A SENSITIVITY ANALYSIS APPLIED TO MORPHOLOGICAL COMPUTATIONS

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INTRODUCTION

In river engineering morphological predictions have to be made to study the implications of changes in a river system due to natural causes or human interference.

It regards here *time-depending processes*. Characteristic parameters of the river have to be forecasted both in time and space. The morphological processes, however, are extremely complex and therefore a substantial degree of schematization is required before e.g. mathematical models can be applied to obtain the predictions wanted. Information about available mathematical models can e.g. be obtained from Jansen (1979) and Klaassen *et al* (1982).

The present physical-mathematical formulation of the morphological problems involved is incomplete. For instance the variation of the width B(x,t) cannot yet be predicted. Therefore in this paper the restriction is made that (relatively) unerodible banks are present.

The description of the problem in two (horizontal) space dimensions has not yet led to mathematical models that are used on a routine basis. During the last two decades or so, however, one-dimensional models have been developed gradually to useful tools for practical problems. In these models average values across the width of the river are predicted for the waterlevel h(x,t), the bedlevel z(x,t) and consequently for the water depth a(x,t).

There is concern, however, regarding the accuracy of these predictions. Very few possibilities exist to calibrate and verify the mathematical models for a particular river. There are a large number of error sources of which here basicly two will be discussed.

Sediment transport s(x,t) has to be predicted from the local hydraulic conditions.

Alluvial roughness, for instance expressed in the Chézy coefficient C(x,t), also has to be forecast locally.

The available *transport predictors* and *roughness predictors* are based on the presence of *steady uniform flow*. Hence there is already a potential source of errors in applying these predictions in a mathematical model with non-steady and non-uniform flow.

The two types of predictions are linked. In many transport predictors (transport formulae) the alluvial roughness has to be known. As future conditions are considered, this roughness also has to be predicted.

Hence the predicted C(x,t) is used twice in the morphological forecasts:

(i) *Explicitly* in order to incorporate properly the hydraulic friction term in the basic hydrodynamic equations.

(ii) *Implicitly* in the transport prediction.

It is the aim of this study to demonstrate the influence of the predictions of s(x,t) and C(x,t) on the results of the morphological predictions.

Therefore in Chapter 2 some general morphological problems are described mathematically and solved analytically. Analytical solutions have the advantage that the sensitivity analysis (Chapter 3) can easier be made than for numerical solutions. In Chapter 4 the results are discussed and some conclusions are given.

2. MORPHOLOGICAL MODELS

2.1 General

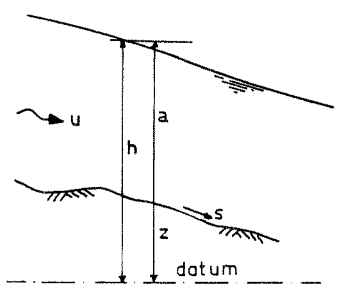
For the formulation of some morphological time-depending problems it is necessary in the first place to discuss the basic-equations. This will here only be done along broad lines (Section 2.2) as a more complete analysis is readily available (Jansen, 1979).

For this study analytical solutions of the basic equations are attractive in order to arrive at some general conclusions. The price to be paid, however, is that in most cases the basic equations have to be linearized. This may reduce the validity of the general conclusions.

2.2 Basic Equations

2.2.1 General

Consider a wide alluvial river of which the unit of width is taken. The independent variables are the x-coordinate in the stream direction and the time t.



The dependent variables are

flow velocity	u(x,t)
water depth	a(x,t)
bed level	z(x,t)
sediment transport	s(x,t)
Chézy roughness	C(x,t)

Fig. 1 Definition sketch

The following equations for the dependent variables are available.

Watermovement

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial a}{\partial x} + g\frac{\partial z}{\partial x} = -g\frac{u|u|}{C^2 a} \qquad (1)$$

$$\frac{\partial a}{\partial t} + u\frac{\partial z}{\partial x} + a\frac{\partial u}{\partial x} = 0 \qquad (2)$$

Sediment movement

$$\frac{\partial z}{\partial t} + \frac{\partial s}{\partial x} = 0 \qquad (3)$$

$$s = f\{u, C, \Delta, D \text{ etc.}\} \qquad (4)$$

The grain size D is supposed to be constant (no grain sorting). This is also assumed for the relative density $\Delta = (\rho_s - \rho)/\rho$ of the grains. In fact one equation is lacking to solve the 5 dependent variables. This is of course a predictor for the alluvial roughness. For the time being C is supposed to be known and to make in the following Section analytical solutions possible C is even assumed to be constant.

Equation (4) is the transport predictor written in a very general way. Later on this equation will be specified.

It has been shown (de Vries, 1959, 1965) that for rivers with slow variations of the discharge the terms $\partial u/\partial t$ and $\partial a/\partial t$ from Eqs. (1) and (2) respectively can be neglected with respect to the other terms in their equations.

The equations can then be reduced to a system of three differential equations for the dependent variables u(x,t); a(x,t) and z(x,t).

$$u\frac{\partial u}{\partial x} + g\frac{\partial a}{\partial x} + g\frac{\partial z}{\partial x} = -g\frac{u^2}{C^2 a} = -g\frac{u^3}{C^2 q} \qquad (5)$$

$$u\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial x} = 0 \quad \text{or} \quad q(x,t) = q(t) \qquad (6)$$

$$\frac{\partial z}{\partial t} + \left[\frac{ds}{du}\right]\frac{\partial u}{\partial x} = 0 \qquad (7)$$

The derivative ds/du is known from Eq. (4).

For moderate Froude numbers $Fr = u/\sqrt{ga}$ the term $u\ \partial u/\partial x$ in Eq. (5) can be neglected with respect to the term $g\ \partial a/\partial x$ as

$$u \frac{\partial u}{\partial x} + g \frac{\partial a}{\partial x} = \frac{\partial}{\partial x}\left[\tfrac{1}{2}u^2 + ga\right] = \frac{\partial}{\partial x}\left[ga\ \{\tfrac{1}{2}Fr^2 + 1\}\right] \approx g \frac{\partial a}{\partial x} \qquad (8)$$

It has been shown (de Vries, 1959, 1965) that Eqs. (5) ... (7) contain the celerity c of a small disturbance at the river bed with

$$c = u \frac{ds/du}{a} \left\{ \frac{1}{1 - Fr^2} \right\} \approx u \frac{ds/du}{a} \qquad (9)$$

The last approximation is based on the assumption $Fr \ll 1$, made here in order to make further analytical manipulations with the simplified basic equations easily possible.

In the following subsections three possible schematizations are used for the Eqs. (5), (6) and (7). The main characteristics for these three cases are:

Case I: The *hyperbolic* character of the basic equations is maintained if neither the backwater effects nor the hydraulic friction term are neglected.
The differential equations can then, however, only be solved analytically after *linearization* and for a *constant discharge*.

Case II: A *parabolic* model is obtained by neglecting the backwater effects in the hydrodynamic equations. Problems for *time-depending discharges* can be solved. A restriction for analytical solutions is that *linearization* is necessary and only *relatively large values of* x (and/or t) can be considered.

Case III: A *simple wave* model which is suitable for analytical solutions is obtained when the hydraulic friction term is negligible.
The case considered here regards a *constant discharge*. However, *no linearization* is necessary. Neglecting the friction implies that *relatively short distances* (x) can be considered only.

In the following subsections for each case an analytical solution is given for a suitable problem.

2.2 Case I

For a constant discharge the basic equations can be combined into one equation in the bed level (z) (Vreugdenhil and de Vries, 1973)

$$\frac{\partial z}{\partial t} - K\frac{\partial^2 z}{\partial x^2} - \frac{K}{c}\frac{\partial^2 z}{\partial x \partial t} = 0 \qquad (10)$$

in which c has the same meaning as before.

The equation can be used to study changes in an originally uniform flow with constant bed slope i_o. The Eq. (10) is obtained after linearization and for the constant coefficient K holds

$$K = \frac{\{ds/du\}_o}{\{di/du\}_o} \qquad (11)$$

As Eq. (10) is *hyperbolic*, the originally hyperbolic character of the basic equations is maintained.

Instead of Eq. (10) it is easier to use two first order partial differential equations in bed level and water level (z and h).
Taking the z-axis along the initial bed slope (i_o) positive upstream and considering water level variations η from the original one (h_o) these equations can be formulated in z and η.

Linearization is then carried out by assuming $u(x,t) \approx u_o$ and $\eta(x,t) \ll h_o$ (for details see de Vries, 1980).

The result is

$$\frac{\partial \eta}{\partial x} - \beta\{\eta - z\} = 0 \qquad (12)$$

$$\frac{\partial z}{\partial t} + c\frac{\partial z}{\partial x} - c\frac{\partial \eta}{\partial x} = 0 \qquad (13)$$

with

$$\beta = \left[\frac{di}{du}\right]_o \cdot \frac{u_o}{h_o} \qquad (14)$$

and

$$c = \left[\frac{ds}{du}\right]_o \cdot \frac{u_o}{h_o} \qquad (15)$$

Apparently $K = c/\beta$.

This hyperbolic model is applied to the following case (Fig. 2)

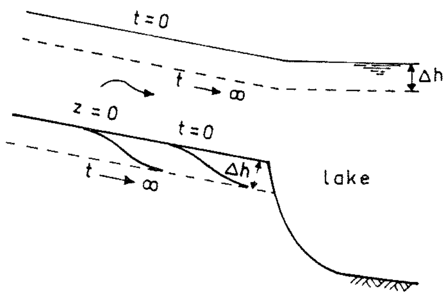


Fig. 2 Case I

A river with a constant discharge q is flowing into a lake. At t = 0 the lake level is dropped over a certain distance Δh. Solutions for $\eta(x,t)$ and $z(x,t)$ are sought. For $\Delta h \ll h_o$ Eqs. (12) and (13) apply with the initial conditions:

$$z(x,0) = 0 \quad \text{and} \quad \eta(x,0) = 0 \tag{16}$$

The boundary conditions are:

$$\lim_{x \to \infty} z(x,t) = 0 \quad ; \quad \lim_{x \to \infty} \eta(x,t) = 0 \tag{17}$$

and

$$\eta(0,t) = -\Delta h \cdot H(t) \tag{18}$$

in which H(t) is the Heaviside function (unit-step function).

It is possible to arrive at an analytical solution for the relative variation of the depth at the river mouth $\xi(0,t)$ (de Vries, 1980).

$$\xi(0,t) = -\frac{\Delta a(0,t)}{\Delta h} = e^{-\tau}\left[I_o(\tau) + I_1(\tau)\right] \tag{19}$$

in which

$\tau = 2\beta ct$ = dimensionless time

I_ν = modified Bessel function of the first kind and the ν^{th} order.

Figure 3 gives a dimensionless representation of Eq. (19).

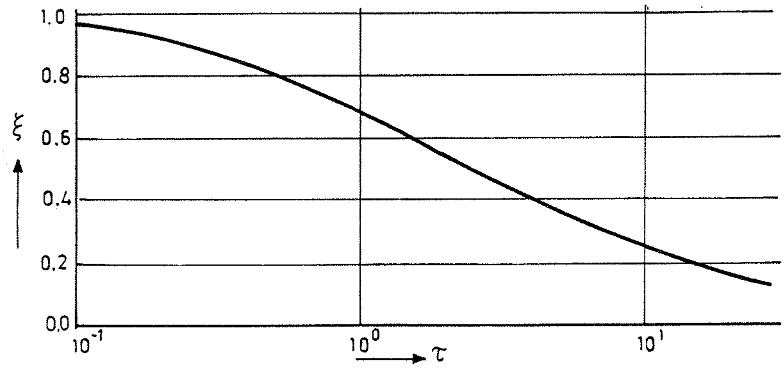


Fig. 3 Dimensionless depth reduction $\xi = \xi(0,\tau)$

In Chapter 3 the solution of Eq. (19) will be used to study the sensitivity of the result to errors in the prediction of s(x,t) and C(x,t).

2.3 Case II

The basic equations given in Eq. (5) ... (7) can further be simplified by assuming the absence of backwater effects. Equation (5) then becomes

$$\frac{\partial z}{\partial x} = - \frac{u^3}{C^2 q} \tag{20}$$

Combining Eqs. (6), (7) and (20) leads to a *parabolic model* (de Vries, 1973, 1975).

$$\frac{\partial z}{\partial t} - K(t) \frac{\partial^2 z}{\partial x^2} = 0 \tag{21}$$

in which the 'diffusion coefficient' has after linearization the same meaning as in the hyperbolic model of Case I (see Eq. (11)). The simplification obtained in neglecting the backwater effects that only validity for relative large distances from the point where backwater effects are induced, can be considered. This is contrary to the hyperbolic model used for Case I. On the other hand contrary to the hyperbolic model in the parabolic model K may vary in time. Both models have in common that they have been linearized.

Case II consists in applying the parabolic model to the same problem as in Case I (Fig. 4). The x-axis is again taken along the initial straight bed level with a slope (i_o); x is positive in the upstream direction. This does not change Eq. (21).

The initial condition is again

$$z(x,0) = 0 \qquad (2$$

The boundary conditions read

$$\lim_{x \to \infty} z(x,t) = 0 \qquad (2$$

and

$$z(0,t) = -\,\Delta h \cdot H(t) \qquad (2$$

in which H(t) is again the Heaviside function.

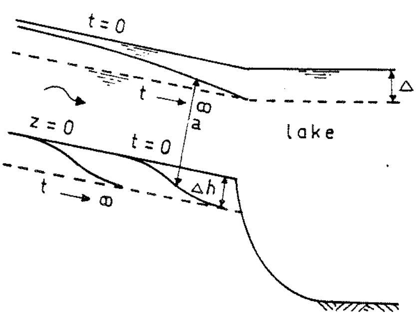


Fig. 4 Case II

The solution reads (see for details de Vries, 1975)

$$z(x,t) = -\,\Delta h \text{ erfc} \left[\frac{x}{2\sqrt{Kt}} \right] \qquad (2$$

in which

$$\text{erfc } y = \frac{2}{\sqrt{\pi}} \int_{y}^{\infty} \exp\{-\xi^2\} d\xi \qquad (2$$

This solution holds as a constant discharge (and hence a constant K value) is assumed. For a varying discharge the solution is

$$z(x,t) = -\,\Delta h \text{ erfc} \left[\frac{x}{2\sqrt{\int_{0}^{t} K(t')dt'}} \right] \qquad (2$$

2.3 Case III

The Eqs. (5) ... (7) can also be combined by eliminating the dependent variables u(x,t) and a(x,t) into the *simple-wave equation*

$$\frac{\partial z}{\partial t} + c\,\frac{\partial z}{\partial x} = R \cdot \frac{c}{g} \tag{28}$$

in which for $Fr^2 \ll 1$ again

$$c = u\,\frac{ds/du}{a} \tag{29}$$

and

$$R = -g\,\frac{u^3}{C^2 q} \tag{30}$$

For practical computations it is not attractive to apply Eq. (28) in stead of Eq. (5) ... (6). However, for relatively short distances the friction term i.e. the right-hand term of Eq. (5) becomes negligible. This means that in Eq. (31) $Rc/g \to 0$ or

$$\frac{\partial z}{\partial t} + c\,\frac{\partial z}{\partial x} = 0 \tag{31}$$

In this fashion the simple-wave equation is attractive to be used to study the deformation of a trench dredged across a river (Fig. 5). The case of bedload transport only is considered. Moreover like in the Cases I and II the discharge q is assumed to be constant as well as the assumption $Fr^2 \ll 1$.

Now it is not necessary to linearize the equation. Hence $c = c(x,t)$.

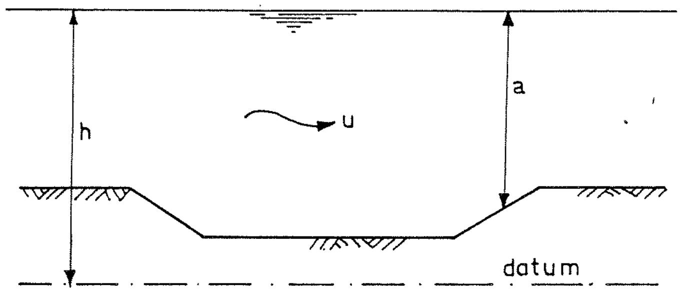


Fig. 5 Case III

The equation of motion for water reduces here to

$$\frac{\partial a}{\partial x} + \frac{\partial z}{\partial x} = \frac{\partial h}{\partial x} = 0 \tag{32}$$

Hence h(x,t) = constant (*rigid-lid approximation*).

It is attractive now to rewrite Eq. (31) with a(x,t) as the dependent variable. With Eq. (32), (29) and (6) this leads to

$$\frac{\partial a}{\partial t} + c(a)\,\frac{\partial a}{\partial x} = 0 \tag{33}$$

in which

$$c(a) = -\frac{ds}{da} \tag{34}$$

As an approximation for the transport formula the power law $s = mu^n$ can be taken in which m and n are not dependent on x and t.
Hence

$$\frac{\partial a}{\partial t} + \left[\, nmq^n \cdot a^{-(n+1)} \,\right] \cdot \frac{\partial a}{\partial x} = 0 \tag{35}$$

For the purpose of this paper the special attention goes to the behaviour of the downstream slope of the trench (Fig. 6).

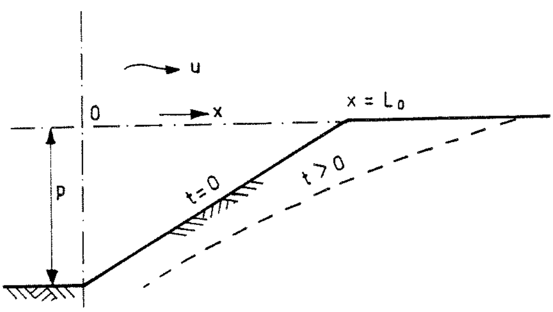


Fig. 6 Downstream slope

Taking the original bedlevel as the x-axis, the downstream slope is for t = 0 given by:

$$a(x,0) = a_o + p\left[\frac{L_o - x}{L_o}\right] \qquad 0 < x < L_o \tag{36}$$

$$a(x,0) = a_o \qquad x > L_o$$

A certain depth a with $a_o < a < a_o + p$ is for $t > 0$ present at a location which is a distance $c(a) \cdot t$ more downstream than at $t = 0$.

The question can be raised at what time t a prescribed depth will be present at a certain location $x = L < L_o$.
This simply leads to

$$t(a,L) = \frac{L - p^{-1} L_o \{a_o + p - a\}}{c(a)} \tag{37}$$

Hence for selected values of a and L the accuracy by which the time $t(a,L)$ can be predicted depends on the accuracy of $c(a)$.

3. SENSITIVITY ANALYSIS

3.1 General

In Chapter 2 deterministic forecasts have been discussed for some morphological processes. These forecasts are based on data for $s(x,t)$ and $C(x,t)$.

In Section 3.2 some discussions on the transport predictors and roughness predictors are given. In Section 3.3 the sensitivity of the results of the analytical models to the accuracy of the predictions of $s(x,t)$ and $C(x,t)$ is treated.

3.2 Prediction of Roughness and Transport

Roughness predictors and transport predictors postulate the presence of *steady uniform flow*. The problem can be formulated as follows.

Consider a wide river with constant width. Given is the discharge per unit width (q); the bed slope (i) and the composition of the bed material (Δ and D). The question is now to estimate the alluvial roughness $C(x,t)$ and the transport $s(x,t)$.

The available predictors have a strong empirical character. Is is therefore not surprising that new predictors are less inaccurate than the older ones; it seems simply because the newer ones are based on more experimental evidence.

It was H.A. Einstein who made the first integrated approach to this problem (Einstein, 1950 and Einstein and Barbarossa, 1952). Now, three decades later, more accurate methods are available. However, the accuracy of the predictions is still limited.

The writer has the impression that in the literature confusion was raised by introducing the phrase 'modified Einstein procedure' (Colby and Hembree, 1955). This 'modified Einstein procedure', however, has a completely different goal *viz.* the determination of s for an *existing* channel, based on measurements of flow and sediment concentration.

Here it regards the prediction of s(x,t) and C(x,t) from data on q, i, Δ and D. However, to be able to separate the influence of s(x,t) and C(x,t) as far as the accuracy of the morphological predictions is concerned two types of predictions will be treated:

(i) Determination of C and s from data on q, i, Δ and D

(ii) Determination of s from data on q, i, Δ, D and C.

The following two predictors are used here:

a) The Engelund (1966) method to predict the roughness, combined with the Engelund-Hansen (1967) transport formula (details are given in Annex I).

b) The Ackers-White (1973) transport formula together with the White *et al* (1980) roughness predictors (details are given in Annex II).

There are a number of reasons to select these two groups of predictors. In the first place these predictors are newer ones and are based on a fair amount of experimental data. Moreover in the second place in these two cases both transport- and roughness prediction is covered by the same group of investigators.

In order to be able to carry out the sensitivity analysis of Section 3.3 some experiments have been used to get some information on the accuracy in the prediction of the alluvial roughness (C) and the transport (s). In fact field data should be used. However, then also the *measured* values of C and s contain errors. Therefore some flume data are used. The data used are the "Fort Collins data' (see Guy *et al*, 1966) and the data of Table 1. This regards data obtained in the sand-flume of the Delft Hydraulics Laboratory. A description of this sand-flume is given by Wijbenga and Klaassen (1981).

For all flume data it is supposed that measured values (C_m and s_m respectively) have negligible errors.

The predicted values are:

(i) The predicted roughness C_p

(ii) The predicted transports

- s_{p1} when based on the *measured* roughness C_m
- s_{p2} when based on the *predicted* roughness C_p

Discharge m^3/s	Sediment transport $\times 10^{-5}$ m^2/s	Flume width m	Depth m	Slope (meas.) $\times 10^{-3}$	C-value (from meas.) $m^{1/2}/s$
0.151	1.239	1.50	0.1980	1.571	30.1
0.151	1.242	1.50	0.1978	1.582	29.9
0.225	1.241	1.50	0.2837	1.250	29.5
0.076	1.208	1.50	0.1108	2.400	28.7
0.0755	1.163	1.50	0.1097	2.340	29.4
0.301	1.232	1.50	0.3626	0.960	31.7
0.340	1.268	1.50	0.3974	0.890	32.7
0.4316	1.283	1.50	0.4897	0.740	33.9
0.552	1.309	1.50	0.5915	0.630	36.2
0.175	0.2391	1.50	0.2777	0.587	34.9
0.269	2.354	1.50	0.3007	1.620	28.4
0.0885	2.437	1.50	0.1146	3.297	27.0
0.476	2.437	1.50	0.4875	1.101	30.4
0.062	0.2591	1.50	0.1051	1.091	38.0
0.305	0.2609	1.50	0.4510	0.377	38.8
0.310	11.77	1.50	0.2626	3.980	25.1
0.342	1.328	1.50	0.4052	0.860	32.2
0.635	11.91	1.50	0.4930	2.700	25.1
0.100	12.06	1.50	0.0870	6.900	31.8
0.225	1.229	1.50	0.2908	1.170	29.4
0.075	1.227	1.50	0.1111	2.361	27.8
0.147	1.239	1.50	0.1961	1.622	28.8
0.220	2.379	1.50	0.2620	1.905	26.5
0.265	2.440	1.50	0.3017	1.708	27.1
0.146	1.237	1.50	0.2002	1.595	28.3
0.265	2.342	1.50	0.3012	1.553	28.5
0.146	1.234	1.50	0.2003	1.654	27.7
0.401	3.351	1.50	0.4051	1.530	28.1
0.146	1.227	1.50	0.2012	1.607	27.9
0.265	2.255	1.50	0.3005	1.677	27.4
0.148	1.205	1.50	0.2002	1.642	28.2
0.116	1.208	1.125	0.2082	1.595	28.5
0.112	1.239	1.125	0.2040	1.641	28.0
0.119	2.322	1.125	0.3060	1.690	27.4
0.048	1.326	0.50	0.2106	1.658	27.0
0.088	1.997	0.50	0.3280	1.693	26.2
0.048	1.419	0.50	0.2090	1.670	27.0
0.025	0.7088	0.50	0.1200	1.616	32.5
0.048	1.305	0.50	0.2090	1.617	27.7
0.133	2.658	0.50	0.4360	1.712	26.7

Sediment data $D_{35} = 0.750$ mm
$D_{50} = 0.780$ mm
$D_{90} = 0.840$ mm

Table 1 Measurements from sand-flume Delft Hydraulics Laboratory

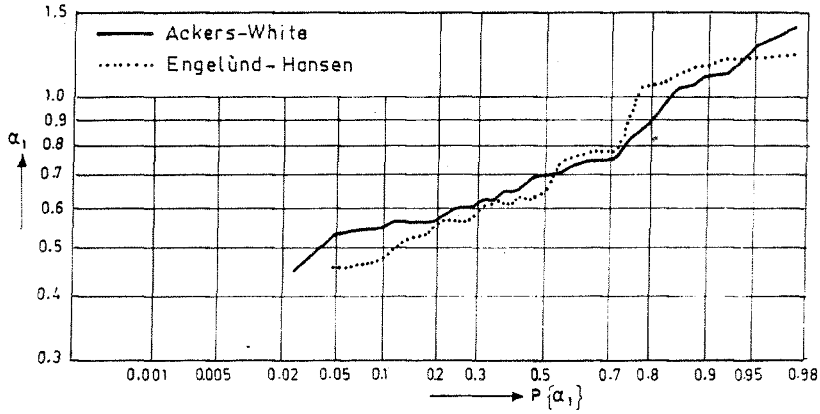


Fig. 7 P{α_1} for CSU data

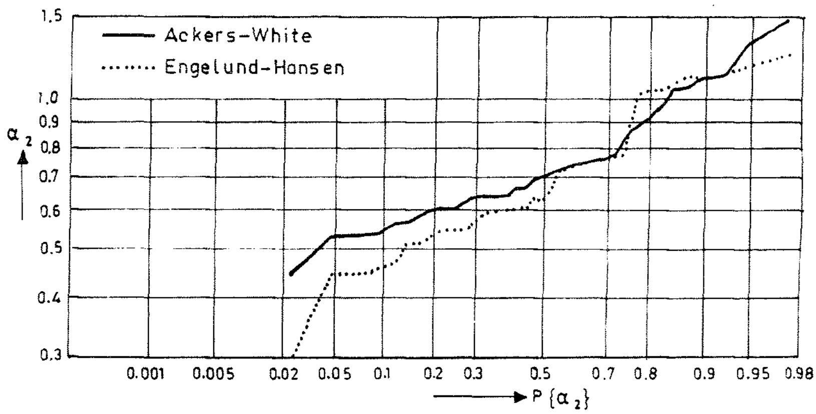


Fig. 8 P{α_2} for CSU data

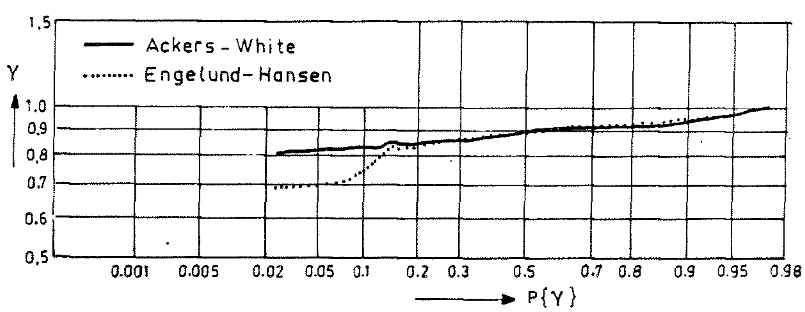


Fig. 9 P{γ} for CSU data

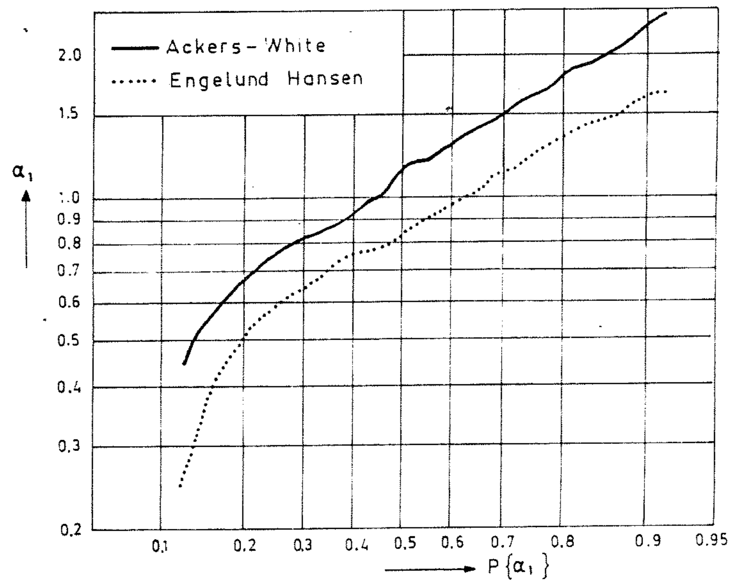


Fig. 10 P{α_1} for sandflume data

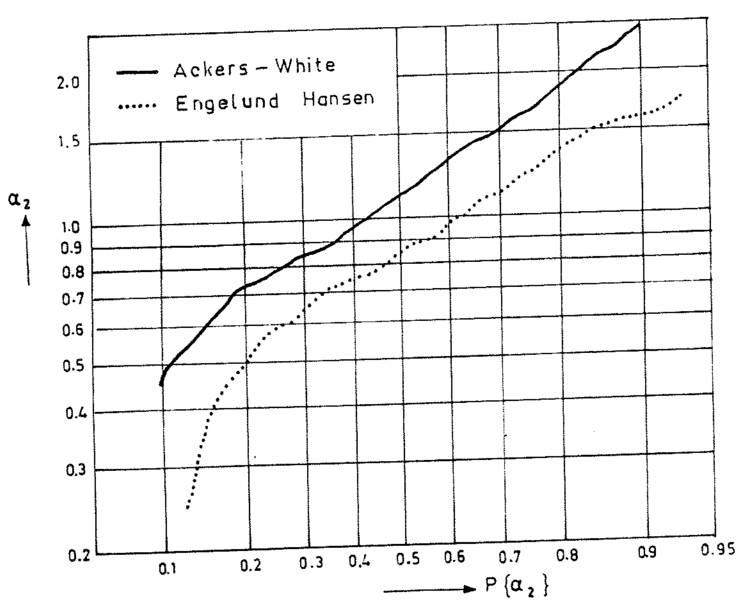


Fig. 11 P{α_2} for sandflume data

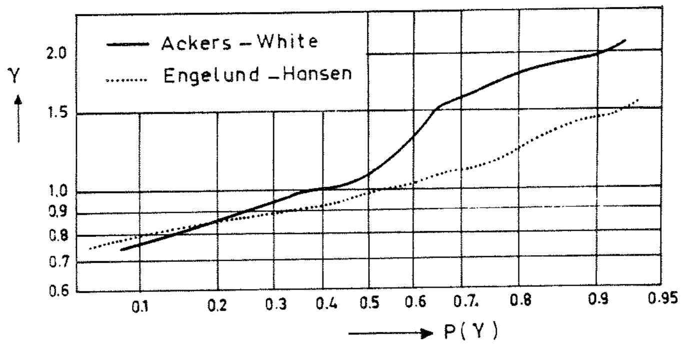
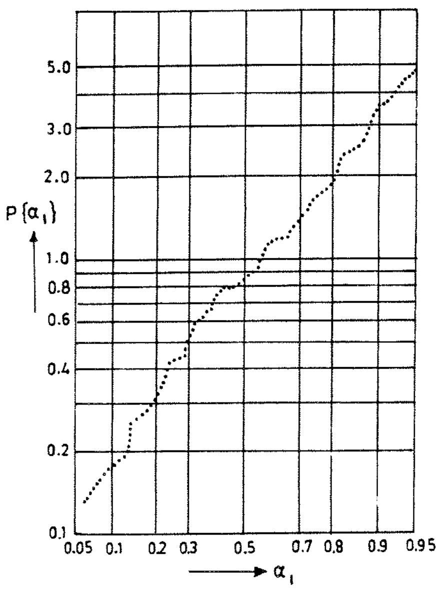


Fig. 12 P{γ} for sandflume data



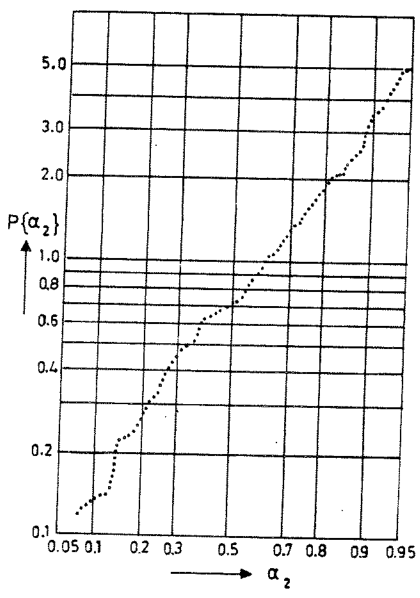


Fig. 13 P{α_1} for Yangtze River

Fig. 14 P{α_2} for Yangtze River

Moreover the following ratio's have been computed:

Roughness ratio: $\gamma = C_m/C_p$

Transport ratios: $\alpha_1 = s_m/s_{p1}$, thus for *measured* roughness

$\alpha_2 = s_m/s_{p2}$, thus for *predicted* roughness

To give some idea about the values for α_1, α_2 and γ that can be obtained for natural rivers some data for the Yangtze River (China) have been used. The writer obtained these data (that only are sufficiently complete to apply the Engelund-Hansen procedure) in 1980 during his visit to Nanjing.

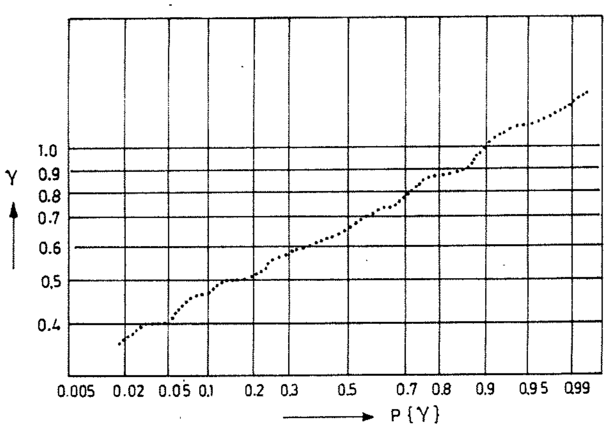


Fig. 15 P{γ} for Yangtze River

This information is added as it has not been used so far in the design of predictors. This is contrary to the CSU-data.

From the series of data probability distributions of α_1, α_2 and γ have been computed. The results are plotted in Figs. 7 ... 15. For α_1, α_2 and γ a log-scale is used. The cumulative probabilities P{ } are plotted on a gaussian scale. As will be discussed later on especially the standard deviations of these distributions are of importance.

Inspection of the Figs. 7 ... 15 seems to allow the following observations:

(i) Considering the standard deviations of α_1, α_2 and γ the differences between the Engelund-Hansen and the Ackers-White method are marginal. There is an (unexplained) exception for part of P{γ} of the Fort Collins (CSU) data and the sand flume data.

(ii) The difference between $P\{\alpha_1\}$ and $P\{\alpha_2\}$ is small. According to Annex I and II this is logical.

(iii) It seems fair to assume that α_1, α_2 and γ have a log-normal distribution.

(iv) Logically the scatter in α_1, α_2 and γ is larger for a natural river (the Yangtze River in this case) than for flume data.

3.3 Propagation of Errors

For the three cases treated in Chapter 2 the morphological predcitions can be interpreted as a *prediction of the time* at which a certain event (depth or bed level) occurs at a certain location.

It will now be studied how errors present in the prediction of roughness and sediment transport manifest themselves in the predicted times. Therefore the information of Section 3.2 is used. Two remarks have to be made in this respect:

(i) It is assumed for the flume data used in Section 3.2 that the errors in the measured values (c_m and s_m) are small compared to the errors in the predicted values (C_p, s_{p1} and s_{p2})

(ii) The aim is to separate the influence from the errors in C and s on the time predicted.

For the three cases of Chapter 2 the following analysis can be given.

Case I

In Eq. (19) the predicted time t_I is incorporated in the dimensionless time τ_I. The values of C and s determine the relation between τ_I and t_I.

$$\tau_I = 2\beta\, ct_I \sim s\, C^{4/3} \cdot t_I \tag{38}$$

as

$$\beta = \left[\frac{di}{du_o}\right] \cdot \frac{u_o}{a_o} \sim C^{2/3} \quad \text{because } q = C\, a_o^{3/2}\, i_o^{1/2} \tag{39}$$

and

$$c = \left[\frac{ds}{du}\right]_o \cdot \frac{u_o}{a_o} \sim s\, C^{2/3} \quad \text{with } s = m\, u^n \tag{40}$$

The proportionality factors contain the known values of q, i_o, Δ, D etc.

Case II

In Eq. (25) the predicted time t_{II} is incorporated in the factor Kt_{II}. This leads to

$$Kt_{II} = \frac{\{ds/du\}_o}{\{di/du\}_o}\, t_{II} = \frac{c}{\beta}\, t_{II} \sim s t_{II} \tag{41}$$

Case III

For this case the predicted time t_{III} is according to Eq. (37) in the factor ct_{III} with

$$ct_{III} = \frac{\{ds/du\}_o}{a_o} \; u_o \, t_{III} \sim s \, C^{2/3} \; t_{III} \tag{42}$$

The influence of the error of C on the error in s can be separated by considering the definitions and using $s \sim C^{\varepsilon}$ (see Annexes I and II)

$$s_m = \alpha_1 \; s_{p1} = \alpha_1 \cdot \gamma^{\varepsilon} \cdot s_{p2} \tag{43}$$

The errors in C and s can now be seen as errors in t_i (i = I, II or III).

Case I

In stead of the 'correct' values s_m and C_m use is made of s_{p2} and C_p. This creates the error in t_I:

$$s_m \, C_m^{4/3} \; t_I = s_{p2} \cdot C_p^{4/3} \cdot \tau_I \cdot t_I$$

with

$$\tau_I = \left[\frac{s_m}{s_{p2}}\right] \cdot \left[\frac{C_m}{C_p}\right]^{4/3} = \alpha_1 \cdot \gamma^{4/3} + \varepsilon \tag{44}$$

Case II

Similarly here

$$s_m \, t_{II} = s_{p2} \cdot \tau_{II} \cdot t_{II}$$

with

$$\tau_{II} = \alpha_1 \cdot \gamma^{\varepsilon} \tag{45}$$

Case III

For this case

$$s_m \, C_m^{2/3} \; t_{III} = s_{p2} \cdot C_p^{2/3} \cdot \tau_{III} \cdot t_{III} \tag{46}$$

with

$$\tau_{III} = \alpha_1 \cdot \gamma^{2/3 + \varepsilon} \tag{47}$$

Hence the errors in the predicted times can be studied by considering the distribution of

$$\underline{\tau}_i = \underline{\alpha}_1 \cdot \left[\gamma_i \right]^{\delta_i} \tag{48}$$

with

$$\delta_I = {}^4/_3 + \varepsilon \quad ; \quad \delta_{II} = \varepsilon \quad \text{and} \quad \delta_{III} = {}^2/_3 + \varepsilon$$

As defined in Annex I the value of ε indicates the influence of the roughness predictions on the transport predictions according to $s \sim C^{\varepsilon}$. For the Engelund-Hansen predictors in Annex I it is shown for a relatively wide river $\varepsilon = {}^1/_3$. This value is indeed found for the Yangtze River in China. For the CSU-data this value is not reached as shown in Annex I (Fig. I-1).

The CSU-data regard not very wide flumes, making wall corrections necessary in the prediction of the roughness. This makes $\varepsilon \neq {}^1/_3$ possible.
The value of ε for the Ackers-White case cannot be deduced so easily in an analytical way. The deduction in an indirect way for the CSU-data with wall-corrections (Annexes I and II) shows that on average $\varepsilon_{AW} < \varepsilon_{EH}$.
Considering the cases I, II and III it can be stated that case I is the most general case. This means that it is realistic to take in Eq. (48) a value $\delta \approx {}^4/_3 + \varepsilon$ or $\delta = {}^4/_3$ to ${}^5/_3$.
With such a fixed value of δ the probability distribution of τ_I can be simulated according to Eq. (48) from the original experimental data. As an approximation (which seems sufficient for this general consideration) a simple analysis is possible. It can be assumed that $\underline{\alpha}_1$ and $\underline{\gamma}$ are log-normally distributed and δ_1 is constant.

Define the normal distributions:

$\underline{y}_1 = \ell n\, \underline{\alpha}_1$ with mean μ_1 and standard deviation α_1

$\underline{y}_2 = \ell n\, \underline{\gamma}$ with mean μ_2 and standard deviation α_2

Then $\underline{\zeta} = \ell n\, \tau_1$ is normally distributed with mean μ and standard deviation σ.
The following relations apply:

$$\mu = \mu_1 + \delta\, \mu_2 \tag{49}$$

$$\sigma^2 = \alpha_1^2 + (\delta\, \sigma_2)^2 \tag{50}$$

From inspection of the figures for the CSU-data the following table (Table 2) has been composed.

	μ_1	μ_2	σ_1	σ_2	μ	σ
Engelund-Hansen	0.85	0.98	0.50	0.26	2.48	0.85
Ackers-White	1.1	1.0	0.67	0.5	3.77	1.49

Table 2 Mean (μ) and standard deviation (σ) of $\zeta = \ell n\ \underline{\tau}_I$ CSU-data with $\delta_I = {}^4/_3 + \varepsilon = {}^5/_3$

The uncertainty of τ_I (as expressed by σ) seems then for the Ackers-White method somewhat larger than for the Engelund-Hansen method. If, however, from Fig. I.1 it is concluded that for the Ackers-White method an average value $\varepsilon \approx 0$ (and hence $\delta = {}^4/_3$) is more justified, then in Table 2 the value $\sigma = 1.49$ reduces to $\sigma = 0.85$. The two methods Ackers-White and Engelund-Hansen thus lead to the same results.
The Engelund-Hansen method has in general some advantages because of its simplicity. This simplicity is also present apparently when the influence of the predicted roughness on the predicted transport is considered.

4. APPLICATION TO MORPHOLOGICAL COMPUTATIONS

4.1 General

The interpretation of the results obtained above will be given in this chapter with respect to morphological computations for practical river-problems. It is therefore necessary to discuss these computations in general terms. The river engineer in charge with the one-dimensional computation has to make a selection for the particular river of the *transport predictor* and the *roughness predictor* to be applied. In the ideal case he will base his selection on measurements in the particular river. Some remarks are made in Sections 4.2 and 4.3.
For a good morphological prediction the entire numerical model has to be calibrated and verified. In general in the *calibration phase* the model is tuned by means of measurements until the best agreement between calculated and measured results is obtained. In the *verification phase* for another set of measurements without additional tuning of the model, calculated and measured results are compared.

For hydrodynamic computations for which the bed is (supposed to be) fixed it is already in many cases not easy to find sufficient field data to carry out a calibration phase and a verification phase independently. For morphological computations the situation is considerably worse. This is mainly due to the fact that morphological processes in rivers are much slower than hydrodynamic ones. Therefore only as an exception calibration and verification of the entire numerical model for a particular river can be carried out.

In order to guarantee the reliability of the morphological predictions it is essential to test the components of the numerical models if an integral calibration and verification is not possible due to the above given reasons. The experimental verification in the particular river of the predictors applied in the numerical model can be seen as testing components of the model.

4.2 Selection of Predictors

In the ideal case the selection of the transport predictors should be carried out by means of transport measurements in the river to be studied. This brings, however, forward another problem. Contrary to what is done for the flume data of Chapter 3 it can now certainly *not* be assumed that the measured transports contain no error. For prototype cases the errors in transport measurements are substantial components in the total errors of the morphological predictions.

For the river the ratio $\alpha_1 = s_m/s_p$ can be calculated for a number of prospective transport predictors. From the data the cumulative probability distributors $P\{\alpha_1\}$ can be determined. It is advisable to select the transport predictor via the *variance* of α_1 rather than with *mean* of α_1. When the mean differs from unity it is quite possible to take this into account.

For the roughness predictor a similar procedure can be followed. There is, however, a difference between the two predictors. In an alluvial river the roughness can be much more accurately measured than the sediment transport. Equation (48) suggests that the influence of $\underline{\alpha}_1$ on $\underline{\tau}_i$ is much larger than the influence of $\underline{\gamma}$ (as $\delta > 1$).

For flume data this is true as then s_m and c_m can be determined with the same order of accuracy. For prototype measurements as stated above this is certainly not the case. This brings forward an important question to practical morphological computations.

It is wise what Chollet and Cunge (1980) did (see also Cunge *et al* 1980) to incorporate predictors for both s and c in the morphological computations?

Both types of predictors have a strong empirical character. With some physical background they are adjusted with experimental data. Why not use predictions that ara based only on the river to be studied?

The writer has the impression that for the two types of predictions the answer has to be different.

(i) *Transport predictors* require time-consuming measurements usually having low accuracy. It is recommended to use a transport predictor that has been controlled with many data of different rivers. It is advisable to carry out some transport measurements in the river involved in order to select the most promising transport predictor.

(ii) *Roughness predictors* do not require time-consuming measurements. The accuracy is higher. Therefore it is not necessary to use a general roughness predictor which may introduce a large systematic error. It seems wiser to recommend the use of an empirical roughness predictor especially designed from data of the river involved.

This line of reasoning leads to the conclusion that the difference in nature of s(x,t) and C(x,t) to use a generally approved *transport predictor*. However for C(x,t) a special-purpose *roughness predictor* is advisable.

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Main symbols

a	depth	[L]
h	waterlevel	[L]
z	bedlevel	[L]
t	time	[T]
C	Chézy coefficient	$[L^{\frac{1}{2}}\ T^{-1}]$
s	sediment transport for unit width	$[L^2\ T^{-1}]$
u	flow velocity	$[L\ T^{-1}]$
x	distance in flow direction	[L]
g	acceleration of gravity	$[L\ T^{-2}]$
Δ	$= (\rho_s - \rho)/\rho$ = relative density	-
D	gram diameter	[L]
ρ	density of water	$[ML^{-3}]$
ρ_s	density of sediment	$[ML^{-3}]$
Fr	$= u/\sqrt{ga}$ = Froude number	-
c	celerity of bed disturbance	$[LT^{-1}]$
K	'diffusion' coefficient	$[L^2\ T^{-1}]$
q	discharge per unit width	$[L^2\ T^{-1}]$
i	slope	-
γ	$= C_m/C_p$	-
α	$= s_m/s_p$	-
P{ }	cumulative probability	-
μ	mean	-
δ	standard deviation	-

Subscripts

m	measured value
p	predicted value

Annex I: THE ENGELUND-HANSEN METHOD

- *Determination of C and s*

For the unit of width the discharge (q) is given together with the slope (i). From the bed material the relative density $\Delta = (\rho_s - \rho)/\rho$ is given together with the mean sedimentation diameter (D).

For the *roughness prediction* the following iteration procedure can be applied: Estimate the depth (a') which would be present without bedforms (Engelund, 1966). Compute the flow parameter θ' from

$$\theta' = \frac{a'i}{\Delta D} \qquad \text{(I-1)}$$

Calculate θ from

$$\theta' = 0.06 + 0.4\,\theta^2 \qquad \text{(I-2)}$$

Determine the depth (a) from

$$\theta = \frac{ai}{\Delta D} \qquad \text{(I-3)}$$

Determine u from $u = q/a$.

Compute a new estimate for a' from

$$\frac{u}{\sqrt{ga'i}} = 6 + 2.5\,\ell n\,\frac{a'}{2 \cdot 5\ D} = 2.5\,\ell n\,\frac{11a'}{2 \cdot 5\ D} \approx 9.48\left[\frac{a'}{2 \cdot 5\ D}\right]^{1/8} \qquad \text{(I-4)}$$

Repeat until a' is sufficiently accurate; compute again a and u and determine C from the Chézy equations.

$$u = C\sqrt{ai} \qquad \text{(I-5)}$$

The transport s (bulk volume per unit of width and time) is determined by the Engelund-Hansen (1967) formula

$$\frac{s}{\sqrt{g\Delta D^3}} = \frac{0.05}{1 - \varepsilon'} \cdot \frac{C^2}{g} \cdot \left[\frac{ai}{\Delta D}\right]^{5/2} \qquad \text{(I-6)}$$

in which ε' is the porosity of the settled sediment ($\varepsilon' \approx 0.4$)

Remarks

i) If in addition C or a is given, then the a or C can be determined from the Chézy equation

$$q = C\ a^{3/2}\ i^{1/2}$$

and Eq. (I-6) can be used directly to determine s.

ii) The influence of C on s can be established in a simple way if q, i, Δ and D are given.
From Eq. (I-6) follows

$$s \sim C^2 \cdot a^{5/2} \tag{I-7}$$

The Chézy equation $q = C\ a^{3/2}\ i^{1/2}$ gives

$$C \sim a^{-3/2} \tag{I-8}$$

Hence

$$s \sim C^2 \cdot \left\{ C^{-2/3} \right\}^{5/2} \tag{I-9}$$

or

$$s \sim C^{1/3} \tag{I-10}$$

Thus if it is defined $s \sim C^{\varepsilon}$ then according to the Engelund-Hansen predictor $\varepsilon = 1/3$.
It has also been tried to derive ε from measurements for which the CSU-data have been used. According to the definitions used here it follows (see also Eq. 43):

$$\varepsilon = \frac{\ln s_{p1} - \ln s_{p2}}{\ln \gamma} \tag{I-11}$$

In Fig. I.1 the cumulative probability $P\{\varepsilon\}$ is given as a result of the application of the Engelund-Hansen predictors to the CSU-data and using Eq. (I-11).
Figure I.1 shows that the median of $P\{\varepsilon\}$ differs somewhat from the theoretical value $\varepsilon = 1/3$ found above. This is not surprising as Eq. (43) shows that for γ values close to $\gamma = 1$ the ε-value becomes indeterminate.

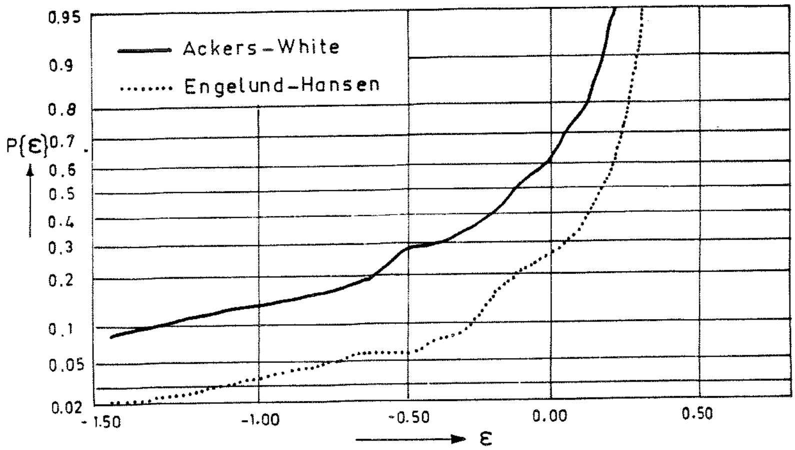

Fig. I.1 P{ε} (CSU-data)

(iii) Matching the Engelund-Hansen transport formula with the general expression $s = m\ u^n$ (power law) simply leads to n = 5.

(iv) The complete predictors for C and s contain 7 experimental constants in the Engelund-Hansen case.

nnex II: THE ACKERS-WHITE METHOD

Determination of C and s

or the unit of width the discharge (q) is given together with the slope (i).
rom the bed material the relative density $\Delta = (\rho_s - \rho)/\rho$ is given together
ith the grain size $D_{(35)}$
or the *roughness prediction* the following iteration procedure can be applied.
stimate the depth (a).
ompute

$$u_* = \sqrt{g\ a\ i} \qquad \text{(II-1)}$$

alculate D_{gr} from

$$D_{gr} = D \left[\frac{g\Delta}{\nu} \right]^{1/3} \qquad \text{(II-2)}$$

alculate the parameters n' and A':

$$\left. \begin{aligned} n' &= 0.0 \\ A' &= 0.17 \end{aligned} \right\} \ D_{gr} \geq 60 \qquad \text{(II-3)}$$

$$\left. \begin{aligned} n' &= 1.0 - 0.56\ {}^{10}\log D_{gr} \\ A' &= 0.23\ D_{gr}^{-1/2} + 0.14 \end{aligned} \right\} \ 1 \leq D_{gr} < 60 \qquad \text{(II-4)}$$

alculate F_{fg} from

$$F_{fg} = \frac{u_*}{\sqrt{g\Delta D}} \qquad \text{(II-5)}$$

etermine F_{gr} from

$$\frac{gr - A'}{fg - A'} = 1.0 - 0.76 \left[1.0 - \frac{1}{\exp\left[\left\{ {}^{10}\log D_{gr} \right\}^{1.7} \right]} \right] \qquad \text{(II-6)}$$

alculate u from

$$gr = \frac{u_*^{n'}}{\sqrt{g\Delta D}} \left[\frac{u}{\sqrt{32}\ \log\ \{10\ a/D\}} \right]^{1-n'} \qquad \text{(II-7)}$$

Estimate a new value for the depth via $a = q/u$ and repeat until a is sufficiently accurate.

Compute C from

$$C = \frac{u}{u_*}\sqrt{g} = u\{ai\}^{-1/2} \qquad \text{(II-8}$$

The transport s can now be calculated as follows

Calculate m':

$$\left.\begin{aligned} m' &= \frac{9.66}{D_{gr}} + 1.34 \quad \text{for} \quad 1.0 \leqslant D_{gr} \leqslant 60 \\ m' &= 1.50 \qquad\qquad\quad \text{for} \quad D_{gr} > 60 \end{aligned}\right\} \qquad \text{(II-9}$$

Calculate C' from

$$\left.\begin{aligned} {}^{10}\log C' &= 2.86\ {}^{10}\log D_{gr} - \left\{ {}^{10}\log D_{gr} \right\}^2 - 3.53 \quad \text{for} \quad 1.0 \leqslant D_{gr} \leqslant 60 \\ C' &= 0.025 \qquad\qquad\qquad\qquad\qquad\qquad\qquad \text{for} \quad D_{gr} \geqslant 60 \end{aligned}\right\} \qquad \text{(II-1}$$

Compute

$$G_{gr} = C' \left[\frac{F_{gr}}{A'} - 1 \right]^{m'} \qquad \text{(II-1}$$

Determine

$$X = \frac{(\Delta+1) D_{35}\ G_{gr}\ C^{n'}}{\left\{ \sqrt{g} \right\}^{n'}} \qquad \text{(II-1}$$

and

$$s = \frac{X\ q}{\rho_s (1-\varepsilon')} * 10^3 \quad \text{with} \quad \varepsilon' \approx 0.4 \qquad \text{(II-1}$$

Remarks

) If in addition C or a is given then the computation of s is shorter and no iteration procedure is necessary (see Ackers *et al*, 1978).

i) The influence of C on s (if q, i, Δ and D are given) cannot be established for the Ackers-White case in such a simple way as is the case for the Engelund-Hansen predictors. Trying to establish ε from $s \sim C^{\varepsilon}$ from the CSU-data using Eq. (43) or Eq. (1-11) led for the Ackers-White predictors to results that seem to vary around $\varepsilon = 0$. Also here indeterminancy of ε around $\gamma = 1$ may have played a role.

ii) Matching the Ackers-White transport formula with the power law $s = m\, u^n$ (m and n being constant) gives a result.

$$n = 1 + m' \frac{F_{gr}/A'}{\left\{ F_{gr}/A' - 1 \right\}} \qquad \text{(II-14)}$$

v) The complete predictors for C and s contain some 20 experimental constants in the Ackers-White case!