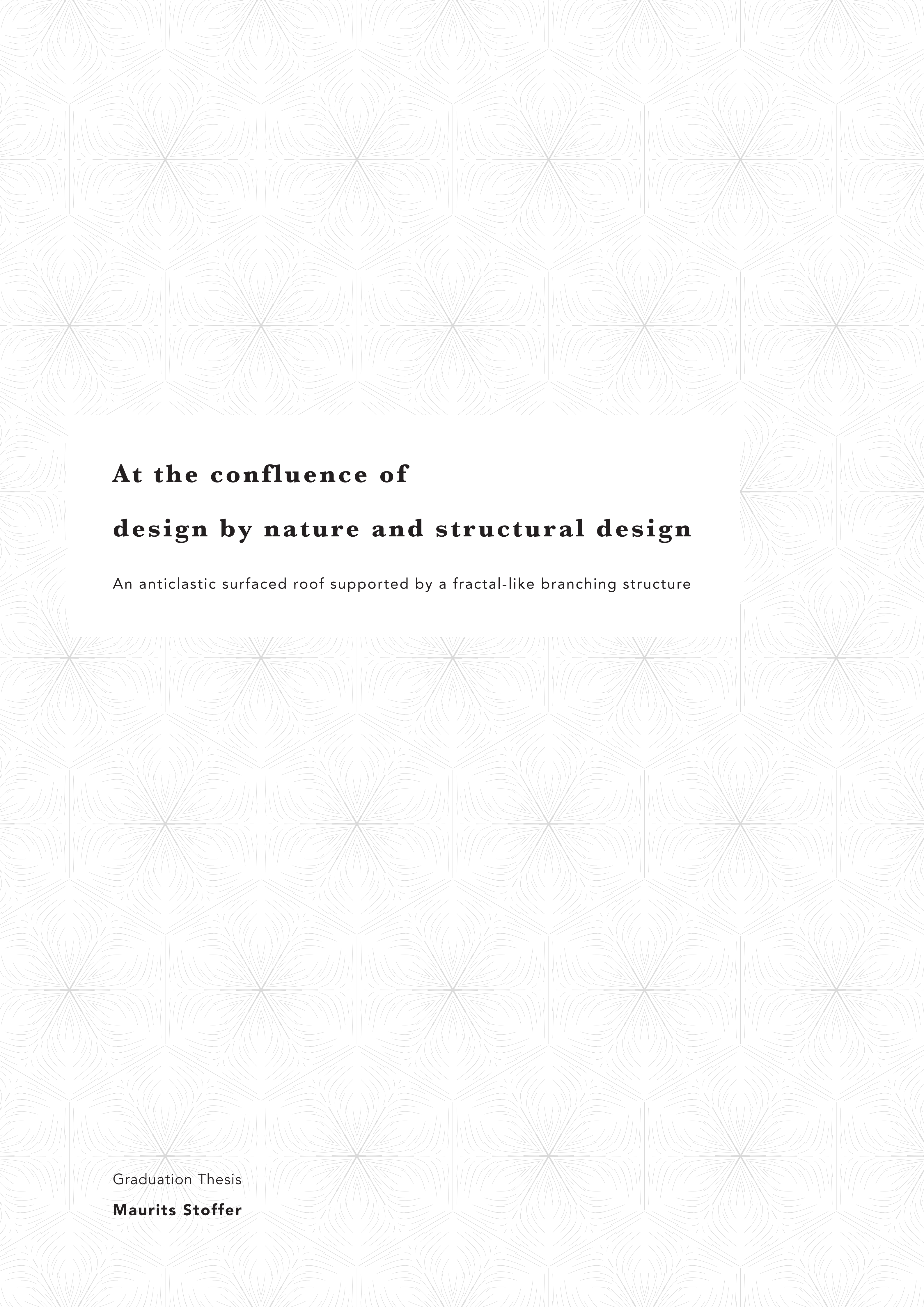

At the confluence of design by nature and structural design

An anticlastic surfaced roof supported by a fractal-like branching structure

Graduation Thesis

Maurits Stoffer

Student:

Maurits Jan Stoffer

Student number:

4098056

Contact:

mauritsstoffer@gmail.com

Members of the Graduation Committee:

Ir. Andrew Borgart

Architectural Engineering + Technology

Department of Structural Mechanics

Ir. Koen Mulder

Architectural Engineering + Technology

Department of Building Technology

Delft University of Technology

Faculty of Architecture and the Built Environment

Master Track of Building Technology

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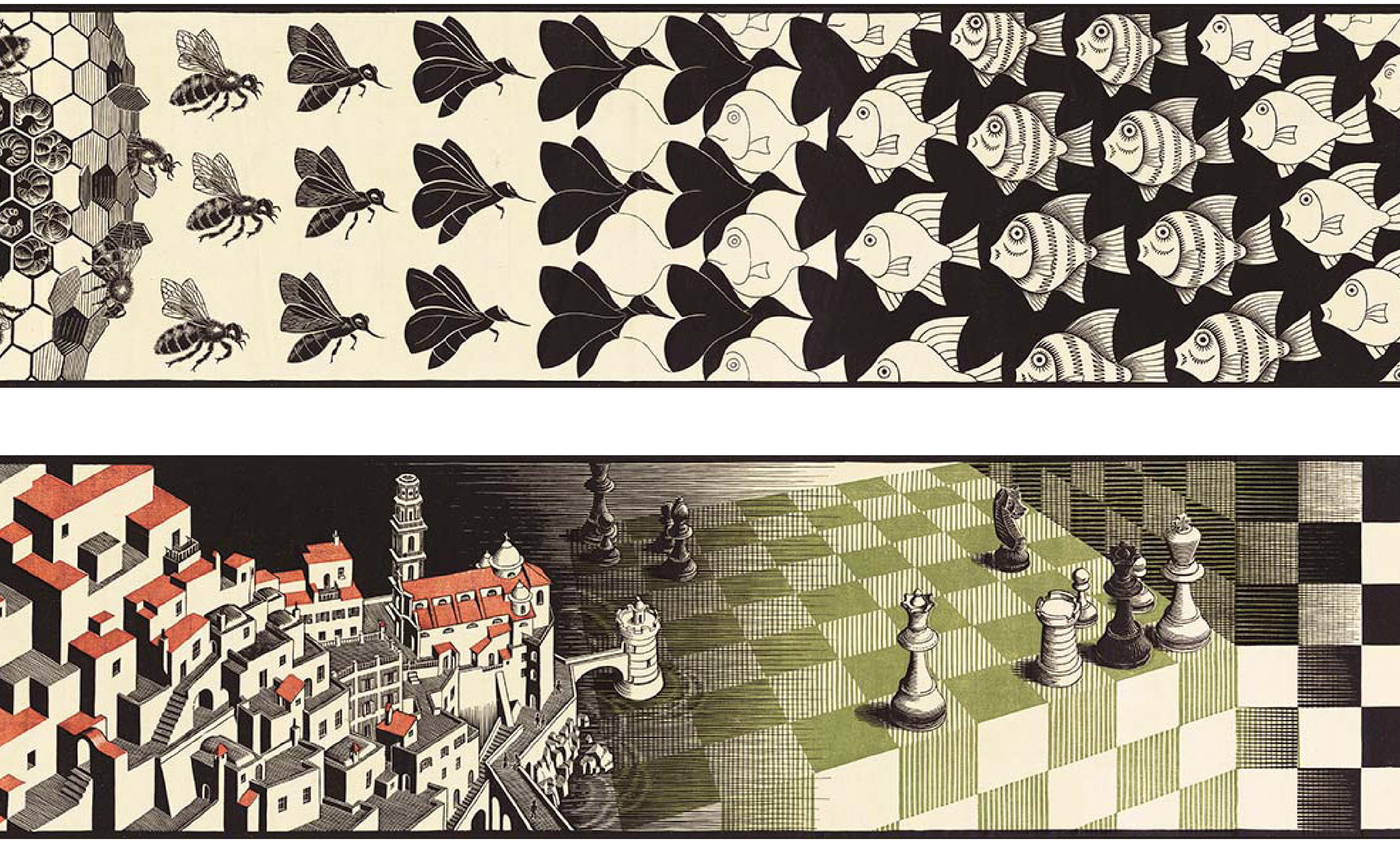

[1] METAMORPHOSIS II - 1940 WOODCUT - MAURITS CORNELIS ESCHER

ACKNOWLEDGEMENT

‘At the confluence of design by nature and structural design’ is the graduation thesis of the master track Building Technology at the University of Technology in Delft. This thesis is made by Maurits Stoffer with the help of many others. First of all my two tutors who have been guiding me during this process. My first mentor Ir. Andrew Borgart guided me from the very beginning. Finding a topic based on fascination that was worth doing further research on was a challenging task and Andrew helped me with this. He helped me by starting off with the basics of structural mechanics and from there on finding the right approach for the next stages of this graduation topic. After a few months, once the phase of the preliminary design was finished, Ir. Koen Mulder started helping me with the building method and the feasability of the idea as a whole. Towards the final phases of the project, they both encouraged me to not only stick to the digital work, but also to start building models and to begin with simple physical tests. Sometimes Andrew gave me a redirection when it comes to the structural design of the application and Koen was there with his expertise for brainstorm sessions on smart building technology solutions. Great thanks goes out to both.

Next, I would like to thank my fellowstudents, especially Tom Scholten and Niels Lok, for not only making this process more sociable, but also for helping me with some challenges I came accross. Whether it was on computational design, on detailing, or on physical modelling, the discussions with them were of great value. Besides, the expertise of Industrial Design Engineer Bart Bossenbroek also helped me with finding solutions on the building method and/or production methods of various components. For the modelling phase I need to thank Jan van der Streek for lending me his tube bending tools. Lastly my great thanks goes out to my parents who were always loving and encouraging no matter what.

INTRODUCTION

> “Why does nature appear to use only a few fundamental forms in so many different contexts?
> Why does the branching of trees resemble that of arteries and rivers?
> Why do crystal grains look like soap bubbles and the plates of a tortoise shell?
> Why do some fronds and fern tips look like spiral galaxies and hurricanes?
> Why do meandering rivers and meandering snakes look like the loop patterns in cables?
> Why do cracks in mud and markings an giraffe arrange themselves
> like films in a froth of bubbles?”
> Peter S. Stevens, 1974

This thesis is about the confluence of the design of the natural world and the design of structures. It seeks to find the relationship between biology in architecture by analysing nature’s time tested patterns and strategies. This will be done by observing physical phenomena, as well as describing these phenomena with mathematics. John A. Adam states in his book ‘Mathematics in Nature’ that the act of “asking questions of nature” can lead to many fascinating “thought trials,” even if we do not always come up with the correct answers. “There will always be “displays” or phenomena in nature that any given individual will be unable to explain to the satisfaction of everyone, for the simple reason that none of us is ever in possession of all the relevant facts, physical intuition, mathematical techniques, or other requirements to do justice to the observed event. However, this does not mean that we cannot appreciate the broad principles that are exemplified in a rainbow, a lenticular cloud, river meander, mud crack or animal pattern.” (Adam 2006)

For now, it is necessary to state that also in this graduation thesis, I do not necessarily hope to find a final answer on the question why nature produces certain forms and why it prefers them to other conceivable forms. In fact, it is extremely likely that there is no final answer, at all. Most scientists have given up the hope of ever finding a complete answer, an ultimate truth. The modest approach of realising that nature’s principles are extremely complex, has led to scientific progress. Throughout history, people have tried to discover such principles and it is fascinating to see how our understanding of nature has grown by means of this process. (Hildebrandt and Tromba 1984)

It is given that nature often repeats certain forms and patterns. There are many similarities that are observable; the resemblance between the spiral pattern in the heart of a sunflower, and that of a seashell, or the similarity between the branching pattern of a river and that of a tree. We can observe many more if we look carefully. According to Stewart, this is the first step of understanding these similarities – to see clearly. This might be even the most difficult one.

ABSTRACT

Biomimicry can be seen as an approach to innovation with the goal to find sustainable solutions to human challenges by emulating nature's time-tested patterns and strategies. In the last few decades various biological processes have been analysed in terms of structure. Several architects and engineers have already studied the confluence of nature and engineering, the confluence of biology and structural design: Frei Otto with his tensile and membrane structures, Félix Candela with his anticlastic thin shell structures and Buckminster Fuller with the geodesic domes are a few examples.

The main objective of this graduation was to find a new structural principle that is emulated by the design by nature. Observing, analysing and transforming the natural design principles and laws could derive a new application in the field of structural design. There are many concepts, approaches and directions to translate biology to architecture. The way to get from biological observations, to physical phenomena, describing these mathematically in the hope to come up with an integrative innovative structural principle for architecture is done through different levels of mimicry.

The observation phase led to a categorisation of biological processes into physical phenomena. It is a necessity to mention that firstly, the twelve phenomena are definitely not the only twelve and secondly, these twelve are not in a specific order or hierarchy. Some phenomena overlap others. To come up with a proper design direction based on natural examinations, a transformation methodology is proposed based on partial mimicry. Every level has its unique grade of abstraction. In the end, after having walked through all the separate levels of the methodology, the derived information will form the basis for the design direction. There are various ways to interpret the methodology. Looping, skipping, going back a few steps or even starting at a totally different level can therefore often be very useful, resulting in more in depth concepts.

The combination of the physical phenomena 'minimal surface' and 'fractals' formed the basis for my design concept. These phenomena were translated into architectural forms; the anticlastic ruled surface and the branching structure. An anticlastic ruled surface is a negatively double curved surface that can be described using straight lines. This is called a 'hyperbolic paraboloid' - in short 'hypar'. The branching structure is based on the natural fractal of a tree. A split of the main 'trunk' is called an iteration. A branching structure defines itself by having one, two, three or even four or more iterations.

Using nature as inspiration combined with mathematics enables us to come up with structurally rational designs. These rational designs are because of its simplicity appropriate for smart solutions on detailing as well. Besides, the research on the structural performance of natural dendriforms contributed to the

sustainable development in such a way that material usage was minimized. Ludwig Glaeser rightfully state in his book on the work of Frei Otto that by applying his minimal theories to support elements and space frames, Otto arrived at lighter structures by reducing the buckling lengths of their compression members. This reducement of buckling lengths is also applied to branching structures.

The design development was done one the branching connections, the assembly of the structural components and the water management of the structure. After having elaborated on these facets briefly, the next step was the building method. There are basically three stages when it comes to the building method. The prefabrication off-site, the transportation to the site, and the assembly on-site. For the stage of assembly a so called ‘method statement’ with building sequence is visualised. Finally, the detailling was done, incorporating the design functionalities and the desired design freedom.

There is still a lot to gain from our observations towards nature and phenomena that are all around us. Today, each element of nature continues to be studied for creating more lightweight, durable, flexible, economical and high-performance architectural structures. With both of the selected phenomena separately several structures are built and have been proven to work. However the final product of this graduation project; ‘an anticlastic surfaced roof supported by a fractal-like branching structure’ has never been done before and therefore the main objective proposed in the beginning is achieved.

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“From my designer’s perspective, I ask:
Why can’t I design a building like a tree? A building that makes
oxygen, fixes nitrogen, sequesters carbon, distils water, builds soil,
accrues solar energy as fuel, makes complex sugars and food, creates
microclimates, changes colours with the seasons and self replicates.
This is using nature as a model and a mentor, not as an inconvenience.
It’s a delightful prospect...”

McDonough and Braungart, 1998

intro

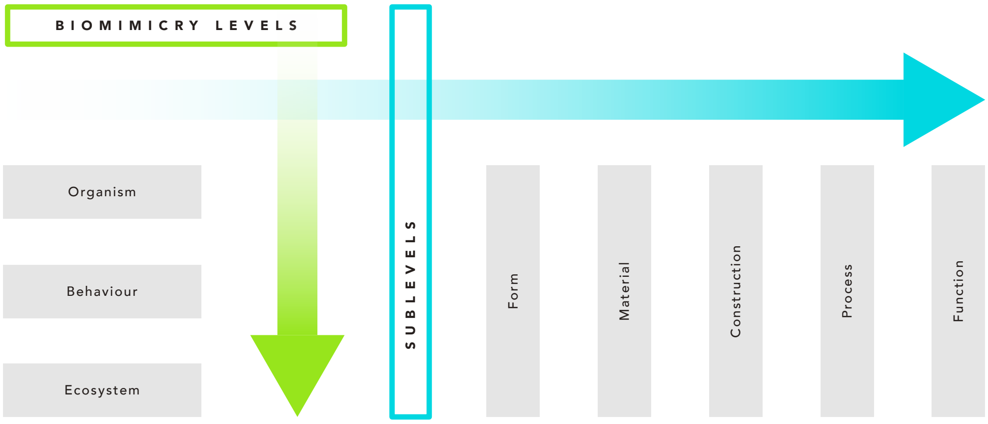
1. RESEARCH FRAMEWORK

1.1 INTRODUCTION

1.1.1 Biomimicry

> “It is a common misconception that naturally-inspired solutions are the engineers’ default response when asked to be creative. I think it is actually the opposite. It is not about the designer being at the centre of the process and generating new ideas; biomimicry to me is about taking a step back and humbly accepting that nature may be ahead of us.”
>
> Michele Mak, 2016

From the very beginning, humans have always been learning from its environment. By imitating, interpreting, and using opportunities of nature man has experienced adaption and has developed skills to provide for his needs (Arslan and Sorguc 2004). This tendency started in the early age, and is still growing nowadays. This approach has gained interest by many architects and engineers in the field of design as well. The last few years the focus of studies in various branches of science is nature. Biomimicry is an approach to innovation that seeks sustainable solutions to human challenges by emulating nature’s time-tested patterns and strategies. First of all, the concept of using ideas from nature has been given several different names over the last couple of years; biomimicry, biomimetics, biomimesis, biognosis and bionics, all of which has the same definition. This concept has a lot of advantages and is by many architects and biologists seen as the answer to many global challenges human face nowadays. There are many directions in which humans can learn by imitating and emulating natural patterns and processes. You can think of reusability, recyclability, sustainability, reducibility of materials, energy usage and waste amount.



[2] LEVELS OF BIOMIMICRY

The article 'Influences of the living world on architectural structures' gives a clear workflow of how biological processes can be analysed and what dimensions are related to this process in order to grasp the complexity of the specific phenomenon. In 2007, P. Zari stated that through an examination of existing biomimetic technologies it is apparent that there are three levels of mimicry; the organism, behavior and ecosystem. He goes on noting that within each of these levels, five possible dimensions of mimicry exist: 1: Form; what does it looks like? 2: Material; what it is made out of? 3: Construction; how it is made? 4: Process; how does it work. 5: Function; what is it able to do? (Zari 2007)

From the beginning of the 1970's Werner Nachtigall, one of the writers of the book 'Biomimetics for Architecture & Design' defined bionic/biomimetic work as follows: "Learning from nature for self-sufficient, engineerable design." Nature provides inspirations that the engineer should not simply copy, but incorporate into the structural design. In 1993, a convention of the Association of German Engineers for the "analysis and evaluation of future technologies" was held, which was about the "Technology Analysis Bionics". The attendants were mainly technical biologists, biomimetics and scientists agreed on the following definition of bionics: Bionics / Biomimetics as scientific discipline is concerned with the technological implementation and application of structural, procedural, and developmental principles of biological systems. (Pohl, 2015)

1.1.2 Nature, Science & Philosophy

> "I think the biggest innovations of the 21st century will be
> at the intersection of biology and technology. A new era is beginning."
> Steve Jobs, 2011

Patterns in nature, it is all around us. Rainbows, halos, waves in air, oceans, rivers, lakes and puddles, cloud formation, tree and leaf branching patterns, the proportions of the trees, the wind in the trees, mud-crack patterns, butterfly markings, leopard spots and tiger strips. The visible world is full of patterns that can be described mathematically.

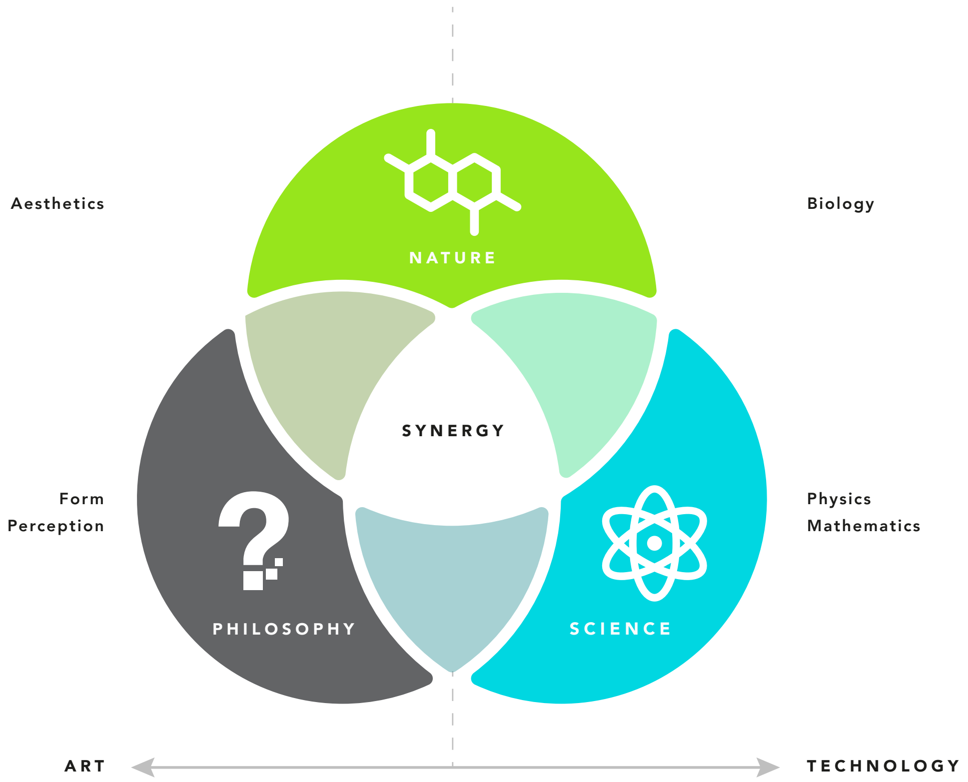
In the last few centuries, many mathematicians and physicists have studied nature and its laws. The passion of the intersection between biology and science has given a lot of development in various fields we know nowadays. This passion began about 3000 years ago with the search to describe natural phenomena mathematically. The Greek made elementary observations of remarkable regular forms and patterns in nature. Their observations led to the beginning of mathematics. The Greek word mathema – which means knowledge, cognition, understanding, and perception – suggests that the study of mathematics began with asking questions about the world. (Hildebrandt and Tromba 1984)

Humans have never let go of the desire to understand nature. And accordingly to several mathematicians and physicists, there lies great value in growing in being able to grasp the ways of nature. Many also acknowledge the importance of mathematical descriptions and physical explanations to biological observations. Karl Pearson stated in 1901 that he believes the day must come when the biologist – without being a mathematician – does not hesitate to use mathematical analysis when he requires it. Another fundamental and philosophical question is how it can be that mathematics, which is of course a product of human thought independent of experience, is so adapted to the objects of reality? (Adam 2006). Already in 1623, Galilei Galileo, gave and answer to this as well as an insight in how to get familiar with describing and analysing natural processes and patterns:

> "The universe cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word."
>
> Galilei Galileo, 1623

Ian Stewart, the writer of the book 'Nature's Numbers' supports Galileo's statement: There is a formal system of thought for recognizing, classifying and exploiting patterns. It is called mathematics. Mathematics helps us to organize and systemize our ideas about patterns; in so doing, not only can we admire and enjoy these patterns, but also we can use them to infer some of the underlying principles that govern the world of nature. (Stewart 2014)

In the book 'On growth and form', which first edition was published exactly 100 years ago and was over 1000 pages long, D'Arcy W. Thompson stated that the entire book was an introduction and an invitation to people trained in mathematics, trained in physics and trained in engineering to look at biology from a perspective associated with geometry. Thompson makes the plea that in order to understand biology it would be useful to try and take ideas from mathematics and physics. The book is a set of analogies between phenomena that happens in biology and similar phenomena that he and others have observed connecting to form. A design principle on which will be reviewed later.



[3] WORKFLOW TRILOGY

1.2 PROBLEM STATEMENT

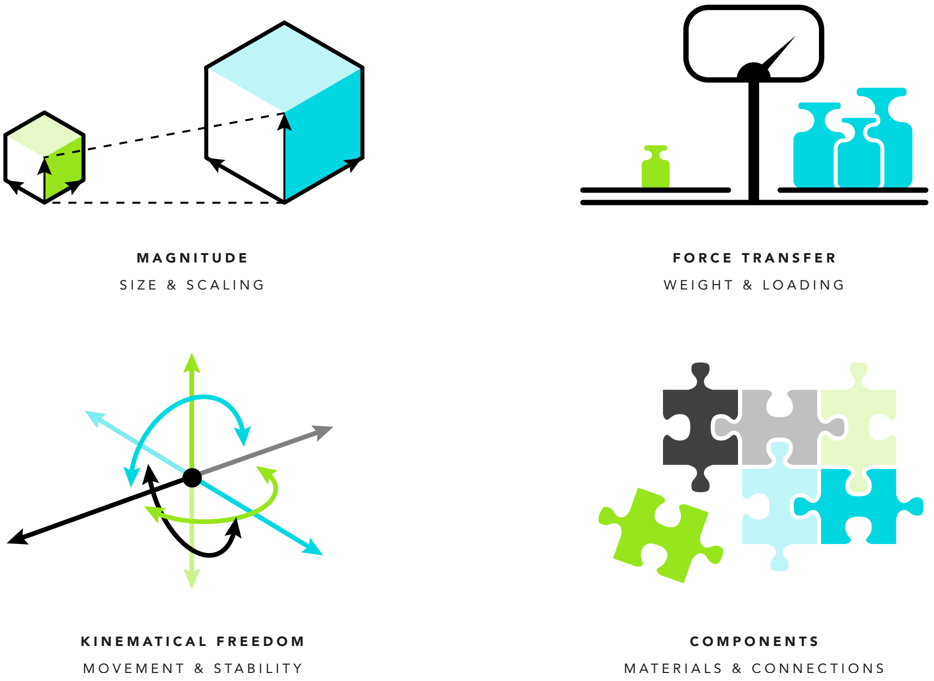
Research on patterns of nature for architecture has revealed the potential of looking at growth in biology to inspire new planning and building processes. The disadvantages of contemporary planning and building practices are manifold. Building construction is often a messy, mostly centrally controlled process that requires exhaustive organization and resources that are transported over global distances as well as being produced unsustainable, often toxic, and also socially irresponsible processes. (Gruber and Imhof 2017)

> “You could look at nature as being like a catalog of products, and all of those have benefited from a 3.8 billion year research and development period. And given that level of investment, it makes sense to use it.”
>
> Michael Pawlyn, 2011

In nature there are laws, which are not triangulated and not regularly articulated and tessellated, however, nature-inspired configurations can be very stable. The way that nature grows and evolves is through a process of 'natural selection', a mechanism of trial-and-error. From the beginning of the living world, nature has had this mechanism, resulting in having had a great amount of time to improve.

As mentioned before, the concept of biomimicry seems to be able to give answers to questions that several branches of engineering currently face on a global level. However, the idea should be carefully taken, since it is not always in a direct way that nature can give solutions to scientific challenges. This is called the problem of 'technology transfer.' In my belief, due to this 'technology transfer' we do not make fully use of the research and development period nature has had over the last billion years. Therefore the integration of biology and technological area's of architecture is not widely applicable yet. We see inspiration from nature in many aspects, but biomimetic architecture is surely not yet fully explored.

One of the main reasons for this is that it is challenging to design structures that are inspired by nature, since nature has different design principles than architecture. One of the main aspects of this difficulty is the one to one copy conflict. It is not possible to directly copy nature's strategies and patterns to architecture. The reasons for this copy conflict, but also the possibility of extrapolation, in the case of structures for architecture or engineering, should be carefully studied attending to four issues: magnitude, force transfer, kinematical freedom and components.



MAGNITUDE
SIZE & SCALING

FORCE TRANSFER
WEIGHT & LOADING

KINEMATICAL FREEDOM
MOVEMENT & STABILITY

COMPONENTS
MATERIALS & CONNECTIONS

[4] PROBLEM STATEMENT ASPECTS

Magnitude: this is about the size of the structure and the possibility of scaling. Not in every situation a pattern can be directly scaled. Secondly, sometimes the scale in which the natural structure is form is so small, the gravitational forces can be neglected. Thirdly, accordingly to D'Arcy W. Thompson does scale have a marked effect on physical phenomena, and that increase or diminution of magnitude might mean a complete change of statical- or dynamical equilibrium. This so called 'problem of scaling' will be further discussed in the chapter on 'Natural Laws'.

Force transfer: this includes the weight and the loading of structures. The load-case should be studied, since a lot of nano-arrangements work in zero-gravity medium, or they work in water, or have to load-bear live changing loads. Secondly, the kind of loads can be different and thirdly, the proportion between the structure's self weight and the loading that is applied can have a total different ratio in comparison to man-made structures.

Kinematical freedom: The freedom of movements is the third aspect that has to be carefully considered. Natural structures are normally designed for having much more kinematical freedom than man-made ones, which should not have large movements or deformations. In comparison to nature, in architecture the translations and rotations along the x-, y-, and z-axis is something that often must be avoided.

Components: this topic contains the diversity of materials with which nature design, as well as the fact that nature mostly doesn't design using components. Architecture is due to fabrication and transportation restricted to designing with components or structural members that will be assembled in a later stage of the to be realised design. For this reason the way we design the fixings, joints and connections between the components is crucial, not only for the assembly, but also for the structural performance. Nature doesn't "think" like this. Nature grows and therefore doesn't design with components.

1.3 RESEARCH OBJECTIVE

> "When we look at what is truly sustainable, the only real model that has worked over long periods of time is the natural world."
>
> Janine Benyus, 2010

In short, the main objective for this graduation thesis is to provide a framework on how to design structures that are inspired from the design of nature. Besides, the aim beforehand is to get a better understanding on how nature designs and how the natural design principles can or may be translated to architecture and especially structural design. The goal is hereby to contribute to a more sustainable built environment.

The study aims, on one hand, to increase structural performance and, on the other hand, to search for nature-originated structures that create a different architectural language. One of the main intentions of this research is to find what humans can learn from developing ideas that are based on natural phenomena. The dualities and similarities with a confluence in several branches of science will be analysed and it will be aimed to find generic elements that can be translated to sustainable solutions on structural design.

The ambition is to end up with a physical model. The final product should represent a proof of concept for the design that is derived from a physical phenomenon or from a biological structural principle. Since biology is an extremely large field, there is not a lot of time to go into depth of the various branches of nature. A focus, a direction, a diverging target should be chosen after already a brief analysis of the design principles and after a given overview of physical phenomena and mathematical description.

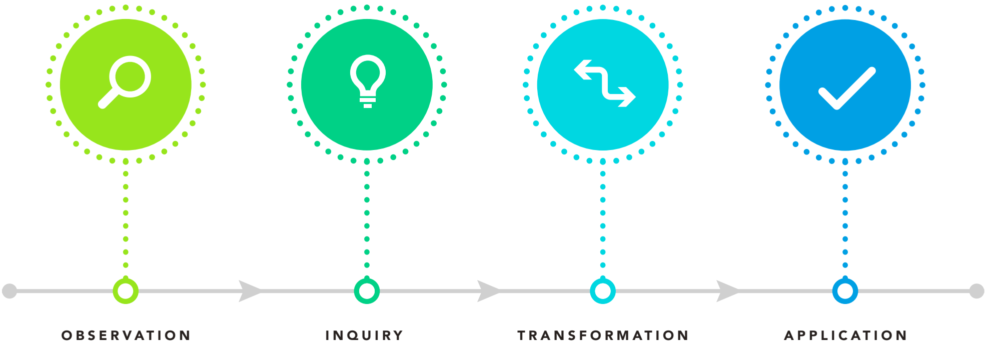
Derived from the problem statement that was described earlier, the objective can be formulated as follows:

1. 'A framework for structural design emulated by design by nature'.
2. 'A framework for transferring technology between biology and architectural structural design'.
3. 'A new structural application that is derived from physical phenomena'.

1.4 RESEARCH QUESTIONS

Derived from the problem statement and research objective, the main research question can be set up.

- How to translate principles of design by nature to structural design?
- How to design a structure that is inspired by natural design principles?
- How to find a new structural application derived from natural phenomena?



[5] RESEARCH QUESTIONS IN PHASES

To structure all the various sub-questions, an overview of questions subdivided into the phases that will form the research of this graduation is made:

Observation

- What are the main natural laws with which nature designs?
- What is the problem statement for this technology transfer?
- What is the 'problem of scaling'?
- What is the difference in scale in which nature designs and the scale of architecture?
- Is there a way to emulate the evolution of nature to the way we interpret structural design?
- In what way can the technology transfer from nature to structural design contribute to sustainable architecture?

- Which part of biology gets the focus for the observations?
- How to observe biological- and physical models?
- How to describe the observed models mathematically?

Inquiry

- Which physical phenomena describe biological- and natural processes?
- How can the broad range of nature's time tested patterns and strategies be categorised in several physical phenomena?
- How to filter this overview to the design principles of architecture?
- What are the precedents of naturally inspired architecture?
- Are there case studies in architecture of combined physical phenomena?
- How to explore new combinations of physical phenomena?
- What are the structural principles we know nowadays that are emulated by nature?

Transformation

- How to come up with a focus of further research and transformation?
- How to transfer new combinations of phenomena to a structural principle?
- How to translate this combination to a new application?
- How to diverge from all the possible directions of transformation to one principle?
- How to translate a mathematical description to useful structural design guidelines?

Application

- How can a naturally-inspired tessellation be applicable in structural design?
- In what way can the structural characteristics of biological processes be translated to

construction?

- How to perform the structural analysis of the application?
- How to build a physical model?
- How to design a feasible building method of this new application?

Besides this list of questions that are mostly related to the workflow of the overall project, there are also secondary questions that are related to natural processes and their similarities. These questions are more abstract and likely to have no final answer, no absolute truth:

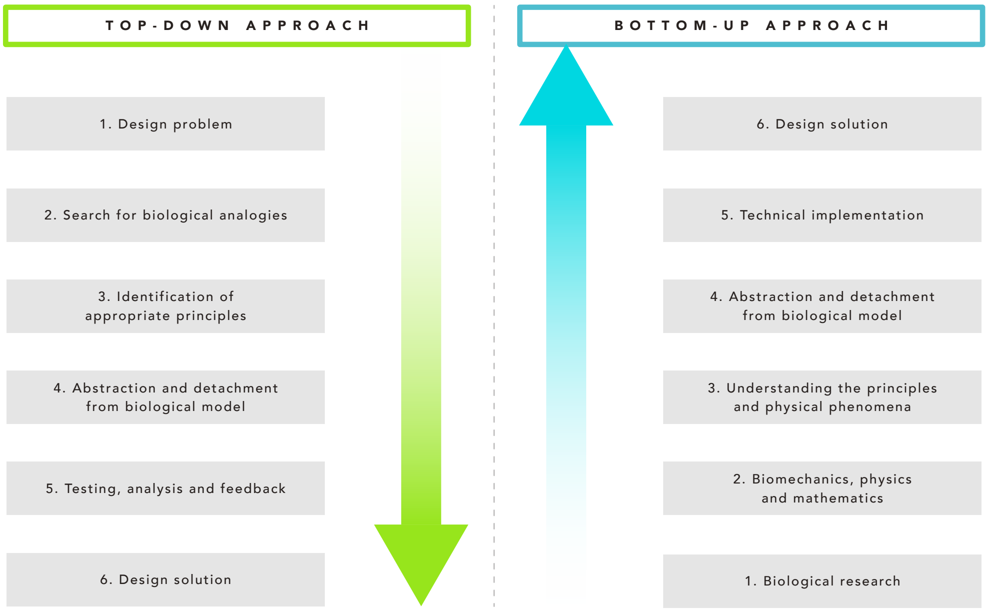
- What happens as things get bigger?
- Why does nature prefer certain forms to other conceivable forms?
- Why does nature appear to use only a few fundamental forms in so many different contexts?
- Why are the celestial bodies arranged in spheres and circular trajectories?
- Why are crystals made out of geometric pyramids and cubes?
- Why do crystal grains look like soap bubbles and the plates of a tortoise shell?
- Why does the branching of trees resemble that of arteries and rivers?
- Why do some fronds and fern tips look like spiral galaxies and hurricanes?
- Why do meandering rivers and meandering snakes look like the loop patterns in cables?
- Why do cracks in mud and markings an giraffe arrange themselves like films of bubbles?

1.5 RELEVANCE

Biomimicry and the translation from natural design to structural design will be a new point of view. In the last few decades various living organisms have been analysed in terms of structure. Several architects and engineers have already studied the confluence between nature and technique, the confluence between biology and structural design: Frei Otto with his tensile and membrane structures, Félix Candela with his anticlastic thin shell structures and Buckminster Fuller with the geodesic domes are a few examples. In this sense, there is still a lot to gain from our observations towards nature and phenomena that are all around us.

Today, each element of nature continues to be studied for creating more lightweight, durable, flexible, economical and high-performance architectural structures. (Stach 2010) states that structural optimization driven by limited resources, environmental impacts, and the technological race is targeted to maximize the performance of a structure or structural component.
Garcia P. and Gomez F. (2009) indicate that this combination of knowledge about structures in nature and the possibility of constructing new structural prototypes made architects and engineers turn back their eyes to nature to learn about optimal morphology, extreme lighting, functional integration and efficiency.

The relevance of this study is clearly articulated in the article 'Influences of the living world on architectural structures.' *"Structures in the nature motivate innovation in architectural and engineering disciplines in terms of aesthetical, functional and structural advantages. Using efficient, lightweight structural forms similar to those in nature reduces material and energy usage and waste amount. In this sense, it can be clearly seen that based on learning from nature in relation to meeting gradually increasing and changing requirements through limited resources and creating modern structural designs, biomimicry will provide much more contribution on architecture and related fields."*



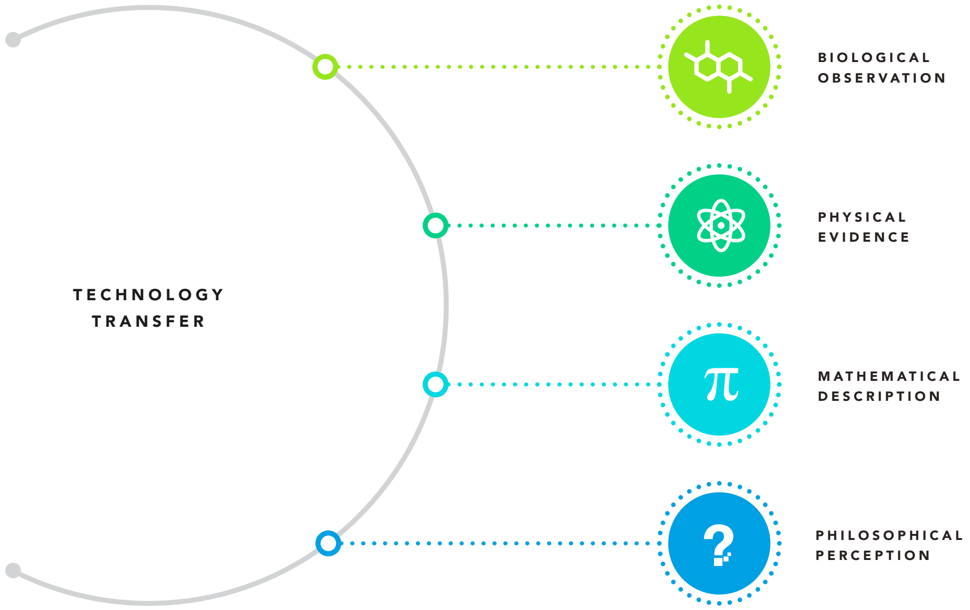
[6] TYPES OF APPROACHES

1.6 RESEARCH METHODOLOGY

The methodology for this research will be based on a bottom-up approach. This approach in the field of biomimetics is often referred to as 'Solution-Driven Biologically Inspired Design'. (El Ahmar 2011) state that when biological knowledge influences human design, the collaborative design process is initially dependent on people having knowledge of relevant biological or ecological research rather than on determined human design problems. A popular example is the scientific analysis of the lotus flower emerging clean from swampy waters, which led to many design innovations as detailed by Baumeister, including Sto's Lotusan paint which enables buildings to be self cleaning.

An advantage of this approach therefore is that biology has the ability to influence humans in ways that might be outside a predetermined design problem. This may result in unthought-of technologies, or even new approaches to design problems. The potential for true shifts in the way humans design and what is focused on as a solution to a problem, exists with such an approach to biomimetic design. (Vincent et al., 2005)

To translate observations and discoveries derived from natural models towards sustainable architecture and structural design, the information transfer needs to be guided. The methodology of biomimetics can be seen as 'technology transfer' from the life sciences to innovative sustainable solutions. In his article, Yeler, 2015, came up with a transformation process of structural design knowledge that exists in nature.

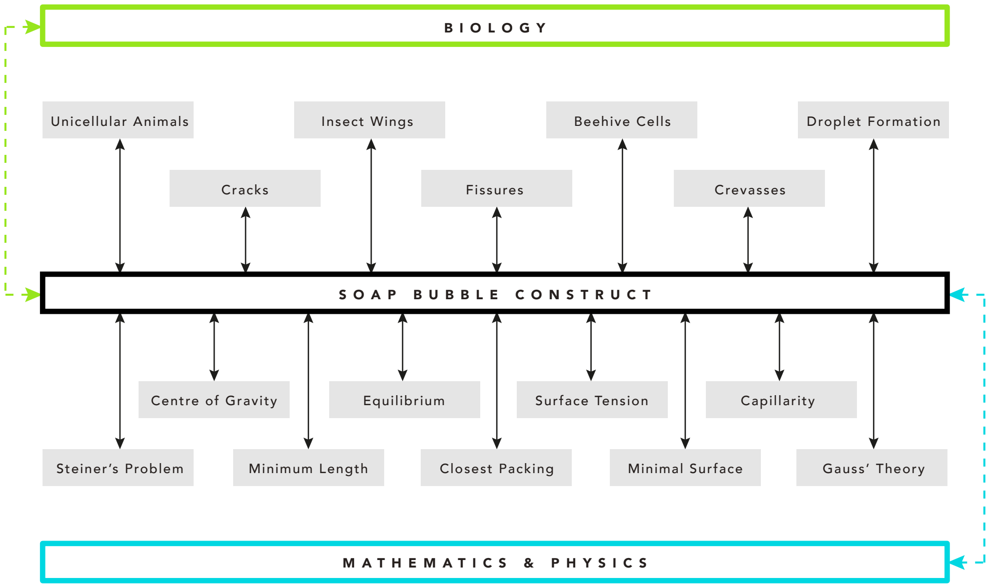


[7] WORKFLOW OF THE TECHNOLOGY TRANSFER

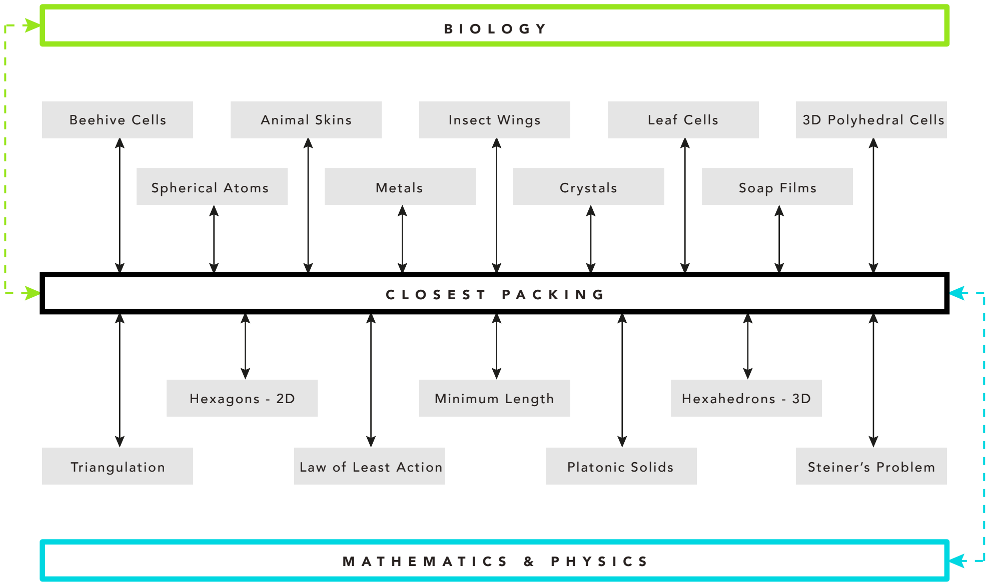
Göran Pohl and Werner Nachtigall state in their book 'Biomimetics for Architecture & Design' that in the 1960's I. Rechenberg and his colleagues already had shown that one can translate the principles of biological evolution for optimizations in technology, by integrating accidents (mutation, recombination) and subsequent testing strategies (selection) in design development. This is in general what will be aimed for in this research as well; by mutating and finding recombinations of biological principles to find a new structural application.

It first starts with the vision that structural design information exists in nature. The first step is the belief to find inspiration from nature. Linked to this step is the observation of natural patterns and processes, which may lead to the discovery of phenomena. These phenomena will be categorised and dualities and similarities will be found that can link biology to architecture. These dualities should give a clear insight on the physical phenomena. This will be followed by analysing and determining these correctly and realising the best and sustainable applications by providing information transformation in terms of structural design. On the next page two examples of visualisations of dualities and similarities are illustrated.

The phase to go from analysis to application is challenging. This process is multidisciplinary and contains fields like biology, mathematics, physics and philosophy. In the scheme on the next page you can see that the approach for the first part of the research will be a constant relation between the observation of biology and the description of biological phenomena with physics and mathematcis. Philosophy is as mentioned earlier an important field that gives input when it comes to form, function and perception.



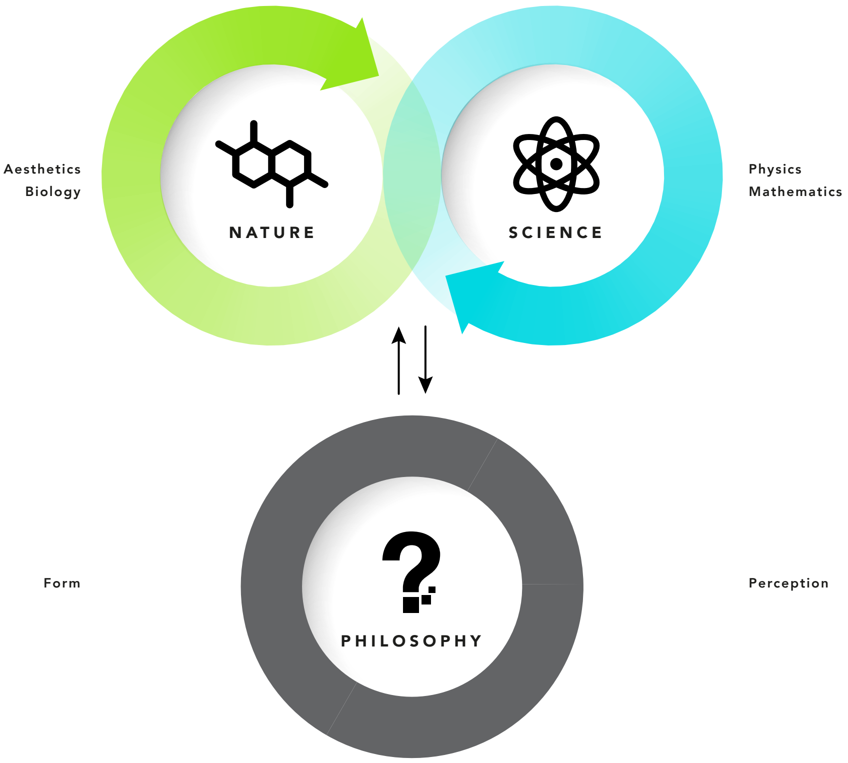
[8] DUALITIES OF THE 'SOAP BUBBLE CONSTRUCT'



[9] DUALITIES OF 'CLOSEST PACKING'

The observation, discovery and analysis part can be subdivided into a systematic workflow. In nature there are universal laws. The laws to describe a phenomenon are in the same for multiple branches of science, however, the thing that is described is differently. After having observed the universal laws, physical evidences need to be found, which will be connected to mathematical theorems. Once the mathematical descriptions are found, the physical phenomena can then be explained. Together with philosophical perception, this will provide similarities and dualities of natural models and processes that can be found all around us. This method should provide key elements and guidelines that will be the fundament of the transformation and translation to structural design.

The first phase is to set up a research framework. This will be done by simultaneously focussing on literature, on formulating the problem statement and research questions and on biological-, physical- and mathematical observations. From this study certain conclusions and discoveries when it comes to dualities and similarities in nature can be derived. These discoveries and following dualities will then be analysed and categorised into physical phenomena. After the observation and inquiry phase, a few of the physical phenomena will be selected to do further research on. This is done for narrowing down the scope on which will be elaborated a bit more. After this selection process, the transformation from the physical phenomena to mathematical models needs to be made. This will be done by changing the constraint and mutating the mathematical description, in the hope to find generic elements, solutions and variants that can be translated to structural design. In the application phase that will follow directly from the transformation, first several concepts will be developed. Then the performance criteria together with indicators and parameters will be set up, resulting in a design development that will be followed by the technical detailing and the structural analysis of the object. A final physical product is what is aimed for.



[10] FIELD RELATION SCHEME

“The reasoning about the wonderful and intricate operations of nature are so full of uncertainty, that, as the wise-man truly observes, hardly do we guess aright at the things that are upon earth, and with labour do we find the things that are before us.”

Stephen Hales, 1738

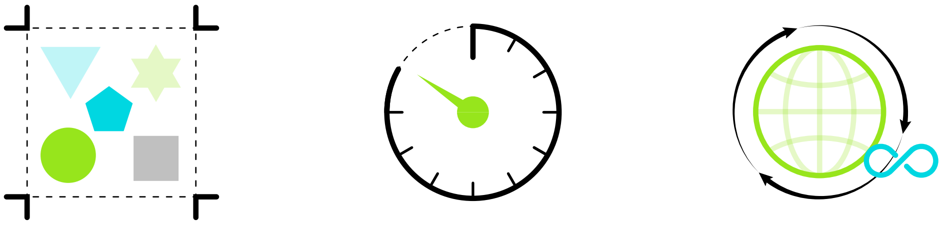
research

2. NATURAL LAWS

2.1 DESIGN PRINCIPLES

In this chapter a short overview of the main principles of design by nature will be given. In the following pages, each of the elements shown below will be briefly discussed.

ENERGY **GROWTH & REPRODUCTION** **COMPONENTS**



DIVERSITY **TIME** **CIRCULARITY**

[11] NATURAL DESIGN PRINCIPLES

Energy efficiency is one of the main design principles of nature. This principle is crucial for natural processes to survive. From the point of view that all natural constructions and processes are optimised for energy use, Werner Nachtigall states nine rules which are valid for the design of natural construction.

- Integrated instead of additive construction
- Optimisation of the whole instead of maximising single elements
- Multifunctionality instead of monofunctionality
- Fine adjustment with regard to the environment
- Direct or indirect use of solar energy
- Temporary limitation instead of useless durability
- Complete recycling instead of waste accumulation
- Integration instead of linearity
- Development by trial and error

(Pohl and Nachtigall 2015)

NATURAL CONSTRUCTION	TECHNICAL CONSTRUCTION
Round form	Right angle
Few parts, varied properties	Many parts, homogenous
Diffusion, laminar flow	Gravity, turbulent flow
Strength	Stiffness
Toughness	Brittleness
Bending, twisting	Sliding
Flexible	Streamlined
Tension	Compression
Big product compared to factory	Small products
Continuous rebuilding	Minimal maintenance
Wet	Dry

[12] COMPARISON BETWEEN NATURAL AND TECHNICAL CONSTRUCTIONS, VOGEL 1998.

2.2 ENERGY

Throughout history, people have searched for laws to describe the phenomena of our physical world. In 1744 the French scientist Pierre-Louis Moreau de Maupertius was the very first in putting forth his observation and vision on the universe, the so called grand scheme of the universe. His proposal to encompass all phenomena of the physical world is nowadays known as the law of least action. (Hildebrandt and Tromba 1984)

Think of a mailman, and consider how to describe his action. If he travels 2 kilometers in 1 hour, you would say that he has carried out twice as much “action” as he would in traveling 2 kilometers in 2 hours. However, you would also say that he carries out twice as much “action” in traveling 2 kilometers in 2 hours than in traveling 1 kilometers in 1 hour. Altogether then, your mailman, by traveling 2 kilometers in 1 hour carries out four times as much “action” as he would in traveling 1 kilometer in 1 hour.

$$action = mass \cdot distance \cdot velocity$$

$$energy = \frac{1}{2} \cdot mass \cdot (velocity)^2$$

$$time = \frac{distance}{velocity}$$

$$action = energy \cdot time$$

Now we have a quantitative definition of action. This is necessary if we want to give a mathematical formulation of a natural law uses the concept of action. Maupertuis' principle can be paraphrased as:

> "Nature always minimises action."
>
> Maupertius

2.3 DIVERSITY

Pearce talks in his book 'Structure in Nature is a Strategy for Design' about nature as "maximum diversity, minimum inventory." He uses the snowflake as the most graphic example in nature of this principle. All planar snow crystals have the symmetry of a regular hexagon. However, within this six-fold form, no two snowflakes have ever been known to be exactly alike. Ian Stewart supports this statement in his book about 'Nature's numbers.' He noted "We live in a universe of patterns… No two snowflakes appear to be the same, but all possess six-fold symmetry." The form of a snowflake is based on certain physical, geometrical and chemical constraints and its variety results from a least-energy interaction with the environmental condition of temperature, humidity, wind velocity, and atmospheric pressure under which it is formed, accordingly to Bentley and Humphreys.

Can we build physical models and analogies? This question will lead to mathematics. Mathematics allows you to abstract ideas from one system and move to a different system. From a biological point of view, it is about figuring out what the principle behind these mathematical abstractions is. Evolution through time has accidentally stumbled on a whole bunch of solutions. The question is what these solutions are and if we can understand these solutions through mathematical and physical principles. Can we take this large diversity of shapes and can we connect them in terms of simple morphospace?

The diversity and complexity is large, so that without a tight grip on reality it becomes hard to find the relation between form and function and it is likely to get lost. The great complexity of nature gives rise to questions on the relevance of the entire approach. Nevertheless, in the complexity there seems to be harmony, in the chaos a certain order. In their book 'By Nature's Design' Murphy, Neill et al. explain this clearly: *"Nature modifies and adapts these basic patterns as needed, shaping them to the demands of a dynamic environment. But underlying all the modifications and adaptations is a hidden unity. Nature*

SIX-FOLD SYMMETRY AND DIVERSITY OF SNOWFLAKES [ALEXEY KLJATOV] *invariably seeks to accomplish the most with the least – the tightest fit, the shortest path, the least energy expended."*

This statement is supported by Pearce's analysis of nature's design:

> "Maximum diversity with minimum inventory."
>
> Peter Pearce, 1990

2.4 ECONOMY

In the article 'Natural structures, strategies for geometrical and morphological optimizations' García and Martinez state that the topic nature's economy is directly related to saving material. Nature mostly doesn't have a deadline when it comes to time, because the "manpower" and the "runtime" to form structures and to design systems are virtually infinite. To some extend nature is bound to time, this in case of the seasons, temperature deviations and available resources, despite this, nature only emphasizes in optimal form-finding, which is related to the formal concept of "continuity". (García and Martinez 2010)

2.5 GROWTH

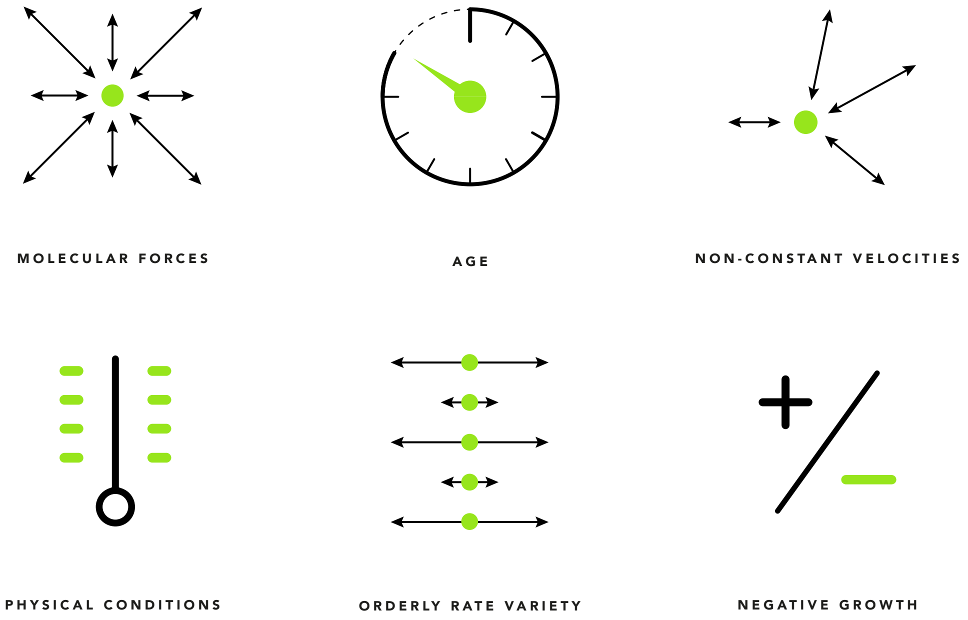
Research on patterns of nature for architecture has revealed the potential of looking at growth in biology to inspire new planning and building processes. (Gruber and Imhof 2017) Growth is simply spoken the increase in size, a development in magnitude. However, growth is an immensely complex phenomenon which is hard to explain in just a few sentences. In his book 'On Growth and Form', D'Arcy W. Thompson points out that on the topic of growth, some basic physical phenomena are associated; for example the principle of minimal surface, but also surface energy, capillarity and equilibrium, which is the condition of minimum potential energy in the system.

Growth can also be seen in many different analogies. Waves in the ocean grow when the wind is getting stronger, the heap grows when sand is poured out of a sack, the crystal grows when the right molecules fall into their appropriate places. He states: *"In all these cases, very mush as in the organism itself, is growth accompanied by change of form and by a development of definite shapes and countours."*

At the end of the second chapter 'The rate of growth' he gives a summary on the principles of the rate of growth:

1. Growth is an phenomena due to the direct action of the molecular forces, where the form of an organism can be seen as a “function of growth”, or an “event in space-time”.
2. The rate of growth depends on the age of the organism. The rate is maximal early in life, and slowly declining afterwards.
3. The ratio of velocities in different directions is not constant, but tends to alter in in course of time or to fluctuate in an orderly way.
4. The rate of growth is directly affected by temperature and by other physical conditions.
5. Growth varies in rate in an orderly way or is subject to definite laws. This is due to the fact that the form of the organism is in general regular and constant.
6. Growth can be negative, meaning that the organism is growing smaller.

(Thompson 1942)

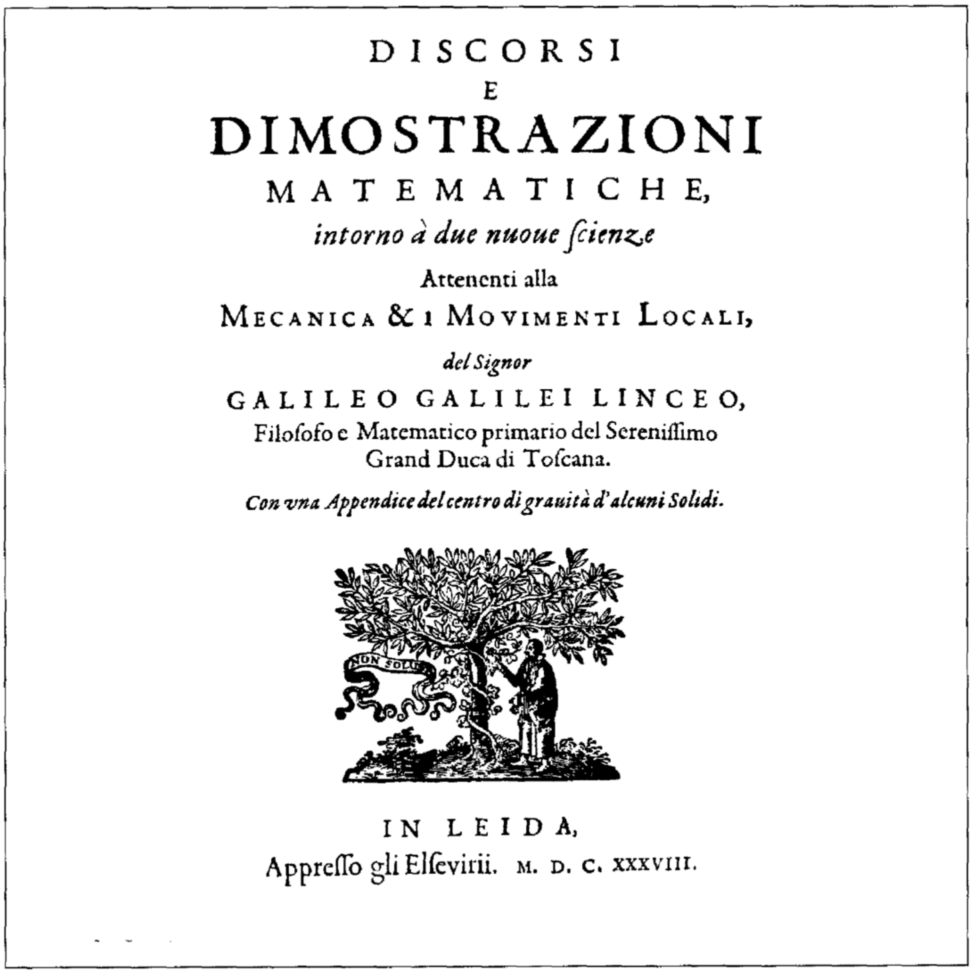


[14] PRINCIPLES OF GROWTH

2.6 SCALE

Scaling is a challenging factor for copying certain patterns or principles from nature to structural design. The knowledge on what happens when things get bigger is crucial to have in order to be successful in this technology transfer.

Accordingly to D'Arcy W. Thompson scale has a *"marked effect on physical phenomena, and that increase or diminution of magnitude might mean a complete change of statical- or dynamical equilibrium."* The challenge of scaling with regards to biomimicry, but also with regards to explaining natural processes in a mathematical way, is in various journals referred as 'the problem of scale'.



DISCORSI
E
DIMOSTRAZIONI
MATEMATICHE,
intorno à due nuoue ſcienze
Attenenti alla
MECANICA & I MOVIMENTI LOCALI,
del Signor
GALILEO GALILEI LINCEO,
Filoſofo e Matematico primario del Sereniſſimo
Grand Duca di Toſcana.
Con vna Appendice del centro di grauità d'alcuni Solidi.

IN LEIDA,
Appreſſo gli Elſevirii. M. D. C. XXXVIII.

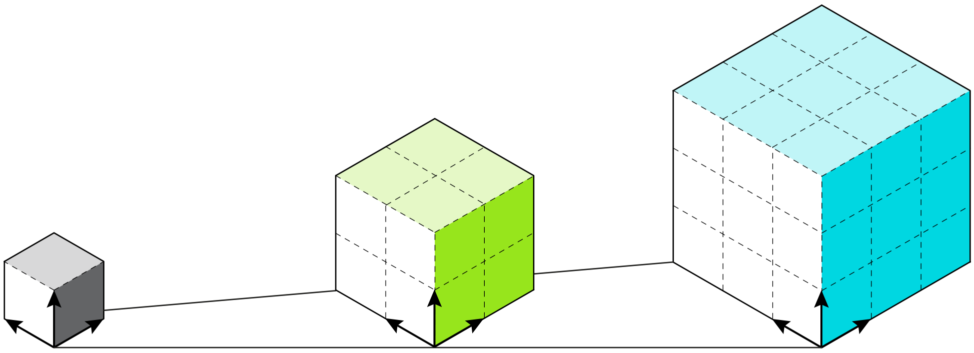
[15] 'TWO NEW SCIENCES' BY GALILEO GALILEI IN 1638

John A. Adam dedicates an entire chapter in his book 'Mathematics in nature' on this problem. *"Consider a simple three-dimensional object: a cube of side L. The surface are of the cube is* $6L^2$ *square units and the volume is* L^3 *cubic units. Doubling the linear dimensions of the cube will result in a new area of* $6(2L^2)$

= $24L^2$ and a new volume of $(2L)^3 = 8L^3$. Thus the area has increased by a factor of 4 and the volume by a factor of 8. Furthermore, the ratio of the area to the volume has decreased from a numerical value of $6L^2/L^3 = 6L^{-1}$ to $24L^2/8L^3 = 3L^{-1}$. Indeed, if the cube dimension is changer by a factor x, the area changer by a factor x^2 and the volume by a factor x^3." This law is called the square-cube law and was first mentioned in 'Two New Sciences' in 1638 by Galileo Galilei; *"When an object undergoes a proportional increase in size, its new surface area is proportional to the square of the multiplier and its new volume is proportional to the cube of the multiplier."*

$$A_2 = A_1 \cdot (\frac{l_2}{l_1})^2$$

$$V_2 = V_1 \cdot (\frac{l_2}{l_1})^3$$



$CrossArea = 1$ $C = 2^2 = 4$ $C = 3^2 = 9$

$Volume = 1$ $V = 2^3 = 8$ $V = 3^3 = 27$

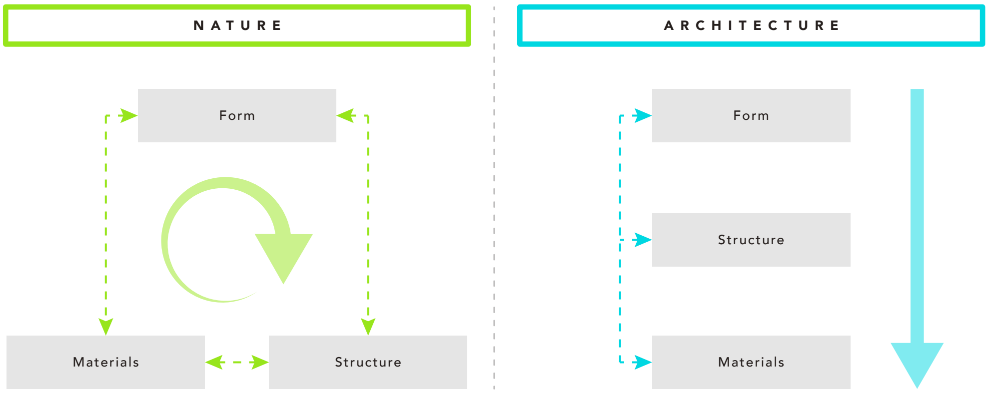
[16] THE PROBLEM OF SCALE

2.7 MORPHOLOGY

2.7.1 Form & Function

According to biology-online, morphology is the study of configuration or the structure of animals and plants. Morphology explains the shapes and arrangements of parts of organisms in terms of such general principles as evolutionary relations, function, and development.

Stach (2010) states in his article 'Structural morphology and self-organization' that in nature, the form-following process, structural extension (growth) and material placement happen simultaneously and are controlled by a constant feedback loop. Whereas architecture is predominantly design focused, resulting in the fact that form determines the structure and the materiality. This process is not continuous as is the case with natural design, but linear.



[17] THE FORM-FOLLOWING PROCESS OF NATURE VS. ARCHITECTURE

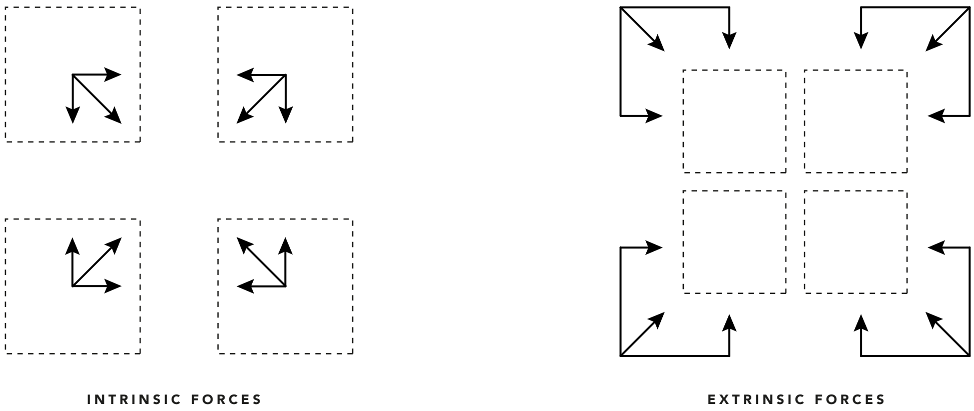
The desire to find mathematical descriptions to physical evidences of the universal laws can also be found in architecture. One approach to bridge the transfer between nature and architecture is analysing form and function. Questioning what determines form rises the question how function is connected to function.

Already in 1896, Architect Louis Sullivan stated the following: *"Whether it be the sweeping eagle in his flight, or the open apple-blossom, the toiling work-horse, the blithe swan, the branching oak, the winding stream at its base, the drifting clouds, over all the coursing sun, form ever follows function, and this is the law. Where function does not change, form does not change. The granite rocks, the ever-brooding hills, remain for ages; the lightning lives, comes into shape, and dies, in a twinkling.*

> It is the pervading law of all things organic and inorganic, of all things physical and metaphysical, of all things human and all things superhuman, of all true manifestations of the head, of the heart, of the soul, that the life is recognizable in its expression, that form ever follows function. This is the law."
>
> Louis Sullivan, 1896

Function and form are very tightly coupled. The reason why form is so interesting in various fields, but mainly biology, even though form is extremely diverse and complex, is that it gives the possibility to learn how form rises, which starts to address the question how function rises. If form and function is understood, a few more questions can be addressed, for example how they have moved hand in hand through time and how they gave rise to the enormous diversity that is observable in the natural world.

Peter Pearce speaks about the determination of the form of any structure. This is due to the interaction of two fundamental classes of forces; intrinsic forces and extrinsic forces.



[18] INTRINSIC- VS. EXTRINSIC FORCES

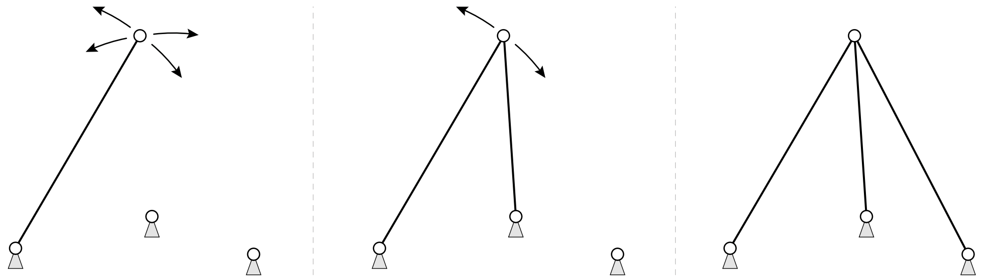
"Intrinsic forces are those governing factors which are inherent in any particular structural system; that is, the internal properties of a system which govern its possible arrangements and its potential performance. Extrinsic forces are those governing influences which are external to any particular structural system. They are the inventory of factors, largely environmental, which give direction to the form options allowed by the inherent combinatorial or form-giving properties of a given structural system." (Pearce 1990)

To illustrate this with a natural example; the snow crystal, which diversity was shown earlier – is perfectly suitable in this case. The intrinsic forces of snow crystals are its molecular structure, which results in the characteristic of infinitely varied patterns and arrangements. The extrinsic forces of snow crystals are based on physical conditions and environmental aspects, such as temperature, humidity, wind velocity, and atmospheric pressure. All these factors interact with the molecular structure to synthesize form.

2.7.2 Form & Stability

Ture Wester illustrates in his book 'Structural Order in Space' the dualism between lattice- and plate structures. The lattice structures contains of two basic elements – the bar and the joint. The bar ensures a constant distance between the two joints it links together. Before going into more depth on this dualism, the basic degree of freedom of these two basic elements is illustrated on the next page.

T. Wester explains this as follows: *"A bar, supported at one end, can only be axially loaded otherwise* *it will move. Two bars, each supported at one end and jointed together, can still only be axially loaded otherwise the system will become movable. As an external force on the free joint lying in the plane given by the two bars can be resolved along the two bar axes, the system will be stable under these forces. It is movable for all others. Three bars, which are not coplanar, are each supported at one end and jointed together at the other end, can still only be axially loaded. As any external force acting on the free joint can always be resolved along the direction of the three bars, the system will be stable."*



[19] FORM STABILITY OF LATTICE STRUCTURES

The rigidity theory is presented in a book called 'Structural Concepts and Their Theoretical Foundations' by Koryo Miura and Sergio Pellegrino. They present the question whether a pin-jointed structure contains a sufficient number of members to be rigid. The given answer is *"to count the total number of degrees of freedom of its joints and to subtract the number of degrees of freedom suppressed by applying kinematic constraints, i.e. foundation constraints, to the joints, and by connecting pairs of joints by means of bars."*

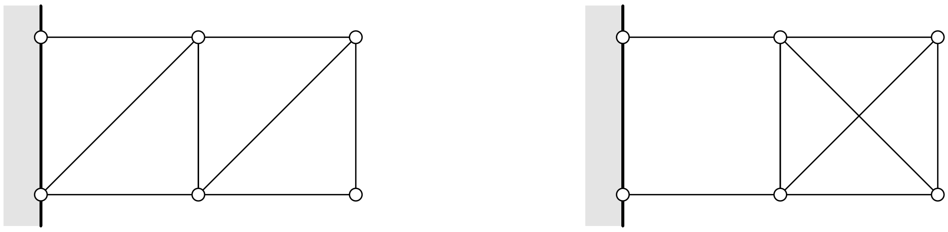
In two dimensions, each joint has two degrees of freedom, along the x-axes and the y-axes. The joints have two independent translation components. Therefore for a structure with j joints the total number of degrees of freedom is 2j. Say that k is the total number of kinematic constraints, where - for example - connecting a joint to a foundation counts as two because it suppresses both translation components, again along the x- and the y-axes. The total number of pin-jointed bars is b. Each bar has two joints.

$$2j - k - b \leq 0$$

This is equation is known as Maxwell's equation (Maxwell, 1864). Consider, for example, the structure that is illustrated in the top left of the figure on the next page. It is a structure that is connected to a foundation and consists of four triangles. The first illustration is a rigid structure. In this case the amount of joints are 6, the amount of kinematic constraints is 4 and the number of bars is 8. Substituting the above gives:

$$2 \cdot 6 - 4 - 8 = 0$$

Pellegrino goes on by explaining that it is important to realise that a structure that has enough bars to be rigid may not, in fact, be rigid, as its bars may be "incorrectly" placed. This is shown in the top right illustration. The bar-bracing of the left square is relocated, so that the right-hand square is now doubly-braced. This structure still satisfies Maxwell's equation, since the number of joints, kinematical constraints and bars hasn't changed. However, this structure is not rigid. In this case we have a so called single-degree-of-freedom mechanism.



A

B

[20] EXAMPLES OF TWO-DIMENSIONAL PIN-JOINTED STRUCTURES

C

D

[21] MECHANISM AND THE STATE OF SELF-STRESS

A structure that admits no mechanisms is called kinematically determinate. Note that the doubly-braced square on the right-hand side of the structure in the figure above admits a state of self-stress, i.e. there is a set of non-zero bar forces that are in equilibrium with zero external forces. A structure that admits no states of self-stress is called statically determinate. Denoting by m the number of independent mechanisms of a structure, and by s the number of states of independent states of self-stress. For structure A we have s = 0 and m = 0 (statically and kinematically determinate), whereas for structure B we have s = 1 and m = 1 (statically and kinematically indeterminate).

Concluding, Maxwell's equation in the form of the equation presented earlier is only a necessary condition for the kinematic determinacy of pin-jointed structures, but not a sufficient condition. In general, the most useful way of writing Maxwell's equation is:

$$dj - k - b = m - s$$

The analysis of the structural systems and form stability of polyhedra starts with the most characteristic plane faced three dimensional solids in existence, i.e. the five Platonic solids. These five polyhedra were already known to the ancient Greeks, and described by Plato in his Timaeus ca. 350 BC (www.mathworld.wolfram.com). According to Peter R. Cromwell, Plato equated the tetrahedron with the "element" fire, the cube with earth, the icosahedron with water, the octahedron with air, and the dodecahedron with the stuff of which the constellations and heavens were made (Cromwell 1997). The polyhedra are named after the Greek word for the number of faces. In the table below, an overview is given on the characteristics of each Platonic Solid as well as the statical and internal kinematical determinacy of the trusses:

Platonic Solid	# faces	# edges	# vertices	s	m'
Tetrahedron	4	6	4	0	0
Hexahedron	6	12	8	0	6
Octahedron	8	12	6	0	0
Dodecahedron	12	30	20	0	24
Icosahedron	20	30	12	0	0

[22] PLATONIC SOLIDS AND THEIR STATICAL AND KINEMATICAL DETERMINACY

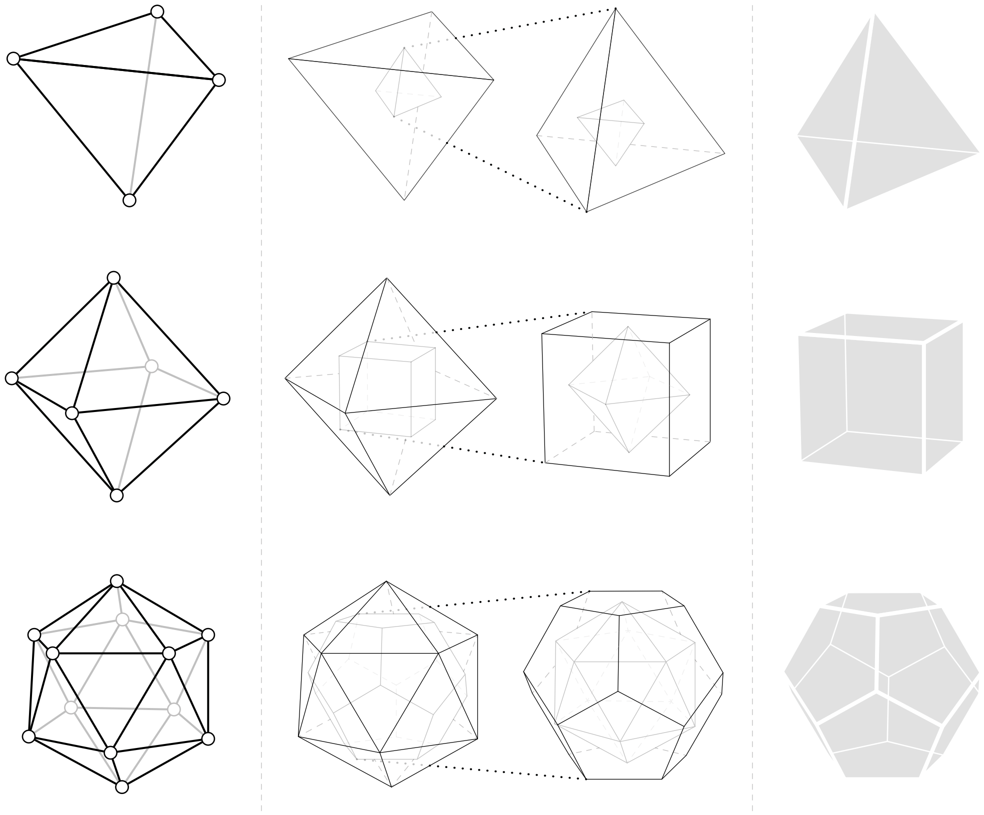
As all other plane faced, convex, closed, singularily coherent polyhedra they obey Euler's theorem for polyhedra:

$$\mathrm{f} + \mathrm{v} = \mathrm{e} + 2$$

Where f is the number of faces, v is the number of vertices and e is the number of edges.

In his book 'Structural Order in Space' Ture Wester clearly presents and illustrates the plate-lattice dualism. He begins by showing the geometric dualism between the five platonic solids. These five are the only three dimensional solids that all comply with the following geometrical requirements:

- All the faces are plane regular polygons.
- All the faces and vertices are congruent.
- All the vertices lie on the surface of the same circumscribed sphere. All the faces are tangents to the surface same inscribed sphere and the midpoints of the faces are the tangent points.



Stable Lattice Structures: Δ

Geometrical Duals

Stable Plate Structures: Y

[23] GEOMETRIC DUALITIES

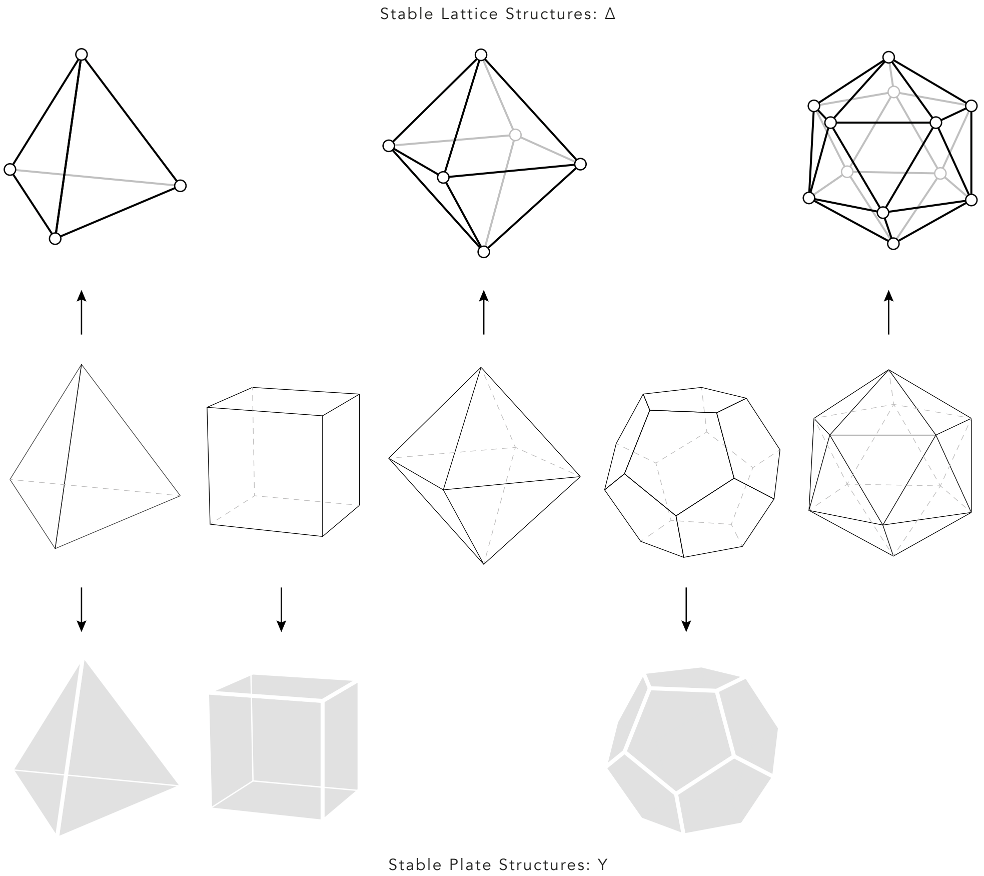
He goes on by explaining the dualism:

"If the midpoints of the tetrahedron's faces are joined up by edges, the result is a new tetrahedron. The tetrahedon's dual solid is thus the tetrahedron. If the midpoints hexahedron's faces are joined up by edges, the result is the octahedron and if the midpoints of the octahedron's faces are joined up by edges, the result is the hexahedron. The hexdual solid thus the octahedron whose dual solid is the hexahedron, etc. If the midpoints of the dodecahedron's faces are joined up by edges, the result is the icosahedron and if the midpoints of the icosahedron's faces are joined up by edges, the result is the dodecahedron. The dodecahedron's dual solid is thus the icosahedron whose dual solid is the dodecahedron, etc."

This can be expressed symbolically:

$$\frac{\Delta}{\mathrm{Y}} = \frac{\text{Lattice}}{\text{Plate}} \qquad \leftrightarrow \qquad \frac{\text{Tetra}}{\text{Tetra}} = \frac{\text{Octa}}{\text{Hexa}} = \frac{\text{Icosa}}{\text{Dodeca}}$$

The tetrahedron is stable as a lattice structure as well as a plate structure. This Platonic Solid is therefore rightfully called the Platonic master solid.



[24] STABLE LATTICE- VS. STABLE PLATE STRUCTURES

T. Wester divides the Platonic polyhedra into two groups, each with extreme geometrical characteristics:

- Those that consist of faces with the least possible number of edges, i.e. triangles, symbolized by Δ.
- Those that consist of vertices with the least number of adjacent edges, i.e. called 3-way vertices in the following, symbolized by Y.

Koryo Miura and Sergio Pellegrino note that the five trusses based on the platonic polyhedra can all be regarded as tessellations of triangles, squares and pentagons on a sphere. With regards to the rigidity of the solids, it can be seen that only the tessellations of triangles have turned out to be rigid – the tetrahedron, octahedron and the icosahedron. The solids consisting of tessellations of squares and pentagons – the hexahedron and dodecahedron respectively— have many mechanisms.

The rigidity of a truss consisting of a tessellation of triangles that lie on a sphere follows from a theorem proved by Cauchy, together with several other theorems for polygons and polyhedra.

Theorem 13 of Cauchy (1813) states that:

> In a convex polyhedron with invariable faces the angles at the edges are also invariable, so that with the same faces one can build only a polyhedron symmetrical to the first one.
>
> Cauchy, 1813

Thus, every convex polyhedron with rigid faces will be rigid and, since the simplest way of forming a rigid face with pin-jointed bars is to use a triangle, Cauchy's theorem can also be stated in the specialised form:

> Every convex polyhedral surface is rigid if all of its faces are triangles.
>
> Cauchy, 1813

“The mathematicians are well acquainted with the difference between pure science, which has to do only with ideas, and the application of its laws to the use of life, in which they are constrained to submit to the imperfections of matter and the influence of accident.”

Dr. Johnson, 1750

research

3. PHYSICAL PHENOMENA & MATHEMATICAL DESCRIPTIONS

3.1 INTRODUCTION

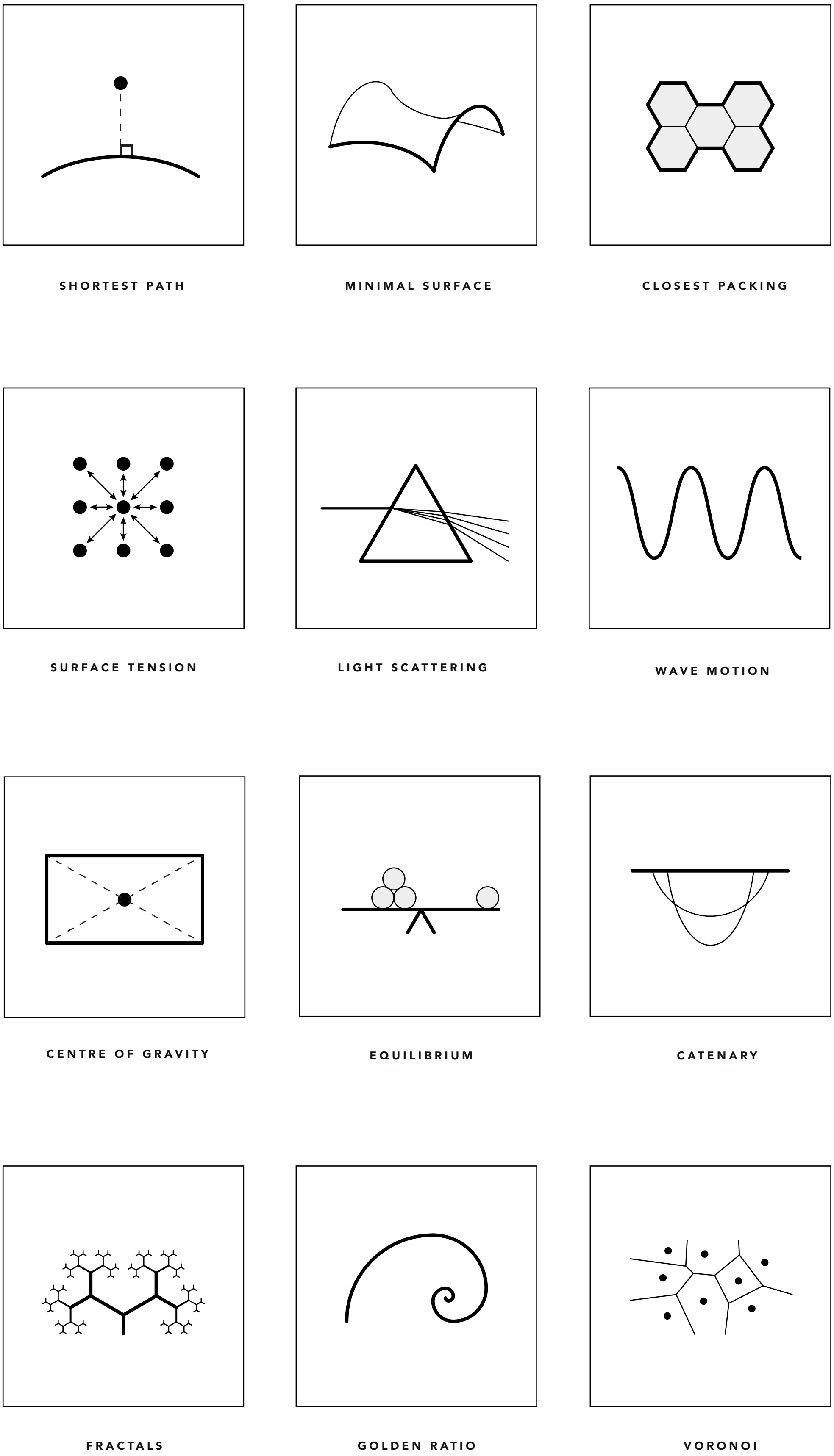
In nature there are universal laws. The laws to describe a phenomenon are the same for multiple branches of science, however, the thing that is described is differently. After having briefly discussed a few of the most important universal laws with which nature designs, it is possible to translate these to physical phenomena, which will later be connected to mathematical theorems.

The biological models and examination of processes in nature are divided into 12 physical principles:

- Shortest path
- Minimal surface
- Closest packing
- Surface tension
- Light scattering
- Wave motion
- Centre of gravity
- Equilibrium
- Catenary
- Fractals
- Golden ratio
- Voronoi

Although there are plenty of other natural laws that can be discribed or categorised, in this thesis the focus will be on these twelve phenomena that illustrates a physical morphology. Each of these laws have to do with shapes and a specific morphology. Vocabulary.com defines a physical phenomenon as a natural phenomenon involving the physical properties of matter and energy. A list of 164 phenomena is distinguished – also containing subdivisions of greater laws as light scattering and wave motion.

The twelve phenomena mentioned above will be explained briefly, as well as described mathematically in this chapter. For each phenomenon a few natural evidences will be given, showing the connection to the great realm of nature. By showing these examples, the mathematical and physical forms, that will be illustrated in diagrams and infographcis, are easily understandable. It is a necessity to mention that firstly, these are definitely not the only twelve and secondly, these twelve are not in a specific order or hierarchy. Some phenomena overlap others. To give an example, the core natural principle of symmetry can be found in many of these phenomena.



SHORTEST PATH

MINIMAL SURFACE

CLOSEST PACKING

SURFACE TENSION

LIGHT SCATTERING

WAVE MOTION

CENTRE OF GRAVITY

EQUILIBRIUM

CATENARY

FRACTALS

GOLDEN RATIO

VORONOI

[25] PHYSICAL PHENOMENA

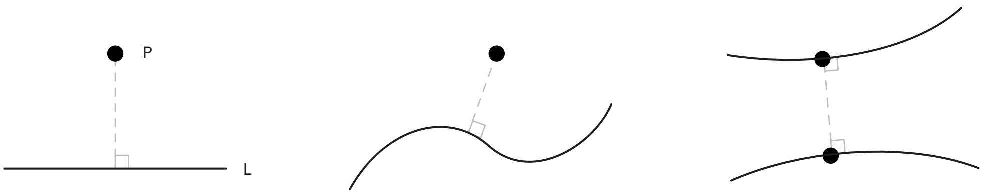
3.2 SHORTEST PATH

This is a mathematical questions that goes back to the era of the Romans. This question were of interest for them, because they had the insight that it is beneficial to have an excellent system of roads. Good roads allowed rapid movements of the Roman legions to suppress any rebellion. This question of shortest and quickest connections became especially important to the European powers during the fifteenth and sixteenth century, when they were searching for the best routes to the Far East and to the New World. Faster sailing router promised greater profits. The well-known expeditions of Vasco da Gama and of Christopher Columbus must be seen mainly in economic terms.

In nature we can see this mathematical principle occurring in various phenomena. The veins in a leaf to transfer water and other sources of life is built on this principle. Water that falls due to the gravitational force is seeking the shortest path. The reflection of light is in definition the shortest path.

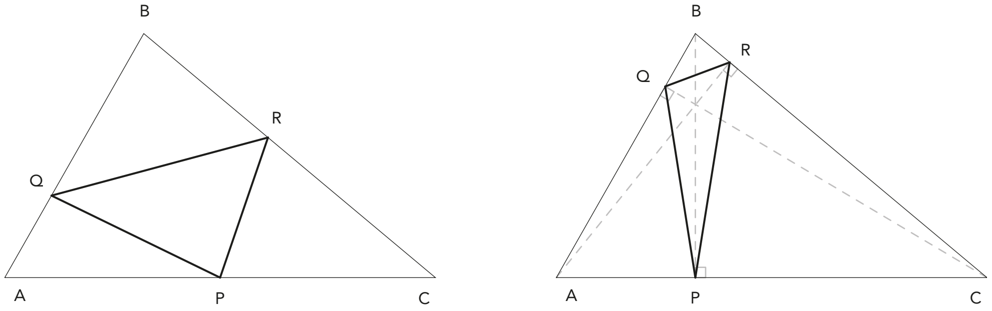
[26-29] NATURAL EVIDENCES FOR THE 'SHORTEST PATH' PHENOMENON

The brothers Jacob and Johann Bernoulli were the first, already in 1697 that founded the mathematical theory of shortest lines on surfaces. As in intro to the theory, a less complex question will be considered beforehand. The first and most basic minimum problem is to find the shortest path between a point and a curve and the shortest path between two curves. Accordingly to the theory of Pythagoras the solution can be found by dropping a line from point P to line L in such a way that the line is perpendicular to line L. It will be the intersection with line L at an angle of 90°.



[30] SHORTEST PATH CONSTRUCTION

The problem to be examined was posed and solved by Hermann Amandus Schwarz. The definition of the problem is as followed: Given an acute triangle, that is, one in which all angles are less than 90°, find an inscribed triangle with the smallest possible perimeter. Schwarz discovered that the inscribed triangle of shortest perimeter is given by the altitude triangle. The vertices P, Q, R of the altitude triangle are obtained by dropping the perpendicular from each vertex A, B, C to its opposite side.

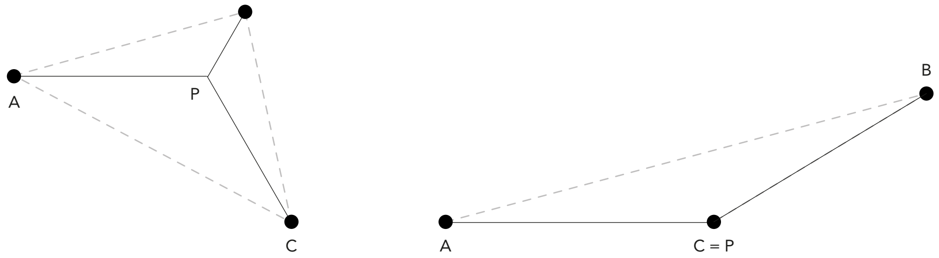


[31] THE ALTITUDE TRIANGLE

The reason why this triangle is the solution of Schwarz's problem comes from Heron's principle. Every solution must be a light triangle. A light triangle is the way light would reflect if the vertex A, B, C are walls of mirrors. The light triangle represents the closed path of travel for a ray of light in the room. The altitude triangle is the only inscribed light triangle. (Hildebrandt and Tromba 1984)

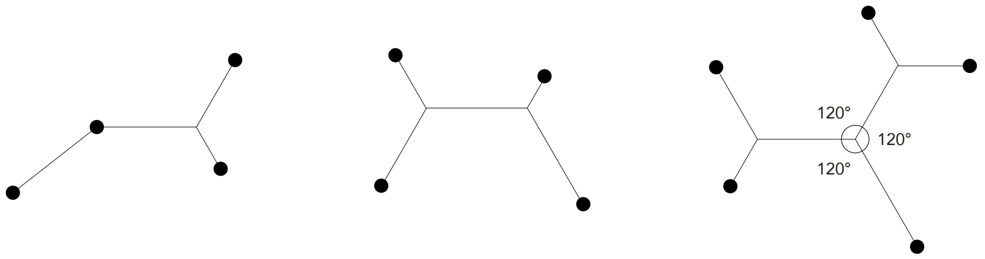
A second problem that will be illustrated is that of Jakob Steiner, who was a professor at the university of Berlin. The mathematical description of this problem is as followed: we are given three points A, B, and

C in a plane and are required to find a point P, and paths that join P to A, B, and C, such that the total length of the paths is a minimum. In the book 'Mathematics and Optimal form' a clear solution is given: *"The nature of the solution depends on how the points are situated. If all angles of triangle ABC are less than 120°, then P is the point within this triangle at which the angles APC, CPB, and BPA are equal, and hence each is equal to 120°. However, if one of the angles (say, the angle at C) in the triangle ABC is greater than 120°, then the solution point P must be point C."*

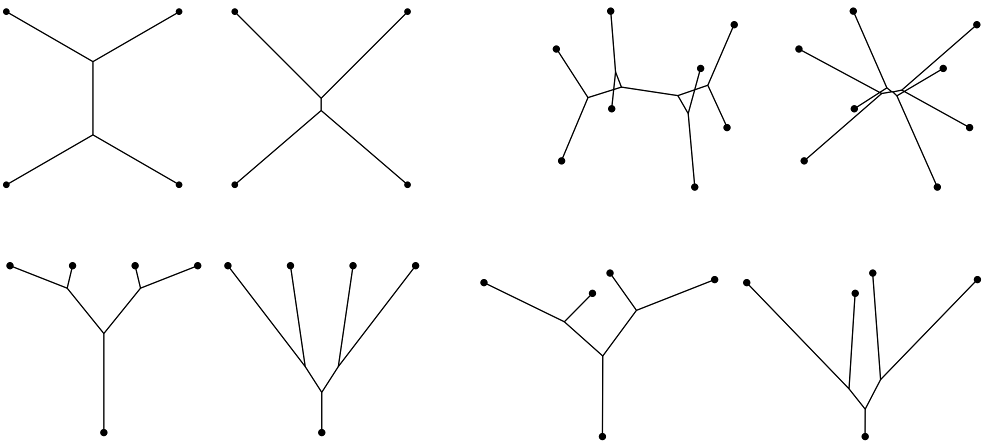


[32] MINIMAL TOTAL PATH LENGTHS

A general version of the Steiner problem could for example be that many points in a plane need to be connected by a system of lines in such a way that the total length is smallest. This is mostly a unique solution, but there are also generalized Steiner problems that have multiple possibilities.



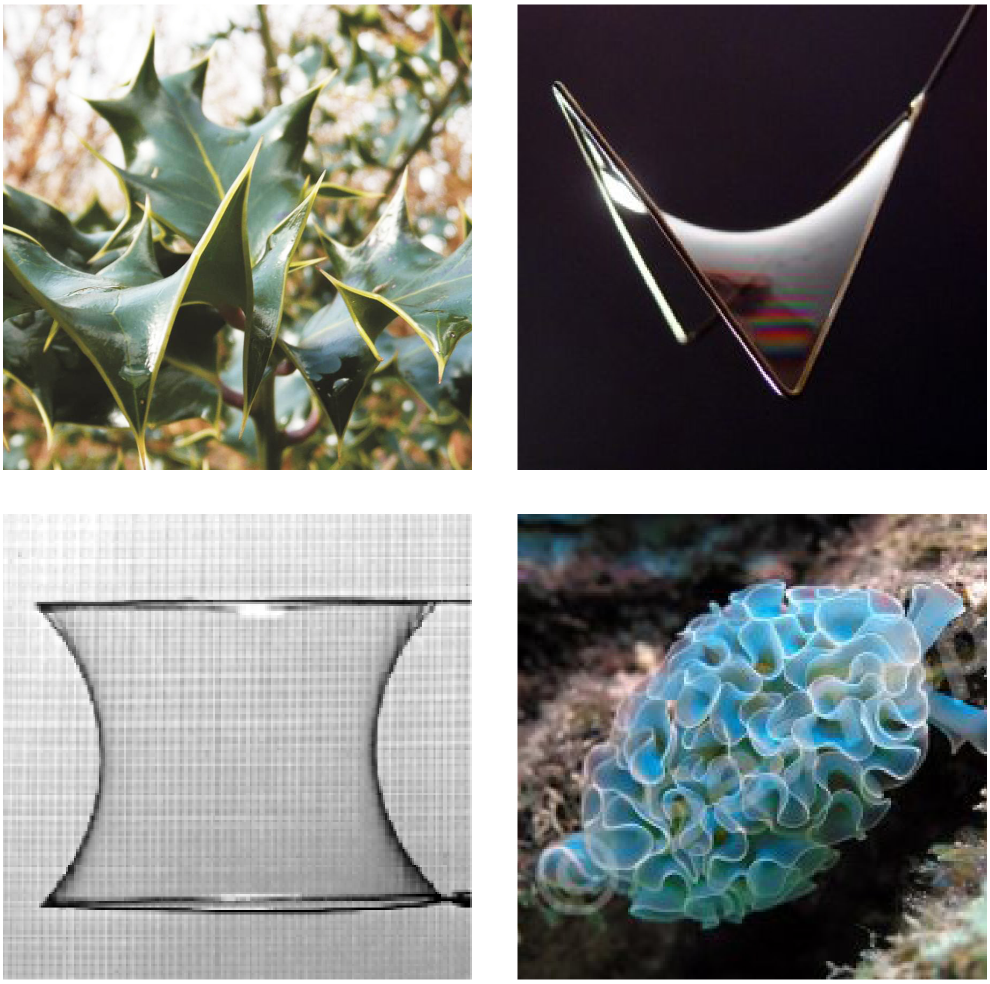
[33] SHORTEST PATH ILLUSTRATIONS



[34] SHORTEST PATH DUALITIES AND BRANCHINGS

3.3 MINIMAL SURFACE

In fact, there aren't many examples in nature that have the exact mathematical characteristics in order to suffice for a 'minimal surface', however, the analogy of a soap films performs remarkable features that can be used to explain the phenomenon. Soap films perform a fascinating mathematical theory. Soap films are in stable equilibrium – which phenomenon will be discussed later - so they have a laminae of minimal potential energy. The surfaces that soap films model are surfaces of least area. In mathematics, a surface of least area is called a minimal surface.



[35-38] NATURAL EVIDENCES FOR THE 'MINIMAL SURFACE' PHENOMENON

It was Lagrange who succeeded in deriving a minimal surface equation in 1760. This theorem will provide the geometric characterisation of least-area surfaces that we are seeking: At each regular point, a surface of minimal area must have a mean curvature of zero. This means that surfaces of minimal area satisfies the equation:

$$(1+h_v^2)h_{uu} - 2h_u h_v h_{uv} + (1+h_u^2)h_{vv} = 0$$

Minimal surfaces are defined as surfaces with zero mean curvature.

$$H = 0$$

A mean curvature is the average Gaussian curvature of a surface. The Gaussian curvature of a surface is the product of the principle curvatures in a point.

$$K = k_1 \cdot k_2$$

$$H = \frac{k_1 + k_2}{2}$$

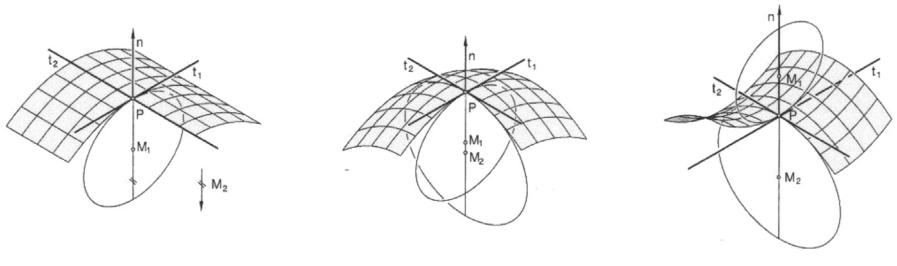
$$k^2 - 2Hk + K = 0$$

Hildebrandt and Tromba state that there is another physical interpretation of this equation that the mean curvature is zero. It can also be described with the Laplace equation:

$$p = T \cdot H$$

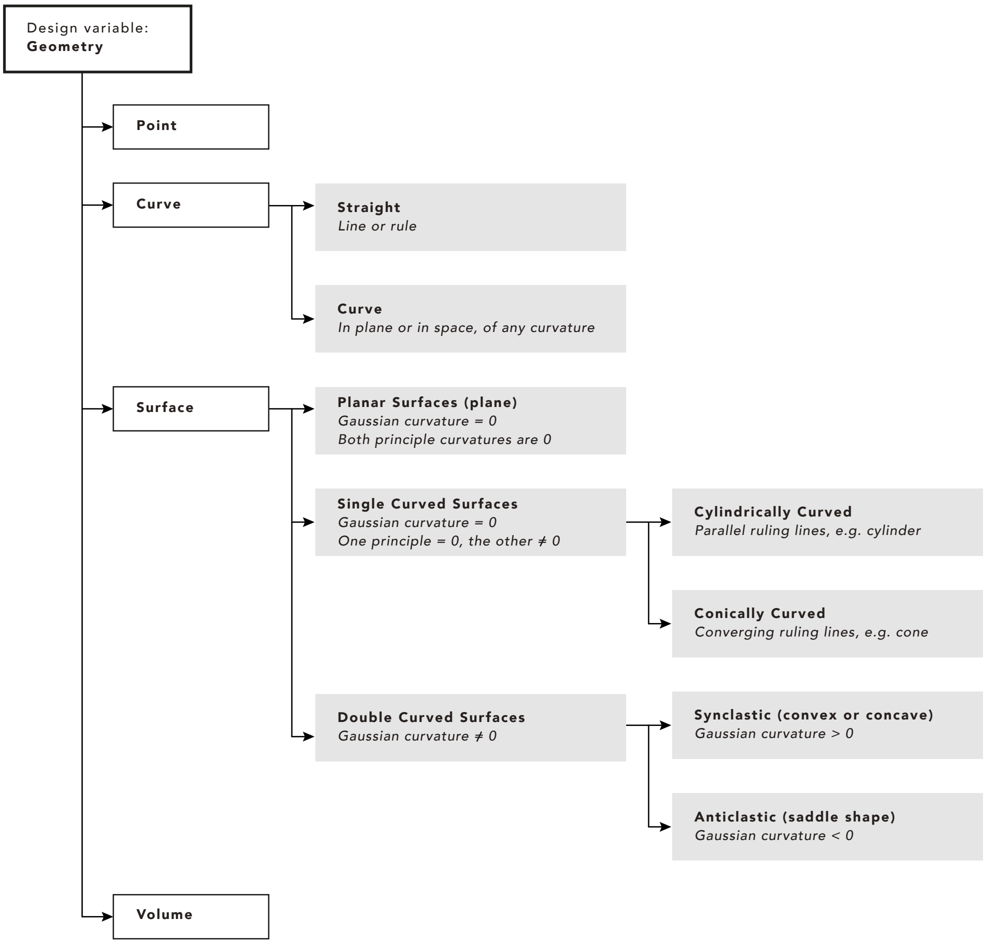
Where T denotes the surface tension of the liquid. Physically can be stated that if the mean curvature is zero, this is equivalent to having the same pressure on both sides of a minimal surface. This immediately explains why a closed soap film, like a soap bubble is no minimal surface, because inside there is a higher pressure than outside. The surface of a soap bubble has a constant mean curvature, but it is not equal to zero. A sphere is a "minimal surface" in the sense that it minimises the surface area-to-volume ratio, it does not qualify as a minimal surface in the sense used by mathematicians.

If a surface has zero mean curvature at each point, then at each point is curving both away and toward a given perpendicular direction. Minimal surfaces are either flat or look like saddle figures – that is anticlastic.



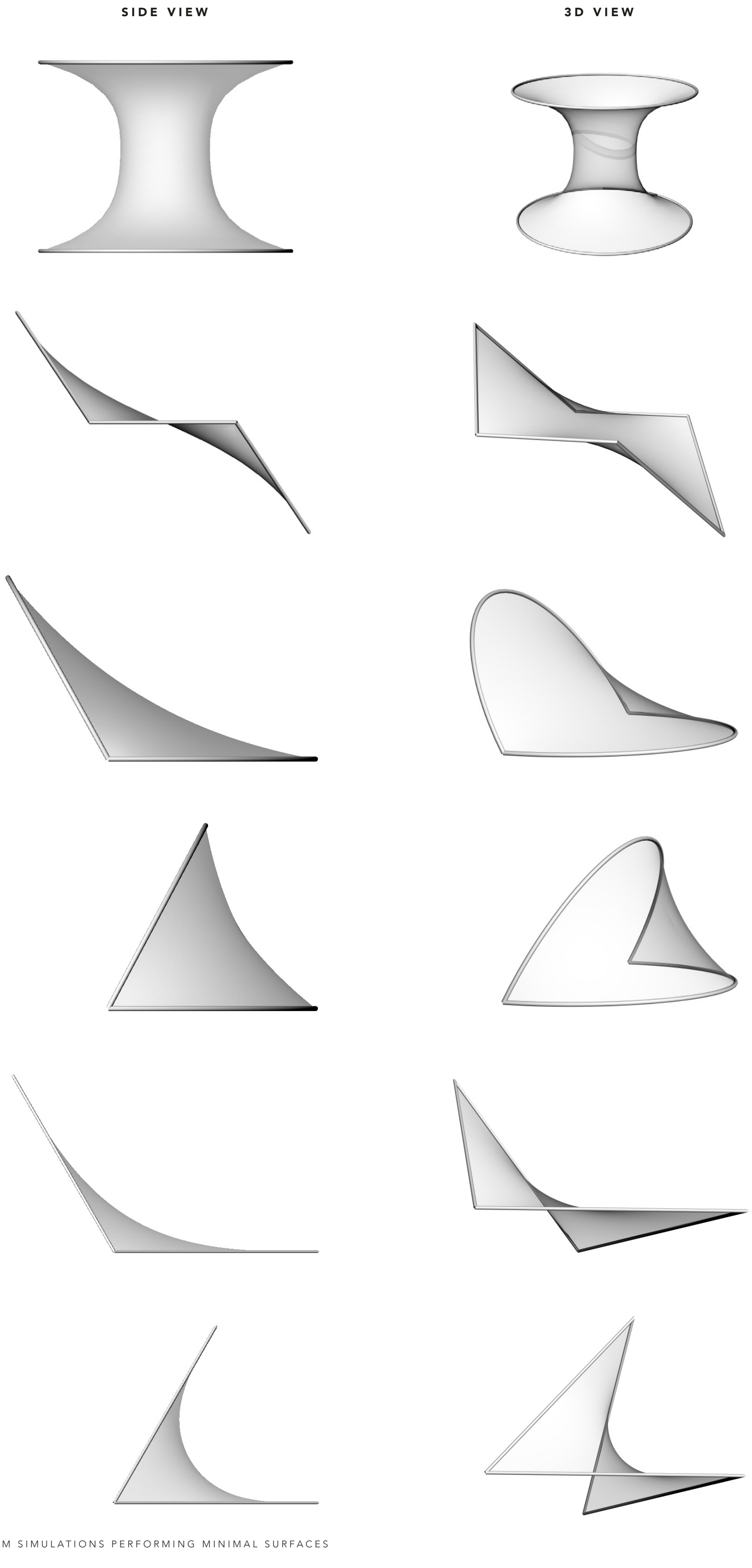
SINGLE CURVED **SYNCLASTIC** **ANTICLASTIC**

[39] TYPES OF CURVATURES



[40] GEOMETRY CLASSIFICATION

Above a classification of geometries is illustrated. A surface can be described in different ways. In this case it is classified in type of curvature and linked to the Gaussian curvature. Planar surfaces, as mentioned before have a mean curvature of zero, but also a Gaussian curvature that is zero, due to the fact that both principle curvatures are zero. For single curved surfaces, for example the cylinder and the cone, one of the principle curvatures is zero, resulting in a Gaussian curvature of zero as well. There is alteration in the double curved surfaces. This category can be divided into synclastic surfaces and anticlastic surfaces. An example of a synclastic surface is a sphere, an egg, the Hoover dam. Both principle curvatures are positive, so the Gaussian curvature is always greater than zero. Anticlastic surfaces are better known as saddle-shaped surfaces. One of the principle curvatures is positive, whereas the other is negative, resulting in a Gaussian curvature that is in every case smaller than zero.



[41] SOAPFILM SIMULATIONS PERFORMING MINIMAL SURFACES

3.4 CLOSEST PACKING

Closest packing is a phenomenon that occurs a lot in nature. It is related to the principle of shortest path and is generally speaking a way of space filling a geometry in plane or in space. Perhaps the most familiar example of closest packing, and certainly the most well-known natural evidence of close-packing, and perhaps the most sublime, is the honeycomb of the bee. The principle behind this is that it contains the greatest amount of honey with the least amount of beeswax. Additionally, the structure requires the least energy for the bees to construct. Another interesting occurring example of closest-packing is that of the 'Giants Causeway' in Northern Ireland. It is an area with about 40.000 basalt columns as a result of a volcanic eruption. It was declared a World Heritage Site by UNESCO in 1986, and a national nature reserve in 1987 by the Department of the Environment for Northern Ireland.

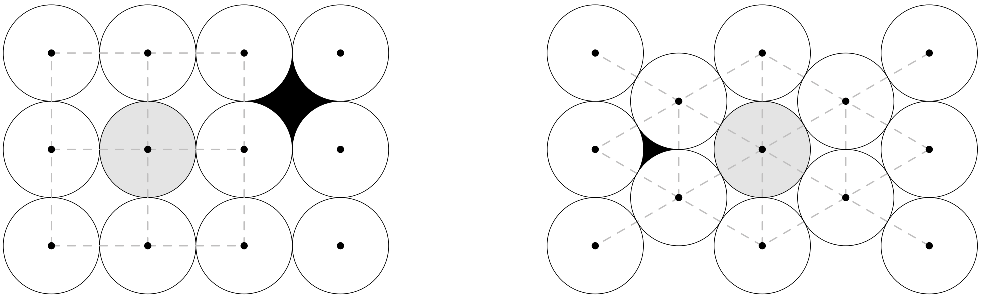
[42-45] NATURAL EVIDENCES FOR THE 'CLOSEST PACKING' PHENOMENON

It can readily seen that the principle of closest packing is equivalent to that of triangulation, and it is well known that triangulated frameworks exhibit inherent geometric stability. Such properties enable framework structures to be built without moment joints, insuring axially loaded members; and this in turn results in high strength-per-weight minimum energy structures. (Pearce 1990)

In 'Mathematics and Optimal Form', Hildebrandt and Tromba give a simple explanation of why a hexagonal arrangement is the most dense we can find. *"Similarly, balls of equal size between two plates, densely packed, arrange themselves in a hexagonal array. Imagine each ball to be a living cell that tries to expand as much as possible, and each by the same amount. Then it seems obvious that a pattern of hexagonal cells will form. In fact, this is what we very often see with cell growth, and exterior pressure may lead to the same results if it is uniformly applied."*

For the mathematical expressions and description for this phenomenon, I would like to refer to the definitions given by Wolfram Mathworld;
The densest packing of circles in the plane is the hexagonal lattice of the bee's honeycomb (right figure; Steinhaus 1999, p. 202), which has a packing density of:

$$\eta_h = \frac{1}{6}\pi\sqrt{3} \approx 0.9069...$$



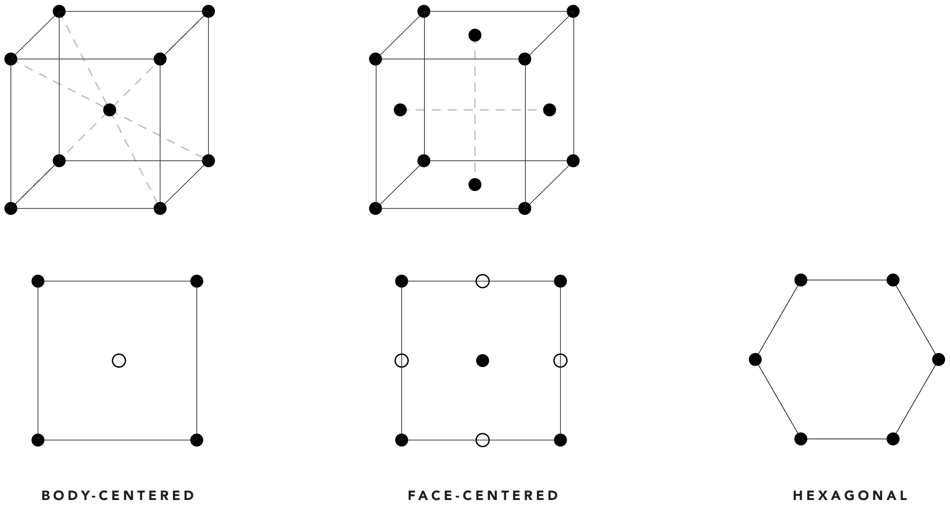
SQUARE PACKING

HEXAGONAL PACKING

[46] TIGHTEST PACKING OF CIRCLES

A gerenal packing density of spheres is defined as n. It can be seen as the greatest fraction of a volume or a space occupied by spheres. In three dimensions, there are three periodic packings for identical spheres:

- Body-centered cubic lattice
- Face-centered cubic lattice
- Hexagonal lattice



[47] THREE PERIODIC PACKING STRUCTURES

It was hypothesized by Kepler in 1611 that close packing (cubic or hexagonal, which have equivalent packing densities) is the densest possible. The problem of finding the densest packing of spheres (not necessarily periodic) is therefore known as the Kepler problem, where

$$\eta_{Kepler} = \eta_{CCP} = \eta_{HCP} = \frac{\pi}{3\sqrt{2}} \simeq 0.74048$$

Steinhaus 1999, p. 202; Wells 1986, p. 29; Wells 1991, p. 237).

In 1831, Gauss managed to prove that the face-centered cubic is the densest lattice packing in three dimensions (Conway and Sloane 1993, p. 9), but the general conjecture remained open for many decades.

For packings in three dimensions, C. A. Rogers (1958) showed that the maximum possible packing density satisfies:

$$\eta_{max} < \sqrt{18} \cdot (\cos^{-1}\frac{1}{3} - \frac{1}{3}\pi) = 0.77964$$

The rigid packing with lowest density known is significantly lower than that reported by Hilbert and Cohn-Vossen (1999, p. 51). To be rigid, each sphere must touch at least four others, and the four contact points cannot be in a single hemisphere or all on one equator.

Hilbert and Cohn-Vossen (1999, pp. 48-50) considered a tetrahedral lattice packing in which each sphere touches four neighbors. This is the lattice formed by carbon atoms in a diamond (Conway and Sloane

1993, p. 113). Random close packing of spheres in three dimensions gives packing densities in the range 0.06 to 0.65 (Jaeger and Nagel 1992, Torquato et al. 2000). Compressing a random packing gives polyhedra with an average of 13.3 faces (Coxeter 1958, 1961).

The packing densities for several types of sphere packings are summarized in the following table.

Packing	Density	Reference
Loosest possible	≈ 0.0555	Gardner (1966)
Tetrahedral packing	$\eta = \frac{\pi\sqrt{3}}{16} \approx 0.3401$	Hilbert & Cohn-Vossen (1999)
Cubical lattice	$\eta = \frac{\pi}{6} \approx 0.5236$	
Hexagonal lattice	$\eta = \frac{\pi}{3\sqrt{3}} \approx 0.6046$	
Random	≈ 0.6400	Jager & Nagel (1992)
Cubic close packing	$\eta = \frac{\pi}{3\sqrt{2}} \approx 0.7405$	Steinhaus (1999), Wells (1986, 1991)
Hexagonal close packing	$\eta = \frac{\pi}{3\sqrt{2}} \approx 0.7405$	Steinhaus (1999), Wells (1986, 1991)

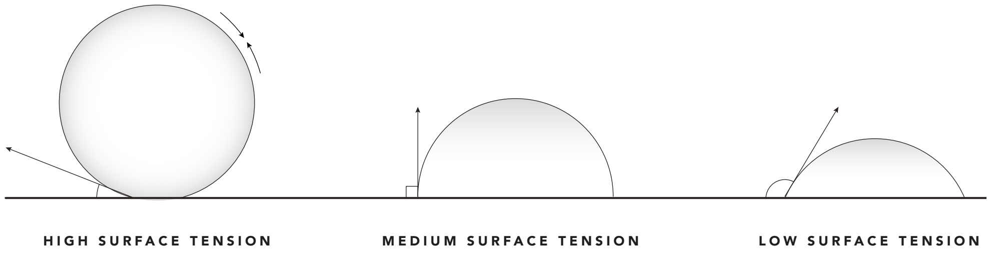
[48] PACKING DENSITIES

3.5 SURFACE TENSION

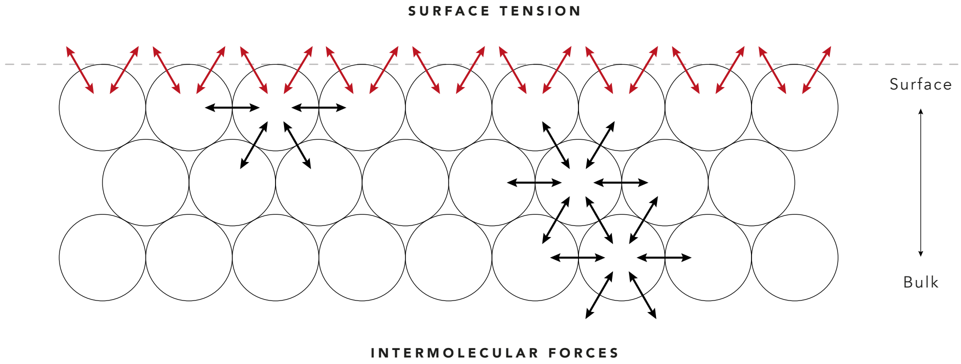
Surface tension is the phenomena that molecules on the surface of a liquid are attracted towards each other, resulting in tensional force. In nature this phenomenon can be observed in how a soap bubble is formed but also how insects float on water, or how a basilisk runs on water by slapping is feet onto the surface.

[49-52] NATURAL EVIDENCES FOR THE 'SURFACE TENSION' PHENOMENON

The molecules on the surface are pulled inwards. This is due to the cohesive forces among liquid molecules. In the bulk of the liquid, the forces acting on the molecules, the so-called intermolucelar forces are equal for every molecule. The net force will therefore be zero. The molecules at the surface are not totally surrounded by other molecules, which results in a net force that doesn't equal zero. The molecules at the surface attract each other and are therefore pulled inwards, which leads to internal pressure. This principle is shown in the illustration on the next page.



[53] SURFACE TENSION VARIATIONS



[54] SURFACE TENSION PRINCIPLE

"The soap bubble is an interesting example of surface tension. However, it is the reduction of surface tension by the soap itself or by other detergents that enable us to produce soap films, which can either be held by a framework consisting of wires, threads, and surfaces or exist without a boundary, in the form of a soap bubble. Without the soap to reduce the tension, a liquid film of water could not persist, but would immediately break."

A precise explanation of surface tension in terms of the action of molecular forces is not easy. Moreover, it must take other phenomena into account – for example, the vapor layer covering a liquid surface. Yet the beautiful and simple concept of characterising stable states as minima of potential energy will enable us to explain the behavior of soap films quite satisfactorily. *"Because a liquid skin behave in many ways like an elastic rubber cloth, it should have higher potential energy the more it is stretched... ...Thus a soap film will be in stable equilibrium if its area is less than that of any other surface satisfying the same restrictions."* (Hildebrandt and Tromba 1984) This principle of potential energy and equilibrium will be discussed later.

The surface is flat if there acts no normal force on the surface. Like already is discussed in the chapter of minimal surface, the curvature of the surface is due to a pressure difference. This pressure difference times the surface area results in a normal force. In order for the surface tension forces to cancel the force due to pressure, the surface must be curved. The mathematical description of the pressure difference is known as the Young–Laplace equation:

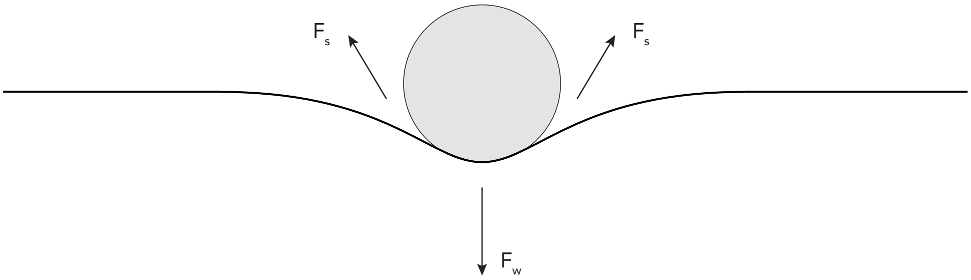
$$\Delta p = \gamma \left(\frac{1}{R_x} + \frac{1}{R_y} \right)$$

where:

Δp	is the pressure difference, known as the Laplace pressure
γ	is surface tension.
R_x, R_y	are radii of curvature in each of the axes that are parallel to the surface.

Another interesting mathematical description is that of floating objects. A natural evidence are the insects floating on a pool. The weight Fw of the mosquito for instance depresses the surface. The surface tension gives an upward force in order to balance for the mosquito's weight. The tensional force Fs that results in an upward force is acting on both sides of pressure. In the illustration a ball is used to show this principle.

$$F_w = 2F_s \cos\theta \qquad \leftrightarrow \qquad \rho A_s L g = 2\gamma L z \cos\theta$$



[55] FLOATING OBJECT PRINCIPLE

3.6 LIGHT SCATTERING

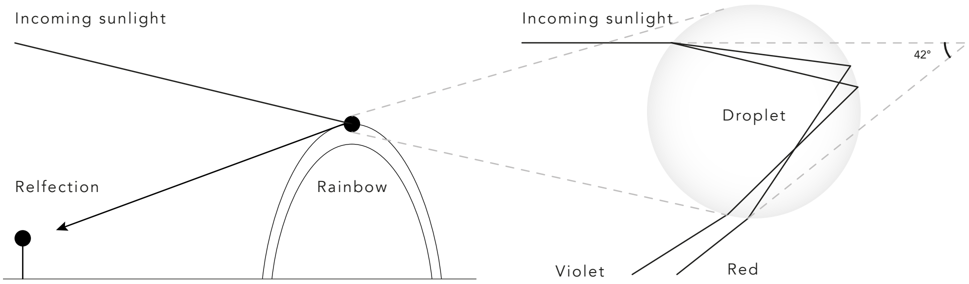
In his book ‘Mathematics in Nature’, John A. Adam states that the two most fundamental and widespread phenomena that occur in the realm of nature are that of the scattering of light and wave motion. Both may occurs almost anywhere given the right circumstances, and both may be described in mathematical terms at varying levels of complexity.

[56-59] NATURAL EVIDENCES FOR THE ‘LIGHT SCATTERING’ PHENOMENON

In the illustrations above a few examples are shown how the scattering of light both by air molecules and by the much larger dust particles give occurring visuals everyday. *“The deep blue sky above and the red glow near the sun at the end of the day are due to molecular scattering of light, though dust or volcanic ash can render the latter quite spectacular at times. The rainbow is formed by sunlight scattering in preferential direction by near-spherical raindrops: scattering in this context means refraction and reflection.”* (Adam, 2006) The most common examples of light scattering that are easily observable are:

- Rainbows
- Glories
- Halos
- Sundogs

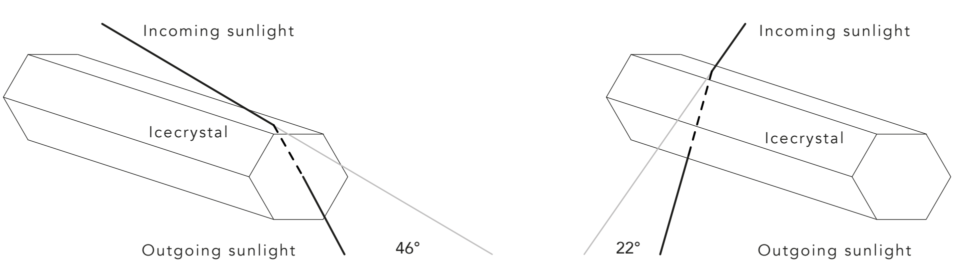
A rainbow is the most well known and is an excellent demonstration of the dispersion of light. Besides, it gives the evidence that visible light is composed of a spectrum of wavelengths, each associated with a distinct color. To view a rainbow, your back must be to the sun as you look at an approximately 40 degree angle above the ground into a region of the atmosphere with suspended droplets of water or even a light mist. Each individual droplet of water acts as a tiny prism that both disperses the light and reflects it back to your eye. As you sight into the sky, wavelengths of light associated with a specific color arrive at your eye from the collection of droplets.



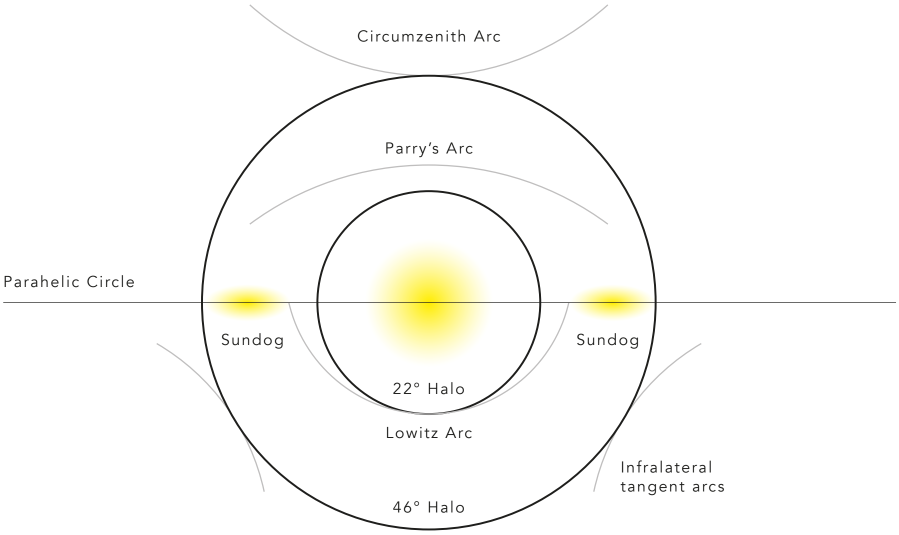
[60] RAINBOW FORMATION

A glory can be seen for example around the shadow of an airplane on a cloud. This effect is called the “backscatter” effect.

The circular arcs around the sun are known as ‘halos’ and are formed by the refraction of sunlight through ice crystals of various shapes in the upper atmosphere. Hexagonal ice crystals can be seen as part of an equilateral prism. If the ice crystals have more or less every possible orientation around the sun, a 22° halo is formed. In the same way a 46° halo can be formed. The way how incoming sunlight is refracted is shown in the image below.



[61] ICECRYSTAL REFRACTIONS



[62] VISABLE CIRCLES AND ARCHES BY LIGHT SCATTERING

Above you can find an illustration of the natural phenomenon of halo's and sundogs. The sundogs are seen on both sides of the sun when high cirrus clouds are present.

In 1637 René Descartes was the first one to be able to "hang the rainbow in the sky". The first one that was able to "paint" the rainbow was Isaac Newton, 30 years later. It is important to state that the bright primary and fainter secondary bows are well described by elementary mathematics, whereas the more subtle principles and observations of the rainbow require some of the most complicated and complex techniques of mathematical physics in order to be explained well.

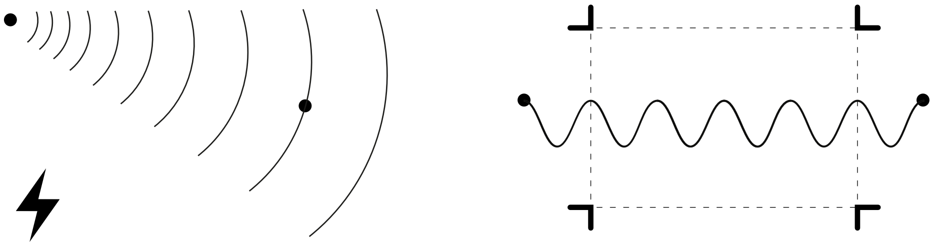
3.7 WAVE MOTION

Wave motion is, as well as light scattering, a physical phenomenon that always occurs. We can observe waves in the ocean, a lake, a pond, etc. But waves are also generated when someone is playing a musical instrument, even the voice generates waves. Clouds are also indicators for the fact that waves are all around us. It is a phenomenon that we are all familiar with. It arises in the fields of acoustics, elektromagnetics and fluid mechanics.

[63-66] NATURAL EVIDENCES FOR THE 'WAVE MOTION' PHENOMENON

John A. Adam states that all the examples wave motion have two very important characteristics in common:

1. Energy is propagated from points near the source of waves to points which are distant from it.
2. The distrurbances travel through the medium, whatever that may be, without giving the medium as a whole any permanent displacement.



ENERGY PROPAGATION

NO DISPLACEMENT OF MEDIUM

[67] WAVE MOTION CHARACTERISTICS

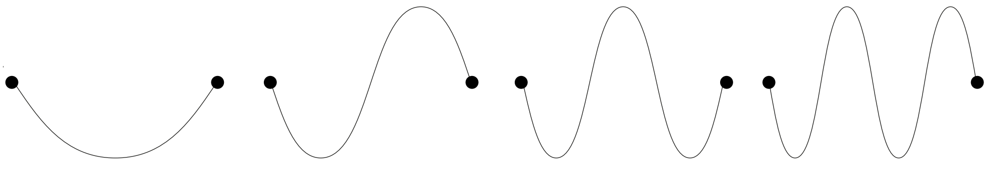
Additionally, the type of medium has no influence on these two fundamental truths on waves, whether the medium is air, a stretched string, a liquid, and electric cable, and so on. The wave equation is an important second-order linear differential equation for the description of waves.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \cdot \frac{\partial^2 u}{\partial x^2}$$

In physics, waves can be separated into two distinct types: Transversal waves and longitudinal waves. For transverse waves the displacement of the medium is perpendicular to the direction of propagation of the wave. A ripple on a pond and a wave on a string are easily visualised transverse waves. It is useful to note that these kinds of waves cannot propagate in a gas or in a liquid, since there is no mechanism for driving motion perpendicular to the propagation of the wave.

In longitudinal waves the displacement of the medium is parallel to the propagation of the wave. Sound waves in air are longitudinal.

(www.hyperphysics.phy-astr.gsu.edu)



TRANSVERSAL WAVES

LONGITUDINAL WAVES

[68] TRANSVERSE- VS. LONGITUDINAL WAVES

3.8 CENTRE OF GRAVITY

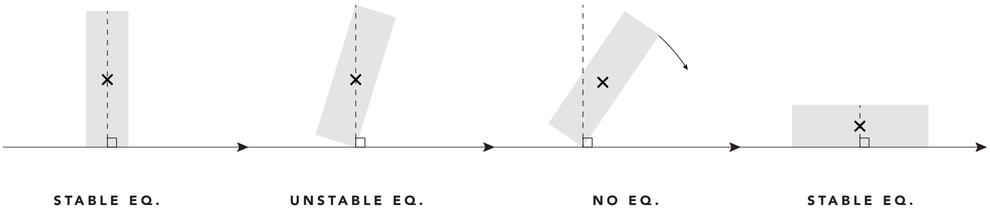
The center of gravity or also called the center of mass is the point at which the entire weight of a body may be considered as concentrated so that if supported at this point the body would remain in equilibrium in any position. Another explanation is that the center of gravity of any body is a point within that body such that, if the body were suspended from the point, the weight carried thereby remains at rest and preserves its original position.

[69-72] NATURAL EVIDENCES FOR THE 'CENTER OF GRAVITY' PHENOMENON

In everyday life there are examples of objects that seek the center of gravity. The most explanatory example is that of a spinning top. The way an athlete jumps as high as possible is by keeping the center over gravity as low as possible throughout the jump. While in the air, there is only a small area of the body above the bar.

About 1800 years after Archimedes' death, scientists like Stevin and Galileo started to build a theory of statics, of the equilibrium of complicated mechanical systems.

Equilibrium is closely related to the principle of the center of gravity. During the time of the Renaissance, the ideas of Archimedes became widely known in Italy. Torricelli, but also Galileo took up the Archimedean concept op the barycenter of a mechanical system. They stated the principle that such a system will be in equilibrium if its barycenter is as low as possible within the given limitations.



[73] EQUILIBRIUM OF A RECTANGULAR BLOCK

Below the basic mathematical formulas are given to find the center of gravity of a body. Knowing the actual weight of the subject or the segments is unnecessary since percentages can also be used.

$$W = \sum \Delta W = \int dW$$

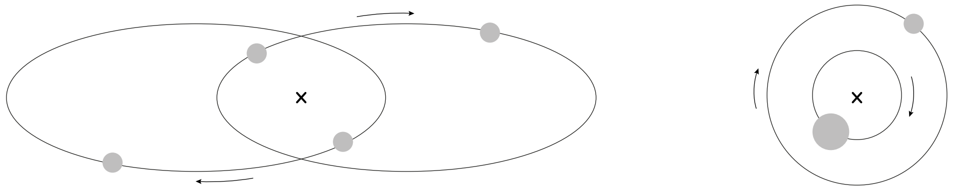
$$\bar{x}W = \sum x_i \Delta W_i = \int x dW \rightarrow \bar{x} = \frac{\int x dW}{\int dW}$$

Another principle where center of gravity needs to be taken into account is that of the barycenter. A barycenter is the center of gravity when two or more bodies orbiting each other. The two-body-problem is a mathematical description on how to find the distance from a body's center of mass to the barycenter. This can be done with the following equation:

$$r_1 = a \cdot \frac{m_2}{m_1 = m_2} = \frac{a}{1 + \frac{m_1}{m_2}}$$

where:

r_1	the distance from body 1 to the barycenter
a	the distance between the centers of the two bodies
m_1, m_2	the masses of the two bodies.

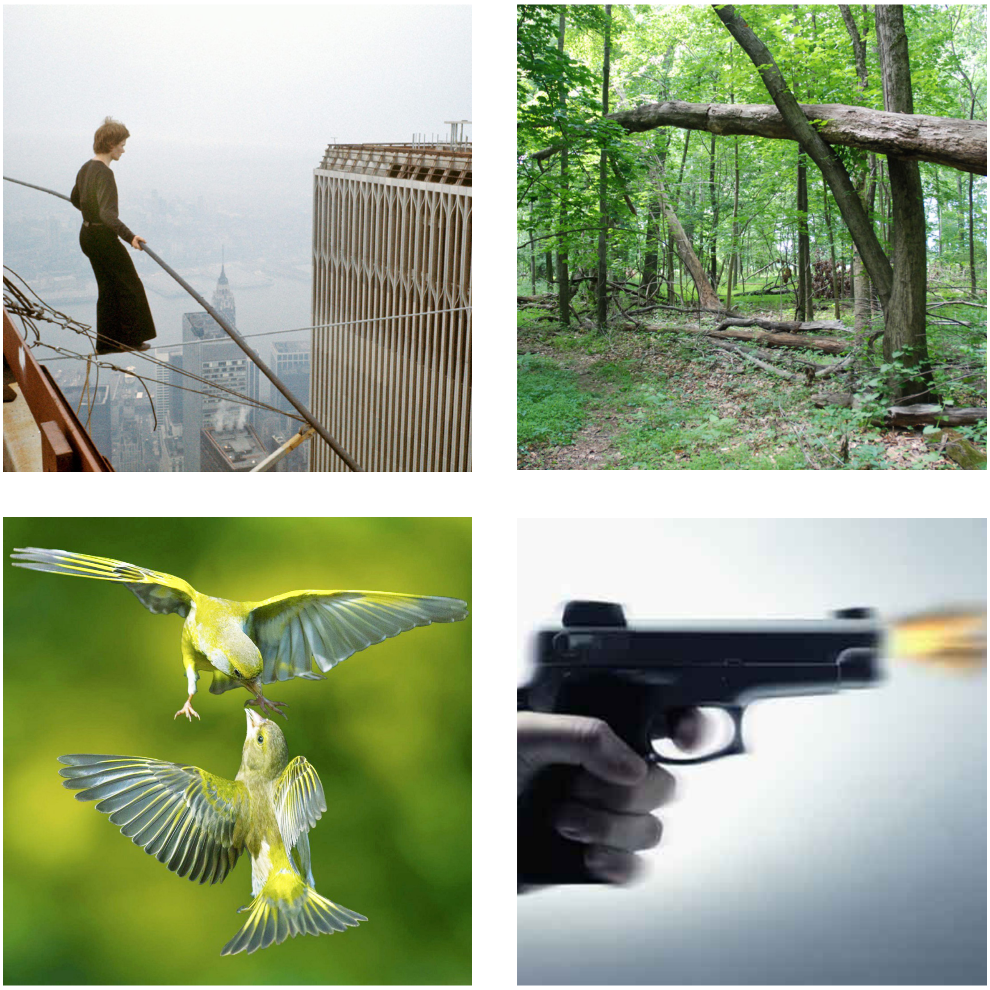


[74] TWO-BODY-PROBLEM

3.9 EQUILIBRIUM

The term equilibrium is quite a broad term, however, in this thesis it will be used as force equilibrium. The definition according to dictionary.com equilibrium is a state of rest or balance due to the equal action of opposing forces. This definition is in line with Newton's Third Law of Motion which states that 'for every action, there is an equal and opposite reaction.'

"In every interaction, there is a pair of forces acting on the two interacting objects. The size of the forces on the first object equals the size of the force on the second object. The direction of the force on the first object is opposite to the direction of the force on the second object. Forces always come in pairs - equal and opposite action-reaction force pairs."



[75-78] NATURAL EVIDENCES FOR THE 'EQUILIBRIUM' PHENOMENON

A variety of action-reaction force pairs are evident in nature. The way a fish uses its fins to push water backwards, which is equal to the size of the force water reacts on the fish, which is in the opposite direction. Action-reaction force pairs make it possible for fish to swim, but also for a bird to fly. The recoiling principle of a rifle due to the explosion pushing forward the bullet is also an example of Newton's Third Law.

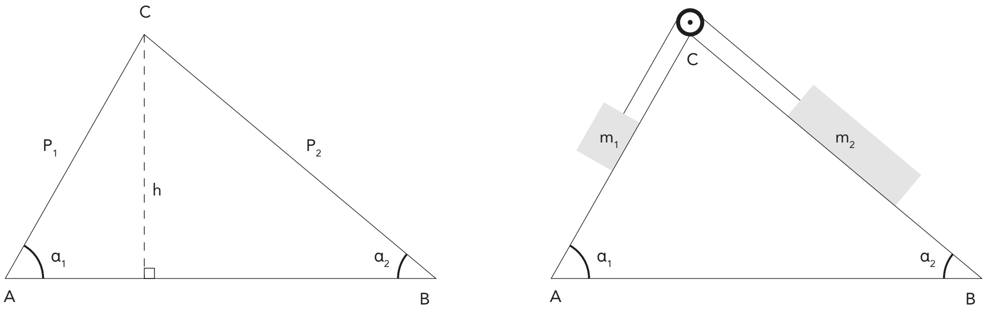
The Dutch engineer Simon Stevin – who lived from 1548 to 1620 – was one of the pioneers when it comes to discoveries on force equilibrium. Stevin focused on an old mechanical problem which was the question 'how much pull is needed to keep a weight that lies on an inclined plane in equilibrium'. He discovered that the two loads of weight m1 and m2 on P1 and P2 respectively, balance each other if and only if:

$$m_1 \cdot \sin\alpha_1 = m_2 \cdot \sin\alpha_2$$

And therefore:

$$\frac{m_1}{m_2} = \frac{\sin\alpha_2}{\sin\alpha_1}$$

This is called the law of the inclined plane.
(Hildebrandt, 1984)



[79] THE INCLINED PLANE EQUILIBRIUM

Hildebrandt goes on by explaining that on the theory of equilibria of mechanical systems great progress was made by Johan Bernoulli. In 1717 he proposed the principle of virtual work. This principle states:

> In equilibrium, no work is needed to achieve an infinitesimal displacement of a given mechanical system.

This rule counts for stable as well as unstable configurations. The number that is connected to this statement is its potential energy. Potential energy is a function of the various states of the system. Peter Gustav Lejeune Dirichlet – who is also know for the Dirichlet Tessellation, better known as the Voronoi

Diagram which will be discussed later – expressed this function in two rules; the Dirichlet's Principle:

Rule 1:

The stable equilibrium states (that is, states of rest) of a physical system are characterized by the condition that, in such a state, the potential energy of the system is less than it would be for any possible (or virtual) close-by rate of the system.

This rule applies for the pits, or the minima, of the range with a stable equilibrium.

Rule 2:

The equilibrium states of a physical system are the stationary states of its potential energy.

This rule applies for the unstable states of rest, for example the tops, or the maxima, of the range, but also the saddle points.

This principle is shown in the illustration below.

[80] STABLE- VS. UNSTABLE EQUILIBRIUM

3.10 CATENARY

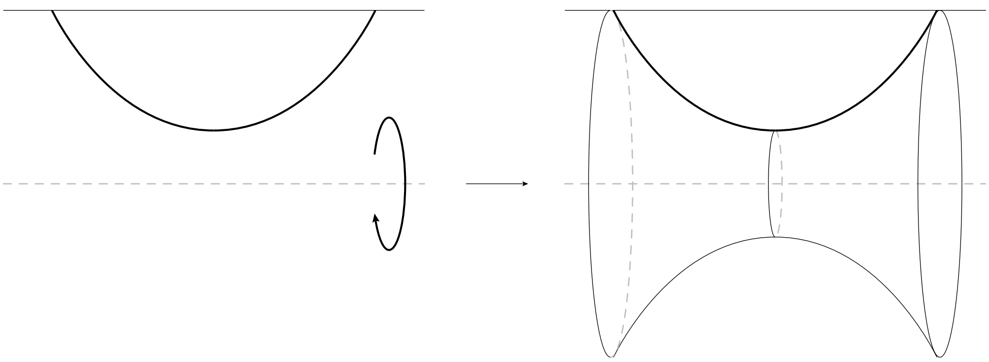
The catenary is a simple principle. The catenary is the shape of a perfectly flexible chain suspended by its ends and acted on by gravity. The word catenary is derived from the Latin word for "chain." Its equation was obtained by Leibniz, Huygens and Johann Bernoulli in 1691. They were responding to a challenge put out by Jacob Bernoulli to find the equation of the 'chain-curve'.

Because of its characteristics, a hanging chain acted on by gravity, this chain-curve occurs all around us. From spider webs to cable- or rope bridges.

[81-84] NATURAL EVIDENCES FOR THE 'CATENARY' PHENOMENON

Huygens was the first to use the term catenary in a letter to Leibniz in 1690 and David Gregory wrote a treatise on the catenary in 1690. Galileo claimed that the curve of a chain haning under gravity would be a parabola. Jungius disproved Galileo's claim.

Hildebrandt and Tromba highlight a remarkable feature of the catenary that is related to the earlier discussed phenomenon of minimal surface. If you revolve the catenary about an axis that is parallel to the stick and below the chain, you will obtain the only kind of non-planar minimal surface that is a surface of revolution.



[85] CATENARY REVOLUTION

The mathematical description of this catenoid is as followed: The height of the lowest point of the catenary above the axis of rotation is:

$$h = \sqrt{b^2 - l^2}$$

L denotes half of the length of the chain and b is the distance between the bar where the chain is suspended from and the axis of rotation. This minimal surface, which is called the 'only curved minimal surface of revolution' was discovered by Euler in 1744.

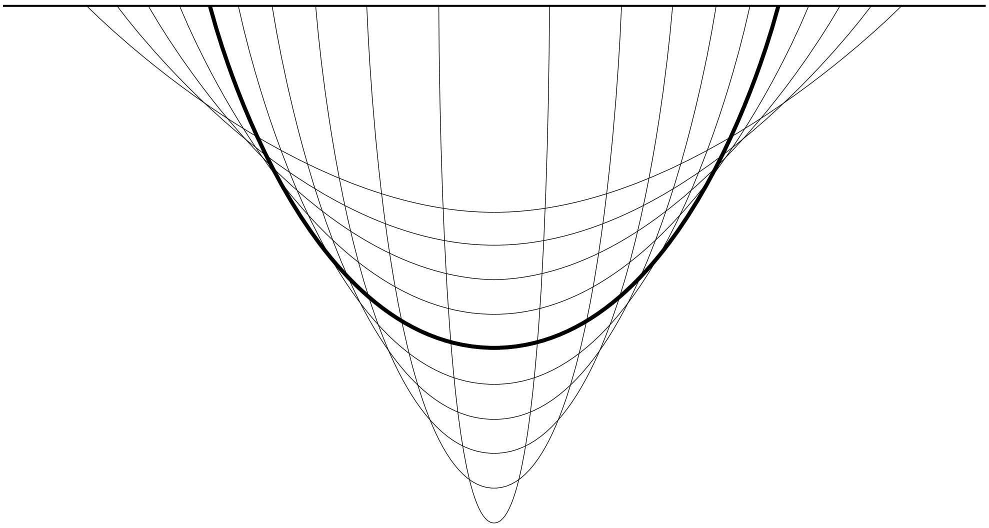
The parametric equations for the catenary are given by:

$$x(t) = t$$

$$y(t) = a\cosh(\frac{t}{a})$$

$$y(t) = \frac{1}{2}a(e^{\frac{t}{a}} + e^{\frac{-t}{a}})$$

where $t = 0$ corresponds to the vertex and a is a parameter that determines how quickly the catenary "opens up." Catenaries for several a values are shown on the next page.



[86] A-VALUE VARYING CATENARIES

The arc length, curvature, and tangential angle for $t > 0$ are given by:

$$s(t) = a\sinh(\frac{t}{a})$$

$$\kappa(t) = \frac{1}{a}\sec\mathrm{h}^2(\frac{t}{a})$$

$$\phi(t) = 2\tan^{-1}\left[\tanh(\frac{t}{2a})\right]$$

3.11 FRACTALS

A fractal can be seen as a pattern that repeats itself at different scales. This property is called "self-similarity." A more mathematical comes from the dictionary that states that a fractal is a curve or geometrical figure, each part of which has the same statistical character as the whole.

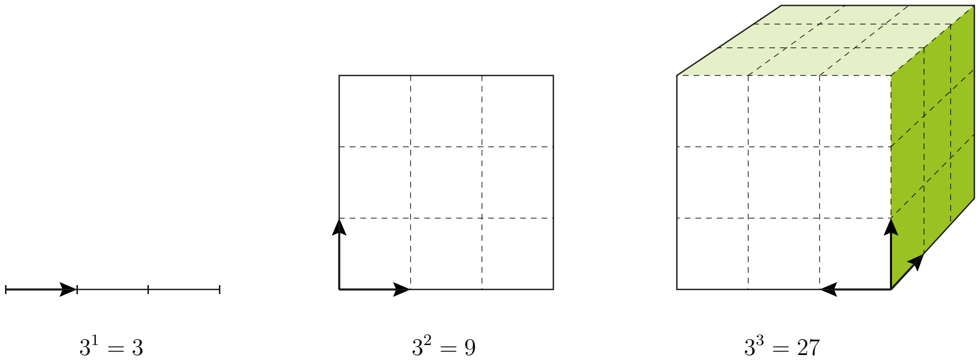
Fractals are extremely complex, sometimes infinitely complex - meaning you can zoom in and find the same shapes forever. Fractals are found all over nature, spanning a huge range of scales. We find the same patterns again and again, from the tiny branching of our blood vessels and neurons to the branching of trees, lightning bolts, and river networks. Regardless of scale, these patterns are all formed by repeating a simple branching process.

[87-90] NATURAL EVIDENCES FOR THE 'FRACTALS' PHENOMENON

According to fractal.org, the two most important properties of fractals are self-similarity and non-integer dimension. The self-similarity property of fractals basically means that it reveals the same configuration at an increasingly smaller scale. The fern leaf for example. Every little individual leaf - part of the bigger one - has the same shape as the whole fern leaf. You can say that the fern leaf is self-similar.

The non-integer dimension is a notable feature. As shown already in the geometry classification, *"classical geometry deals with objects of integer dimensions: zero dimensional points, one dimensional lines and curves, two dimensional plane figures such as squares and circles, and three dimensional solids such as cubes and spheres. However, many natural phenomena are better described using a dimension between two whole numbers. So while a straight line has a dimension of one, a fractal curve will have a dimension between one and two, depending on how much space it takes up as it twists and curves. The more the flat fractal fills a plane, the closer it approaches two dimensions."* www.fractal.org.

To illustrate this dimension, better known as the 'fractal dimension', first the calculation of an object is illustrated below:



[91] DIMENSIONS OF AN OBJECT

1. The line is broken into 3 smaller lines. Each of these lines is similar to the original line, but they are all 1/3 the scale. This is the idea of self similarity.
2. The square below is also broken into smaller pieces. Each of which is 1/3th the size of the original. In this case it takes 9 of the smaller pieces to create the original.
3. As with the others the cube is also broken down into smaller cubes of 1/3 the size of the original. It takes 27 of these smaller cubes to create the original cube.

There is a pattern – which is already shown in the chapter about 'Natural Laws' on the topic of scaling:

$$N = S^D$$

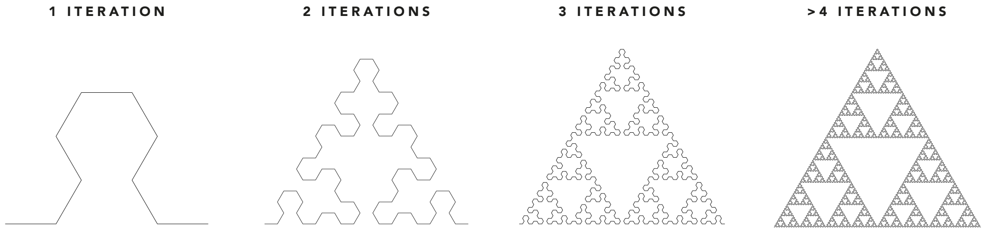
$$D = \frac{\log N}{\log S}$$

Where N is the number of small pieces that go into the larger one, S is the scale to which the smaller pieces compare to the larger one and D is the dimension. D is called the Hausdorff-Besicovitch dimension. (http://davis.wpi.edu/~matt/courses/fractals)

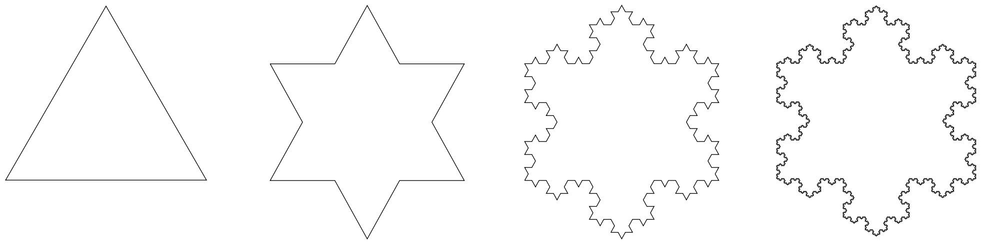
The dimensions of geometrical fractals are displayed in this graph and the following pages.

Fractals	Hausdorff-Besicovitch dimension
Sierprinski Arrowhead Curve	$\log_2(3) \approx 1.5849$
Koch Snowflake	$\log_3(4) \approx 1.2619$
Sierprinski Carpet	$\log_3(8) \approx 1.8928$
Apollonian Gasket	≈ 1.2903
Lévy C. Curve Boundary	≈ 1.9340
Gosper Island Boundary	$2\log_7(3) \approx 1.1292$
Dragon Curve	$\log_{\sqrt{2}}(2) = 2$
Vicsek Fractal	$\log_3(5) \approx 1.4649$

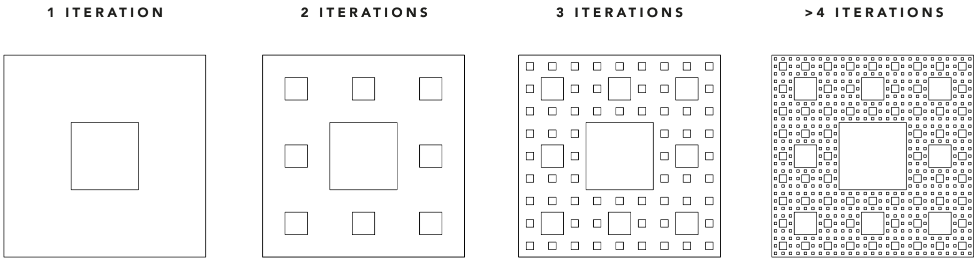
[92] HAUSDORFF-BESICOVITCH DIMENSION



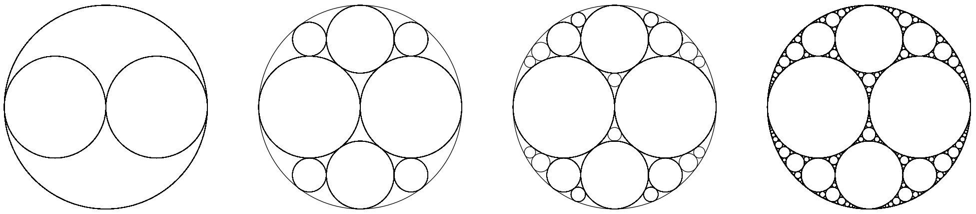
[93] SIERPRINSKI ARROWHEAD CURVE



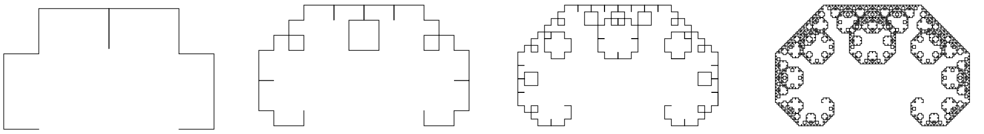
[94] KOCH SNOWFLAKE



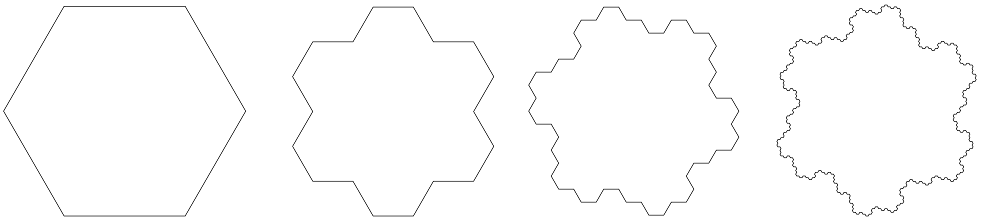
[95] SIERPRINSKI CARPET



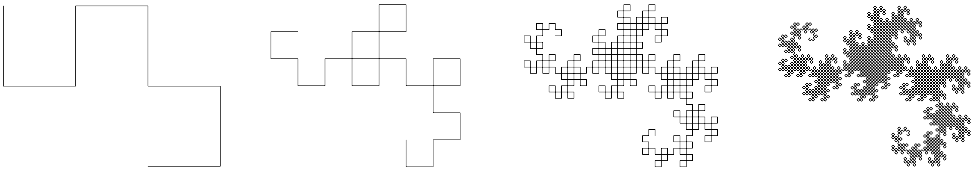
[96] APOLLONIAN GASKET



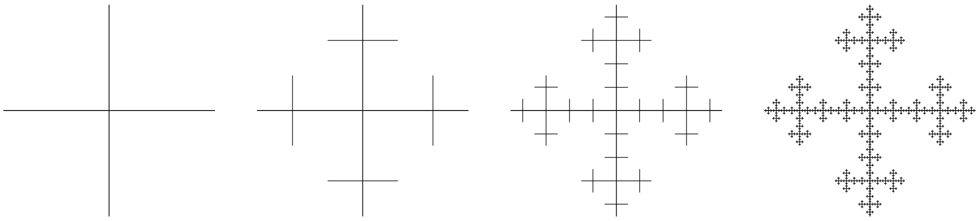
[97] LÉVY C. CURVE BOUNDARY



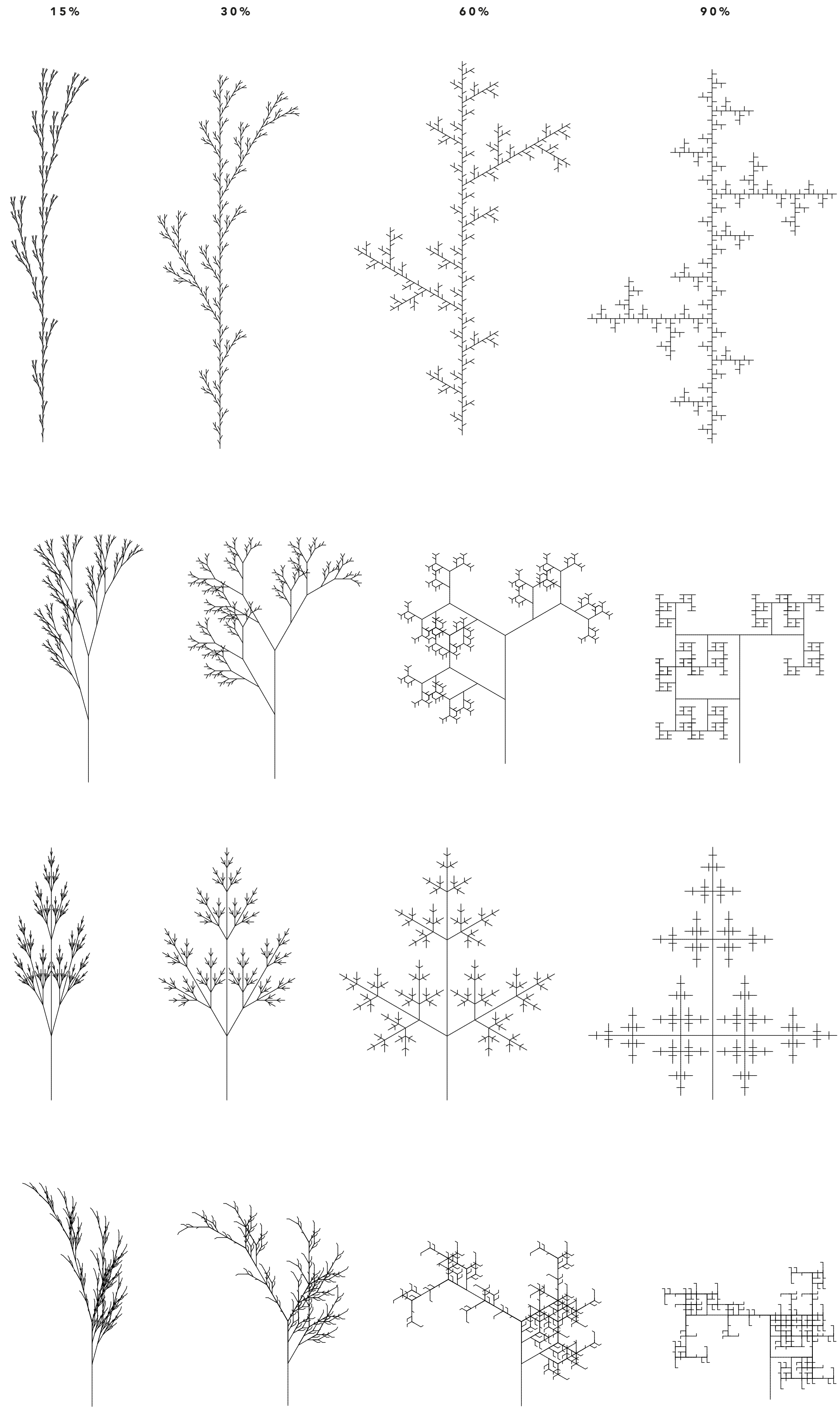
[98] GOSPER ISLAND BOUNDARY



[99] DRAGON CURVE



[100] VICSEK FRACTAL



[101] NATURAL FRACTALS

3.12 GOLDEN RATIO

Accordingly to WolframMathworld, the golden ratio, also known as the divine proportion, golden mean, or golden section, is a number often encountered when taking the ratios of distances in simple geometric figures such as the pentagon, pentagram, decagon and dodecahedron.

The so called phyllotaxis spiral is directly related to the golden ratio. This spiral is another extremely common pattern in nature, and can be seen as another type of fractal, since it is based on a repetition of expansion and rotation. This type of fractal is found over a huge range of scales. Biological spirals are found in the plant and animal kingdoms, and non-living spirals are found in the turbulent swirling of fluids, in storm formations and in the pattern of star formation in galaxies (www.fractal.org)

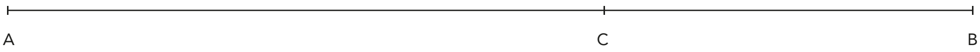
[102-105] NATURAL EVIDENCES FOR THE 'GOLDEN RATIO' PHENOMENON

The first who defined this proportion that was derived from a simple division of a line was Euclid. He called this its "extreme and mean ratio".

His rule states:

> A straight line is said to have been cut in extreme and mean ratio when and only when, as the whole line is to the greater segment, so is the greater to the lesser.

If the ratio of the length of AC to that of CB is the same as the ratio of AB to AC, then the line is cut in extreme and mean ratio, the golden ratio (Livio, 2008).

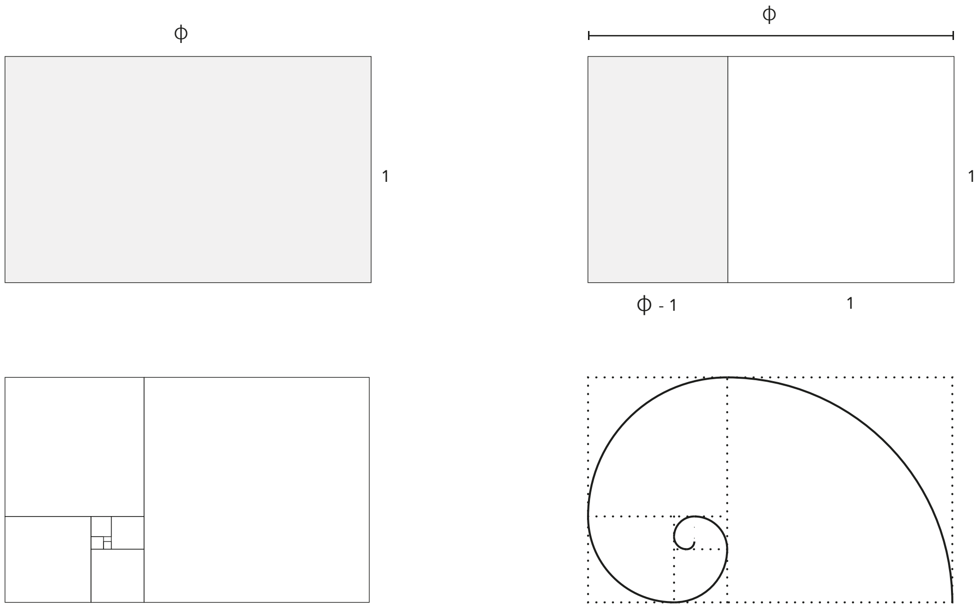


$$\phi = \frac{AC}{CB} = \frac{AB}{AC}$$

This golden division of a line can also be illustrated in a rectangle: WolframMathworld explains this as follows: Given a rectangle having sides in the ratio $1:\phi$ is defined as the unique number ϕ such that partitioning the original rectangle into a square and new rectangle as illustrated above results in a new rectangle which also has sides in the ratio $1:\phi$ (i.e., such that the grey rectangles shown above are similar). Such a rectangle is called a golden rectangle.

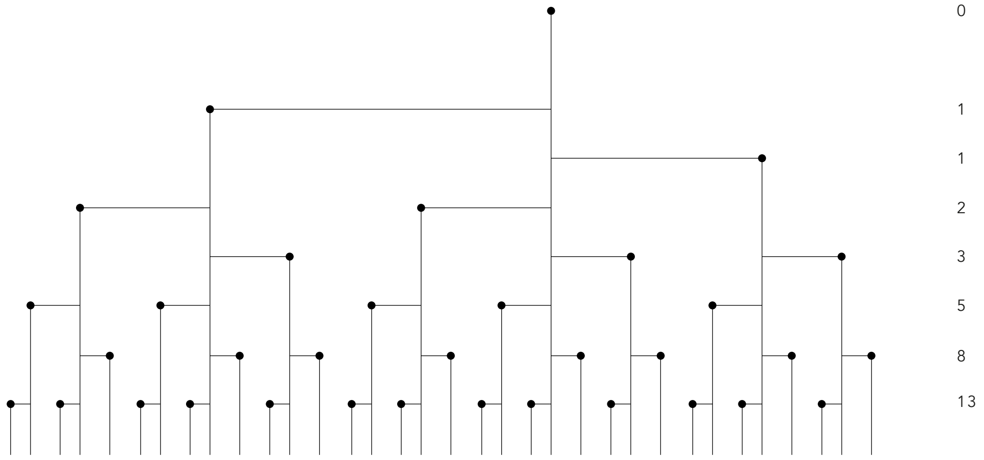
Based on the above definition, it can immediately be seen that:

$$\frac{\phi}{1} = \frac{1}{\phi - 1} \quad \leftrightarrow \quad \phi^2 - \phi - 1 = 0$$



[106] CONTINUED FRACTION - GNOMON

John A. Adam dedicated an entire chapter in his book 'Mathematics in Nature' on the topic of the golden ration and the Fibonacci sequence. In 1202 Fibonacci of Pisa analysed the rabbit population growth and came up with an infinite set of numbers 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144. This set has remarkable and wonderful properties. The tree how Fibonacci based this set of numbers is shown below.



[107] RABBIT POPULATION GROWTH

If one would divide two consecutive terms of this sequence, each new ratio is getting closes and closer to 0.6180339… which is

$$\phi - 1 \quad \leftrightarrow \quad \frac{1}{\phi} \quad \leftrightarrow \quad \phi^{-1}$$

Adam states that with the exception of the first two terms in the above sequence, each term is the sum of the two immediately preceding terms, that is, if x_n represents the nth term in the sequence, then for n > 3.

$$x_n = x_{n-1} + x_{n-2}$$

As will be seen below, it can be shown that:

$$\lim_{n \to \infty} \frac{x_n}{x_{n-1}} = \frac{1+\sqrt{5}}{2} = 1.61803398...$$

Every nth term in the Fibonacci sequence can be found with the following equation:

$$x_n = \frac{\varphi^n - (1-\varphi)^n}{\sqrt{5}}$$

$$x_6 = \frac{(1.618034...)^6 - (-0.618034...)^6}{\sqrt{5}} = 8$$

In geometry, as well as in mathematics, the golden ratio can be derived in multiple ways. As mentioned earlier, the pentagon shows interesting relations to the golden division. In a regular pentagon with unit sides, each diagonal has the length of ϕ. Besides this, the diagonals intersect in such a way that:

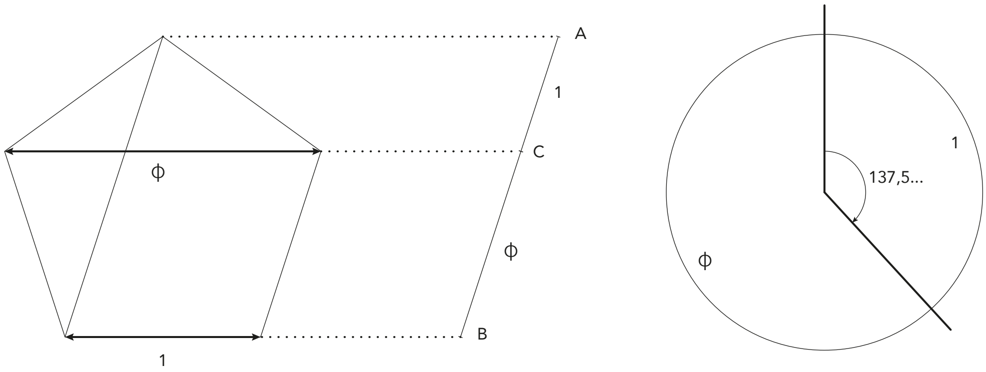
$$BC : CA = \phi$$

A second geometry where the golden ratio can be derived is in a circle. J. Adam states that the golden angle is subtended by an arc that is the equivalent of "1" in the golden section: thus if α is the golden angle in radians then:

$$\frac{2\pi - \alpha}{\alpha} = \frac{2\pi}{2\pi - \alpha}$$

Yielding a quadratic equation in α with the smallest root being $\alpha = (3 - \sqrt{5})\pi$ or $\approx 137.5°$.

$$360° \cdot (1 - 3/5) = 360° \cdot 2/5 = 144°$$
$$360° \cdot (1 - 8/13) = 360° \cdot 5/13 \approx 138.5°$$
$$360° \cdot (1 - 34/55) = 360° \cdot 21/55 \approx 137.5°$$



[108] THE REGULAR PENTAGON AND THE GOLDEN ANGLE

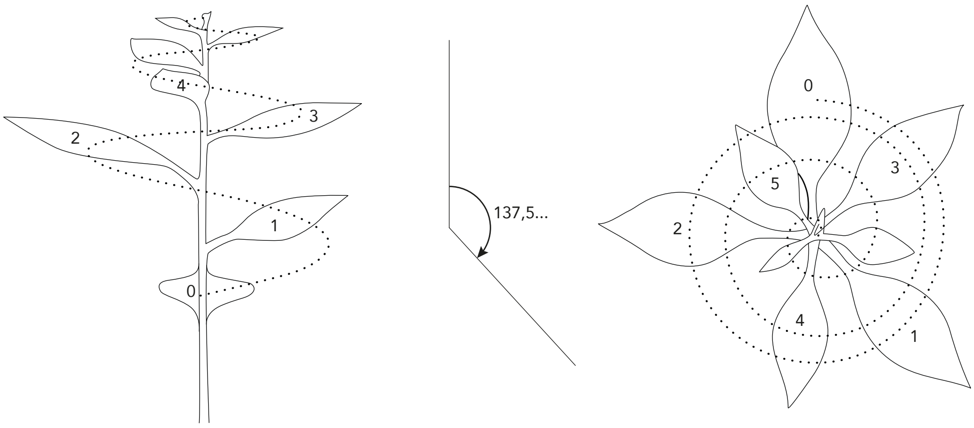
According to J.A. Adam, is the phyllotaxis the distribution or arrangement of leaves on a stem and the mehchanisms that govern it. This type of spiral and the numerical and geometric patterns based on the number of the Fibonacci sequence abound in nature and have been studied for centuries.

Other examples of the Fibonacci Phyllotaxis are the seeds of the sunflower, the pine cone, the petal sequence in a rose or a lotus, the sequence of leaves on a thistle, the fruits partitions of a pineapple, and the succession of twigs branching from the stem of a pear tree. Examine some flower petals as examples of this.

Plant	Number of petals
Lilies	3
Buttercups	5
Delphinium	8
Marigolds	13
Asters	21
Daisies	34, 55, 89

[109] PLANTS AND THEIR NUMBER OF PETALS (ADAM,2006)

The reason for this miraculous fact is well explained by Robert Dixon in his article: 'The mathematical daisy. *"The pattern of leaves growing from a stem reflects an elementary predicament that plants face—that of how to occupy space, collect sunlight and breathe, in the most economic way. To begin with, a plant grows along an axis, extending its occupation of space along one line to gather more and more sunlight. Then periodically it sprouts leaves, which branch out from the stem to occupy the surrounding space. But in which direction do they sprout? At every point on the stem the plant has 360° around the stem to choose from. In response to this choice, plants have evolved several systematic branching patterns, each species following one or other. These patterns represent the relatively few optimum solutions to the geometric problem. One such pattern is the spiral/helical succession of branches at every 137,507...°, or Fibonacci phyllotaxis."* (Dixon, 1981)



[110] LEAVES ARRANGEMENTS ACCORDING TO GOLDEN ANGLE PRINCIPLE

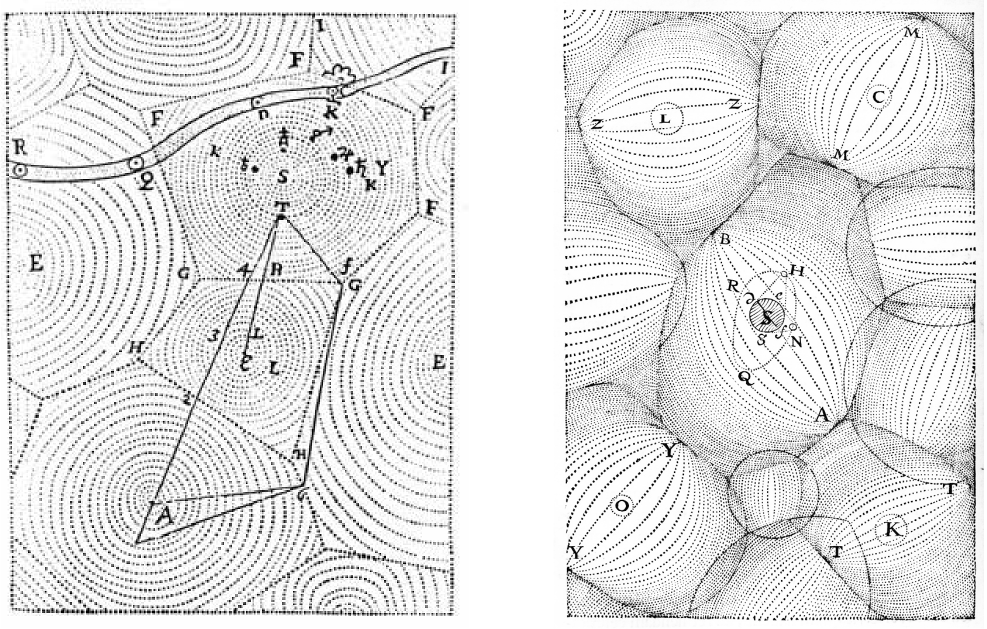
3.13 VORONOI

Professor Roberto Tamassia states in his lecture 'Introduction to Voronoi Diagrams' that Voronoi is a general solution to 2D proximity problems. A sample of the problems that are addressed by this technique include 'closest pair', 'all nearest neighbors', 'Euclidean minimum spanning tree', 'triangulation' and 'nearest neighbor search'. Some of these techniques are already presented in other phenomena.

The phenomenon of the Voronoi tessellation arises naturally when either surfaces crack or cells divide, besides, it can also be found in various skins and patterns of animals – think about the skin of reptiles, turtles, giraffes, or the wings of mosquito's and dragon flies.

[111-114] NATURAL EVIDENCES FOR THE 'VORONOI' PHENOMENON

The origin of Voronoi diagrams dates back to the 17th century. In his book on the principles of philosophy which was published in 1644, René Descartes claims that the solar system consists of vortices. His illustrations show a decomposition of space into convex regions, each consisting of matter revolving round one of the fixed stars.



[115] DESCARTES' DECOMPOSITION OF SPACE INTO VORTICES

Franz Aurenhammer and Rolf Klein state in their book 'Voronoi Diagrams' that the generalized problem and concepts of Voronoi have proven to be useful in various fields of science. Especially in the last thirty years due to the development of the modern fields of science its mathematical description gained interest. Those fields are for example; computation geometry, image recognition, artificial intelligence, molecular biology, radiation physics, terrain modelling, navigation and obstacle avoidance. Different names were given on this concept in various fields, such as medial axis transform in biology and physiology, Wigner-Seitz zones in chemistry and physics, domains of action in crystallography, and Thiessen polygons in meteorology and geography.

F. Aurenhammer and R. Klein go on by noting that the mathematicians Dirichlet and Voronoi in the 19th and 20th century were the first to formally introduce this concept of Voronoi Diagrams. The resulting structure and development mathematically has been called Dirichlet tessellation or Voronoi diagram, which has become its standard name today.

In mathematics, a Voronoi diagram is a partitioning of a plane into regions based on distance to points in a specific subset of the plane. That set of points (called seeds, sites, or generators) is specified beforehand, and for each seed there is a corresponding region consisting of all points closer to that seed than to any other. These regions are called Voronoi cells. The Voronoi diagram of a set of points is dual to its Delaunay triangulation, which will be illustrated later.

In order to further elaborate on the mathematical description of the Voronoi diagram, the general problem that is to be solved needs to be clear. Given a set S of n points in the plane, we wish to associate with each point s a region consisting of all points in the plane closer to s than any other point s’ in S. This can be described formally as:

$$\mathrm{Vor(s)} = \{p : \mathit{distance}(s,p) \leq \mathit{distance}(s',p), \forall s' \in S\}$$

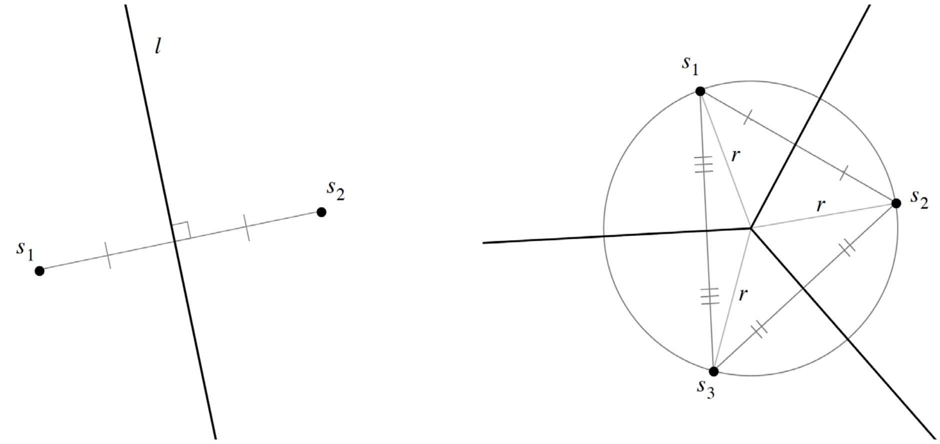
Where $\mathrm{Vor}(s)$ is the Voronoi region for a point s.

In order to understand this generalized problem better, a few theorems with illustration will be presented.

Let us first consider the simplest case for a Voronoi diagram, where S consists of a single point. In this case the Voronoi region for this point is the entire plane. Next, consider a set of two points. The Voronoi diagram for the set $S = \{s_1, s_2\}$ consists of two half-planes divided by the ray l, which is the perpendicular bisector of $\overline{s_1 s_2}$. Note that the two regions are not disjoint, but overlap at the set of points equidistant from both points on the ray l.

Theorem 1:

All points on the half plane containing s1 and delimited by the perpendicular bisector l of s1s2 are closer to s1 than s2.

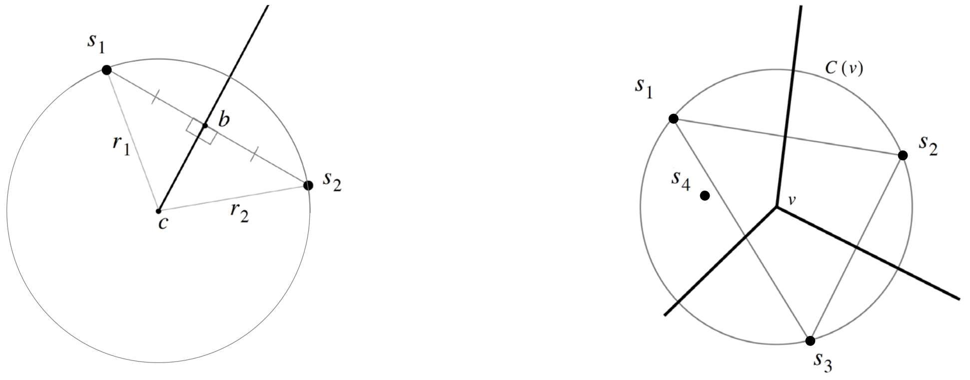


[116] A VORONOI DIAGRAM OF A SET OF TWO POINTS $S = \{s_1, s_2\}$ AND A SET OF THREE POINTS $S = \{s_1, s_2, s_3\}$

The right figure above shows a Voronoi diagram for three points, and the geometry used in its construction. Professor Tamassia explains how to construct this drawing: *“We start by joining each pair of vertices by a line. We then draw the perpendicular bisectors to each of these lines. These three bisectors must* *intersect, since any three points in the plane define a circle. We then remove the portions of each line beyond the intersection and the diagram is complete. The point where the three rays intersect belongs to the Voronoi regions for all three points. This point is also the center of the circle."*

Theorem 2:

The intersection of the 3 perpendicular bisectors of s1; s2 and s3 is the center of the circle containing s1; s2 and s3.



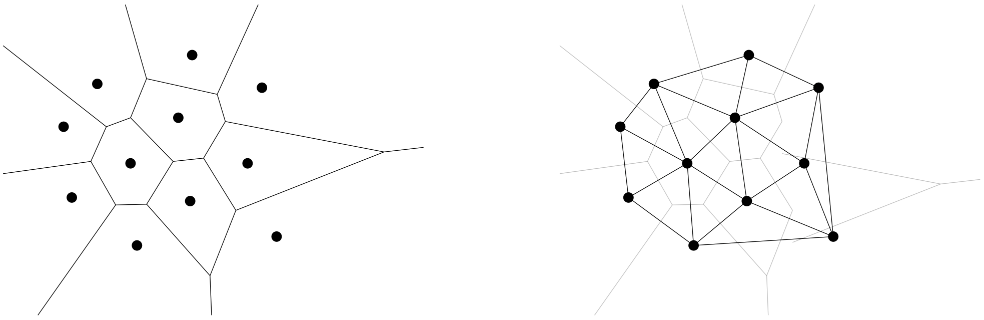
[117] THEOREM 2

Theorem 3:

The circle containing Voronoi vertex v and passing through the three points s1; s2 and s3 is empty.

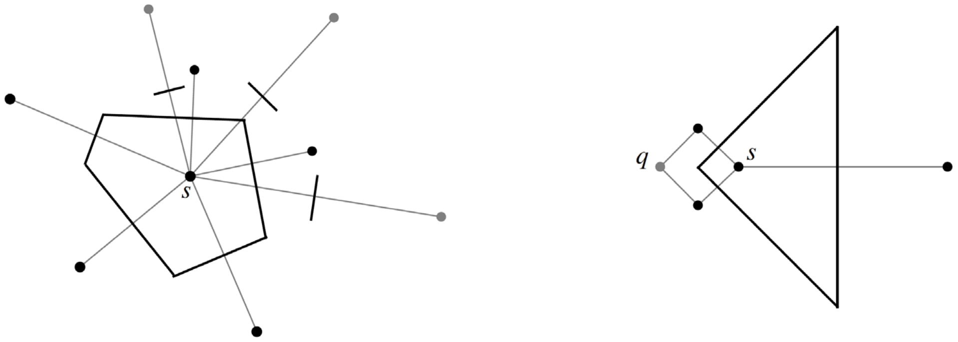
A Voronoi diagram is the union of all the Voronoi regions in the set:

$$\mathrm{Vor}(S) = \bigcup_{s \in S} \mathrm{Vor}(s)$$



[118] VORONOI DIAGRAM AND ITS DUAL DELAUNAY TRIANGULATION

"There are several intuitive methods to construct a Voronoi region for a given point s in set S. First, we can take all of the perpendicular bisectors of the segments connecting s to the remaining members of S. We can then use these rays to delimit half-planes. The intersection of all half planes containing s is the Voronoi region for s. Or, we can start with the segments connecting s to all remaining members of S. We then gradually extend lines outward along the perpendicular bisector of these segments until they intersect. Note that the points which do not contribute to the region are not necessarily the furthest away, as in the right image below." (Aurenhammer, 2000)



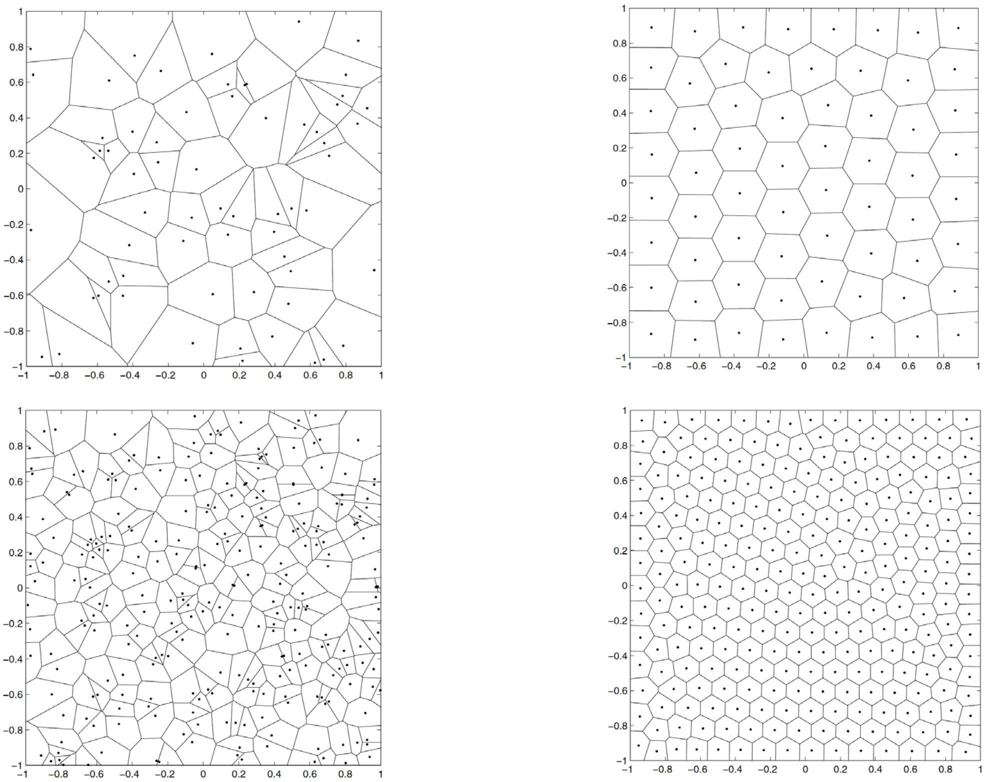
[119] VORONOI DIAGRAM CONSTRUCTION

The dual of the Voronoi diagram is the Delaunay Triangulation. Georgy Voronoi was actually the first who considered this duality, where two seeds or points of the diagram are connected whose regions have a boundary in common. Delaunay obtained the same by defining that two point sites are connected if and only if they lie on a circle whose interior contains no point of S.

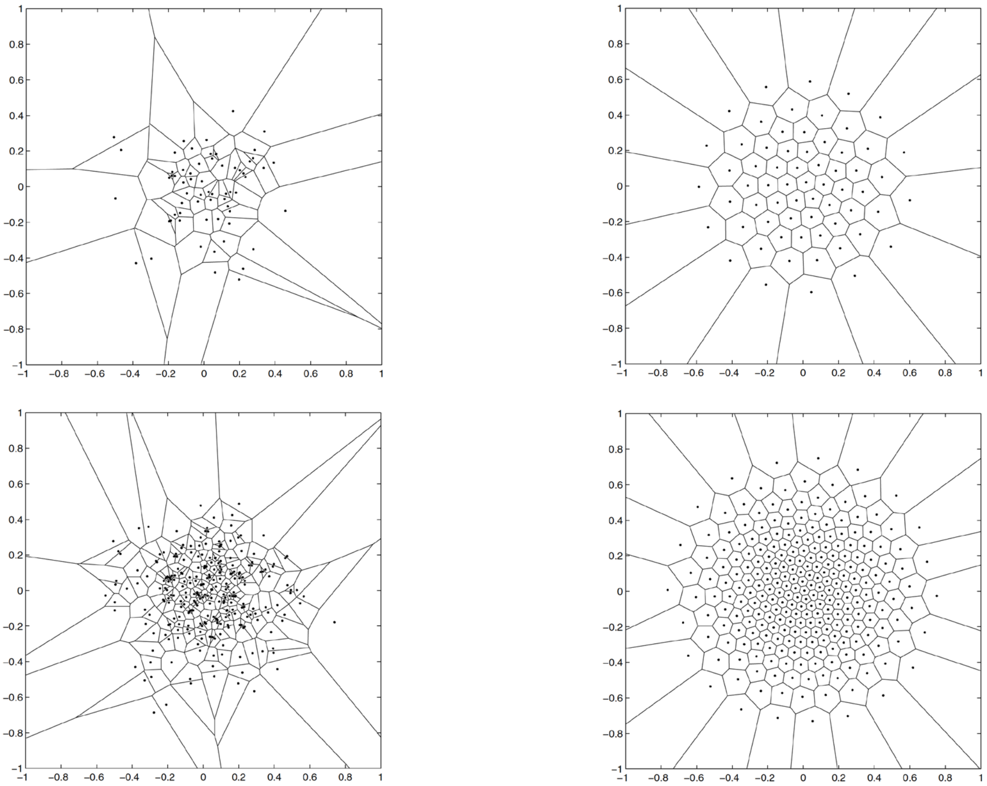
Theorem 4:

The graph constructed by connecting all vertices in a set S across the edges of their Voronoi polygons is a triangulation of S.

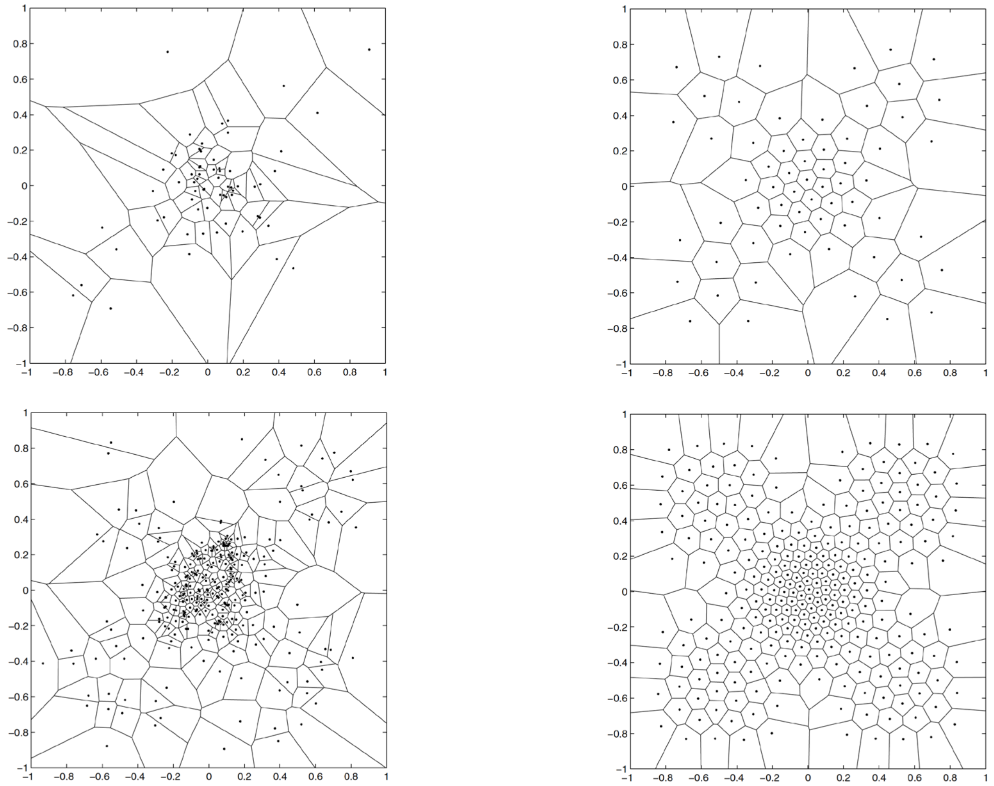
A Centroidal Voronoi Tessellation (CVT) is a Voronoi tessellation whose generating points are the centroids (centers of mass) of the corresponding Voronoi regions. On the following two pages, the CVT's of a square are presented. A comparison is made between a Monte Carlo-based Voronoi tessellation as well as the effects of different density functions. (Du, 1999)



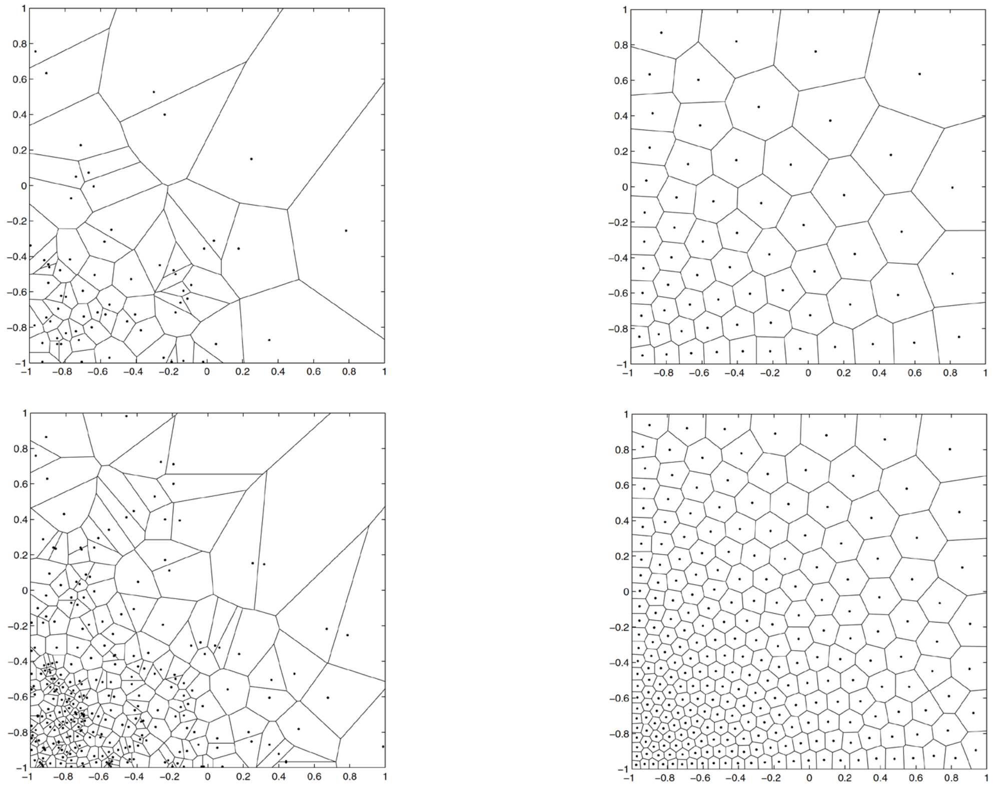
[120] VORONOI DIAGRAM FOR A CONSTANT DENSITY FUNCTION IN $[-1,1]^2$



[121] VORONOI DIAGRAM FOR THE DENSITY FUNCTION $e^{-10x^2-10y^2}$ IN $[-1,1]^2$



[122] VORONOI DIAGRAM FOR THE DENSITY FUNCTION $e^{-20x^2-20y^2} + 0.05\sin^2(\pi x)\sin^2(\pi y)$ IN $[-1,1]^2$



[123] VORONOI DIAGRAM FOR THE DENSITY FUNCTION e^{-2x-2y} IN $[-1,1]^2$

> “Everything man is doing in architecture is to try to go against nature. Of course we have to understand nature to know how far we have to go against nature. The secret, I think, of the future is not doing too much. All architects have the tendency to do too much.”
>
> Frei Otto

transformation

4. PIONEERS & PROTOTYPES

4.1 TRANSFORMATION APPROACHES

There are many ideas on how to transfer the design process from biology to architecture and more specifically structural design. In order to come up with a more combined methodology, several notions and concepts are presented:

First of all, Göran Pohl and Werner Nachtigall give the classification of Building Biomimetics in their book on ‘Biomimetics for Architecture & Design’. According to the VDI guideline 6220, a product is considered biomimetic when it fulfills these three criteria:

1. Biological precedent
2. Abstraction from biological precedent
3. Transfer and application

The VDI definition implies that all three criteria must be fulfilled. If it is only consistent with one or two of the criteria, then it cannot be described as biomimetic. In his book ‘Bionik als Wissenschaft’, which applies the theory of cognition to biomimetics, Werner Nachtigall (2010) signified this process with the subtitle:

Knowledge → Abstraction → Application

They go on stating that the following classification for buildings is to be understood on the basis of an analysis of the development lines and the degree of biological inspiration in the architecture. *“It facilitates the understanding of building biomimetics. This classification of building biomimetics represents the influence of architectural understanding.”* To better understand the biomimetic level on the built environment, the following categories are conceived:

1. Similar to nature: buildings as sculptures similar in appearance to nature
2. Nature analog: building methods analogous to nature
3. Integrative: biomimetic principles as components of architecture

In a similar notion, Grigorian (2014) also indicated the necessity of taking some basic rules into consideration to catch the design information from a living organism (such as a tree) and to transform it. These rules are:

1. Structural applicability

   Geometric and framing similarities, use and behavior of materials
2. Functional similarity

Being subjected to similar loading and environmental conditions

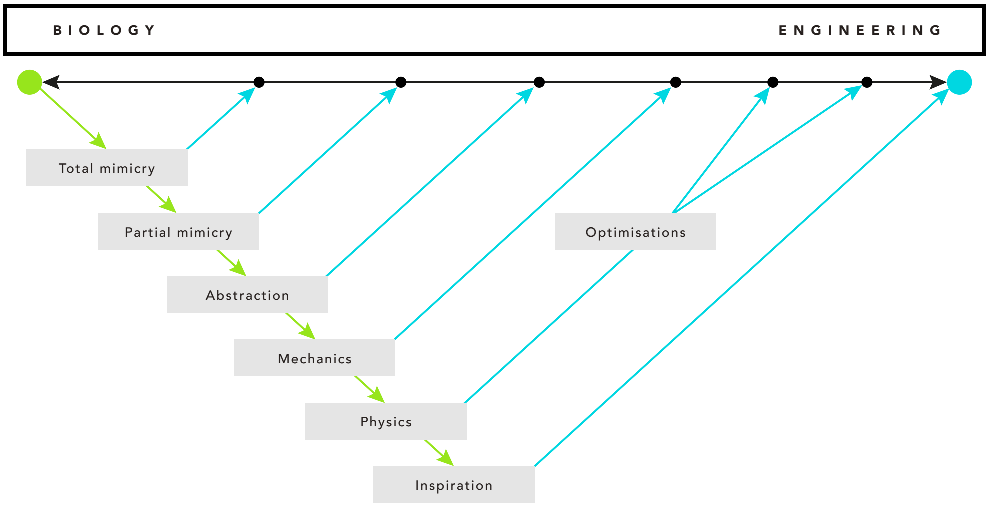
3. Response homology

Behaving the same way against comparable external effects

4. Economic viability

Being as cost effective and as energy efficient as possible

In this stage, nature can be imitated directly or indirectly as a metaphor to solve design problems and to develop environment-friendly functions, systems and solutions. According to the Gruber (2010) abstraction is the key to transferring ideas from one discipline to another. Genrich Altschuller supports this view by saying, and I quote: *"The more abstract a concept is, the more adaptable it is within another discipline."*



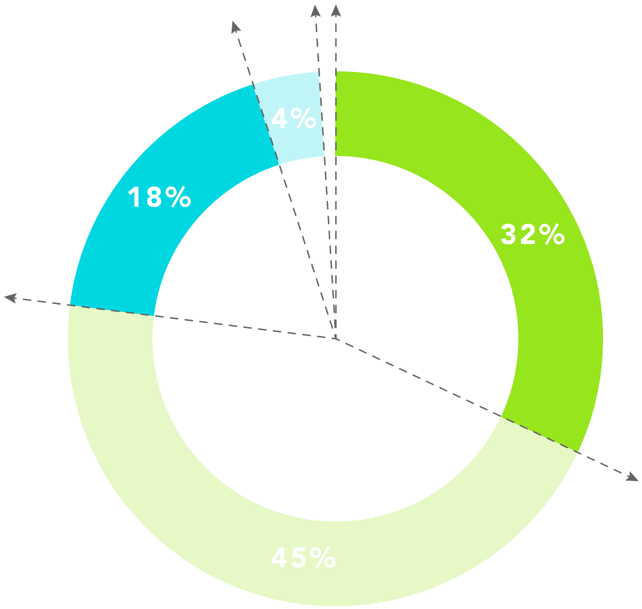
[124] BIOMIMETIC TRANSFORMATION MAP

Julian Vincent presents in his book 'Deployable structures' in chapter 3 on 'Stealing ideas from nature' a biomimetic map that is shown above. This transformation triangle underlines Genrich Altshullers statement. The general concept is that the further down one can move from the natural origin (top left) the more general and therefore more powerful the concept will be.

Like already discussed in the chapter on 'Natural Laws', J. Vincent states that the level of the laws of physics forms a common ground for the transfer of information between the disciplines, because of the fact that the entire world, whether it is the living, or the non-living world, is subject to these laws. Then he gives an analytical view on the topic of 'creativity' in general. *"However, even the most creative person can imagine only what their brain allows, and that brain can work only from the information available - its database. The trick then is the identification of the problem at some basic functional level and the* *marriage of that function with another from a different area."*

In 1988, Genrich Altshuller invented a successful theory called the 'Theory of Inventive Problem Solving (TRIZ). By analysing literally thousands of patents we know nowadays in engineering, he tried to identify the most effective solutions. He came up with a number of different levels of innovation which are listed below, together with the frequency with which they appear in the database of patent literature.

1. A single improvement to a technical system requiring knowledge available within that system (0.32).
2. An improvement that includes the resolution of a technical contradiction requiring knowledge from a related area (0.45).
3. An improvement that includes the resolution of a contradiction at the level of physics requiring knowledge from other industries (0.18).
4. A new technology which involves a "breakthrough" solution requiring knowledge from different fields of science (0.04).
5. Discovery of a new phenomenon (< 0.01).



[125] PERCENTAGES OF BIOMIMETIC INNOVATION-LEVELS

The TRIZ literature by G. Altshuller suggests that the transfer of ideas from biology, and therefore also the transfer of the design process or the transfer of structural applications, can be made at a variety of levels. These levels depend on how far the biological model lies from the technical problem for which a useful paradigm needs to be generated.

4.2 PIONEERS

4.2.1 Antoni Gaudí (1852 - 1926)

> "Those who look for the laws of Nature as a support for their new works collaborate with the Creator."

Antoni Plàcid Guillem Gaudí i Cornet was born in Reus, Catalonia, south of Barcelona on the Mediterranean coast, in June 1852. From the beginning, Gaudí exhibited a great appreciation for nature and especially the environment of his native region of Catalonia. His work as an architect begin in 1874. The artstory talks about a few main key ideas that are visible through all of his work:

- Gaudí's was highly innovative in terms of his explorations of structure, searching through a variety of regional styles before seizing on the parabolic, hyperbolic, and catenary masonry forms and inclined columns that he developed through weighted models in his workshop. These are often integrated with natural and highly symbolic religious imagery that encrust the structure with vibrant, colorful surfaces.

- Gaudí's work is highly personal, in part due to his devout Catholicism, a faith that became increasingly fervent as his career progressed. In part because of this, his work contains many references to religious themes, and he increasingly led an ascetic existence towards the end of his life, even giving up all other commissions to focus on his designs for the church known as the Sagrada Família.

- Gaudí often collaborated with several other Catalan designers, industrialists, artists, and craftsmen on his projects, most prominently Josep María Jujol, who was often responsible for the broken tilework (trencadís) that is common to much of Gaudí's buildings. This helps to explain why Gaudí's structures often feature such a wide variety of materials, used in inventive and clever ways.

http://www.theartstory.org/artist-gaudi-antoni.htm

> "Color in certain places has the great value of making the outlines and structural planes seem more energetic."

4.2.2 Pier Luigi Nervi (1891 - 1979)

> "Concrete is a living creature which can adapt itself to any form, any need, any stress."

This Italian architect, engineer and builder was one of the most inventive exploiters of reinforced-concrete constructions of the 20th century. Nervi graduated from the School of Civil Engineering in Bologna in 1913. The first work that established Nervi's reputation was the Municipal Stadium in Florence. It was a cantilevering grandstand roof. In 1932 he formed his second form, together with Bartoli. They developed a series of airplane hangars. Hangars of which the New York Times described them as "graceful, flying forms of concrete." His most famous work is the Palazzetto dello Sport in Rome, which was design for the Roman Olympics of 1960. This arena has prefabricated diamond-shaped sections descending from an overhead compression ring to exposed Y-shaped piers. The pattern of the structure is a phyllotaxis, a pattern that is described earlier in the subchapter on the 'Golden Ratio'.

Nervi also gained international reputation by several works. In 1964 he received the Gold Medal of the American Institute of Architects, which is the highest honour in American Architecture. In the various articles and books Nervi has written during his life, Nervi reminded the reader that 90 percent of his contracts were awarded in competitions where the governing factors were economy and speed of construction. http://biography.yourdictionary.com/pier-luigi-nervi

> "I have never found this relentless search for economy an obstacle to achieving the expressiveness of form".

4.2.3 Richard Buckminster Fuller (1895 – 1983)

> "Nature is trying very hard to make us succeed, but nature does not depend on us. We are not the only experiment."

Fuller is best known for his work on the Dymaxion House, Dymaxion Bathroom and the Dymaxion Car. Besides, he is the inventor of the geodesic dome – as a means of attaining maximum space related to environment. In his book 'No More Secondhand Gods' he explains: *"My philosophy requires of me that I convert not only my own experiences but whatever I can learn of other men's experiences into statements of evolutionary trending and concomitantly defined problem challenges and responses."* His view on solving a problem is as follows: *"When I am working on a problem, I never think about beauty but when I have finished, if the solution is not beautiful, I know it is wrong."*

Fuller's ideas on the geodesic domes were based on the marine animal called the radiolarian. He started working with the platonic solid the tetrahedron, because of its ratio between material, weight and inner space. For him this component had maximum efficiency to further work with. Later, the geodesic dome seemed to be a popular shape in architecture. The Ford Company built one in Dearborn, Michigan in 1953, the Union Tank Car Company built one in 1958-1959.

Fuller was a pioneer in a sense that his work functioned primarily as a catalyst. He not only gave new inventions, but also upon the new generation. Fuller was the forerunner of concepts of the efficient utilization of materials and of mass production. At the end of his life, Fuller held more than 2000 patents and has written 25 books during his life. http://biography.yourdictionary.com/buckminster-fuller

> "Everything you've learned in school as "obvious" becomes less and less obvious as you begin to study the universe. For example, there are no solids in the universe. There's not even a suggestion of a solid. There are no absolute continuums. There are no surfaces. There are no straight lines."

4.2.4 Félix Candela (1910 – 1997)

> "Nature's most usual way of performing this function is by means of either rigid shells or elastic membranes. Since this second form can hardly be considered as architectonic, "shell" remains a synonym of space enclosure."

Félix Candela was born in Madrid and studied at the Escuela Superior de Arquitectura. He graduated in 1935 and his comment on how his career began is as follows: *"I came to study architecture by coincidence, without the slightest belief in my abilities as an artist or a designer. It might have been the enormous uncertainty on my part that led me toward technical matters and materials […] mathematics and structures […]."*

Accordingly to Enrique X. De Anda Alanís, the writer of the book 'Candela' by Taschen, Candela's approach was clear. His passion for not only mathematics but also for architecture gave him the ability to be a remarkable inventor of a formal system that allowed him to construct concrete membranes based on geometric shapes that were new in such dimensions. In all his work, Candela had a strong inclination towards mathematics; he used this field of science constantly to grasp constructional aspects of doubly-curved surface in space, to complete analysis, and to transfer the results to the concrete construction method.

In 1952, Candela designed the Cosmic Rays Pavilion in Mexico. This was his first design of a roof-shell design that was created with a hyperbolic paraboloid, which is also known as the hypar. This first design marks the start of the most brilliant period of Candela's creative output. In the next thirty years since 1952, he was responsible for nearly 900 constructions. The work that he completed in Mexico embodied two principles:

1. Creating architectural design with spectacular forms and dimensions derived from nature's design.
2. Employing a math-based work method.

Despite the fact that Candela pushed the boundaries of the use of thin-shell constructions, he did not invent it. *"He was the creator of membrane stress equations that made it possible to apply statistical processes to a structure's conception and conduct stress-confirming experiments on its form and thickness. With these equations, Candela could grasp and analyse the distribution of stress on shell structures and attain, as he put it, the "lightest possible result" without wasting materials."* (Alanís, 2008)

> "Every work of art is an interpretation of the world, of what you are contemplating,
> a determination of the perception that creates and tries a different world.
> At the end of the day, a work of art is but a gift to the art."

4.2.5 Frei Otto (1925 – 2015)

> "Everything man is doing in architecture is to try to go against nature.
> Of course we have to understand nature to know how far we have to go against nature.
> The secret, I think, of the future is not doing too much.
> All architects have the tendency to do too much."

Ludwig Glaeser wrote a book about Frei Otto, which was published by the Museum of Modern Art in New York. He states that Frei Otto was working with the basic principles of structural design. His entire work was basically about the one question that is age-old of all construction; how to achieve more with less material and effort. This is in line with fundamental laws with which nature designs: the fact that nature always minimises action and has maximum diversity with minimum inventory. Frei Otto was from his early work concerned with envisioning structures with lightness as well as extreme strength. To achieve this goal he made use of, for that time, new materials as thin cables of steel and thin membranes of synthetic fabric. He introduces designing with one of the principal forces – tension. This was still insignificant in conventional buildings, since most of the materials with which we build involves only compression forces, as well as buckling- and bending moments. Ludwig Glaeser clearly points out that in order to achieve rigidity with a structure in tension, membranes must have specific shapes, which in moest cases are based on anticlastic or saddle-like curvatures. As elaborated on earlier in the chapter on 'physical phenomena and mathematical descriptions' these curvatures can, if they are correctly determined, generate the smallest possible surfaces within the given curvilinear boundaries. This remarkable feature forms the basis for Frei Otto's theory of minimal surfaces.

For him the architect is more a manager of all the energy involved than a designer. His work had basically two objectives:

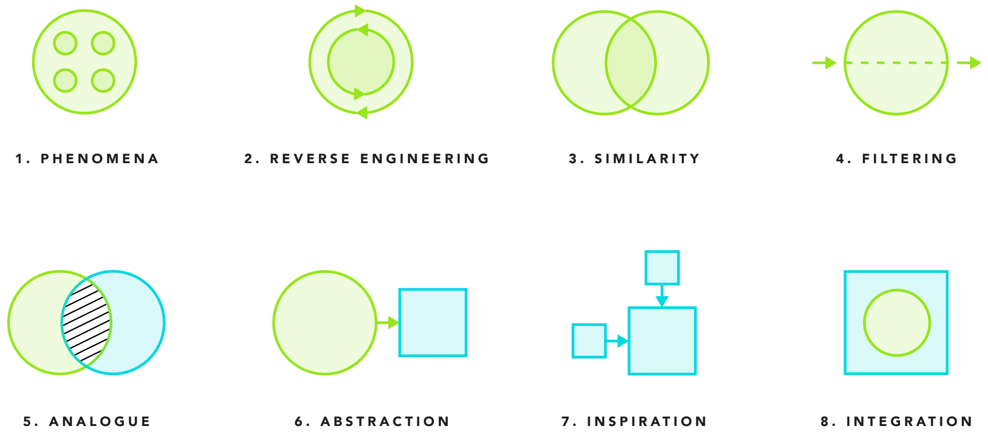
1. Energy and economy: To achieve through maximum efficiency of structure and materials, optimum utilization of the available construction energy.

2. Livability and adaptability: The reduction in construction elements yields increased flexibility and allows the adjustment of interior spaces according to the changing needs of the occupants.

> "…In spite of this, we still build buildings counter to nature from the past epochs. Our times demand lighter, more efficient, mobile, adaptable, or, in brief, natural houses. This consequently leads to the further development of the lightweight structure, of the building of cells, shells, sails, and airborne membranes."

4.3 TRANSFORMATION METHODOLOGY

As described earlier, there are many concepts, directions and ways to translate biology to architecture. It is all depending on what the final design is asking. In my case, the goal is to find a new structural principle. This needs to be integrative with the environmental exposures. The way to get from biological observations, to physical phenomena, describing these mathematically in the hope to come up with an integrative innovative structural principle for architecture is done through different levels of mimicry.



[126] TRANSFORMATION METHODOLOGY LEVELS

The first level in order to find the transformation methodology is to observe biological processes and to come up with categorised physical phenomena, the second level, the level that can be seen as total mimicry, is reversed engineering. This means that the raw materials are used as building components instead of processing the materials into the desired dimensions. The general concept of architectural engineering is that the building is designed in a way that the materials with their exact measurements can be derived, followed by the construction. In reversed engineering the raw materials are used to come up with the design. 3D scanning and other computational methods are therefore important in this type of design process. Raja & Fernandes (2007) state that reverse engineering is also defined as the process of obtaining a geometric CAD-model from 3D-point acquired by scanning and digitising existing parts and products. The following levels of the transformation methodology are based on partial mimicry. Every level has its unique grade of abstraction. Beginning with similarity. This is a classification of several biological categories, which can be mimicked for the transformation to architecture. After the selection of one of two focus groups, the filtering will take place. This will be done on scaling – the ability to overcome the problem of scale mentioned earlier, the form stability, and the constructability when it comes to the ability to build with components. The next level of transformation is to analyse the biological categories and to find analogues. Examples of analogues can be self-optimisation, self-organisation, and self-forming. After the selection of the type of analogue, the level of abstraction and mutation is next. This is the part where the fields of science come into play; this level encloses the structural analysis, the math-based design and material selection. A last potential level can be to learn from pioneers with similar approaches and the analyse precedents. This can be useful but is not a necessity for the transformation. In the end, after having walked through all the separate levels of the methodology, the derived information will form the basis for the design direction.

An important note regarding this methodology is that it is not an absolute truth, or that these steps are the only way for coming up with a new structural principle. In fact the hierarchy between the levels is not very accurate and it should be mentioned that therefore it is completely possible to skip or even to loop several steps. In a design project like this, this is what will be the case. The way how this methodology is used for creating a design direction will be discussed in the next chapter. On the next page you can find a full overview of this transformation methodology.

1

Physical phenomena

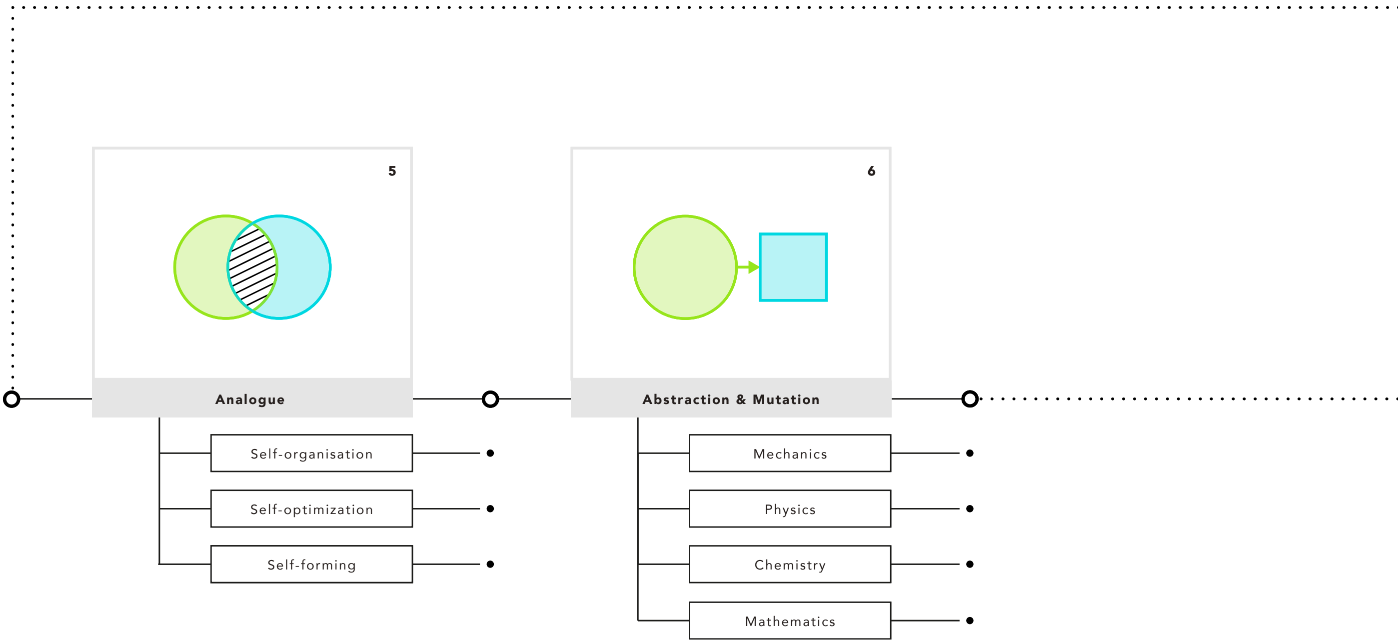
- Shortest path
- Minimal surface
- Closest packing
- Surface tension
- Light scattering
- Wave motion
- Centre of gravity
- Equilibrium
- Catenary
- Fractals
- Golden ratio
- Voronoi

2

Reverse Engineering

- Element topology
- Processing techniques
- Classification
- Translation to virtual
- Comparing
- Design

BIOLOGY - DESIGN BY NATURE



5

Analogue

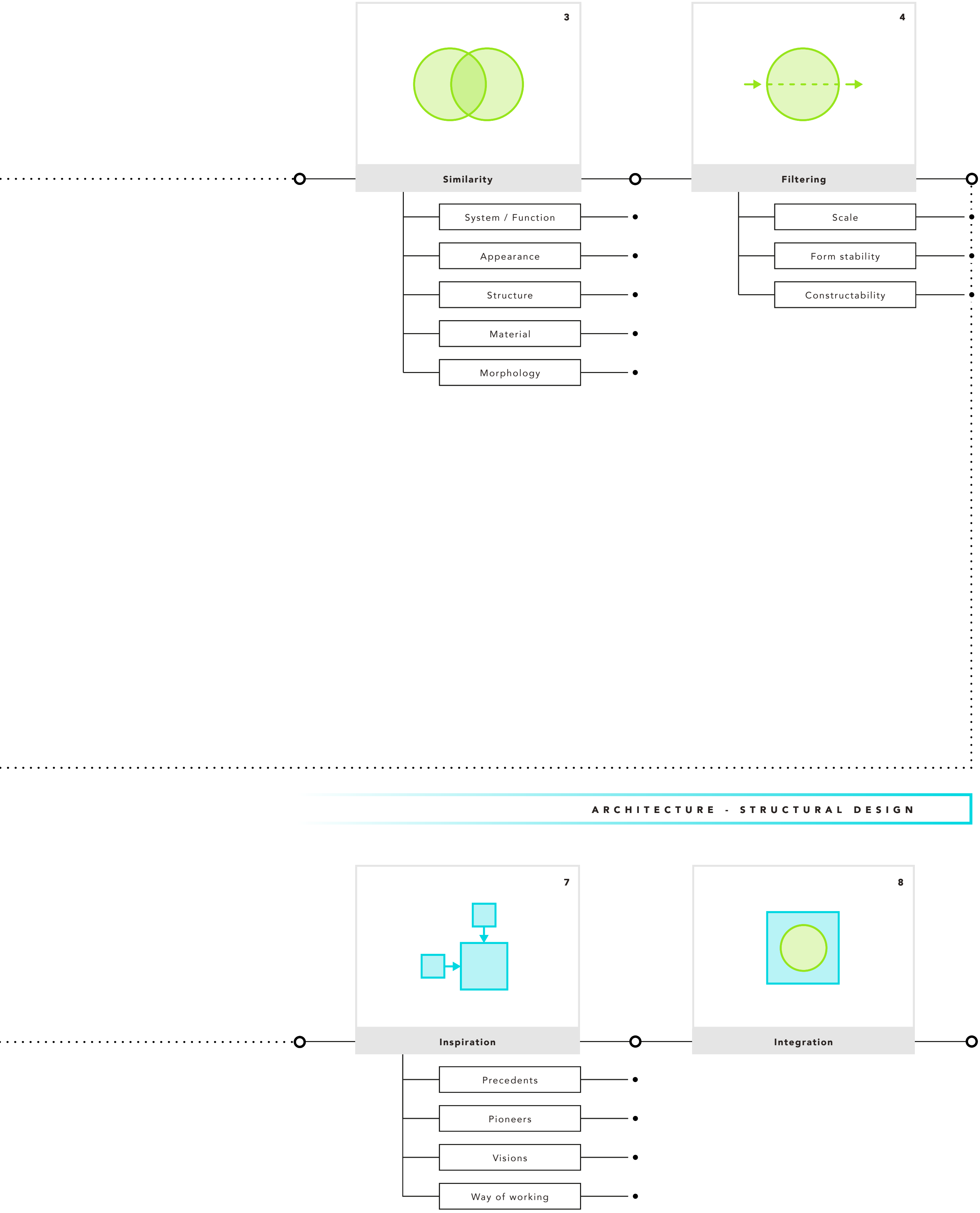
- Self-organisation
- Self-optimization
- Self-forming

6

Abstraction & Mutation

- Mechanics
- Physics
- Chemistry
- Mathematics

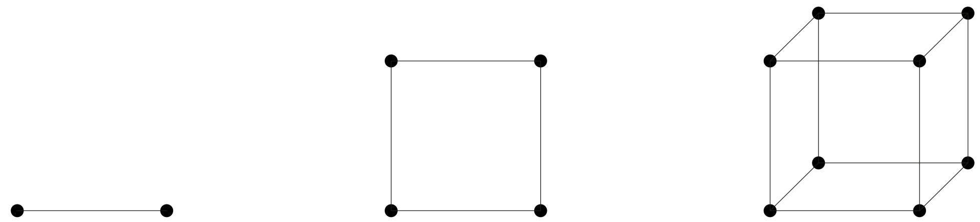
[127] TRANSFORMATION METHODOLOGY - TRANSLATING NATURAL DESIGN TO STRUCTURAL DESIGN


3
Similarity
System / Function
Appearance
Structure
Material
Morphology
4
Filtering
Scale
Form stability
Constructability
ARCHITECTURE - STRUCTURAL DESIGN
7
Inspiration
Precedents
Pioneers
Visions
Way of working
8
Integration

In his book 'Bio-architecture' Javier Senosiain presents a list of biological observations that can be considered as natural structures:

- Cactus
- Skeletons of four-footed animals
- Animal legs
- Trees
- Human body
- Bones
- Insect
- Mollusk shells
- Eggs
- Radiolarians
- Spider webs

In order to classify all the different natural structures, S. Arslan ad A.G. Sorguc (2004) introduced the relation to its load bearing capacities as in the case architects and engineers do when it comes to man-made structures.



ONE-DIMENSIONAL **TWO-DIMENSIONAL** **THREE-DIMENSIONAL**

[128] CLASSIFICATIONS OF NATURAL STRUCTURES

1. One-dimensional

These structures are usually lightweight elements that can only be loaded in one direction. Obviously, the various types of loading are tension, compression and bending. Examples of these types of structures are tension-stressed fibers, hairs, sinews, muscles, intestines and compression- and bending-stressed stalks, trunks, branches, bird feathers and bones.

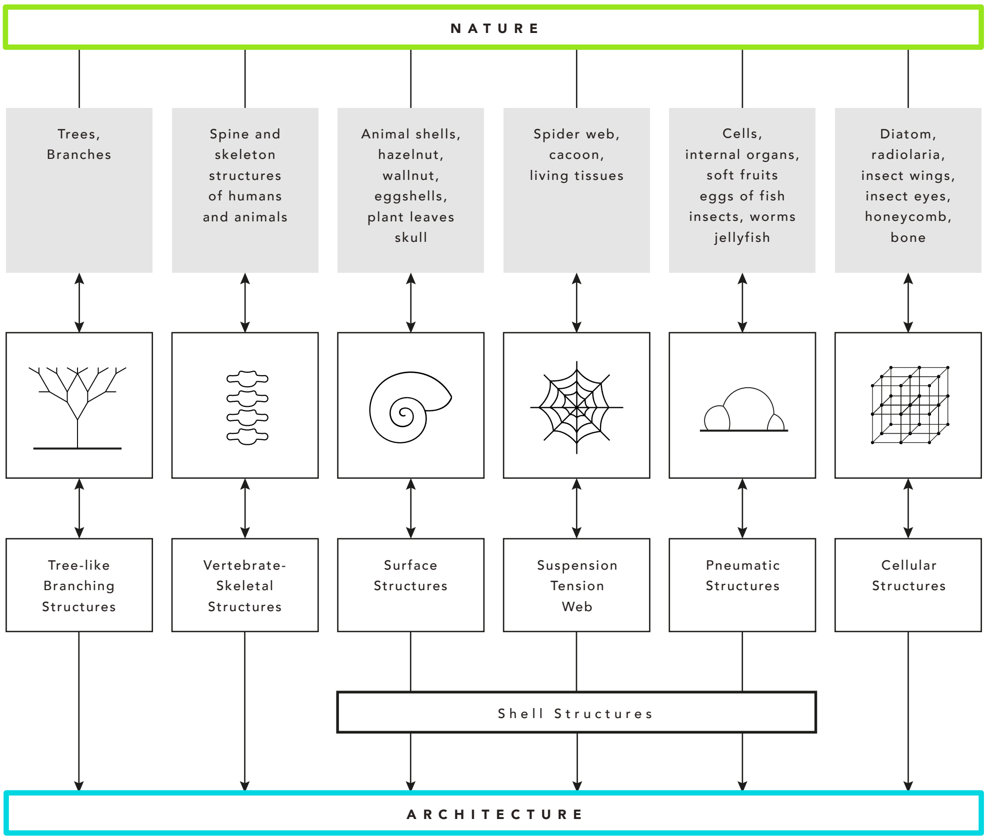
2. Two-dimensional

Two-dimensional structures are also known as membranes. Membranes of cells, skins, intestines and spider webs are examples. According to Arslan and Sorguc, two-dimensional structures are able to resist tension and are able to transmit forces through their surfaces. Tension-stressed as well as compression-stressed composed structures are considered two-dimensional as well, think of the wings of insects, bats and birds.

3. Three-dimensional

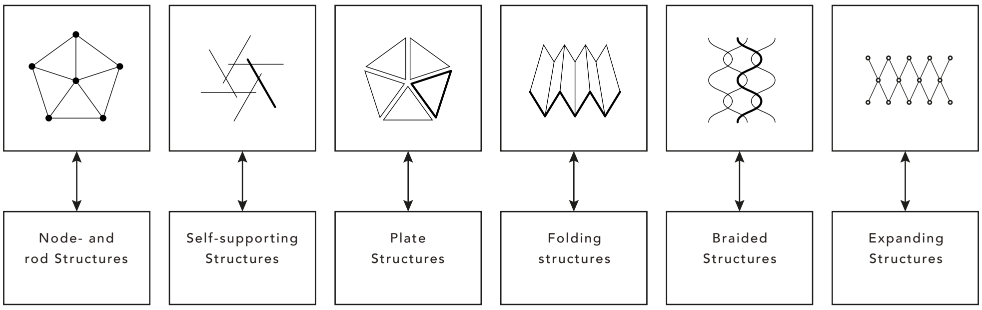
Most structures that can be found in the realm of nature are three-dimensional. This classification has to do with cells, organs, (hollow) bodies and all molluscs. Various skeleton systems are included in this category. Compression- and pressure-stressed structures like our bones as well as the compression- and bending-resistant skeletons of trees and bushes.

In his article 'Influences of the living world on architectural structures; an analytical insight' Gülcan Minsolmaz Yeler also makes a categorisation of the structural systems of nature.



[129] INFLUENCES OF STRUCTURAL SYSTEMS IN NATURE ON ARCHITECTURAL STRUCTURES

Inspired by Senosiain as well as Yeler, I believe that there are six more mentionable structural systems in the realm of nature, ending up with a categorisation of again twelve. In this case not on the physical phenomena, but on natural structural systems.



[130] STRUCTURAL SYSTEMS IN NATURE

“Quoting the older aphorism ‘Function creates the organ’ a well known postulate of functionalism states that ‘Form follow function’. But architecture is not made with words, and in the practical application of both sentences it is often forgotten that the creation of new forms can only take place by means of structure.”

Félix Candela

application

5. DESIGN DIRECTION

5.1 CONCEPT DEVELOPMENT

The transformation methodology that was explained earlier forms the main research and therefore the basis for fulfilling the objective presented at the beginning of this thesis: ‘A new structural application that is derived from physical phenomena’. The main similarity to the design by nature will be on ‘structure’ and ‘appearance’. The filtering will be done on all the three stated in the methodology; the ability to scale beyond the problem of scaling, the morphology and the constructability when it comes to materials and components. This filtering is tightly linked to the problem statement that was presented in the beginning. The next step of ‘analogue’ will be moreover about the self-forming qualities and principles that can be found in biology. By the abstraction- and mutation phase through the four main sciences that are involved in structural design, several core differences in laws between biology and architecture will hopefully be overcome.

Precedents and pioneers originated the main idea for the design direction. Several architects and structural engineers inspired me – Antoní Gaudi, Pier Luigi Nervi, Buckminster Fuller, Eladio Dieste, Frei Otto and Félix Candela. These men all had a similar approach on designing with principles that are derived from natural and physical phenomena. After having elaborated briefly on their way of working, I was able to initiate a selection out of the twelve physical phenomena that were mathematically described earlier. The phenomena that I selected for further research were ‘minimal surface’, ‘equilibrium’, ‘catenary’ and ‘fractals’. This framework gave me a direction to look into the precedents that were designed by the architects and structural engineers mentioned earlier. This selection was based on fascination as well as seeing great potential of combining these phenomena in order to give birth to new structures. With each of the phenomena separately several structures are built and have been proven to work. However, a combination of for example fractals and minimal surface has not been done before, which gained my interest for further research.

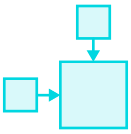
To further grade down the infinite possibilities nature offers, a second phase was done - this time on nature’s similarities and analogues. The German architect Frei Otto was inspired by the analogue of the tree and used this multiple times for designing so-called branching structures. The airport of Stuttgart is an example where this structural principle is applied. This branching structure can be seen as a fractal, which was one of the main physical phenomena presented earlier. Secondly, the Spanish architect Félix Candela has applied double-curved anticlastic surfaces, also known as the hyperbolic paraboloid, in various buildings and structures. As mentioned in chapter 3 on ‘physical phenomena and mathematical descriptions’ a minimal surface is always anticlastic, making the work of Candela worthy of further research. This is greatly inspired by nature as well. The analogue that can be found in the work of Candela is the phenomenon of the soap film, which always seeks its minimal surface.

Next, a filtering on constructability and scaling was made in order to translate the analogies into buildable concepts. The remarkable feature of a double-curved anticlastic surface is that it can be described by straight lines, or straight structural members in case of a structural application. This is called a ruled surface. Mathematically, a ruled surface is a surface that can be swept out by moving a line in space. The rulings are straight and make a double curved surface. This feature contributes to the ability to construct a three-dimensional minimal surface.

Secondly, when it comes to the branching structure, the number of iterations is responsible for the scaling abilities of the structure itself. Ludwig Glaeser rightfully state in his book on the work of Frei Otto that by applying his minimal theories to support elements and space frames, Otto arrived at lighter structures by reducing the buckling lengths of their compression members. The resulting forms are distinctly organic and indicative of Frei Otto's interest in the structural patterns extant in nature. The first concept was to somehow combine anticlastic surfaces with a fractal-like branching structure as the support.

The general idea of how the transformation methodology that was explained earlier is used for the concept development is illustrated on the next two pages.

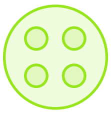
1



Inspiration

- Pioneers
  - Antoni Gaudí
  - Pier Luigi Nervi
  - Eladio Dieste
  - Frei Otto
  - Félix Candela

2

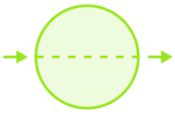


Physical phenomena

- Minimal surface
- Equilibrium
- Catenary
- Fractals

BIOLOGY - DESIGN BY NATURE

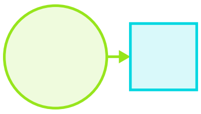
5



Filtering

- Constructability
  - Ruled Surface
  - Branching Structure
- Scaling
  - Number of iterations

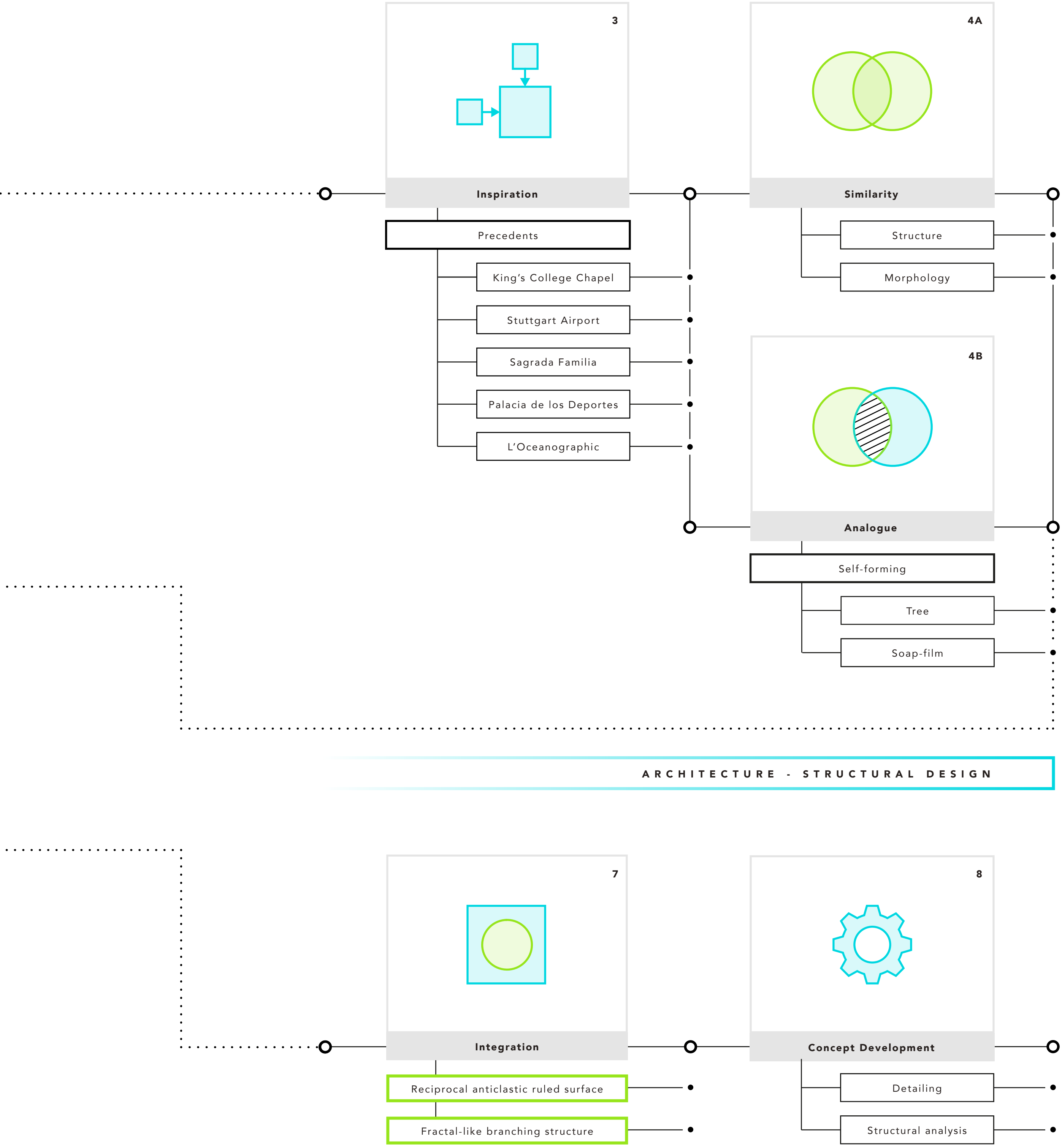
6

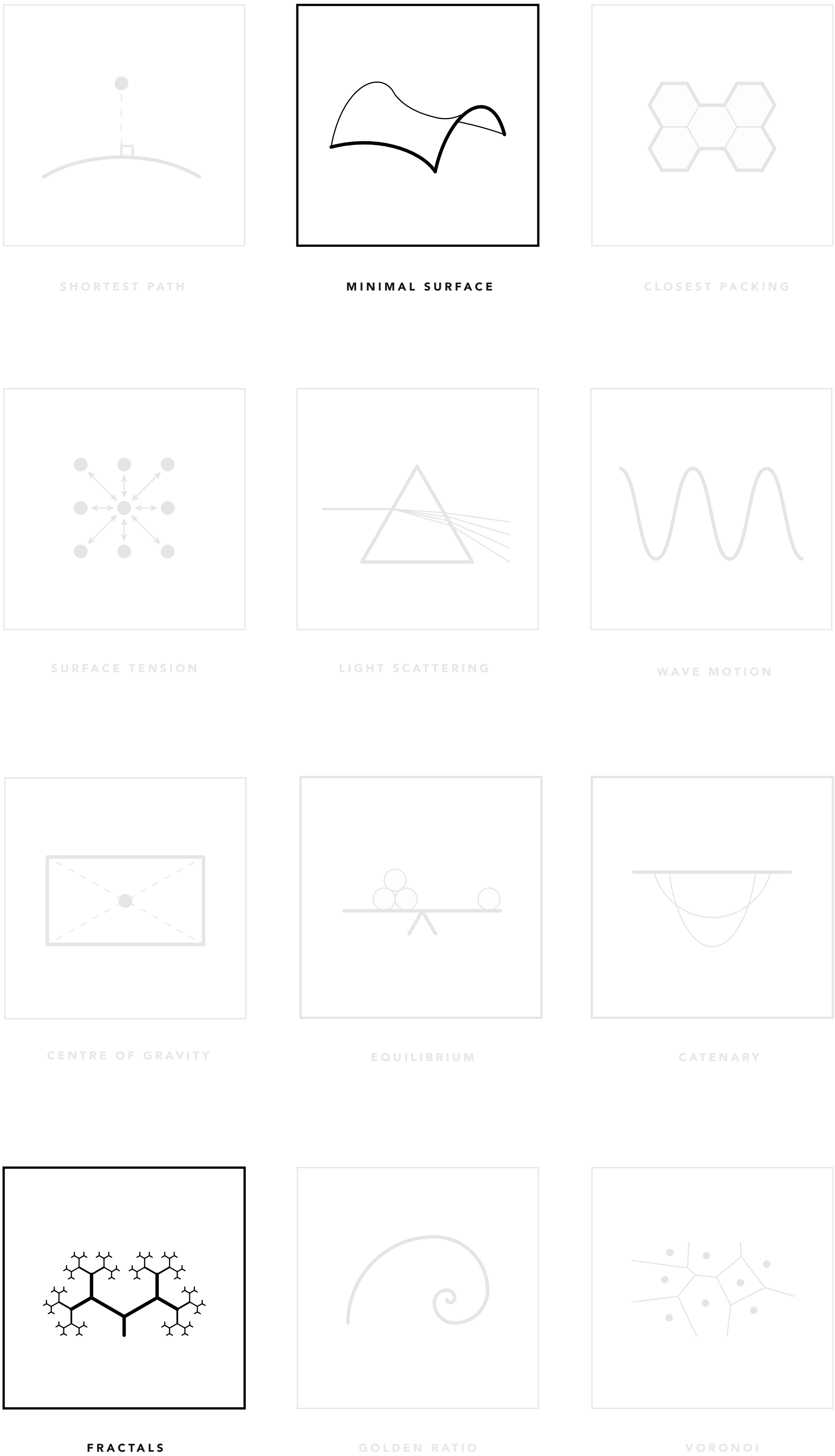


Abstraction & Mutation

- Mechanics
  - Straight members
  - Reduce buckling lengths
  - Less bending moments
  - Large surface spans
- Mathematics
  - Triangular grid
  - Diamond shaped roof
  - Mean Curvature = 0
  - Gaussian Curvature < 0

[131] APPLIED DESIGN BY RESEARCH METHODOLOGY - TRANSLATING NATURAL DESIGN TO STRUCTURAL DESIGN


3
Inspiration
Precedents
King's College Chapel
Stuttgart Airport
Sagrada Familia
Palacia de los Deportes
L'Oceanographic
4A
Similarity
Structure
Morphology
4B
Analogue
Self-forming
Tree
Soap-film
ARCHITECTURE - STRUCTURAL DESIGN
7
Integration
Reciprocal anticlastic ruled surface
Fractal-like branching structure
8
Concept Development
Detailing
Structural analysis



SHORTEST PATH

MINIMAL SURFACE

CLOSEST PACKING

SURFACE TENSION

LIGHT SCATTERING

WAVE MOTION

CENTRE OF GRAVITY

EQUILIBRIUM

CATENARY

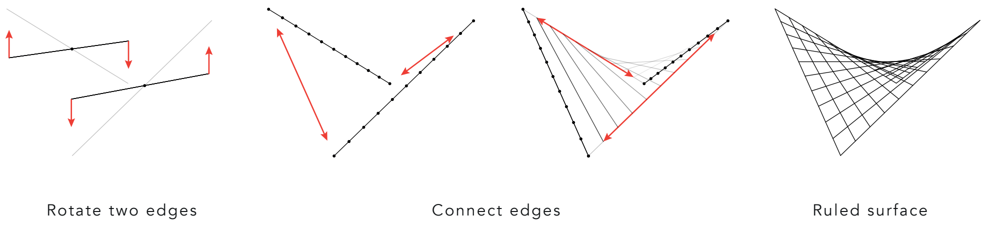
FRACTALS

GOLDEN RATIO

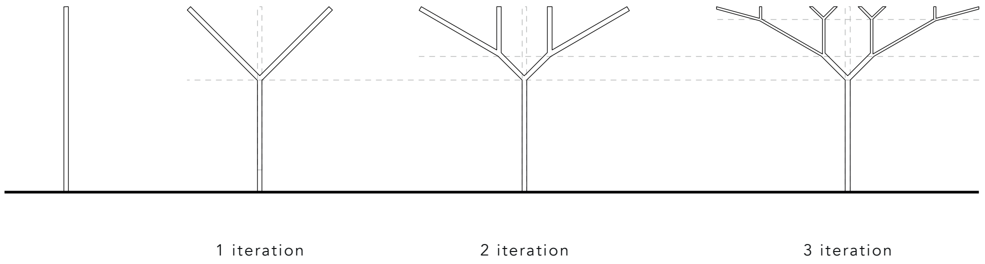
VORONOI

[132] TWO MAIN PHYSICAL PHENOMENA AS CONCEPT DEVELOPMENT

As mentioned before, the first two physical phenomena that were selected based on the inspirations were 'minimal surface' and 'fractals'. Below you can find an illustration on how these phenomena are easily translated into forms that can be used in architecture. Let me briefly explain two typifications; the anticlastic ruled surface and the branching structure. An anticlastic ruled surface is a negatively double curved surface that can be described using straight lines, as mentioned earlier in the chapter on 'physical phenomena and mathematical descriptions'. If one would rotate two opposite edges in opposite direction and connect several points on those curves, the rulers are straight, resulting in a double curved surface. This is called a 'hyperbolic paraboloid' - in short 'hypar'. The branching structure is based on the natural fractal of a tree. A split of the main 'trunk' is called an iteration. This iteration can be done in various ways. A branching structure defines itself by having one, two, three or even four or more iterations.



[133] ANTICLASTIC RULED SURFACE TYPIFICATION



[134] BRANCHING STRUCTURE TYPIFICATION

5.2 FURTHER RESEARCH

5.2.1 Historical background

Yeler (2015) states that throughout history trees and forests have been a source of great interest in architecture. The structural advantages – which will be briefly pointed out later – was the architects and engineers trigger. In the article 'Tree-inspired dendriforms and fractal-like branching structures in architecture: A brief historical overview', Rian & Sassone (2013) made a table on the chronological evolution and development of dendriform sturctures. This table can be found on the next page.

Time period	**Location**	**Building**
1400 BC	Egypt	Ancient Egyptian Architecture
500 BC - 400 AD	Greece & Europe	Classical & Roman Architecture
200 BC - 500 AD	India	Ancient Indian Architecture
771 BC - 476 BC	China	Chinese Tradition Architecture
6th century	Istanbul	Byzantine Architecture
9th to 16th century	Northern Europe	Early Gothic Architecture
1242 - 1248	Paris	Mid Gothic Architecture
1512 - 1515	England	Late Gothic Architecture
1882 - now	Barcelona	Art Nouveau & Gothic Architecture
1890 - 1920	Paris	Art Nouveau Architecture
1st half of 20th century	Worldwide	(Inverted) Umbrella & Mushroom Columns
2nd half of 20th century	Worldwide	Hypar & Umbrella
1980 - 2000	Worldwide	Branching Structures
The 21st century	Worldwide	21st century Architecture

Technology & Significance
Stone-cut, stone-disc and rock-cut architecture: Columns as a fundamental structural element was developed to support the roof and beam loads inspired by the tree trunk that support its crown load. In between the roof and column, capitals were used. Capitals were ornamented with vegetal shapes, later became the prominent design features in architecture as decoration.
Interlocking stiffness; wooden architecture: First known wooden dendriform act as a structural element with its unique interlocking bracket system.
Semi-circular masonry arch vault: More than 300 marble columns of 9 meter high were constructed like a forest to build a huge water reservoir during the Byzantine period.
Pointed-arch and vault action: Gothic style architecture, replicating the forest using a series of treelike columns bundeld with smaller thin columns which become ribs on the upper side for making vaults.
Graphic statics and physical modelling: Graphic statics, apart from physical modelling and equation of equilibrium, was used as a tool for form finding.
Cast iron: Heavily inspired by the curved and spiral forms of vegetal and floral shapes, made by cast irons. These shapes were decorative and partially structural.
Reinforced concrete and steel frames; cantilevers: Reinforced concrete technology had broken the boundary of structural innovations. Long span cantilever allowed architects to build wider and larger mushroom and umbrella structures. Later, with the combination of steel frame, architects achieved very large free indoor spaces with this new minimalist form of dendriforms.
Steel structures: Late 20th century is marked for steel structures. Steel-made branching structures has brought a structural revolution for designing large free-space architecture.
Computer supported form-finding and optimization: This process helped architects to build a new kind of dendriforms, getting shapes similar as naturally growing trees.

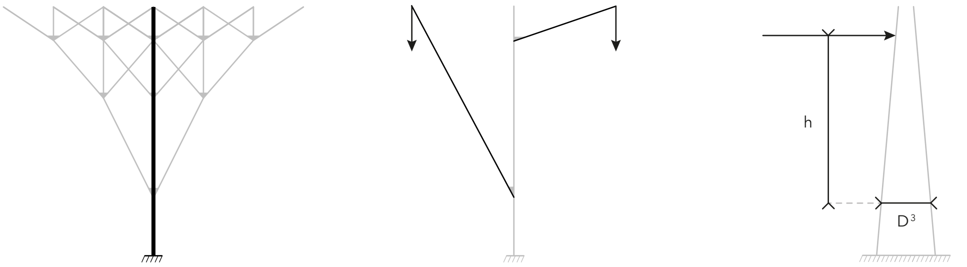
5.2.2 Structural performance natural dendriforms

Mark Grigorian wrote and article about the ‘Biomimicry and Theory of Structures - Design Methodology Transfer from Trees to Moment Frames’. He notes out that for human it is instinctive to try to understand and imitate the underlying design concepts of the most successful natural structure. However, there are quite some structural characteristics of trees and as mentioned earlier. Grigorian (2014) gives a clear list of the structural characteristics of trees.

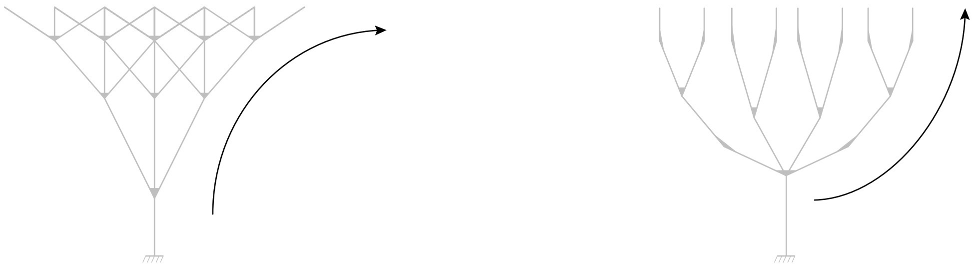
- Trees are three dimensional, structurally determinate natural structures.
- Trees are made out of time tested materials and elements that can adjust themselves for changing environmental conditions, e.g., the leaves can orient themselves in such a way as to absorb/deflect sunlight, high winds and shed snow.
- Trees orient their construction in such a way as to avoid maximum external forces.
- Trees can be classified as upright cantilevers and/or simple moment frames.
- Self weight stresses are minimal in comparison with wind and/or snow induced effects.
- Trees are structures of uniform response, stresses and strains of all sections are nearly the same under constant loading.
- All members of a tree are made out of the same materials with varying strengths as required.
- All tree members are singly connected cantilevered members, there are no simply supported or closed loop elements.
- All cross sections of the stem and the branches are as symmetric as possible, torsional, local and global instability effects are minimized.
- Trees are structures of minimum weight. Each member is optimized for its own function and form.
- Lack of mechanical ductility in trees is compensated by higher flexibility and damping.
- Trees sustain relatively large lateral displacements during extreme wind conditions.
- Tree joints can achieve quasi-plastic response at extreme loading.
- Tree joints possess higher toughness than the stem and the branches.
- Mechanical strength is highly optimized with respect to local form and function.
- Trees are multi-degree of freedom system with high damping characteristics.
- Because of high damping and the multitude of independently vibrating elements (leaves and branches), trees seldom experience resonant vibrations.
- The circular/oval cross-section of tree trunks can withstand greater compressive loads than any other solid cross section with the same amount of material.
- Tree trunks are naturally pre-stressed in both axial and circumferential directions.
- Tree routs are designed to be deformed and uplifted to a certain extent in order to prevent permanent damage to the base of the trunk.

- Trees are known to shed leaves and fruit, even mature branches in order to reduce extreme stresses on the stem and the roots.
- Trees grow on firm foundations with ample access to moisture and nutrition.

Like Grigorian, there are many more writers who made a list of the structural characteristics of trees. Mattheck (1991) states that the phenomenon of needing a main central stem, that leads branches and where the force distribution is centered, is known as Apical Dominance. Besides this, a tree makes compromises between the length and the angle of the branch. Metzger stated already in 1893 that tapering of trees can ensure a uniform distribution of the surface bending stress if the stem diameter (D) has a cubic relation (h-D^3) to the distance (h) from the effective point of wind load. Lastly, another phenomenon trees show is the changing of branching angles with ages. Thomas (2001) explains that the younger branches at the top of the tree are the most upright, while the increasingly older further down tend to become more horizontal. The larger angles of the older branches is due to the load of the newer branches.



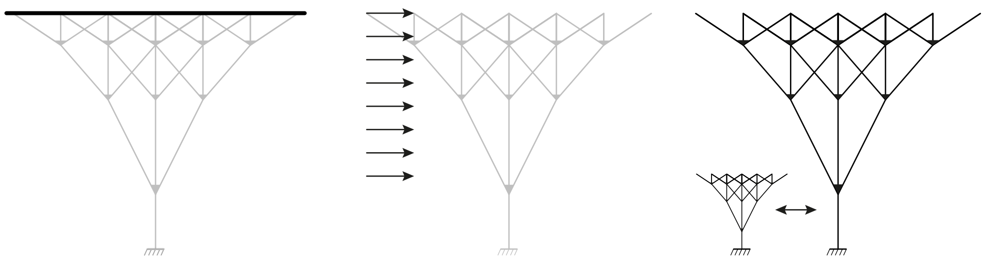
'Apical Dominance' Tree compromises Relation: diameter ~ load



Changing of branching angles with ages

[135] STRUCTURAL PERFORMANCE OF NATURAL DENDRIFORMS

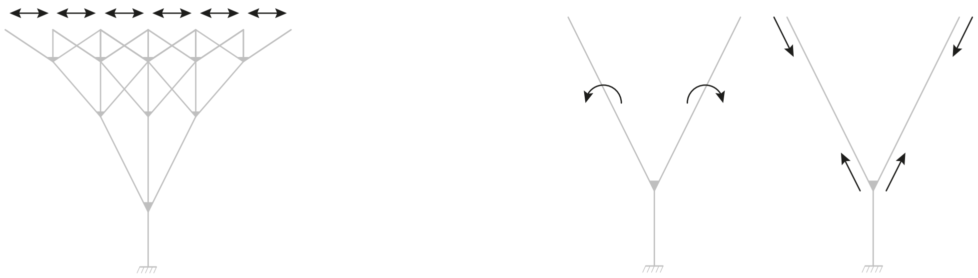
Ahmedi (2007) gives a few characteristics and with the comparison to man-made branching structures. He notes that trees and their branches do not grow equally in all directions while branching structures exhibit a close relationship between the direction and patterns of forces. The situation of a branching structure is a functional combination between the roof construction and the supporting structures. He goes on stating that the advantage of tree-like branching systems is having a short distance between loading points and supports. Rian and Sassone (2014) note that the most inspiring feature of a nature tree is its capacity of carrying a large surface supported by a narrow element, a trunk, through a fractal-like branching configuration. Besides this, the tree's fractal-like branches have great contribution to tackle the wind loads. Another advantage is given by Bovill (1996). He highlights that fractal objects in the general sense show the properties of being exactly or nearly the same at every progressive scale. The last important advantage is based on the difference between naturally growing trees and man-made branching structures. While the branches of naturally growing trees mainly carry bending moments, in man-made structures bending is systematically replaces by axial forces, in order to reduce the internal stresses. Linking the ends of the branches by a stiff roof panel enables the reduction of bending moments.



Large surfaces

Tackle windloads

Scalable for buildable level



Linking ends of the branches

Replace bending moments by axial forces

[136] STRUCTURAL ADVANTAGES OF BRANCHING STRUCTURES

5.2.3 Precedents

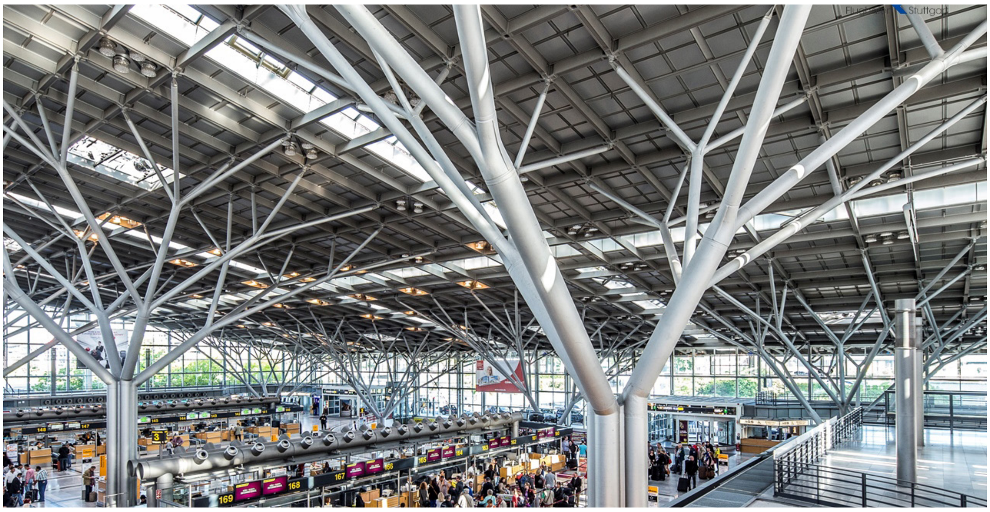
Many precedents can be found on the fusion between architectural language and form, and structural rationality inspired by nature. The first example already dates from the medieval period. In this period a series of fan vaults were designed to replicate the forest from inside, at least that was the goal. One of the best examples of such man-made replication of a forest is the chapel of King's College in Cambridge. The article on fractal-like branching structures by Rian and Sassone states that in the same time period as art nouveau, a unique style of structural dendriforms can be found in the works of Antonio Gaudi. *"His design approach of embedding the forms and structures of trees and plants, including zoological features in architecture was completely different and unique. His quest for finding his own architectural language was heavily inspired by the structural characters of natural forms."* Gaudí also adopted another structural concept of holding a large tree crown. This concept was realised most famously in one of his lasts works, the Sagrada Familia. *"Gaudí imagined this church as if it were the structure of a forest, with a set of tree-like columns divided into different branches to support a structure of intertwined hyperboloid vaults."* After Gaudí many architects adopted this view, for example Frei Otto with his Stuttgart Airport and Santiago Calatrava with his station d'Oriente.

[137] THE CHAPEL OF KING'S COLLEGE, CAMBRIDGE - 1446

[138] GOTHIC RIBBED VAULTS

[139] SAGRADA FAMILIA, BARCELONA - 1982

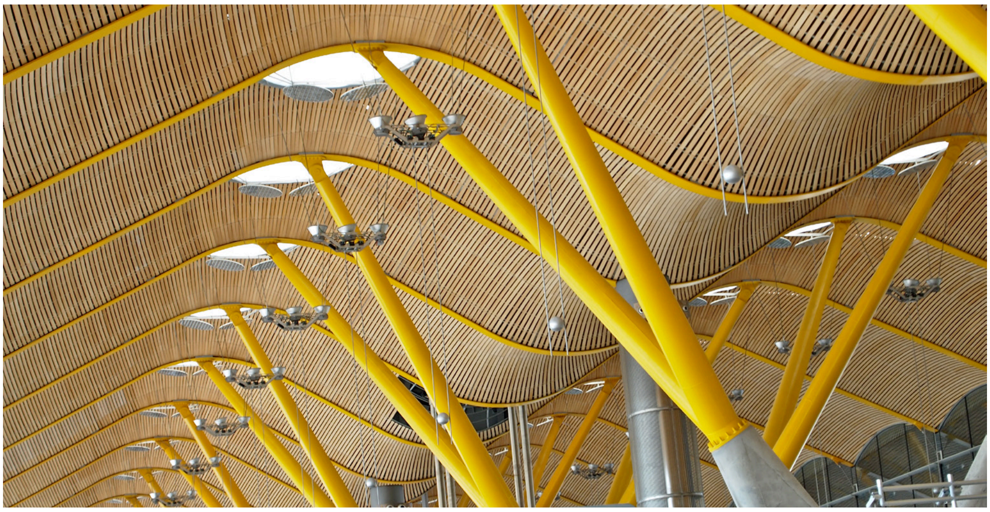
[140] THE TOTE RESTAURANT, MUMBAI - 2009



[141] STUTTGART AIRPORT TERMINAL - 2004

[142] STATION D'ORIENTE, LISBON - 1998

[143] TRAMTERMINAL PLACE FLAGEY, BRUSSELS - 2011



[144] BARAJAS AIRPORT, MADRID - 2004

“The details are not the details, they make the design.”

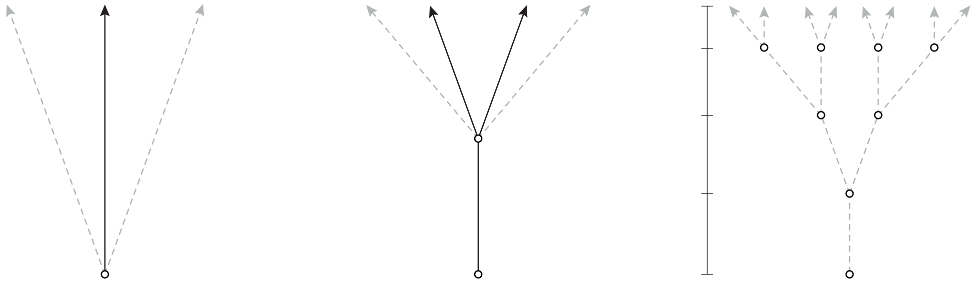
Charles Eames

application

6. DESIGN DEVELOPMENT

6.1 PARAMETERS & VARIATIONS

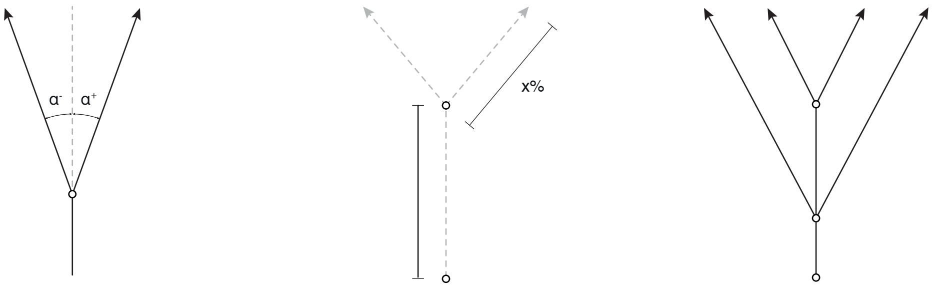
The first step towards the design development concerned setting up several design parameters to get an overview of the possibilities. This was done for the branching structure, as well as for the anticlastic surfaces that needed to cover the branching structure, which was the initial plan. Some of these parameters are tightly linked to each other if the structural performance is taken into account. For example the angle of the iteration is linked to the vectorlength, which is again linked to the branching diameter. Below you can find an overview of the parameters I set up:



Amount of start branches

Number of branching after iteration

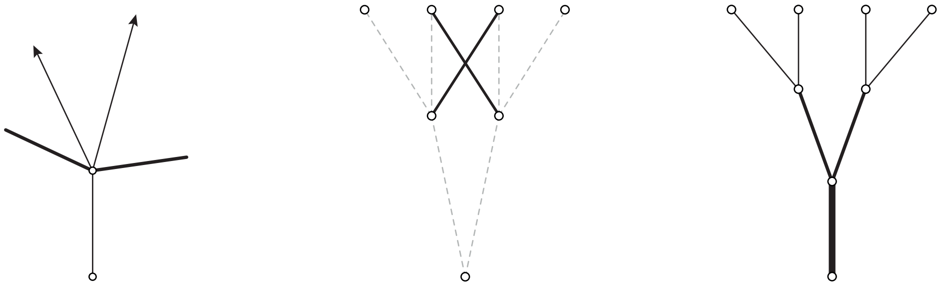
Number of iterations



Iteration angles

Vectorlength - Scaling percentage

Multiple splits per branch



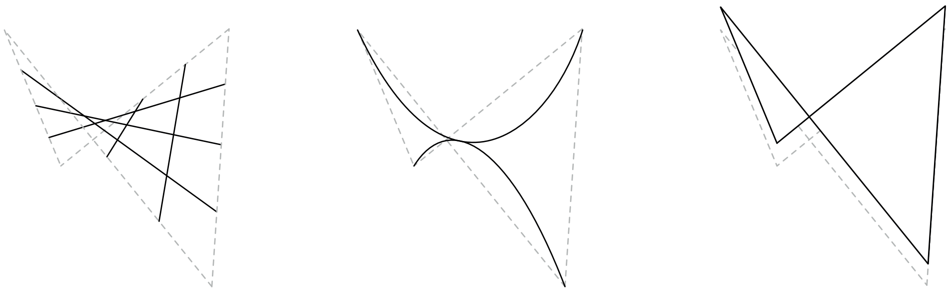
Branching angle deviation

Tension cables addition

Branching diameter

[145] BRANCHING STRUCTURE PARAMETERS

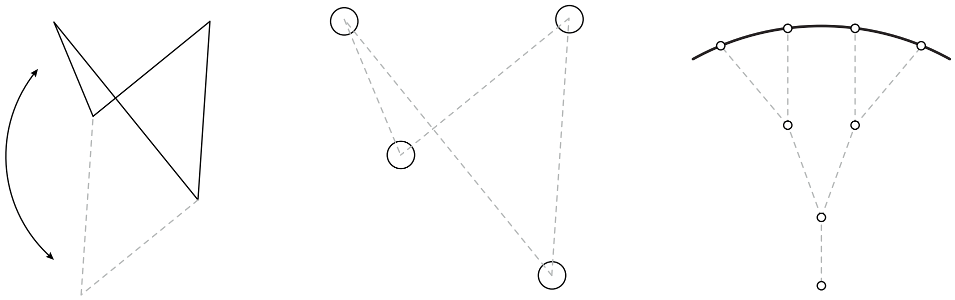
From an early stage it was clear that in order to combine the anticlastic surfaces with a branching structure, the connection was a challenging task. Therefore the parameters for the roof surfaces was about material, curvature, fixings, and so on - all having a different influence on the structural analysis.



Amount of structural members

Amount of curvature

Material



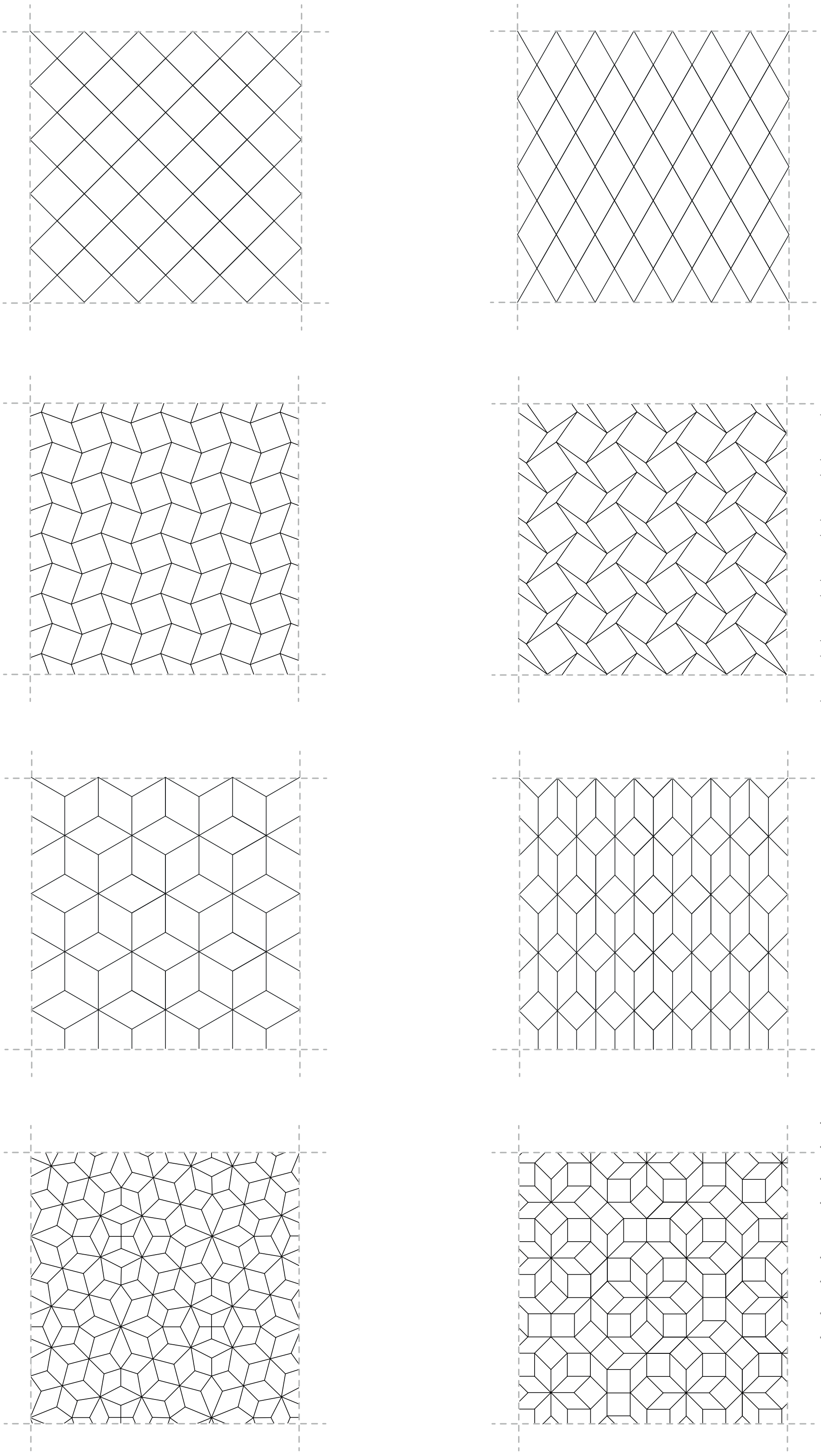
Planar vs. double curved

Fixings and connections

Overall surface curvature

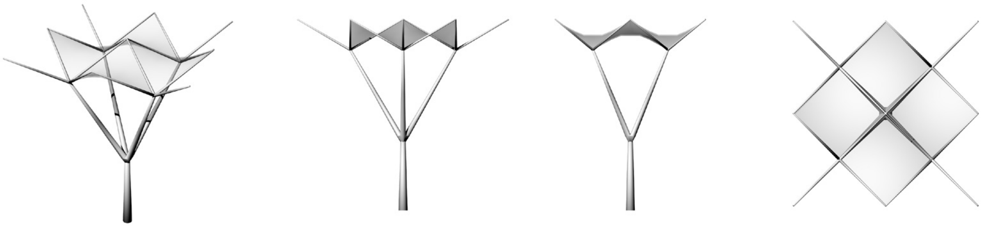
[146] ANTICLASTIC ROOF SURFACES PARAMETERS

With this clear overview of the parameters I wanted to firslty come up with a few basic branching structure variations. Before doing this, research needed to be done on an important constraint in order to be able to fill a certain plane with anticlastic surfaces at all. This is the fact that from above, it needs to be a pattern that consists out of diamonds - a quadrilateral grid, because that is the way how an anticlastic surface is defined. This results in the fact that the branches of the last iteration need to intersect again, they need to make a closed diamond. This does not only have an effect on the last iteration, but also on the iterations below. One could not randomly design splits at a certain angle, because this will affect the constraint of needing a quadrilateral grid on top. There are several ways to fill a plane with diamonds but the most significant categorisation is the squared quadrilateral grid and the hexagonal quadrilateral grid. Besides these two, an aperiodic quadrilateral grid is also possible, resulting in even more challenging irregular branching structures. In the illustration on the next page a few options of quadrilateral grids are given. The first four are squared, the next two are hexagonal, and the last options are aperiodic.

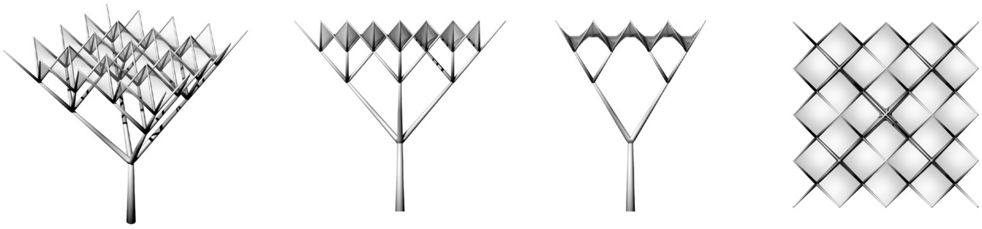


[147] QUADRILATERAL GRID POSSIBILITIES

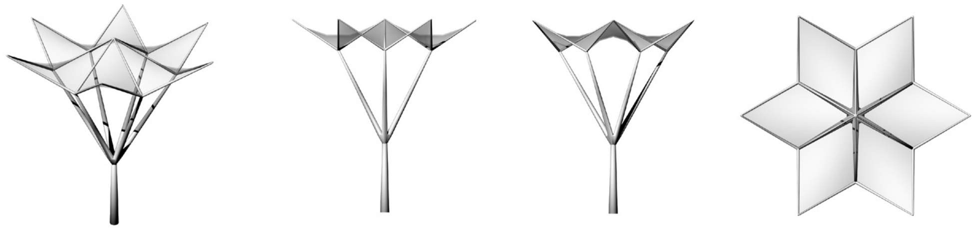
With having done this part of the research, the first preliminary designs of branching variations could be made. I limited myself to only 2 or 3 iterations and to either a squared quadrilateral- or a hexagonal quadrilateral grid. On the images on the right you can find a topview visualising the different grids. For further research and design development I chose the last one since this is the most challenging and appealing to the eye.



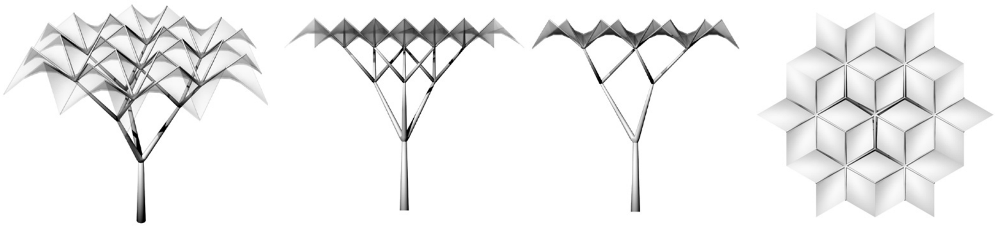
[148] SQUARED QUADRILATERAL TWO ITERATIONS



[149] SQUARED QUADRILATERAL THREE ITERATIONS

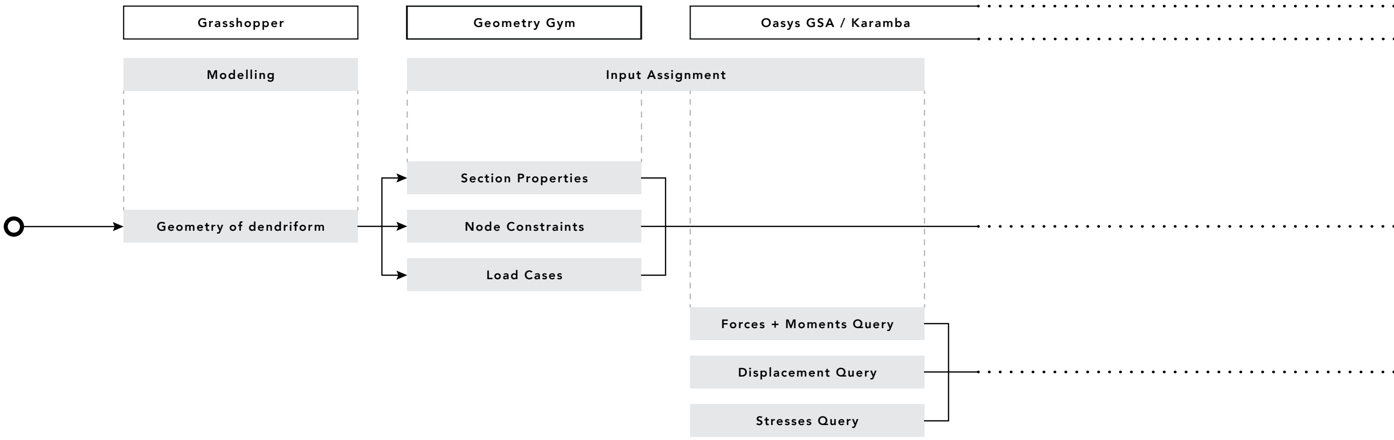


[150] HEXAGONAL QUADRILATERAL TWO ITERATIONS



[151] HEXAGONAL QUADRILATERAL THREE ITERATIONS

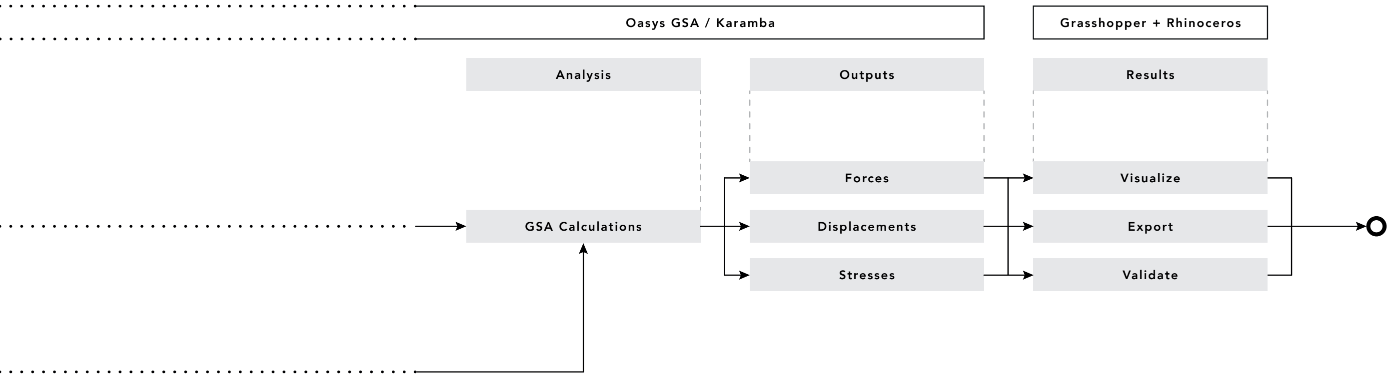
6.2 STRUCTURAL ANALYSIS & VALIDATION

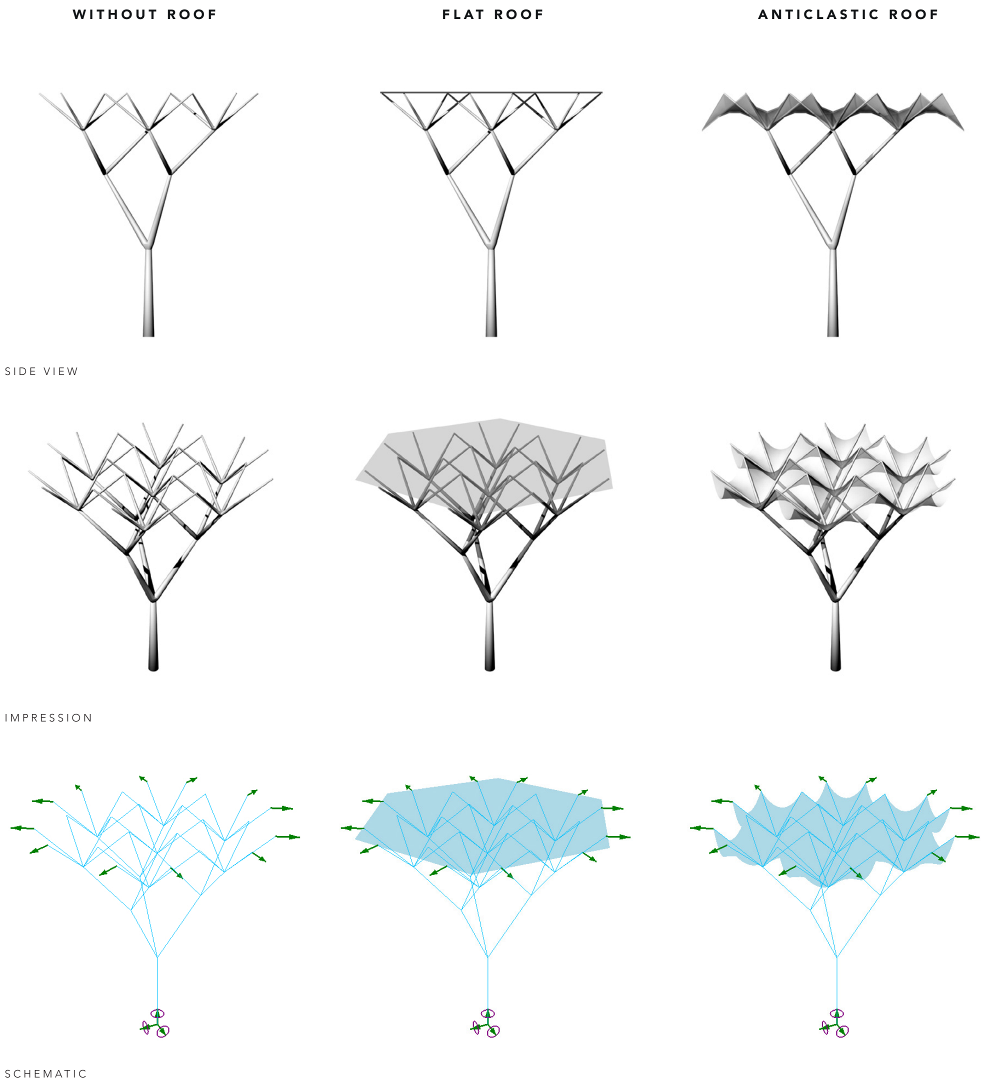


[152] STRUCTURAL ANALYSIS WORKFLOW

After having selected the preliminary design to go further with, I wanted to make a quick validation on the structural performance of a double curved surface on top of a branching structure. The principle of chosing a double curved anticlastic surface was based on the physical phenomenon of minimal surfaces, but in fact, a flat surface has the least amount of surface area possible. Besides this, it is for a reason that a flat roof is more used in architecture than a roof that consists of double curved anticlastic surfaces. A flat roof is cheaper to process, has probably less embodied energie, is easier to construct and is faster to build. The question raised was why one would still go for a double curved surface based on the minimal surface principle. Intuitively this was because the forces due to selfweight of the surfaces would mostly be transfered axial into the branches instead of via bending moments, like was shown earlier in the chapter on the structural performance of trees. This validation was therefore a comparison of a branching structure without a roof, a branching structure with a flat roof and a branching structure with a roof out of hypars - like shown in the image on the next page.

The structural validation was done with Karamba which is a plug-in for Grasshopper. Above you can find a workflow of the various steps that contained obtaining the right results. The first step was to script the overall geometry as seperate line segments using Grasshopper. To perform a certain structural analysis, Karamba as well as GSA Oasys need three types of input; section properties and materials, node constraints and loadcases. If the calculation and analysis are run properly, the output will be forces, displacements and stresses. For this comparison I looked into the resultant bending moments and compared those. I found that there is a significant decrease in bending moments in the branches with the use of a roof of hypars.





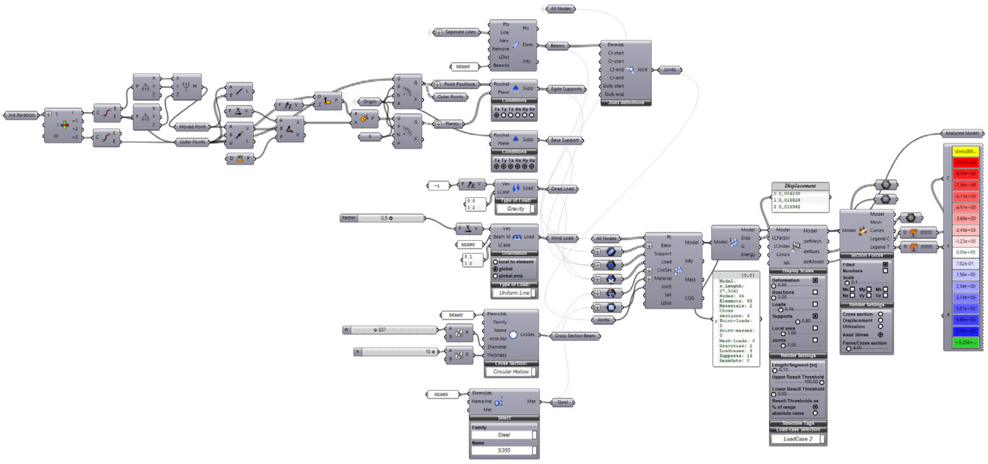
[153] STRUCTURAL ANALYSIS COMPARISON

DIFFERENCE IN PERCENTAGE: FLAT AND ANTICLASTIC

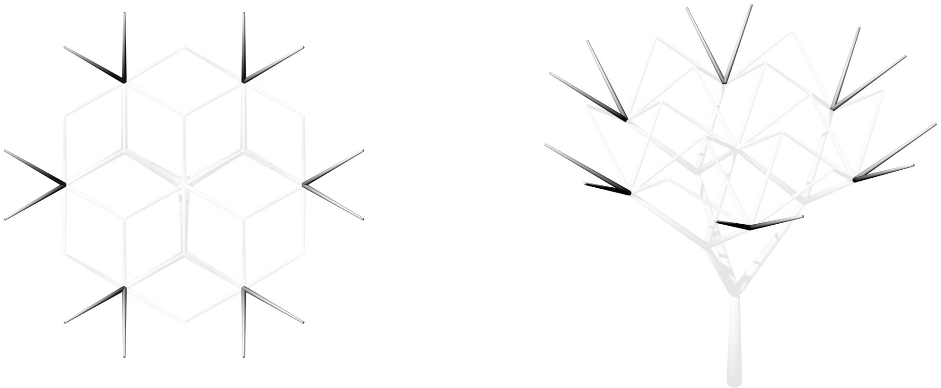
1	173.4	7	54.1	13	48.7	19	35.7	25	171.7	31	48.8	37	54.2
2	196.7	8	54.3	14	48.7	20	35.7	26	175.7	32	48.8	38	54.6
3	177.7	9	54.5	15	48.7	21	35.7	27	194.8	33	48.7	39	54.5
4	97.5	10	35.6	16	53.8	22	34.8	28	53.8	34	35.7	40	98.3
5	85.9	11	35.6	17	53.5	23	34.8	29	53.9	35	35.7	41	88.6
6	88.0	12	35.6	18	53.9	24	34.8	30	53.6	36	35.6	42	86.5

[154a] NUMERIC RESULTANT BENDING MOMENTS OF THE TOPTIER BRANCHES

When a comparison in percentage is done between the resultant bending moments in the branches with a flat roof and the resultant bending moments in the branches with a roof of hypars, it is given that on average the branches of the double curved surfaces have a resultant bending moment of only 70% of that of the branches with a flat roof. In about seventy percent of all the cases, this perecentage is even below 50%. In general, these bending moments are replaced by axial forces, like was stated in the chapter on the structural performance of branching structures. There are twelve branches that have a higher value of the bending moment. These branches are the side branches.

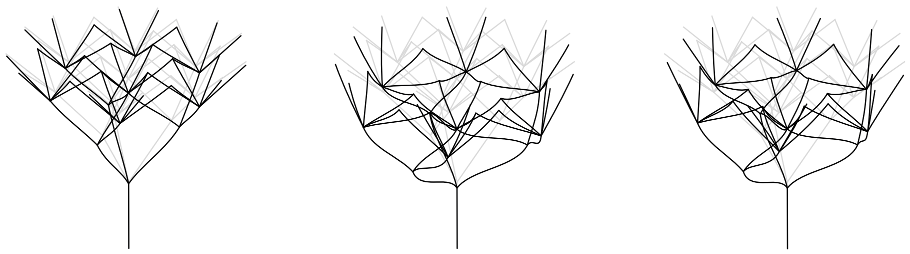


[154b] GRASSHOPPER CODE OF THE STRUCTURAL ANALYSIS

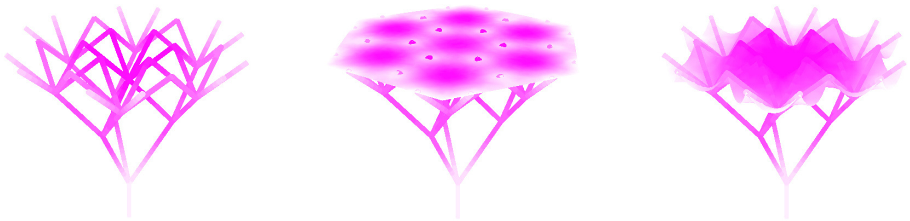


[155] EDGES OF THE TOPTIER BRANCHES

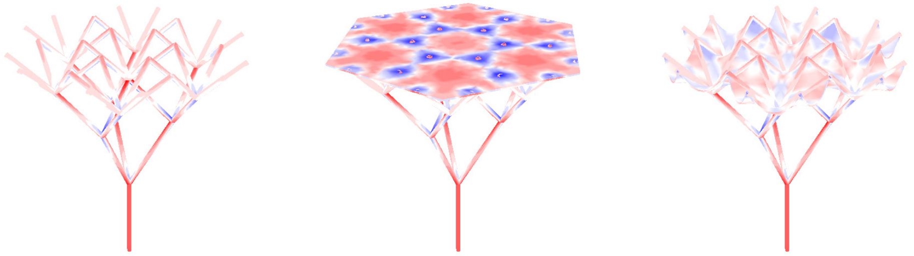
WITHOUT ROOF **FLAT ROOF** **ANTICLASTIC ROOF**



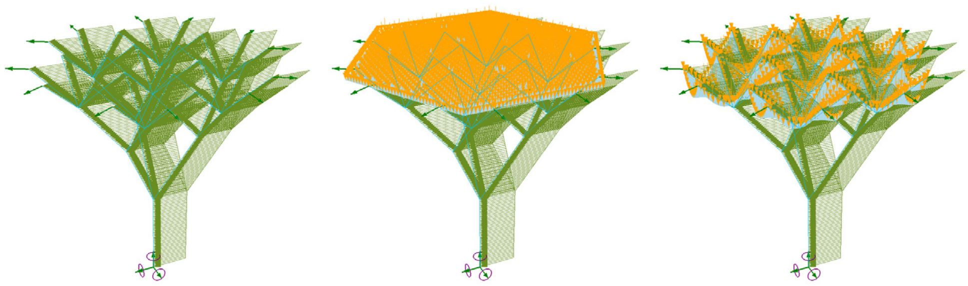
DEFORMATION BY DEADLOAD



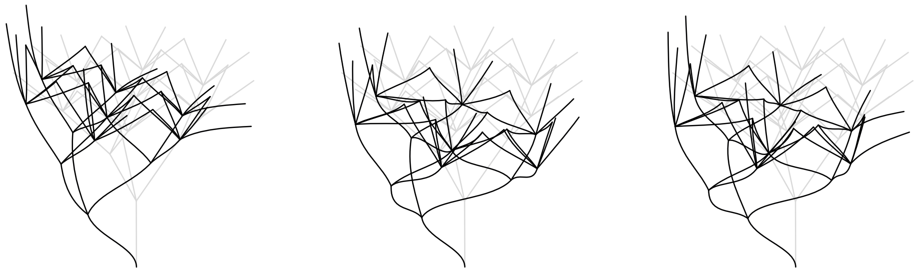
DISPLACEMENT BY DEADLOAD



UTILIZATION BY DEADLOAD



SCHEMATIC OF DEADLOAD + WINDLOAD

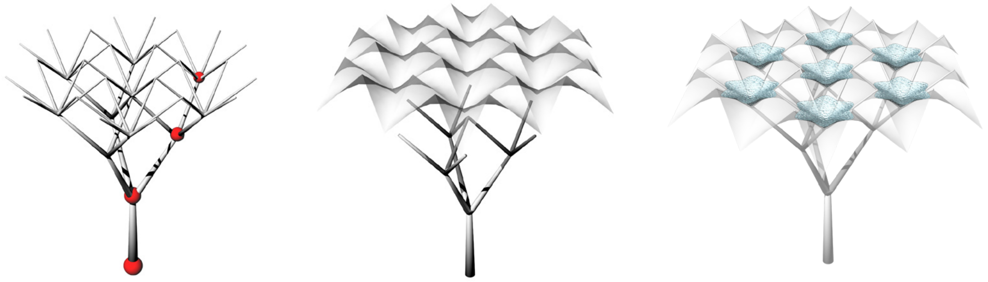


DEFORMATION BY DEADLOAD + WINDLOAD

[156] STRUCTURAL ANALYSIS EXPORTS

6.3 FIRST STEPS TOWARDS THE BUILDING METHOD

Now that it is clear that the double curved surfaces as roof results in the partial replacement of bending moments in axial forces of the branches, further development on the building method could be done. The first inventorisation of this new phase of my graduation. The main objective through all the issues I would come across was on demountability. With the increasing demands on circularity in mind, the goal is to realise a way of detailed design which would contribute to this circularity demand by having demountable connections. Besides this, the inventorisation of the detailing phase of my graduation resulted in having three main issues that needed to be solved. Firstly, the connections between the tubes. Secondly, the connection of the roof structure onto the branching structure. Thirdly, the watermanagement. Each of these elements have a direct influence on the structural performance of the structure as a whole. These three design objectives are not specifically in this order, but are solved in an integrative way, with the effects of a certain choice taken into account over the various topics.

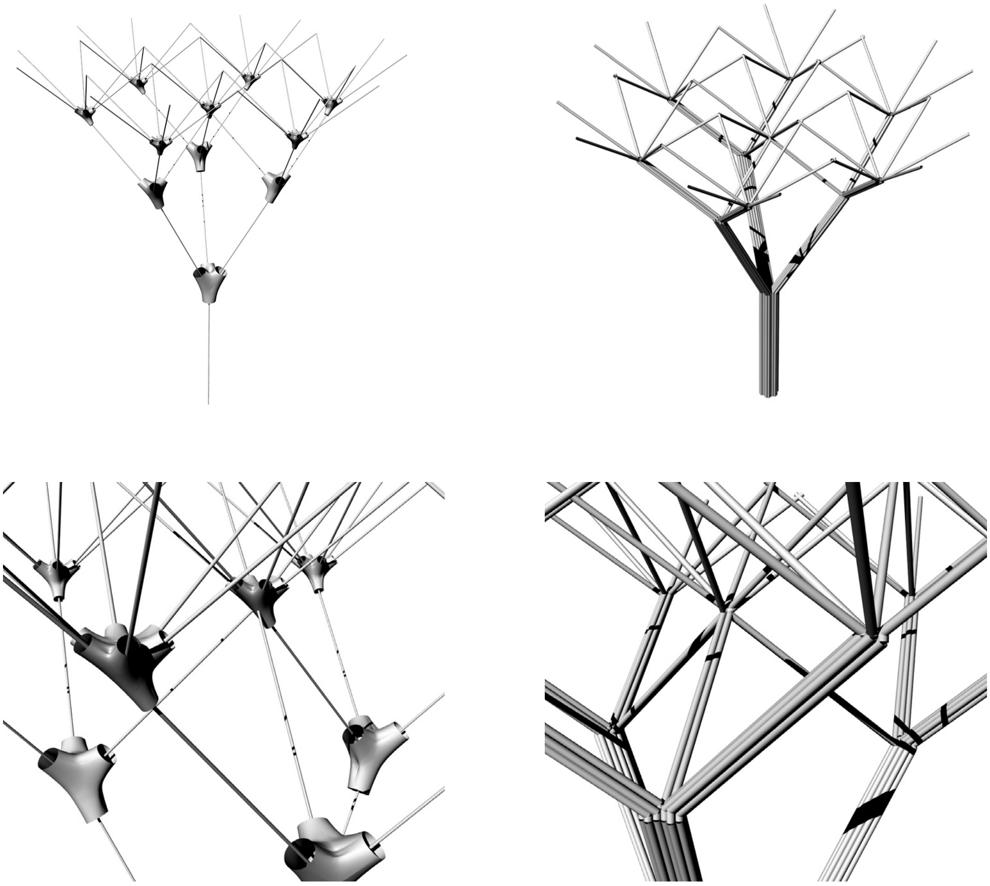


BRANCHING CONNECTION **ASSEMBLY STRUCTURAL COMPONENTS** **WATERMANAGEMENT**

[157] DETAILING OBJECTIVES

6.3.1 Branching connection

Obviously, there are numerous ways to solve the connection between the branches. The first and most used is welding several tubes together. Since this is against my main goal of having demountable connection, this option was quickly eliminated. Two other options I came op with was the use of mounting heads to connect straight tubes together, or the use of uniform-diameter tubes that are combined. Both having advantages as well as disadvantages. For the mounting head the main disadvantage is the production of these connecting components, which are probably unique, making this production even more challenging and expensive, while for the combined tubes it is the transportation since there is a limit of 12 meters in length to transport the tubes to the building site. Besides this the footprint of this particular branching structure will consist out of fourtytwo seperate tubes with all having a unique way of bending, not to mention the mathematics that come into play to maintain having a clean bundle of tubes after a certain split.

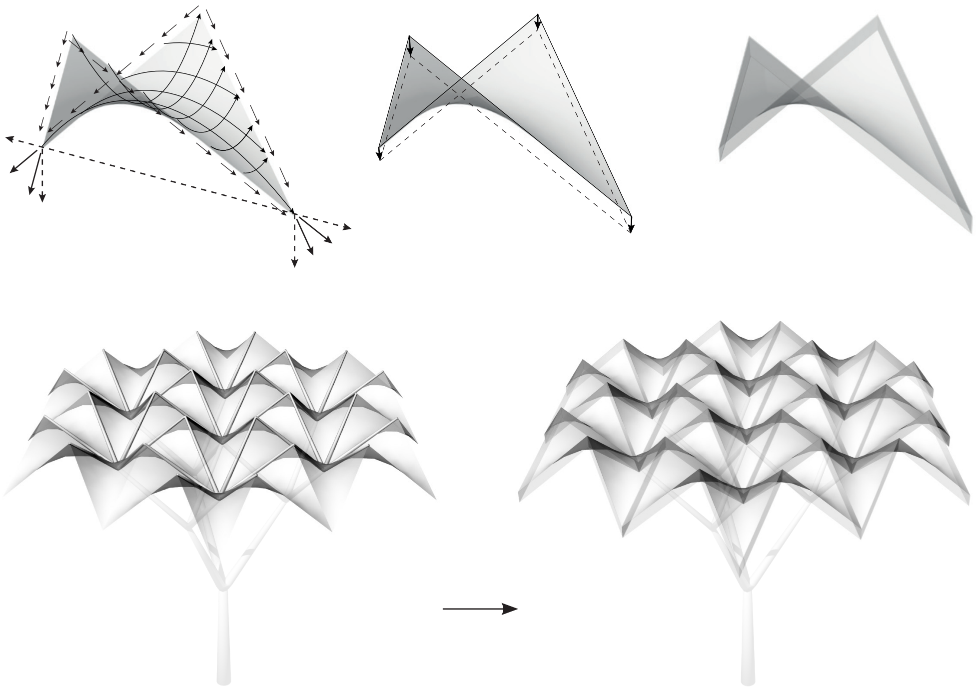


[158] TWO OPTIONS ON THE BUILDING METHOD OF THE BRANCHING STRUCTURE

6.3.2 Assembly structural components

After this overview of possibilities I went on with designing multiple ways on how to connect the anticlastic roof surfaces with the branching structure. The obvious 'cut' so to speak is right before the third and last branching iteration, exaclty the point where the roof starts. With this in mind it is also possible to only use the uniform-diameter combined tubes for the first two iterations. Resulting in not having fourtytwo, but only nine seperate branches. For the assembly of the different structural parts I was inspired by a leaf of the ginkgo biloba tree. This leaf does not have a connection between the branch and the surface of the leaf itself. It is a continuous surface that opens up. On the next page an illustration is shown on the force transfer of an anticlastic surface. There are two forceflows; compression and tension. These flows are directed to the sides and will then be transfered towards the lowest points of the hypar. The branches will be split at the point where the surfaces start, but the effect is that the surfaces would need a way to transfer the forces. With this is mind it is a logical step to introduce ribs, or straight generatrices on the edges of the hypar. These generatrices replace the third branching iteration.

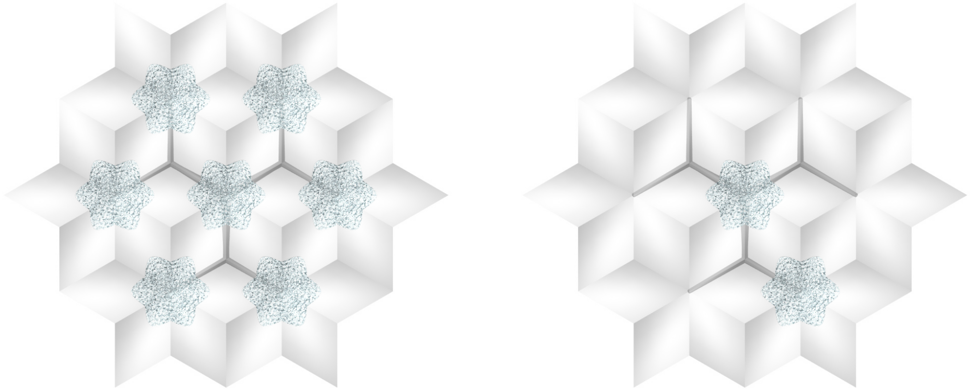
[159] THE GINKGO BILOBA LEAF



[160] ADDING STRAIGHT GENERATRICES OR RIBS TO THE EDGES OF THE HYPAR

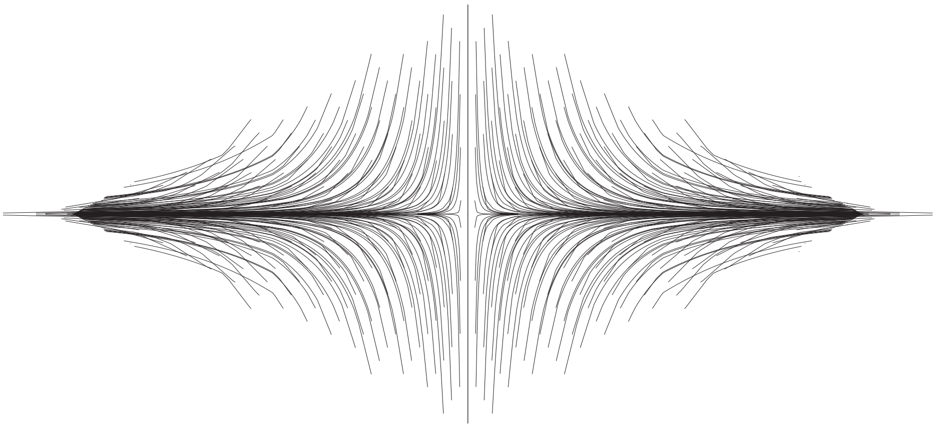
6.3.3 Watermanagement

The third important designelement that needs to be solved is the watermanagement of the structure. Since the hypars are connected together, with the consequence of having low points and high points, there are area's where the water will stream towards to. Every connection between a branch and a group of hypars connected together is such a spot for the water to collect. The most obvious way is to redirect the excess water through the tube. In other words, the connection between branching structure and roof structure becomes even more difficult, since the watermanagement needs to be solved at these connection points as well.



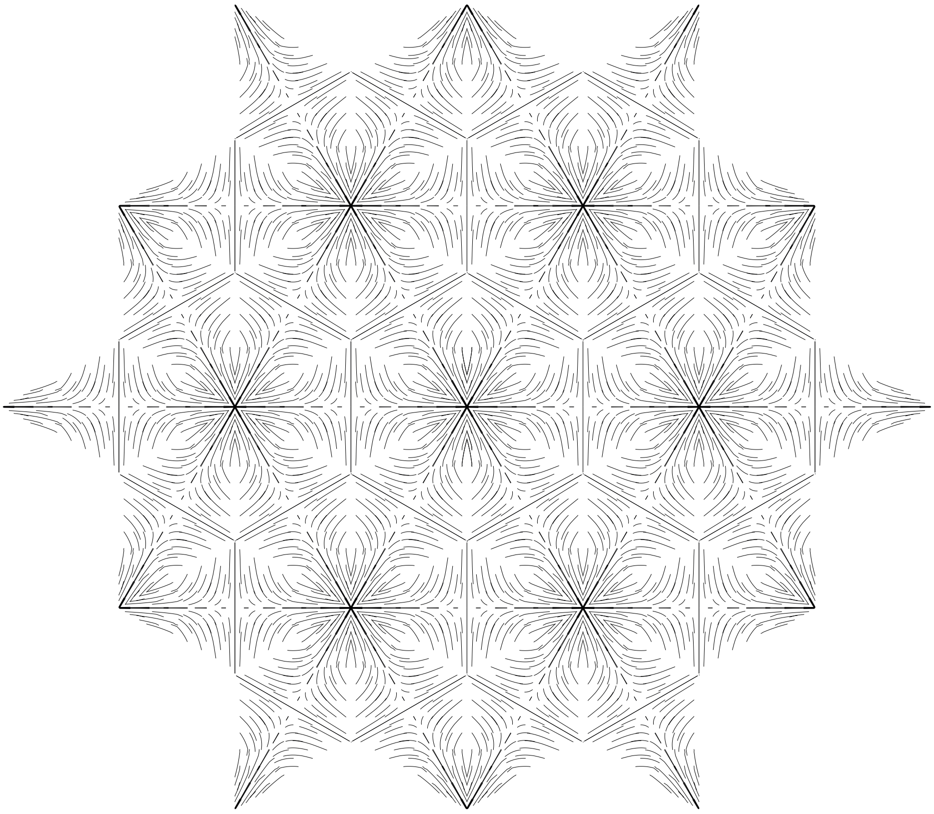
[161] THE COLLECTION OF RAINWATER ON THE HYPARS

The challenge with this is that is can give an excentricity on the branching structure itself. The result when one of the drains gets plugged or obstructed is a rotational force. The branching structure needs to be able to carry and transfer there torsional forces.



[162] RAINWATER FLOW ON A SINGLE HYPAR

To illustrate the rainwater flow, a simulation is made with Kangaroo and Grasshopper. The image above shows how the water will flow towards the sides, which are the lowest points of the hypar and also the locations where the surfaces will be connected onto the branches.



[163] RAINWATER FLOW ON A COLLECTION OF HYPARS

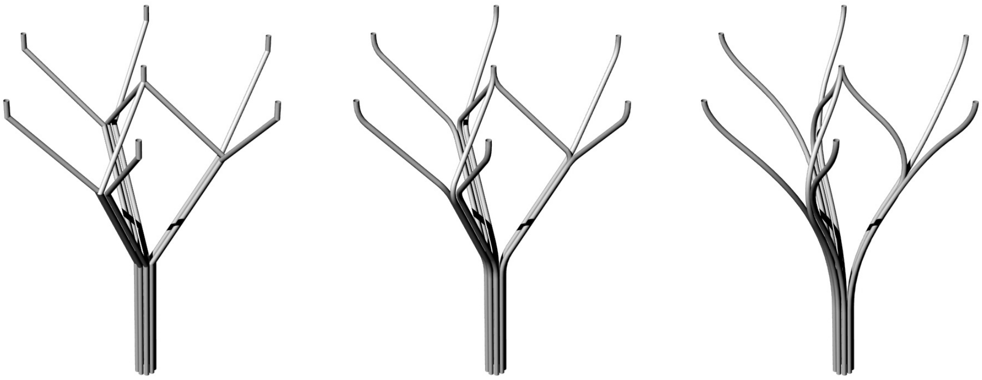
6.3.4 Integration

Finishing of these first steps towards the building method resulted in a integration of the three elements just described. The straight generatrices will be connected to the branching structure that exists out of a bundle of nine seperate tubes of a uniform diameter. There are various ways to bundle nine tubes together. Each with their own second moment of area.

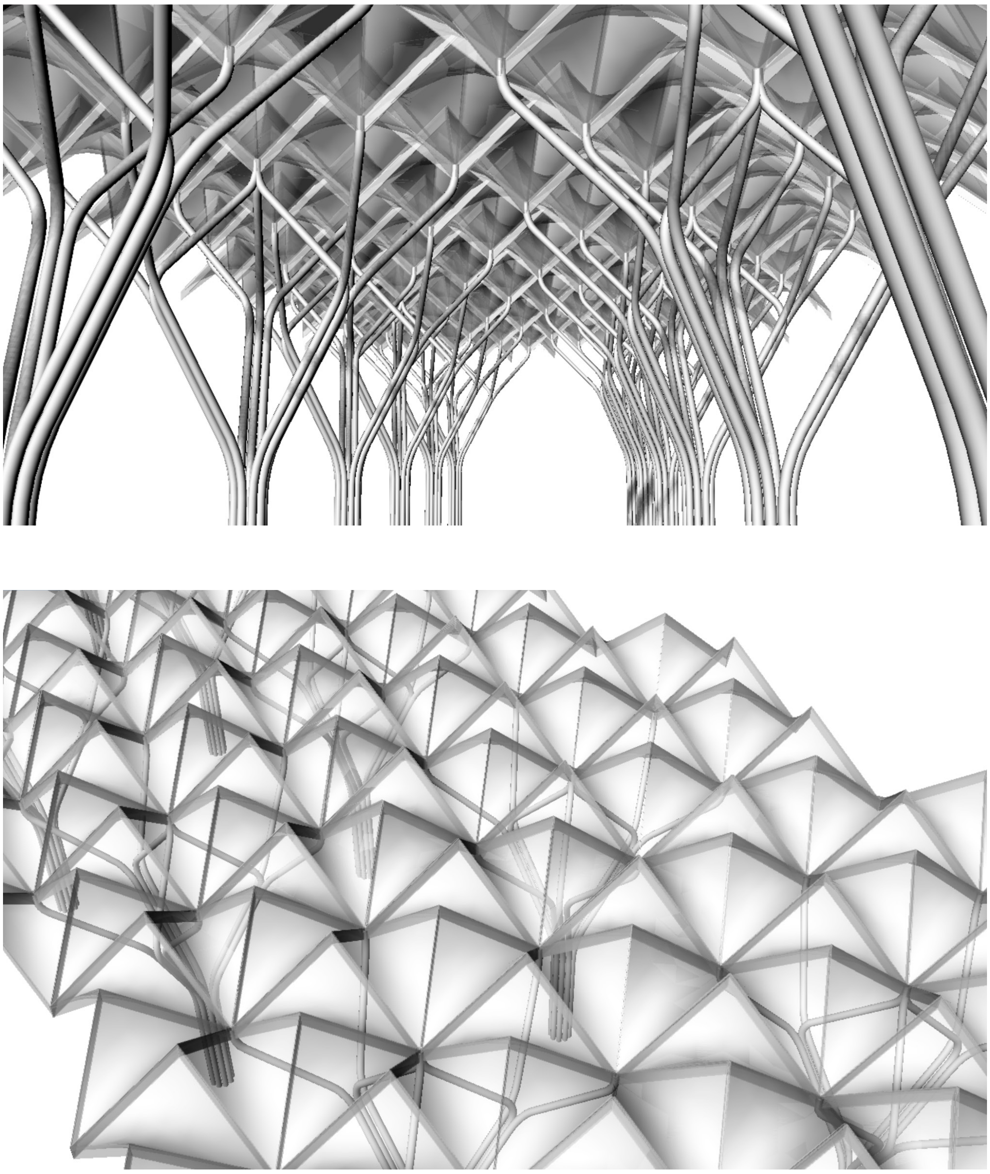
[164] CONFIGURATIONS OF BRANCH-BUNDELING

The choice for the circular configuration was based on the way the branches will be split. Because of this reason the rectangular- as well as the diamond-shaped configuration were eliminated. The triangular configurations are directional. Since I want this new structural principle to be suitable for various applications, the choice fell on the circular configuration.

The next step was to direct the tubes. As is visible in the images below, this can be done with several amounts of curvature. The left image being bend with the least radius and the right image being bend with the maximum amount of curvature possible. For the next phases of this graduation, I went on with the relatively smooth bends as can be seen in the image at the center. The reason for this is that this option has a more natural looking appearance than the kinks of the left image, however, the curvatures are not as strongly followed through as is done with the structure of the right image, which has less structural performance due to the continuous bend. On the next page you can find an impression of how multiple branches with the integration of this way of tube bending combined with the hypars with straight generatrices on the sides.



[165] VARIOUS TUBE BENDING RADII



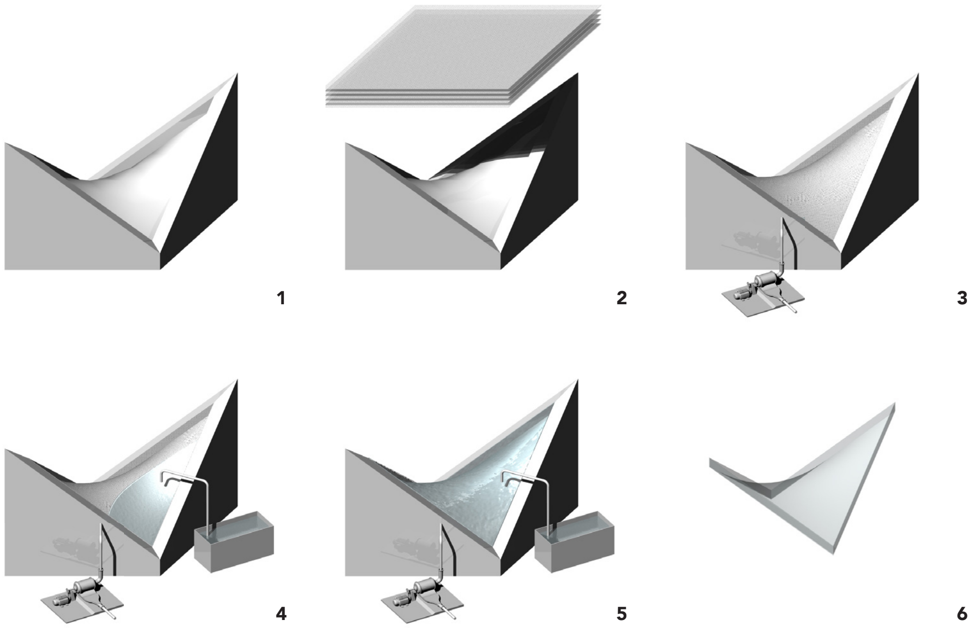
[166] INTEGRATION OF MULTIPLE BRANCHING STRUCTURE

6.4 BUILDING METHOD

There are basically three stages when it comes to the building method. The prefabrication off-site, the transportation to the site, and the assembly on-site. The prefabrication off-site is all about the production, processing, machining and editing of the various components. The entire treatment procedure of the materials will mostly be done off-site. During this prefabrication, the requirements for transportation are taken into account, in order to not exceed the maximum dimensions to transport the components. Once everything is on-site, the assembly can begin. For this stage it is important to write and visualize a so called 'method statement'. This statement for construction is all about controlling specific health and safety risks that could be identified. It basically helps to manage the work and ensures that the necessary precautions have been communicated to those involved. In this graduation project, the method statement is mostly about the building sequence and which cranes, lifts or other tooling are needed for construction.

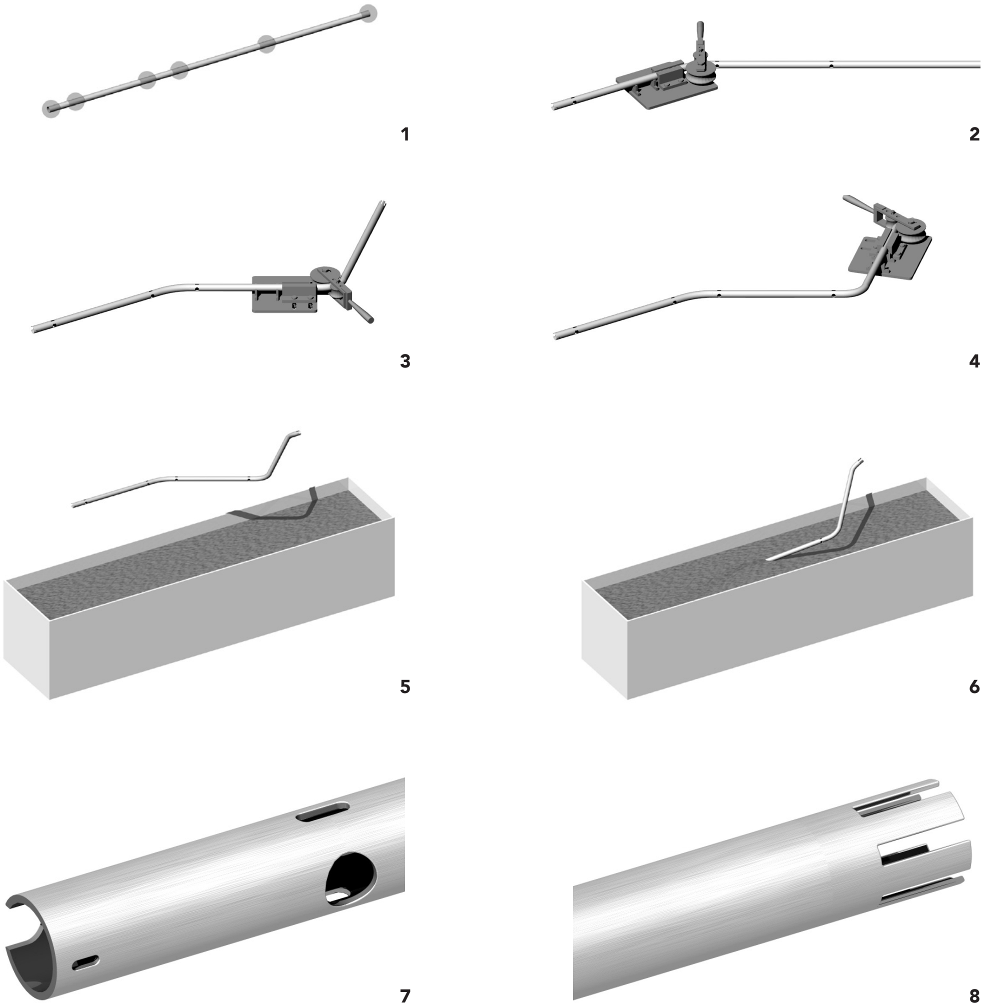
6.4.1 Production

The production and prefabrication will be done off-site. This contains the processing of the steel tubes and the processing of the hypars. Since the hypars need to have significant curvature it is obvious to choose for a reinforced polymer or a sort of metal. For architectural reasons and for the comfort under the structure itself, it would be of great value if the hypars could transmit a certain amount of natural light. This ability made the choice for a fiber reinforced polymer that is translucent: glass fiber reinforced polymer (GFRP).



[167] VACUUM INFUSION - PRODUCTION METHOD OF GLASS FIBER REINFORCED POLYMER

The anticlastic surfaces of GFRP are produced with a method called 'vacuum infusion process' (VIP). The VIP is a technique that uses vacuum pressure to drive resin into a laminate. The first step is to make a mould of for example stainless steel. This mould is a basically a panel upside down, since the side with the least imperfections will be pointing upwards. This is the side where the rainwater will fall on. Secondly, several layers of glass fiber are laid dry into the mould. The vacuum is applied before the resin is introduced. Once a complete vacuum is achieved, resin is literally sucked into the laminate via carefully placed tubing. This method is clean and has a very good fiber-to-resin ratio.

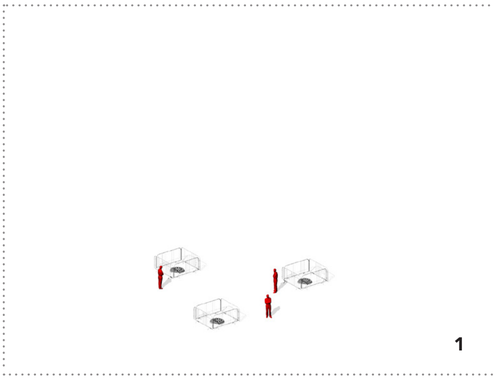


[168] PRODUCTION METHOD OF STEEL TUBES

The structure supporting the GFRP panels are steel tubes. These tubes are produced with a method called extrusion. The first step is to make the whole and cuts with a CNC milling machine. One the whole are at the right spots, the bending can start. There are four ways to bend a steal tube: rotary draw bending, compression bending, ram bending and lastly three-roll bending. The most used methods for bending larger steel tubes is either rotary draw bending of compression bending. The method that is illustrated in the images below is called rotary draw bending. Once the tube is bent, a process that prevents the tube from eroding can begin. This will be done using a bath of molten zinc. This method is called 'hot-dip galvanization'. The last step is to brush the steel to get an accent in vertical direction. Additionally, if the steel tubes are used as interior, the tubes need to be fire resistant. Often, the use of steel is tricky when it comes to the requirements on fire safety. A solution for this can be intumescent paint. This paint is used in buildings as a passive fire resistance measure. According to archtoolbox.com are intumescent products made of a series of chemicals suspended in a binder. "When the binder is exposed to heat it begins to soften, which allows the suspended chemicals to the heat. The chemicals begin to react, which releases vapors that create a foam. A carbonization occurs and the foam solidifies into a black insulating material that is often referred to as char. It is important to note that the product is not burning. Instead, a chemical reaction is taking place that builds up an insulating material that protects the underlying material from the heat." Normally steel structural members are protected with pillows or wood. This is not the finish that I prefer for the branching structure, which I want to be as slim as possible. This method of the use of an intumescent coating provides an aesthetically pleasing finish as well as a structural passive fire protection.

6.4.2 Method statement

To begin with, the detailing and technical drawings of this method statement will be discussed in the next chapter. The first step of the method statement is to install the bearings. Once this is done, the processed steel tubes can be transported with a lowboy. In total there are only three unique tubes, since there is repetition in each branching structure. Meanwhile when the tubes are placed onto the building site, the constructionworkers can begin connecting the tubes together in groups of three tubes - the three unique tubes. Next, a crane will be put in place and with this, the grouped tubes will be mounted on the bearing. Subsequently, if three groups of tubes are mounted onto one bearing, the metal plate can be adjusted with hexagonal bolts. The branching structures are now freestanding structures without the connection between the toptiers.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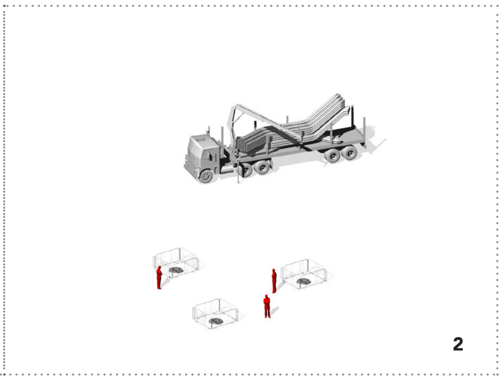
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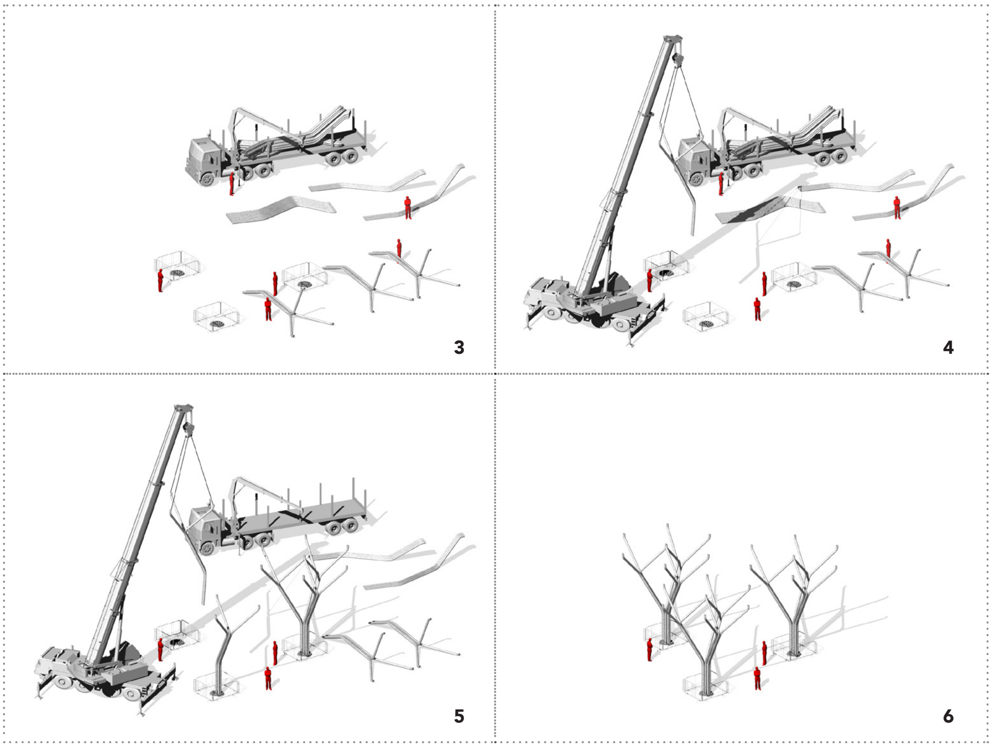
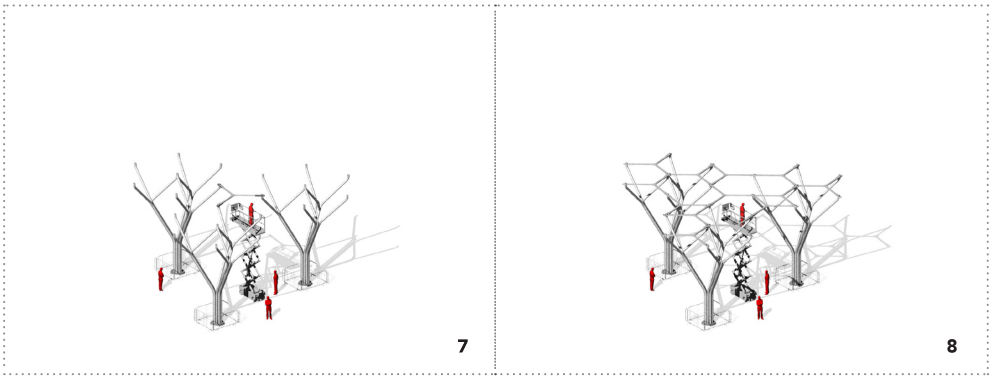


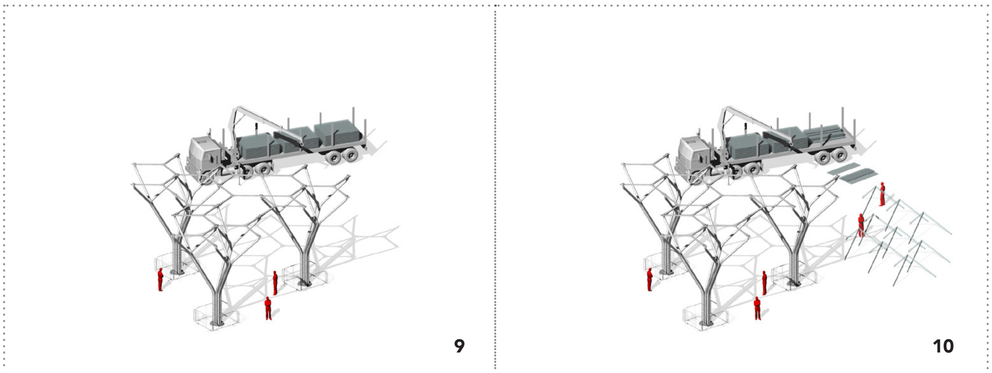
[169] METHOD STATEMENT [1-6]

The next stage of construction is the settlement of the branches. It is expected that the branches will not be perfectly bend, or will be mounted on the bearing with too much tolerances. In other words, the branches will always have a certain deviation. This deviation needs to be overcome. This will be done with the use of a custom made strapping tool. The distance between each branch should be exactly the same, therefore it is possible to produce a tool to settle multiple branches at the same time. This strapping tool will be installed via a scissor crane. Once every branch is strapped into its exactly right location, the fourth stage of the construction can commence.



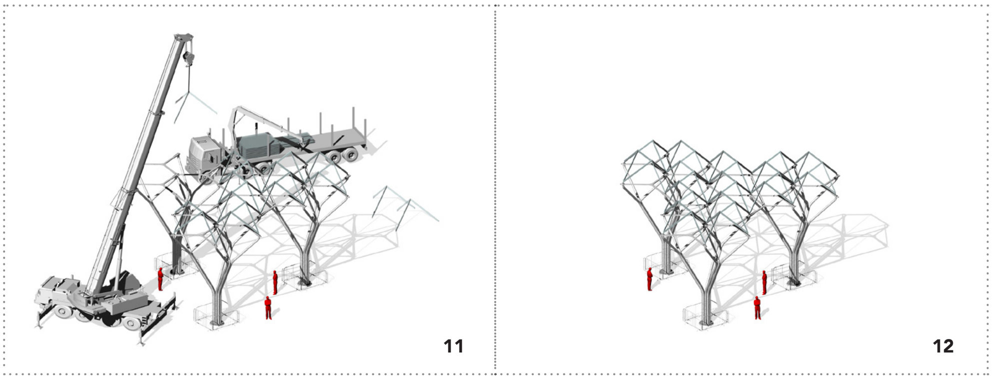
[170] METHOD STATEMENT [7-8]

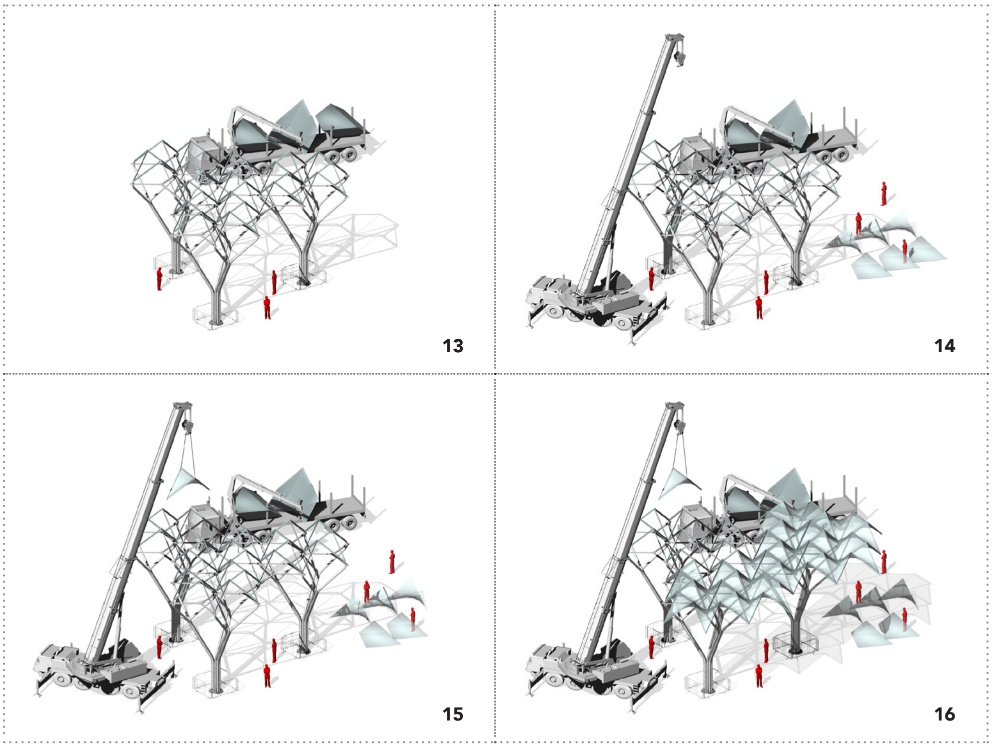
The fourth stage is about mounting the u-profiles. First these profiles will be transported to the building site with a lowboy. The profiles are like the hypars made out of GFRP with a production method called pultrusion. Pultrusion is a way to produce composite materials with constant cross-section. This constant cross-section also enable the profile to be complex and to solve the connection these profiles have to make with the hypars as well. Like was the case with assembling the tubes, the u-profiles will be connected on the ground in groups of three as well. These groups of u-profiles will be places into the grooves of the tubes that are CNC milled. Once six groups are placed onto one tube, a mounting cap can be placed inside the tube. This cap provides for restictions in the x- and y-direction of the u-profiles. The cap will then be bolted onto the steel tube, resulting in the restriction in the z-direction as well. The detailing of how this principle exactly works is shown in the next chapter.



[171] METHOD STATEMENT [9-10]

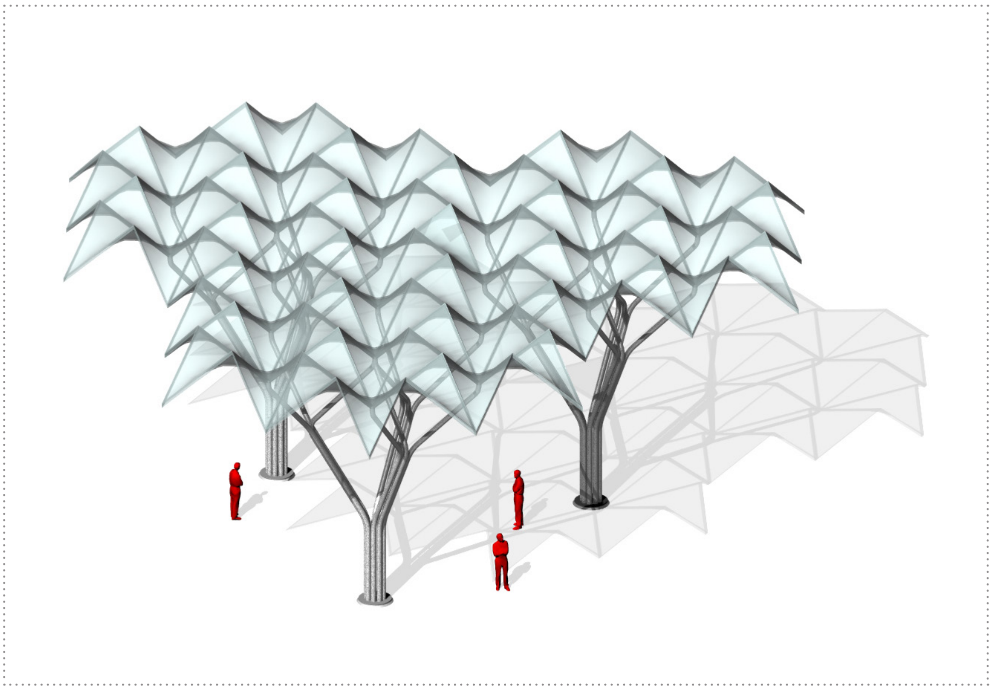
The fifth stage of construction is the last large step of installing the anticlastic surfaces. The same counts for this step as for the last few steps. The transportation will be done using a lowboy. The GFRP panels do not exceed the maximum dimensions for transport. At first it was the idea to connect the panels on site in groups of six with simple nut-and-bolt connections, but in order to have a more simple and generic connection between the u-profiles and the hypars, it is best to mount the panels seperately with a crane. The advantage is that all the fixings are done from the lower side of the roof, accessible for the construction workers. Besides, mounting the panels seperately also gives the ability to remove the panels seperately in case of maintenance or services. Once the entire structure including the anticlastic panels is placed, a few things still need to be done. First of all, the temporary strapping tool can be deinstalled. The combined stiffness of the hypars will accomodate for the branching deviations.





[172] METHOD STATEMENT [11-16]

After realization, the usage phase will commence. With this phase some services are involved as well. Think about maintenance like cleaning or unplugging the drainage when there is debris on the roof. For maintenance, one or more panels can easily be disassembled by a constructionworker on a scissor crane. The easiest way for cleaning is by pressure washing.

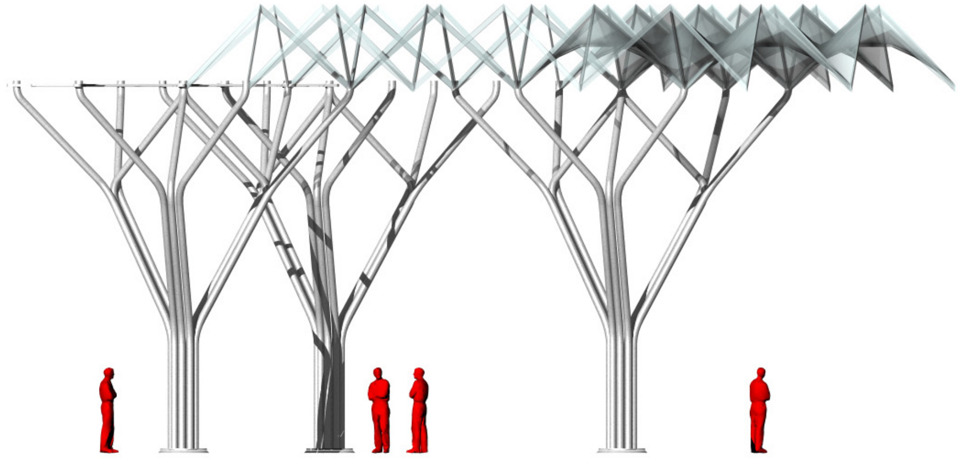


[173] METHOD STATEMENT FINAL IMPRESSION

6.5 DETAILING

After having elaborated on the general constructability with the method statement, the next step is to further zoom in and to focus on the actual detailing of the various connections. Like was mentioned earlier, there are two different structural parts of this structure; the steel tubes as branching structure and the fiber reinforced polymer panels as roof structure. For the detailing I focussed on the connection of those two parts. There are basically five different solutions that can be categorised into the two structural typification just described.

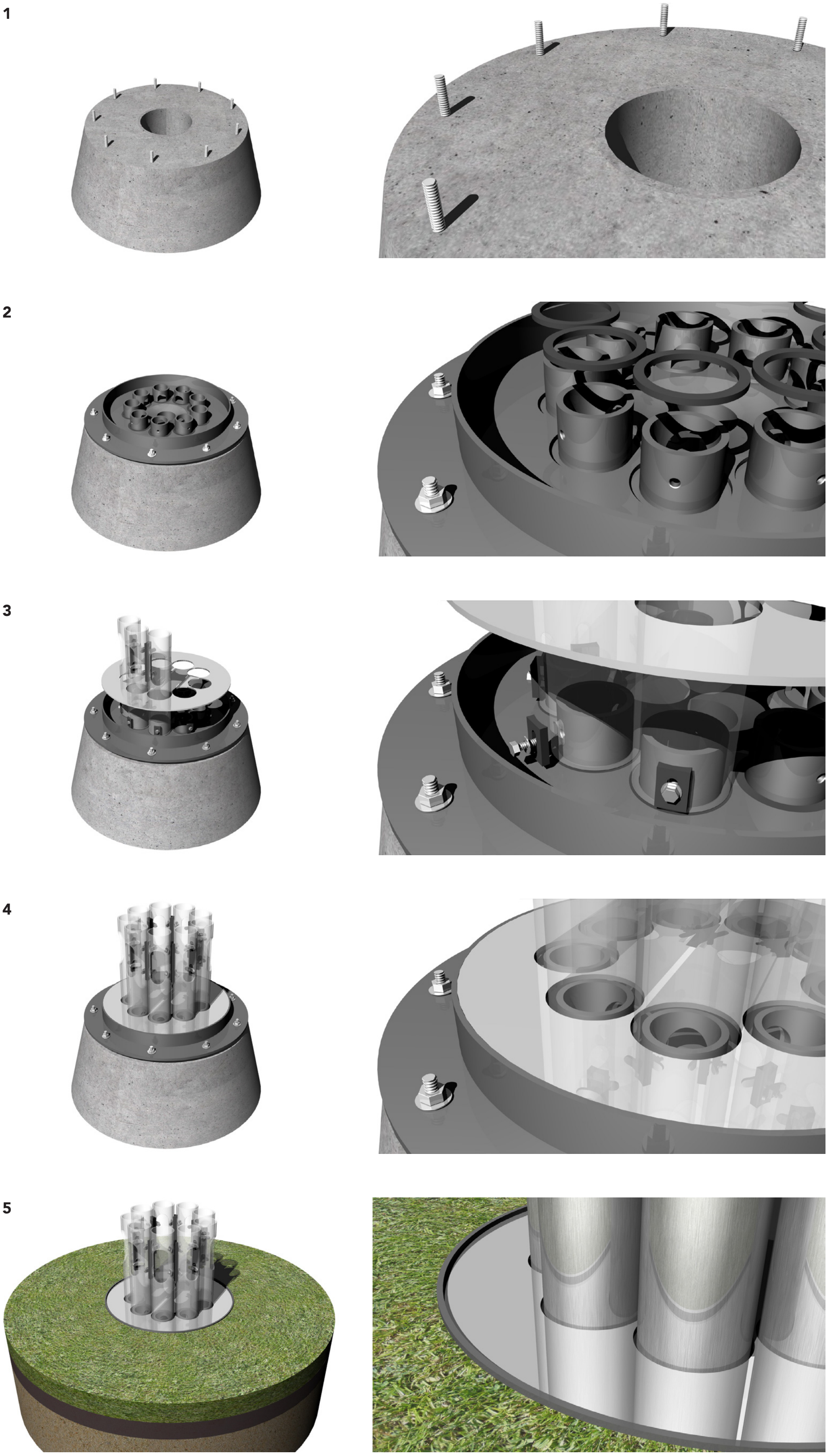
- Tubes as branching structure:
  1. Installing the bearing
  2. Interconnecting the tubes
  3. Adjusting and setting the toptier branch distance
- Panels as roof structure:
  4. Mounting the u-profiles onto the tubes
  5. Placing the anticlastic panels onto the u-profiles



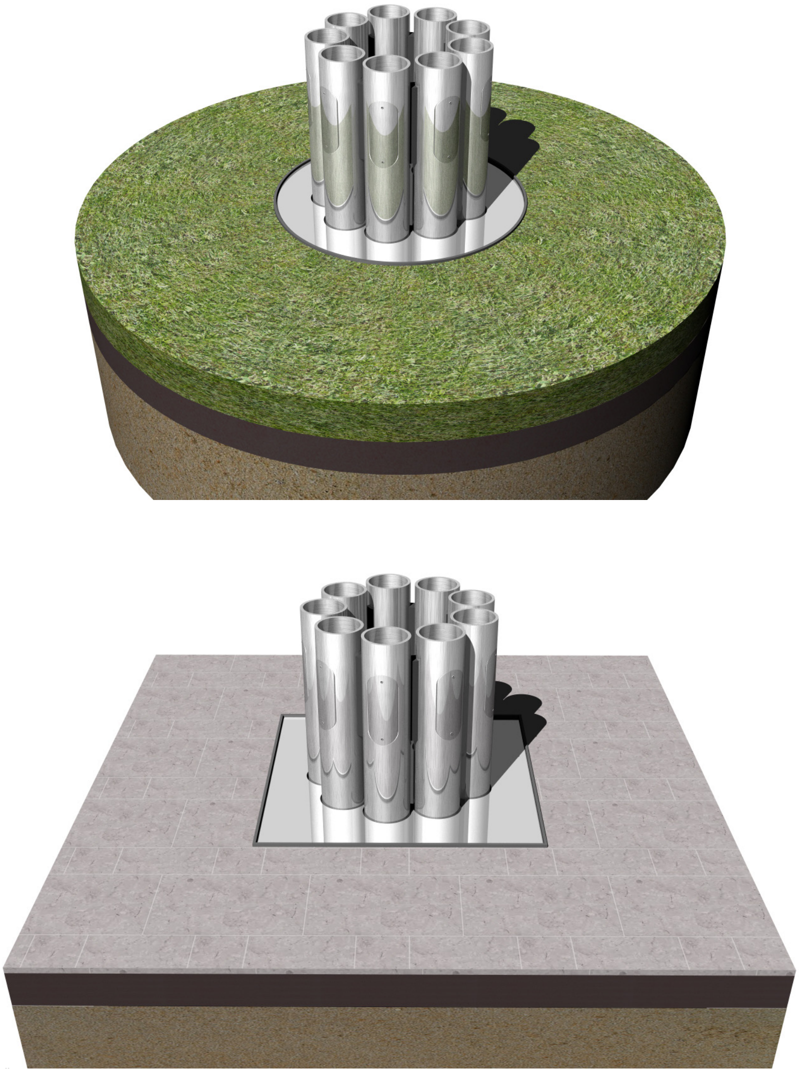
[174] DETAILING CATEGORIES

6.5.1 Bearing

The first step according to the method statement as well is to install the bearing. For this I made two options. One options where the waterdrainage is all covered and one option where the waterdrainage through the pipe is still visible above ground level. Nevertheless, these two types of bearings are quite similar, due to the fact that they both consist of a concrete pedestal including theadends with a drainage hole through the center that will be connected to the sewerage. On top of this goes a metal plate that in both cases is CNC-milled. Onto this metal plate the tubes will be connected with the ability for adjusting the height as well as the angle.

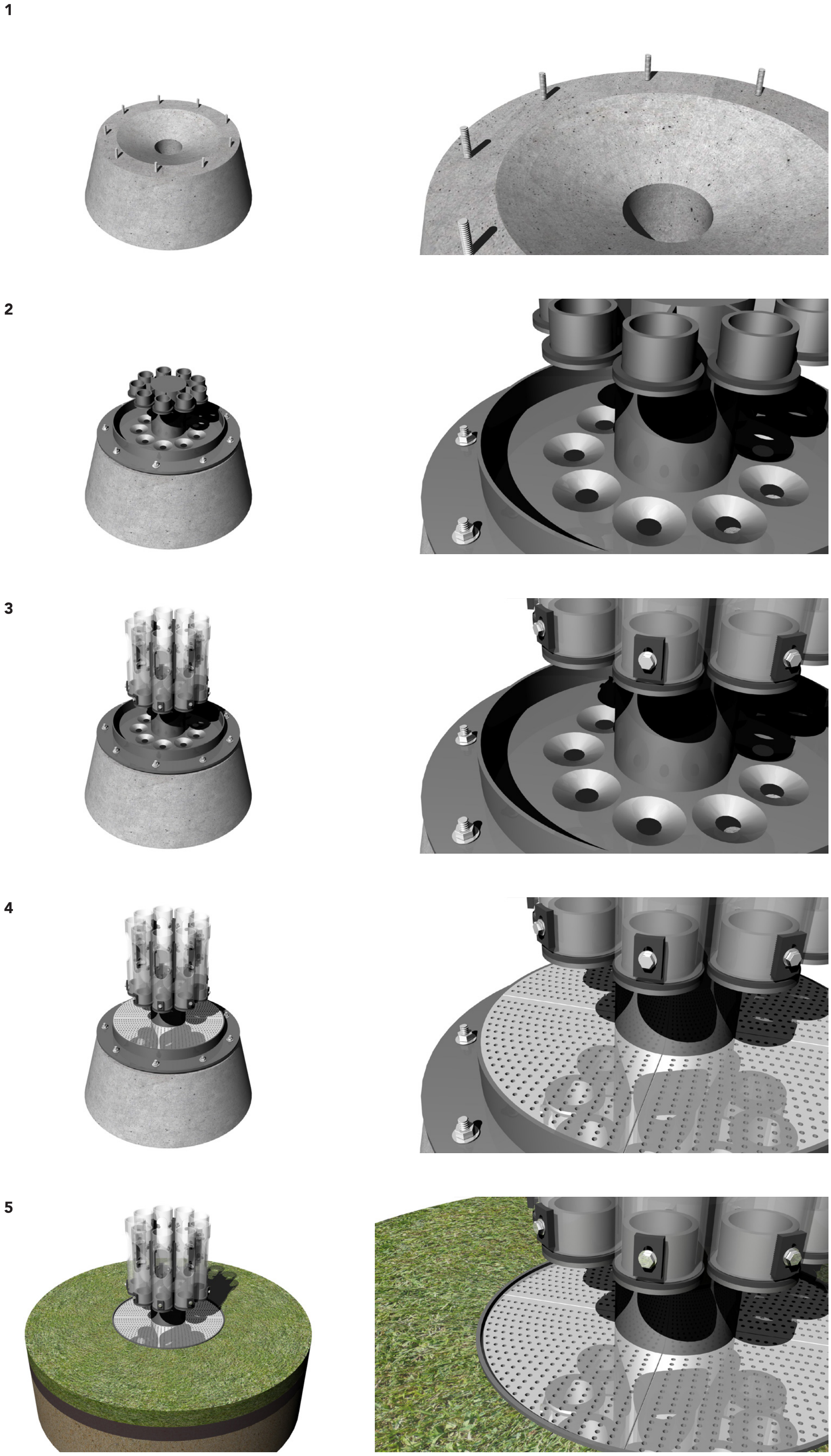


[175] BEARING OPTION ONE

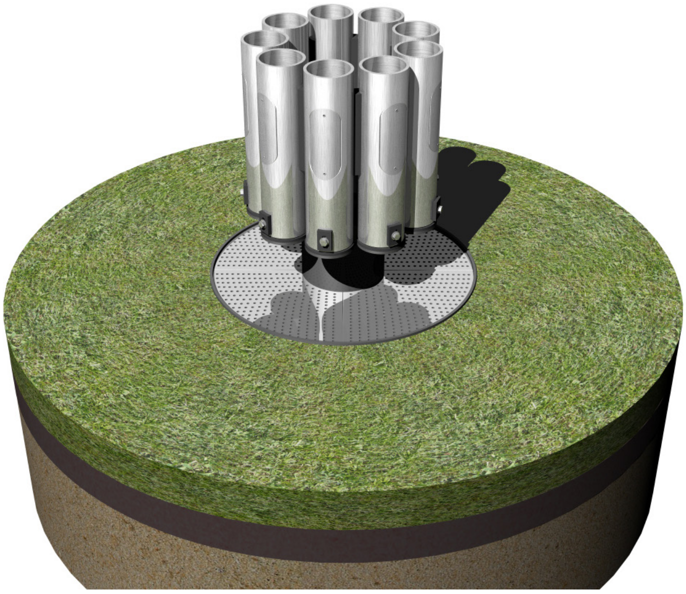


[176] FLOOR FINISHES BEARING OPTION ONE

On the left page the detailing of the first option is visualised. The metal plate consists of a nine seperate consoles - with a hole of a negative threatend for mounting purposes of the tubes - and a circular ring for covering in a later stage of the installation. Between the concrete foundation block and the metal plate is some space for adjusting the plate as well as the follow-up treatment of filling this space up with grout once the plate is correctly set. The next step is to place compression rings over the nine consoles. As explained in the previous chapter, the tubes will be connected together and installed onto the bearing in groups of three. The tubes will be inserted through a singular metal disk that in the end will cover up the space between the bundle of tubes. When all the tubes are placed correctly using the slotted holes, the disk can be lowered which will visually cover the connections. In the images above two types of floor finishes are illustrated; one with a circular disk for a park for example and one with a rectangular cover for a pavement or a city square.



[177] BEARING OPTION TWO



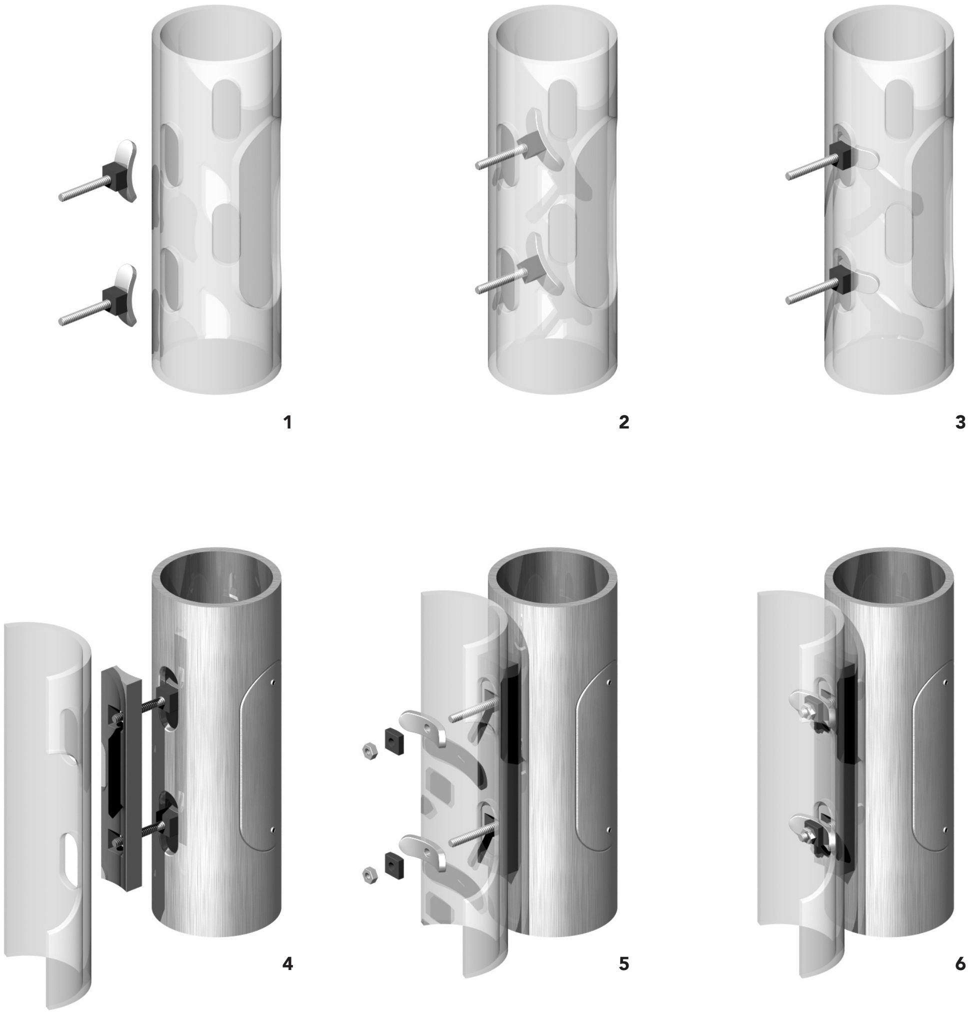
[178] FLOOR FINISH BEARING OPTION TWO

The second option is a more exposed option, where the waterdrainage is visible. The metal plate is highered by a cylinder in the center. This cylinder is the ‘foot’ for the nine consoles on which the tubes will be mounted the same way as is done with the first option. The advantage of this option is that the disk is installed in a much easier way compared to the previous design. This disk is perforated because this is the place where the water will flow via the metal plate and the concrete pedestal into the sewerage. The downside of this design is that the adjustment of the metal plate is more challenging. Besides, in practice the waterflow will not be a clean waterfall, but more likely a dripping tube of rainwater that is a bit filthy due to the roof. However, the design feature that the watermanagement is solved into the tubes of the branching structure is visualised in such a way that it is integrated. Like is the case with the first option, there are of course multiple floor finishes possible with this design.

6.5.2 Tube connection

The second stage of detailing is to mutually connect the tubes. Again, this needs to be demountable and I wanted to solve this connection internally, so that the connection is not visible from outside. The only thing that will be visible is the mounting- and assembly cap to excess this detail. This cap is not only for inserting and fixing this connection, it is also for maintenance. If a tube become plugged, there always needs to be a way to excess the inside of the tube.

The first step of this detail is to insert a wedge that is fixed to a threadend. This wedge will be turned ninety degrees and pulled back like is shown on illustration two and three on the next page. The piece of hardened plastic (POM) prevents the wedge from rotating back. Then a spacer will be placed that will

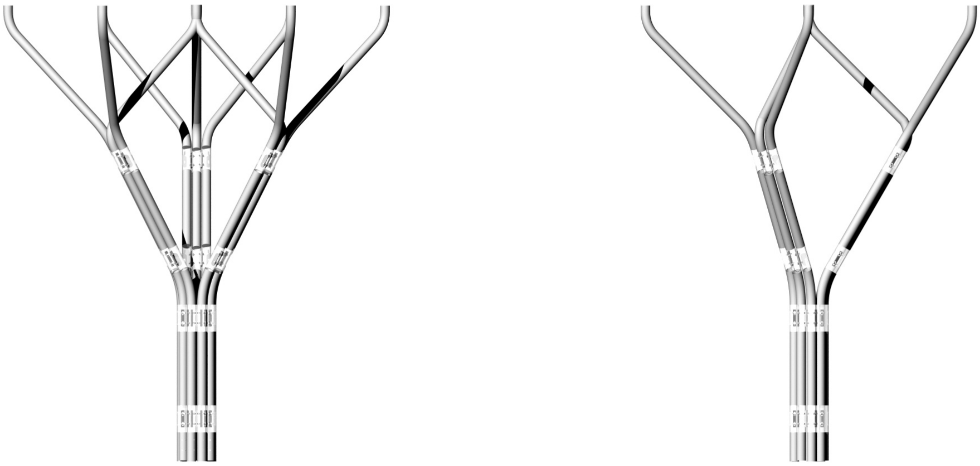


[179] TUBE CONNECTOR DETAILING SEQUENCE

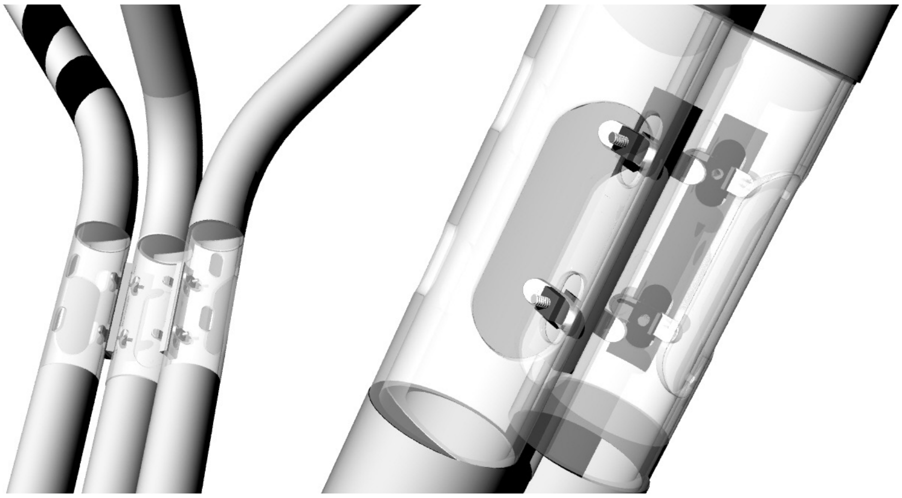
keep the tubes at the exact preferable distance from each other. The connection is finalized by adding another wedge and bolting this together for a tight fix. At first, I modelled the same detail with only one connection. However, in a later stage of the design process I realised that if one of the drainage holes of the roof gets plugged, there will be an excentricity in the structure, resulting in a torsional force. Like is said earlier, the structure needs to be able to carry this torsional forces. The easiest way is to solve this in an integrative way with the detail I already had. Now, this detail was not stiff in the torsional direction, since this connection is performed perpendicular to the tubes at such a small surface. Concluding, the next step was to edit this tube connection is such a way that this force transfer becomes possible. As visible in the images above, a second connection is added so that these connections together work as a torsion-stiff ring of 200 mm. The excess caps are 200 mm by 100 mm which should be enough to insert the components and mount them with tooling. The advantage of this connection is that it is a generic detail that can be used in every configurations of tubes. On the next page you can find an illustration how this connection is used at the base with the bundle of nine.

[180] TUBE CONNECTOR OF BASE BUNDLE

This tube connection is performed multiple times across the length of the tube, resulting in the ability to excess the tube on various points.

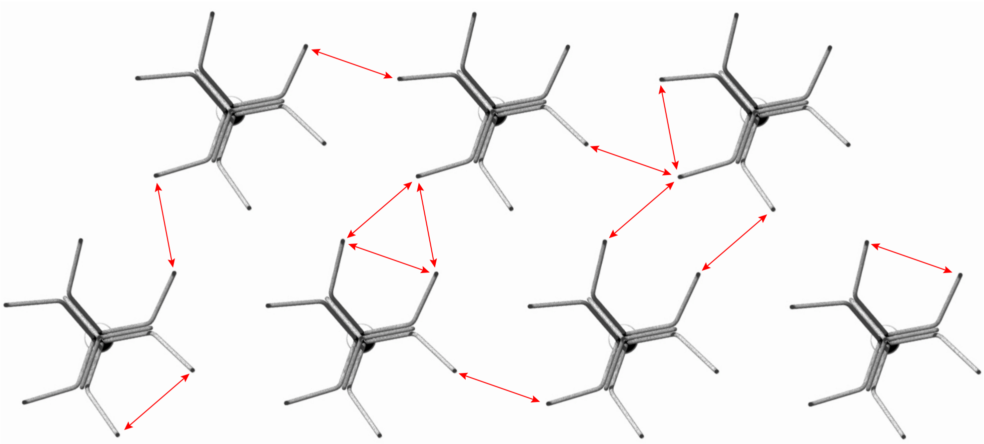


[181] THE FOUR TUBE CONNECTOR LOCATIONS



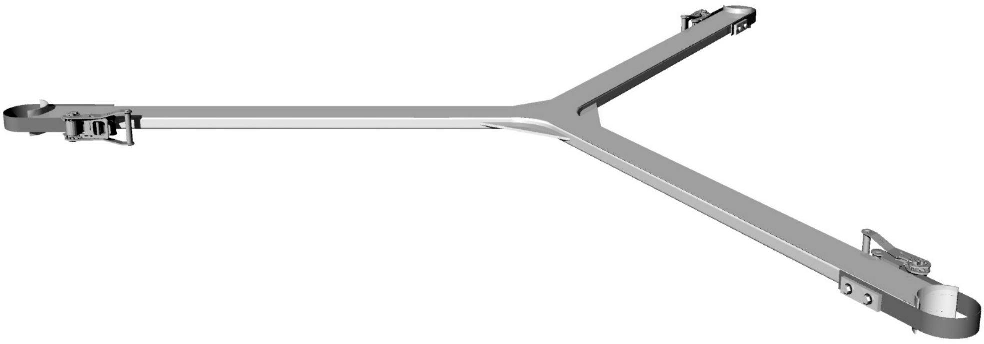
[182] TUBE CONNECTOR IMPRESSION

6.5.3 Strapping tool



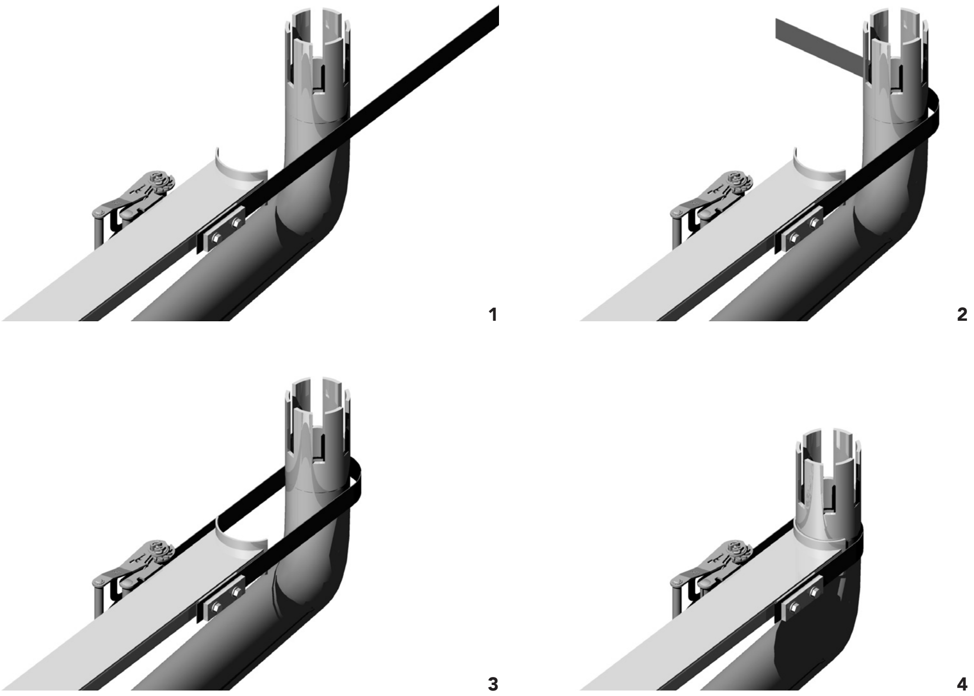
[183] TOPTIER DISTANCES

As is shown in the image above, the distances between the toptier branches is equal on all sides. At least, this should be the case before installing the u-profiles and anticlastic panels. There will always be deviations in bending the tubes and mounting them onto the bearing. To overcome these tolerances, to get from allowance in centimeters to allowance in millimeters, and to adjust them in such a precise manner, I came up with a so called 'strapping tool'. This strapping tool is a threeway steel frame of definite dimensions. In order to set the tubes of a desired distance from each other, multiple tools can be used at once, depending on the level of deviation. However, this is not the only method to bring the tubes closer to each other. After mounting the tubes onto the foundation, the metal console is still adjustable, changing the angle of the entire branching structure.



[184] STRAPPING TOOL

The strapping tool works like a webbing belt and is simple to use. A strap can be wrapped around the top of the branch and can be inserted into the tensioner. This tensioner can haul the strap tighter, as the branch slowly sets into the desired position. How this exactly works is visualised in the illustrations on the next page.

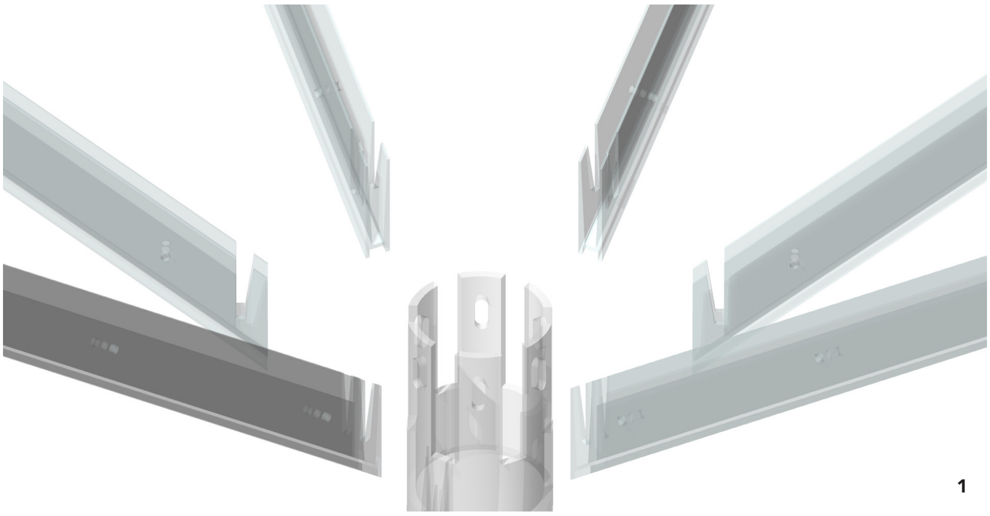


[185] STRAPPING TOOL PRINCIPLE

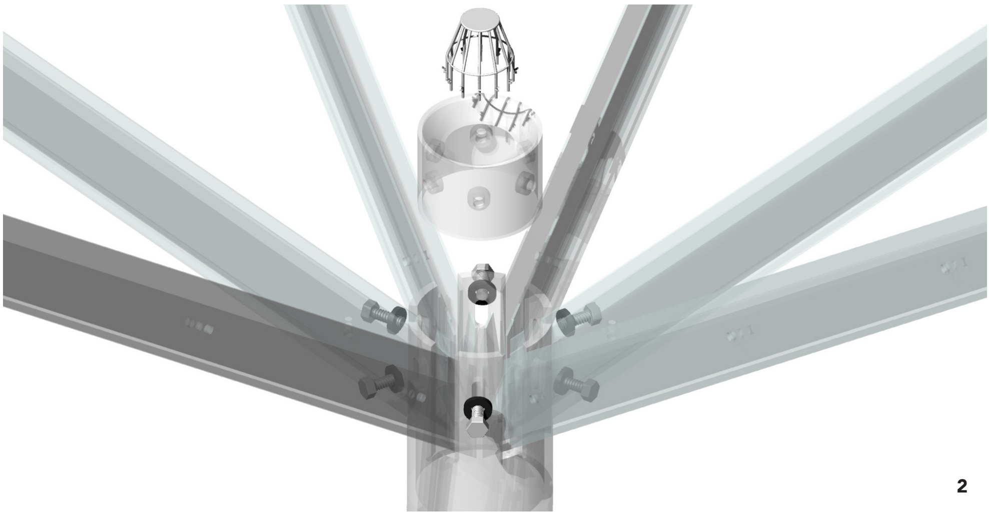
6.5.4 Branch-to-leaf

The last detail concerns the connection between the steel tubes as branching structure and the double curved GFRP roof panels as roof structure. This is a challenging connection, since the watermanagement needed to be solved as well.

As was described in the production of the tubes; grooves are CNC-milled at the very end of the tube. The u-profiles, in groups of three will be placed into these grooves, as was briefly mentioned in the method statement as well. I wanted to come up with a detail that is easy accessible after mounting everything together, so I came up with a mounting ring that will be placed inside the tube. Since the u-profiles also have grooves at the end, this ring restrict the profiles from translating in the x- and y-direction, holding everything together. This ring can then easily be restricted in the z-direction as well by six bolts as shown in the second image of the next page. Besides this, a tube protecter can als be installed inside the mounting ring. This should protect the tube from leaves or other debris. Once the u-profiles and mounthing ring is installed, the anticlastic panels can be lowered into the profiles. The sides of the profiles are tapered, so that the panels will slide into its right position. The u-profiles have a an upright piece at the center, to accommodate for the panels to have the exact distance from each other when the panels are mounted. This mounting is done from beneath, so that in case of a maintenance issue or a leak, a singular panel can easily be disassembled and the problem can be fixed. The panels are mounted on five points on every side, being a distance of about 400 mm. On the next page the sequence of this detailing is visualised.



1



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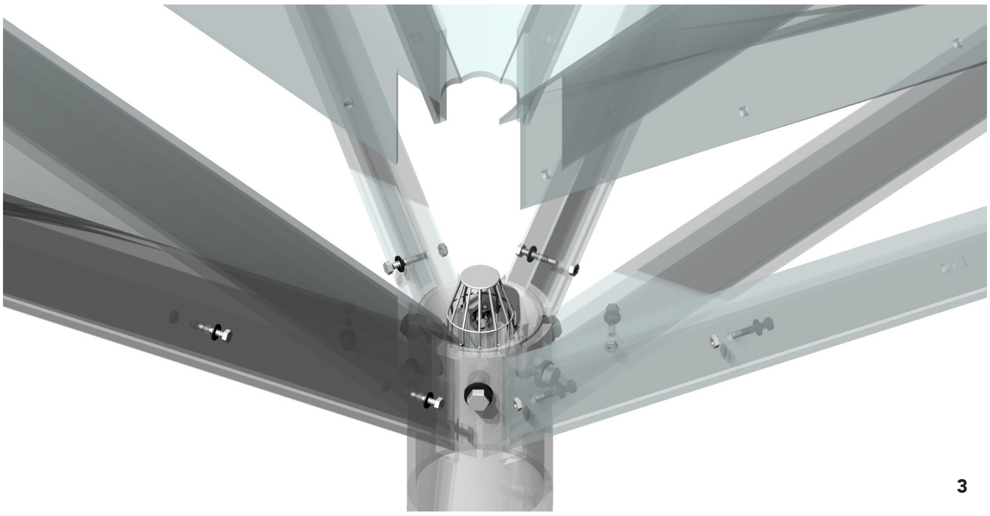

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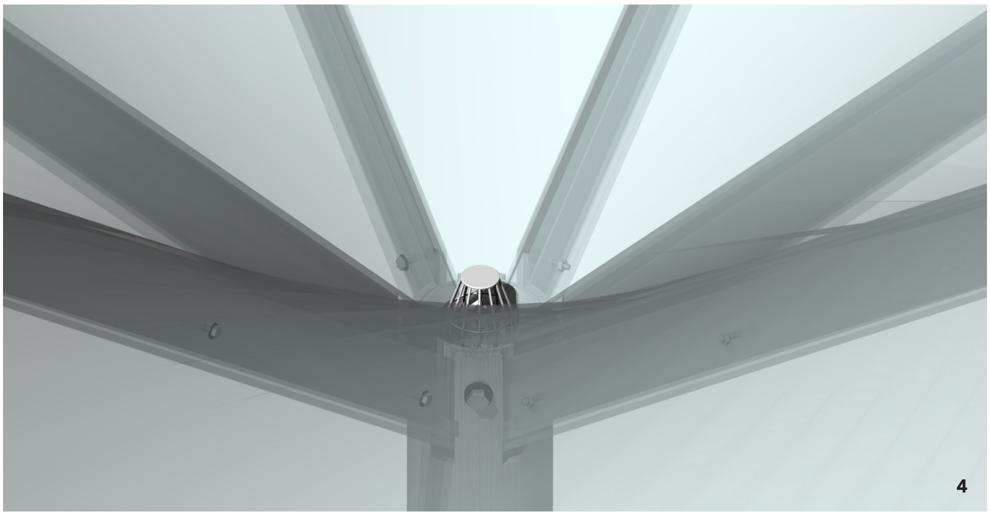
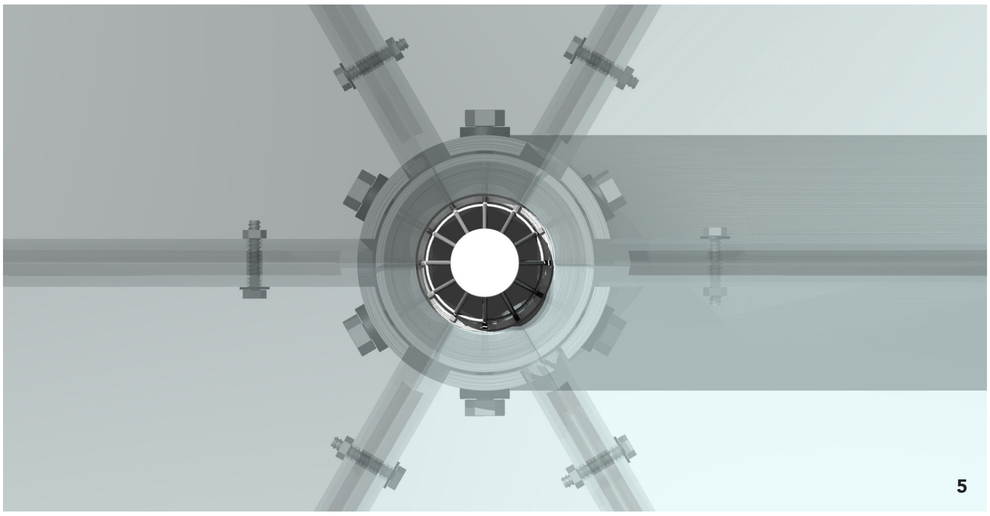
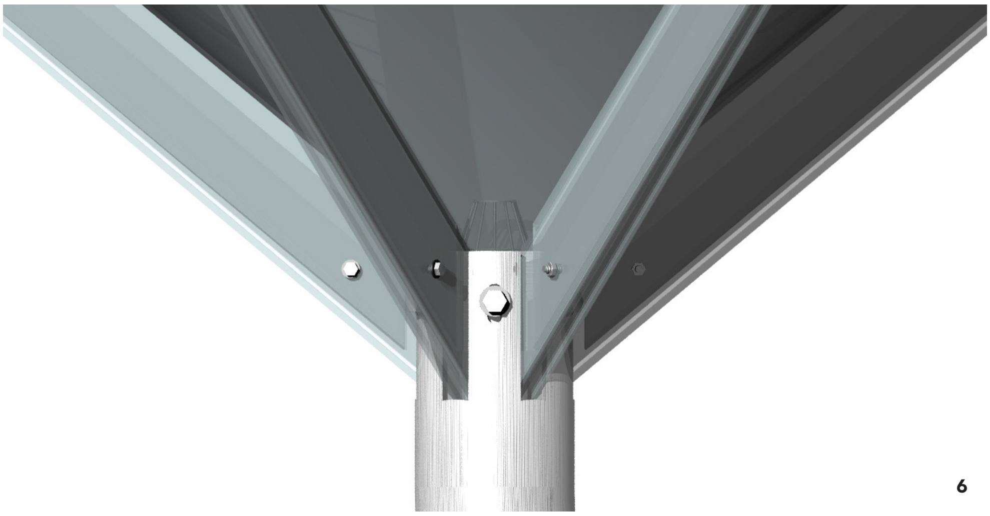






[186] BRANCH-TO-LEAF DETAILING SEQUENCE

“You can look anywhere and find inspiration.”

Frank Gehry

application

7. DESIGN MODELLING

7.1 MODELLING

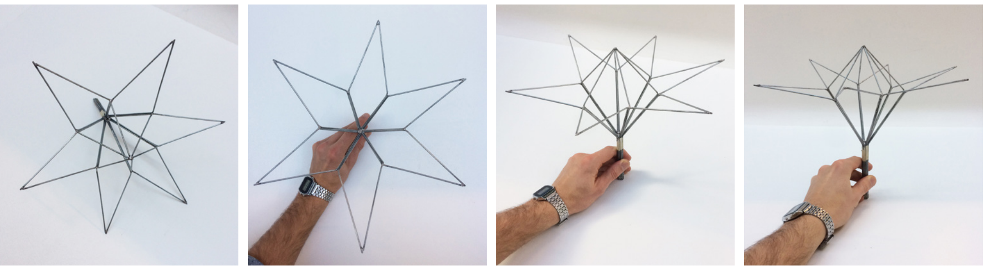
Halfway through the graduation process, it was time to start making physical models in order to better understand the possibilities when it comes to design freedom, structure. I reserved about a month for this and in doing so, I realised that I was gaining a lot of knowledge regarding the design and the issues that I faced.

During this month I explored several production methods as well as different materials. A list of the main models that were made:

- Tube configuration and bending - aluminum and PVC
    - 2 iteration branching structure - steel rod
    - 3 iteration branching structure - steel rod
    - 5 branchbundelings - PVC
    - Tube bending - aluminum
- 3D printing - gypsum and PLA
    - 1 singular branch - gypsum
    - 3 combined branches - gypsum
    - Mould for anticlastic roof - PLA
- XPS Foam cutting rig - steel rod and converter
    - Double curved XPS moulds
    - Glass fiber + Epoxy
- Vacuum forming - polyethylene terephthalate glycol (PET-G)
    - 1 vacuum formed panel (structural performance of ribs)
    - 3 vacuumformed panels
- CNC-milling - wood
    - Wooden mould

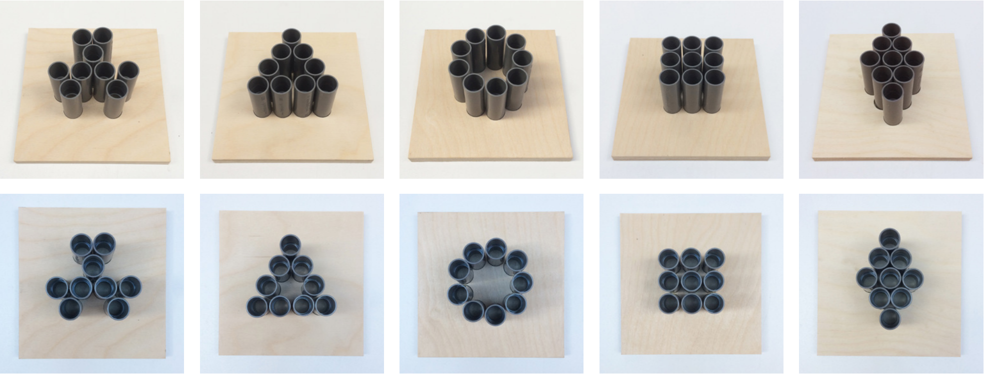
Like was described in the production phase, the methods for making physical models were based on the two different structural parts; the tubes and the anticlastic panels. Below you can find photo's of the models I made during the design development phase. These models were made simultaneously with the design development steps, since these models had great influence on the decisions regarding the building method. The models that are shown below are not in a chronological order, but are categorised between tubes and double curved panels.

7.2 TUBEBENDING & CONFIGURATIONS



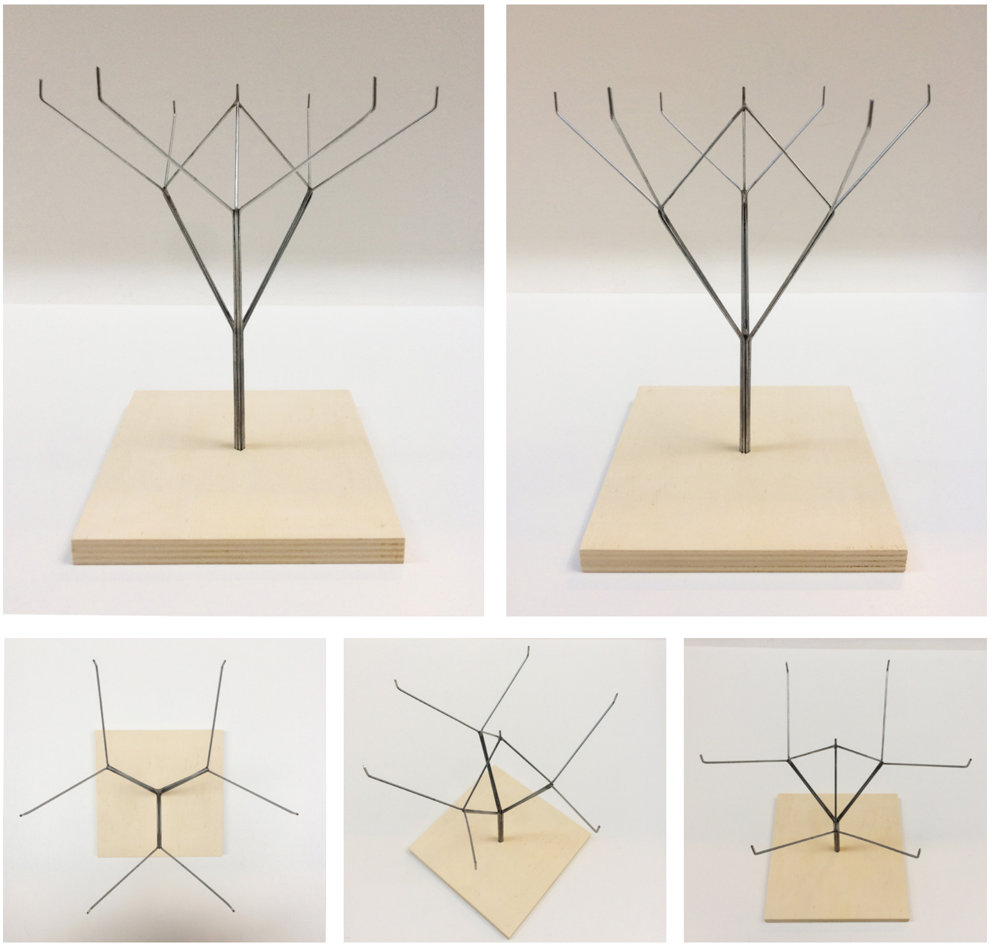
[187] STEEL ROD MODEL TWO ITERATED BRANCHING STRUCTURE

On page 131, the first four branching structures were presented. Two with a squared quadrilateral grid and two with a hexagonal quadrilateral grid. The first trial with a steel rod was the two-iterated hexagonal quadrilateral structure, which can be seen in the images above. This was not an easy model, since eighteen steel rods needed to be joined together at the base of the structure. These eighteen will split up in groups of three which will again split up singulary. The actual production and bending of the tubes in such a way that they can be bundled in a group of eighteen is very challenging and requires exact computational design which results in a lot of unique tubes.
Further development of the design as a whole led to the elimination of the toptier branches of the three-iterated hexagonal quadrilateral structure, which was proposed firstly on page 137. The next step was to find configurations on how the bundle nine seperate tubes together. Simple models were made with small PVC tubes as can be seen below.



[188] BRANCHINGBUNDLE CONFIGURATIONS

The choice for the circular configuration was based on the way the branches will be split. Because of this reason the rectangular- as well as the diamond-shaped configuration were eliminated. The triangular configurations are directional. Since I want this new structural principle to be suitable for various applications, the choice fell on the circular configuration. This circular configuration was then made out of steel rod, like was also done with the two-iterated structure.

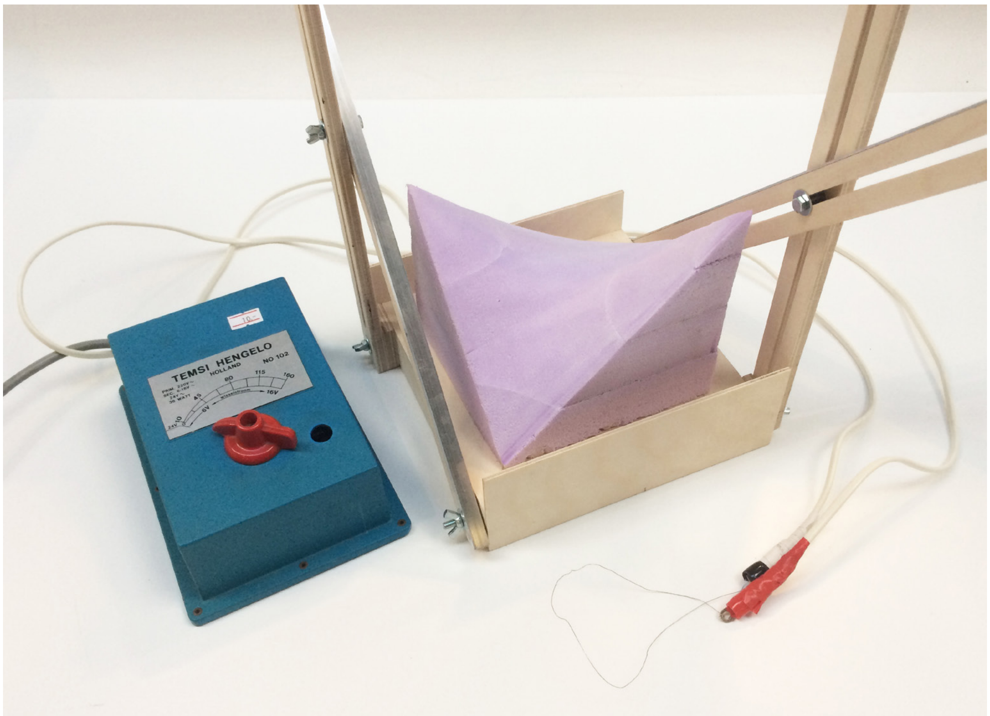


[189] STEEL ROD MODEL THREE ITERATED BRANCHING STRUCTURE

Above you can find the steel rod model of the three-iterated structure, without its toptier. This was eliminated like mentioned earlier. This model was much easier to make, since the base had only nine seperate rods in a circular configuration.

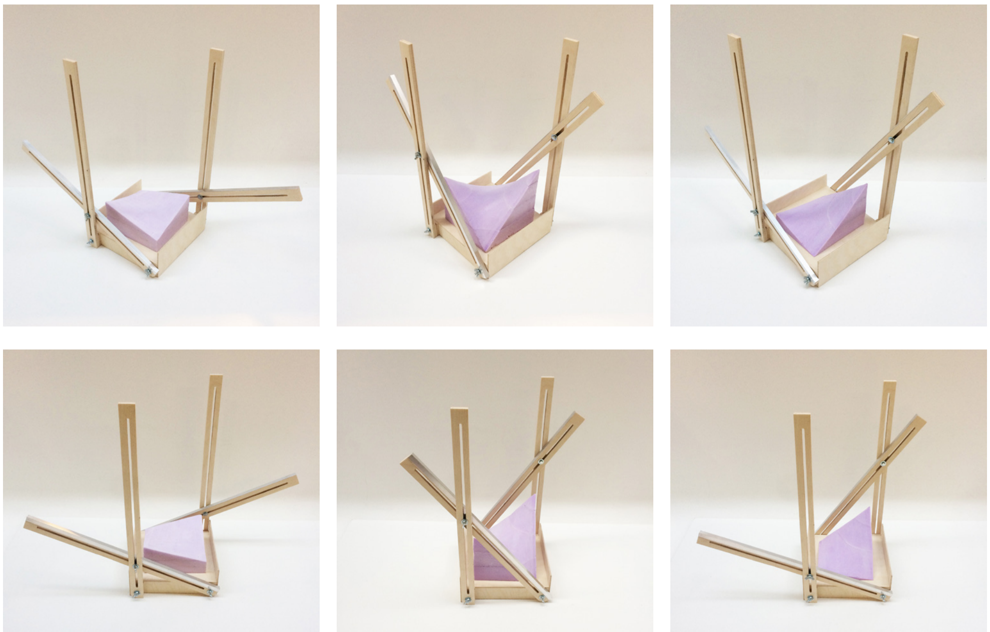
7.3 XPS FOAM CUTTING

Simultaneously, the modelling of the anticlastic surfaces was done as well. I began with making several double curved moulds. The first mould was made out of XPS foam. In the early stage of my graduation I was planning on designing a branching structure where the anticlastic panels could be produced uniquely. Therefore I was looking for a way how unique moulds could easily be produced. The solution was the design of a cutting rig where a steel rod under a small current could cut through the foam. The small current in the rod makes the rod warm and in this way it can melt throught the foam. The rig had the possibility to change the angles of the edges and therefore had the ability to provide for infinite unique moulds made out of XPS. On the next page you can find the set-up of the rig with the converter that provided the small current.

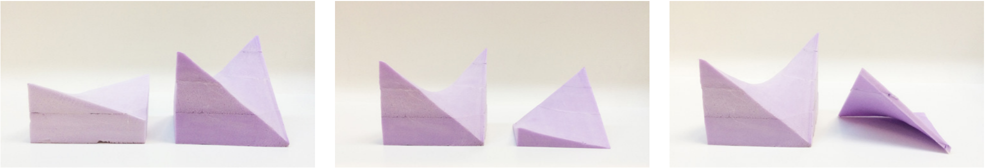


[190] XPS FOAM CUTTER SET-UP

Below you can find three options on how the vertical elements of the rig could be changed in angle, resulting in different XPS moulds. The first and second differ in amount of curvature, whereas the third option has a different angle per side. On the next page you can see the difference of this from the side, as well as the possibility of making a slice of XPS foam.



[191] VARIANTS IN ANGLES AND CURVATURE



[192] SIDE VIEW VARIANTS

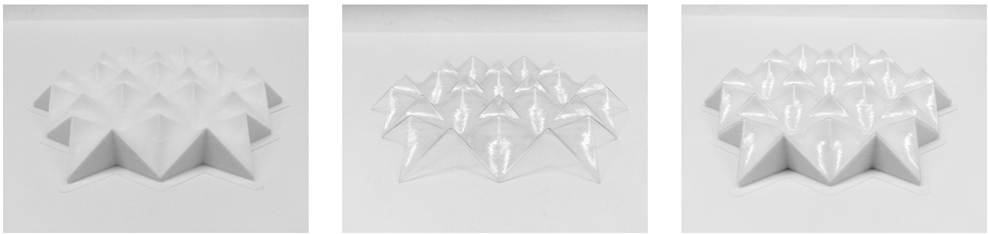
Since the material of the anticlastic panels will be glass-fiber reinforced polymer (GFRP), I wanted to make my own GFRP as well. I bought a sheet of glass-fiber as well as epoxy with a basis of polyester. Below you can find the result. The glass-fiber becomes transparant by the epoxy liquid and once dried, after about an hour, it becomes hard and relatively stiff. In practice, one would need to apply much more layers to get the desired structural performance. In my trial, I only used one singular layer. With the most obvious production method for making GFRP, the layers are pressed together, resulting in dozens of layers per centimeter thickness.

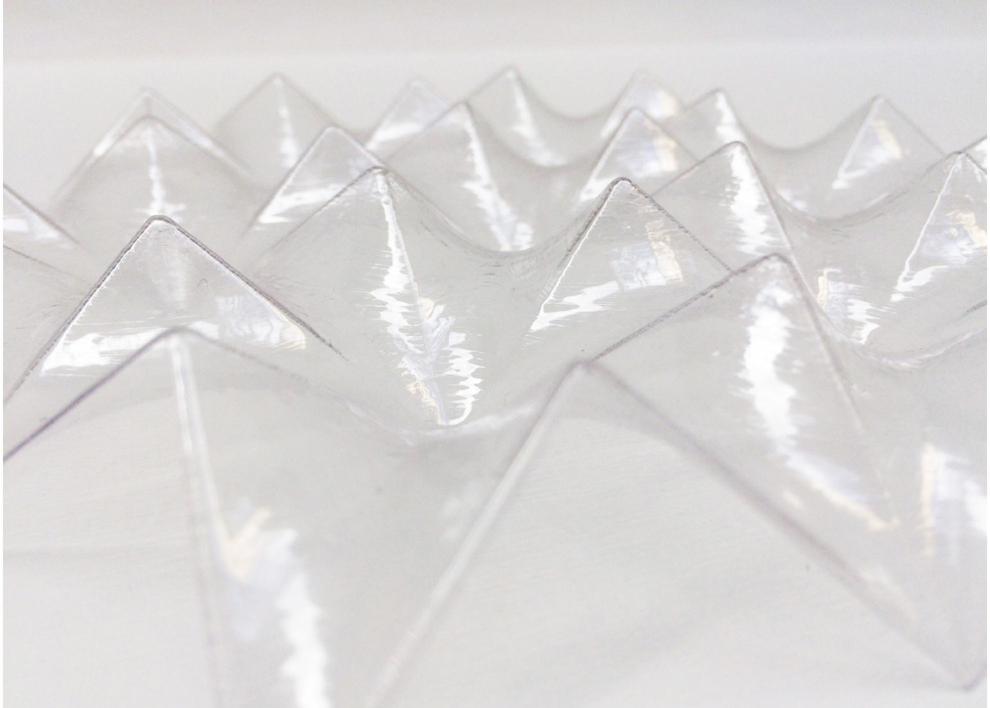
[193] GLASS-FIBER WITH EPOXY MODEL

7.4 MOULDS & VACUUM FORMING

Simultaneously, modelling of anticlastic surfaces was done as well. This mould was then used for vacuum forming with a sheet of polyethylene terephthalate glycol. The result was pleasing, since the 3D printing lines were corresponding with the waterdrainage on the anticlastic surfaces as well.

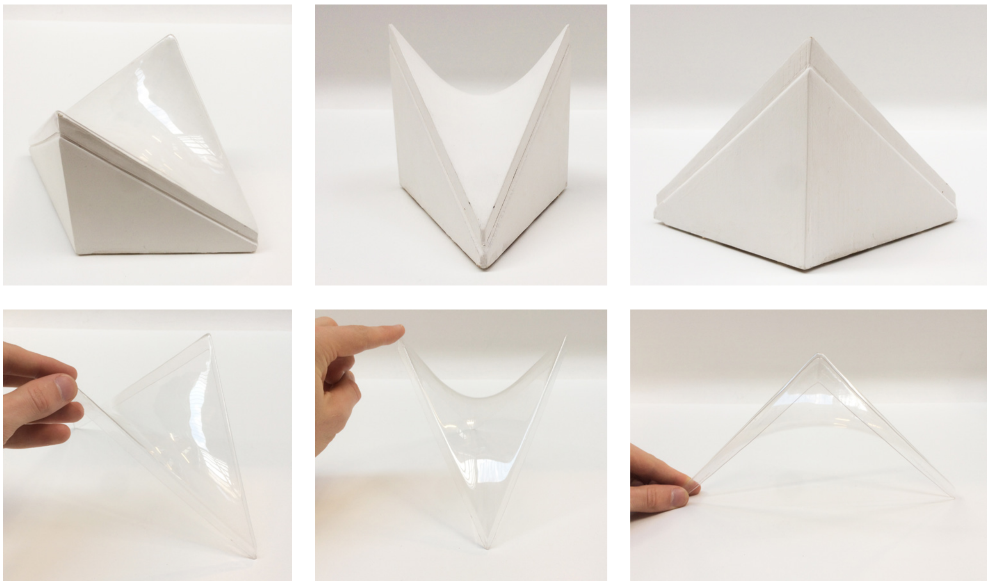
The second mould that was made was one singular anticlastic surface of a larger scale. This mould was made with the production method CNC-milling. The finish of this process was really rough, so it required quite some post-processing and sanding to get the desired smoothness.



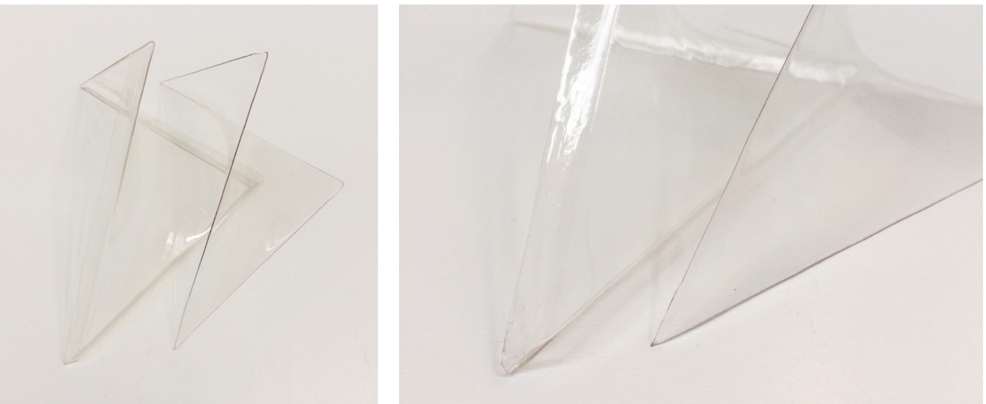


[194] VACUUM FORMED ROOF OF A SINGULAR BRANCHING STRUCTURE

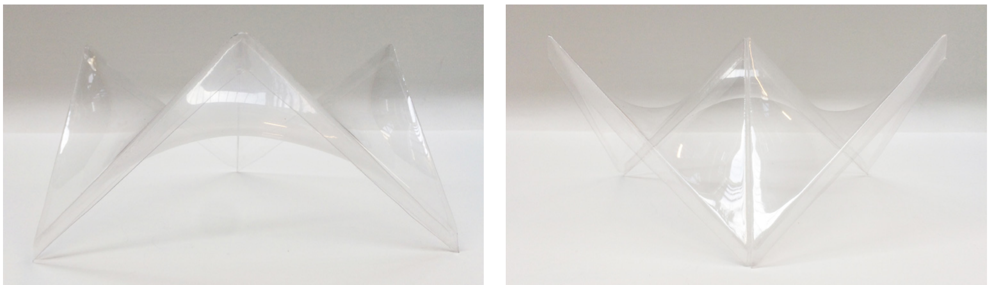
Once this was done, this mould also gave me the ability to test the difference between a surface with and a surface without straight generatrices, or ribs. It gave proper insight in the effect of the addition of ribs. The surface with the ribs is much more stiff compared to the flexible hypar without ribs. Also, combining three of those vacuum-formed surfaces, gave me further understanding on how to connect them together. This can be seen on the next page.



[195] VACUUM FORMED SINGULAR ANTICLASTIC PANEL



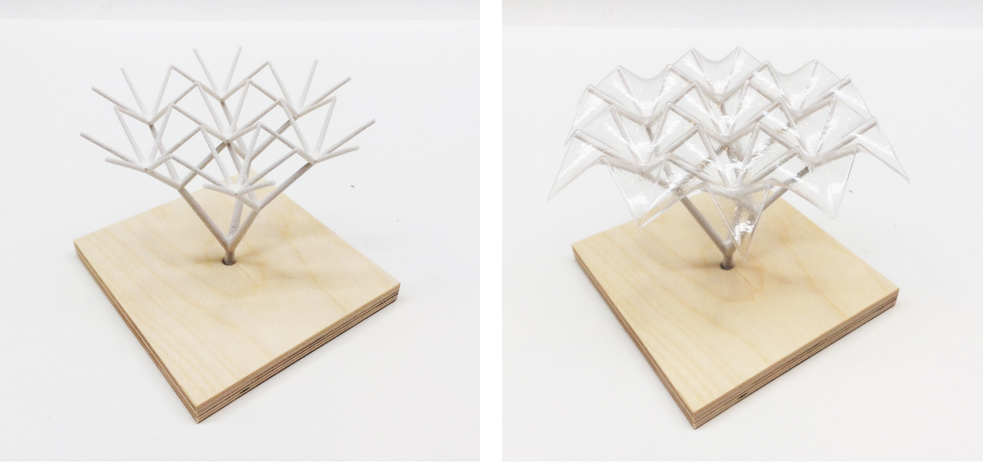
[196] SINGULAR PANEL WITH AND WITHOUT STRAIGHT GENERATRICES



[197] THREE PANELS COMBINED

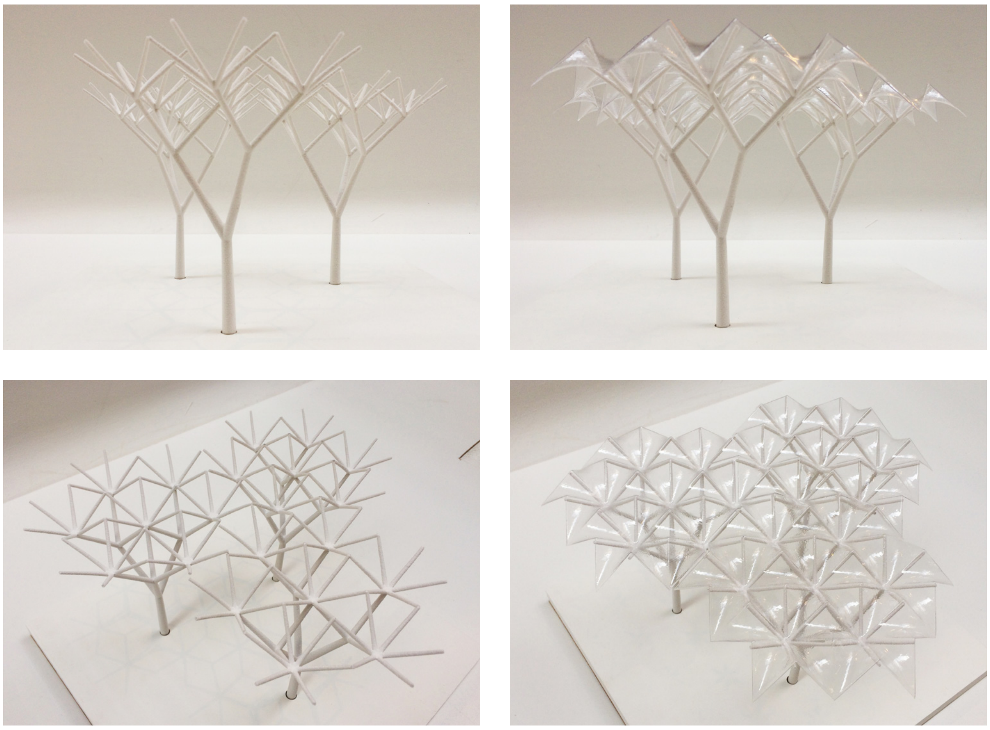
7.5 GYPSUM 3D PRINTING

The most easy way to see the branching structure together with the anticlastic roofsurfaces is to use the production method of 3D-printing. This was done with gypsum. The scale of this 3D printed branching structure is the same as the vacuum-formed sheet, so this would be a perfect fit on top of the branches. For this model, I chose to not eliminate the toptier, since this would give support to the vacuum formed sheet. This can be seen in the next images.

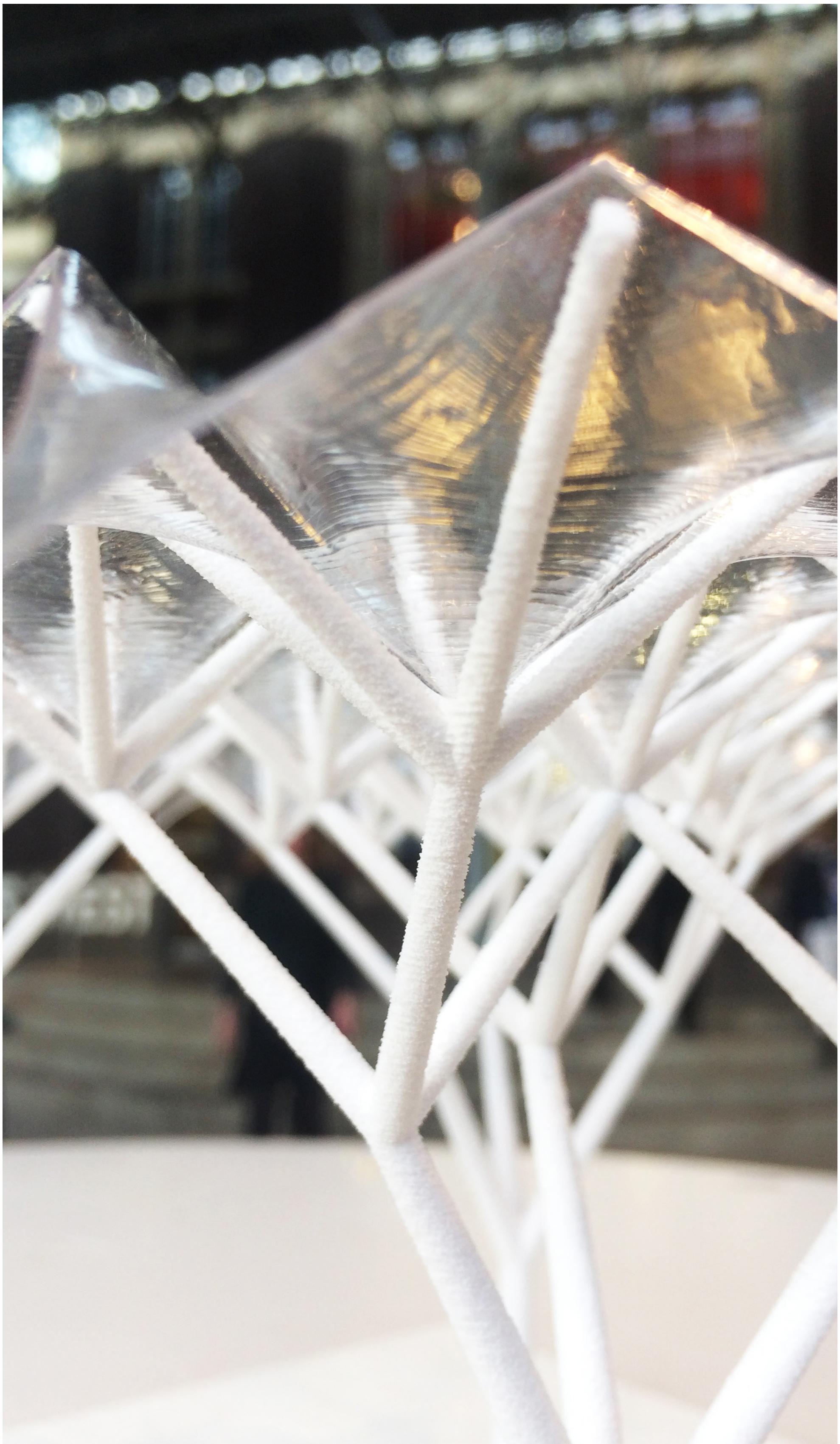


[198] SINGULAR 3D PRINTED GYPSUM BRANCHING STRUCTURE WITH VACUUM FORMED ROOF

The last model before my P4 was to make a group of branching structures, so that the interaction between the singular elements could be seen. Like I did with the method statement, I used a configuration of three branching structures, like shown in the images below.



[199] COMBINED 3D PRINTED GYPSUM BRANCHING STRUCTURES WITH VACUUM FORMED ROOF




WEST


OOST

“Architecture responds to functional outputs and environmental factors. Yet, fundamentally, it is important for me to stress the art in architecture to bring harmony.”

Santiago Calatrava

application

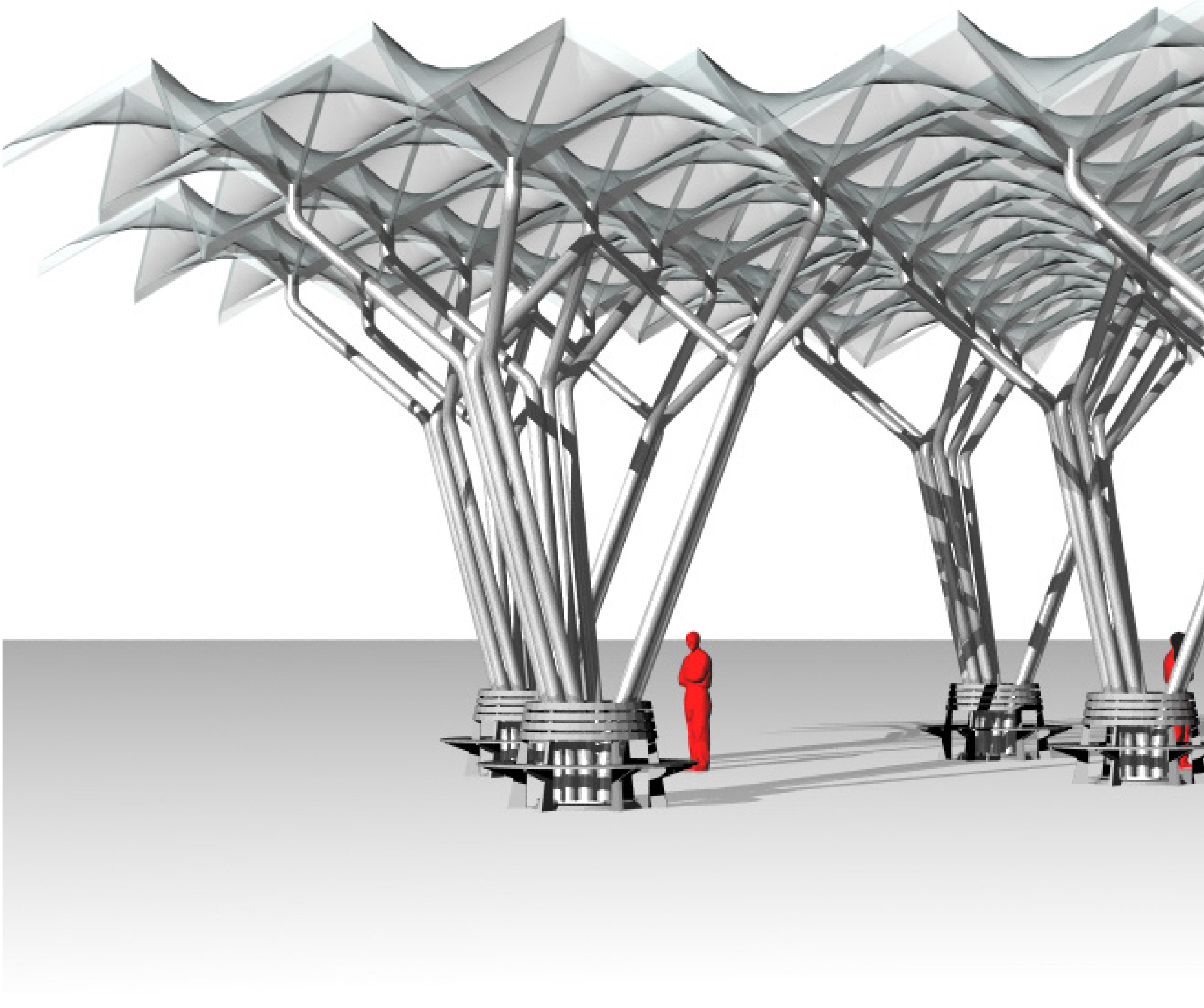
8. DESIGN INTENT

The objective from the early stage of this graduation project was to come up with a new structural principle that is emulated by natural design principles. The structural principle that is designed is a canopy that can be used for various applications.

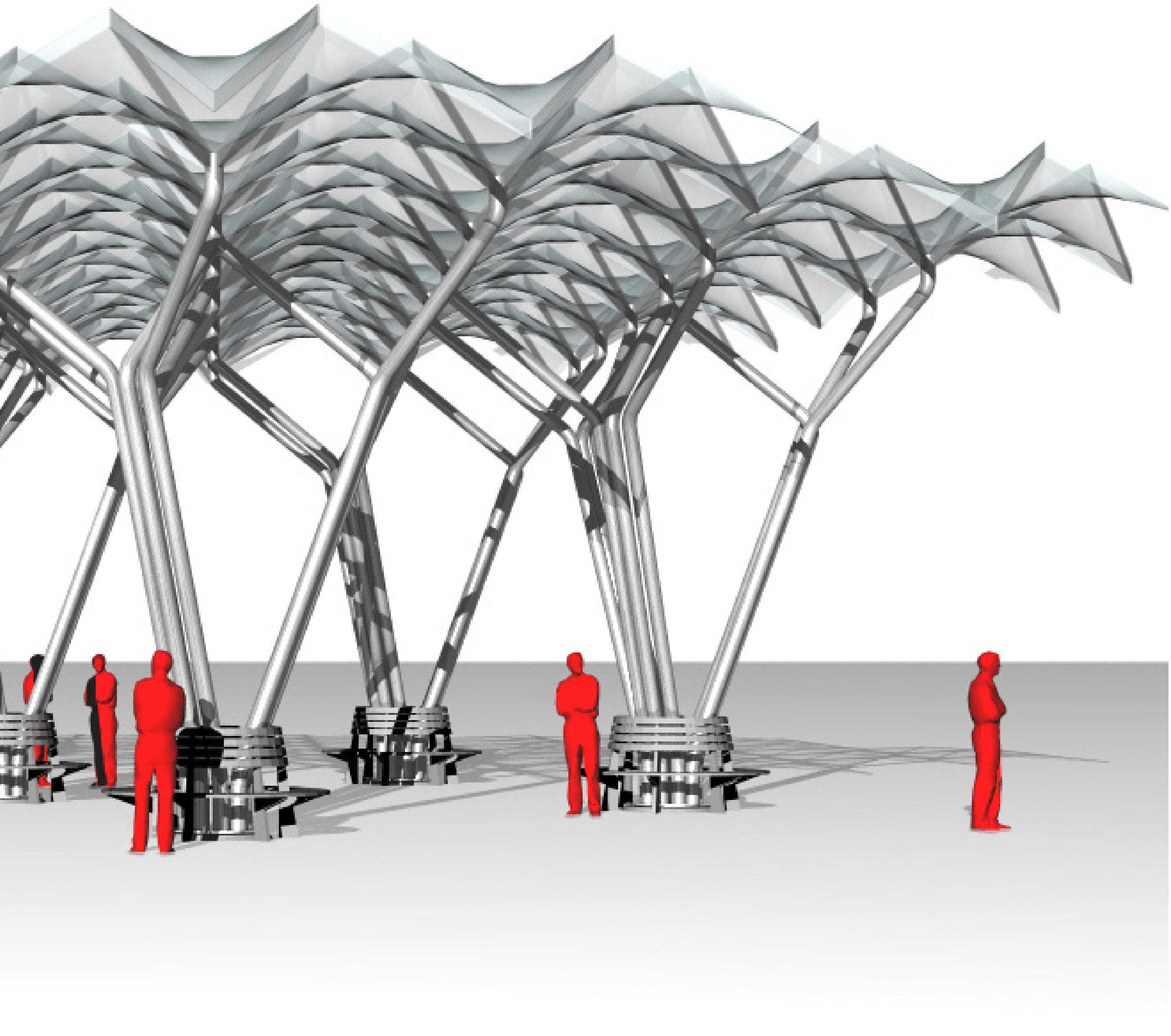
Mainly, there are two different categories:

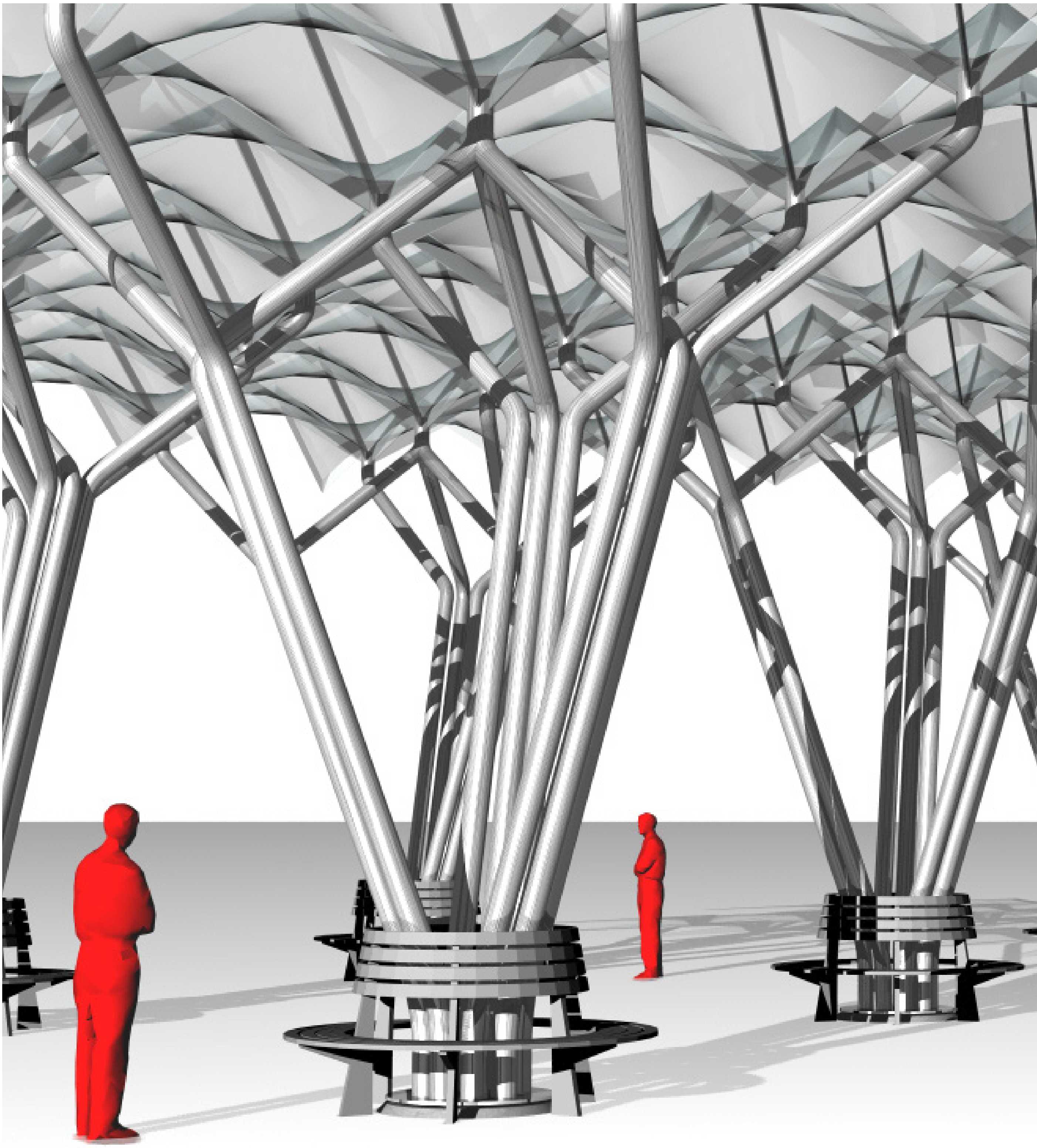
- Cultural Venues
    1. Open-air theaters
    2. Market square canopy
    3. City's icon - as touristic highlight
- Infrastructure facilities
    4. Tram / Busstation
    5. Airport departure halls

On the following pages a few impressions on possible applications are shown.

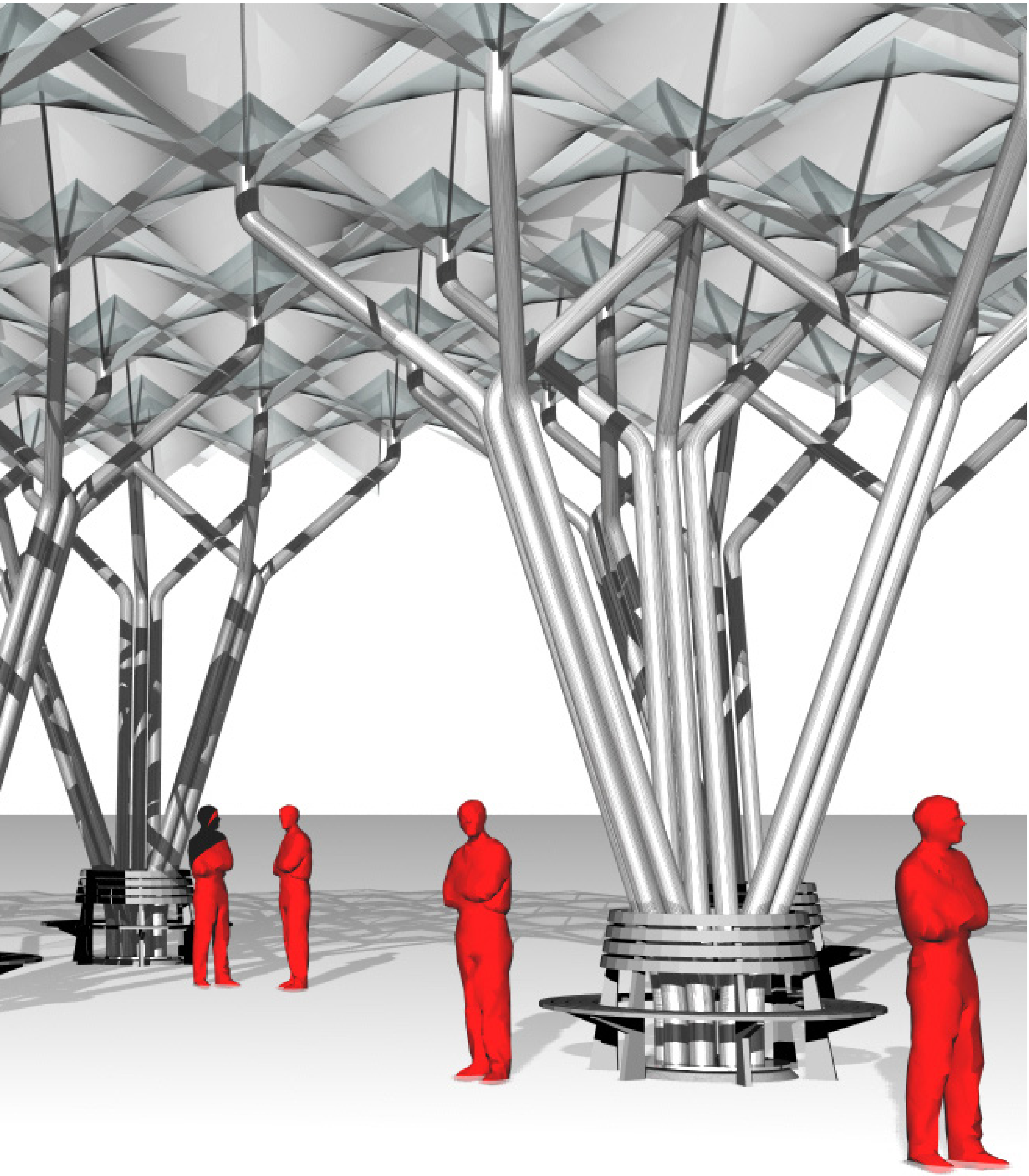


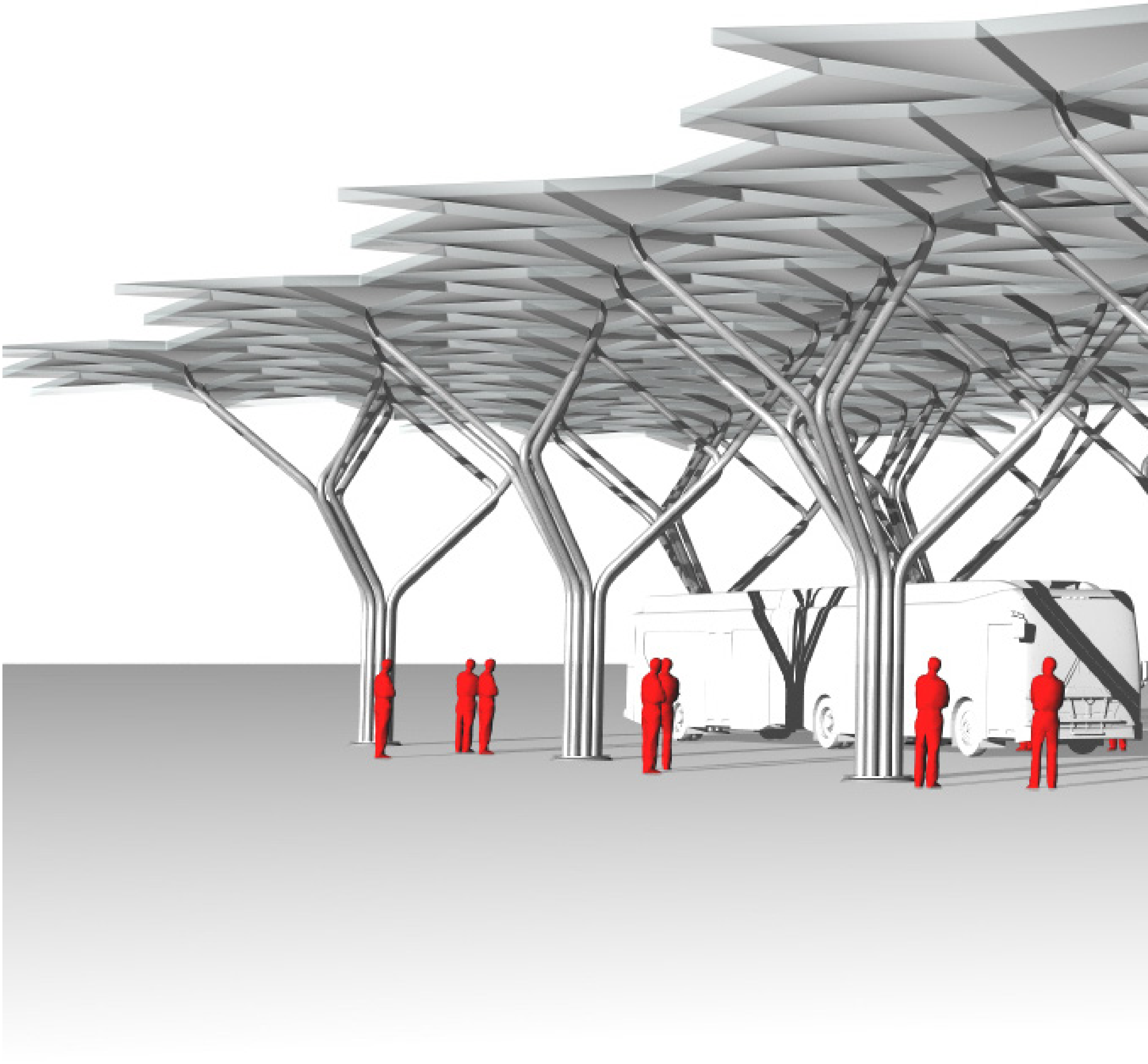
[200] INTENT 1 - OVERVIEW



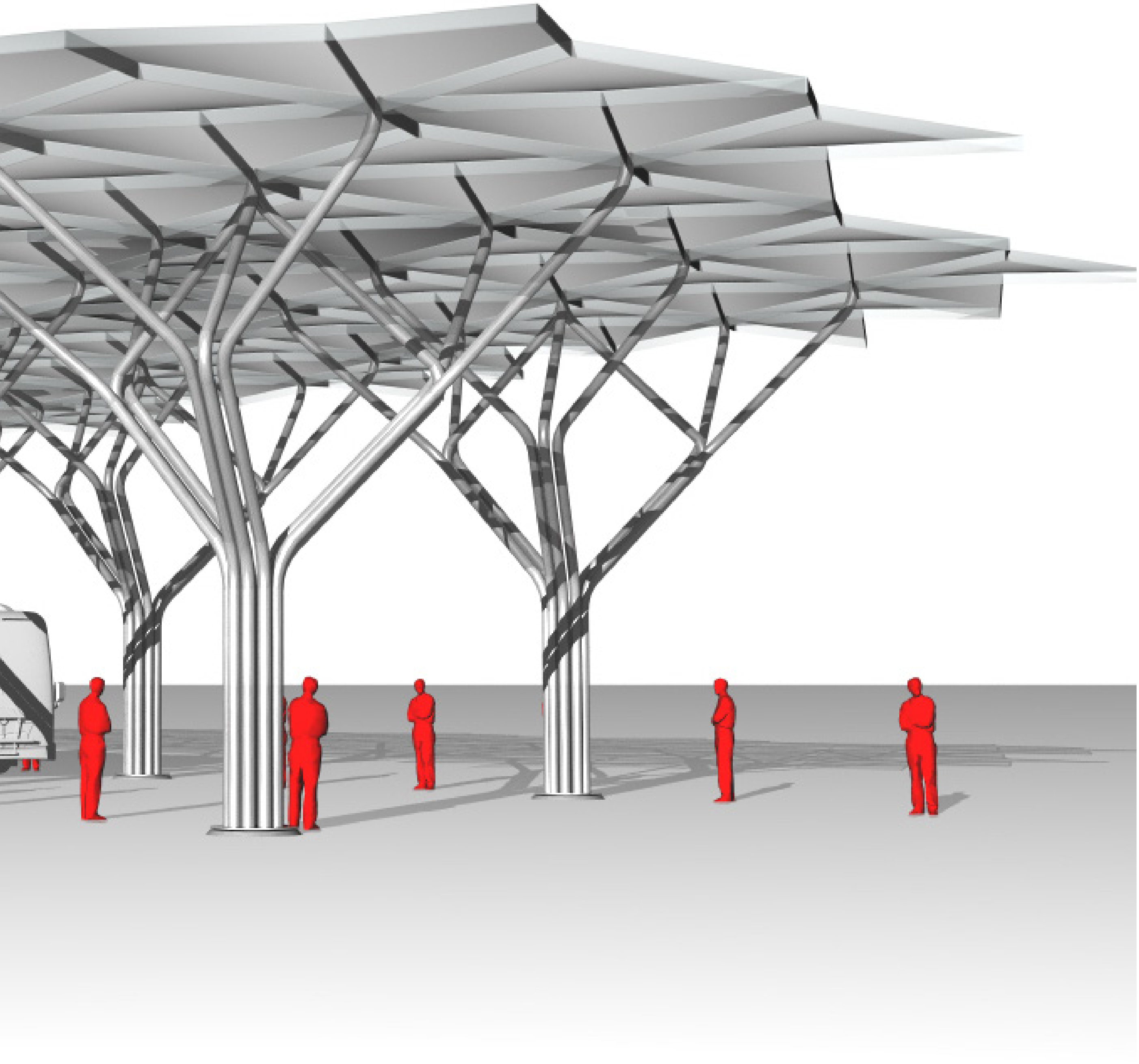


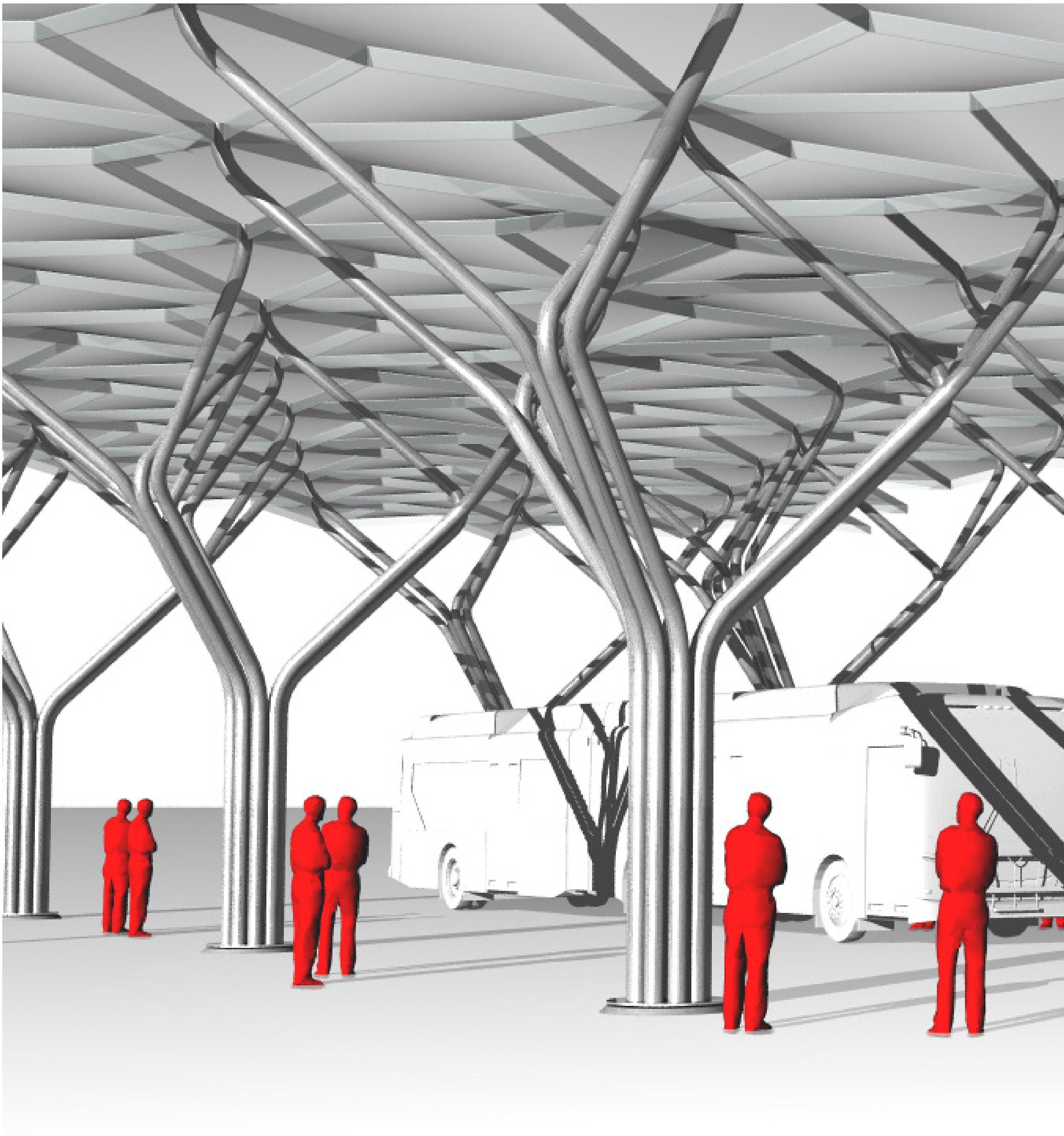
[201] INTENT 1 - ZOOM



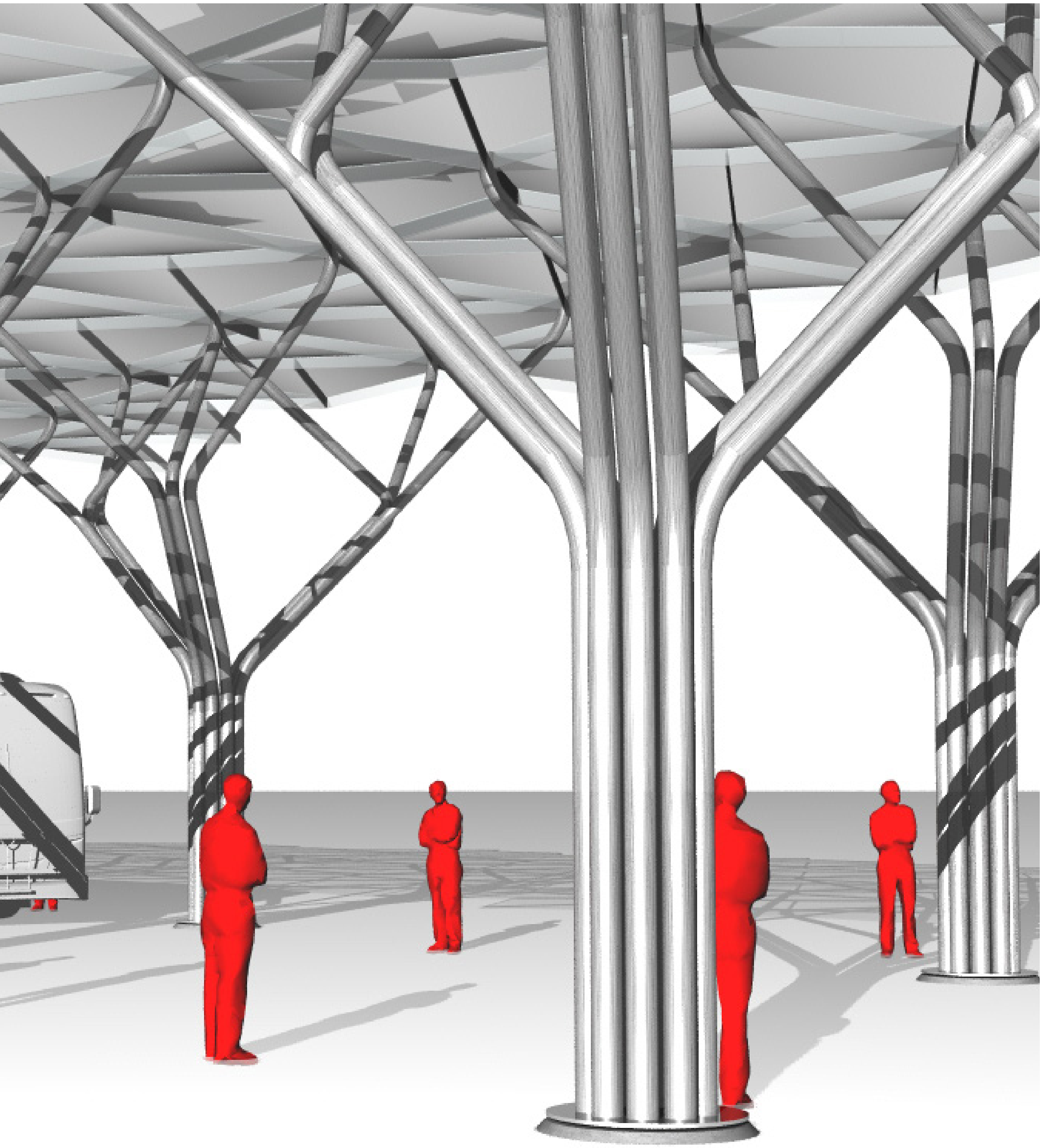


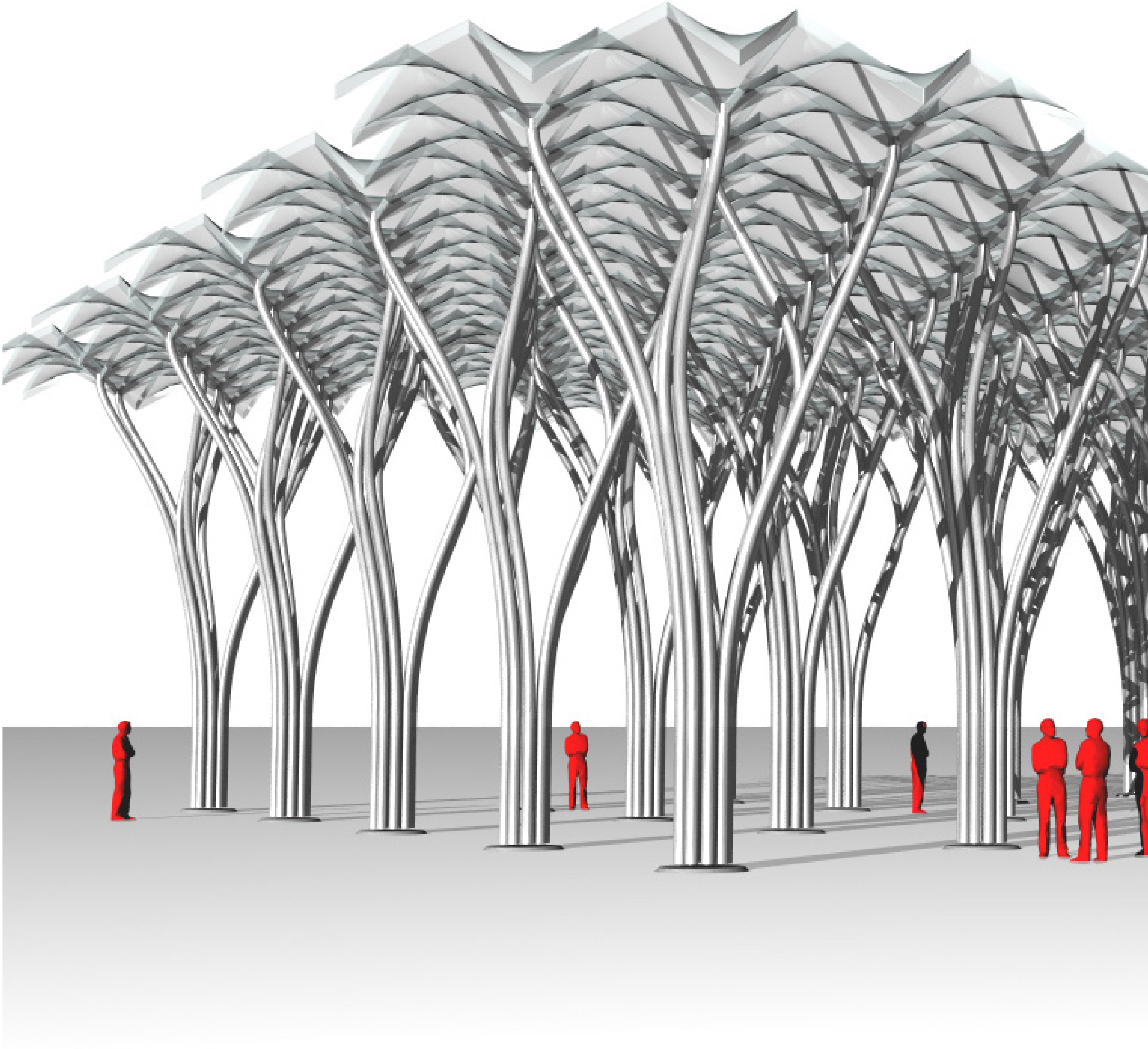
[202] INTENT 2 - OVERVIEW



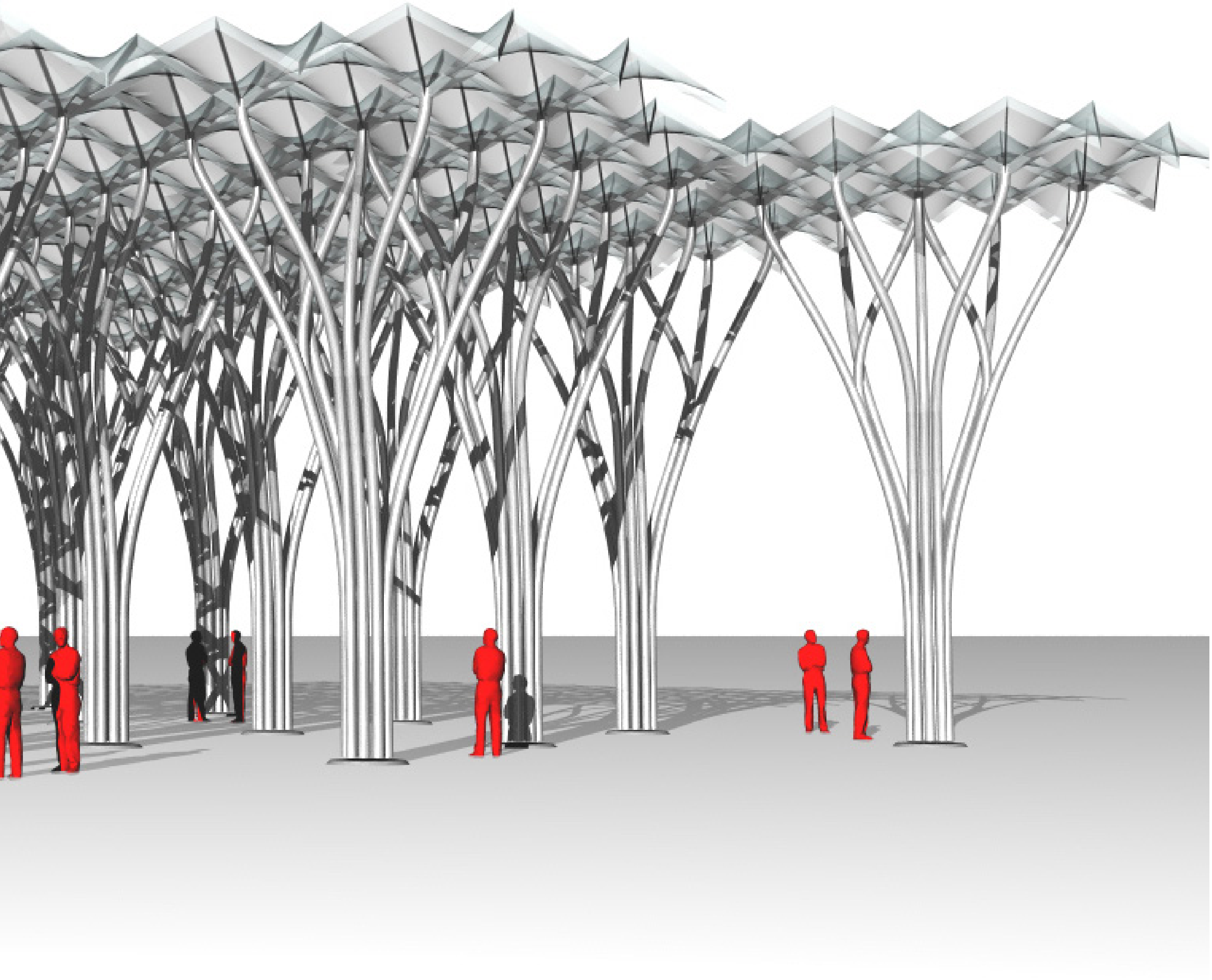


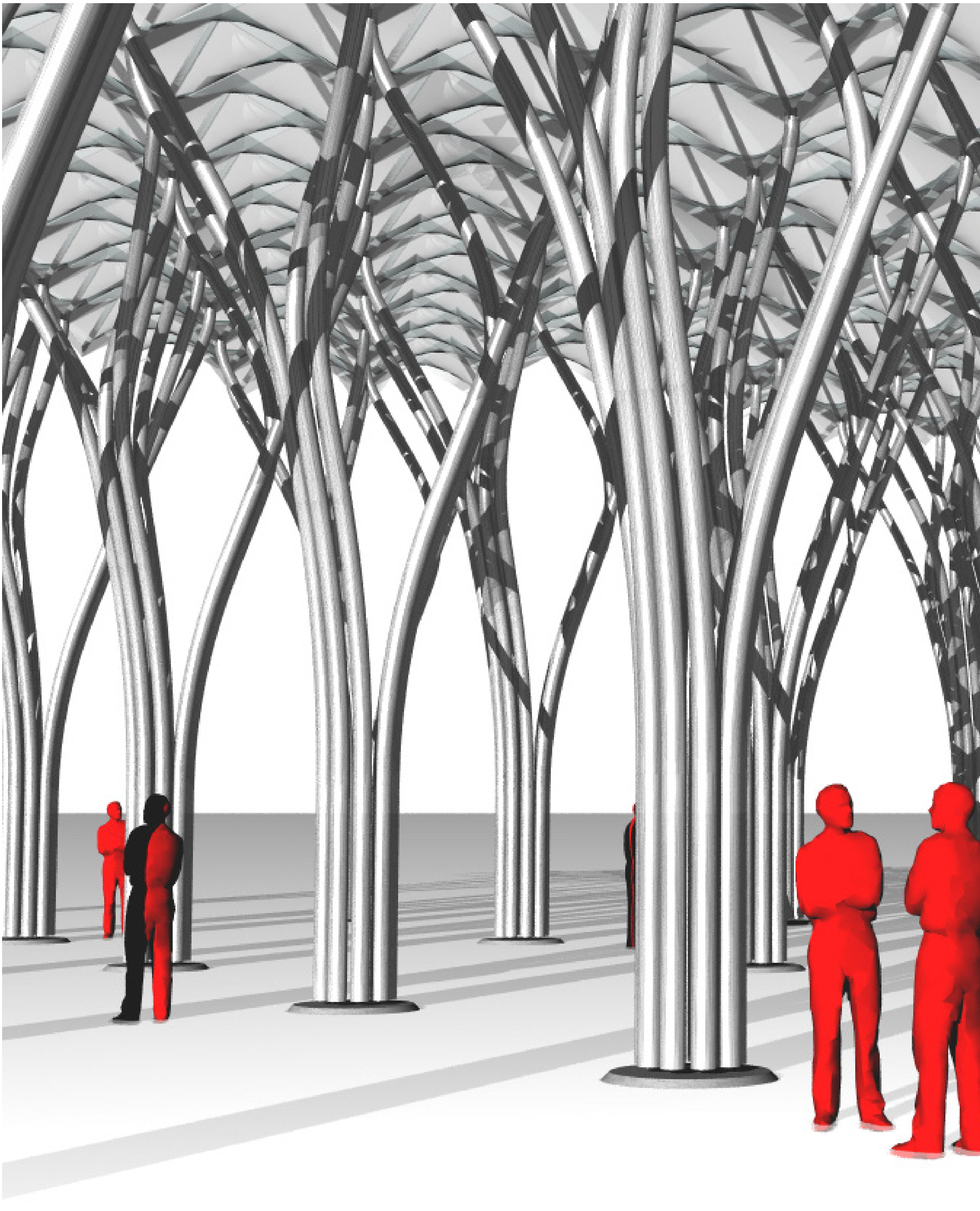
[203] INTENT 2 - ZOOM



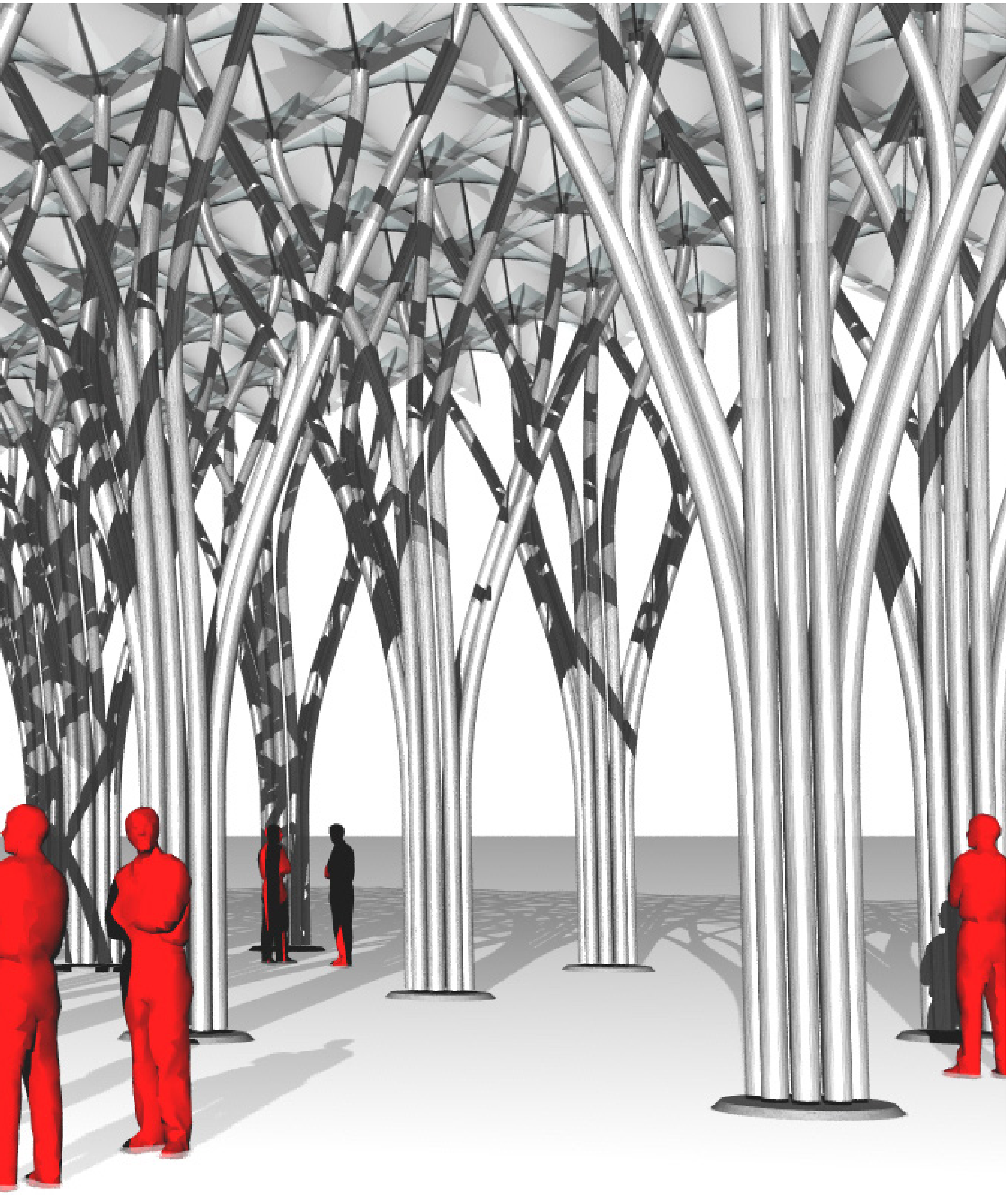


[204] INTENT 3 - OVERVIEW





[205] INTENT 3 - ZOOM



“Don’t fight forces, use them.”

R. Buckminster Fuller

application

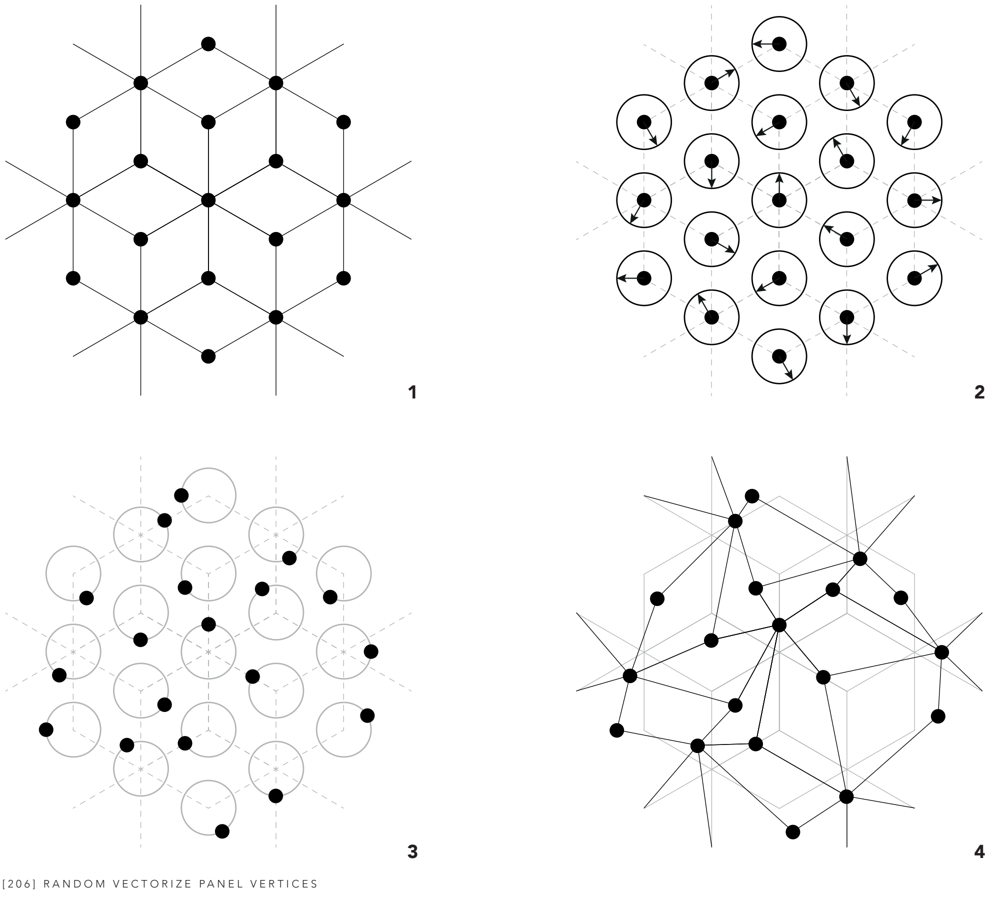
9. DESIGN FUNCTIONALITY

9.1 PARAMETERS

The design intents of the previous chapter are made with only five parameters:

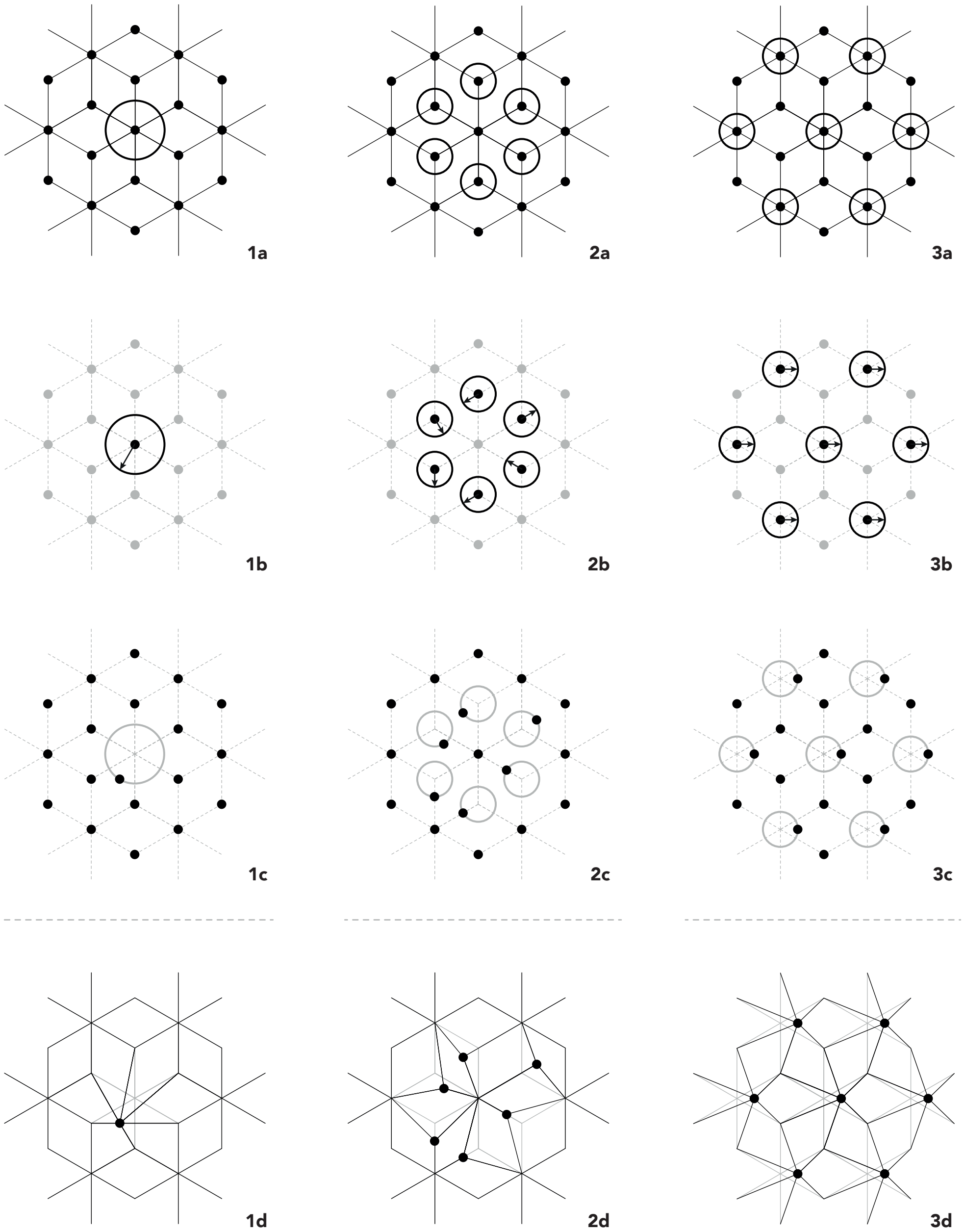
1. Length of the base
2. Length of the first iteration
3. Angle of the first iteration
4. Angle of the second iteration
5. Angle of the third iteration (the roof)

This already gives infinite amount of possible branching structures, however, there are still some more parameters that would give this structure more functionality. The applications that were proposed previously are only location specific to a certain extend. If one would design a canopy with much more requirements when it comes to natural lighting, a more challenging context or specific geotechnical properties, the symmetrical and repetitive structure might not comply with the demands. The first parameter that can be introduced that will result in infinite more varieties is the change of toptier-branch locations. The image below shows a topview of one singular branching structure. The dots represent the corners of the anticlastic panels - which are now all repetitive.

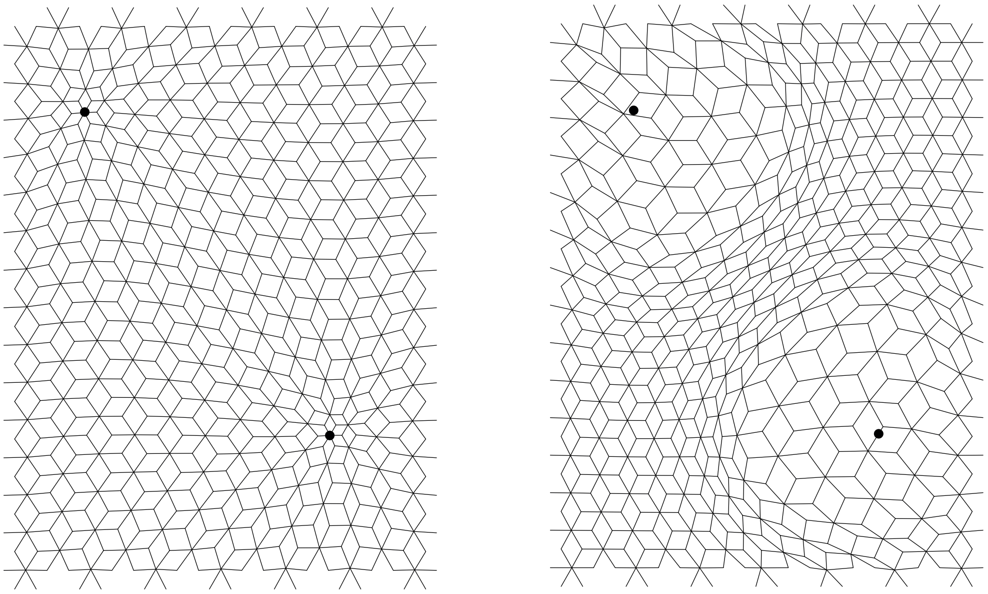


[206] RANDOM VECTORIZE PANEL VERTICES

If one would move the points with a random vector with a certain radius and connect these points again, one would come up with a totally different branching structure. Figure [206] shows a complete randomisation, since every points is moved. You can also think of the requirement that only the column should change in location, for example for archeological purposes. In this case, only the centerpoint should move, resulting in a pattern like shown in the left column of the image below. Another option could be to move a group of points randomly or to even move certain points with the same vector, as visualised in the right column of the image below. This parameter of changing the location of the connectorpoints gives infinite varieties on whatever the design brief might be.



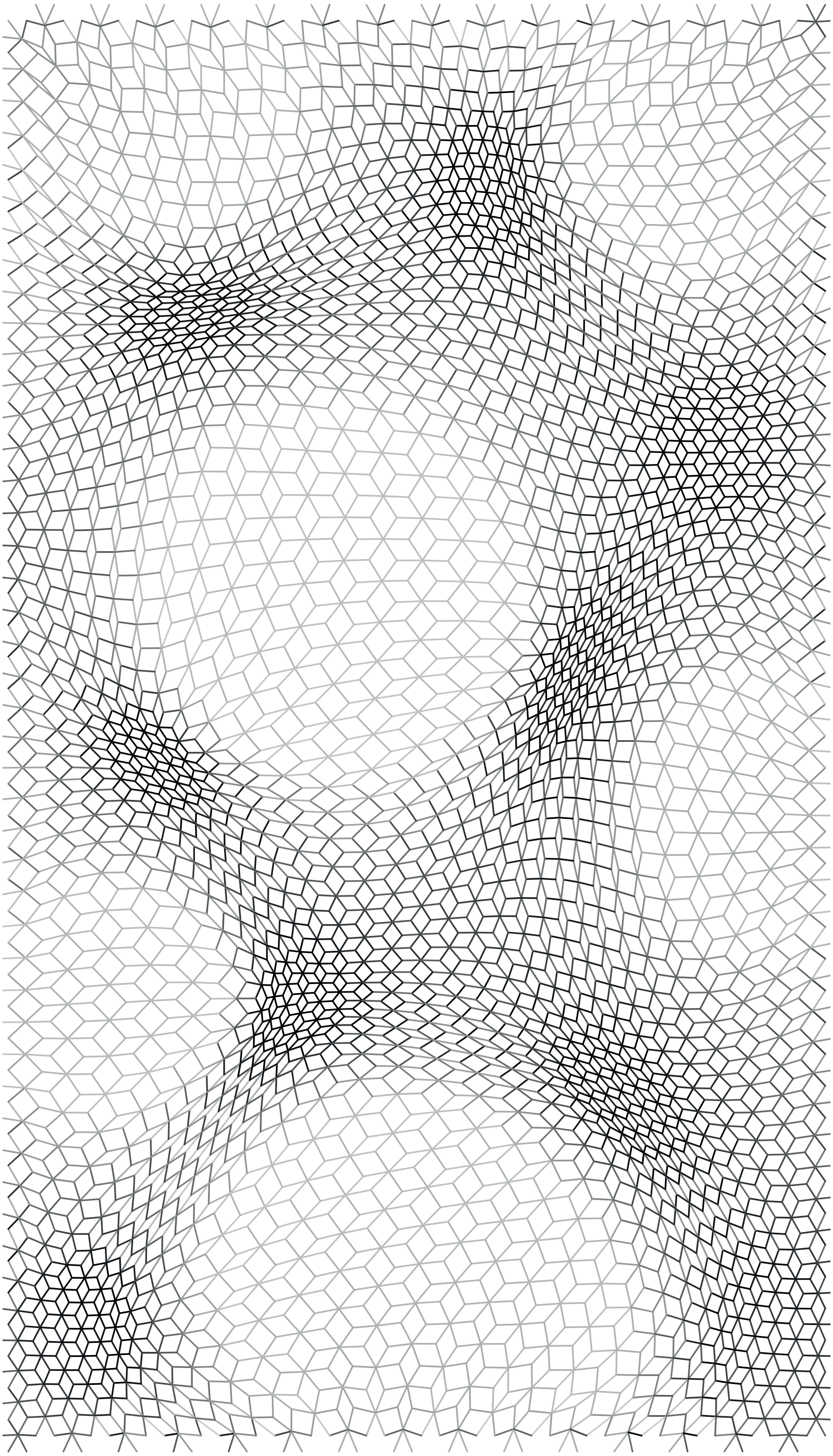
[207] VARIATIONS ON CHANGING THE LOCATION OF PANEL VERTICES



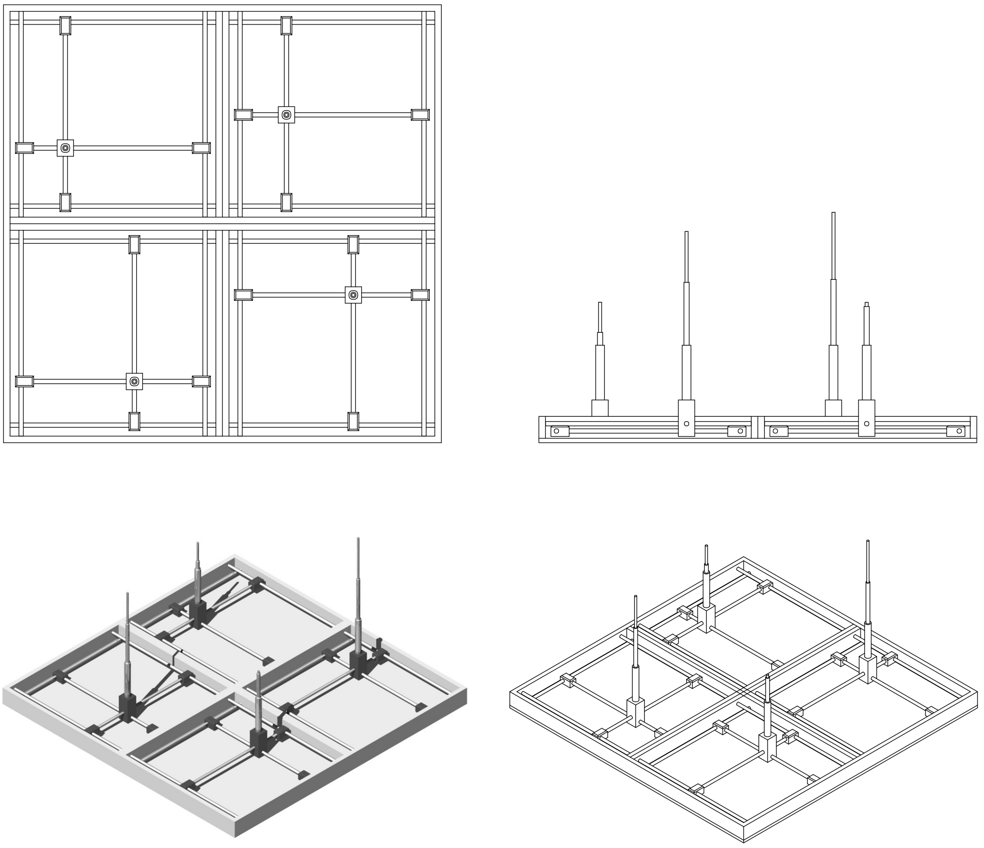
[208] PRINCIPLE OF ATTRACTOR AND DEFLECTORS

This goal as end result was already set since the main design concept was clear. The objective was to get as much design freedom when it comes to the branching structures as possible. This in order to not only use the analogue of a tree in the structural design, but also in the architectural design, the language as a whole. Having set the goal, the approach was top down. The question raised was 'how to come up with a symmetrical and repetitive structure where the detailing leaves possibilities for further development into a more randomised structure?' First the structure with the maximum possible repetition was further developed. This was a structure with only three unique tubes where all the roofpanels are the same. Once this was developed, a phase where the boundaries in this simplicity will be pushed could commence.

With this in mind we can zoom out and imagine a group of branching structures. One way to push the boundaries of the repetitive structure is to incorporate attractorpoints and deflectorpoints, as shown in the image above. An application for this could be to provide some deviation in the amount of daylight that comes in. A combination of this is also possible, resulting in a topview that is appealing to the eye. If one would not only work with attractors or deflectors in the xy-plane, but also incorporate values in the z-direction, this parameters becomes even more functional. With this possibility, this structural principle can be used for some more applications, for example for a dome.



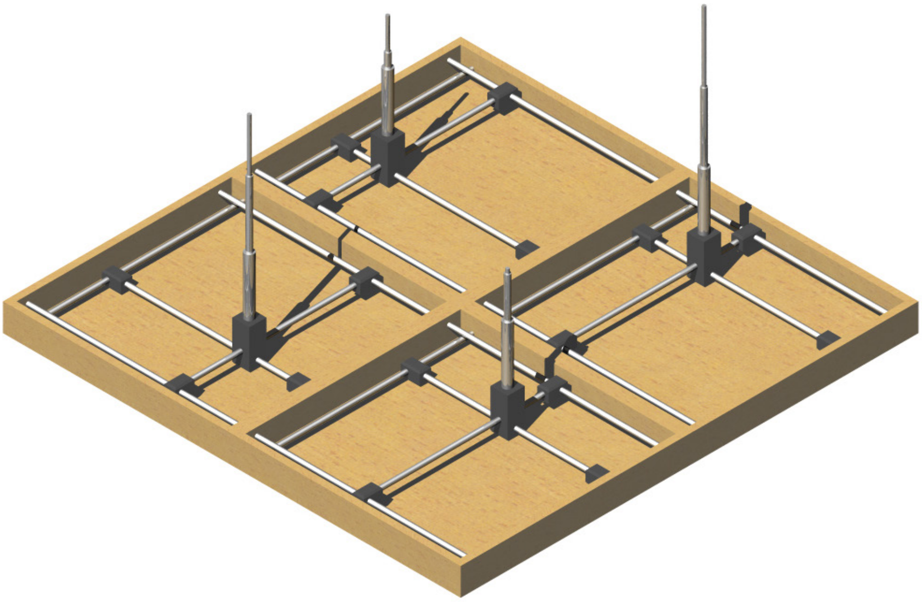
[209] POTENTIAL FLEXIBILITY PATTERN



[210] FLEXIBLE MOULD RIG

9.2 PRODUCTION

The simplicity in detailing as of now that is elaborated on in the chapter on design development is also due to the repetitive nature of the entire structure. There is only one unique element when it comes to the GFRP panels and when it comes to the tubes there or only three unique elements. The result is that it is easy and fast to produce as well as to construct. The challenge with adding the design functionality just described results in having much more unique tubes and besides, every panel is unique as well. The issue with this is not the process of bending the tubes, since this is computer automated, but the production of the anticlastic panels. If the structure has repetitive panels, the production method is vacuum infusion. This uses a static mould that is preferably made of stainless steel. If one would have to produce a certain amount of unique panels, the mould needs to be able to accomodate for this. A flexible mould that easily can be adjusted for the right curvature and dimensions is the challenge. As explained in previous chapters, the double curved panels can be described by straight lines. In other words, every unique panel can be constructed out of four points. If one would need to easily change the geometry of the panels, one would need to be able to move the four vertices in x-, y-, and z-direction. A rig that is able to account for this freedom of movement can be seen above. The size of the panel - the x- and y-direction



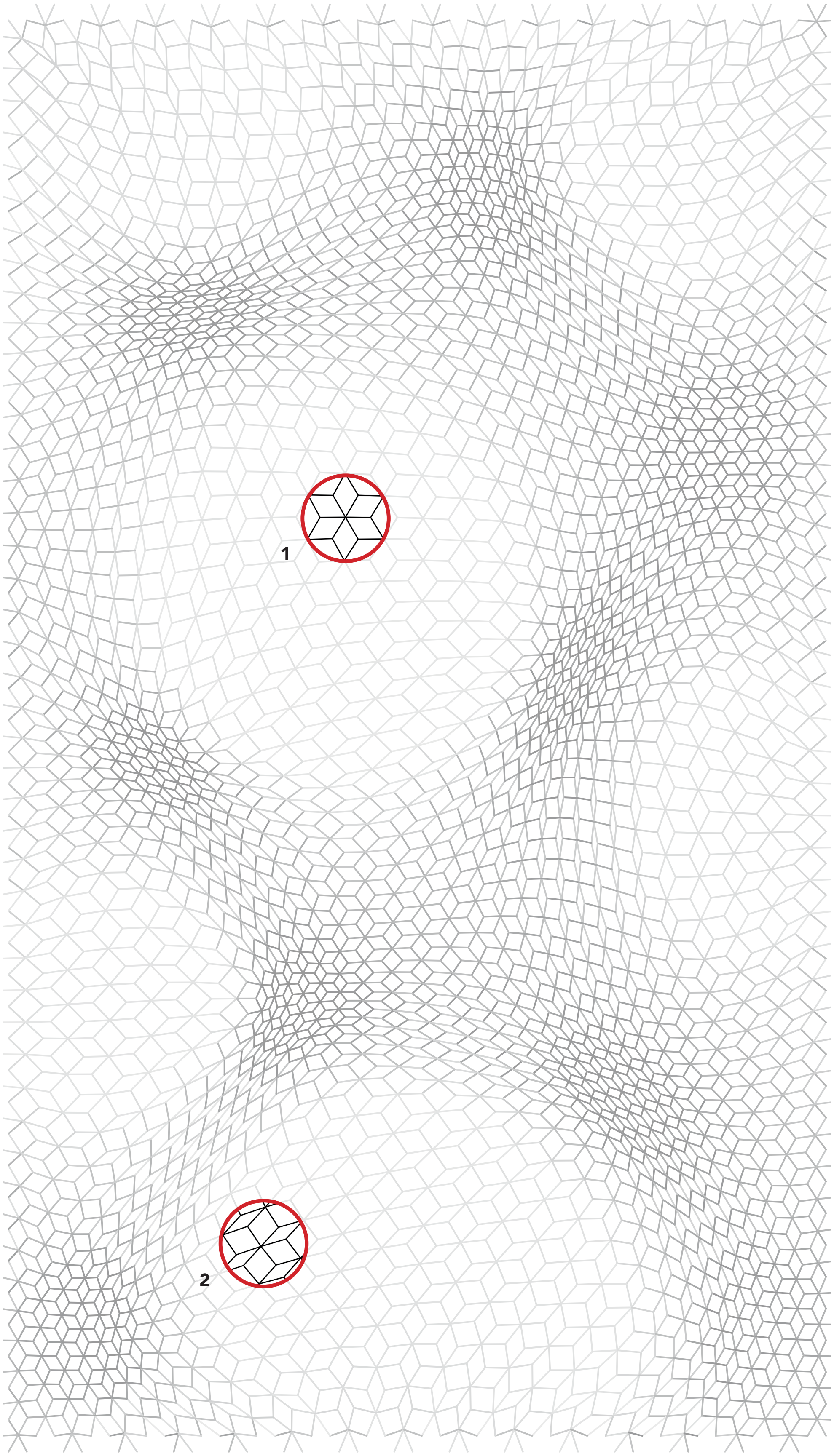
[211] FLEXIBLE MOULD IMPRESSION

- can be changed using the sliders and the curvature of the panel - the z-direction - can be adjusted by a telescopic vertical element. The next step is to connect the points with a certain profile, which will become the straight rulers. There are basically two ways to produce unique panels:

1. By tensing small strips of glass fiber across the length of a rulers. This will result in the desired shape. Possibly this rig can be held upside down and can be dipped in a bath of resin. The disadvantage with this is that the strips will cause a decrease in structural performance. Besides, while drying, the resin shall not be equally absorbed, resulting in a variable thickness along the surface.
2. The advantage of repetitive panels is that a stiff mould can be used for vacuum forming as well, which will result in a much smoother surface once the resin has dried. In order to have such a smooth finish, a flexible, but stiff mould should be designed. This part is the most challenging. One could think of a semi-stiff cloth that will be dropped onto the straight rulers. However, the amount of curvature is hard to control when the size of the panels change.

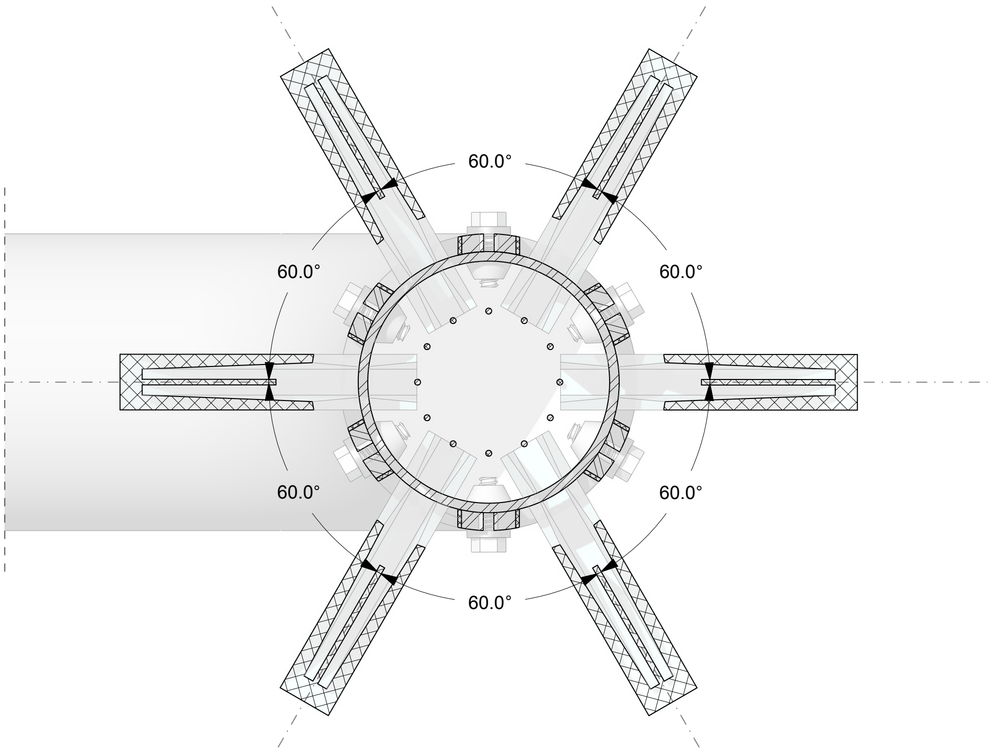
9.3 DETAILING & FLEXIBILITY

Besides the production of the panels, the detailing of the branch-to-leaf connection as I have called it in the chapter on 'Design development' becomes interesting as well. Not only the panels, but also the u-profiles will be of unique length. Secondly, the sequence of construction becomes of great influence and thirdly the angle at which the profiles and panels will come together at the branch is deviating. An easy solutions for this could be to broaden the grooves at the end of the branch, so that there is some allowance for the deviations in angle.

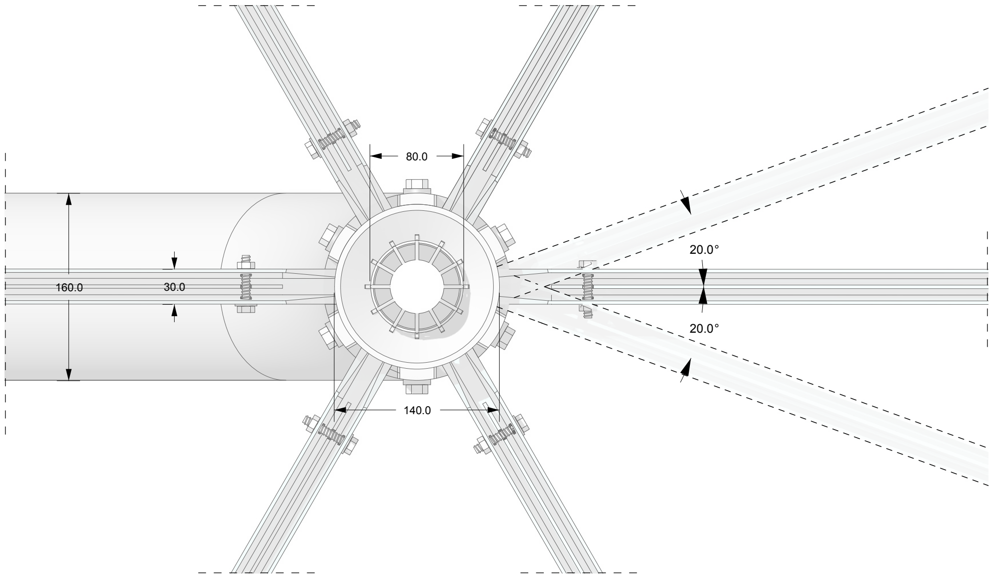


[212] REGULAR VS. IRREGULAR ANGULATION

As stated before, the u-profiles that interconnect the panels need to be able to incorporate this flexibility. The first step towards a more randomised structure is based on the angle deviation in the so-called 'branch-to-leaf' detail. In the potential flexibility pattern, the topview, on the left page two different branch-to-leaf connections are highlighted; one with a regular angulation, and one with an irregular one. In the repetitive situation, the angle between the profiles is sixty degrees - shown below.

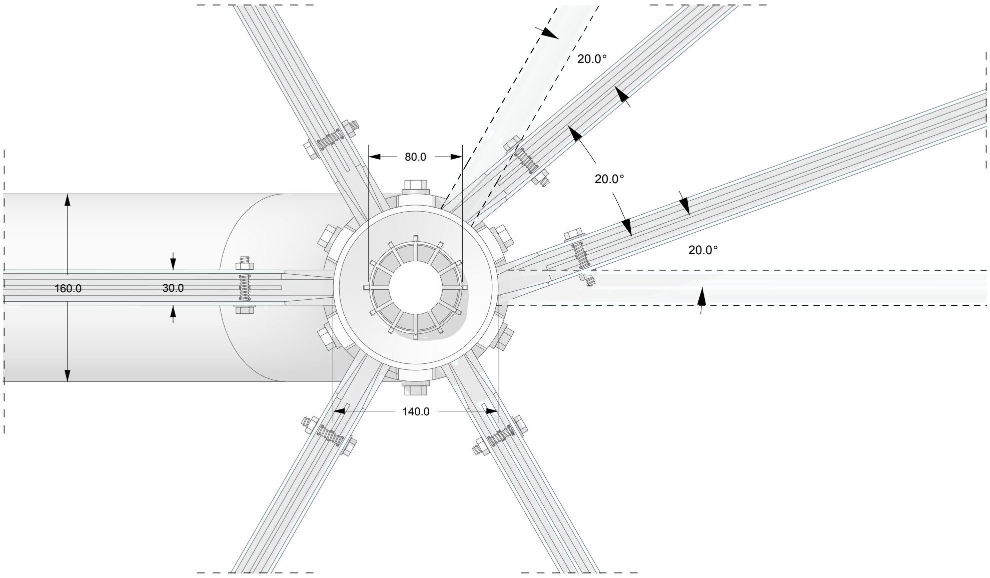


[213] REGULAR ANGULATION - HORIZONTAL

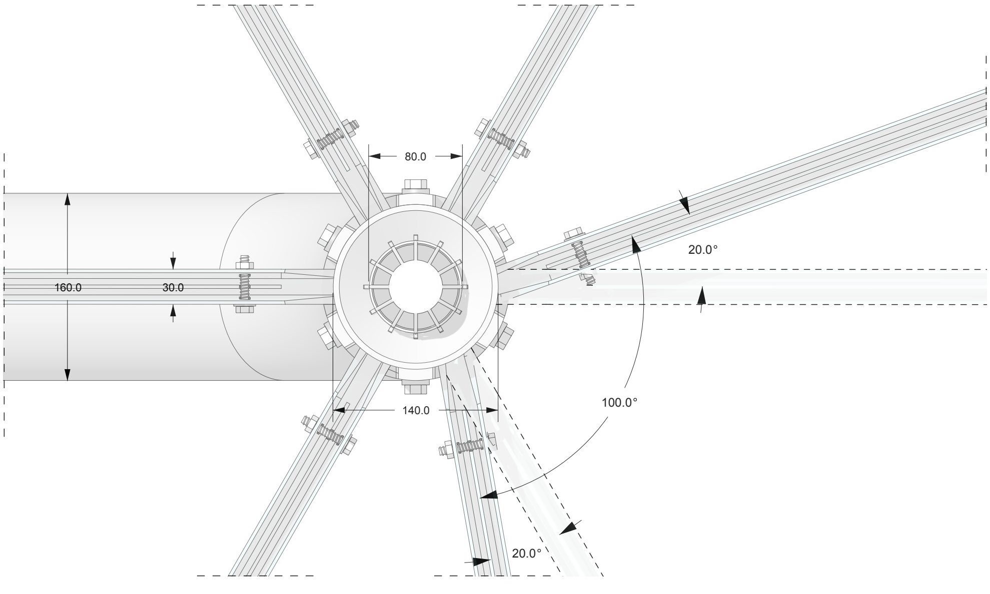


[214] RANGE OF ANGLE DEVIATION - HORIZONTAL

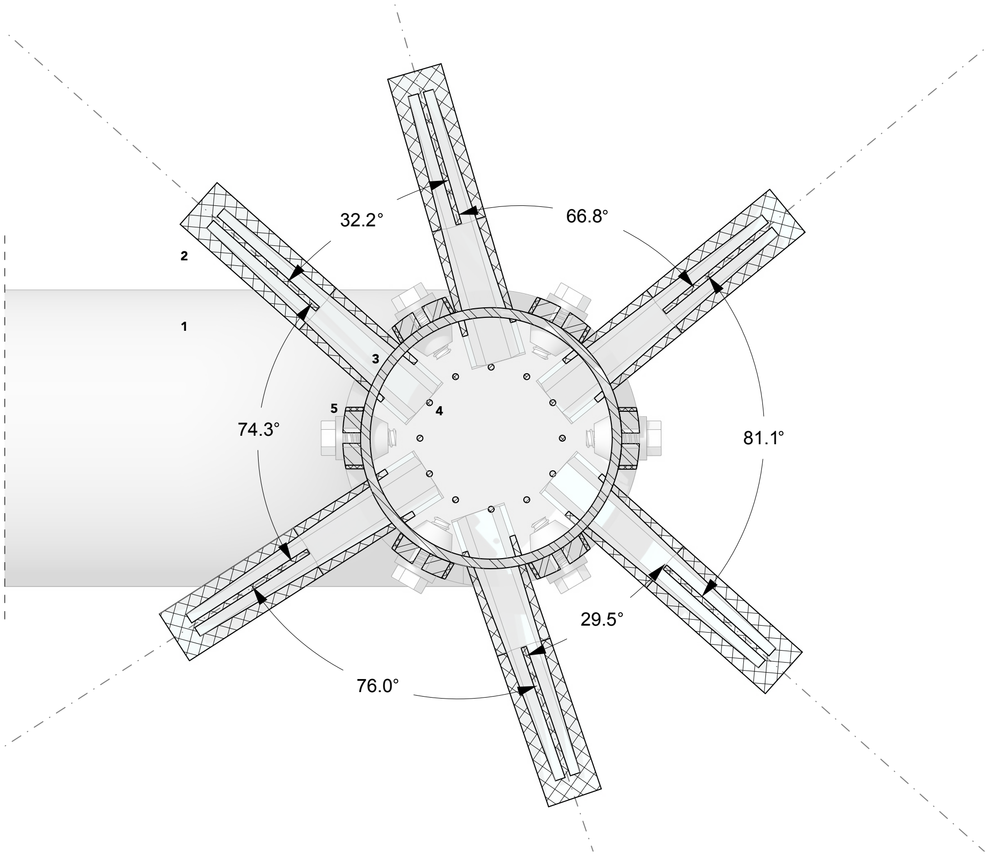
As can be seen in the image on the previous page, the maximum possible angle deviation is twenty degrees in either direction. This amount is based on both the constructability as well as the relevance of the amount of possible deviation. With this design functionality the minimal angle between two u-profiles is twenty degrees, whereas the maximal angle is hundred degrees. These numbers give plenty of flexibility to account for the alteration in natural lighting, a more challenging context and the diversion of geotechnical properties. In the details below the minimum and maximum angle deviation is visualised.



[215] MINIMAL ANGULATION - HORIZONTAL



[216] MAXIMAL ANGULATION - HORIZONTAL

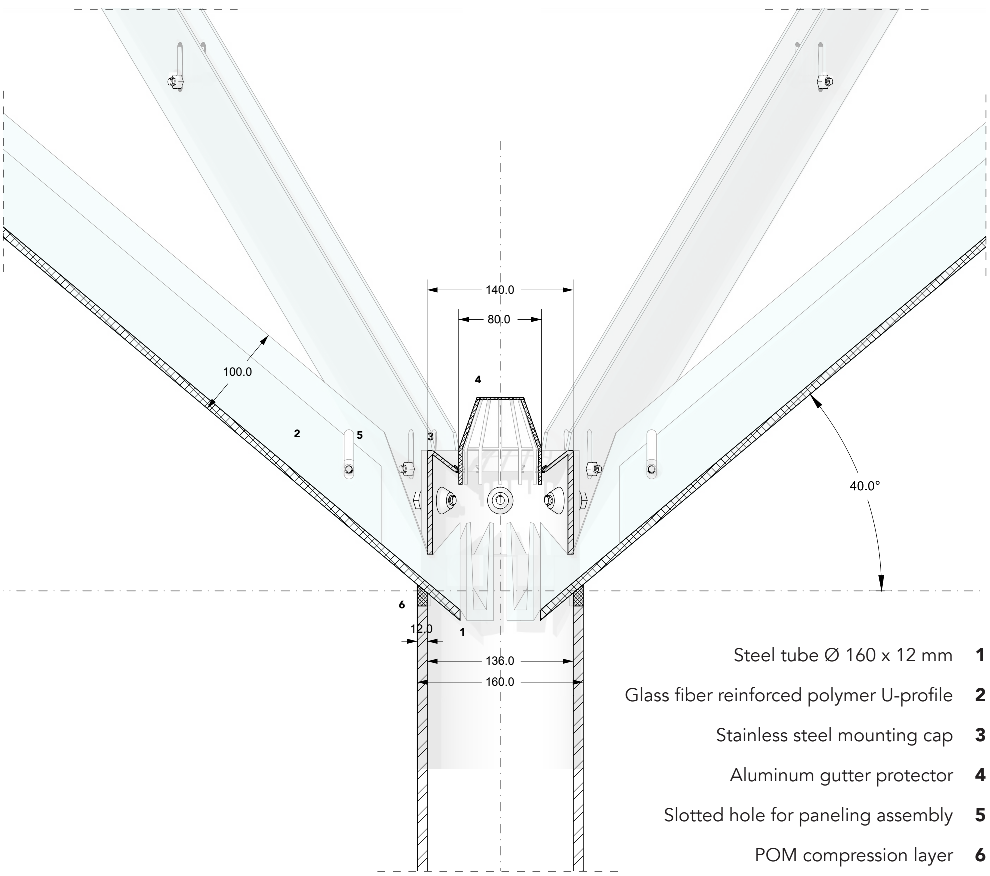


[217] IRREGULAR ANGULATION - HORIZONTAL

Steel tube Ø 160 x 12 mm	**1**
Glass fiber reinforced polymer U-profile	**2**
Stainless steel mounting cap	**3**
Aluminum gutter protector	**4**
POM compression layer	**5**

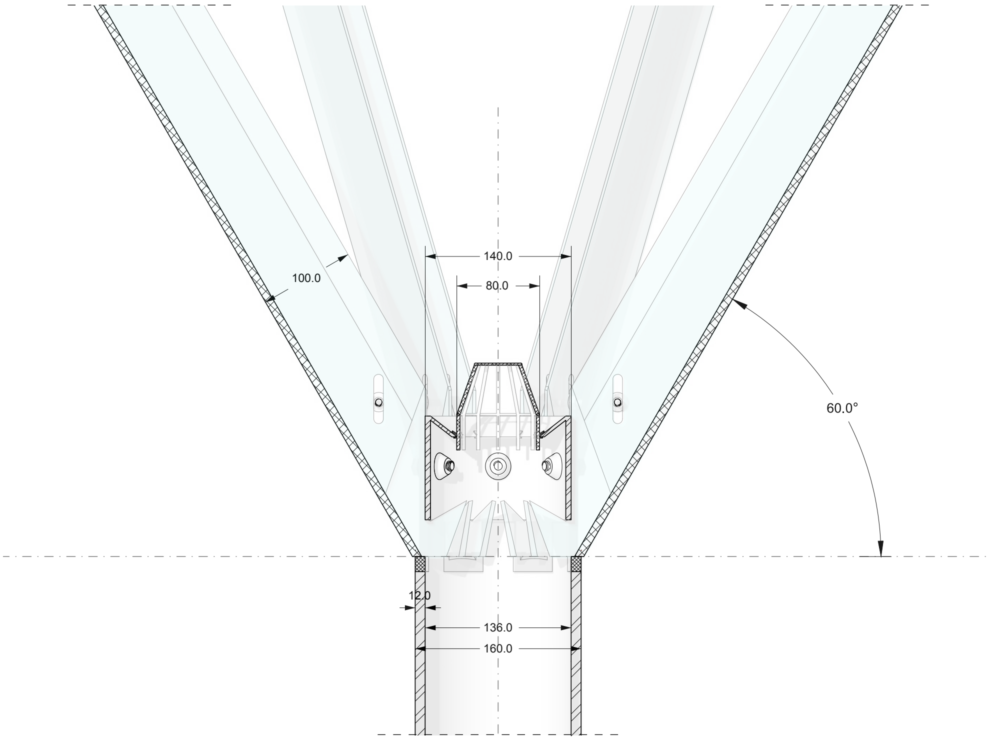
If we go back to the ‘potential flexibility plan’, we can now see that option two, the irregular angulation is also possible since all the angles are not smaller than twenty degrees and not greater than hundred degrees.

The randomisation of the structure is not provided by the flexibility of the horizontal plane alone. The goal was to also provide adjustability in the vertical plane, in height. On the following pages, the detailing of this can be found. The ‘normal’ situation, the design of the physical models, has a curvature of fourty degrees. The maximal possible curvature is sixty degrees, whereas the minimal possible curvature is set to be five degrees. This angle is based on the fact that this structure cannot be applicable when the roof is flat, due to watermanagement issues. The angle of five degrees results in proper waterdrainage as well as keeping the anticlastic shape of the panels. As you might imagine, there is still some possibility for movement, even when the mounting cap is installed. This freedom of movement will be accommodated by the anticlastic panels while working together to get a rigid structure.

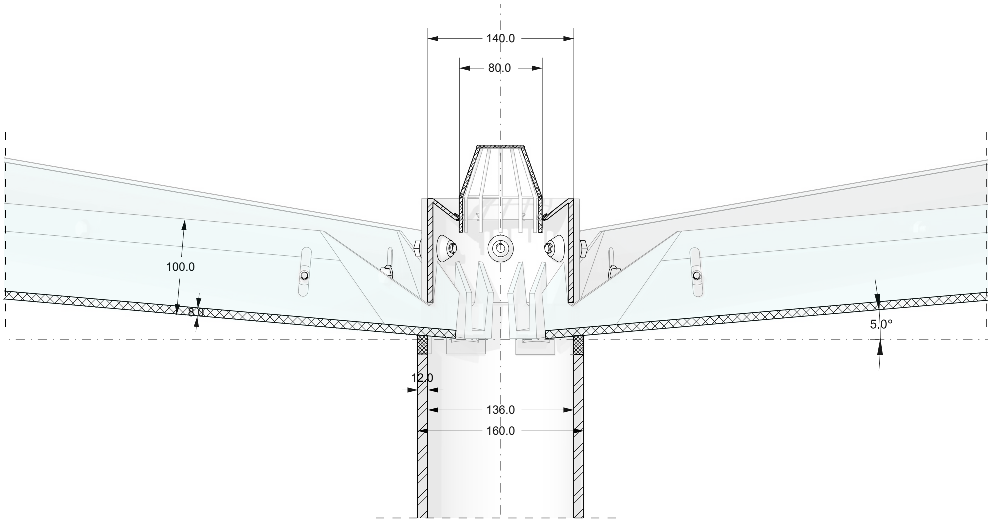


Steel tube Ø 160 x 12 mm	**1**
Glass fiber reinforced polymer U-profile	**2**
Stainless steel mounting cap	**3**
Aluminum gutter protector	**4**
Slotted hole for paneling assembly	**5**
POM compression layer	**6**

[218] MODELLED ANGULATION - VERTICAL

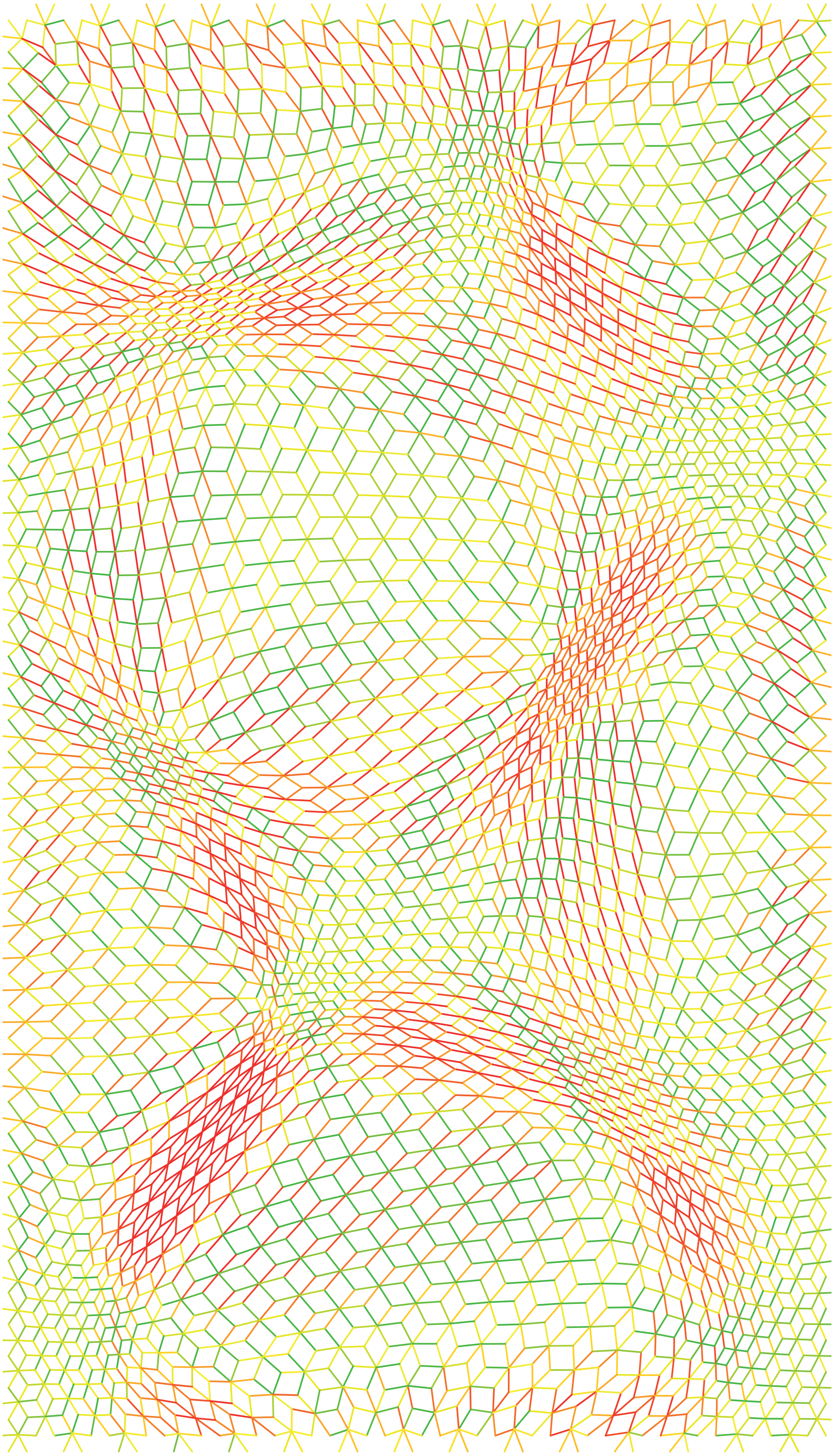


[219] MAXIMAL ANGULATION - VERTICAL



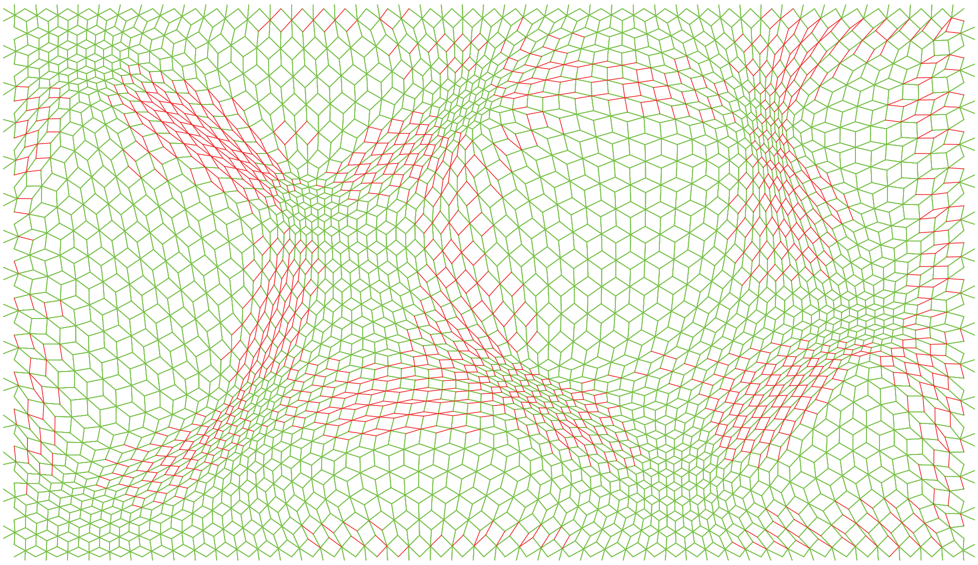
[220] MINIMAL ANGULATION - VERTICAL

You could ask yourself to what extent it is still relevant to have flexibility. Is it useful to have this much freedom or should this be based on the demands of the application, whether this is based on natural lighting or on geotechnical properties. To get a useful inventorisation on the constructability of the entire plan, the angles between the branches should be analysed. In the plan on the next page, this is done gradually. The median and regular angle is sixty degrees and illustrated as green. The more the angle is deviated from this base case of sixty degrees, the more the color turns into red. As mentioned earlier the detailed adjustability of the u-profiles and emerging from this the amount of design freedom is based on the desired amount of flexibility. More flexibility than detailed is not beneficial or relevant for the application where this new structural principle is developed for. An increased density of the roofpanels, leads to a dense columngrid, which again results in inconvenient and less useful floorplans.

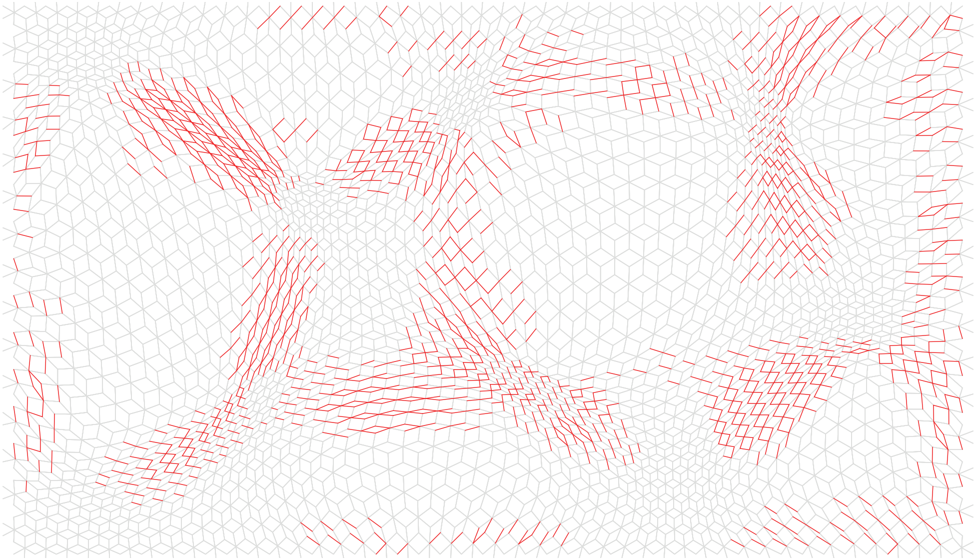


[221] GRADUAL ANGULATION ANALYSIS

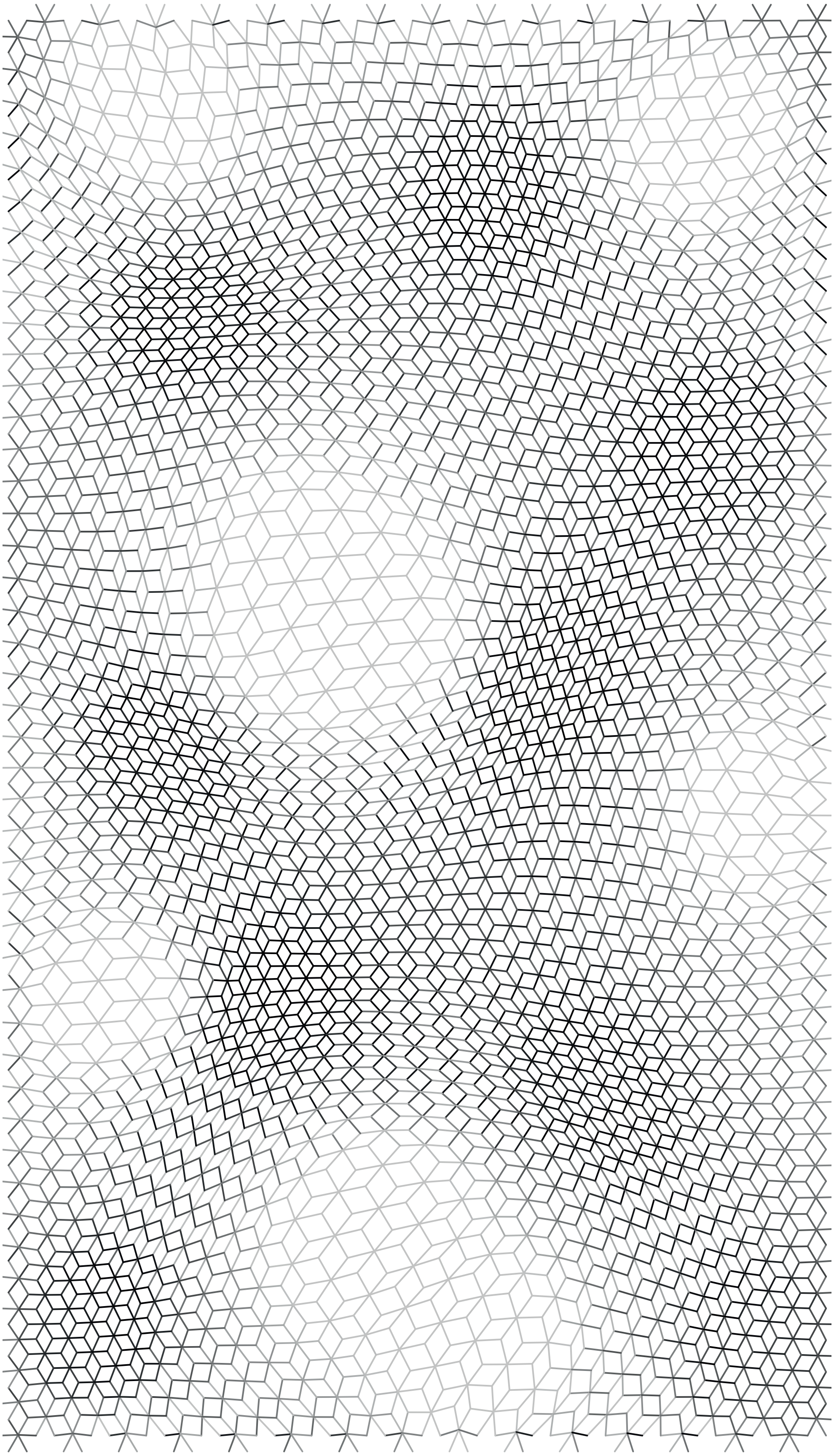
The final step to match this potential flexibility plan with the designed and desired amount of design freedom is to translate the gradual angulation analysis into an absolute angulation analysis. This analysis can be seen below. The result is a plan with green (positive and constructable) lines and red (negative and not constructable) lines. Figure [223] illustrates that about 25% of the potential flexibility plan has an angle that is not relevant or useful for the desired application. The way to coincide the flexibility plan with the detailed possible adjustability is to change the "force" of the attractorpoints and deflectorpoints. This results in a new pattern that is completely constructable with the desired amount of flexibility. This can be seen on the next page.



[222] ABSOLUTE ANGULATION ANALYSIS



[223] ABSOLUTE EXCEEDING ANGULATION



[224] CONSTRUCTABLE PATTERN WITH DESIRED FLEXIBILITY

outro

10. CONCLUSIONS & RECOMMENDATIONS

10.1 INTRO

Throughout the graduation process, many conclusions and lots of design decisions were made. Below you can find the most important conclusions on the bigger scale of this graduation; finding a new structural principle that is derived from nature's design. These conclusions are categorised in a few main elements that have had a great influence on the final design.

10.2 SCOPING & TIME-FRAMING

In the beginning of the graduation process, I knew that I wanted to focus on nature and its design principles. My main interest was in structural design and how nature solves structural design tasks. In the beginning this was challenging. There was an ocean of possibilities and directions that I could take. There was plenty of information available on the broad topic of 'biomimicry'. Analysing natural processes and examining biology gave infinite subtopics for further research. Quite radically, I chose to make a categorisation of general physical phenomena that occur often in the realm of nature, without having done profound research on these phenomena. I chose to make a selection out of the huge amount of phenomena nature has to offer that are familiar to everyone - at least once the terminology is explained. Once this categorisation was made, I started looking for precedents on how these phenomena were used in architecture. I was inspired by many architects and structural engineers who had a simliar approach on designing with nature's help.

In line with the element scoping is the time framing. An important note on the proposed methodology is that this method has practically infinite possibilities of combinations, all resulting in a different design direction. A different outcome of the first step – selecting a few of the categorised physical phenomena, gives already a total different input for the following levels in the transformation. Besides, there is a pretended hierarchy in the levels and definitions, but this does not assume to be the absolute truth. There are various ways to interpret the methodology. Looping, skipping or going back a few steps can therefore often be very useful, resulting in more in depth concepts. Due to a time-issue, I went on with the design direction that was the result of walking through the methodology. Once I had an idea on how to combine a few of the categorised natural phenomena, I felt the need to develop this further so that there was still time to finish this graduation project with a pleasing level of detailing.

10.3 ONE-TO-ONE COPY CONFLICT

As mentioned before, the concept of biomimicry seems to be able to give answers to questions that several branches of engineering currently faces on a global level. However, the idea should be carefully taken, since it is not always in a direct way that nature can give solutions to scientific challenges. This is called the problem of 'technology transfer.' One of the main reasons for this is that it is challenging to design structures that are inspired by nature, since nature has different design principles than architecture. One of the main aspects of this difficulty is the one-to-one copy conflict. It is not possible to directly copy nature's strategies and patterns to architecture. A categorisation of this copy conflict brings up four issues: magnitude, force transfer, kinematical freedom and components.

- Magnitude: Accordingly to D'Arcy W. Thompson does scale have a marked effect on physical phenomena, and that increase or diminution of magnitude might mean a complete change of statical- or dynamical equilibrium. This so called 'problem of scaling'.
- Force transfer: this includes the weight and the loading of structures. The load-case should be studied, since a lot of nano-arrangements work in zero-gravity medium, or they work in water, or have to load-bear live changing loads.
- Kinematical freedom: Natural structures are normally designed for having much more kinematical freedom than man-made ones, which do not need larger movements or deformations.
- Components: Architecture is due to fabrication and transportation restricted to designing with components or structural members that will be assembled in a later stage of the to be realised design. For this reason the way we design the fixings, joints and connections between the components is crucial, not only for the assembly, but also for the structural performance.

10.4 DESIGN PROCESS: SIMPLICITY & FREEDOM

After the observation- and transformation phase, the design development could commence. Once the main design concept was clear, the design development started for me by setting a goal for the end result. The objective was to get as much design freedom when it comes to the branching structures as possible. This in order to not only use the analogue of a tree in the structural design, but also in the architectural design, the language as a whole. Having set the goal, the approach was top down. The question raised was 'how to come up with a symmetrical and repetitive structure where the detailing leaves possibilities for further development into a more randomised structure?'

First the structure with the maximum possible repetition was further developed. This was a structure with only three unique tubes where all the roofpanels are the same. Once this was developed, a phase started where with this simplicity, a more flexible structure could be developed. The detailed adjustability of the u-profiles and emerging from this the amount of design freedom is based on the desired amount of flexibility. More flexibility than detailed is not beneficial or relevant for the application where this new structural principle is developed for.

10.5 FINAL STATEMENT

Finally, I would like to finish off by proposing a technical philosophical statement on the topic 'the confluence of design by nature and structural design.'

1. Nature designs in an unrelenting and ruthless way. It has no mercy for things that have proven not to work. It will be completely rejected. If we want to build more sustainably, I feel that we have to follow this design principle as well. Nowadays, we still build structures that in essence are not rational resulting in adding exceedingly amounts of material, unsustainable materials with a high embodied energy. Using nature as inspiration combined with mathematics enables us to come up with structurally rational designs. These rational designs are because of its simplicity appropriate for smart solutions on a more sustainable built environment.

2. Biology has the ability to influence humans in ways that might be outside a predetermined design problem. This may result in unthought-of technologies, or even new approaches to design problems.

3. There are in my opinion two ways to look at nature. We can observe it and learn how we as architects and structural designers should design. However it can be as useful to observe and learn what not to do. I see great potential in learning how nature rejects principles and laws and how it always seeks for a workaround that is more durable and sustainable in many aspects.

4. For every design phase, nature has solutions available. However, every phase has its own way of how we can be inspired by natural design principles. For the concept forming, the solutions that nature offers is different than in the final detailing phase. The level of filtering, the level of abstraction, the level of mutation should be done adapting the possibilities and the laws.

5. In nature, the form-following process, structural extension (growth) and material placement happen simultaneously and are controlled by a constant feedback loop. Whereas architecture is predominantly

design focused, resulting in the fact that form determines the structure and the materiality. This process is not continuous as is the case with natural design, but linear. Because of nature’s constant feedback loop, the final design is much more integrative than architectural designs. One way of trying to mimic this feature or design principle if you will, is to constantly alternate between digital models and physical models. By doing so, this iterative process can form the basis for an integrative architectural design.

I hope that this methodology and specifically this new structural principle will be a precedent for architects and structural designers on how to approach this type of design task.

outro

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[167] Vacuum infusion - production method of glass fiber reinforced polymer – by author

[168] Production method of steel tubes – by author

[169] Method statement [1-6] – by author

[170] Method statement [7-8] – by author

[171] Method statement [9-10] – by author

[172] Method statement [11-16] – by author

[173] Method statement final impression – by author

[174] Detailing categories – by author

[175] Bearing option one – by author

[176] Floor finishes bearing option one – by author

[177] Bearing option two – by author

[178] Floor finish bearing option two – by author

[179] Tube connector detailing sequence – by author

[180] Tube connector of base bundle – by author

[181] The four tube connector locations – by author

[182] Tube connector impression – by author

[183] Toptier distances – by author

[184] Strapping tool – by author

12.7 Chapter 7: Design modelling

12.8 Chapter 8: Design intent

12.9 Chapter 9: Design functionality

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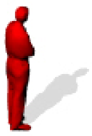
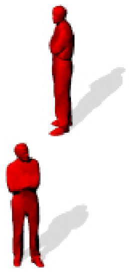
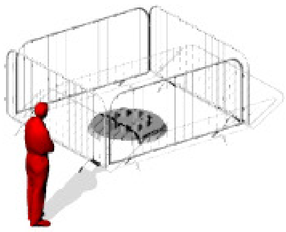
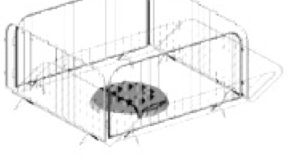
13. APPENDICES

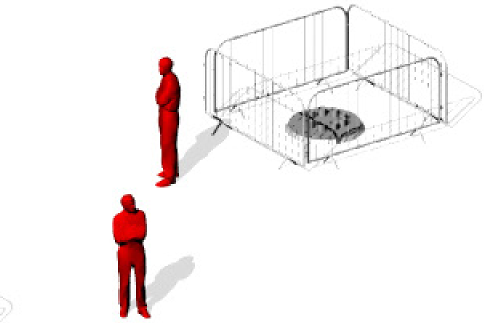
13.1 METHOD STATEMENT VISUALISATIONS

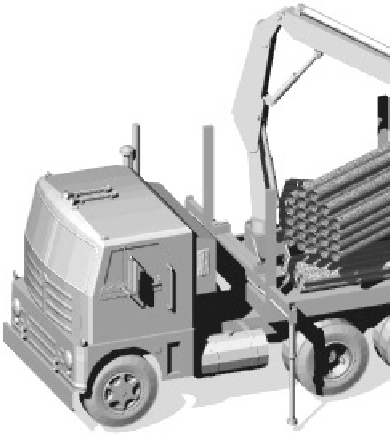


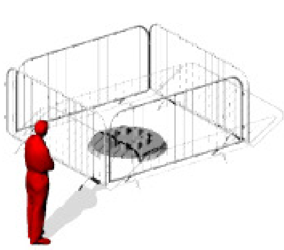


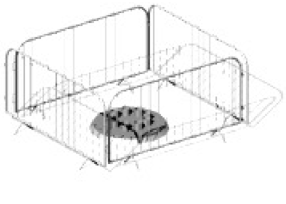


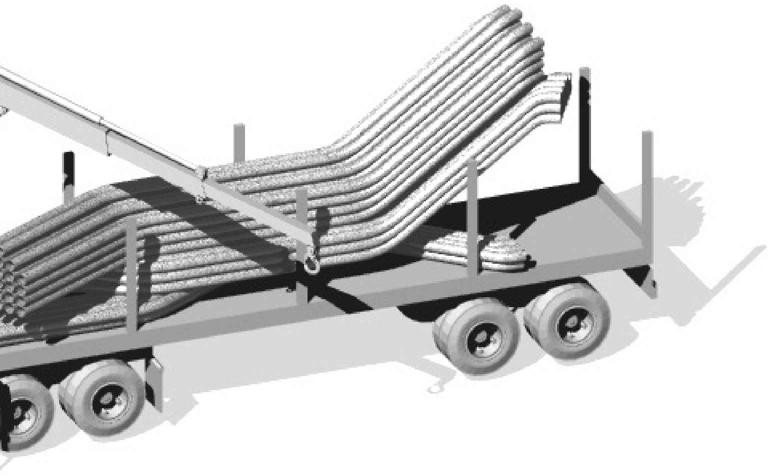


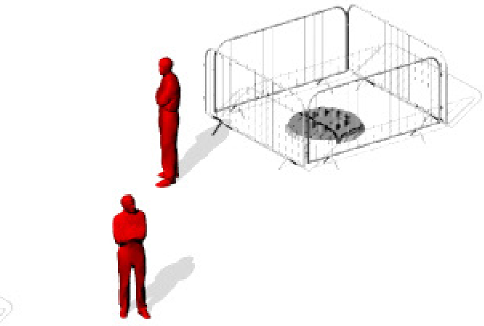


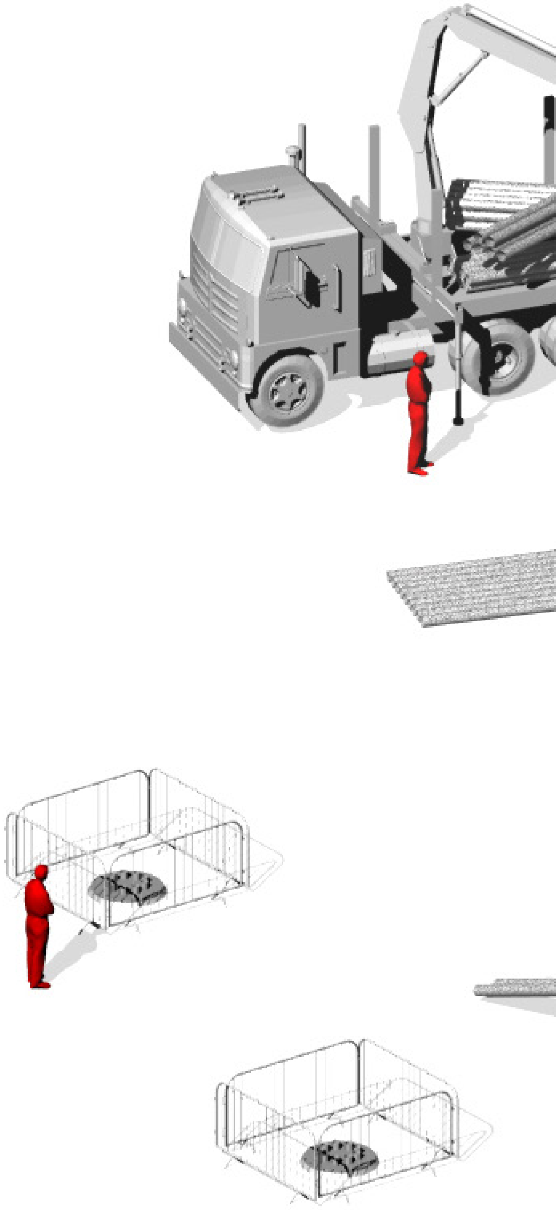


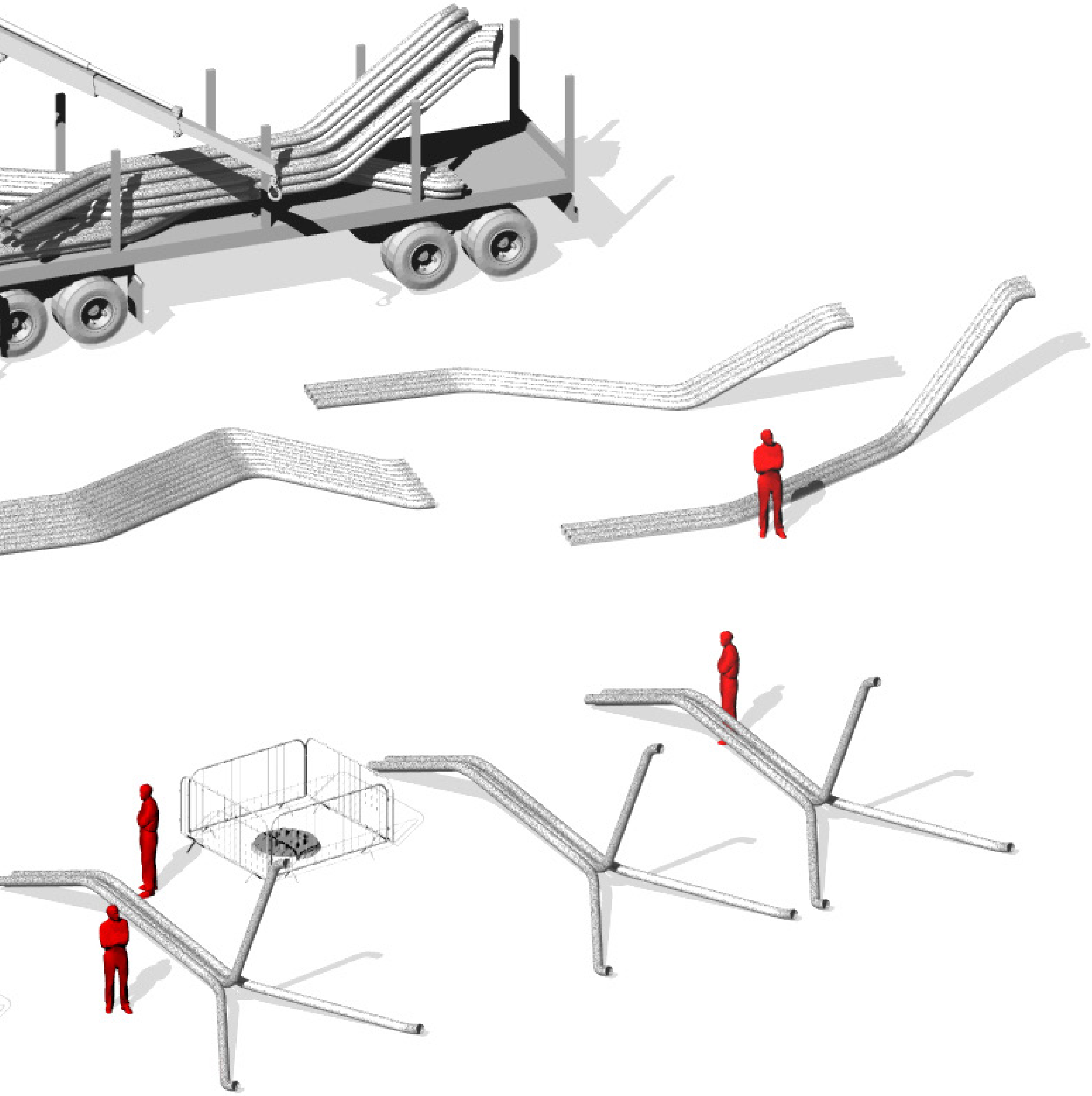


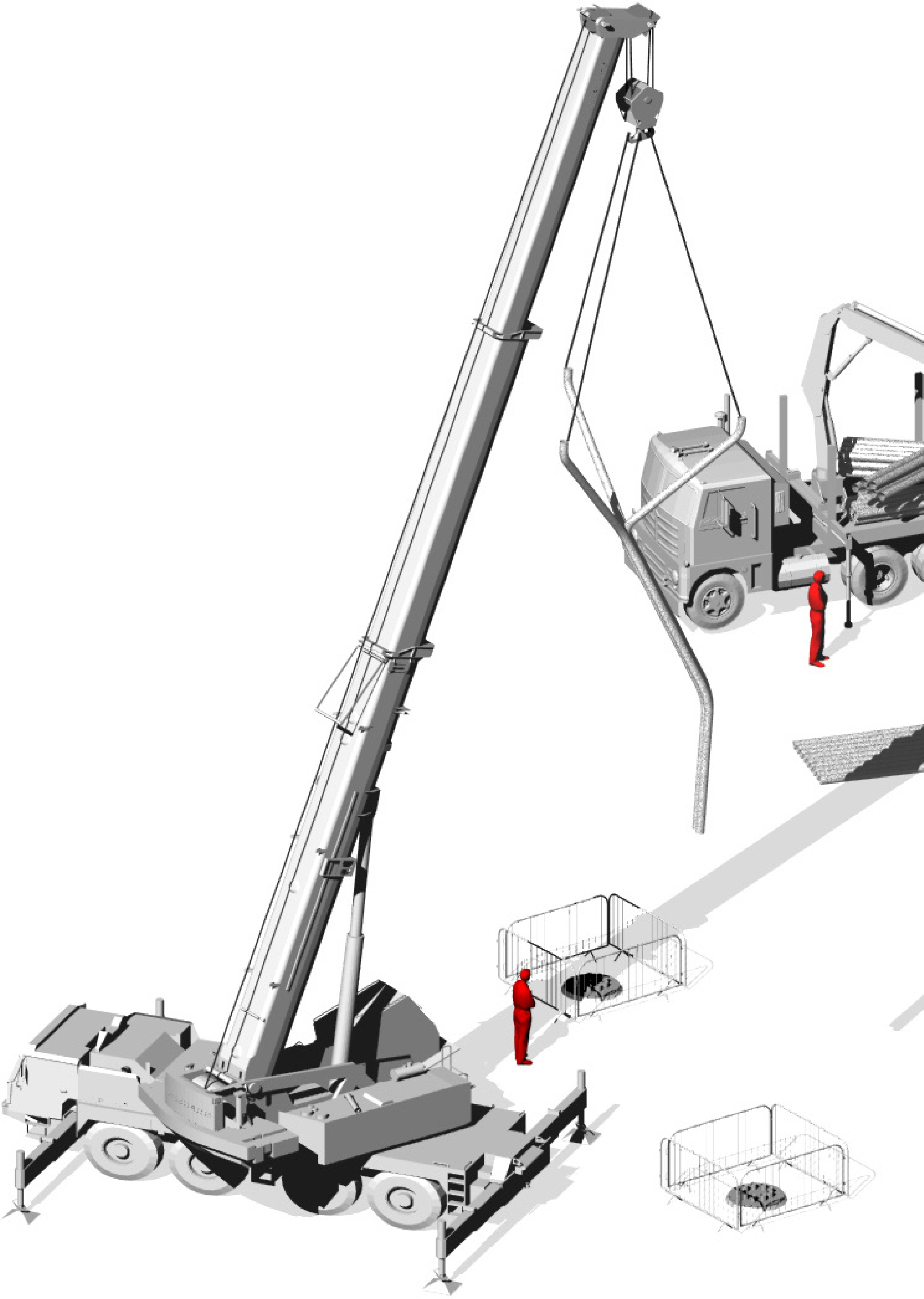


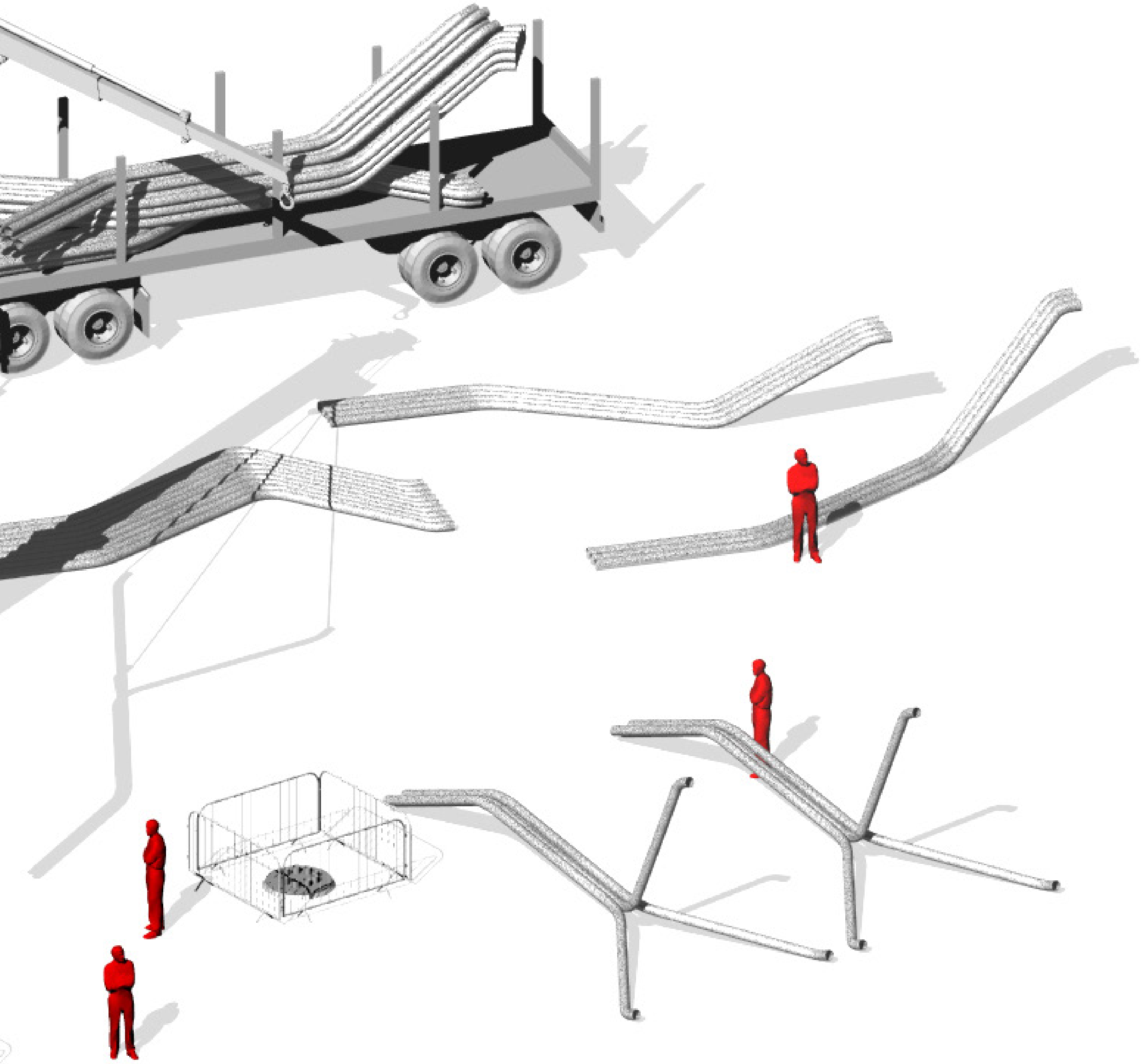


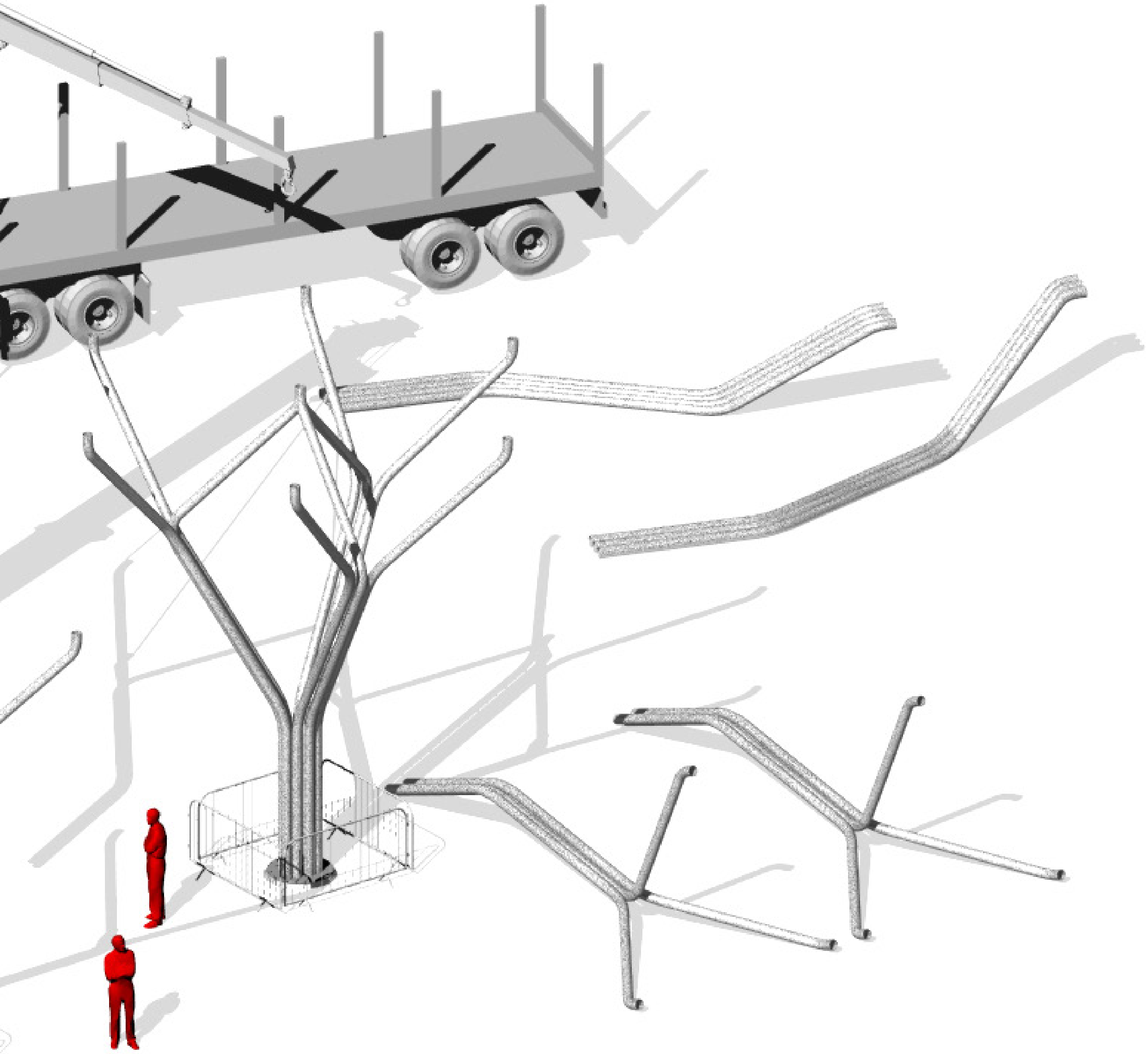


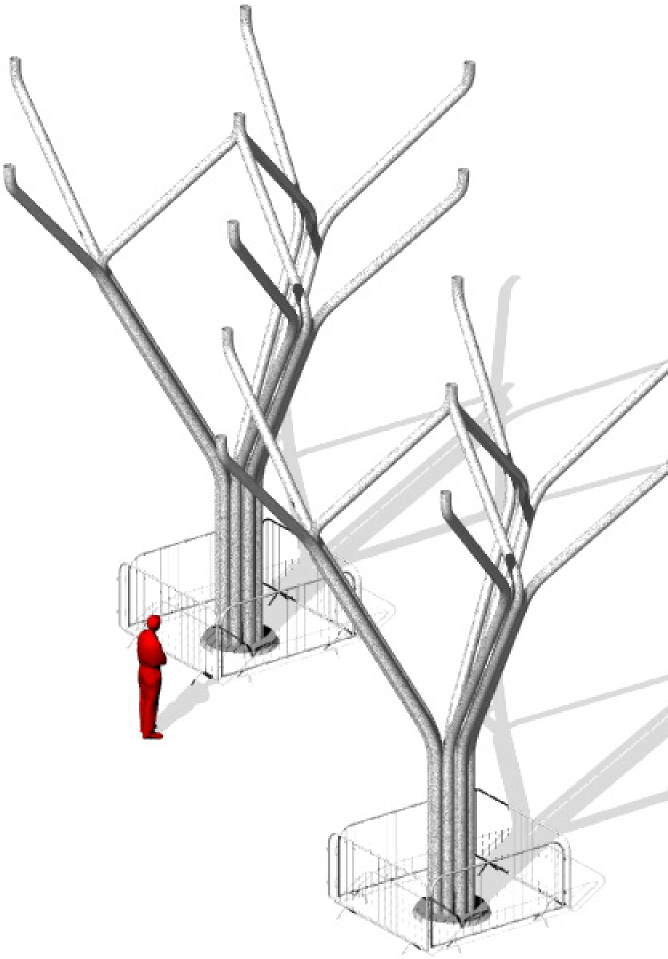


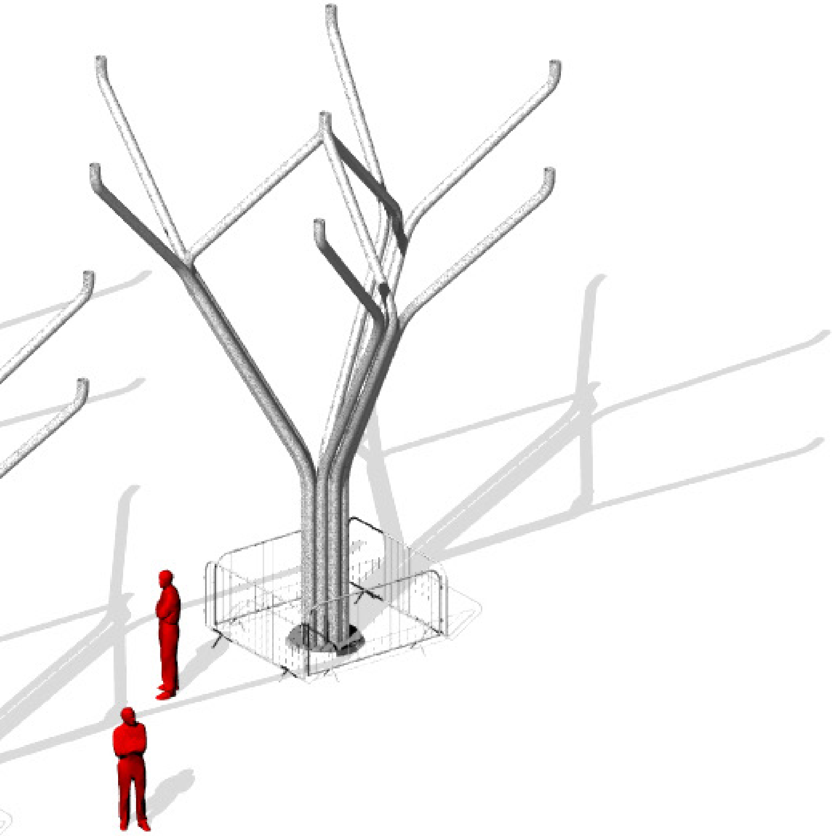


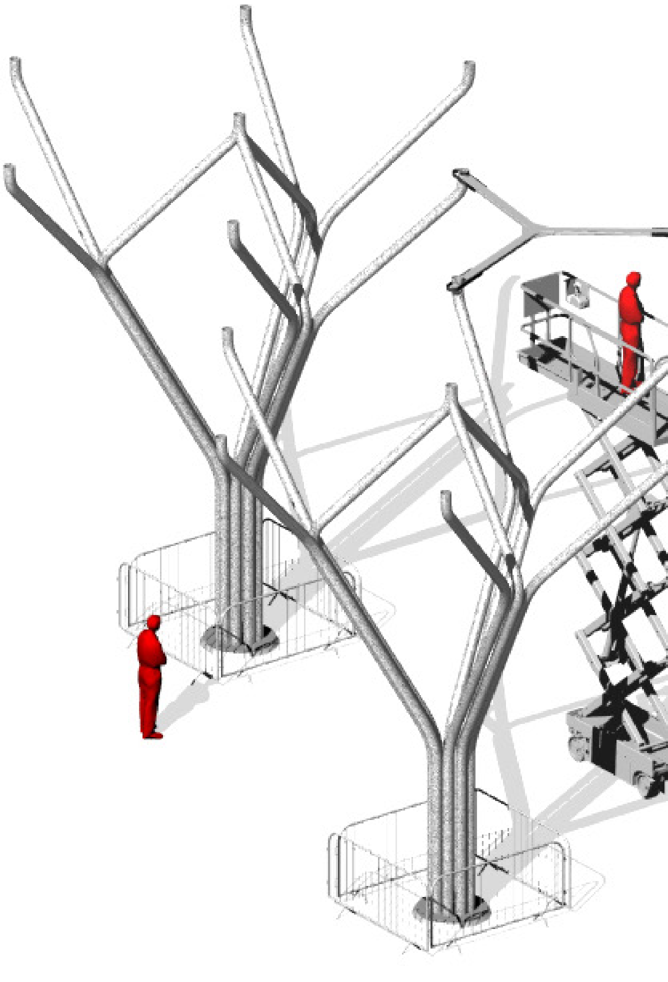


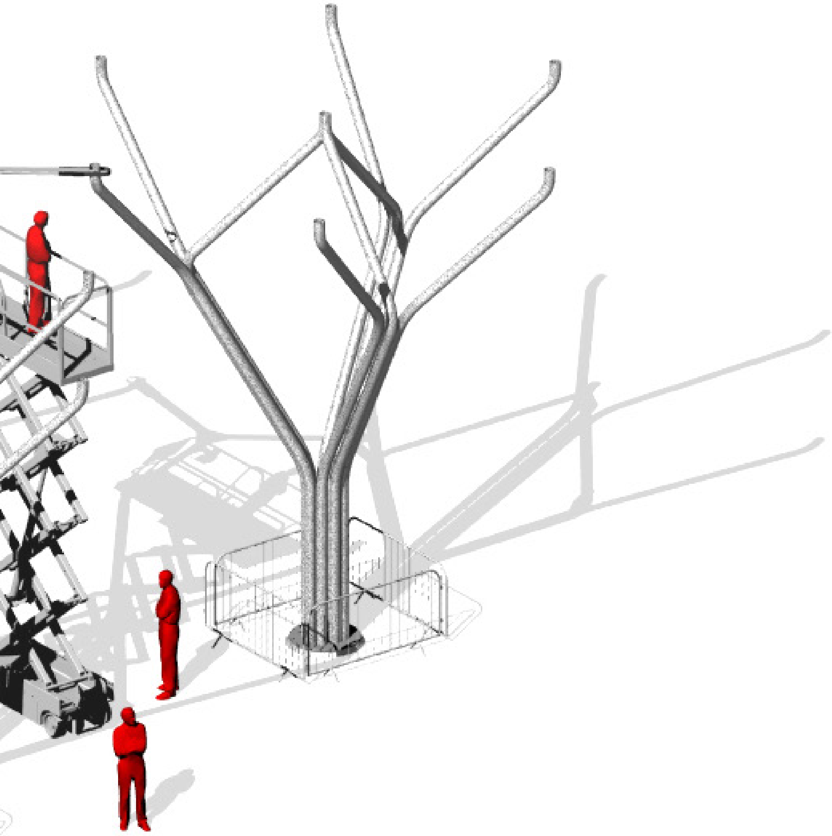


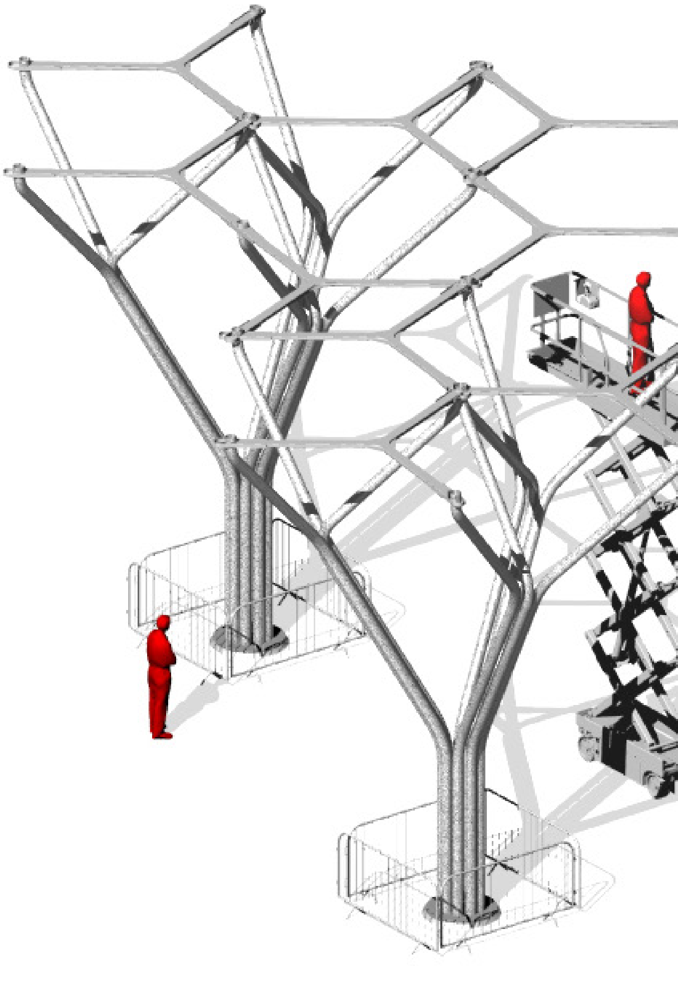


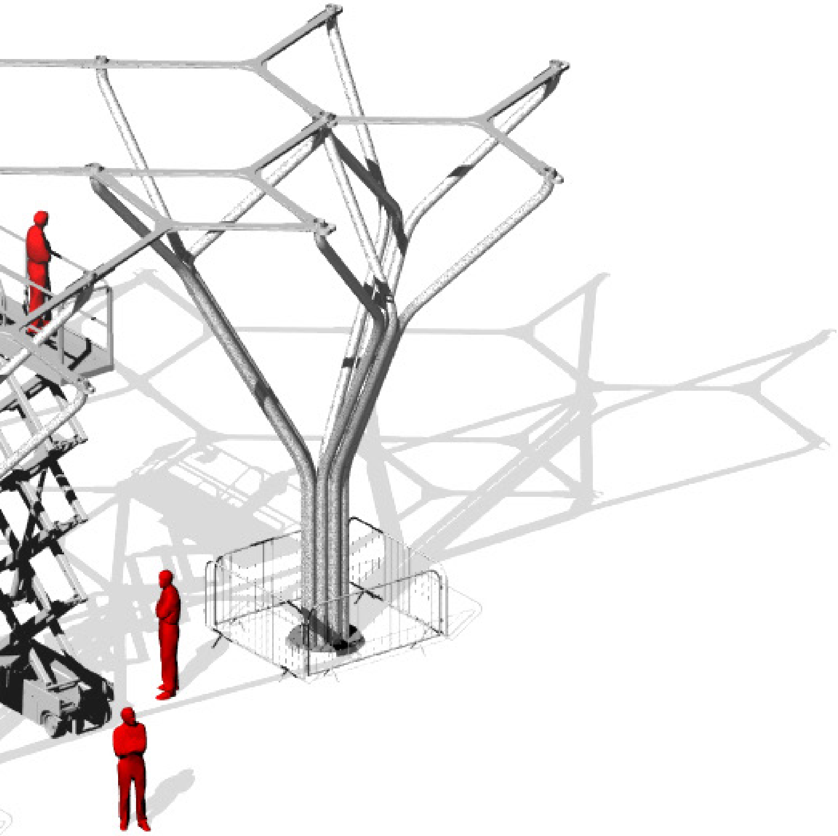


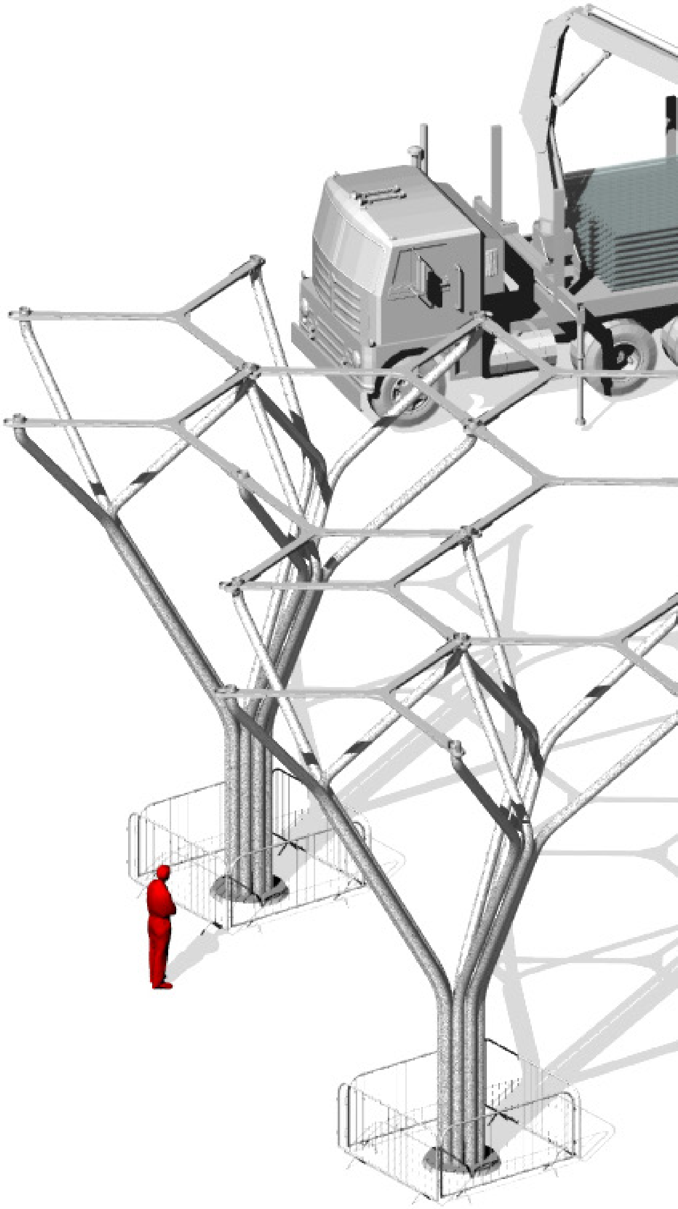


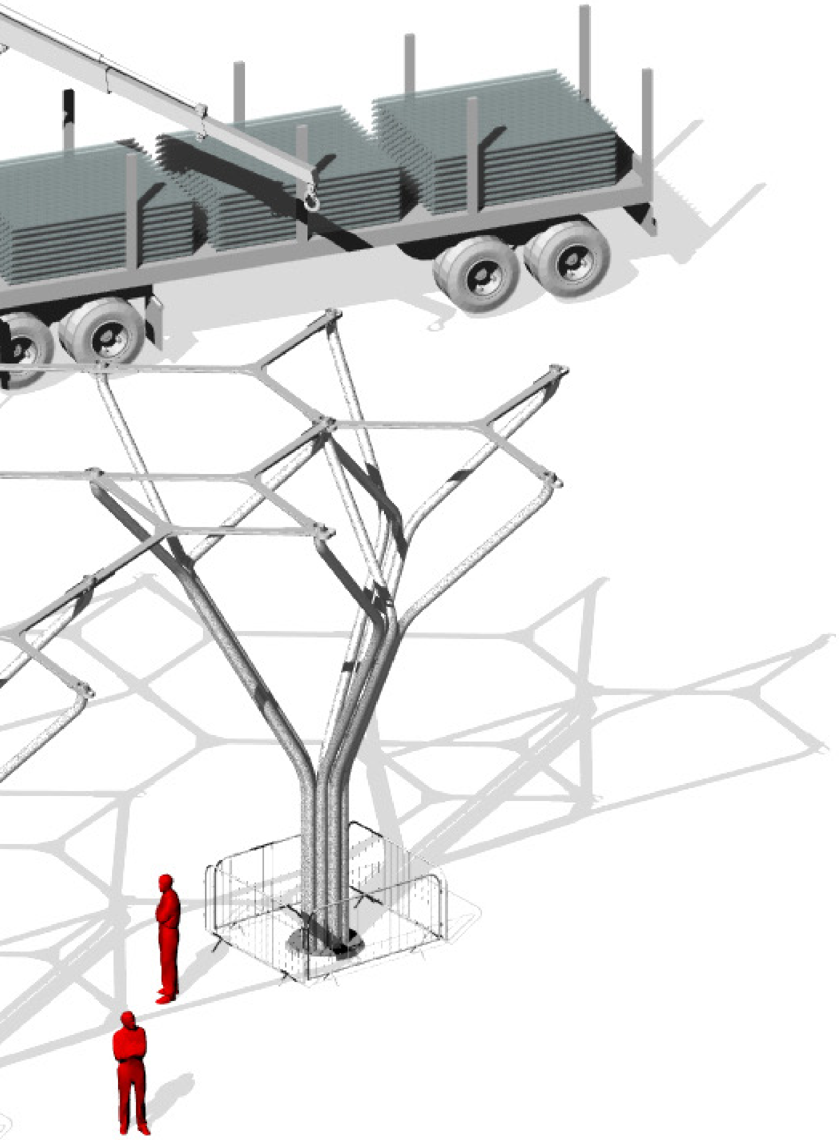


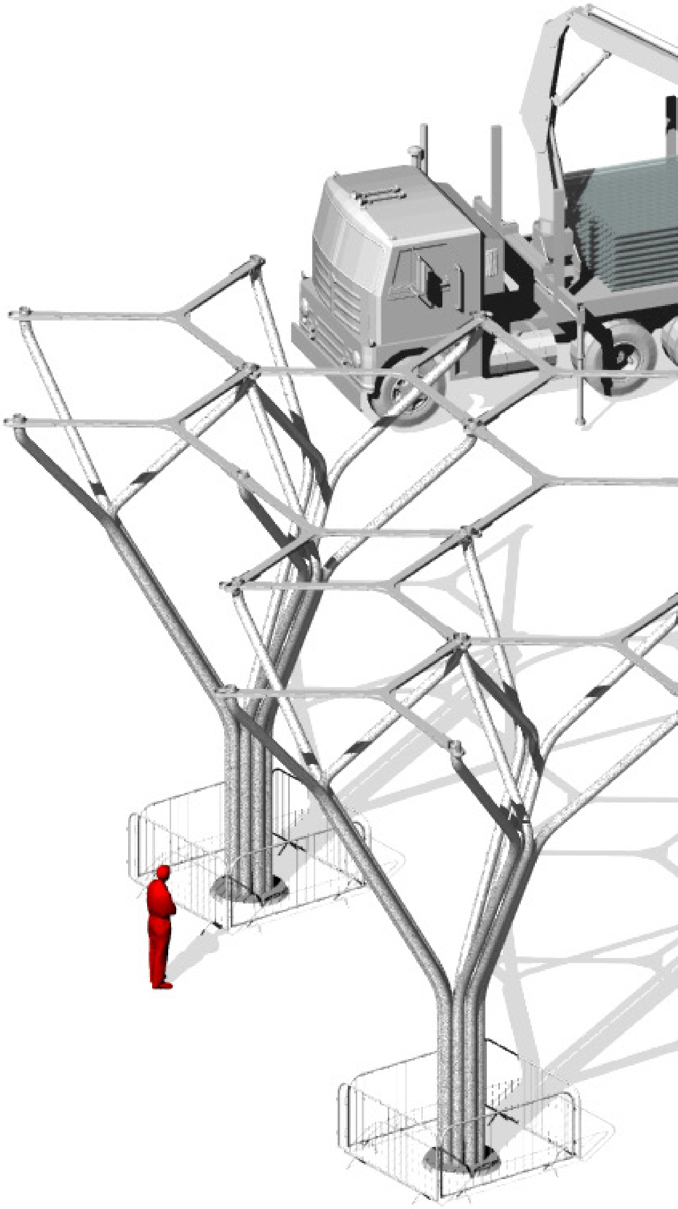


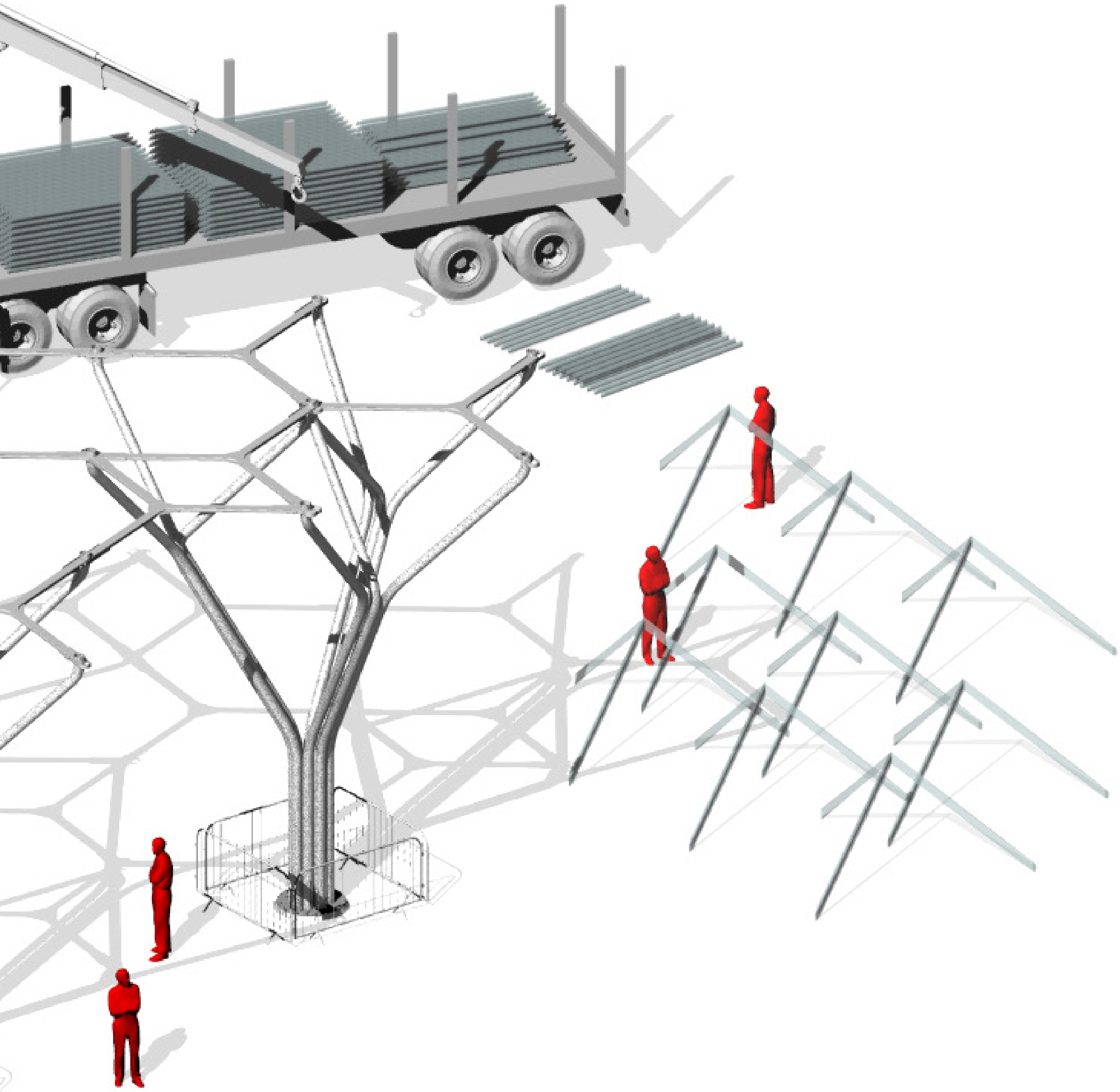


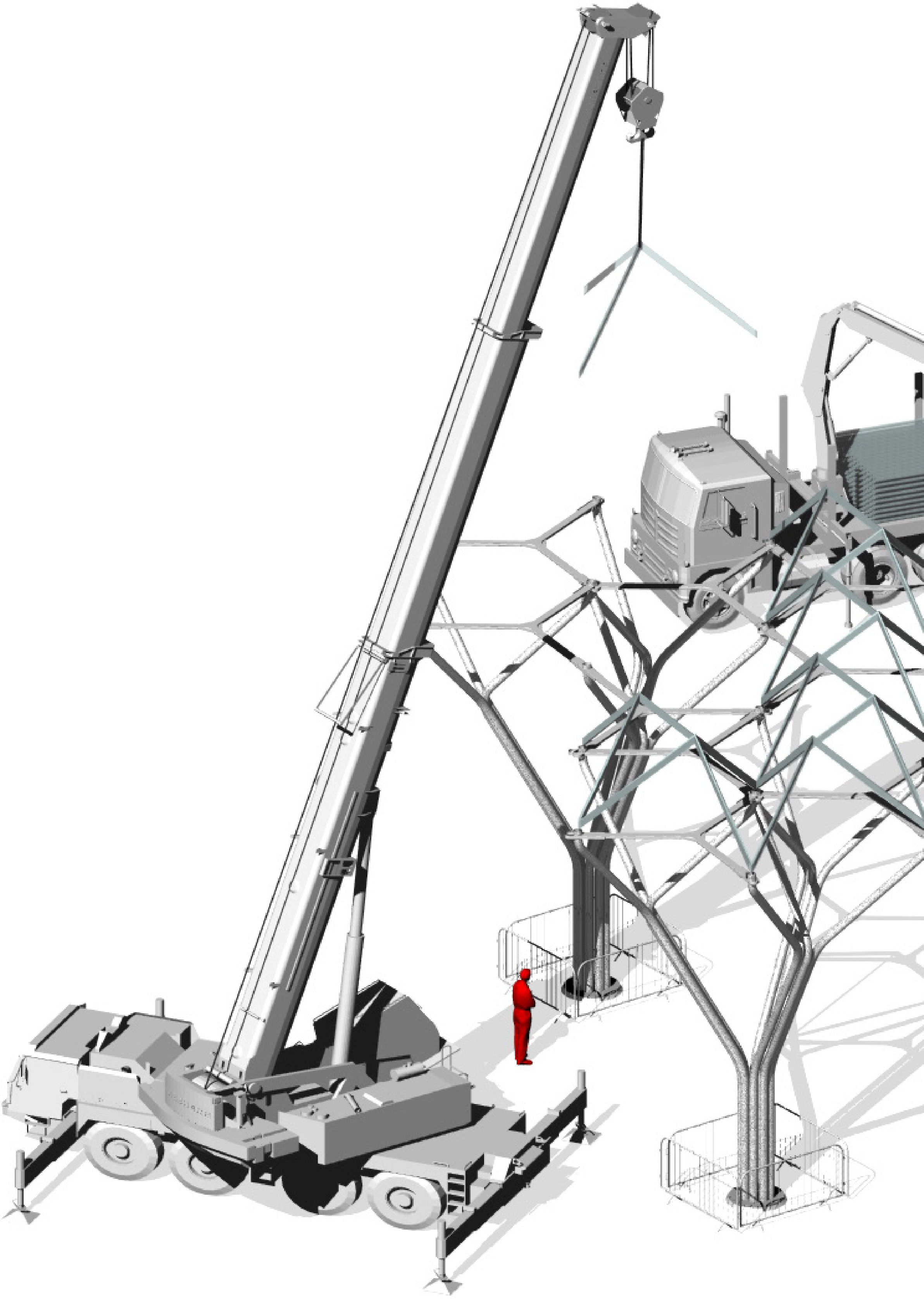


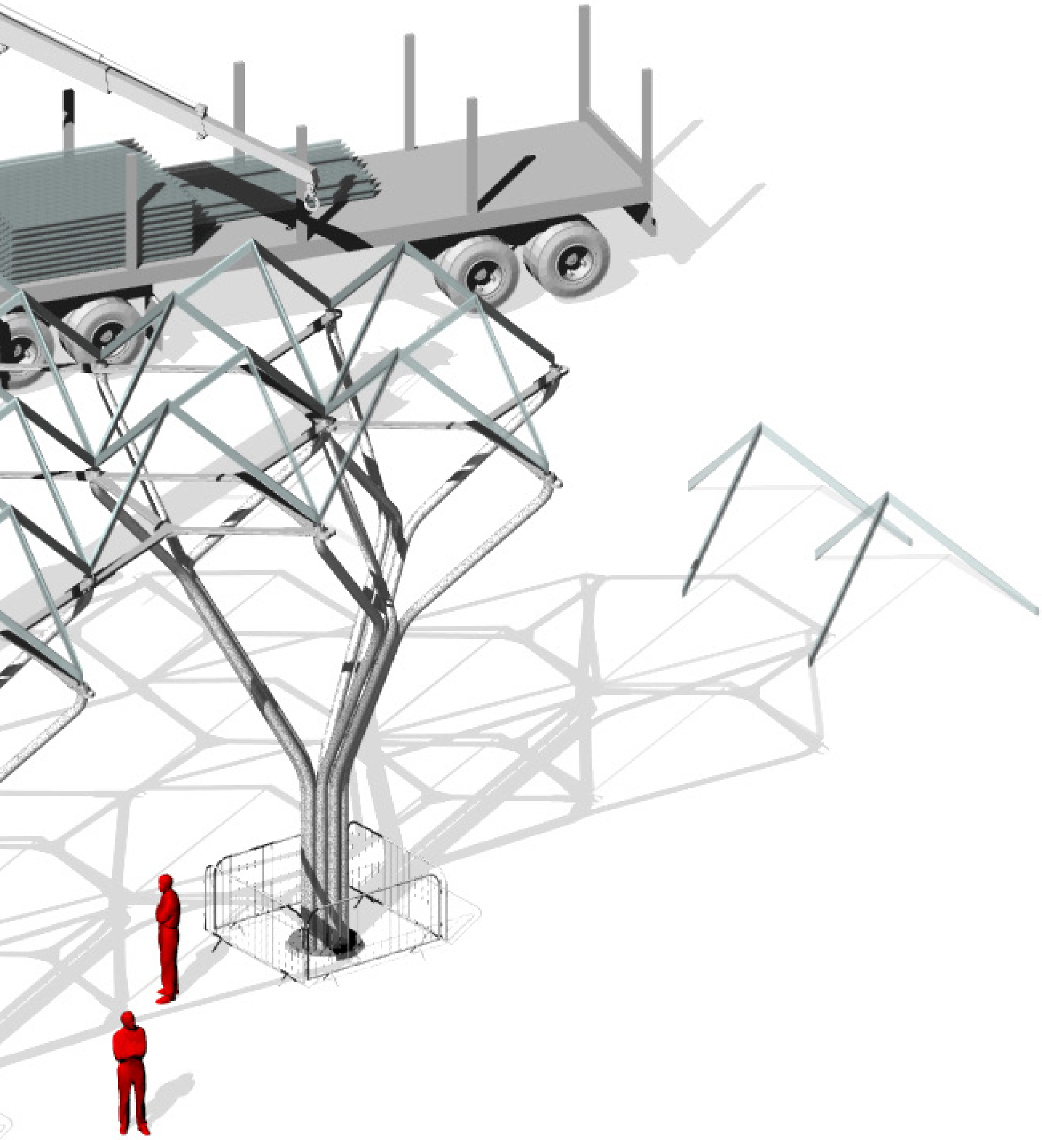


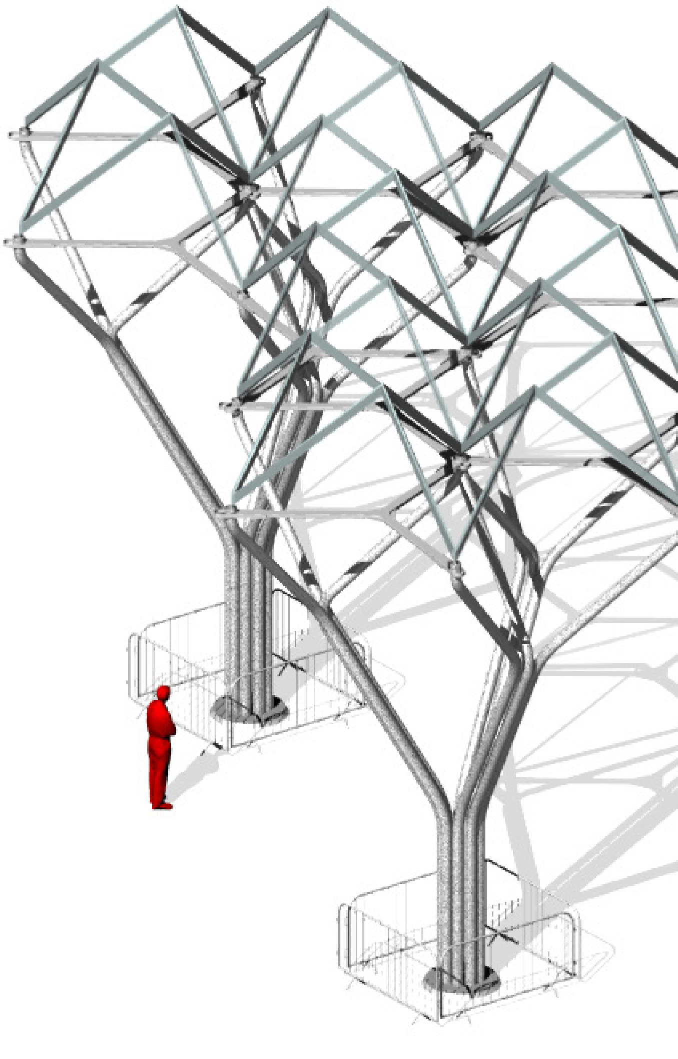


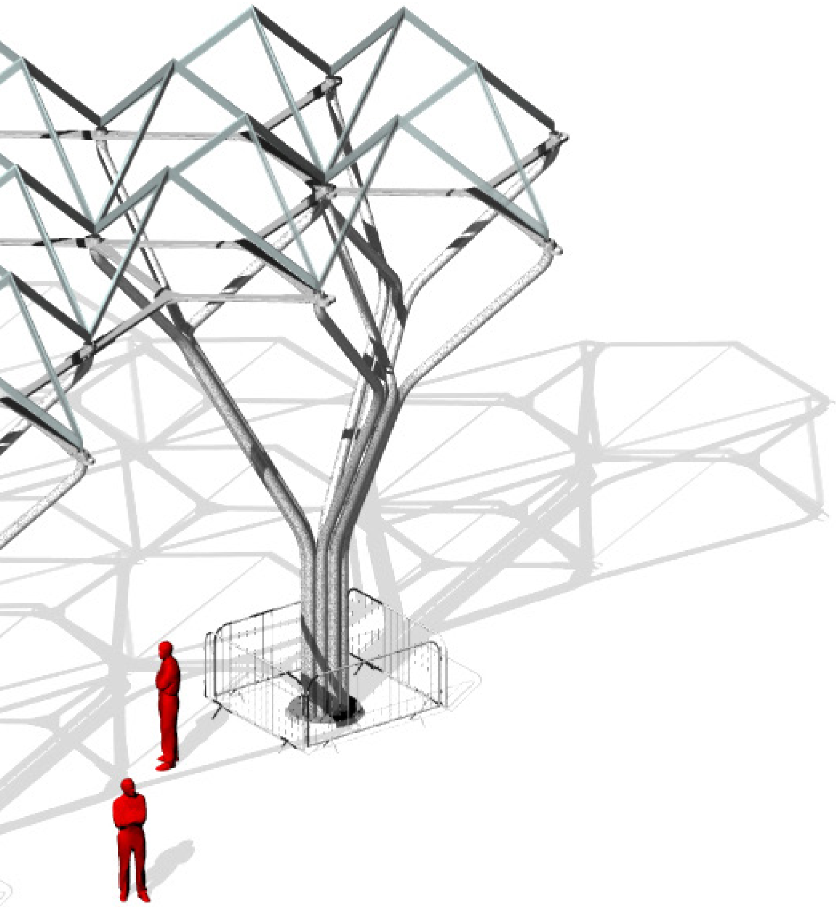


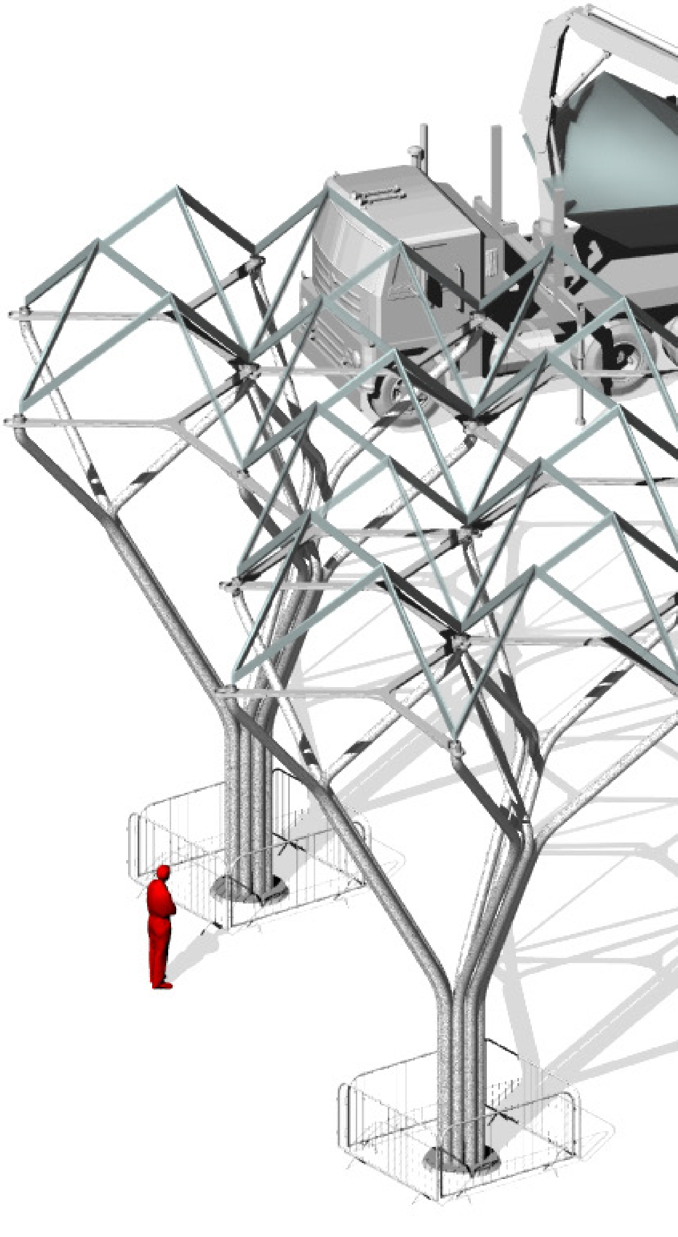


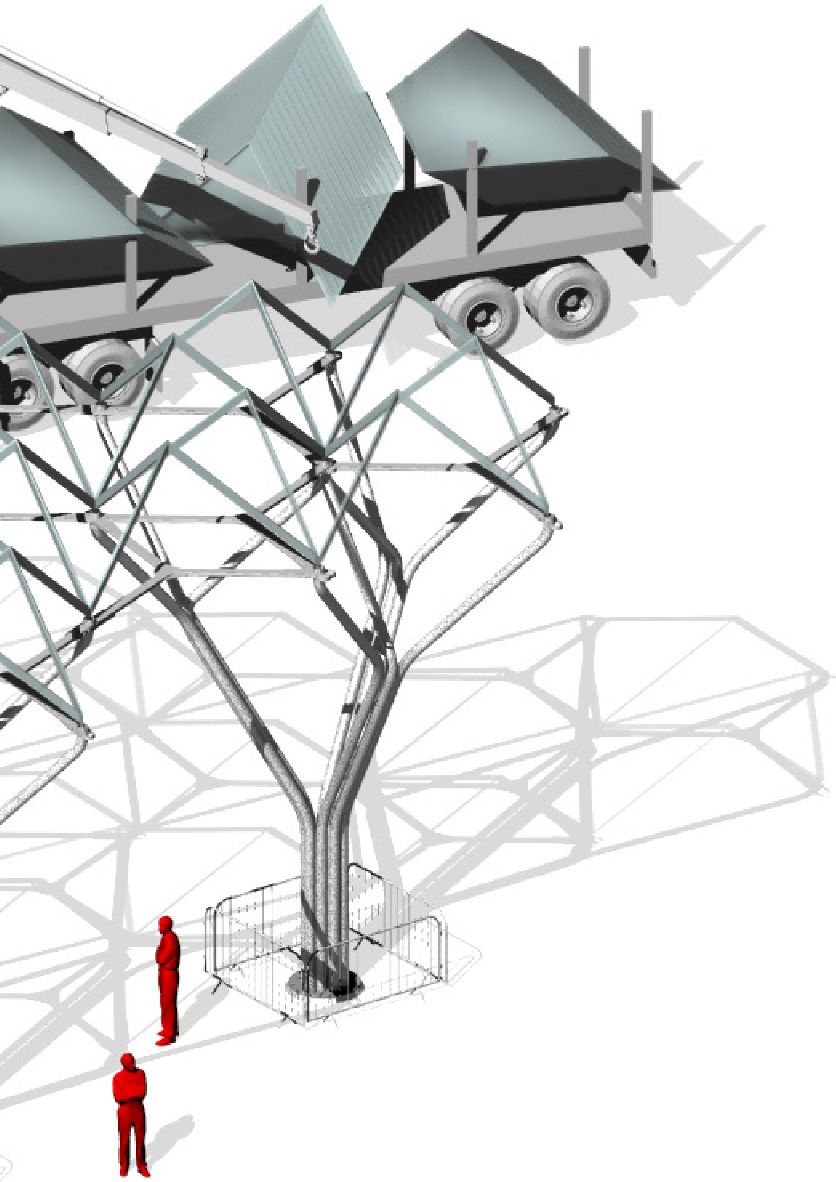


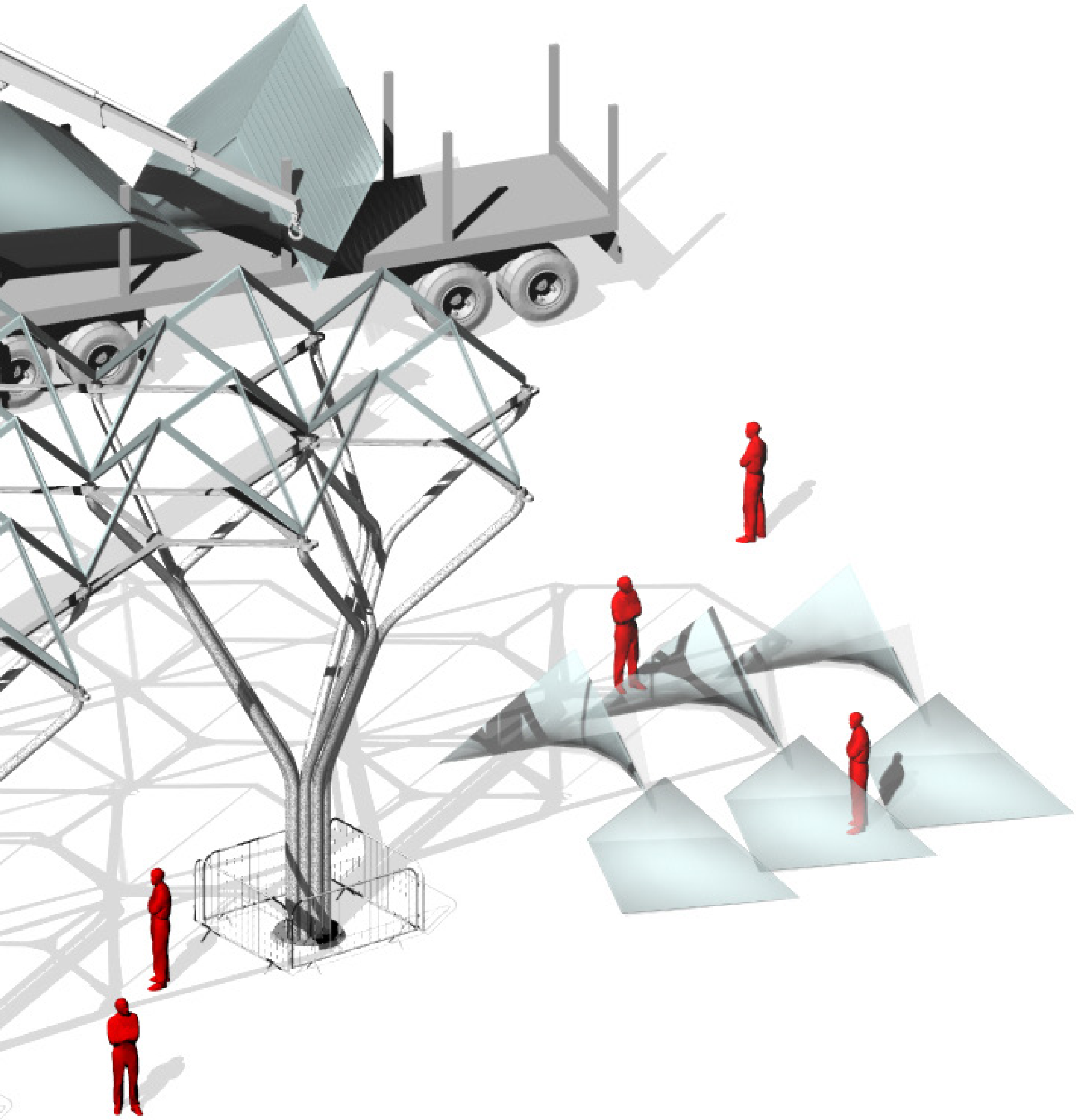


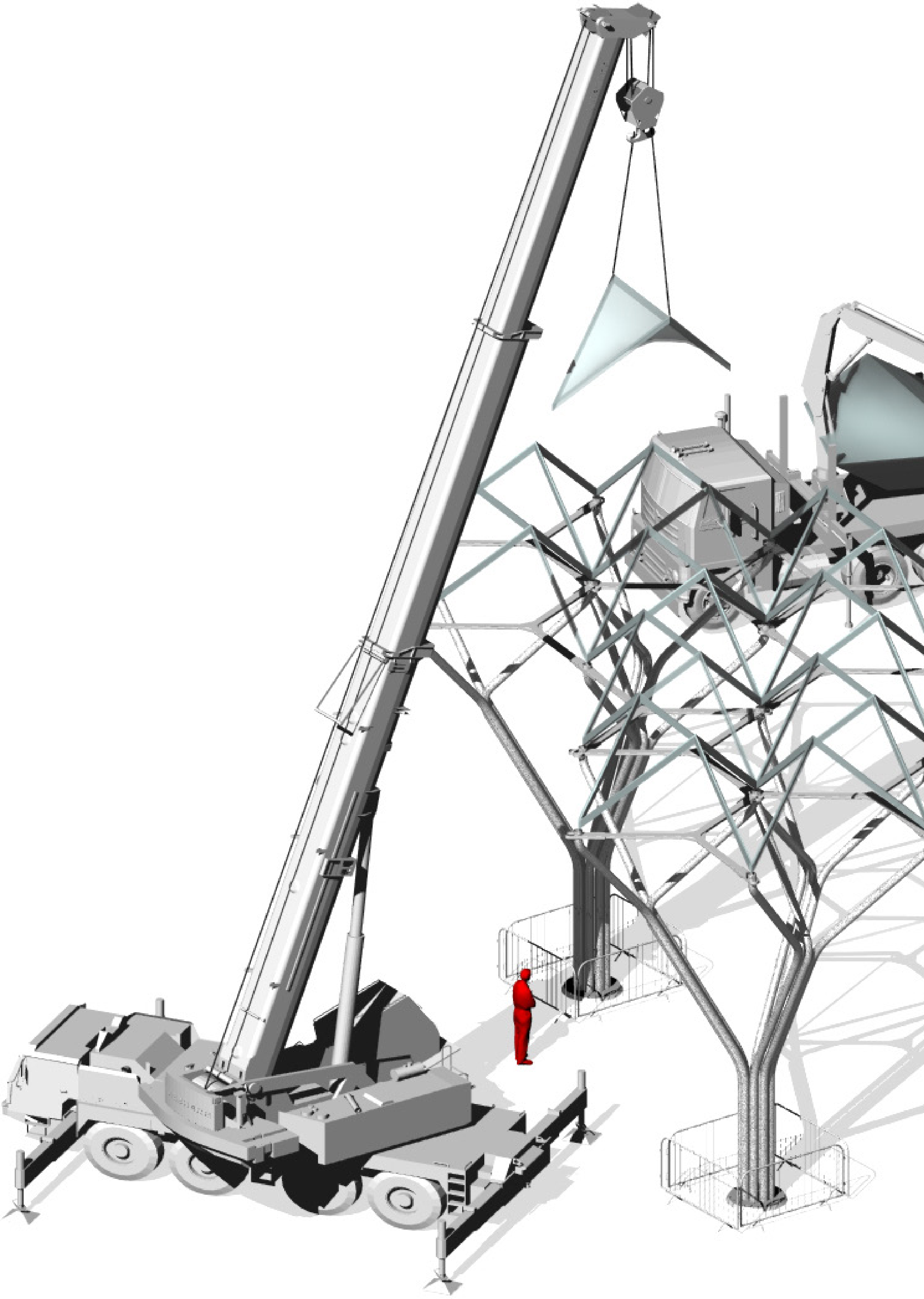


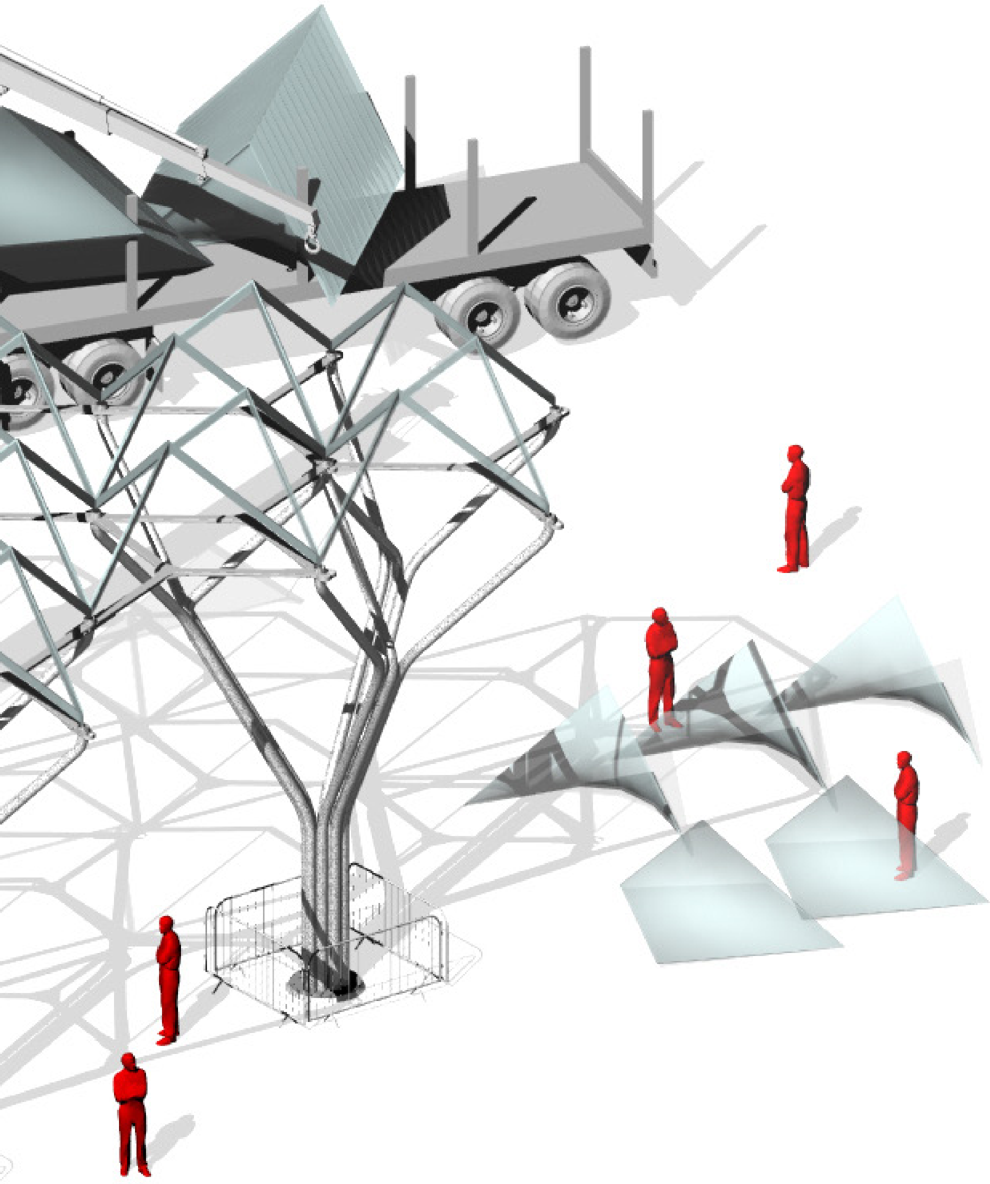


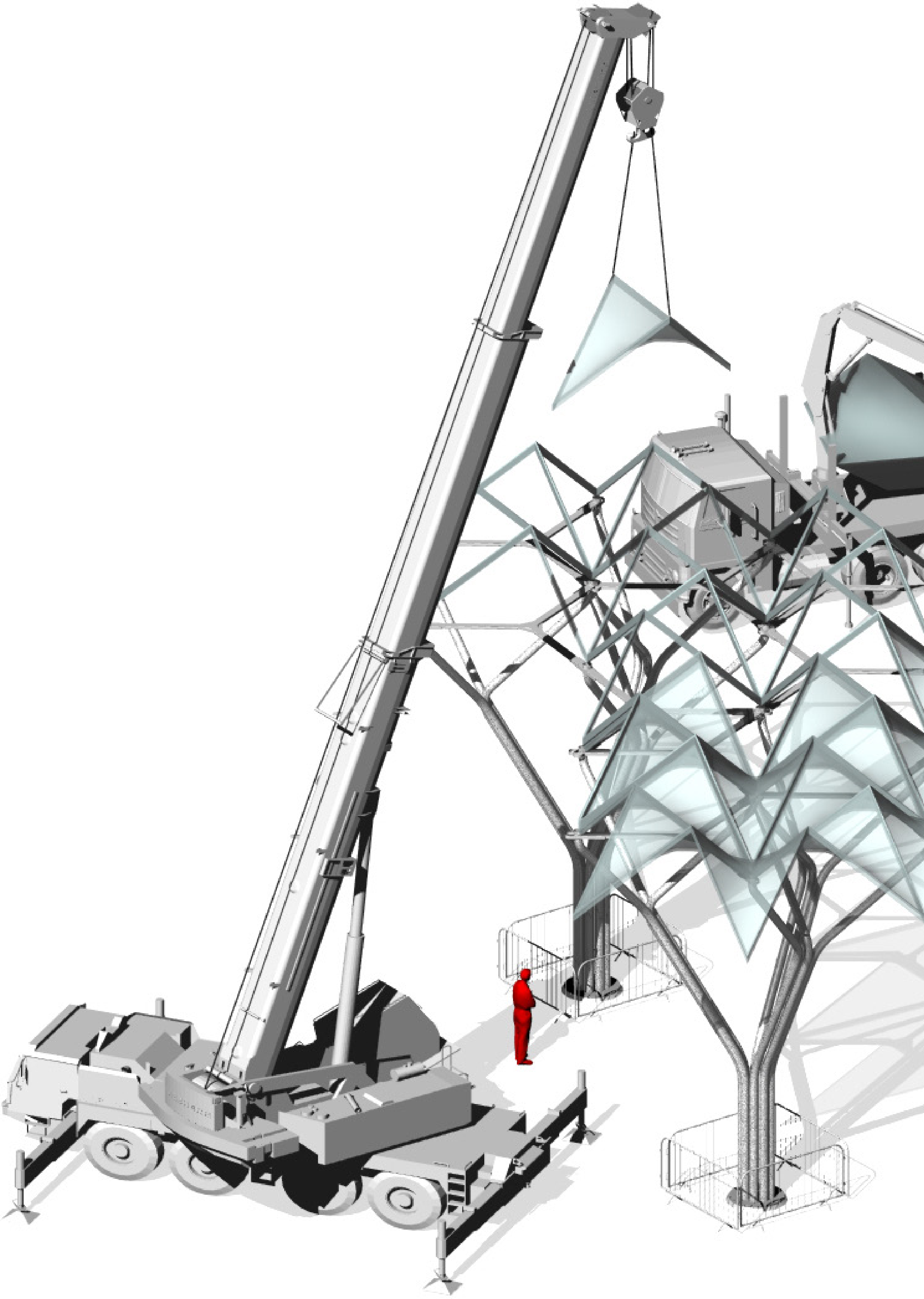


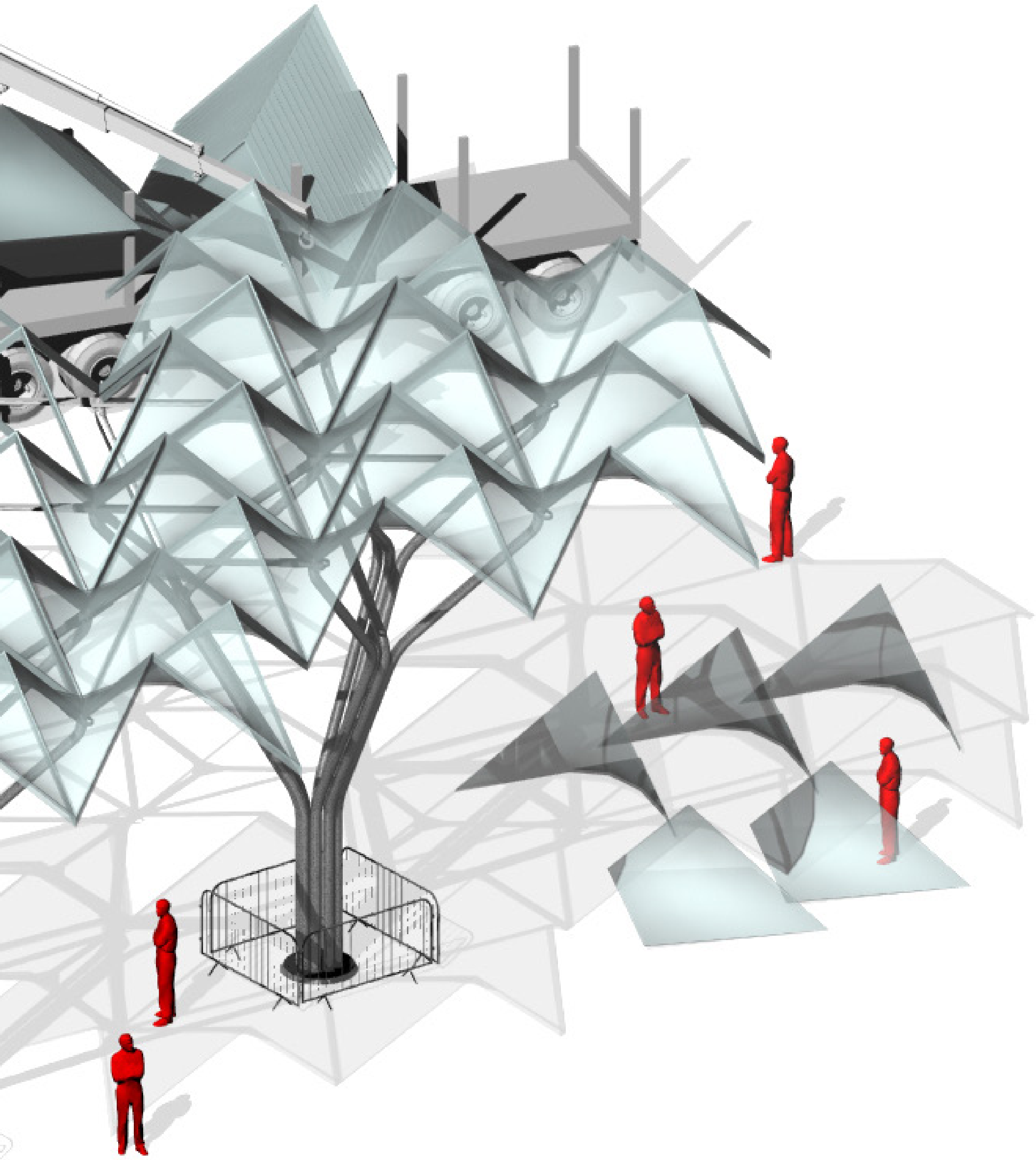


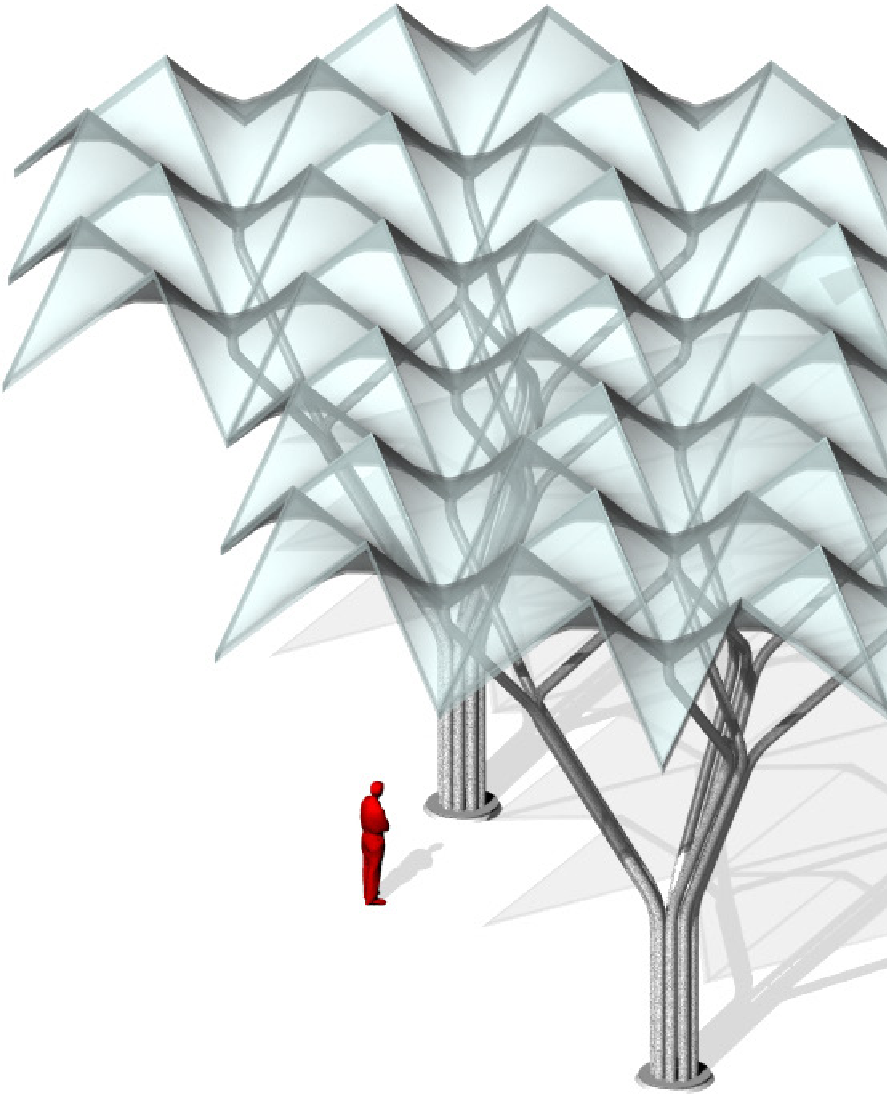


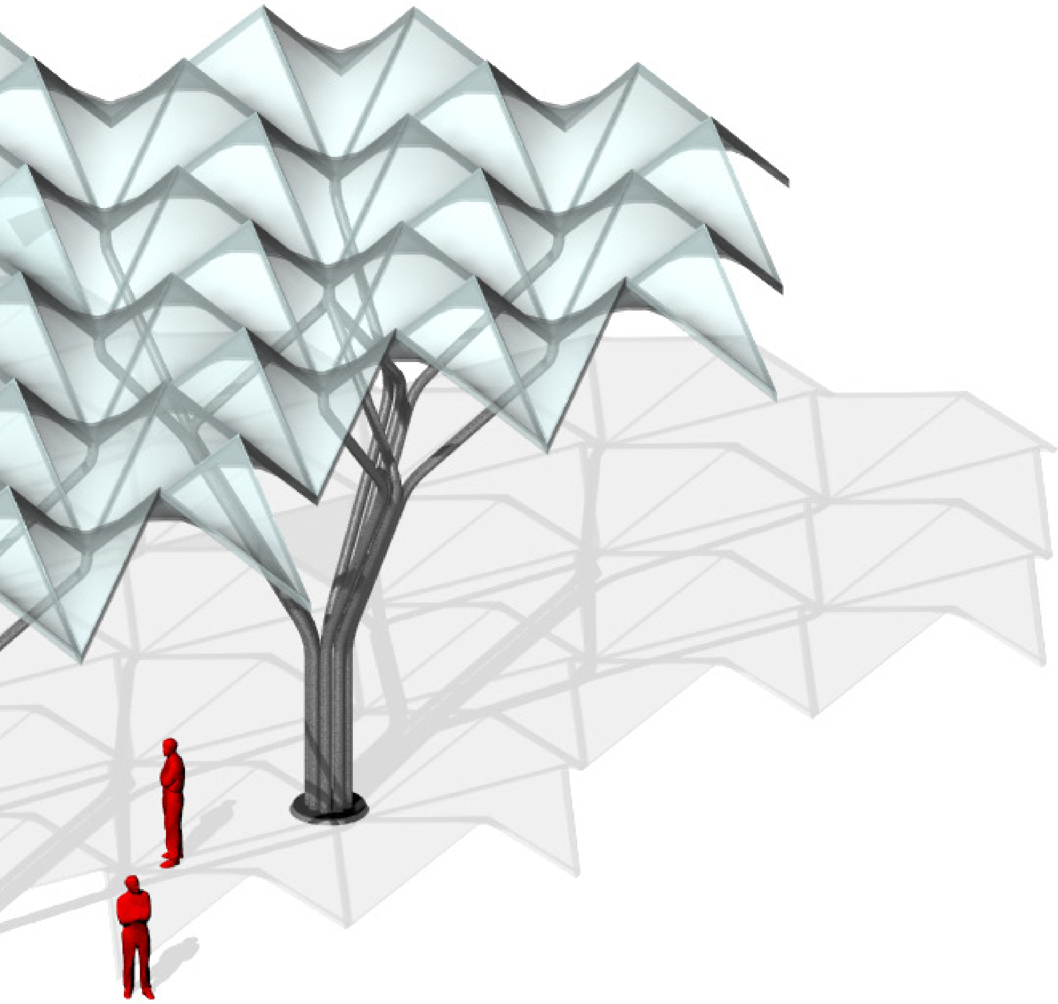












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