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Contents

1. Introduction.

Most steel constructions such as cranes, ships, bridges and building frames experience compressive loads and are thus susceptible to buckling. Buckling of construction members is usually a very sudden process without much warning in advance. Therefore it is important to account for the effect of buckling in the design of a construction.

Buckling analysis can be performed by Finite Element Method(FEM) packages. However, these analyses usually cost a lot of engineering time. This because for the analysis to make any sense, it must often be done on individual plate fields. This brings a lot of uncertainties about boundary conditions into the calculation. Therefore buckling analysis in FEM is very labour intensive and costly.

A much simpler way to check for buckling is by means of a standard, such as Eurocode 3(EC3). Engineers use standards to validate whether the dimensions of a design will be able to resist all loads acting on a structure. These standards are made up from mathematical models that describe buckling combined with empirical data from real life experiments. The recommendations ensure, when followed correctly, that a structure will be strong enough to resist certain applied loads.

Buckling analysis would become less time consuming when the stress results from an FEM analysis can be compared to these standards. However, often there is a discrepancy between FEM results and input parameters used in standard recommendations.

This gap between FEM results and standard input parameters has been researched[1]. The aim of the research was to convert FEM results into design factors that were needed to check for buckling of plated structures. Two different standards were chosen for an in-depth analysis. The American Bureau of Shipping (ABS) guide for buckling assessment for offshore structures, and Det Norske Veritas (DNV) recommended practices for buckling strength of plated structures. These standards for offshore and ship building were chosen because of the common use of plated structures in these industries.

This research will make an analysis of the buckling recommendations as presented in EC3. This because of the broad use of this standard in construction. In the first part of this research, the theory behind recommendations from EC3 will be reviewed. These include beam buckling, plate buckling, inelastic buckling and also the effect of shear lag, which can be observed in thin-walled members loaded in bending.

The second part will look at the recommendations in EC3 themselves. Recommendations from two volumes will be reviewed.

- NEN-EN 1993-1-1 Design of steel structures – Part 1-1: General rules and rules for buildings
- NEN-EN 1993-1-5 Design of steel structures – Part 1-5: Plated structural elements

The first volume will give recommendations for buckling of columns loaded in compression and bending as well as for build-up construction members. The second volume is concerned with recommendations for buckling of plated structures. Furthermore, the recommendations are presented for the critical length, shear lag an inclusion of imperfections.

[1] Aberkrom, B: Defining parameters for buckling checks of plated structures in finite element software packages. Delft, 2014.

In the final part a comparison will be made between two different standards for plate buckling. The DNV recommended practices have been used in order to evaluate buckling results from finite element software packages. In order to safe up on computing and modelling time, FEM models with beam elements would be best. However, to deal with transverse stresses, more complicated plate elements are required. Since the EC3 recommendations deal with plate buckling without the use of transverse stresses, a comparison between the two standards is made.

2. Theory Of Buckling

The buckling recommendations made in EC3 are a combination of mathematical models and factors from empirical data. The theory behind the mathematical models are presented in this part. First, beam buckling is discussed (2.1), with the concept of stability, Euler buckling, torsional buckling and lateral torsional buckling due to bending forces. Next plate buckling is discussed (2.2). Also the ideas behind Inelastic buckling (2.3) and shear lag (2.4) are presented in this part.

2.1. Beam Buckling

Beams subjected to compressive loadings have the tendency to deflect laterally. This lateral deflection is called buckling. A beam will fail under the influence of compressive loadings when the stresses caused by lateral deflection are greater than the materials yield strength. Failure due to buckling is often a process without much warning. Therefore it is required to give special attention to buckling in the design process of beams and columns.

This chapter will discuss a couple of theoretical subject concerning buckling. First the stability of a beam under influence of compressive loadings will be discussed. The stability will dictate the load before a beam will buckle. This load is discussed in Euler buckling. Euler buckling is only concerned with elastic buckling. Therefore the subject of inelastic buckling is discussed next. Finally this chapter will give a discussion about torsional buckling and warping.

2.1.1. Stability

Failure of a beam due to buckling can be a sudden process without warning. A beam transforms from a stable equilibrium to an unstable equilibrium with increasing compressive load. The point where stable will turn to unstable is called the bifurcation point and is depicted in the following figure.

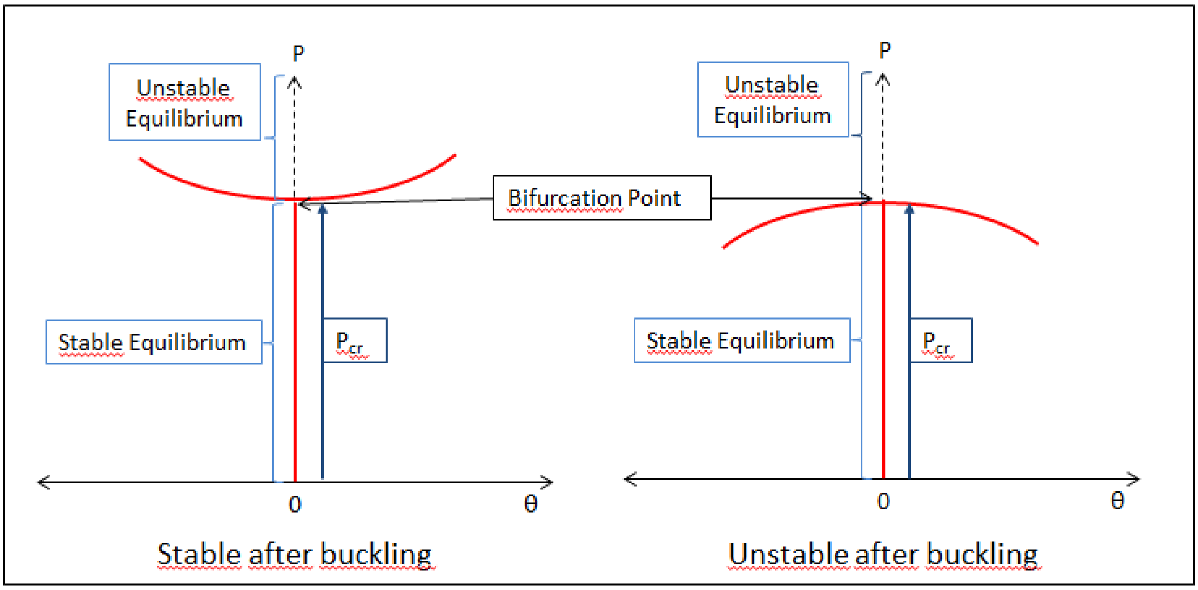


Figure 2.1 Bifurcation point with a stable path after buckling (left) and an unstable path after buckling (right)

After this bifurcation point, a new path is followed. This new path determines the failure of a member with respect to buckling. The left figure allows for an increase in load after buckling. This indicates that despite plastic deformation of a beam, it will not fail completely. The right figure shows an unstable path that cannot even resist the critical load. Such a beam will fail completely due to buckling.

The load that belongs to this tipping point is called the critical load. This critical load can be explained by considering the following mechanism.

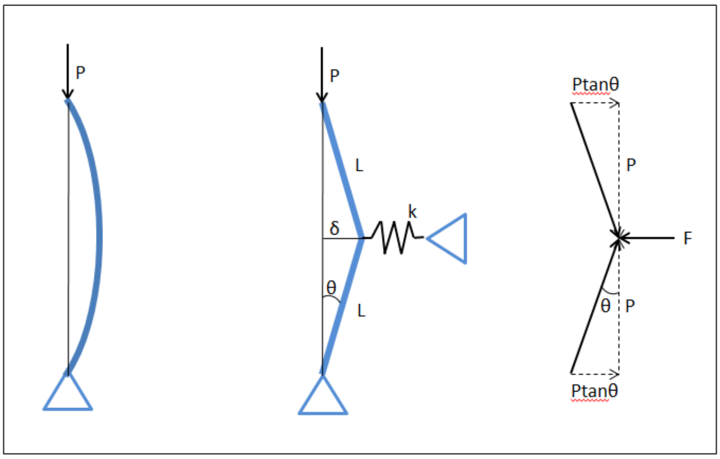


Figure 2.2 Buckling mechanism represented by two bars and a spring

The critical load is that load that will bring the *disturbing* force P in equilibrium with the *restoring* force F. The *disturbing* force P is related to the external loads acting on the beam while the *restoring* force F is related to the bending stiffness of the beam.

2.1.2. Euler Buckling

The Swiss mathematician Leonard Euler was the first to solve the linear buckling problem for ideal columns in 1757. The ideal column considered to solve this problems has the following properties.

- Initially perfectly straight
- Made of homogeneous material
- Material behaves linear-elastic
- External load is applied exactly through the cross section centroid
- Buckling will occur only in a single plane

As mentioned under stability, a column will remain stable when internal resistance is greater than external compressive forces. The internal resistance is related to bending, where the external bending moment is caused by the external load and a small deflection.

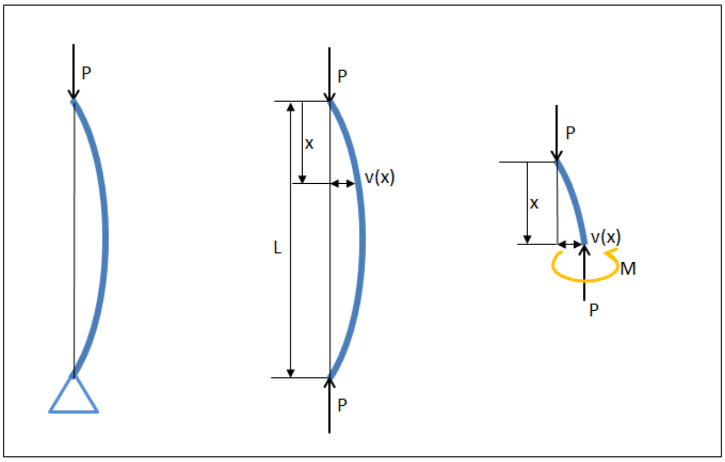


Figure 2.3 Internal forces describing Euler buckling

This leads to the following differential equation by which this buckling problem can be described.

$$\frac{d^2v(x)}{dx^2} + \frac{P}{EI}v(x) = 0 \tag{2.1}$$

Solving this differential equation will provide a set of critical loads belonging to certain buckling shapes. The lowest value, belonging to a half sine wave buckling shape, is of interest for buckling failure, since this is the lowest value for which a column will fail due to buckling.

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{2.2}$$

The strength of columns is usually represented by stresses. Therefore in column design, the *radius of gyration* is introduced, which is defined as

$$r = \sqrt{{}^{I}/_{A}} \tag{2.3}$$

With the radius of gyration, the critical load is transformed into the critical stress.

$$\sigma_{cr} = \frac{\pi^2 E}{(L/r)^2} \tag{2.4}$$

The boundary conditions of a column do influence the value of the buckling load. A column that is considered to be fixed at both ends is able to resist more load than a column which is pinned at both ends. This difference is indicated by the effective column length.

The effective length is defined as the column length between two points of zero moment. Therefore every column can be considered as being pinned at both ends. Other boundary conditions are accounted for by introducing the effective length factor K. The value of this factor can be seen for different boundary conditions in the following figure.

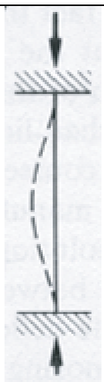
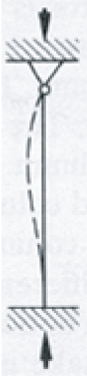
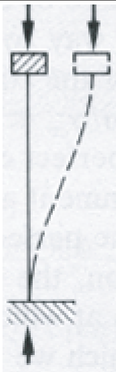
PINNED-PINNED	FIXED-FIXED	FIXED-PINNED	FIXED-FIXED	FIXED-FREE
				
$K = 1$	$K = 0.5$	$K = 0.7$	$K = 1$	$K = 2$

Figure 2.4 Effective length factor K for different boundary conditions

The denominator part in the critical stress formula is called the slenderness ratio.

$$\lambda^2 = \left(\frac{KL}{r}\right)^2 \tag{2.5}$$

This slenderness ratio is most often used to represent a buckling curve. This curve shows critical stress for a column with certain slenderness. The Euler Buckling Curve is represented by the red dotted line in Figure 2.12

2.1.3. Torsional Buckling

It is possible for some thin-walled bars to buckle under the influence of axial compression while its longitudinal axis remain straight. This is called torsional buckling. In particular columns with wide flanges and short lengths are sensitive to this kind of buckling.

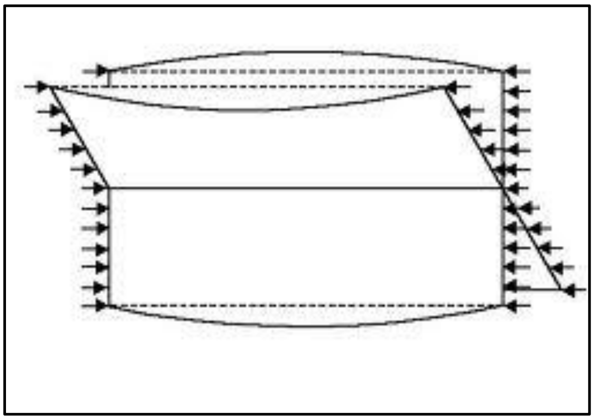


Figure 2.5 Torsional buckling due to compressive load

Torsional buckling is a process where an axial compression force exceeds the resistance of a columns cross section to torsion. This resistance to torsion is build up from two parts. The first being the resistance to shear stresses imposed by pure torsion. These shear stresses are proportional to change of twist angle over the length of a beam loaded in torsion, factorized by the product of shearing modulus of elasticity (G) and the St.Venant torsional constant (J).

The second part is due to lateral deflection of the flanges. This is also known as warping of the stiffener. The lateral deflection increase the torsional moment of the column and is thus proportional to the third order change of twist angle over the length of a beam, and factorized with the warping rigidity. This rigidity is the product of modulus of elasticity (E) and the warping constant (C_w).

These two parts lead to a differential equation for non-uniform torsion.

$$T = T_1 + T_2 = GJ\frac{d\varphi}{dz} - EC_w\frac{d^3\varphi}{dz^3} \tag{2.6}$$

To link the differential equation for torsion to compression loads in the flanges a pin ended strut model is used. Bending moments created by compressive loads about the longitudinal axis represent torsion forces on the column. The torque per unit length is expressed as

$$t_z = -\sigma I_0 C_1\frac{d^2\varphi}{dz^2} \tag{2.7}$$

Where I_0 is the polar moment of inertia of the cross section about the shear centre.

These two expressions for torsion and torque can made into a differential equation that describes the problem of torsional buckling. This differential equation can be solved for different boundary condistions

For a simply supported column, which cannot displace but is free to warp at the ends, the critical compressive stress will be

$$\sigma_{cr} = \frac{GJ}{I_0} + \frac{\pi^2 E C_w}{I_0 l^2} \tag{2.8}$$

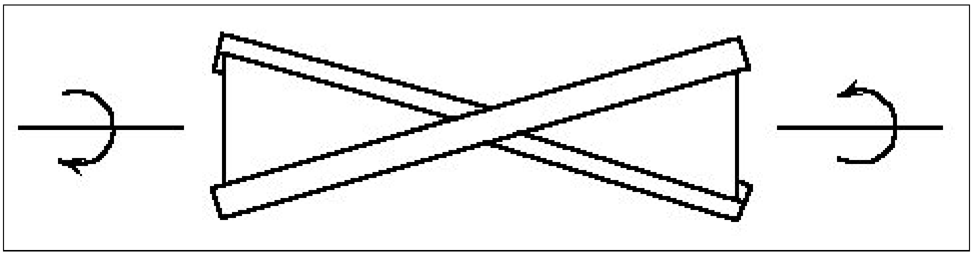


Figure 2.6 Torsional buckling for simply supported column

For a build-in column, which cannot displace and is restricted to warp at the ends, the critical compressive stress will be

$$\sigma_{cr} = \frac{GJ}{I_0} + 4\frac{\pi^2 E C_w}{I_0 l^2} \tag{2.9}$$

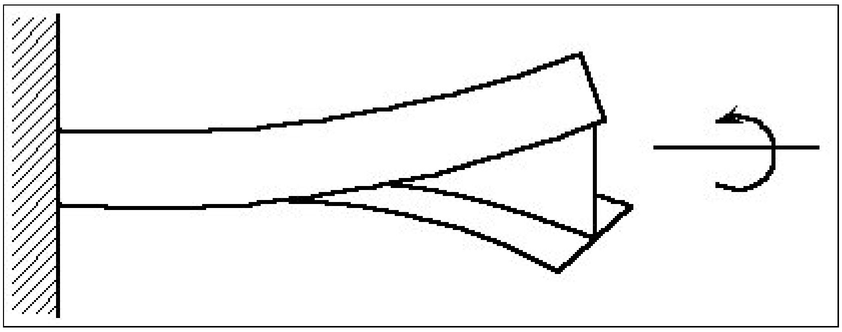


Figure 2.7 Torsional buckling with warping restricted at one side

This shows that the torsional strength of a column will increase when the ends are restricted to warp. Considering a simply supported column is therefore a conservative choice.

2.1.4. Lateral Torsional Buckling

A beam loaded in pure bending can upon reaching a critical value of load buckle laterally. Especially for beams without lateral support, and for which the flexural rigidity (against normal beam buckling) is larger compared to the lateral bending rigidity.

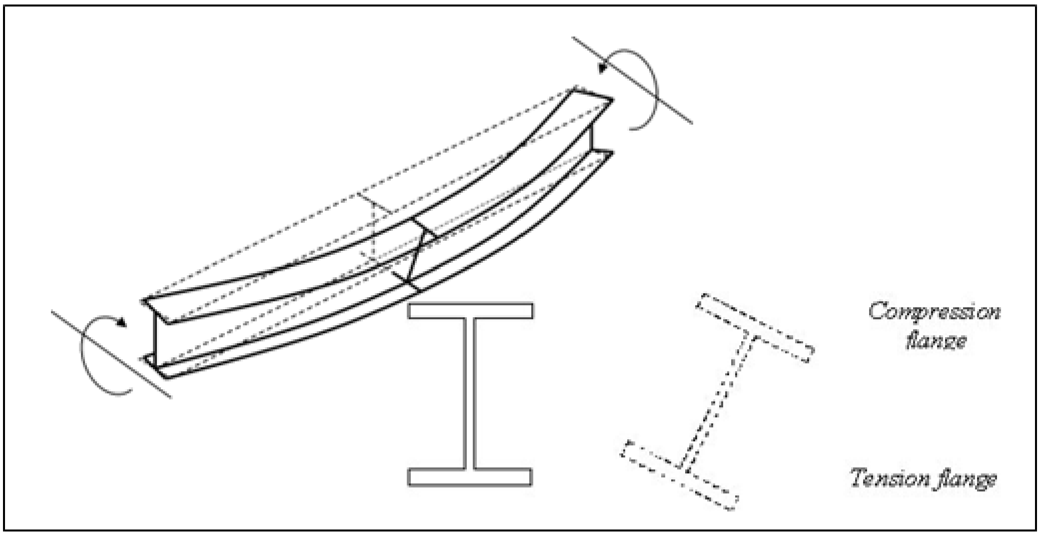


Figure 2.8 Lateral torsional buckling due to bending

This type of buckling is called lateral torsional buckling. This because, as can be seen in Figure 2.8, due to bending, both lateral displacement as well as torsional displacement act simultaneously. However, there is no warping of the cross section.

This type of buckling is due to the difference in stress distributions due to bending. This distribution will cause one flange to be loaded in compression while the other flange is loaded in tension. Failure will occur because the compression flange fails due to buckling.

By preventing lateral displacement of the compression flange, the resistance to lateral torsional buckling can be increased.

2.2.Plate Buckling

Plate buckling can be regarded as a special case of beam buckling. A plate can be regarded as multiple connected beams. When these beams are loaded in compression, they will show the same behaviour with regard to buckling than a single beam does. There are however some differences, as can be seen in the equation for plate buckling load.

$$N_{cr} = K \frac{\pi^2 E t^3}{12b(1-\nu^2)} \tag{2.10}$$

First of all, Poisons ratio will cause the beams to expand laterally. These lateral expansions are prevented by each neighbouring beam in the multiple beam plate model. This has a strengthening effect on the plate since more force is required in order to get the same deformations as for beams.

Another effect has to do with the plate aspect ratio. Wide plates, with relative low aspect ratios, is more resistant against buckling than long plates, with relative high aspect ratios. This effect is represented by the buckling coefficient K. This coefficient is determined by the plates aspect ratio and the number of half-sine buckling waves.

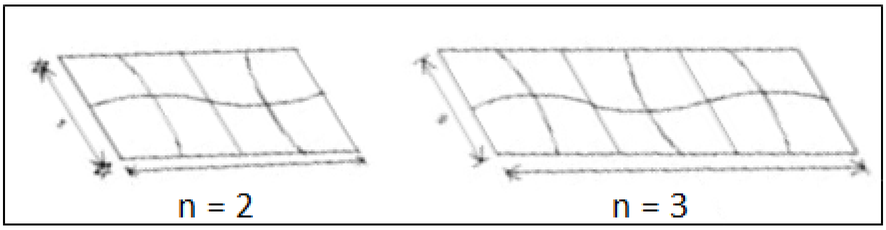


Figure 2.9 Number of half-waves in a buckled plate

Figure 2.10 shows the buckling coefficient for a plate simply supported at all four edges. A special point can be seen at the aspect ratio of $\sqrt{2}$. Here the buckled plate will step from a single half sine wave buckling form, to a double half sine wave buckling form. Also can be seen that for larger aspect ratios, the buckling coefficient will asymptote to 4, which can consequently be regarded as the minimum buckling coefficient for such plates.

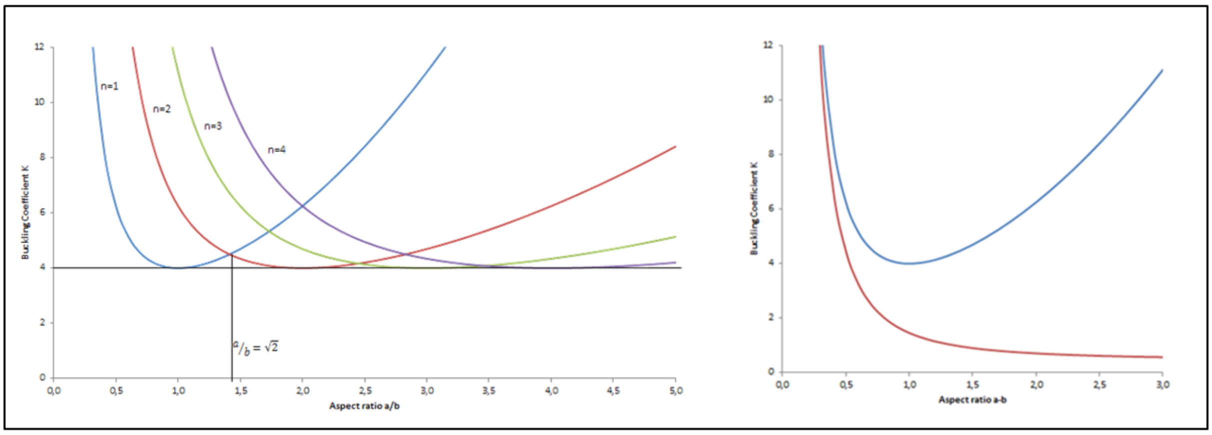


Figure 2.10 Buckling coefficients for a plate simply supported at all edges (left). Difference between buckling coefficients for a plate simply supported on all edges (right blue) and free on one side (right red)

The buckling coefficient depends on the boundary conditions of a plate loaded in compression. A plate which is simply supported on all four sides will have a better buckling coefficient than a plate which has one free edge as can be seen in Figure 2.10. The buckling coefficient is further influenced by the way that load is applied. A plate loaded in compression will yield a different coefficient than a plate loaded in bending.

On more difference between plate and beam buckling is the post buckling stress. Most beams will fail completely after the critical compression load is reached. For plates this is different. Plates are commonly supported at all four edges. Due to these supports, the plate will not fail completely after reaching critical buckling load. After buckling of the middle part of the plate, the edges will be able to resist the compression forces until the materials yield strength is reached. This results in a non-uniform stress distribution as can be seen on the left in Figure 2.11. This non-uniform stress distribution can make it rather difficult to further evaluate the plate load bearing capacity. Therefore the effective width method is often used. This method assumes that the deformed centre of the plate will no longer resist any of the compressive stresses. Instead the stress distribution over the entire width is replaced with an equivalent continues stress over an effective width of the plate. This is shown on the right side in Figure 2.11

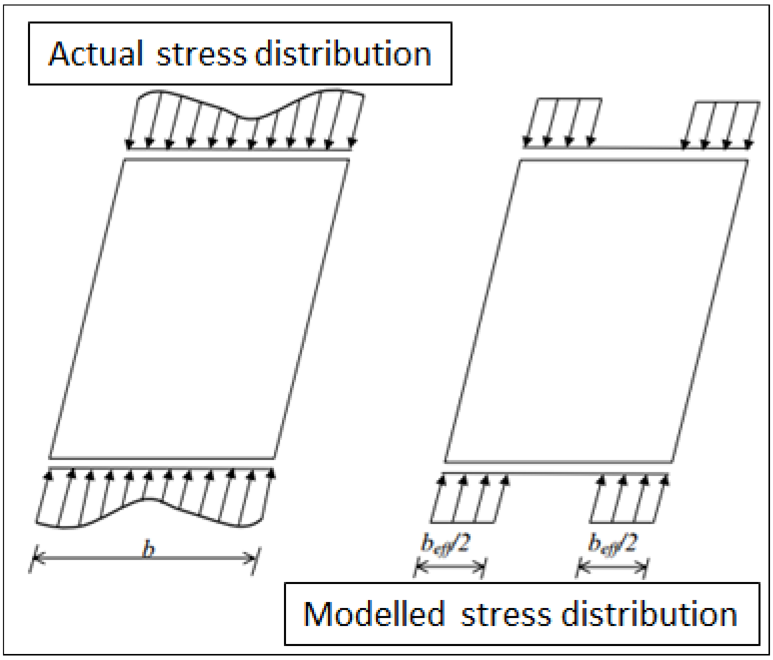


Figure 2.11 The effective width method

2.3.Inelastic Buckling

Columns will buckle when a critical buckling stress is reached. When this stress is in the elastic range of the material, it is called elastic buckling. Most long and slender columns will tend to buckle in this stress range. However, when a column is short and stocky, the critical buckling stress may be greater than the yield stress of the material. In this region the material no longer behaves elastically. Therefore buckling of such regions is called inelastic buckling.

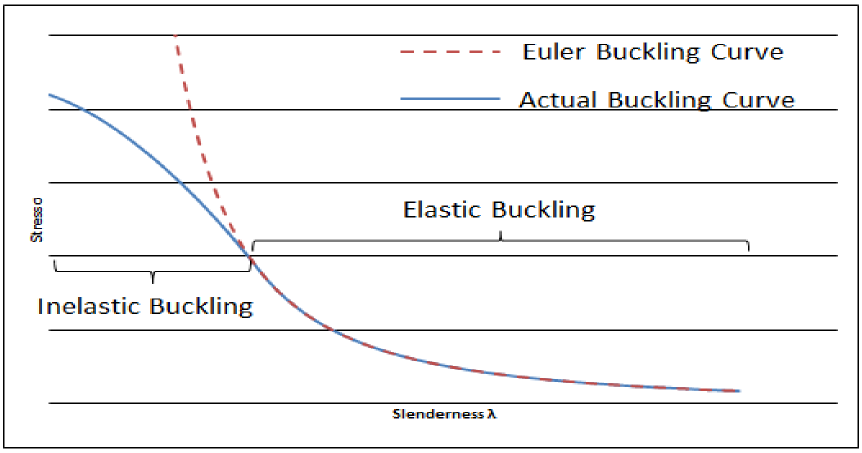


Figure 2.12 Inelastic Buckling Curve

Inelastic buckling of columns is only due to material yielding. Therefore it has a much less predictable shape than elastic buckling has.

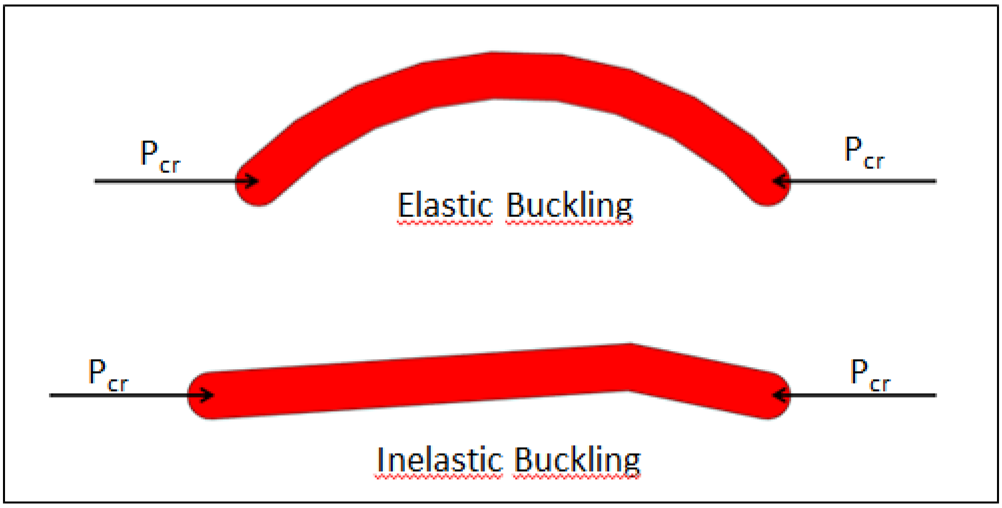


Figure 2.13 Difference in buckling shape for elastic and inelastic buckling

2.4.Shear Lag

In normal linear elastic analysis, plane sections are assumed to remain in shape after bending. This assumption works well for solid columns, however for thin-walled columns, this is not entirely true. Bending forces are normally introduced to a column by means of vertical loads, instead of pure couple. These vertical loads are transferred by the webs to act on the flanges as bending loads. However, they are transferred by shear. Since the connection between web plating and flange has a higher capacity to resist these shear forces than the middle of the flange. This results in a non-uniform stress distribution in the flange plate as can be seen in Figure 2.14.

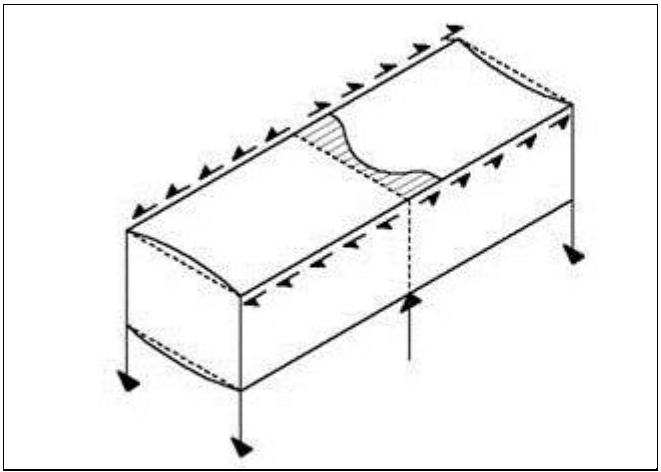


Figure 2.14 Non-uniform stress distribution in flange plate due to transfer of shear forces by web plate.

The shear stresses introduced to the flange by the web plates, combined with the bending forces, will cause the flange plate to deform. Higher deformations can be seen at the connection between web and flange plate than in the middle of the flange. This phenomenon of difference in-plane distortion is referred to as shear lag.

Shear lag is only observed in thin-walled columns where bending forces result from vertical loads. Columns loaded in pure bending will not show the shear lag effect. This because the stress distribution of a pure bending couple will act directly on the flange plate, instead of being transferred by web to flange plate.

There are a number of variables that influence the magnitude of shear lag. These are, plate aspect ratio, distribution of lateral forces, relative proportions of web and flange and stiffener types. Generally, beams with very wide flanges and shallow webs (e.g. aircraft wings) are very susceptible to shear lag. On box girders, the influence of shear lag is usually very low.

3. Buckling Of Steel Structures In Eurocode 3

As mentioned before, the theory of elastic buckling combined with factors from empirical data is transformed into recommendations. The recommendations from EC3 are presented in this part. Two standards are used, that both describe recommendations for the design of steel structures.

- NEN-EN 1993-1-1 Design of steel structures – Part 1-1: General rules and rules for buildings
- NEN-EN 1993-1-5 Design of steel structures – Part 1-5: Plated structural elements

The recommendations in EC3 are very general because it is a European standard. Each country following the EC3 standard may have its own recommendations for certain variables presented in the standard. These are presented in the national annex to the EC3 for each member state. Whenever a reverence to a national annex is made in this research, it is to the Dutch national annexes.

First of all, the differences in cross-section classifications(3.1) is presented, which will yield different results in both column and plate buckling. In the next chapter the recommendations are presented for uniform members in compression (3.2). These contain member stability, buckling curve and relative slenderness. Following is a description for uniform members loaded in bending (3.3). Next are the recommendations for build-up columns (3.4). These columns are still treated as single columns instead of regarding the plates individually. Plate buckling in EC3 is discussed in the next chapter (3.5).

Important for the analysis of both column and plate buckling is the critical length (3.6) of the member. Therefore it is discussed in a separate chapter. The effects of shear lag (3.7) and imperfections (3.8) on buckling strength are also presented. Finally recommendations in Eurocode 3 for the use of FEM (3.9) are presented as well.

3.1. Cross-Section Classification

Slender columns are more susceptible to local buckling than stocky ones. Slender columns will therefore collapse before even the design strength is reached. A cross-section classification is made in Eurocode 3 to indicate the influence of local buckling to the cross-sectional resistance and rotational capacity.

A cross-section is classified according to the highest class of its compression parts.

From NEN-EN 1993-1-1 5.5.2,

Class 1 Plastic cross-sections. Cross-sections are those which can form a plastic hinge with the rotation capacity required from plastic analysis without reduction of the resistance.

Class 2 Compact cross-sections. Cross-sections are those which can develop their plastic moment resistance, but have limited rotational capacity because of local buckling.

Class 3 Semi-compact cross-sections. Cross-sections are those in which the stress in the extreme compression fibre of the steel member assuming an elastic distribution of stresses can reach the yield strength, but local buckling is liable to prevent development of the plastic moment resistance.

Class 4 Slender cross-sections. Cross-sections are those in which local buckling will occur before the attainment of yield stress in one or more parts of the cross section.

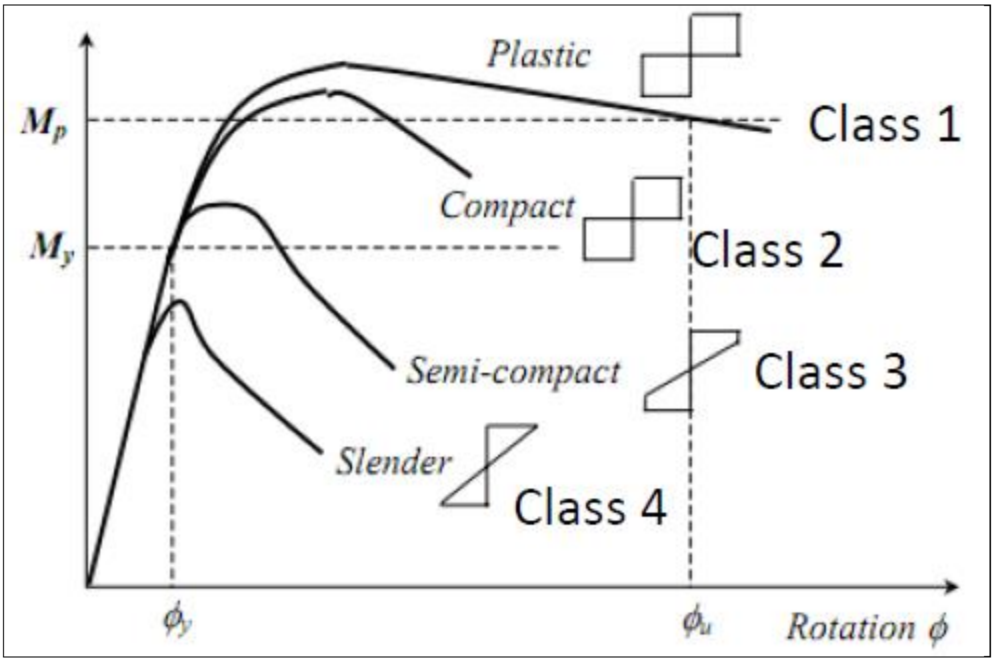


Figure 3.1 Different cross-section classes used in Eurocode 3

As can be seen in Figure 3.1, class 1 cross-sections are able to resist the highest plastic moment (M_p) and will collapse after the ultimate strength rotation capacity (ϕ_u) is reached. Compact class cross-sections can fully develop into a plastic hinge, but has not enough rotational capacity to reach the materials ultimate strength. The Semi-compact class is unable to fully develop a plastic hinge before it will collapse due to local buckling. However the materials yield strength is reached before collapse. Class 4 cross-sections are not able to resist the yield strength before failing due to local buckling.

Because the first three classes are able to reach yield strength before collapsing a distinction is made between these three and class 4. This is represented in Eurocode 3 by the use of the net cross-sectional area for class 1, 2 and 3 and an effective cross section (which is smaller) for class 4. Following is an extract showing the varies ratios for pure compression and pure bending of internal compression parts (Figure 3.2) and outstand flanges (Figure 3.3).

			Compression	Bending
Internal compression parts	Class 1	$c/t \leq$	33 ε	72 ε
	Class 2	$c/t \leq$	38 ε	83 ε
	Class 3	$c/t \leq$	42 ε	124 ε
Outstand flanges	Class 1	$c/t \leq$	9 ε	
	Class 2	$c/t \leq$	10 ε	
	Class 3	$c/t \leq$	14 ε	

f_y	235	275	355	420	460
ε	1.00	0.92	0.81	0.75	0.71

Table 1 Extract from Table 5.2 of NEN-EN 1993-1-1. Width to thickness ratios determining the cross-section class

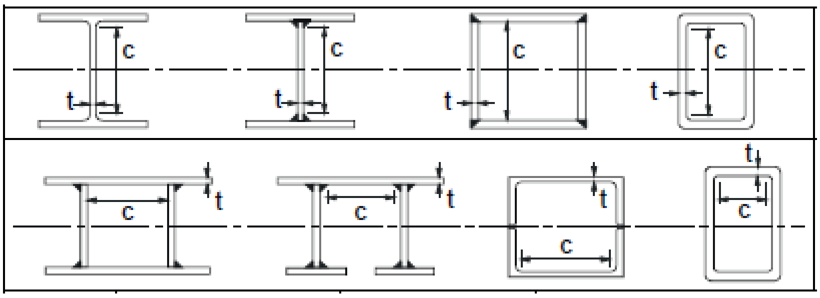


Figure 3.2 Configurations and parameters of internal compression parts

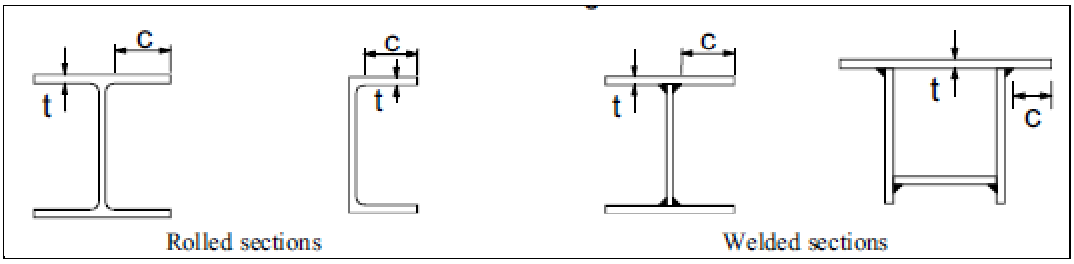


Figure 3.3 Configuration and parameters of outstand flanges

3.2. Uniform Members In Compression

3.2.1. Member Stability

Member stability regulations are prescribed for EC3 in the NEN-EN 1993-1-1 Design of steel structures. Chapter 6.3 of this part describes the criteria for beam buckling stability. It starts with a simple criteria.

6.46 $$\frac{N_{Ed}}{N_{b,Rd}} \leq 1{,}0 \qquad (3.1)$$

This relation insures that the acting compressive force on a member (N_{Ed}) will not exceed the buckling resistance belonging to the cross section of that beam ($N_{b,Rd}$).

The buckling resistance $N_{b,Rd}$ can be calculated for two different situations. The first for class 1, 2 and 3 cross sections and in the second situation for class 4 cross sections. The difference in cross section class will be explained in chapter 3.1, but the main difference is the use of member cross section.

For class 1, 2 and 3 cross sections, the cross section (A) has to be used, while for class 4 cross sections the cross section is replaced by an effective cross section (A_{eff}).

6.47 $$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}} \qquad (3.2)$$

Besides the cross section and materials yield strength is the reduction factor χ incorporated in the beam buckling resistance. This reduction factor is related to a buckling curve and appropriate buckling shape.

The partial factor γ_{M1} relates to the resistance of elements tested for stability. This can be regarded as a safety factor and is in most cases by default taken as 1.

3.2.2. Buckling Curve

As mentioned above, buckling curves are represented is the buckling check by a reduction factor χ. In EC3 there are five different buckling curves that can be used along with the relative slenderness of the member, in order to get the proper value for χ.

These five different buckling curves are shown in Figure 3.5. Each buckling curve is build up from the same equation.

6.49 $$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} \qquad (3.3)$$

The curves are limited to a reduction factor of χ = 1.0. This represents the effect of inelastic buckling as described in chapter 0. The difference in buckling curves comes from an imperfection factor α that is used in the shape function Φ.

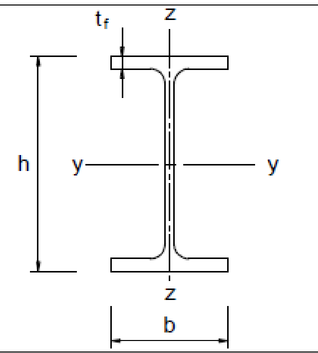
6.49 $$\Phi = 0{,}5\left[1 + \alpha(\bar{\lambda} - 0{,}2) + \bar{\lambda}^2\right] \qquad (3.4)$$

The imperfection values used in this shape function to create the buckling curves a_0 to d, are shown in the following table.

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

Table 2 Imperfection factors used in the shape function Φ to create buckling curves a_0 to d. NEN-EN 1993-1-1-6.3.1.2

The proper buckling curve can be selected with the aid of Table 6.2 in the EC3.



h/b > 1,2	$t_f \le 40$ mm	y – y	a	a_0
		z – z	b	a_0
	40 mm < $t_f \le 100$ mm	y – y	b	a
		z – z	c	a
h/b ≤ 1,2	$t_f \le 100$ mm	y – y	b	a
		z – z	c	a
	$t_f > 100$ mm	y – y	d	c
		z – z	d	c

Figure 3.4 Part of NEN-EN 1993-1-1-6.3.1.2 Table 6.2. Here limitation are given for selecting the proper buckling curve for pressed I-beams. The last two columns indicate which 'best' buckling curve can be used.

The last two columns in Figure 3.4 indicate which buckling curve can be used for this type of cross section. The difference between the two columns is the material type used. The first column represents buckling curves for lower strength steel (S235, S275, S355 and S420). The second column represents the buckling curve that can be taken for higher strength steel (S460).

In order to get the highest allowable compressive force (N_{Ed}), the buckling resistance needs to be as high as possible. Therefore the reduction factor χ needs to be as high as possible, which requires a low imperfection factor α. The best buckling curve therefore is a_0, while curve d gives the poorest performance. This effect can be seen again in Figure 3.4. Here tall cross sections (h/b>1,2) of higher strength steel (S460), which have a high bending resistance, follow the 'best' a_0 buckling curve. However wide cross sections (h/b<1,2), of lower strength steel (S235) and thick flanges (t>100mm), follow the 'worst' buckling curve d.

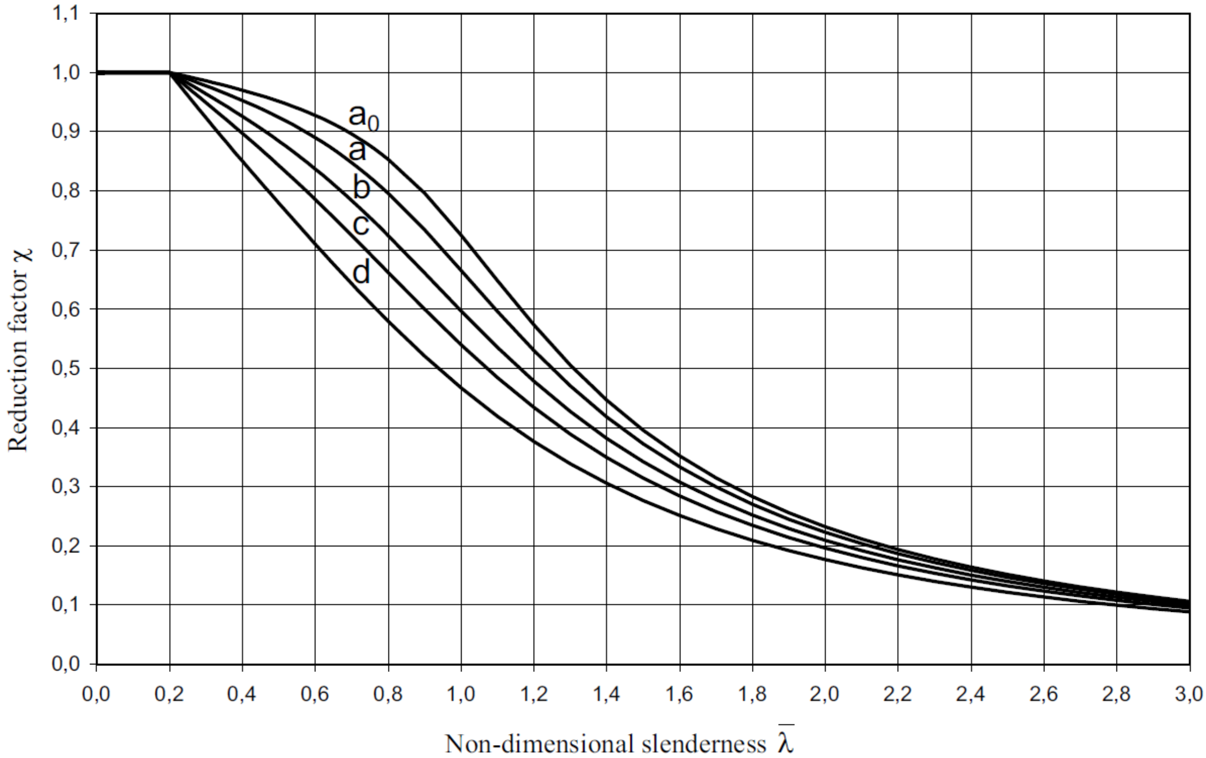


Figure 3.5 Buckling curves as represented in NEN-EN 1993-1-1-6.3.1.2 Figure 6.4

3.2.3. Relative Slenderness

The reduction factor χ is related by the shape function Φ to the relative slenderness $\bar{\lambda}$. Normally the slenderness of a member is a ratio between its critical length and radius of gyration. However in EC3 the material strength is incorporated into the equation as well. Therefore the relative slenderness becomes a ratio between the material yield strength and the critical buckling stress. The relative slenderness is defined by the following equation.

6.50

$$\bar{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} = \frac{L_{cr}}{i}\frac{1}{\lambda_1} \qquad (3.5)$$

Again for cross section classes 1, 2 and 3 the cross section has to be taken as for cross section class 4 the effective cross section has to be taken.

This relative slenderness is the normal slenderness, combined with a material factor λ_1 that relates the elastic modulus, which is the same for all steel materials, to the materials yield strength.

Transformation from the normal slenderness ratio into the relative slenderness ratio is done with the aid of a material factor $\lambda_{1.}$

$$\frac{N_{cr}}{A} = \frac{\pi^2 E}{\lambda^2}$$

$$\bar{\lambda}^2 = \frac{Af_y}{N_{cr}} = \frac{\lambda^2 f_y}{\pi^2 E} = \frac{L_{cr}^2}{i^2}\frac{f_y}{\pi^2 E} = \frac{L_{cr}^2}{i^2}\frac{1}{\lambda_1^2}$$

$$\lambda_1 = \pi\sqrt{\frac{E}{f_y}} \qquad (3.6)$$

The relative slenderness as represented in equation (1.4) is to be used for normal beam buckling. That is to say buckling related to the internal bending moment resistance of the cross section. For torsional buckling EC3 gives another relative slenderness, $\bar{\lambda}_T$.

6.52 $$\bar{\lambda}_T = \sqrt{\frac{Af_y}{N_{cr,TF}}} \quad (3.7)$$

As can be seen, the only difference with the relative slenderness for bending is the use of a different critical compressive force. This $N_{cr,TF}$ is the critical elastic torsional buckling force. In EC3 there are no equations prescribed to calculate the values of $N_{cr,TF}$. In chapter 2.1.3, equations are shown to calculate these critical elastic torsional buckling forces.

The buckling curves show a cut-off point for a relative slenderness value of 0.2. This is because EC3 considers members with a relative slenderness 0.2 or less to be insensitive to buckling. Such members will fail due to yielding of material before buckling will occur. Also for compressive loads that are equal or less than 4% of the critical buckling load can buckling be neglected.

3.3.Uniform Members In Bending

3.3.1. Member Stability

Members loaded in bending will normally have a flange loaded in compression and one loaded in tension. A flange loaded in tension is, of course, unsusceptible for buckling. Therefore buckling under influence of bending is governed by the compression flange. Also a laterally supported member will be unable to buckle because the buckling shape will be restricted.

In EC3, again, a simple criteria is formulated to check for lateral-torsional buckling.

6.54 $$\frac{M_{Ed}}{M_{b,Rd}} \leq 1{,}0 \quad (3.8)$$

This ratio prevents the acting major axis bending moment (M_{Ed}) from exceeding the design buckling resistance moment of the member loaded in bending. As mentioned before, a laterally restraint member will not be sensitive to lateral-torsional bending and therefore has not to be checked. The same is true for members with certain cross-sections, that are designed to withstand torsion, like rectangular hollow sections (RHS), circular hollow sections (CHS) or even fabricated circular tubes and square boxes.

The buckling resistance moment is again calculated for different cross section classes as explained in chapter 3.1.

6.55 $$M_{b,Rd} = \frac{\chi_{LT} W_y f_y}{\gamma_{M1}} \quad (3.9)$$

The appropriate section modulus W_y depends on the cross section class in the following way.

$W_y = W_{pl,y}$	Plastic section modulus	Cross section Class 1 and 2
$W_y = W_{el,y}$	Elastic section modulus	Cross section Class 3
$W_y = W_{eff,y}$	Effective section modulus	Cross section Class 4

3.3.2. Buckling Curve

The design buckling moment is related to buckling curves by means of reduction factor χ_{LT}. EC3 has four different buckling curves for lateral-torsional buckling. Each buckling curve is related to the relative slenderness of the member by means of an imperfection factor. The four buckling curves for lateral-torsional buckling are the curves a, b, c and d in Figure 3.4.

Generally, the buckling curves are represented by the following equation.

6.56 $$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{{\Phi_{LT}}^2 - \bar{\lambda}_{LT}^2}} \quad (3.10)$$

This curve is, like the compression member buckling curve limited to $\chi_{LT} = 1.0$ to represent inelastic buckling effects. However, for rolled sections or equivalent welded sections, other buckling curves must be used.

6.57 $$\chi_{LT} = \frac{1}{\Phi_{LT} + \sqrt{{\Phi_{LT}}^2 - \beta\bar{\lambda}_{LT}^2}} \qquad (3.11)$$

These curves are not only limited to $\chi_{LT} \ = \ 1.0$, but also to $\chi_{LT} = 1/\bar{\lambda}_{LT}^{2}$. The value of β is regulated in the National Annex, which is $\beta = 0.75$ in the Dutch National Annex.

Cross-section	Limits	Buckling curve
Rolled I-sections	h/b ≤ 2 h/b > 2	a b
Welded I-sections	h/b ≤ 2 h/b > 2	c d
Other cross-sections	-	d

Figure 3.6 Selecting the proper buckling curve for members loaded in bending

The reduction factor χ_{LT} can further be modified to account for the fact that the bending moment along the member does not need to be constant. By means of a correction factor k_c the reduction factor is altered to allow for 8 different types of moment distribution.

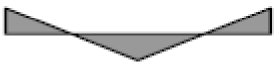
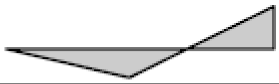
Moment distribution	k_c
 $\psi = 1$	1,0
 $-1 \leq \psi \leq 1$	$\frac{1}{1,33 - 0,33\psi}$
	0,94
	0,90
	0,91
	0,86
	0,77
	0,82

Figure 3.7 Correction factor to account for different moment distributions

3.3.3. Relative slenderness

The relative slenderness of a member used to check for lateral-torsional buckling incorporates the strength of the material by means of its yield strength.

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} \tag{3.12}$$

Here the elastic critical moment for lateral-torsional buckling is included as well. To determine the value of M_{cr}, EC3 refers to the National Annex again. In the Dutch National Annex, a rather large section is dedicated to the exact calculation of M_{cr}. In its most general form, however, the elastic critical moment can be calculated in the following way.

NB.148

$$M_{cr} = k_{red} \frac{C}{L_g} \sqrt{EI_z GI_t} \tag{3.13}$$

The proper value for W_y is again determined by the cross section class, as mentioned earlier.

The effect of lateral-torsional buckling can be neglected for members with relative slenderness of 0.4. Also for bending moments equal to or less than 16% of the elastic critical moment, can lateral-torsional buckling be neglected.

Members loaded in bending, which do have lateral restraint to the compression flanges, are unsusceptible to buckling if the relative slenderness between two consecutive lateral restraints satisfies to following criteria.

6.59

$$\bar{\lambda}_f = \frac{k_c L_c}{i_{f,x} \lambda_1} \leq \bar{\lambda}_{c,0} \frac{M_{c,Rd}}{M_{y,Ed}} \tag{3.14}$$

As can be seen in this criteria, a critical length is introduced again. This is the length between lateral supports for which no lateral-torsional buckling can occur. More on the critical length used in EC3 is discussed in chapter 0.

3.4.Build-Up Compression Members

3.4.1. Build-Up Members

In EC3 a distinction is made between load bearing members and construction members. The main members in a build-up construction are the load bearing chords. These chords are connected by means of construction members, which can be lacing or battening.

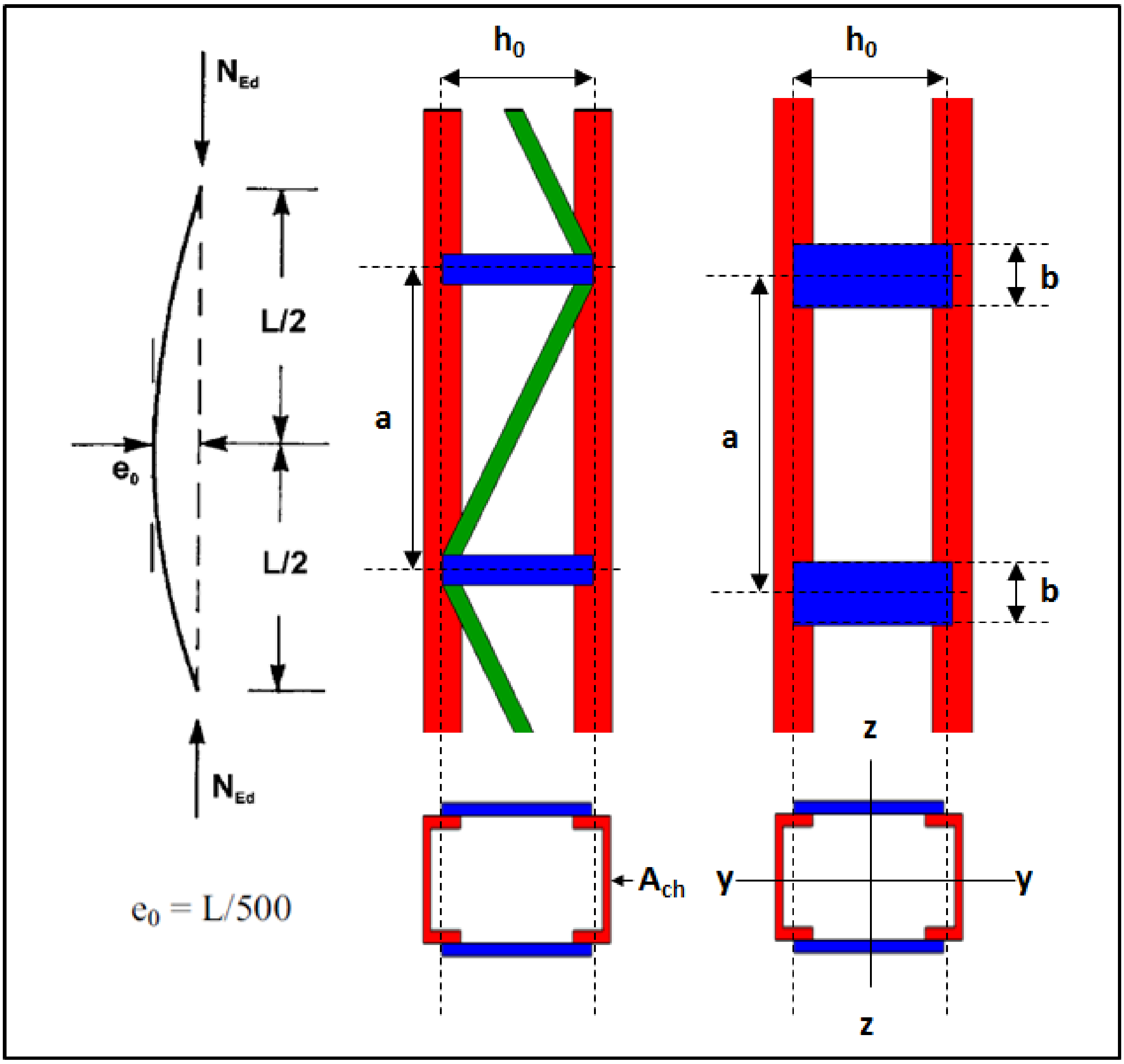


Figure 3.8 Definition of build-up member with important parameters and parts, Chord (Red), Lacing (Green), Battening (Blue)

The use of lacing or battening to build-up a compression member makes for a discrete structure instead of a continue structure. Since it is very labour intensive to perform discretized calculations, EC3 presents two conditions that have to be met in order to assume the build-up member to be continues. It allows the discrete structure to be smeared out into a single continues member when,

1. The member is divided by the lacing or battening in equal modules between parallel chords.
2. The build-up member has at least three of these modules.

Furthermore, it does not matter whether the chords are solid or build-up themselves. When these conditions are met, the build-up member may be considered as a column with an initial imperfection of e_0, due to the manufacturing of the build-up member. The compression force on the member to check for buckling stability (N_{Ed}) has to be transformed to act on the chords. This is done by using a compression force in the chord, $N_{ch,Ed}$, that is constructed from the normal compression force along with moment M_{Ed} at mid span of the build-up member.

6.69 $$N_{ch,Ed} = 0.5N_{Ed} + \frac{M_{Ed}h_0A_{ch}}{2I_{eff}} \tag{3.15}$$

With the bending moment M_{Ed} being the combination of the compression force N_{Ed} acting on the imperfection added to an already present bending moment.

$$M_{Ed} = \frac{N_{Ed}e_0 + M'_{Ed}}{1 - \frac{N_{Ed}}{N_{cr}} - \frac{N_{Ed}}{S_v}} \tag{3.16}$$

In this last equation the shear stiffness of the lacings or battings is included. Next to having enough shear stiffness, the lacings or battings have to be checked for their capacity to resist a shear force acting in the end panel V_{Ed}.

6.70 $$V_{Ed} = \pi\frac{M_{Ed}}{L} \tag{3.17}$$

As mentioned above, a distinction is made between lacing and battening. This distinction is made because of the different way to evaluate the shear stiffness and the effective moment of inertia of the build-up member. Next to these differences, EC3 makes recommendations about the constructional details for both laced and battened members.

3.4.2. Laced Compression Members

Verification for buckling of laced compression members is done with the same check as for normal prismatic member buckling, with the exception of the compression force acting on the chord $N_{ch,Ed}$ instead of the normal compression force acting on the prismatic member N_{Ed}.

6.71 $$\frac{N_{ch,Ed}}{N_{b,Rd}} \leq 1.0 \tag{3.18}$$

The buckling resistance can be calculated in the same way as for prismatic beams, using the same buckling curves and relative slenderness. However the one difference for laced members is the use of another critical length. This buckling length, L_{ch}, is presented for three different configurations and are show in the following figure.

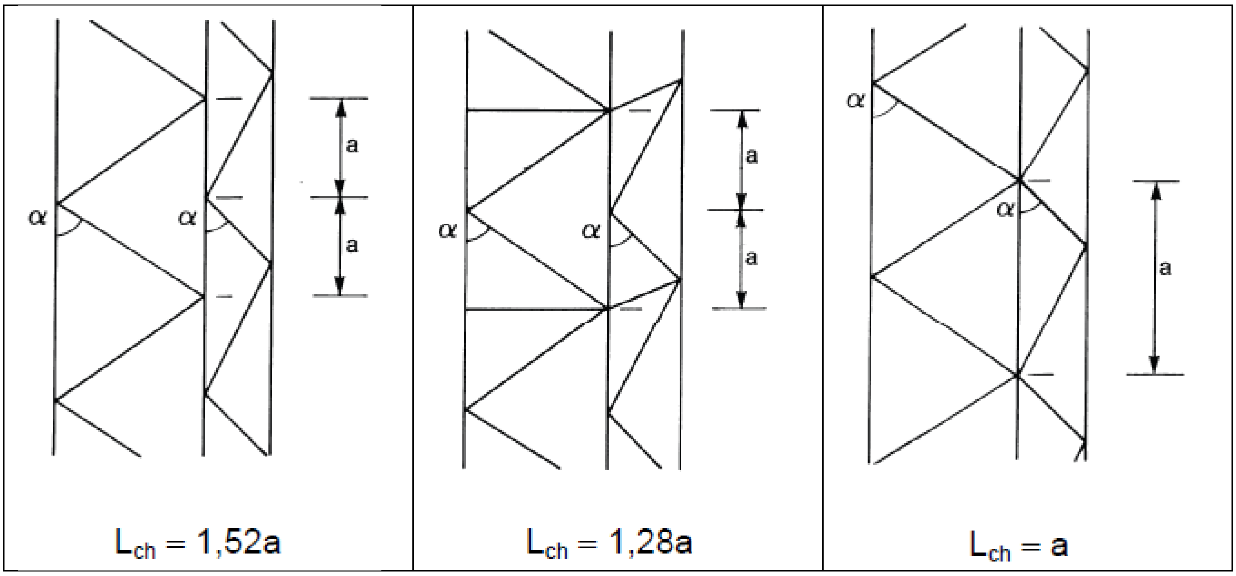


Figure 3.9 Critical chord length for laced build-up compression members

A laced compression member has to follow three construction recommendations. First of all, the lacing system of a build-up member has to be arranged in such way that opposite faced lacings are "in shadow" of each other (see Figure 3.10 A). Due to this shadowing, there will be no additional torsional effects present in the build-up member.

If, for whatever reason, the opposing faces are mutually opposed instead of "in shadow" (see Figure 3.10 B), the appropriate torsional effects should be taken into account.

Finally, tie members, which can be compared to battening, are to be provided at special sections of the build-up member. These special sections are, the ends of lacing systems, where the lacing system is interrupted, and at the joints with other members.

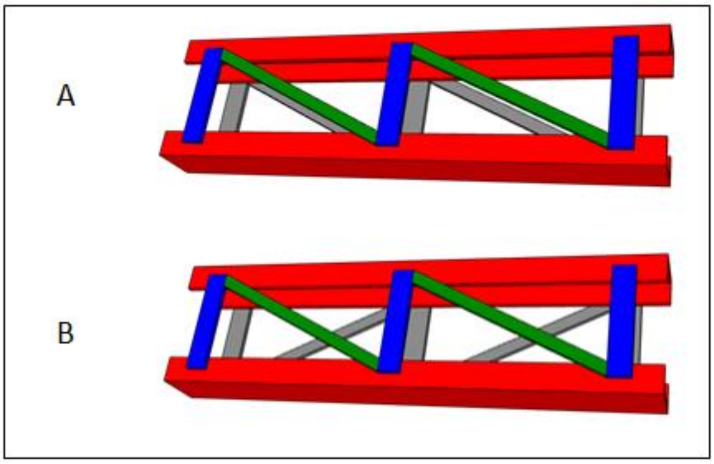


Figure 3.10 Build-up compression members. A) laced with faces "in shadow". B) laced with mutually opposed faces.

3.4.3. Battened Compression Members

A battened compression member is more susceptible to buckling than a laced compression member. Because of this, the cords of a build-up member with battening have to be checked for the actual moments and forces in end panels and at mid-span according to the following figure.

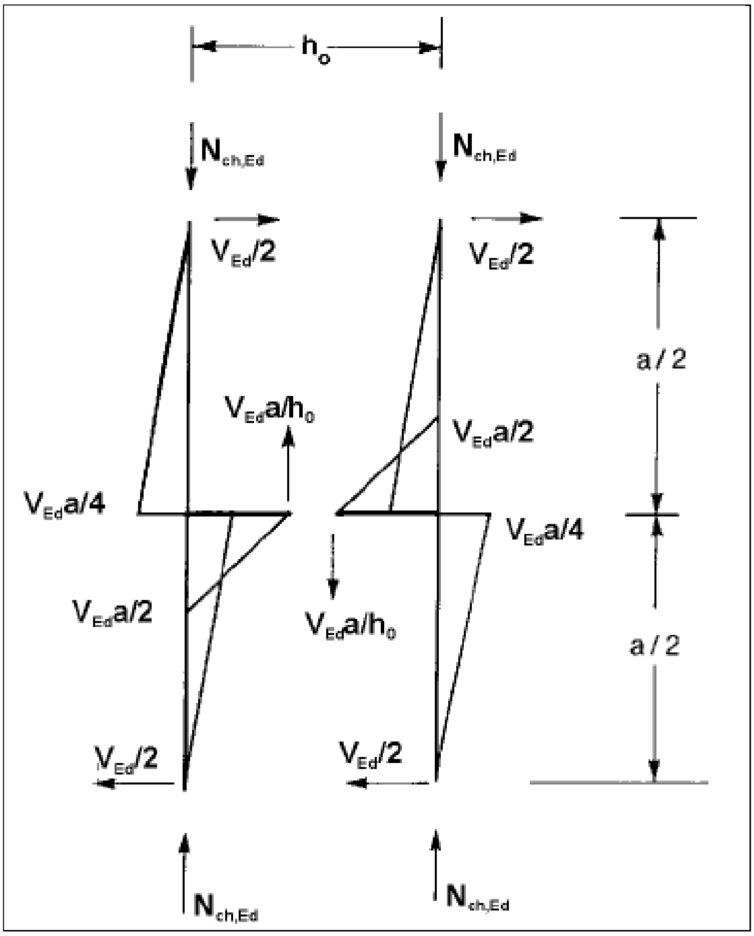


Figure 3.11 Moments and forces acting on a build-up member with battening. NEN-EN 1993-1-1-6.4.3 figure 6.11.

For battened compression members, again three construction recommendations are presented in EC3. First of all, battening is to be provided at each end of a member. Secondly, for members with opposing battened faces, the battening should be arranged opposite each other. And finally, battening should be provided at locations where loads and/or lateral restraint can be applied.

3.5. Buckling of Plated Structures

Members with cross section classification of 1, 2 and 3 will not fail due to buckling before the materials yield strength is reached. However, class 4 members will fail due to local buckling before this material limit has been reached. This local buckling is especially of interest for members build-up from plated structures. Buckling requirements for these plated structures are found in NEN-EN 1993-1-5: Plated structural elements.

This chapter will show the requirements for plate buckling of both stiffened and unstiffened plates. Furthermore, interaction between axial force, bending force and transverse force will be discussed. Numbers in front of every equation refer to the EC3 equations presented in NEN-EN 1993-1-5

3.5.1. Member Stability

The criteria for plated member stability is as a combination of compression and bending moment.

4.14
$$\frac{N_{Ed}}{\frac{f_y A_{eff}}{\gamma_{M0}}} + \frac{M_{Ed} + N_{Ed} e_N}{\frac{f_y W_{eff}}{\gamma_{M0}}} \leq 1.0 \quad (3.19)$$

Instead of the buckling check for uniform members as presented in chapter 3.2, the requirements for plate buckling are not tested against a buckling resistance belonging to a certain cross section. The acting compression force and bending moment are tested against the load bearing capacity of an effective cross section.

4.1
$$A_{c,eff} = \rho A_c \quad (3.20)$$

The effective cross section is the alteration of the normal cross section with a reduction factor ρ. This reduction of cross section represents the effect of plate buckling, where failure of the middle of the plate will not immediately result in plate failure. The plate edges will be able to refrain the entire member from buckling. The plate member will fail completely when the edges fail due to yielding. This effect can be seen in the following figure.

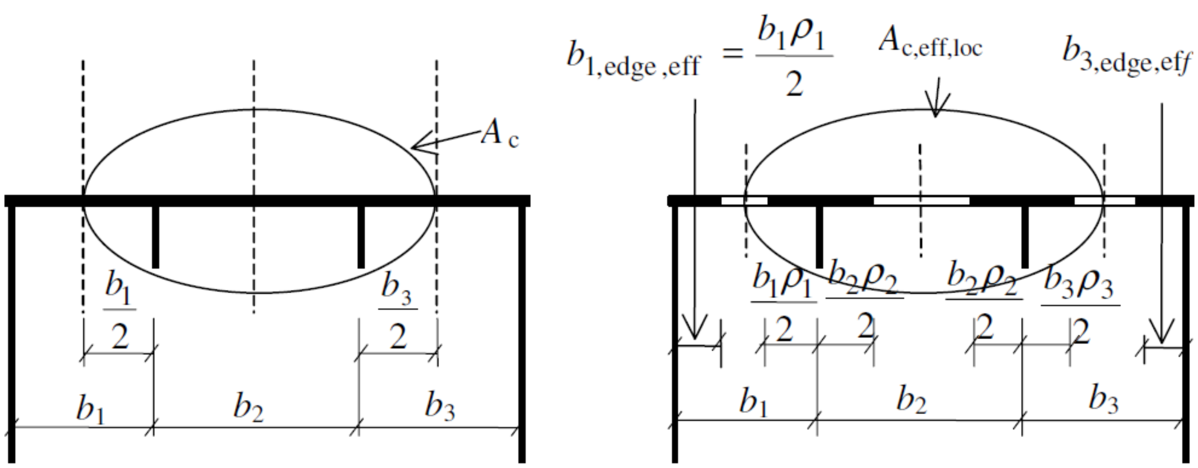


Figure 3.12 The effect of plate buckling. The black areas of the effective cross section (right) resist the compression load acting on the member. The white areas represent part of the plate that has already yielded and is thus unable to resist the compressive load. NEN-EN 1993-1-5-4.5 figure 4.4.

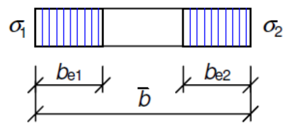
A plate member is able to fail due to buckling in two different ways. First of all, as mentioned above due to failure of the plate edges to resist the compressive load. This is referred to as plate buckling and is represented in Figure 3.12 between the stiffeners. For stiffened plates, as can be seen in Figure 3.12, the middle part of the plate is reduced. The plate will now fail only when the buckling capacity of the stiffener is reached. These stiffeners can be seen as column members. Therefore the second form of buckling failure is referred to as column buckling.

3.5.2. Unstiffened Plates

The effective cross section for unstiffened plates is determined by factorizing the plate cross section with reduction factor ρ. This reduction factor, like with uniform members in compression and bending are buckling curves (see FIGURE). As such they are functions of a relative plate slenderness.

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28.4\varepsilon\sqrt{k_\sigma}} \qquad (3.21)$$

In these tables the buckling factor for intern compression parts and outward flanges can be determined depending on the stress distribution.



$\underline{\psi = 1}$:

$b_{eff} = \rho\ \bar{b}$

$b_{e1} = 0{,}5\ b_{eff}$ $\qquad b_{e2} = 0{,}5\ b_{eff}$

$\psi = \sigma_2/\sigma_1$	1	$1 > \psi > 0$	0	$0 > \psi > -1$	-1	$-1 > \psi > -3$
Buckling factor k_σ	4,0	$8{,}2 / (1{,}05 + \psi)$	7,81	$7{,}81 - 6{,}29\psi + 9{,}78\psi^2$	23,9	$5{,}98\,(1 - \psi)^2$

Figure 3.13 Extract from Table 4.1 NEN-EN 1993-1-5-4.4. Stress distribution over the effective cross section for internal compression part loaded in pure compression. Stress ratio ψ determines the buckling factor k_σ.

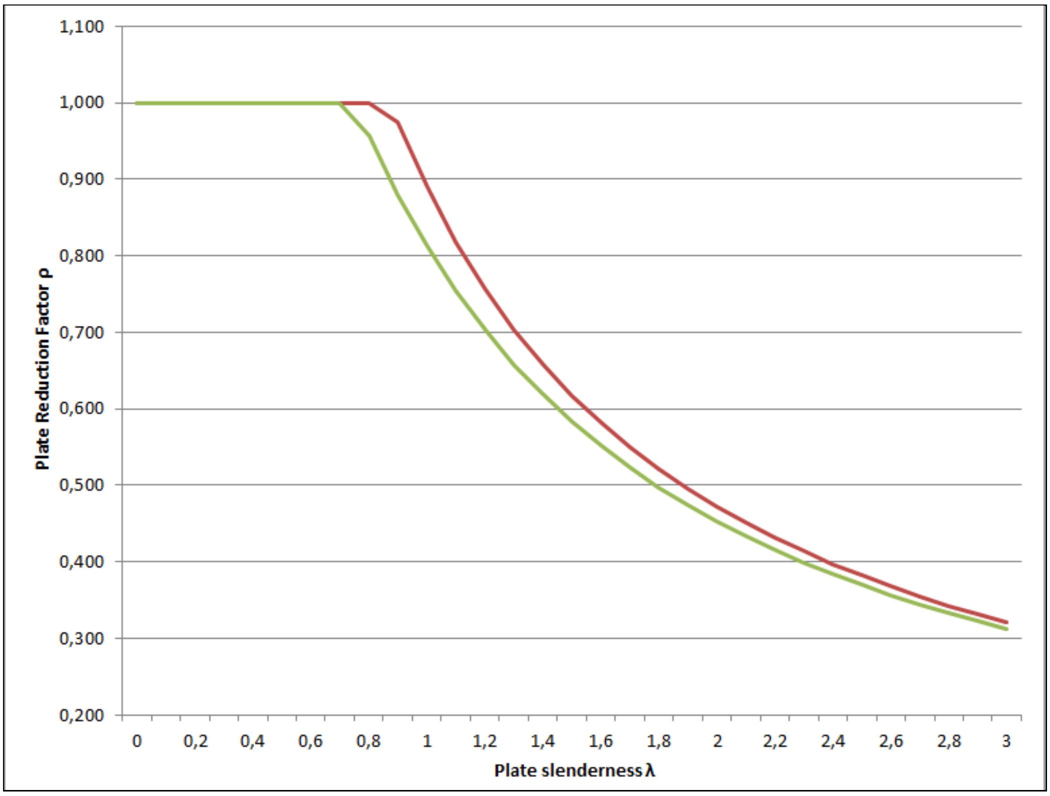


Figure 3.14 Buckling curves for unstiffened plates in pure compression. The red curve represents internal parts of the plated member, while green represents the buckling curve for outward flanges.

3.5.3. Stiffened Plates

Stiffened plates will buckle due to a combined effect of plate buckling between the stiffeners and column buckling of the stiffeners themselves. Due to this combined effect, an interpolation has to be made between the reduction factor for plate buckling ρ, and the reduction factor for column buckling χ_c.

Also unstiffened plates with small plate ratios $a/_b < 1.0$, will fail due to column buckling instead of plate buckling. This because the plate edges of such plates are relatively small, and thus have far less capacity to resist compression loads after the middle of the plate has buckled. Therefore the entire plate will fail after buckling of the middle part of the plate.

Because column buckling is important for these plate types, the reduction factor for uniform members χ_c, from chapter 3.2, has to be taken into account. However, since class 4 cross sections are regarded in this section, a slight different relative plate slenderness is calculated. The difference lies in the fact that for plates the Euler critical stress for plates is used instead of the critical stress for beams. The proper imperfection factor α (Table 2), that is used in the buckling curve shape function is prescribed according to the following table.

Unstiffened plates		α = 0.21	Buckling curve a
Stiffened plates			
Open cross section stiffener	$\alpha_e = \alpha + \dfrac{0.09}{i/e}$	α = 0.34	Buckling curve b
Closed cross section stiffener		α = 0.49	Buckling curve c

Table 3 Imperfection factors used to establish the proper buckling curve for column buckling

The imperfection factor for stiffened plates depends on the ratio between stiffener radius of gyration and the distance between the centroid of the plate and the neutral axis of the effective column. Since this ratio is always greater than zero, the imperfection factor will always be bigger than those belonging to buckling curve b or c. As can be seen in Figure 3.5, this will result in a smaller reduction factor, which will in turn result in a lower buckling load resistance. In order to get the best result, the stiffener i/e ratio needs to be increased. This can be done by using larger stiffeners or by using L- and T-stiffeners of the same size.

For the plate buckling effect between stiffeners, the reduction factor for plates ρ is used. However, another relative plate slenderness is used as well. To account for the added strength of stiffeners the plate is compared with an equivalent orthotropic plate. That is to say that the added area of the stiffeners is smeared out over the plate. This results in an increased plate thickness. Therefore, a slightly different relative plate slenderness has to be used to obtain the plate reduction factor ρ.

Both reduction factors (ρ and χ_c) are interpolated to get the final reduction factor for stiffened plates.

4.13
$$\rho_c = (\rho - \chi_c)\xi(2 - \xi) + \chi_c \qquad (3.22)$$

This reduction factor is used in order to get the proper effective cross section. This cross section is build-up from the effective plate cross section combined with its stiffeners and added to the edges of the plate.

4.5
$$A_{c,eff} = \rho_c A_{c,eff,loc} + \sum b_{edge,eff} t \qquad (3.23)$$

3.5.4. Transverse Forces

When a structural member is not only loaded longitudinally, but transversely as well, the lateral expansion due to Poisons ratio is restraint. This will increase the stress levels in the member, resulting in a lower overall buckling strength. Therefore the effect of transverse forces is regulated in EC3 as well.

Transverse forces in EC3 are considered to be forces, Fs, acting over a certain section of the plate. The section length depends upon the stiff bearing on the flange. Three different situations are considered. First of all, the transverse force is introduced over an area that is effectively distributed over an angle of 45° (Figure 3.15 A). A common example of such a situation is a force being transferred by a perpendicular plate being welded to the flange. The second situation is where a series of concentrated forces are closely spaced. Here the section of plate that has to be checked is the distance from centre-to-centre between the outer loads (Figure 3.15 B). Finally a situation is described where the contact surface between applied load and flange plate is at an angle. In such situation a contact point is created (Figure 3.15 C). In this case, the length of plate section should be taken as zero.

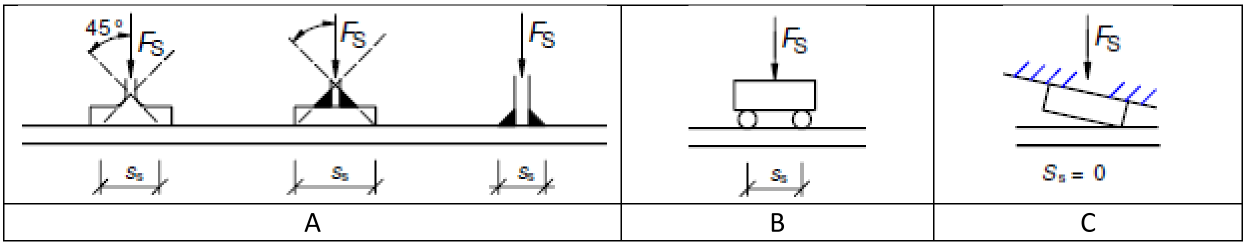


Figure 3.15 Length of stiff bearing used to determine the effective length over which the transverse load will influence the plate buckling strength. NEN-EN 1993-1-5-6.3 figure 6.2.

The stiff bearing length is used to calculate an effective loaded length of plate (l_y), which combined with reduction factor (χ_F), yields the effective length for resistance to transvers forces (L_{eff}). This effective length is required for the design resistance.

6.1 $$F_{Rd} = \frac{f_{yw} L_{eff} t_w}{\gamma_{M1}} \qquad (3.24)$$

With the plate yield strength (f_{yw}), plate thickness (t_w), and partial factor for member instability (γ_{M1}=1.0).

Verification of transverse strength of members is done by the efficiency ratio between acting transverse forces and transverse design resistance.

6.14 $$\eta_2 = \frac{F_{Ed}}{F_{Rd}} \leq 1.0 \qquad (3.25)$$

To get the reduction factor for effective length (χ_F), a plate slenderness is used ($\bar{\lambda}_F$).

6.4 $$\bar{\lambda}_F = \sqrt{\frac{l_y t_w f_{yw}}{F_c}} \qquad (3.26)$$

In which F_c is Euler elastic buckling load for plates (see equation 2.10). As seen in Plate Buckling, there are many different buckling coefficients for different plate boundary conditions. In EC3, the buckling coefficient (k_F), is determined different for unstiffened and longitudinally stiffened plates. For unstiffened plates, EC3 considers three different types of load introduction to the plate.

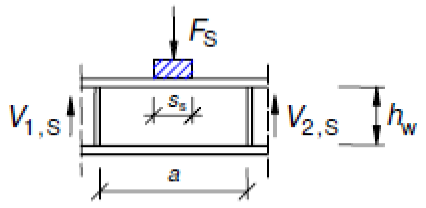
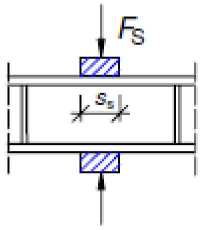
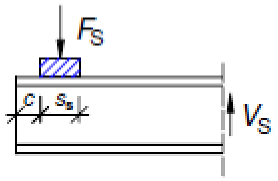
Type A	Type B	Type C
		
$k_F = 6 + 2\left(\frac{h_w}{a}\right)^2$	$k_F = 3{,}5 + 2\left(\frac{h_w}{a}\right)^2$	$k_F = 2 + 6\left(\frac{s_s + c}{h_w}\right) \leq 6$

Figure 3.16 Different plate buckling coefficients for three types of load introduction to the plate. NEN-EN 1993-1-5-6.1 figure 6.1.

Type A is when the introduced force (F_S) is transferred through the plate and resisted by shear forces on both ends of the plate. Type B is when the introduced force is transferred through the plate directly to the other side. Type C is when the introduced force is only resisted by shear forces on one side of the plate.

For stiffened plates, a single more intricate buckling factor is used that considers the plate between plate edge and first longitudinal stiffener. This factor is furthermore influenced by the second order moment of inertia of the stiffener itself.

As mentioned at the beginning of this paragraph, the combination of both longitudinal an transverse loads will influence the buckling strength of a plate. This is illustrated in EC3 by an interaction expression that combines transverse force, bending moment and axial force.

7.2

$$\eta_2 + 0.8\eta_1 \leq 1.4 \qquad (3.27)$$

Where η_2 is the verification expression for transverse loads (equation 3.25), and η_1 is the verification expression for axial forces and bending moments combined (equation 3.19a).

3.6. Critical Length

In order to determine the relative slenderness of a member, its critical length must be determined first. According to EC3, the critical length has to be determined according to the National Annex. This National Annex allows for different countries to implement their own alternative procedures, values and recommendations into EC3. For this study, the Dutch National Annex will be used in order to assess the critical length NEN-EN 1993-1-1/NB-Annex C.

The National Annex presents an evaluation of critical length for four different situations. These are prismatic members, spring supported members, crossing members and non-prismatic members. In this study the evaluation of buckling length will be limited to prismatic and non-prismatic members, however. In other literature, critical length can also be referred to as being the buckling length. In the following chapter the terms critical length and buckling length can be used interchangeably.

3.6.1. Prismatic Members

For prismatic members, the buckling length is determined for two types of beams. Firstly, beams that are part of a non-sway frame and secondly, beams that are part of a sway frame. The difference between these two types of frames is shown in Figure 3.17.

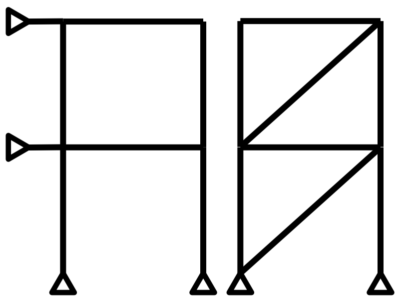
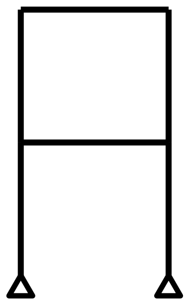
BRACED FRAMES (NON SWAY)	UNBRACED FRAMES (SWAY)
	

Figure 3.17 Non-sway braced frames and Sway unbraced frames

There are two methods presented in the National Annex to determine the buckling length. The first is by regarding the boundary conditions of columns in non-sway and sway frames.

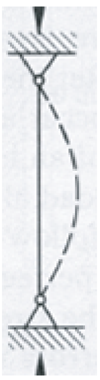
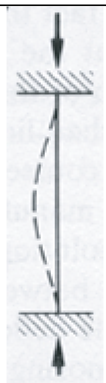
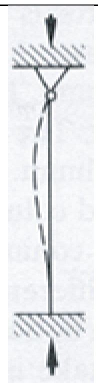
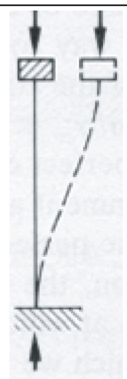
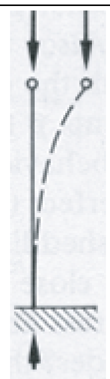
NON-SWAY	NON-SWAY	NON-SWAY	SWAY	SWAY
PINNED-PINNED	FIXED-FIXED	FIXED-PINNED	FIXED-FIXED	FIXED-FREE
				
$L_{cr} = L$	$L_{cr} = \frac{1}{2}L$	$L_{cr} = \frac{L}{\sqrt{2}}$	$L_{cr} = L$	$L_{cr} = 2L$

Table 4 Critical length for columns with different boundary conditions

In order to use these length values, the boundary conditions have to be right. In reality, boundary conditions will be somewhere between the pinned-pinned and fixed-fixed situation. Where the pinned-pinned condition will yield very conservative buckling lengths, the fixed-fixed condition might yield to high buckling lengths. This will result in buckling failure of members before the calculated allowable load is reached.

Therefore a second method is presented in the National Annex, that takes the stiffness of the boundary conditions into account. This method makes use of so-called nomographs. The buckling length can be read from these graphs by drawing a line between the flexibility parameters of both ends. The flexibility parameters range from zero to infinity. This represents a fixed or pinned boundary condition respectively.

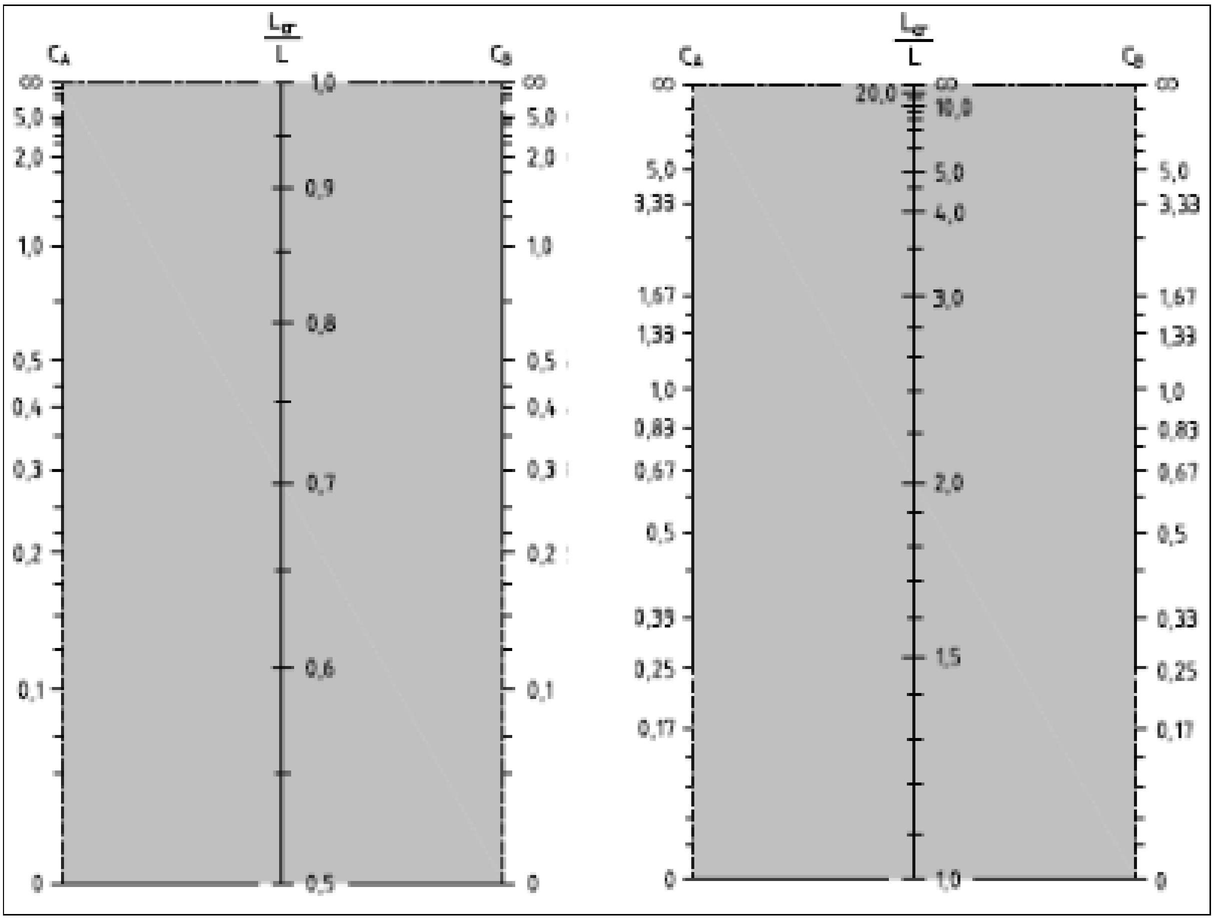


Figure 3.18 Nomographs for Non-sway frames (left) and Sway frames (right). NEN-EN 1993-1-1/NB-C.1.2 and C.1.3

As can be seen in Figure 3.18, bracing a frame has a huge influence on the buckling length of its elements. An element in a braced frame has a buckling length that is much less than an element with the same flexibility parameters that is part of an unbraced frame. Therefore braced frames can resist more compressive forces in its members than unbraced frames.

3.6.2. Non-Prismatic Members

Not all members are constructed with constant cross section. These members are referred to as being non-prismatic members. Because non-prismatic members have a varying cross section along its length, the capacity to resist bending forces varies as well along its length. Therefore members tend to buckle earlier at the narrower part of the beam. However, since these parts are most likely at the ends of the beam, the bending forces will be lower than at the middle. This does however influence the overall buckling length of the member, which means that the critical length cannot be determined with the same method as for prismatic members.

The National Annex shows that the buckling length has to be taken as the largest of either the entire length of the beam, or its effective buckling length. This effective buckling length is a product of the members length with a relative buckling length factor. This factor is a function of three different parameters.

The first parameter is a moment of inertia ratio between the narrowest and widest part of the beam. Another parameter takes the length of flaring into account. That is to say the length ratio between the widest part of the beam and its total length. The third parameter that influences the relative buckling length factor is a parameter that describes the change in thickness of the beam. There are three different types of thickness changes considered in the National Annex, these are shown in the following figure.

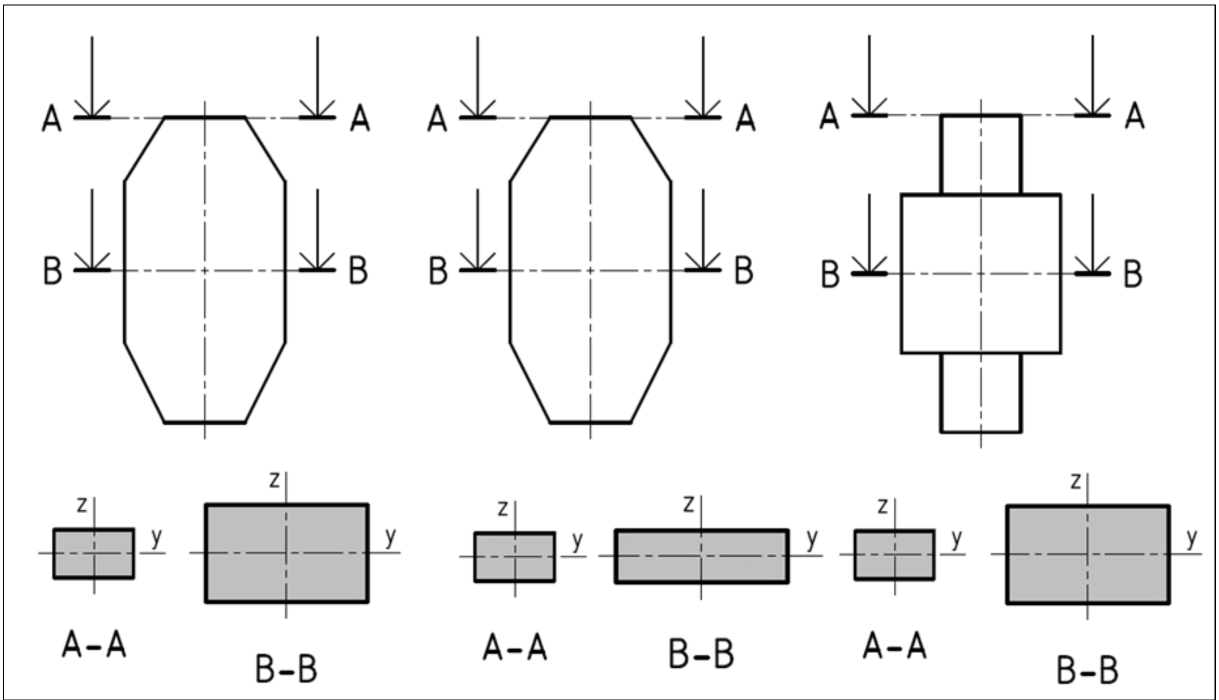


Figure 3.19 Representations of non-prismatic members from the Dutch National Annex to Eurocode 3 NEN-EN 1993-1-1/NB-C.4

The first member in Figure 3.19 represents a change in thickness in both y and z-direction (n = 4). The second shows a change of thickness in only one direction (n = 1 for I_y and n = 3 for I_z), while the third member shows a sudden change in thickness (n = 0).

These three parameters result in five graphs where the proper relative buckling length factor can be found by interpolating between the different lines. The graphs represent the different non-prismatic members from Figure 3.19 by its n-value.

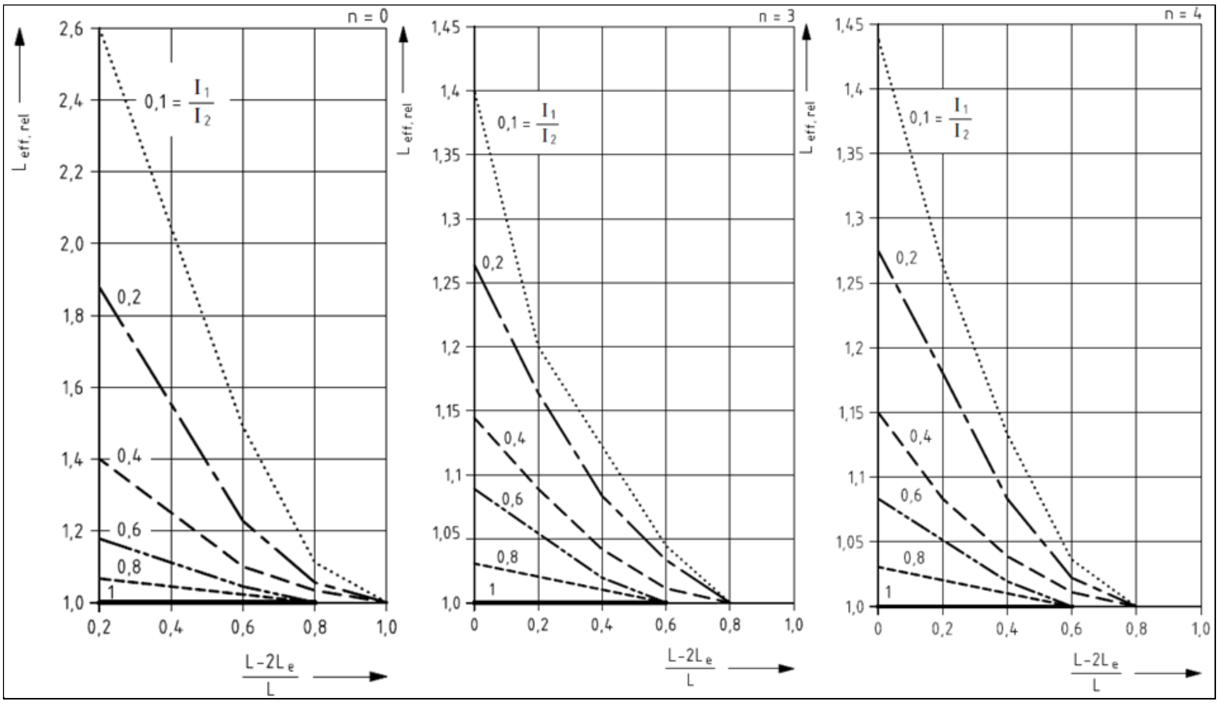


Figure 3.20 Relative buckling length factor graphs from the National Annex to Eurocode 3. NEN-EN 1993-1-1/NB-C.4

The length and moment of inertia ratios represent a measurement by which a member can be called non-prismatic. A member that can be considered to be very non-prismatic will have low ratio values, whereas a member with high ratio values will be almost similar to a normal prismatic member. In the graphs from Figure 3.20 it can be seen that a member which is highly non prismatic (i.e. very low ratio values), will result in a larger buckling length. This means that non-prismatic members are less capable to resist buckling than prismatic members.

3.6.3. Build-Up Compression Members

The critical length for build-up compression members is regulated both in the National Annex as well as in annex BB in the EC3. This latter annex gives very global descriptions for the buckling length of build-up members.

The critical length is related to the system length of a member multiplied by a certain buckling factor. This system length for in-plane buckling is the length between two connections (length a in Figure 3.8). For out-of-plane buckling, the system length is considered to be the length between lateral supports. Therefore out-of-plane system length is generally greater than the in-plane system length.

Generally, the critical length for cord members has to be taken equal to the system length of the members. Also for out-of-plane buckling of web members, such as lacing and battening, should this length be taken. For critical length for in-plane buckling of I or H section chord members may be taken as 0.9L. For out-of-plane buckling of these section chord members a critical length of L should be taken. For web members, the critical length may be taken as 0.9L for in-plane-buckling under the condition that appropriate end restraint and end fixity is applied. In EC3, appropriate end fixity is considered to be at least a 2 bolt connection if bolted.

For members made of hollow sections, other critical lengths are recommended. For hollow section chords, a critical length of 0.9L for both in- and out-of-plane buckling can be used. Web members that are bolted to the chords have a critical length equal to the system length L for both in- and out-of-plane buckling. For hollow section web members, welded around its perimeter to hollow section chords, the critical length may be taken as 0.75L for both in- and out-of-plane buckling.

The critical lengths presented above are all taken very conservative. In every situation a smaller critical length is justified with the proper analysis done. An overview of the default critical lengths is presented in the following table.

	Member type	In-plane	Out-of-plane
Chord Members	General Sections	1.0 $L_{i,ch}$	1.0 $L_{o,ch}$
	I/H Sections	0.9 $L_{i,ch}$	1.0 $L_{o,ch}$
	Hollow Sections	0.9 $L_{i,ch}$	0.9 $L_{o,ch}$
Web Members	General Pinned Sections	1.0 $L_{i,web}$	1.0$L_{o,web}$
	General Fixed Sections	0.9 $L_{i,web}$	1.0 $L_{o,web}$
	Bolted Hollow Sections	$L_{i,web}$	$L_{o,web}$
	Welded Hollow Sections	0.75 $L_{i,web}$	0.75 $L_{o,web}$

Table 5 Default critical length for build-up compression members

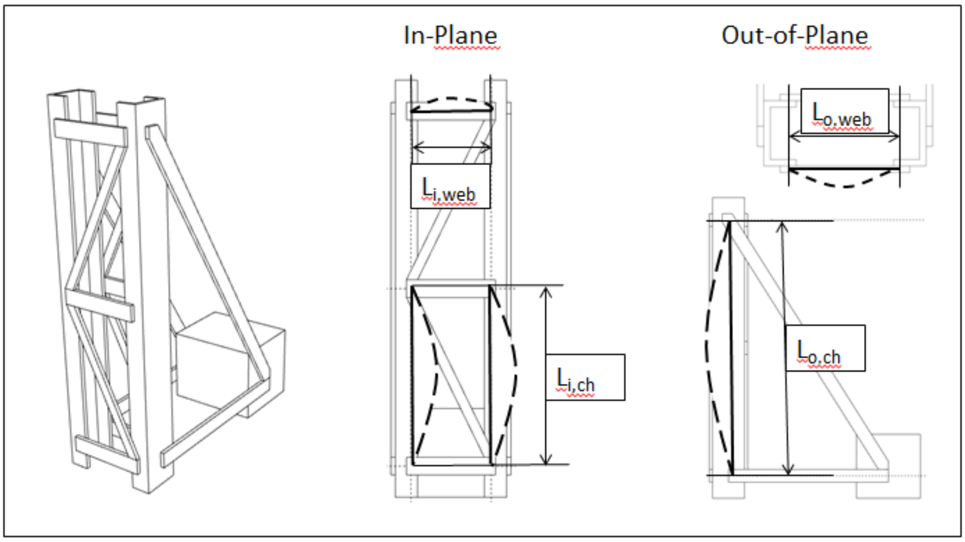


Figure 3.21 Definition of lengths, as used in Table 5, of a build-up member

Furthermore, according to the Dutch National Annex, the entire build-up member should be checked for buckling according to recommendations for uniform members in compression. The critical length that is required to calculate the relative slenderness is given by a combination of system length and buckling length due to shear.

NB.76
$$L_{cr} = \sqrt{L^2 + \frac{\pi^2 EI_{eff}}{S_v}} \tag{3.28}$$

3.7.Shear Lag

The effect of shear lag will be presented in this chapter. Shear lag, as already mentioned in chapter 0, has an influence on the buckling strength. This is also true for class 1 to 3 members. Since NEN-EN 1993-1-1 refer to the requirements for plated structures to deal with the effect of shear lag, it is presented in this chapter.

Shear stresses cause inconstant stress distributions over the cross section of a member. The effect of shear lag is influenced by the width-to-length ratio of a plate. The wider a plates ratio, the less effect shear lag has on that plate. In EC3, the effect of shear lag may be neglected for plate width-to-length ratios $b_0/L_e < 0.02$.

For greater plate ratio's, the effect of shear lag is taken into account by transforming the non-uniform stress over the entire width of the cross section, into a uniform stress over an effective width of the cross section (see Figure 3.22). This effective width for shear stress is obtained with a factor β.

3.1 $$b_{eff} = \beta b_0 \quad (3.29)$$

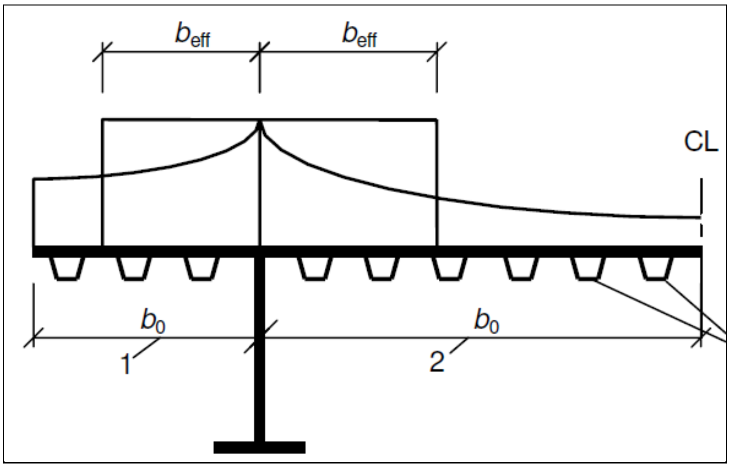


Figure 3.22 Transformation of actual stress over width b_0 to continues stress over effective width b_{eff}. For outstand flanges, b_0 is the entire width of the outstand. For internal flanges, b_0 is half the width between web plates. NEN-EN 1993-1-5-3.2.1 figure 3.2

This β factor can be determined by different equations that include the plate width-to-length ratio, which is compensated with a factor α_0 in order to take longitudinal stiffeners into account. These different equations are represented in EC3 by Table 3.1, of which the following figure is an excerpt.

κ	Verification	β – value
$\kappa \leq 0{,}02$		$\beta = 1{,}0$
$0{,}02 < \kappa \leq 0{,}70$	sagging bending	$\beta = \beta_1 = \dfrac{1}{1 + 6{,}4\,\kappa^2}$
	hogging bending	$\beta = \beta_2 = \dfrac{1}{1 + 6{,}0\left(\kappa - \dfrac{1}{2500\,\kappa}\right) + 1{,}6\,\kappa^2}$

Figure 3.23 Excerpt of Table 3.1 from NEN-EN 1993-1-5-3.1. Equations for β factor in order to take effect of shear lag into account.

The difference between sagging and hogging bending can be best explained by Figure 3.24. Sagging bending is considered over a length between supports (β_1), while hogging bending is considered to be over a length along a support(β_2). Furthermore is the limit represented in this table for which shear lag has to be taken into account. As can be seen for values of $\kappa \leq 0.02$ the β factor is equal to 1.0. This represents the effective width for shear lag to be taken as the width of the plate, thus neglecting shear lag effects.

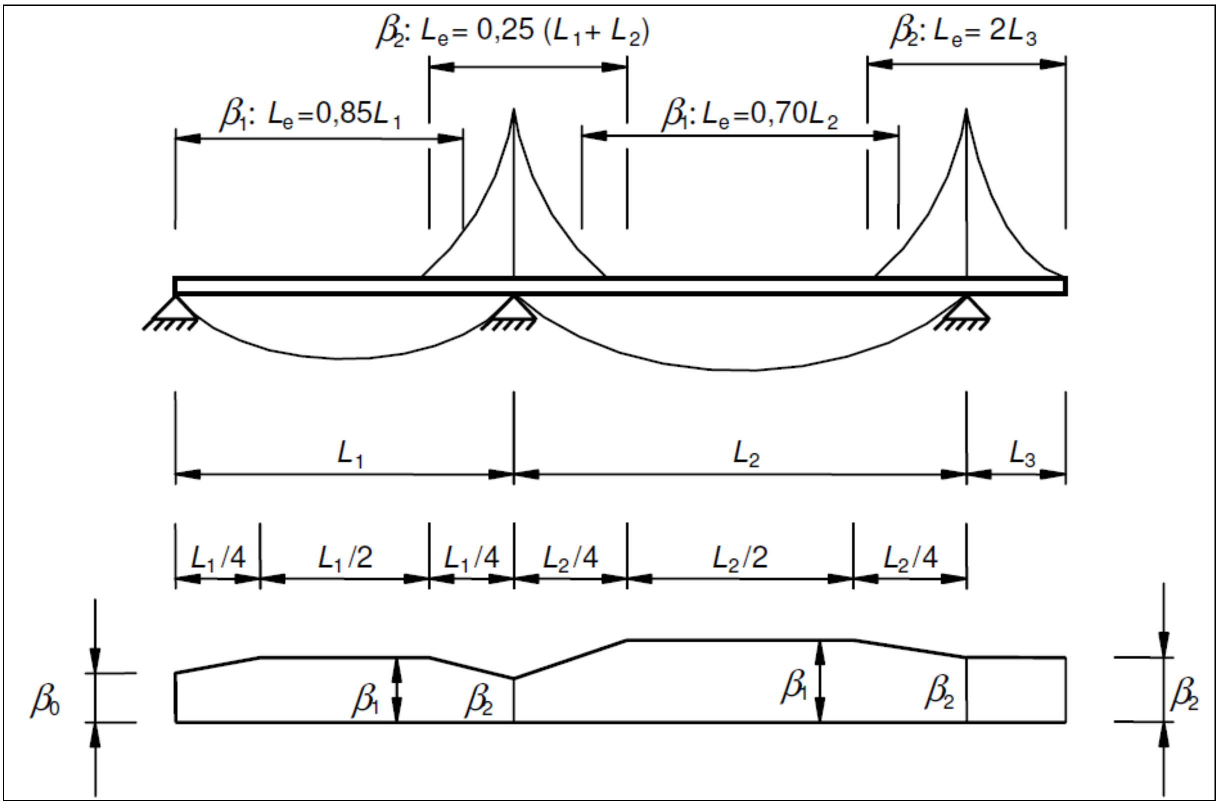


Figure 3.24 Effective length for shear lag. Sagging bending can be seen between supports, while hogging bending is seen over the supports. NEN-EN 1993-1-5-3.2.1 figure 3.1.

Also from Figure 3.24 can be seen that the effective length for sagging and hogging bending is overlapping. The bottom figure shows the β_i factor that has to be taken at certain points along the members length. It can be seen that at some points this β factor has to be interpolated between hogging and sagging.

In EC3 the ultimate state shear lag effect can be determined by three different methods. However, the Dutch National Annex, recommends the method of elastic-plastic shear lag effects allowing for limited plastic strain. This means that the effective cross section as shown in equation (3.20), is adapted to fulfil the following equation.

3.5

$$A_{eff} = A_{c,eff}\beta^{\kappa} \geq A_{c,eff}\beta \tag{3.30}$$

Here the factors β and κ are taken from table 3.1 which is represented in Figure 3.23. This means that the effective cross section that accounts for plate buckling becomes even less when shear lag is taken into account. This effective cross section A_{eff} is used in equation (3.19) for member stability.

The above expression for effective cross section with included shear lag effect is only valid for compressive loads. However, contrary to plate buckling, shear lag also has effect on plates loaded in tension. For these plates, $A_{c,eff}$ has to be replaced with the gross cross section of a plate.

3.8. Imperfections

Most checks and equations presented in EC3 are derived from theory. The theoretic models for elastic buckling imply the use of a perfect member without any imperfections. However, in reality it is impossible to achieve such perfect members. Apart from material imperfections that may occur during fabrication of a construction member, there are construction imperfections which occur during construction. These imperfections result in an additional bending moment. This additional bending moment has to be resisted by the same cross section as for a perfect member. Therefore, the buckling resistance of a member will become lower when imperfections are taken into account. In EC3, imperfections are dealt with in NEN-EN 1993-1-1 chapter 5.3.

To account for imperfections in EC3, a distinction is made between global and local imperfections. Globally, imperfections due to construction occur. Which can be due to lack of fit and minor eccentricities in joints. Local imperfections of members can be seen as lack of straightness of a member, geometrical imperfections and even residual stresses. Because it is often unknown in which direction the imperfections will cause an additional bending moment, the effect has to be taken into account in the most unfavourable direction.

The effect of imperfection in single members is already accounted for in the requirements for member stability. However, this is only true for first order analysis. When member stability is checked with a second order analysis, the initial bow imperfection has to be taken into account as presented below.

3.8.1. Sway Frames

For frames in sway mode (Figure 3.17) the imperfections may be determined by considering the initial sway imperfections and relative initial local imperfections of members, also referred to as initial bow imperfections.

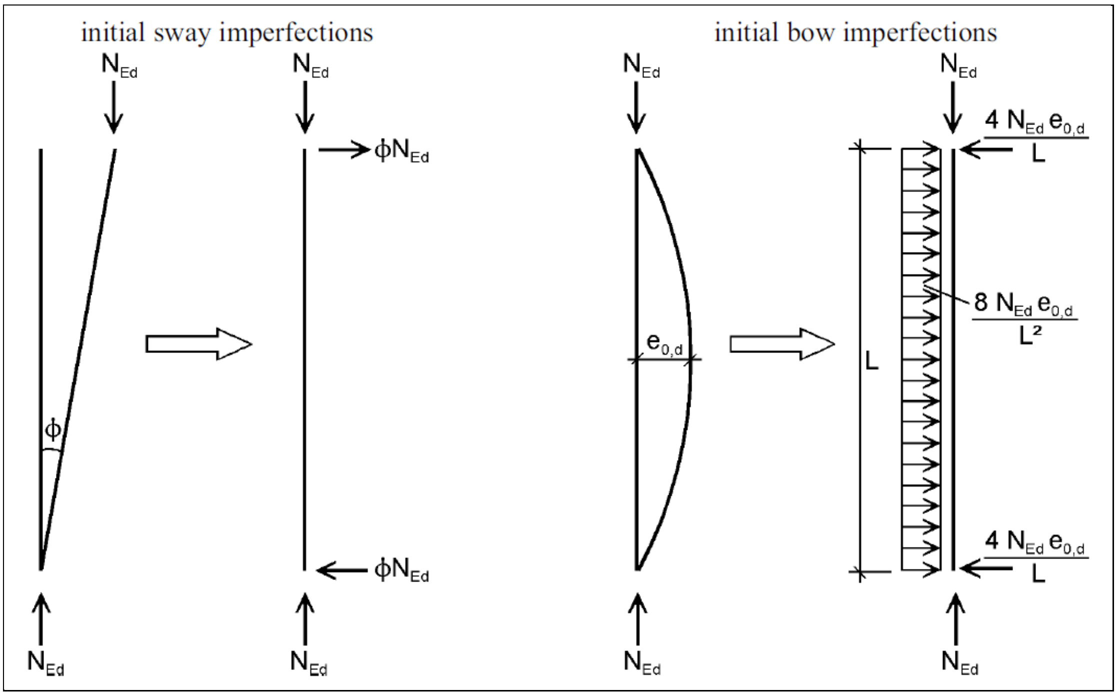


Figure 3.25 Initial sway imperfections and initial bow imperfections for sway frames with the additional forces that represent these imperfections. NEN-EN 1993-1-1-5.3.2 figure 5.4.

The angle accounting for initial sway imperfections is determined by a basic value of ϕ_0=0.005 which is factorized in order to take the height of the frame into account and the numbers of load carrying columns in a row of the frame.

5.5 $$\varphi = \varphi_0 \alpha_h \alpha_m \quad (3.31)$$

The factors α_h and α_m influence the initial sway angle in such way that the angle will decrease with increasing frame height and increasing number of load carrying columns. This is rather logic considering the fact that higher frames will be easier to construct in a straight line. This due to the fact that the same eccentricity for tall frames will result in a lower sway angle, than for short frames. Multiple columns in a frame row will be able to correct each other with respect to their straightness. Therefore it will decrease the initial sway angle with increasing number of columns.

The initial bow imperfection is represented in EC3 by the ratio e_0/L.. Recommendation for the initial displacement e_0 is given in the Dutch National Annex to be

NB.20 $$e_0 = \alpha(\bar{\lambda} - 0.2)\frac{M_{c,Rd}}{N_{c,Rd}} \quad (3.32)$$

The initial sway angle and initial bow displacement are used to transform the imperfections of frame and members into additional forces according to Figure 3.25. The initial sway imperfection influence each member in the frame. However, since all members are connected, the additional force need only be considered in one direction at a time.

3.8.2. Non-Sway Frames

Frames in non-sway mode (Figure 3.17) do not have to take the initial sway mode into account. This due to the fact that the additional horizontal force is easily transferred by the horizontal stability member. As a result only an initial bow imperfection has to be taken into account.

5.12 $$e_0 = \frac{\alpha_m L}{500} \quad (3.33)$$

Here the initial displacement is influenced by the number of columns to be restrained. The more columns which are restraint, the less the initial bow displacement will be. The length L is considered to be the span of the bracing system as can be seen in Figure 3.26

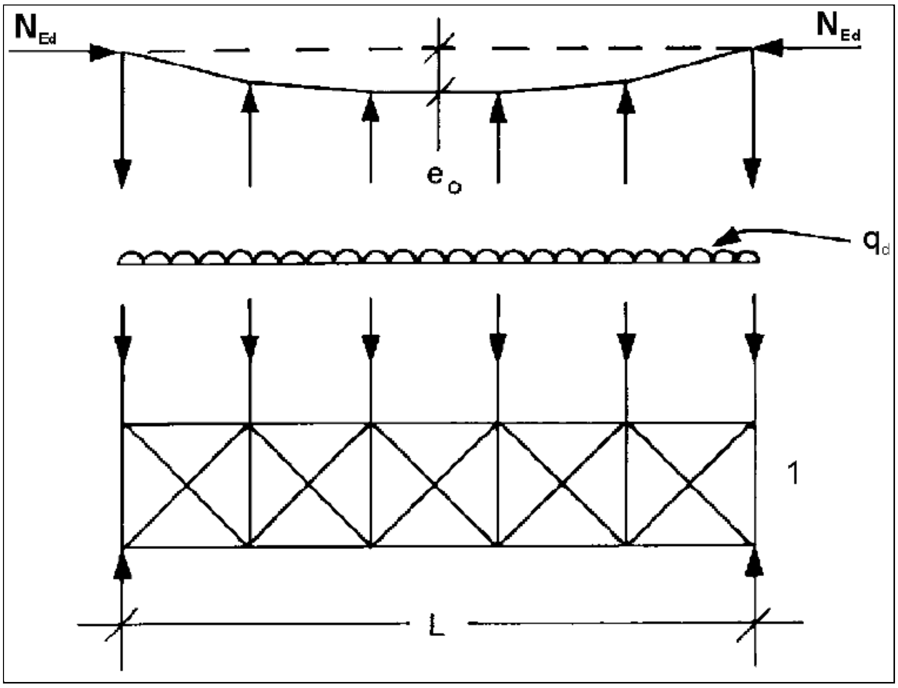


Figure 3.26 Initial bow imperfection of a non-sway frame. The imperfection can be represented as an additional load q_b over the braced span. NEN-EN 1993-1-1-5.3.3

The initial bow imperfection can again be modelled as an additional force acting on the supported frame. This by means of a distributed force.

5.13
$$q_d = \sum N_{Ed} 8 \frac{e_0 + \delta_q}{L^2} \qquad (3.34)$$

This distributed force is, besides the initial bow imperfection, based on the already acting forces. The value of δ_q is the beam displacement due to initial loads on the frame. However, this displacement due to external forces does not have to be included when a second order analysis is made. This due to the fact that deformation implied loads will already be included in the analysis.

3.9.Recommendations for FEM in EC3

Guidelines on the use of FEM analysis op plated structures is presented in NEN-EN 1993-1-5 Annex C. These guidelines are quite straight forward and is intended for engineers who are experienced in the use of FEM.

First of all, a couple of assumptions are given that influence the choice of FEM analysis. These assumptions are presented in the following table.

No	Material behaviour	Geometric behaviour	Imperfections, see section C.5	Example of use
1	linear	linear	no	elastic shear lag effect, elastic resistance
2	non linear	linear	no	plastic resistance in ULS
3	linear	non linear	no	critical plate buckling load
4	linear	non linear	yes	elastic plate buckling resistance
5	non linear	non linear	yes	elastic-plastic resistance in ULS

Table 6 NEN-EN 1993-1-5 Annex C.1 table C.1: Assumptions for FEM analysis

As can be seen, for buckling analysis, assumptions No. 3 and 4 apply. Buckling can be observed when the materials yield limit is reached. Therefore, buckling analysis is within the linear material behaviour range. For post-buckling analysis plastic material behaviour becomes important. The geometric behaviour, however is non-linear. This is because the buckling curve is presented by half-sine waves, which are by definition non-linear.

EC3 gives a couple of guidelines for modelling in a FEM analysis. Because the choice of element types and mesh size can influence the test results, recommendations are made to carry out sensitivity checks with successively refined mesh, in order to validate the result. Also, boundary conditions for supports, interfaces and applied loads should be applied in such way that conservative results are obtained.

Where imperfections are to be included, both geometric and structural imperfections should be included simultaneously. The direction of imperfections have to be chosen in such way that the lowest resistance is obtained. That is to say that the least favourite combinations of loads and imperfections is chosen. The geometric imperfections may be represented by applying equivalent geometric imperfections, which are presented in the following figure.

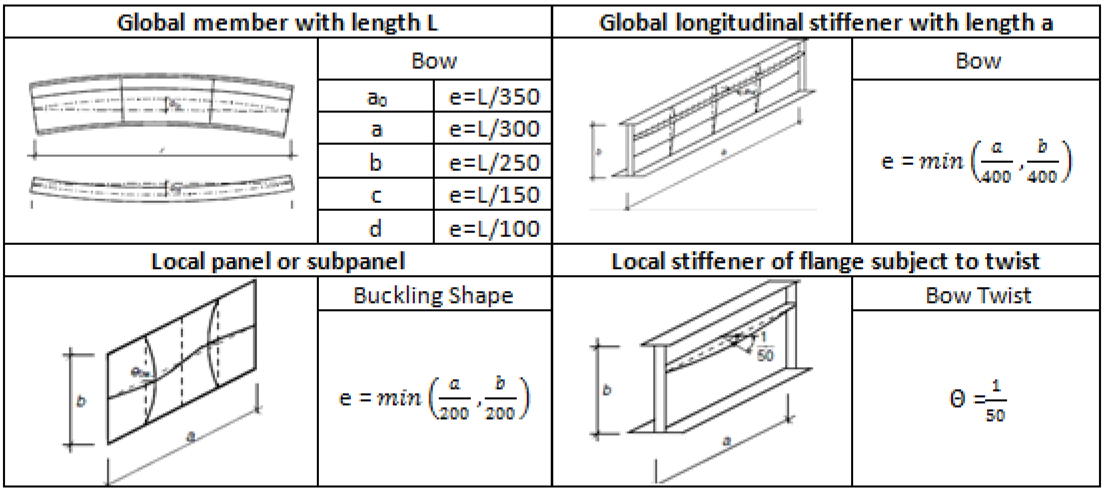


Figure 3.27 Equivalent geometric imperfections. NEN-EN 1993-1-5 Annex C.5 table C.2 combined with figure C.1.

Imperfections can be combined by taking one lead imperfections and combining this with the accompanying imperfections with reduced values to 70%. This combined imperfection can then be applied to the model by applying a fictitious force that results in the proper deformation.

4. Plate Buckling In Eurocode3 And DNV

Most of the recommendations from the previous chapter can be used in order to evaluate a FE model with beam elements. However, for the evaluations of plate buckling this is different. For plate buckling evaluation, stress in more than one direction needs to be calculated per element. Beam elements cannot give these results since they only calculate stress results in one direction.

Research has been done in order to translate plate stresses into beam stresses. With this translation, recommendations from standards can be used to evaluate the FEM results. The recommendations used were that of DNV and ABS, which focus on ship and offshore building.

In this chapter a comparison between the DNV and Eurocode3 recommendations will be made regarding buckling of plates. In the first paragraph the differences for unstiffened plates will be evaluated for axial compression (4.1.1) and transverse compression of plates (4.1.2).

The second paragraph will look at the difference between standards for stiffened plates. Here the buckling check formulas will be split into an axial compression part (4.2.1) and an bending moment part (4.2.2).

References to relative equations in the standards are put in front of the equations in this chapter. They refer to either the Eurocode3 standard or DNV standard.

- EC3: NEN-EN 1993-1-5 Design of steel structures – Part 1-5: Plated structural elements
- DNV: DNV-RP-C201 Buckling strength of plated structures

4.1. Unstiffened Plates

In the EC3 recommendations, the buckling check for stiffened and unstiffened plates is the same. The difference is only made by the effective cross section that has to be used. The DNV standard deals with stiffened and unstiffened plates in different ways. A comparison between the two method follows in this paragraph.

4.1.1. Axial Compression

The recommendations for unstiffened plate buckling in EC3 are already presented in 3.5.2. The plate strength against buckling is tested according to equation (3.19)

EC3 4.14
$$\frac{N_{Ed}}{\frac{f_y A_{eff}}{\gamma_{M0}}} + \frac{M_{Ed} + N_{Ed} e_N}{\frac{f_y W_{eff}}{\gamma_{M0}}} \leq 1.0 \qquad (3.19)$$

The recommendations presented in the DNV standards do not consider effect of bending moments for unstiffened plates. Any bending moment due to misalignment of the load line and neutral axis of the plate is considered to be very small compared to the axial stress. Therefore, for unstiffened plates, the bending moment effects are neglected. The test against plate buckling for axially loaded plates becomes

DNV 6.4
$$\frac{\sigma_{x,Sd}}{\frac{f_y C_x}{\gamma_M}} \leq 1.0 \qquad (4.1)$$

The difference between these two tests can be done by comparing the effective surface area considered. Since EC3 regards axial forces and DNV axial stresses, the C_x factor in equation (4.1), has to be compared with the reduction factor ρ from equation (3.20), that results in the A_{eff} used in EC3.

The reduction factors C_x and ρ are, for both internal compression as well as outstand compression elements, in both standards exactly the same.

	EC3	DNV
Reduction Factor Internal Elements	$\rho = 1{,}0$ for $\bar{\lambda}_p \leq 0{,}673$ $\rho = \frac{\bar{\lambda}_p - 0{,}055(3+\psi)}{\bar{\lambda}_p^2} \leq 1{,}0$ for $\bar{\lambda}_p > 0{,}673$	$C_x = 1$ when $\bar{\lambda}_p \leq 0.673$ $C_x = \frac{\bar{\lambda}_p - 0.055 \cdot (3+\psi)}{\bar{\lambda}_p^2}$ when $\bar{\lambda}_p > 0.673$
Reduction Factor Outstand Elements	$\rho = 1{,}0$ for $\bar{\lambda}_p \leq 0{,}748$ $\rho = \frac{\bar{\lambda}_p - 0{,}188}{\bar{\lambda}_p^2} \leq 1{,}0$ for $\bar{\lambda}_p > 0{,}748$	$C_x = 1$ when $\bar{\lambda}_p \leq 0.749$ $C_x = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p^2}$ when $\bar{\lambda}_p > 0.749$
Plate Slenderness	$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28{,}4\,\varepsilon\sqrt{k_\sigma}}$	$\bar{\lambda}_p = \sqrt{\frac{f_y}{f_{cr}}} = \frac{s}{t} \cdot \frac{1}{28.4\,\varepsilon\sqrt{k_\sigma}}$
Buckling Factor k_σ	Annex A1	Annex A2

Table 7 Comparison between EC3 and DNV recommendations regarding axial plate buckling. Recommendations in both standards are exactly the same. DNV-RP-C201-6.6 and NEN-EN 1993-1-5-4.4.

The only difference between the two recommendations is partial material factor used. Here the DNV recommendations are more conservative with $\gamma_M = 1.15$, as for EC3 $\gamma_{M0} = 1.00$.

4.1.2. Transverse Compression

Verification for plate buckling due to transverse compression is approached very differently in EC3 and DNV. In the recommendations by DNV, the plate is considered to be loaded along the long plate edge with a certain normal stress as depicted in Figure 4.1.

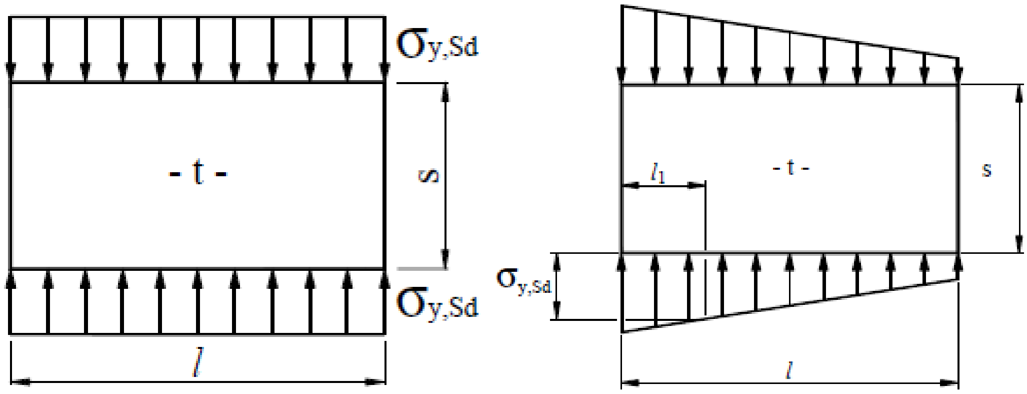


Figure 4.1 Transversely loaded plates with constant stress (left) and varying stress (right) according to DNV

For both situations the plate stability is tested according to the following test.

DNV 6.12
$$\frac{\sigma_{y,Sd}}{\frac{\sigma_{y,R}}{\gamma_M}} \leq 1.0 \qquad (4.2)$$

Which is quite similar to the bucking test for axially loaded plates. However, the buckling resistance ($\sigma_{y,R}$) is calculated with a different expression for the effective width.

DNV 6.6
$$\sigma_{y,R} = C_y \cdot f_y = \left[\frac{1.3t}{l}\sqrt{\frac{E}{f_y}} + \kappa\left(1 - \frac{1.3t}{l}\sqrt{\frac{E}{f_y}}\right)\right] k_p \cdot f_y \qquad (4.3)$$

The value of κ is determined by the plate slenderness. Although slightly more complicated than the effective width factor for axially loaded plates, this equation yields an effective width factor for the length of the plate according to the same principle. Buckling strength for a varying transverse stress is checked by taking the stress value $\sigma_{y,Sd}$ at a length l_1 and half plate width, and regarding that stress value to be constant over the entire length of the plate.

In EC3 the transverse compression forces are regarded to be concentrated forces acting on a flange plate. This force is transferred through the flange plate thickness into a distributed stress in the web plate as can be seen in Figure 4.2.

The transfer of stress is considered to be over an angle of 45°. When the flange thickness is assumed to be half the plate length, the entire edge is loaded with a compressive stress. With this assumption the plate will be loaded according to the left picture from Figure 4.1.

Buckling due to transverse forces is checked by the following equation.

EC3 6.14

$$\eta_2 = \frac{F_{Ed}}{\frac{f_y L_{eff} t}{\gamma_{M0}}} \leq 1.0 \qquad (4.4)$$

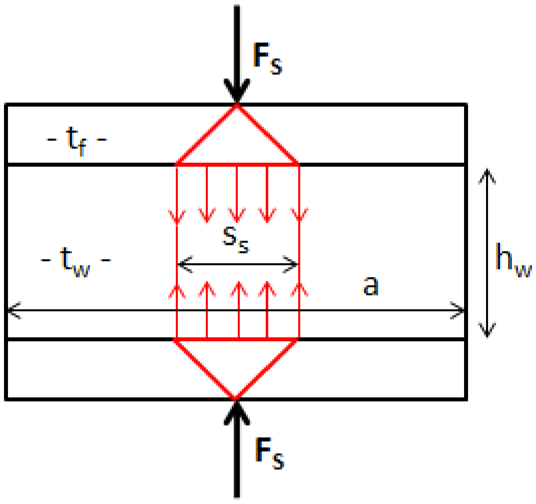


Figure 4.2 Transfer of concentrated force from flange plate into web plate

The effective length (L_{eff}) is a combination of effective load length, which is the length of the plate in this instance, and a reduction factor for the effective length (χ_F). It is this χ_F that should be comparable to the reduction factor from DNV (C_y).

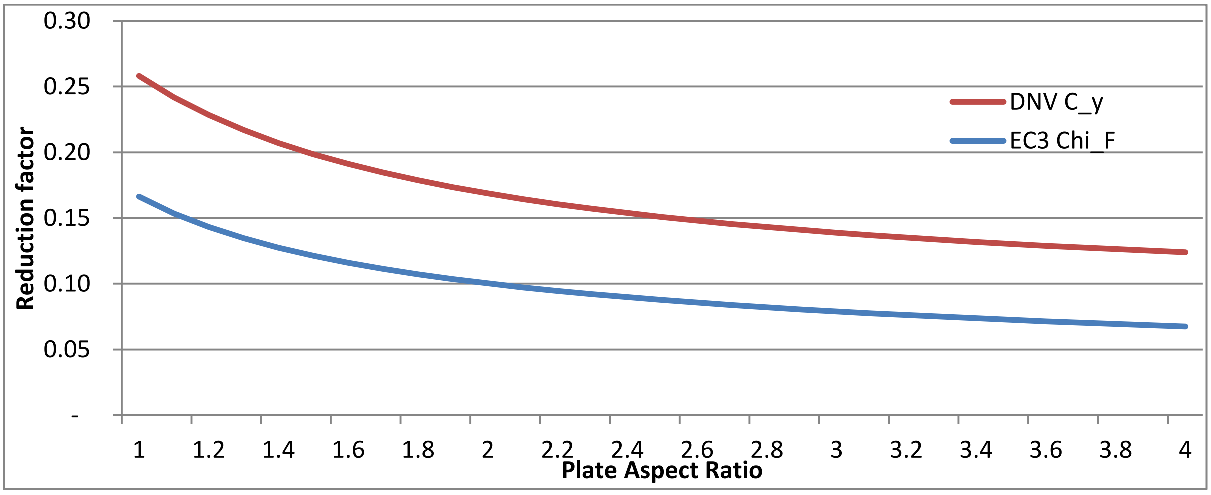


Figure 4.3 Reduction factors for transverse plate buckling from DNV (red) and EC3 (blue)

The graphic shows that the reduction values from the EC3 are about 40% lower than those from the DNV recommendations. Even with the partial factors γ_M and γ_{M0} taken into account, the difference is still about 30%. That would indicate that plates under the EC3 regulations would be able to resist about 30% lower stresses. Any small differences due to safety factors can be expected. A discrepancy of 30% in strength result would sooner indicate that the comparison cannot be made in this way.

The large difference in buckling strength between EC3 and DNV indicates that the recommendations in EC3 for transverse forces must be taken as intended, i.e. as concentrated forces along the edge of the plate instead of compressive stress along the entire length of the plate.

4.2. Stiffened Plates

In both EC3 and DNV recommendations, plate buckling of stiffened plates is considered to be an interaction between beam buckling of the stiffener and connected plate, and plate buckling of the plate between stiffeners. The stiffened plate is considered, in both standards, to be between two girders, as can be seen in the following figure.

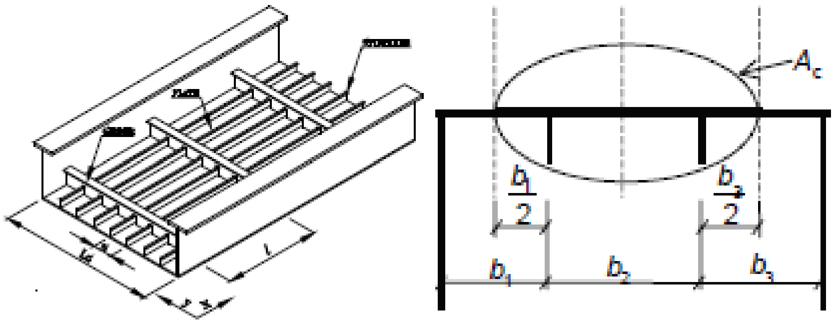


Figure 4.4 Stiffened plate considered to be between girders as presented in DNV (left) and in EC3 (right)

In EC3 recommendations member verification is given for under the influence of uniaxial and biaxial bending. Both expressions are general expressions and should be satisfied along the entire plates length. Verification for uniaxial bending members is done with the following equation, which is already presented before.

EC3 4.14

$$\frac{N_{Ed}}{\frac{f_y A_{eff}}{\gamma_{M0}}} + \frac{M_{Ed} + N_{Ed} e_N}{\frac{f_y W_{eff}}{\gamma_{M0}}} \leq 1.0 \tag{3.19}$$

In the DNV recommendations for buckling of stiffened plates, verification formulas are presented for four different positions. These positions can be seen in [FIGURE INTERACTION POSITIONS] and represent different situations. At the girder side 1 and 2 represent positions for buckling checks of plate and stiffener, respectively. Position 3 and 4 represent the same checks for buckling at midsection.

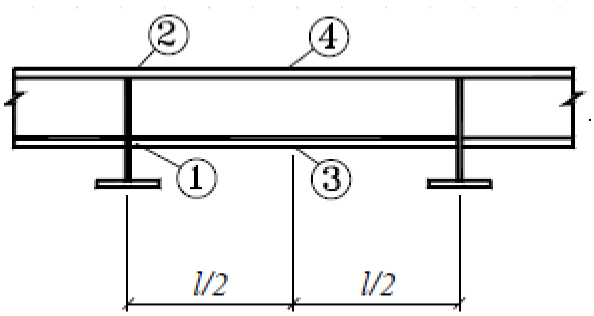


Figure 4.5 Four positions that have to be checked for buckling in the DNV regulations. DNV-RP-C201 figure 10-1.

Because buckling is most likely to occur at position 4, the interaction formulas for that position will be used to compare with the EC3 recommendation.

DNV 7.53

$$\frac{N_{Sd}}{N_{kp,Rd}} + \frac{M_{2,Sd} + N_{Sd} \cdot z^*}{M_{p,Rd}\left(1 - \frac{N_{Sd}}{N_E}\right)} + u \leq 1.0 \tag{4.5}$$

4.2.1. Axial Compression

The first part in both verification formulas is the check for buckling against axial compression. The following equation is the buckling check for axial compression forces from the EC3 recommendations.

$$\frac{N_{Ed}}{\frac{f_y A_{eff}}{\gamma_{M0}}} \tag{4.6}$$

An important factor in the EC3 formula is the effective surface area of the stiffened plate (A_{eff}).

$$A_{eff} = \beta_{ult}\left\{[(\rho - \chi_c)\zeta(2-\zeta) + \chi_c]A_{c,eff,loc} + \sum b_{edge,eff} \cdot t\right\} \tag{4.7}$$

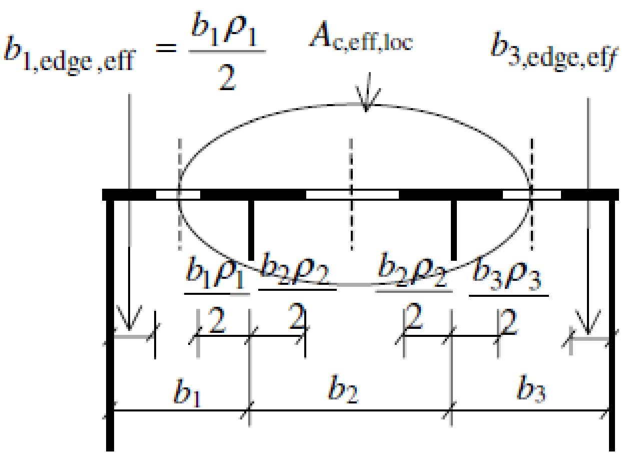


Figure 4.6 The effective cross sectional area of a stiffened plate, $A_{c,eff,loc}$. NEN-EN 1993-1-5-4.5 figure 4.4.

This reduced cross sectional area takes multiple effects into account such as, shear lag (β_{ult}), beam buckling (χ_c) and plate buckling (ρ). What is important to notice, however, is that only axial forces are considered (i.e. only forces along the direction of the stiffeners).

The first part of the validation formula from DNV can be rewritten in the following way.

$$\frac{N_{Sd}}{\frac{f_k A_e}{\gamma_M}} \tag{4.8}$$

This can be compared to the first part from the EC3 formula. Both are ratios between the acting axial force and the resistance against buckling, represented by the material yield strength and an effective cross section. The difference lies in the way the effective cross section is determined.

$$A_e = A_s + s \cdot C_{xs} \cdot C_{ys} \cdot t$$
$$C_{ys} = \sqrt{1 - \left(\frac{\sigma_{y,Sd}}{\sigma_{y,R}}\right)^2 + c_i\left(\frac{\sigma_{x,Sd} \cdot \sigma_{y,Sd}}{C_{xs} \cdot f_y \cdot \sigma_{y,R}}\right)} \tag{4.9}$$

In DNV, the effective cross section (A_e) is influenced not only by the axial stress ($\sigma_{x,Sd}$) but also by the transverse stress ($\sigma_{y,Sd}$).

4.2.2. Bending Moment

The second part in the buckling check from both standards regards buckling under the influence of all bending moments. This includes external bending moments acting on the stiffened plate, bending moments introduced through an eccentricity between load line and neutral axis, but also the bending moment introduced due to lateral pressure on the plates surface.

In EC3, the influence of bending moment is split between the misalignment between load line and neutral axis, and all other bending moments.

$$\frac{M_{Ed} + N_{Ed} \cdot e_N}{\frac{f_y W_{eff}}{\gamma_{M0}}} \tag{4.10}$$

The effective elastic section modulus (W_{eff}) is determined under the assumption that the cross section is only subjected to bending stresses. Non-effective zones (e.g. due to effective width), are excluded when determining the W_{eff}. For Biaxial bending buckling checks, this effective elastic section modulus has to be determined about both main axes.

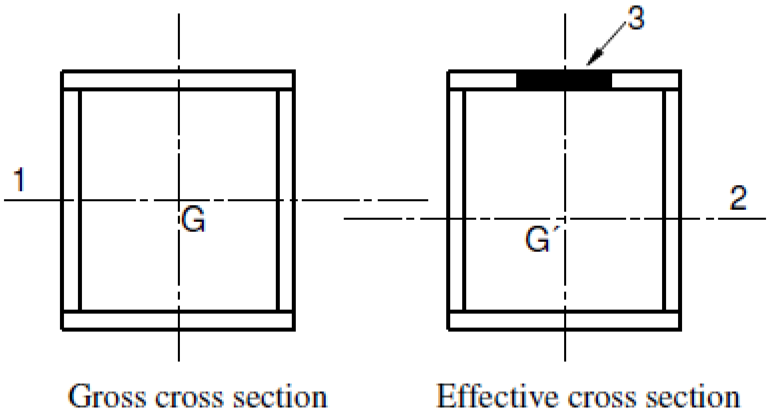


Figure 4.7 Shift of effective cross section centroid from G to G'. Section modulus W_{eff} should be calculated without the non-effective zone (3). NEN-EN 1993-1-5-4.3 figure 4.2.

Again, in EC3, only axial forces are required for the buckling check.

In the DNV regulations the buckling check has the following bending moment part.

$$\frac{M_{2,Sd} + N_{Sd} \cdot z^*}{\frac{f_y W_e}{\gamma_M}\left(1 - \frac{N_{Sd}}{N_E}\right)} \tag{4.11}$$

As can be seen, both DNV and EC3 split the second part of the verification equation into a misalignment part ($N_{Sd} \cdot z^*$), and an external bending moment part ($M_{2,Sd}$). However, unlike EC3, an equation is given in order to calculate the external bending moment.

$$M_{2,Sd} = \left|\frac{q_{Sd} l^2}{24}\right| \tag{4.12}$$

Where the equivalent lateral line load (q_{Sd}) is determined by both the external lateral pressure (p_{Sd}) and the pressure due to buckling under the influence of transverse stress (p_0).

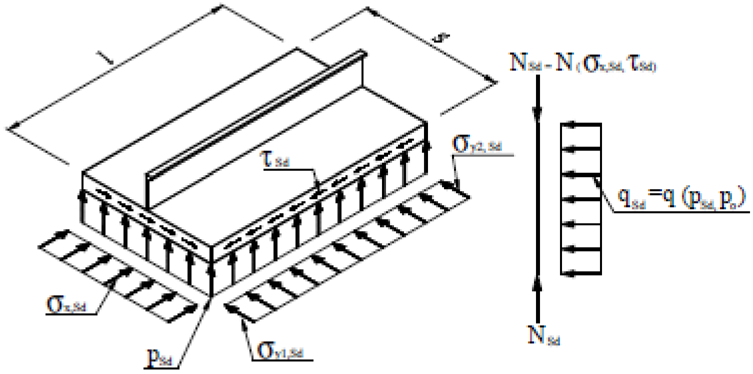


Figure 4.8 Equivalent lateral line load used in DNV to determine buckling strength of stiffened plates. DNV-RP-C201-7.2 figure 7-1.

4.3. EC3 and FEM

As seen in the previous two chapters, the big difference between DNV and EC3 standard is the use of transverse stresses in the buckling check for both stiffened and unstiffened plates. The EC3 recommendations do not use these transverse stresses in the buckling analysis. They only use stresses introduced by axial and bending forces in one directions. This would make the recommendations from EC3 suitable to check FEM beam models for buckling.

However, the effect of transverse stresses cannot simply be ignored. EC3 checks buckling in the direction of the highest stresses. Interaction with concentrated transverse forces is accounted for as can be seen in 3.5.4. Using the equations from these recommendations yield a plate strength that is much lower compared to the results from DNV requirements.

Besides the buckling check for plates, EC3 also makes recommendations for the elastic limit of each plate. In these checks the transverse stresses are an important factor. Even when for plate buckling the transverse stresses can be ignored, for other checks they do become important. Therefore, the EC3 recommendations might be used in order to get quick first results, but for more reliable results, detailed plate models are still required in order to evaluate a structural design with the aid of finite element software

5. Literature

Used Standards

- NEN-EN 1993-1-1 Design of steel structures – Part 1-1: General rules and rules for buildings
- NEN-EN 1993-1-1/NB – National Annex to NEN-EN 1993-1-1
- NEN-EN 1993-1-5 Design of steel structures – Part 1-5: Plated structural elements
- NEN-EN 1993-1-5/NB – National Annex to NEN-EN 1993 1-5
- DNV-RP-C201 – Buckling strength of plated structures

Theory

- Timoshenko, S.P. and Gere, J.M: Theory of Elastic Stability, Second Edition, New York, 2009

Related Reseaches

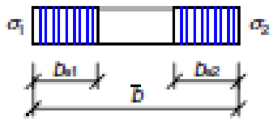
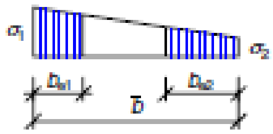
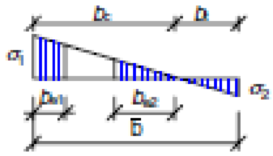
- Kaandorp, R: Plate buckling calculation with the help of FEM-model according to the DIN- and DNV-standard. Delft, 2009
- Coppoolse, J: Semi automated buckling check and stiffener placement optimization. Delft, 2010
- Aberkrom, B: Defining parameters for buckling checks of plated structures in finite element software packages. Delft, 2014

6. Annex A1

EC3 Buckling factor for different stress distributions

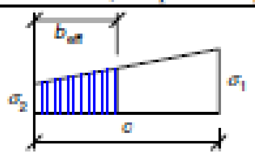
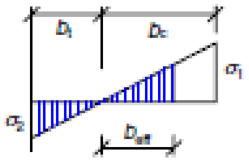
EN 1993-1-5: 2006 (E)

Table 4.1: Internal compression elements

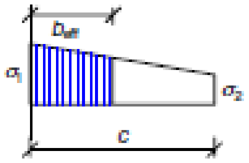
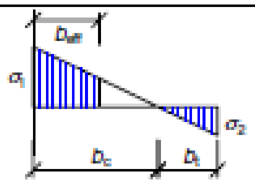
Stress distribution (compression positive)	Effective[p] width b_{eff}
	$\psi = 1$: $b_{eff} = \rho\ \bar{b}$ $b_{e1} = 0{,}5\ b_{eff}$ $\quad b_{e2} = 0{,}5\ b_{eff}$
	$1 > \psi \geq 0$: $b_{eff} = \rho\ \bar{b}$ $b_{e1} = \frac{2}{5-\psi} b_{eff}$ $\quad b_{e2} = b_{eff} - b_{e1}$
	$\psi < 0$: $b_{eff} = \rho\ b_c = \rho\ \bar{b} / (1-\psi)$ $b_{e1} = 0{,}4\ b_{eff}$ $\quad b_{e2} = 0{,}6\ b_{eff}$

$\psi = \sigma_2/\sigma_1$	1	$1 > \psi > 0$	0	$0 > \psi > -1$	-1	$-1 > \psi > -3$
Buckling factor k_σ	4,0	$8{,}2 / (1{,}05 + \psi)$	7,81	$7{,}81 - 6{,}29\psi + 9{,}78\psi^2$	23,9	$5{,}98\ (1-\psi)^2$

Table 4.2: Outstand compression elements

Stress distribution (compression positive)	Effective[p] width b_{eff}
	$1 > \psi \geq 0$: $b_{eff} = \rho\ c$
	$\psi < 0$: $b_{eff} = \rho\ b_c = \rho\ c\ /\ (1-\psi)$

$\psi = \sigma_2/\sigma_1$	1	0	-1	$1 \geq \psi \geq -3$
Buckling factor k_σ	0,43	0,57	0,85	$0{,}57 - 0{,}21\psi + 0{,}07\psi^2$

Stress distribution (compression positive)	Effective[p] width b_{eff}
	$1 > \psi \geq 0$: $b_{eff} = \rho\ c$
	$\psi < 0$: $b_{eff} = \rho\ b_c = \rho\ c\ /\ (1-\psi)$

$\psi = \sigma_2/\sigma_1$	1	$1 > \psi > 0$	0	$0 > \psi > -1$	-1
Buckling factor k_σ	0,43	$0{,}578 / (\psi + 0{,}34)$	1,70	$1{,}7 - 5\psi + 17{,}1\psi^2$	23,8

7. Annex A2

DNV Buckling factor for different stress distributions

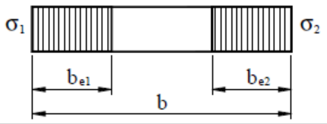
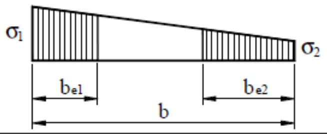
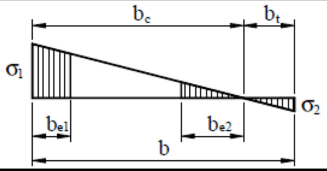
Table 6-1 Effective width for internal compression plate elements		
Stress distribution (compression positive)		Effective width b_{eff}
	$\psi = 1$	$b_{eff} = C_x \cdot b$ $b_{e1} = 0.5\, b_{eff}$ $b_{e2} = 0.5\, b_{eff}$
	$1 > \psi > 0$	$b_{eff} = C_x \cdot b$ $b_{e1} = \frac{2}{5 - \psi} b_{eff}$ $b_{e2} = b_{eff} - b_{e1}$
	$\psi < 0$	$b_{eff} = C_x \cdot b_c = \frac{C_x \cdot b}{1 - \psi}$ $b_{e1} = 0.4\, b_{eff}$ $b_{e2} = 0.6\, b_{eff}$

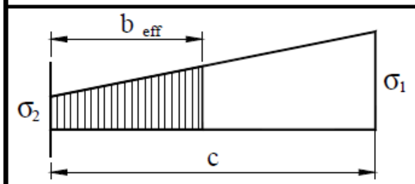
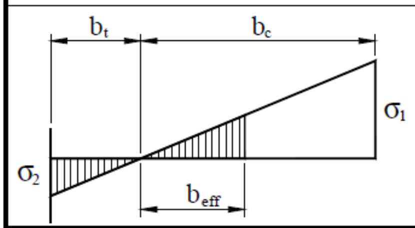
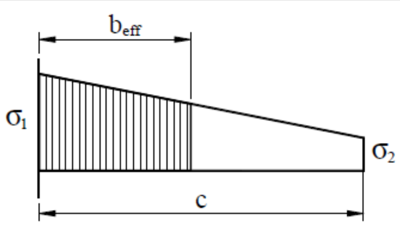
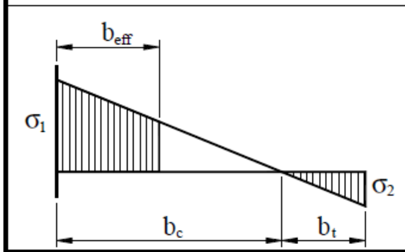
Table 6-2 Effective width for outstand compression plate elements with largest stress at free edge		
Stress distribution (compression positive)		Effective width b_{eff}
	$0 < \psi \leq 1$	$b_{eff} = C_x \cdot c$
	$\psi < 0$	$b_{eff} = C_x \cdot b_c = \frac{C_x \cdot c}{1 - \psi}$

Table 6-3 Effective width for outstand compression plate elements with largest stress at supported edge		
Stress distribution (compression positive)		Effective width b_{eff}
	$0 < \psi \leq 1$	$b_{eff} = C_x \cdot c$
	$\psi < 0$	$b_{eff} = C_x \cdot b_c = \frac{C_x \cdot c}{1 - \psi}$

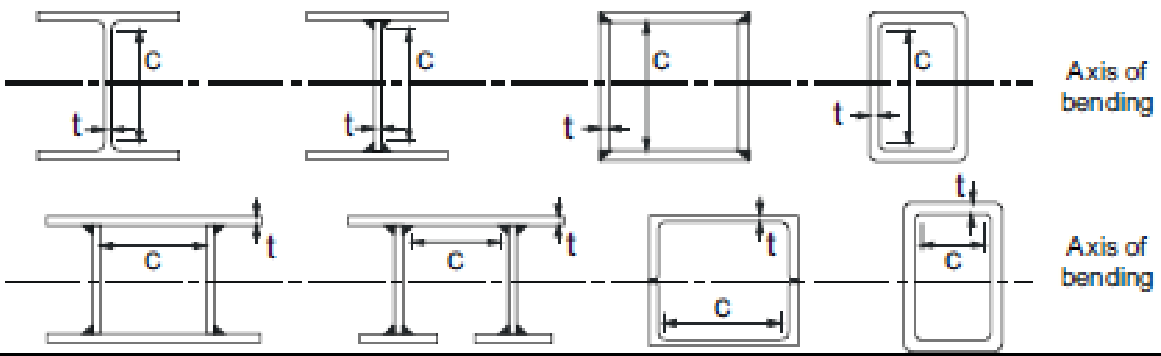
8. Annex B

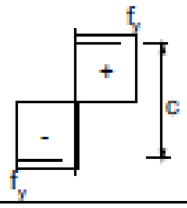
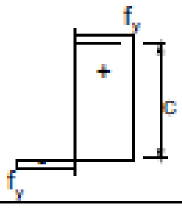
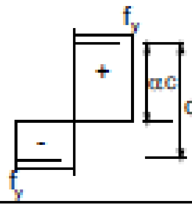
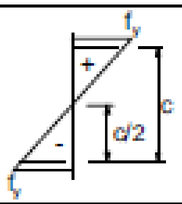
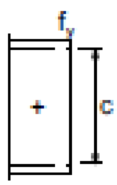
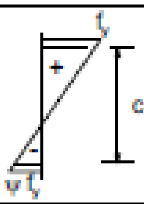
EC3 Cross Section Classes Limitations

EN 1993-1-1: 2005 (E)

Table 5.2 (sheet 1 of 3): Maximum width-to-thickness ratios for compression parts

Internal compression parts



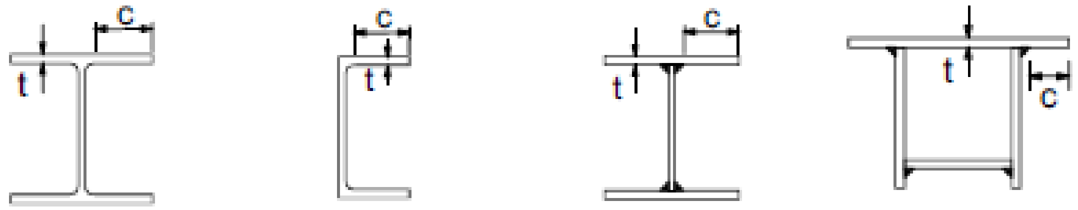
Class	Part subject to bending	Part subject to compression	Part subject to bending and compression
Stress distribution in parts (compression positive)			
1	$c/t \le 72\varepsilon$	$c/t \le 33\varepsilon$	when $\alpha > 0,5$: $c/t \le \dfrac{396\varepsilon}{13\alpha - 1}$ when $\alpha \le 0,5$: $c/t \le \dfrac{36\varepsilon}{\alpha}$
2	$c/t \le 83\varepsilon$	$c/t \le 38\varepsilon$	when $\alpha > 0,5$: $c/t \le \dfrac{456\varepsilon}{13\alpha - 1}$ when $\alpha \le 0,5$: $c/t \le \dfrac{41,5\varepsilon}{\alpha}$
Stress distribution in parts (compression positive)			
3	$c/t \le 124\varepsilon$	$c/t \le 42\varepsilon$	when $\psi > -1$: $c/t \le \dfrac{42\varepsilon}{0,67 + 0,33\psi}$ when $\psi \le -1$ *) : $c/t \le 62\varepsilon(1-\psi)\sqrt{(-\psi)}$

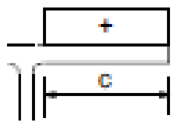
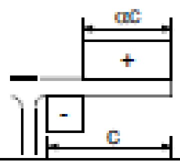
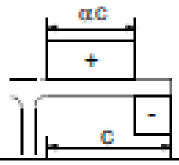
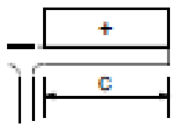
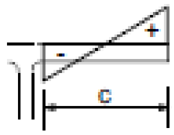
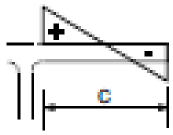
$\varepsilon = \sqrt{235/f_y}$	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71

*) $\psi \le -1$ applies where either the compression stress $\sigma \le f_y$ or the tensile strain $\varepsilon_y > f_y/E$

Table 5.2 (sheet 2 of 3): Maximum width-to-thickness ratios for compression parts

Outstand flanges

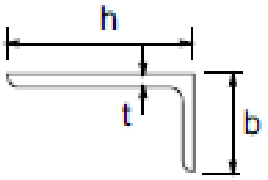


Class	Part subject to compression	Part subject to bending and compression: Tip in compression	Part subject to bending and compression: Tip in tension
Stress distribution in parts (compression positive)			
1	$c/t \le 9\varepsilon$	$c/t \le \frac{9\varepsilon}{\alpha}$	$c/t \le \frac{9\varepsilon}{\alpha\sqrt{\alpha}}$
2	$c/t \le 10\varepsilon$	$c/t \le \frac{10\varepsilon}{\alpha}$	$c/t \le \frac{10\varepsilon}{\alpha\sqrt{\alpha}}$
Stress distribution in parts (compression positive)			
3	$c/t \le 14\varepsilon$	$c/t \le 21\varepsilon\sqrt{k_\sigma}$ For k_σ see EN 1993-1-5	

$\varepsilon = \sqrt{235/f_y}$	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71

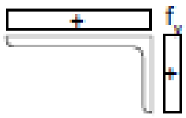
Table 5.2 (sheet 3 of 3): Maximum width-to-thickness ratios for compression parts

Angles

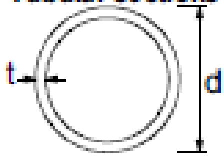


Refer also to "Outstand flanges" (see sheet 2 of 3)

Does not apply to angles in continuous contact with other components

Class	Section in compression
Stress distribution across section (compression positive)	
3	$h/t \leq 15\varepsilon : \dfrac{b+h}{2t} \leq 11{,}5\varepsilon$

Tubular sections



Class	Section in bending and/or compression
1	$d/t \leq 50\varepsilon^2$
2	$d/t \leq 70\varepsilon^2$
3	$d/t \leq 90\varepsilon^2$ **NOTE** For $d/t > 90\varepsilon^2$ see EN 1993-1-6.

$\varepsilon = \sqrt{235/f_y}$	f_y	235	275	355	420	460
	ε	1,00	0,92	0,81	0,75	0,71
	ε^2	1,00	0,85	0,66	0,56	0,51