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# A parametric study on supersonic/hypersonic flutter behavior of aero-thermo-elastic geometrically imperfect curved skin panel

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Abstract In this paper, the effect of the system parameters on the flutter of a curved skin panel forced by a supersonic/hypersonic unsteady flow is numerically investigated. The aeroelastic model investigated includes the third-order piston theory aerodynamics for modeling the flow-induced forces and the Von Kármán non-linear strain-displacement relation in conjunction with the Kirchhoff plate hypothesis for the panel structural modeling. Structural non-linearities are considered and are due to the non-linear coupling between out-of-plane bending and in-plane stretching. The effects of thermal degradation and Kelvin's model of structural damping independent on time and temperature are also considered. The aero-thermo-elastic governing equations are developed from the geometrically imperfect non-linear theory of infinitely long two-dimensional curved panels. Computational analysis and discussion of the finding along with pertinent conclusions are presented.

### Notations

a	Panel length, m
b	Panel width, m
$c_{\infty}$	Speed of sound of undisturbed flow, m/s
$D_0$	Flexural panel stiffness [= $Eh^3/12(1-v^2)$ ], N.m
$E, E_0, E_1, e_T$	Elastic moduli, N/m <sup>2</sup> ; rate of change in elastic moduli/rate of change
	in temperature, $1/C^{\circ}$
$g_s, g_{\rm sm}, g_{\rm sb}$	Structural damping, membrane and bending coefficients, respectively
Н	Camber of the curved panel, m
$h, \bar{h}$	Panel thickness, m; its dimensionless counterpart ( $\equiv h/a$ ), respectively
$M, N_x$	Bending moment resultant, N.m; and axial stress force resultant measured
	per unit length, N/m, respectively
$M_F, V_F$	Mach number at flutter and flutter speed, respectively, m/s
$M_{\infty}, q_{\infty}$	Undisturbed flight Mach number and dynamic pressure ( $\equiv \rho_{\infty} U_{\infty}^2/2$ ), N/m <sup>2</sup> , respectively
n	Number of modes, $n = 1, 2, \ldots \leq \infty$
$p_{\infty}$	Free stream pressure of the undisturbed flow, N/m <sup>2</sup>
$P_z$	Distributed load in the normal direction, N/m <sup>2</sup>

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$R_e$ T(x, z), T	Residual or error term Temperature increment from a stress-free reference temperature $T_0$ , Temperature, C°,
	respectively
$t, \bar{t}$	Time, s, and its dimensionless counterpart ( $\equiv t\Omega_0$ ), respectively
и	Panel longitudinal deflection, m
$U_{\infty}$	Air speed of undisturbed flow, m/s
$v_z$	Normal transverse velocity (downwash), m/s
$\hat{w},ar{ar{W}}$	Initial undeformed shape of the panel mid-plane surface, m;
	its dimensionless counterpart ( $\equiv \hat{w}/a$ ), respectively
w,  ar w	Panel transverse deflection, m, and its dimensionless counterpart
	$(\equiv w/a)$ , respectively
<i>x</i> , <i>z</i> ; ξ	In-plane and out of plane coordinates, m, and dimensionless coordinate
	$(\equiv x/a)$ , respectively
$\alpha, \alpha_0, \alpha_1, \alpha_T$	Coefficients of thermal expansion and rate of change
	in thermal expansion coefficient/rate of change in temperature, 1/C°
$\hat{\delta}, \delta_{ajk}, \delta_{em}, \delta_{\alpha}, \delta_{e}$	Tracer identifying the geometrical imperfection; tracers identifying
	aerodynamic effect, linear and non-linear aerodynamic terms, the derivatives of the $\bar{W}$
	with respect to $\bar{t}$ or $\xi$ , $i = a$ ; $j = 1, 2, 3$ ; $k = \bar{t}$ or $\xi$ ; tracer identifying the edge degree
	of movability; and tracers identifying the thermal degradation effect of the elastic modulus
	and coefficient of thermal expansion, respectively
ε	Panel mid-plane strain change
γ	Isentropic gas coefficient
η	Glauert's aerodynamic correction factor
$\rho_{\infty}, \rho_m, \bar{\rho}$	Free stream density of undisturbed flow, panel mass density, kg/m <sup>3</sup> , and its
	dimensionless counterpart $(\rho_m/\rho_\infty)$ , respectively
$\sigma_{\chi}$	In-plane stress component, N/m <sup>2</sup>
υ	Poisson's ratio
$\psi_j$	Generalized coordinates; $j = 1, 2, \cdots$
8	Curvature change of the mid-plane surface
$\Re_x, \hat{h}$	Constant radii curvature, m, and its dimensionless counterpart (curvature ratio)
	$(\equiv h/\Re_x)$ , respectively
$\Delta, \Delta_x$	Laplace operator; end-shortening in the <i>x</i> direction, respectively
$\Omega_0, \Omega$	First natural frequency of a plate panel, rad/s, and its dimensionless counterpart
	$(\equiv \Omega_0 a/c_\infty)$ , respectively

# Subscript/Superscript

$\infty$	Free stream
$(\cdot)^A$	Unsteady aerodynamic load
$(\cdot)^{\text{stat}}$	Initial static load
$(.)_{,t}, (.)_{,tt}, (.)_{,x}, (.)_{,xx}$	$\partial(.)/\partial t; \ \partial^2(.)/\partial^2 t; \ \partial(.)/\partial x; \ \partial^2(.)/\partial^2 x$
+,-	Quantity evaluated on the upper and lower surfaces of the panel
(_)	Dimensionless quantities

## Abbreviations

- DNIT Direct Numerical Integration Technique
- DOF Degrees-of-freedom
- LCO Limit cycle oscillation
- LFQ Lyapunov first quantity
- PDE Partial differential equation
- PTA Piston theory aerodynamics

## **1** Introduction

The outer surfaces of all flight vehicles are generally supported by internal structural members that divide the surface into individual panels. These panels are subjected to in-plane loads and normal aerodynamic loads,

and it is well known that unstable oscillatory motions of the panels can be caused by the coupling of elastic, inertia, and acting aerodynamic forces. This phenomenon is known as "panel flutter" [4,9-11]. Dynamic aeroelastic instabilities, such as the panel flutter, are of great concern for aerospace designers, since these instabilities can lead to immediate failure or long-term fatigue failure. The outer lifting surfaces and panels of supersonic aircraft and missiles, not designed to carry primary structural loads, such as control surface fairings and wing-fuselage interconnections, can be subjected to panel flutter being very slender and thin. For these and other structural components, panel flutter is one of the primary design constrains, and it must be evaluated. A further consequence of supersonic flight, viz., aerodynamic heating, escalates this problem, since large thermal stresses may arise, and if the panel experiences high compressive in-plane loads, its susceptibility to panel flutter will be critically increased [1].

Jordan [2] was the first to identify such problem. The use of linear structural theory indicates that there is a critical dynamic pressure above which the panel motion becomes unstable. For large deflections, the induced in-plane forces restrain the panel motion to bounded limit cycle oscillations (LCO) [3–5]. At supersonic speeds, the skin panel temperatures can reach several hundred degrees due to aerodynamic heating (e.g., 85–108°C for the Concorde cruising at Mach 2.0, and 141–189°C for the Quiet Supersonic Platform at Mach 2.4). Due to the combined load of airflow and heating, the flexible skin panels might exhibit large aerothermal deflections [6].

Available panel flutter models are divided into linear and non-linear categories. Much of the early work on panel flutter was restricted to supersonic speeds (1.5 <  $M_{\infty}$  < 3). An in-depth discussion of the fundamental aspects of panel flutter was given by Dugundji [7] using a linear, isotropic plate theory and a linear aerodynamic theory. One essential limitation of the linearized analysis of the problem is that it gives information only up to the point of instability, e.g., the flutter speed. Furthermore, the linearized analysis is restricted to cases where the aeroelastic response is small. Often, this assumption is violated before the onset of instability. Thus, to study the behavior of aeroelastic systems near the flutter instability boundary, or even in the post-instability region, the inherent non-linearities of structural and aerodynamic nature must be accounted for. It was recognized that geometrical non-linearities due to moderate plate deflection (mainly mid-plane stretching forces) represented following the Von Kármán large deflection plate theory combined with the linear piston theory, one of the popular unsteady aerodynamic theories used, can play an important role in panel flutter [8]. This led to a number of studies [9-16] which showed that, when geometrically nonlinearities are included in the model, the linear stability boundary can be exceeded, thereby inducing stable LCO with finite amplitudes, having an order of magnitude equal to the thickness of the panel. Librescu et al. [66–68] presented the analytical results of simply supported single-layer and three-layer flat and curved panels made from transversely isotropic materials. The authors solved the non-linear boundary-value problem using Airy's stress function and one-term Galerkin's approximation.

In spite of the deterministic nature of the panel equation, due to large deformations and mutual interaction between the aerodynamic loading and high-order panel modes, the non-linear panel response changes from oscillatory, limit cycle, quasi periodic to random-like irregular chaotic motion. This problem was examined in the context of chaos theory [17]. The chaos of beam and shell elements has been studied by many researchers. Dowell [18,19] has shown that non-linear elastic panel flutter is capable of producing chaotic behavior, while related results were obtained in the presence of inertial forces due to a pull-up maneuver by Sipcic [20] and by Sipcic and Morino [21]. Yamaguchi et al. [22] investigated chaotic vibrations of an elastic shell-panel (without aerodynamic loading) by using the Lyapunov exponents and Lyapunov dimension tools. Due to frequency constraints and limitations on the number of degrees-of-freedom (DOF) retained in analytical solution methods such as perturbation and harmonic balance [23], these methods are only appropriate for simple limit cycle and bifurcation boundary analysis. Also, analytical methods become inapplicable in remote post-critical domains dealing with secondary and further bifurcations and chaos attractors. Bifurcation study can be utilized as a tool for the determination of static (divergence), dynamic (flutter), as well as chaotic instability boundaries [24,25]. Bolotin et al. [26] treated the secondary bifurcation and global instability of a two degrees-of-freedom approximation of an aeroelastic panel. The influence of initial conditions on the post-critical behavior of a nonlinear aeroelastic panel was studied by Bolotin et al. [27]. Their investigation focused on bifurcation boundary and chaotic attractors of an elastic panel. In recent years, there has been renewed interest in supersonic and hypersonic flight vehicles. A thorough review of the non-linear panel flutter was presented by Mei et al. [28], whereas a review of the finite element method within a linearized approach of the supersonic panel flutter was given by Bismarck-Nasr [29].

In case of thermal effects, aero-thermo-elastic considerations are important in the design of space reentry vehicles and high-speed aircraft [30], and these effects may produce deformations, thermomechanical stresses,

and changes in material properties that can dramatically affect their aeroelastic behavior. In this sense, the structural panels of supersonic/hypersonic flight vehicles can experience, among others, the thermal flutter instability generated by the combined influence of the thermal field, unsteady aerodynamic loads, elasticity of structures, and the dynamic effects [31]. The effect of panel heating is twofold. First, there is the reduction in stiffness due to softening of the panel material; second, thermal stress is generated due to mismatch in thermal expansion coefficients of the panel and support structure. These effects, in turn, affect the static and dynamic behavior of the panel [32]. Among the investigations on flat and curved panels flutter dealing with the thermal effect, the bulk of literature mainly discusses models based on stress–strain relationships, quasi-steady first-order, higher-order piston theory aerodynamics, Euler equations for unsteady aerodynamic, shear wall effect, etc. [31–41]. There is a recent bulk of knowledge being developed in the area of functionally graded flat and curved panels exposed to supersonic and hypersonic flows [69–71].

Previous investigations have suggested that detailed studies are needed to better understand the complex motions that can be encountered in the presence of various coupled non-linearities. These studies are also needed when it comes to system identification and damage detection, since the vibration behavior of the system needs to be clearly understood. There are several papers on the flutter boundary of aero-thermo-elastic panels as already discussed earlier, but this paper provides new insight into a specific phenomenon that has not been well investigated in the past. The goals of this paper are, among others, to advance the understanding of the implications of a number of effects such as the structural and aerodynamic non-linearities, thermal degradation, structural damping, and imperfect geometry that contribute to the occurrence of the catastrophic aeroelastic failure of structural panels. Therefore, this paper compliments the bulk of literature on the topic. A better understanding of this issue will contribute to a safer design with evident beneficial implications.

## **2** Formulations

To derive the aero-thermo-elastic governing equations of a curved panel, the geometrically non-linear theory of infinitely long two-dimensional panels with some small initial curvature is considered. The aeroelastic model investigated is based on the third-order piston theory aerodynamics for modeling the flow-induced forces, Von Kármán non-linear strain-displacement relation in conjunction with the Kirchhoff plate hypothesis for modeling the panel. Furthermore, the effects of thermal degradation and Kelvin's model of structural damping independent of time and temperature are also considered in this model.

#### 2.1 Model of the structure

An isotropic infinitely long curved panel model with width a, thickness h, maximum rise height H, and constant radii curvature  $\Re_x$  is considered in Fig. 1. It is assumed that the thickness h is small as compared to the length a. The panel is supported at the ends x = 0 and x = a and is fixed with respect to the longitudinal displacements. The displacements from the unstressed state of the panel's mid-plane surface in the x and z



Fig. 1 Two-dimensional panel with initial curvature

directions are denoted by u and w, and the total transverse displacement of a given mid-plane surface point after deformation is

$$w_{\text{total}}(x,t) = \hat{w}(x) + w(x,t). \tag{1}$$

Herein,  $\hat{w}(x)$  indicates the initial undeformed shape (initial geometric imperfection) of the mid-plane surface, while w(x, t) corresponds to the transverse displacement of the mid-plane surface relative to its undeformed configuration. The strain  $\varepsilon_x$  of the mid-plane surface in the x-direction, based on the Von Kármán assumption, is given by [8]

$$\varepsilon_x = u_{,x} + \frac{1}{2} (w_{,x})^2 + w_{,x} \hat{w}_{,x} - w / \Re_x.$$
<sup>(2)</sup>

The subscript  $(\cdot)_x$  denotes the differentiation with respect to x. The bending equation of motion is

$$M_{,xx} + N_x(w_{,xx} + 1/\Re_x) + P_z = 0$$
(3)

where  $N_x$  represents the axial stress resultant and M is the bending moment; furthermore,  $M \equiv D\aleph$  where D is the panel stiffness  $\equiv Eh^3/12(1-v^2)$ , E is the modulus of elasticity, v is Poisson's ratio, and  $\aleph$  is the curvature change of the mid-plane surface ( $\equiv -w_{,xx}$ ). In Eq. (3),  $P_z$  is the distributed load on the panel and can be expressed as follows:

$$P_{z} = -\rho_{m}hw_{,tt} + P_{z}^{A}(x,t) + P_{z}^{\text{stat}}(x) + \Delta B^{T}.$$
(4)

The first term in the Eq. (4) corresponds to the transverse inertial load, while the superscript  $(\cdot)^A$  indicates an unsteady aerodynamic load and the superscript  $(\cdot)^{\text{stat}}$  indicates an initial static load.  $B^T$  is the thermal load and is defined as [31]

$$B^{T} = \frac{E\alpha}{(1-\upsilon)} \int_{-h/2}^{h/2} T(x,z) \ z \ dz$$
(5)

where  $\alpha$  is the linear thermal expansion coefficient, T(x, z) is the temperature increment from a free-stress temperature  $T_0$ , and  $\Delta$  in Eq. (4) is the Laplace operator. It is assumed that the material properties of the panel, E and  $\alpha$ , are influenced by the thermal field as follows:

$$E = E_0 + E_1 T = E_0 (1 + e_T T), \quad \alpha = \alpha_0 + \alpha_1 T = \alpha_0 (1 + \alpha_T T)$$
(6.1)

where

$$e_T = E_1/E_0 < 0, \quad \alpha_T = \alpha_1/\alpha_0 > 0.$$
 (6.2)

In Eq. (6.2),  $e_T$  and  $\alpha_T$  are the coefficients associated with the thermal degradation. A linear temperature distribution *T* throughout the panel thickness is considered,

$$T(x, z) = T_0(x) + z T_1(x).$$
(6.3)

Note that this temperature distribution was obtained via an exact analysis by Bolotin [9]. As a result of the temperature dependence of the thermoelastic material properties and of the spatially distributed temperature field, the thermoelastic coefficients of the material become also functions of the space variables, that is, E = E(x, T) and  $\alpha = \alpha(x, T)$ . This results in an induced non-homogeneity of the structural panel. A typical aerospace panel, such as a fuselage section and wing and empennage panels, are usually solidly connected to structural members of the airframe. For this reason, it has been assumed that  $\sigma_x = N_x/h = (E(x)/(1 - v^2))(\varepsilon_x + v\varepsilon_y) \rightarrow \sigma$ , that is, the tangential stresses act only in the *x*-direction. Physically, this stress is generated by the constraint of the panel with the members of the airframe. Moreover, when the flight vehicle travels at high flight speed regimes, due to aerodynamic heating, the skin panel temperature can potentially reach the high values of several hundred degrees. This effect can result in a lower value of the flutter instability boundary or in a larger limit cycle amplitude at the same dynamic pressure. This also implies that the effect of the temperature should be carefully considered for more accurate results. This effect is added by including an in-plane tension  $\sigma_x^T = -(E(x)/(1 - v))\alpha(x) T_0(x)$ , acting in the *x*-direction, due to the temperature [42].

Consequently, Eq. (3) becomes

$$(D/h)w_{,xxxx} - \sigma_{x\_total}(w_{,xx} + 1/\Re_x) - P_z/h = 0$$
(7)

where  $\sigma_{x\_total} = \sigma_x + \sigma_x^T$  is the total in-plane stress, in the *x*-direction, evaluated using the average end-shortening  $\Delta_x$  [8,46] in the case of immovable edges, x = (0, a), i.e.  $\Delta_x = 0$ ,

$$\sigma_{x\_\text{total}} = \left[\frac{1}{(1-\upsilon^2)\int_0^a E(x)^{-1} dx}\right] \left[\frac{1}{2}\int_0^a (w_{,x})^2 dx + \int_0^a w_{,x} \hat{w}_{,x} dx - \int_0^a \frac{w}{\Re_x} dx - \int_0^a \alpha(x) (1+\upsilon) T_0(x) dx\right].$$
(8)

#### 2.2 Model of the structural damping independent of time and temperature

Structural damping for panels consists of both material damping and frictional damping acting at the panel supports. The most widely used material-damping models are the linear viscous and hysteresis models. It has been proved that the damping contribution can significantly modify the flutter boundaries. The modification is extremely dependent on the type of structural damping model employed. If only linear damping is considered, the work by Ellen [43] provides a useful classification of this mechanisms and its influence on the flutter boundaries. From the mathematical point of view, structural damping independent of time and temperature can be introduced into the system by adding a term of the form  $(g_{sb}\partial^{j+1}w/\partial t\partial x^j)$  to the bending terms of Eq. (7) and  $(g_{sm}\partial^{j+1}w/\partial t\partial x^j)$  to the membrane terms of Eq. (8). Herein,  $g_s$  is a structural damping coefficient, and it is constant for viscous damping.  $g_{sb}$  and  $g_{sm}$  are the bending and membrane coefficients, respectively. Based on Kelvin's model of elastic materials, E(x) is replaced with the operator  $E(x)(1 + g_s\partial/\partial t)$  [44,45]. Substituting Eq. (8) in Eq. (7), the aero-thermo-elastic bending governing equation becomes

$$D\left(1+g_{sb}\frac{\partial}{\partial t}\right)w_{,xxxx} - \left(1+g_{sm}\frac{\partial}{\partial t}\right)\left[\frac{h}{(1-\upsilon^2)\int\limits_0^a E(x)^{-1}dx}\right]$$
$$\times \left[\frac{1}{2}\int\limits_0^a (w_{,x})^2 dx + \int\limits_0^a w_{,x}\hat{w}_{,x}dx - \int\limits_0^a \frac{w}{\Re_x}dx - (1+\upsilon)\int\limits_0^a \alpha(x)T_0(x)dx\right](w_{,xx} + 1/\Re_x)$$
$$+\rho_m hw_{,tt} - P_z^A(x,t) = P_z^{stat}(x).$$
(9)

Support damping has not been considered here, and therefore, conservative results are likely to be obtained, that is, a lower value of the flutter speed and larger LCO than the one would occur if this additional damping component would be accounted for.

#### 2.3 Model of the aerodynamic loading

The fluid-structure interaction used in the present study is based on the non-linear piston theory [12]. According to this theory, the radial aerodynamic pressure p applied to the surface of the shell can be obtained by analogy with the instantaneous isentropic pressure on the face of a piston moving with velocity  $v_z$  into a perfect gas which is confined in a one-dimensional channel; this pressure is given by

$$p^{+}(x,t)/p_{\infty} = \{1 + [(\gamma - 1)/2](v_{z}/c_{\infty})\}^{2\gamma/(\gamma - 1)}.$$
(10)

In the analogy, the local transverse piston velocity (downwash velocity)  $v_z$  normal to the panel and the undisturbed speed of sound  $c_{\infty}$  may be expressed in terms of the panel transverse displacement w(x, t) in order to obtain the radial aerodynamic pressure applied to the surface of the shell as a consequence of the external supersonic flow

$$v_z = w_{,t} + U_{\infty}[\hat{w} + w]_{,x}; \quad c_{\infty}^2 = \gamma p_{\infty} / \rho_{\infty}.$$
 (11)

Herein,  $p_{\infty}$ ,  $\rho_{\infty}$ ,  $U_{\infty}$ , and  $\gamma$  are the pressure, air density, and air speed of the undisturbed flow and the isentropic gas coefficient ( $\gamma = 1.4$  for dry air), respectively. To study the non-linear panel flutter, in addition to the inclusion of geometrical non-linearities, a non-linear piston theory aerodynamics (PTA) model is used. PTA is a popular modeling technique for supersonic and hypersonic aeroelastic analyses. Retaining, in the binomial expansions of Eq. (10), the terms up to and including  $(v_z/c_{\infty})^3$  yields the pressure formula for the PTA in the third-order approximation

$$p^{+}(x,t)/p_{\infty} = 1 + \gamma \left( v_{z}/c_{\infty} \right) \eta + \left[ \gamma \left( \gamma + 1 \right)/4 \right] \left[ \left( v_{z}/c_{\infty} \right) \eta \right]^{2} + \left[ \gamma \left( \gamma + 1 \right)/12 \right] \left[ \left( v_{z}/c_{\infty} \right) \eta \right]^{3}.$$
 (12)

Consider the flow only on the upper surface of the panel  $U_{\infty}^+ \equiv U_{\infty}$  and  $M_{\infty} = U_{\infty}/c_{\infty}$ , that is, consider  $U_{\infty}^- = 0$  and  $p^- = p_{\infty}$ ; from Eqs. (10) and (12), the aerodynamic pressure difference can be expressed as

$$P_{z}^{A}(x,t) = p^{+} - p_{\infty} = \delta p|_{\text{PTA}} - (2q_{\infty}/M_{\infty}) \eta \left\{ (1/U_{\infty})w_{,t} + (\hat{w} + w)_{,x} + [(1+\gamma)/4]\eta M_{\infty} \right.$$
$$\times [(1/U_{\infty})w_{,t} + (\hat{w} + w)_{,x}]^{2} + [(1+\gamma)/12]\eta^{2}M_{\infty}^{2}[(1/U_{\infty})w_{,t} + (\hat{w} + w)_{,x}]^{3} \right\}$$
(13)

where the undisturbed dynamic pressure  $q_{\infty} = \rho_{\infty} U_{\infty}^2/2$ .

Using Eq. (5) yields the thermal moment given by  $E\alpha h^3/12(1-\upsilon)T_{1,xx}$ . A membrane temperature distribution  $T_0(x)$  (implying  $T_1(x) = 0$ ) will be considered. This temperature distribution can correspond to the steady-state flight regime of a high-speed aerospace vehicle. Such a representation of the temperature field is adopted here to reduce the problem to an eigenvalue one. Specifically,  $T_0(x)$  is expressed as

$$T_0(x) = {\operatorname{T}}^* \sin(\pi x/a)$$
 (14)

where  $\stackrel{*}{T}$  is the temperature amplitude at x = a/2.

## 2.4 Non-linear aero-thermo-elastic governing equations

The following dimensionless variables are defined:

$$\begin{split} \bar{W} &= w/a, \quad \hat{\bar{W}} = \hat{w}/a, \quad \xi = x/a, \quad \bar{t} = t\,\Omega_0, \quad \Omega_0 = (\pi/a)^2 \sqrt{D_0/\rho_m h}, \quad \bar{\Omega} = \Omega_0 a/c_\infty, \\ \bar{h} &= h/a, \hat{h} = h/\Re_x, \quad P_z^{\text{stat}} = \Delta P_z^{\text{stat}}(x) a^4/D_0 h, \quad T_{cr} = D_0/Eha^2\alpha_0, \quad \bar{\rho} = (\rho_m/\rho_\infty), \quad (15) \\ H &\approx a^2/(8\Re_x), \quad \overset{*}{\tau} = \overset{*}{\mathrm{T}}/T_{cr}, \quad \overset{*}{\bar{\mathrm{T}}} = \overset{*}{\tau}\sin(\pi\xi). \end{split}$$

Inserting the non-dimensional variables from Eq. (21) into Eqs. (9), (13), and (14), one can obtain the geometrically non-linear aero-thermo-elastic governing equations of infinitely long curved panels in the form of  $Q\{\hat{W}(\xi), \bar{W}(\xi, \bar{t})\} = 0$ , [46]

 $Q\{\hat{\bar{W}}(\xi), \bar{W}(\xi, \bar{t})\}$ 

viscoelastic bending resistance with thermal degradation effect

$$-\delta_{\rm em} \left(1+g_{\rm sm}\Omega_{0}\frac{\partial}{\partial \bar{t}}\right) \left[1 / \int_{0}^{1} \frac{d\xi}{\left(1+\delta_{e}e_{T}\overset{*}{\bar{T}}T_{cr}\right)} \right] \frac{12}{\bar{h}^{2}} \left[\frac{1}{2} \int_{0}^{1} (\bar{W}_{,\xi})^{2} d\xi + \int_{0}^{1} \bar{W}_{,\xi} \hat{W}_{,\xi} d\xi - \frac{\hat{h}}{\bar{h}} \int_{0}^{1} \bar{W} d\xi\right] \left(\bar{W}_{,\xi\xi} + \frac{\hat{h}}{\bar{h}}\right)$$

$$+ \delta_{\rm em} \left(1+g_{\rm sm}\Omega_{0}\frac{\partial}{\partial \bar{t}}\right) \left[1 / \int_{0}^{1} \frac{d\xi}{\left(1+\delta_{e}e_{T}\overset{*}{\bar{T}}T_{cr}\right)} \right] \underbrace{\frac{1}{\left(1-\upsilon\right)} \left[\int_{0}^{1} (1+\delta_{\alpha}\alpha_{T}T_{cr})\overset{*}{\bar{T}} d\xi\right]}_{\text{thermal degradation effect}} \left(\bar{W}_{,\xi\xi} + \frac{\hat{h}}{\bar{h}}\right) + \underbrace{\pi^{4}\bar{W}_{,\bar{t}\bar{t}}}_{\text{inertia effect}} + \underbrace{\frac{M_{\infty}\pi^{4}}{\bar{h}\bar{\rho}\bar{\Omega}^{2}} \eta \left[\frac{\delta_{a1\bar{t}}\frac{\bar{\Omega}}{M_{\infty}}\bar{W}_{,\bar{t}} + \delta_{a1\xi}(\hat{W}_{,\xi} + \bar{W}_{,\xi}) + \\ + \underbrace{\frac{1+\gamma}{12}\eta^{2}M_{\infty}^{2}\left(\delta_{a3\bar{t}}\frac{\bar{\Omega}}{M_{\infty}}\bar{W}_{,\bar{t}} + \delta_{a3\xi}(\hat{W}_{,\xi} + \bar{W}_{,\xi})\right)^{2}}_{pressure on plate} - \underbrace{\frac{1}{2}\int_{0}^{1} (1+\delta_{\alpha}\bar{U}_{T})^{2} d\xi + \int_{0}^{1} \bar{W}_{,\xi} d\xi - \frac{\hat{h}}{\bar{h}} \int_{0}^{1} \bar{W}_{,\xi} d\xi - \frac{\hat{h}}{\bar{h}} \int_{0}^{1} \bar{W}_{,\xi} d\xi - \frac{\hat{h}}{\bar{h}} \int_{0}^{1} \bar{W}_{,\xi} d\xi + \frac{\hat{h}}{\bar{h}}\right] \left(\bar{W}_{,\xi\xi} + \frac{\hat{h}}{\bar{h}}\right) + \underbrace{\frac{1}{2} \int_{0}^{1} (1+\delta_{e}e_{T})^{2} d\xi + \int_{0}^{1} (1+\delta_{\alpha}\bar{U}_{,\xi})^{2} d\xi + \int_{0}^{1} \bar{W}_{,\xi} d\xi - \frac{\hat{h}}{\bar{h}} \int_{0}^{1} \bar{W}_{,\xi} d\xi + \frac{\hat{h}}{\bar{h}}\right] + \underbrace{\frac{\pi^{4}\bar{W}_{,\bar{t}\bar{t}}}_{inertia} \frac{\hat{H}_{,\xi}}{\hat{U}_{,\xi}} + \frac{\hat{H}_{,\xi}}{\hat{U}_{,\xi}} d\xi + \frac{\hat{H}_{,\xi}}{\hat{$$

aerodynamic loads (3rd PTA)

The imperfection of the curved panel, in non-dimensional form, may be approximated by a sinusoidal function [47], in this case

$$\hat{\bar{W}} = \hat{w}/h = \hat{\delta} \left\{ \sum_{p=1}^{n} q_p \sin(p\pi\xi) \right\}.$$
(17)

To identify the effects of geometrical imperfection, edge movability, aerodynamic, and thermal terms, various tracers have been adopted in the Eqs. (16) and (17). The tracers  $\delta_e$  and  $\delta_\alpha$  identify the terms associated with the thermal degradation of the elastic modulus and the coefficient of thermal expansion, respectively.  $\delta_{\rm em} \in [0, 1]$  identifies the degree of edge movability, where  $\delta_{\rm em} = 1$  indicates immovable edges. Movable edges can be simulated by assuming that the panel is supported at the edges  $\xi = 0$  and  $\xi = 1$  by springs. The tracer  $\hat{\delta} \in [0, 1]$  indicates the implication of geometrical imperfection. The tracer  $\delta_{ajk}$  has three indices, the first index (a) identifies aerodynamic contribution, the second index (j) identifies the degrees of linearity, (1  $\equiv$  linear, 2  $\equiv$  quadratic, and 3  $\equiv$  cubic), while the third index (k) represents the derivatives of the  $\overline{W}$  with respect to  $\overline{t}$  or  $\xi$ .

#### 3 Galerkin method and direct numerical integration technique

Several techniques have been used to solve the non-linear panel flutter problem. These are the harmonic balance technique [48], perturbation method [49], finite element method [28,29,38,39,50], direct numerical integration technique (DNIT)[26,40,47,51–58], pseudo-arc length continuation method that complements the DNIT [32,59] and recently the Lyapunov first quantity (LFQ) [8,31,60]. Galerkin's method [72] and DNIT will be considered to solve the integro-differential equation (Eq. 16) to evaluate the structural response and the character of the curved panel flutter boundary with thermoelastic properties. The boundary conditions for the simply supported panels on  $\xi = 0$ , 1 require that  $\overline{W} = \overline{W}_{,\xi\xi} = 0$ . For these conditions, we seek a solution in the following form:

$$\bar{W}(\xi,\bar{t}) = \sum_{j=1}^{n} \psi_j(\bar{t}) \; \bar{\phi}_j(\xi) \tag{18}$$

where *n* is the number of harmonic modes,  $n \leq \infty$ ;  $\bar{\phi}_j(\xi)$ , are assumed orthogonal shape functions and  $\psi_j(\bar{t})$  are unknown generalized coordinates that depend on time. The assumed functions  $\bar{\phi}_j(\xi)$  are chosen to satisfy the boundary conditions. To fulfill such conditions, the mode shape functions  $\bar{\phi}_j(\xi) = \sin(\lambda_j \xi)$  and  $\lambda_j = j\pi$ ,  $j = 1, 2, \ldots$  are considered. Clearly, the assumed approximate solution is not exactly the same as the unknown exact solution. Consequently, Eq. (18) will not satisfy the partial differential equations (PDE) (16); there is going to be a residual such that  $Q(\xi, \bar{t}) = Q\{\hat{W}, \sum_{j=1}^n \psi_j(\bar{t})\bar{\phi}_j(\xi)\} = R_e \neq 0$ , where  $R_e$  is the

residual or error that results from the use of the approximate solution. Multiplying the residual by the basic function  $\bar{\phi}_r(\xi) = \sin(r\pi\xi)$  with  $r = 1, 2, ..., n \le \infty$  and integrating over the panel length,  $\xi$  from 0 to 1, and imposing the result to be 0, a set of non-linear, simultaneous ordinary differential equations with respect to the series in Eq. (18), and function of geometrical imperfection Eq. (17) can be obtained:

$$\frac{d^2\psi_r}{d\bar{t}^2} + g\frac{d\psi_r}{d\bar{t}} + F_r(\psi_j, M_\infty, \bar{\bar{T}}) = 0, \qquad j, r = 1, 2, 3, \dots.$$
(19.1)

The  $F_r(\psi_j, M_{\infty}, \dot{\mathbf{T}})$  functions can be represented as

$$F_r(\psi_j, M_{\infty}, \overset{*}{\bar{T}}) = F_r^{(l)}(\psi_j, M_{\infty}, \overset{*}{\bar{T}}) + F_r^{(a)}(\psi_j, M_{\infty}) + F_r^{(th)}(\psi_j, M_{\infty}, \overset{*}{\bar{T}}) + F_r^{(s)}(\psi_j, M_{\infty})$$
(19.2)

where  $F_r^{(l)}(\psi_j, M_\infty, \tilde{\bar{T}})$  are linear functions, and  $F_r^{(a)}(\psi_j, M_\infty)$ ,  $F_r^{(th)}(\psi_j, M_\infty, \tilde{\bar{T}})$  and  $F_r^{(s)}(\psi_j, M_\infty)$  are functions including the *a*erodynamic, *thermal*, and *s*tructural non-linearities, respectively.

#### 4 Numerical results and discussion

Numerical investigations to highlight the flutter characteristics of the proposed aero-thermo-elastic model are presented next. Equation (19.1) is integrated numerically using a Gear's BDF solver provided by the IMSL routine DIVPAG [61]. Four and eight modes have been considered [26,62,63]. All the formulations and computations have been performed using an in-house FORTRAN computer program. All symbolic formulations have been developed and verified using Mathematica<sup>®</sup> ver. 5.

#### 4.1 numerical validation

A numerical validation of the proposed approach has been made. Figure 2 evaluates the effect of the curvature ratio on the normalized flutter dynamic pressure of the infinitely long cylindrical panel and provides a comparison with the panel finite length counterpart,  $\lambda_F \equiv 2q_{\infty}a^3/D_0$ . The results obtained from the present



Fig. 2 Comparison of flutter dynamic pressure versus the curvature ratio



Fig. 3 Effect of the temperature and the thermal degradation on the flutter Mach and eigenfrequency

analysis using four and eight modes are compared with the four-mode solution of Dowell [56,59], and very good agreement is reached. With the increase in the curvature ratio, the four-mode solution and eight-mode solution provide a second minimum. As was indicated in the literature, a solution containing a large number of modes probably would indicate a minimum of the flutter dynamic pressure at all intersections of modal frequencies.

#### 4.2 Linear aero-thermo-elastic analysis

In this Section, the system parameters are varied to infer about their effects on the flutter of a curved skin panel forced by a supersonic/hypersonic unsteady flow. Numerical simulations based on Eq. (19) are performed by discarding the non-linear terms. Unless specified otherwise, monolithic titanium (Ti-6Al-4V) is the material used for the cylindrical panel. The mechanical properties (at T = 294.15 K) [64] and geometric parameters and flow field characteristics (sea level) are  $E_0 = 110.352 \times 10^9$  Pa, v = 0.31,  $\alpha_0 = 4.85 \times 10^{-5}/c^{\circ}$ ,  $\rho_m = 4430$  kg/m<sup>3</sup>, a = 1 m,  $\Re_x = 10$  m, h = 0.01 m,  $\rho_{\infty} = 1.225$  kg/m<sup>3</sup>,  $c_{\infty} = 340.4$  m/s,  $\gamma = 1.4$ ,  $\eta = 1$ ,  $P_z^{stat} = 0$ ,  $\delta_{em} = 1$ ,  $e_T = -6.5764 \times 10^{-4}$ /K,  $\alpha_T = 3.07085 \times 10^{-4}$ /K. As a result, the following dimensionless parameters are obtained:  $\bar{h} = 0.01$ ,  $\hat{h} = 0.001$ ,  $\bar{\Omega} = 0.439$ ,  $\bar{\rho} = 3616.32$  and H/h = 1.25. For this test case, the Mach flutter is  $M_F = 6.614$ , and the flutter frequency is  $\omega_F = 13.563$  (rad/s). As it is mentioned in [31,65], in the case of immovable edges, due to the induced compressive stresses as a result of the edge constraints, a decrease in the thermomechanical buckling range, e.g.,  $\overset{*}{\tau} \in [0, 10]$ .

Figures 3a, b highlight the influence of the temperature in conjunction with the thermal degradation of thermomechanical properties of the material of the panel on flutter speed  $M_{\rm F}$  and flutter frequency  $\omega_{\rm F}$ . In this simulation,  $\bar{h} = 0.01$  and  $\hat{h} = 0.001$  are considered. It clearly appears that with the increase in the temperature amplitude, a decrease in  $M_{\rm F}$  and  $\omega_{\rm F}$  is experienced. Moreover, with increasing values of the temperature distribution amplitude  $\tilde{\tau}$ , the thermal degradation of the thermal expansion coefficient has reduced influence as compared to the thermal degradation of the elastic modulus which becomes prevalent in reducing  $M_{\rm F}$  and  $\omega_{\rm F}$ .

For constant panel thickness ( $\bar{h} = 0.01$ ), Fig. 4 reveals the implications of the curvature ratio  $\hat{h}$  on flutter characteristics  $M_{\rm F}$  and  $\omega_{\rm F}$ . In this case, simulations are performed with/without thermal field with and without thermal degradation. For larger  $\hat{h}$ , the effect of the thermal field in the presence of thermal degradation ( $\tilde{\tau} = 10$ ;  $\delta_e = 1$ ;  $\delta_{\alpha} = 1$ ) is more prominent.



Fig. 4 Effect of the temperature with/without thermal degradation as a function of curvature ratio on the flutter a Mach and b eigenfrequency



Fig. 5 Effect of the thermal field with thermal degradation on the flutter Mach as a function of curvature ratio and mass ratio

Figure 5 shows the effects of the mass ratio  $\bar{\rho}$ . It is clear that for larger values of the mass ratio a decrease in the flutter speed is experienced when ( $\tilde{\tau} = 10$ ;  $\delta_e = 1$ ;  $\delta_{\alpha} = 1$ ). This effect becomes more prevalent when increasing the curvature ratio.

Figure 6 shows the implication of  $\hat{h}$ , considered in conjunction with that of  $\hat{h}$ , on the flutter Mach number  $M_{\rm F}$  with/without thermal field in the presence of thermal degradation. The results reveal that, at relatively small values of the curvature ratio, the panels characterized by larger thickness ratios exhibit an increase (without



Fig. 6 Effect of the thermal field with thermal degradation on the flutter Mach as a function of curvature ratio and panel thickness



Fig. 7 Effect of the thermal field with thermal degradation on the flutter Mach as a function of curvature ratio and panel imperfection

thermal field) in flutter speeds. Indeed, due to the presence of a thermal effect, a reduction in the flutter speed is expected. With the increase in the curvature ratio, the flutter speed decreases (with/without thermal field) and reaches a minimum that strongly depends on the particular value of the thickness ratio and the thermal field with thermal degradation. The value of  $\hat{h}$  where the minima occur depends on the panel thickness ratio and thermal effect, and with the increase in  $\bar{h}$ , these minima shift toward larger values of  $\hat{h}$ . For larger  $\hat{h}$ , a severe reduction in the flutter speed is noted for ( $\tilde{\tau} = 10$ ;  $\delta_e = 1$ ;  $\delta_{\alpha} = 1$ ).

In Fig. 7, the effects of the geometric imperfection on the flutter boundary are highlighted along with the variation of the curvature ratio. The results reveal that the effect of increasing the imperfection, represented in terms of  $q_1$ , depends on the curvature ratio, and a large reduction is exhibited in flutter speed. In this case, simulations have been conducted for ( $\overset{*}{\tau} = 10$ ;  $\delta_e = 1$ ;  $\delta_{\alpha} = 1$ ).

The effect of the degree of edge movability ( $\delta_{em}$ ) on the linear flutter Mach number for a system geometrically perfect is emphasized in Fig. 8a. The parameter identifying the condition of partial movability  $\delta_{em}$  was chosen to vary from 0.1 (partially movable edges) to 1.00 (immovable edges). From the present analyses, it appears that for curved panels the effect of movable or immovable panel edges is more complex, in the sense that the behavior for a specific  $\bar{h}$  strongly depends on  $\hat{h}$  and the thermal effect. Figure 8b shows the frequency coalescence with/without thermal effect. The flutter speed is obtained from the coalescence of the two consecutive eigenfrequencies, and this speed increases when the degree of edges movability increases, implying



Fig. 8 a Effect of the thermal field with thermal degradation on the flutter Mach as a function of curvature ratio and edge degree movability. **b** Frequency coalescence for different values of edge degree movability with/without the thermal field with thermal degradation ( $\hat{h} = 0.01$ ;  $\hat{h} = 0.001$ )

lower values of  $\delta_{em}$ . The edge constrain effect can induce earlier flutter. This is due to the reduction in the in-plane forces, thermal field with thermal degradation, and the panel curvature effect.

Figure 9a displays the effect of structural damping for a thermally insulated system. It is seen that the damping can destabilize the system, particularly for small values of the curvature ratio ( $\hat{h} \leq 0.001$ ). The system exhibits different trends, such stabilizing or destabilizing, for  $0.001 < \hat{h} \leq 0.002$  according to the damping values. This behavior is similar when considering the effect of the thermal load and thermal degradation (Fig. 9b). It can be concluded that structural damping increases or decreases the curved panel domain of stability in linear analysis depending on the curvature ratio and thermal field with thermal degradation effect.

#### 4.3 Non-linear aero-thermo-elastic analysis

For the dynamic analysis, the non-dimensional time integration was carried out from  $\bar{t} = 0$  to  $\bar{t} \approx 750$  time units, and only the last 50 units have been retained for the bifurcation representation. The linear Mach flutter (without thermal degradation) is  $M_F = 6.6$ , as shown in Fig. 10. At higher Mach numbers, the system will



Fig. 9 Effect of structural damping on the flutter Mach as a function of curvature ratio.  $\mathbf{a}$  Without,  $\mathbf{b}$  with effect of the thermal field and thermal degradation

exhibit LCO. To consider the effect of a heated panel, a wall temperature has been computed as follows:  ${}^*_{T} = T_w = T_\infty + R_f[(\gamma - 1)/2]M_\infty^2 T_\infty$  where  $R_f = \sqrt{Pr} \approx 0.3$  [44]. The maximum material temperature was limited to  ${}^*_{T} = T_w \approx 810$  K [64] to prevent thermal buckling. Within this constraint, in the case of heated panels, the time simulation was interrupted at  $M_\infty = 5.4$ . Figure 10 shows the bifurcation diagram when the thermal degradation has been considered. It clearly appears that the thermal degradation reduces the flutter speed. Furthermore, limit cycles appear at speeds as low as  $M_\infty \approx 2.6$  due to the temperature-dependent material degradation effect, while an unheated panel will exhibit LCOs at  $M_\infty > 6.6$  (linear flutter Mach number). In addition, in the case of heated panels, LCOs with large amplitude are present, as compared to the case of an unheated panel, and are growing at faster rate.

## **5** Conclusions

Supersonic/hypersonic flutter behavior of infinitely long two-dimensional curved panels has been investigated using Galerkin method. In this context, aerodynamic and structural non-linearities are considered. Numerical studies of the aero-thermo-elastic system parameters including curvature ratio, panel thickness ratio, mass ratio, movability of panel edges, geometric imperfections, and structural damping are conducted to examine the effect of a high-temperature field with thermal degradation of thermoelastic characteristics of the material on the flutter characteristics. It is concluded that a severe reduction in the eigenfrequency and flutter boundary



Fig. 10 Bifurcation diagram of the aero-thermo-elastic curved panel with respect to the variation of flight Mach number (without/ with thermal field and thermal degradation)

will occur when the temperature field leads to a thermal degradation of the elastic modulus. This is more significant than the effect produced by the thermal degradation of the thermal expansion coefficient.

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