STRESS ANALYSIS, UNBONDED PIPES

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CONTENTS

SUMMARY

Nonbonded flexible pipes are built up by layers made of polymers (sheaths) and helically wound armours (helices). Each layer has a specific function in loadcarrying or leakproofness assurance and is not bonded to each other (as opposed to a bonded flexible pipe).

Methods for computing stresses in the layers of a nonbonded pipe due to axisymmetric loading (F, M_t, P_{int} and P_{ext}) and bending (M_b or C) are elaborated. The main equations are given in the enclosed paper by Feret and Bournazel[1]. The stresses are combined as

For a helix :	σ_a	axial (helix direction) stress due to axisymmetric load, pipe bending and friction - $\sigma_a = \sigma_t + \sigma_B + \sigma_f$
	σ_r	radial stress
	(σ_b)	binormal stress - normally taken as zero
For a sheath :	σ_a	axial (pipe direction) stress due to axisymmetric load and pipe bending - $\sigma_a = \sigma_t + \sigma_B$
	σ_c	hoop stress
	σ_r	radial stress

The computaton of stresses are a general basis for capacity, stiffness and lifetime assessments of nonbonded flexible pipes, the latter being based on fatigue and wear of the tendons[3].

REFERENCES

The presentation is based on the included paper :

1 Feret J.J and Bournazel C.L : "Calculations of Stresses and Slip in Structural Layers of Unbonded Pipes", Journal of Offshore Mechanics and Arctic Engineering, Volume 109, August 1987.

Other publications and reports (both public and "not public") are also used as a basis for this presentation:

2 "CAFLEX - A Computer Program for Capacity Analysis of Flexible Pipes - manuals", SINTEF/IFP 1989

3 Feret J.J and Bournazel C.L : "Evaluation of Flexible Pipes´ Life Expectancy Under Dynamic Condition", OTC 5230, Proceedings of the 18th OTC, Houston 1986

4 McCone A. : "Derivation of Equation for Slip of wire upon Bending in Unbonded Flexible Risers", Wellstream corporation technical note #120, February 1990.

5 McNamara J.F, Harte A.M : "Three Dimensional Analytical Simulation of Flexible Pipe Wall Structure", OMAE 89-744, Proceedings of the OMAE conference 1989.

6 Often O. : "FLEXPIPE Technical Description", Lecture Notes 1990.

1 INTRODUCTION

A nonbonded pipe is characterized by a layered structure with layers of two main types :

- sheaths - plastic "tubes" made of polyamid etc
- armour - metallic layers consisting of tendons or helices wound with an angle to the pipe core

Each layer has a specific function in the pipe design. The helical layers are the loadcarrying elements while the plastic sheaths are introduced to ensure low friction between helices that can experience wear and provide pressure barriers towards the inner and outer fluids.

The main manufacturer of nonbonded flexible pipes are Coflexip, Furukawa and Wellstream, and their pipes are basically built up on the same principles.

A typical nonbonded flexible pipe structure is given in section 1 of [1]. When dealing with high pressure nonbonded flexible pipes for dynamic riser applications, normally all of the mentioned layers will be present except for the outer protective carcass. Detailed layer structure for the carcass, the Z-spiral and the tendons of a typical Coflexip design is given in figure 1.

2 BEHAVIOUR OF FLEXIBLE PIPES UNDER AXISYMMETRICAL LOADING

2.1 Deformations

A flexible pipe subjected to axisymmetric forces will undergo the following deformations:

ΔL	axial elongation
$\Delta\theta$	torsion
Δr	a change in radius

The stresses in the tendons will be split into

σ_a	axial (helix direction) stress
σ_r	radial stress

and in the sheaths

σ_a	axial (pipe direction) stress
σ_c	hoop stress
σ_r	radial stress

Note that due to the lateral gaps between the helices in a nonbonded structure, σ_b - the binormal stress component in the helices - will be zero as they are free to move laterally and thereby change lay angle during pipe deformations. This will not be the case for bonded type pipes in which the filling (bonding) material will resist this movement.

2.2 *Solutions*

There exist several ways of establishing the equations for solving the unknowns of an axisymmetric load case.

Sheaths:

Normally the sheaths are treated as isotropic materials, the classical stress-strain equations being -

$$\varepsilon = \frac{1}{E}\begin{bmatrix} 1 & -\nu & -\nu & 0 \\ -\nu & 1 & -\nu & 0 \\ -\nu & -\nu & 1 & 0 \\ 0 & 0 & 0 & \end{bmatrix} \sigma$$

with

$$\varepsilon = \begin{bmatrix} \Delta L/L_0 \\ \Delta r/r \\ \Delta t/t \\ \Delta\theta/L \end{bmatrix}$$

and

$$\sigma = \begin{bmatrix} F/2\pi r t \\ (p_i r_i - p_o r_o)/t \\ -(p_i + p_o)\frac{1}{2} \\ M_t/GJ \end{bmatrix}$$

Orthotropic material (different E-moduli in different directions) can be useful in bonded pipes (fabric layers).

Helices:

The corresponding relation for the helices in a nonbonded pipe (plane stress) will be:

$$\begin{bmatrix} \varepsilon_\lambda \\ \varepsilon_r \end{bmatrix} = \frac{1}{E}\begin{bmatrix} 1 & -\nu \\ -\nu & 1 \end{bmatrix}\begin{bmatrix} \sigma_\lambda \\ \sigma_r \end{bmatrix}$$

The linearization of the strains follows from Figure 2 :

$$ds/s = \cos^2\alpha \cdot dL/L + \sin^2\alpha \cdot dr/r + r \cdot \sin\alpha \cdot \cos\alpha \cdot d\theta/L$$

In addition we must require that the sum of axial force and moments taken by each layer add up to the applied external values and that we obtain continuity in contact pressures between the layers. A proper assembly and solution (non-symmetric) of these relations for all layers of a flexible pipe will cover a general solution as described in 2.1 of [1]. The system of equations can be written as

$$\mathbf{Kr} = \mathbf{R}$$

K being a nonsymmetric "stiffness" matrix
r the unknown deformations
R the load vector

Special effect which are likely to influence the stresses in nonbonded flexible pipes can also be handled with this set of equations:

- Temperature gradient deformations
- Nonlinear geometrical effects such as gaps in the structure and other geometrical changes

The latter has to be implemented through an iterative procedure.

Although simpler formulaes exist for computing pipe capacities and stiffnesses, the above solution (or similar) is a good basis for the computation of important theoretical values of -

- Burst pressure
- Collapse pressure - combined with a buckling formulae for the different layers
- Ultimate tension
- Ultimate torsion
- Axial and torsional stiffnesses

3 PIPE BEHAVIOUR UNDER BENDING

3.1 General mathematics

For a pipe bent with constant curvature radius R, the generated surface is described by a torus or anchor ring[4]. A point on this surface given in Figure 3 is described by the coordinates

$$x = R \cdot v \cdot \cos\varphi \qquad \text{with } v = 1 + \varepsilon\cos\theta$$
$$y = R \cdot \varepsilon \cdot \sin\theta \qquad \varepsilon = r/R$$
$$z = R \cdot v \cdot \sin\varphi \qquad \theta, \varphi \text{ - refer to Figure 8 in } ^1$$

The infinitisemal distance ds is given by

$$ds^2 = dx^2 + dy^2 + dz^2 = (r \cdot d\theta)^2 + (R \cdot v \cdot d\varphi)^2$$

3.2 The geodesic

A path between two points A and B on a curved surface is called a geodesic if it is the shortest such path between the two points. Another property of the geodesic is that the normal to geodesic is the normal to the surface.

The relevance to unbonded flexible pipes is the helical behaviour under bending. For such a pipe the geodesic length is computed as[4] - linearized

$$ds_g = \frac{r\,d\theta}{\sin\alpha}\left(1 - \frac{\varepsilon\cos\theta}{\tan^2\alpha}\right) \qquad \varepsilon = \frac{r}{R}$$

3.3 Slip of tendons

During bending, the helices will slip towards the geodesic. This slip can be related to the pipe core behaviour under bending and the (decomposed) values computed as

$$\Delta p = \frac{2r\varepsilon\sin\theta}{\tan\alpha}$$ paralell to the core

$$\Delta c = \frac{r\varepsilon\sin\theta}{\tan^2\alpha}$$ circumferencial

and the total slip (as given[1])

$$\Delta = (\Delta p^2 + \Delta c^2)^{1/2} = \frac{r^2\sin\theta}{R\tan\alpha}\sqrt{4 + \frac{1}{\tan^2\alpha}}$$

and the direction given by

$$\tan\gamma = \Delta c/\Delta p = (2\cdot\tan\alpha)^{-1}$$

Note that $\gamma \doteq \alpha$ for $\alpha = 35°$, ie the slip is in axial direction of the helices. Note also that the slip will be zero at the inside and outside of the curvature and reach a maximum value at the "sides" of the torus (for $\theta \doteq 90°$). The total slip between two crosswound layers will be

$$\Delta = \Delta\cdot\cos\gamma = 2\cdot\Delta c = 2\cdot r^2\cdot\sin\theta/(R\cdot\tan\alpha) \quad \text{and } \Delta p = 0$$

The evaluation of the relative slip between layers is important for computing stresses in the tendons as well as for wear computations[3] due to cyclic loading.

3.4 Stresses due to bending

Bending of a nonbonded flexible pipe gives rise to axial stresses in the plastic sheaths as

$$\sigma_B = \varepsilon E = rE/R$$

and to the helices due to a change in curvature and friction. The curvature of a helix due to an applied pipe curvature can be evaluated from

$$C_N = \frac{\vec{n}}{\rho}\vec{N}$$ $\vec{n}$ main normal to the tendon, $\vec{N}$ normal to the torus

and applying what we know of the geodesic length (and keeping in mind that the normal to the geodesic is identical to the normal of the pipe surface) -

$$C_N = C_N' + \Delta C_N = \frac{\sin^2\alpha}{r} + \frac{\cos^2\alpha}{R} \cdot \frac{\cos\theta}{1+\varepsilon\cdot\cos\theta}$$; C_N' being the initial curvature of the helix

The axial stress in a tendon due to this curvature change is then computed as

$$\sigma_B = \varepsilon E = 0.5 \cdot t \cdot \Delta C_N$$

with t - thickness of helix
E - Youngs modulus of elasticity

The tendons will also undergo torsion due to the given curvature as show in [1].

Slip between tendons will also give rise to friction forces and stresses. Two phenomena are pointed out in [1].

- axial slip induced friction stresses
- transverse stresses due to a change in C_B due to that friction to some extent will prevent the helix to slip into a geodesic line

4 COMBINED LOADING AND NONLINEARITIES

With linearized analyses the stresses can be superimposed. The contribution to axial stress in tendons and sheaths arising from bending of the pipe adds to the axial stresses due to axisymmetric loading. Note that accounting for transverse friction stresses in the helices (item 2 above) introduces an σ_b - binormal stress - to them.

Also under axisymmetrical loading, a nonbonded flexible pipe will experience slip between layers (due to a change in lay angles) and thereby friction stresses. As mentioned in section 2, and iterative procedure can account for nonlinear geometric effects and friction stresses can then be computed. This will still be a linearized analysis.

Material nonlinearities, especially in the plastic sheaths[1], are likely to affect the stresses in such pipes. This can be taken into account by an incremental loading procedure and a more refined material description. Such an algorithm can also supply an improved treatment of the more general 3D case - a combination of axisymmetric loading and bending.

As pointed out[1], special conciderations must still be done for short length pipes and stress assessments close to end fittings and in bending stiffeners.

5 FLEXIBLE PIPE CAPACITIES

As mentioned in section 2, flexible pipe capacities and stiffnesses can be computed on the basis of a good stress analysis model. The capacities are found by introducing unit loads and simply extrapolate to yield or ultimate stress when dealing with a linearized model. Stiffnesses can be computed in the same way - apply a unit force and divide by the deformation.

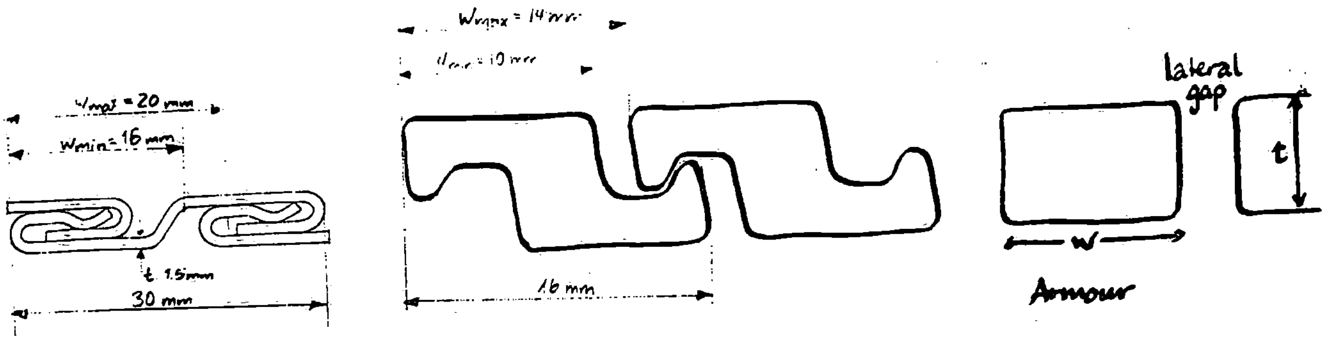

Fig 1 Components of a nonbonded flexible pipe

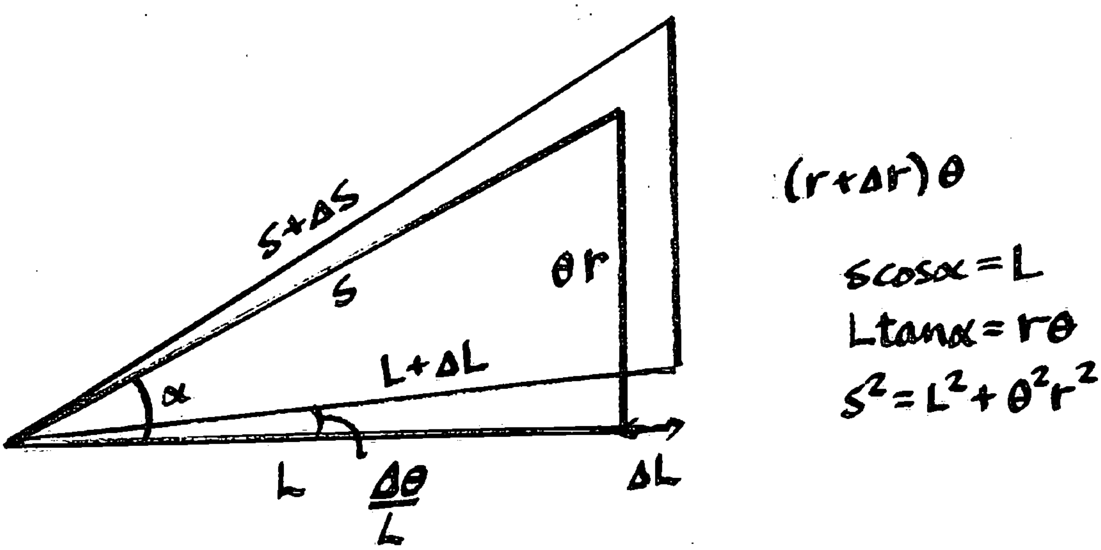

Fig 2 Linearization of the strains
Part of helix layed flat