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# Hybrid cyclicity: Combining the benefits of cyclic and non-cyclic timetables <sup>☆</sup>



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## ABSTRACT

We propose a new type of timetable that would combine both the regularity of the cyclic timetables and the flexibility of the non-cyclic ones. In order to do so, several combinations of the two timetables are considered. The regularity is incorporated in their design and the flexibility is evaluated using the passenger satisfaction (in monetary units). Each of the tested timetables is constructed using the Passenger Centric Train Timetabling Problem (PCTTP), that is solved using a simulated annealing heuristic. Note that the PCTTP, unlike the traditional Train Timetabling Problem (TTP), does not take into account the conflicts among trains. The aim of the PCTTP is to design such timetables that the passengers' satisfaction is maximized and it remains the aim of the TTP to remove any potential conflicts. The performance of each of the considered timetables is assessed on the real network of Israeli Railways. The results of the case study show that our proposed hybrid cyclic timetable can provide the benefits of the cyclic and the non-cyclic timetable simultaneously. This timetable consists of 75% of cyclic trains (securing the regularity of the service) and of 25% of non-cyclic trains (deployed as supplementary trains during the peak hours and capturing the demand fluctuation). The level of the passenger satisfaction of the hybrid cyclic timetable is similar to the level of the non-cyclic one, which has about 18.5% of improvement as compared to the purely cyclic one.

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## 1. Introduction

A train timetable is the main product of a Train Operating Company (TOC). It consists of the departure times of each train from each of its stopping stations. When creating a timetable the TOCs usually focus on its operational aspects rather than the actual passenger demand that it is supposed to serve. According to the literature, two types of timetables (in passenger railway service) exist: cyclic and non-cyclic.

The cyclicity originates from the Periodic Event Scheduling Problem (PESP), where a given set of events is scheduled in equally spaced intervals (first defined in [Serafini and Ukovich, 1989](#)). In the case of passenger railway service, a special type of cyclic timetables is the clock-faced timetable, where the cycle is one hour. It is especially popular within railways,

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following the assumption that such timetables are easy to remember and thus preferred by the passengers. Some studies confirm that indeed a regularity of a timetable leads to an increase in the passenger demand (Wardman et al., 2004; Johnson et al., 2006). However, such a timetable design provides an inefficient operation system as there is a mismatch between the supply (determined by the timetables) and the demand (characterized by the time dependent passengers arrival rate).

In the non-cyclic timetable, on the other hand, no special rule is imposed on the departure time of the trains (see Caprara et al., 2002 for instance). This makes the non-cyclic timetable more flexible in accounting for the passenger demand. A recent study of Robenek et al. (2016) shows that this is especially true for high volumes of passengers. The flexibility was compared based on the passenger satisfaction. But given the structure of a non-cyclic timetable (no repeating pattern), it might discourage some passengers from choosing the train as their mode of transport.

Given the above, one cannot make a statement about the superiority of one timetable over the other. A combination of the two is needed. In this paper, we investigate hybrid timetables that combine the benefits of both - the regularity of the cyclic timetable and the flexibility of the non-cyclic one. The approach consists in imposition of various levels of cyclicity and evaluation of the corresponding flexibility.

The timetables are obtained by solving the Passenger Centric Train Timetabling Problem (PCTTP) through using a large neighborhood search heuristic combined with simulated annealing. The performance of each newly proposed timetable is assessed and compared to the cyclic and non-cyclic timetables on the real network of Israeli Railways.

The manuscript is structured as follows: in the literature review (Section 2), a survey on train timetabling models is presented. The PCTTP model to construct a timetable while maximizing the passenger satisfaction is introduced in Section 3. The definitions of the existing types of the timetables and the constraints that they impose on the PCTTP model are given in Section 4. Similarly in Section 5, the hybrid timetables and their impacts on the PCTTP are discussed. Section 6 provides the insights about the solution methodology that is used to obtain the results for our case study in Section 7. The paper is finalized by drawing some conclusions and discussions of possible extensions in Section 8.

## 2. Literature review

Since the goal of this study is to propose a new type of timetable, the literature review focuses on the timetable design, which is typically done by solving the Train Timetabling Problem (TTP). The main goal of the TTP is to resolve any potential track occupation conflicts among the trains, so as to construct an operational timetable. Two versions of this problem exist in the literature: cyclic and non-cyclic. The difference between the two versions is that the cyclic TTP imposes additional rules on the departure times.

### 2.1. Non-cyclic TTP

This version of the problem is either formulated as an Integer Linear Programming (ILP) or as a Mixed Integer Linear Programming (MILP). The ILP model uses discretized time, whereas the MILP model uses continuous time.

#### 2.1.1. Model

One of the main differences, between the two formulations, is the way how they handle the conflicts among the trains. The ILP model discretizes the time and splits the lines into blocks (Brannlund et al., 1998). One train at a time can occupy a block. Clique constraints are then imposed to secure the safety (Caprara et al., 2002, 2006, etc.). They are growing exponentially in number with the size of the problem. Several other ILP re-formulations exist (Cacchiani et al., 2010a). The ILP formulation can also allow for the scheduling of extra freight trains within the planning of the passenger service (Cacchiani et al., 2010b).

The MILP model, on the other hand, is considering the departure times as continuous variables. The minimum headway between two trains is secured by using binary variables that indicate the order of the trains in which they leave from a given station (Carey and Lockwood, 1995; Higgins et al., 1997, etc.).

#### 2.1.2. Objective function

For the ILP formulations, it is assumed that ideal timetables are known a priori. The ideal timetable is defined as the most profitable one. The objective of the problem is to minimize the changes made to the ideal timetables (Caprara et al., 2002, 2006, 2007, etc.). Note that we did not find in the literature a methodology to create such an ideal timetable.

The MILP formulations have various objective functions: minimization of the overall cost associated with the allocation of a train path (Carey and Lockwood, 1995), minimization of the total weighted travel time (Higgins et al., 1997), minimization of the delays on the arrival and departure times (Harrod, 2012), minimization of the deviation from the originally planned timetable expressed as a delay (Oliveira and Smith, 2000), minimization of the feasibility violation by penalization (Burdett and Kozan, 2010), etc.

A recent approach consists in relaxing the assumptions that the passengers would always take their shortest path and thus use the total travel time minimization as the objective (Schmidt and Schöbel, 2015). This approach is further extended by taking into account the whole passenger satisfaction, which is to be maximized (Robenek et al., 2016).

### 2.1.3. Solution approach

A common methodology to solve the ILP formulation is the Lagrangian relaxation of the track capacity constraints (Brannlund et al., 1998; Caprara et al., 2002, 2006; Cacchiani et al., 2012). However, the column generation framework seems to find better bounds than the afore-mentioned relaxation (Cacchiani et al., 2008). One may also use dynamic programming to solve the clique constraints (Cacchiani et al., 2013).

For the MILP formulation, the literature agrees on using a heuristic as a common solution approach: a heuristic that considers one train at a time to solve the MILP based on the already scheduled trains (Carey and Lockwood, 1995), as well as local search, tabu search, genetic and hybrid heuristics (all presented in Higgins et al., 1997), among others.

## 2.2. Cyclic TTP

The aim of the cyclic TTP is to resolve any potential track occupation conflicts and to make the departure times of all trains cyclic, *i.e.* the departure time of each train on a given line is equally spaced with an interval of a cycle. This class of problems is referred to as Periodic Event Scheduling Problems (PESP). The PESP were first defined by Serafini and Ukovich (1989). Due to the nature of the problem (repeating pattern), it is sufficient to solve the cyclic TTP for one cycle only (typically the peak hour as it is the most dense part of the day) and repeat the solution in every other cycle. A special case of a cyclic timetable is the clock-faced timetable, where the cycle is equal to one hour (typical for railways).

### 2.2.1. Model

The majority of the cyclic TTP formulations are based on the PESP. In some cases, the PESP is extended with additional constraints such as train synchronization (Peeters, 2003) or symmetry (Liebchen, 2004). The symmetry can be described as follows: two trains of the same line traveling in opposite directions meet at time 0, *i.e.* if a train leaves at 14th minute of the cycle in one direction, then the train in the opposite direction leaves from the same station at time 46 for a cycle of one hour. The sum of the departure times at any station is equal to the cycle. The symmetry is a popular measure in Switzerland and Germany.

The basic PESP formulation considers one universal cycle, known a priori, for the whole railway network. However, one may consider to allow for varying sizes of the cycle over the planning horizon (Zhong et al., 2013) or to make the size of the cycle a decision variable (Heydar et al., 2013). The proposed measures have the intention of securing better transfers or to maximize the utilization of the infrastructure respectively.

### 2.2.2. Objective function

While the model of the cyclic TTP is well defined and recognized, the same can not be said about the objective function of the problem. In the very beginning, there were models without any objective (Odijk, 1996), thus an ad-hoc feasible solution would be generated. Later on, an objective function that would minimize the total waiting time was introduced (Nachtigall, 1996; Nachtigall and Voget, 1996; Yang et al., 2010, *etc.*). In some studies, the waiting time was further decomposed among the different types of passengers such as transferring, throughput and others (Vansteenwegen and Oudheusden, 2006, 2007). The focus on the waiting time only came from the assumption that the passengers would always take their shortest path between their origin and destination, thus rendering the total travel time irrelevant. In the recent literature, this assumption has been released and the cyclic TTP integrates the routing of the passengers. One can either minimize the total travel time of the passengers (Hoppmann et al., *in press*), to maximize the number of transported passengers (Cordone and Redaelli, 2011) or to maximize the passenger satisfaction (Robenek et al., 2016).

### 2.2.3. Solution approach

Similarly to the objective function, one superior methodology for solving the problem does not exist. The applied techniques span from the semi/exact methods, such as branch and bound (Nachtigall, 1996), constraint generation (Odijk, 1996), modulo simplex (Nachtigall and Opitz, 2008), cycle periodicity reformulation (Peeters, 2003; Liebchen and Peeters, 2009), satisfiability reformulation (Großmann et al., 2012), *etc.*, to heuristic methods, such as genetic algorithm (Nachtigall and Voget, 1996), and simulated annealing with particle swarm optimization (Jamili et al., 2012).

### 2.2.4. Remark

Due to the restrictive (cyclic) constraints, a feasible solution might not always be obtained. Two options for correction exist: the adjustment of the underlying assumptions (to keep the cyclicity intact) or allowing for some degrees of irregularity in the timetable. One of the assumptions that can be changed is the running time of a train in between two stations. Typically, the cyclic TTP treats it as a fixed input, but it can be turned into a decision on an interval of a minimal and a maximal value and possibly result into having a feasible solution (Kroon and Peeters, 2003). The other option is to affect the cyclicity itself. When a railway network is highly dense (such as in China), one can decide to have some lines cyclic and the others non-cyclic. This framework was proposed by Yang et al. (2010), where the cyclicity is primarily given to the most busy lines in terms of the transported passengers. This approach would make a whole train

line non-cyclic, but sometimes it might be sufficient to allow only for a small deviation from the cyclic departure time of only a few trains. Such a framework is proposed by Caimi et al. (2011), where the problem has a periodic service as an intention and not as a hard constraint.

### 2.3. Summary

From the above survey, we have learned that most often the goal is to provide either a fully non-cyclic or a fully cyclic timetable. A partially cyclic timetable is considered, only when no feasible solution can be obtained. However, a new direction of relaxing the cyclicity to achieve the benefits of the non-cyclic timetables is emerging. In this paper, we are going to further exploit this concept.

### 3. Timetable construction

A timetable, in the passenger railway service, is formally defined as a set of arrival and departure times of each train at each of its stopping stations. The frequency (number of trains) and the stopping patterns are given by the Line Planning Problem (LPP, Schöbel, 2012) and are fixed. By imposing rules on these times, one can obtain different types of timetables. At first, we present a timetable design approach (irrespective of the timetable type) which is based on the overall passenger satisfaction. The existing types of timetables are presented in Section 4 and our proposed types of timetables are presented in Section 5. For the convenience of the reader, we organize all mathematical notation by its type and order of appearance in Table 1 at the end of this section.

**Table 1**  
Mathematical notations.

| Name              | Description   | Units                        | Type      |
|-------------------|---|------------------------------|-----------|
| $L$               | Set of operated train lines given by the LPP  | –                            | Set       |
| $V^\ell$          | Set of available trains for the line $\ell$ (frequency)                                       | –                            | Set       |
| $I$               | Set of origin-destination pairs   | –                            | Set       |
| $P_i$             | Set of possible paths between OD pair $i$   | –                            | Set       |
| $L^p$             | Set of lines in the path $p$  | –                            | set       |
| $T_i$             | Set of preferred arrival times for OD pair $i$  | –                            | Set       |
| $(\ell, v)$       | A train $v$ serving the line $\ell$   | –                            | Index     |
| $(i, t)$          | A passenger group traveling between OD pair $i$ with a preferred arrival time $t$             | –                            | Index     |
| $K$               | Set of cycles within the planning horizon   | –                            | Set       |
| $h$               | Duration of the planning horizon  | min                          | parameter |
| $n_i^t$           | Number of passengers wishing to travel between OD pair $i$ at time $t$                        | –                            | Parameter |
| $a_i^t$           | Preferred arrival time of a passenger group $(i, t)$ to its destination                       | min                          | Parameter |
| $\mathcal{P}_i^t$ | Penalty path when not serving passenger group $(i, t)$ within $h$                             | monetary                     | Parameter |
| $m$               | Minimum transfer time from one train to another   | min                          | Parameter |
| $r_i^p$           | Running time for OD pair $i$ on path $p$ using line $\ell$                                    | min                          | Parameter |
| $ set $           | Size of the set   | –                            | Parameter |
| $( L^p  - 1)$     | Is the number of transfers in path $p$  | –                            | Parameter |
| $\beta_W$         | Coefficient of the waiting time in the relation to the in-vehicle-time                        | –                            | Parameter |
| $\beta_L$         | Coefficient of being late in the relation to the in-vehicle-time                              | –                            | Parameter |
| $\beta_E$         | Coefficient of being early in the relation to the in-vehicle-time                             | –                            | parameter |
| $\beta_T$         | Penalty for having a train transfer   | min                          | Parameter |
| $VOT$             | Value of (in-vehicle-) time (VOT)   | monetary/min                 | Parameter |
| $c$               | Size of the cycle   | min                          | Parameter |
| $\theta$          | Maximum allowed deviation from a cyclic departure time  | min                          | Parameter |
| $\xi$             | Percentage of cyclic trains within a timetable  | %                            | Parameter |
| $\eta$            | Number of cyclic trains per line  | –                            | Parameter |
| $d_v^\ell$        | The departure time of a train $v$ on the line $\ell$ (from its first station)                 | min                          | Decision  |
| $w_i^p$           | Waiting time of a passenger group $(i, t)$ using path $p$                                     | min                          | decision  |
| $S_i^{dp}$        | Satisfaction of a passenger group $(i, t)$ using path $p$                                     | monetary                     | Decision  |
| $\delta_i^{dp}$   | The schedule passenger delay of being early in path $p$ of a passenger group $(i, t)$         | min                          | Decision  |
| $\gamma_i^{dp}$   | The schedule passenger delay of being late in path $p$ of a passenger group $(i, t)$          | min                          | Decision  |
| $a_i^t$           | Actual arrival time of a passenger group $(i, t)$ to its destination                          | min                          | Decision  |
| $z_v^\ell$        | Dummy variable to help modeling the cyclicity corresponding to a train $v$ on the line $\ell$ | $\mathbb{N} \setminus \{0\}$ | decision  |
| $\Delta_v^\ell$   | Deviation of a train $v$ on the line $\ell$ from its cyclic departure time                    | min                          | Decision  |
| $q_v^\ell$        | 1 - if a train has a cyclic departure time, 0 - otherwise                                     | binary                       | Decision  |
| $y_k^\ell$        | 1 - if there is a cyclic train scheduled in the cycle $k$ on the line $\ell$ , 0 - otherwise  | binary                       | decision  |
| $z_{v\ell}^k$     | Dummy variable to help modeling the cyclicity among the cyclic trains                         | $\mathbb{N} \setminus \{0\}$ | Decision  |
| $d_v^{mod c}$     | Modulo time (remainder after integer division) of a train $v$ on the line $\ell$              | min                          | Decision  |

The passenger satisfaction serves as the indicator of the flexibility of a timetable. In order to construct a timetable, we use the model of the Passenger Centric Train Timetabling Problem (PCTTP) proposed by Robenek et al. (2016). In PCTTP, the objective is to construct a timetable (either cyclic or non-cyclic) that would maximize the overall passenger satisfaction. The model combines the timetable design with the routing of the passengers. Unlike the traditional TTP, this model does not deal with the conflicts among trains. The safety of the network is achieved by solving of the TTP on the result of the PCTTP. That is the PCTTP timetables are the ideal timetables on the input of the TTP. Subsequently, the TTP removes any conflicts through shifting of the departure times while minimizing the changes affected to the ideal timetables. The time is discretized into minutes.

The inputs of the PCTTP can be categorized into two groups: the network layout and the passenger demand data. The network layout consists in the train lines given by the LPP. A line  $\ell \in L$  is an ordered set of stopping stations, that includes the dwell times at the stations and the travel times in between. Each line is having a frequency that is assigned by the LPP, i.e. the set of available trains  $v \in V^\ell$  serving the line  $\ell$  during a given time horizon  $h$  in minutes, which remains fixed. From now on, we refer to a train as a combination of indices  $(\ell, v)$ . Based on the lines, the set of paths  $p \in P_i$  for all Origin-Destination (OD) pairs  $i \in I$  is given. An example of an algorithm to construct the set of paths is presented in Appendix A. A path consists in an ordered set of lines  $L^p$  to be traversed from an origin to a destination with a given train of each line  $(\ell, v)$ . Different trains on the same line constitute different paths. Each line can be further decomposed into unique segments. A segment is the part of the infrastructure, where a train does not stop. A train  $(\ell, v)$  is characterized by its line  $\ell$ , departure time  $d_\ell^v$  from the origin of the line in minutes (the travel times and dwell times are fixed) and its capacity that cannot be exceeded on any of the traversed segments. Unlike the TTP, the PCTTP model considers the dwell times fixed, hence a decision on the departure time from the origin station of each line is sufficient.

As for the passengers, they can form groups that share the same OD pair  $i \in I$  and preferred arrival time to the destination  $t \in T_i$ . A passenger group is denoted as  $(i, t)$  and contains  $n_i^t$  passengers. The model does not allow splitting of the groups. However, one may create groups of size one, in order to treat the passengers individually. The actual value of the preferred arrival time to the destination is given by the parameter  $a_i^t$ . The routing of each passenger group is determined based on the available paths  $p \in P_i$  from the origin to the destination. Within each path, a passenger group is given a minimum transfer time  $m$ , in order to be able to realize a transfer from one train to another in the transferring stations. Any other additional time spent in the transferring stations is counted as the waiting time  $w_i^{tp}$ . A passenger group has to use exactly one path to get from an origin to a destination. When no path is available within the planning time horizon  $h$ , due to the train capacity issues, a so-called penalty path  $\mathcal{P}_i^t$  is offered. The penalty path consists in the shortest path between the given OD pair realized after the end of the planning horizon  $h$ .

The satisfaction of a passenger group  $(i, t)$  using a path  $p$  is denoted as  $S_i^{tp}$ . The value of  $S_i^{tp}$  depends on four attributes considered by the passengers: the in-vehicle-time, the waiting time at transfers, the number of transfers and the schedule passenger delay. The in-vehicle-time is the sum of the running times  $r_i^{p\ell}$  of all train lines  $\ell$  used in the path  $p$ , when traveling between OD pair  $i$ . Note that the running times have the same value for different trains of the same line, but differ among the OD pairs. The different OD pairs get on and off the train line at different stations. Similarly for the paths, different paths within the same OD pair use either different lines or use the same lines, but transfer from one train to another in different stations. The waiting time  $w_i^{tp}$  is the sum of all waiting times within the path. The number of transfers is equal to the number of train lines in the path minus one  $(|L^p| - 1)$ . The schedule passenger delay indicates whether the passenger group has arrived early to the destination ( $\delta_i^{tp} = \max(a_i^t - a_i^t, 0)$ ), on time ( $= 0$ ) or late ( $\gamma_i^{tp} = \max(0, a_i^t - a_i^t)$ ). Note that the schedule passenger delay is related to the preferred arrival time to the destination and that it is not a delay of the train.

Each of the above attributes is transformed into in-vehicle-time units using parameters collected from the literature ( $\beta_W, \beta_T, \beta_E$  and  $\beta_L$ ). Details are provided in Appendix A. The sum of these transformed attributes constitutes the generalized time (in minutes), that can be transformed into the generalized cost (in monetary units) by multiplying it with the Value-Of-Time (VOT). The VOT is an indicator of how many of monetary units are the passengers willing to pay, in order to achieve given savings in time. For instance in Israel, the commuting passengers are willing to pay 21.12 New Israeli Shekels (estimated in 2012 by Shifftan et al., 2008) for a saving of one hour. The lower the generalized cost is, the higher is the passenger satisfaction. Based on the above description, the passenger satisfaction of a path  $p$  with respect to the passenger group  $(i, t)$  is formulated as follows:

$$S_i^{tp} = -VOT \cdot \left( \sum_{\ell \in L^p} r_i^{p\ell} + \beta_W \cdot w_i^{tp} + \beta_T \cdot (|L^p| - 1) + \beta_E \cdot \delta_{ip}^t + \beta_L \cdot \gamma_{ip}^t \right), \quad \forall i \in I, \forall t \in T_i, \forall p \in P_i, \quad (1)$$

By solving the PCTTP model, one obtains the exact timetable as well as the subsequent routings of the passengers through the network. The decision of which passenger group is assigned to which path is carried out by the model. This decision is driven by the objective function (passenger satisfaction maximization). The result is a system optimum. The PCTTP as such generates a non-cyclic timetable. It is needed to include additional cyclicity constraints (described in the next section) to obtain a cyclic timetable. Since the PCTTP provides the routing of the passengers, the profit of a train operating company

associated with the resulting timetable can be estimated. However, given that it is assumed that the passengers have railways as their only mode of transport (the penalty path consists in trains), the demand is inelastic. Thus the profit for different timetables is similar. For this reason, we omit the profit estimation in this study.

#### 4. Existing timetables

As mentioned in the previous section, we define a timetable as a set of departure times  $d_v^\ell$  of each train  $v \in V^\ell$  from its origin station on every line  $\ell \in L$ . Even though the lines consist of several stopping stations, in the PCTTP it is sufficient to decide only the departure time of each train from the origin station of a line, as the arrival times to the subsequent stations are derived from the fixed travel times and dwell times. By imposing rules on train departure times, one can obtain two types of timetables: cyclic and non-cyclic.

##### 4.1. Cyclic timetable

The cycle  $c$  represents the difference between the departure times of two consecutive trains on every line. It is given in minutes and its value holds for the whole timetable. When varying size of the cycle is needed, one can either solve a TTP model of variable cycles (Zhong et al., 2013) or (more often) to conceptually split a line into several ones. However, the split of a line is viable only when the various sizes of the cycle are its multiples. In railways, the typical value of the cycle  $c$  is 60 min. The regularity is enforced by including the cyclicity constraints in the model. They can take the following form:

$$d_v^\ell - d_{v-1}^\ell = c, \quad \forall \ell \in L, \forall v \in V^\ell : v > 1. \quad (2)$$

Adding cyclicity constraints to the model creates a *strictly cyclic* timetable, i.e. once the operation of a line  $\ell$  has started, there would be a train departing in each cycle. An example of such a timetable is shown in Table 8 (in Appendix B). This type of timetable is not always desirable. The passenger demand is time dependent and is lower in between the peak hours. Therefore, operators might choose to interrupt their operation for the duration of the off-peak hours. In this case, constraints (2) can be modified to a version where the departures of two consecutive trains are spaced in multiples of cycles  $z_v^\ell$ . In other words, the difference between two consecutive trains can now have values of 60, 120, 180, etc. minutes. The values of these variables are integers with the lowest value being 1. The constraints (2) are reformulated as:

$$d_v^\ell - d_{v-1}^\ell = c \cdot z_v^\ell, \quad \forall \ell \in L, \forall v \in V^\ell : v > 1. \quad (3)$$

In this manuscript, we name a PCTTP containing constraints (3) as *cyclic*. An example of such a timetable is presented in Table 7 (in Appendix B).

##### 4.2. Non-cyclic timetable

In the non-cyclic timetable, no rule is enforced on the departure times of trains. However, one can add the below constraints, in order to avoid the symmetry in the model.

$$d_v^\ell \leq d_{v+1}^\ell - 1, \quad \forall \ell \in L, \forall v \in V^\ell : v < |V^\ell|. \quad (4)$$

No other buffer time between two consecutive trains, than the above one minute, is needed. The decision, if two trains are being scheduled close to each other, is driven by the objective function. Note that the PCTTP model does not handle potential conflicts among trains. Since no rule on the departure times is enforced, the non-cyclic timetables are better in accordance with the time dependency of the demand. Therefore, the non-cyclic timetable is more flexible than the cyclic one. An example of such timetable is shown in Table 10 (in Appendix B).

#### 5. Hybrid timetables

In this section, we introduce the concept of hybrid timetables which are a combination of cyclic and non-cyclic timetables. Each type of hybrid timetable is described in the form of the additional constraints that are imposed on the original PCTTP formulation. Since the regularity of a timetable cannot be quantified, it is taken care of by design and the flexibility is evaluated using the passenger satisfaction upon solving the respective PCTTP formulation. Later on (in Section 7), we test and evaluate these timetables against each other (including the existing ones) using real case data.

##### 5.1. $\theta$ shifted cyclic timetable

The first hybrid timetable is inspired by Caimi et al. (2011). They allow small deviations from the cyclic departure times, in order to obtain a feasible solution. We introduce a variable  $\Delta_v^\ell$  capturing the deviation from the cyclic departure time of a train  $v$  on line  $\ell$  given in minutes. This deviation is restricted within the bounds  $[-\theta, \theta]$  in minutes, in order to control it.

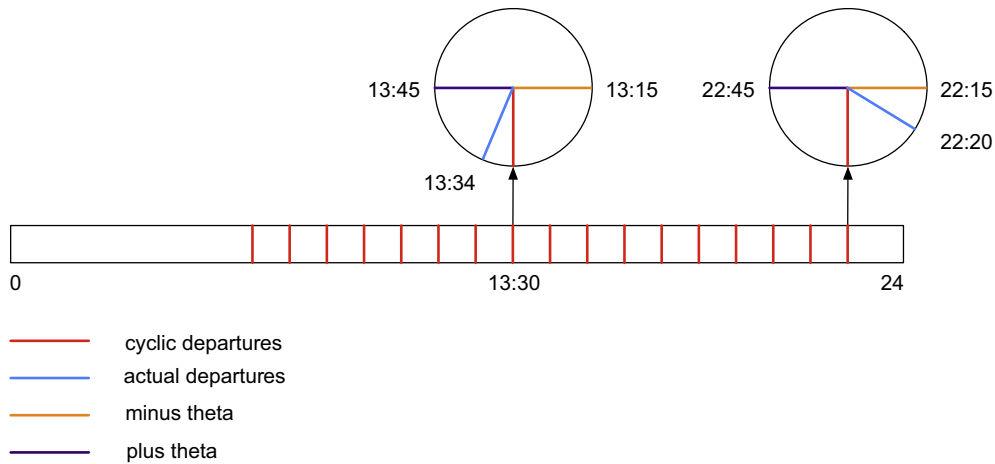


Fig. 1. Example of a  $\theta(= 15)$  shifted timetable for a single line.

For illustration, consider the example of a  $\theta(= 15)$  shifted timetable for a single line in Fig. 1. The cyclic departures of this line are scheduled every xx:30. For each of them a deviation of  $[-15, 15]$  minutes is allowed. Hence, a train at 13:30 might depart at 13:34 and a train originally scheduled at 22:30 is now departing at 22:20 instead. This framework applies to all trains. Note that other lines might have different cyclic times, but their actual departure times follow the same rule of  $[-15, 15]$  minutes within its cyclic departure time. To obtain such a timetable for a given  $\theta$ , we add constraints (5) and (6) in the model of the PCTTP.

$$\left(d_v^\ell - \Delta_v^\ell\right) - \left(d_{v-1}^\ell - \Delta_{v-1}^\ell\right) = c \cdot z_v^\ell, \quad \forall \ell \in L, \forall v \in V^\ell : v > 1, \tag{5}$$

$$-\theta \leq \Delta_v^\ell \leq \theta, \quad \forall \ell \in L, \forall v \in V^\ell. \tag{6}$$

Constraints (5) impose that the original cyclic departure times must respect the cycle. Constraints (6) set the maximum shift from the cyclic departure times. 0 min shifted cyclic timetable is equivalent to the cyclic one. For a cycle of one hour, 30 min shifted cyclic timetable denotes the highest possible deviation, otherwise the trains would overlap. Due to the case, where two consecutive trains might have +30 min and -30 min deviations, constraints (6) have to be adjusted to the form:

$$-\theta \leq \Delta_v^\ell \leq \theta - 1, \quad \forall \ell \in L, \forall v \in V^\ell. \tag{7}$$

Therefore, we test this type of timetable for all values of  $\theta$  between 3 and 30 in 30 min intervals.

### 5.2. $\xi$ partially cyclic timetable

This hybrid timetable is constructed by allowing a percentage of trains on a given line to be non-cyclic. The degree of the regularity is then expressed as  $\xi\%$ , where  $\xi$  is a parameter decided a priori. Since different lines are served by a different amount of trains, applying the  $\xi$  to each line separately might significantly disrupt the regularity of the service. Instead, we propose to treat the  $\xi$  as a percentage of the number of trains of the most frequent line ( $\max(|V^\ell|)$ ) denoted as  $\eta$ . The  $\eta$  number of trains of each line are having a cyclic departure time and the rest of the trains is having a non-cyclic departure time. The decision on which trains are to be cyclic and non-cyclic is arbitrary. Indeed, the order of appearance within the set  $V^\ell$  does not have any impact. The optimal solution will consist in the same values of the departure times no matter the internal order of the trains. For a notation convenience, we assign the first  $\eta$  trains to be cyclic.

For illustration, consider the example of a  $\xi(= 50)$  partially cyclic timetable in Fig. 2. The number of trains per line ( $|V^\ell|$ ) is 16, 12 and 6, respectively. The most frequent line being line 1, i.e.  $\eta = 16 \cdot 0.5 = 8$ . Therefore, each line has to have at least 8 trains with a cyclic departure or to have all trains with cyclic departure, if  $|V^\ell|$  is smaller than 9. The rest of the trains can be scheduled at any time throughout the horizon (1 day in this case). To obtain such a timetable for a given  $\xi$ , we add constraints (8) in the model of the PCTTP.

$$d_v^\ell - d_{v-1}^\ell = c \cdot z_v^\ell, \quad \forall \ell \in L, \forall v \in V^\ell : 1 < v \leq \eta = \max(|V^\ell|) \cdot \frac{\xi}{100}. \tag{8}$$



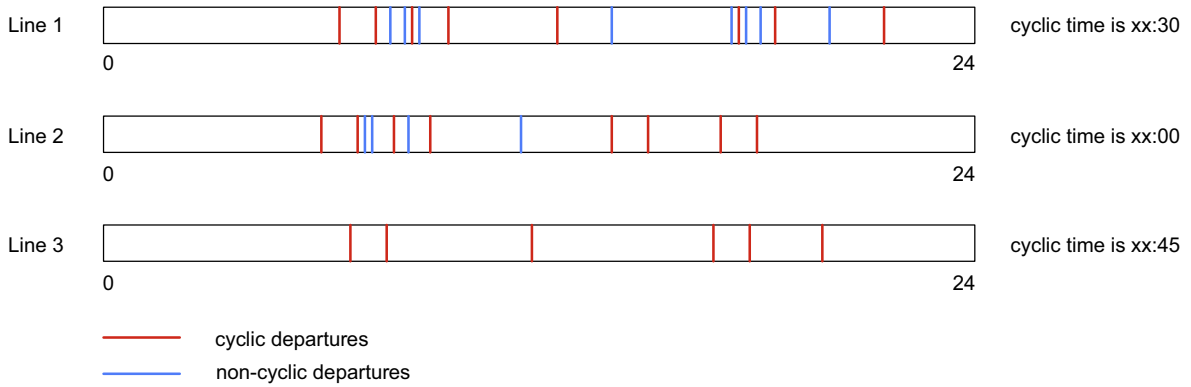


Fig. 2. Example of a  $\zeta(= 50)$  partially cyclic timetable.

We test this timetable for all values of  $\zeta$  between 10% and 90% in 10% intervals. The 100% partially cyclic timetable is equivalent to the cyclic one and the 0% partially cyclic timetable is equivalent to the non-cyclic one. Note that the  $\zeta$  of 0% is a special case. Since zero divided by any number is equal to zero, the above constraint does not hold. However, since the 0% partially cyclic timetable is equivalent to the non-cyclic one, it is sufficient to solve the non-cyclic version of the PCTTP. The non-cyclic trains are most likely to be scheduled within the high demand density periods of the day.

This hybrid timetable is inspired by Yang et al. (2010). They allow a certain number of lines to be non-cyclic. The motivation is again the infeasibility of the problem due to the high density of the trains.

### 5.3. Hybrid cyclic timetable

Even though both,  $\theta$  shifted cyclicity and  $\zeta\%$  partial cyclicity, keep some degree of regularity, the newly created patterns might be too disorganized for the passengers to see. Therefore we propose one additional type of a hybrid timetable. The hybrid cyclic timetable schedules non-cyclic trains only in the hours/cycles where there is already a cyclic train being scheduled. Note that according to constraints (3), not every cycle is required to have a cyclic train scheduled. With such pattern, all of the passengers would obtain the same level of service as a cyclic timetable, with more flexibility. Within a given cycle, the passenger can decide to use the cyclic train or the non-cyclic one (if there is any). The ratio, between the amount of cyclic and non-cyclic trains, is a decision of the model.

For illustration, consider the example of a hybrid cyclic timetable in Fig. 3. In this Figure, each box represents a cycle. Each cycle can either have no train, one cyclic train or one cyclic train and one or more non-cyclic trains within. The cyclic departure times, in this case, follow the pattern of xx:30. To obtain such a timetable, the constraints (9)–(11) need to be included in the PCTTP model.

$$q_v^l \cdot q_{v'}^l \cdot d_v^l - q_v^l \cdot q_{v'}^l \cdot d_{v'}^l = q_v^l \cdot q_{v'}^l \cdot (c \cdot z_{vv'}^l), \quad \forall l \in L, \forall v, v' \in V^l : v > 1, v \neq v', \tag{9}$$

$$y_k^l \leq \sum_{v \in V^l : d_v^l / c = k} q_v^l, \quad \forall k \in K, \forall l \in L, \tag{10}$$

$$(1 - q_v^l) \cdot d_v^l \leq y_{d_v^l / c}^l \cdot h, \quad \forall l \in L, \forall v \in V^l. \tag{11}$$

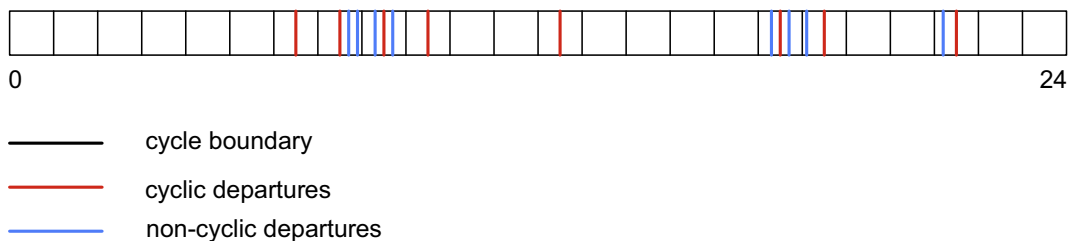


Fig. 3. Example of a hybrid cyclic timetable.

The cyclicity of a train is modeled through the binary decision  $q_v^t$ . Value 1 indicates that the train is a cyclic one, 0 otherwise. The cyclicity pattern, only among the departure times of the cyclic trains, is enforced by constraints (9). The planning horizon consists of  $K = h/c$  cycles. For a planning horizon of one day, the value of  $k$  is on the interval from 1 to 24. Since the non-cyclic trains can be scheduled only in the cycles, where there is a cyclic train running, the binary decision  $y_k^t$  indicates whether there is such a train in the cycle  $k$  (equals to 1) or not (equals to 0). Since there is at most one cyclic train per cycle, the right hand side of constraints (10) is either equal to 1 (when there is a cyclic train scheduled in cycle  $k$ ) or equal to 0 (otherwise). Lastly, constraints (11) allow for non-cyclic trains to exist only in the cycles, where there is a cyclic train scheduled. Note that the constraints (9)–(11) introduce non-linearity in the model.

## 6. Solution methodology

We use the Simulated Annealing (SA) heuristic to solve the PCTTP. Since the SA is a well known heuristic (defined by Kirkpatrick et al., 1983), we focus only on the description of the specifics of the heuristic with respect to the PCTTP. The general pseudocode of the heuristic is shown in Algorithm 1.

### Algorithm 1. Simulated Annealing

---

**Data:**  $N, x_c, \rho, T_c, T_f$   
**Result:** Best timetable  $x^*$

```

1 begin
2   Initialize
3   repeat
4     reset operators
5     for  $n \in N$  do
6       select a neighborhood move
7       impose this move on  $x_c$  and obtain  $x'_c$ 
8       perform passenger assignment on  $x'_c$ 
9       apply acceptance criterion on  $x'_c$ 
10      update weights  $\rho$ 
11     cooling of the  $T_c$ 
12  until  $T_c \leq T_f$ 

```

---

#### 6.1. Solution representation

A solution of the PCTTP problem consists in the departure times  $d_v^t$  for each train  $(\ell, v)$ , that constitute the timetable, and its underlying flexibility. We distinguish among four types of a solution: the initial solution, the current solution  $x_c$ , the potential new solution  $x'_c$  and the best found solution  $x^*$ . Any cyclic timetable can be used as an initial solution. Indeed, the other types of timetable have less constraints, so that a cyclic timetable verifies their constraints as well. A cyclic timetable can be constructed for instance by randomly generating the departure time of the first train of each line and setting the departure times of each subsequent train on each line by adding the value of the cycle to the departure time of the previous train. The other three types of a solution follow the standard SA logic.

The flexibility of a solution is its passenger satisfaction that is estimated using the value function (see Section 6.2).

#### 6.2. Value function and its estimation

The value function of the problem is the overall passenger satisfaction that is to be maximized. In order to estimate its value, the passenger assignment to the trains is carried out on the associated timetable. We design our own passenger assignment procedure. The typical assignment in the literature is not suitable for our value function. The classical assignment approach is based on the first come first serve (FCFS) policy. Such an assignment provides user optimum whereas the goal of the PCTTP is the global system optimum. Moreover, since the demand elasticity is not modeled, the FCFS might give priority to a passenger that might not realize her journey and another passenger that would realize her journey, is either left out or re-routed on a worse path due to the capacity issues. Whereas in the PCTTP the goal is to take care of the demand elasticity implicitly by maximizing the overall passenger satisfaction.

**Algorithm 2.** Passenger Assignment

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**Algorithm 2:** Passenger Assignment

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**Data:**  $I, T_i, P_i, n_i^t$   
**Result:**  $\mathcal{S}$

```

1 begin
2    $\mathcal{S} = 0$ 
3   for  $i \in I$  do
4     for  $t \in T_i$  do
5       for  $p \in P_i$  do
6         calculate  $\mathcal{S}_i^{tp}$ 
7       sort paths  $P_i$  according to  $\mathcal{S}_i^{tp}$  descending
8   sort  $(i, t)$  according to  $\mathcal{S}_i^{|P_i|}$  ascending
9   for  $i \in I$  do
10    for  $t \in T_i$  do
11      for  $j = 1 \dots n_i^t$  do
12        for  $p \in P_i$  do
13          if path  $p$  does not violate capacity of any of its trains then
14            assign passenger  $j$  of the group  $(i, t)$  to path  $p$ 
15             $\mathcal{S} = \mathcal{S} + \mathcal{S}_i^{tp}$ 
16            break

```

---

The general pseudocode of the proposed passenger assignment is shown in [Algorithm 2](#). In the first stage of the algorithm, the satisfaction of each path of each of the passenger groups is calculated and sorted in the descending order. This part of the algorithm is parallelized. In the second stage of the algorithm, the passenger groups are sorted according to their last (worst) path (index  $|P_i|$ ) in the ascending order. This means that, in the next stage, the first processed passenger is having the worst possible satisfaction of the whole problem within her choice set. However, since the paths are sorted in the descending fashion, she will be first offered her best possible path. The algorithm splits the passenger groups into individuals, in order to achieve better values of the overall satisfaction. For each of the passengers, the algorithm iterates through the respective set of possible paths and assigns the first realizable path. Such a path does not violate the capacity constraints of the trains. No passenger remains unassigned as each set  $P_i$  contains the respective penalty path. The algorithm terminates once all of the passengers are assigned to the paths.

### 6.3. Neighborhood structure

Each type of a timetable is having a different neighborhood structure. A neighborhood is defined as a set of candidate solutions that can be reached by a modification of the current solution  $x_c$  using a single neighborhood move. A timetable type can have more than one neighborhood that is characterized by its move. An overview of all the moves per type of a timetable is shown in [Table 2](#).

The columns represent the moves and the rows represent a stage of a move. The moves are categorized by the type of a timetable they belong to. The first column denoted N. stands for the non-cyclic timetable. The last column shows the distributions from which to draw randomly, in order to obtain the respective attribute's value. Only the cells with a grey background constitute a move. Each move is decomposed into 3 parts: selection, modification and application. Two entities can be selected: a specific train  $(\ell, v)$  or the whole line  $\ell$ . The selection of a specific train might be conditioned by its type  $(q_v^\ell)$ . If no condition is specified, any train can be selected.

The aim of the modification is to replace the value of one of the 4 attributes with a new one. The only not yet defined attribute is  $d_v^\ell \bmod c$ . This attribute represents the modulo time within any cycle. For instance, when the cycle  $c$  is equal to one hour and the departure time of a train is 5:45, then the modulo time is 45 min. This time is the same among all cyclic trains of the same line.

The application of the modification is always performed on the pre-selected entity. However, its application might be conditioned. Some applications are made only to the cyclic trains of the given line  $(\forall q_v^\ell = 1)$ . Other applications are conditioned that the new cycle  $k$  does not already contain a cyclic train  $(y_{k/c}^\ell = 0)$  or that the new departure time  $d_v^\ell$  is in a cycle that does already contain a cyclic train  $(y_{d_v^\ell/c}^\ell = 1)$ . If the newly generated values of attributes fail to fulfill the last two conditions (when they are required), the modification is repeated (until they do).

**Table 2**  
Overview of the neighborhood moves by the type of a timetable.

|        | N.                        | Cyclic | $\theta$ Shifted C. | $\xi$ Partially C. | Hybrid Cyclic | Distribution                                    |
|--------|---------------------------|--------|---------------------|--------------------|---------------|---|
| Select | $(\ell, v)$               | ■      | ■                   | ■                  | ■             | $\mathcal{U}(1,  L ), \mathcal{U}(1,  V^\ell )$ |
|        | $\ell$                    |        | ■                   | ■                  | ■             | $\mathcal{U}(1,  L )$                           |
|        | $q_v^\ell = 0$            |        |                     |                    | ■             |   |
|        | $q_v^\ell = 1$            |        |                     |                    | ■             |   |
| Modify | $d_v^\ell$                | ■      |                     |                    | ■             | $\mathcal{U}(0, h - 1)$                         |
|        | $d_v^\ell \bmod c$        |        | ■                   | ■                  | ■             | $\mathcal{U}(0, c - 1)$                         |
|        | $k$                       |        | ■                   | ■                  | ■             | $\mathcal{U}(0, h/c - 1)$                       |
|        | $\Delta_v^\ell$           |        |                     | ■                  |               | $\mathcal{U}(-0, 0)$                            |
| Apply  | $\forall q_v^\ell = 1$    |        |                     | ■                  | ■             |   |
|        | $y_{k/c}^\ell = 0$        |        | ■                   | ■                  | ■             |   |
|        | $y_{d_v^\ell/c}^\ell = 1$ |        |                     |                    | ■             |   |

For illustration, consider the first move of the hybrid cyclic timetable: at first a train is selected by drawing the line  $\ell$  from  $\mathcal{U}(1, |L|)$  and subsequently drawing the  $v$  from  $\mathcal{U}(1, |V^\ell|)$ . The drawing from the distribution(s) is repeated until the condition  $q_v^\ell = 0$  is satisfied. Hence, a non-cyclic train is selected. In the second phase, a new departure time  $d_v^\ell$  of this train is drawn from  $\mathcal{U}(0, h - 1)$ . In the third phase, this new departure time is applied given that it is in the cycle where there is a cyclic train scheduled ( $y_{d_v^\ell/c}^\ell = 1$ ). Otherwise the modification phase is repeated until this condition is complied.

When a timetable type has more than one neighborhood move  $j$ , each of the moves is associated with a weight  $\rho_j$ . Each  $\rho_j$  is initialized to 1 at the beginning of every new temperature  $T_c$ . Based on the performance of the move, its weight is being updated according to the below scheme ( $f(x)$  is the value function):

- If  $f(x_{c'}) \geq f(x^*) \rightarrow \rho_j = \rho_j + 3$
- If  $f(x_{c'}) \geq f(x_c) \rightarrow \rho_j = \rho_j + 2$
- If  $f(x_{c'}) < f(x_c)$  and the new solution is accepted with a probability  $r < \exp(-(x_c - x_{c'})/T_c)$ , where  $r$  is drawn from  $\mathcal{U}(0, 1)$ , then  $\rho_j = \rho_j + 1$

In each iteration of the SA heuristic, a move is selected using the roulette wheel mechanism based on the weights  $\rho$  of the currently solved type of a timetable.

6.4. Values of the parameters

We set the value of the final temperature  $T_f$  to be zero and the initial temperature  $T_0$  to be a function of the passenger satisfaction value of the initial timetable  $f(x_0)$  where  $T_0 = 10^{-5} \cdot f(x_0)$ . The other two parameters, which values need to be tested, are the cooling scheme and the total number of iterations  $N$  per temperature  $T_c$ . The cooling scheme provides the information on how to decrease the temperature  $T_c$ . We test the cooling schemes of 5%, 10% and 20% of the initial temperature  $T_0$  and the values of  $N$  of 100, 500 and 1000 iterations.

The tests were performed on 5 realistic instances of the S-train network of Canton Vaud in Switzerland as described by Robenek et al. (2016) for cyclic and non-cyclic timetables. Since the hybrid timetables are a combination of the neighborhood moves contained in the two timetables, there is no need for their explicit testing. We have performed 10 runs of each combination of the parameter settings for both types of the timetables. The cooling scheme of 5% and the number of iterations  $N = 1000$  provided on average the best values of the passenger satisfaction. Given that the solution time was fast (on average 5 min for both cyclic and non-cyclic timetables), we have selected it as our final parameter setting.

6.5. Validation

In this section, we validate the SA algorithm as compared to CPLEX on a small artificial network. The considered instances vary in the distribution and the number of passengers, and the capacity of the trains. The precise information of each of the instances is incorporated in their names: 0 - if the demand is evenly distributed across the planned horizon (the horizon is 3 h), 1 - otherwise; d - provides the total number of the passengers in the network; c - is the capacity of a single train (in total there are 16 trains).

**Table 3**  
Validation of SA as compared to the optimal solutions.

| Instance | Cyclic timetable |          |         |          |         | Non-cyclic timetable |          |         |          |         |
|----------|------------------|----------|---------|----------|---------|----------------------|----------|---------|----------|---------|
|          | Optimal          | Time [s] | SA      | Time [s] | Gap [%] | Optimal              | Time [s] | SA      | Time [s] | Gap [%] |
| 0-d35-c1 | -3269            | 4        | -3358   | 11       | 2.72    | -3208                | 2        | -3272   | 12       | 1.98    |
| 0-d41-c2 | -2504            | 59       | -2624   | 13       | 4.80    | -2482                | 127      | -2589   | 12       | 4.29    |
| 0-d40-c2 | -2515            | 385      | -2570   | 14       | 2.20    | -2479                | 152      | -2507   | 10       | 1.15    |
| 1-d37-c2 | -2184            | 18       | -2236   | 11       | 2.37    | -2165                | 40       | -2215   | 11       | 2.30    |
| 0-d53-c3 | -3729            | 6177     | -3887   | 13       | 4.25    | -3698                | 716      | -3826   | 13       | 3.45    |
| 1-d36-c3 | -3020            | 4        | -3066   | 11       | 1.50    | -2989                | 3        | -3019   | 11       | 0.99    |
| 0-d52-c4 | -5686            | 20       | -5761   | 12       | 1.32    | -5641                | 7        | -5720   | 13       | 1.41    |
| 1-d63-c4 | -6568            | 2686     | -6656   | 14       | 1.34    | -6531                | 2905     | -6613   | 14       | 1.26    |
| 1-d63-c4 | -6854            | 6853     | -6966   | 12       | 1.63    | -6803                | 1854     | -6904   | 14       | 1.49    |
| 1-d80-c4 | -11,204          | 1457     | -12,021 | 15       | 7.29    | -11,133              | 32       | -11,866 | 15       | 6.58    |
| Avg.     | -4753            | 1766     | -4914   | 13       | 2.94    | -4713                | 584      | -4853   | 13       | 2.49    |

The detailed numerical results can be found in Table 3. They have been categorized by the type of the solved timetable: cyclic or non-cyclic. The first column provides the information about the solved instance. The optimal solution is the passenger satisfaction obtained by CPLEX and SA is the passenger satisfaction obtained by the SA heuristic. Both values are in Swiss Francs. All the methods have solution time reported in seconds. The gap between the optimal solution and the solution found by the SA heuristic is reported in percentages.

The SA heuristic can find solutions with a reasonable gap from the optimal solution. The worst performance is for the instances *0-d41-c2*, *0-d53-c3* and *1-d80-c4*. The first two instances are hard to solve as the demand is almost matching the supply. Therefore the order, in which the passengers are being processed in the assignment procedure, plays a crucial role. The FCFS policy would obtain worse values. The instance *1-d80-c4*, on the other hand, has the most extreme passenger distribution: 90% of the passengers in the first hour and 5% in each of the two subsequent hours. The average gap for the cyclic timetable is 2.94% and for the non-cyclic timetable 2.49%. In terms of the time, the SA heuristic performs better than CPLEX.

## 7. Case study

In order to evaluate the performance of the various timetables, we apply the proposed methodology on the network of Israeli Railways (IR) shown in Fig. 4. The aim is to compare the level of passenger satisfaction for different timetables. The exact procedures, assumptions and information about the data can be found in Appendix A.

We consider two instances in our study: the *2008 demand* and the *2014 demand*. The 2008 demand is build from the ticket selling machines' data of an average working day (from 6 a.m. to 1 a.m.) in 2008 in Israel. This data was kindly provided to us by Mor Kaspí and Tal Raviv, who have used it in their study Kaspí and Raviv (2013). The 2008 demand constitutes in 126,036 passengers. The 2014 demand constitutes in 193,886 passengers and it was created from the 2008 demand multiplied by the constant factor of 1.6 per OD pair (based on the newspaper article in Globes, 2015).

The demand data is available for the year 2008. Therefore, we consider the network layout of 2008: there were 47 stations and the red line in Fig. 4 was operated only between *Hod HaSharon* and *Tel Aviv - HaHagana*. We do not consider the night line (blue with black bordering) as it runs mainly in the period for which we do not have the demand data (*i.e.* between 1 a.m. and 6 a.m.). Even though there are only 18 unidirectional lines visualized in Fig. 4, in reality there are 34 unidirectional lines in the timetable (some lines are operated with higher frequency of 2 or more modulo times). Since some of the trains follow different stopping patterns within a line, we have taken a union of stopping stations for each line (but same colored lines with different modulo times may operate different stopping patterns). Since the OD matrix is given for an average working day, we have removed the trains that operate only during the holidays.

The timetable operated in Israel is cyclic with a cycle of 60 min. We have processed the timetable of 2013/14<sup>1</sup> where 6 out of 388 trains have non-cyclic departure times. The planning horizon  $h$  of this case study is one day. Even though the demand is between 6 a.m. and 1 a.m., we allow the PCTTP model to schedule trains during any time of the day. We solve the PCTTP using the SA heuristic for the following timetables:

- **Non-Cyclic** – no specific rule on the departure times is enforced.
- **Cyclic** – the departure times have to be cyclic according to the constraints (3).
- **$\theta$  Shifted Cyclic** – the departure times are subject to the constraints (5) and (6). The values of  $\theta$  vary between 3 and 30 min in 3 min intervals. Since  $\theta$  of 0 is equivalent to the cyclic timetable, it does not need to be solved.
- **$\xi$  Partially Cyclic** – the departure times are subject to the constraints (8). The values of  $\xi$  vary between 10% and 90% in 10% intervals. Since  $\xi$  of 0 and 100 is equivalent to the cyclic and the non-cyclic timetables respectively, they do not need to be solved.

<sup>1</sup> Unlike Europe, the timetable change in Israel happens during the summer period, *i.e.* the naming 13/14.

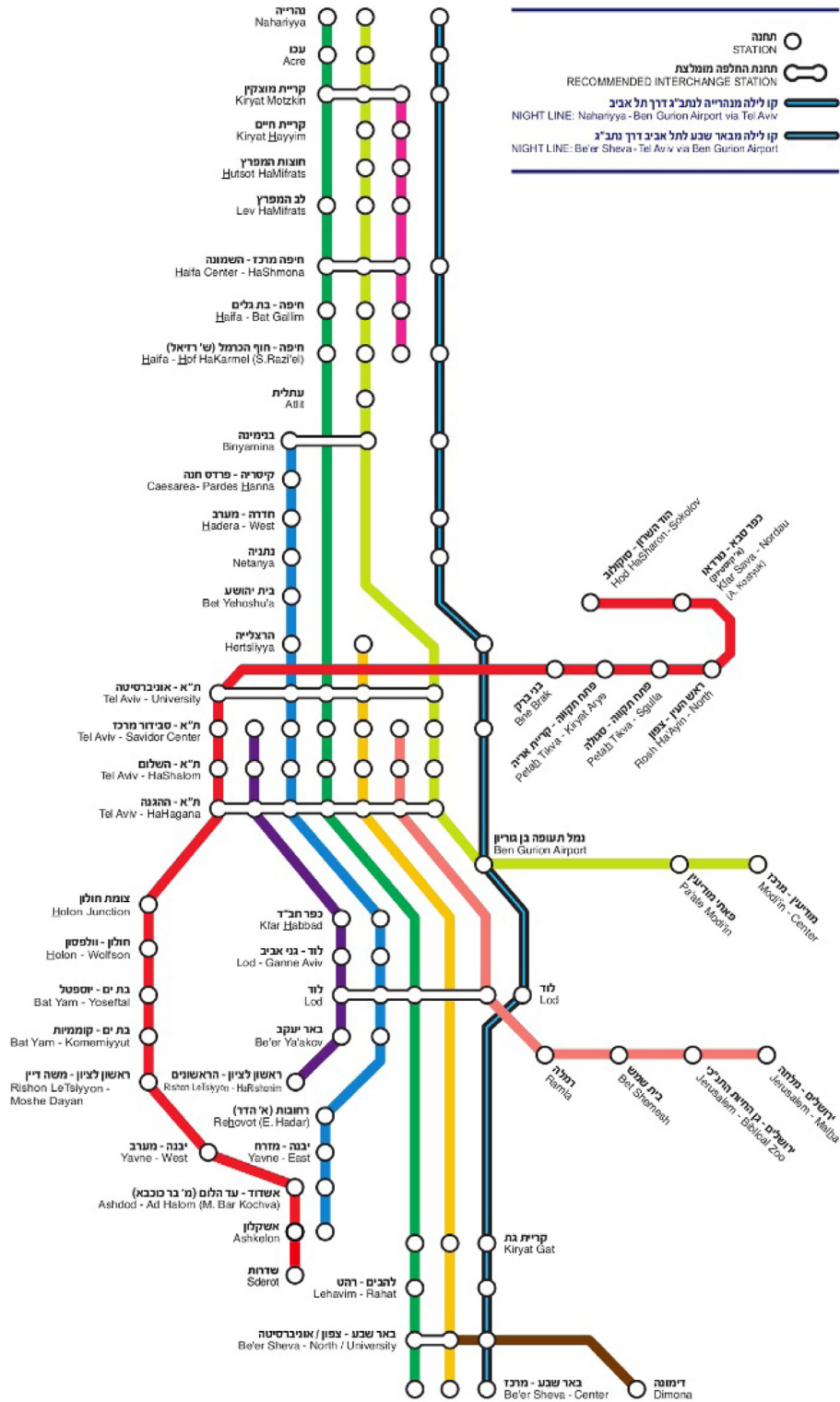


Fig. 4. Network of Israeli Railways ([www.rail.co.il](http://www.rail.co.il)).

- **Hybrid Cyclic** – the departure times have to comply with the constraints (9)–(11).

For the sake of the comparison, we show the performance of the two below timetables as well:

- **IR 13/14** – the departure times are fixed to the ones of IR timetable of 13/14. The IR timetable is cyclic as using the constraints (3), with an exception of having the aforementioned 6 non-cyclic trains.
- **IR 13/14 as Strictly Cyclic** – is the same as IR 13/14, where the 6 non-cyclic trains are fixed to their closest cyclic departure time, and the gaps between the first and the last scheduled train of every line are filled with cyclic trains (in total 82 trains more). This is equivalent to having the constraints (2). The exact difference between the two timetables can be observed on the train distributions in Table 7 and Table 8 in Appendix B.

The passenger satisfaction of each type of timetable is given in New Israeli Shekels (NIS). All of the tested instances have been run in Java on a Unix server with up to 24 cores of 3.33 GHz and 62 GiB RAM. Since we would typically run 3 instances at the same time, the average core usage would be then 8.

### 7.1. Results

In this section, we present the results of our case study. We categorize the results into two groups: existing and hybrid timetables. The existing ones are: strictly cyclic, cyclic and non-cyclic timetables. The hybrid ones are:  $\theta$  shifted cyclic,  $\zeta$  partially cyclic and hybrid cyclic timetables.

At first, we construct the demand distributions over the day (Fig. 5). Each distribution is in fact a network load, where the passengers take their shortest path between their origin and destination with just a minimum transfer time (*i.e.* no waiting time in transfers) and arriving to their destinations exactly on time, thus their schedule passenger delay being zero. Note that this network load does not involve any actual timetable and it is only used as a benchmark. We denote it as the *perfect service*.

The passenger satisfaction of the 2008 demand distribution under the *perfect service* is –2,089,049 NIS and of the 2014 demand distribution –3,171,721 NIS. These two values are upper bounds of the two instances for all the types of the timetables (no better solution exists). The aim of the timetable design should be a network load as close as possible to the perfect service.

#### 7.1.1. Existing timetables

The performance of each timetable can be found in Table 4 for the 2008 demand and in Table 5 for the 2014 demand. For each timetable, we report the underlying passenger satisfaction in New Israeli Shekels (NIS). The perfect service reports the satisfaction as it is and the different timetables report the passenger satisfaction as a relative difference to the perfect service. Apart from the satisfaction, the tables provide information on the number of train drivers needed to realize the operation, as well as the needed number of train units, the percentages of passengers that were able to realize their journeys and the solution time in seconds.

It is interesting to see (from the tables) that even though the IR 13/14 as strictly cyclic timetable offers more trains, its passenger satisfaction is lower than of the IR 13/14 timetable for both demand distributions. The reason behind this, is the fact that there are 6 non-cyclic trains in the IR 13/14 timetable that need to have a cyclic departure time in the strictly cyclic one. The decrease is then caused by the additional waiting time between the original non-cyclic departure and the new (strictly) cyclic one. Overall, running a strictly cyclic timetable is not desirable as the operating cost is higher and the additional benefit is either low or non-existent.

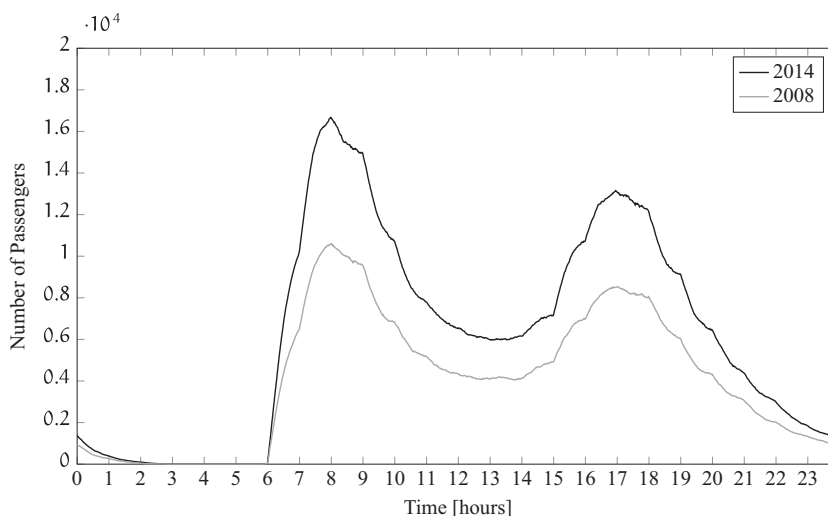


Fig. 5. Demand distributions over the day.

**Table 4**

Computational results of the existing timetables for the 2008 demand.

|                    | IR 13/14 as Strictly Cyclic | IR 13/14 | Cyclic   | Non-cyclic | Perfect service |
|--------------------|-----------------------------|----------|----------|------------|-----------------|
| Satisfaction [NIS] | -704,904                    | -537,503 | -476,774 | -424,529   | -2,089,049      |
| Drivers [-]        | 470                         | 388      | 388      | 388        | 48,960          |
| Rolling stock [-]  | 940                         | 776      | 776      | 776        | 48,960          |
| Covered [%]        | 100                         | 100      | 100      | 100        | 100             |
| Time [s]           | 12                          | 6        | 24,997   | 25,613     | 1               |

**Table 5**

Computational results of the existing timetables for the 2014 demand.

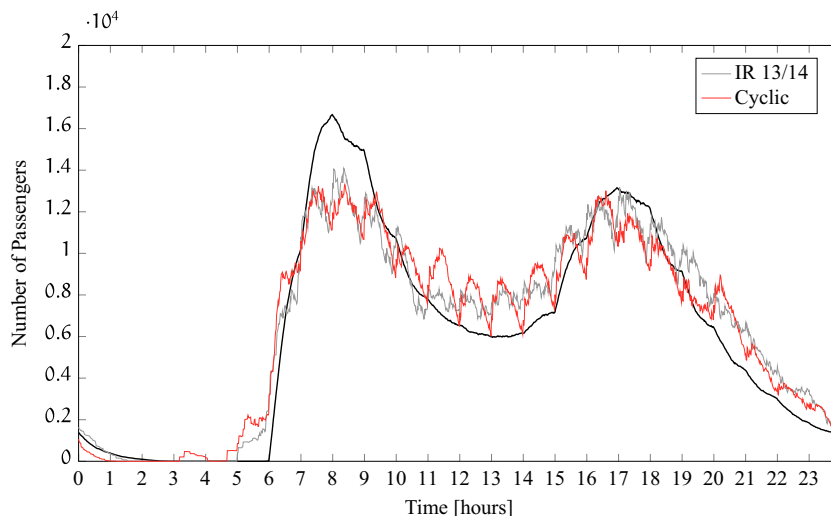
|                    | IR 13/14 as Strictly Cyclic | IR 13/14   | Cyclic     | Non-cyclic | Perfect service |
|--------------------|-----------------------------|------------|------------|------------|-----------------|
| Satisfaction [NIS] | -3,792,733                  | -3,379,596 | -2,392,909 | -1,365,779 | -3,171,721      |
| Drivers [-]        | 470                         | 388        | 388        | 388        | 48,960          |
| Rolling stock [-]  | 940                         | 776        | 776        | 776        | 48,960          |
| Covered [%]        | 99.17                       | 99.32      | 99.32      | 99.23      | 100             |
| Time [s]           | 11                          | 8          | 86,627     | 88,342     | 2               |

When we compare the IR 13/14 timetable with the cyclic timetable, we can see that proportionally large increase in passenger satisfaction can be achieved: approx. 60,000 NIS for the 2008 demand and approx. 1 million NIS for the 2014 demand. As the passenger coverage is 100% and 99.32% respectively, we can assume that the 2014 demand is just about the maximum capacity of the network, whereas the 2008 demand is over supplied.

A similar situation happens for the difference between the cyclic and the non-cyclic timetable. The differences in the passenger satisfaction are approx. 50,000 NIS and approx. 1 million NIS for the two demand distributions respectively. The solution time of the SA heuristic is on average 7 h for the 2008 demand and one day for the 2014 demand. As the differences among the timetables in the 2008 demand are marginal, we will from now on keep our focus only on the 2014 demand.

When we plot the network load for the 2014 case of the IR 13/14 timetable and of the cyclic timetable (Fig. 6), we can see that both timetables are fairly good in addressing the evening peak hours, but fail at addressing the morning ones. The failure propagates into surrounding time periods: early start of the morning peak (*i.e.* some passengers arriving early at their destinations) and a heightened level of passengers in the network during the off peak hours (*i.e.* late arrivals from the morning peak and early arrivals for the evening peak). Both timetables exhibit the nature of the repeating pattern inside the plot.

The improvement of the cyclic timetable over the IR 13/14 timetable is approx. 1 million NIS. According to the breakdown of the passenger satisfaction (in Fig. 8), the improvement is mainly in the passenger schedule delay. This is due to the fact,

**Fig. 6.** Network load of the IR 13/14 and of the cyclic timetable for the 2014 demand.



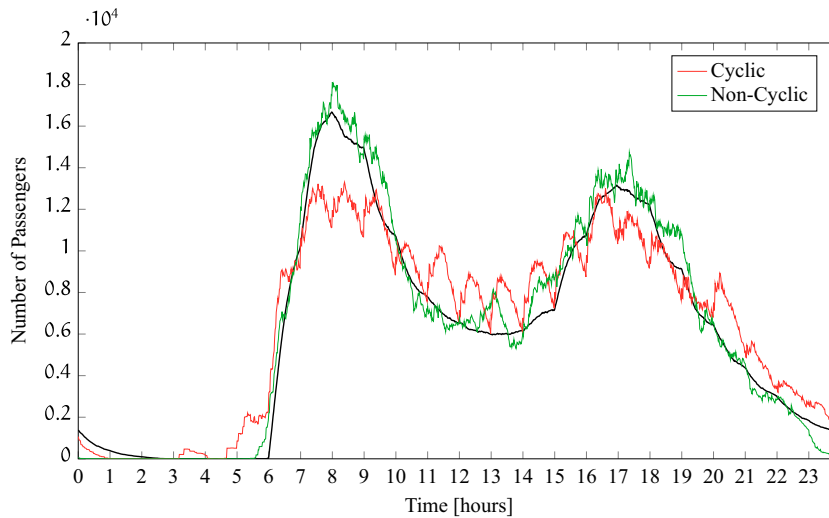


Fig. 7. Network load of the cyclic and of the non-cyclic timetable for the 2014 demand

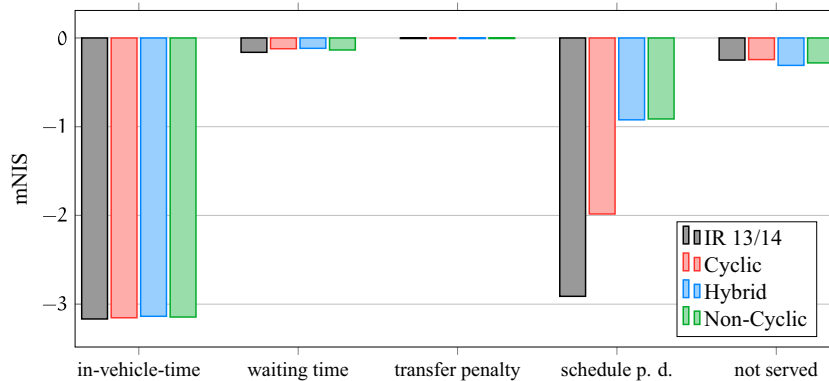


Fig. 8. Breakdown of the passenger satisfaction for various timetables under the 2014 demand.

that the cyclic timetable is having more passengers arriving earlier into their destinations than the IR 13/14 timetable. Indeed, the  $\beta$  estimate of being early has lower value ( $= 0.5$ ) than the  $\beta$  estimate of being late ( $= 1$ ).

When we compare the network load of the cyclic and the non-cyclic timetable (Fig. 7), we can see that the non-cyclic one is indeed more flexible in accounting for the passenger demand. Especially within the peak hours, where there is often more than one train being scheduled (in some cases even 4 trains per hour, Table 10 in Appendix B as compared to Table 9 in Appendix B for the cyclic one). This brings about one more million NIS of improvement in the schedule passenger delay (Fig. 8).

When we look at the breakdown of the passenger satisfaction (Fig. 8), we can see that the most important is the attribute of the schedule passenger delay representing the fact that the passenger demand is time dependent. Thus, when designing a railway timetable, one should always take into account the passengers and their travel time preferences.

### 7.1.2. $\theta$ shifted cyclic timetable

As mentioned before, we solve this timetable for various values of  $\theta$  within the interval between 3 and 30 in 3 min step size (0 being the solution of the cyclic timetable). The plot of the passenger satisfaction as a function of  $\theta$  can be found in Fig. 9.

The 0 min shifted cyclic timetable is equivalent to the cyclic one, which is represented by the dashed line. The 30 min shifted cyclic timetable, on the other hand, is not equivalent to the non-cyclic one (the dotted line), as the maximum number of trains that are scheduled within one hour is 2 (Table 11 in Appendix B) and 4 (Table 10 in Appendix B) respectively. Thus the  $\theta$  shifted cyclic timetable can achieve at most half of the flexibility of the non-cyclic one (around half a million NIS). The trend of the function is rather linear.

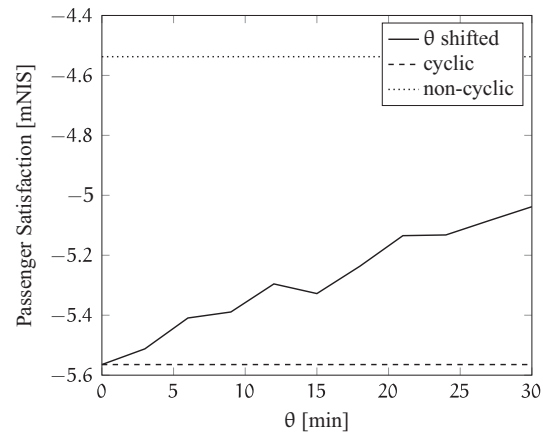


Fig. 9. Passenger satisfaction of the  $\theta$  shifted cyclic timetable for the 2014 demand.

Table 6

Computational results of partially cyclic timetables for different levels of partiality under the 2014 demand.

| $\xi$ [%] | P. Satisfaction [NIS] | $\eta$ | Cyclic | Non-cyclic | Time [s] |
|-----------|-----------------------|--------|--------|------------|----------|
| 0         | +1,027,131            | 0      | 0      | 388        | –        |
| 10        | +996,108              | 2      | 64     | 324        | 48,698   |
| 20        | +998,444              | 4      | 124    | 264        | 59,889   |
| 30        | +1,059,768            | 5      | 152    | 236        | 64,320   |
| 40        | +991,634              | 7      | 207    | 181        | 69,351   |
| 50        | +1,002,523            | 9      | 254    | 134        | 66,584   |
| 60        | +973,353              | 11     | 296    | 92         | 72,352   |
| 70        | +842,934              | 13     | 331    | 57         | 77,017   |
| 80        | +811,925              | 14     | 346    | 42         | 69,147   |
| 90        | +629,754              | 16     | 372    | 16         | 76,766   |
| 100       | –5,564,631            | 18     | 388    | 0          | –        |

Overall, the improvement in flexibility of the newly proposed timetable is promising. It is a sign that new alternative timetables exist and should be taken into account. As this was the more restrictive configuration, we expect even larger impacts from the other two hybrid timetables.

### 7.1.3. $\xi$ partially cyclic timetable

In this section, we present the computational results of the  $\xi$  partially cyclic timetable for different levels of partiality under the 2014 demand (Table 6). The first column represents the level of the partiality as a percentage. The second column shows the passenger satisfaction in NIS. The satisfaction for  $\xi = 100$  is given as it is and all other satisfactions are given as relative to this one. Next columns give information on the  $\eta$  (which is the number of trains per line that have to follow the cyclic pattern, Eq. (8)), the total number of trains that follow cyclic pattern, the total number of trains that are non-cyclic and the solution time of the SA heuristic.

We plot the passenger satisfaction as a function of  $\xi$  in Fig. 10. The function is rather exponential. At the partiality of 30%, it outperforms the non-cyclic timetable. This is caused due to the heuristical solution approach, i.e. the non-cyclic timetable might have gotten trapped in a local optimum.

More importantly, the 90% partially cyclic timetable can achieve slightly better values of the passenger satisfaction than the 30 min shifted cyclic timetable by having only 16 trains non-cyclic. Overall, the partially cyclic timetable achieves the passenger satisfaction values close to the ones of the non-cyclic timetable already around 60% partiality. The ratio, of the cyclic and the non-cyclic trains at this level of the partiality, is approximately 3:1. This means that a large improvement can be achieved by having only small adjustments made to the cyclic timetable. We further support this conclusion on the results of the hybrid cyclic timetable.

### 7.1.4. Hybrid cyclic timetable

The hybrid cyclic timetable as compared to the partially cyclic one, controls the ratio between the number of the cyclic and the non-cyclic trains by itself. The resulting ratio is approximately 3:1, which is the same case as for the 60% partially

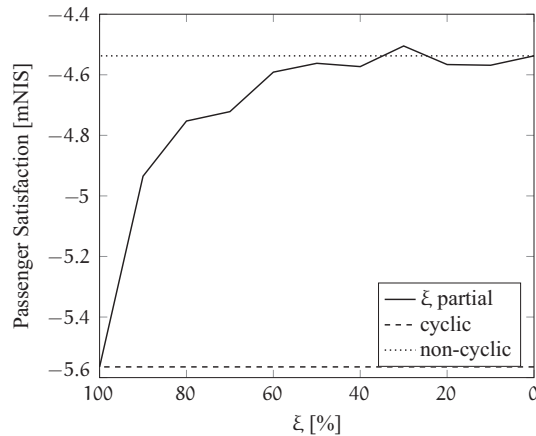


Fig. 10. Passenger satisfaction of the  $\xi$  partially cyclic timetable for the 2014 demand.

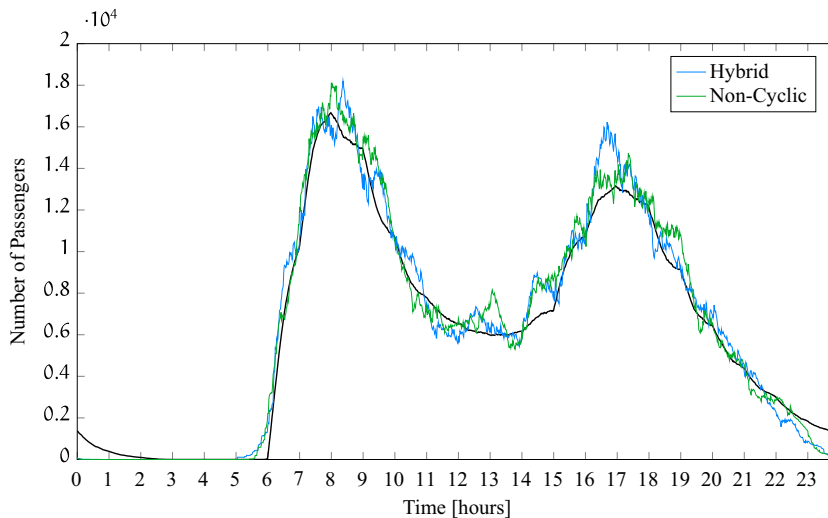


Fig. 11. Network load of the non-cyclic and of the hybrid cyclic timetable for the 2014 demand.

cyclic timetable. The passenger satisfaction is close to the one of the non-cyclic timetable (it is worse by only 13,405 NIS) and of the aforementioned 60% partially cyclic timetable (it is better by 40,373 NIS).

When we consider the network load (Fig. 11), it is having a similar shape as the one of the non-cyclic timetable. In terms of the train distribution (Table 12 in Appendix B), we can see the trend of the non-cyclic trains being mainly scheduled during the morning and evening peak hours. The actual departure times of all of the trains can be found in Table 13 in Appendix C. The solution time of the SA heuristic is 60,113 s. Overall, the hybrid cyclic timetable has proven to be the most suitable configuration as it achieves the flexibility of the non-cyclic one, while keeping a good level of regularity.

## 7.2. Summary

Overall for this case study, the proposed hybrid timetables tend to reduce or to completely diminish the quantitative impact of the regularity (cyclic constraints). The original gap between the flexibility of the cyclic and the non-cyclic timetable is 18.5%. The  $\theta$  shifted cyclic timetable is able to reduce this gap to a half (for  $\theta = 30$ ). The  $\xi$  partially cyclic timetable can further reduce this gap to zero already at a level of  $\xi = 60$ . The ratio of the cyclic and the non-cyclic trains under this  $\xi$  is 75% and 25% respectively. This ratio of trains is further supported by the hybrid cyclic timetable, that aims at providing more even offer of the regularity to the passengers than the other hybrid timetables. Cycles can have either only a cyclic train or both cyclic and several non-cyclic ones. The cyclic trains are used to secure the regularity and the non-cyclic trains are used to improve the service level during high demand periods.

**Table 7**  
Train distribution of the IR 13/14 timetable

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0  | 0  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 0  | 0  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0  | 0  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 0  | 0  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0  | 0  | 0  | 0  | 0  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0  | 0  | 0  | 0  | 0  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0  | 0  | 0  | 0  | 0  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0  | 1  | 0  | 0  | 1  | 0  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 0  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |

**8. Conclusion and future work**

In this research, we consider the quantitative and the qualitative aspects of different passenger railway timetables. The quantitative attributes (the passenger satisfaction) show that the non-cyclic timetable is more flexible to account for the passenger demand than the cyclic one. On the other hand, the cyclic timetable has some qualitative aspects that are favored by the passengers. Namely the regularity of the service, which makes the timetable easy to be memorized by the passengers. Thus a superiority of one over the other is difficult to estimate.

In the light of these findings, we propose several combinations of the two, that would implement the qualitative aspects and allow us to explore and to reduce the quantitative impacts of the regularity. We have considered three types of hybrid timetables. The first two are generalizations of existing methods, where the relaxations of the cyclicity are explicitly modeled. The last one is an extension of the cyclic timetable imposing the non-cyclic trains to be coordinated with cyclic ones. The basic idea is that the passengers can choose between the flexibility and the regularity. We use the Passenger Centric Train Timetabling Problem to design the different timetables and solve it using the simulated annealing heuristic, that has proven to be an efficient instrument. We illustrate our findings on the network of Israeli Railways using the ticket selling machines' data to construct the passenger demand data. The results of our case study suggest that the hybrid cyclic timetable is the most promising approach.

Our findings have important impacts for TOCs. Indeed, cyclicity is desired in order to encourage the passengers to use train as a mode of transportation. Still, its lack of flexibility does not generate a supply configuration that is satisfying for the travelers in terms of level of service. The concept of hybrid timetables that we propose in this research allows to maintain



**Table 9**  
Train distribution of the cyclic timetable under the 2014 demand.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 0  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0  | 1  | 0  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 0  | 1  | 0  | 0  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 0  | 0  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1  | 0  | 0  | 0  | 0  | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0  | 1  | 0  | 1  | 0  | 0  | 1  | 1  | 0  | 0  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
| 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 1  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0  | 0  | 0  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0  | 0  | 0  | 0  | 1  | 0  | 1  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 1  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0  | 0  | 1  | 1  | 0  | 0  | 1  | 0  | 0  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 1  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0  | 0  | 1  | 1  | 0  | 0  | 1  | 0  | 0  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0  | 1  | 0  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 1  |

through all OD pairs, where at first it considers the paths that consist of a single line and then the paths that would transfer from the currently selected line up to two other lines. The transfer from one line to another can be made only at one of the designated transfer points (there are 7 recommended interchange stations in the network of Israel – Fig. 4). Note that the fact, that a transfer is actually possible depends on the operated timetable. Therefore, some paths might be eliminated later on by the PCTTP model itself. When all the possible paths are generated, the algorithm removes the paths that a passenger would not consider (note that these rules are related to the network layout of Israel and might differ for other case studies):

- paths that consist of several lines including a direct line between the given OD pair, where both options travel on the same infrastructure (i.e. the passenger would rather stay on the direct line instead of transferring to another line).
- paths that consist of several lines, where two of them can reach the given destination. Changing one train to another, when both of them are going to the same destination would not make sense (the same does not happen for the origin).
- paths that take 25% longer generalized time (sum of in-vehicle-times and transfer penalties) than the shortest possible path.
- paths that consist of redundant transfers, i.e. transferring from one line to another line that covers the same stations.

**Table 10**  
Train distribution of the non-cyclic timetable under the 2014 demand.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 1 | 1 | 1  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 0  | 1  | 0  | 1  | 2  | 2  | 2  | 2  | 0  | 2  | 0  | 0  | 1  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 4 | 1 | 1 | 0  | 1  | 0  | 1  | 1  | 1  | 0  | 2  | 0  | 1  | 0  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 2 | 1 | 1  | 1  | 1  | 0  | 2  | 2  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1 | 1  | 0  | 0  | 0  | 1  | 0  | 3  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 2 | 2 | 0  | 0  | 0  | 0  | 0  | 1  | 2  | 0  | 0  | 0  | 0  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 0  | 0  | 1  | 0  | 1  | 2  | 2  | 1  | 1  | 1  | 0  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 1 | 1 | 0  | 0  | 0  | 0  | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 1 | 1 | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 0  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 1  | 0  | 1  | 1  | 1  | 2  | 2  | 2  | 1  | 0  | 1  | 0  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 1 | 1 | 1  | 1  | 1  | 0  | 2  | 1  | 3  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 2 | 0 | 1  | 0  | 1  | 2  | 0  | 1  | 1  | 0  | 2  | 0  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0  | 1  | 0  | 0  | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1  | 0  | 1  | 0  | 0  | 0  | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 3 | 2 | 0  | 1  | 1  | 0  | 0  | 1  | 3  | 2  | 1  | 2  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 2 | 1 | 0  | 1  | 0  | 1  | 2  | 1  | 1  | 1  | 2  | 0  | 1  | 0  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 1 | 1  | 0  | 0  | 0  | 0  | 2  | 1  | 1  | 3  | 0  | 0  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 2 | 3 | 1  | 0  | 1  | 0  | 0  | 1  | 0  | 2  | 0  | 1  | 1  | 0  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1  | 1  | 1  | 2  | 1  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 1  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 0 | 0 | 1  | 0  | 0  | 0  | 1  | 0  | 2  | 0  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 1 | 1 | 1  | 0  | 2  | 0  | 1  | 4  | 0  | 1  | 1  | 1  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 3 | 2 | 0  | 1  | 2  | 0  | 2  | 0  | 1  | 2  | 1  | 0  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0  | 1  | 1  | 0  | 1  | 1  | 2  | 2  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 1 | 0 | 2  | 1  | 1  | 0  | 1  | 1  | 1  | 1  | 0  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0  | 0  | 1  | 0  | 0  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1  | 1  | 0  | 0  | 0  | 0  | 2  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 1  | 1  | 0  | 1  | 1  | 0  | 1  | 1  | 0  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 1 | 1 | 1  | 0  | 0  | 0  | 0  | 1  | 0  | 1  | 0  | 1  | 0  | 0  | 0  | 1  |

Each passenger is having one extra path that represents their shortest path between an origin and their destination, when using the first scheduled train after the planning horizon. This path represents a penalty of not being served. The largest number of paths between an origin and a destination is 73 and the total amount of paths in the network is 21,469.

The traveling times have been extracted from the IR's website along with the dwell times at stations that remain fixed (as of the timeTable 2013/14). The minimum transfer time is set to 4 min as in Kaspi and Raviv (2013). The Value Of Time of commuters in Israel as of the year 2012 is 21.12 New Israeli Shekel (NIS)/hour (the updated value was given to us by the author of Shiftan et al. (2008)). The  $\beta$  parameters and their values are as follows:  $\beta_W = 2.5$  (Wardman, 2004),  $\beta_T = 10$  (de Keizer et al., 2012),  $\beta_E = 0.5$  and  $\beta_L = 1$  (Small, 1982).

### A.1. Passenger

The OD flows were kindly provided by Mor Kaspi and Tal Raviv, who have cleaned the ticket selling machines' data for the year 2008 and produced the flows of an average working day in Israel. They have used this data in their study Kaspi and Raviv (2013). The OD matrix consists of hourly passenger rates between 6 a.m. and 1 a.m. The flows were smoothed into minutes by using non-homogenous Poisson process, where the hourly flows per OD pair were used as the arrival rate variable. Since the schedule passenger delay is related to the destination, we have added the time that it takes to get from an origin to a destination using the shortest path (if the path consisted of transfers, we assumed the perfect connection, i.e. only

**Table 11**  
Train distribution of the 30 min shifted cyclic timetable under the 2014 demand.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 0  | 2  | 1  | 1  | 1  | 0  | 1  | 1  | 0  | 1  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 2  | 0  | 1  | 2  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 2 | 0  | 2  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 1 | 1  | 0  | 1  | 1  | 1  | 1  | 2  | 1  | 1  | 1  | 1  | 0  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 0 | 0  | 1  | 0  | 0  | 1  | 1  | 1  | 2  | 0  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 1 | 1 | 1  | 1  | 0  | 0  | 0  | 1  | 1  | 0  | 2  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 0  | 0  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 1 | 1  | 1  | 1  | 0  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 1 | 1  | 0  | 1  | 1  | 0  | 2  | 0  | 2  | 0  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0  | 1  | 0  | 1  | 0  | 2  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 1  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 1 | 1 | 0  | 1  | 1  | 1  | 1  | 2  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 1 | 1 | 2 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 1  | 0  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 1  | 0  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 1  | 0  | 0  | 0  | 0  | 1  | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0  | 1  | 0  | 0  | 0  | 1  | 1  | 1  | 0  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 1 | 1 | 1  | 1  | 1  | 0  | 1  | 0  | 2  | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 1  | 0  | 1  | 1  | 1  | 0  | 2  | 0  | 1  | 1  | 2  | 0  | 0  | 1  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 1 | 1  | 1  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0  | 1  | 1  | 0  | 0  | 1  | 0  | 2  | 1  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 1 | 0 | 0  | 0  | 0  | 0  | 0  | 2  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 0 | 2 | 1  | 1  | 0  | 2  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 0 | 1  | 1  | 1  | 2  | 0  | 2  | 1  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 2 | 1  | 1  | 1  | 1  | 1  | 0  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0  | 1  | 0  | 1  | 0  | 0  | 2  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0  | 1  | 0  | 1  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0  | 1  | 1  | 0  | 1  | 1  | 0  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 2 | 0  | 1  | 1  | 0  | 0  | 0  | 1  | 1  | 0  | 0  | 0  | 1  | 1  | 0  |

the minimum transfer time without any additional waiting at the transfer station). In total there are 1505 out of 2162 OD pairs with 126,036 passengers.

**A.2. Operator**

As no information about the rolling stock fleet of IR is available, we have introduced the following assumptions:

- The fleet is homogenous
- A train unit has a passenger capacity of 250
- Each train can consist of up to 2 train units
- The number of train units remains the same between the start and the end station of a train

In order to verify that the assumed train capacity is reasonable, we have solved the un-capacitated PCTP of the IR 13/14 timetable under the 2008 demand, where the average train occupation was 172 passengers per train per segment (ppts),



**Table 12**  
Train distribution of the hybrid cyclic timetable under the 2014 demand.

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
|---|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 2 | 1 | 2 | 1  | 1  | 0  | 1  | 1  | 1  | 1  | 0  | 2  | 0  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 2  | 1  | 1  | 1  | 1  | 1  | 0  | 1  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 3 | 2 | 2 | 1 | 0  | 0  | 2  | 0  | 1  | 1  | 2  | 1  | 0  | 0  | 0  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 0 | 1  | 1  | 0  | 1  | 1  | 3  | 1  | 2  | 0  | 1  | 1  | 0  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0  | 1  | 1  | 0  | 1  | 1  | 1  | 1  | 0  | 1  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 3 | 2 | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 1  | 2  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 0 | 2 | 0  | 0  | 0  | 0  | 0  | 2  | 2  | 1  | 0  | 2  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 3 | 0 | 2  | 0  | 0  | 0  | 2  | 0  | 1  | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 2 | 1 | 1  | 1  | 0  | 0  | 1  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 0 | 0  | 0  | 2  | 1  | 2  | 1  | 2  | 1  | 2  | 1  | 0  | 0  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 3 | 1 | 1 | 0  | 2  | 1  | 1  | 1  | 2  | 1  | 3  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 2 | 1 | 1  | 1  | 1  | 1  | 1  | 1  | 2  | 1  | 1  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 2  | 1  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 0 | 0 | 1  | 1  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 1 | 1 | 0  | 1  | 0  | 1  | 2  | 1  | 3  | 1  | 1  | 1  | 1  | 0  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 3 | 1 | 1 | 1  | 0  | 2  | 0  | 0  | 1  | 1  | 1  | 2  | 2  | 0  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 2 | 0 | 0  | 1  | 3  | 1  | 0  | 3  | 1  | 1  | 1  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1 | 1 | 1 | 1  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 2  | 0  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 2 | 1  | 1  | 0  | 0  | 0  | 0  | 0  | 1  | 1  | 1  | 0  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 3 | 0 | 0  | 0  | 0  | 2  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 1  |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 1  | 1  | 0  | 1  | 2  | 2  | 1  | 2  | 2  | 1  | 1  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 3 | 1 | 2 | 2 | 1  | 0  | 0  | 1  | 0  | 2  | 1  | 1  | 0  | 1  | 1  | 1  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 2 | 2 | 0  | 1  | 0  | 1  | 1  | 1  | 2  | 0  | 0  | 2  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 3 | 1 | 1 | 2  | 0  | 2  | 0  | 1  | 1  | 1  | 0  | 2  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 1  | 0  | 0  | 1  | 0  | 3  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 1 | 1 | 1 | 0  | 2  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0  | 1  | 0  | 1  | 1  | 1  | 1  | 1  | 0  | 0  | 0  | 1  | 1  | 0  |
| 0 | 0 | 0 | 0 | 0 | 0 | 2 | 2 | 0 | 0 | 0  | 1  | 0  | 1  | 1  | 1  | 0  | 0  | 1  | 0  | 0  | 0  | 0  | 1  |

minimum occupation was 0 pptps, maximum occupation 1188 pptps and median was 124 pptps. Thus the capacity of 250 passengers per train unit offers a good level of service.

**Appendix B. Train distributions**

In the tables of this section, we present the train distributions of various types of timetables. The first row of each table, represents the hour of the day. Each subsequent row, represents each of the lines in the network (in total 34 lines). The number in each cell gives the information about the number of trains scheduled in that given hour on that given line. The minimum value found is 0 and the maximum value found is 4.

**Appendix C. Resulting hybrid cyclic timetable**

Each line of the above table represents a single line. The first column of the table provides the value of the cyclic time ( $d_i^c \text{ mod } c$ ). The following columns show the value of the departure times in minutes. The cyclic trains have a clear background, whereas the non-cyclic trains have a grey background. If a cell is empty, it means that its line is having less trains available than the most frequent line.

**Table 13**  
Resulting hybrid cyclic timetable under the 2014 demand.

| ct |      |     |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
|----|------|-----|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| 43 | 343  | 388 | 403  | 436  | 463  | 523  | 544  | 583  | 643  | 703  | 823  | 883  | 943  | 1003 | 1082 | 1123 | 1243 | 1303 |
| 17 | 377  | 437 | 497  | 557  | 617  | 677  | 737  | 797  | 857  | 917  | 977  | 1000 | 1037 | 1097 | 1157 | 1217 | 1277 | 1397 |
| 38 | 398  |     |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| 9  | 969  |     |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| 11 | 371  | 401 | 417  | 431  | 461  | 488  | 491  | 551  | 731  | 776  | 851  | 911  | 971  | 1015 | 1031 | 1271 |      |      |
| 17 | 377  | 437 | 497  | 498  | 617  | 677  | 797  | 857  | 917  | 949  | 956  | 977  | 1034 | 1037 | 1157 | 1217 | 1337 |      |
| 22 | 382  | 502 | 562  | 682  | 742  | 862  | 922  | 982  | 1042 | 1162 | 1222 |      |      |      |      |      |      |      |
| 11 | 371  | 431 | 482  | 487  | 491  | 551  | 557  | 971  | 1031 | 1091 | 1116 |      |      |      |      |      |      |      |
| 11 | 371  | 431 | 444  | 472  | 551  | 597  | 911  | 948  | 961  | 971  | 1031 | 1151 | 1155 |      |      |      |      |      |
| 14 | 374  | 408 | 434  | 493  | 494  | 522  | 614  | 649  | 849  | 854  | 974  | 1034 |      |      |      |      |      |      |
| 25 | 385  | 418 | 445  | 476  | 505  | 536  | 565  | 625  | 685  | 865  | 925  | 985  | 1045 | 1105 | 1165 |      |      |      |
| 54 | 427  | 474 | 534  | 725  | 774  | 834  | 876  | 894  | 954  | 976  | 1014 | 1074 | 1119 | 1134 | 1194 | 1374 |      |      |
| 0  | 300  | 360 | 420  | 455  | 477  | 480  | 540  | 660  | 711  | 720  | 780  | 840  | 900  | 909  | 960  | 1020 | 1026 | 1063 |
| 20 | 320  | 380 | 440  | 488  | 500  | 560  | 620  | 680  | 740  | 800  | 860  | 920  | 972  | 980  | 1040 | 1100 | 1160 |      |
| 35 | 431  | 445 | 455  | 995  |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| 9  | 429  | 489 | 1089 | 1149 |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| 54 | 414  | 474 | 594  | 954  | 1109 | 1134 | 1194 |      |      |      |      |      |      |      |      |      |      |      |
| 6  | 366  | 426 | 479  | 606  | 666  | 966  | 1206 |      |      |      |      |      |      |      |      |      |      |      |
| 56 | 385  | 416 | 476  | 536  | 596  | 716  | 836  | 856  | 896  | 956  | 984  | 995  | 1016 | 1076 | 1136 | 1196 | 1256 | 1376 |
| 9  | 369  | 382 | 423  | 429  | 452  | 489  | 549  | 609  | 729  | 742  | 909  | 969  | 1029 | 1089 | 1137 | 1149 | 1197 | 1269 |
| 12 | 372  | 432 | 492  | 522  | 672  | 732  | 748  | 775  | 792  | 912  | 940  | 947  | 972  | 1032 | 1092 |      |      |      |
| 41 | 341  | 389 | 401  | 461  | 521  | 581  | 641  | 701  | 761  | 881  | 941  | 1001 | 1061 | 1108 | 1121 | 1241 |      |      |
| 29 | 509  | 569 | 586  | 629  | 689  | 1049 | 1109 | 1169 | 1289 |      |      |      |      |      |      |      |      |      |
| 49 | 469  | 498 | 510  | 529  | 829  | 836  | 1069 | 1369 |      |      |      |      |      |      |      |      |      |      |
| 31 | 331  | 391 | 451  | 511  | 631  | 691  | 811  | 871  | 892  | 902  | 931  | 991  | 1051 | 1070 | 1081 | 1111 | 1171 | 1231 |
| 3  | 303  | 363 | 387  | 390  | 423  | 483  | 515  | 543  | 599  | 603  | 783  | 903  | 941  | 963  | 1023 | 1143 | 1203 | 1263 |
| 30 | 390  | 487 | 510  | 564  | 570  | 690  | 810  | 870  | 930  | 990  | 997  | 1163 | 1170 |      |      |      |      |      |
| 0  | 420  | 442 | 453  | 480  | 540  | 600  | 629  | 720  | 747  | 840  | 900  | 960  | 1080 | 1117 |      |      |      |      |
| 17 | 1337 |     |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| 32 | 932  |     |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
| 38 | 398  | 818 | 998  | 1095 | 1104 | 1118 |      |      |      |      |      |      |      |      |      |      |      |      |
| 39 | 421  | 459 | 519  | 579  | 639  | 726  | 759  |      |      |      |      |      |      |      |      |      |      |      |
| 13 | 493  | 673 | 793  | 853  | 913  | 973  | 1033 | 1273 | 1333 |      |      |      |      |      |      |      |      |      |
| 40 | 374  | 400 | 460  | 479  | 700  | 820  | 880  | 940  | 1120 | 1300 | 1420 |      |      |      |      |      |      |      |

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