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Dynamic and robust timetable rescheduling for uncertain railway disruptions

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ABSTRACT

Unexpected disruptions occur frequently in the railways, during which many train services cannot run as scheduled. This paper deals with timetable rescheduling during such disruptions, particularly in the case where all tracks between two stations are blocked for hours. In practice, a disruption may become shorter or longer than predicted. To take the uncertainty of the disruption duration into account, this paper formulates the timetable rescheduling as a rolling horizon two-stage stochastic programming problem in deterministic equivalent form. The random disruption duration is assumed to have a finite number of possible realizations, called scenarios, with given probabilities. Every time a prediction about the range of the disruption end time is updated, new scenarios are defined, and a two-stage stochastic model computes the optimal rescheduling solution to all these scenarios. The stochastic method was tested on a part of the Dutch railways, and compared to a deterministic rolling-horizon method. The results showed that compared to the deterministic method, the stochastic method is more likely to generate better rescheduling solutions for uncertain disruptions by less train cancellations and/or delays, while the solution robustness can be affected by the predicted range regarding the disruption end time.

1. Introduction

Railway systems are vulnerable to unexpected disruptions caused by for instance incidents, infrastructure failures, and extreme weather. A typical consequence of a disruption is that the tracks between two stations are completely blocked for a few hours. Under this circumstance, trains are forbidden to enter the blocked tracks, and therefore the planned timetable is no longer feasible. Thus, traffic controllers have to reschedule the timetable for which they usually apply a pre-designed contingency plan specific to the disruption. Since the contingency plan is manually designed, its optimality cannot be guaranteed, and sometimes cannot even meet all operational constraints (Ghaemi et al., 2017b). For this reason, increasing attention is being paid to developing optimization models for computing rescheduling solutions. A detailed review can be found in Cacchiani et al. (2014).

Until now, many timetable rescheduling models have been proposed to deal with disruptions, which differ in e.g. the complexity of the network, the infrastructure modelling, the used dispatching measures, the objective, and the number of disruptions considered. For instance, Zhan et al. (2015) propose a Mixed Integer Linear Programming (MILP) model to reschedule the timetable in case of a complete track blockage by delaying, reordering and cancelling trains. They focus on a Chinese high-speed railway corridor where seat reservations are necessary for passengers, and therefore the measure of short-turning trains is not applicable. Veelenturf et al. (2015) propose an ILP model to handle partial or complete track blockages focusing on a part of the Dutch railway network where short-turning trains is commonly used during disruptions. They assign each train with the last scheduled stop before the blocked

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track as the only short-turn station. If the short-turn station lacks capacity to short-turn a train then it has to be cancelled completely. To reduce complete train cancellations, Ghaemi et al. (2018a) propose an MILP model to decide the optimal time and station of short-turning a train by assigning two short-turn station candidates. This has also been implemented in Ghaemi et al. (2017a) where the infrastructure is modelled at a microscopic level to improve solution feasibility in practice. The aforementioned papers aim to minimize train cancellations and delays. To reduce passenger inconveniences during disruptions, Zhu and Goverde (2019b) propose an MILP model where more short-turn station candidates are given for each train and also the stopping patterns of trains can be changed flexibly (i.e. skipping stops and adding stops). Binder et al. (2017) integrate passenger rerouting and timetable rescheduling into one ILP model where limited vehicle capacity is taken into account. While most literature focus on a single disruption, Zhu and Goverde (2019a) propose an MILP model to deal with multiple disruptions that have overlapping periods and are pairwise connected by at least one train line. Most literature share the assumption that the disruption duration is known and will not change over time. However in practice, a disruption may become shorter or longer than predicted (Zilko et al., 2016), thus dynamic adjustments are required.

To deal with the uncertainty of the disruption duration, Zhan et al. (2016) embed their rescheduling model into a rolling horizon framework where the timetable is adjusted gradually with renewed disruption durations taken into account. Ghaemi et al. (2018b) develop an iterative approach to reschedule the timetable in each iteration when a new disruption duration is updated. In both cases, deterministic models are used for the rescheduling. Meng and Zhou (2011) propose a stochastic programming model that takes the uncertainty of the disruption duration into account. The model reschedules the timetable dynamically by a rolling horizon approach for single-track railway lines using two dispatching measures: delaying and reordering. Quaglietta et al. (2013) also propose a rolling horizon approach to manage stochastic disturbances (small train delays) using retiming and reordering, where at regular rescheduling intervals the current delays are measured and the associated conflicts are predicted over a prediction horizon of fixed length. Then rescheduling solutions are generated for the entire prediction horizon but only the first part is implemented in the next rescheduling interval.

This paper deals with uncertain disruptions using two methods. We implemented a deterministic rolling-horizon approach based on the deterministic timetable rescheduling model of Zhu and Goverde (2019b). Also, we propose a stochastic rolling-horizon approach based on a two-stage stochastic timetable rescheduling model. Different from the existing literature, both methods are devoted to more complicated conditions, where (1) single-track and double-tack railway lines both exist; (2) a wide range of dispatching measures is allowed: delaying, reordering, cancelling, adding stops and flexible short-turning; (3) rolling stock circulations at terminal stations are considered, and (4) station capacity is taken into account. The rescheduling solution is computed until the normal schedule has been recovered.

The main contributions of this paper are summarized as follows:

- A rolling horizon two-stage stochastic timetable rescheduling model is proposed to handle uncertain disruptions.
- The proposed model allows delaying, reordering, cancelling, adding stops and flexible short-turning, and considers station capacity and rolling stock circulations at terminal stations.
- We test the stochastic method on a part of the Dutch railways, and compare it to a deterministic rolling-horizon method.

The remainder of the paper is organized as follows. Section 2 introduces the deterministic and stochastic methods. Both methods are tested with real-life instances in Section 3. Finally, Section 4 concludes the paper.

2. Methodology

A brief introduction is given to the basics considered in the deterministic and stochastic methods. After that, both methods are explained.

2.1. Basics

2.1.1. Event-activity network

The rescheduling model is based on an event-activity network. An *event* e is either a train departure or arrival that is associated with the original scheduled time o_e, station st_e, train line tl_e, train number tr_e, and operation direction dr_e. All departure (arrival) events constitute the set E_{de} (E_{ar}). An *activity* is a directed arc from an event to another. Multiple kinds of activities are established, including running activities A_{run}, dwell activities A_{dwell}, pass-through activities A_{pass}, headway activities A_{head}, short-turn activities A_{turn}, and OD turn activities A_{odturn}. We refer to Zhu and Goverde (2019b) for the details.

2.1.2. Decision variables

Any event $e \in E_{\mathrm{de}} \cup E_{\mathrm{ar}}$ corresponds to the following decision variables: (1) the rescheduled time x_e, (2) the delay d_e, (3) and the binary decision c_e with value 1 indicating that e is cancelled. Particularly for an event $e \in E^{\mathrm{turn}}_{\mathrm{de}} \cup E^{\mathrm{turn}}_{\mathrm{ar}}$, a binary decision y_e is needed, of which value 1 indicates that train tr_e is short-turned at station st_e. Here, $E^{\mathrm{turn}}_{\mathrm{de}}$ ($E^{\mathrm{turn}}_{\mathrm{ar}}$) is the set of departure (arrival) events that have short-turning possibilities. To deal with station capacity, for any arrival event $e \in E_{\mathrm{ar}}$, two binary decision variables are needed: (1) $u_{e,i}$ with value 1 indicating that train tr_e occupies the ith platform of station st_e, (2) and $v_{e,j}$ with value 1 indicating that train tr_e occupies the jth pass-through track of station st_e.

A short-turn (OD-turn) activity $a \in A_{\mathrm{turn}}$ ($a \in A_{\mathrm{odturn}}$) corresponds to a binary decision variable m_a with value 1 indicating that a is selected. A pass-through activity $a \in A_{\mathrm{pass}}$ corresponds to a binary decision variable s_a with value 1 indicating that a is added

Table 1
Sets and parameters.

Notation	Description
E_{ar}	Set of arrival events
E_{de}	Set of departure events
E	Set of events: $E = E_{\mathrm{ar}} \cup E_{\mathrm{de}}$
$E_{\mathrm{ar}}^{\mathrm{turn}}$	Set of arrival events that have short-turning possibilities
$E_{\mathrm{de}}^{\mathrm{turn}}$	Set of departure events that have short-turning possibilities
E^{turn}	Set of events that have short-turning possibilities: $E^{\mathrm{turn}} = E_{\mathrm{ar}}^{\mathrm{turn}} \cup E_{\mathrm{de}}^{\mathrm{turn}}$
o_e	The original scheduled time of event $e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}}$
p_w	The occurrence probability of scenario $w \in \{1,\dots,W\}$
$p_{w_{k,n}}$	The occurrence probability of scenario $w_{k,n}, n \in \{1,\dots,W_k\}$
r_e^{k-1}	The rescheduled time of event e determined at stage $k-1$, which is a known value at stage k
R_k	The recovery time length at stage $k \in \{1,\dots,K\}$
$R_k^{w_{k,n}}$	The recovery time length of scenario $w_{k,n}, n \in \{1,\dots,W_k\}$ at stage $k \in \{1,\dots,K\}$
st_e	The station corresponding to event $e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}}$
tr_e	The train corresponding to event $e \in E_{\mathrm{ar}} \cup E_{\mathrm{de}}$
t_{start}	The actual disruption starting time
t_{end}	The actual disruption ending time
$t_{\mathrm{end}}^{\mathrm{min}}$	The predicted minimal disruption ending time
$t_{\mathrm{end}}^{\mathrm{max}}$	The predicted maximal disruption ending time
t_{end}^{w}	The predicted disruption ending time of scenario $w \in \{1,\dots,W\}$: $t_{\mathrm{end}}^{\mathrm{min}} \le t_{\mathrm{end}}^{w} \le t_{\mathrm{end}}^{\mathrm{max}}$
$t_{\mathrm{end}}^{k,\mathrm{min}}$	The predicted minimal disruption ending time at stage $k \in \{1,\dots,K\}$
$t_{\mathrm{end}}^{k,\mathrm{max}}$	The predicted maximal disruption ending time at stage $k \in \{1,\dots,K\}$
$t_{\mathrm{end}}^{w_{k,n}}$	The predicted disruption ending time of scenario $w_{k,n}, n \in \{1,\dots,W_k\}$: $t_{\mathrm{end}}^{k,\mathrm{min}} \le t_{\mathrm{end}}^{w_{k,n}} \le t_{\mathrm{end}}^{k,\mathrm{max}}$
$w_{k,n}$	The nth scenario defined at stage k, where $n \in \{1,\dots,W_k\}, k \in \{1,\dots,K\}$
W_k	The total number of scenarios defined at stage k
X	Set of the 1st-stage decisions in the two-stage stochastic model
X_k	Set of the 1st-stage decisions in the two-stage stochastic model formulated at update stage $k \in \{1,\dots,K\}$
$Y(w)$	Set of the 2nd-stage decisions of scenario $w \in \{1,\dots,W\}$ in the two-stage stochastic model
$Y_k(w_{k,n})$	Set of the 2nd-stage decisions of scenario $w_{k,n}, n \in \{1,\dots,W_k\}$ in the two-stage stochastic model formulated at update stage $k \in \{1,\dots,K\}$
Z^{I}	Set of constraints for the 1st-stage decisions X
$Z^{\mathrm{II}}(X,w)$	Set of constraints for the 2nd-stage decisions given X in scenario $w \in \{1,\dots,W\}$
ℓ	A given time period ensuring a timely implementation of a new rescheduling solution
β_c	The penalty of cancelling a train run between two adjacent stations

with a stop. For any two different events $e, e' \in E_{\mathrm{de}} \cup E_{\mathrm{ar}}$, we have a binary decision variable $q_{e,e'}$ with value 1 indicating that e occurs before e'.

Note that due to our formulation, once the decisions regarding x_e, d_e, c_e and y_e are determined, the other decisions are also determined.

2.1.3. Disruptions

This paper considers a disruption that occurs at t_{start} and is predicted to end within the period $\left[t_{\mathrm{end}}^{\mathrm{min}}, t_{\mathrm{end}}^{\mathrm{max}}\right]$. The disruption duration is a stochastic variable that is assumed to have a finite number of possible realizations, called scenarios, $1,\dots,W$, with corresponding probabilities, $p_1,\dots,p_W$, satisfying $\sum_{w=1}^{W} p_w = 1$. Each scenario w has a unique disruption duration $\left[t_{\mathrm{start}}, t_{\mathrm{end}}^{w}\right]$ where $t_{\mathrm{end}}^{\mathrm{min}} \le t_{\mathrm{end}}^{w} \le t_{\mathrm{end}}^{\mathrm{max}}$.

During a disruption, the range of the disruption end time may change when new information is received from the disruption site. Therefore, we define the concept of *stages* at which the estimated range of the disruption end time is updated, which triggers a rescheduling model to compute a new solution based on the updated range. The range of the disruption end time updated at stage k is defined as $\left[t_{\mathrm{end}}^{k,\mathrm{min}}, t_{\mathrm{end}}^{k,\mathrm{max}}\right]$, where $t_{\mathrm{end}}^{k,\mathrm{min}}$ ($t_{\mathrm{end}}^{k,\mathrm{max}}$) refers to the minimal (maximal) disruption end time predicted at stage k with $t_{\mathrm{end}}^{k,\mathrm{max}} \ge t_{\mathrm{end}}^{k,\mathrm{min}}$. It is assumed that $t_{\mathrm{end}}^{k,\mathrm{min}} \ge t_{\mathrm{end}}^{k-1,\mathrm{min}}$, while $t_{\mathrm{end}}^{k,\mathrm{max}}$ is allowed to be equivalent to, smaller, or larger than $t_{\mathrm{end}}^{k-1,\mathrm{max}}$. This paper is also based on the following assumptions:

- At stage $k = 1$, the range of the disruption end time $\left[t_{\mathrm{end}}^{k,\mathrm{min}}, t_{\mathrm{end}}^{k,\mathrm{max}}\right]$ is obtained at the disruption start time t_{start}
- At stage $k \in [2, K-1]$, the range of the disruption end time $\left[t_{\mathrm{end}}^{k,\mathrm{min}}, t_{\mathrm{end}}^{k,\mathrm{max}}\right]$ is updated before time $t_{\mathrm{end}}^{k-1,\mathrm{min}} - \ell$
- At final stage K, the exact disruption end time t_{end} is received at time $t_{\mathrm{end}}^{K-1,\mathrm{min}} - \ell$, and $t_{\mathrm{end}} \ge t_{\mathrm{end}}^{K-1,\mathrm{min}}$

Here, ℓ is a given parameter relevant to the update time, which must ensure a timely implementation of a new rescheduling solution based on the updated information. The value of ℓ is relevant to the traffic density of the considered network and the extent of the deviation from the planned timetable. A network that has a denser traffic and the corresponding rescheduled timetable has more deviations than the planned one may need longer time for implementing the rescheduled timetable.

The notation of parameters and sets can be found in Table 1.

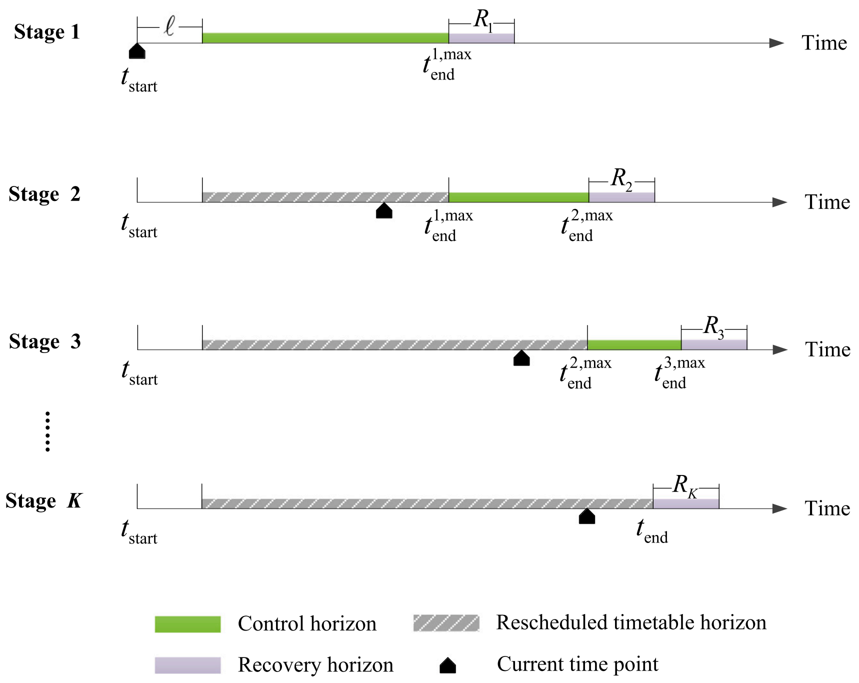

Fig. 1. The rolling horizon approach based on a deterministic rescheduling model using a pessimistic strategy.

2.2. Deterministic rolling-horizon method

A deterministic rescheduling model can only consider *one* possible disruption duration $\left[t_{\text{start}}, t_{\text{end}}^{w_{k,n}}\right]$ at stage k, where $t_{\text{end}}^{k,\min} \leq t_{\text{end}}^{w_{k,n}} \leq t_{\text{end}}^{k,\max}, w_{k,n} \in \left\{w_{k,1}, \ldots, w_{k,W_k}\right\}$. Here, $w_{k,n}$ refers to the nth scenario defined in stage k, and $1 \leq n \leq W_k$, where W_k is the total number of scenarios defined in stage k. The choice of $t_{\text{end}}^{w_{k,n}}$ depends on the adopted strategy. For example, the value of $t_{\text{end}}^{w_{k,n}}$ is chosen as (1) $t_{\text{end}}^{k,\min}$ in an optimistic strategy, (2) $t_{\text{end}}^{k,\max}$ in a pessimistic strategy, (3) or $\sum_{n=1}^{W_k} p_{w_{k,n}} t_{\text{end}}^{w_{k,n}}$ in an expected-value strategy.

In the remainder of this section, we give an example of a rolling horizon approach for a deterministic rescheduling model with a pessimistic strategy, see Fig. 1. Note that a new stage starts when a new prediction about the range of the disruption ending time is updated.

At stage $k \in [1, K-1]$, the prediction $\left[t_{\text{end}}^{k,\min}, t_{\text{end}}^{k,\max}\right]$ is updated. Using a pessimistic strategy, a control horizon is then defined as $\left[t_{\text{start}} + \ell, t_{\text{end}}^{k,\max}\right]$ if $k = 1$, where ℓ is a time period ensuring the decisions determined for the control horizon at stage 1 to be successfully implemented. It is assumed that the planned timetable is applied for the period $\left[t_{\text{start}}, t_{\text{start}} + \ell\right)$ during which some trains may have to wait at the last stations before the blocked tracks. A recovery horizon is defined as $\left(t_{\text{end}}^{k,\max}, t_{\text{end}}^{k,\max} + R_k\right]$ if $k = 1$. Here, R_k represents the recovery time length after $t_{\text{end}}^{k,\max}$, which is not a given input to the rescheduling model but an output that can only be known after the rescheduling solution has been computed. The deterministic rescheduling model computes a rescheduling solution over the combined control and recovery horizons. When $k \geq 2$, the rescheduling solution respects the previous disruption management decisions up to (1) $t_{\text{end}}^{k-1,\max}$ if $t_{\text{end}}^{k,\max} \geq t_{\text{end}}^{k-1,\max}$ or (2) $t_{\text{end}}^{k,\max}$ if $t_{\text{end}}^{k,\max} < t_{\text{end}}^{k-1,\max}$, and thus $\left[t_{\text{start}} + \ell, t_{\text{end}}^{k-1,\max}\right]$ or $\left[t_{\text{start}} + \ell, t_{\text{end}}^{k,\max}\right]$ is regarded as the rescheduled timetable horizon. Fig. 1 is an example of case (1). The proposed rolling-horizon approach also applies to case (2) in which the current time point (the update time) is ensured to be before $t_{\text{end}}^{k,\max}$ because it is assumed that the update at stage k occurs before $t_{\text{end}}^{k-1,\min} - \ell$ that holds for $t_{\text{end}}^{k-1,\min} - \ell \leq t_{\text{end}}^{k,\min} \leq t_{\text{end}}^{k,\max}$. A rescheduling solution is constituted by a set of disruption management decisions (e.g. cancelling trains and short-turning trains) that were introduced in Section 2.1.

At the final stage K, an exact disruption end time t_{end} is assumed to be known. If $t_{\text{end}} = t_{\text{end}}^{K-1,\max}$, the rescheduling solution obtained at stage $K-1$ is used without any further adjustments. If $t_{\text{end}} \neq t_{\text{end}}^{K-1,\max}$, the rescheduling model is solved again by respecting the previous disruption management decisions up to (1) $t_{\text{end}}^{K-1,\max}$ if $t_{\text{end}} \geq t_{\text{end}}^{K-1,\max}$, or (2) t_{end} if $t_{\text{end}} < t_{\text{end}}^{K-1,\max}$. In case (1) the control horizon is $\left[t_{\text{end}}^{K-1,\max}, t_{\text{end}}\right]$, while in case (2) the control horizon is zero. In both cases, the recovery horizons are $\left(t_{\text{end}}, t_{\text{end}} + R_K\right]$.

This paper uses the rescheduling model of Zhu and Goverde (2019b) for the deterministic rolling-horizon method, where the dispatching measure of skipping stops is removed due to the new objective of minimizing train cancellation and delay, and the station capacity part is reformulated as in Zhu and Goverde (2019a) for faster computation.

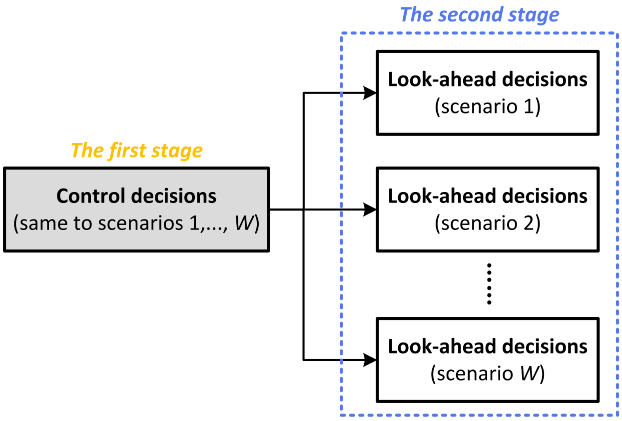

Fig. 2. Illustration of the two stages in the stochastic timetable rescheduling model.

2.3. Stochastic rolling-horizon method

The timetable rescheduling problem taking into account the uncertainty of the disruption duration is formulated as a rolling horizon two-stage stochastic program in deterministic equivalent form (Birge and Louveaux, 2011). For clarity, the stochastic timetable rescheduling model is introduced first without considering different update stages of the disruption durations, which are explicitly included later when describing the corresponding rolling horizon approach.

2.3.1. Stochastic timetable rescheduling model

The stochastic rescheduling model considers multiple possible disruption durations at each computation as follows. The set of disruption management decisions are divided into two groups: (1) the 1st-stage decisions that have to be taken before the exact scenario with a given disruption duration is known are called *control decisions* and the horizon when these decisions are applied is called *control horizon*, and (2) the 2nd-stage decisions that could be taken after the exact scenario with a given disruption duration is known are called *look-ahead decisions* with corresponding *look-ahead horizon*. Recall that we have an estimated range of disruption end time $\left[t_{\text{end}}^{\min}, t_{\text{end}}^{\max}\right]$ to represent the stochastic part of disruption duration, and each scenario $w \in \{1, \dots, W\}$ is defined with a unique disruption duration $\left[t_{\text{start}}, t_{\text{end}}^{w}\right]$ where $t_{\text{end}}^{\min} \leq t_{\text{end}}^{w} \leq t_{\text{end}}^{\max}$.

In each scenario w, $\left[t_{\text{start}} + \ell, t_{\text{end}}^{\min}\right]$ is defined as the control horizon, while $\left(t_{\text{end}}^{\min}, t_{\text{end}}^{w} + R^{w}\right]$ is defined as the look-ahead horizon, where ℓ refers to a time period ensuring the control decisions to be timely implemented, and R^{w} represents the recovery time to the planned timetable. The planned timetable is applied for the period $\left[t_{\text{start}}, t_{\text{start}} + \ell\right)$ where some trains might be forced to wait at the last stations before the blocked tracks. Recall that R^{w} can only be known after the disruption management decisions for scenario w are determined, and so the value may vary across scenarios. A look-ahead horizon consists of a *disruption horizon* $\left(t_{\text{end}}^{\min}, t_{\text{end}}^{w}\right]$ in which the disruption is ongoing, and a *recovery horizon* $\left(t_{\text{end}}^{w}, t_{\text{end}}^{w} + R^{w}\right]$ that goes from the end of the disruption until completely resuming to the planned timetable. The 1st-stage control decisions are scenario independent and are thus the same over all scenarios. The 2nd-stage look-ahead decisions are scenario dependent, which can be different among scenarios. As shown in Fig. 2, determining the control decisions up to $t_{\text{end}}^{\min}$ is the first stage of the stochastic timetable rescheduling model, and determining the look-ahead decisions within the period $\left(t_{\text{end}}^{\min}, t_{\text{end}}^{w} + R^{w}\right]$ for any scenario w is the second stage. The control decisions determined at the first stage affect the look-ahead decisions determined at the second stage.

The two-stage stochastic timetable rescheduling model can be formulated in a more *compact* form as

$$\min \ Q^{\text{I}}(X) + E_{w}\left[\min \ Q^{\text{II}}(Y(w))\right], \tag{1}$$

$$\text{s.t.} \quad X \in Z^{\text{I}}, \tag{2}$$

$$Y(w) \in Z^{\text{II}}(X, w), \qquad w \in \{1, \dots, W\} \tag{3}$$

where X are the **1st-stage decisions** defined as the scenario-independent disruption management decisions associated with the train arrival and departure events e of which the original scheduled times o_e are in the control horizon $\left[t_{\text{start}} + \ell, t_{\text{end}}^{\min}\right]$,

$$X = \left\{\{c_e, d_e, x_e\} : o_e \in [t_{\text{start}} + \ell, t_{\text{end}}^{\min}], e \in E\right\} \cup \left\{y_e : o_e \in [t_{\text{start}} + \ell, t_{\text{end}}^{\min}], e \in E^{\text{turn}}\right\},$$

and $Y(w)$ are the **2nd-stage decisions** of scenario w, which are defined as the disruption management decisions associated with the train arrival and departure events e of which the original scheduled times o_e are in the look-ahead horizon $\left(t_{\text{end}}^{\min}, t_{\text{end}}^{w} + R^{w}\right]$ of scenario w,

$$Y(w) = \left\{\{c_e^{w}, d_e^{w}, x_e^{w}\} : o_e \in (t_{\text{end}}^{\min}, t_{\text{end}}^{w} + R^{w}], e \in E\right\} \cup \left\{y_e^{w} : o_e \in (t_{\text{end}}^{\min}, t_{\text{end}}^{w} + R^{w}], e \in E^{\text{turn}}\right\}, w \in \{1, \dots, W\}.$$

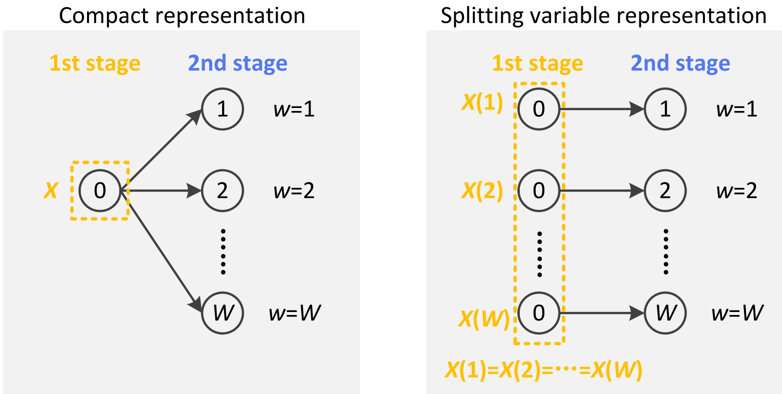

Fig. 3. Illustration of scenarios.

$Y(w)$ is dependent on X since X and the $Y(w)$ are jointly optimized in (1)–(3). Here, c_e represents the decision to cancel event $e \in E$, d_e represents the delay of event $e \in E$, x_e represents the rescheduled time of event $e \in E$, and y_e represents the decision to short-turn train tr_e at station st_e considering event $e \in E^{\text{turn}}$. Recall that E is the set of arrival and departure events, and E^{turn} is the set of arrival and departure events that have short-turning possibilities. In the 2nd stage the same notation is used with a superscript w to indicate the scenario. The developed two-stage stochastic timetable rescheduling model includes more decision variables (see Section 2.1.2) than those shown in the formulation of (1)–(3). We only show the event-related decision variables with respect to cancelling, delaying, re-timing, and short-turning in the compact formulation because once these decisions are determined the other decisions will be determined implicitly as well. $Q^{\text{I}}(\cdot)$ is the cost function for X, and $Q^{\text{II}}(\cdot)$ is the cost function for $Y(w)$, which are formulated respectively as follows:

$$Q^{\text{I}}(X) = \beta_c \sum_{e\in E_{\text{ar}}:\ c_e\in X} c_e + \sum_{e\in E_{\text{ar}}:\ d_e\in X} d_e,$$
$$Q^{\text{II}}(Y(w)) = \beta_c \sum_{e\in E_{\text{ar}}:\ c_e^w\in Y(w)} c_e^w + \sum_{e\in E_{\text{ar}}:\ d_e^w\in Y(w)} d_e^w, \qquad w \in \{1,\dots,W\},$$

where parameter β_c refers to the cost of cancelling a train run between two adjacent stations. The cost function $Q^{\text{I}}(\cdot)$ ($Q^{\text{II}}(\cdot)$) measures the train cancellations and arrival delays within the control horizon (look-ahead horizon) relevant to the first stage (the second stage) of the stochastic timetable rescheduling model. The objective (1) is to minimize the train cancellations and arrival delays within the control horizon plus the expectation of the train cancellations and arrival delays within the look-ahead horizons of all scenarios. The expectation $E_w[\cdot]$ is defined as $\sum_{w=1}^{W} p_w \cdot Q^{\text{II}}(Y(w))$, where p_w represents the occurrence probability of scenario w. In (2), Z^{I} refers to the constraint set for X. In (3), $Z^{\text{II}}(X,w)$ refers to the constraint set for $Y(w)$ given X under scenario w. $Y(w)$ is required to be consistent with X. For any scenario $w \in \{1,\dots,W\}$, the decisions X and $Y(w)$ together constitute a feasible rescheduling solution satisfying the constraints in $Z^{\text{I}} \cup Z^{\text{II}}(X,w)$ for the time horizon $\left[t_{\text{start}} + \ell, t_{\text{end}}^w + R^w\right]$.

The two-stage stochastic timetable rescheduling model of (1)–(3) is based on a compact representation of scenarios as shown in the left part of Fig. 3, where each root-to-leaf path refers to a specific scenario w. For simplicity, we used a splitting variable representation (Escudero et al., 2013) as shown in the right part of Fig. 3. In this way, the first-stage decisions X is duplicated for each scenario $w \in \{1,\dots,W\}$ as $X(w)$. Based on the splitting variable representation, we reformulated the two-stage stochastic timetable rescheduling model of (1)–(3) with explicit *nonanticipativity* constraints considering stage $k = 1$ (the range of the disruption end time is updated for the first time),

$$\min \sum_{n=1}^{W_1} p_{w_{1,n}} \cdot \left(\left(\beta_c \sum_{e\in E_{\text{ar}}:\ c_e^{w_{1,n}}\in X_1(w_{1,n})} c_e^{w_{1,n}} + \sum_{e\in E_{\text{ar}}:\ d_e^{w_{1,n}}\in X_1(w_{1,n})} d_e^{w_{1,n}} \right) + \left(\beta_c \sum_{e\in E_{\text{ar}}:\ c_e^{w_{1,n}}\in Y_1(w_{1,n})} c_e^{w_{1,n}} + \sum_{e\in E_{\text{ar}}:\ d_e^{w_{1,n}}\in Y_1(w_{1,n})} d_e^{w_{1,n}} \right) \right), \tag{4}$$

$$\text{s.t.}\quad X_1(w_{1,n}) \in Z_1^{\text{I}}(w_{1,n}), \qquad n \in \left\{1,\dots,W_1\right\}, \tag{5}$$
$$Y_1(w_{1,n}) \in Z_1^{\text{II}}\left(X_1(w_{1,n}), w_{1,n}\right), \qquad n \in \left\{1,\dots,W_1\right\}, \tag{6}$$
$$X_1(w_{1,n}) = X_1(w_{1,m}), \qquad n,m \in \left\{1,\dots,W_1\right\} : n \neq m, \tag{7}$$

where the first-stage decisions $X_1(w_{1,n})$ of scenario $w_{1,n}$ is

$$X_1(w_{1,n}) = \left\{ \{c_e^{w_{1,n}}, d_e^{w_{1,n}}, x_e^{w_{1,n}}\} : o_e \in [t_{\text{start}} + \ell, t_{\text{end}}^{1,\min}], e \in E \right\} \cup \left\{ y_e^{w_{1,n}} : o_e \in [t_{\text{start}} + \ell, t_{\text{end}}^{1,\min}], e \in E^{\text{turn}} \right\},$$
$$n \in \{1,\dots,W_1\},$$

Table 2
Part of decision variables.

Notation	Description
c_e^w	Binary variable with value 1 indicating that event e is cancelled in scenario w, and 0 otherwise
d_e^w	Delay of event e in scenario w
x_e^w	Rescheduled time of event e in scenario w
y_e^w	Binary variable with value 1 indicating that train tr_e is short-turned at station st_e in scenario w, and 0 otherwise

and the second-stage decisions $Y_1(w_{1,n})$ of scenario $w_{1,n}$ is

$$Y_1(w_{1,n}) = \left\{ \{c_e^{w_{1,n}}, d_e^{w_{1,n}}, x_e^{w_{1,n}}\} : o_e \in (t_{\text{end}}^{1,\min}, t_{\text{end}}^{w_{1,n}} + R_1^{w_{1,n}}], e \in E \right\} \cup \left\{ y_e^{w_{1,n}} : o_e \in (t_{\text{end}}^{1,\min}, t_{\text{end}}^{w_{1,n}} + R_1^{w_{1,n}}], e \in E^{\text{turn}} \right\},$$
$$n \in \{1, \ldots, W_1\}.$$

Here, $w_{1,n}$ represents the nth scenario defined at stage 1, W_1 refers to the number of scenarios defined at stage 1, and $t_{\text{end}}^{1,\min}$ is the minimal disruption end time update at stage 1. Note that $X_1(w_{1,n}) = X_1$ for some optimally determined X_1 for all $w_{1,n}, n \in \{1, \ldots, W_1\}$. The formulation of (4)–(7) can be seen as W_1 separate deterministic Mixed-Integer Linear Programming (MILP) timetable rescheduling models linked together by the so-called nonanticipativity constraints (7) (Escudero et al., 2010), which force the 1st-stage decisions $X_1(w_{1,n})$ to be the same in any scenario $w_{1,n}, n \in \{1, \ldots W_1\}$. To be more specific, (7) requires each decision of $X_1(w_{1,n})$ to be equivalent to the same type of decision corresponding to the same event in $X_1(w_{1,m})$ considering two different scenarios $w_{1,n}$ and $w_{1,m}$. For example, $c_e^{w_{1,n}} = c_e^{w_{1,m}}$, where $c_e^{w_{1,n}} \in X(w_{1,n}), c_e^{w_{1,m}} \in X(w_{1,m}), n \neq m$. In (5), $Z_1^{\text{I}}(w_{1,n})$ refers to the constraint set for $X_1(w_{1,n})$. In (6), $Z_1^{\text{II}}\left(X_1(w_{1,n}), w_{1,n}\right)$ refers to the constraint set for $Y_1(w_{1,n})$ given $X_1(w_{1,n})$ under scenario $w_{1,n}$. The objective (4) is to minimize the expected consequences measured in train cancellations and arrival delays both in the 1st stages and 2nd stages of all scenarios.

To establish (4)–(7), we construct, for each scenario $w_{1,n}, n \in \{1, \ldots, W_1\}$, an independent deterministic MILP timetable rescheduling model by the method of Zhu and Goverde (2019b), of which the variables are $\{X_1(w_{1,n}), Y_1(w_{1,n})\}$, and the constraints are $\{Z_1^{\text{I}}(w_{1,n}), Z_1^{\text{II}}(X_1(w_{1,n}), w_{1,n})\}$ that ensure feasible rescheduling solutions adjusted by delaying, reordering, cancelling, adding stops and flexible short-turning trains. For a detailed MILP constraint formulation we refer to Zhu and Goverde (2019b). For all scenarios the variables $\bigcup_{n\in\{1,\ldots,W_1\}} \{X_1(w_{1,n}), Y_1(w_{1,n})\}$ and constraints $\bigcup_{n\in\{1,\ldots,W_1\}} \{Z_1^{\text{I}}(w_{1,n}), Z_1^{\text{II}}(X_1(w_{1,n}), w_{1,n})\}$ are established in the stochastic timetable rescheduling model with also nonanticipativity constraints (7).

The notation of the decision variables are described in Table 2.

The rescheduling solution formed by X_1 will be delivered to the traffic controllers directly. As for the scenario-dependent 2nd-stage decisions $Y_1(w_{1,n}), n \in \{1, \ldots, W_1\}$, only one of them will be delivered at time $t_{\text{end}}^{1,\min} - \ell$ when the exact scenario is foreseen to be a specific scenario $w_{1,n}$. ℓ is set to an appropriate value (e.g. 10 min) to ensure that the 2nd-stage decisions can be implemented in time. If none of the defined scenarios correspond to the exact scenario, the rescheduling model computes a new solution considering one single scenario with disruption duration $\left[t_{\text{end}}^{1,\min}, t_{\text{end}}\right]$, which should be consistent with the 1st-stage decisions up to $t_{\text{end}}^{1,\min}$. Here, t_{end} represents the exact disruption end time. Note that in this case, nonanticipativity constraints are not needed.

2.3.2. Rolling horizon approach based on a two-stage stochastic model

During the disruption, the range of the disruption end time $[t_{\text{end}}^{\min}, t_{\text{end}}^{\max}]$ may change several times. Under this circumstance, we have a multiple-stage stochastic timetable rescheduling problem. We solve this problem by a rolling horizon approach with successive application of the two-stage stochastic timetable rescheduling model every time an estimated range of the disruption end time is updated in a new stage. The rolling horizon approach is based on the assumptions given in Section 2.1.3. An example of the rolling-horizon stochastic method is shown in Fig. 4.

At stage $k \in [1, K-1]$, the prediction $\left[t_{\text{end}}^{k,\min}, t_{\text{end}}^{k,\max}\right]$ is updated. Thus, W_k scenarios are defined where each has a unique disruption duration $\left[t_{\text{start}} + \ell, t_{\text{end}}^{w_{k,n}}\right]$, and $t_{\text{end}}^{k,\min} \leq t_{\text{end}}^{w_{k,n}} \leq t_{\text{end}}^{k,\max}, w_{k,n} \in \left\{w_{k,1}, \ldots, w_{k,W_k}\right\}$. Recall that $w_{k,n}$ refers to the nth scenario defined at stage k, and the planned timetable is applied for the period $\left[t_{\text{start}}, t_{\text{start}} + \ell\right)$. Based on the new scenarios defined at stage k, the two-stage stochastic optimization is performed, and the 1st-stage decisions X_k from the optimization are delivered to the traffic controllers directly. The 1st-stage decisions X_k are for the period $\left[t_{\text{start}} + \ell, t_{\text{end}}^{k,\min}\right]$ if $k = 1$ or the period $\left[t_{\text{end}}^{k-1,\min}, t_{\text{end}}^{k,\min}\right]$ if $k \geq 2$, which will no longer change at later stages. This is why the period $\left[t_{\text{start}} + \ell, t_{\text{end}}^{k-1,\min}\right]$ is regarded as the *rescheduled timetable horizon* when $k \geq 2$. The 2nd-stage decisions $Y_k(w_{k,n})$ of scenario $w_{k,n}$ is for the period $\left(t_{\text{end}}^{k,\min}, t_{\text{end}}^{w_{k,n}} + R_k^{w_{k,n}}\right]$ that consists of the disruption horizon $\left(t_{\text{end}}^{k,\min}, t_{\text{end}}^{w_{k,n}}\right]$ and the recovery horizon $\left(t_{\text{end}}^{w_{k,n}}, t_{\text{end}}^{w_{k,n}} + R_k^{w_{k,n}}\right]$.

The two-stage stochastic timetable rescheduling model is then used for each following stage where new scenarios are defined according to the updated range of disruption end time. The two-stage stochastic timetable rescheduling model with *nonanticipativity*

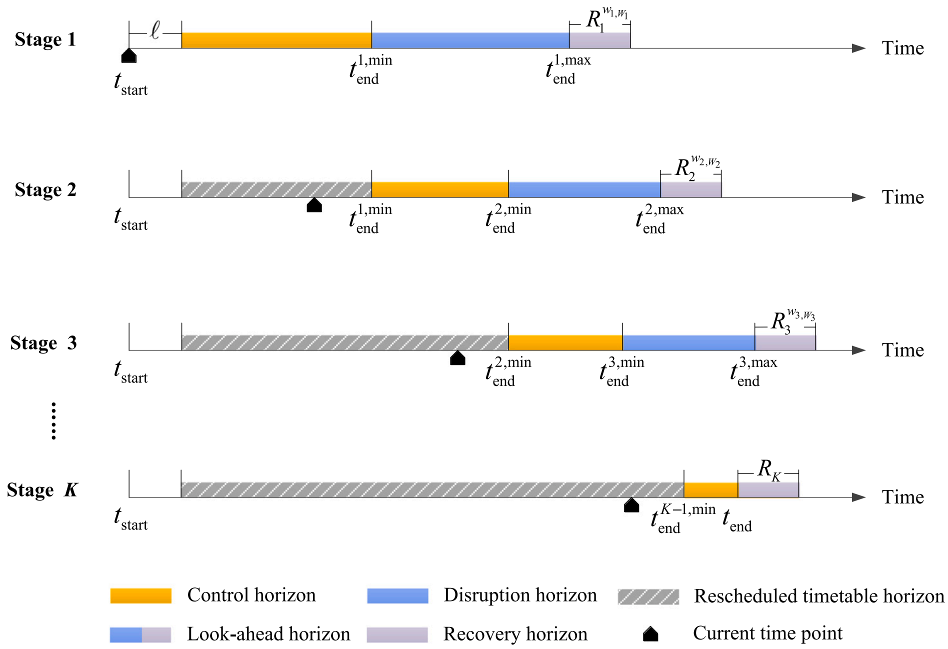

Fig. 4. The rolling-horizon two-stage stochastic timetable rescheduling model to solve the multiple-stage stochastic timetable rescheduling problem.

constraints for stage $1 \leq k \leq K-1$ then is

$$\min \sum_{n=1}^{W_k} p_{w_{k,n}} \cdot \left\{ \left(\beta_c \sum_{e \in E_{\text{ar}}:\ c_e^{w_{k,n}} \in X_k(w_{k,n})} c_e^{w_{k,n}} + \sum_{e \in E_{\text{ar}}:\ d_e^{w_{k,n}} \in X_k(w_{k,n})} d_e^{w_{k,n}} \right) + \left(\beta_c \sum_{e \in E_{\text{ar}}:\ c_e^{w_{k,n}} \in Y_k(w_{k,n})} c_e^{w_{k,n}} + \sum_{e \in E_{\text{ar}}:\ d_e^{w_{k,n}} \in Y_k(w_{k,n})} d_e^{w_{k,n}} \right) \right\}, \tag{8}$$

$$\text{s.t. } X_k(w_{k,n}) \in Z_k^{\text{I}}(w_{k,n}), \quad n \in \{1,\ldots,W_k\}, \tag{9}$$

$$Y_k(w_{k,n}) \in Z_k^{\text{II}}(X_k(w_{k,n}), w_{k,n}), \quad n \in \{1,\ldots,W_k\}, \tag{10}$$

$$X_k(w_{k,n}) = X_k(w_{k,m}), \quad n,m \in \{1,\ldots,W_k\} : n \neq m, \tag{11}$$

where the first-stage decisions

$$X_k(w_{k,n}) = \left\{ \{c_e^{w_{k,n}}, d_e^{w_{k,n}}, x_e^{w_{k,n}}\} : o_e \in [t_{\text{start}} + \ell, t_{\text{end}}^{k,\min}], e \in E \right\} \cup \left\{ y_e^{w_{k,n}} : o_e \in [t_{\text{start}} + \ell, t_{\text{end}}^{k,\min}], e \in E^{\text{turn}} \right\},$$
$$n \in \{1,\ldots,W_k\}, \text{if } k = 1,$$

$$X_k(w_{k,n}) = \left\{ \{c_e^{w_{k,n}}, d_e^{w_{k,n}}, x_e^{w_{k,n}}\} : r_e^{k-1} \in [t_{\text{end}}^{k-1,\min}, t_{\text{end}}^{k,\min}], e \in E \right\} \cup \left\{ y_e^{w_{k,n}} : r_e^{k-1} \in [t_{\text{end}}^{k-1,\min}, t_{\text{end}}^{k,\min}], e \in E^{\text{turn}} \right\},$$
$$n \in \{1,\ldots,W_k\}, \text{if } 2 \leq k \leq K-1,$$

and the second-stage decisions

$$Y_k(w_{k,n}) = \left\{ \{c_e^{w_{k,n}}, d_e^{w_{k,n}}, x_e^{w_{k,n}}\} : o_e \in (t_{\text{end}}^{k,\min}, t_{\text{end}}^{w_{k,n}} + R_k^{w_{k,n}}], e \in E \right\} \cup \left\{ y_e^{w_{k,n}} : o_e \in (t_{\text{end}}^{k,\min}, t_{\text{end}}^{w_{k,n}} + R_k^{w_{k,n}}], e \in E^{\text{turn}} \right\},$$
$$n \in \{1,\ldots,W_k\}, \text{if } k = 1,$$

$$Y_k(w_{k,n}) = \left\{ \{c_e^{w_{k,n}}, d_e^{w_{k,n}}, x_e^{w_{k,n}}\} : r_e^{k-1} \in \left(t_{\text{end}}^{k,\min}, t_{\text{end}}^{w_{k,n}} + R_k^{w_{k,n}}\right], e \in E \right\} \cup \left\{ y_e^{w_{k,n}} : r_e^{k-1} \in \left(t_{\text{end}}^{k,\min}, t_{\text{end}}^{w_{k,n}} + R_k^{w_{k,n}}\right], e \in E^{\text{turn}} \right\},$$
$$n \in \{1,\ldots,W_k\}, \text{if } 2 \leq k \leq K-1,$$

in which o_e is the original scheduled time, r_e^{k-1} is a known value representing the rescheduled time of event e determined at the previous stage $k-1$, and $w_{k,n}$ refers to the nth scenario defined at stage k. Note that $X_k(w_{k,n}) = X_k, n \in \{1,\ldots,W_k\}, 1 \leq k \leq K-1$. In (9), $Z_k^{\text{I}}(w_{k,n})$ refers to the constraint set for $X_k(w_{k,n})$. In (10), $Z_k^{\text{II}}(X_k(w_{k,n}), w_{k,n})$ refers to the constraint set for $Y_k(w_{k,n})$ given $X_k(w_{k,n})$ under scenario $w_{k,n}$.

For the final stage K, the exact disruption end time t_{end} is received. If a disruption end time of a scenario $w_{K-1,n}$ defined at the previous stage is equal to t_{end} (i.e. $t_{\text{end}}^{w_{K-1,n}} = t_{\text{end}}$), then the corresponding 2nd-stage decisions $Y_{K-1}(w_{K-1,n})$ will be delivered to the traffic controllers directly. If none of the previous scenarios corresponds to the exact scenario, the rescheduling model can simply

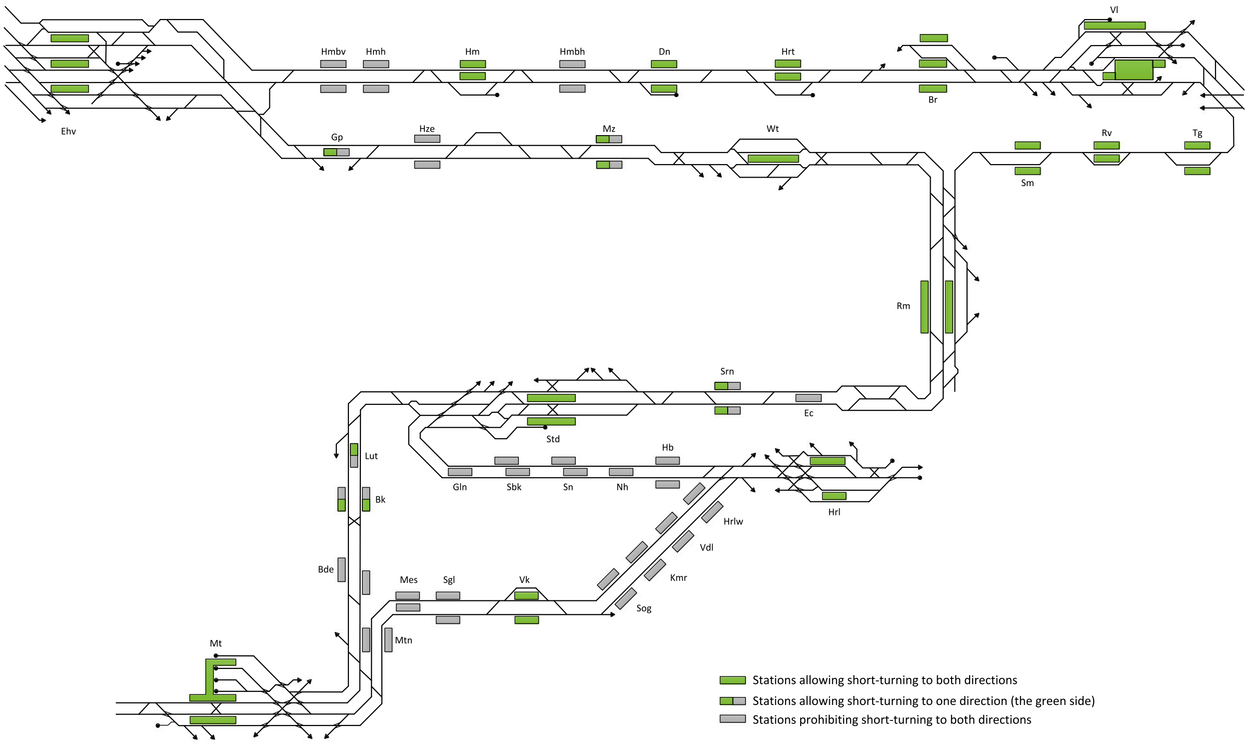

Fig. 5. The schematic track layout of the considered network.

Table 3
Train lines in the considered network.

Train line	Terminals in the considered network
IC800	Maastricht (Mt)
IC1900	Venlo (Vl)
IC3500	Heerlen (Hrl)
SPR6400	Eindhoven (Ehv) and Wt
SPR6800	Roermond (Rm)
SPR6900	Sittard (Std) and Hrl
SPR9600	Ehv and Dn
SPR32000	–
IC32100	Mt and Hrl
SPR32200	Rm

compute a new solution considering the single scenario with the disruption duration $\left[t_{\text{end}}^{K-1,\min}, t_{\text{end}}\right]$, which should be consistent with the previous control decisions up to $t_{\text{end}}^{K-1,\min}$. In this case, nonanticipativity constraints are not needed in the rescheduling model.

3. Case study

The deterministic and stochastic methods are tested on a part of the Dutch railway network. Section 3.1 investigates the impact of the range of the disruption end time, and Section 3.2 analyses the computation performances of both methods.

Fig. 5 shows the schematic track layout of the considered network with 38 stations and both single-track and double-track railway lines.

In the considered network, 10 train lines operate half-hourly in each direction. Fig. 6 shows the scheduled stopping pattern of each train line. Table 3 lists the terminals of the train lines that are located in the considered network, while the terminals outside the considered network are neglected. The deterministic and stochastic rescheduling models both consider trains turning at the terminals to operate the return direction (i.e. OD turnings). We distinguish between intercity (IC) and local (called sprinter (SPR) in Dutch) train lines. Both rescheduling models were developed in MATLAB and solved using GUROBI release 7.0.1 on a desktop with Intel Xeon CPU E5-1620 v3 at 3.50 GHz and 16 GB RAM.

The penalty β_c of cancelling a train run between two neighbouring stations is set to 100 min, and the time period ℓ that ensures a new rescheduling solution to be implemented is set to 10 min. Besides, we set the minimum duration required for short-turning or OD

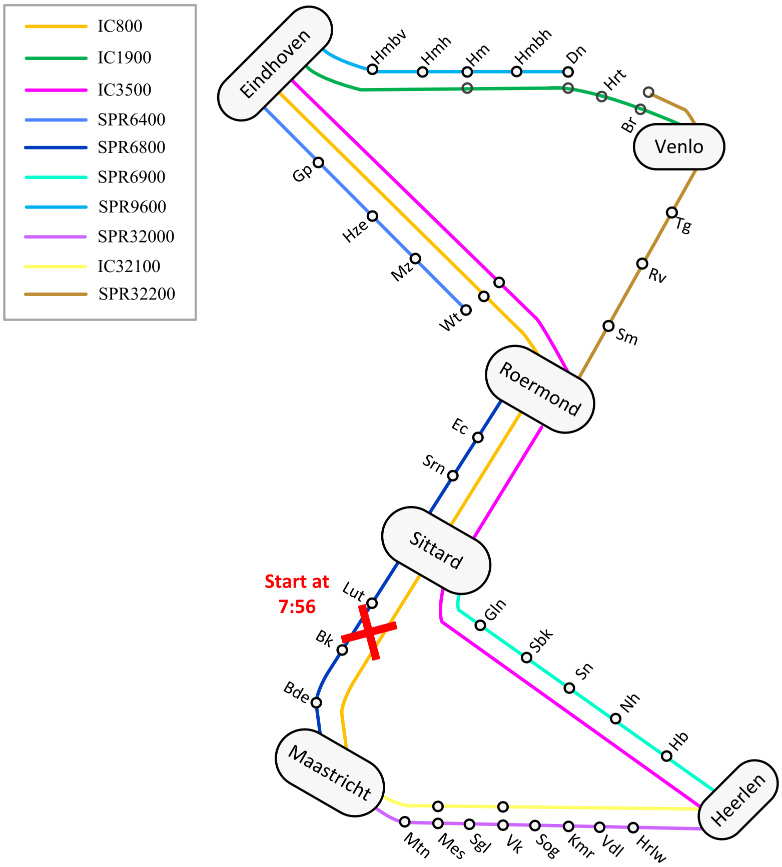

Fig. 6. The train lines operating in the considered network.

Table 4
The predicted disruption end times at each stage of three cases.

Case	Stage k	Disruption end time $t_{end}^{w_{k,1}}$	$t_{end}^{w_{k,2}}$	$t_{end}^{w_{k,3}}$	$t_{end}^{w_{k,4}}$	$t_{end}^{w_{k,5}}$	$t_{end}^{w_{k,6}}$	$t_{end}^{w_{k,7}}$
I	1	9:51	9:56	10:01	10:06	10:11	10:16	10:21
	2	10:36	10:41	10:46	10:51	10:56	11:01	11:06
II	1	10:06	10:11	10:16	10:21	10:26	10:31	10:36
	2	10:36	10:41	10:46	10:51	10:56	11:01	11:06
III	1	10:06	10:11	10:16	10:21	10:26	10:31	10:36
	2	10:51	10:56	11:01	11:06	11:11	11:16	11:21

Optimistic; Expected-value; Pessimistic

turning to 300 s, the minimum duration required for each headway to 180 s, the maximum delay allowed for a train departure/arrival to 15 min, and the minimum dwell time of an extra stop to 30 s.

We consider a complete track blockage between station Bk and station Lut starting at 7:56 (see Fig. 6). The range of the disruption end time update at each stage is indicated by Table 4, which is uniformly distributed to 7 scenarios with the same probabilities: 1/7. Three cases are considered: cases I and II differ in the range of the disruption end time update at stage 1, and cases II and III differ in the range of the disruption end time update at stage 2. At stages 1 and 2, the stochastic method considers 7 disruption scenarios simultaneously, whereas the deterministic method considers one single disruption scenario of which the corresponding end time using optimistic, expected-value, and pessimistic strategies are coloured in green, blue and red, respectively. Recall that the optimistic strategy considers the minimum disruption end time $t_{end}^{k,\min}$, the pessimistic strategy considers the maximum disruption end time $t_{end}^{k,\max}$, and the expected-value strategy considers the expected disruption end time $\sum_{n=1}^{W_k} p_{w_{k,n}} t_{end}^{w_{k,n}}$ at stage k.

Table 5
Results of the rescheduled timetables by the deterministic method at stage 1.

Approach	Case I				Case II or III			
	Predicted end time	Obj [min]	# cancelled services	Total train delay [min]	Predicted end time	Obj [min]	# cancelled services	Total train delay [min]
Optimistic	9:51	2,967	26	367	10:06	3,078	28	278
Expected-value	10:06	3,078	28	278	10:21	3,641	32	351
Pessimistic	10:21	3,641	32	441	10:36	3,751	34	351

Table 6
Results of the rescheduled timetables by the stochastic method at stage 1.

Approach	Case I				Case II or III			
	Predicted end time	Obj [min]	# cancelled services	Total train delay [min]	Predicted end time	Obj [min]	# cancelled services	Total train delay [min]
	9:51	3,078	28	278	10:06	3,394	30	394
	9:56	3,078	28	278	10:11	3,394	30	394
	10:01	3,078	28	278	10:16	3,399	30	399
Stochastic	10:06	3,078	28	278	10:21	3,751	34	351
	10:11	3,122	28	322	10:26	3,751	34	351
	10:16	3,192	28	392	10:31	3,751	34	351
	10:21	3,641	32	441	10:36	3,751	34	351

3.1. The influence of the range of the disruption end time

Table 5 shows the results of the deterministic method at stage 1, including the objective values, the numbers of cancelled services, and the total train delays. Cases II and III have the same result since the range of the disruption times are the same to both cases at stage 1. No matter which case, at stage 1 the optimistic strategy generated the best solution, the pessimistic strategy generated the worst solution, and the expected-value strategy was in between. It is obvious that for the deterministic method the optimal solution considering one disruption duration satisfies the shorter the better.

Table 6 shows the results of the stochastic method at stage 1. In each case, 7 rescheduled timetables are obtained, where the services rescheduled up to 9:51 are forced to be the same in case I, and the services rescheduled up to 10:06 are forced to be the same in case II and III. In case I, the first 4 scenarios have the same result, although the corresponding disruption end times are different. The reason is that no further train services were affected when the disruption end time was extended from 9:51 up to 10:06, due to the service pattern of the planned timetable. In this paper, we use a cyclic planned timetable that has a cycle time of 30 min, which is why we observed a similar phenomenon in case II and III that no changes happened to the results when the disruption end time was extended from 10:21 up to 10:36.

At stage 1, the stochastic method generated solutions that were no better than the deterministic method, due to the anticipation towards longer disruptions considered. Just because of the anticipation, at later stages when the ranges of the disruption end times are updated, better solutions can be obtained by the stochastic method compared to the deterministic method. The results of both methods at the final stage are shown in Tables 7–9 for cases I, II, and III, respectively, including the average performances.

We consider 7 different actual disruption end times, 10:36, 10:41, 10:46, 10:51, 10:56, 11:01, 11:06, in cases I and II that have the same range of the disruption end time at stage 2. As for case III which has a different range of the disruption end time at stage 2, the considered actual disruption end times are: 10:51, 10:56, 11:01, 11:06, 11:11, 11:16, 11:21. Recall that the actual end time t_{end} updated at the final stage K is not smaller than the minimum end time $t_{\text{end}}^{K-1,\min}$ updated at the previous stage. Under such settings of actual end times, the stochastic method obtained the final rescheduled timetables at stage 2, while in most situations the deterministic method needed to recompute new solutions based on the solutions from stage 2 and thus the final stages were stage 3 (see Tables 7–9). In Tables 7–9, also the value of the stochastic solution (*VSS*) is shown, which quantifies the cost of ignoring uncertainty in decision making. It is calculated as *VSS*=*EEV*-*RP*, where *EEV* is the expected result of using the expected-value solution and *RP* is the optimal solution of the two-stage stochastic model (Birge and Louveaux, 2011). In our case (a minimization problem), the higher the *VSS* is, the better the stochastic solution will be. The improvement percentages with respect to *VSS* were also calculated, which were between 6.1% and 10.2% in our cases, demonstrating the benefit of the stochastic formulation. The relevant results can be found in Tables 7–9.

In case I (Table 7), the optimistic strategy performed better than the stochastic method when the actual disruption end time was from 10:36 up to 10:51, whereas the stochastic method performed no worse than any deterministic strategy when the actual disruption end time was from 10:56 up to 11:06. On average, the stochastic method is the best, which is slightly better than the optimistic strategy which is the best among all deterministic strategies.

Compared to case I (Table 7), in case II (Table 8) the stochastic method performed much better than the deterministic method: for each considered actual disruption end time (except 10:36), the stochastic method was better than any deterministic strategy. This is because the ranges of the disruption end times update at stage 1 are different in cases I and II, and thus result in different solutions by the stochastic method at stage 1, which further affect the solutions at stage 2. The pessimistic strategy resulted in

Table 7
Results of the final rescheduled timetables in Case I.

Actual end time	Approach	Obj [min]	# cancelled services	Total train delay [min]	Final stage
10:36	**Stochastic**	4,452	40	451	2
	Optimistic	4,135	38	335	2
	Expected-value	4,135	38	335	3
	Pessimistic	4,452	40	451	3
10:41	**Stochastic**	4,452	40	451	2
	Optimistic	4,180	38	380	3
	Expected-value	4,667	42	467	3
	Pessimistic	4,808	44	408	3
10:46	**Stochastic**	4,457	40	457	2
	Optimistic	4,250	38	450	3
	Expected-value	4,685	42	485	3
	Pessimistic	4,808	44	408	3
10:51	**Stochastic**	4,808	44	408	2
	Optimistic	4,698	42	498	3
	Expected-value	4,698	42	498	2
	Pessimistic	4,808	44	408	3
10:56	**Stochastic**	4,808	44	408	2
	Optimistic	5,193	48	393	3
	Expected-value	5,509	50	509	3
	Pessimistic	4,808	44	408	3
11:01	**Stochastic**	4,808	44	408	2
	Optimistic	5,193	48	393	3
	Expected-value	5,509	50	509	3
	Pessimistic	4,808	44	408	3
11:06	**Stochastic**	4,808	44	408	2
	Optimistic	5,193	48	393	3
	Expected-value	5,509	50	509	3
	Pessimistic	4,808	44	408	2
Average	**Stochastic**	**4,656**	**42**	**428**	–
	Optimistic	4,691	43	406	–
	Expected-value	4,959	45	473	–
	Pessimistic	4,757	43	414	–
VSS	$4,959 - 4,656 = 303$				
Improvement	$303/4,959 = 6.1\%$				

the best solution when the actual end time was 10:36, because it was the optimal solution obtained at stage 1 where 10:36 is the considered disruption end time for the pessimistic strategy (see Table 4).

The stochastic method also performed much better than any deterministic strategy for each considered actual disruption end time in case III (Table 9), which has the same range of the disruption end time at stage 1 as in case II. The average performance of the stochastic method in case III (Table 9) is even better than the one in case I (Table 7), although case III considers longer actual disruption end times. The reason is related to the solution obtained at stage 1, which is affected by the corresponding range of the disruption end time. In case III (Table 9) the result of the stochastic method is all the same when the actual end time is 10:51 up to 11:06, and the result of any deterministic strategy is all the same when the actual end time is 10:56 up to 11:06. These also happen in case I (Table 7) or case II (Table 8). The reason is that no further train services were affected when the disruption end time was extended from 10:51 up to 11:06 for the stochastic method, or from 10:56 up to 11:06 for the deterministic method. Recall that this is due to the service pattern of the timetable.

Tables 7–9 indicate that compared to the deterministic method, the stochastic method is more likely to generate better rescheduling solutions for uncertain disruptions by less cancelled train services and/or train delays. This is mainly because the stochastic method generates solutions that are flexible to the short-turning patterns under different disruption durations. We explain this by the example of the actual disruption end time of 10:36 in case II as follows.

Figs. 7 and 8 show the time-distance diagrams of the rescheduled timetables obtained by the deterministic method using the optimistic strategy at stages 1 and 2 in case II, respectively. The dashed (dotted) lines represent the original scheduled services that are cancelled (delayed) in the rescheduled timetables, while the solid lines represent the services scheduled in the rescheduled timetables. The red triangles indicate extra stops. Compared to stage 1 (Fig. 7), more services were cancelled at stage 2 (Fig. 8) due to the extended disruption. At stage 1, the operation of a dark blue train from stations Mt to Bk is cancelled (Fig. 7), which is why the operation of another dark blue train from stations Bk to Mt has to be cancelled at stage 2 (Fig. 8) to keep consistent control decisions.

Figs. 9 and 10 show the time-distance diagrams of the rescheduled timetables obtained by the stochastic method at stages 1 and 2 in case II, respectively. Compared to the solution of the optimistic strategy at stage 1 (Fig. 7), more services were cancelled/delayed in

Table 8
Results of the final rescheduled timetables in Case II.

Actual end time	Approach	Obj [min]	# cancelled services	Total train delay [min]	Final stage
10:36	**Stochastic**	4,067	36	467	2
	Optimistic	4,135	38	335	2
	Expected-value	4,452	40	452	3
	Pessimistic	3,751	34	351	3
10:41	**Stochastic**	4,067	36	467	2
	Optimistic	4,180	38	380	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
10:46	**Stochastic**	4,073	36	473	2
	Optimistic	4,250	38	450	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
10:51	**Stochastic**	4,424	40	424	2
	Optimistic	4,698	42	498	3
	Expected-value	4,808	44	408	2
	Pessimistic	4,808	44	408	3
10:56	**Stochastic**	4,424	40	424	2
	Optimistic	5,193	48	393	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
11:01	**Stochastic**	4,424	40	424	2
	Optimistic	5,193	48	393	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
11:06	**Stochastic**	4,424	40	424	2
	Optimistic	5,193	48	393	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	2
Average	**Stochastic**	**4,272**	**38**	**443**	–
	Optimistic	4,691	43	406	–
	Expected-value	4,757	43	415	–
	Pessimistic	4,657	43	400	–
VSS	$4{,}757 - 4{,}272 = 485$				
Improvement	$485/4{,}757 = 10.2\%$				

the solution of the stochastic method at stage 1 (Fig. 9) due to the anticipation towards longer disruption durations in consideration. Just because of the anticipation, at stage 2, the solution of the stochastic approach resulted in less cancelled services and train delays, compared to the solution of the optimistic strategy (Fig. 10).

It is found that the flexibility of the solution by the stochastic method can be affected by the range of the disruption end time update. An example is given as follows. Figs. 11 and 12 show the time-distance diagrams of the rescheduled timetables obtained by the stochastic method at stage 1 and 2 in case I, respectively. Recall that cases I and II have different ranges of the disruption end times at stage 1, but the same range of the disruption end times at stage 2 (see Table 4).

At stage 1, compared to the solution of case II (Fig. 9) that considered the end time range of [10:06,10:36], the solution of case I (Fig. 11) resulted in less cancelled services and train delays due to an earlier end time range of [9:51,10:21] considered. In case II (Fig. 9) the cancelled operation of a dark blue train from stations Mt to Bk was after the minimum end time of stage 1, 10:01, and thus this cancellation decision was a look-ahead decision at phase 1, which did not need to be respected at stage 2 (see Fig. 10); while in case I (Fig. 11) the cancelled operation of a dark blue train from stations Mt to Bk was before the minimum end time of stage 1, 9:51, and thus this cancellation decision was a control decision at stage 1, which had to be respected at stage 2 (see Fig. 12) causing the operation of another dark blue train from stations Bk to Mt cancelled at stage 2.

This shows that the range of the disruption end time affects the flexibility of a solution, which is relevant to short-turning patterns. Smooth short-turning patterns for possible longer disruptions like in case II (Figs. 9 and 10) help to reduce cancelled train services. Case II has an later range of the disruption end time at stage 1 than case I, while both cases have the same range of the disruption end time at stage 2. In that sense, compared to case I, case II considers that longer disruption durations are more likely to happen at stage 1, which turns to be true due to another range update at stage 2. From the results of both cases, we infer that in the situations where longer disruption durations are more likely to happen, short-turning the last train services approaching to the predicted minimum disruption end time (e.g. Fig. 9 corresponding to case II) rather than cancelling them (e.g. Fig. 11 corresponding to case I) might be helpful to improve solution flexibility.

Table 9
Results of the final rescheduled timetables in Case III.

Actual end time	Approach	Obj [min]	# cancelled services	Total train delay [min]	Final stage
10:51	**Stochastic**	4,424	40	424	2
	Optimistic	4,698	42	498	2
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
10:56	**Stochastic**	4,424	40	424	2
	Optimistic	5,509	50	509	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
11:01	**Stochastic**	4,424	40	424	2
	Optimistic	5,509	50	509	3
	Expected-value	4,808	44	408	3
	Pessimistic	4,808	44	408	3
11:06	**Stochastic**	4,424	40	424	2
	Optimistic	5,509	50	509	3
	Expected-value	4,808	44	408	2
	Pessimistic	4,808	44	408	3
11:11	**Stochastic**	4,469	40	469	2
	Optimistic	5,509	50	509	3
	Expected-value	4,853	44	453	3
	Pessimistic	5,340	48	540	3
11:16	**Stochastic**	4,539	40	539	2
	Optimistic	5,514	50	514	3
	Expected-value	4,923	44	523	3
	Pessimistic	5,358	48	558	3
11:21	**Stochastic**	4,987	44	587	2
	Optimistic	5,866	54	466	3
	Expected-value	5,371	48	571	3
	Pessimistic	5,371	48	571	2
Average	**Stochastic**	**4,527**	**41**	**470**	–
	Optimistic	5,445	49	502	–
	Expected-value	4,912	45	454	–
	Pessimistic	5,043	46	472	–
VSS	$4{,}912 - 4{,}527 = 385$				
Improvement	$385/4{,}912 = 7.8\%$				

Table 10
Computation times [s] at each update stage.

Approach	Case I		Case II		Case III	
	Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2
Stochastic	**234**	**66**	**244**	**51**	**244**	**51**
Optimistic	10	3	9	3	9	3
Expected-value	10	3	11	3	11	3
Pessimistic	11	3	10	2	11	3

3.2. Computation analysis

Table 10 shows the computation times for the stochastic method and the deterministic method using different strategies at stages 1 and 2 for all cases. In each case, the computation time of each approach to stage 1 is longer than the one to stage 2. This is because at a later stage only the dispatching decisions for the new control and look-ahead horizons (for the extended duration) need to be made. The deterministic method for each strategy take much shorter computation time than the stochastic method, as it considers a single disruption scenario at each computation. Although the stochastic method is relatively time-consuming, the rescheduling solutions are better. Table 11 shows the numbers of variables, binary variables and constraints required respectively by the stochastic method and the deterministic method using a pessimistic strategy. We only show the pessimistic strategy in Table 11, because it needs more variables and constraints compared to the optimistic or expected-value strategy due to longer disruption duration considered. Because the stochastic method handled 7 scenarios at a stage, the required variables and constraints (see Table 11) were longer than the ones of the deterministic method using a pessimistic strategy, which handled only 1 scenario at a stage.

Among all cases, the longest computation time of a stochastic solution was around 4 min. This shows the applicability of applying the proposed stochastic approach assuming that the range of the disruption end time prediction update is provided at least 10 min before the current minimal end time prediction ($\ell = 10$ min).

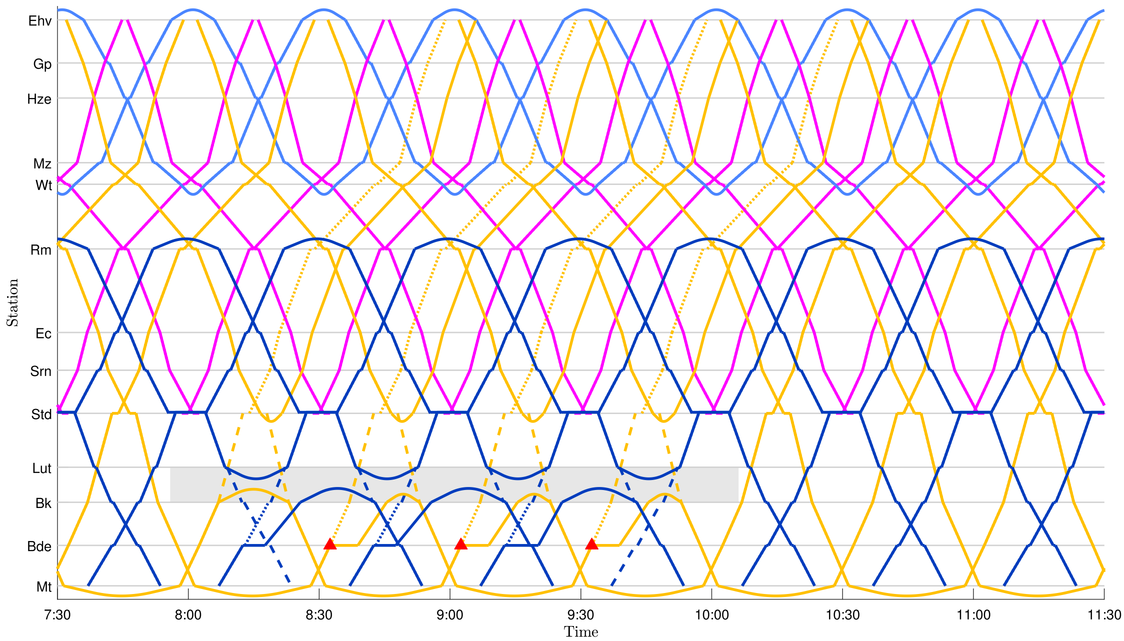

Fig. 7. The rescheduled timetable by the optimistic strategy at stage 1 in case II (disruption end time: 10:06).

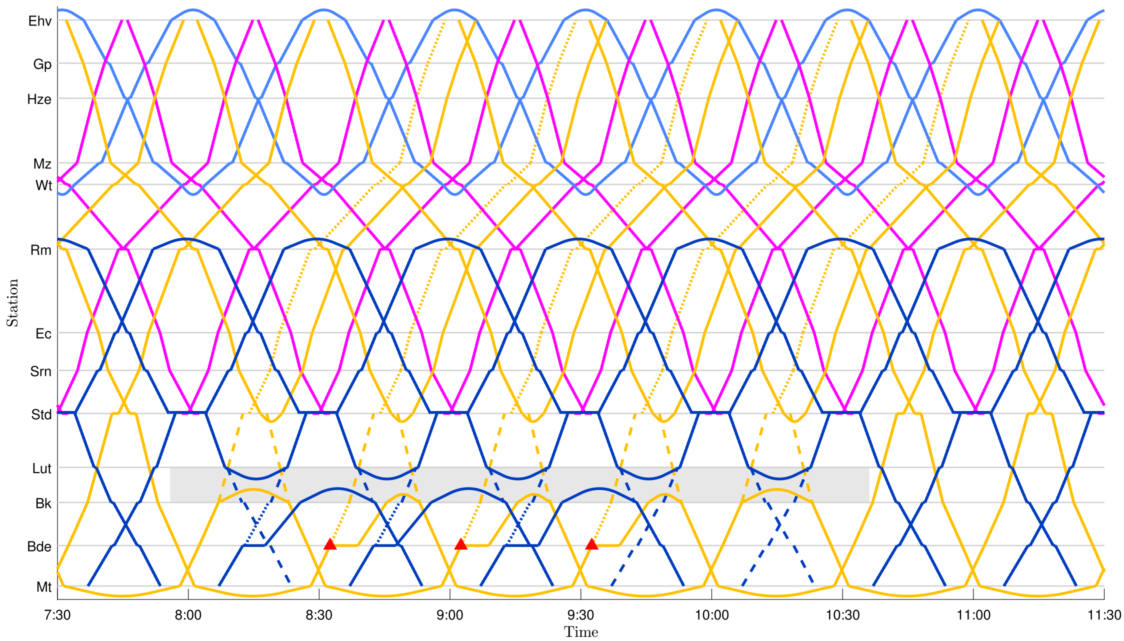

Fig. 8. The rescheduled timetable by the optimistic strategy at stage 2 in case II (disruption end time: 10:36).

4. Conclusions

This paper proposed a rolling horizon two-stage stochastic timetable rescheduling model to manage uncertain disruptions with better solutions. It was tested on a part of the Dutch railways and compared to a deterministic rolling horizon timetable rescheduling model. The results showed that compared to the deterministic method, the stochastic method is more likely to generate better rescheduling solutions for uncertain disruptions by less train cancellations and/or delays, due to the flexibility towards the short-turning patterns under different disruption durations. The flexibility of a solution by the stochastic method can be impacted by the range of the disruption end time. From the results we infer that in the situations where longer disruption durations are more likely

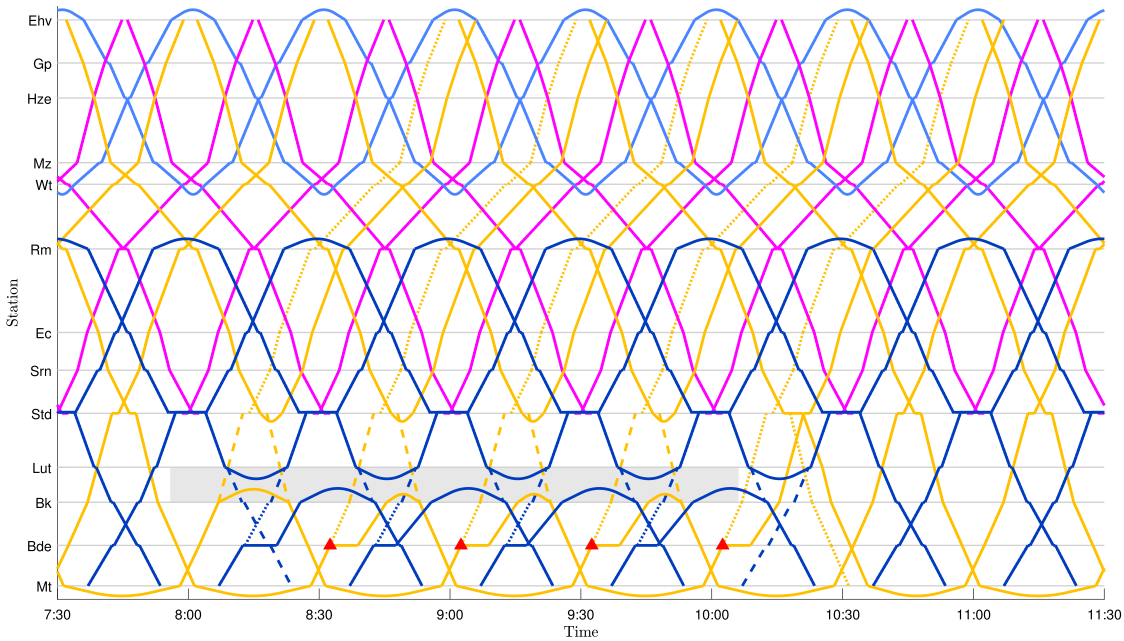

Fig. 9. The rescheduled timetable by the **stochastic** approach at stage 1 in case II (disruption end time: 10:06).

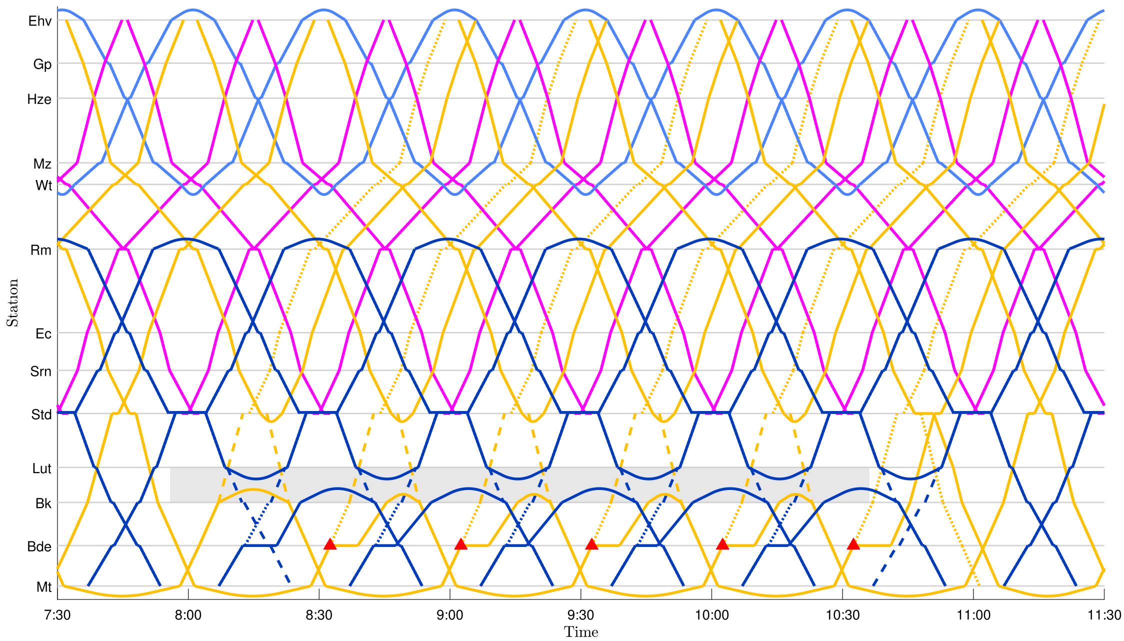

Fig. 10. The rescheduled timetable by the **stochastic** approach at stage 2 in case II (disruption end time: 10:36).

to happen, short-turning the last train services approaching to the predicted minimum disruption end time rather than cancelling them might be helpful to improve solution flexibility. This will be examined in near future. The stochastic programming model considers several scenarios simultaneously, is therefore larger and thus takes longer computation time. The computation time might be reduced without affecting the solution quality by optimizing the number of scenarios, the size of the network, the length of the look-ahead horizon, or exploiting the periodic structure of the (rescheduled) timetable.

This paper used a discrete uniform distribution over the range of the estimated disruption end to define scenarios with the same occurrence probabilities. From the case study results we found that although some scenarios had different disruption durations the rescheduling solutions to these scenarios were the same. The scenario estimation method can be improved by identifying various

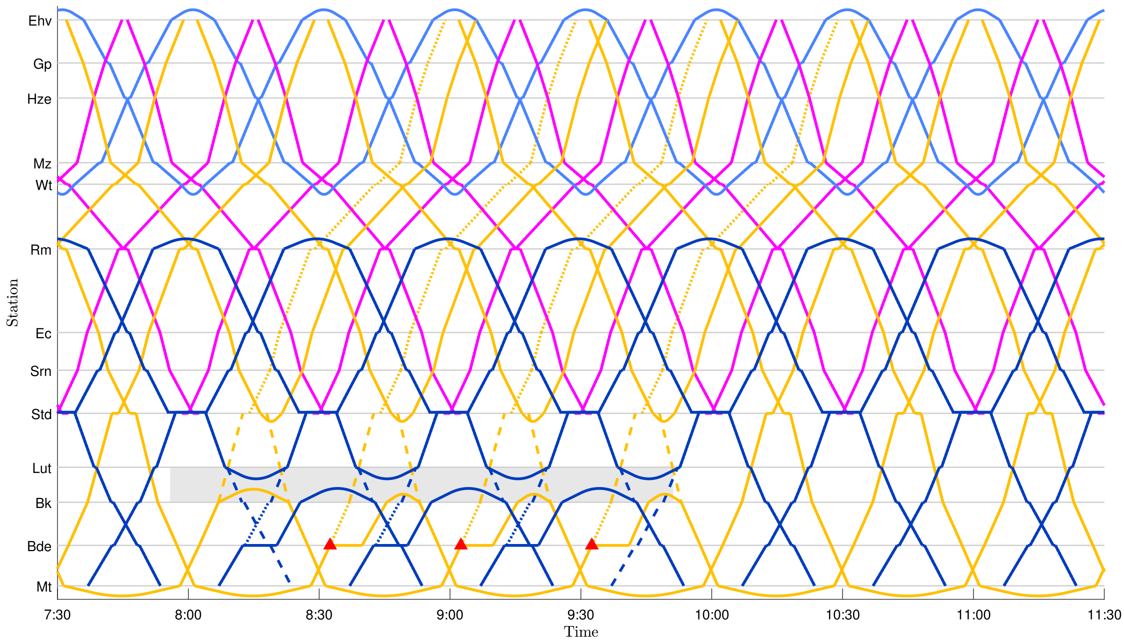

Fig. 11. The rescheduled timetable by the **stochastic** approach at stage 1 in case I (disruption end time: 9:51).

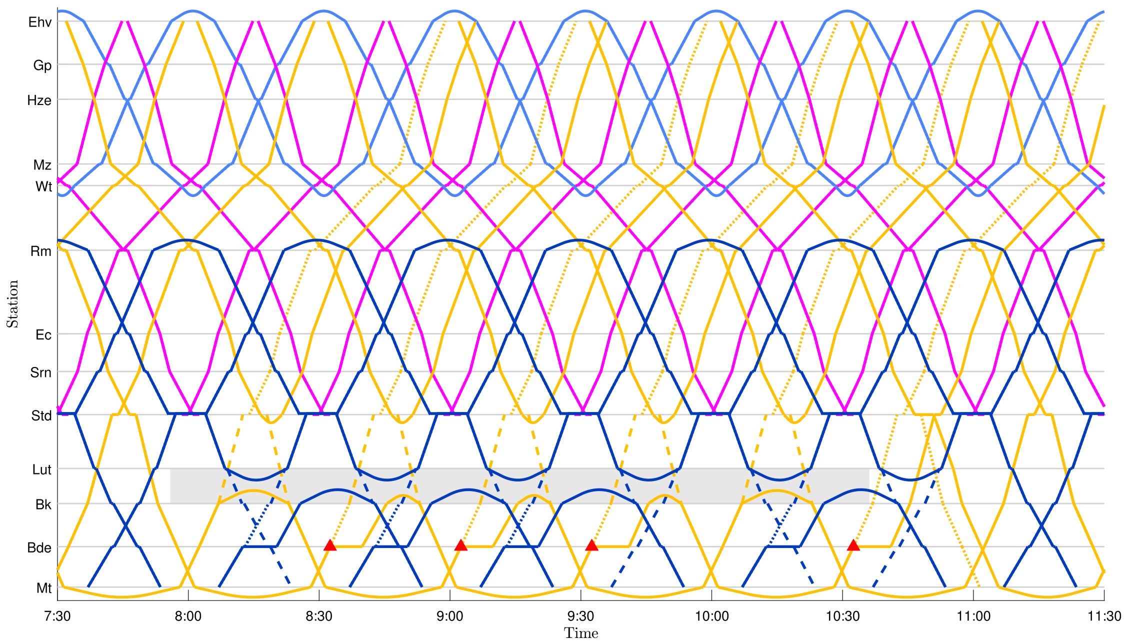

Fig. 12. The rescheduled timetable by the **stochastic** approach at stage 2 in case I (disruption end time: 10:36).

different scenarios with essentially different outcomes to find a rescheduling solution. As we rely on a periodic planned timetable there should be a finite number of discrete scenarios that lead to essentially different outcomes. It is beneficial to identify these representative scenarios, of which the probabilities can be assigned based on the relative sub-range that they would occur. This will be part of future work.

Table 11
The problem sizes of the stochastic and deterministic models.

Approach	Indicators	Case I		Case II		Case III	
		Stage 1	Stage 2	Stage 1	Stage 2	Stage 1	Stage 2
Stochastic	# variables	476,105	476,994	476,378	476,994	476,378	477,337
	# binary variables	423,017	423,906	423,290	423,906	423,290	424,249
	# constraints	2,200,666	2,323,312	2,229,771	2,325,510	2,229,771	2,361,890
Pessimistic	# variables	68,015	68,142	68,054	68,142	68,054	68,191
	# binary variables	60,431	60,558	60,470	60,558	60,470	60,607
	# constraints	293,467	306,538	296,046	306,853	296,046	310,299

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