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Combining Reset Control with Active Damping Control to improve Nanopositioning System Performance

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Abstract

Piezoelectric nan positioning systems are widely used in applications demanding high-speed and high-precision motion. Their closed-loop performance is often constrained by lightly damped modes and strong inherent nonlinearity which limit achievable bandwidth and tracking accuracy. Conventional approaches such as notch filters can mitigate resonances but struggle to provide robust damping without sacrificing bandwidth.

This thesis investigates the combination of active damping control and nonlinear reset control to overcome the limitations of linear time-invariant (LTI) feedback methods. A dual-loop control architecture is proposed, employing an active damping controller to suppress dominant modes and enable higher bandwidths, together with a nonlinear CgLP reset controller to recover the phase lag introduced by the damping control. Unlike typical applications, the high nonlinearity of piezoelectric systems makes introducing additional nonlinearity from reset control challenging, particularly when aiming for large phase lead. To address this, a dedicated shaping filter is designed to limit higher-order harmonics and prevent multiple resets, enabling effective phase compensation of up to 20° .

The control architecture is optimized in the frequency domain and validated through simulation and experiments on a piezoelectric nan positioning platform. Results demonstrate significant improvements in both open- and closed-loop bandwidth and tracking performance compared to conventional LTI controllers, while maintaining stability and robustness. The shaping filter effectively reduces harmonic distortion, ensuring the practical viability of combining reset control with active damping in highly nonlinear, lightly damped systems.

This work shows that the integration of reset control with shaping filter, within a dual-loop active damping framework, can push the performance of nan positioning systems beyond the traditional LTI control limits.

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1 Introduction

1.1 Background

Nanopositioning stages are indispensable in fields where ultra-precise positioning is non-negotiable, with motions ranging from sub-nanometers to several hundred micrometers. They form the backbone of critical applications in semiconductor manufacturing, data storage, life sciences, optics, and photonics, enabling innovations that define modern technology.

In semiconductor fabrication, for example, nanopositioning systems enable wafer inspection and overlay alignment [1], ensuring that features are placed with nanometer-level accuracy. In the data storage industry, they control the read/write head position with extreme precision [2], directly impacting storage density and reliability. In life sciences, they enable live-cell imaging and DNA sequencing [3], where the smallest deviation can compromise results. In optics and photonics [4], they facilitate sub-micron alignment of lasers and optical components, which is essential for bandwidth filtering and signal integrity.

A typical single-axis nanopositioning stage (Figure 1.1) consists of piezoelectric actuators (PZTs), which are chosen for their high stiffness, fast response, and strong actuation force. The motion is guided by flexure mechanisms that act as frictionless, backlash-free guides, ensuring smooth displacement without mechanical play. High-resolution position sensors, such as capacitive sensors, provide accurate feedback over small displacements [5, 6].

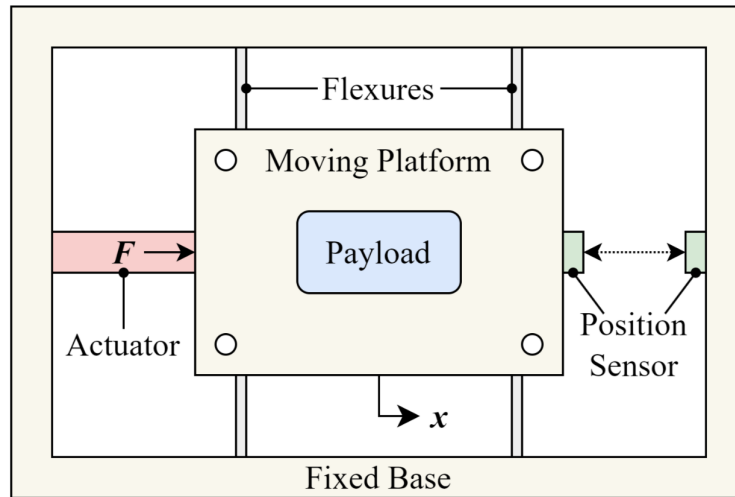


Figure 1.1: Nanopositioning system with piezo actuators. Source: [5]

The physics of PZT-based nanopositioners is far from trivial. They exhibit lightly damped resonant modes in the kHz range, significant nonlinearities including hysteresis, creep, and actuator saturation, as well as sensitivity to external disturbances and time delays in sensing and computation. These challenges make high-speed, high-accuracy tracking a non-trivial control problem.

1.2 Motivation and Problem Statement

Over the past two decades, the demand for higher throughput and tighter tolerances has driven the push towards control designs that can achieve closed-loop bandwidths close to the first resonance frequency. One widely adopted strategy is Active Damping Control (ADC), where a controller introduces additional damping to suppress resonance peaks without sacrificing low-frequency performance. This allows the closed-loop bandwidth (CL BW) to be pushed up to the resonance.

Yet, in practice, CL BW alone does not tell the full story. The open-loop bandwidth (OL BW), which directly influences the achievable tracking error, disturbance rejection, and sensitivity—remains limited by fundamental constraints of Linear Time-Invariant (LTI) control theory. The Bode gain-phase relation and the waterbed effect [7] dictate that increasing low-frequency gain for better tracking inevitably reduces phase margin or high-frequency robustness. Moreover, resonant peaks and time delays introduce additional phase lag, forcing conservative bandwidth limits to maintain stability.

As illustrated in Figure 1.2, time delay and high-order resonances cause substantial phase loss even before the first resonance. These problems significantly restrict the OL BW.

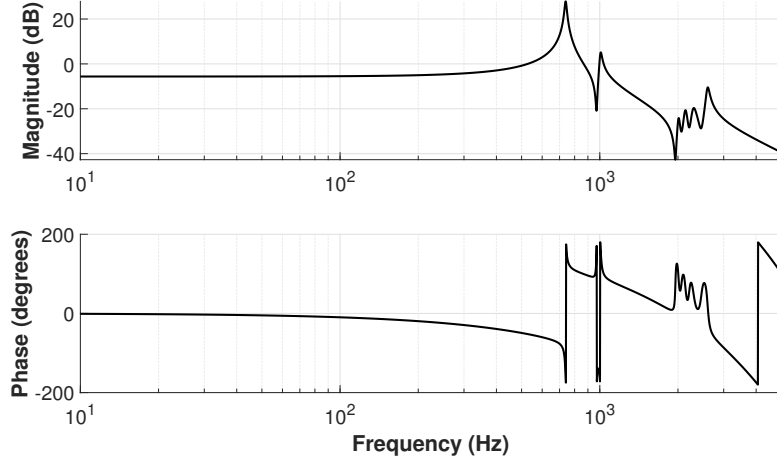


Figure 1.2: Bode plot of a plant with a time delay and higher-order resonances.

Applying active damping control can help to shape the closed loop frequency response to tackle part of these challenges and improve the reference tracking, noise and disturbance rejection performance as shown in Figure 1.3. The OL BW and CL BW can be effectively increased with fine-tuned ADC and tracking control which play a crucial role in determining the real performance. However, the fundamental trade-offs imposed by LTI control still apply.

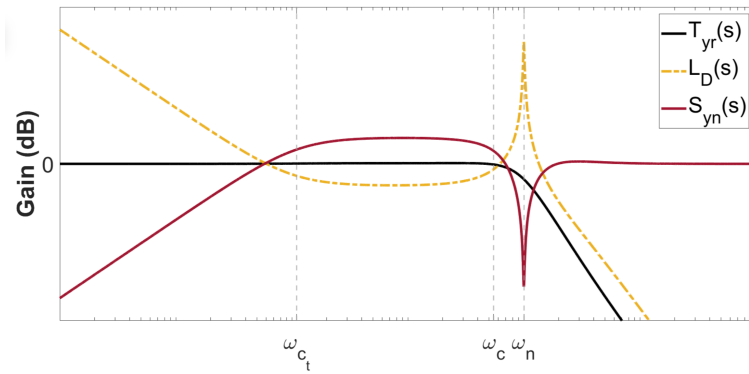


Figure 1.3: Illustration of the dual closed-loop shaping for a 2nd-order lightly damped system [5]

Reset control has emerged as a promising nonlinear alternative for overcoming these LTI performance barriers. By resetting the controller state at certain instants, reset elements can generate phase lead without the associated gain penalty of LTI system. This approach has been successfully combined with active damping control in nanopositioning systems to extend OL BW and improve tracking performance [8].

However, applying reset control to PZT stages is not straightforward. The large intrinsic nonlinearity of the plant means that aggressive reset elements—especially those producing high phase lead—can introduce undesirable nonlinear artifacts, potentially degrading stability and precision. The interaction between actuator hysteresis

and controller nonlinearity is complex, making simple implementations risky.

This thesis addresses these challenges by introducing a novel reset controller with shaping filter architecture that enables the safe application of high-phase-lead reset elements in highly nonlinear PZT systems. A systematic tuning methodology is proposed to balance bandwidth improvement, robustness, and nonlinearity suppression. Experimental validation demonstrates significant improvements in open-loop bandwidth and subsequent performance over state-of-the-art ADC-based designs.

In summary, the work pushes nanopositioning control beyond the CL BW $\approx$ first resonance paradigm, showing that with the right architecture, OL BW can be effectively increased while maintaining stability and precision, even in the face of plant nonlinearities.

1.3 Contributions of This Thesis

In this thesis, we propose a dual-loop reset control architecture that integrates a active damping controller (NRC) with a reset controller (CgLp) in dual loop architecture to enhance the performance of nanopositioning stages. The objective is to improve OL BW, CL BW and tracking accuracy beyond the limitations of linear control methods. The influence of higher-order harmonics introduced by the reset controller on the dual-loop system is systematically analyzed. High phase lead CgLp elements, providing 15° and 20° phase compensation, are employed and assessed using higher-order sinusoidal input describing function (HOSIDF)-based frequency domain analysis. To manage the nonlinear effects associated with high phase lead CgLp, a dedicated shaping filter is designed. The complete control design is validated through both simulation and experimental studies, showing strong agreement with theoretical predictions and demonstrating notable improvements in performance compared to conventional LTI controllers.

1.4 Thesis Outline

This thesis is organized as follows:

- Chapter 1: Introduction, providing the background, previous work, contributions, and outline of the thesis.
- Chapter 2: Preliminaries, covering the necessary theoretical foundations and definitions.
- Chapter 3: Control System Design and Tuning, detailing the methodologies for design and tuning of the reset control system.
- Chapter 4: Simulation and Discussion, presenting the simulation results and discussions on the performance of the reset control system.
- Chapter 5: Experimental Validation, presenting the experimental setup, implementation, and results of the reset control system.
- Chapter 6: Conclusion and Future Work, summarizing the findings and proposing directions for future research.

2 Preliminaries

2.1 Linear Tracking Control

2.1.1 Feedback Control System

Feedback control is a widely used method to control a Linear Time-Invariant (LTI) system with state-space representation:

$$\begin{cases} \dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}u(t) \\ y(t) = \mathbf{C}x(t) + \mathbf{D}u(t) \end{cases} \quad (2.1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, and $y(t) \in \mathbb{R}^p$ is the output vector. The matrices $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and $\mathbf{D}$ are constant matrices that define the system dynamics. The transfer function of the system can be derived from the state-space representation as:

$$P(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D} \quad (2.2)$$

The general feedback control system structure is shown in Figure 2.1. The system is controlled by a tracking controller $C_t(s)$ that takes the error signal $e(t) = r(t) - y(t)$ as input, where $r(t)$ is the reference signal. The controller generates the control signal $u(t)$ to drive the system output $y(t)$ to follow the reference signal $r(t)$. The feedback loop is closed by feeding back the output signal $y(t)$ to the controller.

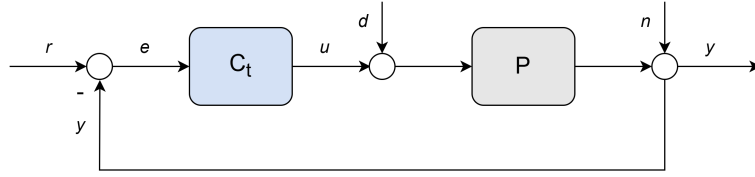


Figure 2.1: Block diagram of feedback control system

2.1.2 Frequency Analysis

Frequency Response In the design of a tracking controller, the frequency response of the system can be a crucial criteria for predicting the system's behavior in response to reference signals. The open-loop transfer function of the system can be represented as:

$$L(s) = C_t(s)P(s) \quad (2.3)$$

Based on the signals shown in Figure 2.1, 4 closed-loop transfer functions: 'gang of four' can be obtained to represent the performance of a feedback system[2]:

The **Sensitivity** can be written as follows in full Laplace notation:

$$S(s) = \frac{y}{n} = \frac{e}{r} = \frac{1}{1 + P(s)C_t(s)} \quad [-]. \quad (2.4)$$

The **Control Sensitivity** can be written as follows in full Laplace notation:

$$CS(s) = \frac{u}{r} = \frac{u}{n} = \frac{C_t(s)}{1 + P(s)C_t(s)} \quad [-]. \quad (2.5)$$

The **Process Sensitivity** equals in full Laplace notation:

$$GS(s) = \frac{x}{d} = \frac{y}{d} = \frac{P(s)}{1 + P(s)C_t(s)} \quad [\text{m/N}]. \quad (2.6)$$

Written in full Laplace notation the **Complementary Sensitivity** equals:

$$T(s) = \frac{x}{r} = \frac{y}{r} = -\frac{x}{n} = \frac{P(s)C_t(s)}{1 + P(s)C_t(s)} \quad [-]. \quad (2.7)$$

Stability Margins To ensure a motion control system remains stable and resistant to becoming unstable, specific criteria are used to analyze the feedback loop's behavior.

A system is considered marginally stable when its total phase lag reaches exactly 180 degrees at the point where the loop gain equals 1 (the unity-gain crossover frequency). In this state, even a small additional delay or phase lag can cause the system to become unstable. To maintain stability and robustness, the total phase lag in the feedback loop—which includes both the controller and the system being controlled—must stay below 180 degrees. This ensures the system has enough margin to handle small disturbances or changes without becoming unstable.

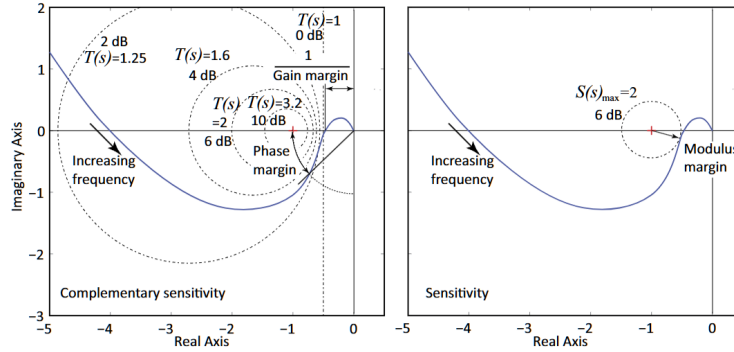


Figure 2.2: Nyquist plot of the feedback-loop response of a stable feedback controlled motion system. Source: [2]

The most appropriate method to visualize the robustness on stability of a feedback system is Nyquist plot. Three values are shown in Figure 2.2 related to the robustness of the closed-loop feedback system, the stability based gain margin and phase margin and the Sensitivity $S(s)$ based modulus margin or vector margin [2].

The **gain margin** determines by which factor the feedback loop-gain additionally can increase before the closed-loop system goes unstable. It is defined by the distance between the loop-gain $L(s)$ and unity-gain at the frequency where the phase-lag of $L(s)$ becomes more negative than -180° . The gain margin can have values between one and infinity. With first- and second-order transfer functions where the phase does never become more negative than -180° , the gain can be increased theoretically to infinite, corresponding to an infinite gain margin.

The **phase margin** determines how much additional phase-lag at the unity-gain cross-over frequency is acceptable before the closed-loop system becomes unstable. It is defined by the difference between the actual phase-lag of $L(s)$ and -180° at the unity-gain cross-over frequency.

The **modulus margin** is defined by the closest distance in a Nyquist plot between the graph and the -1 point. It determines the highest level of the Sensitivity $S(s)$ in the frequency range where the error is increased by feedback.

2.1.3 Loop Shaping for Feedback Control

Loop shaping is a design technique used in control systems to shape the frequency response of the open-loop transfer function $L(s)$ to achieve desired closed-loop performance. It involves modifying the open-loop transfer function to meet specific performance criteria, such as stability, bandwidth, and sensitivity. The goal is to design a controller that shapes the loop gain to achieve desired closed-loop characteristics while maintaining stability margins. The objectives of loop shaping are to achieve a desired system performance and stability by

adjusting the shape of the open-loop transfer function and the closed-loop transfer functions. As follows are the objectives and guidelines for loop shaping in feedback control systems:

- **Open loop Bandwidth:** Maximize the open-loop bandwidth ω_c to ensure that the system can respond quickly to the reference signal. The open loop bandwidth is defined as [5]:

$$\omega_c := \{\omega_c \in \mathbb{R} \mid \omega_c > 0 \text{ and } |L(j\omega_c)| = 1\} \quad (2.8)$$

- **Stability Margins:** The gain and phase margins should be sufficient to ensure that the system remains stable under disturbances and uncertainties. The gain margin should be greater than 6dB, the phase margin should be greater than 60° and the modulus margin should be less than 6dB.
- **Sensitivity Shaping:** Minimize the sensitivity gain $|S(j\omega)|$ at low frequencies to improve the tracking performance of the system.
- **Complementary Sensitivity Shaping:** Ensure the complementary sensitivity gain $|T(j\omega)|$ is flat and as close to 1 as possible and push the flat region of the complementary sensitivity function $T(j\omega)$ to high frequencies which indicates that the system can track the reference signal accurately.

2.1.4 Limitations of Linear Control

The linear control methods, such as PID control have been widely used in motion control systems due to their simplicity and effectiveness. However, they have limitations. Fundamental limitations of linear control methods, such as Bode's gain-phase tradeoff and the waterbed effect, further restrict the performance potential of traditional controllers. The Bode's gain-phase relation describes the limitation of LTI system from the open loop view. According to Bode's gain-phase relationship, a slope of $-20n$ dB/decade ($n \in \mathbb{R}$) in the magnitude plot corresponds to a phase lag of $n \times 90^\circ$ [7]. An increase in the low-frequency gain or attenuation at high frequencies for a fixed bandwidth can lead to an increased phase lag. This negatively impacts the system's robustness (phase margin) and may even render the system unstable.

The waterbed effect is another limitation of LTI controllers. It describes the trade-off between the low frequency gain and high frequency gain of sensitivity functions. This effect could be mathematically explained by Bode's sensitivity integral [7] with the following equation:

$$\int_0^\infty \ln S(j\omega) d\omega = 0 \quad (2.9)$$

where $S(j\omega)$ is the sensitivity function of a PID-feedback controlled mass-spring system. Figure 2.3 visualizes the sensitivity integral by dividing the sensitivity function into two parts: the low-frequency gain and high-frequency gain. The property of the integral implies that when the gain at low frequencies is decreased, the gain at high frequencies will increase, and vice versa. This trade-off limits the achievable performance of the system and also highlights the necessity of introducing nonlinear elements to the control system to overcome these limitations.

2.2 Active Damping Control

Piezo ceramic nanopositioning systems often have several low-frequency resonances with weak damping. These resonances limit how quickly the system can track signals and may cause it to fail if not managed properly. In fact, piezo actuators in open-loop mode can usually only follow signals accurately up to about 1% of their first resonance frequency[9]. Therefore, in applications that require fast scanning, these resonances must be controlled or reduced. This chapter discusses the loop shaping technology for damping control and reviews common methods used to actively damp these resonances in nanopositioning systems.

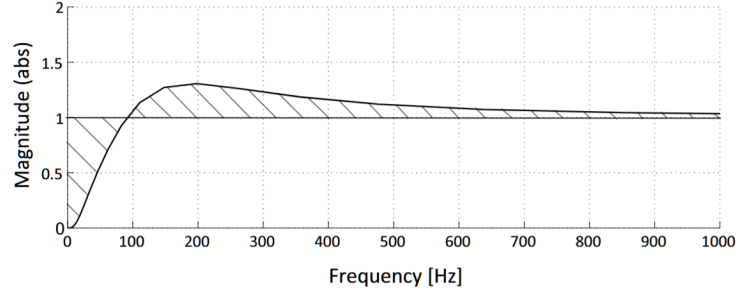


Figure 2.3: The Sensitivity $S(f)$ of the PID-feedback controlled mass-spring system plotted on a linear scale. Source: [2]

2.2.1 Conventional Control to Suppress Resonances

Notch Filter The most common control in nanopositioning systems to suppress the resonance the notching filter, also known as notch, which can be used to suppress the the peaks in the frequency response due to resonating eigenmodes. It is a typical method for the frequency range above the unity-gain cross-over frequency[2]. The notch filter can be regarded as a band-stop filter that attenuates the gain in a narrow frequency band. One can design the notch filter simply by taking the inverse of the resonance to achieve pole-zero cancellation at the resonance frequency. The transfer function of a notch filter based on pole-zero cancellation is given by:

$$C_{\text{notch}}(s) = \frac{\left(\frac{s}{\omega_n}\right)^2 + \frac{s}{Q_1\omega_n} + 1}{\left(\frac{s}{\omega_n}\right)^2 + \frac{s}{Q_2\omega_n} + 1} \quad (2.10)$$

where ω_n is the resonance frequency that needs to be suppressed, and Q_1 and Q_2 are the 2 quality factors of the notch filter. The ratio $(Q_1/Q_2 > 1)$ shows the depth of the notch. The width of the notch can be changed by adjusting the quality factors separately. It sees in Figure 2.4 that the notch filter has a distinct dip at ω_n .

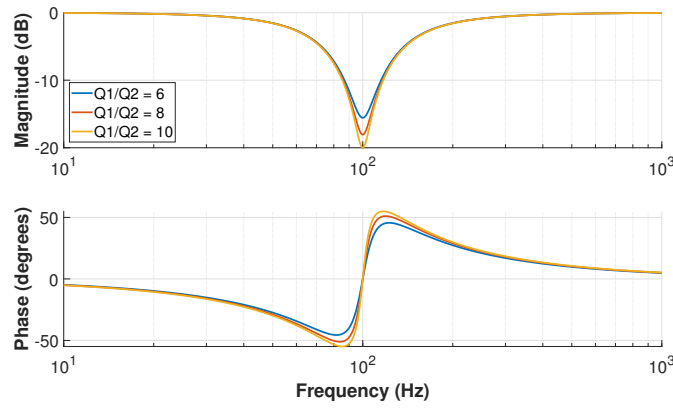


Figure 2.4: Notch filters with different Q_1/Q_2 ratios.

Limitations of Notch Filter As shown in Figure 2.4, there is a significant phase shift around ω_n which can be a problem when the bandwidth of the system is close to the resonance frequency. Another limitation of the notch filter is that it can only cancel the resonance peak at exactly the resonance frequency. In practical systems, the resonance frequency may vary due to temperature changes, aging, or other factors. Therefore, the notch filter may not be effective in suppressing the resonance peak when the resonance frequency changes. This is illustrated in Figure 2.5 that when a 5% of frequency uncertainty is added to the plant ,the notch filter is no longer effective in damping the resonance peak.

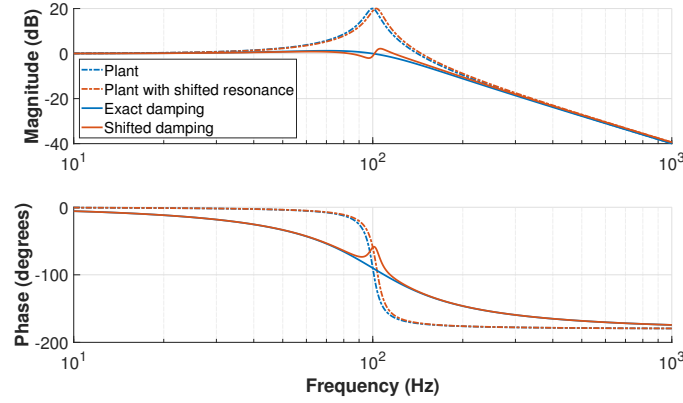


Figure 2.5: A system with resonance uncertainty damped by a notch filter

2.2.2 Active Damping Control Using Dual Loop

Positive Position Feedback Positive Position Feedback (PPF) is a widely used method in nanopositioning systems for it is robust in dealing with a plant with multiple higher order modes and resonance uncertainty. It is first introduced by Goh and Caughey [10]. The structure is shown in Figure 2.6, where the PPF controller measured the displacement signal and sent it back through a second order low-pass filter positively. It is implemented in the inner loop L_i with the following transfer functions:

$$C_d(s) = \frac{\Gamma \omega_d^2}{s^2 + 2\zeta_d \omega_d s + \omega_d^2} \quad (2.11)$$

$$L_i(s) = \frac{P(s)}{1 - C_d(s)P(s)} \quad (2.12)$$

where Γ is the feedback gain, ω_d is the cut-off frequency, and ζ_d is the damping ratio of PPF. The PPF controller is normally designed as a second order low-pass filter with a relatively high damping ratio (0.3 ~ 0.7). Figure 2.7 shows the frequency response of the inner loop of a weakly damped plant $\zeta_n = 0.05$ dampened by PPF ($\omega_d = \omega_n$, $\zeta_d = 0.6$, $\Gamma = 0.6$).

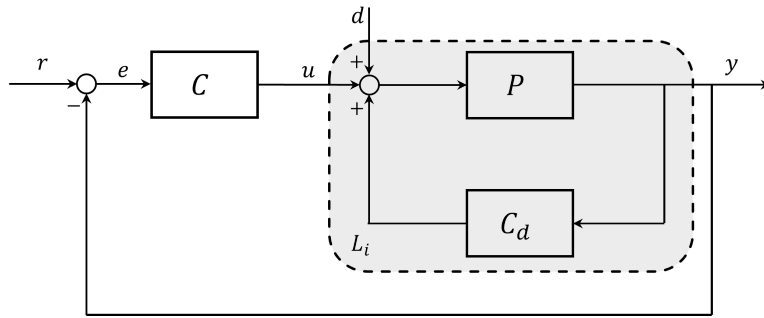


Figure 2.6: Dual-loop control system with PPF

Figure 2.7 illustrates that the high damping ratio of PPF significantly reduces the resonance peak in the open-loop system. This effect primarily depends on the plant's gain, as the plant itself contributes to damping the resonance. Consequently, the dual-loop architecture demonstrates robust performance against uncertainties in the resonance characteristics of the plant. For instance, as shown in Figure 2.8, with a 5% frequency uncertainty introduced in the plant, PPF effectively suppresses the resonance peak, maintaining system stability and performance.

There are 3 tunable parameters in PPF: ω_d , Γ and ζ_d . Normally, the cut-off frequency ω_d is set to the resonance frequency ω_n to maximize the damping effect and reduce the tuning complexity. The feedback gain Γ is set

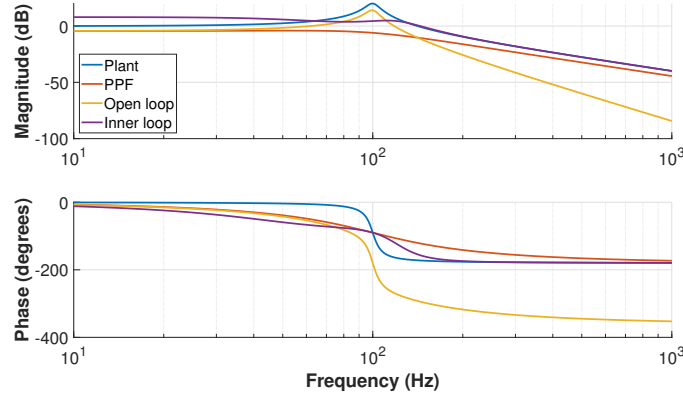


Figure 2.7: Inner loop response of a system with PPF

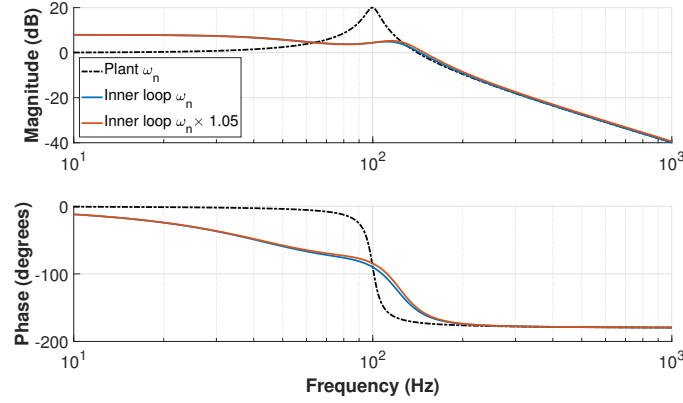


Figure 2.8: A system with resonance uncertainty damped by PPF

between $0 < \Gamma < 1$ to ensure the stability[11]. The effect of the feedback gain is shown in Figure 2.9a. It can be seen that higher gains can suppress the resonance peak more effectively, but as a trade off, higher gain at low frequencies and more phase lag before the resonance are observed. A new resonance peak at slightly higher frequency is also gradually seen as increasing the gain. Figure 2.9b shows the effect of the damping ratio ζ_d from 0.3 to 0.7. It can be seen that smaller damping ratio can suppress the peak more at the resonance frequency. However, the resonance pole is split into two poles around the resonance frequency when the damping ratio is too small. It sees higher gain around the resonance frequency that may influence the robustness against the resonance uncertainty.

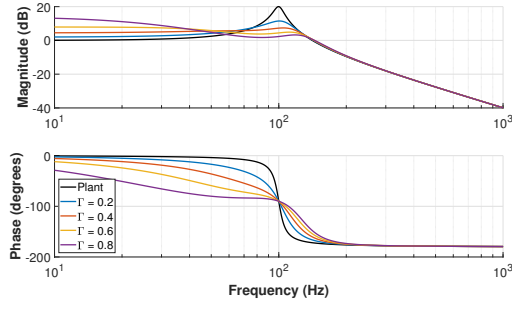
Integral Resonant Control Another widely used damping control method in industry is Integral Resonant Control (IRC). This method has also been widely used in the industry as it is able to effectively dampen multiple resonance modes while keeping high stability margins[12].

Constant Feed-Through:

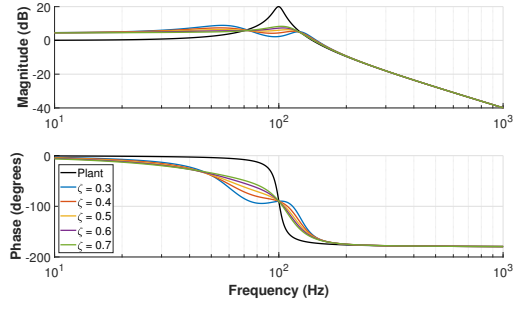
The system transfer function can be represented as the sum of the second order modes with the following form:

$$G(s) = \sum_{i=1}^M \frac{k_i}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} \quad (2.13)$$

where M is the number of resonances, k_i is the gain. In practice, limited bandwidth can be achieved, $N(N < M)$ modes would fit in the bandwidth. The out-of-bandwidth modes can not be easily truncated as the in-bandwidth zeros of the system are highly dependent on the out-of-bandwidth poles. To address this issue, Clark [13] proofed



(a) Different feedback gains with $\zeta = 0.6$



(b) Different damping ratios with $\Gamma = 0.4$

Figure 2.9: A system damped by PPF with different parameters

that a constant feed-through is sufficient to model the effect of high-frequency modes on low-frequency zeros. The truncated model can be represented as:

$$\tilde{G}(s) = \sum_{i=1}^N \frac{k_i}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} + D_f \quad (2.14)$$

where $D_f \in \mathbb{R}$ is the constant feed-through. A typical pole-zero plot of the collocated transfer function before and after addition of the feed-through term D is shown in Figure 2.10. Note that the pole locations remain the same even after adding the feed-through term.

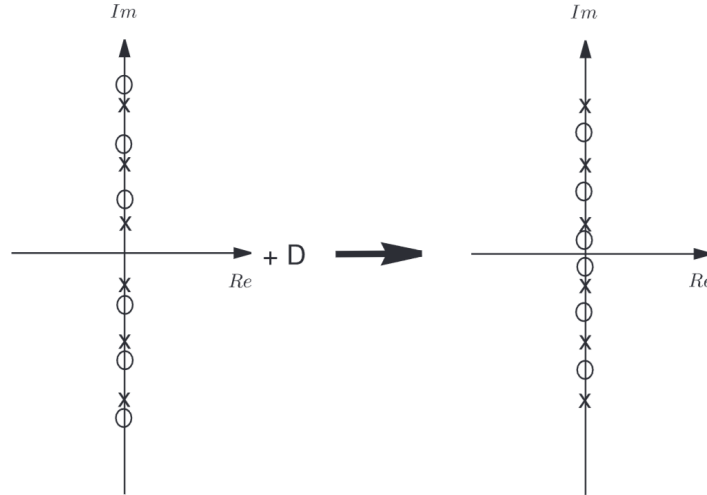


Figure 2.10: Poles (x) and zeros (o) of the collocated transfer function, before and after the addition of the feed-through term (D_f). Source: Clark [13]

IRC Control:

The Integral Resonant Controller (IRC) proposed by Aphale et al. [14] introduced a feed-through constant D_f and a feedback integral control C_d to the system as shown in Figure 2.11. C_d is designed as a simple integrator with a negative gain represented as:

$$C_d(s) = \frac{-k_i}{s} \quad (2.15)$$

where k_i can be determined by root locus analysis. The frequency response of the system damped by IRC is shown in Figure 2.12. It sees an additional zero pair added to the system before the resonance due the

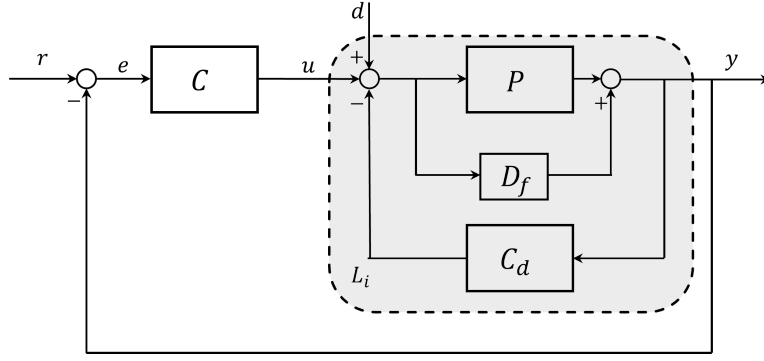


Figure 2.11: Dual-loop control system with IRC

constant feed-through. The closed loop response in Figure 2.12b shows that the resonance peak is significantly reduced by the IRC controller. It also indicates the the main drawback of this application: the unnecessarily high sensitivity at low frequencies, which may lead to actuator saturation. As for this a lossy integrator and band-pass filter are introduced as controllers in [14]. Both reduce the low frequency gain, but this also leads to a slightly reduced phase margin. The IRC control has been implemented in multiple applications including both piezoelectric nanopositioning stages and flexible beams [12].

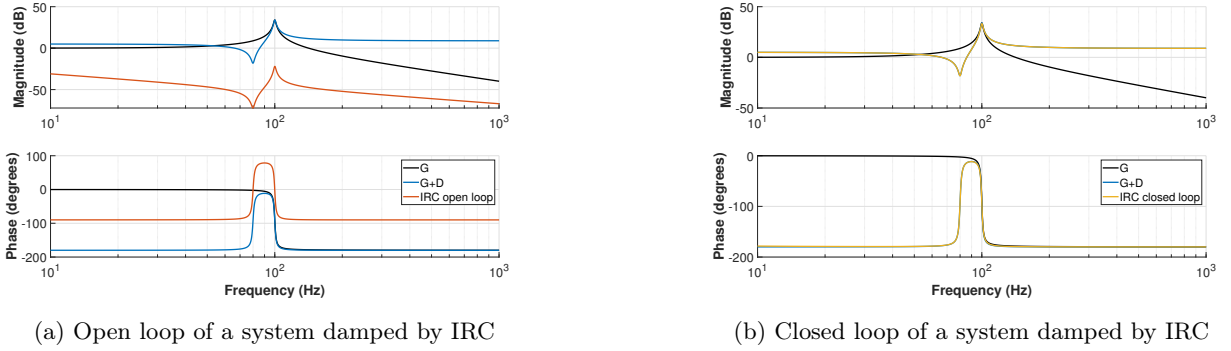


Figure 2.12: A system damped by IRC

Non-Minimum-Phase Resonant Controller A novel active damping controller, Non-minimum-phase Resonant Controller (NRC), is proposed by Natu and HosseinNia [5], described by the following transfer function:

$$C_d(s) = k \cdot \left(\frac{s - \omega_a}{s + \omega_a} \right) \quad (2.16)$$

where ω_a signifies the tuned corner frequency of the controller, and k represents the controller's gain. A zero at $s = \omega_a$ is placed at right half plane (RHP) resulting in a non-minimum-phase characteristic. The NRC controller is implemented in the inner loop with a negative feedback as shown in Figure 2.13.

The frequency response is depicted in Figure 2.14a. It sees constant gain but the phase drops 180° at the corner frequency ω_a due to the sign change and provides -90° phase at the corner frequency. Figure 2.14b shows the frequency response of the inner loop of a system damped by NRC ($\omega_a = 3\omega_n$, $k = |G(s)|_{s=0}^{-1}$). It can be seen that the resonance peak is significantly reduced by the NRC controller.

The NRC controller has two tunable parameters which are defined as:

$$\omega_a = n \cdot \omega_n \quad (2.17)$$

$$k = \gamma |G(s)|_{s=0}^{-1} = \gamma g^{-1} \quad \forall \gamma \in (0, 1] \quad (2.18)$$

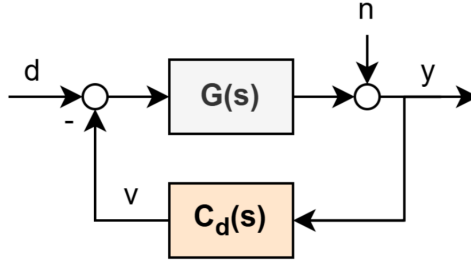
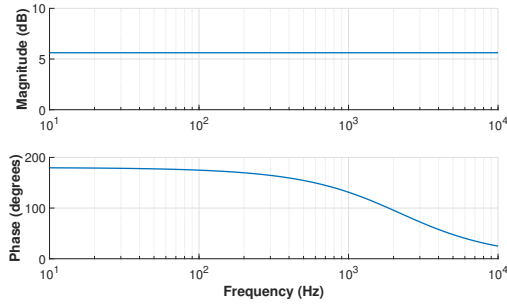
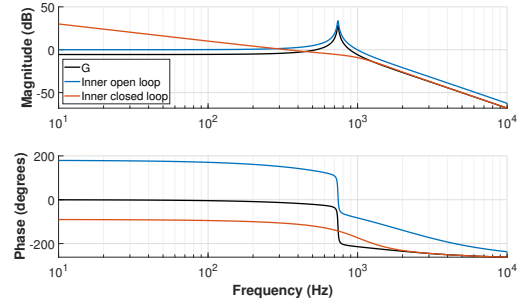


Figure 2.13: Dual-loop control system with NRC



(a) Frequency response of NRC controller



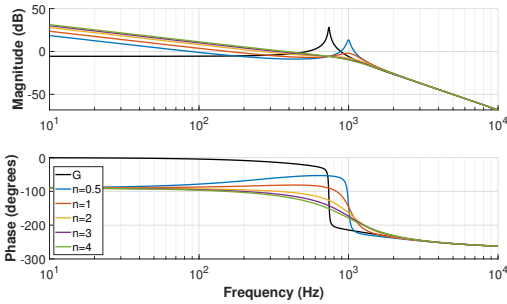
(b) Frequency response of the NRC inner loop with $\omega_a = 3\omega_n$, $k = |G(s)|_{s=0}^{-1}$

Figure 2.14: A system damped by NRC

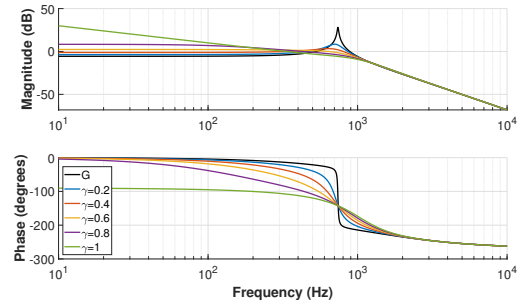
where n represents the normalized corner frequency with respect to the natural frequency of the plant. It is notable that when $\gamma = 1$, a pole of the inner closed loop will be placed at the origin, similar to a pure integrator, making the closed-loop marginally stable, but a tracking controller in the outer loop can be tuned to guarantee sufficient stability margins.

Figure 2.15 shows the frequency response of the inner loop of a system damped by NRC with different parameters. For $\gamma = 1$, it can be seen that larger corner frequency ω_a can suppress the resonance peak more effectively. Higher gain at low frequencies and more phase lag before the resonance are observed.

Figure 2.15b shows the effect of the feedback gain γ from 0.2 to 1. Higher γ results in more efficient resonance peak suppressing and as a trade off more phase lag occurs. And it implies that for $\gamma \in (0, 1)$ the gain ends up with a constant value at low frequencies, but for $\gamma = 1$ the gain has a slope of -20dB/dec at low frequencies which is the effect of placing the pole at the origin.



(a) Frequency response of NRC dual loop with different corner frequencies ($\gamma = 1$)



(b) Frequency response of NRC dual loop with different feedback gains ($\omega_a = 3\omega_n$)

Figure 2.15: A system damped by NRC with different parameters

2.2.3 Loop Shaping for Active Damping Control

The loop shaping method for damping control is to attenuate the magnitude peak at the resonance frequency by shaping the open-loop transfer function, as the high gain at resonance frequency will lead to limited bandwidth and performance. The resonance can also be seen in the pole-zero map as shown in Figure 2.16. The pole of a weakly damped system is closer to imaginary axis, where the angle ϕ between the pole and the imaginary axis denotes the damping ratio of the resonance $\zeta = \sin \phi$. The object of loop shaping is to move the pole to the left in the complex plane to increase the damping ratio. There are several commonly used tools to achieve this, such as pole-zero map, pole-placement and root locus [15].

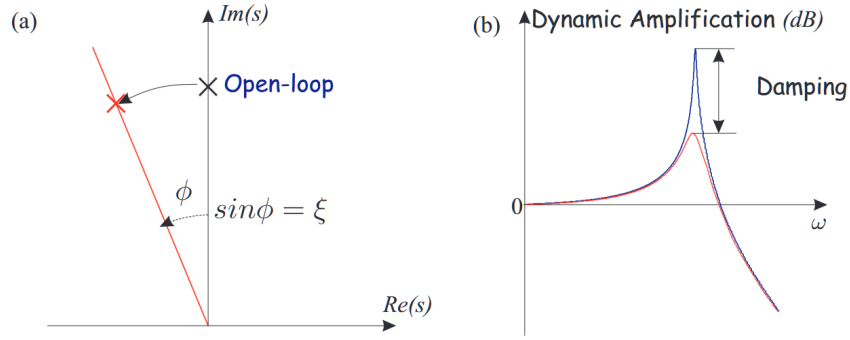


Figure 2.16: Role of damping (a) System poles. (b) Dynamic amplification. Source: Preumont [15]

In dual loop control with the inner loop as shown in Figure 2.17, the root locus method is effectively applied to tune the negative feedback gain of the inner loop. The root locus of a dynamic system contains the closed-loop pole trajectories as a function of the feedback gain k (assuming negative feedback). Root locus are used to study the effects of varying feedback gains on closed-loop pole locations. In turn, these locations provide indirect information on the time and frequency responses.

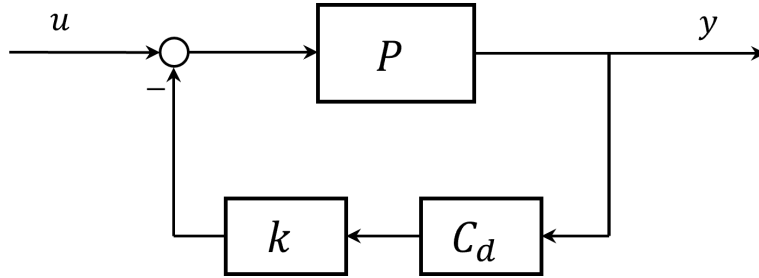


Figure 2.17: Inner loop of a dual loop system with negative feedback

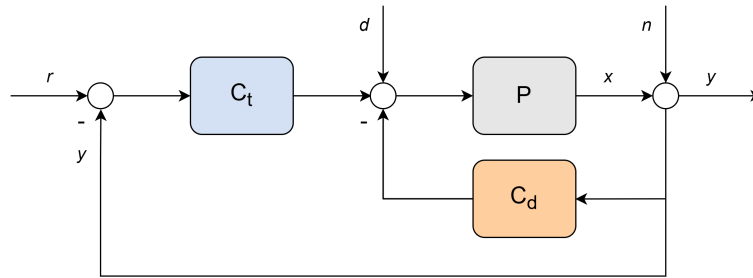


Figure 2.18: Dual-loop control system with negative feedback

In a dual loop control system with negative feedback as shown in Figure 2.18, the closed-loop transfer function

is different from the single loop system as discussed in Section 2.1.3. The closed inner loop transfer function can be expressed as:

$$L_i(s) = \frac{P(s)}{1 + P(s)C_d(s)} \quad (2.19)$$

Based on the closed inner loop transfer function, the closed-loop transfer function can be expressed as[5]: The tracking performance of the system can be evaluated by the closed-loop transfer functions mapping from the inputs r to output y and error e as follows:

$$T_{yr}(s) = \frac{Y}{R} = \frac{C_t(s)P(s)}{1 + P(s)(C_t(s) + C_d(s))} \quad (2.20)$$

$$S_{er}(s) = \frac{E}{R} = \frac{1 + P(s)C_d(s)}{1 + P(s)(C_t(s) + C_d(s))} \quad (2.21)$$

The disturbance rejection and noise rejection performance can be evaluated by the closed-loop transfer functions mapping from the disturbance d and noise n to output y as follows:

$$PS_{yd}(s) = \frac{Y}{D} = \frac{P(s)}{1 + P(s)(C_t(s) + C_d(s))} \quad (2.22)$$

$$S_{yn}(s) = \frac{Y}{N} = \frac{1}{1 + P(s)(C_t(s) + C_d(s))} \quad (2.23)$$

In reality, the actual positioning error $e_r = r - x$ is of greater importance than the control error $e = r - y$, where x is the real position of the system as shown in Figure 2.18. Therefore, the closed-loop transfer functions mapping from the inputs r , d and n to the real position x are worthwhile to be discussed, which are defined as follows:

$$T_{xr}(s) = \frac{X}{R} = \frac{C_t(s)P(s)}{1 + P(s)(C_t(s) + C_d(s))} \quad (2.24)$$

$$PS_{xd}(s) = \frac{X}{D} = \frac{P(s)}{1 + P(s)(C_t(s) + C_d(s))} \quad (2.25)$$

$$S_{xn}(s) = \frac{X}{N} = \frac{-P(s)(C_t(s) + C_d(s))}{1 + P(s)(C_t(s) + C_d(s))} \quad (2.26)$$

2.3 Reset Control

2.3.1 Definition

Reset control system is defined as a system that resets its state when a certain condition is met. Since the introduction of the first reset controller by Clegg [16] in 1958, extensive research has been conducted on reset control systems. This paper adopts the definition of reset controllers provided by Banos and Barreiro [17], with subsequent studies extending the governing equations of reset controllers as follows:

$$\mathcal{R} \begin{cases} \dot{x}(t) = Ax(t) + Be(t) & e(t) \neq 0 \\ x(t^+) = A_r x(t) & e(t) = 0 \\ y(t) = Cx(t) + De(t) \end{cases} \quad (2.27)$$

Where $x(t)$ is the state, $e(t)$ is the error input and $y(t)$ is the output of the system $\mathcal{R}$. $x(t^+)$ is the state after resetting. In a continuous system, $x(t^+) = \lim_{\delta \rightarrow 0^+} x(t + \delta)$. A_r , the reset matrix, is a diagonal matrix including the reset factor(s) $\gamma \in (-1, 1)$ which indicates how aggressive the reset is: factor further from 1 leads to more significant changes in the state. Specifically, the state $x(t^+)$ is reset to $A_r x(t)$ when the error input crosses 0 as depicted in Figure 2.19. And this reset action brings non-linearity to the system. Outside the reset event, when $e(t) \neq 0$, the system behaves like a linear time-invariant (LTI) system with state-space representation (A, B, C, D) .

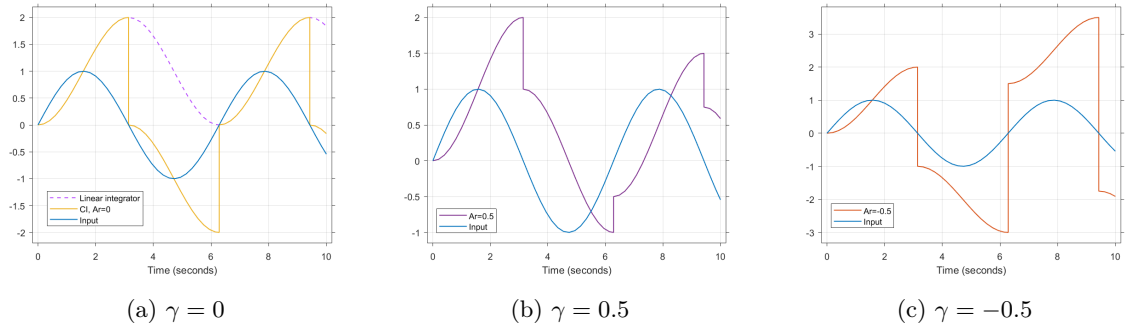


Figure 2.19: General Clegg integrator with different reset factors γ

2.3.2 Sinusoidal Input Describing Function

Loop shaping in frequency domain is a widely used method to tune a controller. However, the frequency response of a reset control system is not straightforward to obtain due to the nonlinear reset action. To overcome this challenge, Guo et al. [18] proposed a method to use a family of sinusoidal waves to approximate the frequency response of a nonlinear system. Figure 2.20 shows how the describing function approaches the Clegg integrator using a single sinusoidal signal. The sinusoidal input describing function (SIDF) of the reset control system 2.27 is defined as [18]:

$$\mathcal{N}(j\omega) = \frac{Y_{ss}(j\omega)}{E(j\omega)} \quad (2.28)$$

where $E(j\omega)$ and $Y_{ss}(j\omega)$ is the input sinusoidal wave and the steady-state response of one period in frequency domain respectively. This definition results in the following expression for the SIDF of the reset control system:

$$\mathcal{N} = C(j\omega I - A)^{-1}(I + j\Theta(\omega))B + D \quad (2.29)$$

where $\mathcal{N}$ is the describing function of the reset control system, and $\Theta(\omega)$ is defined as:

$$\begin{aligned} \Lambda(\omega) &= \omega^2 I + A^2, \quad \Delta(\omega) = I + e^{\frac{\pi}{\omega} A}, \quad \Delta_r(\omega) = I + A_r e^{\frac{\pi}{\omega} A}, \\ \Gamma_r(\omega) &= \Delta_r^{-1}(\omega) A_r \Delta(\omega) \Lambda^{-1}(\omega), \\ \Theta(\omega) &= \frac{-2\omega^2}{\pi} \Delta(\omega) [\Gamma_r(\omega) - \Lambda^{-1}(\omega)] \end{aligned}$$

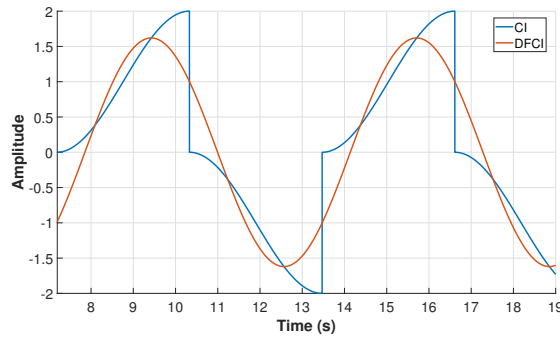


Figure 2.20: Sinusoidal input describing function (SIDF) of Clegg integrator in time domain

With SIDF, the frequency response of reset control system can be approximated. For instance, the SIDF of Clegg integrator [16] with state-space representation $(A, B, C, D, A_r) = (0, 1, 1, 0, 0)$ can be derived as:

$$CI(j\omega) = \frac{1}{j\omega} \left(1 + j\frac{4}{\pi} \right) = \frac{1.62}{\omega} \angle -38.2^\circ \quad (2.30)$$

As indicated in equation 2.30, the phase lag of Clegg integrator is 38.2° , which is less than the linear integrator (90°). It sees different shift in the gain of Clegg integrator from the linear integrator, but its slope remains the same. Besides, different reset factors γ lead to gain shift and phase lag, as shown in Figure 2.21. The gain of the system increases and the phase lag decreases as the reset factor γ decreases from 1 to -1 . This property shows that, due to nonlinearity of reset action, the reset control system is no longer following the gain-phase relation of linear systems.

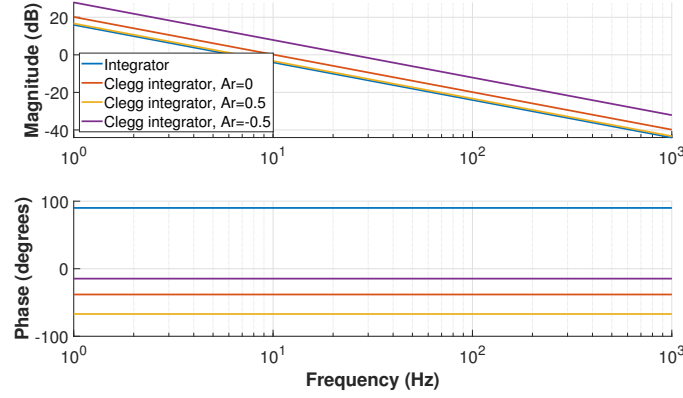


Figure 2.21: DF of Clegg integrator with different reset factors γ

2.3.3 Higher Order Sinusoidal Input Describing Function

In a nonlinear system, the signal frequency can be distorted by the nonlinear action. When a single sinusoidal wave is sent through a reset element, the output signal contains not only the fundamental frequency but also different harmonics. As illustrated in Figure 2.22, the top figure shows the FFT of the sinusoidal input signal which contains only the fundamental frequency 10 Hz. The bottom figure shows the FFT of the output signal of the Clegg integrator with reset factor $\gamma = 0$, where it can be observed that the output signal contains higher frequency components.

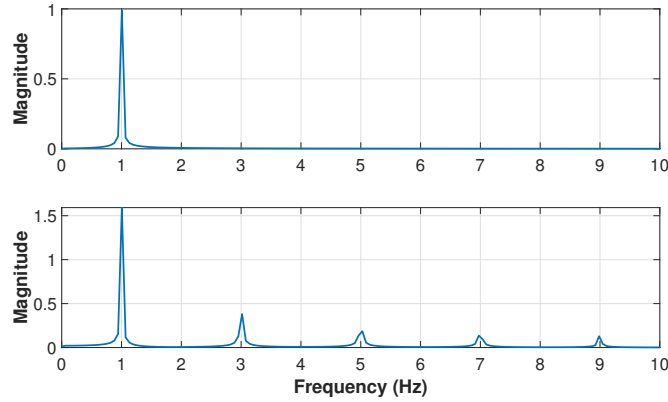


Figure 2.22: Normalized power spectrum of sinusoidal input signal and Clegg integrator output signal

Virtual Harmonic Generator As discussed in the previous section, the SIDF method is effective in approximating the frequency response of a reset control system. However, the SIDF method is limited to first-order

harmonics and assumes that the higher-order harmonics are negligible. Nuij et al. [19] extended the SIDF method to include higher-order harmonics by introducing the concept of virtual harmonics generator (VHG).

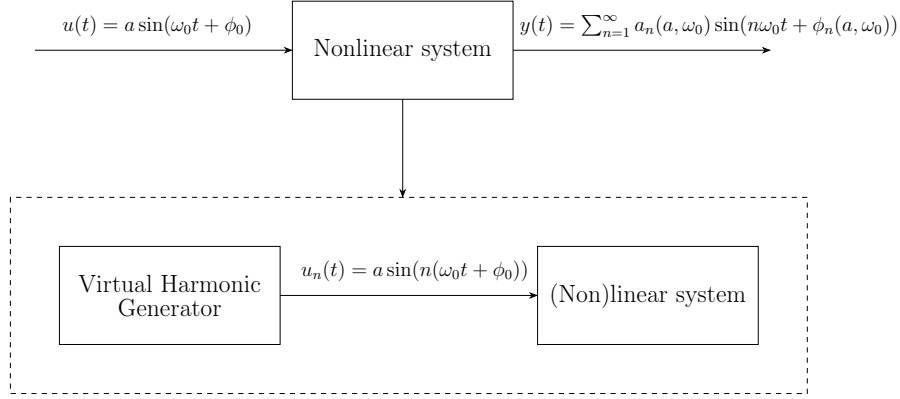


Figure 2.23: Virtual Harmonic Generator

The concept discussed in [19] is illustrated in Figure 2.23. When a nonlinear system is subjected to a sinusoidal input signal $u(t) = a \sin(\omega_0 t + \phi_0)$, the output contains multiple harmonics of the input frequency. Additionally, both the amplitude and phase of the output signal depend on the input signal's amplitude a and frequency ω_0 . To make the analysis of nonlinearity in the reset control system more straightforward, the system is divided into two parts: the virtual harmonic generator and the (non)linear system. The input signal is sent through the virtual harmonic generator first, and it only generate higher order harmonics while the amplitude and phase remains the same as the input signal. The rest of the system can be a linear system or a nonlinear system and it change the amplitude and phase of all the harmonics generated by the virtual harmonic generator and add them up to form the output signal.

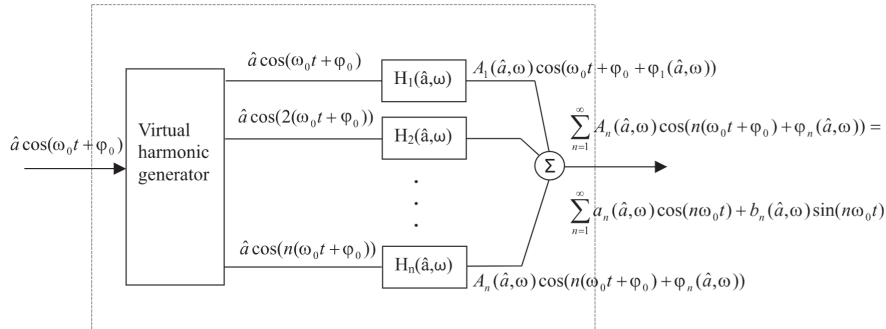


Figure 2.24: Representation of Higher-order sinusoidal-input describing function for reset controller. Source: Saikumar et al. [20]

Higher Order Sinusoidal Input Describing Function With the virtual harmonic generator, the reset control system can be significantly simplified and it is possible to calculate its frequency response for each harmonic. Saikumar et al. [20] represented the higher-order sinusoidal input describing function (HOSIDF) of the reset controller as shown in Figure 2.24. The HOSIDF model takes a single sinusoidal signal as input. The higher order harmonics generated by the virtual harmonic generator are then the input of the describing function $\mathcal{N}(jn\omega)$ provided in Eq.2.28, which is a linear approximation of the reset controller so it changes only the amplitude and phase of the higher order harmonics signal. [20] give the following expression for the HOSIDF of the reset control system $\mathcal{R}$ (Eq.2.27):

$$H_n(\omega) = \begin{cases} C(j\omega I - A)^{-1}(I + j\Theta_D(\omega))B + D & \text{for } n = 1, \\ C(jn\omega I - A)^{-1}\Theta_D(\omega)B & \text{for odd } n \geq 2, \\ 0 & \text{for even } n \geq 2 \end{cases} \quad (2.31)$$

with

$$\begin{aligned} \Lambda(\omega) &= \omega^2 I + A^2, \\ \Delta(\omega) &= I + e^{\frac{\pi}{\omega} A}, \\ \Delta_r(\omega) &= I + A_r e^{\frac{\pi}{\omega} A}, \\ \Gamma_r(\omega) &= \Delta_r^{-1}(\omega) A_r \Delta(\omega) \Lambda^{-1}(\omega), \\ \Theta_D(\omega) &= \frac{-2\omega^2}{\pi} \Delta(\omega) [\Gamma_r(\omega) - \Lambda^{-1}(\omega)]. \end{aligned}$$

The HOSIDF shows that, the output of reset control system contains only odd harmonics of the input signal. Figure 2.25 shows the HOSIDF of the proportional Clegg integrator with reset factor $\gamma = 0$, whose base linear system (BLS) is a PI controller with following transfer function:

$$\text{PI}(s) = 1 + \frac{2\pi}{s} \quad (2.32)$$

Comparing to a linear PI controller, the SIDF of PCI has less phase lag and slightly higher gain at low frequencies. The HOSIDFs of PCI ($n = 3, n = 5$) is lower in gain than the SIDF and have a constant slope of gain. Whereas the phase remain at 0° for all frequencies. The gain of the HOSIDF of PCI is not approaching unit gain at high frequencies, which indicates that the higher order harmonics are negligible at high frequencies. But at low frequencies, the higher order harmonics are not significantly smaller than SIDF and it is necessary to consider them in the frequency response analysis.

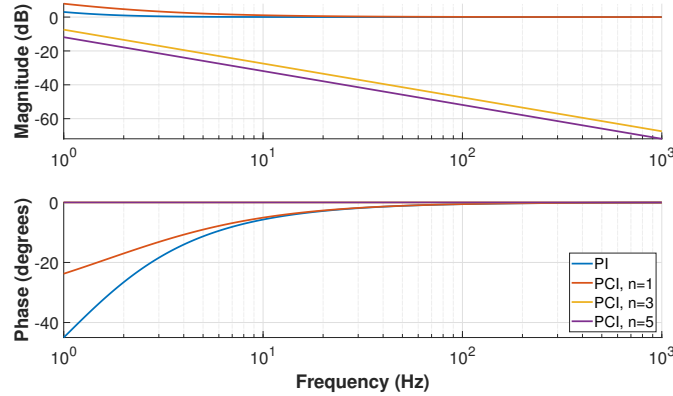


Figure 2.25: HOSIDF of PCI ($\gamma = 0$)

2.3.4 Open Loop HOSIDFs of Reset Systems

Figure 2.26 illustrates the open-loop higher-order sinusoidal input describing functions (HOSIDFs) of a reset control system. When a reset controller is used before the plant, it introduces higher-order harmonics into the system. In Figure 2.27, a weakly damped second-order plant is controlled by a PCI controller with $\gamma = 0$. This shifts the resonant peak of the HOSIDFs to the left. The figure shows that the gain of higher-order harmonics can match or even exceed the gain of the sinusoidal input describing function (SIDF), particularly near the resonant frequency. This highlights that higher-order harmonics can significantly impact the system's frequency response.

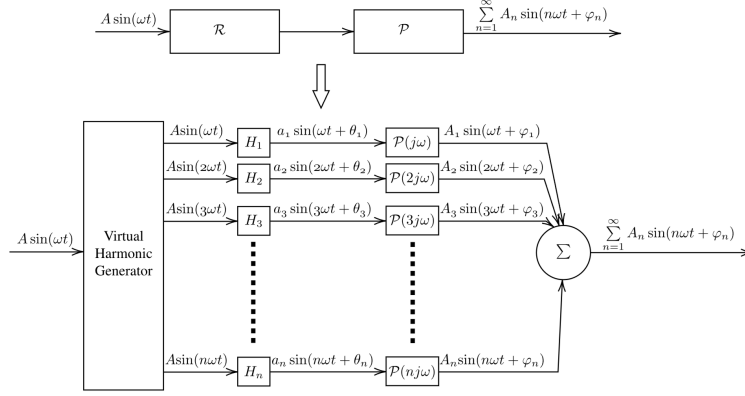


Figure 2.26: Open loop HOSIDFs of reset control system. Source: Saikumar et al. [20]

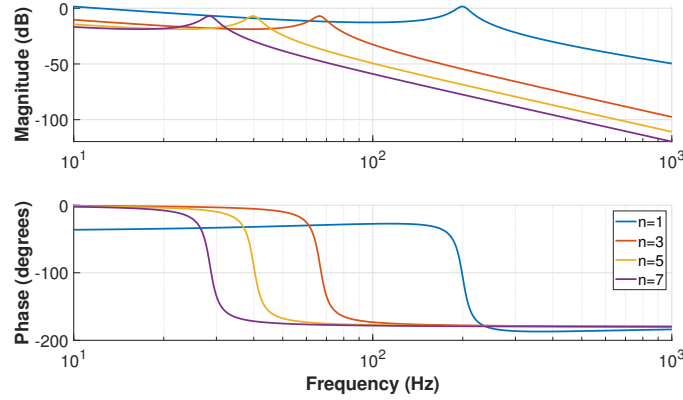


Figure 2.27: Open loop of a weakly damped system with PCI ($\gamma = 0$)

2.3.5 Stability Analysis of Reset Control System

There has been a variety of research on the stability of reset control systems. A stable base linear system (BLS) within an RCS does not inherently ensure the stability of the overall system, as the introduction of a reset action can destabilize the BLS[17]. This necessitates the development of stability criteria specifically designed for nonlinear reset systems. After establishing bounded-input bounded-output (BIBO) stability using the β -condition, a more generalized method H_β -condition was subsequently introduced in [21].

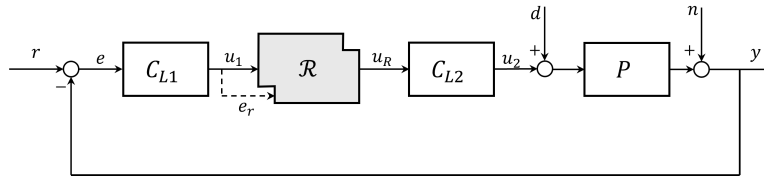


Figure 2.28: Feedback system with reset controller

Figure 2.28 shows a feedback system with a reset controller. Take the external disturbance $d(t)$ into account, the overall state-space form, including the reset element and the linear part, can be represented as:

$$\begin{cases} \dot{x}(t) = \bar{A}x(t) + \bar{B}r(t) + \bar{B}_d d(t) & e(t) \neq 0 \\ x(t^+) = \bar{A}_r x(t) & e(t) = 0 \\ y(t) = \bar{C}x(t) \end{cases} \quad (2.33)$$

Let

$$C_0 = \begin{bmatrix} \rho & \beta C \end{bmatrix}, \quad B_0 = \begin{bmatrix} 1 \\ 0_{n_p \times 1} \end{bmatrix}, \quad \rho > 0, \quad \beta \in \mathbb{R} \quad (2.34)$$

where C is the dynamics matrix of the linear part in system 2.33. The H_β -condition states that the reset control system 2.33 with $-1 < \gamma < 1$ and $r(t) = d(t) = 0$ is quadratically stable if and only if there exists $\rho > 0$ and β such that the transfer function

$$H(s) = C_0(sI - \bar{A})^{-1}B_0 \quad (2.35)$$

is Strictly Positive Real (SPR).

2.3.6 Closed Loop Analysis

In section 2.3.2 and 2.3.3, the open loop response of reset control system has been discussed which provides insight to tune a reset controller. However, in a feedback system shown in Figure 2.28, the loop shaping method also necessitates the analysis of the closed loop response due to the existence of disturbance and noise. Besides, the SIDF and HOSIDF consider only a signal sinusoidal signal as the input the reset controller and neglect that the higher order harmonics generated by the reset element can be feedback to the system. In this section, the HOSIDF based sensitivity functions and pseudo sensitivity functions are discussed to analyze the closed loop response of a reset control system.

HOSIDF based Sensitivity Functions In a linear feedback system, three sensitivity functions are frequently used to determine the performance of the system[2]: sensitivity $S(\omega)$, complementary sensitivity $T(\omega)$ (closed loop response), and control sensitivity $CS(\omega)$. The sensitivity function is defined as the ratio of the error signal $e(t)$, output signal $y(t)$ and the controller output signal $u(t)$ to the reference signal $r(t)$ in frequency domain. The 3 sensitivity functions are defined as follows:

$$S(\omega) = \frac{E}{R} = \frac{1}{1 + CP} \quad (2.36)$$

$$T(\omega) = \frac{Y}{R} = \frac{CP}{1 + CP} \quad (2.37)$$

$$CS(\omega) = \frac{U}{R} = \frac{C}{1 + CP} \quad (2.38)$$

where C is the linear controller transfer function and P is the plant transfer function. E , Y , U and R is the signal in frequency domain. However, in a reset control system, the higher order harmonics will be sent back to the reset element and so that the describing function is no longer valid since it takes only a single sinusoidal signal as input. It is straightforward to send the error signal which contains all the harmonics to the VHG for each order and calculate the sensitivity functions, but is computationally expensive and not practical because the higher order errors will result in another series of higher order harmonics in each loop. To address this issue, Saikumar et al. [20] introduced the virtual harmonic separator to divide the error signals into the fundamental frequency signal and the higher order signals. The fundamental frequency signal is then sent to the reset element and the higher order signals are sent to the base linear system (BLS) of the reset control system. This method is valid based on the following assumptions:

- **Assumption 1:** The reset control system is input-to-state convergent. [22]
- **Assumption 2:** Reset occurs twice in a period evenly. [21] & [20]
- **Assumption 3:** The first harmonic dominates the error signal. $|E_1| \gg (|E_n| \forall \text{odd } n > 1)$. [20]

Based on these assumptions, the closed loop representation of the reset control system can be simplified as shown in Figure 2.29. The virtual harmonic generator creates higher order harmonics exclusively for e_1 , which will be sent through the describing functions for each order in parallel. The virtual harmonic separator separates the higher order harmonics from the error signal and sends them to the BLS of $\mathcal{R}$, represented as $\mathcal{R}_{bl}$.

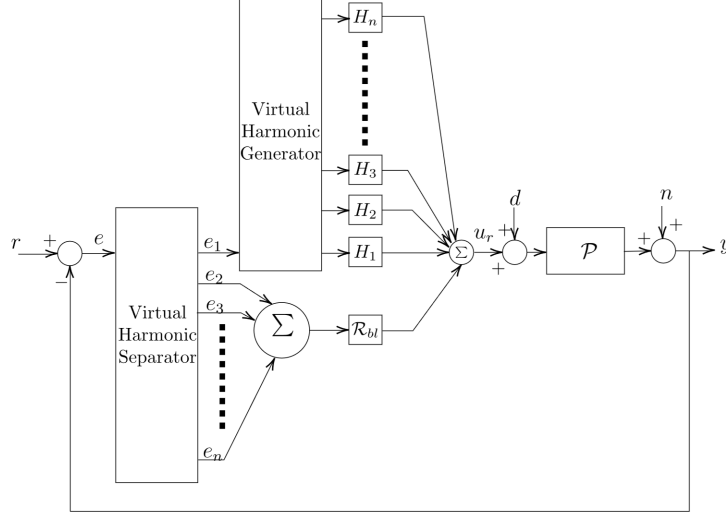


Figure 2.29: Representation of HOSIDF of reset control system in closed loop. Source: Saikumar et al. [20]

With the representation of the closed loop system, Saikumar et al. [20] derived the HOSIDF based sensitivity functions for the reset control system. For convenience, the following notations are used:

$$L_n(\omega) = H_n(\omega)P(n\omega) \quad (2.39)$$

$$L_{bl}(\omega) = \mathcal{R}_{bl}(\omega)P(\omega) \quad (2.40)$$

$$S_{bl}(\omega) = \frac{1}{1 + L_{bl}(\omega)} \quad (2.41)$$

The sensitivity DFs and HOSIDFs of the reset control system are then defined as:

$$S_1(\omega) = \frac{E_1(\omega)}{R(\omega)} = \frac{1}{1 + L_1(\omega)} \quad (2.42)$$

$$S_{n>1}(\omega) = \frac{E_n(\omega)}{R(\omega)} = -L_n(\omega)S_{bl}(n\omega) \left(|S_1(\omega)|e^{jn\angle S_1(\omega)} \right) \quad (2.43)$$

$$T_1(\omega) = \frac{Y_1(\omega)}{R(\omega)} = \frac{L_1(\omega)}{1 + L_1(\omega)} \quad (2.44)$$

$$T_{n>1}(\omega) = \frac{Y_n(\omega)}{R(\omega)} = L_n(\omega)S_{bl}(n\omega) \left(|S_1(\omega)|e^{jn\angle S_1(\omega)} \right) \quad (2.45)$$

$$CS_1(\omega) = \frac{U_1(\omega)}{R(\omega)} = H_1(\omega)S_1(\omega) \quad (2.46)$$

$$CS_{n>1}(\omega) = \frac{U_n(\omega)}{R(\omega)} = H_n(\omega) (1 - L_{bl}(n\omega)S_{bl}(n\omega)) \left(|S_1(\omega)|e^{jn\angle S_1(\omega)} \right) \quad (2.47)$$

Figure 2.30 shows the the first and higher order sensitivity functions of a weakly damped system ($\omega_n = 200\text{Hz}$) with PCI ($\gamma = 0$), whose open loop HOSIDFs are depicted in Figure 2.27. The resonance peak is seen at 200Hz

both in the first and higher order sensitivity functions. The higher order sensitivity functions also contain the open loop pole which shifted to left as illustrated in Eq.2.43, after which the gain is significantly lower than the first order sensitivity function. Whereas the gain around the shifted resonance frequencies is close to the first order sensitivity function which is not consistent with the **Assumption 3**. This indicates that at these frequencies, the first order sensitivity functions may not be trustworthy.

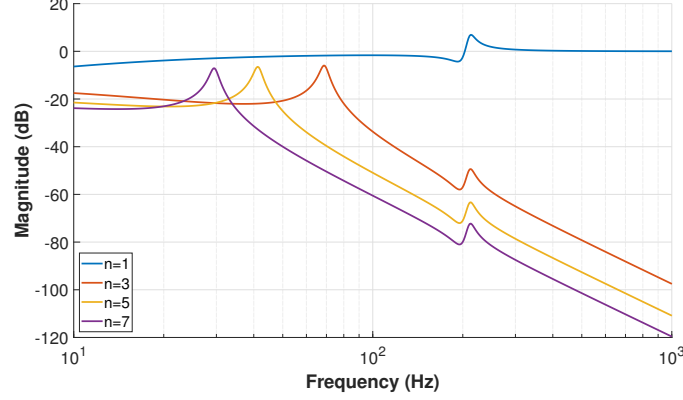


Figure 2.30: Sensitivity functions of a weakly damped system with PCI ($\gamma = 0$)

Pseudo Sensitivity Functions In the previous section, the HOSIDF based sensitivity functions are discussed to analyze the closed loop response of a reset control system. However, the higher order sensitivity functions are not always negligible at certain frequencies. Dastjerdi et al. [23] introduced the pseudo sensitivity S_∞ to provided a more accurate representation of the closed loop response of a reset control system by combining all the harmonics in one sensitivity function. It has been proven in 2.3.5 that the error of the reset control system with a sinusoidal input are periodic with same period as the input signal. The pseudo sensitivity S_∞ is defined as the ratio of the maximum error of the RCS to the amplitude of the reference signal at each frequency. The pseudo sensitivity for all ω can be expressed as:

$$S_\infty(j\omega) = e_{\max}(\omega)e^{j\phi_{\max}(\omega)} \quad (2.48)$$

where

$$e_{\max}(\omega) = \frac{\max_{t_s \leq t \leq t_s + 2q} (r(t) - y(t))}{r}, \quad \phi_{\max} = \frac{\pi}{2} - \omega t_{\max} \quad (2.49)$$

with

$$t_{\max} \in \{t_{\text{ext}} \mid \dot{e}(t_{\text{ext}}) = 0, t_s \leq t_{\text{ext}} \leq t_s + 2q\} \cup \{t_{s+i} \mid i \in \mathbb{Z}, 0 \leq i \leq 2q\} \quad (2.50)$$

$$t_{\text{ext}} \in (t_{s+i}, t_{s+i+1}], i = \{i \in \mathbb{Z}^+ \mid i < 2q\} \quad (2.51)$$

The pseudo sensitivity function provides a more accurate way to evaluate the performance of a reset control system in closed-loop operation. Figure 2.31 illustrates the pseudo sensitivity of a weakly damped system with PCI ($\gamma = 0$) and compares it to the first-order sensitivity function and higher-order sensitivity.

The pseudo sensitivity closely follows the first-order sensitivity function at frequencies beyond the open-loop pole in the higher-order sensitivity. However, at these frequencies, it identifies additional poles. This shows that the pseudo sensitivity offers a better representation of the closed-loop behavior of a reset control system and helps in fine-tuning the reset controller to reduce the impact of higher-order harmonics.

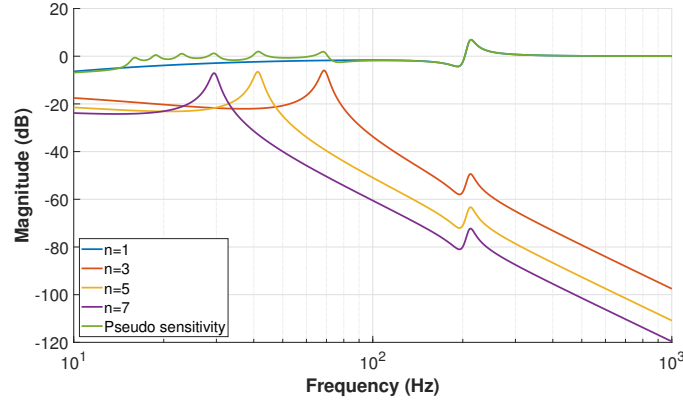


Figure 2.31: Pseudo sensitivity of a weakly damped system with PCI ($\gamma = 0$)

2.3.7 Reset Elements

There has been a variety of research on reset control elements since the introduction of the first reset controller by Clegg [16]. The nonlinear property can be a powerful tool to improve the performance of a control system. In the previous sections, multiple methods are introduced to analyze the frequency response of a reset control system. With these methods, some reset elements will be studied in this section.

Generalized First Order Reset Element FORE (First Order Reset Element) was first introduced by Horowitz and Rosenbaum [24] and then generalized by Guo et al. [18]. GFORE is a first order lag filter with corner frequency ω_r and reset matrix A_r which can be represented as:

$$\text{GFORE} = \frac{1}{\frac{s}{\omega_r} + 1} A_r \quad (2.52)$$

The state-space model of GFORE is given as:

$$A = -\omega_r, \quad B = \omega_r, \quad C = 1, \quad D = 0, \quad A_r = \gamma \quad (2.53)$$

The frequency response can be approximated by SIDF and HOSIDF. Comparing to a linear lag filter, the GFORE has a phase lag less than -90° after the corner frequency and as γ gets closer to -1 , more nonlinearity is added and it results in less phase lag as shown in Figure 2.32a. The HOSIDFs of GFORE depicted in Figure 2.32b show that at low frequencies, the gain of the higher order harmonics is significantly lower than the first order harmonics, while after the corner frequency, the gain of the higher order harmonics is close to the first order harmonics.

Hazeleger et al. [25] has extended the GFORE to GSORE (Generalized Second Order Reset Element) which is a second order lag filter with reset matrix A_r . As compared to GFORE, GSORE has more design freedom. but as a trade off, A_r contains 2 reset factors which increases the nonlinearity and complexity of the system.

Constant in Gain Lead in Phase CgLp (Constant in Gain Lead in Phase) was recently introduced by Saikumar et al. [26] as a reset element with a constant gain and a lead in phase at a certain frequency region. It then became a popular reset element due to its ability to provide broadband phase compensation with a constant gain. CgLp combines the reset lag filter GFORE/GSORE and the lead filter in series to achieve this property. In this section, the CgLp using GFORE will be mainly discussed. The GFORE $\mathcal{R}$ and the lead filter L can be represented as:

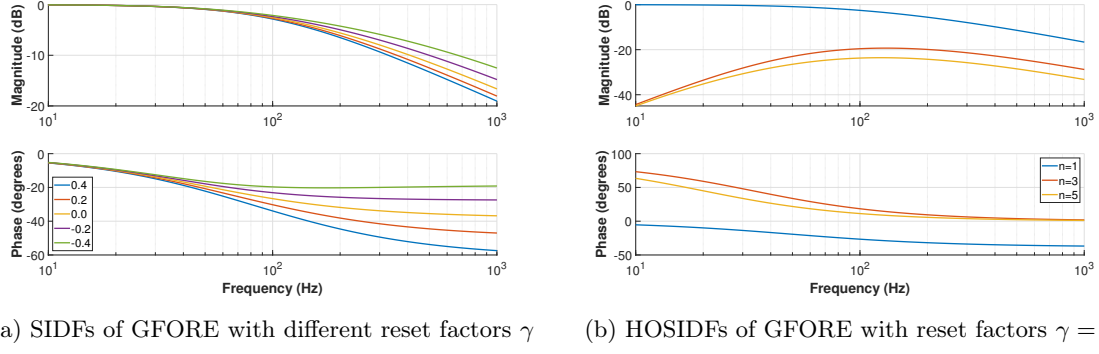


Figure 2.32: SIDFs and HOSIDFs of GFORE

$$\mathcal{R}(s) = \frac{1}{s/\omega_{r\alpha} + 1} \xrightarrow{A_r} L(s) = \frac{s/\omega_r + 1}{s/\omega_f + 1} \quad (2.54)$$

where the parameter $\omega_{r\alpha} = \omega_r/\alpha$ represents the corner frequency of the GFORE. The correction factor α is used to ensure that the gains of the two filters are accurately canceled. The reset state-space model of CgLp is given as:

$$A = \begin{bmatrix} -\omega_{r\alpha} & 0 \\ \omega_f & -\omega_f \end{bmatrix}, \quad B = \begin{bmatrix} \omega_{r\alpha} \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} \omega_f & \left(1 - \frac{\omega_f}{\omega_r}\right) \end{bmatrix}, \quad D = [0], \quad A_r = \begin{bmatrix} \gamma & 0 \\ 0 & 1 \end{bmatrix} \quad (2.55)$$

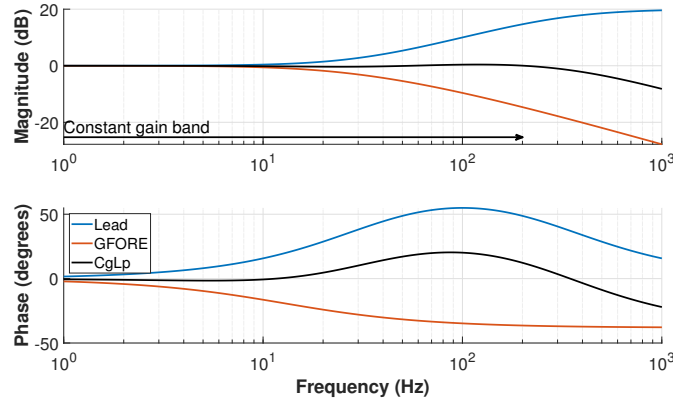


Figure 2.33: SIDs of CgLp using GFORE with reset factor $\gamma = 0$ and $\omega_f = 10\omega_r$

Figure 2.33 shows the combination of GFORE ($\gamma = 0$) with a lead filter to form CgLp. The CgLp filter create 20° of phase at the desired frequency, that the constant gain band can be pushed further. This property is useful in adding phase margin at the bandwidth in open loop without changing the gain and the bandwidth remains the same. When the bandwidth is limited by large phase lag due to time delay or higher order modes, the CgLp can be used to add extra phase to ensure enough phase margin.

The SIDFs of CgLp with different reset factors γ are shown in Figure 2.34a. As discussed in section 2.3.7, the closer to -1 reset factor γ of GFORE, the less phase lag is obtained, which results in more phase lead of CgLp. The HOSIDFs of CgLp with different filters in different order are shown in Figure 2.34b. The higher order harmonics show the same property as GFORE, where the gain of the higher order harmonics is getting closer to the first order harmonics after the corner frequency. The HOSIDFs also indicate that the order of placing the filters does effect the response of the reset system. As shown in Figure 2.26, the reset element creates higher

order harmonics which will be sent through the following filters. Figure 2.34b shows that when the GFORE is placed at first, the gain of higher order harmonics are magnified by the lead filter. However, when take the closed loop response into account, the higher order errors will then be magnified by the lead if placing it before GFORE. This implies that the sequencing of filters in a reset control system is important to ensure the stability and the approximation accuracy of the system.

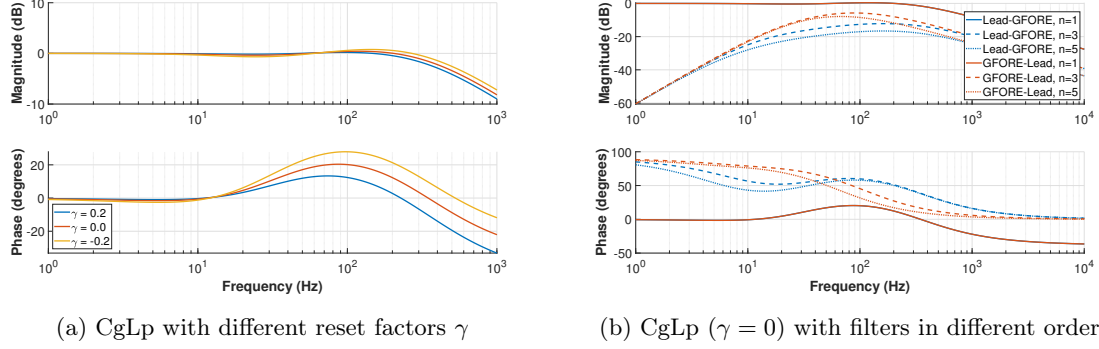


Figure 2.34: SIDFs and HOSIDFs of CgLp

2.3.8 Architecture of Reset Control System

The reset control system can be implemented in different architectures. The nonlinearity of the reset element can make significant changes to the system's response as compared to a linear system. In this section, the architecture of the reset control system will be discussed.

Sequence for Reset Controllers The HOSIDF method shows that the the controller parts, as well as the plant placed after the reset element, will affect the higher order harmonics, while the controller parts placed before the reset element also effect the higher order errors. Changing the sequence of the elements in RCS will result in changes both in frequency response approximation by HOSIDF and time response. Cai et al. [27] studied the effects of different sequences of controller parts on the performance of CgLp in a reset control system using HOSIDF method. The CgLp is split into 3 parts: the GFORE, the lead filter and the lag filter. 4 different sequences are considered in n th harmonic as shown in the following table:

No.	Sequence	n th order harmonic
1	Lead-Reset-Lag	$\text{lead}(j\omega)H_n(j\omega)\text{lag}(nj\omega)G(nj\omega)$
2	Lag-Reset-Lead	$\text{lag}(j\omega)H_n(j\omega)\text{lead}(nj\omega)G(nj\omega)$
3	Reset-Lead-Lag	$H_n(j\omega)\text{lead}(nj\omega)\text{lag}(nj\omega)G(nj\omega)$
4	Lead-Lag-Reset	$\text{lead}(j\omega)\text{lag}(j\omega)H_n(j\omega)G(nj\omega)$

Table 1: n th Order Harmonic Sequences. Source: Cai et al. [27]

Cai et al. [27] proposed an optimal sequence by comparing the gain of the open loop HOSIDFs. Besides, the closed loop response with noise signal is also compared by transforming the time response into sensitivity and control input. The results show that that No. 1: Lead-Reset-Lag provided the best tracking performance with up to 3% noise and it has the smallest control input , which provides greater flexibility for actuator choice and design.

Continuous Reset CgLp Reset control systems are known for having instantaneous jump and peaks in their control output which can be a limiting factor in practical application. Karbasizadeh and HosseinNia

[28] proposed a new architecture called Continuous Reset CgLP (CRCgLP) for CgLP element which has a continuous output to smooth the jump action without jeopardizing the steady-state performance and the higher order harmonics are meanwhile reduced. The new architecture adds a first order lag filter $R(s)$ after the reset element and adding the inverse of it which is a lead filter $L(s)$ before the reset element as shown in Figure 2.35.

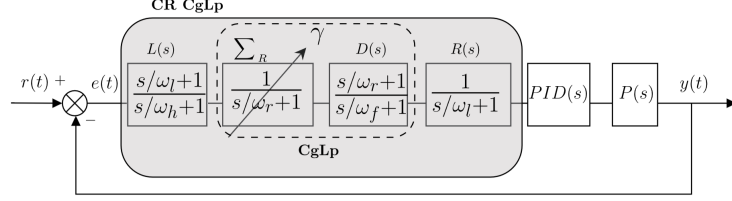


Figure 2.35: Block diagram of the continuous reset CgLP. Source: Karbasizadeh and HosseinNia [28]

The SIDF and HOSIDFs of the CRCgLP are compared with the conventional CgLP in Figure 2.36. The SIDF of CRCgLP remains basically the same as the CgLP since $\omega_h \gg \omega_c$. Whereas the HOSIDF shows that, the higher order harmonics of CRCgLP are drastically lower than the conventional CgLP. The time response of the CRCgLP is depicted in Figure 2.37, where it shows that the CR architecture is continuous as opposed to the reset element in the conventional CgLP. The response is much more smooth and ensure the availability of practical implementation.

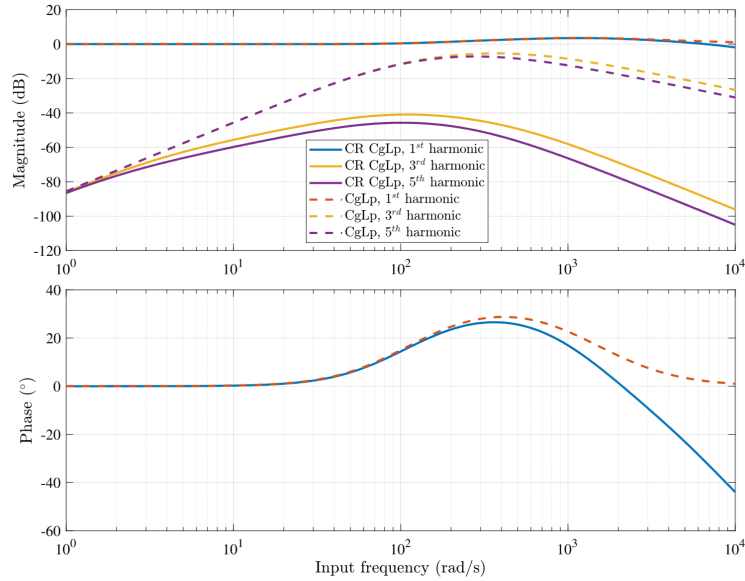


Figure 2.36: Frequency response of CRCgLP comparing with conventional CgLP. Source: Karbasizadeh and HosseinNia [28]

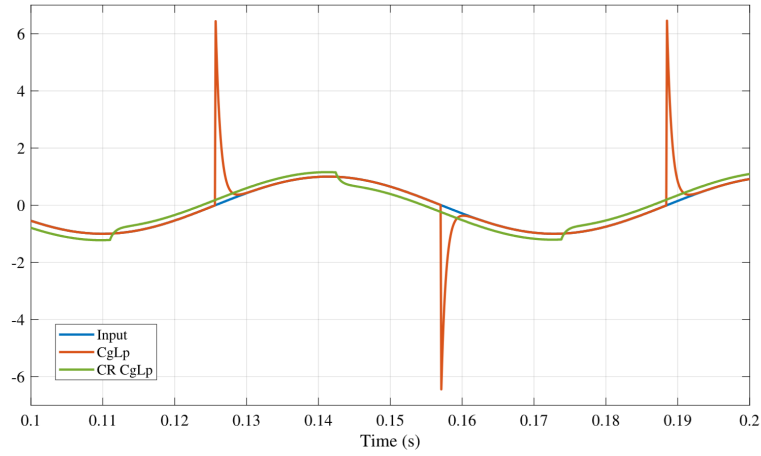


Figure 2.37: Sinusoidal response of the CRCgLp comparing with conventional CgLp. Source: Karbasizadeh and HosseinNia [28]

Shaping Filter As discussed in section 2.43, the higher order harmonics can make significant difference in the performance of reset control system and the frequency response using HOSIDF can be less accurate at certain frequencies. To reduce the effect of higher order harmonics, Karbasizadeh et al. [29] introduced a method called "phase shaping" to shape the nonlinearity of the reset element and attenuate the effect of higher order harmonics.

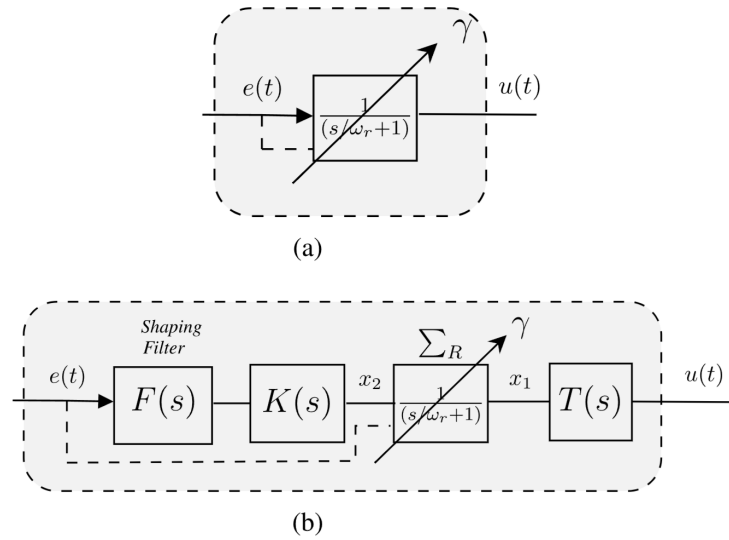


Figure 2.38: Block diagrams of a conventional FORE and a single-state reset element including a shaping filter. (a) Conventional FORE. (b) Single-state reset element including a shaping filter. Source: Karbasizadeh et al. [29]

As illustrated in Figure 2.38, the conventional reset element takes the input signal $e(t)$ as the same as the reset signal which determines the reset instant. Karbasizadeh et al. [29] introduced a set of filters to create a phase difference between the input signal and the reset signal. Assuming the error signal $e(t) = \sin(\omega t)$, the reset condition changes to $\sin(\omega t - \varphi) = 0$ with the phase difference $\varphi = \angle F(j\omega)K(j\omega)$. The state-space model of the reset system with shaping filter is given as:

$$\Sigma_R \begin{cases} \dot{x}(t) = Ax(t) + Be(t) & \sin(\omega t - \varphi) \neq 0 \\ x(t^+) = A_r x(t) & \sin(\omega t - \varphi) = 0 \\ y(t) = Cx(t) + De(t) \end{cases} \quad (2.56)$$

Karbasizadeh et al. [29] defined the phase difference ψ as: $\psi(\omega) = \angle F(j\omega)K(j\omega)R(j\omega)$, where $R(j\omega)$ is base linear system of the reset controller $\Sigma_R(j\omega)$. The HOSIDF[19] of the reset system with shaping filter can be expressed as:

$$G_{\varphi, n}(\omega) = \begin{cases} f(n, \omega)(1 - e^{-j2\psi}) + C(j\omega I - A)^{-1}B + D, & n = 1 \\ f(n, \omega)(1 - e^{-j2\psi}), & \text{odd } n > 2 \\ 0, & \text{even } n \geq 2. \end{cases} \quad (2.57)$$

where

$$f(n, \omega) = C(A - j\omega n I)^{-1} \frac{\omega e^{-j \tan^{-1}(\frac{\omega}{\omega_r})}}{\pi \sqrt{1 + \left(\frac{\omega}{\omega_r}\right)^2}} \times \left((1 - \gamma) \left(1 + \gamma e^{\frac{-\pi \omega_r}{\omega}} \right)^{-1} \left(1 + e^{\frac{-\pi \omega_r}{\omega}} \right) \right) \quad (2.58)$$

The HOSIDF $G_{\varphi, n>2}(\omega) = f(n, \omega)(1 - e^{-j2\psi})$ indicates that when the phase $\psi(\omega)$ is close to 0, the gain of the higher order harmonics is significantly low. By tuning the shaping filter, one can achieve to limit the nonlinearity inside the desired frequency band and reduce the gain of the higher order harmonics outside the band. As illustrated in Figure 2.39, phase lead of CgLp with shaping filter is only observed in the frequency

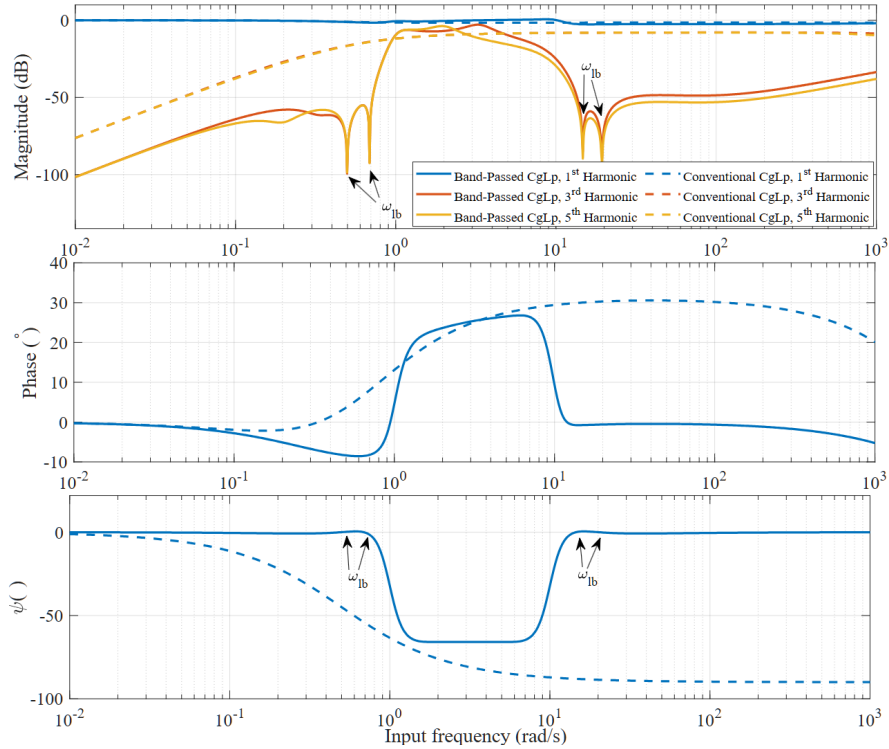


Figure 2.39: Comparison of HOSIDF of a band-passed CgLp with a conventional one, along with their ψ plot. $\omega_r = 0.5 \text{ rad/s}$ and γ is 0.2 and 0.35 for band-passed CgLp and conventional one, respectively. Source: Karbasizadeh et al. [29]

band where the phase difference ψ is less than 0. When the phase difference ψ is close to 0, the phase lead is also significantly reduced to around 0. The 3rd and 5th order harmonics of the band-passed CgLp are significantly

lower than the conventional CgLP at the frequencies outside the band. Meanwhile, the first order harmonic gain is not affected by the shaping filter.

Another way to implement the shaping filter is to place the shaping filter in the reset line [30] as show in Figure 3.13. In this case, the shaping filter will only affect the signal to determine the reset instances by introducing gain and phase difference, but not the input to the tracking controller. Extra filters to compensate the effect of shaping filter on the 1st harmonic are no longer needed.

Therefore, by tuning the phase of shaping filter, one can achieve a desired frequency band of phase lead and limit the nonlinearity of the reset element outside the band. The shaping filter provides more freedom in tuning the reset controller and can be used to improve both the tracking performance and disturbance/noise rejection of the reset control system.

3 Control System Design and Tuning

3.1 Introduction

In this chapter, we will discuss the design and tuning of the control system for the nanopositioning system based on the property of plant characteristics and the requirements of the control system.

Figure 3.1 shows the bode plant of the nanopositioning system provided by PI. The following properties can be observed in the frequency domain:

- The dcgain of the plant is -5.6dB .
- The dominant resonance occurs at 739.3Hz . The second resonance at 1009.3Hz is smaller but close to the first one. An anti-resonance is observed between these two resonances.
- Higher order resonances are present between 2013.7Hz and 2606.2Hz , with the highest peak at 2606.2Hz .
- High delay is observed in the phase plot. The exact time delay is 0.215ms . This phase delay limits the open loop bandwidth before the first resonance to ensure enough phase margin.

These properties indicate the objectives of the control system design:

1. A robust active damping controller should be designed to damp the first resonance and should not amplify the second resonance.
2. The bandwidth should be as high as possible but lower than the first resonance frequency to ensure enough phase margin. Therefore, a proportional integral (PI) controller can be implemented to achieve a high bandwidth.
3. Reset elements can be used to compensate the phase delay and further increase the bandwidth.
4. The controllers should maintain the robustness and stability of the system by ensuring enough margins: gain margin $> 6\text{dB}$, phase margin $> 60^\circ$ and modulus margin $< 6\text{dB}$.

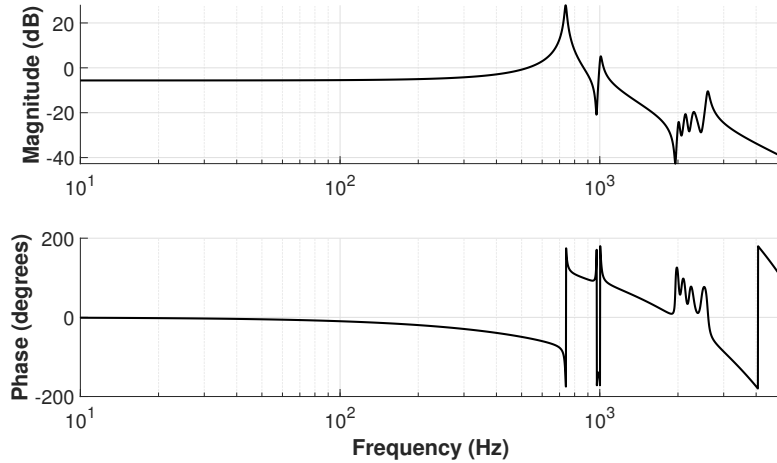


Figure 3.1: Frequency response of the plant

3.2 Active Damping Controller Design

As discussed in Section 2.2.2, the NRC controller can be an effective and robust solution for the active damping of the first resonance. It is low in order and easy to implement. Besides, it can be used to damp the second

resonance without extra filters. The NRC controller is defined as:

$$C_d(s) = k \cdot \frac{s - n\omega_n}{s + n\omega_n} \quad (3.1)$$

where ω_n is the dominant resonance frequency, $k = \gamma \cdot |G(s)|_{s=0}^{-1}$ is the gain of the controller. Two parameters n and $\gamma \in (0, 1]$ are used to tune the controller. It has been proved that $n > 3$ will be sufficient to damp the resonance[5]. The following figures show the effect of different parameters of the NRC controller to damp the target plant.

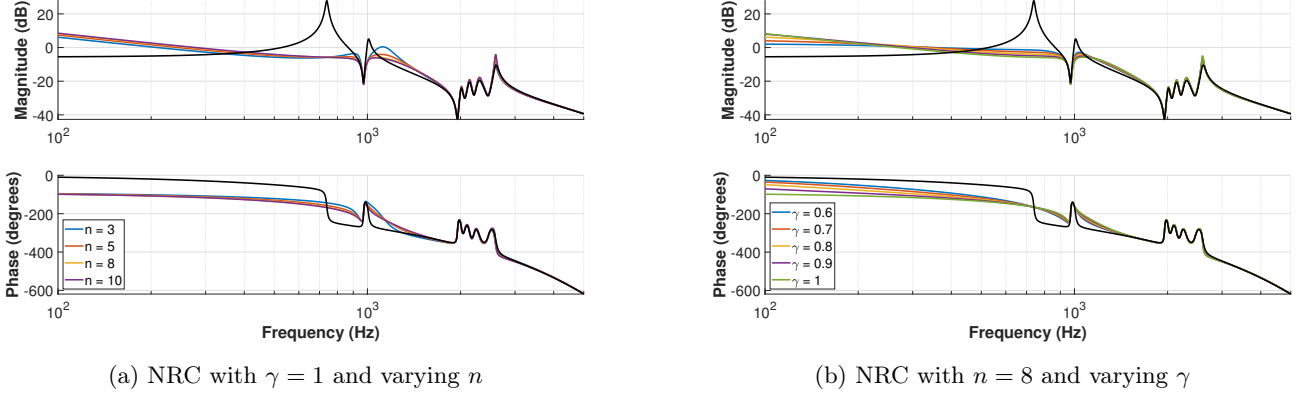


Figure 3.2: Closed inner loop response of the NRC controller with different parameters

The left Figure 3.2a shows that any $n > 3$ can damp the first resonance effectively. Higher n helps to damp the second resonance but also increases the phase delay. The right Figure 3.2b shows that higher γ can suppress the resonance peak more effectively. For $\gamma = 1$, the gain has a slope of -20dB/decade at low frequencies, which is the effect of placing the pole at the origin.

Based on the above analysis, we choose $n = 8$ and $\gamma = 1.0$ for the NRC controller.

Figure 3.3a illustrates the damping effect of the NRC controller compared to the notch controller. It can be observed that with a sharp notch filter, the 1st resonance is damped effectively but the second resonance still remains. The right Figure 3.3b shows the process sensitivity of the notch controller and the NRC controller which indicates the output response to the disturbance. It sees that even though the notch filter can suppress the first resonance in open loop, it still amplifies the disturbance at the resonance frequency. In contrast, the NRC controller can suppress the both first 2 resonance and also reduce the disturbance response.

To summarize, as compared to the notch filter, the NRC controller is a robust solution for the active damping. It can damp the first resonance effectively and suppress the second resonance without extra filters. Also, it can improve the disturbance rejection performance.

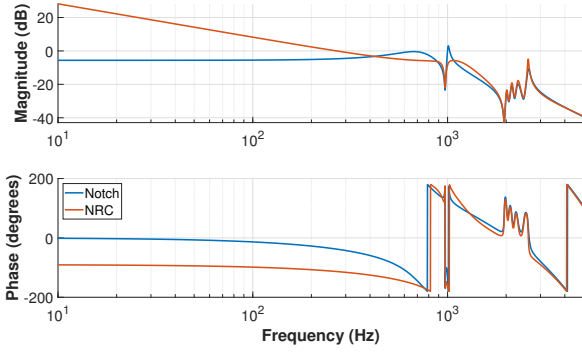
3.3 PI Controller Design

The tracking controller aims to achieve a high bandwidth and good tracking performance. The proportional gain is used to increase the bandwidth. It reacts to the error signal and helps to track the reference signal and stabilize the system.

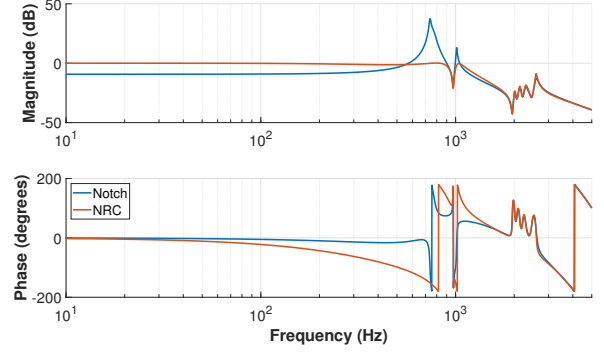
As shown in Figure 3.2b, the NRC controller with $\gamma = 1$ has a slope of -20dB/decade at low frequencies. So, a low cutoff frequency integrator can be used to further increase the low frequency tracking performance.

As discussed in Section 3.1, the bandwidth should be lower than the first resonance frequency to ensure enough phase margin. Therefore, the derivative is not needed in the tracking controller to add extra phase lead. The PI tracking controller is defined in frequency domain as:

$$C_{PI}(s) = k_p \left(1 + \frac{\omega_i}{s} \right) \quad (3.2)$$



(a) Open loop response comparison of Notch and NRC damping controllers



(b) Process sensitivity comparison of Notch and NRC damping controllers

Figure 3.3: Comparison of Notch and NRC damping controllers

In addition, filters can be used to further shape the frequency response of the PI controller. A low pass filter C_{lpf} can be used to limit the high frequency noise and flatten the T_{xr} response. A smooth notch filter C_{notch1} is used to shape the frequency response around the first 2 resonances to slightly improve the stability margins. As discussed in Section 2.3.3, the higher order harmonics of a reset system squeeze the frequency response to lower frequencies. Therefore, the higher order modes at around 2600Hz in the plant as shown in Figure 3.1 will increase the gain in the higher order harmonics. To avoid this, a sharp notch filter C_{notch2} will be placed at the high frequency modes with highest peak. In order to ensure the consistency of comparison, this high frequency notch filter will also be included in the linear control system as the baseline.

To summarize, the linear tracking controller C_t is defined as:

$$C_t(s) = C_{PI}(s) \cdot C_{lpf}(s) \cdot C_{notch1}(s) \cdot C_{notch2}(s) \quad (3.3)$$

where:

$$\begin{aligned} C_{PI}(s) &= k_p \left(1 + \frac{\omega_i}{s} \right) \\ C_{lpf}(s) &= \frac{\omega_{lpf}}{s + \omega_{lpf}} \\ C_{notch1}(s) &= \frac{\frac{s^2}{\omega_{n1}^2} + \frac{s}{Q1\omega_{n1}} + 1}{\frac{s^2}{\omega_{n1}^2} + \frac{s}{Q2\omega_{n1}} + 1} \\ C_{notch2}(s) &= \frac{\frac{s^2}{\omega_{n2}^2} + \frac{s}{Q1\omega_{n2}} + 1}{\frac{s^2}{\omega_{n2}^2} + \frac{s}{Q2\omega_{n2}} + 1} \end{aligned}$$

By tuning the parameters of the linear elements, one can achieve higher bandwidth with insufficient phase margin. The CgLp will be used to compensate the phase lag to be compared with the baseline linear system.

3.4 Reset Controller Design

3.4.1 CgLp Using Proportional FORE

As discussed in Section 2.3.7, the CgLp can be used to increase phase without changing the gain. However, the FORE element ends up with a phase lag and minus gain slope at high frequency. By adding a constant feed-through term to the FORE, one can achieve proportional gain at high frequency. The CgLp with proportional FORE has constant gain at all frequencies and a phase lead at desired frequencies while maintaining 0 phase at other frequencies. The CgLp using proportional FORE is defined as:

$$C_{gLp} = k_c \cdot \mathcal{R} \cdot C_{lead-lag} \quad (3.4)$$

where k_c is the correction value, $C_{\text{lead-lag}}$ is a lead-lag filter with transfer function:

$$C_{\text{lead-lag}}(s) = \frac{1 + \frac{s}{\omega_r}}{1 + \frac{s}{\omega_f}} \quad (3.5)$$

$\mathcal{R}$ is the proportional FORE element defined as:

$$A = -\omega_\alpha, \quad B = 1, \quad C = \omega_\alpha, \quad D = \frac{\omega_r - \omega_f}{\omega_f}, \quad A_r = \gamma \quad (3.6)$$

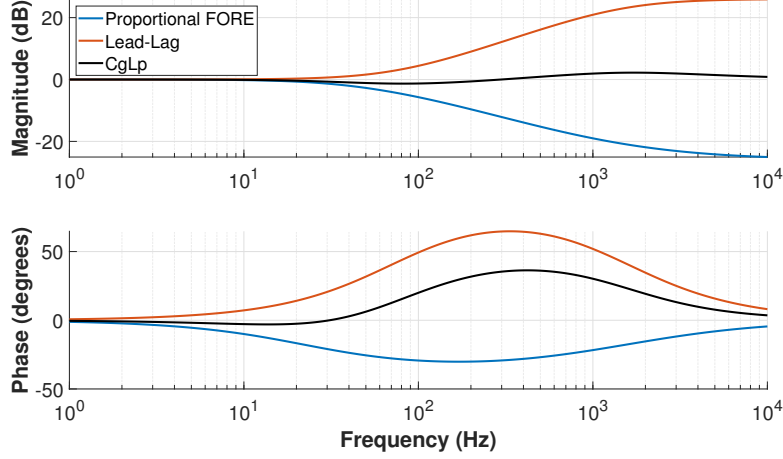


Figure 3.4: Describing function of CgLp using proportional FORE

As compared with conventional CgLp using FORE shown in Figure 3.5, the conventional CgLp has a minus gain slope at high frequency, since the phase lag of the FORE element is not compensated. And the cutoff frequency of the lead-lag filter in conventional CgLp is decisive to the maximum phase lead. The closed loop response will be affected by the phase lag and the minus gain slope at high frequency. Therefore, the CgLp using proportional FORE is used in this thesis which provides more freedom in loop shaping by introducing a separate low pass filter.

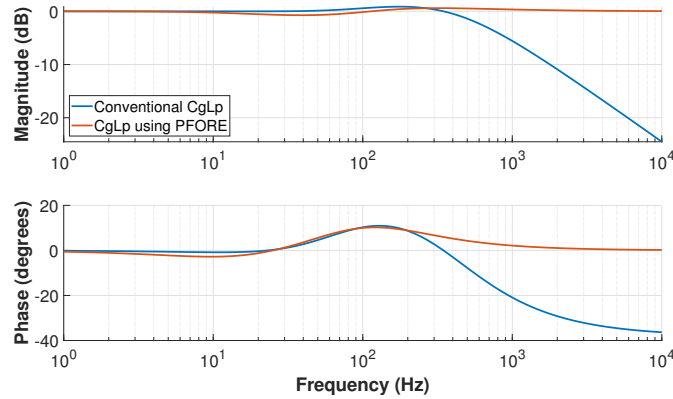


Figure 3.5: Describing function of CgLp using proportional FORE

Figure 3.6 shows the effect of different γ on the CgLp with same desired phase lead and corner frequency ω_r . The SIDF as depicted in Figure 3.6a shows that larger γ results in wider bandwidth of phase lead which indicates the higher order harmonics. Higher gain peak is also observed which reduces the gain margin. For smaller γ , larger jump of reset results in a narrower bandwidth of phase lead but also introduces more phase lag below corner frequency. Also, more gain dip is observed which shifts the bandwidth to lower frequency. The

3rd HOSIDF as depicted in Figure 3.6b shows that larger γ leads to higher maximum gain of the higher order harmonics.

Therefore, choosing $\gamma = 0$ is a good trade-off to achieve flat gain and compromise the higher order harmonics distribution. And γ is set to 0 for all CgLP designs to ensure the consistency of comparison.

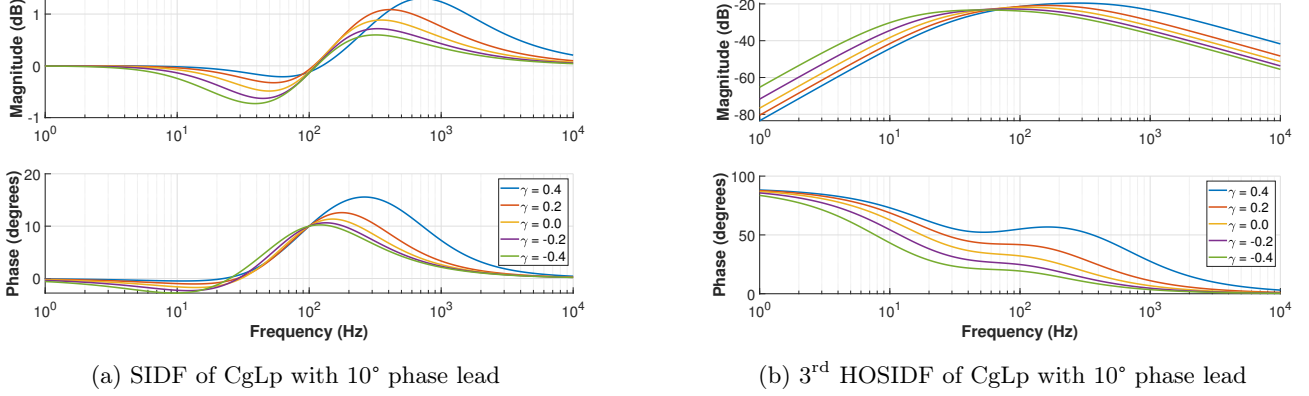


Figure 3.6: HOSIDF of CgLP with 10° phase lead for different γ

3.4.2 CgLP with 5°, 10°, 15° & 20° Phase Lead

In this thesis, we will design the CgLP with 5°, 10°, 15° and 20° phase lead and implement it in the linear system to study the effect of CgLP on the system performance respectively. There are 4 parameters to tune in the CgLP: ω_r , ω_f , ω_α and γ . And k_c is the correction value to ensure the gain is 1. Based on the findings in [31], to achieve a corner frequency of CgLP as close as possible to ω_r , ω_α is designed as:

$$\omega_\alpha = \frac{\omega_r}{\sqrt{1 + \Theta_{D,\infty}^2}} \quad (3.7)$$

where $\Theta_{D,\infty} := \lim_{\omega \rightarrow \infty} \Theta_D(\omega) = \frac{4(1-\gamma)}{\pi(1+\gamma)}$.

ω_r is tuned to place the phase lead at the desired frequency and ω_f is tuned to achieve the desired phase lead.

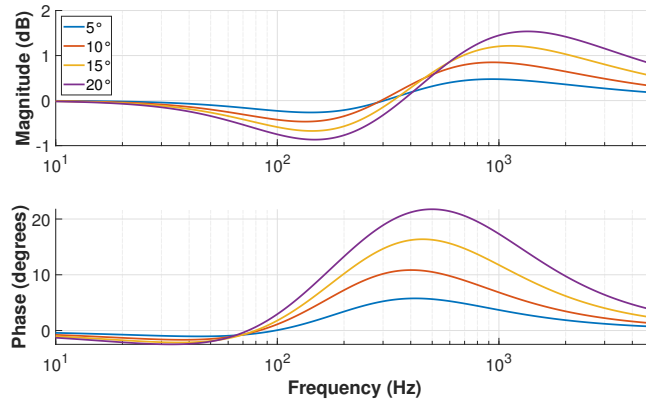


Figure 3.7: Describing function of CgLP with 5°, 10°, 15° and 20° phase lead

Figure 3.7 shows the describing function of CgLP with 5°, 10°, 15° and 20° phase lead respectively. It can be observed that the gain of CgLP has a slight dip followed by a rise fluctuation within (−1dB, 2dB) at the desired phase lead frequency. Higher phase lead results in a higher gain dip and a higher gain fluctuation. Besides, before the phase lead frequency, it sees a phase lag less than −3deg. These properties indicate that when

implementing the CgLp in a linear system, the linear elements should be slightly tuned to compensate these effects.

To study how the CgLp can improve the system performance, the linear elements in the reset system have the same structure as the linear system. Whereas the PI controller is re-tuned to achieve higher bandwidth and the CgLp is placed at the bandwidth to compensate the phase lag to ensure enough phase margin. The notch filter C_{notch1} is also re-tuned to maintain the stability margins.

The following table shows the detailed parameters of the CgLp with 5° , 10° , 15° and 20° phase lead respectively. As shown in Figure 3.7, the bandwidth of the system will be shifted when implementing the CgLp due to the gain fluctuation. To avoid significant shift in the bandwidth, one can place the CgLp by matching the gain crossover frequency with the desired bandwidth. Therefore, the cut-off frequency ω_r should be adjusted accordingly. It is also observed that to achieve a higher phase lead, the difference between ω_r and ω_f should be larger, which results in a wider region with nonlinearity.

Table 2: Frequencies Corresponding to Deflection Angles

Parameter	5°	10°	15°	20°
$\omega_r(\text{Hz})$	261.6517	219.3114	205.5480	181.2853
$\omega_a(\text{Hz})$	161.6139	135.4616	126.9604	111.9741
$\omega_f(\text{Hz})$	405.6638	486.4864	672.3213	882.7832

The maximum phase lead of the CgLp is higher than the desired phase lead and it is not utilized. This is due to necessity to place the nonlinearity at higher frequency. As shown in Figure 3.8, gain of higher order harmonics is high at frequency with phase lead. To reduce the impact of the higher order harmonics at low frequency which is more important in tracking, the maximum phase lead is placed at higher frequency.

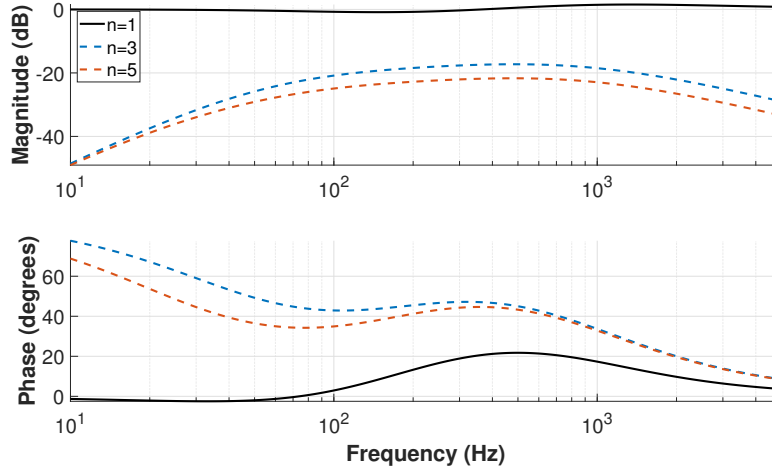


Figure 3.8: HOSIDF of CgLp of $n = 1, 3, 5$

3.4.3 CgLp Implementation in Linear System

To study the effect of the CgLp, 5 different configurations were tested. Case 1 is the baseline case with only linear controllers. Case 2 to 5 are the cases designed with insufficient phase margin from -5° to -20° at the desired open loop bandwidth where the CgLp will be placed to compensate the phase margin. With the extra phase lead, cases 2 to 5 can achieve higher bandwidth and higher cut-off frequency of the PI controller. The first notch filter C_{notch1} is re-tuned to maintain the stability margins. Whereas the second notch filter

C_{notch2} and the low pass filter C_{lpf} are kept the same for all cases to ensure the consistency of comparison. The detailed parameters of the controllers for cases 1 to 5 are shown in Table 3.

Table 3: Controller Parameters for Cases 1 to 5

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5
k_p	0.9879	1.0537	1.1028	1.2129	1.3170
w_i (Hz)	10	10	15	15	20
ω_{n1} (Hz)	1100.0	1100.0	1050.0	1000.0	1000.0
Q_{1n1}	1.0500	0.9680	1.0125	1.0571	1.1970
Q_{2n1}	1.0000	0.8000	0.7500	0.7000	0.7000
ω_{n2} (Hz)	2582.0	2582.0	2582.0	2582.0	2582.0
Q_{1n2}	40.0	40.0	40.0	40.0	40.0
Q_{2n2}	5.0	5.0	5.0	5.0	5.0
w_{lpf} (Hz)	5000.0	5000.0	5000.0	5000.0	5000.0

3.5 HOSIDF Analysis

3.5.1 Dual Loop HOSIDF Analysis

As discussed in Section 2.3.3, when a single-frequency sinusoidal input ω_0 is applied to a reset-based controller, the nonlinearity introduced by the reset elements generates higher-order harmonics at frequencies $\omega_i = i\omega_0$. These components appear in the control output $u(t)$.

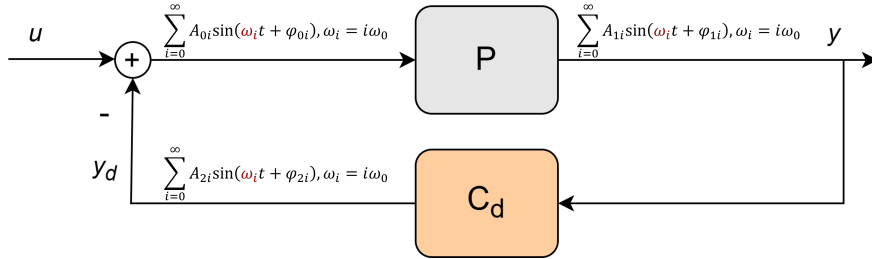


Figure 3.9: Block diagram of the inner loop

Referring to Figure 3.9, the control output $u(t)$ serves as the input to the LTI plant P , and the plant output is denoted by $y(t)$, while the damping controller output is $y_d(t)$. The signal $u(t)$ can be expressed as a sum of harmonics:

$$u(t) = \sum_{i=1}^{\infty} A_{0i} \sin(\omega_i t + \varphi_{0i}), \quad \omega_i = i\omega_0 \quad (3.8)$$

Since the plant P is linear and time-invariant, it acts independently on each sinusoidal component by modifying its amplitude and phase:

$$\begin{aligned} y(t) &= \sum_{i=1}^{\infty} P(j\omega_i) A_{0i} \sin(\omega_i t + \varphi_{0i}) \\ &= \sum_{i=1}^{\infty} A_{0i} |P(j\omega_i)| \sin(\omega_i t + \varphi_{0i} + \angle P(j\omega_i)) \\ &= \sum_{i=1}^{\infty} A_{1i} \sin(\omega_i t + \varphi_{1i}) \end{aligned}$$

Similarly, since the damping controller C_d is also LTI, it processes $y(t)$ without introducing new frequencies:

$$y_d(t) = \sum_{i=1}^{\infty} A_{2i} \sin(\omega_i t + \varphi_{2i}) \quad (3.9)$$

Thus, despite the presence of nonlinear reset elements in the control path, the inner-loop signals $y(t)$ and $y_d(t)$ remain composed solely of the harmonic components $\omega_i = i\omega_0$, as would be the case for LTI systems in series. Therefore, to study the closed-loop HOSIDF, the dual-loop system can be reformed as a single-loop system by calculating the equivalent transfer function of the closed inner loop.

To study the HOSIDF to predict tracking performance with absence of disturbance and noise, the higher order harmonics generated by the reset element $\mathcal{R}$ will be sent to the equivalent transfer function of the inner loop with the following transfer function:

$$L_{i,n>1}(\omega) = \frac{P(jn\omega)}{1 + P(jn\omega)C_d(jn\omega)} \quad (3.10)$$

For conventional damping controllers notch filter placed after the reset element, the transfer function for higher order harmonics can be expressed as:

$$L_{notch,n>1}(\omega) = C_{notch}(jn\omega) \cdot P(jn\omega) \quad (3.11)$$

Figure 3.10 shows the frequency response of the dominant higher order harmonics ($n = 3$) in the inner loop as compared to the notch filter. It can be observed that the higher order harmonics in the inner loop will not be amplified. The gain of damped 2nd resonance and the higher order modes is close to the 1st harmonic response, but it can be further suppressed by linear elements placed after the reset element. This shows that the inner loop is also robust for the higher order harmonics and not amplifying the higher order harmonics.

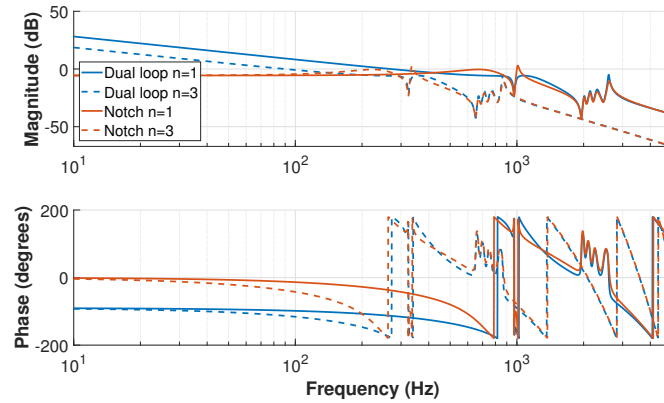


Figure 3.10: Frequency response of higher order harmonics in inner loop as compared to notch filter

3.5.2 HOSIDF based Open loop and Closed loop Describing Function

To study the effect of the higher order harmonics on reference tracking performance, the HOSIDF method can be used based on the equivalent transfer function to calculate the open loop and closed loop describing function. The HOSIDF of the reset element can be expressed as[20]:

$$H_n(\omega) = \begin{cases} C(j\omega I - A)^{-1}(I + j\Theta_D(\omega))B + D & \text{for } n = 1, \\ C(jn\omega I - A)^{-1}\Theta_D(\omega)B & \text{for odd } n \geq 2, \\ 0 & \text{for even } n \geq 2 \end{cases} \quad (3.12)$$

Open Loop Describing Function The following open loop describing function can be calculated based on the HOSIDF of the reset element:

- The dual loop gain describing function is defined as:

$$L_{D1}(j\omega) = P(j\omega)(H_1(j\omega)C_t(j\omega) + C_d(j\omega)) \quad (3.13)$$

$$L_{Dn>1}(j\omega) = P(nj\omega)(H_n(j\omega)C_t(jn\omega) + C_d(jn\omega)) \quad (3.14)$$

- The feedback open loop describing function is defined as:

$$L_1(j\omega) = H_1(j\omega)C_t(j\omega) \frac{P(j\omega)}{1 + P(j\omega)C_d(j\omega)} \quad (3.15)$$

$$L_{n>1}(j\omega) = H_n(j\omega)C_t(jn\omega) \frac{P(jn\omega)}{1 + P(jn\omega)C_d(jn\omega)} \quad (3.16)$$

Closed Loop Describing Function The closed loop describing function mapping from the reference $r(t)$ to the output $y(t)$ and the error $e(t)$ can be calculated as:

- The HOSIDF based sensitivity mapping from the reference to the error is defined as:

$$S_1(j\omega) = \frac{E_1}{R} = \frac{1}{1 + L_1(j\omega)} \quad (3.17)$$

$$S_{n>1}(j\omega) = \frac{E_n}{R} = -L_n(j\omega)Sl_{bl}(jn\omega)(|S_1(j\omega)|\angle(n\angle S_1(j\omega))) \quad (3.18)$$

where

$$Sl_{bl}(jn\omega) = \frac{1}{1 + L_{bl}(jn\omega)}$$

$$L_{bl}(jn\omega) = \mathcal{R}_{bl}(jn\omega) \frac{P(jn\omega)}{1 + P(jn\omega)C_d(jn\omega)}$$

- The HOSIDF based closed loop transfer mapping from the reference to the output is defined as:

$$T_{yr1}(j\omega) = \frac{Y_1}{R} = L_1(j\omega)S_1(j\omega) \quad (3.19)$$

$$T_{yrrn>1}(j\omega) = \frac{Y_n}{R} = L_n(j\omega)Sl_{bl}(jn\omega)(|S_1(j\omega)|\angle(n\angle S_1(j\omega))) \quad (3.20)$$

- The HOSIDF based process sensitivity mapping from the disturbance to the output is defined as:

$$PS_1(j\omega) = \frac{P(j\omega)}{1 + P(j\omega)C_d(j\omega)} S_1(j\omega) \quad (3.21)$$

$$PS_{n>1}(j\omega) = L_{n>1}(j\omega)Sl_{bl}(jn\omega)(|-PS_1(j\omega)|\angle(n\angle -PS_1(j\omega))) \quad (3.22)$$

3.6 Shaping Filter Design

3.6.1 Motivation of Shaping Filter

As discussed in Section 2.3.3, CgLp with higher phase lead results in higher gain of the higher order harmonics. There has not been a systematic study on the implementing the CgLp using proportional FORE with phase lead higher than 10° . As shown in Figure 3.11, the pseudo-sensitivity of the reset system with 15° and 20° CgLp shows that the gain of the higher order harmonics is slightly higher than the linear system at frequency between 100Hz and 200Hz. Whereas the main purpose of introducing the CgLp is to provide extra phase lead at around the open loop and closed loop bandwidth. This indicates that by shaping the phase lead of the CgLp at the desired frequency, one can reduce the gain of the higher order harmonics without losing the improvement on the bandwidth.

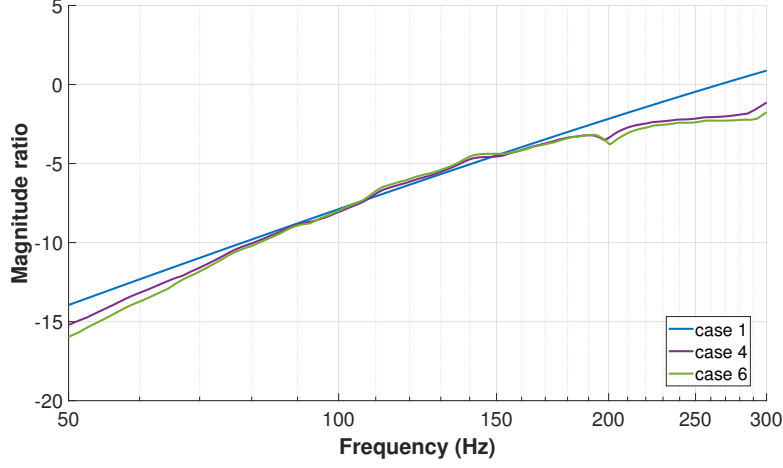
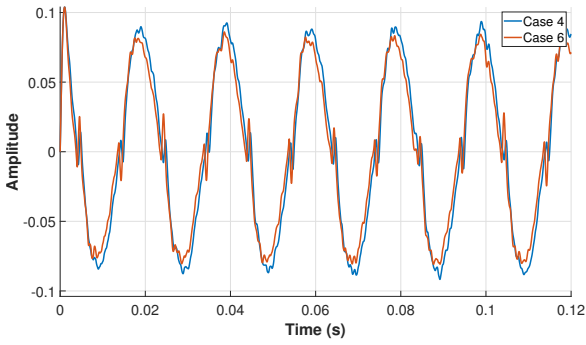
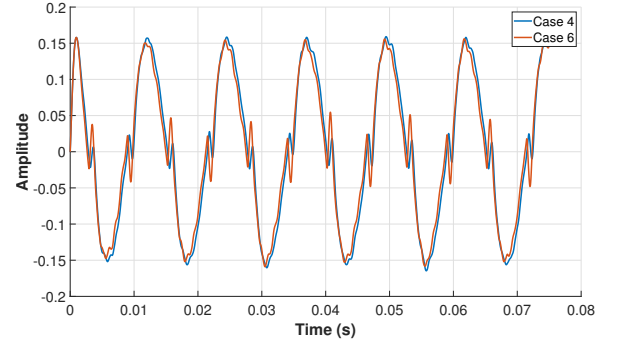


Figure 3.11: Pseudo-sensitivity of the reset system with 15° and 20° CgLp as compared with the linear system

The other important aspect to consider is the multiple resets in the reset system which results in the invalidity of the SIDF and HOSIDF methods. As depicted in Figure 3.12, multiple resets can be observed in the error response which is the reset signal at frequency of 50 Hz and 80 Hz. The shaping filter can be used to prevent the multiple resets by introducing phase and gain difference on the higher order harmonics.



(a) Error response of 50 Hz reference tracking



(b) Error response of 80 Hz reference tracking

Figure 3.12: Error response of reset system with 15° and 20° CgLp

3.6.2 Reset Line Shaping Filter Design

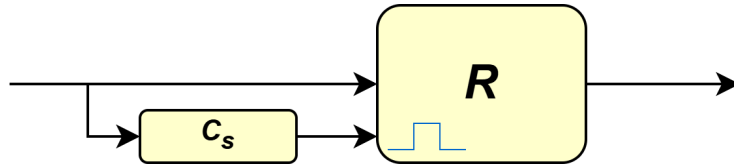


Figure 3.13: Block diagram of reset signal shaping filter

When placing the shaping filter in the input line, an extra filter $T(s)$ is needed to compensate the effect of the shaping filter $C_s(s)$ on 1st order harmonic[29]. As proposed by Zhang et al. [30], the shaping filter can also be placed at the reset signal to provide phase difference as depicted in Figure 3.13. In this case, the shaping filter will only affect the reset condition. The tracking controller transfer function remains unchanged. Placing the shaping filter in the reset line eliminates the need for an additional compensating filter and also provides a clearer view on how it interacts with the reset signal.

The shaping filter is defined as:

$$C_s = \frac{\mathcal{R}_{bl}}{\text{Notch1} \cdot \text{Notch2} \cdot \text{LF}} \quad (3.23)$$

where $\mathcal{R}_{bl}$ is the base linear system of the FORE element, Notch1 and Notch2 are the notch filters with reciprocal $q = Q1/Q2$ of each other and they are placed at the edges of the desired phase lead frequency. The notch filters are defined as:

$$\text{Notch1}(s) = \frac{\frac{s^2}{\omega_l^2} + \frac{s}{\omega_l} + 1}{\frac{s^2}{\omega_l^2} + \frac{s}{q\omega_l} + 1} \quad (3.24)$$

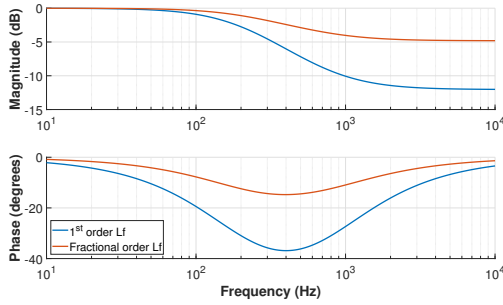
$$\text{Notch2}(s) = \frac{\frac{s^2}{\omega_h^2} + \frac{s}{q\omega_h} + 1}{\frac{s^2}{\omega_h^2} + \frac{s}{\omega_h} + 1} \quad (3.25)$$

The fractional order lead-lag filter LF is used to shape the phase difference to achieve desired properties with transfer function:

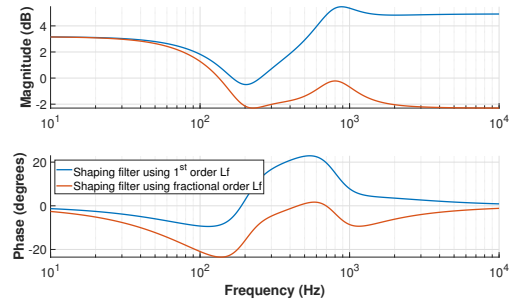
$$\text{LF}(s) = \left(\frac{s/\omega_l + 1}{1 + s/\omega_h} \right)^\lambda \quad (3.26)$$

Figure 3.14 illustrates the necessity of implementing the shaping filter with fractional order lead-lag filter. The cut-off frequency of lead-lag filter is dependent on the desired phase lead frequency range. For 1st order lead-lag filter, the phase lead is completely dependent on the cut-off frequency. Therefore, the phase of the shaping filter is not able to be shaped to achieve desired phase difference. As shown in the right Figure 3.14b, the shaping filter with 1st order lead-lag filter is significantly high in lead but less phase lag which results in increasing the gain of the higher order harmonics. Whereas the fractional order lead-lag filter is able to provide any desired phase lead with fixed cut-off frequency. It can be tuned to suit any PFORE design.

In reality, the fractional order lead-lag filter does not have a physical implementation. However, it can be approximated with a rational frequency response using a nonlinear least squares algorithm. By selecting the specific order of nominator and denominator, MATLAB function `invfreqs` can find the best fit of the fractional order lead-lag filter. In practice, the specific order is set to 2 for both nominator and denominator which ensures reliable approximation and avoids numerical issues due to the high order of the transfer function.



(a) Fractional order Lf components as compared with 1st order Lf

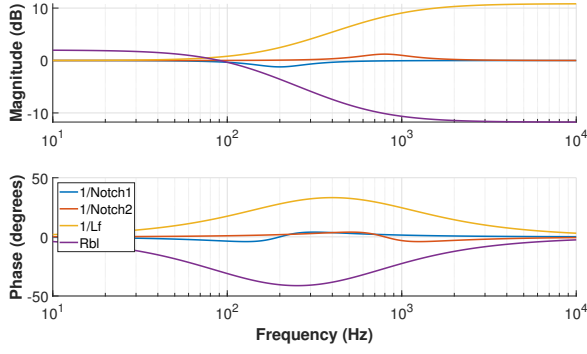


(b) Shaping filter with fractional order Lf

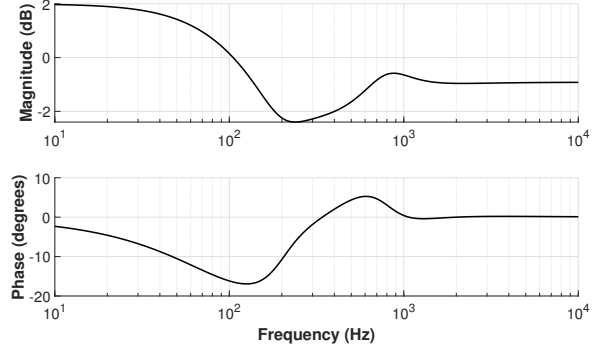
Figure 3.14: Bode plot of shaping filter with fractional order Lf and 1st order Lf

Figure 3.15 shows the bode plot of the shaping filter components and the overall shaping filter for 20° CgLp respectively. According to the discussion in [30], the higher phase lag of the shaping filter will result in less phase lead of the CgLp where the higher order harmonics will be attenuated.

For reference at frequency from 50Hz to 100Hz, the higher order harmonics (mainly 3rd from 150Hz to 300Hz) will cause multiple resets as depicted in Figure 3.12. The shaping filter will introduce a phase difference between the 1st and 3rd harmonics to place the 3rd harmonic peak before or after the 1st harmonic crossing. Meanwhile, the gain of the 3rd harmonic is also reduced due to the gain properties of the shaping filter. This also helps to avoid the 3rd harmonic peak crossing 0 which results in the multiple resets.



(a) Bode plot of shaping filter components



(b) Bode plot of shaping filter

Figure 3.15: Bode plot of shaping filter for 20° CgLp

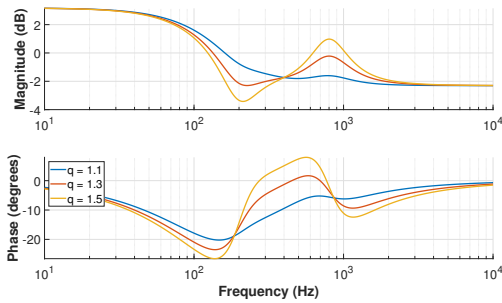
3.6.3 Shaping Filter Tuning

To attenuate the higher order harmonics below 200Hz and maintain the benefit of phase lead for both open loop and closed loop bandwidth, the corner frequency of the shaping filter is set to $\omega_l = 200\text{Hz}$, $\omega_h = 800\text{Hz}$. The parameters order λ and q are tuned to shape the CgLp with following properties:

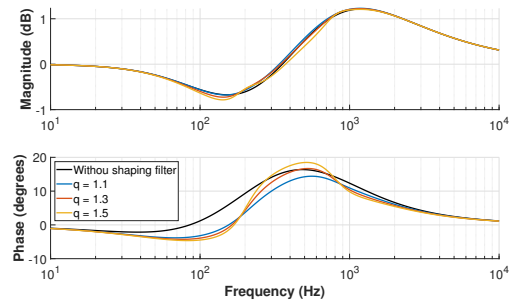
- Suppress the phase lead at frequency below 200Hz.
- Remain the same phase lead at desired bandwidth.
- Avoid more phase lead between 200Hz and 800Hz which introduces more higher order harmonics.

Based on the above requirements, the phase of shaping filter should be as close to 0 as possible to remain same phase lead of CgLp. Other than (200Hz, 800Hz), a phase lag should be carefully tuned in the shaping filter to suppress the phase lead of CgLp effectively and avoid introducing more phase lag of CgLp.

Figure 3.16a shows that q changes the maximum phase difference between 200Hz and 800Hz. It indicates the effect of CgLp phase lead attenuation. While λ plays a role in shifting the phase response of the shaping filter as shown in Figure 3.17a. It can be used to ensure 0 phase of shaping filter to remain phase lead of CgLp at the desired bandwidth.



(a) Shaping filter of different q

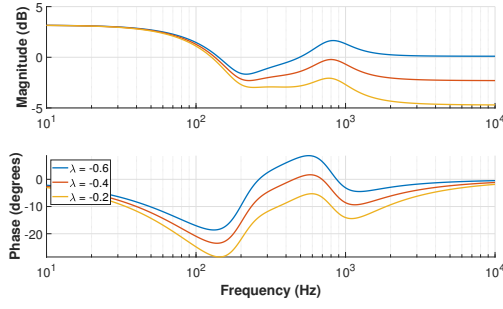


(b) SIDF of CgLp with shaping filter of different q

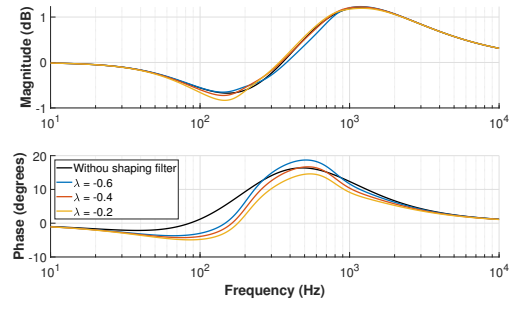
Figure 3.16: CgLp with shaping filter of varying q

Therefore, a tuning guideline is proposed to achieve the desired shaping filter characteristics based on the above observations.

1. Select the frequency range of the shaping filter ω_l and ω_h where inside the range the phase lead of CgLp is needed whereas outside the range the phase lag of CgLp is suppressed.



(a) Shaping filter of different λ



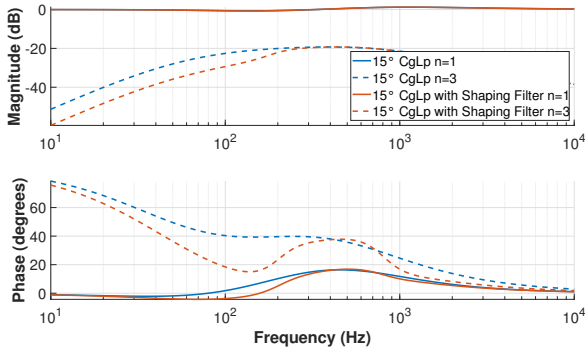
(b) SIDF of CgLp with shaping filter of different λ

Figure 3.17: CgLp with shaping filter of varying λ

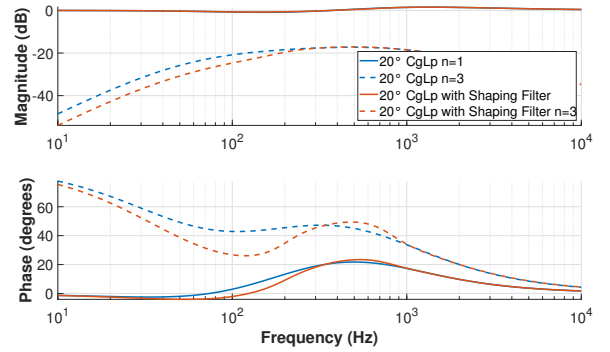
2. Adjust the parameter q to change the maximum phase difference between inside the range and outside the range.
3. Adjust the parameter λ to shift the phase response of the shaping filter to ensure as close to 0 phase as possible inside the frequency range.
4. Calculate the SIDF of the CgLp with shaping filter to check if the phase lead is unchanged inside the frequency range and if there is too high phase lag outside the frequency range.
5. Repeat step 2,3 and 4 until the desired shaping filter characteristics are achieved.

3.6.4 HOSIDF of Reset System with Shaping Filter for 15° and 20° CgLp

The HOSIDF of the CgLp with shaping filter is shown in Figure 3.18 as compared with the CgLp without shaping filter. It can be observed that the gain of the CgLp is slightly changed while the phase lead at frequency below 200Hz is reduced as expected. The 3rd harmonic gain is also reduced as compared with the CgLp without shaping filter.



(a) HOSIDF of shaping filter for 15° CgLp



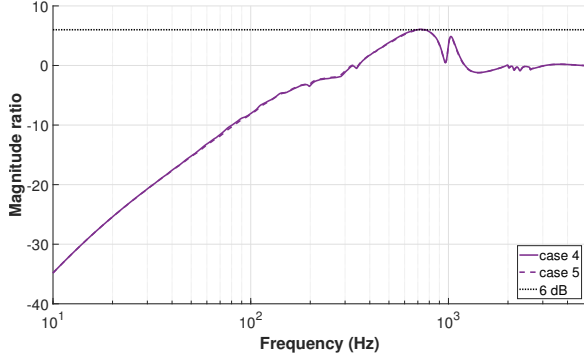
(b) HOSIDF of shaping filter for 20° CgLp

Figure 3.18: HOSIDF of 15° and 20° CgLp with shaping filter

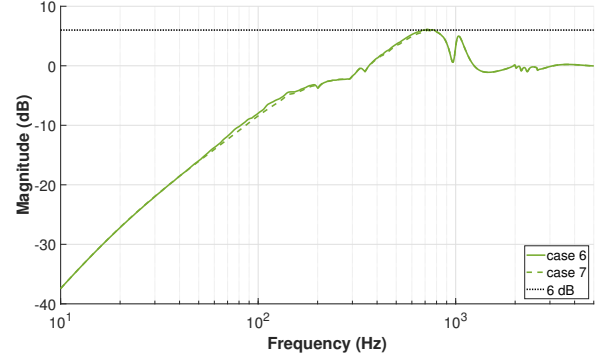
The shaping filter is able to attenuate the higher order harmonics by reduce the phase lead at the desired frequency. However, the phase difference also results in the change of the closed loop response.

Figure 3.19 compares the sensitivity of the reset system with and without the shaping filter. It can be observed that the shaping filter slightly reduces the sensitivity between (50Hz,200Hz) where the CgLp phase lead is suppressed. This is more significant for 20° CgLp (Case 6&7).

As depicted in Figure 3.20, the closed loop HOSIDF of the reset system with shaping filter sees a peak at the frequency where the phase lead is reduced as compared with the reset system without shaping filter. This is due



(a) Sensitivity Comparison With and Without the Shaping Filter (Case 4&5)



(b) Sensitivity Comparison With and Without the Shaping Filter (Case 6&7)

Figure 3.19: Effect of Shaping Filter on System Sensitivity

to the phase reduction of the CgLP at the desired frequency without gain compensation. Therefore, the tuning of the shaping filter should be done carefully to ensure the closed loop response is not significantly affected.

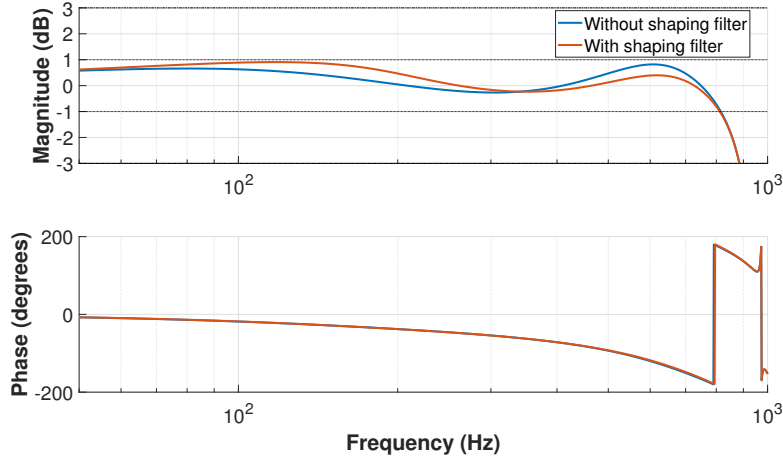


Figure 3.20: Closed loop HOSIDF of reset system with shaping filter

The detailed parameters of the shaping filter for 15° and 20° CgLP are shown in Table 4.

Table 4: Shaping Filter Parameters

	w_l (Hz)	w_h (Hz)	λ	Q
CgLP 15	200	800	-0.4	1.3
CgLP 20	200	800	-0.9	1.15

3.7 HOSIDF Based Frequency Response Analysis

3.7.1 Controller Parameters Across Different Cases

To summarize the controller parameters across different cases, Table 5 presents the detailed parameters used in the design of the controllers from Case 1 to Case 7. The tracking performance of the system is improved by increasing the proportional gain k_p and integral frequency w_i .

It is important to note that Case 5 and Case 7 have the same parameters as compared with Case 4 and Case 6,

respectively. The only difference is that Case 5 and Case 7 include the shaping filter to limit the higher order harmonics.

Table 5: Controller Parameters Across Different Cases

Parameter	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
k_p	0.9879	1.0537	1.1028	1.2129	1.2129	1.3170	1.3170
w_i (Hz)	10	10	15	15	15	20	20
ω_{n1} (Hz)	1100.0	1100.0	1050.0	1000.0	1000.0	1000.0	1000.0
Q_{1n1}	1.0500	0.9680	1.0125	1.0571	1.0571	1.1970	1.1970
Q_{2n1}	1.0000	0.8000	0.7500	0.7000	0.7000	0.7000	0.7000
ω_{n2} (Hz)	2582.0	2582.0	2582.0	2582.0	2582.0	2582.0	2582.0
Q_{1n2}	40.0	40.0	40.0	40.0	40.0	40.0	40.0
Q_{2n2}	5.0	5.0	5.0	5.0	5.0	5.0	5.0
w_{lpf} (Hz)	5000.0	5000.0	5000.0	5000.0	5000.0	5000.0	5000.0
$CgLp$	No CgLp	5°	10°	15°	15° with SF	20°	20° with SF

3.7.2 HOSIDF Based Frequency Response Comparison

The following Table 6 summarizes the performance metrics across different cases, including the cutoff frequency of PI, the open-loop bandwidth and improvement in bandwidth comparing with linear control system, as well as the closed-loop bandwidth crossing ± 1 dB and ± 3 dB. For comparison consistency, all the cases are designed with enough stability margins: gain margin > 6 dB, phase margin $> 60^\circ$ and modulus margin < 6 dB and are fine tuned to achieve as high bandwidth as possible.

Table 6: Performance Metrics Across Different Cases

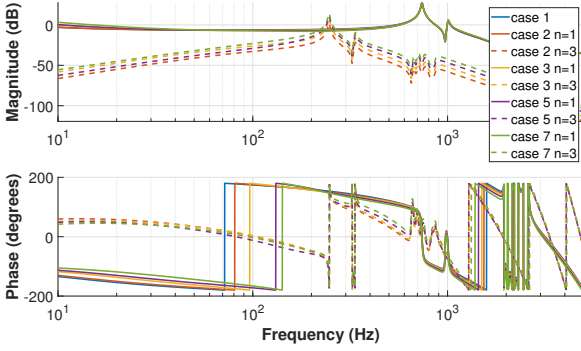
Metric	Case 1	Case 2	Case 3	Case 4	Case 5	Case 6	Case 7
CgLp Phase Lead	–	5°	10°	15°	15°	20°	20°
PI Cut-off Frequency (Hz)	10	10	15	15	15	20	20
Open-Loop BW (Hz)	265.5	278.0	288.9	309.3	311.0	329.3	330.5
Improvement (Hz)	–	12.5	23.4	43.8	45.5	63.8	65.0
Closed-Loop BW (± 1 dB) (Hz)	791.4	803.2	795.8	815.1	815.1	816.6	821.1
Closed-Loop BW (± 3 dB) (Hz)	872.6	872.6	871.0	883.9	882.3	885.5	885.5

As presented in Table 6, the reset systems achieve a open loop bandwidth improvement of up to 65.0Hz compared to the linear control system. The closed-loop bandwidth crossing ± 1 dB and ± 3 dB are also improved. The extra phase lead introduced by the CgLp also helps to achieve a higher PI cut-off frequency up to 20Hz, which results in better tracking performance at low frequency.

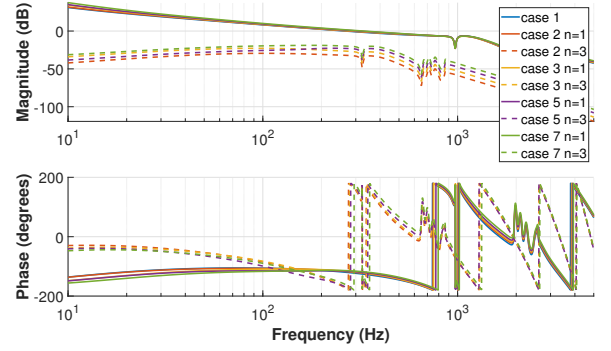
3.7.3 HOSIDF Based Frequency Response Visualization

Reference Tracking The following figures present the HOSIDF based frequency response of the reset system across different cases to illustrate in detail the improvement of introducing the CgLp and the shaping filter. For clarity, Cases 1, 2, 3, 5 and 7 are presented in the figures to highlight the effect of different CgLp phase leads.

The comparison with and without the shaping filter is also presented in the figures to illustrate the effect of the shaping filter on the sensitivity of the system.



(a) Dual loop gain describing function (1st, 3rd) across different cases

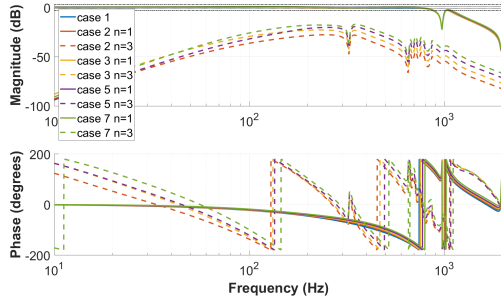


(b) Open loop describing function (1st, 3rd) across different cases

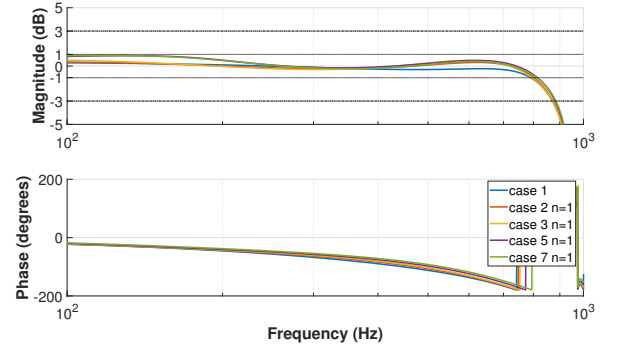
Figure 3.21: HOSIDF based open loop response of the reset system

The dual loop gain describing function (L_D) as shown in Figure 3.21a illustrates the damping performance of dual loop system to avoid excessive excitation of disturbances at that frequency [5]. It sees high peak at the first resonance frequency both in SIDF (ω_n) and HOSIDF (ω_n/n), which has π phase difference with the resonance of the plant. This peak implies effective disturbance rejection at the resonance frequency both for the dominant 1st harmonic and the higher order harmonics.

As shown in Figure 3.21b, the 1st open loop SIDF of the reset system across different cases shows that the bandwidth is pushed to higher frequency with the increase of extra phase margin. At low frequency, it also sees a gain rise due to higher integral frequency of the PI controller. The 3rd HOSIDF shows that higher phase lead CgLp results in a higher gain at the higher order harmonics.



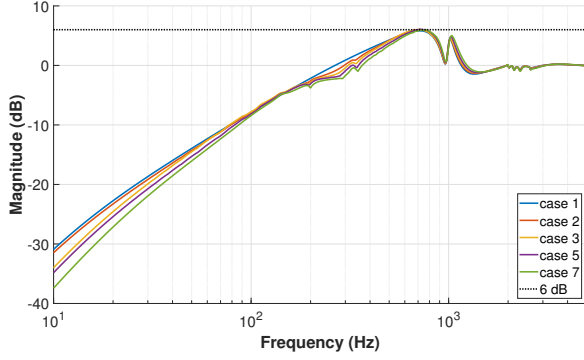
(a) Closed loop describing function across different cases



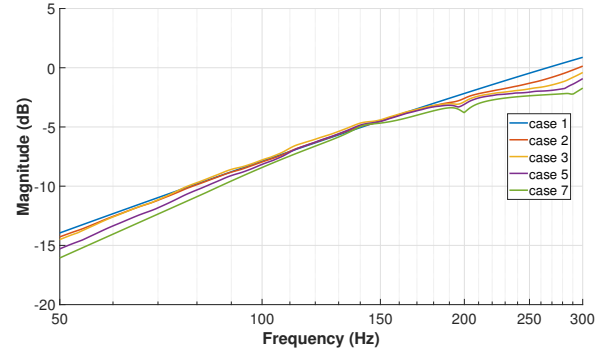
(b) Closed loop describing function across different cases (zoomed)

Figure 3.22: HOSIDF based closed loop response of the reset system

Figure 3.22 shows the closed loop SIDF and HOSIDF of the reset system across different cases which indicates the tracking performance mapping output y to the reference input r . It can be observed that the closed loop SIDF gain is limited in between $(-1\text{dB}, 1\text{dB})$ across all cases up to frequency higher than the first resonance. As shown in Figure 3.22b, the closed loop bandwidth crossing $\pm 1\text{ dB}$ and $\pm 3\text{ dB}$ is improved with the increase of phase lead. The 3rd HOSIDF presents similar properties as the open loop HOSIDF, where higher phase lead CgLp results in a higher gain at the higher order harmonics. However, It can be seen that with the shaping filter, the gain of the 3rd HOSIDF at low frequency is reduced, which helps to reduce the effect of the higher order harmonics on the low frequency tracking performance.



(a) Sensitivity describing function across different cases



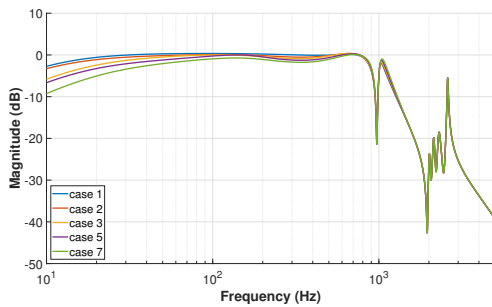
(b) Sensitivity describing function across different cases (zoomed)

Figure 3.23: HOSIDF based sensitivity of the reset system

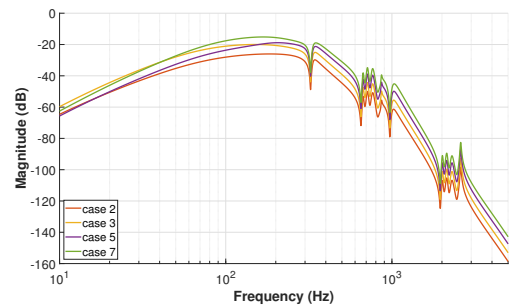
The Pseudo-sensitivity as shown in Figure 3.23 illustrates the tracking performance mapping from the reference input r to the error output e . It highlights the improvement in attenuation of the tracking error especially at low frequency ($f < 100\text{Hz}$). This sensitivity improvement is also observed at frequency between (200Hz, 500Hz). At higher frequencies ($f > 700\text{Hz}$), the waterbed effect does not show significant increase in the sensitivity as compared with Case 1 and for all cases, the sensitivity is limited below 6dB as required.

In the frequency range of 100Hz to 200Hz, the influence of higher-order harmonics becomes more pronounced. Within this range, Case 3 exhibits slightly higher sensitivity compared to Case 1. However, the sensitivity levels in Case 5 and Case 7 are effectively attenuated by the shaping filter. As a result, despite the presence of a higher phase lead in the CgLp structure, their sensitivities remain lower than that of Case 3.

Disturbance Rejection Figure 3.24 illustrates the HOSIDF based process sensitivity of the reset system which highlights the system's ability to reject disturbances. Dual loop helps to suppress the peak of the target resonance as previously discussed. The SIDF PS as depicted in Figure 3.24a shows that the reset system is able further improve the disturbance rejection at the frequencies below the first resonance frequency without losing high frequency performance. As shown in Figure 3.24b, the 3^{rd} order PS demonstrates that as the phase lead of the CgLp increases, the gain of the higher order harmonics is also slightly increased.



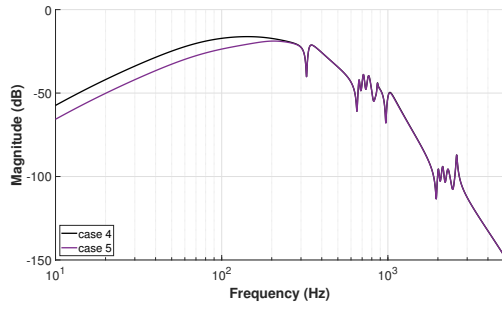
(a) Process sensitivity SIDF across different cases



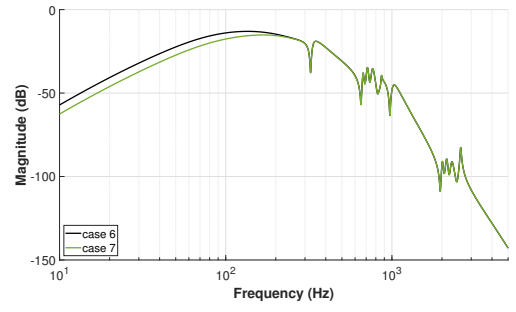
(b) Process sensitivity HOSIDF across different cases

Figure 3.24: HOSIDF based process sensitivity of the reset system

The peak of 3^{rd} order PS is observed between 100Hz and 200Hz where the shaping filter is implemented to suppress the effect of the higher order harmonics. As shown in Figure 3.25, the shaping filter effectively reduces the gain of the higher order harmonics below 200Hz which attenuates the effect of higher order harmonics on the disturbance rejection performance.



(a) Process sensitivity HOSIDF of Case 4,5



(b) Process sensitivity HOSIDF of Case 6,7

Figure 3.25: HOSIDF based process sensitivity of reset system with and without shaping filter

To summarize, the reset system with CgLP and shaping filter achieves higher open loop bandwidth and closed loop bandwidth. It also reduces the tracking error at low frequency and middle frequency without significantly increasing the sensitivity at high frequency. The disturbance rejection performance is also improved at frequencies below the first resonance frequency. The shaping filter helps to suppress the effect of higher order harmonics both in reference tracking and disturbance rejection at desired frequency.

4 Simulation and Discussion

This chapter presents the simulation results of the reset control system designed in the previous chapter. The simulations are conducted using MATLAB and Simulink to analyze the time response. The focus is on the sinusoidal reference response. The following features are discussed:

- Prediction validation of HOSIDF based frequency response.
- Effect of Multiple resets.
- Effect of shaping filter on reset signal.

4.1 Simulink Setup

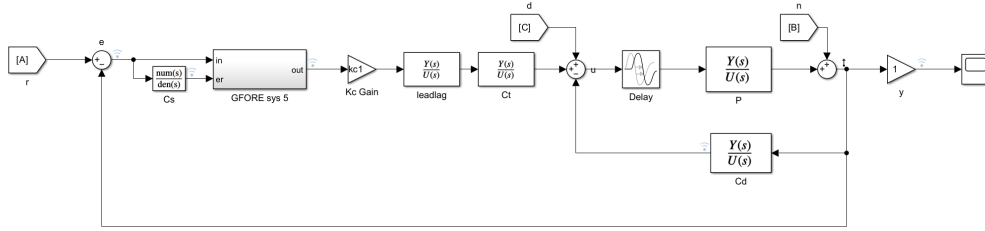
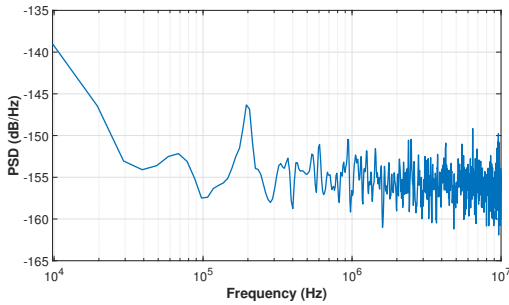
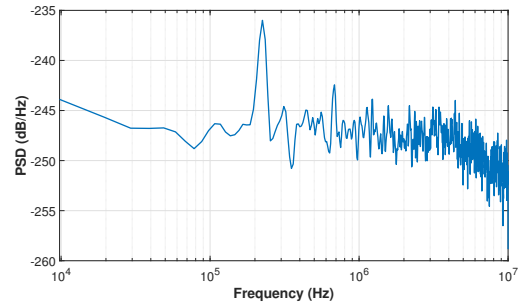


Figure 4.1: Simulink setup for the reset control system

Figure 4.1 depicts the Simulink setup for the reset control system. Input A is the sinusoidal reference signal, input B and C is the noise and disturbance signal respectively which are measured data provided by PI. The noise and disturbance data are implemented as timeseries objects in MATLAB and imported into Simulink. The PSD of the noise and disturbance signals are depicted in Figure 4.2.



(a) Power Spectral Density of Disturbance



(b) Power Spectral Density of Noise

Figure 4.2: Power Spectral Density of Disturbance and Noise

The linear elements including tracking controller C_t , damping controller C_d , lead-lag filter $C_{leadlag}$, shaping filter C_s and the plant are modeled as transfer functions. Whereas the reset element FORE is modeled in a subsystem by state space representation as shown in Figure 4.3. The reset action is triggered by block Hit Crossing.

4.2 Time Response Results and Discussion

Sinusoidal Reference Response The following figures show the time response of the reset control system to a sinusoidal reference signal at various frequencies. The output response on left and the error response on

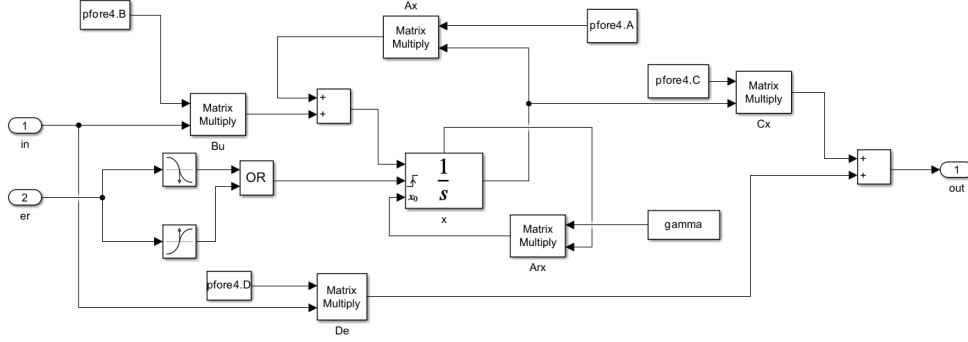


Figure 4.3: Simulink setup for the reset element FORE

the right are plotted for each frequency. The output shows a significant delay at high frequency reference due to the phase lag which can be observed in closed loop frequency response. This can be improved by introducing feed-forward compensation in the tracking controller or advanced input reference for the exact phase lag.

It is notable that at extremely low frequencies such as 1Hz, the error sees numerous resets. This is because at low frequencies, sensor noise constitutes the dominant component of the error signal. The RMS tracking error for 1Hz reference is 0.0022, whereas the RMS of the noise signal is 0.0019. Therefore, the multiple resets are not triggered by the higher order harmonics but by the noise. This is also why it does see a significant improvement in tracking performance at low frequencies.

The error response depicted on the right side of the figures shows that the error is reduced by the reset control system as compared to the linear control system. Cases with higher phase lead CgLP show a more significant reduction in error at frequencies other than (100Hz, 150Hz) which is as predicted by the Pseudo-sensitivity function. To summarize, the time response to sinusoidal reference signal shows the improvement in tracking performance and the validation of the HOSIDF based frequency response prediction.

Effect of Shaping Filter on Reset Signal Figure 4.6 compares the reset signal with and without the shaping filter at selected frequencies where multiple resets occur. For CgLP without the shaping filter (Case 4 & Case 6), the reset signal is the error signal, whereas with the shaping filter (Case 5 & Case 7), the reset signal is the output of the shaping filter. As illustrated in the bottom row of figures, the peak of the higher order harmonics which shown as the sub peak is significantly suppressed by the shaping filter as expected. It shows that the multiple resets are effectively prevented by introducing the shaping filter without affecting the tracking performance.

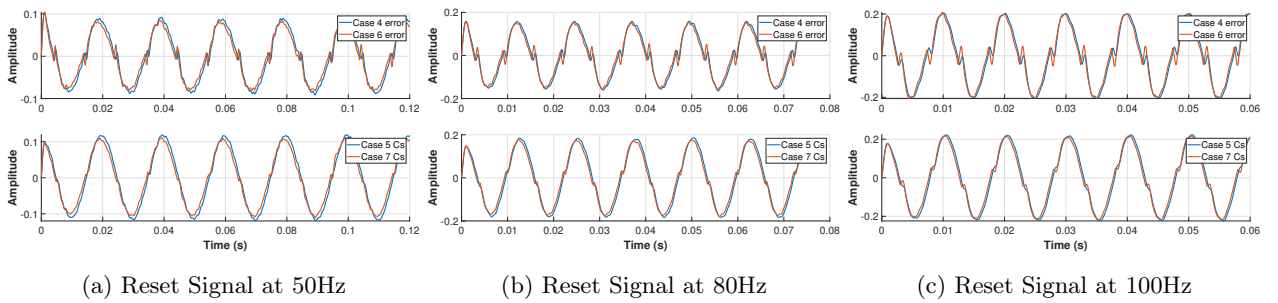
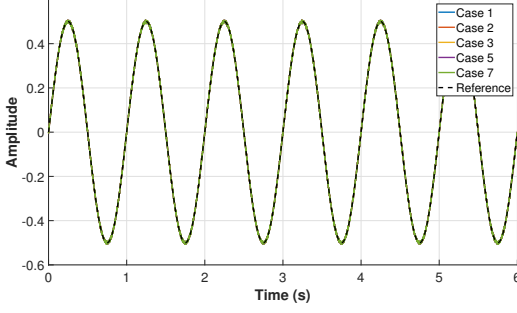
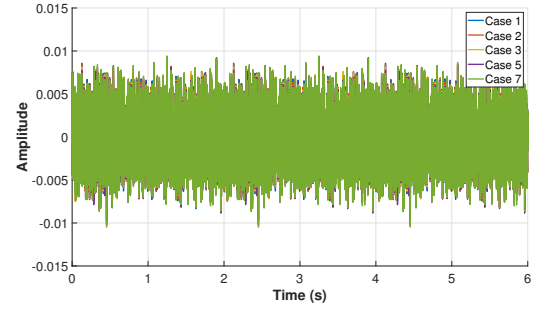


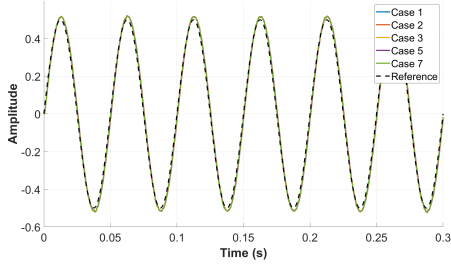
Figure 4.6: Reset signal comparing with and without the shaping filter



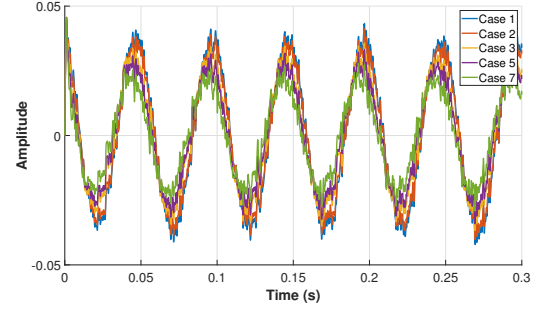
(a) Output Response at 1Hz



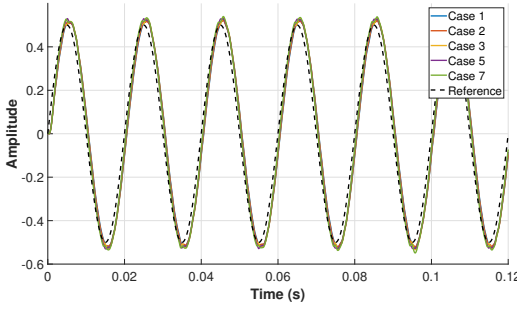
(b) Error Response at 1Hz



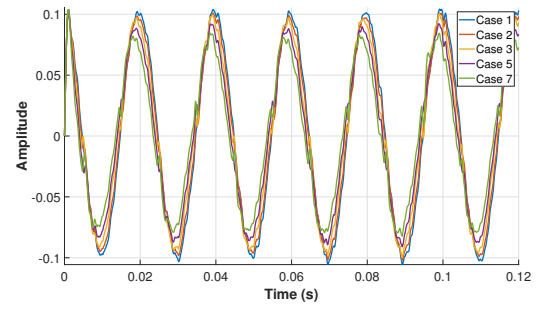
(c) Output Response at 20Hz



(d) Error Response at 20Hz



(e) Output Response at 50Hz



(f) Error Response at 50Hz

Figure 4.4: Time response to sinusoidal reference signal

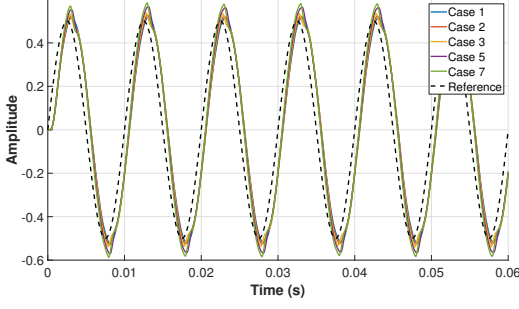
5 Experimental Validation

In this chapter, we present the experimental validation of the reset control system designed in the previous chapters. The experiments are conducted on the P621.1CD PIHera linear precision positioner. The focus is on the validation of the reset control system performance in a real-world application as compared to the HOSIDF based frequency response analysis. The following sections introduce the experimental setup, the implementation of the reset controller in LabVIEW, and the results obtained from the experiments.

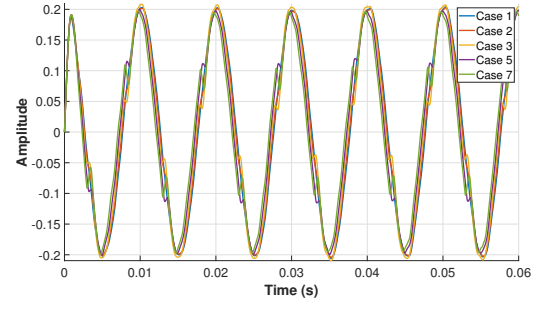
5.1 Experiment Setup

The experimental setup, depicted in Figure 5.1, employs a commercial P-621.1CD PIHera linear precision nanopositioning system, offering a $100\mu\text{m}$ travel range. This single-axis stage integrates a ceramic-insulated multilayer piezo-stack actuator, guided by a flexure-based mechanism and monitored by a high-resolution capacitive sensor. The system is driven by an E-712 piezo-controller, which houses both the voltage amplifier and sensor signal conditioning modules.

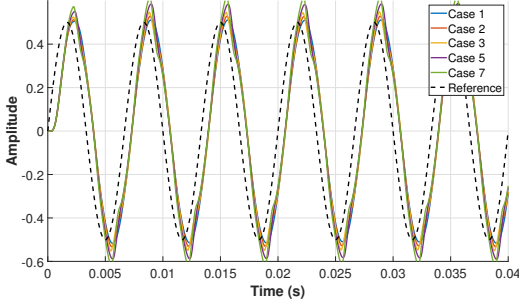
To enable advanced external control, the nanopositioning system is interfaced with an NI CompactRio chassis featuring an embedded FPGA. This setup includes multiple analog input-output modules that facilitate the transmission and acquisition of actuation and sensing signals. The control strategy is implemented in NI



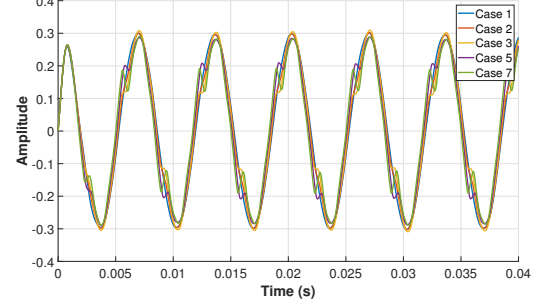
(a) Output Response at 100Hz



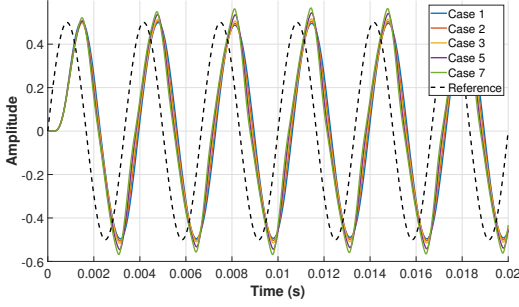
(b) Error Response at 100Hz



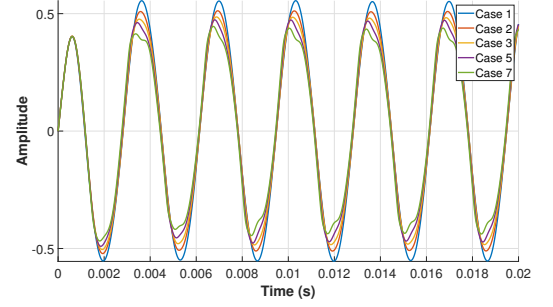
(c) Output Response at 150Hz



(d) Error Response at 150Hz



(e) Output Response at 300Hz



(f) Error Response at 300Hz

Figure 4.5: Time response to sinusoidal reference signal

LabVIEW, which bridges the host computer and the piezo-stage. The actuation voltage is limited to a range of 0 – 10V, and the FPGA-based control loop operates at a high sampling rate of $30\mu\text{s}$, ensuring precise real-time control.

5.2 LabVIEW Implementation

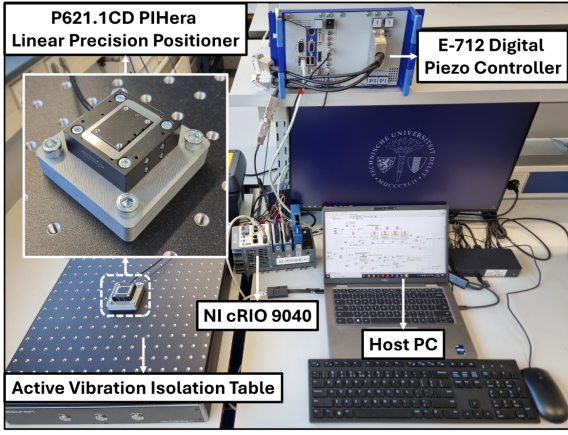
The control system is implemented in LabVIEW, utilizing the NI CompactRIO platform for real-time control and signal processing. The architecture consists of three main components: the FPGA VI, the Real-Time (RT) Main VI, and the Host VI. Each component plays a crucial role in the overall system operation:

- **FPGA.vi**

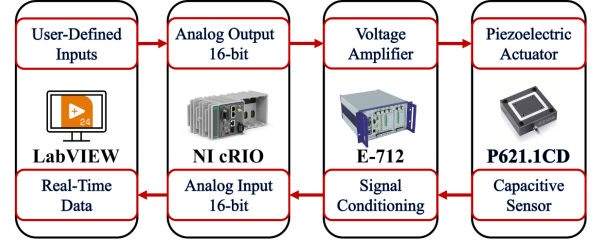
The *Field Programmable Gate Array (FPGA)* implementation encapsulates the complete signal generation and control logic. By leveraging the parallel processing capabilities of the FPGA, this module ensures deterministic and high-speed execution of time-critical operations, such as real-time signal generation, feedback control, and low-level hardware interfacing.

- **RT_Main.vi**

The *Real-Time (RT) Main VI* serves as an intermediary layer between the user's laptop (host system)



(a) Experimental setup of a piezo-actuated nanopositioning system



(b) System workflow of the piezo-actuated nanopositioning system

Figure 5.1: Experimental setup and system workflow

and the CompactRIO hardware. It is responsible for managing communication with the FPGA and maintaining system-wide global variables through `Globals.vi`. These variables can be dynamically updated (hot-fixed) during operation, allowing flexible adjustments without the need for system reboot.

- **Host.vi**

The *Host VI* operates on the user's operating system and primarily facilitates data acquisition, logging, and visualization. It communicates with the real-time controller to retrieve relevant signals and presents them in a user-friendly interface, supporting both monitoring and offline analysis.

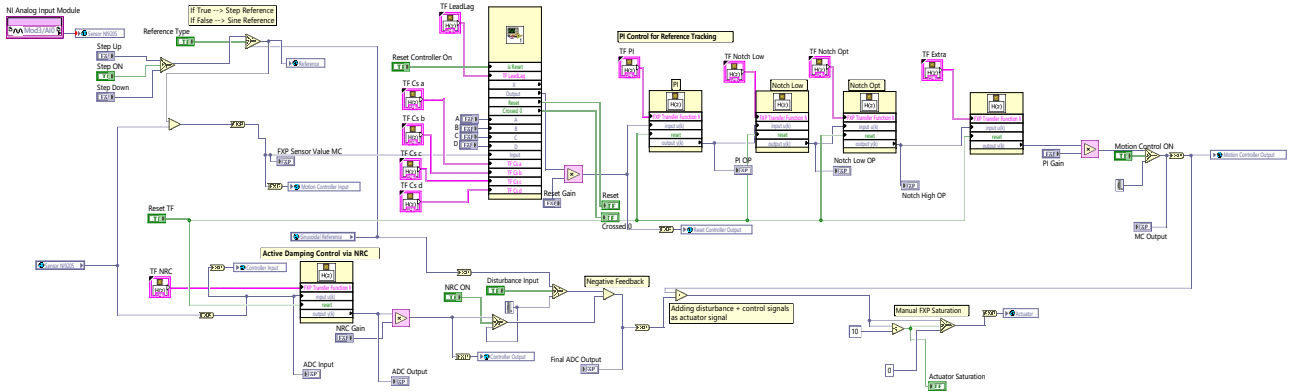


Figure 5.2: Block diagram structure of the FPGA implementation

Figure 5.2 illustrates the main structure of feedback control system implemented on the FPGA with a global sampling period $30\mu\text{s}$. The tracking controllers and damping controller are implemented by discrete transfer function blocks. The reset controller is encapsulated in a sub-VI `Reset Controller.vi`, which handles the reset logic and ensures smooth transitions during operation. Figure 5.3 shows the sub-VI block diagram of the triangular wave generator, which is used to generate the reference signal.

5.2.1 Reset Controller Implementation in LabVIEW

The reset controller is implemented as a sub-VI within the FPGA architecture. It is designed to handle the reset logic based on the reset condition defined in Equation 2.27. In discrete time, the reset condition is based

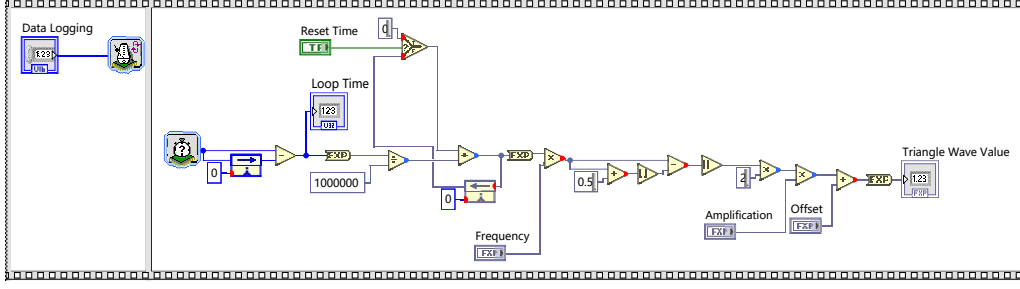


Figure 5.3: Block diagram of the triangular wave generator

on the previous step which is stored in a **Feedback Node** in LabVIEW. The crossing 0 condition is detected by multiplying the current state with the previous state and checking if the product is negative or equal to 0, indicating a sign change. Another **Feedback Node** is used to avoid consecutive resets, ensuring that the reset action is only triggered once per crossing event. The PFORE element is implemented as a discrete state space representation based on the definition. Whereas the lead-lag filter and shaping filter are implemented as discrete transfer function blocks. It is notable that the shaping filter is divided into 4 parts to avoid high order transfer function implementation in LabVIEW.

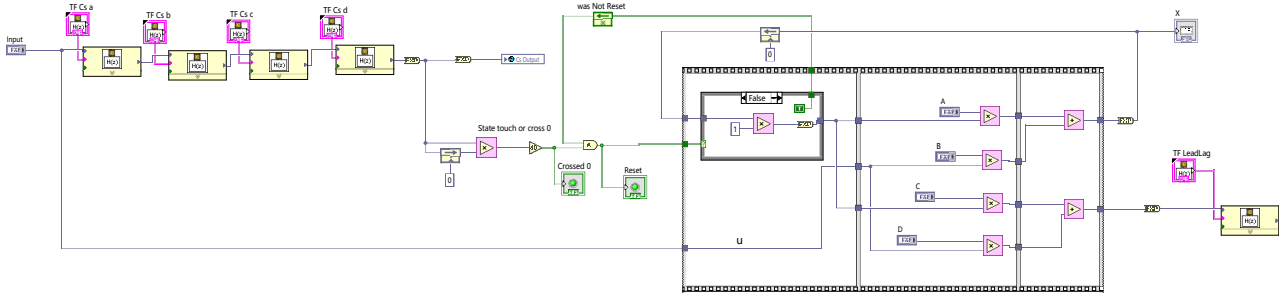


Figure 5.4: Block diagram of the reset controller

5.2.2 Controller Discretization and Implementation

The controller is implemented in discrete time with a sampling period of $30\mu s$. Bilinear transformation (Tustin's method) is used to discretize the continuous-time controller. The discretization process involves transforming the continuous transfer function into a discrete transfer function using the following formula:

$$H(z) = H(s) \Big|_{s=\frac{2}{T} \frac{z-1}{z+1}} \quad (5.1)$$

where $H(s)$ is the continuous transfer function, $H(z)$ is the discrete transfer function, and T is the sampling period. The resulting discrete transfer function is then implemented in LabVIEW using the **discrete transfer function** block. The frequency range ($[1\text{Hz}, 10^4\text{Hz}]$) of interest is significantly lower than the sampling frequency ($3.33 \times 10^4\text{Hz}$), ensuring that the discretization has minimal impact on the system performance.

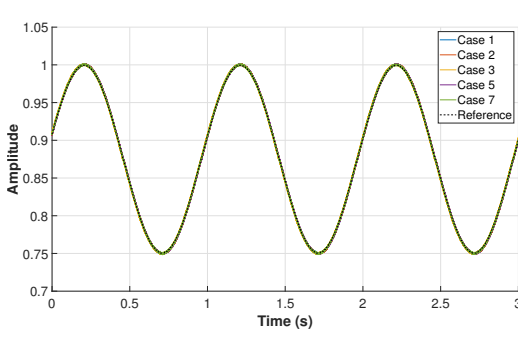
5.3 Results and Discussion

5.3.1 Time Response

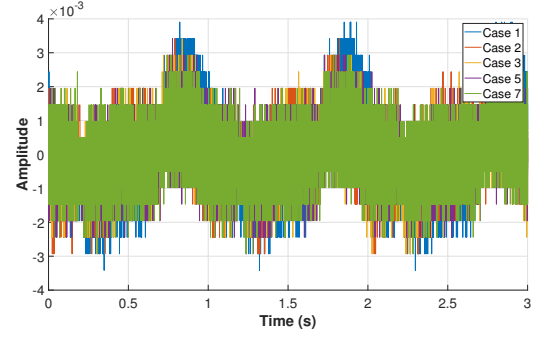
Sinusoidal Reference Response Figure 5.5 and 5.6 show the time response of the reset control system to sinusoidal reference signals at various frequencies. The left column displays the output response as compared with the reference signal, while the right column shows the error response.

The error response implies a good prediction of HOSIDF based frequency response analysis, as the error is reduced as compared to the linear control system. Reset control system with higher phase lead CgLP is able to improve the low frequency tracking performance more without losing the high frequency tracking performance. Meanwhile, there is no multiple reset events observed due to higher order harmonics which ensures the validity of HOSIDF method.

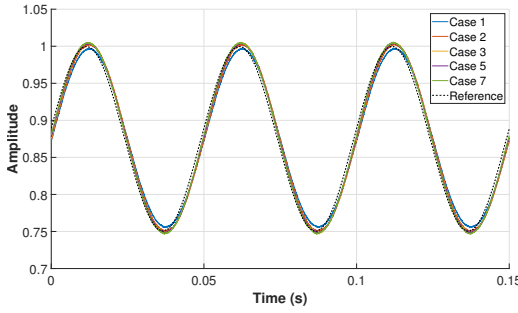
Similarly as discussed in Chapter 4.2, extremely low frequency tracking error are mainly caused by sensor noise. The multiple resets are not considered as the problem of higher order harmonics.



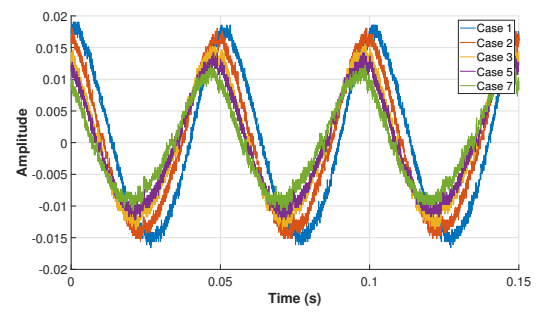
(a) Output at 1Hz



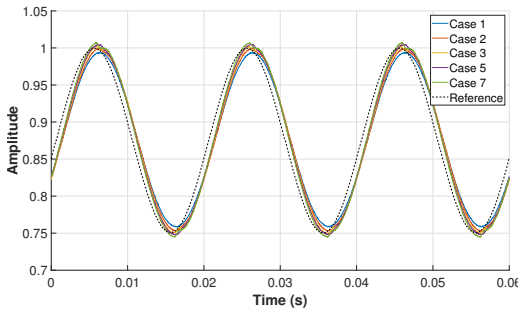
(b) Error at 1Hz



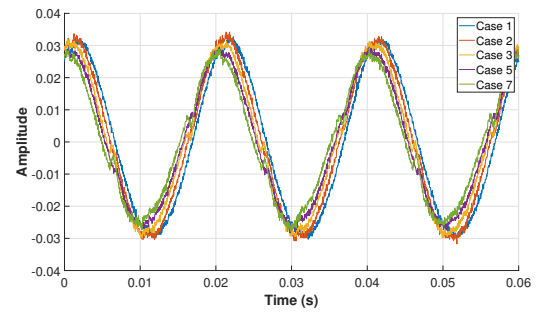
(c) Output at 20Hz



(d) Error at 20Hz



(e) Output at 50Hz



(f) Error at 50Hz

Figure 5.5: Time response to sinusoidal reference signals at various frequencies

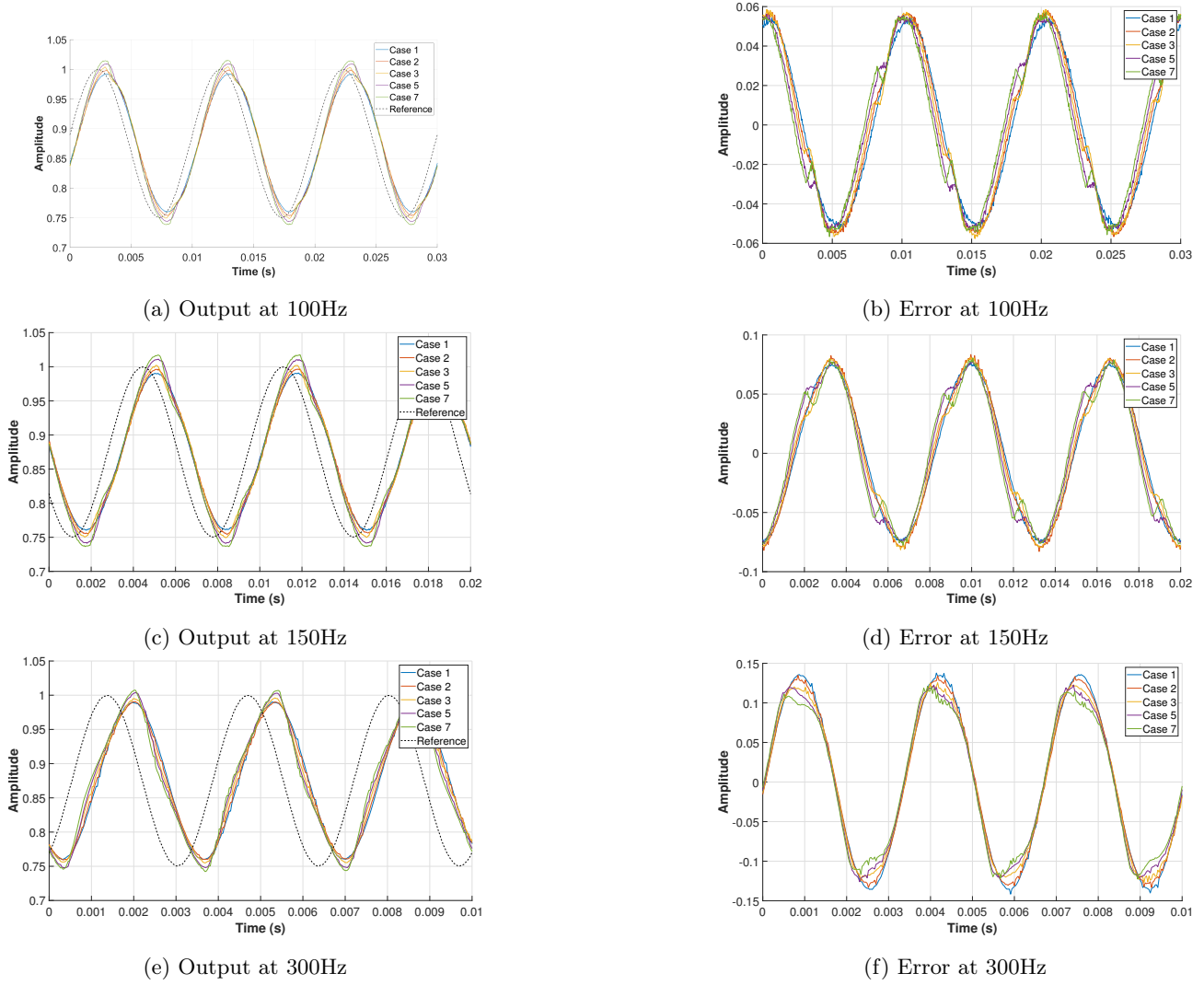
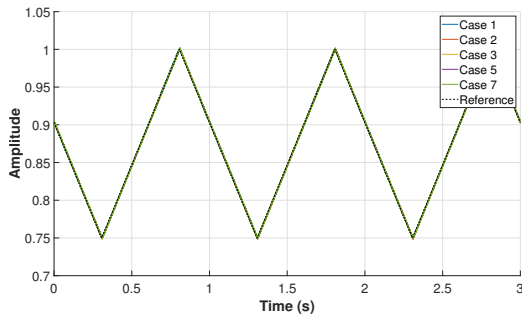


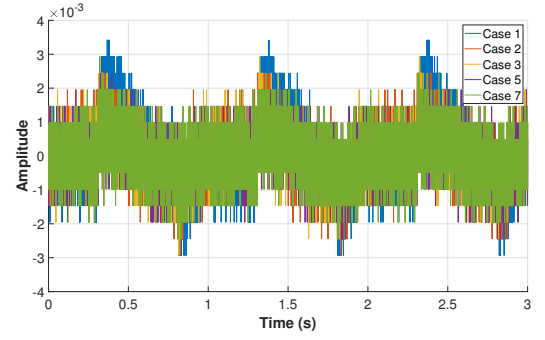
Figure 5.6: Time response to sinusoidal reference signal at various frequencies

Triangular Reference Response Other than sinusoidal reference signals, triangular reference signals are also used to validate the reset control system. Triangular wave is a periodic waveform whose Fourier transform consists of discrete odd harmonics (1, 3, 5, ...) and the fundamental harmonic account for 81.1% power contribution. The odd harmonics of the triangular wave will add up to the higher order harmonics generated by the reset control system which are also odd harmonics. Therefore, it amplifies the effect of higher order harmonics and allows us to observe the reset control system performance in a more challenging scenario.

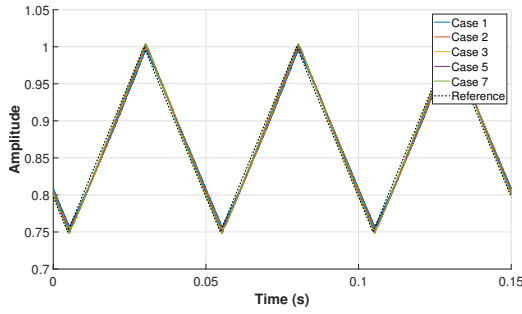
The following figures show the time response of the reset control system to triangular reference signals at various frequencies. The left column displays the output response as compared with the reference signal, while the right column shows the error response. It shows that the error response of triangular reference is also as well predicted by the HOSIDF based frequency response analysis. The reset control system is able to improve the low frequency tracking performance without losing the high frequency tracking performance. And high frequency tracking performance is also improved as compared to the linear control system. Multiple resets due to higher order harmonics are not observed in the triangular reference response.



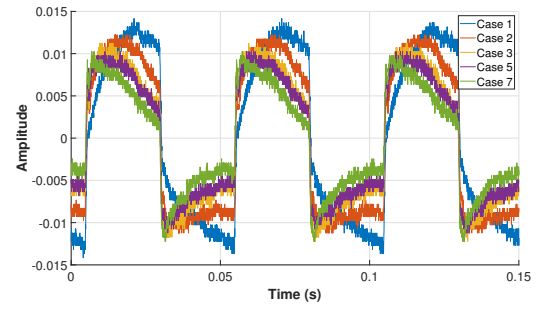
(a) Output at 1Hz



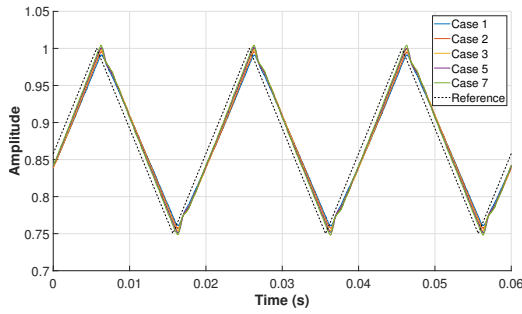
(b) Error at 1Hz



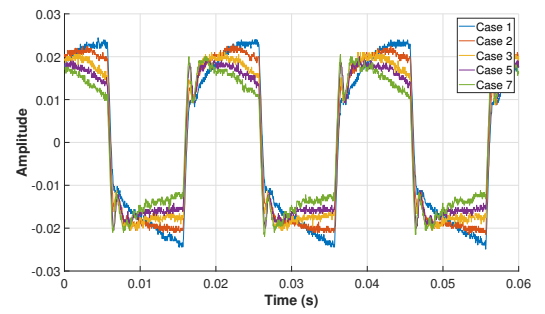
(c) Output at 20Hz



(d) Error at 20Hz

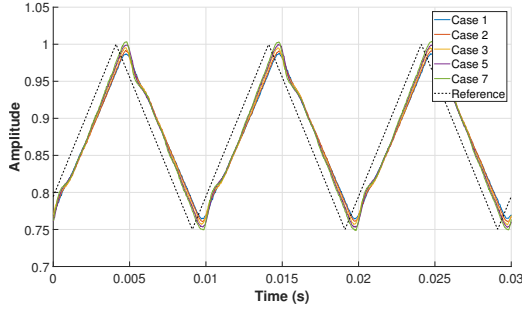


(e) Output at 50Hz

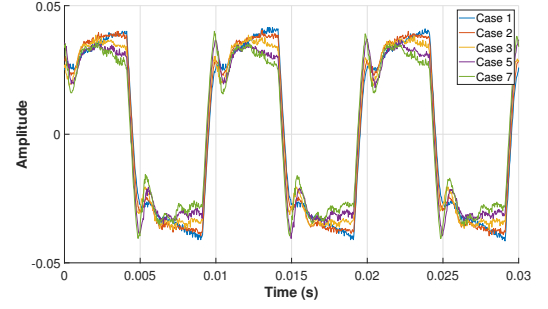


(f) Error at 50Hz

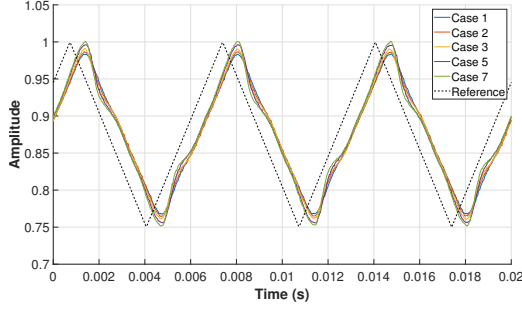
Figure 5.7: Time response to triangular reference signals at various frequencies



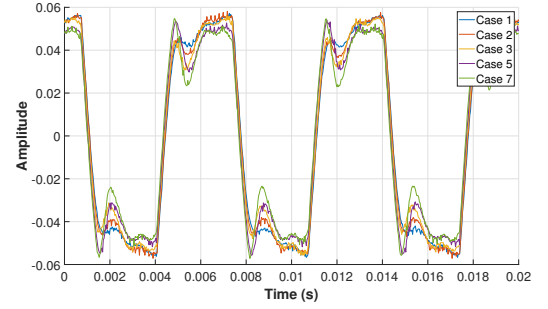
(a) Output at 100Hz



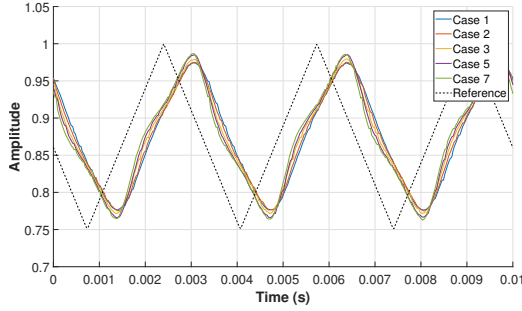
(b) Error at 100Hz



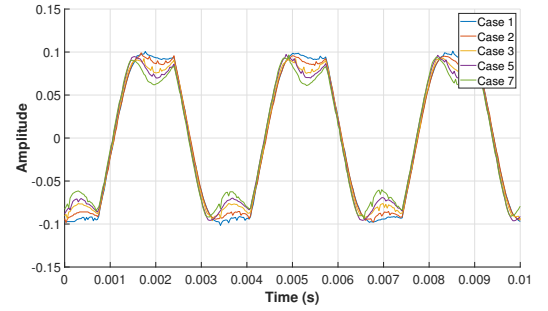
(c) Output at 150Hz



(d) Error at 150Hz



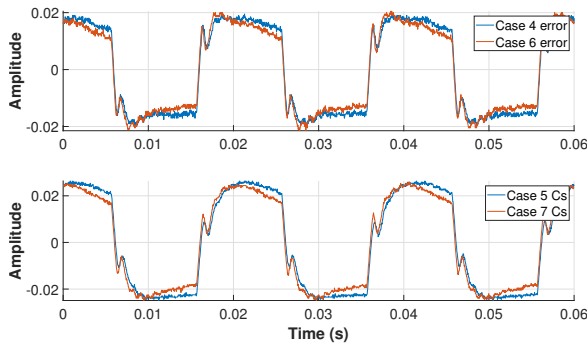
(e) Output at 300Hz



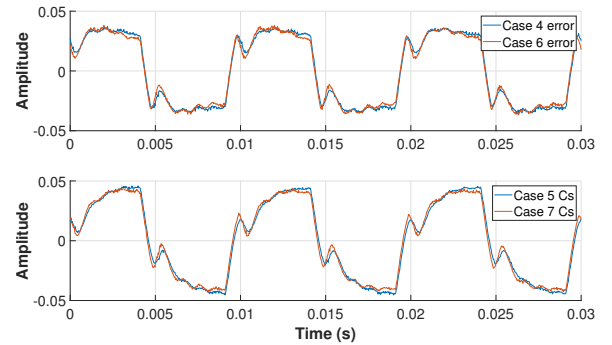
(f) Error at 300Hz

Figure 5.8: Time response to triangular reference signal at various frequencies

Effect of Shaping Filter on Reset Signal The shaping filter is also validated in the experimental setup. The shaping filter is designed to shape the reset signal to attenuate the higher order harmonics at specific frequencies. As shown in Figure 5.9, the shaping filter is able to prevent multiple reset by suppressing the peak of 3^{rd} harmonic effectively. This ensures the validity of the HOSIDF based frequency response analysis.



(a) Output Response to Sinusoidal Reference



(b) Error Response to Sinusoidal Reference

Figure 5.9: Time response to sinusoidal reference signal at 50Hz

5.3.2 Frequency Estimation

Time response is also used to estimate the frequency response function. By applying a chirp signal as the reference input which sweeps through a range of frequencies, the frequency response can be estimated using `tfestimate` in MATLAB. Figure 5.10 shows the estimated sensitivity response across all cases. It sees that the estimation is consistent with the HOSIDF based frequency response analysis and the time response results.

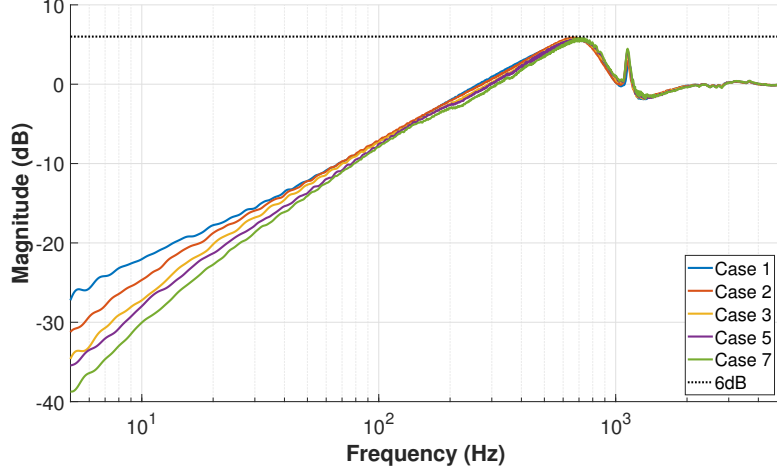


Figure 5.10: Sensitivity response estimation across all cases

Estimation by `tfestimate` is based on FFT and best linear approximation which is can be accurate for LTI systems. However, for reset control systems, higher order harmonics are generated which are not present in the input. `tfestimate` will try to fit a linear transfer function based on the dominant fundamental frequency component. In other words, it will not be able to capture the higher order harmonics.

Therefore, to capture the higher order harmonics, the L_∞ -Norm based frequency response estimation method can be used[20]. The L_∞ -Norm based frequency response estimation method is based on the idea of pseudo-sensitivity. It takes a single sinusoidal input and calculate the ratio between the maximum error and the maximum input in a complete period to estimate the transfer function at each frequency. The sensitivity transfer function can be calculated as:

$$S_\infty(\omega) = \frac{\max e_\omega(t)}{\max r_\omega(t)} \quad (5.2)$$

where $e_\omega(t)$ is the error signal at frequency ω and $r_\omega(t)$ is the sinusoidal input at frequency ω . By the prediction of HOSIDF based frequency response analysis, one can achieve a good $S_\infty(\omega)$ estimation by selecting dense frequencies between 100Hz and 200Hz.

As shown in Figure 5.11, the $S_\infty(\omega)$ estimation is able to capture the higher order harmonics and matches the prediction of HOSIDF based frequency response analysis better than the `tfestimate` method.

The L_∞ -Norm based sensitivity estimation is also able to illustrate the effect of shaping filter in frequency domain. As shown in Figure 5.12, the sensitivity response estimation comparing with and without shaping filter shows that the shaping filter is able to reduce the error due to higher order harmonics at the desired frequency range as predicted.

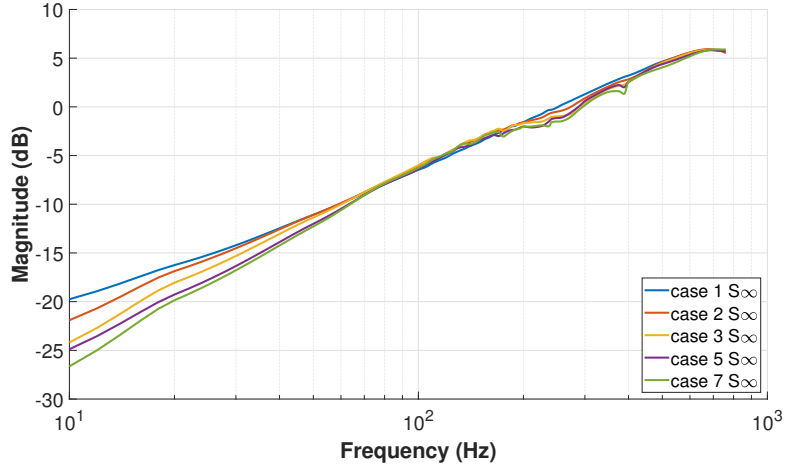
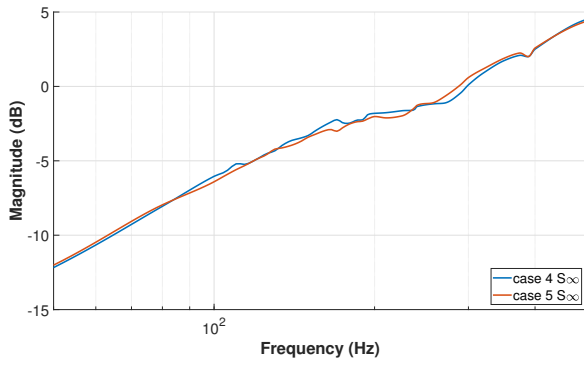
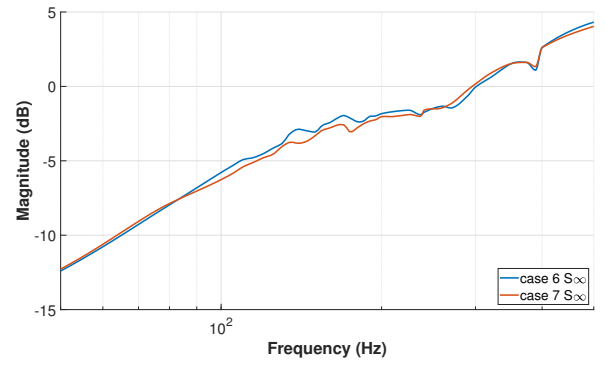


Figure 5.11: Sensitivity response estimation by L_∞ -Norm across all cases



(a) Closed loop response estimation comparing Case 4 and Case 5



(b) Sensitivity response estimation comparing Case 6 and Case 7

Figure 5.12: Sensitivity response estimation by L_∞ -Norm comparing with and without shaping filter

6 Conclusion and Recommendations

6.1 Conclusion

This thesis investigated the design and implementation of reset controller in dual loop damping control system. Through theoretical analysis, simulation, and experimental validation, the work demonstrated how reset elements can overcome fundamental performance limitations of linear controllers, particularly in terms of phase compensation and transient response improvement.

The research reveals the potential of implementing high phase lead CgLp in dual loop damping architecture to enhance the tracking performance of the lightly damped nanopositioning system, P-628.1CD. The 1st dominant resonance was significantly suppressed by applying NRC damping controller. A linear tracking controller PI with extra optimization filters was designed to significantly improve both the open loop and closed loop bandwidth while maintaining enough stability margins. The proposed dual loop control architecture enhanced the reference tracking and disturbance rejection performance of the nanopositioning system.

We pushed the boundaries of LTI control by integrating reset elements. The novel reset controller CgLp using proportional FORE was designed to provide phase lead compensation at any specific frequency range with flat gain. With the extra phase lead, the system was able to achieve a higher open loop bandwidth without losing stability. Meanwhile, the cut-off frequency of the integrator, which plays a crucial role in the low frequency performance, was improved without losing high frequency performance. CgLp with phase lead 5°, 10°, 15° and 20° was designed based on the HOSIDF analysis. The effect of higher order harmonics is observed in high phase lead CgLp (15°, 20°) which resulted in multiple resets at low frequency and inevitable gain increase of higher order harmonics. To address these challenges, a dedicated band-passing shaping filter placed at reset line was designed to limit the nonlinearity inside a specific frequency range. The gain of higher order harmonics was reduced outside the frequency range, and the multiple resets were effectively prevented.

The design was validated through extensive simulations and experiments, confirming its effectiveness in enhancing the performance of the nanopositioning system in practice. Sinusoidal reference and triangular wave reference signals were used in experiments to evaluate the HOSIDF based frequency analysis prediction and the systems' tracking performance with real noise and disturbance. Furthermore, the frequency response estimation was calculated based on the experimental transient response data. Infinity norm was used to capture the nonlinearity of the reset system to provide more accurate frequency response estimation.

6.2 Future Work Recommendations

Future research could explore more potential shaping filter designs to further manage the nonlinearity of CgLp. The phase shaping can be further optimized to achieve desired CgLp phase lead characteristics and the effect of the shaping filter gain to tackle multiple resets can be investigated. Additionally, other reference signal types, such as step or impulse signals, could be tested to evaluate the robustness of the proposed reset control strategy under different operating conditions. The frequency response estimation of the reset system can be further refined to improve accuracy and reliability which helps in the tuning of reset elements.

Appendix

A Time Response of Simulation

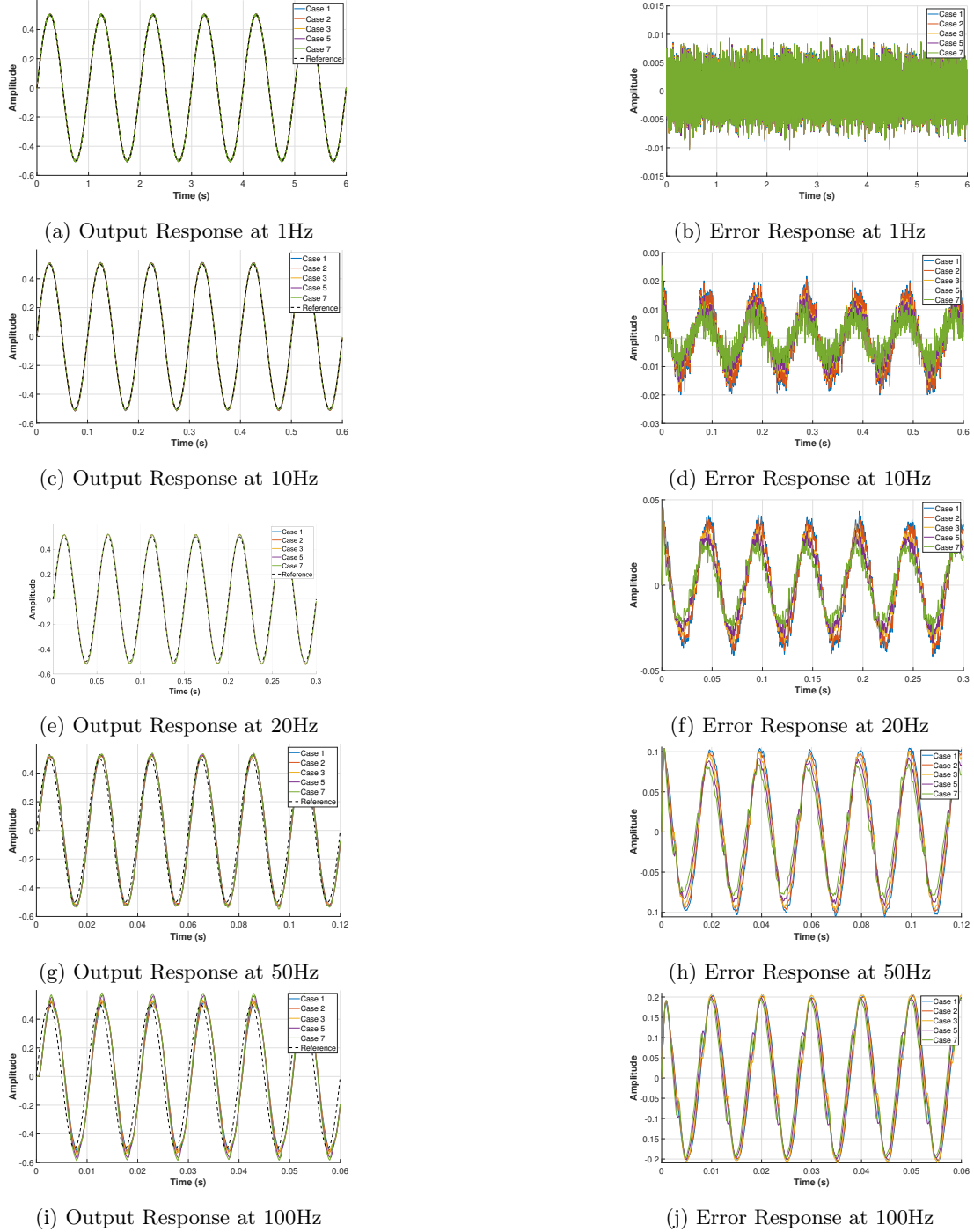
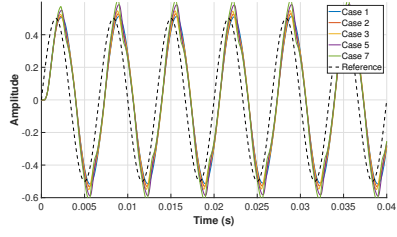
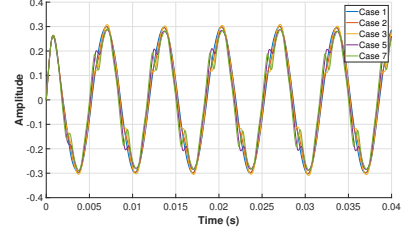


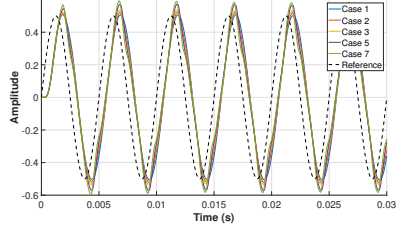
Figure A.1: Time response to sinusoidal reference signal



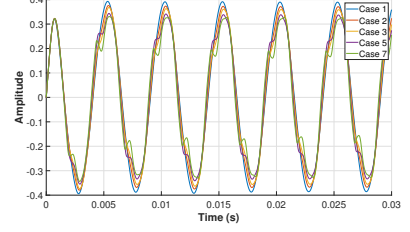
(a) Output Response at 150Hz



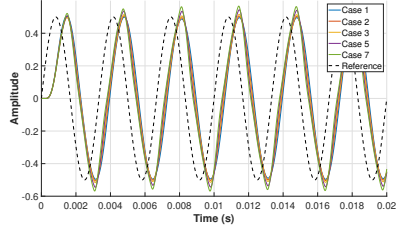
(b) Error Response at 150Hz



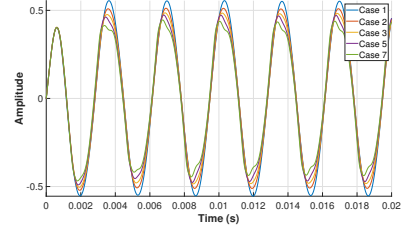
(c) Output Response at 200Hz



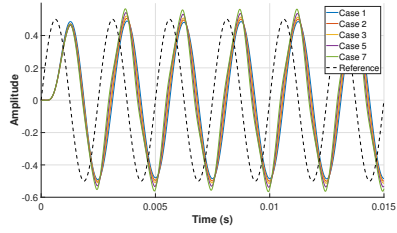
(d) Error Response at 200Hz



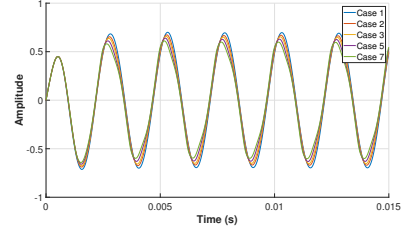
(e) Output Response at 300Hz



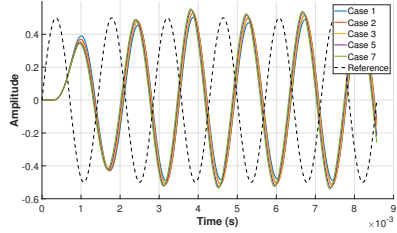
(f) Error Response at 300Hz



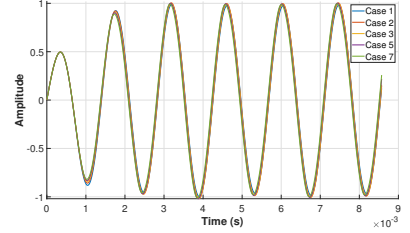
(g) Output Response at 400Hz



(h) Error Response at 400Hz



(i) Output Response at 700Hz



(j) Error Response at 700Hz

Figure A.2: Time response to sinusoidal reference signal

B Time Response of Experimental Data

B.1 Sinusoidal Reference Response

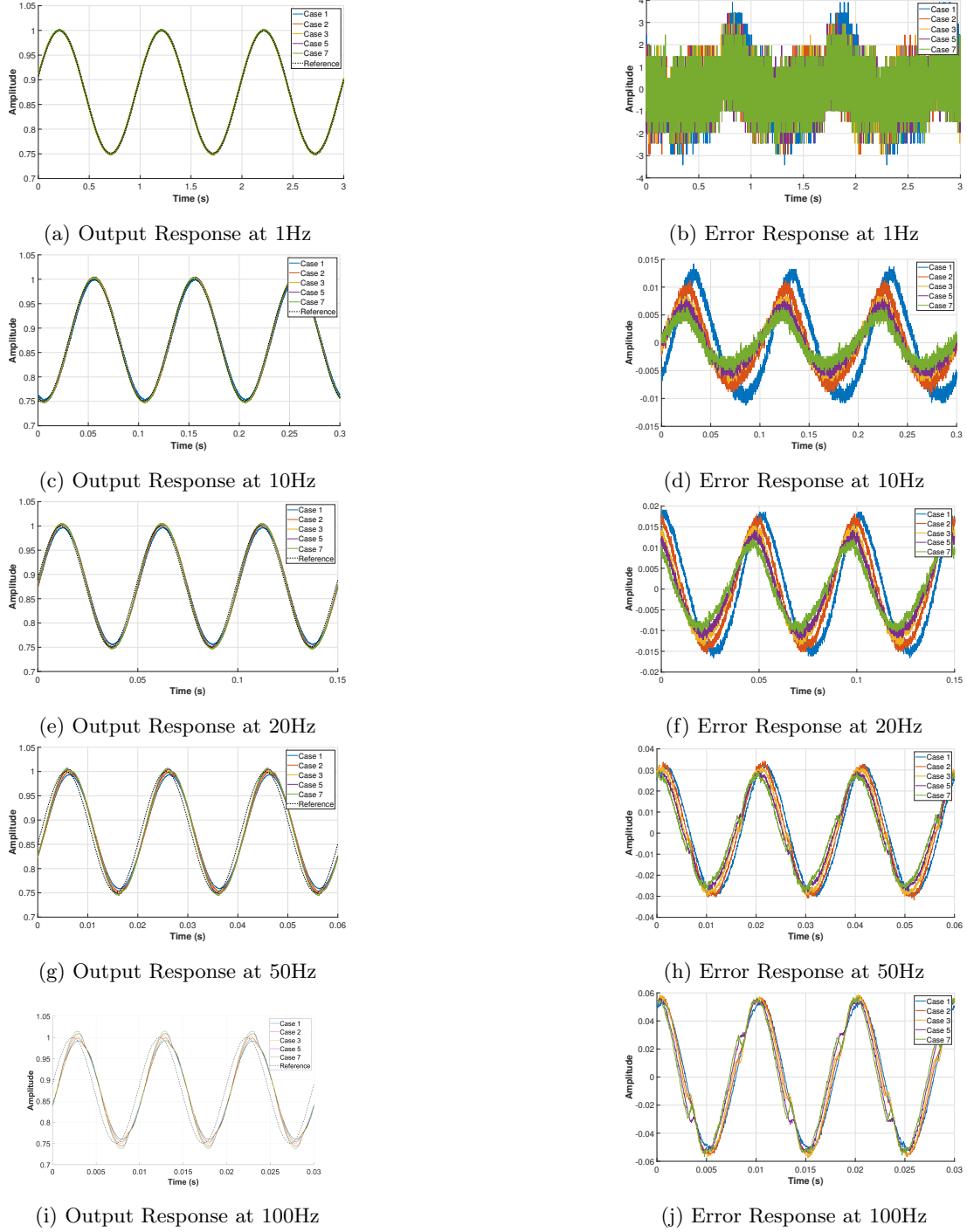
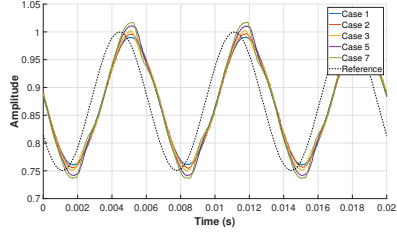
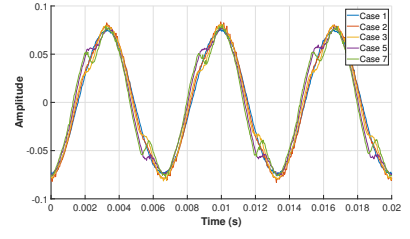


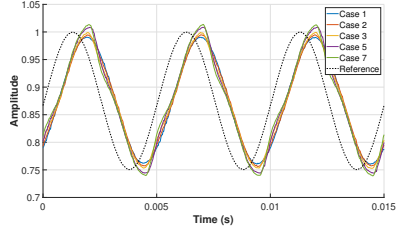
Figure B.1: Time response to sinusoidal reference signal



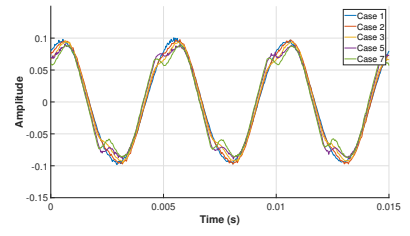
(a) Output Response at 150Hz



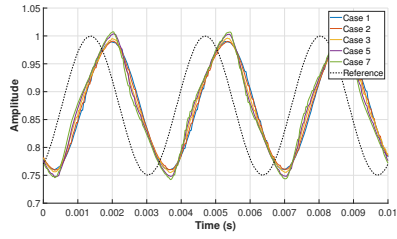
(b) Error Response at 150Hz



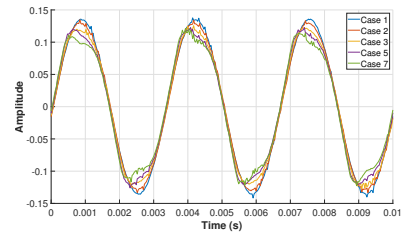
(c) Output Response at 200Hz



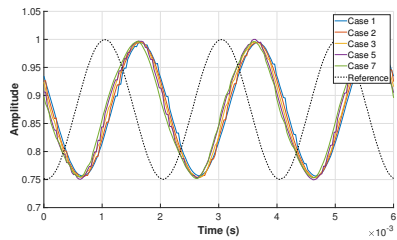
(d) Error Response at 200Hz



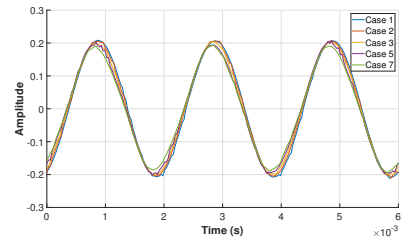
(e) Output Response at 300Hz



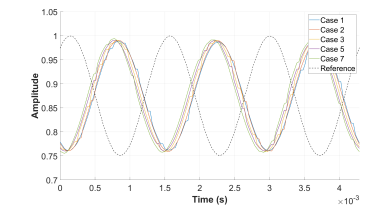
(f) Error Response at 300Hz



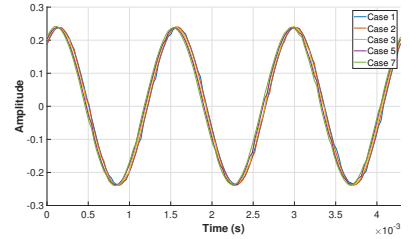
(g) Output Response at 500Hz



(h) Error Response at 500Hz



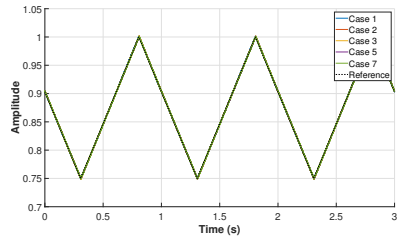
(i) Output Response at 700Hz



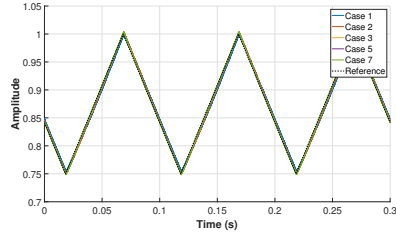
(j) Error Response at 700Hz

Figure B.2: Time response to sinusoidal reference signal

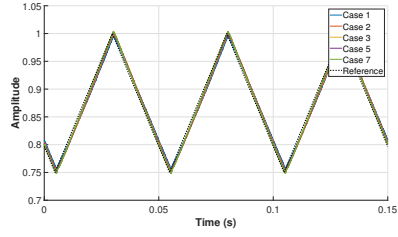
B.2 Triangular Reference Response



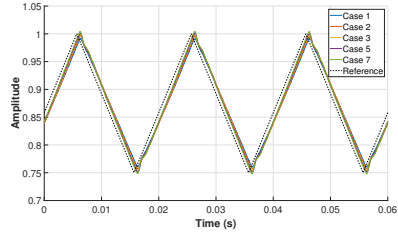
(a) Output Response at 1Hz



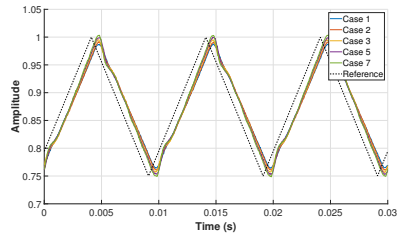
(c) Output Response at 10Hz



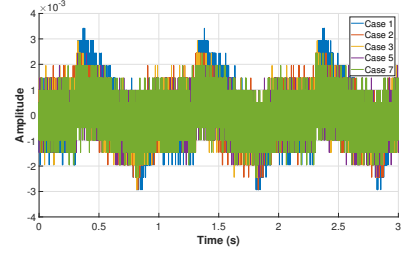
(e) Output Response at 20Hz



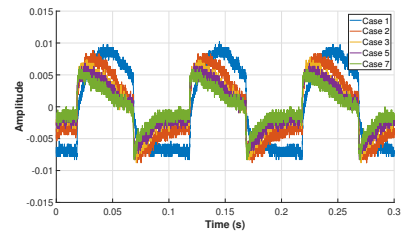
(g) Output Response at 50Hz



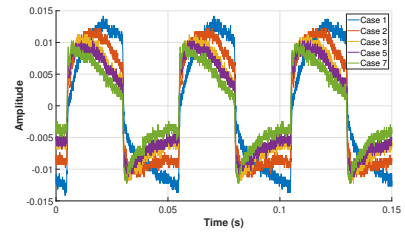
(i) Output Response at 100Hz



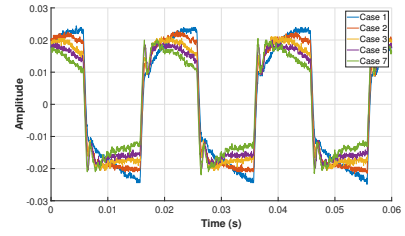
(b) Error Response at 1Hz



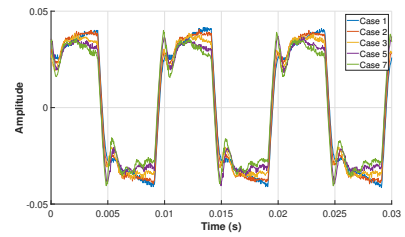
(d) Error Response at 10Hz



(f) Error Response at 20Hz

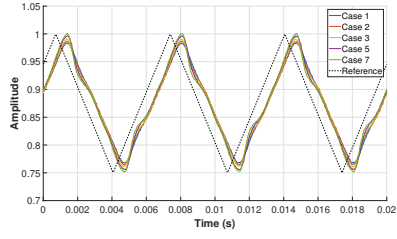


(h) Error Response at 50Hz

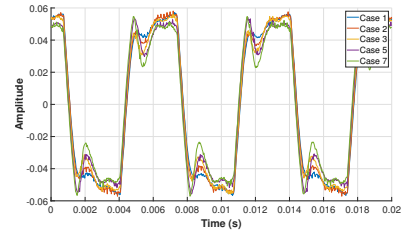


(j) Error Response at 100Hz

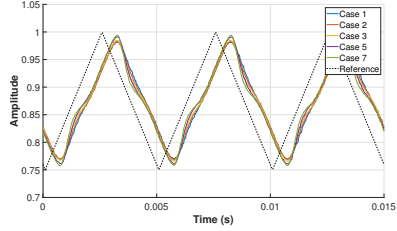
Figure B.3: Time response to sinusoidal reference signal



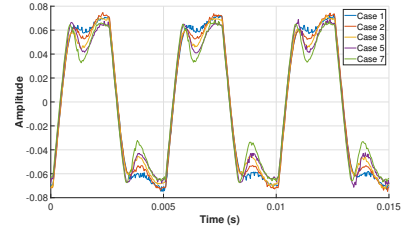
(a) Output Response at 150Hz



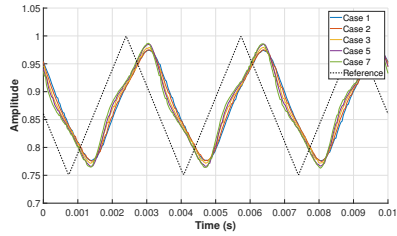
(b) Error Response at 150Hz



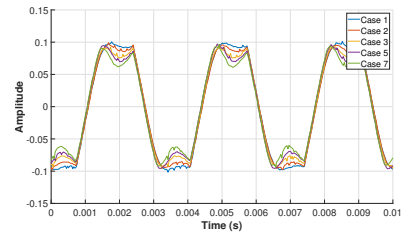
(c) Output Response at 200Hz



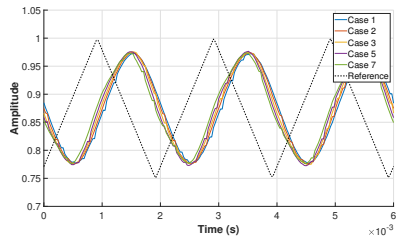
(d) Error Response at 200Hz



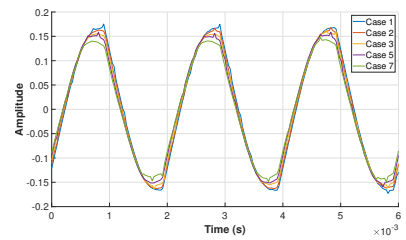
(e) Output Response at 300Hz



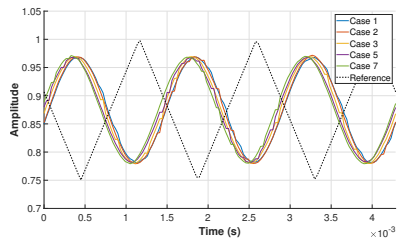
(f) Error Response at 300Hz



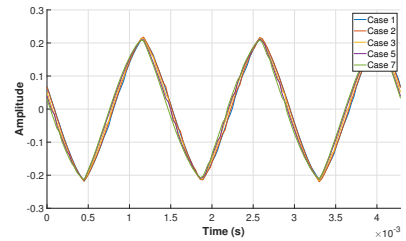
(g) Output Response at 500Hz



(h) Error Response at 500Hz



(i) Output Response at 700Hz



(j) Error Response at 700Hz

Figure B.4: Time response to sinusoidal reference signal

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