but is short enough to avoid over-heating of the uncooled PCM walls.
With the PCM idling (small d.c. arc current) in a non-multiplicative mode, the capacitor bank is charged up to the existing anode voltage. The d.c. power supply is turned off and simultaneously replaced by the charged capacitor bank. Each segment is then electronically connected to earth through its own resistive network. The arc current is observed to increase within a few micro-seconds to a peak value determined by the number of active segments and thereupon to decrease with an RC time constant. The typical biasing voltage and arc current dependence on time of any one segment have been photographed (refer to Fig. 1). During the first fifty milliseconds, the PCM is idling at a low arc current density (typically 30 ma/mm²), draining power directly from the capacitor bank. At \( t = 50 \) msec, all segments between the one being monitored and the cathode are biased. Fifty msec after the triggering pulse, the PCM is in the multiplicative mode. The arc current is approximately 30 amps in this case. The discharge extinguishes itself (not seen in this photograph) when the capacitor voltage drops below the minimum voltage necessary to sustain the positive column.

2.2.2 The Vacuum System

The vacuum envelope consists of a fifteen centimeter internal diameter (I.D.) by 1.8 meter long pyrex tube coupled to a pumping system via a 10 cm x 10 cm I.D. glass cross. A two-stage Sargent Welch Duo-Seal type 1402B rotary mechanical pump roughs the vacuum envelope down to about 7 pascals (1 mtorr) in approximately four minutes. The oil vapour diffusion pump is a three stage, 70 litre per second Speedivac unit, capable of achieving an ultimate pressure of approximately 7 x 10⁻⁴ pascals. Backstreaming from the F203 diffusion pump is suppressed by a liquid nitrogen trap. The details of this arrangement are given schematically in Fig. 2.

The pressure is monitored continuously during the experimental run by an Edwards Model 9 Pirani gauge or a Televac, model 2A (thermocouple) gauge. A Balzers IMG-2 ionization gauge monitors the pressures below a tenth of a pascal. All gauge heads were periodically checked against a standard mercury McLeod gauge. Vacua of the order of 10⁻³ pascals could be consistently achieved within the experimental volume.

To minimize the build-up of impurities during experimental runs, the system was always operated in the flow-through mode. Fresh gas introduced at one end of the system, continuously flushed the experimental volume through the pumping system to atmosphere at the opposite end. Gases with purities on the order of 100 ppm were used. Appendix 2 lists the actual impurity levels and constituents for the gases used. The operating pressures (fill pressures) were maintained by the adjustment of the mass flux through a choked orifice.

Each experimental run was preceded by pumping the experimental volume down to at least 7 x 10⁻³ pascals. For a typical operating pressure of a few pascals, the background impurity level was on the order of a few hundred parts per million at the maximum. Ion bombardment of the interior walls and gas pumping at the idling arc current level would condition the interior wall surfaces and minimize any impurity concentration buildup in the discharge volume. Impurities dislodged from the internal discharge surfaces and the background residual gas would be diluted and removed by the flushing gas.
If impurity levels on the order of a few hundred parts per million represent a realistic estimate, the effective ion mass in the discharge region will increase by only a fraction of a percent. The corresponding decrease in the PCM gain (Eq. 1.6) will also be insignificant.

2.2.3 PCM Mechanical Description

The physical arrangement of the plasma current multiplier components have been illustrated in Fig. 3. Each stainless steel cylindrical tube, 2.54 cm long (referred to as an insert) was press-fit into a 5 cm x 5 cm stainless steel supporting block. The primary functions of these supporting blocks were to facilitate the electrical connections to the inserts and to assist in aligning the inserts end to end without touching each other. Typically the cylindrical inserts were 2.5 cm or 2.794 cm long and of three bore diameters: 0.5 cm, 0.46 cm and 0.30 cm. The segments, consisting of an insert and its supporting structure, were aligned in a glass tray and insulated from each other with glass or macor spacers. The inserts were typically separated from each other by 2 mm gaps, which occur inside the supporting structures. Refer to the typical 5.08 mm bore assembly of Fig. 3. The collector was essentially an enlarged insert, 5 cm long with a 1.3 cm bore diameter.

Intersegment arcing outside the active discharge region was minimized by equipping the glass base with glass walls and a close fitting glass top. This entire assembly was finally placed in the vacuum envelope, sealed and electrically connected to the resistive networks via vacuum tight feed-throughs.

2.2.4 Microwave Emission Source

Operation of a PCM requires an initial source of electrons. Thermionic sources are well suited to generating high emission current densities. In high power applications this characteristic can be used to full advantage. However these types of electron sources have a number of inherent drawbacks. Filaments are susceptible to burn-out. Aging adversely affects the surface work function and consequently the emission characteristics. Operation in oxidizing or reducing atmospheres is not possible. At pressures in excess of a few tens of pascals, ion bombardment reduces the filament lifetime and degrades the emission properties rapidly. Microwave sources offer a viable alternate to thermionic sources. With all working parts external to the plasma, these electron sources circumvent many of the problems inherent to filaments.

A typical microwave source consists of a high frequency (upwards of 1 GegaHertz) power amplifier connected to a resonant cavity (refer to Fig. 4). In the present work the power amplifier was a 200 watt 2450 MHz magnetron. The microwave cavity could be tuned to the plasma to match the amplifier-load impedances and maximize the power transfer. To ensure that the power reflected back into the microwave amplifier did not exceed the manufacturer's recommended limit (75 watts RMS) a reflected-power meter was inserted in the transmission line (Fig. 4). Calibration of this custom meter was accomplished by reversing the directional coupler and correlating the attenuated power with the voltage generated by the RC network.
With the microwave cavity positioned about the quartz tube, the gas flow set for a particular operating (fill) pressure and the power amplifier set at approximately 45 watts (power out), the plasma in the cavity region was initiated by a high frequency Tesla coil. This cavity plasma could be sustained indefinitely if the electrons oscillating in resonance with the high frequency (microwave) field could provide the necessary electron-neutral impact ionizations to counterbalance electron ion recombinations at the walls. The size of this plasma could be controlled by fine tuning the cavity impedance, by the microwave generator power and by the ambient pressure; see Fig. 5.

A fraction of the electrons could be extracted from the cavity plasma by an applied axial field. This electric field was established by applying a positive potential to the first PCM segment and by grounding the ion collector. (Refer to Fig. 5.) The extractable electron current was determined by the cavity plasma size, the ion collector surface area, and its distance from the cavity. Experimentally the optimum cavity-probe separation was found to be approximately 6 mm. For separations less than this, microwave heating of the probe was severe.

In the cavity region, the quartz tube was cooled by an air stream. In the coupling block region, both the quartz tube and the ion collector were cooled by a water flow (refer to Fig. 6). Under typical operating conditions, the quartz tube wall temperature rarely exceeded 30°C.

With the geometric arrangement described above, this source was capable of injecting up to 60 ma of electron current into the first segment of the PCM. At electron current levels in the milliampere range, the emissionless PCM was operated continuously in a variety of gases: nitrogen, neon, helium, hydrogen and Argon. The scope of the emissionless PCM was extended to reactive gaseous environments. Continuous operation in oxygen up to twelve hours produced no noticeable deterioration in either the electron source or the PCM. These experiments demonstrated the feasibility of not only replacing high power thermionic sources with emissionless plasma current multipliers in environments which reduce filament lifetimes but also their use in gases which do not allow the use of filaments.

2.2.5 Thermionic Sources

Both ac and dc filament heating methods were employed in the present work. In high emission work (emission currents greater than 0.5 amp), reasonable filament lifetimes could only be realized with large diameter filaments, typically on the order of 0.5 mm diameters. In these cases, the filament heater current necessary to raise thoriated tungsten to suitable emission temperatures was approximately 30 amperes. For this range of heating currents, the ac heating method was the most convenient. In this method, a 1 kVA power transformer controlled the 110 VAC input line voltage into a 20 to 1 step-down voltage transformer (refer to Fig. 7).

For experimental work which required low emission currents (a few ma), 0.3 mm diameter filaments provided acceptable lifetimes (typically 10 hours). In these cases, the required heating currents were supplied by a 400 ampere-hour lead acid battery (refer to Fig. 8).

At low arc current density levels (a few ma/cm²) noise picked up by the measurement lines could severely degrade the quality of the measured
signals. Sixty cycle ripple was a notably troublesome noise source at low arc current density levels, which could be conveniently avoided with the dc heating techniques.

2.2.6 The High Voltage PCM Power Supply

The primary PCM circuit which carried the main arc discharge was powered by a 95 kilojoule, 1 kilovolt capacitor bank. This unit consisted of three banks of electrolytic capacitors stacked in series. Each bank was made up of twenty 33000 microfarad, 350 volt capacitors connected in parallel. This assembly could be charged to a maximum voltage of 1050 volts. Computation of the effective capacitance suggests a value of 0.22 farads. Measurements, however, using an LRC resonant circuit indicated the actual capacitance to be approximately 0.18 farads. The entire assembly was housed in a wooden container approximately 1 cubic meter in volume. The capacitor bank was charged by a 1.5 ampere, 1 kV Sorenson power supply.

To accommodate occasions which required the rapid dissipation of the stored capacitor energy, a capacitor emergency-dump circuit (refer to Fig. 9) was incorporated into the capacitor bank electrical charging system. Once activated, the voltage on the capacitor bank decayed with a 3.3 sec. time constant. The 1.5 kva dc charging power supply remained turned off until this dump circuit was disconnected.

2.2.7 PCM Firing Method

The SCR triggering, scope triggering and power supply isolation from the plasma current multiplier prior to the multiplication mode were electronically sequenced. First the 1.5 kva power supply was decoupled from the primary circuit (see Fig. 10) with an electromagnetic relay. Then a 10 volt pulse was applied to the debouncer circuit (Fig. 11). The shaped single 10 volt step was then simultaneously applied to the scope trigger and a pulse delay circuit. Upon leaving the monostable multivibrator, the delayed pulse was amplified and applied to the SCR gates. The delay period (typically fifty msecs) was adjustable to allow examination of the entire arc current behaviour or just the low to high arc transition phase.

2.2.8 Segment Biasing Voltage Characteristics

Figure 12 illustrates the typical biasing arrangement for any individual segment. The biasing resistor had to be determined by trial and error for each segment once a particular fill pressure and emission current combination were selected. Starting with a large resistance, the SCR would be fired a number of times while monitoring the segment voltage. The biasing resistance was progressively reduced until a voltage drop of approximately 20 volts below the floating potential was attained. Experimentally this condition represented the maximum gain per segment achievable without shorting the plasma. Refer to Fig. 17 to see typical experimental GPC versus resistance saturation curves for a given segment.

If the initial conditions of a particular run: the fill pressure, the emission current, the anode voltage, and the resistance of biased segments between the cathode and the segment of interest remained constant, the arc current density into the monitored segment depended exclusively on
on the total number of active segments. All the SCR's attached to these segments were triggered simultaneously. Figure 13 illustrates the temporal characteristics of the segment voltages of the last four segments (excluding the monitored one) during the firing of the PCM. These transition voltages (V_{float} to V_{bias}) were observed to remain constant during any given experimental run.

Severe biasing of a particular segment would extinguish the plasma. Strong biasing however would cause short duration sporadic arcing within the active region. Figure 14 illustrates the arc current density and the associated biasing voltage of a segment which has been biased too strongly. Increasing the biasing resistance in that particular segment always eliminated the arcing problem.

The absolute segment floating potential of each segment was measured with a 4 digit DVM. During the gain optimizing procedure, the floating potential of the segment of interest was monitored continuously. Its value just prior to firing was noted and recorded. The error in this measurement was less than 1% of the measured value.

Floating-to-biased voltage transitions had to be photographed. At low arc current densities, a 50 V/cm vertical deflection scale was used. At high arc current densities a 20 V/cm scale was more appropriate. The error in the relative transition voltage (zero point being arbitrary) was ±5 volts in the former case and ±2 volts in the latter. Periodic checks ensured the oscilloscope input amplifiers were linear over the deflection range used.

2.2.9 SCR Considerations

The SCR's were subject to three possible failure modes:

(a) exceeding the rated di/dt caused the thyristor junction to conduct excessively high current densities near the gate connection thereby generating a local hot spot and shorting the anode to the cathode;

(b) exceeding the rated stand-off voltage would drive the SCR into failure by electron avalanching across the junction;

(c) exceeding the rated peak current carrying capacity of the semiconductor would destroy it thermally.

The latter two failure channels are either tabulated for the particular SCR in its specification sheets (failure mode 2) or may be estimated by following calculations illustrated in any SCR handbook (failure mode 3).

To limit the di/dt failure mode, the rate of current increase in the circuit must be restricted below the SCR's critical di/dt value. For any individual segment the rate of current increase will be

\[
\frac{dI}{dt} \approx \frac{I_{\text{max}}}{\tau} = \frac{V_{\text{float}}}{R_{\text{seg}}} \cdot \frac{R_{\text{seg}}}{I_T} \\
\approx \frac{V_{\text{float}}}{I_T}
\]
where $R_{\text{seg}}$ is the segment resistance and $L_T$ is the total circuit inductance. Typically the non-inductive resistors used (Dale NH series) had an inductance of approximately 1 microhenry. For a maximum current-time slope of 200 A/s and for floating voltages of 200 volts or higher, it is apparent that additional inductance must be introduced into the line. Excessive inductive values (millihenry range) such as those in conventionally wound power resistors, however, were found to generate voltage spikes which would obscure the segment floating-to-biased voltage transition, thus making qualitative gain measurements impossible. The inductive value of short air core solenoids may be shown to be

$$L \approx N^2 \mu R \left[ \ln \frac{8R}{r} - \frac{r}{4} \right] \mu k$$

where $N$ is the number of turns, $R$ is the internal solenoid radius, and $r$ is the wire radius. For 12 gauge wire wrapped around a 2.54 cm diameter ceramic core, the inductance will be approximately

$$L = 0.033 N^2 \mu k$$

For a safety factor of 2, the required inductance and number of loops as a function of segment voltage have been calculated:

<table>
<thead>
<tr>
<th>$V_f$ volts</th>
<th>$L \mu h$</th>
<th>Number of Turns</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Case A</td>
</tr>
<tr>
<td>200</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>400</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>600</td>
<td>6</td>
<td>11</td>
</tr>
</tbody>
</table>

where case A is for the case in which a particular segment is biased with resistors in series, and case B for the case in which only one resistor is used.

2.2.10 Arc Current Measurements

The arc current was monitored by an oscilloscope. The required scope deflection voltage was obtained by placement of a small resistance in the main power transmission line. (Refer to Fig. 10.) Three different resistors were chosen to cover the range of arc current densities studied in the PCM: 1.27$\Omega$, 66.7 $\Omega$, and 20.0 $\Omega$. In each case, the values of the custom resistors were measured to an accuracy of 1/2%. In each case, the change of resistance of these arc sensing resistors due to self-ohmic heating was measured and found to be negligible over their respective ranges of intended use.

To minimize oscilloscope absolute and drift errors in measuring the arc current, the trace deflection was calibrated against a standard potential
and checked on a regular basis. The maximum absolute deflection above the reference (zero voltage) line represented the peak arc current attained in the PCM. At high current densities, the emission current trace, typically during the first 50 msec, served adequately as the zero reference line thus minimizing the possibility of drift errors. The error in the arc current measurement at the high arc current densities was estimated to be approximately 10% of the peak arc current. At low arc current densities an additional method was available to estimate the peak arc current. The emission current was always known to within 1% of its value at the time of firing the PCM. With the zero reference on the current trace set and checked just prior to every PCM firing, the arc current could be estimated by scaling up the emission current in proportion to the trace deflections on either side of the firing interval. At low arc currents, this method generated arc current estimates within 5% of the first technique discussed.

2.3 Experimental Results

2.3.1 Introduction

While the PCM is in principle a simple discharge device a variety of experimental difficulties were encountered during the course of the acquisition of useful data. This section will briefly review some of these difficulties.

The experimental repeatability of a particular arc current value is dependent on three factors:

1. the constancy of the emission current,
2. the cathode potential,
3. the long term constancy of the pressure.

The large emission currents drawn from the 0.38 mm diameter filaments restricted their useful lifetime to approximately 6 hours of operation. During the latter half of this time, the filament diameter will be reduced substantially and short term drift in the emission current will occur. For a given heater current, the emission current will be fixed, if the filament diameter does not change. However, in the latter half of the filament lifetime the percentage mass-loss rate is increasing. This results in an increase in the emission current which in turn aggravates the shrinking of the filament diameter. This problem can be resolved by the use of larger filaments, typically 0.69 mm in diameter. Unfortunately there is an associated increase in heater current and heat deposition in the cathode end of the PCM. However the short term drift effect was made negligible.

The voltage drop taken up by the positive column is fixed by conditions within the plasma once the neutral density-radius product is selected. The electric field does depend on the arc current but this effect is small. The anode voltage drop is also fixed by plasma conditions once the anode surface area is selected. Consequently any perturbation of the total voltage is absorbed by the cathode fall ($V_c$). Fluctuations in $V_c$ influence the emission current. To minimize variability in the total arc current, $V_c$ must be monitored for all PCM firings and matched to the initially chosen conditions. Short term instabilities were minimized by the introduction of a resistor in the emission circuit of the cathode end of the PCM.
Any tendency to draw more emission current into the PCM is countered by the additional voltage induced by the ballast resistor. The net effect is to reduce the voltage perturbation. The use of such a ballast resistor, typically $\sim 100\Omega$, however, means the PCM must operate approximately

$$100 I_{\text{emission}} \text{ (volts)}$$

above ground potential. Thus the silicon controlled rectifiers (SCR) must have higher stand-off voltages than for operations without the ballast resistor.

Long term drift in the fill pressure remained a vexing problem during the entire duration of the experimental work, despite numerous attempts to use choked orifice flow backed up with pressure regulators. Scatter in the experimental results must be attributed to the poor regulation of the fill pressure. This problem was especially acute at low fill pressures, where the gain is most sensitive to the fill pressure. Experimentally, the pressure was monitored continuously at two points in the vacuum envelope. Both thermocouple gauges were calibrated and periodically checked against a Mercury McLeod gauge. Throughout all experimental runs, the fill pressures were maintained within 30% of the stated values.

Noise posed a special challenge in this work. The physical size of the PCM was large to accommodate the power handling requirements of each segment and the personal safety features. As a result the interconnection of various PCM components resulted in a substantial footage of wiring. Positive columns are excellent random noise generators. The extensive PCM wiring became an effective receiving antenna. This noise was troublesome in two respects:

1. by causing the SCRs to trigger sporadically,
2. by obscuring measurement signals.

Good grounding procedures and careful wiring arrangements reduced noise levels to tolerable magnitudes. Further noise suppression required high frequency filters and electronic isolation circuitry.

Arcing within the vacuum envelope was the single most troublesome problem encountered in the PCM work. The final PCM arrangement consisted entirely of glass walls and a glass base to-curtail intersegment arcing. This type of short circuit usually occurred over distances of several decimeters. Arcing between adjacent segments was not a problem. A complex path at the anode end ensured that electron avalanching would not occur while still permitting gas to flow freely through the PCM to the ambient reservoir at the anode end. Arcing at the cathode end to ground could be minimized by operating at low $V_c$ values, floating metal surfaces where possible and shielding the filament leads from the PCM discharge.

### 2.3.2 Axial Electric Field

The difference in the floating wall potential between adjacent segments was observed to be constant over the entire PCM length except for the immediate vicinity of the anode and cathode. Figure 15 shows a typical segment wall potential versus distance characteristic. This figure indirectly indicates the uniformity of the axial electric field over the bulk of the PCM length.
The electric field of a positive column is sensitive to both the arc current density and the fill pressure (9). A similar dependence was observed in the PCM. At low PCM arc current densities the electric field decreased with increasing arc current density and with decreasing pressure. Refer to Table 1 for typical values.

**Table 1**

Axial Electric Field Dependence on Arc Current and on Fill Pressure in a 2.286 mm Discharge Radius

<table>
<thead>
<tr>
<th>P (Pascals (mTorr))</th>
<th>I (Amps ± 10%)</th>
<th>E (V/cm ± 2%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16 (120)</td>
<td>0.2</td>
<td>6.1</td>
</tr>
<tr>
<td></td>
<td>0.6</td>
<td>5.6</td>
</tr>
<tr>
<td></td>
<td>1.0</td>
<td>5.0</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>4.8</td>
</tr>
<tr>
<td>1.9 (14)</td>
<td>0.6</td>
<td>4.5</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>4.2</td>
</tr>
</tbody>
</table>

The importance of establishing the segment floating potentials over the entire PCM length prior to any quantitative gain measurements is two-fold. Local disturbances such as adjacent segment shorting, insert deformities caused by sporadic local arcing or small current discharges to ground may be detected and eliminated. Secondly, most experimental studies extended over a period of ten to twenty hours in the low arc current work and over a series of days in the high current work. As a result, periodic shut-downs for repairs and adjustments were necessary. The original operating conditions could be regained only if pressure, initial emission current and the original segment floating potentials could be duplicated. Typically this required the simultaneous adjustment of the microwave power or the filament heater current and anode voltage, once the other operating conditions had been duplicated.

2.3.3 Segment Biasing Voltage

Over the arc current range from a few milliamperes to two hundred amperes, the gain per segment was observed to increase rapidly with increasing biasing potential initially and subsequently to plateau (see Fig. 16). At low arc currents (a few milliamperes), this plateau was typically achieved at forty to sixty volts below the segment floating potential. The minimum voltage drop below the floating potential necessary for gain saturation, decreased with increasing arc current density, typically being less than 20 volts for arc currents in the ampere range and less than 10 volts for arc currents in the 100 ampere range.
The segment gain characteristic may be monitored equivalently as a function of biasing resistance. The resistance necessary to achieve segment gains in the plateau region decreased with increasing arc current; refer to Fig. 17.

The general features of the gain-biasing voltage characteristic did not depend on the bore radius. The plateau height however did increase with decreasing bore radius (Fig. 18).

2.3.4 PCM Gain at Intermediate Currents

Over the current discharge range 23 ma to 2.5 amperes, in 2.286 mm radius segments, in 16 pascals of Argon (fill pressure), the gain per unit length (g) was approximately independent of the arc current density magnitude; see Table 2. In this range neither sheath growth at the cathode end nor neutral depletion at the anode end appeared to be important effects.

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Segment No.</th>
<th>Io ma</th>
<th>Iarc ma</th>
<th>Vfloat volts</th>
<th>Vb volts</th>
<th>Total Gain</th>
<th>Gain per cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>56</td>
<td>83</td>
<td>149</td>
<td>24</td>
<td>1.5</td>
<td>1.168</td>
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<tr>
<td></td>
<td>9</td>
<td></td>
<td>125</td>
<td>160</td>
<td>34</td>
<td>2.2</td>
<td>1.175</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td></td>
<td>190</td>
<td>190</td>
<td>23</td>
<td>3.4</td>
<td>1.179</td>
</tr>
<tr>
<td></td>
<td>11</td>
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<td>230</td>
<td>50</td>
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<td>1.203</td>
</tr>
<tr>
<td></td>
<td>13</td>
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<td>260</td>
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<td>9</td>
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<td>10</td>
<td></td>
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<td>220</td>
<td>30</td>
<td>3.2</td>
<td>1.152</td>
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<td>11</td>
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<td>225</td>
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<td>240</td>
<td>48</td>
<td>7.5</td>
<td>1.195</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td></td>
<td>720</td>
<td>256</td>
<td>36</td>
<td>11.3</td>
<td>1.173</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td></td>
<td>1070</td>
<td>272</td>
<td>32</td>
<td>16.7</td>
<td>1.169</td>
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<tr>
<td></td>
<td>15</td>
<td></td>
<td>1600</td>
<td>290</td>
<td>18</td>
<td>25.0</td>
<td>1.172</td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td>23</td>
<td>34</td>
<td>268</td>
<td>23</td>
<td>1.5</td>
<td>1.166</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td></td>
<td>46</td>
<td>285</td>
<td>22</td>
<td>2.0</td>
<td>1.126</td>
</tr>
<tr>
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<td>12</td>
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<td>306</td>
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<td>2.7</td>
<td>1.132</td>
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<td></td>
<td>13</td>
<td></td>
<td>86</td>
<td>325</td>
<td>15</td>
<td>3.7</td>
<td>1.130</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td></td>
<td>123</td>
<td>340</td>
<td>15</td>
<td>5.4</td>
<td>1.151</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td></td>
<td>175</td>
<td>352</td>
<td>27</td>
<td>7.6</td>
<td>1.149</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td></td>
<td>256</td>
<td>365</td>
<td>20</td>
<td>11.1</td>
<td>1.162</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td></td>
<td>380</td>
<td>370</td>
<td>26</td>
<td>16.5</td>
<td>1.168</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td></td>
<td>568</td>
<td>385</td>
<td>25</td>
<td>24.7</td>
<td>1.171</td>
</tr>
<tr>
<td></td>
<td>19</td>
<td></td>
<td>825</td>
<td>400</td>
<td>24</td>
<td>35.9</td>
<td>1.158</td>
</tr>
</tbody>
</table>

Typical error (%) 5 5 5 10 10
Furthermore the GPUL was not sensitive to the absolute value of the segment floating voltage, provided of course it remained constant for any particular run and was sufficient for "plateau" operation. For example, see segments 8 through 13 of run Nos. 4 and 2. On the strength of these observations, it is legitimate to compute a mean \( \bar{g} \) for the entire PCM. Equivalently a mean gain constant \( \bar{\lambda} \) could be computed, where

\[
g = \exp(\bar{\lambda} L)
\]

and \( L \) is the segment length. This value \( \bar{\lambda} \) has been computed for each of the seven runs and subsequently these mean gain constants were averaged over the total number of runs. The resultant averaged mean gain constant was found to be

\[
\bar{\lambda} = 15 \pm 10\% \text{ m}^{-1}
\]

The corresponding \( g \) was 1.160. The Argon fill pressure used during these runs was 16 ± 30\% pascals. At low arc current levels (60 ma) in a 2.54 mm discharge radius, theory predicts a \( g \) of 1.103 and 1.148 respectively for the Argon fill pressures 13.3 and 21.4 pascals. Refer to Appendix K, cases 3 and 4. Adjustment of these two values to account for the smaller experimental tube radius (see Eq. 1.9) increases the gains per unit length to 1.115 and 1.166 respectively. These theoretical values compare reasonably well with the average experimental \( g \) value.

Large bore segments concomitant with lower \( g \) values, afford easier positive column penetration through each segment at low arc currents. A compromise to retain this startup feature without sacrificing the \( g \) so severely, may be found in the composite bore arrangement of each segment (Fig. 19). The major drawback of this approach, however, is determining the effective bore radius.

Assuming a cylinder of length \( L \) and a radius \( r_{\text{eff}} \) such that the surface area is identical to that of the internal composite bore surface area, one may readily compute

\[
r_{\text{eff}} \approx 4.22 \text{ mm}
\]

The validity of this approach, however, requires some experimental confirmation. To this end a series of arcs was struck in 16 pascal Argon gas in the composite bores and the gains per unit length measured. If the very low arc current gains which displayed a strong dependence on the wall sheath thickness are neglected, the mean gain constant \( \bar{\lambda} \) can be computed in a manner analogous to the 2.286 mm case. The resultant values averaged over five different runs in a 16 pascal Argon discharge were:

\[
\bar{\lambda} = 8 \pm 10\% \text{ m}^{-1}
\]

\[
g = 1.081
\]
Assuming once again that the gain constant-radius product \( (\lambda r) \) is a constant for a given gas at low arc current values (refer to Eq. 1.9) and using the average values of the mean gain constants computed in Tables 3 and 4, we have

\[
\text{r}_{\text{eff}} = \frac{15 \pm 10\%}{8 \pm 10\%} \times 2.286 = 4.4 \pm 20\% \text{ mm}
\]

in reasonable agreement with the radius deduced from geometrical arguments.

To test the \( \lambda r \) constancy assumption, a comparison of this product for various radii in different gases is necessary. The effective radius of the composite bore segment is taken to be 4.22 mm. The results of this comparison are available in Table 3.

Theory also predicts the gain-radius-product should be inversely proportional to the square root of the species mass:

\[
\lambda r \alpha M^{-\frac{1}{2}}
\]

**Table 3**

Comparison of the Gain Constant-Radius Product \( (\bar{\lambda}_{\text{exp}}r) \)

for Various Gases in the 60 ma to 2.5 amp Current Range

<table>
<thead>
<tr>
<th>Gas</th>
<th>P ( \pm 30% )</th>
<th>( r ) mm</th>
<th>( \bar{\lambda}_{\text{exp}} ) m(^{-1} )</th>
<th>( \bar{\lambda}_{\text{exp}}r ) ( 10^{-2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O_2 )</td>
<td>6.7</td>
<td>4.22</td>
<td>5.78 ± 0.45</td>
<td>2.44 ± 8%</td>
</tr>
<tr>
<td></td>
<td>13.3</td>
<td>4.22</td>
<td>5.06 ± 0.59</td>
<td>2.14 ± 12%</td>
</tr>
<tr>
<td></td>
<td>13.3</td>
<td>4.96</td>
<td>4.81 ± 0.15</td>
<td>2.39 ± 3%</td>
</tr>
<tr>
<td></td>
<td>13.3</td>
<td>4.96</td>
<td>4.13 ± 0.19</td>
<td>2.05 ± 5%</td>
</tr>
<tr>
<td>( H_2 )</td>
<td>24.0</td>
<td>4.22</td>
<td>13.83 ± 1.05</td>
<td>5.84 ± 8%</td>
</tr>
<tr>
<td>( D_2 )</td>
<td>21.3</td>
<td>4.96</td>
<td>7.52 ± 0.37</td>
<td>3.73 ± 5%</td>
</tr>
<tr>
<td></td>
<td>21.3</td>
<td>4.22</td>
<td>9.27 ± 0.36</td>
<td>3.91 ± 4%</td>
</tr>
<tr>
<td>Neon</td>
<td>13.3</td>
<td>4.22</td>
<td>7.75 ± 0.41</td>
<td>3.27 ± 5%</td>
</tr>
<tr>
<td></td>
<td>13.3</td>
<td>4.96</td>
<td>6.79 ± 0.84</td>
<td>3.37 ± 12%</td>
</tr>
<tr>
<td>Helium</td>
<td>26.7</td>
<td>4.22</td>
<td>15.5 ± 1.24</td>
<td>6.54 ± 8%</td>
</tr>
<tr>
<td></td>
<td>80.0</td>
<td>4.22</td>
<td>15.0 ± 1.08</td>
<td>6.33 ± 7%</td>
</tr>
<tr>
<td></td>
<td>80.0</td>
<td>4.96</td>
<td>10.4 ± 0.99</td>
<td>5.16 ± 10%</td>
</tr>
<tr>
<td></td>
<td>26.7</td>
<td>4.96</td>
<td>13.1 ± 1.04</td>
<td>6.50 ± 8%</td>
</tr>
</tbody>
</table>
Table 4

Comparison of $\lambda_{exp} r \sqrt{\dot{M}}$ Product

for Different Gases used in the PCM

<table>
<thead>
<tr>
<th>Gas</th>
<th>$\lambda_{exp} r \sqrt{\dot{M}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>N₂</td>
<td>0.135 ± 24%</td>
</tr>
<tr>
<td>O₂</td>
<td>0.128 ± 7%</td>
</tr>
<tr>
<td>D₂</td>
<td>0.076 ± 4%</td>
</tr>
<tr>
<td>Neon</td>
<td>0.149 ± 9%</td>
</tr>
<tr>
<td>Helium</td>
<td>0.123 ± 8%</td>
</tr>
<tr>
<td>Argon</td>
<td>0.211 ± 4%</td>
</tr>
<tr>
<td>H₂</td>
<td>0.082 ± 8%</td>
</tr>
</tbody>
</table>

Although Table 4 suggests the $\lambda_{exp} \sqrt{\dot{M}}$ is approximately constant for a wide range of species masses, the exact gain dependence on mass is obscured by the species dependence of the $\alpha/\beta$ ratio and to some degree by possible pressure dependence as well. See Sections 3.3.2 and 3.3.3. A more precise mass-gain corroboration can be established for a gas pair such as H₂ and D₂ for which the $\alpha/\beta$ ratio can be expected to be almost identical. For nearly identical pressures it was found that:

$$\frac{\lambda_{H₂}}{\lambda_{D₂}} = \frac{13.8 \pm 8\%}{9.3 \pm 4\%} = 1.5 \pm 11\%$$

a value well within experimental limits of the square root of the D₂ to H₂ mass ratio (1.414).

2.3.5 PCM Gain at Low Currents

At very low arc current densities, a few ma/cm², the $g$ exhibits a strong dependence on the arc current. Refer to Fig. 20. At these current density levels, the sheath thickness is no longer a negligible fraction of the discharge radius. If the low current $g$ is $G_o$, in a tube of radius $r_o$, the effective radius ($r_{eff}$) will be:

$$r_{eff} = r_{tube} - \text{sheath thickness}$$

and the $g$ at very low arc currents will be:

$$G_e = (G_o)^{r_o/r_{eff}}, \quad r_o = r_{tube}$$

At these arc current densities, the electron temperature will be specified by the fill pressure-tube radius product in accordance with the T-L
model (8). Thus both \( T_e \) and the ion generation rate \( \xi \), also a function of \( T_e \), will be constant for a given radius and fill pressure. Consequently the parameter \( B^2 \) (Eq. A.9) will depend only on the axial electron concentration \( n_{eo} \), for a given gas. At the low arc current densities of interest

\[
j_a \propto n_e \propto f(B^2)n_{eo} \quad [50]
\]

where \( f(B^2) = n_e/n_{eo} \) and \( B^2 \) has been defined by Eq. A.9 (see Fig. 21). For the 21 pascal case mentioned in Appendix A, a 300 ma arc current would correspond to \( B^2 \approx 10^5 \). For the approximately equal pr conditions considered here \( (pr \approx 68 \text{ pascal-mm}) \), an arc current of 1.6 ma would correspond to \( B^2 \approx 500 \). At this \( B^2 \) value, the sheath thickness is approximately half the tube radius (Fig. 21). If the \( g \) in a 4.22 mm, 16 pascal Argon discharge and at arc currents which make sheath considerations negligible is 1.081, the expected increase in the \( g \) of 1.6 ma should be:

\[
g_e \approx (1.181)^2 \approx 1.17
\]

This value is representative of the experimental \( g \) at the 1.5 ma arc current level, shown on Fig. 26.

2.3.6 PCM Gain at High Currents

The cumulative gain dependence on the PCM length is examined in Fig. 22 for arc current densities above 2.5 amp/cm². With the exception of the initial emission current for the two cases presented, the experimental conditions were identical. Increasing the PCM length, according to Eq. 1.6, should result in an exponential increase in arc current density.

\[
I_a \propto \ell^z
\]

Contrary to the intermediate-current PCM-gain behaviour and to the low current PCM theory, the slope of these gain-distance curves exhibits a dependence on the arc current. At any axial position, the slope for the \( I_0 = 1 \text{ amp case} \) is lower than the slope for the \( I_0 = 0.5 \text{ case} \). The arc current in the \( I_0 = 1 \text{ amp case} \) is roughly double that for the \( I_0 = 0.5 \text{ case} \) for a given axial location. At locations of equal arc current, however, the slopes of the gain-distance curves are similar:

<table>
<thead>
<tr>
<th>( z ) cm</th>
<th>( I_a ) amp</th>
<th>( I_0 ) amp</th>
<th>( m^{-1} ) ± 10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>35.5</td>
<td>50</td>
<td>0.5</td>
<td>6.8</td>
</tr>
<tr>
<td>32.5</td>
<td>50</td>
<td>1.0</td>
<td>6.1</td>
</tr>
</tbody>
</table>
Comparison for the $g$ for the two cases over the range of arc current densities encountered (see Fig. 23) indeed suggests that within experimental error ($\pm 2\%$) the $g$ depends solely on the local current density, assuming the fill pressure and tube dimensions are fixed.

The cumulative gain dependence on the PCM length has also been measured for a range of Argon fill pressures while simultaneously maintaining a constant emission current and a fixed tube radius (refer to Fig. 24). The cumulative gain for a given PCM length is found to increase with increasing fill pressure. Furthermore the change in the total gain-distance slope is seen to be more dramatic for the higher fill pressure cases than for the lower ones. To investigate the $g$ dependence on the arc current density, the $g$ was computed from the curve fit to the experimental points shown in Fig. 24. These values have been tabulated in Appendix I and plotted in Figs. 25 and 26. In each case the $g$ exhibits a strong decline with increasing arc current observed in Fig. 23. In addition, however, for any given arc current density the $g$ is seen to increase with increasing fill pressure. This dependence becomes less pronounced in the high (greater than 2 amps/mm$^2$) range. Comparing the $g$ at the 0.5 and 2 amps/mm$^2$ levels, for example, we have from Figs. 25 and 26:

<table>
<thead>
<tr>
<th>$j_{\text{arc}}$ (amp/mm$^2$)</th>
<th>$g$ (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.08</td>
</tr>
<tr>
<td>2.0</td>
<td>1.06</td>
</tr>
</tbody>
</table>

Thus the $g$ depends on both the arc current density and on the neutral density.

Figure 26 suggests the increase in $g$, for a given gas and discharge radius, saturates with increasing fill pressure. This behaviour was further substantiated by gain measurements at two different fill pressures while keeping all other PCM variables constant (Fig. 27). As the PCM operation approaches the short free path regime, plasma parameters such as the electron temperature, electric fields, etc., exhibit steadily weakening dependence on the neutral pressure. With reference to Fig. 29, for example, the slope $dT_e/d(pr)$ is seen to decrease with increasing $pr$. Since

$$g \propto f(T_e, E/n)$$

a similar behaviour in $g$ can be expected.

In view of the $g$-fill pressure saturation phenomenon, there will be an optimum operating pressure for the PCM. At low pressures, the $g$ is low. At high pressures, the $g$ saturates with increasing pressure. Energy losses at the emission source, however, increase with increasing neutral pressure. In addition, the probability of arcing along the PCM (external to the active discharge) increases as the pressure-distance product (a criterion for voltage breakdown) approaches the Paschen minimum. For Argon discharges,
operation in pressures in the 20 pascal range yielded g values close to the high pressure values without encountering many of the high pressure difficulties.

Physically, the gain dependence on the arc current density has been attributed to the local neutral depletion through gas heating. The mechanism responsible for gas heating, however, has not been clarified. Electron-neutral and electron-ion elastic energy exchanges cannot account for the neutral energies observed in the arc discharge (see Section A.3 and Appendix L). Kitaeva (48) has noted that ion-neutral resonant charge exchange may play an important role in determining the neutral temperature in high arc current density Argon plasmas. There is similar evidence in the present work. At pressures of about one pascal \((r = 2.54 \text{ mm})\), the PCM gain does not depend on the arc current (see Fig. 24) in contrast to the example cited at the higher fill pressure (see Fig. 22). In the low pressure case, the ion-neutral mean free path is on the order of the tube dimensions (see Section 3.3.3). Infrequent ion-neutral collisions would imply no strong energy coupling between ions and neutrals, little neutral heating and rarefaction, and consequently little sensitivity to the arc current density. Also see Appendix L.

In the PCM, the arc current density has been shown to increase with increasing multiplying length. The neutral density therefore decreases with distance. Schemes which favour axial neutral density profiles that increase with distance during the idling mode may increase the GFUL during the multiplicative mode, by providing neutral particles where they are most needed, i.e. at the high arc current density end of the PCM. Forcing the gas flow through the anode end, down the capillary and out at the cathode end is one such possibility. In this arrangement the neutral concentration gradient will be required for a flow down the capillary. Total gain measurements for this arrangement were compared against the case in which the gas entered the capillary by diffusion (Fig. 28). This particular approach did not appear to be encouraging. Evidently the dynamic forces responsible for the neutral rarefaction are stronger than the pressure gradient required to force the gas flow down the tube.

CHAPTER 3. MODELING

3.1 Introduction

The Argon experimental results presented in Chapter 2 span a wide range of pressures and currents. While the PCM theory of Stangeby and Allen appears to properly describe the gain dependence on the tube radius and the gas species, it is clear that this theory is only an approximation of the actual PCM behaviour, most applicable to the intermediate current range. This theory for instance provides no indication of the strong influence of pressure on the PCM gain nor does it predict the experimentally observed decrease in gain per unit length at high currents. A number of phenomena associated with high current discharges may influence the PCM operation:

(a) gas heating and the resultant neutral rarefaction,
(b) gas pumping due to electron-neutral momentum transfer,
(c) cumulative ionization from excited levels,
(d) self-magnetic field effects,
(e) radial inhomogeneity established by the outgoing ions and returning neutrals.

A survey of the literature uncovered an extensive body of experimental and theoretical information of direct relevance to high current PCM operations. Arc discharges are completely characterized by five independent parameters: discharge radius, arc current magnitude, fill pressure, wall temperature and gas species. Argon ion lasers operate in the identical pressure, radius and current regime employed in the present study. Laser discharges differ from those in the PCM in that the former do not involve current multiplication and consequently have constant currents along the arc. It was easily demonstrated experimentally that the PCM characteristics are dependent on local conditions, i.e. local to a given segment. The PCM operation was shown to be identical at a particular arc current density whether or not the segment under consideration was preceded by a length of constant arc current. On the strength of these observations, it was concluded that the extensive findings of the Argon ion laser researchers could be directly applied to explain the high current PCM characteristics.

The review of the ion laser literature has revealed two salient features:

(1) no fundamental theory exists for high-current-density argon-ion laser-plasmas,
(2) all authors use an empirical neutral-density arc-current relation coupled with the T-L model (8) to explain laser operation. Some authors include radiative losses in the power balance equation.

This model, however, is deficient in a number of respects:

(1) it neglects the contribution of electron and ion pressures,
(2) pressure balance of neutrals between the reservoir and the arc is assumed whereas flux balance may be more appropriate since the neutral mean free path is of the order of the tube radius.

In the present study a comparison of the experimental data was made with:

(a) the most accepted plasma model used by ion laser researchers,
(b) a model which included both electron and ion pressure contributions,
(c) a model which employed the flux balance relation instead of pressure balance.

As will be noted below, the PCM operation is best described by the model which has also been most successful in ion laser studies.
These models in the present work have also been employed to establish that the high current phenomena mentioned earlier could be eliminated, at least as the principal explanation of the high current PCM gain behaviour. These effects have been discussed in some detail in the appendices:

(a) current limitation in Appendix D,
(b) gas pumping in Appendix E,
(c) self-magnetic field effects in Appendix F,
(d) cumulative ionization in Appendix G.

Nevertheless it was noted that some of these phenomena, in particular (a) and (c) were beginning to influence the PCM behaviour at the highest arc current densities (~ 6 amp/mm²).

The principal finding of the present study on high current PCM gain is that gas heating is the primary cause of the gain fall-off at the high currents. The mechanisms responsible for this gas heating are probably non-resonant charge exchange and electron elastic energy exchange, the former mechanism being dominant. Decrease of neutral density leads to higher electron drift velocities and thus to a large drift to random current ratio (β). Correspondingly the \( g \propto \exp[1/β] \) decreases. The influence of neutral density (fill pressure) on the \( g \) may be similarly explained. As the fill pressure increases, the neutral density (especially at the cathode end of the PCM where gas heating is the least important) increases, thereby lowering the electron drift velocity \( V_D \propto E/P \) and hence \( β \). Correspondingly a smaller \( β \) implies a larger \( g \). The above explanations are only valid to first order. The influence of both pressure and arc current is more complex than indicated, for instance the change in neutral density will affect the electron temperature which in turn will affect the electron random velocity and hence \( β \). These interactions have been taken into account by the simultaneous solution of the positive column equations to obtain the dependence of various parameters on the current density and by the subsequent application of these values to the equation which describes the high current PCM gain.

3.2 Extension of Arc Theory to High Current Density

3.2.1 Introduction

Interest in high current-density low pressure glow discharges increased dramatically with the discovery of electron-impact-excited ion lasers in 1961. Bridges (45), Bennett (21), Gordon (1) and Kitaeva (4), among numerous other investigators, established the cylindrical positive column operating in or near the medium mean-free-path regime to be a convenient and viable environment to generate pulsed, quasi-continuous, or continuously operating (CW) noble gas ion lasers. Typical CW plasma conditions for ion lasers span current densities of the order of 1 to 5 amperes/mm², bore radii of 1 to 10 mm and neutral fill pressures in the 1 to 100 pascal range. Clearly the applicability of the Tonks-Langmuir positive column theory to this regime would require assessment. To this end a flood of research papers appeared in the open literature. Over the subsequent ten years following the discovery of the ion laser, many phenomena of the high current density positive column were separated and identified. Some of these phenomena: gas pumping (39), self-magnetic field (53), current limitation (40, 41) were resolved on a fundamental level. Other properties, neutral temperature and concentration, ionic temperature, continue to elude basic theoretical formulation. Typically
the discharges operate in the transitional free path regime in which both inter-particle collisions and boundary collisions play a critical role in establishing the specie energy and velocity distributions. For these properties, empirical relations have been deduced and have been found capable of providing explanations of higher order effects, e.g. populations for laser transitions.

3.2.2 Electron Temperature

In the Tonks-Langmuir model the electron temperature is uniquely specified by the neutral concentration-radius product.

Simultaneous solution of the ion generation rate equation with the plasma balance equation yields the following electron temperature relation, for the free fall case (9, p. 292):

\[
\frac{\eta_1}{1 + 2/\eta_1} = 0.469 \, aV_1 \, pr \sqrt{M}
\]

where \( \eta_1 = eV_1/kT_e \),

- \( V_1 \) is the ionization potential,
- \( M \) is the gaseous atomic weight,
- \( p \) is the pressure in pascals,
- \( r \) is the discharge radius in cm,
- \( a \) is the ionization efficiency coefficient (the slope of the ionization curve of Fig. 46).

The electron temperature dependence on the reduced radius is illustrated in Fig. 29. This behaviour has been well confirmed experimentally (20).

At high current densities, however, the electron temperature appears to depend on the reduced radius as in the T-L model but also on the electron concentration. Experiments by Kitaeva (43), Donin (3), Pleasance (2) and others indicate the electron temperature decreases with increasing pressure and increases with increasing current density. Increased current density raises the atomic temperature and thereby reduces the neutral concentration in the positive column. Herziger and Seelig (17) verified the constancy of the electron temperature with increasing arc current experimentally by maintaining the neutral concentration in the column at the fill pressure while increasing the current density. It is clear, therefore, that the electron temperature \( T_e \) must be related to the local neutral density in the positive column, rather than to the fill pressure.

3.2.3 Ionic and Atomic Temperature

The velocity distributions of the ions and atoms are different. Atoms exhibit a near-Maxwellian velocity distribution, although their "temperature" does depend on the operating pressure, discharge radius and the arc current density (48). Ions, on the other hand, are strongly anisotropic in velocity
space with radial velocities at least an order of magnitude greater than those longitudinally. Despite the non-Maxwellian character of these profiles, all investigators reporting noble ion laser plasma properties deduce effective temperatures from Doppler line widths according to the relation

$$\Delta \lambda_{1/2} = 7.16 \times 10^{-7} \lambda \sqrt{\frac{T}{M}}$$

where \(\Delta \lambda\) and \(\lambda\) are in Å and \(T\) is in °K. This equation assumes a Maxwellian velocity distribution in its derivation. All theoretical treatments further simplify the picture by assuming identical ion and atom "temperatures".

The line-width temperatures are dependent on the macroscopic parameters: gas pressure, arc current density and radius. Both along the cylindrical axis and radially, the ionic temperature decreases with increasing pressure-radius product. The atomic temperature, on the other hand, increases. Consequently for \(pr\) values as low as 43 pascal-mm (322 mtorr-mm) (48), heavy particle collisions must play an increasingly significant role in the energy exchange process between "hot" ions and "cold" atoms. Increases in the arc current density cause both ionic and atomic temperatures to rise. For narrow bore tubes in the range 1 to 5 mm, Chester (39) obtained the empirical temperature-current relation:

$$\frac{T_a}{300} = 1 + 8.95 \times 10^{-5} j_a r^{1/2}$$  \hspace{1cm} (3.2)

where \(T_a\) is in °K, \(r\) is in meters and \(j_a\) is in amperes/m². This fit is based upon data obtained by Chester (39), Kitaeva (44, 4), and Labuda (1). Herziger and Seelig, on the other hand, working with larger bore tubes, up to 11 mm in diameter, favour a \(j_{ar}\) dependence. Near optimum laser conditions they find in MKSA units:

$$T_a = 1500 \left[1 + 5 \times 10^{-8} (j_a r)^2\right]$$  \hspace{1cm} (3.3)

This relation arises from a curve fitting to data obtained by Kitaeva (47, 4, 48, 33), Donin (3), and Bridges (45). It should be noted that no theory working from basic principles has yet been published relating the atomic temperature to the arc current density and radius.

3.2.4 Local Neutral Number Density

At the low current densities and the large discharge radii considered in the Tonks-Langmuir positive column, neither neutral radial dependence nor tube-reservoir coupling mechanisms play an important role in the operation of the discharge. The local neutral density in this regime is well represented by the ideal gas law. Allen and Thonne mann (40), Stangeby and Allen (41), Caruso and Cavaliere (49), among other investigators, have considered the growth of radial atomic inhomogeneities at large discharge radii in moderate current densities. The physical picture developed by these researchers is treated in Appendix D.
For pr values typical of both laser and plasma current multiplier operations, the neutral mean free path is of the order of the tube dimensions. Assuming smooth rigid elastic spherical molecules, Chapman and Enskog (25) deduced the viscosity to be approximately:

\[ \eta_v = \frac{0.4989 \ p \ c_n}{\sqrt{\frac{2}{\pi}} n \delta^2} \]  \hspace{1cm} (3.4)

where \( c_n \) is the neutral random velocity,
\( \rho \) is the neutral mass density,
\( n \) is the neutral number density,
\( \delta \) is the neutral atomic diameter.

The mean free path (\( \lambda \)) however is related to the molecular diameter (25):

\[ \lambda = \frac{1}{\sqrt{\frac{2}{\pi}} n \delta^2} \]  \hspace{1cm} (3.5)

from which it follows that

\[ \lambda = \frac{6.44 \times 10^4}{p} \eta_v \sqrt{\frac{T}{M}} \text{ cm} \]

where \( \eta_v \) is in poise,
\( p \) is in pascals.

For argon at 25°C,

\[ \lambda = \frac{39.8}{p} \left( = \frac{5.3}{p_{\text{torr}}} \right) \text{ cm} \]  \hspace{1cm} (3.6)

where \( p \) is in pascals. For the case of \( r = 2.54 \text{ mm}, p = 21 \text{ pascals (160 mtorr)}, \lambda \approx 0.33 \text{ mm}, \lambda/r \approx 0.13 \). The local neutral concentration is coupled to the reservoir concentration by slip flow, a regime which must account for both inter-atomic collisions and boundary effects.

Any comprehensive treatment of the local neutral concentration must also account for the electron partial pressure. With increasing current and decreasing neutral densities (see Section 3.3.1), the electron pressure becomes comparable to the neutral pressure.

Despite all these strong dynamic influences, ion laser researchers proposing theoretical explanations: Kitaeva (4, 16, 33, 43, 44), Herziger and Seelig (17, 26), Lin and Chen (46), Labuda (1), Gur'ev (32), Webb (36), Boscher (13), have exclusively used the simple gas law for neutrals:
in the regime of present interest. \( n \) and \( T \) are the local neutral number density and temperature, respectively, and \( n_f \) and \( T_f \) the reservoir number density and temperature. Absorption of X-rays along the cylindrical axis indicates that the neutral concentration decreases with increasing atomic temperature in keeping with the simple gas law \((45)\). Calculated number densities using this relation appear to be consistently 20 to 40% lower than the measured values \((45)\). Interferometric measurements by Stepanov \((32)\) halfway between the cylindrical axis and the wall also correspond to calculations using this relation. The experimental work of Miller \((45)\) further strengthens the validity of using this relation as a reasonable working approximation.

3.2.5 Solution of the Positive Column Equations

The equations which govern the plasma discharge have been presented in the previous sections. In total there are six equations. The dependent parameters are local neutral number density \( n \), electron number density \( n_e \), local neutral atomic temperature \( T \), ion current density \( j_w \), and electric field \( E \). The independent parameters list five in total: discharge radius \( r \), arc current density \( j_a \), fill neutral number density \( n_f \), the fill neutral temperature \( T_f \), and the gas species used in the discharge. In summary the equations to be solved have been compiled below in a condensed form:

(1) Ion generation (Eq. A.15):

\[
j_w = \frac{e}{2} n n_e r c_e \bar{q}_{ei} \tag{3.8}
\]

where \( c_e \) is the random electron velocity,
\( \bar{q}_{ei} \) is the velocity averaged ionization cross-section.

(2) Ion Wall Current (Eq. A.12):

\[
j_w = e s n_e v_B \tag{3.9}
\]

where \( s \) may assume values other than \( s_0/\sqrt{2} \) (see Section 3.3.3).

(3) Mobility equation (Eqs. B.25 through B.30):

\[
j_a = \sigma E \tag{3.10}
\]

\[
\sigma = \frac{2}{3} \sqrt{\frac{2}{m_e}} \frac{n_e e^4}{(kT_e)^{2.5}} \int_{0}^{\infty} \frac{e^{e/e_{de}}}{n Q_{ea} + n Q_{ei}} \tag{3.11}
\]

and the cross-section \( Q_{ea} \) is bounded by the ranges specified in Eqs. B.26 and B.27.
Neutral Density-Temperature Relation:

\[ n = n_f \frac{T_f}{T} \]  

(3.12)

Energy Balance (Eq. A.30):

\[ j_a E_r = 2 j_w (a T_e + V_1) \]  

(3.13)

where

\[ a = \frac{k_e}{e} \left\{ \ln \left[ \frac{1}{2 \sigma_0 \hbar} \sqrt{\frac{M}{m_e}} \right] + 1.7 \right\} = 6.336 \times 10^{-4} \]

for Argon and \( \sigma_0' = 0.7722 \) and \( h_0 = 0.35 \) are taken to be constant (8).

Neutral Temperature-Arc Current Dependence:

\[ T = 300 \left( 1 + 8.95 \times 10^{-5} \sqrt{\frac{E}{j_a}} \right) \]  

(3.14)

The species concentrations represent cross-sectionally averaged values. To simplify the computational procedure, the electron temperature is treated as the independent parameter. Given an electron temperature, the electron number density may be computed by a Newton-Raphson iteration scheme. Once these two values are known all other parameters may be uniquely calculated. This process is repeated for a series of electron temperatures and the data stored in matrix form. The data can be re-arranged to make the arc current density the independent variable. Figures 30 through 32 illustrate the typical dependence of electron temperature, electron number density and ionization fraction, respectively, on arc current and fill pressure in Argon. The electron temperature at low current values is seen to approach the Tonks-Langmuir values, as expected a priori. Donon (3) has observed experimentally the saturation of the electron number density with increasing arc current in Argon discharges.

3.3 New Models for Plasma Current Multiplier Operation

3.3.1 Extension of the Model to High Current Densities

In the following sections, the models developed in the Argon ion laser studies have been applied to establish a more comprehensive PCM theory. Basically, where Stangeby and Allen treated \( \alpha, \beta \), and \( \gamma \) as constants, the models presented in this section will establish a current and pressure dependence of these quantities. The drift to random electron current density ratio (\( \beta \)) will be shown to explicitly depend on the electron temperature, the electric field, and the momentum-transfer collision cross-section. The variation in \( \alpha \) and \( \gamma \) is treated in the subsequent section.
Consider a cylindrical metal vessel of unit length and cross-sectional area \( \pi r^2 \) in which electrons constituting the majority charge carriers are moving with an axial drift velocity \( v_D \). It is assumed that the wall biasing is such that all electrons created in the plasma are prevented from reaching the walls. The ion current to the wall \( j_w \), however, is the same as for floating walls. The net result is an injection of electrons into the sample volume equal to the ion loss to the walls. The sheath thickness is assumed to be negligible in comparison to tube dimensions. In the plasma the ion concentration is taken to be equal to the electron number density. The ion axial drift velocity is taken to be at least an order of magnitude smaller than that of the electrons. With reference to Fig. 33, the conservation of charge requires

\[
\pi^2 \frac{\partial}{\partial x} (n_e v_D) = 2\pi \frac{j_w}{e} \frac{\partial}{\partial z}
\]

or

\[
\frac{\partial}{\partial z} j_a = \frac{2}{r} j_w
\]  

(3.15)  

(3.16)

The ion current density, Eq. 1.1, and the random electron current density at the sheath's edge, are related by the sheath charged particle concentration,

\[
j_w = \frac{\hbar \alpha}{\gamma} \frac{v_B}{c_e} j_e
\]

\[
j_a = \frac{4v_D}{c_e}
\]  

(3.17)  

(3.18)

gives

\[
j_w = \frac{\hbar \alpha}{\beta \gamma} \frac{v_B}{c_e} j_a
\]

or

\[
j_w = \frac{\alpha}{\beta \gamma} \sqrt{\frac{2m_e}{M}} j_a
\]

(3.19)

Substitution in Eq. 3.16 yields:

\[
\frac{1}{j_a} \frac{d}{dz} j_a = \frac{2}{r} \frac{\alpha}{\beta \gamma} \sqrt{\frac{2m_e}{M}}
\]

(3.20)

If \( \alpha, \beta \) and \( \gamma \) are independent of current, the integration of this equation would yield the low current gain equation (1.6) derived in Ref. 50. In the present work, however, variation in \( \alpha, \gamma \) and most especially \( \beta \) will be shown to be critical to the explanation of high current PCM operation.

An expression for the drift velocity has been derived in terms of the microscopic parameters \( T_e, n_e \) and \( n \), all of which depend on the arc
current density (see Appendix B). Assuming for the moment that $\alpha/\gamma$ is a constant, fixed at the Tonks-Langmuir value of 0.546, Eq. 3.20 becomes:

$$\int \frac{j_a(l)}{j_a(0)} \frac{d}{\gamma} = 2 \sqrt{\frac{2m_e}{M}} \frac{\gamma}{r} \left( \frac{\alpha}{\gamma} \right)$$

(3.21)

where $\ell$ is the effective multiplying length of the current multiplier, $j_a(0)$ is the initial emission current density and $j_a(l)$ is the current density at the distance $l$, and

$$\beta = \frac{4v_D}{c_e}$$

(3.22)

$$\beta = \frac{4}{3} \left( \frac{e}{kT_e} \right)^3 E \int_0^\infty \frac{e^{-\eta_d \epsilon}}{nq_e + ne^q_{e1}}$$

(3.23)

Values for $q_{ea}$ and $q_{e1}$ are given in Appendix B. The gain per unit length $(g)$ follows directly from Eq. 3.20:

$$g = \frac{j_a(\ell)}{j_a(0)} = \exp(\lambda)$$

(3.24)

where

$$\lambda = \frac{3}{2r} \frac{\alpha}{\gamma} \sqrt{\frac{2m_e}{M}} \left( \frac{kT_e}{e} \right)^3 \frac{1}{E} \int_0^\infty \frac{e^{-\eta_d \epsilon}}{nq_e + ne^q_{e1}}$$

(3.25)

is the gain exponent per unit length. Note the explicit dependence of $\lambda$ on $n$ through the $\beta$ term. Such dependence was not previously incorporated in PCM theory since $\beta$ was treated as a constant. The general tendency for increasing $n$ is to diminish $\beta$ and since $\lambda$ varies as $1/\beta$, the gain increases with $n$. In reality the situation is more complex since $\beta$ also depends on $T_e$ and thus all of the column equations, 3.8 to 3.14, must be solved simultaneously.

Once the arc discharge equations, 3.8 through 3.14, have been solved, where $j_{arc}$ is now dependent on $n$, the parameters $n$, $n_e$, $T_e$ and electric field $E$ are known for a range of arc current densities. The current multiplier total gain at any specified length may be computed by numerical integration of Eq. 3.20, updating the values of $n$, $n_e$, $T_e$ and $E$ for each incremental value of arc current. This particular parameter however is subject to cumulative errors both in experiment and theory. A more accurate comparison is available through the gain per unit length versus current dependence.

With regard to establishing the local neutral density in Section 3.2.4, it was suggested that a pressure equality law should be favoured over a flux equality relation. The atomic temperature on the other hand could be determined by either the Chester (39) or the Herziger (57) relations. The effect of each of these expressions on the gain per unit length may be explored by constructing four different positive column
model provides a convenient means of investigating the effect of various postulated phenomena which may be active in the current multiplier.

Figure 34 compares experimental data against the four theoretical models generated by the various combinations of the neutral density-temperature and temperature-current equations. In the comparison with experiment, it is apparent that the model which uses flux balance and the Chester relation predicts the wrong gain per centimeter (GPC) dependence on current density. The model which uses flux balance and the Herziger relation yields a decreasing GPC curve. However the rate of decrease is too slow. The two models which use the pressure balance relation follow experiment values reasonably well at the higher current densities.

Comparison of the predicted electron temperatures at low currents (Fig. 35) suggests that the Chester relation is preferable to the Herziger one. The model using the latter relation yields electron temperatures which do not match those predicted by the low current Tonks-Langmuir model.

Figure 36 shows the effect of expanding the pressure balance law to incorporate the electron pressure.

The contribution of the ion pressure will be small since $n_i < n$, but may be included for completeness:

$$n_a T_f = n_a T_a + n_e T_e + n_i T_i$$

(3.26)

where

$$T_i \approx T_a$$

$$n_e \approx n_i$$

and subscript $f$ indicates reservoir conditions:

$$n_a = n_f \frac{T_f}{T_a} - n_e \left( \frac{T_e}{T_a} + 1 \right)$$

(3.27)

The inclusion of the electron pressure in the gas law causes the GPC curves to drop dramatically. To attain current densities of the order of 1 amp/mm² one must assume unreasonably high electron temperatures. If electron pressure is of the order of the neutral pressure as simple calculations suggest, this straightforward application of a partial pressure concept is evidently inappropriate in the positive column model. This unexpected behaviour undoubtedly arises from use of the empirical relations for the neutral gas properties. Presumably the empirical relations already include the effect of gas reduction due to the electron pressure, making the explicit introduction of the electron pressure term incorrect. It should be noted that such terms have not been explicitly incorporated in the laser studies (13, 16, 39).
It is the conclusion of this section, therefore, that as with the laser studies, the most useful model for describing PCM operation is that employing pressure balance plus the Chester relation in calculating the neutral gas properties in the arc.

3.3.2 Applicability of the Free Fall Criterion

In this section the variation of the $\alpha/\gamma$ ratio and its effect on the gain per unit length are considered.

If neutral and ion temperatures are equal, the average momentum transfer cross-section for ion-neutral collisions in Argon because of atomic polarizability may be shown to be (6):

$$\tilde{\sigma}_D = \frac{16.94 \times 10^{-14}}{\sqrt{T}} \text{ cm}^2$$  \hspace{1cm} (3.28)

Since in general for our case $T_i > T_n$, this cross-section represents an upper bound on the actual cross-section.

In addition to this non-resonant collision process, the ions and neutrals may undergo a charge exchange collision. The average momentum transfer cross-section for this process is (6):

$$\tilde{\sigma}_D = 2[A + 3.96 B - B \log\Gamma] \frac{x}{A^2}$$  \hspace{1cm} (3.29)

where $A$ and $B$ are in $\text{Å}$ and $\Gamma$ is the sum of ion and neutral temperatures. The derivation of this relation assumes that both species possess a Maxwellian distribution. The coefficients $A$ and $B$ are obtained by a curve fit of the predicted charge exchange cross-section dependence on energy (6) to Cramer's experimental data (29); see Fig. 37. With these coefficients Eq. 3.29 becomes:

$$\tilde{\sigma}_D = [14.4 - 1.2 \log\Gamma]^2 \times 10^{-16}$$  \hspace{1cm} (3.30)

where $\Gamma = T_n + T_i$ in °K and $\tilde{\sigma}_D$ is in cm$^2$. To evaluate the relative importance of these two processes, the cross-sections are plotted against the total temperature of the two colliding species; see Fig. 38. It is apparent that the transition from polarizing collisions to charge exchange collisions has been completed below room temperature. Charge exchange collisions will be the most important heavy species collisions. Under typical operation conditions, the local pressure and consequently the ion mean free path to radius ratio will vary with arc current density. For an Argon discharge this dependence is tabulated below. In this case the initial fill pressure, at room temperature, was 21.4 pascals (160 microns). This corresponds to a neutral particle concentration of 51.8 $\times$ $10^{20}$ particles/M$^3$. The discharge radius was 2.54 mm.

As is evident from this table (5) or any theoretical gain-per-centimeter comparison with experiment (see Fig. 40, curve labelled $s = 0.546$), the most
Table 5
Comparison of the Arc Discharge Magnitude to the Ion Knudsen Number

\( p = 21 \) pascals, \( r = 2.54 \) mm

<table>
<thead>
<tr>
<th>Arc Current Density A/mm²</th>
<th>Local Neutral Concentration 10²⁰ particles/M³</th>
<th>Ion Knudsen Number ( \lambda_i/r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.03</td>
<td>45.7</td>
<td>0.11</td>
</tr>
<tr>
<td>0.1</td>
<td>35.9</td>
<td>0.14</td>
</tr>
<tr>
<td>0.5</td>
<td>16.3</td>
<td>0.31</td>
</tr>
<tr>
<td>1.0</td>
<td>9.4</td>
<td>0.54</td>
</tr>
<tr>
<td>2.0</td>
<td>5.2</td>
<td>0.97</td>
</tr>
<tr>
<td>3.0</td>
<td>3.5</td>
<td>1.4</td>
</tr>
<tr>
<td>4.0</td>
<td>2.7</td>
<td>1.8</td>
</tr>
<tr>
<td>5.1</td>
<td>2.1</td>
<td>2.3</td>
</tr>
<tr>
<td>6.9</td>
<td>1.6</td>
<td>3.0</td>
</tr>
<tr>
<td>8.0</td>
<td>1.4</td>
<td>3.5</td>
</tr>
</tbody>
</table>

Serious deviations from free fall will occur at the low current densities. Moreover, the condition for free fall

\[
\frac{\lambda_i}{r} \gg 1 \tag{3.31}
\]

is never achieved at any current density, suggesting the use of a modified short-free-path ion current equation may be appropriate. The Tonks-Langmuir paper (8) provides an expression for the short-free-path ion wall current:

\[
j_w = s_1^2 n_e eq \frac{kT_e}{r} \tag{3.32}
\]

where

\[
q = \frac{0.895}{\sqrt{kT_M \lambda_1}} \tag{3.33}
\]

\[
s_1 = 2.405 \tag{3.34}
\]

\[
\lambda_1 = 1/n_0D \tag{3.35}
\]

Rearranging this equation into a form similar to the long free path case we have:
\[ j_w = \frac{\alpha}{\gamma} e^{-n_e \gamma B} \]  

(3.36)

where

\[ \frac{\alpha}{\gamma} = \frac{2.588}{n r \delta_D} \sqrt{\frac{T_e}{T_g}} \]  

(3.37)

For a typical operating condition, \( p = 21.4 \) pascals, \( I_o = 0.6 \) amp and \( r = 2.54 \) mm, the plasma parameters at low arc currents will be:

- \( n \approx 4.57 \times 10^{21} \) P/M³
- \( T_e \approx 36500 \) °K
- \( T_g \approx 340 \) °K
- \( \delta_D \approx 9 \times 10^{-15} \) cm²

where the neutral density is given in Table 5, the neutral temperature is specified by the Chester relation (Eq. 3.14) and the electron temperature base on the T-L model (refer to Fig 29). For these numbers, Eq. 3.37 yields

\[ \frac{\alpha}{\gamma} \approx 2.6 \]

This value is seen to represent almost a five-fold increase in \( \alpha/\gamma \) over the long mean free path value (0.546). The corresponding increase in the PCM gain is even more dramatic since

\[ \text{Gain } \alpha \exp \left( \frac{\alpha}{\gamma} \right) \]

At this stage two factors require emphasis:

1. the \( \alpha/\gamma \) is dependent on the neutral density and this influence has a strong effect on the PCM gain,
2. a quite accurate value is required for the \( \alpha/\gamma \) ratio if a meaningful gain is to be calculated.

Unfortunately the \( \alpha/\gamma \) ratio is not amenable to an exact theoretical treatment. Uncertainties in the values which make up Eq. 3.37, e.g. \( \delta_D \), preclude an evaluation of the \( \alpha/\gamma \) sufficiently accurate for present purposes. Furthermore this approach is restricted to the high fill pressure cases in which

\[ \frac{\lambda_l}{r} \ll 1 \]
if short free path conditions are to be fulfilled. The 21.4 pascal case (160 mtorr) for instance must be considered as transitional at the low arc current values and the use of Eq. 3.37 is not strictly valid.

To circumvent these difficulties, a low current gain study was undertaken to experimentally determine the ratio applicable in the short free path case. The 40 pascal (300 mtorr) case with an ion Knudsen number of

\[ \text{Kn} = \frac{\lambda_i}{r} \approx 0.04 \]

was chosen since it satisfies the short free path criterion. For this pressure an estimate of \( \alpha/\gamma \) parallel to that given above for the 21.4 pascal case yields

\[ \frac{\alpha}{\gamma} \approx 1.3 \]

where

\[ n = 8.6 \times 10^{21} \text{ /m}^3 \]
\[ T_n = 340 \text{ °K} \]
\[ T_e = 32500 \text{ °K} \]

(Fig. 29)

On the other hand, an experiment carried out at very low currents and for \( P = 40 \text{ Pa} \) indicated a value of

\[ \frac{\alpha}{\gamma} = 1.99 \]

The difference between this experimental value and that obtained theoretically, \( \alpha/\gamma = 1.3 \), is small and is well within the uncertainties with which the factors in Eq. 3.37 can be estimated. The experimental value of \( \alpha/\gamma \) (= 1.99) was used in the following to represent the short mean free path value of \( \alpha/\gamma \).

For \( \lambda_i/r = 1 \) (which occurs at 1.8 pascals) it is taken that \( \alpha/\gamma \) assumes the long free path value

\[ \frac{\alpha}{\gamma} = 0.546 \]

Matching these two values through the transition regime requires a complex theoretical treatment which was not attempted here. Rather, the simple assumption was made that \( \alpha/\gamma \) varies linearly with neutral density between these two values. This results in:

37
where \( n \) is in \( M^{-3} \).

\[
\frac{\alpha}{\gamma} = s(n) = 0.478 + 1.56 \times 10^{-22} n
\]  

(3.38)

The positive column equations, 3.8 through 3.14, were then solved for the \( g \) dependence on arc current density using the \( \alpha/\gamma \) defined by Eq. 3.38. Comparison between this model and experiment is made in Fig. 40, where the theoretical result assuming \( \alpha/\gamma = 0.546 \) is also shown.

As can be seen from Fig. 40, the inclusion of the \( \alpha/\gamma \) dependence on local neutral number density brings theory and experiment into general agreement. Indeed it is possible to make the theory and the experiment fit precisely, simply by choosing a suitable nonlinear dependence for \( s(n) \), rather than the arbitrarily assumed linear dependence of Eq. 3.38. Without a proper transitional treatment, however, such curve fitting would prove little. The general result stands however, namely that the dependence of \( \alpha/\gamma \) on \( n \) is a principal influence in the initial rapid fall-off of GPUL at currents up to ~1 amp/mm²; see Fig. 40. The use of Eq. 3.38 for the 40 pascal case, but for significant currents, yielded the result shown in Fig. 39. The agreement between theory and experiment is excellent in this case. That theory and experiment agree at very low arc currents is not significant of course, since the \( g \) at 40 pascals and \( I \approx 0 \) was used to obtain Eq. 3.38. However, the agreement between theory and experiment for larger arc currents is not forced and thus constitutes a further experimental test of the model, and the assumption that \( s \) depends on the local neutral density.

Consider next the 6.7 pascal case illustrated in Fig. 41. Here the experimental form of the \( g \) dependence on arc current curve is reproduced by the model, but the model predicts a higher gain fall-off with decreasing pressure than that found experimentally.

A principal factor in the under-estimate of gain for the 6.7 pascal case can be attributed to the use of the pressure balance relation for neutral density rather than the flux balance relation which becomes increasingly more relevant as pressures are lowered to this region. A recalculation of the theory assuming flux balance raised the predicted \( g \) values by a substantial factor, although primarily at the higher currents.

Considering the lower currents for 6.7 pascals, it must be acknowledged that a further important, and at present unidentified, factor is playing a role. Basically, ion wall currents and thus PCM gains, are anomalously high for Argon compared with other gases. This can be noted by reference to Table 4 in which gains for various gases are compared. This anomaly was also noted in the earlier work of Stangeby and Allen (24), and as indicated earlier in this report, was one of the practical factors influencing the choice of Argon as the principal gas for study. The Tonks-Langmuir theory appears to give values of \( \alpha/\gamma \) which are too low for this particular gas. As was noted in this section, for short mean free paths T-L theory predicts \( \alpha/\gamma = 1.3 \) while a value of 1.99 is experimentally observed. At long mean free paths, as can be noted in Fig. 40, the T-L value of \( \alpha/\gamma = 0.546 \) is clearly too low as well. As with the short mean free path case, it is possible to experimentally establish a value of \( \alpha/\gamma \) for low arc currents and for low pressures, and to then use this in
the model to predict the behaviour for larger arc currents. This particular approach was not pursued further, since the low pressure behaviour of the PCM is of no practical interest due to the very low gains.

To conclude this section, it is found that by inclusion of the dependence of $\alpha/\gamma$ on local neutral density in the model, the high current behaviour of the PCM can be explained in its basic aspects.

3.3.3 Dependence of Gain on Biasing Voltage

The fraction of electrons in the sheath region with sufficient energy to overcome the wall potential will be:

$$\frac{n_w}{n_s} = \int_{\eta_w}^{\infty} f_e^M(\eta)d\eta$$

(3.44)

where $n_w$ is the number at the interface capable of reaching the wall,
$n_s$ is the total number of electrons at the sheath plasma interface,
$\eta_w$ is the potential drop between the interface and wall,
$f_e^M(\eta)$ is the electron energy distribution:

$$f_e^M(\eta) = \frac{2}{\sqrt{\pi}} \eta^{\frac{1}{2}} e^{-\eta}$$

(3.45)

The ion wall current has been shown to be insensitive to changes in the wall potential. Furthermore, for floating walls the total electron current to the wall must just balance the ion current. Depressing the wall potential below its floating value, however, will reduce the number of electrons capable of reaching the wall from the plasma. Electrons injected from an external source will be required to maintain the original balance. The number of electrons required from the external source will depend on the magnitude of the wall potential change. This number is:

$$\frac{2}{\sqrt{\pi}} \left[ \int_{\eta_w}^{\infty} \eta^{\frac{1}{2}} e^{-\eta} d\eta - \int_{\eta_w}^{\infty} \eta^{\frac{1}{2}} e^{-\eta} d\eta \right]$$

(3.46)

where the first term equals the ion wall current and the second term represents the fractional number of electrons reaching the wall when the wall potential is depressed to

$$\eta = 5\eta_w$$

Simplifying Eq. 3.36 and plotting the result:
as a function of $\delta$, Fig. 42 indicates a saturation in the number of electrons which can be injected. Physically this situation implies that all the plasma electrons have been reflected.

From the wall boundary condition we have:

$$\eta_w = \ln \left[ \frac{M}{4\pi m e} \sqrt{\frac{s_w}{e}} \int_0^{\eta(\sigma)} e^{-\eta(\sigma)} d\sigma \right]$$

For the condition in which $B^2 \to \infty$, the wall potential in an Argon discharge has been calculated (8, 51) to be approximately $\eta_w = 5.6$. For decreasing $B^2$, $\eta_w$ is found to increase slightly for a given gas species (51).

Figure 42 indicates that saturation has been completely attained by $\delta \approx 2$ for the $\eta_w = 5$ and $\eta_w = 6$ cases. Experimentally therefore, one should expect a similar gain per unit length versus biasing voltage curve for a wide range of arc current densities. Furthermore the maximum gain per unit length should be achievable by depressing the floating potential by approximately:

$$25 \approx 10 \text{ to } 15 \text{ volts}.$$ 

CHAPTER 4. CONCLUSIONS AND SUMMARY

The Plasma Current Multiplier (PCM) is a simple and robust gas discharge device with the capacity to amplify input currents, both ac and dc. The single most important characteristic of this device is its gain. Previous work in this area suggested the potential of the PCM to multiply the input currents to high levels. However neither the influence of pressure nor current level on the PCM gain was established. Accordingly an extensive experimental program was undertaken to explore the device's operating potential and limitations.

One of the major and unexpected experimental finds of this work was the PCM gain dependence on arc current density. Contrary to the previous theoretical explanation, which predicted a constant PCM gain for all currents, the gain is found to be a decreasing function of arc current density. At low arc current densities (few ma/mm$^2$ for Argon), however, this decrease is small and the proposed exponential behaviour of the Stangeby and Allen model is a reasonable approximation. For larger current densities the rate of gain decrease increases with increasing current densities until in the case of Argon at approximately 7 amp/mm$^2$ the gain per unit arc length approaches unity, i.e. no further gain.

The research on Argon ion lasers was found to be directly applicable to the present work. Unfortunately the model developed by these workers lacks a solid fundamental basis in two respects:
(1) the exact nature of the capillary-reservoir coupling,
(2) the mechanisms responsible for gas heating.

To rectify these shortcomings, empirical relations were deduced from experimental work and introduced into their analyses. In addition electron pressure is not generally included in the pressure balance equation, although this has been criticised on theoretical grounds. Notwithstanding these theoretical difficulties, this model has adequately explained many features of argon ion plasmas. Cognizant of both the limitations of the model and its success in predicting the correct ion laser plasma behaviour, this model has adequately explained many features of argon ion plasmas. Cognizant of both the limitations of the model and its success in predicting the correct ion laser plasma behaviour, this model was applied to the present device to deduce the PCM gain characteristics. As a result, it was established that gain fall-off at high arc currents was primarily due to neutral rarefaction because of local gas heating.

The PCM gain dependence on mass and radius was further tested by extending the range of gas species used. The closely matched pair of H₂ and D₂ provided an especially sensitive and conclusive test of the theory.

The PCM requires some initiating source of electrons. Although this source may be thermionic, filaments have inherent drawbacks:

(1) limited lifetimes in any plasma environment,
(2) unusable in reactive environments.

Accordingly an emissionless source was developed. A microwave discharge provided the required plasma from which electrons could be drawn off by the appropriate introduction of a collecting surface and an accelerating electric field. Continuous operation in the reactive gas O₂ as well as a wide variety of other gases was demonstrated. The combination of such an emissionless source plus the PCM effectively constitutes a power cathode, however, one without the normally associated filament problems. Such power cathodes may find application in fusion energy devices, for instance, where filament replacement in a radioactive atmosphere is inconvenient or operating conditions dictate minimum down time. From the general viewpoint, this series of experiment clearly indicate the separability of the PCM from the initiating source. Consequently any appropriate source can be used providing care is taken to properly interface the two devices.

As with any amplifying device, the PCM may find application as a current control unit, although the gain fall-off does present an obstacle to transforming this device into a HVDC breaker, for example. Experimental work in this direction must explore two avenues:

(1) arrangements to minimize the gain fall-off,
(2) the response of the PCM to the cessation of the emission current while a high arc current density is present in the device.

Positive results in both areas are necessary if the PCM is to be applicable to the high current control field. Further research will be required to focus specifically on the question of introducing extra gas into the rarefied regions of the PCM, if gain fall is to be avoided.
The model used in the present study to describe the PCM behaviour not only predicts the current gain but also a number of other quantities which could, at least in principle, be measured:

- electron temperature
- neutral temperature
- electric field
- electron number density
- neutral number density

The most relevant quantity and also the most difficult to measure is the neutral density. A radial neutral concentration and its dependence on current could be instrumental in providing a more fundamental understanding of the processes taking place in the intense plasma regions of a high current PCM discharge. Such future research would not only further test the model in the present work, but also could provide the solution to the gain-fall-off problem.
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FIGURE 1: TYPICAL OSCILLOGRAM OF ARC CURRENT AND BIASING VOLTAGE IN A MULTIPLYING SEGMENT. DURING THE SWITCHING INTERVAL THE PRE-SELECTED BIASING RESISTOR IS CONNECTED TO THE FLOATING SEGMENT. THE SEGMENT VOLTAGE DROPS WHILE THE TOTAL ARC CURRENT INCREASES.
Figure 2: Vacuum system schematic layout. Details of the PCM in the experimental volume are given in Figures 3 and 5.

Figure 3: Longitudinal cross-section of the current multiplier segment assembly. Composite bore details are given in Figure 19.
FIGURE 5: MICROWAVE EMISSION SOURCE-PCM COUPLING ARRANGEMENT. DETAILS OF THE ION COLLECTOR ASSEMBLY ARE GIVEN IN FIGURE 6. ELECTRON EXTRACTION FIELD APPLIED BETWEEN THE FIRST PCM SEGMENT AND THE COUPLING BLOCK.
FIGURE 6: EXPLODED VIEW OF THE MICROWAVE COLLECTOR

FIGURE 7: AC - POWERED FILAMENT HEATER CIRCUIT
FIGURE 3: DC - POWERED FILAMENT HEATER CIRCUIT

FIGURE 9: HIGH VOLTAGE PCM POWER SOURCE. THE EMERGENCY-ENERGY-DUMP CIRCUIT IS ACTIVATED BY THE DPDT TOGGLE AND THE ELECTROMAGNETIC (E-M) RELAY R1. THE E-M RELAY R2 CONNECTS THE 1.5 KVA POWER SUPPLY (NOT SHOWN) TO THE CAPACITOR BANK. THE CAPACITOR BANK IS CHARGED VIA THIS LINE.
FIGURE 10: SCHEMATIC OF THE PCM ELECTRICAL CIRCUIT. THE CAPACITOR EMERGENCY DUMP CIRCUIT IS DETAILED IN FIGURE 9. DIFFERENT FILAMENT HEATER CIRCUITS ARE SHOWN IN FIGURES 4, 5 AND 6 (MICROWAVE TYPES) AND FIGURES 7 AND 8 (THERMIonic TYPES). THE TRIGGER CIRCUITRY IS GIVEN IN FIGURE 11. THE 1.5 KVA POWER SUPPLY DISCONNECTING RELAY IS NOT SHOWN.
FIGURE 11: SCR-SCOPE TRIGGER CIRCUIT. THE 1.3V SQUARE PULSE IS APPLIED TO EACH SEGMENT SCR THROUGH A 47 OHM RESISTOR. THE PULSE DELAY WAS DETERMINED BY THE CHOICE OF THE RESISTORS IN THE CD4047 CHIPS.
FIGURE 12: ARRANGEMENT USED TO SWITCH A TYPICAL PCM SEGMENT FROM THE IDLING MODE INTO THE MULTIPLICATIVE MODE. TEMPORAL VOLTAGE CHARACTERISTICS ARE GIVEN IN FIGURE 13.

\[ GI_a = I_a + I_{seg} \]

\[ V_{Bias} = I_{seg} R_{seg} \]

To Trigger Circuit
FIGURE 13: TYPICAL BIASING - VOLTAGE TIME CHARACTERISTICS OF FOUR CONSECUTIVE MULTIPLYING SEGMENTS. THE ZERO-REFERENCE IS ARBITRARY FOR EACH FLOATING SEGMENT. X REPRESENTS ANY ARBITRARY MULTIPLYING SEGMENT NUMBER.
FIGURE 14: TYPICAL ARC CURRENT AND BIASING VOLTAGE TEMPORAL CHARACTERISTICS FOR AN OVERBIASED MULTIPLYING SEGMENT.
FIGURE 15: TYPICAL WALL FLOATING POTENTIAL ALONG PCM FROM CATHODE TO ANODE IN ARGON

- $r = 2.54$ mm
- $p = 2$ pascals
- $I = 500$ mA

$E = 4.33$ Volts/cm

$E/p = 2030$ Volts/pascal-cm

(270 Volts/torr-cm)
FIGURE 16: DEPENDENCE OF SEGMENT GPUL ON BIASING VOLTAGE AT VARIOUS ARC CURRENTS IN A 21 PASCAL ARGON DISCHARGE \( r = 2.54 \text{ mm} \).

The maximum experimental error was ± 10%.
FIGURE 17: DEPENDENCE OF SEGMENT GPUL ON BIASING RESISTANCE AT VARIOUS ARC CURRENTS IN A 21 PASCAL ARGON DISCHARGE \( r = 2.54 \text{ mm} \).

The maximum experimental error was \( \pm 10\% \).
FIGURE 18: GAIN PER UNIT LENGTH DEPENDENCE ON SEGMENT RADIUS IN A NEUTRAL ARGON PRESSURE OF 16 PASCALS WITH AN INPUT CURRENT OF 600 MA. THE EXPERIMENTAL ERROR = ± 5%.

FIGURE 19: CROSS-SECTION OF A COMPOSITE BORE SECTION
FIGURE 20: PCM GAIN DEPENDENCE ON INITIAL CURRENT IN A 16 PASCALS ARGON DISCHARGE \( r = 2.54 \text{ mm} \).
FIGURE 21: THE DEPENDENCE OF THE CROSS-SECTIONALLY AVERAGED TO CENTRELINE ELECTRON CONCENTRATION RATIO ON THE ARC CURRENT DENSITY IN AN ARGON DISCHARGE. (AFTER PARKER 50)
FIGURE 22: CUMULATIVE GAIN DEPENDENCE ON THE PCM LENGTH FOR TWO DIFFERENT INITIAL CURRENTS, 13 PASCAL ARGON FILL PRESSURE, RADIUS = 2.54 MM. TOTAL EXPERIMENTAL UNCERTAINTY ± 10%.
FIGURE 23: COMPARISON OF THE GAIN PER UNIT LENGTH OVER A RANGE OF ARC CURRENT DENSITIES FOR THE 13 PASCAL ARGON, 2.54 MM BORE CASE. THE EXPERIMENTAL ERROR IS ± 2%. 

○ $I_0 = 0.5$ amp
△ $I_0 = 1.0$ amp

ARC CURRENT DENSITY (Amp/mm$^2$)
FIGURE 24: CUMULATIVE GAIN DEPENDENCE ON NEUTRAL ARGON FILL PRESSURES IN A 2.54 MM BORE DISCHARGE TUBE. EXPERIMENTAL UNCERTAINTY = ± 10%.

- △,○ REPRESENTS AVERAGE OF 8 MEASUREMENTS
- △,△ SINGLE MEASUREMENTS
- ○ REPRESENTS AVERAGE OF 16 MEASUREMENTS
- $I_0 = 0.5$ Amps EXCEPT AS NOTED
FIGURE 25: EXPERIMENTAL GAIN PER UNIT LENGTH DEPENDENCE ON ARC CURRENT DENSITY FOR THE 1.5 AND 6.7 PASCAL ARGON FILL PRESSURE CASES, RADIUS = 2.54 MM, EXPERIMENTAL UNCERTAINTY = ± 2%.
FIGURE 26: EXPERIMENTAL GAIN PER UNIT LENGTH DEPENDENCE ON ARC CURRENT DENSITY FOR THE 21 AND 40 PASCAL ARGON FILL PRESSURE CASES, RADIUS = 2.54 MM. EXPERIMENTAL UNCERTAINTY = ± 2%.
FIGURE 27: GAIN COMPARISON AT TWO HIGH FILL PRESSURES FOR AN ARGON DISCHARGE

\( r = 2.54 \text{ mm.} \quad I_o = 0.5 \text{ amps} \)

- 67 PASCALS
- 30 PASCALS

EXPERIMENTAL DATA \( \pm 10\% \)

AXIAL DISTANCE (cm.)

<table>
<thead>
<tr>
<th>CUMULATIVE CURRENT GAIN ( \frac{I_a}{I_o} )</th>
<th>0</th>
<th>1</th>
<th>3</th>
<th>6</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>AXIAL DISTANCE (cm.)</td>
<td>0</td>
<td>10</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
FIGURE 28: EFFECT OF GAS FLOW THROUGH PCM ON THE TOTAL GAIN IN ARGON

$r = 2.54$ mm.

$I_o = 0.5$ amps

$p = 60$ pascals

- DIFFUSIVE FLOW THROUGH PCM
- X FORCED FLOW FROM ANODE TOWARD CATHODE

EXPERIMENTAL UNCERTAINTY $\pm 10\%$
FIGURE 29: ELECTRON TEMPERATURE DEPENDENCE ON THE REDUCED RADIUS IN ARGON (EQUATION 3.1)
FIGURE 30: PREDICTED ELECTRON TEMPERATURE DEPENDENCE ON THE ARC CURRENT DENSITY FOR VARIOUS ARGON FILL PRESSURES, RADIUS = 2.54 MM.
FIGURE 31: PREDICTED ELECTRON NUMBER DENSITY DEPENDENCE ON ARC CURRENT DENSITY FOR VARIOUS ARGON FILL PRESSURES. RADIUS = 2.54 MM.
FIGURE 32: PREDICTED IONIZATION FRACTION OF ARGON PLASMA AT VARIOUS FILL PRESSURES. RADIUS = 2.54 MM.
FIGURE 33: CURRENT FLUX THROUGH AN ELEMENT OF PCM
FIGURE 34: COMPARISON OF FOUR THEORETICAL MODELS WITH EXPERIMENT. INFINITE ION MEAN FREE PATH IS ASSUMED IN ALL MODELS. EXPERIMENTAL CONDITIONS ARE 21 PASCAL ARGON FILL PRESSURE AND 2.54 MM TUBE RADIUS. EXPERIMENTAL UNCERTAINTY IS ± 2%.

$O = $ EXPERIMENT
$C = $ CHESTER RELATION FOR $T$
$H = $ HERZIGER RELATION FOR $T$
$\phi = $ FLUX BALANCE FOR PARTICLES
$P = $ PRESSURE BALANCE FOR NEUTRALS

GAIN PER CENTIMETER

ARC CURRENT DENSITY (Amp/mm²)
FIGURE 35: COMPARISON OF THE PREDICTED ELECTRON TEMPERATURE DEPENDENCE ON THE ARC CURRENT DENSITY FOR THE FOUR THEORETICAL MODELS. TUBE RADIUS = 2.54 MM. ARGON FILL PRESSURE = 21 PASCALS.

- $O =$ EXPERIMENT
- $C =$ CHESTER RELATION FOR T
- $H =$ HERZIGER RELATION FOR T
- $\phi =$ FLUX BALANCE FOR PARTICLES
- $P =$ PRESSURE BALANCE FOR NEUTRALS

**Graph Details:**
- **Y-axis:** Electron Temperature ($10^3$ K)
- **X-axis:** Arc Current Density (Amp/mm$^2$)
- **Lines:**
  - $PH$
  - $PC$
  - $\phi H$
  - $\phi C$
- **Note:** Low current electron temperature predicted by T-L theory.
FIGURE 36: PRESSURE BALANCE MODELS ARE EXTENDED TO INCLUDE ELECTRON AND ION PRESSURE AND THEIR EFFECT ON THE GAIN IS COMPARED WITH EXPERIMENT FOR THE 21 PASCAL ARGON FILL PRESSURE CASE. RADIUS = 2.54 MM. THE THEORETICAL MODELS ASSUME AN INFINITE ION MEAN FREE PATH. EXPERIMENTAL ERROR = ± 2%.
FIGURE 37: ENERGY DEPENDENT CROSS-SECTION FOR CHARGE EXCHANGE COLLISIONS IN ARGON (REFERENCE 29)

\[ \sqrt{\sigma(\epsilon)} = A - B \log_{10}(\epsilon) \]

\(\sigma\) = EXPERIMENT (CRAMER, 1959)

A = 6.8 \times 10^{-8} \text{ cm.}

B = 0.85 \times 10^{-8} \text{ cm.}

FIGURE 38: ION-NEUTRAL MOMENTUM TRANSFER CROSS-SECTION IN ARGON
FIGURE 39: COMPARISON OF THE PREDICTED AND EXPERIMENTAL GAIN PER UNIT LENGTHS. THE MODEL ASSUMES PRESSURE BALANCE OF NEUTRALS ONLY, AND FINITE ION MEAN FREE PATH. THEORY IS MATCHED TO EXPERIMENT AT 0.02 AMP/MM². FILL PRESSURE = 40 PASCALS (300 TORR) ARGON. RADIUS = 2.54 MM. EXPERIMENTAL ERROR IS ± 2%.
FIGURE 40: COMPARISON OF THE GAIN DERIVED FROM THE
FINITE AND INFINITE ION MEAN FREE PATH MODELS FOR
THE 21 PASCAL ARGON FILL PRESSURE CASE. RADIUS = 2.54
MM. MODELS ASSUME PRESSURE BALANCE OF NEUTRALS ONLY
AND CHESTER'S NEUTRAL TEMPERATURE RELATION. EXPERI­
MENTAL ERROR IS ± 2%.
FIGURE 41: COMPARISON OF THE THEORETICAL GAIN WITH EXPERIMENT FOR THE 6.7 PASCAL ARGON FILL PRESSURE CASE. RADIUS = 2.54 MM. MODEL ASSUMES: NEUTRAL PRESSURE BALANCE, CHESTER'S RELATION AND FINITE ION MEAN FREE PATH. EXPERIMENTAL ERROR IS $\pm$ 2%. 

- **GAIN PER CENTIMETER**
  - 1.14
  - 1.10
  - 1.05
  - 1.00

- **ARC CURRENT DENSITY (Amp/mm$^2$)**
  - 0
  - 1
  - 2
  - 3

- **Graph**
  - **○** EXPERIMENT
  - **-** THEORY
FIGURE 42: PREDICTED SEGMENT GAIN DEPENDENCE ON THE WALL BIASING
POTENTIAL FOR VARIOUS FLOATING WALL POTENTIALS IN AN ARGON DISCHARGE.
$\eta_w$ IS THE DIMENSIONLESS WALL POTENTIAL ($= \frac{\eta}{kT}$).

FIGURE 43: DIMENSIONLESS RADIAL POTENTIAL PROFILE FOR VARIOUS ARC CURRENT DENSITIES IN ARGON. (AFTER PARKER 1964)
$n_{eo} = \text{AXIAL ELECTRON CONCENTRATION}$

$s_{w} = \text{DIMENSIONLESS WALL LOCATION}$

**Figure 44:** RADIAL ELECTRON CONCENTRATION PROFILE FOR AN ARGON PLASMA AT VARIOUS ARC CURRENT DENSITIES.
FIGURE 45: ENERGY DEPENDENT MOMENTUM TRANSFER CROSS-SECTION OF ELECTRONS IN ARGON

○ EXPERIMENTAL DATA BY BARBIERE (ref. 10)
(SLOPE = $1.44 \times 10^{-16}$ cm$^2$/eV)

SEMI-EMPIRICAL CROSS-SECTION OF FLETCHER AND BURCH
ref. 11
FIGURE 46: ELECTRON IMPACT IONIZATION CROSS-SECTION FOR ARGON

○ EXPERIMENTAL DATA BY RAPP & GOLDEN

(SLOPE = 0.1345 Å²/eV)
Figure 47: Gas Pumping in the PCM

Case A (Closed Anode Port)

Case B (Opened Anode Port)

Figure 48: Gain Dependence on Intersegment Flow. Glass spacers around the test segment were removed to accommodate the intersegment flow. Input arc current density was 2.5 AMP/MM².
FIGURE 49: EXCITATION CROSS-SECTION OF METASTABLE ARGON

EXPERIMENTAL RESULTS (ref. 23)
LINEAR APPROXIMATION

FIGURE 50: ABSOLUTE EXCITATION CROSS-SECTION OF THE TWO LOWEST RESONANT LEVELS OF ARGON (ref. 19)
FIGURE 51: IONIZATION CROSS-SECTION OF METASTABLE ARGON AFTER VRIENS (ref. 28)

LINEAR APPROXIMATION AFTER VRIENS (ref. 28)

FIGURE 52: GAIN DEPENDENCE ON CUMULATIVE IONIZATION FOR THE 21 PASCAL ARGON FILL PRESSURE CASE, RADIUS = 2.54 MM. THEORETICAL MODEL INCLUDES NEUTRAL PRESSURE BALANCE, CHESTER'S TEMPERATURE RELATION AND AN INFINITE ION MEAN FREE PATH.
FIGURE 53: SCHEMATIC OF A TYPICAL IDLING PCM. IN THIS MODE ANODE CURRENT EQUALS THE EMISSION CURRENT.

FIGURE 54: AXIAL VOLTAGE PROFILE ALONG THE CENTERLINE OF A CYLINDRICAL PCM. THE CATHODE FALL IS DEPENDENT ON THE TOTAL APPLIED VOLTAGE. THE POSITIVE COLUMN LENGTH IS ARBITRARY.
FIGURE 55: THE RADIAL VOLTAGE PROFILE OF A TYPICAL CYLINDRICAL PCM SEGMENT FOR THE LONG ION MEAN FREE PATH CASE. THE DIMENSIONLESS VOLTAGE IS THE RATIO OF THE MEASURED VALUE TO THE LOCAL PLASMA POTENTIAL. THE SHEATH REGION HAS BEEN ENLARGED FOR CLARITY.
FIGURE 56: SCHEMATIC OF A TYPICAL MULTIPLYING PCM
APPENDIX A

REVIEW OF THE TONKS-LANGMUIR MODEL

A.1 Introduction

The analysis of the low pressure positive column originally developed by Tonks and Langmuir was published in 1929. The prevalent view at the time assumed that both the electrons and ions could be characterized by Maxwellian distributions with temperatures orders of magnitude higher than the neutral temperature. In contrast to this picture, the authors proposed a model in which the ionic motion was orderly and highly non-Maxwellian. The ions, assumed to be at rest upon formation, were drawn towards the non-conducting, confining walls under the influence of internal electric fields. During the initial stages of establishing a positive column, more electrons than ions reach the wall. A negative potential, relative to the centreline potential, develops on the wall to restrict the number of electron arrivals. Assuming the wall is electrically isolated from the ground, the wall potential stabilizes at a value which permits just enough electron current to balance the ion current. Thus the internal fields arise from the requirement that the net current to the wall vanish. Once at the wall, the ions are neutralized, thermally accommodated and re-emitted into the discharge as cold neutrals. The background gas is taken to be spatially isotropic and homogeneous and in thermal equilibrium with the walls. The walls are assumed to be at room temperature. At the instant of formation the ions are considered to have zero velocity in comparison to that attained upon arrival at the sheath's edge. The concomitant error in the calculated radial potential distribution as a result of this approximation is shown to be of the order of the ratio of the neutral to electron temperatures ($T_n/T_e$), in the long mean free path case (8). The neutral concentration in this case is low, permitting the ions access to the walls without any intermediate heavy particle collisions. This condition is usually termed "free-fall". The charge production mechanism assumed by this model is electron impact ionization:

$$e + A \rightarrow A^+ + 2e$$

Assuming low neutral pressures and arc current densities, this model neglects cumulative ionizations.

The mathematical description of the T-L model of the positive column begins with evaluation of the local electron and ion number densities. These in turn are fed into the Poisson equation, the solution of which yields the radial potential profile. Given this profile, the macroscopic characteristics of the arc discharge, wall potential, arc current, ion wall current, are calculable. One of the important contributions of this theory arises from the nondimensionalization procedure of the plasma-sheath equation. In the process of generalizing this equation, a second equation is constructed: the plasma-balance equation. Physically this relation equates the volume ion generation rate to the ion loss rate at the plasma boundary. Mathematically this equation provides the last outstanding equation necessary to completely determine all the positive column parameters in closed form. If the tube radius $r$, the neutral fill pressure $p$, the boundary wall temperature $T$, the gas species and the arc current density comprise the independent variables, the dependent variables number five in total:
the longitudinal electric field $E$, the axial electron concentration $n_{eo}$, the electron temperature $T_e$, the ion current density at the wall $j_w$, and the local ion generation rate $R(r)$. Assuming only single ionization events are possible, $R(r)$ also specifies the electron generation rate. Thus five equations are required for complete solution of the problem:

- the ion wall current
- the plasma-balance equation
- the energy balance equation
- the mobility equation
- the ion generation equation.

### A.2 The Plasma Balance and Ion Current Equations

Given a Maxwellian electron distribution, the electron concentration across a cylindrical positive column will follow the Boltzmann distribution. Since the collision integral must be zero and for equilibrium there can be no time dependence, the Boltzmann equation reduces to

$$\nabla \cdot \frac{\partial f}{\partial x} + \frac{F}{m} \cdot \frac{\partial f}{\partial v} = 0$$  \hspace{1cm} (A.1)

where symbols have their usual meaning, and

$$f_m = n(r) \left( \frac{m}{2\pi kT_e} \right)^{3/2} e^{-\eta}$$

is the velocity distribution

$$\eta = \frac{m v^2}{kT_e}$$  \hspace{1cm} (A.2)

$$n(r) = \int \int \int f(\vec{r}, \vec{v}) d\vec{v}$$

is the electron spatial distribution. The Boltzmann equation becomes

$$n_e(r) = n_{eo} e^{-eV(r)/kT_e}$$  \hspace{1cm} (A.3)

where

$$\vec{F} = \nabla [V(r)]$$

The electron density at any radial position ($r$) will be determined exclusively by the axial concentration, and the reduced local potential $\eta$ [$= eV(r)/kT_e$].

If $\xi$ represents the number of ions generated by an electron in one second, and $n_e(r)$ is the local electron number density, then

$$R(r) = \xi n_e(r)$$  \hspace{1cm} (A.4)
will be the local number of ions generated per second and per unit volume at radius \( r \).

An ion generated at \( r' < r \), and moving radially outward, will pass the radius \( r \) with a velocity

\[
v(r) = \sqrt{\frac{2e}{M} [V(r') - V(r)]}
\]

or defining

\[
\eta(r) = -\frac{eV(r)}{kT_e}
\]

\[
v(r') = \sqrt{\frac{2kT_e}{M} [\eta(r) - \eta(r')]}
\]

The contribution to the ion density at \( r \) will be inversely proportional to \( v(r') \), and proportional to the number of ions generated in the region between \( r' \) and \( r' + dr' \). This region contributes \( 2\pi r' n_e(r') dr' \) ions per second. The total number density of ions at \( r \) will be:

\[
2\pi r n_i(r) = \int_0^r 2\pi \xi n_e(r') \frac{r'dr'}{v(r')}
\]

\[
n_i(r) = \frac{1}{r} \int_0^r \frac{\xi n_e(r') r'dr'}{v(r')}
\]

\[
n_i(r) = \sqrt{\frac{M}{2kT_e}} \frac{1}{r} \int_0^r \frac{\xi n_e(r') r'dr'}{\sqrt{\eta(r) - \eta(r')}}
\]  

Having determined the radial electron and ion number density functional relations, the authors (T-I) proceeded to solve the Poisson equation for the radial potential profile.

\[
\nabla \cdot \nabla W = -\frac{e}{\varepsilon_0} [n_i(r) - n_e(r)]
\]

In cylindrical coordinates:
Introducing the dimensionless distance $s$,

$$s = \xi \sqrt{\frac{M}{2kT_e}} r$$  \hspace{1cm} (A.7)

The above reduces to:

$$\frac{1}{s} \frac{\partial}{\partial s} \left[ s \frac{\partial \eta(s)}{\partial s} \right] = B^2 \left[ \frac{1}{s} \int_{0}^{s} \frac{e^{-\eta(\sigma)} \sigma d\sigma}{\sqrt{\eta(s) - \eta(\sigma)}} - e^{-\eta(s)} \right]$$  \hspace{1cm} (A.8)

where

$$B^2 = \frac{2e^2 n_{eo}}{\epsilon_0 \xi M}$$  \hspace{1cm} (A.9)

$$\sigma = \xi \sqrt{\frac{M}{2kT_e}} r'$$

The solution of this equation must proceed from the cylindrical axis of symmetry where

$$\eta = 0 \quad \text{at} \quad s = 0$$

and

$$\frac{\partial \eta}{\partial s} = 0 \quad \text{at} \quad s = 0$$

outward to the confining electrically floating wall. At this point the net wall current must be identically zero, thus providing the third boundary condition. The total number of ions reaching the wall area per unit time will be:

$$2\pi s_w J_w = 2\pi \int_{0}^{s_w} e^{\xi \sqrt{\frac{M}{2kT_e}}} \frac{n_e \sigma d\sigma}{\xi \sqrt{\frac{M}{2kT_e}}}$$

or

$$J_w = \frac{e}{s_w} \int_{0}^{s_w} \sqrt{\frac{2kT_e}{M}} n_{eo} e^{-\eta(\sigma)} \sigma d\sigma$$  \hspace{1cm} (A.10)
where \( j_w \) is the ion current density at the wall and

\[
\eta_w = \frac{\xi r_w}{\sqrt{2kT_e/M}}
\]

since the electron current density at \( s_w \) will be

\[
\eta_e = n_e \frac{e}{2m_e} \sqrt{\frac{kT_e}{e}}
\]

The third boundary condition will be satisfied when

\[
\frac{4\pi}{M} \int_0^{s_w} \frac{\eta(s)}{s_w} e^{-\eta(\sigma)} d\sigma = 1
\]

(A.11)

A typical cylindrical positive column consists of two regions: the central plasma and the enveloping sheath. The plasma is characterized by near charge neutrality and small radial electric fields. The sheath on the other hand has strong electric fields which repel all but the most energetic electrons back into the plasma. This region consists primarily of positive charged particles. The sheath thickness is approximately ten Debye lengths (5), where the Debye length is defined as (5):

\[
\delta_D = 6\sqrt{\frac{T_e}{n_e}} m
\]

in MKSA units. For typical positive columns, the sheath thickness is usually a small fraction of the discharge radius, except for very small arc current densities (few mA/mm² range). The details of the sheath thickness dependence on arc current density will be discussed later in this section.

Recognizing these general features of the positive column, the authors (T-L) argued the Poisson differential term of Eq. 1.7 must be negligible for the plasma region:

\[
\frac{1}{B^2} \frac{1}{s} \frac{\partial}{\partial s} \left[ s \frac{\partial}{\partial s} [\eta(s)] \right] \approx 0
\]

and solved the simpler integral equation:

\[
\frac{1}{s} \int_0^s \frac{e^{-\eta(\sigma)}}{\sqrt{\eta(s) - \eta(\sigma)}} d\sigma - e^{-\eta(s)} = 0
\]

for the radial potential profile by a power series expansion of \( s \) in terms of \( \eta \). The electric field \((\partial/\partial s)\) however must remain finite for all radial positions. Violation of this condition will reflect that the contribution of the Poisson differential term can no longer be neglected. Physically the
authors interpreted this criterion as representing the maximum radial extent of the plasma. Consequently, the physical concept of a plasma-sheath interface could be mathematically located at the value of $s$ at which $\frac{\partial s}{\partial \eta} = 0$. For the case of long mean free paths with ion generation proportional to electron density and in cylindrical configuration, Tonks and Langmuir computed this critical condition to occur at $s = s_0 = 0.7722$ and $\eta = \eta_0 = 1.155$.

Assuming negligible contribution to the ion current outside the plasma region, the authors computed the ion current (1.10):

$$j_w = \frac{e}{s_0} \int_0^{s_0} \sqrt{2} v_B n_{e0} e^{-\eta(\sigma)} d\sigma$$

(A.12)

$$j_w = \frac{s_0}{\sqrt{2}} e n_e v_B$$

where $n_e$ is the cross-sectional averaged electron number density and $v_B$ is the Bohm speed ($= \sqrt{\frac{kT_e}{m}}$).

To the extent that the sheath thickness is negligible, the value $s_0$ may be identified with the discharge radius according to the relation (see Eq. A.7):

$$\frac{r_w}{s_0} = \sqrt{2} \frac{v_B}{\xi}$$

(A.13)

Rearranging this equation into the form

$$\xi r_w = \sqrt{2} s_0 v_B$$

(A.14)

T-L noted that this relation specifies the average ionization rate of neutrals by electron impact. This is the plasma balance equation. It prescribes a unique temperature to the plasma electrons once the discharge parameters: radius, neutral number density and mass species are given.

Since $e n_e \pi r_w^2$ represents the total volume charge generated by electron ionization per unit length of positive column, and $2\pi r_w j_w$ the total charge loss per unit column length, equilibrium will exist if

$$e n_e \pi r_w^2 = 2\pi r_w j_w$$

(A.15)

Using the ion equation (A.12), this equality reduces to Eq. (A.14). Physically, therefore, the plasma balance equation states that volume ion generation must exactly balance ion wall losses.

If $<v_{ei}>$ is the velocity averaged electron impact collision cross-section for ionization, then

$$\xi = n <v_{ei}> \eta s_I$$

(A.16)
where \( n \) is the local neutral concentration. Equation A.15 becomes:

\[
e_n n <v_{ei}^e> r_w = 2j_w \tag{A.17}
\]

the plasma balance equation in its most useful form.

Implicit in the T-L approach to solve the integro-differential equation is the requirement of a plasma sufficiently dense to restrict the sheath region to a small fraction of the discharge radius. An exact solution for the complete collisionless plasma-sheath equation which describes the cylindrical positive column (Eq. A.8) was published in 1964 (51). Following the example set by Tonks and Langmuir, Parker expanded each of the terms of the complete plasma-sheath equation in terms of a power series of the form:

\[
s^2 = \frac{n}{a} - \frac{b}{a} \left( \frac{n}{a} \right)^2 + ... \tag{A.18}
\]

and numerically integrated point from the centre outward to the vessel walls. The radial potential profile is improved by successive iterations until the projected boundary potential agrees with that obtainable from the third boundary condition within prescribed error limits. The nondimensional potential profiles for different degrees of ionization obtained by Parker have been reproduced in Fig. 43. The curve labelled \( B^2 = \infty \) corresponds to the Tonks-Langmuir case. From these profiles, the radial electron number density distribution is readily obtainable by use of Eq. A.3. These profiles have been calculated for the case of an Argon plasma and are illustrated in Fig. 44.

The electron number density has been scaled with respect to the axial concentration \( (n_{eo}) \). The independent distance parameter \( (s) \) has also been scaled with respect to the wall radius \( (s_w) \), where from Eq. A.13,

\[
s_w = r_w \frac{\xi}{\sqrt{2} v_B} \tag{A.18}
\]

or

\[
s_w^2 = \left( \frac{r_w}{\xi_B} \right)^2 \frac{1}{B^2} \tag{A.19}
\]

\( \xi_B \) being the Debye length corresponding to the electron number density on axis. As expected a priori, the sheath thickness grows with decreasing electron concentration, i.e. with decreasing \( B^2 \). Physically this means the plasma discharge radius no longer approximates the tube dimensions but must be corrected for the thickening sheath. Herein is embedded a second problem. As the sheath grows the plasma-sheath transition becomes less abrupt and for conditions with \( B^2 \) less than about \( 10^4 \) the interface concept becomes progressively more meaningless. The electron concentration changes increasingly more gradually and smoothly across the column. As a bench marker, \( B^2 \approx 10^5 \) corresponds to a 15 ma/mm² arc current density in a 2.54 mm radius tube in Argon at a fill pressure of 21.3 pascals (160µ). The corresponding electron temperature is approximately 35500°K (see Fig. 29).
A.3 Energy Balance Equation

The input energy per unit time and length of positive column \( m^2 j_0 E \) joules/sec-m must counter-balance all of the energy loss channels. Some of the more important loss channels are:

1. **Ions gain:**
   \[ e(V_s - V_w) \text{ joules} \]  
   \[ (A.20) \]
   in falling from the plasma sheath boundary, at a potential \( V_s \), to the wall, at a potential \( V_w \). This kinetic energy is deposited on the floating wall.

2. **Ions release:**
   \[ eV_i \text{ joules/ion} \]  
   \[ (A.21) \]
   at the wall surface upon recombining with electrons.

3. **Electrons strike the wall with an average energy of** \( 2kT_e \) joules/electron.

4. **The energy per unit column length carried to the walls by metastables will be**
   \[ 2n j_m e V_m \text{ joules/sec-m} \]  
   \[ (A.22) \]
   where \( j_m \) is the wall metastable flux and \( V_m \) the metastable potential energy.

5. Excited atoms radiate energy out of the plasma.

6. **Excited atomic states are de-excited by wall collisions.**

7. "Hot" electrons transfer energy by elastic collisions to both ions and neutrals. This energy is dissipated at the wall during the thermal accommodation interval.

The first three of the above mechanisms were treated by Tonks and Langmuir (8), and shown to be dependent on the electron temperature and the gas species. If \( j_w \) is the ion wall current, these loss modes may be expressed in the form:

\[ 2n j_w r \left[ V_i + (V_s - V_w) + \frac{4}{3} \frac{V}{V_i} \right] \]

\[ = 2n j_w r \left[ \frac{kT_e}{e} \ln \left( \frac{1}{2s o h_o} \sqrt{\frac{M}{m_e}} \right) + 1.7 \right] + V_i \]  

\[ (A.23) \]

\[ = 2n j_w (aT_e + V_i) \]  

\[ (A.24) \]

where \( a = 6.336 \times 10^{-4} \) for Argon, \( V_i = 15.7 \) eV, \( s_o, h_o \) are constants treated in Section 3.3.2.
Banks (6) derives a general equation for the rate of energy transfer between two colliding partners undergoing an elastic encounter. If $E_g$ is the mean gas energy and $n_{ea}$ the energy-averaged momentum transfer collision frequency, the rate at which the cold neutral gas gains energy from a Maxwellian electron cloud will be:

$$\frac{dE_g}{dt} = 3n_e \frac{m_e}{(m_e + M)^2} k(T_e - T_a) n_{ea}$$

$$= 4n_e n_a \frac{m_e}{M} k c \tilde{Q}_{ea}(T_e - T_a) \text{ joules/m}^3\text{-sec} \quad (A.25)$$

where

$$n_{ea} \triangleq \frac{4}{3} c \tilde{Q}_{ea} \quad (6) \quad (A.26)$$

and $\tilde{Q}_{ea}$ is the electron-atom collision cross-section for momentum transfer, averaged over the Maxwellian electron velocity distribution.

Using a similar approach to that given in the case of neutral gas heating, Banks also shows the energy exchange rate between electrons and ions both having a Maxwellian distribution to be of the form (6):

$$\frac{dE_i}{dt} \approx 3n_e \frac{m_e}{M} kT_e c \tilde{Q}_{ei} \text{ watts/m}^3 \quad (A.27)$$

where

$$\tilde{Q}_{ei} = 6\pi b_0^2 (\ln \Lambda - 1.37)$$

$$b_0 = 5.56 \times 10^{-6} \frac{T_e}{T_e}$$

$$\Lambda = 1.24 \times 10^7 \frac{T_e^3}{n_e}$$

Therefore,

$$\frac{dE_i}{dt} \approx 2.04 \times 10^{-33} \frac{n_e^2}{\sqrt{T_e}} (\ln \Lambda - 1.37) \text{ watts/m}^3 \quad (A.28)$$

Herziger and Seelig (17) show that radiation loss from the positive column of a cylindrical configuration, if Doppler broadening is dominant, will be of the form

$$\frac{E}{m e^2 n_e^2 \tilde{Q}_{re} \tilde{V}} \text{ watts/m} \quad (A.28)$$

where $\tilde{V}$ is the mean excitation voltage of all the excited states considered, and $\tilde{Q}_{re}$ is the average sum of inelastic cross-sections averaged over the
APPENDIX B

MOBILITY EQUATION

B.1 Introduction

Both Mitchner (5) and Von Engel (9) treat the phenomenon of electron mobility in weakly ionized gases in some detail. A summary of the concepts of mobility based on their work is presented in this section. Charged particles in a plasma subject to an electric field ranging from weak to moderately strong, will experience a diffusive motion, with the electrons drifting in a direction opposite to the ions and counter to the field direction. In the mean mass velocity reference frame, the instantaneous electron velocity will obey

\[ \frac{d\vec{v}_e}{dt} = - e \vec{E} + \vec{F}_e(t) \]  

(B.1)

where \( \vec{F}_e(t) \) represents the effective drag force applied to the electrons through collisions with the other plasma constituents. On the average, the effect of this force may be approximated as the average rate of momentum exchange between the collision partners.

\[ \vec{F}_e = \frac{1}{\tau} \int_0^\tau \vec{F}_e(t) dt \]  

(B.2)

\[ \vec{F}_e \approx -m_e [\nu_{ei} (\vec{u}_e - \vec{u}_i) + \nu_{en} (\vec{u}_e - \vec{u}_n)] \]  

(B.3)

where \( \tau \) is a time interval much larger than \((\nu_{ei} + \nu_{en})^{-1}\), \( \nu_{ei} \) is the average momentum transfer collision frequency between electrons and ions, \( \vec{u}_x \) the drift velocity of specie \( x \). For plasmas of interest both \( \vec{u}_i \) and \( \vec{u}_n \) are negligible in comparison to the electron drift velocity \( \vec{u}_e \), thereby simplifying Eq. B.3 to

\[ \vec{F}_e \approx -m_e \nu_e \vec{v}_e \]

where

\[ \nu_e = \nu_{ei} + \nu_{en} \]  

(B.4)

If the average electron acceleration in the field is zero, the average momentum lost per collision must just balance that gained during the collision interval \( (\nu_e^{-1}) \).

\[ eE = -m_e \nu_e \vec{u}_e \]  

(B.5)

where the electric field vector is antiparallel to the electron drift velocity vector. The electron current density is defined as:
\[ j_2 = -e n_e u_e = e n_e \mu E = \sigma E \]  

(B.6)

where the electron mobility \( \mu \) is

\[ \mu = \frac{e}{m_e v_e} \]  

(B.7)

and the electron conductivity \( \sigma \) is

\[ \sigma = \frac{2 e n_e}{m_e v_e} \]  

(B.8)

For cold plasmas such as positive columns (8):

\[ T_e \gg T_i \]

\[ n_e \approx n_i \]

These conditions make the ion mobility negligible in comparison to the electron mobility. The arc current density therefore is essentially equal to the electron current density:

\[ j_a = \frac{e^2 n_e}{m_e v_e} \]  

(B.9)

The mean free path formulation of the mobility equation (B.9) is generally accurate to within a factor of two or three (5). Fundamental to this approach lies the assumption that the sum of the mean electron-ion and mean electron-neutral momentum transfer cross-sections is identical to the mean of the sum of the electron-ion and electron-neutral cross-sections, i.e.,

\[ \bar{Q}_{ea} + \bar{Q}_{ei} = \bar{Q}_{ea} + \bar{Q}_{ei} \]  

(B.10)

With reference to Eq. (B.2) the average electron momentum loss is more correctly written as:

\[ \bar{p}_e = -m_e \bar{\nu} u_e \]  

(B.11)

where once again the velocity of the other species is taken to be negligible in comparison to the electron velocity, and \( \bar{\nu} \) is the average collision frequency for momentum transfer collisions in the mixture of neutrals and
ions. The velocity dependent collision frequency is by definition (5):

$$v_{12}(g) = n_2 \ g \ Q_{12}(g) \quad \text{(B.12)}$$

where \( n_2 \) is the target particle concentration, \( g \) the relative encounter speed \( (\approx c_e) \), and \( Q_{12}(g) \) the speed dependent probability for encounter. Prior to any averaging procedure to obtain a mean collision frequency, a summation over all field particle types will be necessary. For such mixtures, the average collision frequency can be shown to be:

$$\bar{v} = \frac{R}{n_e} = c_e \int_0^\infty \eta e^{-\eta} \sum_s n_s Q_{es} \ d\eta \quad \text{(B.13)}$$

where \( R \) is the reaction rate for the process, \( n_e \) the electron concentration, \( c_e \) the mean electron speed, and \( \eta_T \) the dimensionless threshold energy \( (= eT_e/kT_e) \). For a discussion on the construction of this equation for the case of a single species, refer to Appendix C.

A more accurate method of calculating the electron conductivity requires the use of the Boltzmann equation instead of the mean free path formulation. For the case of a weakly ionized gas, the first Chapman-Enskog approximation of this more rigorous approach produces a relatively simple formula for the conductivity (5):

$$\sigma = \frac{4\pi}{3} \frac{n_e^2}{kT_e} \int_0^\infty \frac{4}{\nu_e(c)} f_e^M(c) \ dc \quad \text{(B.14)}$$

where \( f_e^M(c) \) is the Maxwellian electron velocity distribution function, \( c \) is the random electron velocity in the mean velocity reference frame. \( \nu_e(c) \) is the total speed dependent collision frequency \( = c/m \). Thus to construct a mobility equation, the velocity dependent momentum transfer collision cross-section must be evaluated.

### B.2 Charged Particle Momentum Transfer

The charged particle momentum transfer collision frequency is amenable to a theoretical treatment. For charged particles in a plasma the influence of many particles on the test particle is accounted by introduction of a screening factor into the Coulombic force law (6):

$$\phi(r) = \frac{-e^2}{4\pi\epsilon_0 r^2} e^{-r/\delta_D} \quad \text{(B.15)}$$

where \( r \) is the collision partner separation distance and \( \delta_D \) is the Debye shielding length (5):

$$\delta_D = \frac{\sqrt{\frac{e kT_e}{\epsilon_0 n_e}}}{e n_e^{1/2}} = 69.1 \sqrt{\frac{T_e}{n_e}} \ m \quad \text{(B.16)}$$

B-3
The Rutherford differential scattering cross-section (6,5):

\[ \sigma(\theta, g) = \left[ \frac{e^2}{4\pi\varepsilon_0 (2\mu g^2)} \right] \frac{1}{\sin 4\theta/2} \]  

(B.17)
describes charged particle encounters. Here the single charged species are approaching each other with a relative velocity \( g \) and scattering at the angle \( \theta \) in the centre of mass frame. \( \mu \) is the reduced mass of the collision partners.

Integrating over the scattering angle, the energy dependent momentum transfer cross-section will be (5):

\[ q(g) = 2\pi \int_{\theta_m}^{\pi} (1 - \cos\theta) \sin\theta \sigma(\theta, g) d\theta \]  

(B.18)

which reduces to (5):

\[ q(g) = 4\pi \bar{b}_0^2 \ln \left[ 1 + \left( \frac{\lambda_0}{\bar{b}_0} \right)^2 \right]^{1/2} \]  

(B.19)

where

\[ \bar{b}_0 = \frac{e^2}{8\pi\varepsilon_0 \varepsilon} = \frac{1.15 \times 10^{-28}}{\varepsilon} \text{m} \]

for singly charged particles and where \( \varepsilon \) is the electron energy in joules. Expressing Eq. B.19 in terms of the electron temperature and average electron number density, the energy dependent momentum cross-section will be:

\[ q(g) = \frac{1.67 \times 10^{-55}}{\varepsilon^2} \ln \left\{ 1 + 3.61 \times 10^{59} \frac{T_e}{n_e} \varepsilon^2 \right\}^{1/2} \]  

if \( \varepsilon \) is in joules, or

\[ q(g) = \frac{6.50 \times 10^{-18}}{\varepsilon^2} \ln \left\{ 1 + 9.24 \times 10^{21} \frac{T_e \varepsilon^2}{n_e} \right\}^{1/2} \]  

(B.20)

if \( \varepsilon \) is in electron volts and \( q(g) \) in \( \text{m}^2 \).

B.3 Neutral Momentum Transfer

No satisfactory theoretical momentum cross-section is known to exist for the noble gases over the electron energy range 0 to 30 eV. Fletcher and Burch in 1972 performed a computer curve fit to the experimental ionization functions available for the noble gases. This procedure allowed them to generate a functional relationship between the electron energy and the momentum cross-section for electron-neutral elastic impacts.
If a negligible fraction of the electrons are found below the Ramsauer minimum, the momentum transfer cross-section increases linearly with electron energy from 1/4 eV to approximately 10 eV (10). A maximum in the cross-section occurs in the 10 to 15 eV range and subsequently the momentum cross-section decreases with increasing electron energy (11). Within the context of the present calculations, the experimental results of Barbiere (10) are used over the range 1/4 to 10 eV whereas the semi-empirical work of Fletcher and Burch (11) is used for the higher electron energy range:

\( q_{ea} = 1.44 \times 10^{-20} \epsilon m^2 \quad 0 < \epsilon < 10 \text{ eV} \) (B.21)

\( q_{ea} = 1.41 \times 10^{-20} \epsilon m^2 \quad 0 < \epsilon < 10 \text{ eV} \) (B.22)

\( q_{ea} = [14.1 - 0.279 (\epsilon - 10)]10^{-20} m^2 \quad 10 < \epsilon < 40 \text{ eV} \) (B.23)

B.4 The Mobility Equation

Given the functional relations for the electron-neutral and electron-ion energy dependent momentum transfer collision cross-section, the mobility equation follows easily from Eqs. B.6 and B.14:

\[
\mathbf{j}_a = \frac{4\pi}{3} \frac{n_0 e^2 E}{kT_e} \int_0^\infty c \frac{f_e M(c) dc}{v_e(c)}
\]

where

\[
f_e M(c) = \left( \frac{\ell}{\pi} \right)^{3/2} e^{-\frac{\ell c^2}{2}}
\]

\[
\ell = \frac{m}{2kT_e}
\]

\[
v_e(c) = c(n_{q_{ea}} + n_1 q_{ei})
\]

c is the random electron velocity in the mean mass reference frame. Making the substitutions and defining

\[
Q_e = n_{q_{ea}} + n_1 q_{ei}
\]

\[
\ell c^2 = \eta
\]

the mobility equation becomes:

\[
\mathbf{j}_a = \frac{2e^2}{3} \sqrt{\frac{2}{m_e}} \frac{n_0 e^2 E}{\sqrt{kT_e}} \int_0^\infty \frac{n_e e^{-\eta d} d\eta}{Q_e}
\]

(B.25)
where
\[ q_{ea1} = 1.44 \times 10^{-20} \epsilon \text{ m}^2 \quad 0 < \epsilon < 10 \text{ eV} \]  
(B.26)
\[ q_{ea2} = [16.9 - 0.279 \epsilon]10^{-20} \text{ m}^2 \quad 10 < \epsilon < 40 \text{ eV} \]  
(B.27)
\[ q_{ei} = \frac{6.50 \times 10^{-18}}{\epsilon^2} \ln \left\{ 1 + 9.24 \times 10^{21} \frac{T_e}{n_e} \epsilon^2 \right\}^{1/2} \]  
(B.28)

and for the ranges:

(A) 0 < \epsilon < 10 \text{ eV:} \quad Q_e = n_a q_{ea1} + n_1 q_{ei} \quad (B.29)

(B) 10 < \epsilon < 40 \text{ eV:} \quad Q_e = n_a q_{ea2} + n_1 q_{ei} \quad (B.30)

Hernqvist and Fendly (14), in 1967, were among the first ion laser researchers to apply the Tonks-Langmuir model to estimate ion laser plasma parameters. Although they discussed the applicability of each of the five low-current positive column equations to ion-laser plasmas, no general studies were undertaken. Their mobility equation as in the Tonks-Langmuir model was dependent on the Langevin mobility expression (7):

\[ \mu = \frac{0.75 e \lambda_c}{N_e \frac{m_e c}{N_L} e \epsilon} \]

which accounted for persistence of motion but neglected the ion contribution to the mobility. The parametric analyses of the ion laser plasma and the applicability of the T-L model were investigated by numerous investigators, notably: Herziger and Seelig (17), Hattori and Goto (15), Boscher et al (13) and Kitaeva et al (16). The method of deriving a mobility relation is common to all these investigations. Typically the speed-dependent Ramsauer cross-section (Fig. 45) and the speed-dependent charged-particle collision cross-section are integrated over the electronic distribution function separately to yield the velocity averaged cross-sections \( \bar{Q}_{ea} \) and \( \bar{Q}_{ei} \) respectively. Since

\[ \bar{v}_{es} = n_e \bar{q}_{es} Q_{es} \approx n_e c \bar{q}_{es} \]

for a target species \( s \), assuming the electron speed \( c_e \) is much greater than the random spece speed, a mobility relation of the form

\[ \mu = \frac{e}{m_e c_e} \frac{1}{n_e \bar{q}_{ea} + n_1 \bar{q}_{ei}} \]

is constructed (17,13,16).
APPENDIX C

ELECTRON-IMPACT NEUTRAL IONIZATION RATE COEFFICIENT

Consider two monoenergetic beams, of number density \( n_1 \) and \( n_2 \) respectively, approaching each other with a relative speed \( g \). If the rate at which a reaction occurs is \( R_{12} \), then the cross-section for this encounter will be defined by the relation,

\[
R_{12} = n_1 n_2 g Q_{12}(g) \quad \text{meters}^3/\text{sec} \quad (C.1)
\]

In general both collision entities possess their own respective velocity distributions. Further the collision cross-section is dependent on the relative velocity. Considering two such gases with average velocities \( c \) and \( w \) relative to the mean mass velocity of the fluid, the total reaction rate per unit volume and time is:

\[
r_{12} = n_1 n_2 \int_{-\infty}^{\infty} f_1(c) f_2(w) |c - w| Q_{12}(|c - w|) d^3c d^3w \quad (C.2)
\]

where \( f(c) \) represents the velocity distribution function of the gas with mean velocity \( c \), \( |c - w| \) is the relative velocity magnitude. Assuming the two distributions are Maxwellian with \( T_e \gg T_n \) with no angular bias and

\[
f_e = \left( \frac{m_e}{2\pi kT_e} \right)^{3/2} \exp \left[ -\frac{m_e v_e^2}{2kT_e} \right]
\]

the reaction rate may be reduced to the form (5,6):

\[
r = n_e n_1 c_e \int_{\eta_1/kT_e}^{\infty} \eta e^{-\eta} Q_{el} d\eta \quad (C.3)
\]

or

\[
r = n_e n_1 s_{el}
\]

where

\[
c_e = \sqrt{\frac{8kT_e}{m_e}}
\]

\[
\eta = \frac{c}{kT_e}
\]

\( \varepsilon_1 \) = the threshold energy to a given process if one exists, and \( s_{el} \) = the rate coefficient for process 1 via electron impact (\( = \langle v_e Q_{el} \rangle \)).
Lotz (12) has generated an empirical formula for the electron-impact ionization cross-section of Argon over the electronic energy range 15.7 eV to 10^4 eV. The author maintains that most experimental cross-sections obtained to date can be approximated within 10% by this formula over the entire energy range, considering of course the experimental results in conjunction with their experimental errors. Defining, for the case of Argon,

\[ a = 4 \times 10^{-18} \text{ m}^2\text{-eV}^2 \quad P_1 = 15.8 \text{ eV} \]

\[ b = 0.62 \quad P_2 = 29.2 \text{ eV} \]

\[ c = 0.40 \quad U = \frac{\epsilon}{P_1} \]

the Lotz formula becomes:

\[ \sigma(\epsilon) = a(1 - be^{-c(U-1)}) \left[ \frac{6\ln U}{eP_1} + \frac{2\ln(e/P_2)}{eP_2} \right] \quad (C.5) \]

if \( \epsilon \) is greater than 29.9 eV and where the second term vanishes if \( \epsilon \leq P_2 \) but is greater than \( P_1 \). The rate coefficient follows.

\[ s_{eI} = \Delta s_{eI} = c_\epsilon \int_{\eta_1}^{\infty} \frac{6a_\epsilon e^{-\eta}}{P_1} (1 - be^{-c(U-1)}) \frac{\ln U}{\epsilon} \, d\eta \]

\[ + c_\epsilon \int_{\eta_2}^{\infty} \frac{2a_\epsilon e^{-\eta}}{P_2} (1 - be^{-c(e/P_2-1)}) \frac{\ln(e/P_2)}{\epsilon} \, d\eta \quad (C.6) \]

or

\[ s_{eI} = c_\epsilon e^1 \int_{T_1}^{\infty} e^{-T/T_e} \ln \left( \frac{T}{T_1} \right) \left[ 1 - be^{-c(T/T_2-1)} \right] \, dT \]

\[ + c_2 c_\epsilon e^2 \int_{T_2}^{\infty} \ln \left( \frac{T}{T_e} \right) e^{-T/T_e} \left[ 1 - be^{-c(T/T_2-1)} \right] \, dT \quad (C.7) \]

where

\[ T_1 = \frac{eP_1}{k} = 1.832 \times 10^5 \text{ K} \]

\[ T_2 = \frac{eP_2}{k} = 3.386 \times 10^5 \text{ K} \]
\[ c_1 = \frac{6a}{kT_e^2 p_l} = \frac{1.761 \times 10^{-14}}{T_e^2} \text{ m}^2 \]

\[ c_2 = \frac{2a}{kT_e^2 p_2} = \frac{3.177 \times 10^{-15}}{T_e^2} \text{ m}^2 \]

and \( T_e \) is the electronic temperature.

For rapid estimates of the neutral ionization rate coefficient at low electron mean energies, a linear approximation of the collision cross-section may be employed. Using the experimental data of Rapp-Golden, the straight line (56):

\[ \sigma = a \left( \frac{kT_e}{e} \right) (\eta - \eta_1) \quad (c.8) \]

where

\[ a = 1.345 \times 10^{-17} \text{ cm}^2/\text{eV} \]

\[ V_1 = 15.5 \text{ eV} \]

is a reasonable approximation to the ionization collision cross-section curve. Refer to Fig. 46. Integration by parts yields the rate coefficient:

\[ s_1 = aV_1 c_e e^{-\eta_1} \left[ 1 + \frac{2}{\eta_1} \right] \quad (c.9) \]

where

\[ aV_1 c_e = 1.295 \times 10^{-16} \sqrt{T_e} \text{ m}^3/\text{sec} \]

A comparison of these two rate coefficients for different electron temperatures is given in Table A.1. The Lotz function rate coefficient is used throughout this work. The linear approximation on the other hand is convenient for estimates.

<table>
<thead>
<tr>
<th>Electron Temperature</th>
<th>( s_1 ) using the Lotz Function ( x 10^{-15} \text{ m}^3/\text{sec} )</th>
<th>( s_1 ) using the Linear Approx. ( x 10^{-15} \text{ m}^3/\text{sec} )</th>
<th>% Diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>31000</td>
<td>0.1114</td>
<td>0.0926</td>
<td>17</td>
</tr>
<tr>
<td>35000</td>
<td>0.2336</td>
<td>0.1973</td>
<td>16</td>
</tr>
<tr>
<td>40000</td>
<td>0.4826</td>
<td>0.4169</td>
<td>13</td>
</tr>
<tr>
<td>50000</td>
<td>1.360</td>
<td>1.234</td>
<td>9</td>
</tr>
<tr>
<td>60000</td>
<td>2.765</td>
<td>2.638</td>
<td>5</td>
</tr>
<tr>
<td>70000</td>
<td>4.649</td>
<td>4.665</td>
<td>3</td>
</tr>
<tr>
<td>80000</td>
<td>6.932</td>
<td>7.305</td>
<td>-5</td>
</tr>
<tr>
<td>90000</td>
<td>9.524</td>
<td>10.53</td>
<td>-10</td>
</tr>
</tbody>
</table>

Table A.1

Comparison of the Two Rate Coefficients
APPENDIX D

CURRENT LIMITATION

At the low-current densities and large discharge radii considered in the Tonks-Langmuir positive column, neither neutral radial dependence nor tube-reservoir coupling mechanisms play an important role in the operation of the discharge. The local neutral density in this regime is well represented by the ideal gas law. Allen and Thonemann (40), Stangeby and Allen (41), Caruso and Cavaliere (49), among other investigators, have considered the growth of radial atomic inhomogeneities at large discharge radii in moderate current densities. The physical picture developed by these researchers postulated the development of a local neutral depletion at elevated current densities. As the ionization intensity increases, more ions are drawn from the central region where the electron concentration is the highest. Neutral replenishment however, depending primarily on the wall temperature, remains approximately constant. Any increase in arc current therefore will strengthen the radial neutral gradient. At the current limit, all the neutrals will be "pumped" to the wall region. This condition will determine the maximum steady state current supportable by a particular pr-combination.

In the case of the PCM, however, an evaluation of the current limit in the plasma discharge is hampered by the indeterminacy of the neutral flux from the wall. Consider the simple Allen-Thonemann current limit theory (40) which prescribes the maximum discharge arc current density to be:

\[ j_{AT} = \frac{\beta \gamma}{\alpha} \sqrt{\frac{M}{2 k T_e}} e v_o \]  

where \( \beta \gamma / \alpha \) are a set of constants whose ratio is of order unity. Refer to Sections 3.3.2 and 3.3.3 for a discussion of these constants. \( v_o \) is the neutral flux leaving the wall. Equation D.1 is based on the assumption that the ion wall current grows with increasing arc current until this value reaches the Allen-Thonemann \( j_{AT} \) limit. At this point, the ion wall current just balances the neutral flux from the wall \( (v_o) \) which is assumed to be fixed. Hence this neutral flux must be known if \( j_{AT} \) is to be determined. Experimentally, current limit researchers (40,41) fixed the neutral flux by use of condensible gases such as mercury. Regulation of the wall temperature conveniently controlled the neutral emission rate from the liquid film condensed on the discharge walls, since this film was in thermal equilibrium with the walls.

In wall-stabilized noble gas discharges however this simplifying technique is not viable. The neutral flux from the walls will depend on the neutral population at the walls, the accommodation time and the neutral energy upon leaving. Both the ion-wall and the neutral-wall impingement rates will determine the maximum number of neutrals which could be deposited on the walls. The actual sorption rate however will depend on the wall temperature and the wall material. The energy of the impacting ion will modulate the wall temperature locally and hence strongly influence the neutral desorption rate and energy. In addition the mechanism and degree of coupling between
the reservoir and the capillary discharge will determine the extent to which this neutral replenishing source will assist in reducing the neutral radial inhomogeneity. Consequently theoretical predictions of a current limit for discharges using non-condensing walls will be highly uncertain. In fact experimental work is required to establish the parameters which govern current limit for this situation. Since the neutral flux from the walls \( (v_0) \) must in steady state precisely balance the heavy particle flux (ions and neutrals) to the walls, it is not immediately apparent that a current limit does exist for such a discharge.

If for some reason \( v_0 \) were restricted to equal the neutral random flux in the gas reservoir, then for the 21 \( P_a \) case in Argon, a current limit may be readily computed. Given \( r_{\text{wall}} = 2.54 \, \text{mm}, \) this value is: \( j_{\text{AT}} = 8.9 \, \text{amp/mm}^2 \). This is fairly close to the maximum current densities achieved in the 21 Pascal case (\( \approx 7 \, \text{amp/mm}^2 \)).

A discharge close to the current limit is characterized by unusually high axial electric fields and consequently rapidly increasing arc voltages. Such increments in arc voltage were not encountered in the present work. Sample axial electric field measurements at high arc current densities were found to be of the same order of magnitude as those at low arc densities, implying the current limit was not approached.

It is therefore concluded that current limit effects did not play a significant role in the present work. Operation at still higher current densities, however, could subject the PCM to these effects.
APPENDIX E

GAS PUMPING

When an arc current burns in a positive column, a pressure difference between the anode and cathode has been experimentally observed to develop. In a discharge tube with no bypass tube connecting the anode and cathode, the cathode tends to be pumped out while the anode pressure increases above the ambient (fill) level. Physically this behaviour depends on the net momentum imparted to the neutrals by the two charged species. Both electrons and ions gain momentum from the axial electric fields, although in opposite directions. Electrons, drifting towards the anode, undergo frequent momentum exchange encounters with the neutrals on the average imparting all of their anode directed momentum to the neutrals. Ions however, in falling to the wall, collide infrequently with the neutrals and transmit their cathode directed momentum gains primarily to the walls. On the average, therefore, the neutrals will experience an anode directed force.

For an arc discharge operating in the free fall regime, Rosa and Allen (55) have obtained an expression for the induced neutral flux in terms of the electron flux and collision frequencies between three gas species: electrons, ions, neutrals, and between each of these components and the walls. Calculations based on their theoretical model, for the 21.4 pascal (160 mtorr) Argon and 2.54 mm discharge radius case, suggest that the induced neutral flux in the anode direction is comparable to the random neutral flux. These estimates were based on the plasma parameter values predicted by the model discussed in Section 3.3. Gas pumping theories in the free fall and transition regimes however are very approximate. Of these the Rosa-Allen model is the most comprehensive. However, this work lacks experimental verification for the high arc current densities of present interest. The uncertainty of the importance of gas pumping to the performance of the PCM consequently requires experimental verification.

Experiments were arranged in which gas pumping effects, if they were present, would be substantially modified by the PCM configurations used. The result of these changes in the gas pumping should be reflected in the PCM gain. To increase the measurement sensitivity, the cumulative gain of ten segments was used rather than that of a single segment. In addition an initial emission current of 2.5 amp/mm² was selected to insure strong gas pumping effects. With the anode port blocked (see Fig. 47) the total gain for ten segments (Case A) was measured several times. These measurements were subsequently repeated with the anode opened to the vacuum envelope (Case B, Fig. 47). If gas pumping did alter the axial neutral concentration profile in the ten segment PCM, the total gain for Case A should be enhanced over Case B. There was, however, no discernible change in the cumulative gain for the two cases. Gas pumping evidently was negligible under these conditions.

A second test for the influence of gas pumping effects was arranged in the following manner. Refer to Fig. 48. Under normal operating conditions the gas flow induced by gas pumping will be forced down the entire length of the PCM bore to the large external reservoir and subsequently through the large conductance reservoir to the cathode end of the PCM. In such an
arrangement any neutral density changes induced by gas pumping should be maximized by the long narrow-bore (low conductance) path. Direct linkage of a segment near the centre of the PCM (a region with arc currents at the order of 2 A/mm²) to the large reservoir by a short high conductance path should effectively short-circuit any gas pumping-induced neutral density perturbation. Experimentally this short-circuit should manifest itself by a change in the PCM segment gain. No such change was observed however. Regardless of whether the short-circuit was included or not, the segment gain remained the same.

On the strength of the above mentioned tests, it has been concluded that gas pumping effects introduce a negligible perturbation to the neutral density of the PCM in comparison to other phenomena, such as gas heating. Consequently, for the regime studied, gas pumping effects can be neglected.
APPENDIX F

SELF-MAGNETIC FIELD EFFECTS

The experimental law of Biot-Savart states that at any point \( \vec{R} \) located away from a filamentary conductor, the magnetic field intensity \( \vec{H} \) produced by the conductor is proportional to the product of the current \( I \), the magnitude of the differential length \( d\vec{L} \) and the distance separating the conductor and the observation point \( \vec{R} \) (54):

\[
\frac{d\vec{H}}{d\vec{L}} = \frac{I \times \vec{R}}{4\pi R^3} \quad \text{in Amperes/M (F.1)}
\]

In the integral form this relation will be:

\[
\vec{H} = \int_{\text{Loop}} \frac{I \times \vec{R}}{4\pi R^3} \quad \text{(F.2)}
\]

where the integration is performed along a path circumventing the conductor. The direction of the magnetic field intensity vector, in a cylindrical system, will be circumferential if the current flows axially. Electrons in the range \( r \) and \( r + dr \) with an axial drift velocity \( v_D \) which is independent of radial position, will experience an inward force:

\[
e v_D H(r) \quad \text{(F.3)}
\]

because of the presence of a filamentary conductor at \( r = 0 \). This force will increase with increasing arc current densities. In well-stabilized, high arc current density discharges such as the current multiplier, this self-magnetic field will retard the radial electron motion to the walls. Ions on the other hand, experiencing less influence from the field, will continue to arrive at the walls at the rate determined by the plasma balance equation. The radial electric field which was originally established to reduce the electron flux to the wall will be gradually reduced with increasing arc current density. In the limit, the equilibrium charge density distribution will be established by the balance between the magnetic forces and the diffusion rates to the wall due to the concentration gradient.

The radial force \( F_e \) acting on an electron will be:

\[
F_e = eE - e v_D H(r) \quad \text{(F.4)}
\]

By the substitution of this equation into the Boltzmann equation instead of the simpler electrostatic potential treated in Appendix A, Thonemann and Cowhig (53) show the radial concentration \( [n(r)] \) to be related to the axial value \( (n_e) \) by:
\[ n(r) = n_{eo} \exp \left[ -\frac{e}{kT_e} \left( V(r) + V_m \right) \right] \]  

where \( V(r) \) is the electrostatic potential treated in Appendix A and \( V_m \) is the "magnetic potential" defined by Thonemann and Cowhig as

\[ V_m = \int_0^r H(r)v_D dr \]  

Solution of this equation (F.6) provides the concentration gradient in terms of the arc current \( (I_a) \), and the drift velocity:

\[ n(r) = n_{eo} \left[ 1 + (A - 1)^{-1} \frac{r^2}{r_0^2} \right]^{-2} \]  

where

\[ A = \frac{2 \times 10^7 kT_e}{eI_a v_D} \]  

\( r_0 \) is the tube radius, \( v_D \) is the electron drift velocity. Langmuir probe measurements in large bore, high current density mercury discharges support the author's predicted concentration gradients (53). The value of \( A \) can be used as a measure of the importance of self-magnetic effects in the PCM. For \( A \) near unity, the self-magnetic field will perturb the electron radial concentration, in general, causing the arc to contract somewhat. For the condition \( A >> 1 \) this influence will be negligible. The importance of self-magnetic effects is examined with the aid of Table A.2 which lists \( A \) for the range of arc currents used in the 21.4 pascal, 2.54 mm tube radius, Argon PCM. As can be noted, at the very highest current used in that particular study, self-magnetic fields are starting to play a role. For the majority of the regime studied, however, the self-magnetic influence was negligible. This conclusion has experimental support. Measurements undertaken by Kitaeva et al, up to 1 Amp/mm² in an Argon discharge for a range of fill pressures which span the higher fill pressures used in the present study, indicate that the electron radial concentration follows the Tonks-Langmuir predicted profile and gives no evidence of contractions due to self-magnetic fields (33).

On the basis of both the theoretical work of Thonemann and Cowhig (53) and on the experimental observation of Kitaeva et al (33), self-magnetic field influences are concluded not to be a dominant effect in modifying the PCM gain characteristics for the range of currents studied. For arc current densities in excess of 7 Amp/mm², however, some account of this effect will be necessary both in the \( \gamma \) term in that the electron profile has been perturbed from the electrostatic case and in the change in \( \alpha \). With regard to the latter factor, at low currents, the radial electric field is established to match the electron flux to that of the ions at the walls. This field in fact repels electrons from the wall regions. At the higher current densities however, the magnetic potential restricts the electron flux to the walls, thereby reducing the need for the radial electric field. The ions, on the other hand, are virtually unaffected by the magnetic field. Consequently the
ions must diffuse towards the walls because of concentration gradients. With the resultant radial electric field flatter, the mean ion energy at the walls will therefore also be lower (α smaller, see Eq. 1.2) than in the electrostatic case.

With the arc contracting with increasing current density γ (refer to Eq. 1.5) will also increase, subsequently causing the α/γ factor to decrease and the PCM gain to deteriorate even more rapidly, than that observed at lower arc current densities.

Table A.2
Calculation of the "A" Factor for Various Arc Currents
for the 21 Pascal, Argon Case

<table>
<thead>
<tr>
<th>$J_a$ A/mm$^2$</th>
<th>A (Eq. F.8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>.03</td>
<td>632</td>
</tr>
<tr>
<td>.1</td>
<td>173</td>
</tr>
<tr>
<td>.5</td>
<td>37</td>
</tr>
<tr>
<td>1.0</td>
<td>21</td>
</tr>
<tr>
<td>2.0</td>
<td>10</td>
</tr>
<tr>
<td>4.0</td>
<td>4.8</td>
</tr>
<tr>
<td>6.0</td>
<td>3.2</td>
</tr>
<tr>
<td>6.8</td>
<td>3.0</td>
</tr>
</tbody>
</table>
APPENDIX G

CUMULATIVE IONIZATION

The effect of cumulative ionization on the behaviour of the Plasma Current Multiplier is investigated in this section.

Under typical operating conditions, the main source of both metastable and resonant state population densities is electron impact excitation of neutral Argon ground state atoms via two channels:

(A) Sudden perturbation directly into the $3s_{2-5}$ levels (21).

(B) Cascading from higher short-lived excited states (19, 18).

The two resonant excited states at 11.623 eV and 11.828 eV, i.e. $3P_1$ and $1P_1$ respectively, though optically connected to ground must be included in the metastable category. Experimental monitoring of spontaneous emission from these two states as a function of both pressure and current density indicates that in narrow bore capillary discharges at the pressures used here radiation trapping effectively increases the state lifetimes to that comparable with the metastable levels (36).

In general, any list of metastable atom loss mechanisms may exclude neutral-metastable and ion-metastable encounters in the low pressure regime, primarily on the basis of the large energy difference between the heavy plasma species and electrons.

For current densities greater than approximately 0.3 amp/mm$^2$ in Argon capillary discharges, current dependent loss mechanisms dominate all other modes (1,3,4,34). Possible loss routes are tabulated:

(1) diffusion to the wall and subsequent collisional de-excitation (1),

(2) collisional excitation to a neighbouring radiating state via two or three body collisions,

(3) collisional de-excitation to an adjacent radiating level (34),

(4) formation of diatomic molecules which dissociate either collisionally or radiatively (31),

(5) ionization via electron-metastable or metastable-metastable collisions (35).

Of these, the metastable decay by radiation into the vacuum ultraviolet continuum via a rare gas molecule with a cross-section of the order of 10-20 cm$^2$ (31) may be excluded. Contributions to ionization from collisionally dissociated diatomic molecules are also insignificant in relation to other channels (31). Similarly, metastable-metastable collisions are not important relative
to other cumulative ionization modes (35). Collisional de-excitation of metastables at the wall is important only at low current densities.

The inclusion of ionization into the arc discharge theoretical model requires the addition of the metastable concentration to the:

1. ion generation equation,
2. ion wall equation,
3. energy balance equation.

Following Section 3.2.5, the atomic temperature will be assumed to follow the Chester relation (39). Since the metastable concentration is at maximum only a few percent of the neutral density, the mobility equation will be considered unaffected. The metastable population density as a function of the discharge parameters may be obtained by application of the equation of continuity to the metastables. The two resonant states and the two metastable states are assumed to form the single intermediate level in a simplified atomic model containing solely three levels: ground, metastable and ionized. With reference to the atom level diagram (Fig. G 1), the metastable population density will depend on:

\[
n \cdot n_{EM} = n \cdot n_{DEM} + n \cdot n_{IM} + n \cdot n_{W}
\]

(Fig. G.1) Simplified Argon Grothian

where

- \( S_{EM} \) = metastable excitation rate coefficient
- \( S_{DEM} \) = metastable de-excitation rate coefficient
- \( S_{IM} \) = metastable ionization rate coefficient
- \( S_{W} \) = metastable wall loss rate coefficient

The wall loss term accounts for the wall diffusive losses:

\[
S_{W} = \frac{c_{N}}{r} = \frac{1}{r} \sqrt{\frac{8kT_{N}}{\pi M}}
\]

(G.2)
The equation of continuity (G.1) becomes:

\[ n_m = E_{xf} n \]  

(G.4)

where

\[ E_{xf} = \left[ \frac{\eta_0}{12} + \frac{S_w}{2n_e} + \frac{S_{EM}}{2S_{EM}} \right] \]  

(G.5)

The production cross-section for metastable neutral Argon atoms by electron impact has been obtained by Borst in 1974 (23). To separate the excited optical states from the desired metastable signal a time of flight method was used. With this arrangement the ultraviolet radiation decays well in advance of the metastable arrival time at the detector. The metastable count is affected by Auger emission from a CuBeO coated dynode of an electron multiplier. The excitation function is a total cross-section in that cascading from high neutral levels into the metastable states, \(3^2P_2\) at 11.5 eV and \(3^2P_0\) at 11.7 eV, is included.

The author estimates the degree of certainty in specifying the absolute level of the metastable excitation cross-section to be within a factor of two. The author determines the peak cross-section value via two different methods based on different experimental work and discovers the peak values to be separated by only 8%. Consequently he suggests the absolute production cross-section may have an actual error much lower than the estimated factor of 2.

A reasonably accurate approximation to the experimental metastable excitation curve may be obtained by partitioning the cross-section into two linear sections:

\[ \sigma(\eta) = a k T_e (\eta - \eta_0) \quad 11.6 < \epsilon < 20 \text{ eV} \]
\[ \sigma(\eta) = Q_{MAX} - b k T_e (\eta - \eta_1) \quad 20 < \epsilon < 50 \text{ eV} \]

Integrating over the electron Maxwellian energy distribution and defining:

\[ A_0 = [(2 + \eta_0) e^{-\eta_0} - (\eta_1^2 + (2 - \eta_0)(\eta_1 + 1)) e^{-\eta_1}] a k T_e \]
\[ A_\infty = e^{-\eta_1} [Q_{MAX}(\eta_1 + 1) - b k T_e (\eta_1 + 2)] \]
\[ A_{\infty 0} = e^{-\eta_1} [b(\eta_1^2 + 2\eta_1 + 2) k T_e - (\eta_1 + 1)(Q_{MAX} + b k T_e \eta_1)] \]

where

\[ a = 4.52 \times 10^{-18} \text{ cm}^2/\text{eV} = 2.82 \times 10^{-3} \text{ M}^2/\text{joule} \]
b = 1.27 \times 10^{-18} \text{ cm}^2 / \text{eV} = 7.93 \times 10^{-4} \text{ m}^2 / \text{joule}

Q_{\text{MAX}} = 0.38 \times 10^{-16} \text{ cm}^2

\eta_0 = 11.6 \, \text{e} / kT_e = 6.346 \times 10^5 / T_e

\eta_1 = 20 \, \text{e} / kT_e = 2.321 \times 10^5 / T_e

\eta_7 = 50 \, \text{e} / kT_e = 5.803 \times 10^5 / T_e

The excitation rate coefficient of the metastable levels by electron impact may be shown to be:

\[ S_M = c_e [A_0 + A_{oo} + A_{ooo}] \]  

(G.6)

McConkey and Donaldson (19) have measured the electron impact excitation cross-section of the lowest (3P_1, 1P_1) resonant states in atomic Argon over the 10 to 100 eV electron energy range. Both channels, direct level excitation from the ground state and cascading from higher excited states, are responsible for populating the two resonant states. The total excitation function of neutral Argon atoms will be the sum of the two individual cross-sections; see Fig. 50.

Subdividing the electron energy range into two regions in which the excitation may be approximated by linear functions, one obtains:

\[ \sigma = 2.69 \times 10^{-3} \, kT_e (\eta - \eta_0) \quad 11.6 < \epsilon < 22.5 \, \text{eV} \]

\[ \sigma = 0.47 \times 10^{-20} - 1.36 \times 10^{-4} \, kT_e (\eta - \eta_9) \quad 22.5 < \epsilon < 100 \, \text{eV} \]

where \( \sigma \) is in m\(^2\) and \( \eta_9 = 22.5 \, \text{e} / kT_e \). The velocity-averaged resonant level excitation rate coefficient may be readily shown to be:

\[ S_{eo} = c_e [R_o + R_{oo} + R_{ooo}] \]  

(G.7)

where

\[ R_o = 2.69 \times 10^{-3} \, kT_e \left( (2 + \eta_9)e^{-\eta_0} - [\eta_9^2 + (2 - \eta_0)(\eta_9 + 1)]e^{-\eta_9} \right) \]

\[ R_{oo} = e^{-\eta_9} \left[ Q_{\text{MAX}}(1 + \eta_9) - 1.36 \times 10^{-4} kT_e (\eta_9 + 2) \right] \]
The total excitation rate coefficient, including both the two resonant optical levels and the two metastable levels, will be:

\[ R_{\text{oo}} = \exp( -\eta \ln[1.36 \times 10^{-4} kT_e (\eta_{10}^2 + 2 \eta_{10} + 2) - (\eta_{10} + 1)] ) \]

\[ Q_{\text{MAX}} = 0.47 \times 10^{-20} \text{ m}^2 \]

Atoms in the metastable level (Fig. G.1) may be de-excited by collisions of the second kind. In this type of collision, the stored potential energy of the atom is converted into any other form of energy except radiation (9). With electrons being the most active collision partner in all types of encounters, one of the dominant metastable depopulating processes will be superelastic collisions between excited atoms and electrons:

\[ e^- \text{ (slow)} + A^* \rightarrow A + e^- \text{ (fast)} \]

For Argon the degeneracies of the four levels contributing to the metastable level of Fig. G.1 are:

\[ \begin{array}{ll}
3P_2 & 5 \\
3P_0 & 1 \\
3P_1 & 3 \\
1P_1 & 3 \\
\end{array} \]

The total de-excitation rate coefficient will be:
or from Eq. G.8,

\[
S_{\text{DEM}} = \frac{\eta_m}{6} \left[ S_{\text{EO}} + S_m \right]
\]

where

\[
\eta_m = 11.6 \frac{e}{kT_e} = \eta_0
\]

Experimental cross-sections for the ionization of metastable Argon atoms by electron impact have not been found in the literature. Employing a theory developed by Gryzinski, Vriens (28) has estimated the absolute ionization cross-section of various monatomic neutral and metastable atoms without resorting to any adjustable free parameters. Based on the acceptable agreement between his work and available experimental results for the cases of neutral rare gases and mercury, the theoretical ionization cross-sections of metastable Argon are assumed to be realistic approximations.

Subdividing the electron energy domain of Fig. 51 into two regions, linear functions which approximate the true behaviour, the metastable ionization rate coefficient may be obtained in a fashion similar to the metastable excitation rate coefficient:

\[
S_{\text{IM}} = c_e[A_1 + A_{11} + A_{111}]
\]

where

\[
A_1 = \frac{d}{kT_e} \left\{ (2 + \eta_2) e^{-\eta_2} - [\eta_5^2 + (2 - \eta_2)(\eta_5 + 1)] e^{-\eta_5} \right\}
\]

\[
A_{11} = e^{-\eta_5} [Q_{\text{PEAK}}(\eta_5 + 1) - f kT_e (\eta_5 + 2)]
\]

\[
A_{111} = e^{-\eta_8} [f kT_e (\eta_8^2 + 2\eta_8 + 2) - (\eta_8 + 1)(Q_{\text{PEAK}} + f kT_e \eta_5)]
\]

\[
\eta_2 = 4.2 \frac{e}{kT_e} = 4.874 \times 10^4 \frac{1}{T_e}
\]

\[
\eta_5 = 12 \frac{e}{kT_e} = 1.393 \times 10^4 \frac{1}{T_e}
\]

\[
\eta_8 = 68 \frac{e}{kT_e} = 7.892 \times 10^5 \frac{1}{T_e}
\]

\[
da = 1.2 \times 10^{-16} \text{ cm}^2/\text{eV} = 7.5 \times 10^{-2} \text{ M}^2/\text{joule}
\]

\[
Q_{\text{PEAK}} = 9 \times 10^{-20} \text{ M}^2
\]
and the applicable cross-sections are:

\[
\sigma_{IM} = \frac{dkT_e}{(\eta - \eta_2)} \quad 4.2 < \epsilon < 12 \text{ eV}
\]

\[
\sigma_{IM} = Q_{PEAK} - \frac{dkT_e}{(\eta - \eta_5)} \quad 12 < \epsilon < 68 \text{ eV}
\]

Taking account of the metastable level (Fig. G.1), the equations for the positive column will become:

(1) Ion Wall Equation:

\[
j_w = esn_e v_B \quad (G.12)
\]

(2) Ion Generation:

\[
j_w = \frac{e}{2} \frac{n_e n_r (s_I + E x f s_{IM})}{v_m} \quad (G.13)
\]

(3) Energy Balance:

\[
r_{r_{a}} E = 2j_{w}(s_{T_e} + V_1) + \frac{en_m c v_m}{2} \quad (G.14)
\]

(4) Mobility:

\[
j_a = \sigma E \quad (G.15)
\]

where \(\sigma\) has been defined by Eq. B.14.

(5) Ideal Gas Law:

\[
n + n_m = n_f \frac{T_f}{T} \quad (G.16)
\]

(6) Chester's Relation:

\[
T = 300(1 + 8.95 \times 10^{-5} \sqrt{r a}) \quad (G.17)
\]

The solution of this system of equations is compared against both the simpler case which excludes cumulative ionizations and experiment. The major difference between the two theoretical curves lies not in the gain per unit length curves (see Fig. 52) but in the electron temperatures in the 0-7 amp/mm² range. In general \(T_e\) is approximately 20% lower for the case which includes metastables. Since the present study of PCM gain does not distinguish between the two models, the simpler one will be used in all calculations. It should be noted that Argon laser research has indicated that cumulative ionization can be neglected (16) for the regime employed in the present work.
## APPENDIX H

### IMPURITY CONCENTRATION AND CONSTITUENTS IN GASES USED IN THE PCM

<table>
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<th>Min. Purity Concentration %</th>
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APPENDIX I

EXPERIMENTAL GPUL

The total current gain for the PCM was plotted as a function of the multiplier length for a given fill pressure and emission current. The GPUL was subsequently calculated from the slope of the curve which best fit the experimental total-gain-distance data. These GPUL values are estimated to have an experimental error of ± 5%. For all fill pressures except the 21 pascal case, the emission current (I_o) was 0.5 amperes. In the 21 pascal case, I_o was 0.6 amperes. Although the emission current (I_o) does not affect the GPUL measurements it has been included to indicate the arc current level at which the PCM total gain measurements were initiated. For all fill pressures tabulated herein, the bore discharge radius was 2.54 mm.

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<th>Fill Pressure</th>
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<th>GPC**</th>
<th>J_{arc}</th>
<th>GPC</th>
<th>J_{arc}</th>
<th>GPC</th>
<th>J_{arc}</th>
<th>GPC</th>
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<td>2.5</td>
<td>1.105</td>
<td>0.030</td>
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<tr>
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<td>3.1</td>
<td>1.102</td>
<td>0.047</td>
<td>1.148</td>
<td>4.2</td>
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</table>

*J_{arc} in Amp/mm^2

**GPC - Gain per Centimeter

The experimental error in the arc current was ± 5% and ± 2% in the GPC.
APPENDIX J

COMPUTER PROGRAM LISTING

This section contains a listing of the program used to model the PCM behaviour. The program was designed to operate on the IBM 1130 computer. The program must be initiated by specifying the discharge radius, the fill pressure, the initial current and electron temperature. The T-L model is useful for indicating practical combinations of parameters.

The corresponding electron number density is computed by a Newton-Raphson iteration procedure in the Raph subprogram. Subsequently the arc current density (ja), the electric field (e), the electron drift to random current ratio (β) and the GPUL are computed. At this point the electron temperature is incremented and the computations repeated. The values are stored in a matrix form and resorted to provide the Te, ne, E, β and GPUL dependence on the arc current density.

The later part of the main program computes the total gain-distance characteristic taking into account the plasma parametric dependence on the arc current density.
**APPENDIX K**

**THEORETICAL PLASMA PARAMETER DEPENDENCE ON ARC CURRENT FOR VARIOUS FILL PRESSURES**

The plasma parameter dependence on the arc current density in a 2.54 mm discharge radius is tabulated in this appendix for various fill pressures of Argon. The theoretical model assumes $s$ to be adjustable and is matched to experiment in the 40 pascal case at the low arc current densities. Electron pressure, cumulative ionization, self-magnetic field and gas pumping effects are excluded. The positive column equations used to predict these values have been treated in Section 3.2.5, where $s$ has been defined by Eq. 3.38.

**Case 1: 1.5 Pascals (11 Mtorr)**

<table>
<thead>
<tr>
<th>$J_{\text{arc}}$ (Amp/mm$^2$)</th>
<th>$N_e$ (10$^{18}$/m$^3$)</th>
<th>$N$ (10$^{20}$/m$^3$)</th>
<th>$T_e$ (°K)</th>
<th>GPC</th>
<th>$E$ (V/cm)</th>
<th>$\beta$</th>
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### Case 2: 6.7 Pascals (50 Mtorr)

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<th>N (10²⁰#/M³)</th>
<th>$T_e$ (°K)</th>
<th>GPC</th>
<th>E (V/cm)</th>
<th>$\beta$</th>
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<th>N (10²⁰#/M³)</th>
<th>$T_e$ (°K)</th>
<th>GPC</th>
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GAS HEATING MECHANISMS

The atomic temperature has been observed to increase with increasing arc current density. The results of experimental efforts in this direction have been discussed in Section 3.2.3. Mechanisms responsible for this phenomenon however (with one exception found in the open literature) have been neither offered nor discussed by the various researchers. Kitaeva et al. (48) have obtained experimental evidence for regimes of interest to the present study that charge exchange may be important in modifying the neutral velocity distribution and temperature. A similar line of reasoning appears to be valid in the present study. Refer to Section 2.3.6.

Neutrals may gain energy from a number of different channels, however two dominant processes must be:

(1) electron neutral collisions because of the large number of elastic collisions,

(2) radially moving ion-neutral collisions because of the large collision cross-section.

Assuming the rate of energy transfer between the neutrals and the walls is:

\[(2kT_n) \left( \frac{nc_n}{4} \right) \frac{2\pi r}{2\pi} \text{ watts} \quad (L.1)\]

per unit length of wall and that this loss channel is dominant, an estimate may be made of the average energy gained by the neutrals as a result of each of the two energy gaining encounters mentioned above.

Equating energy gained due to elastic electron impacts (Eq. A.25) to the energy carried to the walls by neutrals, it follows:

\[4n_e n \frac{m_e}{M} \langle v_e q_e \rangle k(T_e - T_n) \frac{m^2}{2} = 2kT_n \frac{nc_n}{4} 2\pi \text{ watts} \quad (L.2)\]

where

\[T_e \gg T_n\]

\[\langle v_e q_e \rangle \approx \int q_e \eta e^{-\eta} d\eta = c_e \tilde{q}_e \text{ (see Eq. C.3)} \quad (L.3)\]

\[c_e = \sqrt{\frac{3kT_e}{m_e}}\]
\[
\bar{Q}_e = 2AKT_e
\]
\[
q_e = Ae \quad \text{(Fig. 45)}
\]
\[
A \approx 0.1 \text{ m}^2/\text{joule (Fig. 45)}
\]

Applying the above information to Eq. L.2, that equation may be reduced to:

\[
T_{n}^{3/2} \approx 8 \frac{m_e}{M} \alpha n r \frac{T_e^{5/2}}{T_n}
\]

\[
T_n \approx 2.2 \times 10^{-19} n_e^{2/3} T_e^{5/3}
\]

where \( r = 2.54 \text{ mm} \). The increase in neutral temperature due to this process is compared with that given by the empirical Chester relation (3.14) for a range of currents for the 2.14 pascal case in Argon.

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As can be seen, this process is not capable of explaining the experimentally observed neutral temperatures.

For the case of neutral-ion collisions, equating neutral energy gains,

\[
n_{i} n \frac{5}{2} v_B m_i^2 \frac{M v_B^2}{2}
\]

and their losses to the walls,

\[
n_c n \frac{2kT_n}{4} 2\pi r
\]
it follows that

\[ T_n^{3/2} \approx \frac{\pi}{2} \frac{r}{4} n_e \bar{q} T_e^{3/2} \]  

(L.5)

where

\[ n_e \approx n_i \]

\[ \bar{q} \approx 9 \times 10^{-19} \text{ m}^2 \text{ (Fig. 38)} \]

and the ions are assumed to have the Bohm velocity as their relative encounter velocity. This of course would represent an upper estimate in that the average ion velocity (radially) would be less than \( v_B \) within the plasma where the majority of the ion-neutral encounters would occur.

The neutral temperature increment due to this process is compared below with that given by the Chester relation for a range of arc currents for the 21.4 pascal case in Argon.

<table>
<thead>
<tr>
<th>( J_a ) (A/mm²)</th>
<th>( T_n ) (°K) ( \text{Ja} ) ( \text{(Eq. L.4)} )</th>
<th>Chester Relation ( \text{E}q. \text{ 3.2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>.03</td>
<td>340</td>
<td>41</td>
</tr>
<tr>
<td>.1</td>
<td>711</td>
<td>135</td>
</tr>
<tr>
<td>1.0</td>
<td>4060</td>
<td>1350</td>
</tr>
<tr>
<td>2.0</td>
<td>6440</td>
<td>2710</td>
</tr>
<tr>
<td>4.0</td>
<td>11000</td>
<td>5550</td>
</tr>
<tr>
<td>8.0</td>
<td>19200</td>
<td>10830</td>
</tr>
</tbody>
</table>

It is apparent from this table that ion-neutral collisions do play an important role in the PCM discharge. The foregoing calculation gives an overestimate of the heating due to charge exchange since no account has been taken of the loss of energy from the neutrals to the ions by charge exchange and, as noted, the maximum ion energy has been taken to be the average energy. With these adjustments in mind it is clear that the charge exchange process rather than electron heating is the relevant influence, and is numerically of the right magnitude.
APPENDIX M

POTENTIAL PCM APPLICATIONS

Three examples of potential PCM applications are:

1. power cathode
2. robust amplifier
3. power switch

Experimental confirmation of the last application is outstanding.

Thermonic emitters have limited lifetimes. As summarized in the table at the end of this appendix, operation of filaments at elevated pressures further diminishes their lifetime. Furthermore, filament operation in reactive gases (such as O₂) is not possible. Emissionless electron sources, on the other hand, have indefinite lifetimes in all gaseous environments, although low current densities are characteristic of these sources. Despite lifetime problems, two factors make thermionic emitters more popular than emissionless ones:

1. the simplicity of operation,
2. the ease with which high emission current densities can be achieved.

The emissionless PCM provides a bridge between the two types of sources. The PCM is a device capable of yielding a high current density with good lifetime characteristics, although at the expense of simplicity of operation. For situations which demand minimum down time, this combination could be ideal. Under such circumstances, the device's efficiency will be secondary to its prime advantages: operability in reactive environments and longevity at higher pressures.

Experiments in which the initial current was modulated illustrated the PCM could behave as a current amplifier (55). In these experiments the gain-frequency curve was found to be constant from DC to approximately 100 kHz. This upper limit is significant for it reflects the mean ion transit time across the discharge radius to be the PCM's operating frequency limit. The amplification feature of this device can be readily integrated with the power cathode. The device efficiency can be estimated from the illustration given below.

H.T. Line

Knife Switch

PCM

Ground
Typical segment gains are of the order of 110% at the high current densities. Refer to Appendix K. The device efficiency therefore will be low. In addition the load impedance will have to be properly matched to the PCM to ensure the maximum extractable gain. However, an emissionless PCM amplifier offers not only operability but also longevity in both reactive and high pressure environments.

The PCM also offers the possibility of breaking HVDC currents. The figure below illustrates schematically the procedure in which a PCM would act as a circuit breaker.

For initial conditions take the knife switch closed and the PCM between the high voltage line and ground to be off. As the switch is opened and begins to arc, a potential will develop across the switch terminals and therefore across the PCM. If an initial PCM current exists, an arc discharge will develop within the PCM. As the switch is drawn to its fullest extent, the inter-terminal arc should transfer from the knife switch to the PCM discharge. At this point the PCM control current can be turned off forcing the positive column to decay. Providing no arcing occurs between the PCM anode and its neighbouring segments, the line current will also be interrupted. In this capacity PCM efficiencies are irrelevant. Instead the device's response to the decaying plasma will be paramount. Research in this area will establish the criteria important to applying the PCM to HVDC current interrupting.
## Comparison of Electron Sources

<table>
<thead>
<tr>
<th></th>
<th>Thermionic</th>
<th>Emissionless</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lifetime in vacuum</td>
<td>Good</td>
<td>Not applicable</td>
</tr>
<tr>
<td>Lifetime in low pressures</td>
<td>Poor</td>
<td>Indefinite</td>
</tr>
<tr>
<td>Lifetime in reactive gases</td>
<td>None</td>
<td>Indefinite</td>
</tr>
<tr>
<td>Current density capacity</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>Simplicity in operation</td>
<td>Excellent</td>
<td>Good</td>
</tr>
</tbody>
</table>
The Plasma Current Multiplier (PCM) is a wall-stabilized low pressure arc discharge with the capability of multiplying the initial electron current injected into the device. Experimentally the PCM gain per unit length \( g \) was found to decrease with increasing arc current at very low arc current densities, to remain constant at moderate arc current densities, and to decrease asymptotically towards unity at extremely high current densities. Theoretically, sheath thickening and neutral rarefaction due to local gas heating have been identified as the phenomena responsible for the PCM gain behaviour at the very low and high arc current densities respectively. Experimental confirmation of the gain dependence on mass and tube radius originally predicted by the Stangeby and Allen theory was extended to several atomic species. An emissionless source of electrons was constructed and tested, thus expanding the operating capacity of this device to reactive gases.