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Abstract. Since the first commercial projects, the development of vertical-axis wind turbines (VAWTs) has been impeded by the limited understanding and inability to accurately model VAWTs. This paper investigates and compares different aerodynamic modelling techniques for VAWTs in 3D. All considered models are using the same blade-element characteristics but use different descriptions to determine the induced velocity field. The H- and Φ-rotor are studied with various aspect ratios and rotor loadings. Both instantaneous azimuthal parameters as well as integral parameters, such as the thrust and power are investigated. The paper concludes that capturing the 3D effects of VAWTs is challenging and the trends to be expected are not straightforward due to the complex vortex system created by VAWTs. All model assumptions affect the results both at the mid-plane of the rotor as well as at the blade tips.

1. Introduction

Vertical axis-wind turbines (VAWTs) have witnessed a long-standing, yet fluctuating, interest from the renewable energy community. In the last two decades of the previous century, confidence in the viability of this concept was particularly high, and VAWTs were considered suitable alternatives to horizontal-axis wind turbines (HAWTs). Unfortunately, however, many of these early VAWTs suffered from unanticipated fatigue issues and severely underperformed. This led to a declining interest in vertical-axis concepts and enabled their horizontal-axis counterparts to become the dominant wind energy conversion systems in today's landscape. Despite the substantial technology gap between both concepts, the interest in VAWTs is currently resurging, in part driven by the push towards floating, far-offshore wind energy systems where VAWTs could provide critical benefits.

Developing VAWTs to become competitive alternatives to HAWTs, however, requires several challenges to be overcome. In particular, the ability to accurately model their aerodynamic behaviour is necessary to uncover more about the complex flow field around these turbines.

1.1. Background

Many studies considering VAWTs have been performed in 2D. Examples are studies related to airfoil design, load optimisation, or turbine configuration. However, depending on the shape and aspect ratio of the turbine, it could be of upmost importance to include 3D effects into the aerodynamic analysis. Although research performed on 3D effects has been limited, some researchers have attempted to elucidate the underlying principles. De Vries[1] proposed

a three-dimensional vortex theory for VAWTs, in which the vortex system is described to be a combination of trailing vorticity (due to spanwise lift variation) and shed vorticity (due to orbital lift variation). Ferreira[2] provided a discussion on the 3D near wake with a main focus on the tip vortex and its convection. More recently, we have performed a study with a particular interest in the 3D effect on rotor and wake induction as a result of various predefined load configurations and aspect ratios [3].

In the literature, VAWTs have been modelled using various modelling techniques. In 2D, a model comparison study has been conducted by Ferreira et al.[4]. In this study, a comparison is presented between six models which use different formulations of the actuator and the wake/induction system. However, such a study has not yet been performed in 3D.

1.2. Objective

The objective of this study is to present a blind comparison of various aerodynamic models, which employ different formulations for the induction field around the rotor and require different computational resources. A range of test cases is considered, including both the H-rotor and Φ-rotor. Various rotor aspect ratios and thrust coefficients are studied. It is the purpose of this paper to present a comparison and explain differences based on the assumptions made by the models. However, it is not the purpose to improve the models nor to compare them to real data.

This paper is structured as follows. In section 2, the models included in this comparison study are presented, followed by a detailed description of the baseline turbine and test cases. In section 3, the models are compared. Not only the instantaneous (azimuthal distribution) results are presented, but the integral (rotor average) results, such as the thrust and power coefficient, are also considered. In the end, an outlook towards future research directions is provided.

2. Methodology

2.1. Models

The benchmark paper will compare six different aerodynamic models. These models are: 2D actuator cylinder model, 2D actuator cylinder model with near wake correction, fixed-wake vortex model, free-wake vortex model, actuator-line OpenFOAM model and the actuator-line OpenFOAM model with end-effect correction. To ensure that the models are simulating similar flow conditions, the flow is assumed to be inviscid and unsteady effects such as dynamic stall are neglected. All models use blade-element theory to determine the forces on each individual small blade element but have a different description for the induced velocity field.

- **2D actuator cylinder model:** The Actuator Cylinder model, developed by Madsen[5], is a 2D flow model extending the actuator disk concept. The solution of the velocity field around the actuator cylinder builds on the 2D, steady, incompressible Euler equations and the equation of continuity. The linear solution is combined with the *ModLin*[6, 7] correction to compensate the non-linear part of the solution. The 2D actuator cylinder model can be used to model the 3D rotor by decomposing the rotor into 2D slices. The 2D actuator cylinder model is a steady model and assumes infinite number of blades. There is no aerodynamic coupling between the two-dimensional slices and it does not include any correction for tip losses.

- **2D actuator cylinder with near wake correction:** The HAWC2 model for VAWTs is based on the actuator cylinder model. HAWC2-NW provides a correction to include the tip loss effect by applying a lifting line based near wake model for trailing vorticity. It assumes straight vortex filaments behind each blade that are convected downstream at the local flow angle at each blade section. Thus, the wake curvature is neglected. This dynamic tip loss correction provides an aerodynamic coupling between the different blade sections, but every blade can only see its own trailed wake. The method is developed by [8] and recently modified by [9, 10]. The VAWT version of the model is based on the straight wake extension presented in [11]. The shed vorticity due to the time variation in the bound circulation induces a downwash at the three-quarter chord of the airfoil. This is modelled

in HAWC2-NW using the inviscid part of the unsteady airfoil aerodynamics model by [12]. It treats the shed vorticity effects as a time lag on the angle of attack according to Jones' function for a flat plate[10]. Note that the HAWC2-NW model only works for straight blades and thus cannot be applied to the Φ-rotor.

- **Free- and fixed-wake vortex model:** The Code for Axial and Cross-flow TUrbine Simulation, CACTUS[13], is a three-dimensional vortex model. The blades are represented by the lifting line approximation. The wake is constructed using a vortex lattice structure where the induced velocity is calculated based on the Biot-Savart law. The wake convection velocity is either calculated based on the induced velocity at every time step (free-wake) or kept constant in time (fixed-wake). A vortex core model is included to avoid instabilities near the vortices. The vortex core size is set as 1/8 of the chord over radius of the baseline turbine.

- **Actuator line OpenFOAM model:** The actuator line model uses the open-source TurbineFoam library[14] in OpenFOAM. This model is based on the classical blade element theory and uses a Navier-Stokes description to solve the flow field in space and time. The blades are represented as lines at the quarter chord point. The blade loads are introduces as body forces or a momentum source (per unit density, assuming incompressible flow).[15] The loads are smeared out by means of a spherical Gaussian function. A correction method for end-effects based on a lifting-line approach is implemented, however, it only works for straight blades with equally spaced spanwise elements.[15]

2.2. Test cases

For the comparison study, two baseline turbines are considered: a H-shaped turbine and a Φ-shaped turbine. Both turbines are lift-driven. The baseline turbine has a solidity ($\sigma = Bc/2R$) of 0.085 and spins at a tip speed ratio ($\lambda = \omega R/V_\infty$) of 3. This choice is motivated as it represents a medium-loaded turbine. Note that these parameters are defined with respect to the rotor midsection. In Figure 1 an artistic impression of the baseline turbines is presented, however, for the simulations only the main torque-producing elements are considered. Other elements such as the struts or tower are disregarded. The baseline turbine has an aspect ratio ($AR = H/2R$) of 1 and consists of 3 blades. The H-rotor has straight blades with a fixed cross-section. For the Φ-rotor, the radius follows a parabolic distribution. The maximum radius of the H- and Φ-rotor are the same and fixed to $2.5m$. The incoming velocity is $1m/s$.

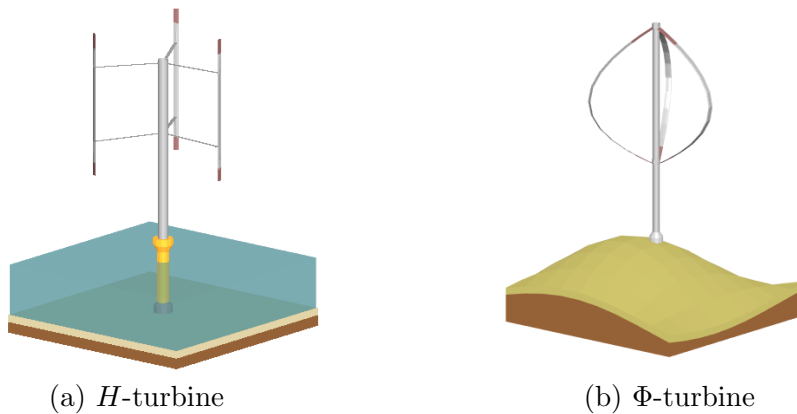(a) H-turbine   (b) Φ-turbine

Figure 1: Artistic impression of the baseline turbines.

All models use the blade-element theory to determine the blade loads from the velocity field. The blade-element characteristics are prescribed by $C_l = 1.11 \cdot 2\pi \cdot sin(\alpha)$ and $C_d = 0$, to represent the characteristics of an 18% thick inviscid airfoil. No unsteady 2D airfoil aerodynamics effects, such as dynamic stall, or flow curvature, are included. No shear is considered in the inflow and all simulations are run assuming inviscid and laminar flow. All models use a mounting point corresponding to the aerodynamic point, at 25% of the chord. All these simplifications are

justified since the model comparison focusses only on the definition of the induced velocity. It is not the purpose to necessarily represent and model a realistic VAWT turbine but to compare the various 3D descriptions of the induced velocity field.

The baseline turbine is altered to analyse various test cases. A study is performed on the aspect ratio. The aspect ratio is altered between 0.5 and 5 and this is realised by stretching the turbine in spanwise direction. As such, the radius distribution and maximum radius remain constant. To quantify how the models perform and compare at different loadings (different thrust coefficient), two other solidities (besides the baseline solidity) are studied. A small solidity to represent a lowly loaded rotor with minimal wake expansion and one to represent a highly loaded rotor with large wake expansion and induction. The different solidities are realised by altering the chord length. The radius and number of blades are kept constant to avoid the need to change the rotational speed and exclude the finite number of blades effect. The test cases are summarised in Table 1.

The comparison of the different models is performed on instantaneous, azimuthal results such as the angle of attack, blade loads and relative velocity at the blades. Also, integral parameters or rotor averaged results such as the thrust and power coefficient are studied.

Table 1: Overview of the test cases

	Baseline turbine	Study of aspect ratio	Study of thrust coefficient
Solidity, σ [-]	0.085	0.085	0.05, 0.085, 0.12
Tip speed ratio, λ [-]	3	3	3
Aspect ratio, AR [-]	1	0.5, 1, 2, 5	0.5, 1, 2, 5
Number of blades, B [-]	3	3	3
Rotor shape	H, Φ	H, Φ	H, Φ
Output parameters	instantaneous, integral	instantaneous, integral	integral

3. Results and discussion

3.1. Baseline turbine

In Figure 2 the tangential blade force is presented as a function of the azimuth angle and spanwise direction for the H-shaped baseline turbine. Note that an azimuth angle from 0 to 180 deg corresponds to the upwind part of the rotor and from 180 to 360 deg to the downwind part. For the AC2D model there is no spanwise variation, since this is a 2D model where no interaction between the different slices is considered. The near-wake correction implemented in HAWC2-NW, causes the loads at the edges of the rotor to reduce to zero. A similar behaviour is predicted by the CACTUS free- and fixed-wake models. The actuator line TurbineFoam model shows an increased loading at the edges of the rotor since it predicts a larger angle of attack at the blade tips. This is not following expectations and is a common problem that is identified for actuator line models in CFD. The smearing of the forces in the flow field leads to a viscous core of the vorticity and this reduces the velocity near the vortex centre compared to the inviscid solution of the lifting line.[16, 17] Correction models are proposed to account for this. In TurbineFoam, a correction method is implemented causing the loads at the blade tips to reduce to zero. However, the currently used method is not related to the vortex core issue and thus it is difficult to justify from a physical point of view.

In Figure 3 the same figures are presented for the baseline Φ-rotor. Since the HAWC2-NW and the TurbineFoam model with end-effects only work for straight blades, both models do not allow a prediction for this egg-shaped rotor. The overall behaviour of the AC2D, CACTUS fixed-wake and CACTUS free-wake is very similar. High loads appear at the rotor centre plane and the loads decrease at the edges. This is true because of the panel inclination angle and lower relative velocity. Note that the angle of attack at the rotor edges becomes very large (due to the low rotational velocity ωR wrt the incoming velocity V_∞) and since this paper is using inviscid

polars ($C_L = 1.11 \cdot 2\pi sin(\alpha)$), loads might appear to be relatively high. Although this paper does not necessarily focus on the modelling a realistic VAWT, this might cause the operational conditions to be out of range of interest. This should be taken into account when drawing conclusions. The actuator line model TurbineFoam predicts a significantly different behaviour than the AC2D and vortex models. The loads decrease less abrupt at the rotor edges. This can be attributed to the fact that this model, in its current implementation, does not account for the panel inclination angle. As such the relative velocity component due to the incoming velocity (and induced velocity) is over-predicted resulting in a larger angle of attack and thus blade forces. Since the panel inclination is zero at the rotor centre, TurbineFoam does make a relatively good prediction at the rotor centre that complies well with the other models.
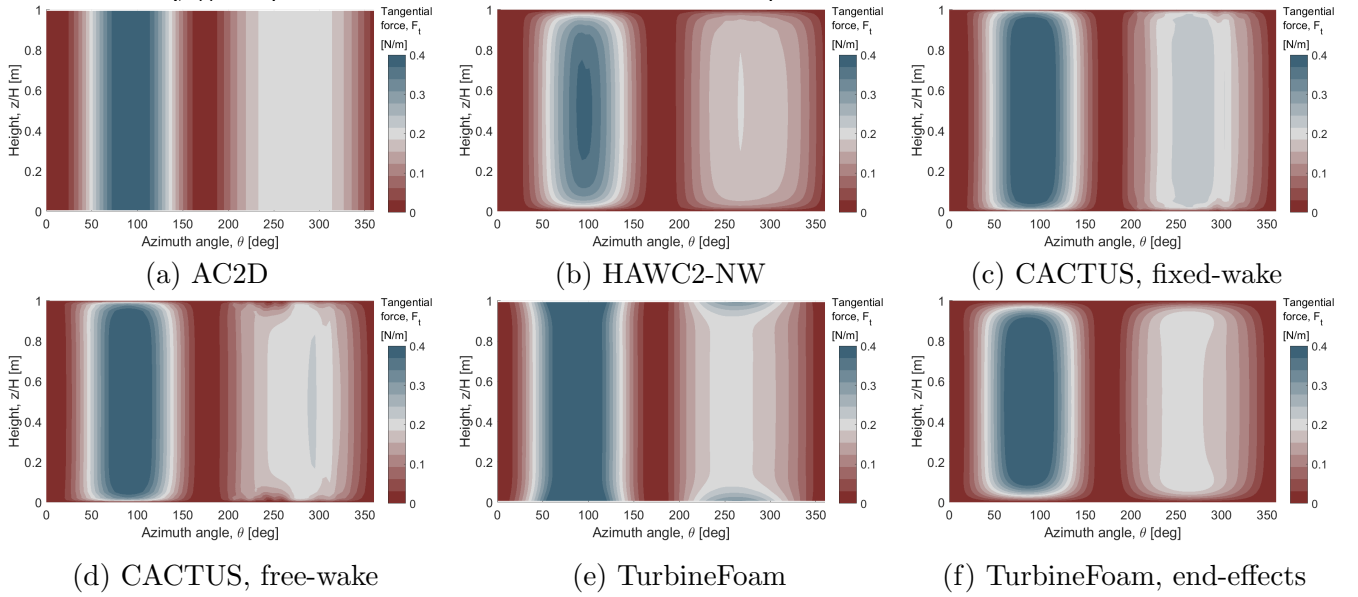

Figure 2: Tangential blade force vs azimuth angle and spanwise direction for the 3-bladed H-shaped baseline turbine using six different models. The rotor solidity is 0.085, the aspect ratio is 1 and the tip speed ratio is 3.
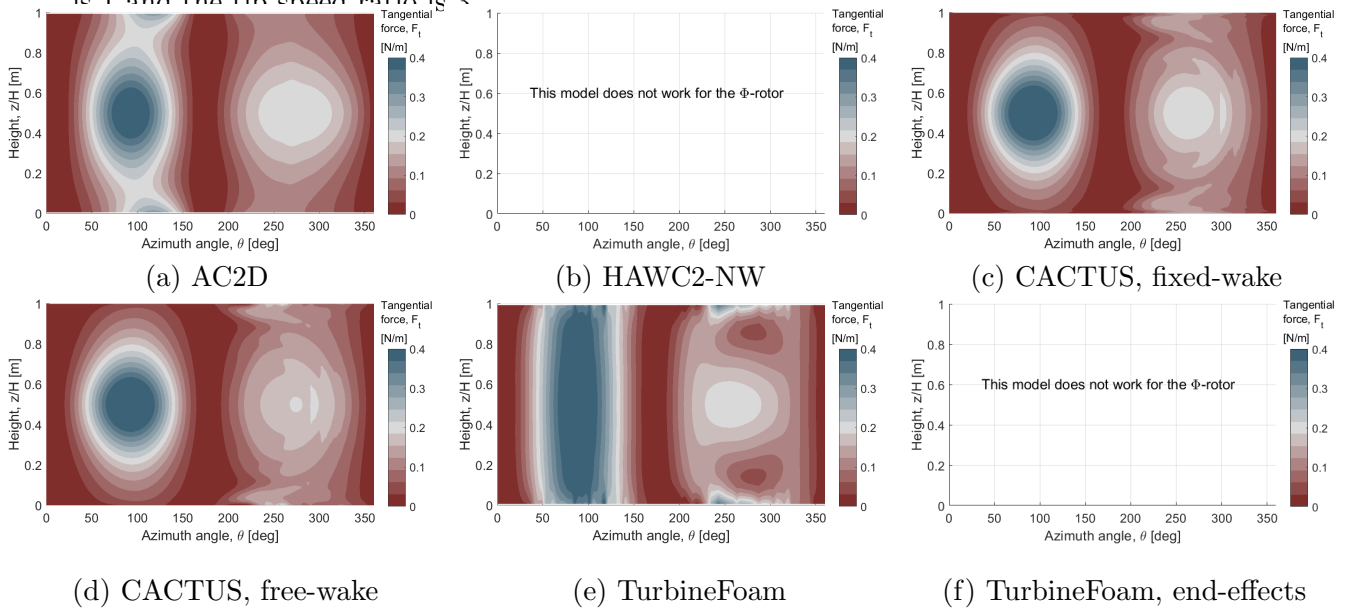

Figure 3: Tangential blade force vs azimuth angle and spanwise direction for the 3-bladed Φ-shaped baseline turbine using six different models. The rotor solidity is 0.085, the aspect ratio is 1 and the tip speed ratio is 3.

Integrating the tangential and normal loading on the turbine allows to determine the power and thrust of the baseline turbines. In Table 2, this information is summarised. A distinction is made between the power and thrust at the rotor mid-plane ($C_{P,mid}$ and $C_{T,mid}$) and of the full rotor ($C_{P,tot}$ and $C_{T,tot}$). The power coefficient and thrust coefficient are normalised with $0.5\rho V_\infty^3 2R_{max}H$ and $0.5\rho V_\infty^2 2R_{max}H$ respectively, for both the H- and Φ-rotor. Since the frontal area of the Φ-rotor is smaller than for the H-rotor, a lower power coefficient is expected. At the mid-plane all models predict a power coefficient around 0.5. HAWC2-NW remains as only model below 0.5. For the full turbine, HAWC2-NW and the TurbineFoam model with end-effects predict the lowest power. The CACTUS fixed-wake and AC2D model predict very similar values. For the Φ-rotor, the power and thrust of the various models remain very close. The induction at the rotor tips (and thus the tip effects) are of less importance since the relative velocity is very small at these locations. TurbineFoam significantly over-predicts the power and thrust with respect to the other models which is again a result of neglecting the panel inclination angle.

Table 2: Power and thrust at the rotor mid-plane and averaged over the rotor for the H- and Φ-baseline turbine using six different models.

Model	H-rotor				Φ-rotor			
	$C_{P,mid}$	$C_{T,mid}$	$C_{P,tot}$	$C_{T,tot}$	$C_{P,mid}$	$C_{T,mid}$	$C_{P,tot}$	$C_{T,tot}$
AC2D	0.510	0.653	0.510	0.653	0.510	0.651	0.256	0.388
HAWC2-NW	0.442	0.604	0.400	0.570	-	-	-	-
CACTUS, fixed-wake	0.535	0.680	0.509	0.647	0.525	0.671	0.241	0.367
CACTUS, free-wake	0.515	0.669	0.486	0.643	0.512	0.664	0.236	0.365
TurbineFoam	0.523	0.660	0.522	0.660	0.498	0.639	0.298	0.397
TurbineFoam, end-effects	0.524	0.661	0.469	0.578	-	-	-	-

3.2. Study of aspect ratio

To illustrate better the effect of aspect ratio, Figure 4 to Figure 7 are presented. In Figure 4, the spanwise variation of the tangential force is plotted for the H-rotor with various aspect ratios. The blade is located at a 90 deg azimuth position, close to the point where the tangential force is maximum. As expected, the AC2D model does not show any dependency on spanwise direction nor aspect ratio. HAWC2-NW, the CACTUS fixed- and free-wake and the TurbineFoam model with end-effects correction, show a clear decrease of the loads at the blade tips, known as tip losses. For larger aspect ratios, the tip losses are concentrated to a smaller portion of the blade and the mid-section is approaching 2D conditions. The CACTUS models and the TurbineFoam models show an increase in loading at the rotor centre for smaller aspect ratios. Note that in case of a single rotating blade instead of three, this behaviour is not observed by the CACTUS models nor TurbineFoam.

Figure 5, where the tangential force is plotted against the azimuth angle for the mid-plane for various aspect ratios reveals that the CACTUS free- and fixed-wake as well as the TurbineFoam models show a redistribution of the loads due to the tip effects. A gain in power (higher tangential force) is observed upwind while a loss in power (lower tangential force) is observed downwind. This also corresponds with the findings in [3], where the 3D effects of an actuator cylinder with prescribed loads are studied. The results of HAWC2-NW do not observe this behaviour but predict an overall loss both upwind and downwind. This could most probably be attributed to the fact that HAWC2-NW only accounts for the vorticity trailed of that blade and not the trailing vorticity shed by the other rotor blades. Note that the current tip correction implementation will lead to no tip correction in case of infinite number of blades. Also HAWC2-NW assumes a straight wake and does not account for the curvature in the wake.

Similar plots are made for the Φ-rotor. The overall trend of the tangential force as a function of the spanwise direction, as given by Figure 6, are similar for the AC2D, CACTUS fixed-wake and CACTUS free-wake models. The TurbineFoam model again deviates from the others

because the panel orientation is not considered in this model. The AC2D model now does show a dependency on the aspect ratio. This is true because the panel orientation changes when scaling the aspect ratio. A smaller panel inclination is observed for an increasing aspect ratio leading to a larger relative velocity and thus a larger tangential force. However, notice that the AC2D model does not account for the force acting in spanwise direction. This violates the assumption of independent aerodynamic slices in the AC2D model more.
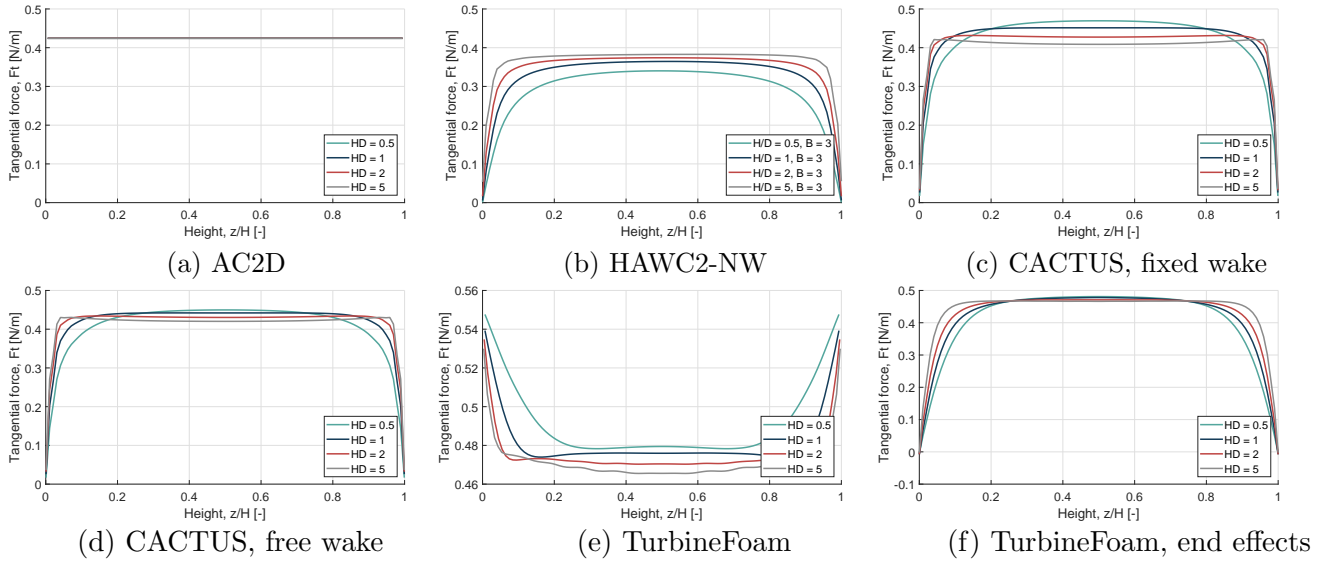

Figure 4: Tangential blade force vs spanwise direction using six different models for the three-bladed H-turbine with an aspect ratio of 0.5, 1, 2 or 5. The rotor solidity is 0.085 and the tip speed ratio is 3. The blade is located at a 90 deg azimuth angle.
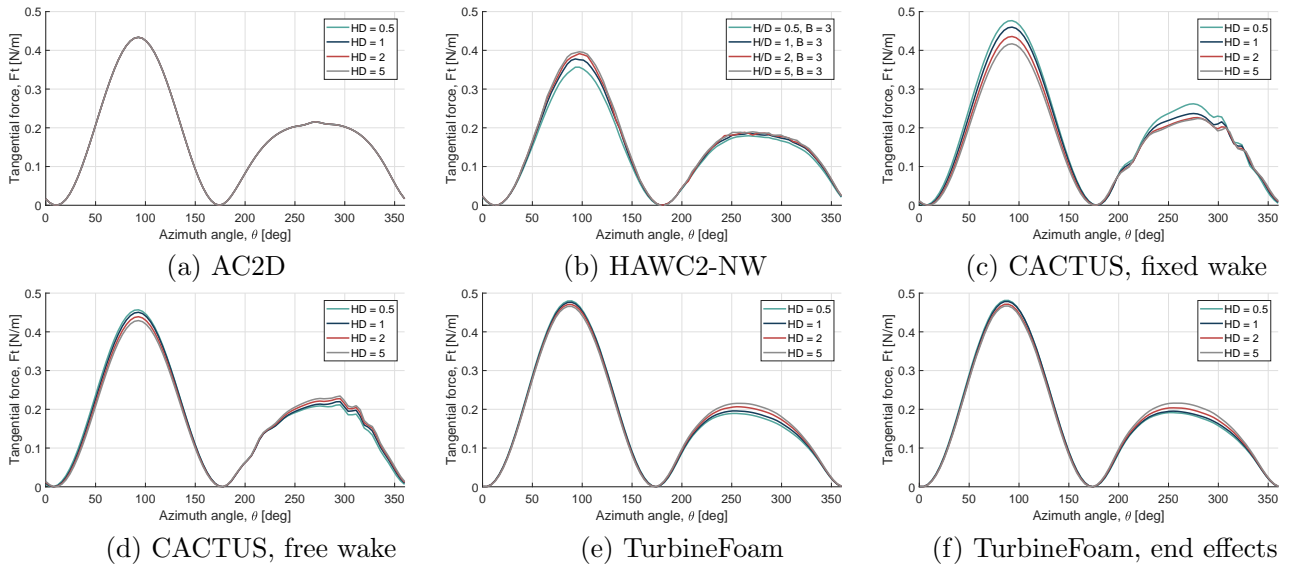

Figure 5: Tangential blade force vs azimuth angle using six different models for the three-bladed H-turbine with an aspect ratio of 0.5, 1, 2 or 5. The rotor solidity is 0.085 and the tip speed ratio is 3. The loads are shown for the mid-plane.

The Φ-rotor still shows that the tip effects cause a redistribution of the loads. In Figure 7, again a power gain upwind and power loss downwind can be identified, in a similar way as was identified for the H-rotor. Although that the TurbineFoam model fails in calculating the loads

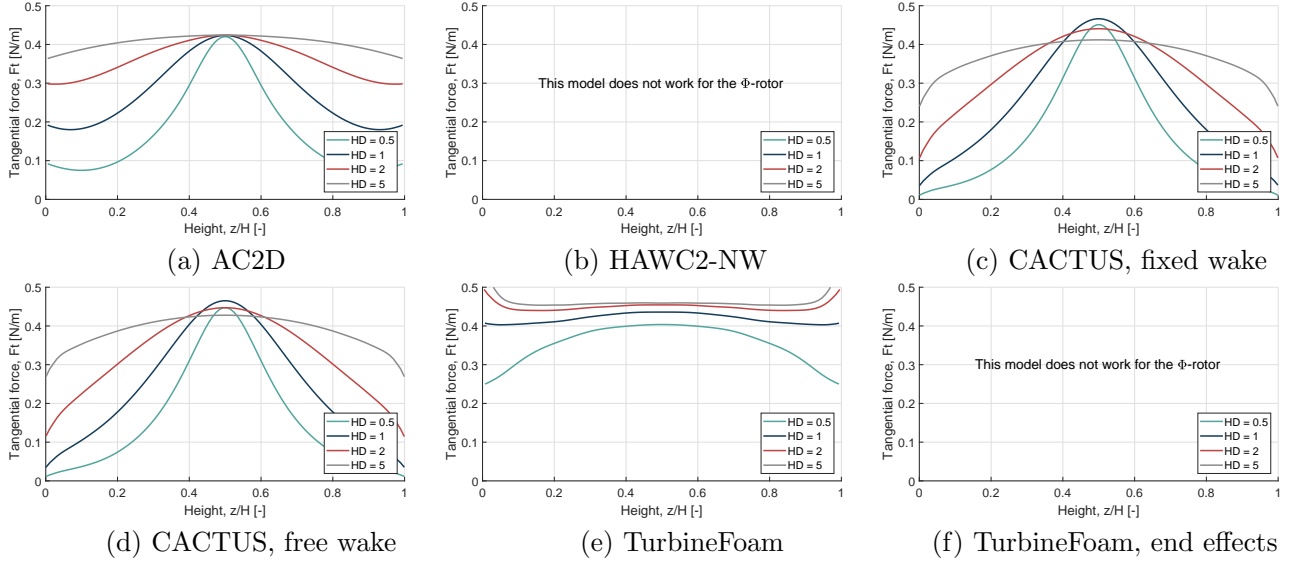at the tips, at the centre of the rotor the tangential force is matching fairly well with the other models.

Figure 6: Tangential blade force vs the spanwise direction using six different models for the three-bladed Φ-turbine with an aspect ratio of 0.5, 1, 2 or 5. The rotor solidity is 0.085 and the tip speed ratio is 3. The blade is located at a 90 deg azimuth angle.
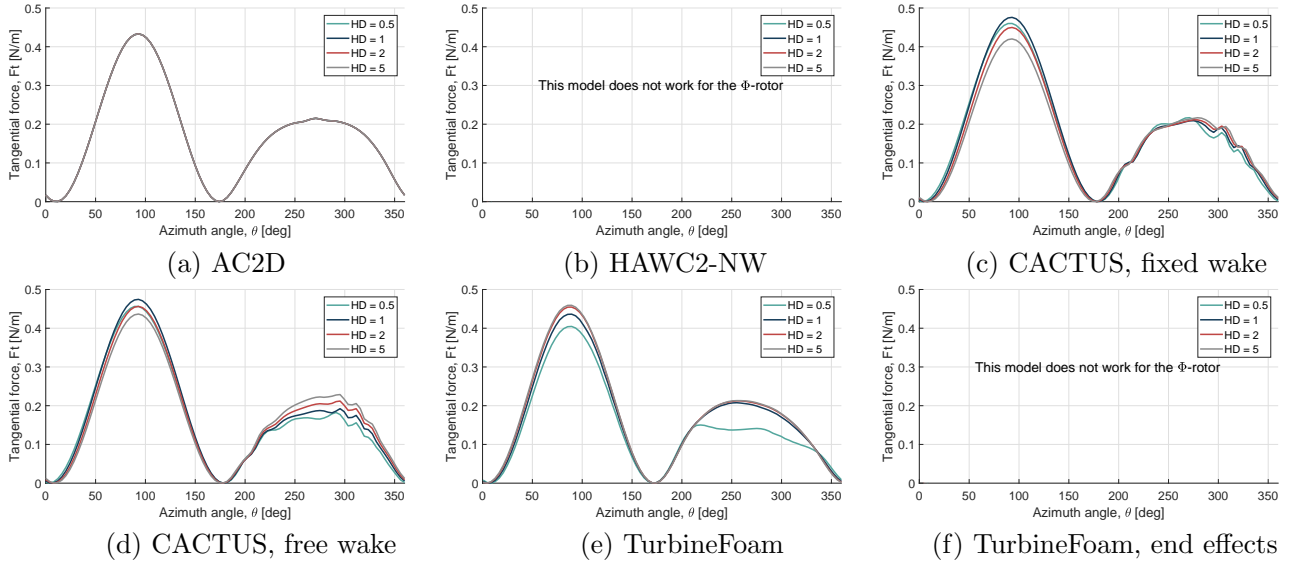

Figure 7: Tangential blade force vs the azimuth angle using six different models for the three-bladed Φ-turbine with an aspect ratio of 0.5, 1, 2 or 5. The rotor solidity is 0.085 and the tip speed ratio is 3. The loads are shown for the mid-plane.

3.3. Study of turbine loading

In Figure 8, the effect of aspect ratio and solidity on the power coefficient according to the various models for the H-rotor is identified. HAWC2-NW, CACTUS with free-wake (and to some extend with a fixed-wake) as well as TurbineFoam with end-effects correction, all predict a power loss for smaller aspect ratios. The power loss identified by HAWC2-NW is larger than for the other models since HAWC2-NW is the only model predicting a power loss both in the upwind and downwind part of the rotor. For the other three models the loss of power downwind is to a certain extend compensated by the gain upwind. The AC2D model does not show any dependency on

the aspect ratio and TurbineFoam without end-effects correction shows an increase in power for smaller aspect ratios, which can be attributed to the unrealistic behaviour of the loads at the tips. For the Φ-rotor, all models indicate a similar trend. Even the AC2D model, follows well the trend of the other models. The power of the TurbineFoam model (especially at lower aspect ratios) is higher then what is predicted by the other models but this is again due to the assumption of all panels being perpendicular to the incoming flow. Including 3D effects for the Φ-rotor seem to be less crucial than for the H-rotor. Similar trends are identified for the different solidities for both the H- and Φ-rotor. Finally, note that integral parameters such as the power coefficient might present misleading results since error cancellation may occur.

Figure 8: Power coefficient of the total rotor vs aspect ratio for the 3-bladed H-turbine using 6 different models. Comparison shown for three different rotor solidities. Tip speed ratio is 3.
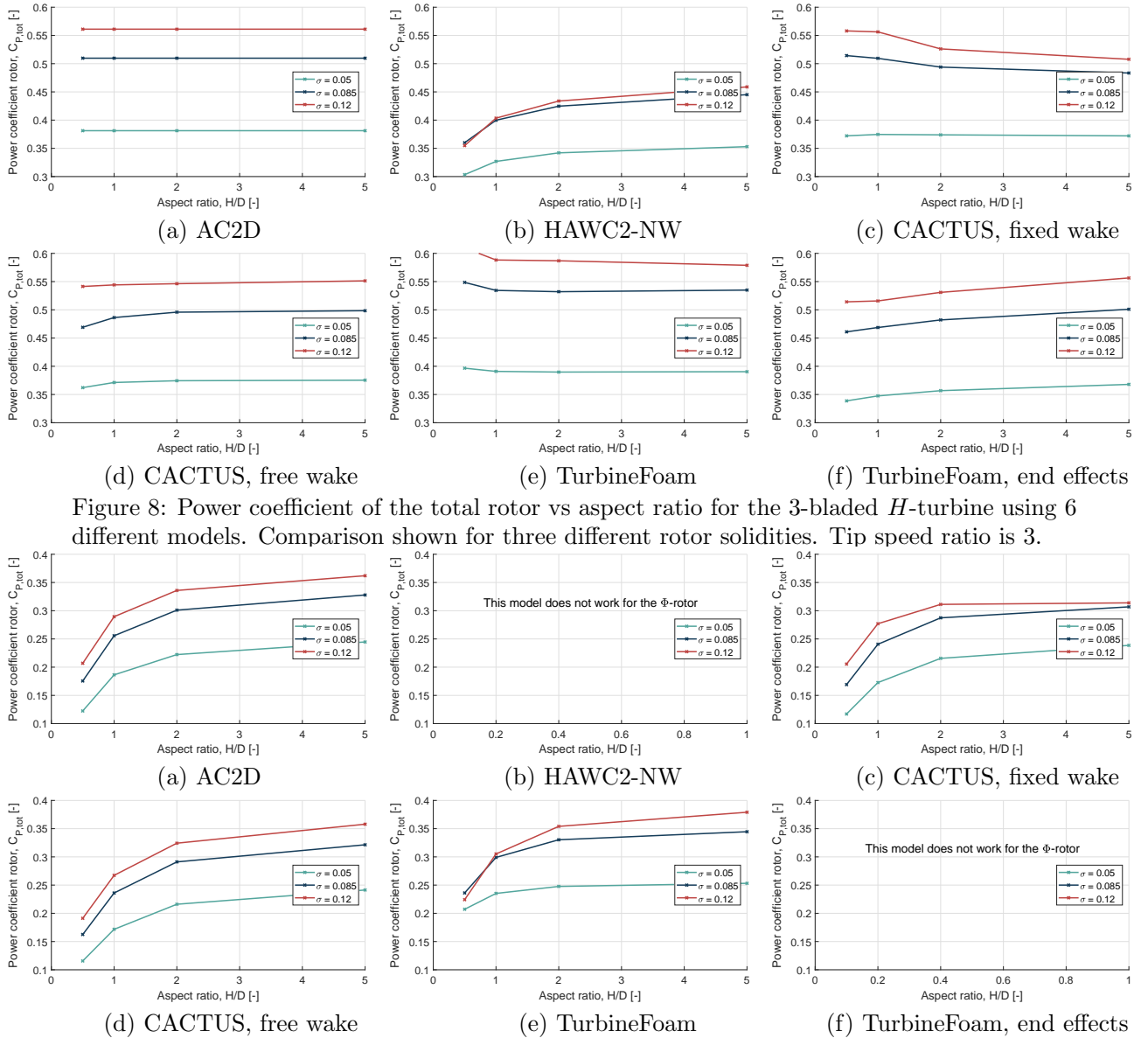

Figure 9: Power coefficient of the total rotor vs aspect ratio for the 3-bladed Φ-turbine using 6 different models. Comparison shown for three different rotor solidities. Tip speed ratio is 3.

4. Conclusion

This paper presents a blind comparison of 6 different aerodynamic models to determine the performance of a 3D VAWT: (1) 2D actuator cylinder model, (2) 2D actuator cylinder model

with near wake correction, (3) fixed-wake vortex model, (4) free-wake vortex model, (5) Actuator line OpenFOAM model and (6) Actuator line OpenFOAM model including an end-effect model. All models are using the same blade-element information but have a different description for the induced velocity field. The study is performed on two turbine shapes, H-turbine and the Φ-turbine with various aspect ratios. A baseline turbine is defined with a rotor solidity of 0.085 and tip speed ratio of 3, representing a medium-loaded turbine.

From this study it can be concluded that modelling VAWTs in 3D is not straightforward. The vortex system of VAWTs is rather complex making it hard to identify the true trends of 3D effects. The models all have assumptions, that limit there capabilities to capture the 3D flowfield. The actuator cylinder model is 2D and per definition does not model 3D effects. The HAWC2-NW model does compensate for the 3D effects using a lifting-line approach, however, it is assuming a straight wake behind each blade section, only includes the trailing vorticity of the blade itself causing that for infinite blades, the tip effects correction goes to zero. The TurbineFoam model has difficulties in capturing the physical behaviour of the loads at the tips due to the smearing of the forces. A correction method for these end-effects is introduced in TurbineFoam, however, the currently implemented model is difficult to justify from a physical point of view. Also TurbineFoam does not account for the panel orientation making the simulations of the Φ-rotor unreliable. The CACTUS fixed-wake model does account for the vortex path to a certain extend but does not account for any wake roll-up effects. This is captured by the CACTUS free-wake model, however the tip effects are significantly influenced by the adopted vortex core model. The vortex models seem to present the most expected results with a limited list of assumptions which might suggest that these results are the most reliable.

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