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Improving sustainability under uncertainty

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A robust-fuzzy multi-objective optimization approach for a supplier selection and order allocation problem: Improving sustainability under uncertainty

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ABSTRACT

Attaining sustainability objectives has received wide attention in the supplier selection and order allocation (SSOA) literature. This paper aims to investigate an SSOA problem under multiple items, multiple suppliers, multiple price levels, and multiple period using a robust-fuzzy multi-objective programming in which: (a) transportation cost, delay penalty cost, and demand are uncertain; (b) four objectives are proposed to minimize total costs and the number of defective items and to maximize environmental and social impacts; and (c) all objectives of the problem have a fuzzy membership degree that is determined by the decision-makers. A robust optimization approach is elaborated as a solution procedure to address the uncertainty of the decision variables. The significance of each objective in practice is discussed based on seven distinct scenarios that produce a specific membership degree to help practitioners make efficient decisions in selecting the suppliers and allocating the orders. Two numerical examples with different sizes are conducted to validate the mathematical model. Thereafter, the sensitivity of each scenario on objectives and total satisfaction degree is analyzed. The results of the numerical solution compare the value of four objective functions under each developed scenario to provide a trade-off insight between different objectives for practitioners. Eventually, the credibility and efficiency of the proposed solution procedure are evaluated to validate the findings.

1. Introduction

Over the last decades, one of the debating issues has been about mitigating environmental pollution that can have irrevocable damages to ecosystems such as global warming and climate change (Micheli, Cagno, Mustillo, & Trianni, 2020). Besides, communities expect to find a more positive impact of industries' evolution on their social life. For instance, it can create new job opportunities and enhance the community's living conditions (Bektur, 2020). Incorporating sustainability into supply chain management (SCM) has gained wide attention, which has motivated both academic and industry experts to focus on sustainable supply chain management (SSCM) (Rezaei Vandchali, Cahoon, & Chen, 2020). The sustainability concept is often defined as "economic practices which meet the needs of the present without compromising the ability of future generations to meet their own needs" (Imperatives, 1987). SSCM can only be successful if all operations of the supply chain including supplier selection, order allocation, production scheduling, lot-sizing, and logistics follow sustainable development considerations (Sepehri, 2021). In this context, developing sustainability in supply chain operations has become a prominent objective for practitioners (Isaloo & Paydar, 2020; Vandchali, Cahoon, & Chen, 2021).

The supply chain environment is becoming more dynamic and complex as it faces globalization, technological advancement, and growing customer responsiveness (Jahani, Sepehri, Vandchali, & Tirkolaee, 2021). To survive in this intense competition, supply chain practitioners need to consider uncertainty in their supplier selection and

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order allocation operations (Govindan, Mina, Esmaeili, & Gholami-Zanjani, 2020). From the sustainability perspective, the pressure toward incorporating sustainable development objectives into the supply chain has been raised. Moreover, firms are consistently encouraged to add social and environmental aspects to the traditional economic aspect in selecting suitable suppliers in an uncertain environment (Sepehri, Mishra, & Sarkar, 2021). In addition, since companies deal with different internal and external factors in real-world problems, considering uncertainties provides valuable insights into the application of suppliers' sustainability practices (Stević, Pamučar, Puška, & Chatterjee, 2020). Accordingly, many firms have attempted to operationalize sustainable development objectives under uncertainty in their supplier selection process and allocate the optimal quantities to their suppliers (Jia, Liu, & Bai, 2020).

Selecting suppliers based on the sustainability objectives of the firms is a challenging task and can be affected by various uncertain variables such as transportation cost, delay penalty cost, demand parameters, and aspiration levels for objective functions. Despite the importance of this issue, few attempts have been made to provide a comprehensive mathematical model that addresses sustainability considerations in the SSOA problem. In order to fill this gap, three research questions are outlined in this research: (1) How to formulate a mathematical model that addresses sustainability in SSOA problems? (2) How to consider the uncertain environment in SSOA problems, and which approaches are applied to solve an uncertain mathematical model? (3) What approach can be utilized to determine the significance of a multi-objective SSOA problem? In order to address the above-mentioned questions, this paper develops a multi-objective programming model which can help firms to make efficient decisions regarding SSOA by minimizing the total relevant costs and the number of defect items and maximizing the environmental and social impacts. The model is also equipped with uncertain variables where a fuzzy programming approach is employed to obtain the satisfaction degree of each objective function, and a robust optimization approach is adopted to address uncertain parameters.

The rest of this paper is structured as follows. Section 2 provides a concise review of the related literature. Section 3 mainly focuses on the problem description and the required elements for model development. An initial mathematical model is also developed. Section 4 focuses on proposing a robust-fuzzy optimization approach as a solution procedure to address the uncertainties of parameters and objective functions. Section 5 elaborates numerical examples for seven different scenarios. Section 6 analyzes the impact of each scenario and conservatism levels on the optimal value of objective functions and the total satisfaction degree. Section 7 discusses the findings and evaluates the credibility and efficiency of the proposed robust-fuzzy optimization. Section 8 concludes the paper and outlines some directions for future research.

2. Literature review

The supplier selection problem is one of the pivotal issues discussed in SCM literature, and numerous approaches and models have been developed to address this problem (Feng & Gong, 2020). The primary goal of this problem is to select the best suppliers among the potential candidates and allocate the right order quantities (Nasr, Tavana, Alavi, & Mina, 2021). This section is divided into three subsections. The first subsection focuses on the papers that have studied the SSOA problem. The focus of the second subsection is on the sustainable SSOA problem, and the third section reviews the studies addressing the uncertainty in the parameters and objectives of the sustainable SSOA problem.

2.1. SSOA problem

Mafakheri, Breton, and Ghoniem (2011) proposed the integration of the analytic hierarchy process (AHP) and dynamic programming model for addressing the SSOA problem aiming to maximize firm utility function and minimize the total costs. Amin, Razmi, and Zhang (2011) suggested the combination of fuzzy linear programming and fuzzy SWOT analysis to address the vagueness of input parameters and human judgments in the SSOA problem. A multi-objective model for a single-item and single-period SSOA problem accompanied by AHP and a goal programming model as the solution approach was presented by Erdem and Göçen (2012). Nazari-Shirkouhi, Shakouri, Javadi, and Keramati (2013) proposed the use of an interactive two-phase fuzzy multi-objective linear programming model for solving an SSOA problem with multi-price levels and items. A multi-objective optimization model, aiming to minimize rejected items, total cost, and lead-time, was presented by Jadidi, Zolfaghari, and Cavalieri (2014). They developed a normalized goal programming model for solving their model. Torabi, Baghersad, and Mansouri (2015) proposed stochastic programming to solve a single-period and single-item SSOA problem considering the risk factor. Scott, Ho, Dey, and Talluri (2015) developed a model by integrating AHP, quality function development (QFD), and chance-constrained optimization to address an SSOA problem. Prasanna Venkatesan and Goh (2016) analyzed the disruption risk factor in an SSOA problem by presenting a multi-objective mixed-integer linear programming model and using fuzzy AHP, PROMETHEE, and the particle swarm optimization as the solution approach. Çebi and Otay (2016) proposed the use of fuzzy MULTIMOORA and fuzzy goal programming for evaluating an SSOA problem with multiple items and suppliers and quantity discounts. A multi-objective model considering stochastic parameters in an SSOA problem was presented by Moheb-Alizadeh, Mahmoudi, and Bagheri (2017). They used the minimum deviation method and a genetic algorithm as the solution approach. Mirzaee, Naderi, and Pasandideh (2018) proposed a multi-objective mixed-integer linear programming model for an SSOA problem with multiple periods, items, and suppliers with quantity discounts. They developed the preemptive fuzzy goal programming approach to solve their mathematical model. Hosseini et al. (2019) proposed a stochastic bi-objective mixed-integer programming model to analyze the disruption risk factor in a single-item and multi-period SSOA problem. Esmaeili-Najafabadi, Nezhad, Pourmohammadi, Honarvar, and Vahdatzad (2019) studied the risk factor and proposed alternatives for reducing the effect of disruption in an SSOA problem through a mixed-integer non-linear programming model.

2.2. Sustainable SSOA problem

Azadnia, Saman, and Wong (2015) proposed the integration of a rule-based weighted fuzzy method, fuzzy AHP, and multi-objective mathematical programming for addressing a sustainable SSOA with multiple periods and items. Ghadimi, Toosi, and Heavey (2018) developed a model focusing on facilitating and automating a cooperative partnership in a sustainable SSOA problem using a Multi-Agent Systems approach. Gören (2018) proposed a three-stage decision framework combining fuzzy DEMATEL, Taguchi loss functions, and a multi-objective mathematical programming model for sustainable SSOA problems considering lost sales. Vahidi, Torabi, and Ramezankhani (2018) addressed disruption and operational risk in a multi-item sustainable SSOA using the combination of SWOT-GFD and a stochastic programming model. Nourmohamadi Shalke, Paydar, and Hajiaghaei-Keshteli (2018) presented a multi-objective mathematical model for a multi-item, multi-period sustainable SSOA considering quantity discounts. They used the revised multi-choice goal programming approach to solve their model. Park, Kremer, and Ma (2018) proposed a combined multi-attribute utility theory and multi-objective integer linear programming model to solve sustainable SSOA with multiple items and a single period. Cheraghalipour and Farsad (2018) focused on quantity discounts under disruption risks in sustainable SSOA. They used the BWM and the revised multi-choice goal programming as the solution process for their model. Kellner and Utz (2019) extended a mathematical model using the Markowitz portfolio theory to address the risk factor in a multi-item and single-period model. They suggested the use of the ε-constraint approach for solving their model. Rabieh, Modarres, and

Table 1
Summary of selected sustainable SSOA literature.

Author (s)	Uncertainty		Multi-price	Multi-period	Multi-item	Defective items	Sustainability factors			Solution approach
	Variables	Objectives					Economic	Social	Environmental	
Nourmohamadi Shalke et al. (2018)			✓	✓	✓		✓	✓	✓	Multi-choice goal programming
Park et al. (2018)					✓	✓	✓	✓	✓	Weighted-sum method
Cheraghalipour and Farsad (2018)			✓	✓	✓		✓	✓	✓	Multi-choice goal programming
Mohammed et al. (2018)	Fuzzy				✓		✓	✓	✓	ε-constraint and LP-metrics
Arabsheybani et al. (2018)		Fuzzy	✓	✓	✓		✓	✓	✓	Fuzzy goal programming
Rabieh et al. (2018)	Robust	Fuzzy		✓	✓	✓	✓			Robust-fuzzy optimization
Rabieh, Babaee, Fadaei Rafsanjani, and Esmaeili (2019)					✓	✓	✓	✓	✓	ε-constraint, weighted sum
Moheb-Alizadeh and Handfield (2019)			✓	✓	✓		✓	✓	✓	ε-constraint
Mohammed et al. (2019)	Fuzzy						✓	✓	✓	ε-constraint and LP-metrics
Bektur (2020)	Fuzzy		✓	✓	✓	✓	✓	✓	✓	ε-constraint, LP-metrics
Jia et al. (2020)	Fuzzy	Fuzzy		✓	✓	✓	✓	✓	✓	-fuzzy goal programming, ε-constraint and LP-metrics, F-AHP, F-PROMETHEE, TOPSIS
Isaloo and Paydar (2020)	Robust				✓		✓		✓	LP-metric and goal programming
Arabsheybani et al. (2020)			✓	✓	✓		✓			Metaheuristic algorithms
Feng and Gong (2020)				✓	✓		✓		✓	Linguistic entropy weight method
Sontake et al. (2021)			✓		✓	✓	✓	✓	✓	Mixed-integer linear programming
Nasr et al. (2021)		Fuzzy	✓	✓	✓		✓	✓	✓	Fuzzy goal programming, F-BWM
This study	Robust	Fuzzy	✓	✓	✓	✓	✓	✓	✓	Robust-fuzzy optimization

Azar (2018) investigated an innovative robust-fuzzy method for multi-objective, multi-period supplier selection problems under multiple uncertainties. They presented a multi-objective mathematical integer programming model for sustainable SSOA with multiple sourcing. Moheb-Alizadeh and Handfield (2019) developed a multi-objective mixed-integer linear programming model for sustainable SSOA with multiple periods, items, and transportation modals considering shortage and discount. Arabsheybani, Paydar, and Safaei (2020) proposed a mathematical model to study the sustainable SSOA problem with multiple suppliers, items, and periods. The objective of their model was to maximize total profit while minimizing total risk and unsatisfied demand. The model was solved using two different metaheuristic algorithms. Sontake, Jain, and Singh (2021) developed a mixed-integer linear programming model for the sustainable SSOA problem with a single period and multiple suppliers and items. The primary consideration of their study was the selection of transportation alternatives.

2.3. Uncertain sustainable SSOA problem

Mohammed, Setchi, Filip, Harris, and Li (2018) presented a fuzzy multi-objective programming model for the sustainable SSOA problem for a meat supply chain to address the uncertainty in some input parameters. Arabsheybani, Paydar, and Safaei (2018) developed a multi-objective mathematical model addressing the sustainable SSOA problem. They used a fuzzy goal programming approach to cope with the uncertain nature of the specified goals for each objective function. A study aiming to address the uncertainty in input parameters for sustainable SSOA problem was done by Mohammed, Harris, and Govindan (2019). They proposed a fuzzy multi-objective model to cope with the existing uncertainty and solved the model using ε-constraint and LP-metrics approaches. Bektur (2020) proposed the use of a fuzzy multi-objective optimization model for addressing the sustainable SSOA problem with uncertain parameters to find the order quantities for purchasing items from suppliers. Their model was developed for a supply chain with multiple suppliers, periods, items, and price levels considering lost sales. Jia et al. (2020) developed a fuzzy goal programming model to solve the sustainable SSOA problem. They considered demand, cost, and capacity as fuzzy parameters and ranked the suppliers using the TOPSIS method. Isaloo et al. (2020) presented a robust bi-objective mathematical model for supply chain network design (SCND) considering sustainability factors. Feng and Gong (2020) introduced an integrated linguistic entropy weight method and multi-objective programming model for SSOA in a circular economy. Nasr et al. (2021) developed a novel fuzzy goal programming approach to address uncertainty in the value of determining goals for objective functions. They presented a multi-objective mixed-integer programming model for the sustainable SSOA problem with multiple items, periods, and price levels, considering vehicle scheduling and inventory-location-routing. Uncertainty is an inevitable issue in real-world problems; in many studies, researchers attempt to deal with the uncertainty of parameters or objective functions using the existing methods. Some of the parameters that have been mentioned repeatedly are demand (Hatefi & Jolai, 2013; Kim, Do Chung, Kang, & Jeong, 2018), costs including production cost, purchasing cost, delivery cost, delay penalty cost, transportation cost, etc. (RezaHoseini, Noori, & Ghannadpour, 2021; Vali-Siar & Roghanian, 2022), lead time (Thevenin, Ben-Ammar, & Brahimi, 2022), and capacity (RezaHoseini et al., 2021). The summary of the selected literature related to the SSOA problem is presented in Table 1.

In today's competitive world, organizations pay significant attention to selecting suitable suppliers to reduce purchase expenses and enhance the final product or service quality. Hence suppliers' performance has an important effect on the success or failure of a supply chain. Supplier selection is the main issue that is considered a strategic responsibility. According to Table 1, it is clear that sustainability has become a crucial part of supply chain decision-making problems. In recent years, most researchers have considered sustainability factors in their studies to make a trade-off among the sustainability objectives (economic, environmental, and social). However, a more significant portion of the researchers have only focused on environmental and economic aspects of sustainability; recently, they have made endeavors to take the social factor into account as well. The literature on the uncertain sustainable SSOA problem shows that there has been not enough attention to multiple uncertainties and robust optimization methods, specifically Bertsimas and Sim's approach as an efficient method. Moreover, simultaneous consideration of factors like, multiple items, multiple suppliers, multiple price levels, multiple period, as well as multiple objectives can make the model more practical in real world problems. The purpose of this study is to fill the existing gaps by proposing an integrated robust-fuzzy model to identify the best potential supplier considering all three aspects of sustainability simultaneously and to allocate the optimal order quantities to each supplier incorporating factors like multiple items, multiple suppliers, multiple price levels, and multiple period in an uncertain environment for objective functions and a number of parameters.

3. Problem description and mathematical formulation

3.1. Problem description

Uncertainties in SSOA problems occur because of different reasons such as the lack of adequate data to determine the value of parameters or unexpected situations. Suppliers might have unexpected delays in the delivery of items to customers which might impose a certain amount of penalty cost which is charged by the customer at the delivery time. The delay in delivery may occur due to production downtime, unexpected logistic failures, harsh weather conditions, etc. Demand is another factor that copes with uncertainties that result in difficulties to satisfy the demand at the expected time. In most SSOA problems, demand is considered stochastic and follows a certain distribution function based on product types. It can help researchers to formulate mathematical models in a way that is more applicable in real-world cases. Transportation cost is another item that is uncertain due to constantly changing fuel prices and unexpected occurrences such as accidents. Addressing the uncertainties in SSOA problems requires a holistic approach that estimates the uncertain decision variables at the same time and considers the impact of each uncertain parameter on other parameters.

Based on the research gaps derived from the literature, this paper develops a robust fuzzy optimization approach to solve a multi-objective problem. Four objectives are developed for this problem in order to minimize the total costs associated with supplier selection and order allocation, minimize the rate of imperfect manufacturing, maximize the total environmental aspect of suppliers, and maximize the total social aspect of suppliers. In the proposed model, multiple items, suppliers, periods, and price levels are considered when robust variables and fuzzy objectives are developed. A robust counterpart of the model is proposed based on the robust optimization approach proposed by Bertsimas and Sim (2004) to address the existing uncertainty in the input parameters. A payoff matrix is formed to specify the upper and lower bounds of objective functions to determine their membership functions. Thereafter, the value of membership function for different values of objective functions and their piecewise linear membership functions are determined. Later, a hybrid robust-fuzzy optimization is proposed, which is an integration of the robust optimization approach (Bertsimas & Sim, 2004) and the fuzzy goal programming approach (Bellman & Zadeh, 1970). Finally, separate satisfaction levels are assigned to each objective function and an average of these satisfaction levels is considered as a Min-Max objective function.

3.2. Notations

Terminologies applied to develop the multi-objective model are as follows.

Indices

i	Items $(i = 1, 2, 3, \cdots, I)$
j	Suppliers $(j = 1, 2, 3, \cdots, J)$
k	Price levels $(k = 1, 2, 3, \cdots, K)$
t	Periods $(t = 1, 2, 3, \cdots, T)$

Parameters

n_i	Maximum number of potential suppliers for item i
m_j	Maximum number of available price levels for supplier j
D_{it}	Demand for item i in period t
P_{ijkt}	Unit price of purchasing item i from supplier j with price level k in period t ($)
O_{ijt}	Ordering cost of item i from supplier j in period t ($)
Q_{ijkt}	Defect rate of item i purchased from supplier j with price level k in period t (%)
TI_{ijkt}	Rate of delayed delivery for item i purchased from supplier j with price level k in period t (%)
DC_{ijkt}	Delay penalty cost for item i purchased from supplier j with price level k in period t ($)
C_{ijt}	Capacity of supplier j for providing item i in period t (Items)
H_{it}	Holding cost of item i in period t ($)
TC_{ijt}	Transportation cost item i from supplier j in period t ($)
DTC_j	Distance from supplier j (Kilometer)
FE_{ij}	Flexibility of supplier quota allocation of supplier j for item i(%)
F_i	Lower bound of quota flexibility for item i (%)
SE_{ij}	Service level of supplier j for item i (%)
S_i	Lower bound of service level for item i (%)
RE_{ij}	Rating value of supplier j for item i (%)
R_i	Lower bound of rating value for item i (%)
E_{ij}	Environmental score of supplier j in providing item i
SO_{ij}	Social score of supplier j in providing item i
B_{ijk}	Breaking point of price for item i purchased by supplier j (%)
M	A big number

Decision variables

X_{ijkt}	Number of item i supplied by supplier j with price level k in period t
IE_{it}	Inventory of item i at the end of period t
Y_{ijkt}	$\begin{cases} 1, \text{If supplier } j \text{ provides item } i \text{ with price level } k \text{ at period } t \\ 0, \text{Otherwise} \end{cases}$
YP_{ijt}	$\begin{cases} 1, \text{If supplier } j \text{ provides at least one unit of item } i \text{ at period } t \\ 0, \text{Otherwise} \end{cases}$

3.3. Assumptions

The following assumptions are taken into account to develop the multi-objective problem.

1. Multiple suppliers can provide the needs of a single customer during different periods and multiple items can be supplied by each supplier simultaneously.
2. Shortage of item i from supplier j is not permitted in the whole supply chain network neither in terms of backordering nor lost sales. It means that the suppliers cannot deliver the items with a delay due to the lack of enough items in stock. Similarly, the delivery of items in more than a single batch (i.e., partial delivery) is not allowed.
3. Suppliers can offer quantity discounts to stimulate demand and encourage customers to purchase more of their products.
4. A multi-objective mixed-integer linear programming model is developed to quantify four different objectives (costs, defective items, environmental factors, and social factors) of the supply chain. Three parameters including demand, delay penalty cost, and transportation cost are uncertain parameters. Due to the theoretical certitude for robust optimization models, which are more conservative in decision strategies, and quality of their solution that are guaranteed in contrast with the stochastic approach, the robust counterpart model is proposed. Afterwards, with the aim of maximizing the degree of satisfaction, a robust counterpart model is converted to a fuzzy goal programming that led us to employ a robust-fuzzy optimization model to determine the optimal value for the number of shipped items and allocating orders to suppliers. It is

possible to use the proposed model in handling real-world situations where there is a lack of adequate historical data to handle the uncertainty in parameters and accordingly most uncertain parameters cannot be expressed as probability distributions.

5. The performance of each supplier is calculated using environmental and social scores based on the supplies of the items. Besides, the supplier selection and order allocation is optimized based on the number of defective items transported from each supplier to the customer and the total relevant cost associated with the supplying operations.
6. The inventory level of each supplier should satisfy the demand of the customer at a specific period. Moreover, there are service level and order flexibility constraints for each supplier.

3.4. Mathematical formulation

In this section, a multi-objective MILP problem is extended to minimize (1) the total costs associated with supplying the items for each supplier consisting of the costs of purchasing, ordering, holding, transporting, and the penalty cost of deliveries; (2) the proportion of defective items for each supplier when allocating the orders, and to maximize the social and environmental scores of each supplier to optimize the supplier selection.

$$\text{Min } Z_1=\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j}\sum_{t=1}^{T}P_{ijkt}X_{ijkt}+\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}O_{ijt}YP_{ijt}+\sum_{i=1}^{I}\sum_{t=1}^{T}H_{it}IE_{it}+\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j}\sum_{t=1}^{T}TI_{ijkt}Dc_{ijkt}X_{ijkt}+\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}TC_{ijt}DTC_jYP_{ijt} \tag{1}$$

$$\text{Min } Z_2=\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j}\sum_{t=1}^{T}Q_{ijkt}X_{ijkt} \tag{2}$$

$$\text{Max } Z_3=\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j}\sum_{t=1}^{T}E_{ij}X_{ijkt} \tag{3}$$

$$\text{Max } Z_4=\sum_{i=1}^{I}\sum_{j=1}^{n_i}\sum_{k=1}^{m_j}\sum_{t=1}^{T}SO_{ij}X_{ijkt} \tag{4}$$

Subject to

$$IE_{it}=IE_{i,(t-1)}+\sum_{j=1}^{n_i}\sum_{k=1}^{m_j}(1-Q_{ijkt})X_{ijkt}-D_{it}\forall i,t \tag{5}$$

$$IE_{i,(t-1)}+\sum_{j=1}^{n_i}\sum_{k=1}^{m_j}X_{ijkt}\geq D_{it}\forall i,j,k,t \tag{6}$$

$$Y_{ijkt}\leq X_{ijkt}\leq C_{ijt}Y_{ijkt}\forall i,j,k,t \tag{7}$$

$$\sum_{k=1}^{K}Y_{ijkt}\leq 1\forall i,j,t \tag{8}$$

$$\sum_{j=1}^{n_i}\sum_{k=1}^{m_j}\sum_{t=1}^{T}FE_{ij}X_{ijkt}\geq\sum_{t=1}^{T}F_iIE_{it}\forall i,t \tag{9}$$

$$\sum_{j=1}^{n_i}\sum_{k=1}^{m_j}\sum_{t=1}^{T}SE_{ij}X_{ijkt}\geq\sum_{t=1}^{T}S_iIE_{it}\forall i,t \tag{10}$$

$$\sum_{j=1}^{n_i}\sum_{k=1}^{m_j}\sum_{t=1}^{T}RE_{ij}X_{ijkt}\geq\sum_{t=1}^{T}R_iIE_{it}\forall i,t \tag{11}$$

$$B_{ij,(k-1)}Y_{ijkt}\leq X_{ijkt}\leq B_{ijk}Y_{ijkt}\forall i,j,k,t \tag{12}$$

$$YP_{ijt}\leq\sum_{k=1}^{K}Y_{ijkt}\leq MYP_{ijt}\forall i,j,t \tag{13}$$

$$X_{ijkt},IE_{it}\geq 0 \tag{14}$$

$$Y_{ijkt},YP_{ijt}\in\{0,1\} \tag{15}$$

Eq. (1), as the first objective function, aims to minimize the sum of different cost elements consisting of five terms of purchasing, ordering, inventory, the penalty for delayed delivery, and transportation costs, respectively. The second objective function presented in Eq. (2) minimizes the number of defective items to enhance the quality of purchased items. Eqs. (3) and (4) maximize the environmental and social factors as the third and fourth objective functions, respectively. Eq. (5) represents the inventory balance constraint. The demand constraint, shown as Eq. (6), implies that the demand for each item in each period should not exceed the number of ordered items from suppliers and their inventory. Eq. (7) represents the capacity constraint of each supplier, which means the ordering process can only be done when a supplier is selected to provide the required items. Eq. (8) guarantees that each item is only ordered from a specific price level. Specification of the lower limitations for flexibility, service level, and ranking and ensuring that they should not get a value lower than these specified levels are formulated in Eqs. (9) to (11), respectively. Eq. (12) shows that the ordered items from a specific price level would fall in the specified range for that level. Eq. (13) sums all of the items purchased from one particular supplier for calculating their purchasing cost in the first objective function. Finally, the type and characteristics of decision variables are specified in Eqs. (14) and (15).

4. Robust-fuzzy optimization approach

Parameters in mathematical models are considered deterministic to avoid the complexity of real-world problems; however, in practice, most of the input data are uncertain. In the case of mathematical models with uncertainty, slight changes in the value of parameters can change the optimality of the solution or take the problem out of its feasible space. Hence, the focus should be on presenting a solution that would show robustness to these potential changes. Hereof, a robust optimization approach is developed to keep the model away from considerable deviations.

Bertsimas and Sim (2004) revolutionized the robust optimization approach by presenting a robust counterpart model, which is a linear mathematical programming model with an adjustable conservatism level that can be set for different values. The proposed robust counterpart model is not only able to account for various conservatism levels but can also be applied to optimization problems with discrete variables. The robust counterpart model is developed as follows (Bertsimas & Sim, 2004).

$$\max Z=c'x \tag{16}$$

Subject to

$$Ax\leq b \tag{17}$$

$$l\leq x\leq u \tag{18}$$

It is assumed that the uncertainty in parameter only influences the elements of matrix A. Consider the specific row i in matrix A where J_i is the set of uncertain coefficients in row i, we represent $a_ix\leq b_i$. Each of a_{ij}, $j\in J_i$ would be modeled as a symmetrical random independent variable $\widetilde{a}_{ij}$, $j\in J_i$ in the range of $[a_{ij}-\widehat{a}_{ij},a_{ij}+\widehat{a}_{ij}]$. Random variable $\eta_{ij}=\left(a_{ij}-\widetilde{a}_{ij}\right)/\widehat{a}_{ij}$ is defined according to the uncertain nature of the parameter $\widetilde{a}_{ij}$, which has an unknown value but a symmetrical distribution in $[-1,1]$. The conservatism level, which is non-negative and smaller or equal to the number of uncertain parameters in a constraint or objective function, is shown as Γ_i. Parameter Γ_i which is not necessarily an integer number, would be defined for each i with a value in the range of $[0,|J_i|]$. Decision-makers can adjust the robustness of the method against the conservatism of the solution. It seems that there is a slight chance that all a_{ij}, $j\in J_i$ will change simultaneously. Based on these explanations, the presented model by Bertsimas and Sim (2004) would be proposed as follows.

$$\max Z = c^{'}x \tag{19}$$

Subject to

$$\sum_j a_{ij}x_j + \max_{\{S_i\cup\{t_i\}|S_i\subseteq J_i,|S_i|=\lfloor\Gamma_i\rfloor,t_i\in J_i/S_i\}} \left\{\sum_{j\in S_i}\widehat{a}_{ij}y_j + (\Gamma_i-\lfloor\Gamma_i\rfloor)\widehat{a}_{it}y_{it}\right\} \le b_i \tag{20}$$

$$-y_j \le x_j \le y_j \tag{21}$$

$$l \le x \le u \tag{22}$$

$$y \ge 0 \tag{23}$$

Each model in the robust optimization problem has a robust counterpart model. Bertsimas and Sim (2004) proved that the following model is the robust counterpart of the model provided above.

$$\max Z = c^{'}x \tag{24}$$

Subject to

$$\sum_j a_{ij}x_j + z_j\Gamma_i + \sum_{j\in J} p_{ij} \le b_i \tag{25}$$

$$z_i + p_{ij} \ge \widehat{a}_{ij}y_j \tag{26}$$

$$-y_j \le x_j \le y_j \tag{27}$$

$$p_{ij} \ge 0 \tag{28}$$

$$y_j \ge 0 \tag{29}$$

$$z_i \ge 0 \tag{30}$$

$$x \ge X \tag{31}$$

To set the robustness and specify the conservatism level of the mathematical model, three decision variables $\left(z_i,p_{ij},y_i\right)$ are defined. Due to the uncertainties stemming from the real-world solution of the mathematical model, a robust optimization model is proposed to address the uncertainties of parameters and find the optimal solution. It is assumed that the delay penalty cost (DC_{ijkt}), transportation cost (TC_{ijt}), and demand (D_{it}) are uncertain parameters. These parameters are considered to be uncertain because their uncertainty is the case in practical situations. Demand forecasting has always been a huge challenge for practitioners while demand usually does not follow a specific stochastic function and different factors affect the demand. Transportation cost which constitutes a large portion of the total cost is another uncertain parameter that is completely influenced by uncertain fuel prices in competitive markets. Delay which has an uncertain nature and occurs due to the lack of in-stock items or delivery failures is another uncertain parameter considered in the developed model. Also, the uncertainty of these items is based on the problem setting elaborated for solving the numerical example under different scenarios in Sections 5 and 6. The modified model after applying the robustness is summarized as follows.

$$\begin{aligned}\min Z_1^{'} = {} & \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}P_{ijkt}X_{ijkt} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}O_{ijt}YP_{ijt} + \sum_{i=1}^{I} \\ & \times \sum_{t=1}^{T}H_{it}IE_{it} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}TI_{ijkt}Dc_{ijkt}X_{ijkt} + \sum_{i=1}^{I}\sum_{j=1}^{J} \\ & \times \sum_{t=1}^{T}TC_{ijt}DTC_jYP_{ijt} + \Gamma_1 ZB + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{t=1}^{T}PB_{ijkt} + \sum_{i=1}^{I}\sum_{j=1}^{J} \\ & \times \sum_{t=1}^{T}PB_{ijt}^{'}\end{aligned} \tag{32}$$

Objective functions (2)-(4)

Subject to.

Constraints (7)-(15)

$$ZB + PB_{ijkt} \ge \widetilde{DC}_{ijkt}YB_{ijkt}\forall i,j,k,t \tag{33}$$

$$-YB_{ijkt} \le X_{ijkt} \le YB_{ijkt}\forall i,j,k,t \tag{34}$$

$$ZB + PB_{ijt}^{'} \ge \widetilde{TC}_{ijt}YPB_{ijt}\forall i,j,t \tag{35}$$

$$-YPB_{ijt} \le YP_{ijt} \le YPB_{ijt}\forall i,j,t \tag{36}$$

$$\left(\overline{D}-\Gamma_2^{'}\widehat{D}_{it}\right) \le IE_{i,t-1} - IE_{it} + \sum_{j=1}^{n_i}\sum_{k=1}^{m_j}(1-Q_{ijkt})X_{ijkt} \le \left(\overline{D}+\Gamma_2^{'}\widehat{D}_{it}\right)\forall i,t \tag{37}$$

$$IE_{i,t-1} + \sum_{j=1}^{n_i}\sum_{k=1}^{m_j}X_{ijkt} \ge \left(\overline{D}+\Gamma_2^{'}\widehat{D}_{it}\right)\forall i,t \tag{38}$$

$$ZB, PB_{ijkt}, YB_{ijkt}, PB_{ijt}^{'}, YPB_{ijt} \ge 0\forall i,j,k,t \tag{39}$$

Parameter Γ_i is added to the first objective function $Z_1^{'}$ to set a conservatism level for this objective and this term is added to provide a conservatism in the case of the existence of the delay penalty cost (DC_{ijkt}) and transportation cost (TC_{ijt}). Other objective functions are the same as in the previous model. Variables $ZB, PB_{ijkt}, YB_{ijkt}, PB_{ijt}^{'}, YPB_{ijt}$ are added to the model for setting the robustness of the model and specifying the conservatism levels. DC_{ijkt} is the uncertain parameter for the delay penalty cost, and Eqs. (33) and (34) specify the characteristics of this parameter. The value of TC_{ijt}, which represents the uncertain parameter for transportation cost, is determined by Eqs. (35) and (36). D_{it} indicates an uncertain parameter for demand with a value in the interval of $[\overline{D}_{it}-\Gamma_2^{'}\times\widehat{D}_{it},\overline{D}_{it}+\Gamma_2^{'}\times\widehat{D}_{it}]$, where $\overline{D}_{it}$ and $\Gamma_2^{'}\times\widehat{D}_{it}$ show the deterministic value of demand and the maximum deviation from the deterministic value, respectively (See Eqs. (37) and (38)). Since the existing uncertainty in the demand parameter influences the right-hand side of a constraint, the model has been written based on similar cases in the literature (Hatefi & Jolai, 2014; Pishvaee, Rabbani, & Torabi, 2011). $\Gamma_2^{'}$ represents the uncertainty budget, which gets a value in $[0,1]$ and influences the right-hand side of the constraint. $\Gamma_2^{'}=0$ indicates that there is no conservatism against uncertainty, and $\Gamma_2^{'}=1$ indicates that there is full conservatism against uncertainty. The rest of the constraints will not change and remain the same. Finally, Equation (39) shows the non-negativity of robustness variables.

4.1. Piecewise membership function

After determining the ranges of objective functions, the piecewise membership function is determined based on the proposed approach by Hannan (1981). In this paper, the membership degrees are determined by decision-makers which means that they are divided into three equal

Table 2
Instruction for determining membership functions.

For minimized objective functions								
Z_1	$> T_{10}$	T_{10}	T_{11}	T_{12}	⋯	T_{1V_1}	T_{1V_1+1}	$< T_{1V_1+1}$
$f_1(Z_1)$	0	0	u_{11}	u_{12}	⋯	u_{1V_1}	1	1
Z_2	$> T_{20}$	T_{20}	T_{21}	T_{22}	⋯	T_{2V_2}	T_{2V_2+1}	$< T_{2V_2+1}$
$f_2(Z_2)$	0	0	u_{21}	u_{22}	⋯	u_{2V_2}	1	1
For maximized objective functions								
Z_3	$< T_{3V_3+1}$	T_{3V_3+1}	T_{3V_3}	T_{3V_3-1}	⋯	T_{31}	T_{30}	$T_{30} <$
$f_3(Z_3)$	0	0	u_{31}	u_{32}	⋯	u_{3V_3}	1	1
Z_4	$< T_{4V_4+1}$	T_{4V_4+1}	T_{4V_4}	T_{4V_4-1}	⋯	T_{41}	T_{40}	$T_{40} <$
$f_4(Z_4)$	0	0	u_{41}	u_{42}	⋯	u_{4V_4}	1	1

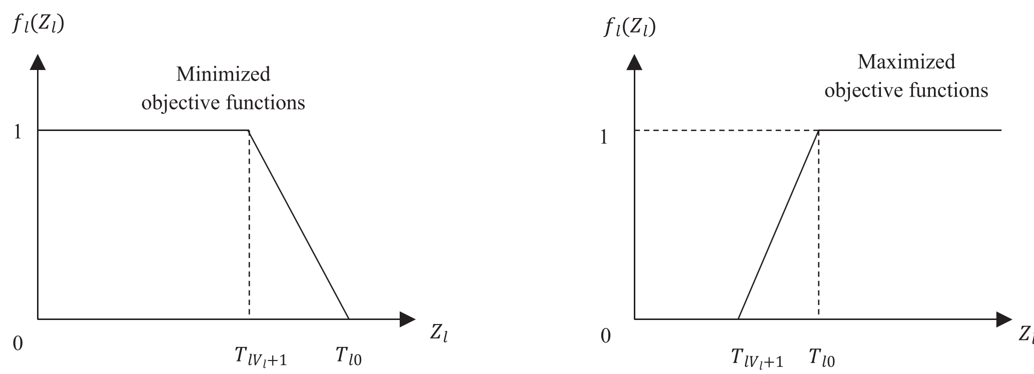

Fig. 1. The general form of membership charts.

parts after specifying the range. Thereafter, the decision-makers determine the membership degree for two of the points based on their experience and the given aspiration levels. The membership degrees are numbers between zero and one. For example, for an objective function with the scope of change for $[T_{l,v_1+1}, T_{10}]$, the membership degree for minimized and maximized objective functions is determined in Table 2.

when.

$(0 \le u_{lb} \le 1)$, $(u_{lb} \le u_{lb+1})$, $(l = 1, 2, 3, 4)$,.$(b = 1, 2, \cdots, V_1)$After determining the membership degrees, the membership function chart is obtained by connecting the dots. A general form of membership function for each type of objective function is illustrated in Fig. 1.

Moreover, the piecewise linear function for each objective function is formulated as follows.

$$f_l(Z_l) = \sum_{b=1}^{P_l} \alpha_{lb} |Z_l - ZT_{lb}| + \beta_l Z_l + \theta_l, l = 1-4 \quad (40)$$

where

$$\alpha_{lb} = -\frac{\gamma_{l,b+1} - \gamma_l}{2}; \beta_l = \frac{\gamma_{l,V_{l+1}} - \gamma_{l1}}{2}; \theta_l = \frac{S_{l,V_{l+1}} - S_{l1}}{2} \quad (41)$$

Assume that $f_l(Z_l) = \gamma_{lr} Z_l + S_{lr}$ for each interval $T_{l,r-1} \le Z_l \le T_{lr}$, where γ_{lr} indicates the slope and S_{lr} is the y-intercept of the line section on $[T_{l,r-1}, T_{lr}]$ in the piecewise linear membership function. After substituting α_{lb}, β_l, and θ_l in equation (41), the following result is obtained.

$$f_l(Z_l) = -\left(\frac{\gamma_{l2} - \gamma_{l1}}{2}\right)|Z_l - T_{l1}| - \left(\frac{\gamma_{l3} - \gamma_{l2}}{2}\right)|Z_l - T_{l2}| - \cdots - \left(\frac{\gamma_{l,V_{l+1}} - \gamma_{lV_l}}{2}\right)|Z_l - T_{lV_l}| + \left(\frac{\gamma_{l,V_{l+1}} - \gamma_{l1}}{2}\right)Z_l + \frac{S_{l,V_{l+1}} - S_{l1}}{2};$$

$$\left(\frac{\gamma_{l,b+1} - \gamma_{lb}}{2}\right) \ne 0, l = 1-4; b = 1, 2, \cdots, V_l, \quad (42)$$

where

$$\gamma_{l1} = \left(\frac{u_{l1} - 0}{T_{l1} - T_{l0}}\right), \gamma_{l2} = \left(\frac{u_{l2} - u_{l1}}{T_{l2} - T_{l1}}\right) \cdots \gamma_{l,V_{l+1}} = \left(\frac{1 - u_{lV_l}}{T_{l,V_{l+1}} - T_{lV_l}}\right) \quad (43)$$

V_l denotes the number of broken points of the lth objective function and $S_{l,V_{l+1}}$ is the y-intercept for the section of the line segment on $[T_{lV_1}, T_{l,V_1+1}]$.

$$Z_l + d_{lb}^- - d_{lb}^+ = T_{lb}; l = 1-4; b = 1, 2, \cdots, V_l, \quad (44)$$

d_{lb}^+ and d_{lb}^- are the positive and negative deviational variables at the lth point, respectively and T_{lb} represents the value of the lth objective function at the lth point. Equation (45) is the result of substituting equation (44) into (42).

$$f_l(Z_l) = -\left(\frac{\gamma_{l2} - \gamma_{l1}}{2}\right)(d_{l1}^- - d_{l1}^+) - \left(\frac{\gamma_{l3} - \gamma_{l2}}{2}\right)(d_{l2}^- - d_{l2}^+) - \cdots - \left(\frac{\gamma_{l,V_{l+1}} - \gamma_{lV_l}}{2}\right)(d_{lV_l}^- - d_{lV_l}^+) + \left(\frac{\gamma_{l,V_{l+1}} - \gamma_{l1}}{2}\right)Z_l + \frac{S_{l,V_{l+1}} - S_{l1}}{2}, l = 1-4. \quad (45)$$

4.2. Hybrid robust-fuzzy optimization

A generic model is proposed in this section which is developed based on the integration of the robust optimization model proposed by Bertsimas and Sim (2004) and the fuzzy approach presented by Bellman and Zadeh (1970). This model aims to maximize the degree of satisfaction (φ_0) using the max(min) function. The hybrid robust-fuzzy model for the proposed model in this paper is summarized as follows.

$$\max \varphi_0 \quad (46)$$

Subject to

$$\varphi_0 \le -\left(\frac{\gamma_{l2} - \gamma_{l1}}{2}\right)(d_{l1}^- - d_{l1}^+) - \left(\frac{\gamma_{l3} - \gamma_{l2}}{2}\right)(d_{l2}^- - d_{l2}^+) - \cdots - \left(\frac{\gamma_{l,V_{l+1}} - \gamma_{lV_l}}{2}\right)(d_{lV_l}^- - d_{lV_l}^+) + \left(\frac{\gamma_{l,V_{l+1}} \mp \gamma_{l1}}{2}\right)Z_l + \frac{S_{l,V_{l+1}} - S_{l1}}{2} \quad (47)$$

$$Z_l + (d_{lb}^- - d_{lb}^+) = Y_{lb}, l = 1-4 \text{and} b = 1, 2, \cdots, V_1 \quad (48)$$

Constraints (7)-(15) and (33)-(39)

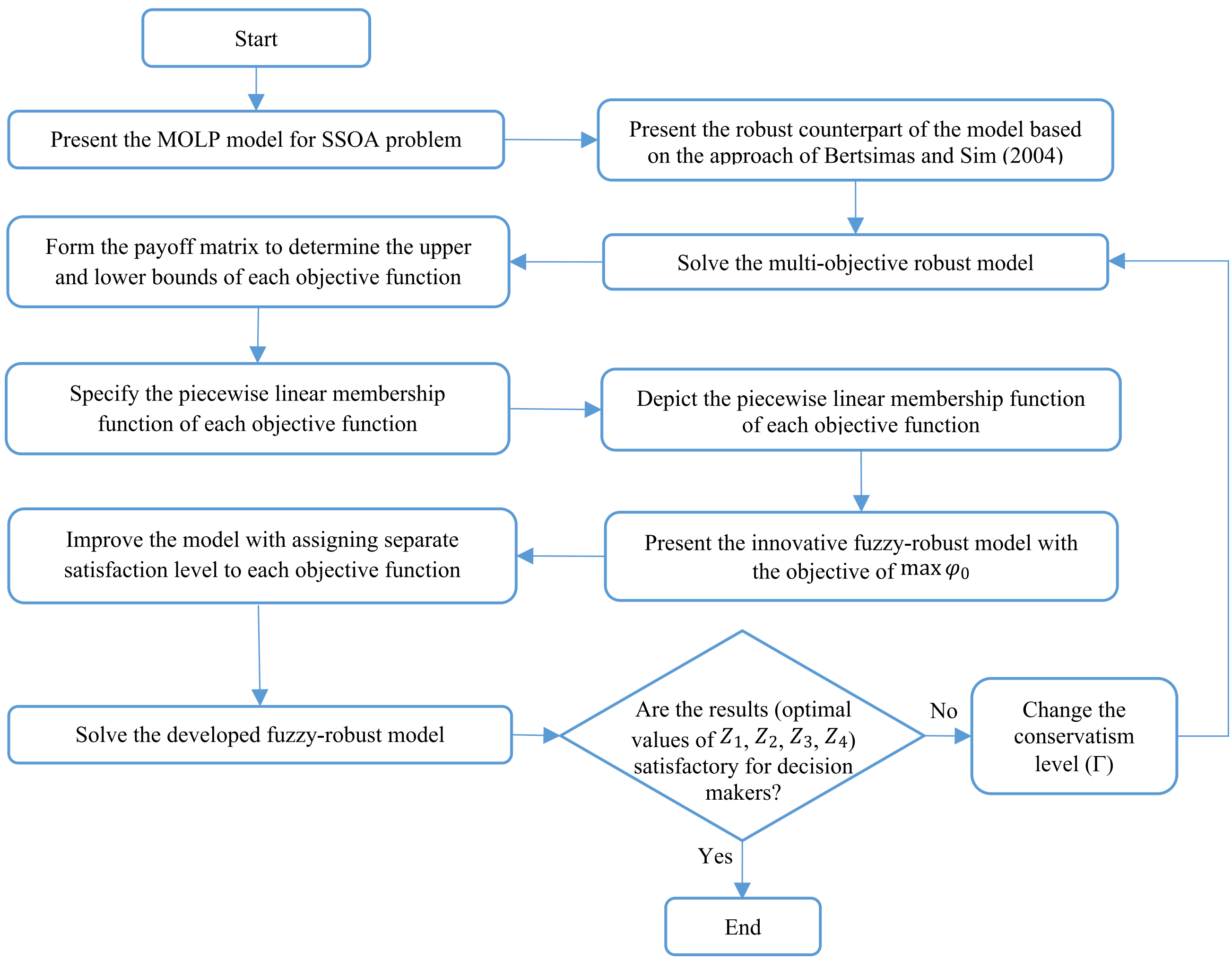

Fig. 2. The flowchart indicating the steps of the solution procedure.

4.3. Robust-fuzzy model improvement

After solving the model presented in Section 4.2 and determining the satisfaction degree φ_0, the results are improved using the method proposed by Nazari-Shirkouhi et al. (2013). Using a $\max(\min)$ objective function, separate membership functions are developed to maximize the average value of satisfaction degrees.

$$\max \frac{1}{4}\sum_{l=1}^{4}\varphi_l \tag{49}$$

Subject to

$$\begin{aligned}\varphi_0 \leq \varphi_l \leq & -\left(\frac{\gamma_{l2}-\gamma_{l1}}{2}\right)\left(d_{l1}^- - d_{l1}^+\right) - \left(\frac{\gamma_{l3}-\gamma_{l2}}{2}\right)\left(d_{l2}^- - d_{l2}^+\right) - \cdots \\ & -\left(\frac{\gamma_{l,V_{l+1}}-\gamma_{lV_l}}{2}\right)\left(d_{l,V_l}^- - d_{l,V_l}^+\right) + \left(\frac{\gamma_{l,V_{l+1}}+\gamma_{l1}}{2}\right)Z_l + \frac{S_{l,V_{l+1}}-S_{l1}}{2}\forall l\end{aligned} \tag{50}$$

$$Z_l + \left(d_{lb}^- - d_{lb}^+\right) = Y_{lb}, l = 1-4; b = 1,2,\cdots,V_l, \tag{51}$$

Constraints (7)-(15), (33)-(39).

The steps of the solution procedure proposed in Section 4 are summarized in Fig. 2.

5. Numerical example

In this section, the proposed model is solved using numerical data. 5 items, 5 periods, and 3 price levels are considered, where the items can be supplied by 4 suppliers. No supplier can provide all 5 items, and some can only supply 2 or 3 items. Suppliers offer discounts on the unit price of items to the customers that buy a specific number of items. Table A1 presents the input data for the multi-objective model consisting of the unit price of purchasing items, demand for items, holding cost, the capacity of suppliers in providing different items, transportation cost, and the lower bound for flexibility, service, and ranking of the suppliers in the 5 periods. The price breakpoints and discounts offered by suppliers are shown in Table A1 (See Appendix A). The process of solving the presented model based on the steps mentioned in Section 4.4 is presented in the following. The numerical values for parameters have been obtained from a case study of the automobile industry by Rabieh et al. (2018). This paper considers uncertainties and ambiguity conditions for parameters. The model proposed by Rabieh et al. (2018) is developed for two certain items while this study is extended for five items. Besides, the numerical experiment conducted in this study is extended for large-scale problems when the number of items is raised from 5 to 30 items and the number of suppliers is increased from 4 to 100 suppliers. Implementing the mathematical model in a large-size setting is done to validate the model and enhance generalizability of the findings. In this way, the numerical study takes advantage of data from a real case study while preserving the flexibility provided by using numerical examples built on top of the case study data. Such an approach is particularly useful in evaluating the proposed model and credibility and applicability of the proposed solution procedure.

Table 3
Results of solving the multi-objective deterministic model.

Objective function	Value	Run time
Z_1(Total cost)	43770.453	00:00:34.134
Z_2(Defect rate)	55.408	00:00:22.423
Z_3(Environmental aspect)	845.387	00:00:23.409
Z_4(Social aspect)	854.354	00:00:22.974

Table 4
Results of solving the robust counterpart of the model with different conservatism levels.

Conservatism levels	$Z_1^{'}$	Z_2	Z_3	Z_4	Run time
$\Gamma_1 = 0, \Gamma_2^{'} = 0$	43770.453	55.408	845.387	854.354	00:00:29.291
$\Gamma_1 = 5, \Gamma_2^{'} = 0.2$	57053.194	55.797	854.837	864.912	00:00:55.897
$\Gamma_1 = 10, \Gamma_2^{'} = 0.4$	66777.353	58.045	863.386	874.106	00:14:41.963
$\Gamma_1 = 15, \Gamma_2^{'} = 0.6$	76279.735	61.460	871.528	882.766	00.17:09.254
$\Gamma_1 = 20, \Gamma_2^{'} = 0.8$	78888.898	64.886	878.629	890.311	00:17:09.500
$\Gamma_1 = 25, \Gamma_2^{'} = 1$	82587.266	68.332	884.995	896.858	00:05:18.325

5.1. Step 1. Solving the deterministic model

In this stage, an initial solution for each objective function can be determined using a mixed-integer linear model. The data to be used in the proposed model lack any uncertainties to avoid further complexity. The model is solved using the CPLEX solver in GAMS (Version 37) for the four single-objective models. The execution time is also indicated for each solution and the results are summarized in Table 3.

5.2. Step 2. Solving the robust optimization model

In this step, uncertainty is considered for the transportation cost, delay penalty cost, and demand. The results of solving the robust counterpart of the model with different conservatism levels (Bertsimas & Sim, 2004) are presented in Table 4, where ($\Gamma_1 = 0$, $\Gamma_2^{'} = 0$) and ($\Gamma_1 = 25$, $\Gamma_2^{'} = 1$) correspond to the lowest and highest conservatism levels, respectively. The role of the protection level parameter, Γ_i, is to adjust the robustness of the proposed method against the level of conservatism of the solution and usually set by the model builder, according to risk preference of the decision maker. The higher the conservatism level for decision making, the more cautious the decision makers in selecting the optimal values for objective functions. Also, it can be interpreted that decision makers are more confident about selecting the optimal values of objective functions. In other words, decision-makers are responsible for selecting the most suitable conservatism level for their situation. According to Bertsimas and Sim (2004) and Hatefi & Jolai (2013), the values of different conservatism levels are defined in 6 different levels for Γ_1 and $\Gamma_2^{'}$. According to Table 4, the level of robustness in objective function is controlled by parameter Γ_1, similarly in constraints the parameter $\Gamma_2^{'}$ adjusts the robustness of the proposed method to the level of conservatism of the solution. Consequently, where $\Gamma_1 = 0$ and $\Gamma_2^{'} = 0$, the conservatism level is in its lowest, that means the model is equivalent to the nominal model. In contrast, $\Gamma_1 = 25$ and $\Gamma_2^{'} = 1$, indicates the highest level of conservatism level for the proposed model.

Fig. 3 illustrates the results of solving the deterministic model and the robust model with different conservatism levels to demonstrate the impact of uncertainty and different conservatism levels.

5.3. Step 3. Developing the payoff matrix

After solving the counterpart robust optimization model, the problem is solved by considering each objective function as a single-objective optimization model and the payoff matrix is developed by substituting optimal values of X and considering $\Gamma_1 = 25$ and $\Gamma_2^{'} = 1$ which is proposed by the decision-makers. The results are summarized as follows.

$\Gamma_1 = 25, \Gamma_2^{'} = 1$	X_1^*	X_2^*	X_3^*	X_4^*
$Z_1^{'}$	82587.264	102645.828	607732.254	623013.107
Z_2	123.1874	68.331	640.583	645.886
Z_3	178.015	94.345	884.995	864.391
Z_4	153.677	128.258	865.670	896.857

After determining the payoff matrix, the scope of change for each

Table 5
The scope of change for each objective function.

82587.264	$\le Z_1^{'} \le$	623013.107
68.331	$\le Z_2 \le$	645.886
94.345	$\le Z_3 \le$	884.995
128.258	$\le Z_4 \le$	896.857

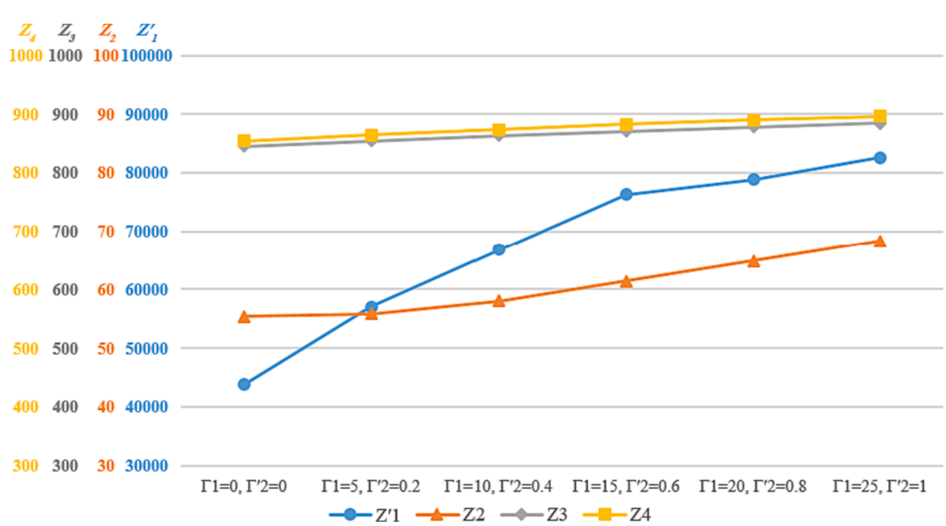

Fig. 3. Comparing the value of objective functions in the deterministic and robust model.

Table 6
The membership degree for each objective function.

$Z_1^{'}$	< 82587.26	82587.26	262729.2	442871.15	623013.1	> 623013.1
$f(Z_1^{'})$	1	1	0.8	0.5	0	0
Z_2	< 68.33	68.33	260.85	453.36	645.88	> 645.88
$f(Z_2)$	1	1	0.7	0.4	0	0
Z_3	< 94.34	94.34	357.89	621.44	884.99	> 884.99
$f(Z_3)$	0	0	0.4	0.7	1	1
Z_4	< 128.25	128.25	384.45	640.65	894.85	> 894.85
$f(Z_4)$	0	0	0.4	0.7	1	1

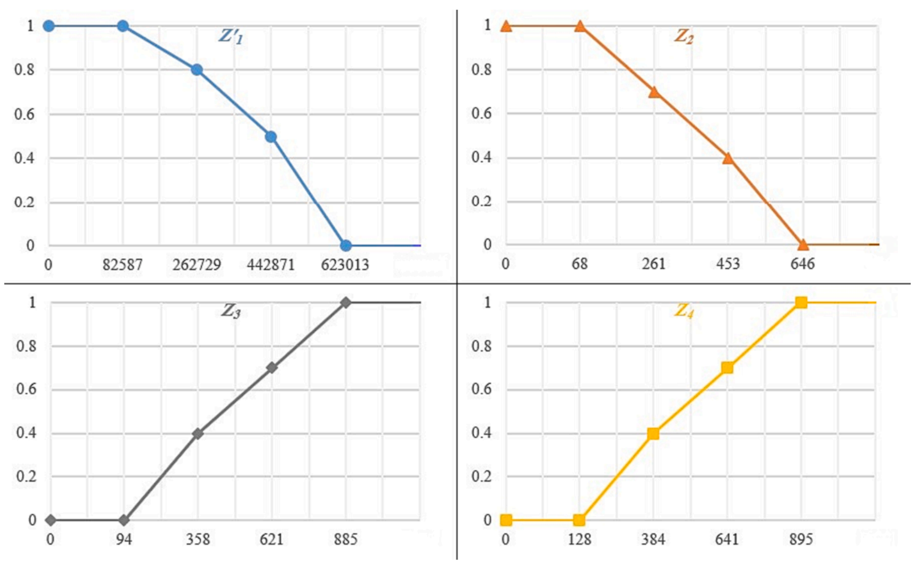

Fig. 4. Membership functions for different objectives.

objective function is illustrated in Table 5.

5.4. Step 4. Determining the piecewise linear membership function

After determining the scope of change for each objective function, the membership degree for each objective function is determined using the piecewise linear membership function proposed by Hannan (1981). The membership degree is determined by decision-makers and provided in Table 6.

The changes in membership degree for each objective are shown in Fig. 4.

5.5. Step 5. Robust-fuzzy optimization

Based on the discussions in Section 4.3, the proposed mathematical model is rewritten as the following.

$$\max \varphi_0 \tag{52}$$

Subject to

$$\varphi_0 \leq 0.000000277559(d_{11}^- - d_{11}^+) + 0.000000555118(d_{12}^- - d_{12}^+) - 0.00000194291Z_1^{'} + 1.410460307 \tag{53}$$

$$\varphi_0 \leq 0.000259716(d_{21}^- - d_{21}^+) - 0.001818009Z_2 + 1.224227812 \tag{54}$$

$$\varphi_0 \leq -0.000189717(d_{31}^- - d_{31}^+) + 0.001328022Z_3 - 0.75293322 \tag{55}$$

$$\varphi_0 \leq -0.00019516(d_{41}^- - d_{41}^+) + 0.001366122Z_4 - 0.125216425 \tag{56}$$

$$Z_1^{'} + (d_{11}^- - d_{11}^+) = 262729.211 \tag{57}$$

Table 7
Results of solving the robust-fuzzy model for both sizes of the problem.

φ_0 **(i = 5, j = 4)**	0.732	**Run time**	00:00:28.578	
d_{11}^-	d_{11}^+	d_{12}^-	d_{12}^+	$Z_1^{'}$
0	96492.185	83649.763	0	359221.397
d_{21}^-	d_{21}^+			Z_2
159.876	0			293.492
d_{31}^-	d_{31}^+			Z_3
0	218.747			576.76
d_{41}^-	d_{41}^+			Z_4
0	212.646			897.218
φ_0 **(i = 30, j = 100)**	0.729	**Run time**	00.16:55.741	
d_{11}^-	d_{11}^+	d_{12}^-	d_{12}^+	$Z_1^{'}$
0	2245723.866	1,479,709	0	20,349,320
d_{21}^-	d_{21}^+			Z_2
12643.29	0			18482.736
d_{31}^-	d_{31}^+			Z_3
0	19268.684			43259.783
d_{41}^-	d_{41}^+			Z_4
0	22277.278			50239.438

Table 8
Results of solving the improved robust-fuzzy model for both sizes of the problem.

φ_{Total} (i = 5, j = 4)	0.764		**Run time**	00:00:35.768	
φ_1	d_{11}^-	d_{11}^+	d_{12}^-	d_{12}^+	Z_1'
0.859	0	50747.246	129394.702	0	313482.5
φ_2	d_{21}^-	d_{21}^+			Z_2
0.732	159.790	0			293.578
φ_3	d_{31}^-	d_{31}^+			Z_3
0.732	0	218.747			576.642
φ_4	d_{41}^-	d_{41}^+			Z_4
0.732	0	212.646			597.104
φ_{Total} (i = 30, j = 100)	0.784		**Run time**	00:26:09.850	
φ_1	d_{11}^-	d_{11}^+	d_{12}^-	d_{12}^+	Z_1'
0.95	0	1710147.322	1,533,267	0	1,919,334
φ_2	d_{21}^-	d_{21}^+			Z_2
0.729	12625.265	0			18504.239
φ_3	d_{31}^-	d_{31}^+			Z_3
0.729	0	19241.213			43250.158
φ_4	d_{41}^-	d_{41}^+			Z_4
0.729	0	22245.517			50228.310

$$Z_1' + (d_{12}^- - d_{12}^+) = 442871.159 \quad (58)$$

$$Z_2 + (d_{21}^- - d_{21}^+) = 453.368 \quad (59)$$

$$Z_3 + (d_{31}^- - d_{31}^+) = 357.895 \quad (60)$$

$$Z_4 + (d_{41}^- - d_{41}^+) = 384.458 \quad (61)$$

Constraints (7)-(15) and (33)-(39).

The values of objective functions, deviations, and run time for both sizes of the problem is obtained from solving the model using the numerical data and presented in Table 7.

5.6. Step 6. Improving the proposed robust-fuzzy model

According to Section 4.4, separate satisfaction degree are considered for each of the objective functions to maximize the average of the satisfaction degree using a $\max(\min)$ function. The proposed mathematical model is rewritten as the following.

$$\max\varphi_{total} = \frac{1}{4}(\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4) \quad (62)$$

Subject to

$$0.732 \le \varphi_1 \le 0.000000277559(d_{11}^- - d_{11}^+) + 0.000000555118(d_{12}^- - d_{12}^+) - 0.00000194291Z_1' + 1.410460307 \quad (63)$$

$$0.732 \le \varphi_2 \le 0.000259716(d_{21}^- - d_{21}^+) - 0.001818009Z_2 + 1.224227812 \quad (64)$$

$$0.732 \le \varphi_3 \le -0.000189717(d_{31}^- - d_{31}^+) + 0.001328022Z_3 - 0.75293322 \quad (65)$$

$$0.732 \le \varphi_4 \le -0.00019516(d_{41}^- - d_{41}^+) + 0.001366122Z_4 - 0.125216425 \quad (66)$$

Constraints (7)-(15), (33)-(39).

The values of objective functions, satisfaction degrees, and deviations obtained from the numerical solution are presented in Table 8.

By comparing the values of deviation from goal levels proposed in Tables 7 and 8, it can be understood that the optimal values of objective functions in the improved fuzzy robust model are closer to the optimality. At this step, the algorithm stops if the obtained solution is convincing for the decision-makers. Otherwise, the proposed six steps will be repeated using different conservatism levels and membership degrees (See Table 9).

The selection of the proposed conservatism levels is based on experts' opinions about the preferences of decision-makers. Also, based on the case study of the automobile industry given by Rabieh et al. (2018), these conservatism levels are selected to indicate different levels of cautiousness for decision makers when selecting suppliers and allocating orders to them.

In order to prove that the proposed robust-fuzzy optimization approach is completely applicable to larger problems, the model is applied to another example, in which the number of items (*i*) is raised from 5 to 30 and the number of suppliers (*j*) is increased from 4 to 100, while the price level (*k*) and periods considered unchanged by 3 and 5, respectively. The results are illustrated in Table 10. Decision-makers can choose the best practice for their situation.

6. Scenario analysis

As described in the following, seven scenarios are defined to analyze the optimal value of each objective function in different conditions. All of the problems are solved considering $\Gamma_1 = 25$ and $\Gamma_2' = 1$.

- Scenario 1: Eliminating objective function 2 and only considering objective functions 1, 3, and 4.
- Scenario 2: Eliminating objective function 3 and only considering objective functions 1, 2, and 4.
- Scenario 3: Eliminating objective function 4 and only considering objective functions 1, 2, and 3.
- Scenario 4: Considering the set of $(Z_2, f(Z_2))$, $(Z_3, f(Z_3))$, and $(Z_4, f(Z_4))$ with their original range and changing the set of $(Z_1', f(Z_1'))$.
- Scenario 5: Considering the set of $(Z_1', f(Z_1'))$, $(Z_3, f(Z_3))$, and $(Z_4, f(Z_4))$ with their original range and changing the set of $(Z_2, f(Z_2))$.

Table 9
Value of satisfaction degree and objective functions for different values of Γ_1 andΓ_2'

Conservatism levels	φ_0	Z_1'	Z_2	Z_3	Z_4	Run time
$\Gamma_1 = 0, \Gamma_2' = 0$	0.749	216691.08	263.90	542.42	547.61	00:00:30.395
$\Gamma_1 = 5, \Gamma_2' = 0.2$	0.637	288287.27	328.94	626.48	574.02	00:00:27.442
$\Gamma_1 = 10, \Gamma_2' = 0.4$	0.635	373729.73	325.44	624.94	550.55	00:00:26.774
$\Gamma_1 = 15, \Gamma_2' = 0.6$	0.634	287198.78	330.33	610.39	546.61	00:00:27.039
$\Gamma_1 = 20, \Gamma_2' = 0.8$	0.635	301893.36	336.16	605.49	545.02	00:00:27.302
$\Gamma_1 = 25, \Gamma_2' = 1$	0.732	374369.18	293.49	576.75	597.21	00:00:28.578

Table 10
Value of satisfaction degree and objective functions for different values of Γ_1 and $\Gamma_2^{'}$ for the large-scale problem (30 items and 100 suppliers).

Conservatism levels	φ_0	$Z_1^{'}$	Z_2	Z_3	Z_4	Run time
$\Gamma_1 = 0, \Gamma_2^{'} = 0$	0.726	19,597,220	18377.267	42417.390	49240.274	00:02:14.522
$\Gamma_1 = 5, \Gamma_2^{'} = 0.2$	0.590	16,869,510	21141.047	34642.750	40275.883	00.06:13.089
$\Gamma_1 = 10, \Gamma_2^{'} = 0.4$	0.729	19,761,330	18343.968	42842.589	49739.926	00.02:21.002
$\Gamma_1 = 15, \Gamma_2^{'} = 0.6$	0.739	21,505,740	18021.615	44239.158	46687.595	00.18:57.124
$\Gamma_1 = 20, \Gamma_2^{'} = 0.8$	0.693	20,315,970	19833.836	40996.304	47686.472	00.11:09.797
$\Gamma_1 = 25, \Gamma_2^{'} = 1$	0.729	20,349,320	18482.736	43259.783	50239.438	00.16:55.741

Table 11
Results of solving scenarios 1, 2, and 3.

Scenario	$Z_1^{'}$	Z_2	Z_3	Z_4	φ
Scenario 1	358099.599	–	682.062	699.584	0.892
Scenario 2	347412.449	277.717	–	618.211	0.765
Scenario 3	354010.042	286.939	579.724	–	0.742

- Scenario 6: Considering the set of $(Z_1^{'}, f(Z_1^{'}))$, $(Z_2, f(Z_2))$, and $(Z_4, f(Z_4))$ with their original range and changing the set of $(Z_3, f(Z_3))$.
- Scenario 7: Considering the set of $(Z_1^{'}, f(Z_1^{'}))$, $(Z_2, f(Z_2))$, and $(Z_3, f(Z_3))$ with their original range and changing the set of $(Z_4, f(Z_4))$.

The results of solving the problem considering different combinations of scenarios are presented in the following. Table 11 provides a solution for each of the objective functions based on the first three scenarios.

A detailed solution of each objective function and the obtained

Table 12
Details of scenario 4.

Z_2	>645.886	645.886	453.368	260.85	68.331	<68.331
$f(Z_2)$	0	0	0.4	0.7	1	1
Z_3	<94.345	94.345	357.895	621.445	884.995	>884.995
$f(Z_3)$	0	0	0.4	0.7	1	1
Z_4	<384.458	384.458	384.458	640.657	896.857	>896.857
$f(Z_4)$	0	0	0.4	0.7	1	1
Run 1 $(Z_1^{'})$	>550956.328	550956.328	370814.380	190672.432	10530.484	<10530.484
	0	0	0.5	0.8	1	1
Run 2 $(Z_1^{'})$	>586984.717	586984.717	406842.77	226700.822	46558.874	<46558.874
	0	0	0.5	0.8	1	1
Run 3 $(Z_1^{'})$	>623013.107	623013.107	442871.159	262729.211	82587.264	<82587.264
	0	0	0.5	0.8	1	1
Run 4 $(Z_1^{'})$	>803155.055	803155.055	623013.107	442871.159	262729.211	<262729.211
	0	0	0.5	0.8	1	1
Run 5 $(Z_1^{'})$	>983297.002	983297.002	803155.055	623013.107	442871.159	<442871.159
	0	0	0.5	0.8	1	1

Table 13
Results of solving scenarios 4, 5, 6, and 7.

Scenario		Run 1	Run 2	Run 3	Run 4	Run 5
Scenario 4	$Z_1^{'}$	345630.355	313407.970	374369.181	539363.279	327871.889
	Z_2	413.646	293.533	293.492	293.492	293.553
	Z_3	611.998	576.702	576.759	576.759	550.608
	Z_4	660.433	597.162	597.218	597.218	574.609
	φ	0.727	0.732	0.732	0.732	0.732
Scenario 5	$Z_1^{'}$	336896.911	367477.406	374369.181	304677.169	301544.778
	Z_2	263.113	278.299	293.492	413.146	433.12
	Z_3	543.047	559.907	576.759	676.507	682.062
	Z_4	564.445	580.835	597.218	694.184	699.584
	φ	0.681	0.707	0.732	0.884	0.892
Scenario 6	$Z_1^{'}$	340617.605	352188.316	374369.181	431599.231	515406.773
	Z_2	279.975	284.349	293.492	390.981	502.137
	Z_3	529.376	556.331	576.759	706.849	818.232
	Z_4	615.206	609.385	597.218	592.56	735.032
	φ	0.76	0.751	0.732	0.53	0.299
Scenario 7	$Z_1^{'}$	386898.7	356594.442	374369.181	492170.822	517856.885
	Z_2	338.859	289.983	293.492	379.107	505.41
	Z_3	632.771	581.563	576.759	620.899	765.23
	Z_4	615.096	569.863	597.218	739.482	827.601
	φ	0.742	0.739	0.732	0.554	0.292

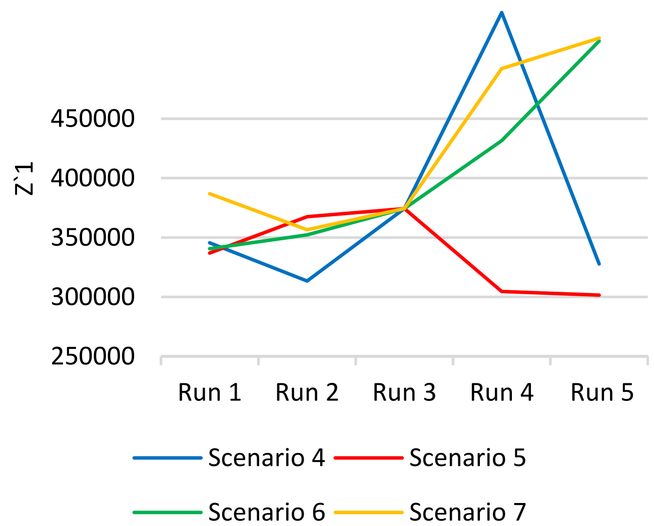

Fig. 5. Changes in the first objective function (total cost) for different scenarios.

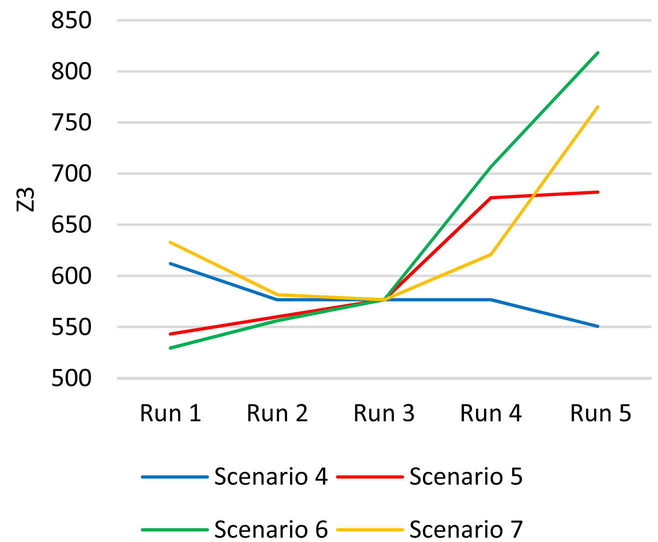

Fig. 7. Changes in the third objective function (environmental factors) for different scenarios.

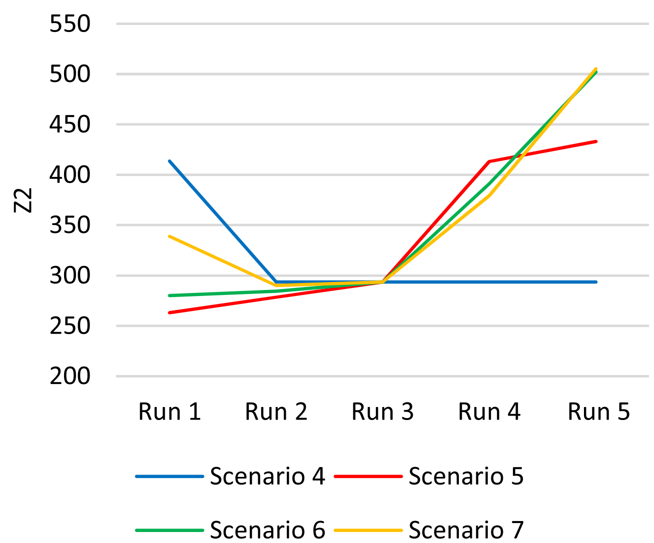

Fig. 6. Changes in the second objective function (number of defective items) for different scenarios.

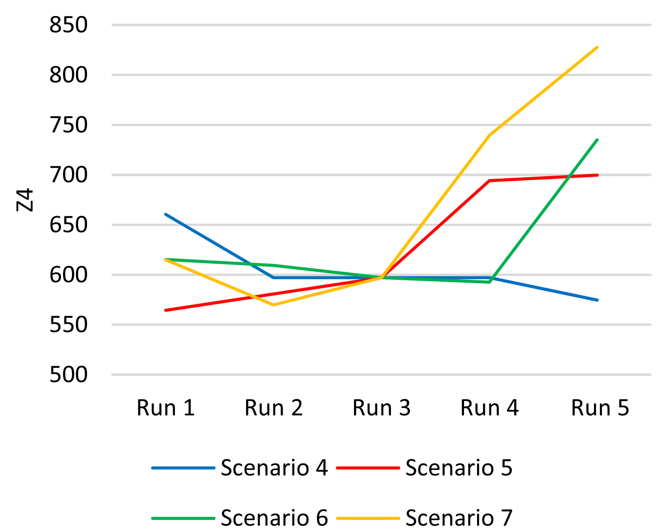

Fig. 8. Changes in the fourth objective function (social factors) for different scenarios.

membership degree is provided in Table 12.

To demonstrate the value of each objective function under scenarios 4, 5, 6, and 7, the numerical model is implemented in GAMS for five runs (with a fixed conservatism level of $\Gamma_1 = 25, \Gamma_2^{'} = 1$) and the sensitivity analysis of each objective is conducted for these four scenarios (See Table 13).

Changes in all four objective functions and the membership degree are illustrated in Figs. 5-9. Fig. 5 shows that the total cost mainly has an increasing trend; however, this increasing trend is not persistent under the fourth and fifth scenarios as the total cost decreases after the third run under these two scenarios. Also, Fig. 5 shows that the lowest cost level occurs under scenario 5 and the total cost is relatively lower in this scenario in comparison to other scenarios.

The lowest number of defective items can be obtained under scenario 4 as it is more consistent in this case and the number of defective items has an increasing trend under other scenarios. Scenario 6 is the best option if the sustainability aspect of suppliers is more important for the managers as it gives the highest amount of environmental factors after the third run (See Fig. 7). For the sake of social responsibility, it is better to select scenario 7 which has the highest amount of social aspects after the third run. Based on the significance of each of the objectives for practitioners, a supplier can be selected to fulfill economic, environmental, or social requirements.

In order to validate the proposed solution for large-size problems, another sensitivity analysis is performed when the number of items is raised from 5 to 30 items and the number of suppliers is increased from 4 to 100 suppliers. The results provided in Table 14 show that the trend of objective functions under different scenarios is almost the same as the small-scale problem.

Also, Figs. 10-14 show the increasing or decreasing trend of different objective functions along with membership degrees. It is observed that the trend is similar to the trend for the small-scale problem. The only difference is the optimal value of objective functions which is assumed to occur because of enlarging the datasets.

7. Discussion

SSOA is a prominent problem for decision-makers in companies that have frequently been addressed in the literature. The problem discussed in this paper highlights a combination of sustainability and quality

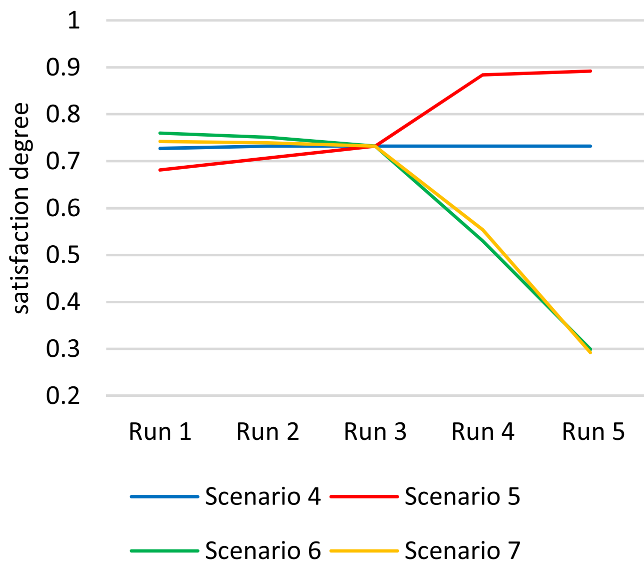

Fig. 9. Changes in the satisfaction degree for different scenarios.

improvement in SSOA and proposes a novel scenario-based robust-fuzzy optimization solution procedure. To the authors' knowledge, this is the first study that discussed the mentioned issues simultaneously to address three research questions in Section 1. The sustainability consideration is addressed by defining two objectives that minimize environmental and social impacts. A scenario-based approach was utilized as a solution procedure that eliminated one objective and solved the model with the rest of the objectives. Then, the value of objectives was compared and a robust-fuzzy approach is adopted to address the uncertainty.

To elaborate on the implications of the mathematical solution under scenarios 1–3, it can be implied that the total satisfaction degree is the highest for scenario 2 when the second objective is eliminated, and the multi-objective model is solved for the rest of the objectives to decrease the complexity of the numerical solution. The second and third scenarios have the next highest total satisfaction degrees respectively which means that eliminating quality improvement function and environmental aspects is less significant than total costs for practitioners. Another important point that can be addressed is that the first objective function which shows the total costs has the lowest amount in scenario 2. A reason for this circumstance is that without considering imperfect quality items (i.e., all items are perfect), more items are produced and the total cost will be reduced. The rate of defective items is on its minimum for scenario 2. However, scenario 1 has the highest environmental and social scores. It can be understood that if practitioners prioritize the minimization of total costs and defect rate of production, scenario 2 will be the one that can be chosen because of the highest satisfaction degree, and if they consider sustainability as the most important issue, scenario 1 is the optimal solution because the maximization of sustainability aspect in scenario 1 is at the highest level. Scenario 3 is not the optimal choice for any of the objectives. The optimal value of objective functions changes in scenarios based on the conservatism levels. Therefore, practitioners will decide which scenario they have to select based on the priority of the objective function. In another word, the objective function with a higher satisfaction degree is the most significant and the practitioners will find a combination of scenario and conservatism levels that optimizes the objective function with the highest satisfaction degree. As it is mentioned earlier, the membership degree of each objective function is determined based on the opinion of decision-makers. Therefore, finding and running the best scenario, which can optimize the total satisfaction degree, is highly dependent on the decision-makers.

After solving the problem with numerical examples, the distance of ideal (optimal) solutions to the given solutions can be determined to evaluate the credibility and efficiency of the proposed approach. The following set of distance functions defined by Abd El-Wahed and Lee (2006) is utilized for this purpose.

$$D_p(W,L) = \left[\sum_{l=1}^{L} w_l^p (1-d_l)^p\right]^{1/p} \tag{67}$$

where d_l is the degree of proximity of the obtained solution vector to the optimal solution vector of the l^{th} objective function. The set of vectors $W = (W_1, W_2, \cdots, W_l)$ indicates the importance of the l^{th} objective function and p is a distance parameter $(1 \le p \le \infty)$. As the sum of all importance vectors for objective functions equals one, $D_p(W,L)$ is written as follows $(1 \le p \le \infty)$.

Table 14
Results of solving scenarios 4, 5, 6, and 7 for large-scale problem.

Scenario		Run 1	Run 2	Run 3	Run 4	Run 5
Scenario 4	Z_1'	21517249.325	19546388.308	24563978.127	32569874.497	20256987.149
	Z_2	182136.96	125937.687	124489.307	125589.756	125156.921
	Z_3	42698.367	39806.239	39756.171	39846.213	36746.369
	Z_4	496.607	476.369	468.693	472.598	463.802
	φ	0.728	0.734	0.733	0.733	0.734
Scenario 5	Z_1'	19536521.054	22364983.367	24569873.025	17569987.207	17356987.106
	Z_2	125976.028	134597.257	140369.367	187609.237	196378.674
	Z_3	39807.692	40267.509	41705.349	45873.643	47893.144
	Z_4	364.209	386.403	394.501	458.720	461.255
	φ	0.693	0.711	0.737	0.854	0.869
Scenario 6	Z_1'	20546391.506	20784632.331	20947369.645	37846931.364	40364820.255
	Z_2	133602.364	135304.361	137802.301	136004.390	152307.991
	Z_3	35914.064	38125.314	41002.671	45106.319	49507.814
	Z_4	391.245	382.607	378.602	375.741	428.307
	φ	0.771	0.768	0.745	0.524	0.315
Scenario 7	Z_1'	22456910.725	20154695.381	21798015.346	26418016.402	27564911.176
	Z_2	139456.647	134605.102	135210.379	150167.223	164597.384
	Z_3	37846.554	34197.677	33954.107	37155.608	45863.001
	Z_4	390.667	371.204	385.851	451.631	476.387
	φ	0.753	0.749	0.745	0.621	0.312

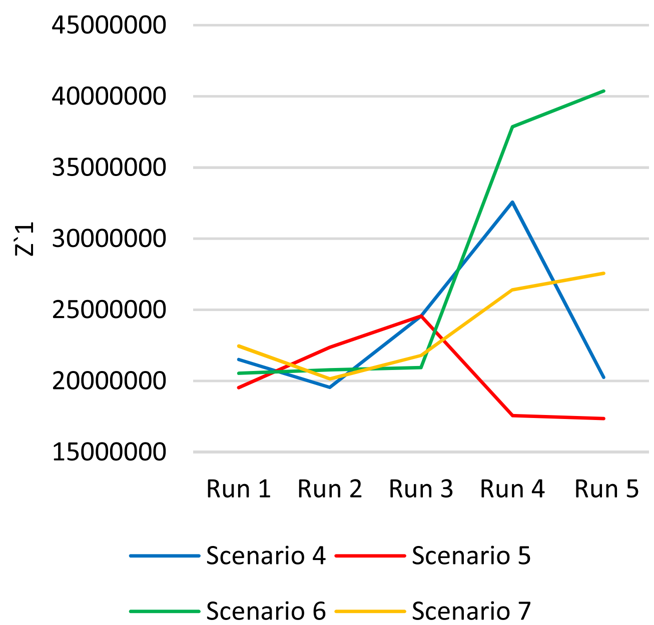

Fig. 10. Changes in the first objective function (total cost) for different scenarios (large-scale problem).

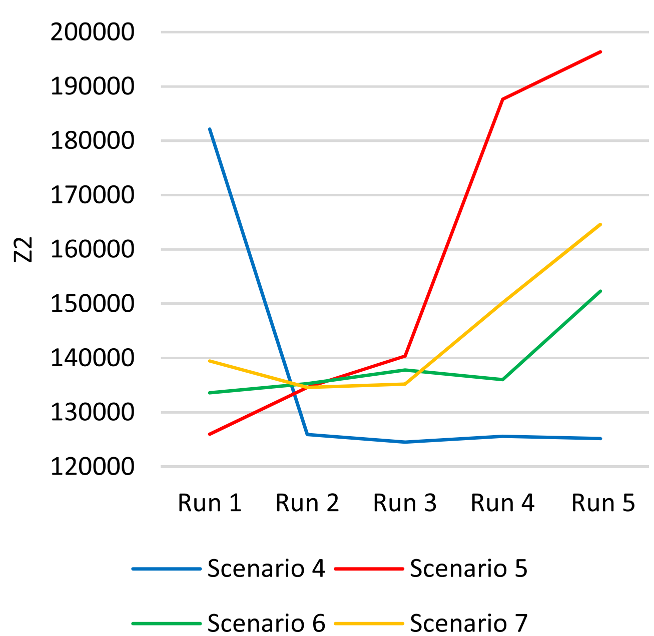

Fig. 11. Changes in the second objective function (number of defective items) for different scenarios (large-scale problem).

$$D_1(W,L) = \left[\sum_{l=1}^{L} w_l(1-d_l)\right] (\text{The Manhattan distance}) \tag{68}$$

$$D_2(W,L) = \left[\sum_{l=1}^{L} w_l^2(1-d_l)^2\right]^{1/2} (\text{The Euclidean distance}) \tag{69}$$

$$D_\infty(W,L) = \max\{w_l(1-d_l)\} (\text{The Chebyshev distance}) \tag{70}$$

The degree of proximity d_l for minimizing and maximizing objective functions is calculated in different ways which are shown as follows.

Minimizing objective function	Maximizing objective function
$d_l = \frac{Z_l^*}{Z_l}$	$d_l = \frac{Z_l}{Z_l^*}$

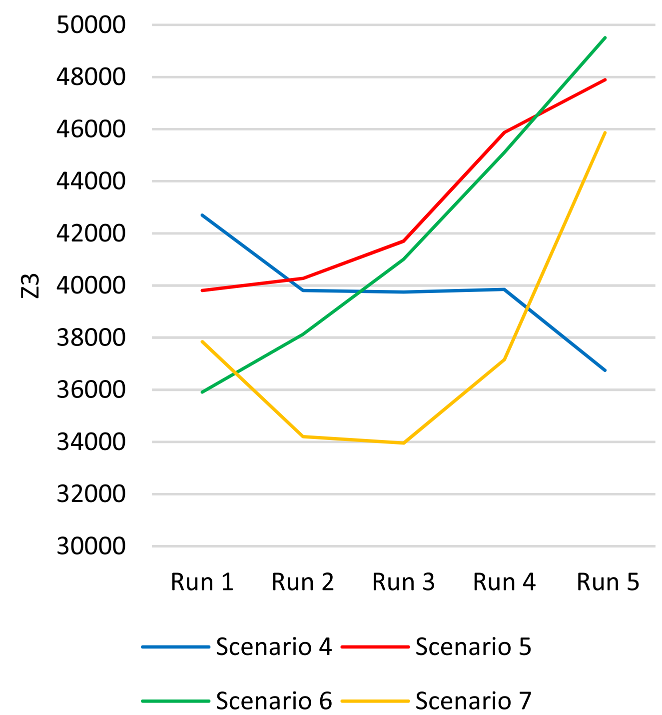

Fig. 12. Changes in the third objective function (environmental factors) for different scenarios (large-scale problem).

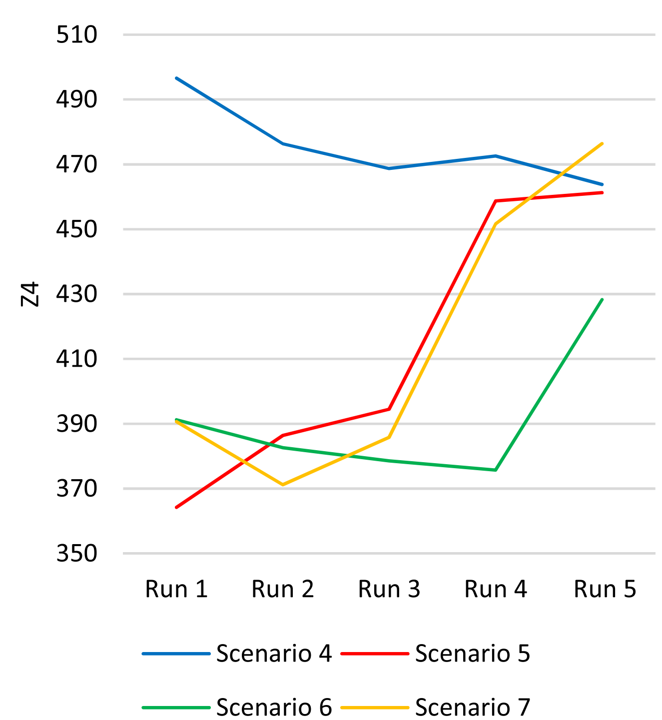

Fig. 13. Changes in the fourth objective function (social factors) for different scenarios (large-scale problem).

Higher degrees of proximity show a more credible and efficient solution approach in comparison to the one proposed by Abd El-Wahed and Lee (2006). Table 15 provides a comparison between the approach proposed by Wang and Liang (2004) which is a basic model for this development and the robust-fuzzy optimization approach proposed in this research.

The proposed approach is applicable for companies to select

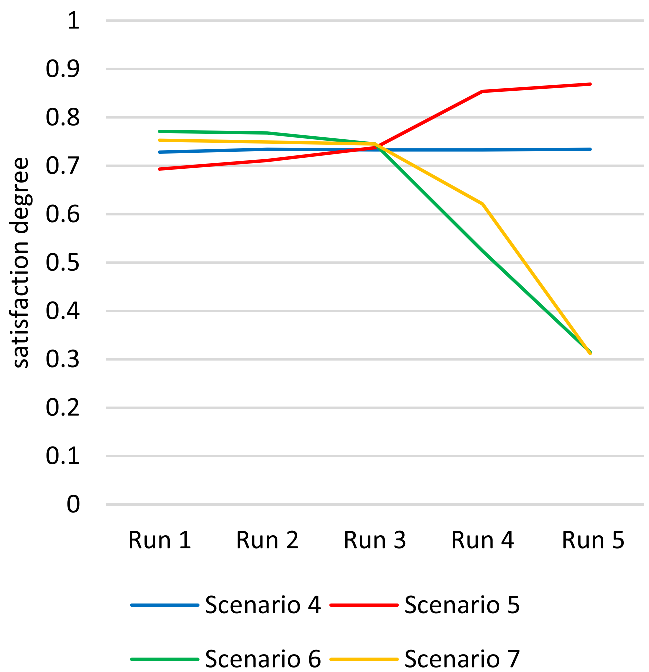

Fig. 14. Changes in the satisfaction degree for different scenarios (large-scale problem).

Table 15
Comparison of the degrees of the proximity.

Degree of proximity	Wang and Liang's (2004) approach		The proposed approach	
	(I = 5, J = 4)	(I = 30, J = 100)	(I = 5, J = 4)	(I = 30, J = 100)
D_1	0.554542	0.683034	0.546238	0.631608
D_2	0.297231	0.371932	0.2919	0.340301
D_∞	0.192523	0.244637	0.191813	0.244195

suppliers and allocate their orders. Besides, with environmental regulations in practice such as carbon emission reduction policies, companies can make benefit from the model proposed in this study to maximize their environmental impact. It has more applications in the case of encouraging policies such as carbon cap-and-trade and carbon subsidies which are incentives for companies to invest in developing green technologies. Supplier evaluation can be another implication of the results of this study in practice. In this regard, evaluation criteria can be measured and the best suppliers are chosen. The suppliers can determine the criteria that can improve their performance and collaborate in a competitive environment. Appropriate procurement decisions can be made using efficiency evaluations by the data envelopment analysis approach.

8. Conclusion

To address the increasing concerns regarding sustainable development objectives, firms are looking to analyze their current supply chain operations and revisit them based on the issues associated with economic, environmental, and social concerns. Besides, the uncertain environment of these operations leads to more complex systems in supply chain. The supplier selection and order allocation process as one of the crucial parts of supply chain operations has received increasing attention due to its ability to reduce environmental damages and promote social impacts on the community. Addressing the sustainability issues under real-world uncertainties is the main contribution of this paper to the extant literature. A multi-objective optimization approach is developed to minimize total costs and the number of imperfect quality items and to maximize environmental and social impacts considering uncertainties for transportation cost, delay penalty cost, and demand. A hybrid robust-fuzzy approach is used to provide a solution procedure. Moreover, numerical examples and sensitivity analysis are developed for validation to find out the impact of different conservatism levels on the optimal value of objective functions and their membership degree. The outcomes of the quantitative analysis indicate that the optimal value of objectives and the applied scenario is mainly dependent on the membership degree selected for each objective by the decision-makers. In other words, the decision-makers can choose the optimal scenario to regulate their lot-sizing, warehousing, production, and transportation decisions which can lead to an integrated management information system. Based on an approach proposed by Abd El-Wahed and Lee (2006), a degree of proximity is defined for multi-objective problems to illustrate the deviation between the obtained solution and the optimal solution. An index is defined to demonstrate this comparison with the value of 0 to 1. When the value of the index approaches 1, a more credible and sufficient algorithm is utilized to solve the multi-objective optimization.

Several future directions of research can be addressed. For instance, new constraints can be added under other circumstances that may make the proposed model an NP-hard model, where metaheuristic algorithms such as genetic algorithm (GA), standard particle swarm optimization (SPSO), or other algorithms can help solve the model. The proposed model can be implemented in circular economy (CE) systems considering all sustainability aspects. Inspection operations can be utilized to detect the number of defective items. The environmental aspect can be discussed in terms of carbon dioxide emissions which can be controlled by regulating carbon emission policies by the government such as carbon cap, carbon tax, carbon trade, and carbon offset. The possibility of investing in technologies that lead to carbon emission mitigation can be outlined as well. Some parameters and variables of the problem can be considered stochastic. For instance, the rate of defective items can follow a probabilistic distribution function such as uniform or Weibull distribution. Another future direction is to create a collaborative model between suppliers and retailers to develop a game-based approach that looks for optimal revenue management. Defining specific inquiries for the suppliers for a certain product and weighting the stakeholders based on their involvement in supplier selection can also be a solution for mitigating risks of long-term decision making. Multiple sourcing is another solution that can be employed to minimize the risk of selecting the best suppliers. Proposing quantifiable and comparable environmental and social indicators for evaluating the suppliers can address the problem of data validation when evaluating the suppliers is done by qualified expert-based surveys.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Appendix A

Table A1.

Table A1
Parameter values, breaking points, discounts, and the imperfect rate of items.

Items	Supplier	Period	O_{ijt}	H_{it}	TC_{ijt}	D_{it}	C_{ijt}	TL_{ijtk}	DTC_j	FE_{ij}	F_i	S_{ij}	SE_i	RE_{ij}	R_i	B_{ij0t}	P_{ij1t}	B_{ij1t}	P_{ij2t}	B_{ij2t}	P_{ij3t}	B_{ij3t}	Q_{ijkt}
1	1	1	80	5	40	391	900,000	1	6	3	2	94	90	92	84	0	18	150	17.5	300	17	400	4
	2	1	75	5	30		800,000	2	5	2		90		95		0	17	170	16.5	320	16	450	3
	3	1	60	5	50		900,000	1	7	2		94		90		0	15	200	14.5	400	14	450	4
	4	1	65	5	20		300,000	2	10	5		94		96		0	16	140	15.5	280	15	350	3
	1	2	80	4.5	40	103	900,000	1	6	3		94		92		0	18	150	17.5	300	17	400	4
	2	2	75	4.5	30		800,000	2	5	2		90		95		0	17	170	16.5	320	16	450	3
	3	2	60	4.5	50		900,000	1	7	2		94		90		0	15	200	14.5	400	14	450	4
	4	2	65	4.5	20		300,000	2	10	5		94		96		0	16	140	15.5	280	15	350	3
	1	3	80	4	40	433	900,000	1	6	3		94		92		0	18	150	17.5	300	17	400	4
	2	3	75	4	30		800,000	2	5	2		90		95		0	17	170	16.5	320	16	450	3
	3	3	60	4	50		900,000	1	7	2		94		90		0	15	200	14.5	400	14	450	4
	4	3	65	4	20		300,000	2	10	5		94		96		0	16	140	15.5	280	15	350	3
	1	4	80	4	40	130	900,000	1	6	3		94		92		0	18	150	17.5	300	17	400	4
	2	4	75	4	30		800,000	2	5	2		90		95		0	17	170	16.5	320	16	450	3
	3	4	60	4	50		900,000	1	7	2		94		90		0	15	200	14.5	400	14	450	4
	4	4	65	4	20		300,000	2	10	5		94		96		0	16	140	15.5	280	15	350	3
	1	5	80	4	40	207	900,000	1	6	3		94		92		0	18	150	17.5	300	17	400	4
	2	5	75	4	30		800,000	2	5	2		90		95		0	17	170	16.5	320	16	450	3
	3	5	60	4	50		900,000	1	7	2		94		90		0	15	200	14.5	400	14	450	4
	4	5	65	4	20		300,000	2	10	5		94		96		0	16	140	15.5	280	15	350	3
2	1	1	80	7	40	379	700,000	2	6	3	2	91	88	95	89	0	6.5	130	6	270	5.5	370	4
	3	1	75	7	50			3	7	3		95		96		0	4	110	3.5	290	3	350	2
	4	1	60	7	20		600,000	2	10	4		96		96		0	5	140	4.5	310	4	400	1
	1	2	65	7	40	496	700,000	2	6	3		91		95		0	6.5	130	6	270	5.5	370	4
	3	2	80	7	50			3	7	3		95		96		0	4	110	3.5	290	3	350	2
	4	2	75	7	20		600,000	2	10	4		96		96		0	5	140	4.5	310	4	400	1
	1	3	60	7	40	142	700,000	2	6	3		91		95		0	6.5	130	6	270	5.5	370	4
	3	3	65	7	50			3	7	3		95		96		0	4	110	3.5	290	3	350	2
	4	3	80	7	20		600,000	2	10	4		96		96		0	5	140	4.5	310	4	400	1
	1	4	75	6.5	40	302	700,000	2	6	3		91		95		0	6.5	130	6	270	5.5	370	4
	3	4	60	6.5	50			3	7	3		95		96		0	4	110	3.5	290	3	350	2
	4	4	65	6.5	20		600,000	2	10	4		96		96		0	5	140	4.5	310	4	400	1
	1	5	80	6.5	40	383	700,000	2	6	3		91		95		0	6.5	130	6	270	5.5	370	4
	3	5	75	6.5	50			3	7	3		95		96		0	4	110	3.5	290	3	350	2
	4	5	60	6.5	20		600,000	2	10	4		96		96		0	5	140	4.5	310	4	400	1
3	2	1	65	9	30	485	650,000	2	5	5	2	95	85	91	91	0	11	125	9.5	280	5	350	2
	3	1	80	9	50		800,000	1	7	4		95		92		0	11	110	10.5	280	7	350	5
	2	2	75	9	30	282	650,000	2	5	5		95		91		0	11	125	9.5	280	5	350	2
	3	2	60	9	50		800,000	1	7	4		95		92		0	11	110	10.5	280	7	350	5
	2	3	65	7	30	149	650,000	2	5	5		95		91		0	11	125	9.5	280	5	350	2
	3	3	80	7	50		800,000	1	7	4		95		92		0	11	110	10.5	280	7	350	5
	2	4	75	7	30	451	650,000	2	5	5		95		91		0	11	125	9.5	280	5	350	2
	3	4	60	7	50		800,000	1	7	4		95		92		0	11	110	10.5	280	7	350	5
	2	5	65	7	30	196	650,000	2	5	5		95		91		0	11	125	9.5	280	5	350	2
	3	5	80	7	50		800,000	1	7	4		95		92		0	11	110	10.5	280	7	350	5
4	1	1	75	4.5	40	434	600,000	3	6	4	3	92	92	93	84	0	8	150	7.5	280	7	350	2
	2	1	60	4.5	30		400,000	3	5	4		92		93		0	12	200	11.5	400	11	500	1
	3	1	65	4.5	50		500,000	4	7	1		95		92		0	10	140	9.5	260	9	400	1
	4	1	80	4.5	20		600,000	2	10	1		94		93		0	13	200	12.5	340	12	420	0

(*continued on next page*)

Table A1 (*continued*)

Items	Supplier	Period	O_{ijt}	H_{it}	TC_{ijt}	D_{it}	C_{ijt}	TL_{ijtk}	DTC_j	FE_{ij}	F_i	S_{ij}	SE_i	RE_{ij}	R_i	B_{ij0t}	P_{ij1t}	B_{ij1t}	P_{ij2t}	B_{ij2t}	P_{ij3t}	B_{ij3t}	Q_{ijkt}
	1	2	75	8.5	40	434	600,000	3	6	4		92		93		0	8	150	7.5	280	7	350	2
	2	2	60	8.5	30		400,000	3	5	4		92		93		0	12	200	11.5	400	11	500	1
	3	2	65	8.5	50		500,000	4	7	1		95		92		0	10	140	9.5	260	9	400	1
	4	2	80	8.5	20		600,000	2	10	1		94		93		0	13	200	12.5	340	12	420	0
	1	3	75	8.5	40	213	600,000	3	6	4		92		93		0	8	150	7.5	280	7	350	2
	2	3	60	8.5	30		400,000	3	5	4		92		93		0	12	200	11.5	400	11	500	1
	3	3	65	8.5	50		500,000	4	7	1		95		92		0	10	140	9.5	260	9	400	1
	4	3	80	8.5	20		600,000	2	10	1		94		93		0	13	200	12.5	340	12	420	0
	1	4	75	8.5	40	211	600,000	3	6	4		92		93		0	8	150	7.5	280	7	350	2
	2	4	60	8.5	30		400,000	3	5	4		92		93		0	12	200	11.5	400	11	500	1
	3	4	65	8.5	50		500,000	4	7	1		95		92		0	10	140	9.5	260	9	400	1
	4	4	80	8.5	20		600,000	2	10	1		94		93		0	13	200	12.5	340	12	420	0
	1	5	75	6	40	126	600,000	3	6	4		92		93		0	8	150	7.5	280	7	350	2
	2	5	60	6	30		400,000	3	5	4		92		93		0	12	200	11.5	400	11	500	1
	3	5	65	6	50		500,000	4	7	1		95		92		0	10	140	9.5	260	9	400	1
	4	5	80	6	20		600,000	2	10	1		94		93		0	13	200	12.5	340	12	420	0
5	1	1	75	7	40	485	700,000	1	6	4	2	91	90	90	88	0	6	170	5.5	320	5	400	2
	2	1	60	7	30		700,000	2	5	4		96		91		0	5	150	4.5	300	4	400	2
	1	2	65	7	40	266	700,000	1	6	4		91		90		0	6	170	5.5	320	5	400	2
	2	2	80	7	30		700,000	2	5	4		96		91		0	5	150	4.5	300	4	400	2
	1	3	75	7	40	189	700,000	1	6	4		91		90		0	6	170	5.5	320	5	400	2
	2	3	60	7	30		700,000	2	5	4		96		91		0	5	150	4.5	300	4	400	2
	1	4	65	7	40	265	700,000	1	6	4		91		90		0	6	170	5.5	320	5	400	2
	2	4	80	7	30		700,000	2	5	4		96		91		0	5	150	4.5	300	4	400	2
	1	5	75	7	40	404	700,000	1	6	4		91		90		0	6	170	5.5	320	5	400	2
	2	5	60	7	30		700,000	2	5	4		96		91		0	5	150	4.5	300	4	400	2

References

Abd El-Wahed, W. F., & Lee, S. M. (2006). Interactive fuzzy goal programming for multi-objective transportation problems. *Omega, 34*(2), 158–166.

Amin, S. H., Razmi, J., & Zhang, G. (2011). Supplier selection and order allocation based on fuzzy SWOT analysis and fuzzy linear programming. *Expert Systems with Applications, 38*(1), 334–342.

Arabsheybani, A., Paydar, M. M., & Safaei, A. S. (2018). An integrated fuzzy MOORA method and FMEA technique for sustainable supplier selection considering quantity discounts and supplier`s risk. *Journal of Cleaner Production, 190*, 577–591.

Arabsheybani, A., Paydar, M. M., & Safaei, A. S. (2020). Sustainable Supplier Selection and Order Allocation Applying Metaheuristic Algorithms. *International Journal of Supply and Operations Management, 7*(2), 164–177.

Azadnia, A. H., Saman, M. Z. M., & Wong, K. Y. (2015). Sustainable supplier selection and order lot-sizing: An integrated multi-objective decision-making process. *International Journal of Production Research, 53*(2), 383–408.

Bektur, G. (2020). An integrated methodology for the selection of sustainable suppliers and order allocation problem with quantity discounts, lost sales and varying supplier availabilities. *Sustainable Production and Consumption, 23*, 111–127.

Bellman, R. E., & Zadeh, L. A. (1970). Decision-making in a fuzzy environment. *Management Science, 17*(4), B-141-B-164.

Bertsimas, D., & Sim, M. (2004). The price of robustness. *Operations research, 52*(1), 35–53.

Çebi, F., & Otay, İ. (2016). A two-stage fuzzy approach for supplier evaluation and order allocation problem with quantity discounts and lead time. *Information Sciences, 339*, 143–157.

Cheraghalipour, A., & Farsad, S. (2018). A bi-objective sustainable supplier selection and order allocation considering quantity discounts under disruption risks: A case study in plastic industry. *Computers & Industrial Engineering, 118*, 237–250.

Erdem, A. S., & Göçen, E. (2012). Development of a decision support system for supplier evaluation and order allocation. *Expert Systems with Applications, 39*(5), 4927–4937.

Esmaeili-Najafabadi, E., Nezhad, M. S. F., Pourmohammadi, H., Honarvar, M., & Vahdatzad, M. A. (2019). A joint supplier selection and order allocation model with disruption risks in centralized supply chain. *Computers & Industrial Engineering, 127*, 734–748.

Feng, J., & Gong, Z. (2020). Integrated linguistic entropy weight method and multi-objective programming model for supplier selection and order allocation in a circular economy: A case study. *Journal of Cleaner Production, 277*, Article 122597.

Ghadimi, P., Toosi, F. G., & Heavey, C. (2018). A multi-agent systems approach for sustainable supplier selection and order allocation in a partnership supply chain. *European Journal of Operational Research, 269*(1), 286–301.

Gören, H. G. (2018). A decision framework for sustainable supplier selection and order allocation with lost sales. *Journal of Cleaner Production, 183*, 1156–1169.

Govindan, K., Mina, H., Esmaeili, A., & Gholami-Zanjani, S. M. (2020). An integrated hybrid approach for circular supplier selection and closed loop supply chain network design under uncertainty. *Journal of Cleaner Production, 242*, Article 118317.

Hannan, E. L. (1981). Linear programming with multiple fuzzy goals. *Fuzzy Sets and Systems, 6*(3), 235–248.

Hatefi, S. M., & Jolai, F. (2014). Robust and reliable forward–reverse logistics network design under demand uncertainty and facility disruptions. *Applied Mathematical Modelling, 38*(9–10), 2630–2647.

Hosseini, S., Morshedlou, N., Ivanov, D., Sarder, M., Barker, K., & Al Khaled, A. (2019). Resilient supplier selection and optimal order allocation under disruption risks. *International Journal of Production Economics, 213*, 124–137.

Imperatives, S. (1987). Report of the World Commission on Environment and Development: Our common future. *Accessed Feb, 10*, 1-300.

Isaloo, F., & Paydar, M. M. (2020). Optimizing a robust bi-objective supply chain network considering environmental aspects: A case study in plastic injection industry. *International Journal of Management Science and Engineering Management, 15*(1), 26–38.

Jadidi, O., Zolfaghari, S., & Cavalieri, S. (2014). A new normalized goal programming model for multi-objective problems: A case of supplier selection and order allocation. *International Journal of Production Economics, 148*, 158–165.

Jahani, N., Sepehri, A., Vandchali, H. R., & Tirkolaee, E. B. (2021). Application of Industry 4.0 in the Procurement Processes of Supply Chains. *A Systematic Literature Review. Sustainability, 13*(14), 7520.

Jia, R., Liu, Y., & Bai, X. (2020). Sustainable supplier selection and order allocation: Distributionally robust goal programming model and tractable approximation. *Computers & Industrial Engineering, 140*, Article 106267.

Kellner, F., & Utz, S. (2019). Sustainability in supplier selection and order allocation: Combining integer variables with Markowitz portfolio theory. *Journal of Cleaner Production, 214*, 462–474.

Kim, J., Do Chung, B., Kang, Y., & Jeong, B. (2018). Robust optimization model for closed-loop supply chain planning under reverse logistics flow and demand uncertainty. *Journal of Cleaner Production, 196*, 1314–1328.

Mafakheri, F., Breton, M., & Ghoniem, A. (2011). Supplier selection-order allocation: A two-stage multiple criteria dynamic programming approach. *International Journal of Production Economics, 132*(1), 52–57.

Micheli, G. J., Cagno, E., Mustillo, G., & Trianni, A. (2020). Green supply chain management drivers, practices and performance: A comprehensive study on the moderators. *Journal of Cleaner Production, 259*, Article 121024.

Mirzaee, H., Naderi, B., & Pasandideh, S. H. R. (2018). A preemptive fuzzy goal programming model for generalized supplier selection and order allocation with incremental discount. *Computers & Industrial Engineering, 122*, 292–302.

Mohammed, A., Harris, I., & Govindan, K. (2019). A hybrid MCDM-FMOO approach for sustainable supplier selection and order allocation. *International Journal of Production Economics, 217*, 171–184.

Mohammed, A., Setchi, R., Filip, M., Harris, I., & Li, X. (2018). An integrated methodology for a sustainable two-stage supplier selection and order allocation problem. *Journal of Cleaner Production, 192*, 99–114.

Moheb-Alizadeh, H., & Handfield, R. (2019). Sustainable supplier selection and order allocation: A novel multi-objective programming model with a hybrid solution approach. *Computers & Industrial Engineering, 129*, 192–209.

Moheb-Alizadeh, H., Mahmoudi, M., & Bagheri, R. (2017). Supplier selection and order allocation using a stochastic multi-objective programming model and genetic algorithm. *International Journal of Integrated Supply Management, 11*(4), 291–315.

Nasr, A. K., Tavana, M., Alavi, B., & Mina, H. (2021). A novel fuzzy multi-objective circular supplier selection and order allocation model for sustainable closed-loop supply chains. *Journal of Cleaner Production, 287*, Article 124994.

Nazari-Shirkouhi, S., Shakouri, H., Javadi, B., & Keramati, A. (2013). Supplier selection and order allocation problem using a two-phase fuzzy multi-objective linear programming. *Applied Mathematical Modelling, 37*(22), 9308–9323.

Nourmohamadi Shalke, P., Paydar, M. M., & Hajiaghaei-Keshteli, M. (2018). Sustainable supplier selection and order allocation through quantity discounts. *International Journal of Management Science and Engineering Management, 13*(1), 20–32.

Park, K., Kremer, G. E. O., & Ma, J. (2018). A regional information-based multi-attribute and multi-objective decision-making approach for sustainable supplier selection and order allocation. *Journal of Cleaner Production, 187*, 590–604.

Pishvaee, M. S., Rabbani, M., & Torabi, S. A. (2011). A robust optimization approach to closed-loop supply chain network design under uncertainty. *Applied Mathematical Modelling, 35*(2), 637–649.

Prasanna Venkatesan, S., & Goh, M. (2016). Multi-objective supplier selection and order allocation under disruption risk. *Transportation Research Part E: Logistics and Transportation Review, 95*, 124–142.

Rabieh, M., Babaee, L., Fadaei Rafsanjani, A., & Esmaeili, M. (2019). Sustainable supplier selection and order allocation: An integrated delphi method, fuzzy TOPSIS and multi-objective programming model. *Scientia Iranica, 26*(4), 2524–2540.

Rabieh, M., Modarres, M., & Azar, A. (2018). Robust-fuzzy model for supplier selection under uncertainty: An application to the automobile industry. *Scientia Iranica, 25*(4), 2297–2311.

Rezaei Vandchali, H., Cahoon, S., & Chen, S.-L. (2020). Creating a sustainable supply chain network by adopting relationship management strategies. *Journal of Business-To-Business Marketing, 27*(2), 125–149.

RezaHoseini, A., Noori, S., & Ghannadpour, S. F. (2021). Integrated scheduling of suppliers and multi-project activities for green construction supply chains under uncertainty. *Automation in Construction, 122*, Article 103485.

Scott, J., Ho, W., Dey, P. K., & Talluri, S. (2015). A decision support system for supplier selection and order allocation in stochastic, multi-stakeholder and multi-criteria environments. *International Journal of Production Economics, 166*, 226–237.

Sepehri, A. (2021). Optimizing the Replenishment Cycle and Selling Price for an Inventory Model Under Carbon Emission Regulation and Partially Permissible Delay in Payment. *Process Integration and Optimization for Sustainability*, 1–21.

Sepehri, A., Mishra, U., & Sarkar, B. (2021). A sustainable production-inventory model with imperfect quality under preservation technology and quality improvement investment. *Journal of Cleaner Production, 127332*.

Sontake, A., Jain, N., & Singh, A. (2021). Sustainable Supplier Selection and Order Allocation Considering Discount Schemes and Disruptions in Supply Chain. In *Operations Management and Systems Engineering* (pp. 61–94). Springer.

Stević, Ž., Pamučar, D., Puška, A., & Chatterjee, P. (2020). Sustainable supplier selection in healthcare industries using a new MCDM method: Measurement of alternatives and ranking according to COmpromise solution (MARCOS). *Computers & Industrial Engineering, 140*, Article 106231.

Thevenin, S., Ben-Ammar, O., & Brahimi, N. (2022). Robust optimization approaches for purchase planning with supplier selection under lead time uncertainty. *European Journal of Operational Research*.

Torabi, S., Baghersad, M., & Mansouri, S. (2015). Resilient supplier selection and order allocation under operational and disruption risks. *Transportation Research Part E: Logistics and Transportation Review, 79*, 22–48.

Vahidi, F., Torabi, S. A., & Ramezankhani, M. (2018). Sustainable supplier selection and order allocation under operational and disruption risks. *Journal of Cleaner Production, 174*, 1351–1365.

Vali-Siar, M. M., & Roghanian, E. (2022). Sustainable, resilient and responsive mixed supply chain network design under hybrid uncertainty with considering COVID-19 pandemic disruption. *Sustainable Production and Consumption, 30*, 278–300.

Vandchali, H. R., Cahoon, S., & Chen, S.-L. (2021). The impact of supply chain network structure on relationship management strategies: An empirical investigation of sustainability practices in retailers. *Sustainable Production and Consumption, 28*, 281–299.

Wang, R.-C., & Liang, T.-F. (2004). Application of fuzzy multi-objective linear programming to aggregate production planning. *Computers & Industrial Engineering, 46*(1), 17–41.