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Silani, Amirreza; Tindemans, Simon H.

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Mean-Field Stackelberg Game for Mitigating the Strategic Bidding of Energy Consumers in Congested Distribution Networks

Amirreza Silani, Simon H. Tindemans

***Abstract*— The sudden proliferation of Electric Vehicles (EVs), batteries and photovoltaic cells in power networks can lead to congested distribution networks. A substitute for upgrading network capacity is a redispatch market that enables the Distribution System Operators (DSOs) to mitigate congested networks by requesting the energy consumers to modify their consumption schedules. However, energy consumers are able to strategically modify their day-ahead market bids in anticipation of the redispatch market outcomes. This behaviour, which is known as increase-decrease gaming, can exacerbate congestion and give arbitrage opportunities to the energy consumers for gaining windfall profits from the DSO. In this paper, we propose an algorithm based on mean-field Stackelberg game to mitigate the increase-decrease game for large populations of energy consumers. In this game, the energy consumers (followers) maximize their individual welfare on the day-ahead market with anticipation of the redispatch market outcomes while the leader maximizes the social welfare of all agents and minimizes the costs of DSO on the redispatch market. We show the convergence of this algorithm to the mean-field leader-follower ε_N-Nash equilibrium.**

I. INTRODUCTION

These days, there are increasing distributed energy resources connected to the electrical distribution grid, including Electric Vehicles (EVs), batteries and photovoltaic cells. The decarbonization of the energy system is the advantage of these resources but they provide additional problems for power networks, namely network congestion. There are several ways to mitigate the congestion problems: Direct Load Control (DLC), Local Flexibility Markets (LFMs), and new forms of distribution tariffs [1], [2]. Among those, we focus on the LFM proposals aimed at addressing congestion management. After the Day-Ahead (DA) market has closed, LFM proposals usually take energy consumption schedules into account. The Distribution System Operator (DSO) requests the energy consumers to redispatch their consumption schedules in the LFM, which can be considered a local Redispatch (RD) market, if the forecast consumption schedules of flexible demand and other demand cause congestion problems [2]. In order to resolve congestion, the DSO pays the consumers who reduce their consumption schedules on the redispatch market [1], [2]. However, the consumers are able to anticipate the outcome of the redispatch market and bid strategically on the day-ahead market in order to maximize their individual welfare, which leads to increase-decrease gaming [3]. The increase-decrease game can exacerbate congestion and give arbitrage opportunities to the energy consumers for gaining windfall profits from the DSO. Recently, modeling and analysis of the increase-decrease game in similar electrical energy markets have attracted increasing research interest (see for instance [3]–[7]). In [3], it is demonstrated that in an inconsistent power market design, producers bid strategically on the spot markets and even in absence of market power, the increase-decrease game is possible. In [4], a two-stage game is designed to investigate the effects of imperfect competition among producers in zonal power markets. A profit decomposition approach is proposed in [5] to assess how different bidding strategies affect overall payoffs. In [6], a deterministic mean field game method is utilized to model the increase-decrease game for large populations of energy consumers in power networks. This was extended in [7] to a stochastic setting. These papers study the increase-decrease game in various electrical energy markets; however, no existing research addresses an effective solution for the increase-decrease game under uncertainty, in particular for the case of congestion in distribution networks. In other related work, mean field game theory was utilized for constrained charging control of large populations of EVs in [8]–[10].

In this paper, we consider a mitigation strategy for increase-decrease gaming in redispatch markets, for large populations of energy consumers. Building on the research in [7], we model this as a mean-field Stackelberg game where the followers, *i.e.*, consumers, aim to maximize their personal welfare on the day-ahead market while anticipating the outcomes of the redispatch market. In the present paper, the leader balances the social welfare of all agents and the cost incurred by the DSO on the redispatch, by adding an offset to the day-ahead price. An iterative algorithm is used to obtain the consumers' day-ahead schedules and the day-ahead price for a number of time steps (e.g., 24 hours). We show the convergence of the algorithm to the mean-field leader-follower ε_N-Nash equilibrium.

Notation: Let $\mathbf{1}$ be the vector of all ones. $\mathbb{I}_M$ denotes the identity matrix of size M. $\|x\|_S$ denotes the weighted L_2 norm of x and $\operatorname{diag}(x)$ denotes the diagonal form of x. Let $[z]^+ := z$ if $z > 0$, 0 otherwise. The mathematical expectation with respect to (w.r.t.) the random variable ν is represented by $\mathbb{E}_\nu[\cdot]$.

A. Silani and S.H. Tindemans are with the Department of Electrical Sustainable Energy, Delft University of Technology, The Netherlands ({a.silani, s.h.tindemans}@tudelft.nl)

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II. Modeling and problem formulation

In this section, we consider the increase-decrease game for large populations of energy consumers and model the objective functions for both the leader and the followers.

A. Problem formulation

We consider the problem of energy consumption for consumers with flexible loads. We consider the set of energy consumers (followers) $\mathcal{N} := \{1, 2, \ldots, N\}$ and the time horizon $\mathcal{T} := \{1, \ldots, T\}$. Each consumer $i \in \mathcal{N}$ assigns a utility $u_i \in \mathbb{R}_{>0}$ to the (optional) consumption of energy. For each $i \in \mathcal{N}$, $t \in \mathcal{T}$, let $p^t_{\mathrm{b},i}$ be the charging/discharging schedule of battery, $p^t_{\mathrm{g},i}$ be the consumption schedule of other flexible loads on the day-ahead market, and c be the network capacity. We consider day-ahead uncertainty in the energy demand, represented by the random variable D^t.

The day-ahead market schedules are investigated by the DSO, taking into account the realisation of the uncertain demand (*i.e.*, with a specific realization $d^t \leftarrow D^t$). Then, if a congestion issue occurs, the DSO requests the consumers to decrease their consumptions on the redispatch market and compensates those who do so. However, consumers can anticipate congestion in the network and adjust their day-ahead market schedules such that the DSO pays them to decrease it on the redispatch market. This is known as increase-decrease gaming [3].

For each $t \in \mathcal{T}$, let π^t_{d} be the day-ahead market price and Π^t_{r} be the stochastic redispatch market price[1]. Then, define $\pi_{\mathrm{d}} := (\pi^1_{\mathrm{d}}, \ldots, \pi^T_{\mathrm{d}})^\top$ and $\Pi_{\mathrm{r}} := (\Pi^1_{\mathrm{r}}, \ldots, \Pi^T_{\mathrm{r}})^\top$. Quadratic cost functions are commonly used to model power generation [8]. Equivalently, we use the affine marginal cost function [9], [11]

$$\pi^t_{\mathrm{d}} = \mu^t \Delta^t + \nu^t, \tag{1}$$

where μ^t and ν^t are price function parameters and Δ^t is the total demand which is price sensitive [12] as $\Delta^t = \sum_{i\in\mathcal{N}} (\delta^t_i + p^t_{\mathrm{b},i} + p^t_{\mathrm{g},i} - \pi^t_{\mathrm{d}} e^t_i)$, with other demand δ^t_i and the demand elasticity $e^t_i \in \mathbb{R}_{>0}$, for $i \in \mathcal{N}$. Now, we define

$$z^t = \frac{1}{N} \sum_{i\in\mathcal{N}} \left(p^t_{\mathrm{b},i} + p^t_{\mathrm{g},i}\right),\ \ \delta^t = \frac{1}{N} \sum_{i\in\mathcal{N}} \delta^t_i. \tag{2}$$

Then, by replacing Δ^t in (1), we have

$$\pi^t_{\mathrm{d}}(z^t, \tilde{\nu}^t) = \tilde{\mu}^{t*}(z^t + \delta^t) + \tilde{\nu}^t, \tag{3}$$

where $\tilde{\mu}^{t*} = \frac{N\mu^t}{1+\mu^t \sum_{i\in\mathcal{N}} e^t_i}$ is assumed to be constant and $\tilde{\nu}^t = \frac{\nu^t}{1+\mu^t \sum_{i\in\mathcal{N}} e^t_i}$ is considered as the leader's variable in the following subsections. For each $t \in \tau$, we consider the constraint

$$\tilde{\nu}^t \in [\underline{\nu}^t, \bar{\nu}^t], \tag{4}$$

where $\underline{\nu}^t$ and $\bar{\nu}^t$ are positive bounds. Practically, $\underline{\nu}^t$ can be considered the uncongested price parameter, and $\tilde{\nu}^t - \underline{\nu}^t$ the price offset applied by the leader. Such a price offset is analogous to the use of Locational Marginal Pricing (LMP) in the deterministic case [5]. Moreover, we define $z := (z^1, \ldots, z^T)^\top$, $\delta := (\delta^1, \ldots, \delta^T)^\top$, $\tilde{\mu}^* := (\tilde{\mu}^{1*}, \ldots, \tilde{\mu}^{T*})^\top$ and $\tilde{\nu} := (\tilde{\nu}^1, \ldots, \tilde{\nu}^T)^\top$.

[1]The redispatch price is stochastic due to the demand uncertainty D^t.

B. Model of Followers

We consider the consumers as the followers of this game. For each $i \in \mathcal{N}$, $t \in \mathcal{T}$, let the battery State of Charge (SoC) be denoted by $\chi^t_i \in \mathbb{R}_{>0}$, whose dynamics is given by

$$\chi^t_i = \chi^{t-1}_i + \frac{1}{\beta_i} p^t_{\mathrm{b},i}, \tag{5}$$

where β_i is the battery capacity. We consider the time step equal to one hour in (5). The set of admissible SoC, the initial and final SoC and the charging/discharging power constraints are expressed as

$$\chi^t_i \in [\chi^{\min}_i, \chi^{\max}_i],\ \chi^0_i = \chi^{\mathrm{T}}_i,\ p^t_{\mathrm{b},i} \in [p^{\min}_{\mathrm{b},i}, p^{\max}_{\mathrm{b},i}], \tag{6}$$

where $\chi^{\min}_i$, $\chi^{\max}_i$ are allowable minimum and maximum SoC and $p^{\min}_{\mathrm{b},i}$, $p^{\max}_{\mathrm{b},i}$ are minimum and maximum charging/discharging rates of follower i, respectively. The battery degradation cost for the follower i is expressed as [9]

$$C_{\mathrm{b},i}(p^t_{\mathrm{b},i}) = a_i \left(p^t_{\mathrm{b},i}\right)^2 + b_i |p^t_{\mathrm{b},i}| + c_i, \tag{7}$$

where a_i, b_i and c_i are constants. Furthermore, we consider the following constraint for other flexible loads

$$p^t_{\mathrm{g},i} \in [0, p^{\max}_{\mathrm{g},i}] \tag{8}$$

where $p^{\max}_{\mathrm{g},i}$ is the maximum consumption schedules of other flexible loads. Let define $p_{\mathrm{b},i} := (p^1_{\mathrm{b},i}, \ldots, p^T_{\mathrm{b},i})^\top$ and $p_{\mathrm{g},i} := (p^1_{\mathrm{g},i}, \ldots, p^T_{\mathrm{g},i})^\top$. Then, the convex set of admissible strategies for the follower i is given by

$$\Psi_i := \{(p_{\mathrm{b},i}, p_{\mathrm{g},i}) |\ (5), (6), (8) \text{ are satisfied}\}. \tag{9}$$

Each follower $i \in \mathcal{N}$ maximizes its individual welfare $\mathcal{J}^{\mathrm{F}}_i\left(p_{\mathrm{b},i}, p_{\mathrm{g},i}, z, \tilde{\nu}, \Pi_{\mathrm{r}}\right)$ by solving the problem

$$\max_{(p_{\mathrm{b},i}, p_{\mathrm{g},i}) \in \Psi_i} \mathcal{J}^{\mathrm{F}}_i\left(p_{\mathrm{b},i}, p_{\mathrm{g},i}, z, \tilde{\nu}, \Pi_{\mathrm{r}}\right) \tag{10}$$

where $\mathcal{J}^{\mathrm{F}}_i\left(p_{\mathrm{b},i}, p_{\mathrm{g},i}, z, \tilde{\nu}, \Pi_{\mathrm{r}}\right) = \sum_{t\in\tau} p^t_{\mathrm{g},i} u_i - (p^t_{\mathrm{b},i} + p^t_{\mathrm{g},i}) \pi^t_{\mathrm{d}}(z^t, \tilde{\nu}^t) + p^t_{\mathrm{g},i} \mathbb{E}_{\Pi^t_{\mathrm{r}}}\left[[\Pi^t_{\mathrm{r}} - u_i]^+\right] - C_{\mathrm{b},i}(p^t_{\mathrm{b},i})$. If we assume that z is a fixed reference, then (10) becomes a linear program in $p_{\mathrm{g},i}$, and its optimal solution becomes discontinuous w.r.t. this fixed reference. Hence, inspired by [8], [10], for each follower $i \in \mathcal{N}$, we consider the following problem

$$\max_{(p_{\mathrm{b},i}, p_{\mathrm{g},i}) \in \Psi_i} \mathcal{J}^{\mathrm{F}}_{\sigma i}\left(p_{\mathrm{b},i}, p_{\mathrm{g},i}, z, \tilde{\nu}, \Pi_{\mathrm{r}}\right) \tag{11}$$

where $\mathcal{J}^{\mathrm{F}}_{\sigma i}\left(p_{\mathrm{b},i}, p_{\mathrm{g},i}, z, \tilde{\nu}, \Pi_{\mathrm{r}}\right) = \mathcal{J}^{\mathrm{F}}_i\left(p_{\mathrm{b},i}, p_{\mathrm{g},i}, z, \tilde{\nu}, \Pi_{\mathrm{r}}\right) - \sigma(p^t_{\mathrm{g},i} - z^t)^2$, with $\sigma \in \mathbb{R}_{\geq 0}$. The extra term in (11) regularizes the problem and numerical experimentation suggests that it can be adjusted to arbitrarily small values.

C. Model of Leader

We consider a leader for this game. The convex set of admissible strategies for the leader is given by

$$\Phi := \{\tilde{\nu} | \text{ (4) is satisfied}\}. \tag{12}$$

The leader can adjust the day-ahead market price to balance two objectives: maximizing the social welfare of all consumers (followers) and minimizing the costs of the DSO on the redispatch market. Therefore, the leader solves the following optimization problem, where the balance between the two is controlled by the constant $\rho \geq 1$:

$$\max_{\tilde{\nu}\in\Phi} \mathcal{J}^{\mathrm{L}}\big(z, \tilde{\nu}, \Pi_{\mathrm{r}}\big) \tag{13}$$

where $\mathcal{J}^{\mathrm{L}}\big(z, \tilde{\nu}, \Pi_{\mathrm{r}}\big) = \sum_{i\in\mathcal{N}} \mathcal{J}_i^{\mathrm{F}}\big(p_{\mathrm{b},i}, p_{\mathrm{g},i}, z, \tilde{\nu}, \Pi_{\mathrm{r}}\big) - \rho \sum_{t\in\tau} \mathbb{E}_{\Pi_r^t}\big[\Pi_r^t[\tilde{\Delta}^t - c]^+\big]\big)$, with $\tilde{\Delta}^t = \Delta^t + D^t$ and D^t is the demand uncertainty at time $t \in \mathcal{T}$.

III. Mean-Field Stackelberg Game Approach

The leader-follower interaction can be modeled as a two-level game. In this game, the leader determines the offset of the day-ahead price function and sends the price function to the followers and the followers respond optimally to it via modifying their consumption schedule strategies. Now, we define the game $\mathcal{G}$ as follows: (i) **players**: followers $\mathcal{N}$ and leader; (ii) **strategies**: follower i: $(p_{\mathrm{b},i}, p_{\mathrm{g},i}) \in \Psi_i$, leader: $\tilde{\nu} \in \Phi$; (iii) **objective functions**: follower i: $\mathcal{J}_{\sigma i}^{\mathrm{F}}\big(p_{\mathrm{b},i}, p_{\mathrm{g},i}, z, \tilde{\nu}, \Pi_{\mathrm{r}}\big)$, leader: $\mathcal{J}^{\mathrm{L}}\big(z, \tilde{\nu}, \Pi_{\mathrm{r}}\big)$. The game $\mathcal{G}$ is a repetitive game with incomplete information (*i.e.*, a game with incomplete information that is played in multiple rounds to determine its equilibrium). In this game, the followers are coupled via z, which is called the mean-field term. The followers are not aware of strategies or objective functions of other followers but they know the leader's strategy and mean-field term and determine their optimal strategies based on them. The leader determines its optimal strategy using the followers' strategies. As we show below, through exchanging moves in multiple rounds, the players iteratively arrive at a desirable solution.

A set of agents' strategies is called a mean-field Nash equilibrium if each agent cannot improve its profit by altering its own strategy whilst the aggregated strategies of the other agents remain unchanged. Following [9, Definition 1], we define the mean-field leader-follower ε-Nash equilibrium as follows.

Definition 1: **(Mean-field leader-follower ε-Nash equilibrium).** A set of strategies $\big(\{(p^*_{\mathrm{b},i}, p^*_{\mathrm{g},i})\}_{i\in\mathcal{N}}, \tilde{\nu}^*\big)$ is called a Mean-field leader-follower ε-Nash equilibrium for the game $\mathcal{G}$ if $\varepsilon > 0$ exists such that for all $i \in \mathcal{N}$, $(p_{\mathrm{b},i}, p_{\mathrm{g},i}) \in \Psi_i$, and $\tilde{\nu} \in \Phi$, we have $\mathcal{J}_{\sigma i}^{\mathrm{F}}\big(p^*_{\mathrm{b},i}, p^*_{\mathrm{g},i}, \frac{1}{N}\sum_{j\in\mathcal{N}}(p^*_{\mathrm{b},j} + p^*_{\mathrm{g},j}), \tilde{\nu}^*, \Pi_{\mathrm{r}}\big) + \varepsilon \geq \max_{(p_{\mathrm{b},i}, p_{\mathrm{g},i})\in\Psi_i} \mathcal{J}_{\sigma i}^{\mathrm{F}}\big(p_{\mathrm{b},i}, p_{\mathrm{g},i}, \frac{1}{N}(p_{\mathrm{b},i} + p_{\mathrm{g},i}) + \frac{1}{N}\sum_{j\in\mathcal{N}-\{i\}}(p^*_{\mathrm{b},j} + p^*_{\mathrm{g},j}), \tilde{\nu}^*, \Pi_{\mathrm{r}}\big)$ and $\mathcal{J}^{\mathrm{L}}\big(\frac{1}{N}\sum_{j\in\mathcal{N}}(p^*_{\mathrm{b},j} + p^*_{\mathrm{g},j}), \tilde{\nu}^*, \Pi_{\mathrm{r}}\big) \geq \max_{\tilde{\nu}\in\Phi} \mathcal{J}^{\mathrm{L}}\big(\frac{1}{N}\sum_{j\in\mathcal{N}}(p^*_{\mathrm{b},j} + p^*_{\mathrm{g},j}), \tilde{\nu}, \Pi_{\mathrm{r}}\big)$.

Algorithm 1: Mean-field Stackelberg game based Algorithm

```
Select z_1 and ν̃_1; set ϵ_1 > ε, m = 1 and z• = z_1;
while ϵ_1 > ε do
    set l = 1 and ϵ_2 > ε;
    while ϵ_2 > ε do
        set z_1 = z•;
        the followers solve (11) and obtain p_{b,i}, p_{g,i};
        z_{l+1} = (1 − α_l) z_l + α_l Λ(z_l, ν̃_m);
        ϵ_2 = ‖z_{l+1} − z_l‖;  l = l + 1;
    end
    set z• = z_l;
    the leader solves (13) based on z• and obtain
      ν̃_{m+1};
    ϵ_1 = ‖z*(ν̃_{m+1}) − z•‖;  m = m + 1.
end
```

A. Mean-field game approach for the followers

We use a mean field approach to focus on collective strategic behaviour without individual market power. In this subsection, an iterative method converging to the mean-field leader-follower ε_N-Nash equilibrium of $\mathcal{G}$, with $\varepsilon_N = \mathcal{O}(\frac{1}{N})$ is used [9]. This method obtains the mean-field game equilibrium which is a function of the leader's variable.

We assume that the mean-field term z is provided to the followers. Then, for a given z, each follower can solve the problem (11) and determine its optimal strategy. Now, at iteration l, z_l is considered as the estimate of mean-field term z. Then, the Mann iteration algorithm [10] is used as

$$z_{l+1} = (1 - \alpha_l) z_l + \alpha_l \Lambda(z_l, \tilde{\nu}), \tag{14}$$

where $\Lambda(z_l, \tilde{\nu}) = \frac{1}{N}\sum_{i\in\mathcal{N}}(p^*_{\mathrm{b},i}(z_l, \tilde{\nu}) + p^*_{\mathrm{g},i}(z_l, \tilde{\nu}))$ and $\alpha_l \in (0, 1)$ is the learning rate that fulfills $\lim_{l\to\infty} \alpha_l = 0$, $\sum_{k=0}^{\infty} \alpha_l = \infty$. The mean-field decentralized method is composed of two iterative steps: optimization and estimation step. In the optimization step, each follower solves the problem (11) for a given mean-field term. In the estimation step, according to (14), the mean-field term is updated, then this will be utilized in the next iteration of the optimization step. These steps continue until the convergence of the mean-field term is achieved. When the algorithm converged, we have $z_{l+1} = z_l = z^*$; following (14), we obtain

$$z^* = \frac{1}{N}\sum_{i\in\mathcal{N}} \big(p^*_{\mathrm{b},i}, (z^*, \tilde{\nu}) + p^*_{\mathrm{g},i}(z^*, \tilde{\nu})\big) \tag{15}$$

Remark 1: **(Existence and uniqueness of the solution to** (11)**).** The followers' objective function given in (11) is concave w.r.t. $(p_{\mathrm{b},i}, p_{\mathrm{g},i})$ over the convex and compact set Ψ_i. Hence, for each realization $\pi_{\mathrm{r}} \leftarrow \Pi_{\mathrm{r}}$, a unique solution to (11) exists in the convex and compact set Ψ_i [13].

B. Stackelberg game approach for the leader

The leader in the Stackelberg game, takes on the role of declaring a leader function, mapping the decision space of followers (Ψ_i) into the decision space of leader (Φ).

Indeed, the leader sends a mapping of the decision space of followers into its own decision space [9]. Therefore, the leader solves the optimization problem (13) based on the mean-field Nash solution of the followers and sends the day-ahead price function to the followers for the next round (see Algorithm 1).

C. Convergence of the proposed method

In this subsection, we study the convergence of Algorithm 1. We prove that this algorithm converges to the mean-field leader-follower ε_N-Nash equilibrium for large populations of consumers. In the following, we first investigate the problem for the case that the redispatch price π_{r} is known and is broadcast to all players (the deterministic case), then we extend the problem for stochastic redispatch price Π_{r} (the stochastic case).

Lemma 1: **(Lipschitz property of the solution to (11)).** Let $\gamma := \max_{i\in\mathcal{N}}\big(\sqrt{\lambda_{\min}(S_i)/\lambda_{\max}(S_i)}\big)\sqrt{\big(\max_{i\in\mathcal{N}}\big(1/a_i\big)\|\tilde{\mu}^*\|\big)^2+\|\mathbb{I}_T-(1/2\sigma)\operatorname{diag}(\tilde{\mu}^*)\|^2}$ and $\bar{\gamma} := \max_{i\in\mathcal{N}}\big(\sqrt{\lambda_{\min}(S_i)/\lambda_{\max}(S_i)}\big)\big(\max_{i\in\mathcal{N}}\big(\frac{1}{a_i}\big)+\frac{1}{2\sigma}\big)$, where $S_i=\operatorname{diag}\left(a_i\mathbb{I}_T,\sigma\mathbb{I}_T\right)$. Then, the solution to the problem (11) for the deterministic case is Lipschitz w.r.t. z and z^* is Lipschitz w.r.t. $\tilde{\nu}$, *i.e.*, for all z, $\hat{z}$, $\tilde{\nu}$, $\hat{\tilde{\nu}}$

$$\big\|\big(p^*_{\mathrm{b},i}(z,\tilde{\nu}),p^*_{\mathrm{g},i}(z,\tilde{\nu})\big)-\big(p^*_{\mathrm{b},i}(\hat{z},\tilde{\nu}),p^*_{\mathrm{g},i}(\hat{z},\tilde{\nu})\big)\big\| \leq \gamma\|(z,\tilde{\nu})-(\hat{z},\tilde{\nu})\|, \tag{16}$$

$$\|z^*(\tilde{\nu})-z^*(\hat{\tilde{\nu}})\|\leq\bar{\gamma}\|\tilde{\nu}-\hat{\tilde{\nu}}\|. \tag{17}$$

Proof: Let $\hat{p}_{\mathrm{b},i}$, $\hat{p}_{\mathrm{g},i}$ be the solution to the problem (11) without constraints and for the deterministic case, *i.e.*, $\big(\hat{p}^*_{\mathrm{b},i}(z,\tilde{\nu}),\hat{p}^*_{\mathrm{g},i}(z,\tilde{\nu})\big)=\arg\max_{(\hat{p}_{\mathrm{b},i},\hat{p}_{\mathrm{g},i})}\mathcal{J}^{\mathrm{F}}_{\sigma i}\big(\hat{p}_{\mathrm{b},i},\hat{p}_{\mathrm{g},i},z,\tilde{\nu},\pi_{\mathrm{r}}\big)$. By getting derivative, for all $t\in\mathcal{T}$, the optimal solution is obtained as

$$\begin{aligned}\hat{p}^{t*}_{\mathrm{b},i}(z^t,\tilde{\nu}^t)&=\frac{1}{2a_i}\big(\big(-\pi^t_{\mathrm{d}}(z^t,\tilde{\nu}^t)-b_i\big)H\big(-\pi^t_{\mathrm{d}}(z^t,\tilde{\nu}^t)\\&\quad-b_i\big)+\big(-\pi^t_{\mathrm{d}}(z^t,\tilde{\nu}^t)+b_i\big)H\big(\pi^t_{\mathrm{d}}(z^t,\tilde{\nu}^t)-b_i\big)\big)\\\hat{p}^{t*}_{\mathrm{g},i}(z^t,\tilde{\nu}^t)&=\frac{1}{2\sigma}\big(u_i-\pi^t_{\mathrm{d}}(z^t,\tilde{\nu}^t)+[\pi^t_{\mathrm{r}}-u_i]^+\big)+z^t,\end{aligned} \tag{18}$$

where $H(\cdot)$ is the heavy-side function. We have for all z, $\hat{z}$

$$\begin{aligned}&\|\hat{p}^*_{\mathrm{b},i}(z,\tilde{\nu})-\hat{p}^*_{\mathrm{b},i}(\hat{z},\tilde{\nu})\|\leq\Big\|\frac{1}{a_i}(\pi_{\mathrm{d}}(\hat{z},\tilde{\nu})-\pi_{\mathrm{d}}(z,\tilde{\nu}))\Big\|\\&\leq\max_{i\in\mathcal{N}}\big(\frac{1}{a_i}\big)\|\tilde{\mu}^*\|\|(z,\tilde{\nu})-(\hat{z},\tilde{\nu})\|\end{aligned} \tag{19}$$

$$\begin{aligned}&\|\hat{p}^*_{\mathrm{g},i}(z,\tilde{\nu})-\hat{p}^*_{\mathrm{g},i}(\hat{z},\tilde{\nu})\|\leq\Big\|\frac{1}{2\sigma}(\pi_{\mathrm{d}}(\hat{z},\tilde{\nu})-\pi_{\mathrm{d}}(z,\tilde{\nu}))\\&+(z-\hat{z})\Big\|\leq\Big\|\mathbb{I}_T-\frac{1}{2\sigma}\operatorname{diag}(\tilde{\mu}^*)\Big\|\|(z,\tilde{\nu})-(\hat{z},\tilde{\nu})\|.\end{aligned} \tag{20}$$

Hence, we have

$$\begin{aligned}&\|(\hat{p}^*_{\mathrm{b},i}(z,\tilde{\nu}),\hat{p}^*_{\mathrm{g},i}(z,\tilde{\nu}))-(\hat{p}^*_{\mathrm{b},i}(\hat{z},\tilde{\nu}),\hat{p}^*_{\mathrm{g},i}(\hat{z},\tilde{\nu}))\|\\&\leq\sqrt{\big(\max_{i\in\mathcal{N}}\big(1/a_i\big)\|\tilde{\mu}^*\|\big)^2+\|\mathbb{I}_T-1/2\sigma\operatorname{diag}(\tilde{\mu}^*)\|^2}\|(z,\tilde{\nu})\\&-(\hat{z},\tilde{\nu})\|.\end{aligned} \tag{21}$$

Consequently, the solution to the problem (11) without constraints is Lipschitz w.r.t. to z. Now, let the projection of the solution without constraints on the convex and compact set Ψ_i be $\operatorname{Proj}^{S_i}_{\Psi_i}\big((\hat{p}^*_{\mathrm{b},i}(z,\tilde{\nu}),\hat{p}^*_{\mathrm{g},i}(z,\tilde{\nu}))\big)=\arg\min_{(p_{\mathrm{b},i},p_{\mathrm{g},i})\in\Psi_i}\big\|\big((p_{\mathrm{b},i}(z,\tilde{\nu}),p_{\mathrm{g},i}(z,\tilde{\nu}))\big)-\big((\hat{p}^*_{\mathrm{b},i}(z,\tilde{\nu}),\hat{p}^*_{\mathrm{g},i}(z,\tilde{\nu}))\big)\big\|^2_{S_i}=\big(p^*_{\mathrm{b},i}(z,\tilde{\nu}),p^*_{\mathrm{g},i}(z,\tilde{\nu})\big)$. Using the non-expansive property of projection [13], $\lambda_{\min}(S_i)\|\cdot\|^2\leq\|\cdot\|^2_{S_i}\leq\lambda_{\max}(S_i)\|\cdot\|^2$ and (21), we obtain

$$\begin{aligned}&\|(p^*_{\mathrm{b},i}(z,\tilde{\nu}),p^*_{\mathrm{g},i}(z,\tilde{\nu}))-(p^*_{\mathrm{b},i}(\hat{z},\tilde{\nu}),p^*_{\mathrm{g},i}(\hat{z},\tilde{\nu}))\|\\&\leq\sqrt{\lambda_{\min}(S_i)/\lambda_{\max}(S_i)}\|(\hat{p}^*_{\mathrm{b},i}(z,\tilde{\nu}),\hat{p}^*_{\mathrm{g},i}(z,\tilde{\nu}))\\&-(\hat{p}^*_{\mathrm{b},i}(\hat{z},\tilde{\nu}),\hat{p}^*_{\mathrm{g},i}(\hat{z},\tilde{\nu}))\|\leq\gamma\|(z,\tilde{\nu})-(\hat{z},\tilde{\nu})\|.\end{aligned} \tag{22}$$

Now, according to (15) and (18), for the large populations of followers, we have for all $\tilde{\nu}$, $\hat{\tilde{\nu}}$

$$\begin{aligned}&\|\hat{p}^*_{\mathrm{b},i}(z^*,\tilde{\nu})-\hat{p}^*_{\mathrm{b},i}(z^*,\hat{\tilde{\nu}})\|\leq\Big\|\frac{1}{a_i}(\pi_{\mathrm{d}}(z^*,\hat{\tilde{h}})-\pi_{\mathrm{d}}(z^*,\tilde{h}))\Big\|\\&\leq\max_{i\in\mathcal{N}}\big(\frac{1}{a_i}\big)\|\tilde{\nu}-\hat{\tilde{\nu}}\|\end{aligned} \tag{23}$$

$$\begin{aligned}&\|\hat{p}^*_{\mathrm{g},i}(z^*,\tilde{\nu})-\hat{p}^*_{\mathrm{g},i}(z^*,\hat{\tilde{\nu}})\|=\Big\|\frac{1}{2\sigma}(\pi_{\mathrm{d}}(z^*,\hat{\tilde{\nu}})-\pi_{\mathrm{d}}(z^*,\tilde{\nu}))\Big\|\\&\leq\frac{1}{2\sigma}\|\tilde{\nu}-\hat{\tilde{\nu}}\|.\end{aligned} \tag{24}$$

By the same procedure and following (15), we have $\|z^*(\tilde{\nu})-z^*(\hat{\tilde{\nu}})\|=\big\|\frac{1}{N}\big(\sum_{i\in\mathcal{N}}(p^*_{\mathrm{b},i}(z^*,\tilde{\nu})+p^*_{\mathrm{g},i}(z^*,\tilde{\nu}))-\sum_{i\in\mathcal{N}}(p^*_{\mathrm{b},i}(z^*,\hat{\tilde{\nu}})+p^*_{\mathrm{g},i}(z^*,\hat{\tilde{\nu}}))\big)\big\|\leq\frac{1}{N}\sum_{i\in\mathcal{N}}\sqrt{\lambda_{\min}(S_i)/\lambda_{\max}(S_i)}\big(\|\hat{p}^*_{\mathrm{b},i}(z^*,\tilde{\nu})-\hat{p}^*_{\mathrm{b},i}(z^*,\hat{\tilde{\nu}})\|+\|\hat{p}^*_{\mathrm{g},i}(z^*,\tilde{\nu})-\hat{p}^*_{\mathrm{g},i}(z^*,\hat{\tilde{\nu}})\|\big)\leq\frac{1}{N}\sum_{i\in\mathcal{N}}\sqrt{\lambda_{\min}(S_i)/\lambda_{\max}(S_i)}\big(\max_{i\in\mathcal{N}}\big(\frac{1}{a_i}\big)+\frac{1}{2\sigma}\big)\|\tilde{\nu}-\hat{\tilde{\nu}}\|\leq\bar{\gamma}\|\tilde{\nu}-\hat{\tilde{\nu}}\|$. Thus, the solution to (11) for the deterministic case is Lipschitz w.r.t. z and z^* is Lipschitz w.r.t. $\tilde{\nu}$. ■

Theorem 1: **(Convergence analysis).** The convergence of Algorithm 1 to the solution of $\mathcal{G}$ is ensured for any initial condition z_1 and the deterministic case if $\gamma\in(0,1)$.

Proof: Following Lemma 1, $\Lambda(z_l,\tilde{\nu})$ is Lipschitz and continuous w.r.t. z_l and relates to a convex and compact set. Hence, according to the Brouwer fixed point theorem [14], $z\mapsto\Lambda(z)$ is contractive and has a fixed point if $\gamma\in(0,1)$. Then, in analogy with the contraction mapping theorem [15, Theorem 1.2.2], the convergence of algorithm (14) to a unique fixed point $z^*(\tilde{\nu})$ is ensured. Moreover, following Lemma 1, z^* is Lipschitz w.r.t. $\tilde{\nu}$; thus, the convergence of Algorithm 1 to the solution of $\mathcal{G}$ is guaranteed. ■

Theorem 2: **(Convergence to the mean-field leader-follower ε_N-Nash equilibrium).** For the finite population of consumers and the deterministic case, the convergence of the strategies of the leader and followers via Algorithm 1 to the mean-field leader-follower ε_N-Nash equilibrium of $\mathcal{G}$, with $\varepsilon_N=\mathcal{O}(\frac{1}{N})$ is ensured if $\gamma\in(0,1)$.

Proof: Since the objective function $\mathcal{J}^{\mathrm{F}}_{\sigma i}\big(p_{\mathrm{b},i},p_{\mathrm{g},i},z,\tilde{\nu},\pi_{\mathrm{r}}\big)$ is quadratic and belongs to a compact set, it is Lipschitz w.r.t. $(p_{\mathrm{b},i}, p_{\mathrm{g},i})$ and z. Thus, a $L > 0$ exists such that

$$\begin{aligned}&\big|\mathcal{J}^{\mathrm{F}}_{\sigma i}\big(p^*_{\mathrm{b},i}, p^*_{\mathrm{g},i}, z^*, \tilde{\nu}, \pi_{\mathrm{r}}\big) - \mathcal{J}^{\mathrm{F}}_{\sigma i}\big(\bar{p}^*_{\mathrm{b},i}, \bar{p}^*_{\mathrm{g},i}, \bar{z}^*, \tilde{\nu}, \pi_{\mathrm{r}}\big)\big| \\ &\leq L\big\|\big(p^*_{\mathrm{b},i}(z^*,\tilde{\nu}), p^*_{\mathrm{g},i}(z^*,\tilde{\nu})\big) - \big(\bar{p}^*_{\mathrm{b},i}(\bar{z}^*,\tilde{\nu}), \bar{p}^*_{\mathrm{g},i}(\bar{z}^*,\tilde{\nu})\big)\big\| \\ &+ L\|z^*(\tilde{\nu}) - \bar{z}^*(\tilde{\nu})\|. \qquad (25)\end{aligned}$$

For follower $i \in \mathcal{N}$, the Nash optimal welfare function is given by $\tilde{\mathcal{J}}^{\mathrm{F}}_{\sigma i}\big(\tilde{p}^*_{\mathrm{b},i}, \tilde{p}^*_{\mathrm{g},i}, \tilde{z}^*, \tilde{\nu}, \pi_{\mathrm{r}}\big) := \mathcal{J}^{\mathrm{F}}_{\sigma i}\big(\tilde{p}^*_{\mathrm{b},i}, \tilde{p}^*_{\mathrm{g},i}, \frac{1}{N}(\tilde{p}^*_{\mathrm{b},i} + \tilde{p}^*_{\mathrm{g},i}) + \frac{1}{N}\sum_{j\in\mathcal{N}-\{i\}}(p^*_{\mathrm{b},j} + p^*_{\mathrm{g},j}), \tilde{\nu}, \pi_{\mathrm{r}}\big) = \max_{(p_{\mathrm{b},i},p_{\mathrm{g},i})\in\Psi_i} \mathcal{J}^{\mathrm{F}}_{\sigma i}\big(p_{\mathrm{b},i}, p_{\mathrm{g},i}, \frac{1}{N}(p_{\mathrm{b},i} + p_{\mathrm{g},i}) + \frac{1}{N}\sum_{j\in\mathcal{N}-\{i\}}(p^*_{\mathrm{b},j} + p^*_{\mathrm{g},j}), \tilde{\nu}, \Pi_{\mathrm{r}}\big)$, where $\tilde{z}^* = \frac{1}{N}(\tilde{p}^*_{\mathrm{b},i} + \tilde{p}^*_{\mathrm{g},i}) + \frac{1}{N}\sum_{j\in\mathcal{N}-\{i\}}(p^*_{\mathrm{b},j} + p^*_{\mathrm{g},j})$ and $(p^*_{\mathrm{b},j}, p^*_{\mathrm{g},j})$ is the optimal strategy of jth follower with $j \in \mathcal{N} - \{i\}$. Moreover, the optimal mean-field welfare function is expressed as $\bar{\mathcal{J}}^{\mathrm{F}}_{\sigma i}\big(\bar{p}^*_{\mathrm{b},i}, \bar{p}^*_{\mathrm{g},i}, z^*, \tilde{\nu}, \pi_{\mathrm{r}}\big) := \mathcal{J}^{\mathrm{F}}_{\sigma i}\big(\bar{p}^*_{\mathrm{b},i}, \bar{p}^*_{\mathrm{g},i}, \frac{1}{N}\sum_{j\in\mathcal{N}}(p^*_{\mathrm{b},j} + p^*_{\mathrm{g},j}), \tilde{\nu}, \pi_{\mathrm{r}}\big) = \max_{(p_{\mathrm{b},i},p_{\mathrm{g},i})\in\Psi_i} \mathcal{J}^{\mathrm{F}}_{\sigma i}\big(p_{\mathrm{b},i}, p_{\mathrm{g},i}, \frac{1}{N}\sum_{j\in\mathcal{N}}(p^*_{\mathrm{b},j} + p^*_{\mathrm{g},j}), \tilde{\nu}, \Pi_{\mathrm{r}}\big)$. Now, we show that a follower can increase its welfare from mean-field optimal solution by ε_N. We define $\hat{\mathcal{J}}^{\mathrm{F}}_{\sigma i}\big(\hat{p}^*_{\mathrm{b},i}, \hat{p}^*_{\mathrm{g},i}, \tilde{z}^*, \tilde{\nu}, \pi_{\mathrm{r}}\big) := \mathcal{J}^{\mathrm{F}}_{\sigma i}\big(\hat{p}^*_{\mathrm{b},i}, \hat{p}^*_{\mathrm{g},i}, \frac{1}{N}(\tilde{p}^*_{\mathrm{b},i} + \tilde{p}^*_{\mathrm{g},i}) + \frac{1}{N}\sum_{j\in\mathcal{N}-\{i\}}(p^*_{\mathrm{b},j} + p^*_{\mathrm{g},j}), \tilde{\nu}, \pi_{\mathrm{r}}\big) = \max_{(p_{\mathrm{b},i},p_{\mathrm{g},i})\in\Psi_i} \mathcal{J}^{\mathrm{F}}_{\sigma i}\big(p_{\mathrm{b},i}, p_{\mathrm{g},i}, \frac{1}{N}(\tilde{p}^*_{\mathrm{b},i} + \tilde{p}^*_{\mathrm{g},i}) + \frac{1}{N}\sum_{j\in\mathcal{N}-\{i\}}(p^*_{\mathrm{b},j} + p^*_{\mathrm{g},j}), \tilde{\nu}, \Pi_{\mathrm{r}}\big)$. Due to the fact $\bar{\mathcal{J}}^{\mathrm{F}}_{\sigma i} \leq \tilde{\mathcal{J}}^{\mathrm{F}}_{\sigma i} \leq \hat{\mathcal{J}}^{\mathrm{F}}_{\sigma i}$, we obtain

$$\begin{aligned}0 \leq&\tilde{\mathcal{J}}^{\mathrm{F}}_{\sigma i}\big(\tilde{p}^*_{\mathrm{b},i}, \tilde{p}^*_{\mathrm{g},i}, \tilde{z}^*, \tilde{\nu}, \pi_{\mathrm{r}}\big) - \bar{\mathcal{J}}^{\mathrm{F}}_{\sigma i}\big(\bar{p}^*_{\mathrm{b},i}, \bar{p}^*_{\mathrm{g},i}, z^*, \tilde{\nu}, \pi_{\mathrm{r}}\big) \\ \leq&\hat{\mathcal{J}}^{\mathrm{F}}_{\sigma i}\big(\hat{p}^*_{\mathrm{b},i}, \hat{p}^*_{\mathrm{g},i}, \tilde{z}^*, \tilde{\nu}, \pi_{\mathrm{r}}\big) - \bar{\mathcal{J}}^{\mathrm{F}}_{\sigma i}\big(\bar{p}^*_{\mathrm{b},i}, \bar{p}^*_{\mathrm{g},i}, z^*, \tilde{\nu}, \pi_{\mathrm{r}}\big) \\ \leq&\big|\mathcal{J}^{\mathrm{F}}_{\sigma i}\big(\hat{p}^*_{\mathrm{b},i}, \hat{p}^*_{\mathrm{g},i}, \tilde{z}^*, \tilde{\nu}, \pi_{\mathrm{r}}\big) - \mathcal{J}^{\mathrm{F}}_{\sigma i}\big(\bar{p}^*_{\mathrm{b},i}, \bar{p}^*_{\mathrm{g},i}, z^*, \tilde{\nu}, \pi_{\mathrm{r}}\big)\big|. \qquad (26)\end{aligned}$$

Hence, according to Lemma 1 and (25), we have

$$\begin{aligned}&\big|\mathcal{J}^{\mathrm{F}}_{\sigma i}\big(\hat{p}^*_{\mathrm{b},i}, \hat{p}^*_{\mathrm{g},i}, \tilde{z}^*, \tilde{\nu}, \pi_{\mathrm{r}}\big) - \mathcal{J}^{\mathrm{F}}_{\sigma i}\big(\bar{p}^*_{\mathrm{b},i}, \bar{p}^*_{\mathrm{g},i}, z^*, \tilde{\nu}, \pi_{\mathrm{r}}\big)\big| \\ &\leq L(\gamma+1)\big\|\tilde{z}^*(\tilde{\nu}) - z^*(\tilde{\nu})\big\| \leq \frac{L(\gamma+1)}{N}\big(\big\|\tilde{p}^*_{\mathrm{b},i}(\tilde{z}^*,\tilde{\nu}) \\ &- \bar{p}^*_{\mathrm{b},i}(z^*,\tilde{\nu})\big\| + \big\|\tilde{p}^*_{\mathrm{g},i}(\tilde{z}^*,\tilde{\nu}) - \bar{p}^*_{\mathrm{g},i}(z^*,\tilde{\nu})\big\|\big) \\ &\leq \frac{L(\gamma+1)(\beta_1+\beta_2)}{N}, \qquad (27)\end{aligned}$$

where $\beta_1 = \max_{\tilde{p}_{\mathrm{b}},\bar{p}_{\mathrm{b}}} \|\tilde{p}_{\mathrm{b}} - \bar{p}_{\mathrm{b}}\|$ and $\beta_2 = \max_{\tilde{p}_{\mathrm{g}},\bar{p}_{\mathrm{g}}} \|\tilde{p}_{\mathrm{g}} - \bar{p}_{\mathrm{g}}\|$. By considering the optimal solution of the leader (*i.e.*, the solution to (13)), the leader–follower mean-field ε_N-Nash equilibrium of this game is obtained via Algorithm 1. ■

Following Theorem 2, ε_N approaches to zero and Algorithm 1 converges to the leader-follower mean-field Nash equilibrium of this game when the population size tends to infinity. Now, we investigate the problem for the stochastic case. The DSO collects the day-ahead market schedules and according to the realization of $D := (D^1, \ldots, D^T)^\top$, if a congestion issue occurs, the DSO requests offers for demand reduction on the redispatch market and the redispatch price is determined based on the total realized flexible and other demand. Thus, stochastic redispatch price Π_{r} depends on D and z. Now, consider the following assumption.

Assumption 1: **(Condition on Π_{r}).** There exists $M > 0$ such that for all $z, \hat{z}$, we have $\big\|\mathbb{E}_{\Pi_{\mathrm{r}}}\big[[\Pi_{\mathrm{r}}(z,D) - u_i\mathbf{1}_T]^+\big] - \mathbb{E}_{\Pi_{\mathrm{r}}}\big[[\Pi_{\mathrm{r}}(\hat{z},D) - u_i\mathbf{1}_T]^+\big]\big\| \leq M\|z - \hat{z}\|$.

Theorem 3: **(Convergence analysis for the stochastic case).** Let Assumption 1 hold. The convergence of Algorithm 1 to the solution of $\mathcal{G}$ is guaranteed for any initial condition z_1 and the stochastic case if $\tilde{\gamma} \in (0,1)$, where $\tilde{\gamma} = \max_{i\in\mathcal{N}}\big(\sqrt{\lambda_{\min}(S_i)/\lambda_{\max}(S_i)}\big)\big(\big\|\mathbb{I}_T - \frac{1}{2\sigma}\operatorname{diag}(\tilde{\mu}^*)\big\| + \max_{i\in\mathcal{N}}\big(\frac{1}{a_i}\big)\|\tilde{\mu}^*\| + \frac{1}{2\sigma}M\big)$. Moreover, if $\tilde{\gamma} \in (0,1)$, for the finite population of consumers and the stochastic case, the convergence of the strategies of the leader and followers via Algorithm 1 to the mean-field leader-follower ε_N-Nash equilibrium of $\mathcal{G}$ is ensured.

Proof: Let $\hat{p}_{\mathrm{b},i}$, $\hat{p}_{\mathrm{g},i}$ be the solution to the problem (11) without constraints and for the stochastic case, *i.e.*, $\big(\hat{p}^*_{\mathrm{b},i}(z,\tilde{\nu}), \hat{p}^*_{\mathrm{g},i}(z,\tilde{\nu})\big) = \arg\max_{(\hat{p}_{\mathrm{b},i},\hat{p}_{\mathrm{g},i})} \mathcal{J}^{\mathrm{F}}_{\sigma i}\big(\hat{p}_{\mathrm{b},i}, \hat{p}_{\mathrm{g},i}, z, \tilde{\nu}, \Pi_{\mathrm{r}}(z,D)\big)$. By getting derivative, for all $t \in \mathcal{T}$, the optimal solution is obtained as follows:

$$\begin{aligned}\hat{p}^{t*}_{\mathrm{b},i}(z^t,\tilde{\nu}^t) &= \frac{1}{2a_i}\big(\big(-\pi^t_{\mathrm{d}}(z^t,\tilde{\nu}^t) - b_i\big)H\big(-\pi^t_{\mathrm{d}}(z^t,\tilde{\nu}^t) \\ &- b_i\big) + \big(-\pi^t_{\mathrm{d}}(z^t,\tilde{\nu}^t) + b_i\big)H\big(\pi^t_{\mathrm{d}}(z^t,\tilde{\nu}^t) - b_i\big)\big) \\ \hat{p}^{t*}_{\mathrm{g},i}(z^t,\tilde{\nu}^t) &= \frac{1}{2\sigma}\big(u_i - \pi^t_{\mathrm{d}}(z^t,\tilde{\nu}^t) + \mathbb{E}_{\Pi^t_{\mathrm{r}}}\big[[\Pi^t_{\mathrm{r}}(z^t,D^t) \\ &- u_i]^+\big]\big) + z^t. \qquad (28)\end{aligned}$$

Therefore, for all z, $\hat{z}$, we have $\|\hat{p}^*_{\mathrm{b},i}(z,\tilde{\nu}) - \hat{p}^*_{\mathrm{b},i}(\hat{z},\tilde{\nu})\| \leq \max_{i\in\mathcal{N}}\big(\frac{1}{a_i}\big)\|\tilde{\mu}^*\|\|(z,\tilde{\nu}) - (\hat{z},\tilde{\nu})\|$ and $\|\hat{p}^*_{\mathrm{g},i}(z,\tilde{\nu}) - \hat{p}^*_{\mathrm{g},i}(\hat{z},\tilde{\nu})\| \leq \big\|\mathbb{I}_T - \frac{1}{2\sigma}\operatorname{diag}(\tilde{\mu}^*)\big\|\|(z,\tilde{\nu}) - (\hat{z},\tilde{\nu})\| + \frac{1}{2\sigma}\big\|\mathbb{E}_{\Pi_{\mathrm{r}}}\big[[\Pi_{\mathrm{r}}(z,D) - u_i\mathbf{1}_T]^+\big] - \mathbb{E}_{\Pi_{\mathrm{r}}}\big[[\Pi_{\mathrm{r}}(\hat{z},D) - u_i\mathbf{1}_T]^+\big]\big\|$. Hence, according to Assumption 1, we have $\|(\hat{p}^*_{\mathrm{b},i}(z,\tilde{\nu}), \hat{p}^*_{\mathrm{g},i}(z,\tilde{\nu})) - (\hat{p}^*_{\mathrm{b},i}(\hat{z},\tilde{\nu}), \hat{p}^*_{\mathrm{g},i}(\hat{z},\tilde{\nu}))\| \leq \big(\big\|\mathbb{I}_T - \frac{1}{2\sigma}\operatorname{diag}(\tilde{\mu}^*)\big\| + \max_{i\in\mathcal{N}}\big(\frac{1}{a_i}\big)\|\tilde{\mu}^*\| + \frac{1}{2\sigma}M\big)\|(z,\tilde{\nu}) - (\hat{z},\tilde{\nu})\|$. Consequently, the solution to the problem (11) without constraints is Lipschitz w.r.t. z. Now, let us define the projection of the solution without constraints on the convex and compact set Ψ_i as $\operatorname{Proj}^{S_i}_{\Psi_i}\big((\hat{p}^*_{\mathrm{b},i}(z,\tilde{\nu}), \hat{p}^*_{\mathrm{g},i}(z,\tilde{\nu}))\big) = \arg\min_{(p_{\mathrm{b},i},p_{\mathrm{g},i})\in\Psi_i} \big\|\big((p_{\mathrm{b},i}(z,\tilde{\nu}), p_{\mathrm{g},i}(z,\tilde{\nu}))\big) - \big((\hat{p}^*_{\mathrm{b},i}(z,\tilde{\nu}), \hat{p}^*_{\mathrm{g},i}(z,\tilde{\nu}))\big)\big\|^2_{S_i} = \big(p^*_{\mathrm{b},i}(z,\tilde{\nu}), p^*_{\mathrm{g},i}(z,\tilde{\nu})\big)$. Now, using the non-expansive property of the projection [13] and $\lambda_{\min}(S_i)\|\cdot\|^2 \leq \|\cdot\|^2_{S_i} \leq \lambda_{\max}(S_i)\|\cdot\|^2$, we obtain $\|(p^*_{\mathrm{b},i}(z,\tilde{\nu}), p^*_{\mathrm{g},i}(z,\tilde{\nu})) - (p^*_{\mathrm{b},i}(\hat{z},\tilde{\nu}), p^*_{\mathrm{g},i}(\hat{z},\tilde{\nu}))\| \leq \sqrt{\lambda_{\min}(S_i)/\lambda_{\max}(S_i)}\|(\hat{p}^*_{\mathrm{b},i}(z,\tilde{\nu}), \hat{p}^*_{\mathrm{g},i}(z,\tilde{\nu})) - (\hat{p}^*_{\mathrm{b},i}(\hat{z},\tilde{\nu}), \hat{p}^*_{\mathrm{g},i}(\hat{z},\tilde{\nu}))\| \leq \tilde{\gamma}\|(z,\tilde{\nu}) - (\hat{z},\tilde{\nu})\|$. By the same procedure and according to (15) and (28), for the large populations of followers and the stochastic case, (17) holds. Thus, the solution to the problem (11) for the stochastic case is Lipschitz w.r.t. z and z^* is Lipschitz w.r.t. $\tilde{\nu}$. Now, with analogous analysis in the proof of Theorem 1, if $\tilde{\gamma} \in (0,1)$ and Assumption 1 hold, the convergence of Algorithm 1 to the solution of $\mathcal{G}$ is guaranteed for any initial condition z_1 and the stochastic case. Moreover, with similar analysis in the proof of Theorem 2, if $\tilde{\gamma} \in (0,1)$ and Assumption 1 hold, for the finite population of consumers and the stochastic case, the convergence of the strategies of the leader and followers via Algorithm 1 to the mean-field leader-follower ε_N-Nash equilibrium of $\mathcal{G}$ is ensured. ■

Following Theorem 3, ε_N approaches to zero and Algorithm 1 converges to the leader-follower mean-field Nash equilibrium when the population size tends to infinity.

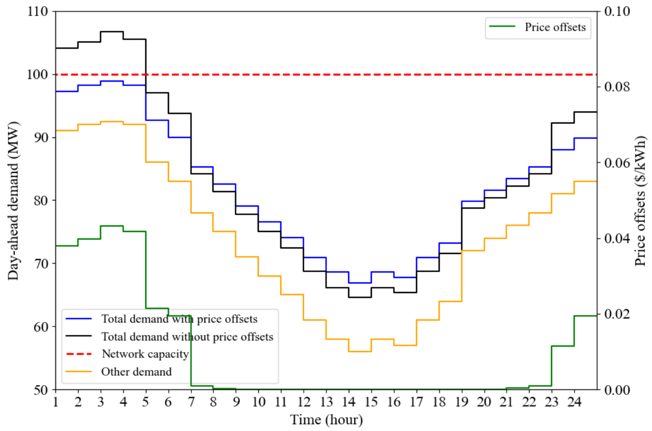

Fig. 1. Total expected demand on the day-ahead market with and without considering the price offsets; and the price offsets.

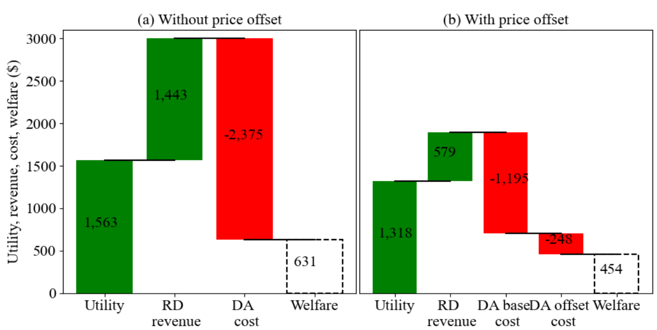

Fig. 2. Total consumers' expected utility, redispatch revenue, day-ahead cost and welfare at time $t = 2$: (a) the price offset is not considered and (b) the price offset is considered.

IV. Simulation Results

This section demonstrates the effectiveness of the suggested approach through computational simulations. The utility is randomly selected from the uniform distributions over the interval $[0.01, 0.3]$ \$/kWh. We consider $c = 100$ MW, $N = 2000$, $\rho = 1.5$, $\chi_i^{\min} = 0.15$, $\chi_i^{\max} = 0.95$ and $\alpha_l = (0.9/l)$. Normal distributions with (mean, sd) were used for the battery capacity β (20 kWh, 2 kWh), initial value of SoC (0.5, 0.2), $p_{\mathrm{g},i}^{\max}$ (9 kW, 1 kW), $p_{\mathrm{b},i}^{\min}$ (-8 kW, 1 kW) and $p_{\mathrm{b},i}^{\max}$ (8 kW, 1 kW). For the anticipated other demand, we utilize the load profile given in [8, Fig. 1] and the demand uncertainties are selected randomly from the normal distribution with the standard deviation 10 MW. The price function parameters are the same as those employed in [9].

We will now apply Algorithm 1 to this simulation scenario. Fig. 1 demonstrates that the congestion occurs at times $t = 1, 2, 3, 4$ when the price offsets are not considered (*i.e.*, we force $\tilde{\nu}^t = \underline{\nu}^t$). More precisely, consumers submit high consumption bids on the day-ahead market at times when they anticipate congestion, aiming to profit from the redispatch market. Fig. 1 indicates that the increase-decrease game is mitigated by applying Algorithm 1. Specifically, we can observe from Fig. 2 that flexible consumers earn less redispatch revenue from the DSO at time $t = 2$ when the price offset is considered.

Across the 24-hour period, the DSO redispatch cost is reduced from $\$7,975$ to $\$3,487$. Although the welfare of flexible consumers is reduced at $t = 2$ when the price offset is imposed (Fig. 2), across the 24-hour period, their welfare improves from $\$10,914$ to $\$11,017$. Thus, exploiting Algorithm 1, the leader is able to balance the welfare of consumers and the costs made by the DSO.

V. Conclusions and Future Work

In this paper, we have proposed a mean-field Stackelberg game-based algorithm to mitigate the increase-decrease game for large populations of energy consumers. We have shown the convergence of this algorithm to the mean-field leader-follower ε_N-Nash equilibrium. Future research will focus on analyzing the increase-decrease game through reinforcement learning techniques.

References

[1] X. Jin, Q. Wu, and H. Jia, "Local flexibility markets: Literature review on concepts, models and clearing methods," *Appl. Energy*, vol. 24, 2020.

[2] R. Hennig, S. H. Tindemans, and L. De Vries, "Market Failures in Local Flexibility Market Proposals for Distribution Network Congestion Management," in *Proc. 18th Conf. Eur. Energy Market (EEM)*, Ljubljana, Slovenia, 2022.

[3] L. Hirth and I. Schlecht, "Market-Based Redispatch in Zonal Electricity Markets: Inc-Dec Gaming as a Consequence of Inconsistent Power Market Design (not Market Power)," ZBW - Leibniz Information Centre for Economics, Kiel, Hamburg, 2019.

[4] M. Sarfati, M. R. Hesamzadeh, and P. Holmberg, "Increase-Decrease Game under Imperfect Competition in Two-stage Zonal Power Markets - Part I: Concept Analysis," JSTOR, 2018.

[5] J. Sun, N. Gu, and C. Wu, "Strategic Bidding in Extended Locational Marginal Price Scheme," *IEEE Control Syst. Lett.*, vol. 5, no. 1, 2021.

[6] A. Silani and S. H. Tindemans, "Mean Field Game for Strategic Bidding of Energy Consumers in Congested Distribution Networks," in *Proc. 62rd IEEE Conf. Decis. Control (CDC)*, Singapore, 2023.

[7] A. Silani and S. H. Tindemans, "Stochastic Mean Field Game for Strategic Bidding of Consumers in Congested Distribution Networks," 2024, *arXiv:2403.11836*. [Online]. Available: https://arxiv.org/abs/2403.11836

[8] Z. Ma, D. S. Callaway, and I. A. Hiskens, "Decentralized Charging Control of Large Populations of Plug-in Electric Vehicles," *IEEE Trans. Control Syst. Technol.*, vol. 21, no. 1, pp. 67-78, 2013.

[9] M. A. Tajeddini, H. Kebriaei, and L. Glielmo, "Decentralized Hierarchical Planning of PEVs Based on Mean-Field Reverse Stackelberg Game," *IEEE Trans. Autom. Sci. Eng.*, vol. 17, no. 4, pp. 2014-2024, 2020.

[10] S. Grammatico, F. Parise, M. Colombino, and J. Lygeros, "Decentralized Convergence to Nash Equilibria in Constrained Deterministic Mean Field Control," *IEEE Trans. Autom. Control*, vol. 61, no. 11, pp. 3315-3329, 2016.

[11] Y. Liu and F. F. Wu, "Prisoner dilemma: Generator strategic bidding in electricity markets," *IEEE Trans. Autom. Control*, vol. 52, no. 6, pp. 1143-1149, 2007.

[12] K. Dehghanpour, M. H. Nehrir, J. W. Sheppard, and N. C. Kelly, "Agent-based modeling of retail electrical energy markets with demand response," *IEEE Trans. Smart Grid*, vol. 9, no. 4, pp. 3465-3475, 2018.

[13] H. H. Bauschke and P. L. Combettes, *Convex Analysis and Monotone Operator Theory in Hilbert Spaces*. vol. 408, New York, NY, USA: Springer, 2011.

[14] K. Border, *Fixed Point Theorems with Applications to Economics and Game Theory*. Cambridge, U.K.: Cambridge University Press, 1985.

[15] D. R. Smart, *Fixed point theorems.* Cambridge University Press Archive, 1974.

[16] V. Berinde, *Iterative approximation of fixed points.* Springer, 2007.