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Mechanical Cloaking of Halftoned Imagery

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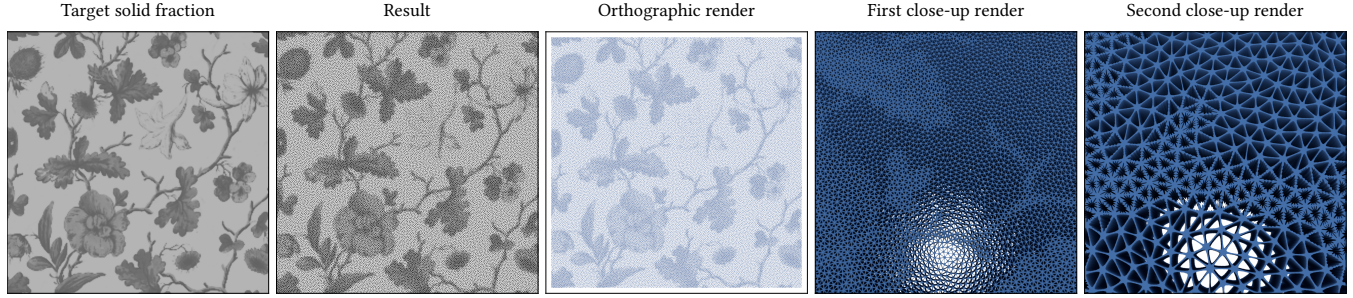


Fig. 1. Our approach takes a grayscale image (target) and generates a porous material (result) that halftones the image while preserving near-isotropic and near-homogeneous linear elastic properties. This enables mechanical cloaking of the input image. The three images on the right show renders of a thin plate representing the physical implementation of our method.

Cloaking objects with metamaterials has been extensively studied to hide internal objects across various physical properties, including optical, acoustic, and thermal. We explore a new direction in mechanical cloaking: halftoning an image using a porous structure that behaves like a uniform, linear, isotropic material and visually matches a target image. For an external observer, this creates the surprising effect where the object appears mechanically isotropic and homogeneous while its porous structure resembles a target image. We introduce a parametric class of porous structures suitable for this problem, as demonstrated by numerical experiments. The structures we define offer a wide range of visual contrasts, enabling effective halftoning while maintaining near isotropic effective mechanical properties.

CCS Concepts: • **Applied computing** → **Computer-aided design**; • **Computing methodologies** → **Computer graphics**.

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1 Introduction

The ability to render objects invisible to an external observer, i.e., *cloaking*, has captivated researchers. Cloaking science seeks to conceal objects across many physical properties, including optical, thermal, electrical, and mechanical. The advent of metamaterials that can effectively grade and optimize multiscale physical properties has significantly fostered cloaking science.

This paper presents a novel research direction: designing a porous metamaterial that simultaneously resembles a given image and exhibits homogeneous linear elastic behavior. Unlike traditional mechanical cloaking, our goal is not to mechanically hide an object. Instead, we strive for mechanical homogeneity while strikingly revealing a porous structure that halftones an image. This is nontrivial because visual contrast is achieved through changes in the solid fraction, which typically affect a material's stiffness. To address this problem, we devised a parametric family of porous structures with pores, ranging from smooth to spiky, drilled into triangles. This family enables us, in turn, to extract a suitable subspace for our cloaking problem to create halftoning patterns that are nearly isotropic from a mechanical standpoint.

Potential applications of our method include architectural panels, fashion elements, and other domains where motifs need to be cut out while preserving the uniform mechanical behavior of the supporting structure (see Figure 20). Our approach also applies wherever visual information must be encoded in metamaterials without altering mechanical properties. The code and data of our method are available at <https://github.com/jonasmb/halftone-mech-cloak>.

2 Related work

Computer graphics has a long history of exploring the interplay of geometry and physics to produce striking visual effects. For example, methods have been proposed for optimizing shapes that cast specific shadows from different angles [Mitra and Pauly 2009], enhancing the perception of relief on low-profile shapes [Schüller et al. 2014; Weyrich et al. 2007], or modifying an object’s geometry to minimize its reflectivity, making it harder to see under a light beam [Tojo et al. 2023]. Another line of work explored the inverse design of caustic patterns, so that optically transparent shapes produce target images when light shines through [Papas et al. 2011; Schwartzburg et al. 2014]. This transforms a seemingly homogeneous transparent glass panel into a device that casts a complex pattern of light and shadow. We pursue a similar objective by introducing a controlled visual effect in a plate while ensuring its mechanical response remains homogeneous, despite visual cues to the contrary. As the changes we introduce are mechanically neutral [Mansfield 1953], we achieve a form of *mechanical cloaking*: in the linear regime, our patterned plates exhibit effective (i.e., average) elastic properties that are both nearly isotropic and close to the target properties.

Mechanical cloaking is an active research direction, part of a more general effort around physical cloaking devices [Fleury et al. 2015; Martínez and Maldovan 2022]. The most common goal in the literature is to hide a region from an external observer. In elasticity, this means preserving the observable stress-strain behavior along the domain’s boundary, so the behavior remains the same before and after inserting the cloaked region.

Many methods in this field follow the so-called transformation approach: a neutral spherical hole or inclusion is created by continuously deforming the surrounding physical medium. The deformation impacts the material properties, and the required properties can then be approximated with metamaterials [Milton et al. 2006; Norris and Shuvalov 2011]. This is theoretically possible for any phenomenon whose form is invariant under coordinate transformation (e.g., electromagnetism [Pendry et al. 2006]). However, this is not the case for elasticity [Milton et al. 2006], so most approaches use approximations [Bückmann et al. 2015] and optimization [Sanders et al. 2021]. Nassar et al. [2020] later showed that *polar* metamaterials – exhibiting asymmetric stresses – are required to achieve perfect elastic cloaking [Xu et al. 2020]. Elastic cloaking has been practically demonstrated in plates [Farhat et al. 2009; Stenger et al. 2012; Xu et al. 2020] and volumes [Bückmann et al. 2014]. Most recent works in the field explore active devices that could recover from local damage [Kundu et al. 2024] or be selectively activated [Zhang et al. 2024]. Lu et al. [2025] perform multi-scale topology optimization to achieve mechanical cloaking under a single load. Senhora et al. [2025] identify a set of worst-case loads and employ a graded porous structure to cloak an inclusion with arbitrary geometry. Yang et al. [2025] consider a growth strategy of periodic tiles that achieves elastic cloaking under varied loads and internal inclusion geometries. While earlier efforts have focused on mechanically cloaking internal regions, to our knowledge, patterning porosity to achieve a halftoning effect while preserving a globally homogeneous isotropic response has not yet been explored.

Coming back to computer graphics, our method builds on both halftoning techniques [Ulichney 1987] to create a visual pattern and mechanical metamaterials. Early works focus on linear elastic periodic [Panetta et al. 2015; Schumacher et al. 2015] or stochastic [Martínez et al. 2016] metamaterials. More recent studies address nonlinear elasticity [Li et al. 2023; Zhang et al. 2023]. Our work tackles a new problem combining both mechanical cloaking and halftoning.

3 Method

This section introduces a parametric porous structure that enables the creation of materials with spatially varying solid fraction and near-constant linear-isotropic elastic properties. We define the geometry of the porous structure in Section 3.1 and analyze its mechanical behavior in Section 3.2.

3.1 Porous structure

We start by introducing a family of porous structures made of solid triangles with perforated pores. Our idea is to modulate the appearance of small spikes in the pores. These spikes do not affect the overall stiffness much, but they progressively increase the relative solid fraction of the pore. This will later allow us to spatially vary the solid fraction of the pores while globally maintaining nearly homogeneous elastic properties. To do so, we distinguish the following steps (see Figure 2):

- (1) **Triangulation**: Given a blue-noise point set $\mathcal{P}$ [Balzer et al. 2009] we compute its Delaunay triangulation $\mathcal{D}$ to obtain evenly distributed triangles.
- (2) **Indicator function $\mathcal{S}$** : We define a global indicator function $\mathcal{S}$ to determine whether a point p in a triangle T of $\mathcal{D}$ is solid or void material.

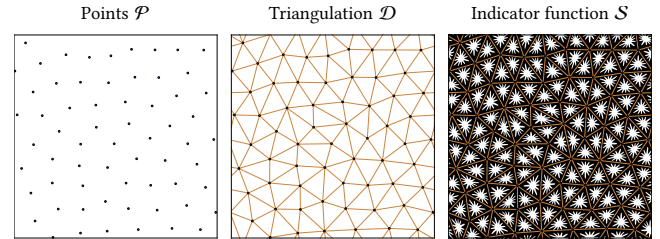


Fig. 2. An overview of the porous structure definition. Starting with a blue noise point set $\mathcal{P}$ (left), we compute its Delaunay triangulation $\mathcal{D}$ (center). We then define the void and solid material phases with a parametric function $\mathcal{S}$ defined for any point in a triangle T (right).

The following steps describe how to obtain the indicator function $\mathcal{S}$. Let $\lambda_1, \lambda_2, \lambda_3$ be the normalized barycentric coordinates of a point p with respect to a triangle T with vertices v_1, v_2, v_3 . The point p can be expressed as

$$p = \lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 \quad (1)$$

where the coordinates satisfy $\lambda_1 + \lambda_2 + \lambda_3 = 1$.

We first define the function $\mathcal{R}$ within the interior of T as

$$\mathcal{R} = 27(\lambda_1 \lambda_2 \lambda_3) \in [0, 1] \quad (2)$$

that vanishes to 0 on the boundary of T and reaches its maximum value at the centroid C where $\lambda_1 = \lambda_2 = \lambda_3 = \frac{1}{3}$. We multiply the product of the barycentric coordinates by 27 to normalize the function so its values range from 0 to 1. Intuitively, the isolines of $\mathcal{R}$ define a continuous contraction from the boundary of the triangle toward its centroid. Next, we define the function $\mathcal{E}$ in T as

$$w = (0, 0) \left(\lambda_1 - \frac{1}{3} \right) + (1, 0) \left(\lambda_2 - \frac{1}{3} \right) + \left(\frac{1}{2}, \frac{\sqrt{3}}{2} \right) \left(\lambda_3 - \frac{1}{3} \right) \quad (3)$$

$$\mathcal{E} = \left| \sin \left(\frac{n}{2} \operatorname{atan2}(w_1, w_0) \right) \right|^d \in [0, 1] \quad (4)$$

where $d \geq 1$, and n is a positive integer. $\mathcal{E}$ is a radial oscillating function originating from the centroid C with period n and exponent d , which results in faster oscillations for larger d . The vector $w = (w_0, w_1)$ from Equation (3) maps the barycentric coordinates onto an equilateral triangle with an origin in C , and is used in Equation (4) to obtain an angle. This mapping ensures the inner shape is symmetric with respect to the triangle's vertices. As shown in Figure 3, three spikes align with the vertices while the rest transition smoothly along the edges. Note that the triangles given by the Delaunay triangulation of a blue noise point set are expected to be close to equilateral. In all results, we use $n = 12$ and $d = 3$. We then define $\mathcal{U}$ in T as

$$\mathcal{U} = \mathcal{R}^{(1+\varphi\mathcal{E})} \in [0, 1] \quad (5)$$

where $\varphi \geq 0$ modulates the effect of $\mathcal{E}$ and, in turn, the triangle solid fraction. The Heaviside function $H(x)$ is defined as

$$H(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases} \quad (6)$$

Finally, with a threshold parameter $\beta \in [0, 1]$, we define the pore indicator function $\mathcal{S}$ to distinguish between the void ($= 0$) and solid phase ($= 1$).

$$\mathcal{S} = H(\beta - \mathcal{U}) \quad (7)$$

See Figure 3 for an illustration of each triangular function, Figure 4 for different combinations of φ and β , and Algorithm 1 for the pseudocode to compute $\mathcal{S}$ for some given barycentric coordinates. In the following, $\mathcal{S}$ may represent either a single triangle or a collection of triangles.

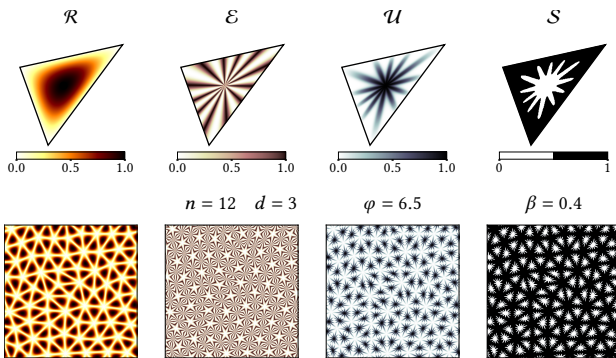


Fig. 3. $\mathcal{R}$ defines a continuous retraction towards the centroid, $\mathcal{E}$ is a radial oscillating function, $\mathcal{U}$ combines both effects, and $\mathcal{S}$ is a binary phase thresholding $\mathcal{U}$.

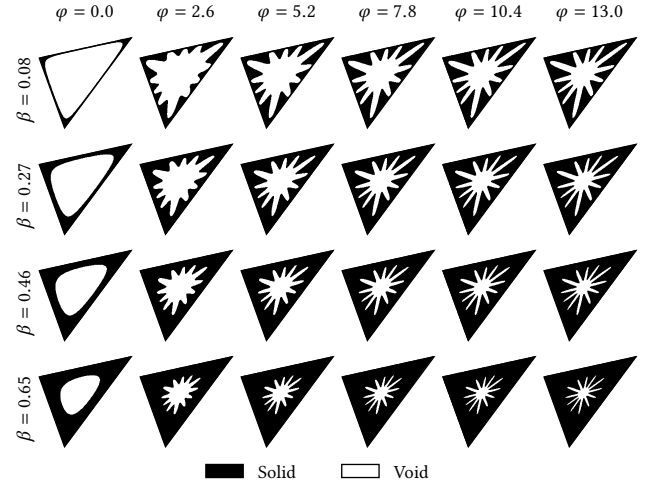


Fig. 4. Illustration of the impact of φ and β . From left to right, the value of φ increases, and from top to bottom, the value of β increases.

ALGORITHM 1: Computation of $\mathcal{S}$

Input: Barycentric coordinates $\lambda_1, \lambda_2, \lambda_3$ and parameters β and φ	
Output: 0 (void phase) or 1 (solid phase)	
Compute $\mathcal{R}$;	/* eq. (2) */
Compute $\mathcal{E}$;	/* eq. (4) */
Compute $\mathcal{U}$ with $\mathcal{R}$ and $\mathcal{E}$;	/* eq. (5) */
return $H(\beta - \mathcal{U})$;	/* eq. (7) */

Computation. The function $\mathcal{S}$ can be approximated using a 2D image $\mathcal{Z}$ with resolution n^2 , as required by the mechanical analysis in Section 3.2. However, please note that $\mathcal{S}$ is not necessarily tied to a grid representation and could be meshed with other techniques.

For each image pixel, we assign the value returned by Algorithm 1. Unless stated otherwise, all results use a supersampling grid of size 2^2 , producing a grayscale image $\mathcal{Z}$ with pixel values in $[0, 1]$, computed as the average of four samples. Each pixel of $\mathcal{Z}$ can be independently computed in parallel.

Properties. Let $A > 0$ be the triangular area. To calculate the solid fraction area of a non-degenerate triangle, consider

$$A_s = 2A\bar{\mathcal{S}} \quad \text{where} \quad \bar{\mathcal{S}} = \int_0^1 \int_0^{1-\lambda_2} \mathcal{S} d\lambda_1 d\lambda_2 \quad (8)$$

The relative solid fraction $\tau \in [0, 1]$ is given by

$$\tau = \frac{A_s}{A} = 2\bar{\mathcal{S}} \quad (9)$$

Thus, τ is independent of triangle geometry, as the integral depends only on barycentric coordinates. Our technique reduces statistical fluctuations compared to standard cellular materials defined by thickened solid edges, as shown in Figure 5. Notably, this lower statistical variance applies to any indicator function parameterized with barycentric coordinates, and is not tied to our particular definition of $\mathcal{S}$.

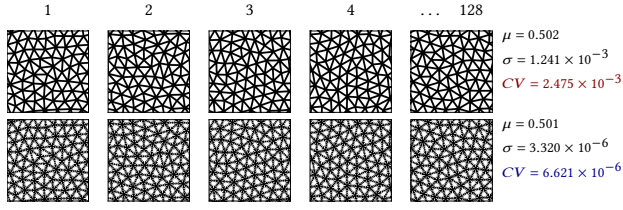


Fig. 5. Comparison of the coefficient of variation (CV) of the solid fraction between a Delaunay triangulation with fixed-thickness edges (top row) and our method (bottom row), both trying to closely match the mean solid fraction. We computed 128 different random realizations with a resolution of $n = 2^{12}$, with statistics given on the right: mean μ , standard deviation σ , and $CV = \frac{\sigma}{\mu}$. While our method theoretically yields $\sigma = CV = 0$, discretization into a grid introduces minor numerical deviations. The conventional approach using fixed-thickness edges exhibits a significantly higher CV, three orders of magnitude greater than ours.

In the following analysis, we assume the triangulation is periodic. Since each triangle always has a solid fraction of τ (see Equation (9)), it follows that the expected value of any periodic triangulation is $E[\mathcal{S}] = \tau$. Since $\mathcal{S}$ is a binary function then $E[\mathcal{S}^i] = E[\mathcal{S}] = \tau$, for $i > 1$. Thus, the variance can be expressed as $V[\mathcal{S}] = E[\mathcal{S}^2] - E[\mathcal{S}]^2 = \tau(1 - \tau)$.

$\mathcal{S}$ is better approximated with higher resolution of $\mathcal{Z}$ and fewer points $\mathcal{P}$. However, a low number of points results in a poor approximation of the macroscopic elastic behavior, as shown in Section 3.2. Conversely, the accuracy of image rasterization of $\mathcal{S}$ rapidly degrades if the number of points is too large relative to the resolution. Thus, there is a delicate balance between the resolution n^2 of $\mathcal{Z}$ and the number of points k that we seek to analyze. To do so, we choose to reason in terms of the mean number of pixels per triangulation point given by $n_k = \frac{n^2}{k}$. From the numerical analysis in Figure 6, we determine that $n_k = \frac{1024^2}{64} = 2^{14}$ reasonably approximates the geometry $\mathcal{S}$ while keeping computational cost moderate.

3.2 Mechanical analysis

In this section, we present the main ingredients of the mechanical analysis and study the mapping between the parameters controlling the porous structure and its macroscopic elastic behavior. Using this inverse mapping, we define a parametric subset of porous structures that halftone a given grayscale input field while preserving the isotropic Young's modulus constant.

We begin by introducing concepts for the multi-scale mechanical analysis of thin plates. We investigate the macroscopic linear elastic behavior of porous structures, assuming they consist of infinitesimally small pores, which enables extrapolation of a globally homogeneous elastic behavior using stochastic homogenization techniques [Blanc et al. 2016; Kanit et al. 2003; Zerhouni et al. 2019].

Technically, we want to approximate the macroscopic effective elasticity tensor $\tilde{\mathbf{C}}$ that relates strains ϵ and stresses σ following the linear relation $\sigma = \tilde{\mathbf{C}}\epsilon$ that holds for infinitesimal strains. We use compressed Mandel notation [Mandel 1965] to represent with an

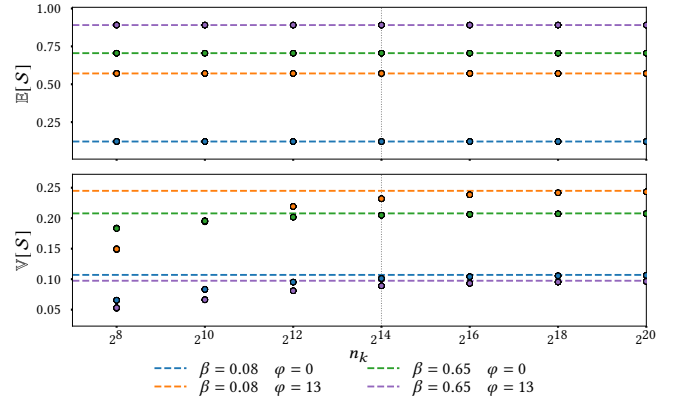


Fig. 6. Analysis of the expectation $E[\mathcal{S}]$ and variance $E[\mathcal{S}]$ with respect to increasing values of n_k . Dashed lines denote the results derived by numerically integrating $\tilde{\mathcal{S}}$ in Equation (8). Note that although $\mathcal{S}$ is a binary function, it is approximated with a grayscale image $\mathcal{Z}$ via supersampling with values between zero and one that more finely capture the boundary transitions. For each n_k , we compute 16 different realizations (properties plotted by points) that consider a periodic triangulation with points $k = 64$ and an increasing n_k . We observe a larger deviation from the expected variance because some pixels in $\mathcal{Z}$ have intermediate grayscale values. Selecting $n_k = 2^{14}$ gave a maximum percent relative error of 0.04% for the mean and 8.64% for the variance among all random realizations.

isomorphism the fourth-order elasticity tensor $\{\tilde{\mathbf{C}}_{ijkl}\}$

$$\begin{pmatrix} \sigma_{11} \\ \sigma_{22} \\ \sqrt{2}\sigma_{12} \end{pmatrix} = \begin{pmatrix} C_{1111} & C_{1122} & \sqrt{2}C_{1112} \\ C_{2211} & C_{2222} & \sqrt{2}C_{2212} \\ \sqrt{2}C_{1211} & \sqrt{2}C_{1222} & 2C_{1212} \end{pmatrix} \begin{pmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \sqrt{2}\epsilon_{12} \end{pmatrix} \quad (10)$$

This work considers two-dimensional plane stress, loosely assuming the stress is negligible in the plane's orthogonal direction. Plane stress is suited for analyzing two-dimensional plate structures where the perpendicular dimension is much smaller than the others (i.e., thin plates), leading to the following decomposition of the elasticity tensor $\mathbf{C}_{iso}$

$$\mathbf{C}_{iso} = 2\kappa\mathbf{J} + 2\mu\mathbf{K} \quad (11)$$

where κ is the bulk modulus, μ is the shear modulus, and the matrices $\mathbf{J}$ and $\mathbf{K}$ are

$$\mathbf{J} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \mathbf{K} = \mathbf{I} - \mathbf{J} \quad (12)$$

where $\mathbf{I}$ is the identity matrix, and $\mathbf{J}$ and $\mathbf{K}$ are the spherical and deviatoric projection matrices, respectively. Recall the following conversion formulae for plane stress between Young's modulus E , Poisson's ratio ν , bulk κ modulus, and shear modulus μ

$$\begin{aligned} \kappa(E, \nu) &= \frac{E}{2(1 - \nu)} & \mu(E, \nu) &= \frac{E}{2(1 + \nu)} \\ \nu(\kappa, \mu) &= \frac{\kappa - \mu}{\kappa + \mu} & E(\kappa, \mu) &= \frac{4\kappa\mu}{\kappa + \mu} \end{aligned} \quad (13)$$

Combining Equations (11) and (13) leads to

$$\mathbf{C}_{iso} = \frac{E}{1-\nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu \end{pmatrix} \quad (14)$$

Closest isotropic elasticity tensor. We conjecture that our family of porous structures behaves in a mechanically isotropic manner and verify this assumption as follows. Let $\langle A, B \rangle$ be the Frobenius inner product between matrices A and B and $\|A\|_F$ be the Frobenius norm of A . We seek to find the matrix $\mathbf{C}_{iso}$ of an isotropic material closest to $\tilde{\mathbf{C}}$ with respect to the Frobenius norm [Moakher and Norris 2006]

$$\min_{\kappa, \mu} \|\tilde{\mathbf{C}} - \mathbf{C}_{iso}\|_F \quad (15)$$

It is known that the minimizing values κ and μ are given by the projection operators [Gazis et al. 1963]

$$2\kappa = \frac{\langle \tilde{\mathbf{C}}, \mathbf{J} \rangle}{\langle \mathbf{J}, \mathbf{J} \rangle} \quad 2\mu = \frac{\langle \tilde{\mathbf{C}}, \mathbf{K} \rangle}{\langle \mathbf{K}, \mathbf{K} \rangle} \quad (16)$$

Since $\langle \mathbf{J}, \mathbf{J} \rangle = 1$ and $\langle \mathbf{K}, \mathbf{K} \rangle = 2$, it follows that

$$\kappa = \frac{\langle \tilde{\mathbf{C}}, \mathbf{J} \rangle}{2} \quad \mu = \frac{\langle \tilde{\mathbf{C}}, \mathbf{K} \rangle}{4} \quad (17)$$

We quantify the deviation from isotropy incurred by projecting to the closest isotropic tensor given in Equation (17). To do so, we consider the normalized criteria of [Zerhouni et al. 2019]

$$\delta_{iso} = \frac{\|\tilde{\mathbf{C}} - \mathbf{C}_{iso}\|_F}{\|\mathbf{C}_{iso}\|_F} \geq 0 \quad (18)$$

A value of δ_{iso} close to zero indicates isotropy.

Hashin-Shtrikman bounds. First, we review the Hashin-Shtrikman (HS) upper bounds for bulk, shear, and Young's modulus [Hashin and Shtrikman 1963] to quantify the coverage of achievable isotropic material properties. Since one of the phases is void, we only consider the upper bound; the lower bound converges to zero for solid fractions lower than one.

Let κ_s , μ_s , and E_s be respectively the bulk, shear, and Young's modulus of the solid phase. Let us recall that $\tau \in [0, 1]$ is the solid fraction. Then, the upper bound of the bulk modulus is [Torquato et al. 1998]

$$0 \leq \kappa \leq \kappa_{up}^{HS} = \frac{\kappa_s \mu_s \tau}{(1-\tau)\kappa_s + \mu_s} \quad (19)$$

for the shear modulus is

$$0 \leq \mu \leq \mu_{up}^{HS} = \frac{\kappa_s \mu_s \tau}{(1-\tau)(\kappa_s + 2\mu_s) + \kappa_s} \quad (20)$$

and for Young's modulus is

$$0 \leq E \leq E_{up}^{HS} = \frac{E_s \tau}{3-2\tau} \quad (21)$$

Computational homogenization. We now detail our computational homogenization method to approximate the macroscopic effective elasticity tensor $\tilde{\mathbf{C}}$. We extract some periodized samples of the stochastic porous structure, usually known as the Representative Volume Elements (RVEs) [Kanit et al. 2003]. Next, we apply the FFT homogenization method from [Brisard and Dormieux 2010] to find the effective elasticity tensor for each RVE sample. The overall effective elasticity tensor, $\tilde{\mathbf{C}}$, is then calculated as the average of all

RVEs [Blanc et al. 2016]. In our implementation, we consider 8 different RVEs, a base isotropic material with solid Young's moduli $E_s = 1$ phase, and a constant Poisson's ratio $\nu_s = 0.3$.

We transform the grayscale values of the grid $\mathcal{Z} \in [0, 1]$, approximating the function $\mathcal{S}$ to a varying field of Young's modulus $\mathcal{Z}_E \in [0, E_s]$. We use the upper bound E_{up}^{HS} from Equation (21) so that $\mathcal{Z}_E = \frac{E_s \mathcal{Z}}{3-2\mathcal{Z}}$. We keep Poisson's ratio ν_s constant everywhere. Since we set the Young's modulus of the solid phase to one, our results directly represent the relative effective Young's modulus, calculated by dividing the effective Young's modulus by that of the solid phase.

Convergence. While increasing the space covered by the RVE enhances the approximation's accuracy [Bourgeat and Piatnitski 2004], we are limited by available computational resources. We aim to determine if the grid resolution of $n = 1024$ and the number of triangulation points $k = 64$ determined in Section 3.1 accurately capture the average elastic properties.

We focus the convergence analysis on the case when β is small and φ is big since, as shown in Figure 6, this case requires more grid resolution to capture the geometry adequately. Given a constant mean number of pixels per point $n_k = 2^{14}$, we accordingly increase the number of triangulation points and resolution until we observe that the spread of κ and μ moduli among random realizations stabilizes. The results shown in Figure 7 also indicate that the porous structure exhibits nearly isotropic behavior since δ_{iso} remains relatively low, and its average value monotonically decreases.

Invariance of effective Young's modulus. To validate our results, we also numerically verify the Cherkashev-Lurie-Milton (CLM) theorem proven in [Cherkashev et al. 1992] and experimentally developed in [Thorpe and Jasiuk 1992], which implies that the effective Young's modulus E under plane stress for a porous material is independent of the solid phase's Poisson's ratio ν_s . Figure 8 shows that for fixed parameters β and φ , the value of ν_s has no impact on the effective Young's modulus, and only a moderate influence on the effective Poisson's ratio as the solid fraction decreases. This means that for small τ , the base Poisson's ratio ν_s has little impact on the effective Poisson's ratio.

3.3 Material space

Equipped with a method to compute the macroscopic elastic moduli, we explore the space of porous structures spawned by β and φ . We sample this space with a homogeneous partition of $\varphi \in [0, 13]$, and $\beta \in [0.08, 0.65]$ in 16 samples, forming a Cartesian product that results in 16^2 combinations. For each parameter combination, we consider 8 random RVEs and compute the effective elasticity $\tilde{\mathbf{C}}$ as the average of the homogenized elasticity tensors from these 8 RVEs. The results of this numerical exploration are shown in Figure 9.

Interpolation. Given the homogenized samples shown in Figure 9, we reconstruct the relationship between the parameters β and φ through interpolation. We assume that physical properties do not vary abruptly for sufficiently closely spaced samples. Figures 10 and 11 show the results using cubic interpolation. The effective

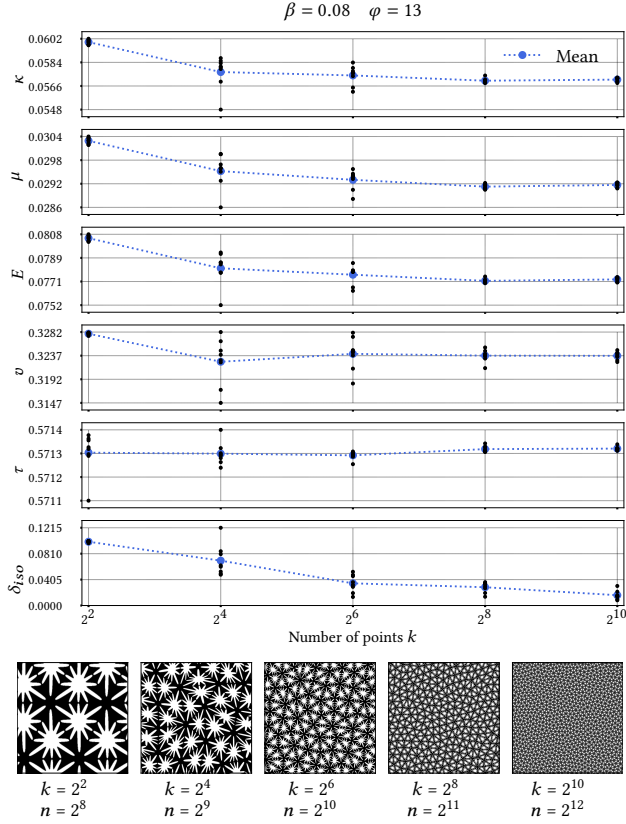


Fig. 7. Convergence analysis of computational homogenization. *Top*: Assessment of properties (κ , μ , E , ν , τ , δ_{iso}) for fixed $n_k = 2^{14}$ and increasing resolution n and number of points k , from left to right. For each combination, we consider 8 different random samples. Note that as the number of points increases, the resolution increases according to the constant value of n_k , seeking to maintain a constant ratio of pixels per triangulation point. *Bottom*: Some sample porous structures with an increasing number of points, shown from left to right.

Young's modulus is in the range $E \in [0.03, 0.507]$, while the solid fraction varies between $\tau \in [0.121, 0.891]$.

In Figure 11 we observe that lower β values are correlated with lower effective Young's modulus E , while higher β values show the opposite trend. Similarly, lower φ values result in lower effective Young's modulus, with higher φ values yielding the reverse effect.

Figure 12 illustrates the spread of effective Poisson's ratio, which remains relatively small with $\nu \in [0.293, 0.357]$, though ideally, it would be constant throughout. The results Section 4 show that this slight deviation from a constant effective Poisson's ratio has a limited impact. We also observe in Figure 11 that small increments from $\varphi = 0$ cause a sudden decrease in Poisson's ratio. This transition corresponds to the case where pores lack spikes ($\varphi = 0$) and then small spikes start to emerge.

Solid fraction coverage. We seek to compactly parameterize a subspace of porous structures with target constant Young's modulus E_t and a slightly varying Poisson's ratio ν_t , spanning a broad range

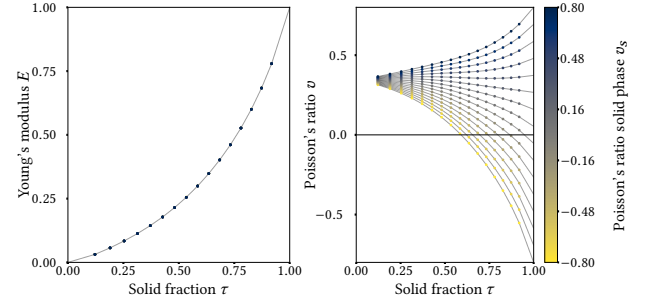


Fig. 8. Numerical verification of the invariance of effective Young's modulus with respect to variations in the Poisson's ratio ν_s of the solid phase. We consider $\nu_s \in [-0.8, 0.8]$ and $\beta = [0.08, 0.9]$, with a fixed solid fraction given by $\varphi = 0.0$ and 16^2 different samples. The Young's modulus on the right appears as a barely distinguishable agglomeration of points for each tested β , as it theoretically remains constant for a fixed solid fraction while ν_s varies. This is numerically confirmed by a maximum coefficient of variation of 5.247×10^{-3} across 16 samples of E for varying ν_s .

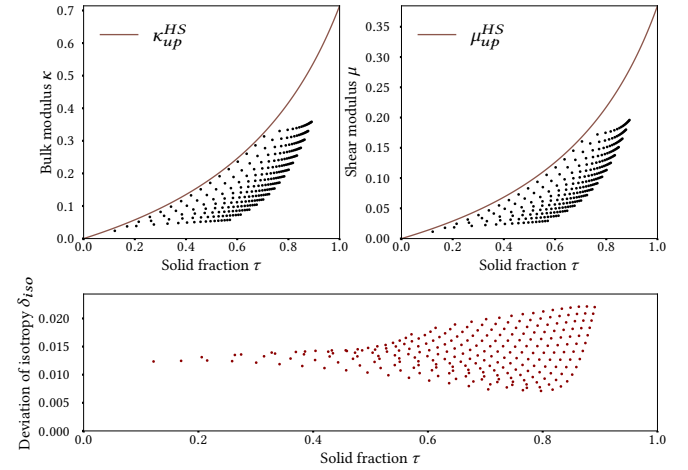


Fig. 9. *Top*: Isotropic bulk and shear modulus samples. The blue line shows the upper Hashin-Shtrikman bounds. *Bottom*: Deviation from isotropy is at most $\delta_{iso} < 0.022$ (with $\delta_{iso} = 0$ describing exact isotropy).

of solid fractions. Let $\tau_{min}(E_t)$ be the inverse function of the upper bound given in Equation (21) that for a given Young's modulus $E_t \in [0, E_s]$ returns the minimum solid fraction to attain E_t

$$\tau_{min}(E_t) = \frac{3E_t}{E_s + 2E_t} \in [0, 1] \quad (22)$$

Let $\tau \in [\tau_{min}^{E_t}, \tau_{max}^{E_t}]$ be an interval of solid fractions achieved with our method. Then, we consider the following normalized measure $\tau_{cov}^{E_t}$ to fairly quantify the coverage of solid fractions for some E_t

$$\tau_{cov}^{E_t} = \frac{\tau_{max}^{E_t} - \tau_{min}^{E_t}}{1 - \tau_{min}(E_t)} \in [0, 1] \quad (23)$$

As shown by the HS bounds in Figure 9, as the Young's modulus target increases, achieving greater contrast in the solid fractions becomes increasingly difficult. Namely, it is possible to construct

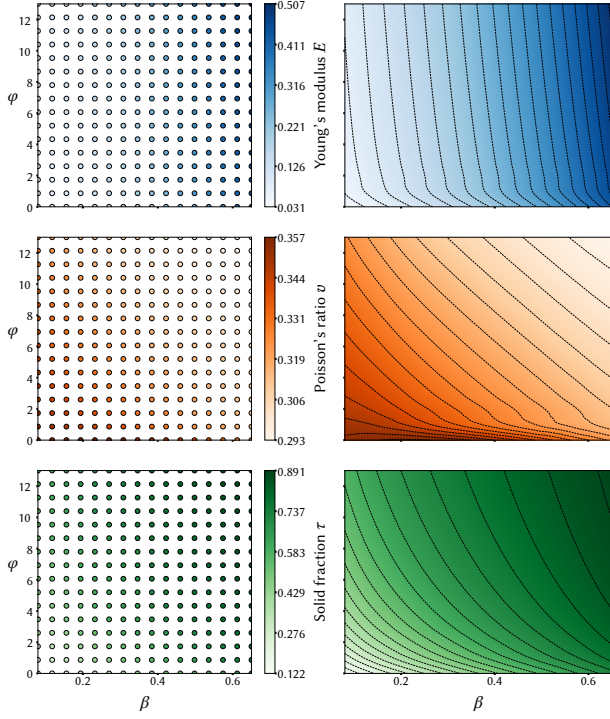


Fig. 10. Left: a discrete sampling of β and ϕ , and the obtained Young's modulus, Poisson's ratio, and solid fraction. Right: continuous interpolation of the sampling.

a porous material with a high solid fraction and a low Young's modulus, but not the other way around: a small solid fraction and a high Young's modulus.

Inverse mapping. We seek to find functions $\beta_{E_t}(\tau)$ and $\phi_{E_t}(\tau)$ that, for a given solid fraction $\tau \in [\tau_{\min}^{E_t}, \tau_{\max}^{E_t}]$, return the parameters leading to a porous structure with constant Young's modulus E_t . To do so, we first densely generate sample points by interpolating with equal Young's moduli and varying solid fraction, and then reconstruct both functions by fitting a polynomial of some given degree to the sample points. The results of this procedure are shown in Figures 13 to 15. All the polynomial coefficients are rounded up to two decimals.

4 Results

This section presents numerical results to assess the effectiveness of our method.

Halftoning. Each triangle is assigned a constant solid fraction when halftoning with a variable solid fraction, so β and ϕ are constant per triangle. The solid fraction per triangle is approximated by integrating over sampled points in the target solid fraction field within the triangle. Figure 21 shows different halftoning results, where an input grayscale field is cloaked into an apparent object that is approximately homogeneous under the linear elasticity assumption. Increasing the number of points k enhances halftoning

quality. The Poisson's ratio variation is small, so its impact on elastic behavior is limited. When the target solid fraction is obtained from an image, we map the grayscale values so that the minimum corresponds to $\tau_{\min}^{E_t}$ and the maximum to $\tau_{\max}^{E_t}$. We also apply a transfer function to enhance contrast in the target solid fraction for the results in Figure 1 and the *Hokusai Wave* result in Figure 21. Note that this preprocessing step is optional and not required by our method, and the target solid fraction shown in the figures has already been preprocessed.

Homogenization of a periodic halftoning. We analyze in Figure 16 the elastic response of a periodic variable halftoning using again the method of [Brisard and Dormieux 2010]. The effective Young's modulus E converges towards the target one as the number of pores increases; even for small numbers of pores, deviations of the effective Young's modulus remain around 1%, consistent with similar findings in the literature [Zerhouni et al. 2019]. The isotropy deviation δ_{iso} does not vanish to zero but instead converges towards a small positive value. This is expected since the porous structures exhibit slight variations in the effective Poisson's ratio ν (see Figure 12), preventing a perfectly isotropic result.

Comparison with finite scale FEM. To verify the mechanical behavior, we perform a finite element analysis to assess whether the elastic behavior of the porous structures approaches isotropic homogeneity as the number of points k increases. The applied boundary conditions are shown in Figure 17, and the results are in Figure 22. The test surrounds the porous structure with a thin solid layer and applies various deformations. The homogeneous material is isotropic, with Young's modulus $E_t = 0.1$ and a Poisson's ratio equal to that of the solid phase of the porous structure $\nu_s = 0.3$. This choice of Poisson's ratio for the homogeneous material is heuristic, but it leads to good agreement as shown in Figures 18 and 22. Since the porous structure is modeled at a finite scale, deviations between the homogeneous and heterogeneous cases are expected to decrease as the number of pores increases, and as shown in Figure 16, we do not expect them to disappear, but rather to eventually converge to a certain non-zero deviation. We further quantify this by considering the displacement error between the homogeneous and heterogeneous cases. Let u_{hom} be the displacements of the homogeneous results and u_{het} those of the heterogeneous porous structure. We use the following relative error measure $u_{\text{err}} \geq 0$ computed at the nodal points

$$u_{\text{err}} = \frac{\|u_{\text{het}} - u_{\text{hom}}\|_2}{\|u_{\text{hom}}\|_2} \quad (24)$$

Results of u_{err} for increasing numbers of points and the results of Figure 22 are shown in Figure 18. As expected, u_{err} decreases as the number of points increases and appears to converge around a small value.

It is important to highlight the difference between effective homogenized results and how finite samples actually perform. Our porous structures converge in an infinite-periodic setting toward the target average Young's modulus (see Figure 16) and exhibit nearly isotropic effective behavior. For finite-size plates, differences between effective heterogeneous and homogeneous results decrease as the number of pores increases, but local heterogeneity of the stress and strain fields from the homogenized average prediction is,

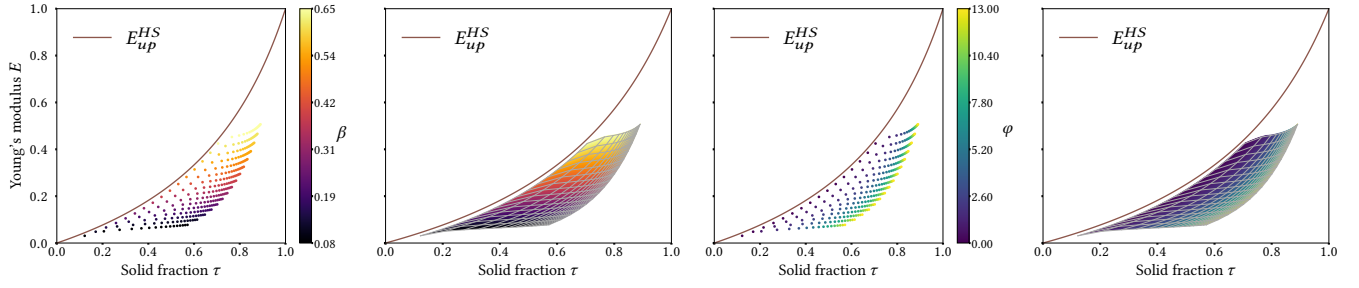


Fig. 11. Discrete sampling (dots) and corresponding continuous interpolation of Young's moduli with respect to the solid fraction and parameters β and φ .

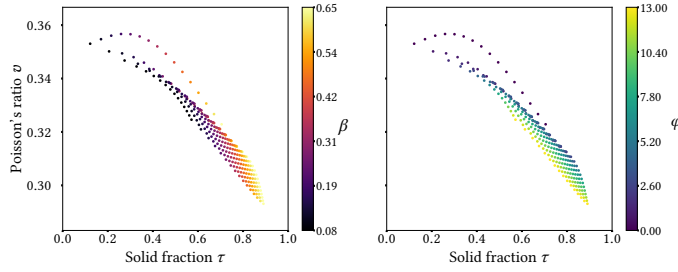


Fig. 12. Discrete sampling (dots) of Poisson's ratio with respect to the solid fraction and parameters β and φ .

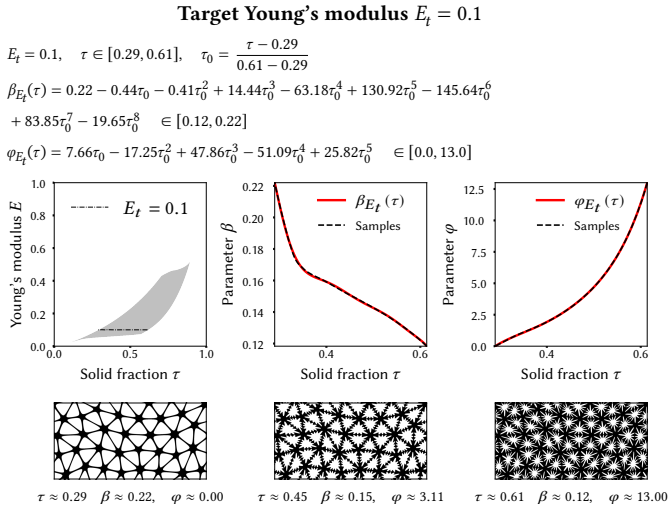


Fig. 13. Top: Interpolation with Young's modulus $E_t = 0.1$ providing a range of solid fractions between $\tau_{\min}^{E_t} \approx 0.29$ and $\tau_{\max}^{E_t} \approx 0.61$ and results in a Poisson's ratio ν_t slightly varying from 0.32 to 0.357, and that gives approximately a coverage of $\tau_{\text{cov}}^{E_t} = 43.18\%$. Bottom: Increasing solid fractions.

by construction, unavoidable due to the contrast in material properties between the solid and void phases. Moreover, as shown in Figure 16, the deviation from isotropy δ_{iso} does not converge to zero because a small amount of anisotropy remains from local changes in the effective Poisson's ratio. These effects are amplified by the

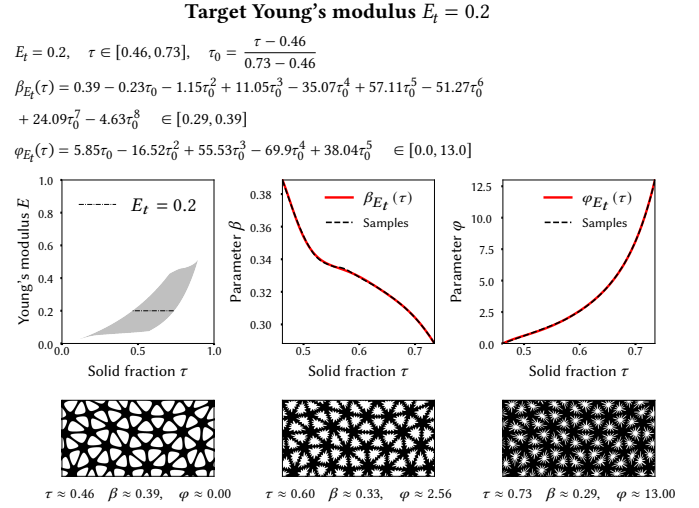


Fig. 14. Top: Interpolation with Young's modulus $E_t = 0.2$ providing a range of solid fractions between $\tau_{\min}^{E_t} \approx 0.46$ and $\tau_{\max}^{E_t} \approx 0.73$ and results in a Poisson's ratio ν_t slightly varying from 0.308 to 0.349, and that gives approximately a coverage of $\tau_{\text{cov}}^{E_t} = 47.73\%$. Bottom: Increasing solid fractions.

solid boundary, which is absent in the periodic homogenization framework, and by the fact that we heuristically set Poisson's ratio of the homogeneous counterpart equal to that of the solid phase, while results in Figure 16 show a slightly higher effective Poisson's ratio.

Comparison with periodic patterns. Using lattice points instead of blue-noise sampling yields a triangulation with a markedly anisotropic power spectrum, whereas our method yields power spectra that are close to isotropic (see Figure 19). Note that periodic patterns may induce visual discomfort due to inefficient neural processing [Wilkins 2016], and are less favored compared to superior isotropic sampling techniques for digital halftoning.

5 Limitations and future work

We have introduced the problem of finding a subspace of porous structures that behave homogeneously in the isotropic elastic regime and cover a wide range of solid fractions. Our approach could be

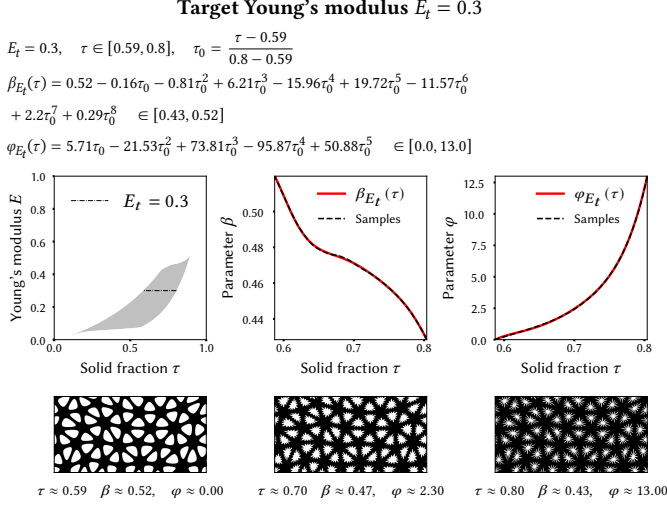


Fig. 15. Top: Interpolation with Young's modulus $E_t = 0.3$ providing a range of solid fractions between $\tau_{\min}^{E_t} \approx 0.59$ and $\tau_{\max}^{E_t} \approx 0.8$ and results in a Poisson's ratio ν_t slightly varying from 0.301 to 0.337, and that gives approximately a coverage of $\tau_{\text{cov}}^{E_t} = 49.54\%$. Bottom: Increasing solid fractions.

useful in applications that require porous structures with homogeneous elastic behavior and seek to encode visual information in the structure, as showcased in the architectural pavilion of Figure 20. Our method paves the way for future work in that direction and has some limitations that we outline below.

First, we have proposed a custom parametric function that has proven effective, although better ones may exist. We carefully designed the indicator function $\mathcal{S}$ by hand to use a few parameters and to show the target mechanical properties, but there is room for improvement. A promising direction for future research is to further optimize a class of parametric functions that increase the coverage described in Equation (23) and minimize the variability of the Poisson ratio shown in Figure 21. The porous structures we study exhibit mechanical behavior that is close to but not perfectly isotropic in an average effective sense. The ultimate goal would be to find an alternative family of porous structures with a constant effective Poisson's ratio that covers a broad range of solid fractions and Young's moduli. Second, extending our method to finite strains would enhance its applicability, as it currently addresses only infinitesimal strains. We also conjecture that similar mechanical cloaking effects could be achieved by considering plane strain rather than plane stress. Third, for plates of finite size, discrepancies between the homogenized prediction and the heterogeneous results are expected. Another direction is to study methods that account for the plate's specific size and shape to achieve more accurate results at a given scale of the porous structures. Finally, this study presents a detailed numerical analysis, but future experimental research exploring different manufacturing technologies and considering their limitations, such as the minimum length scale, would be valuable.

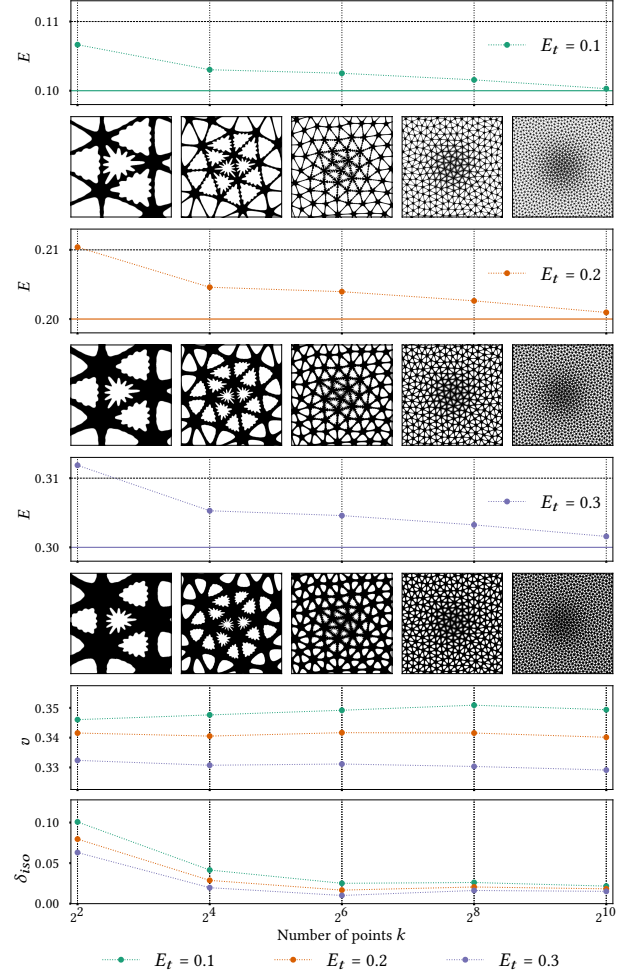


Fig. 16. Halftoning of a periodic pattern with three different target Young's moduli E_t given in Figures 13 to 15 and increasing number of points. The target solid fraction is modulated by the function $\sin(x\pi)^4 \sin(y\pi)^4$ in the domain $x, y \in [0, 1]^2$, linearly clamped between the minimum and maximum achievable solid fraction.

Acknowledgments

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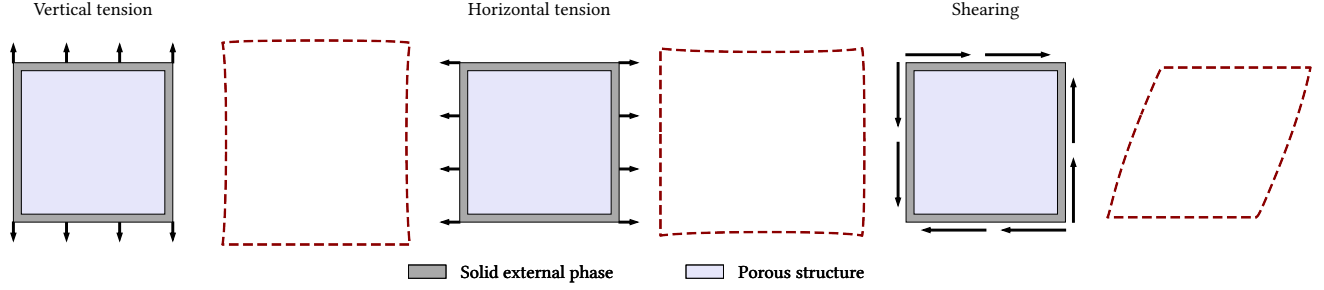


Fig. 17. The three different load cases, from left to right, are uniaxial vertical loading, uniaxial horizontal loading, and shear loading. In dashed red lines is the outline of the resulting deformation of the equivalent homogeneous material. The red outline, also shown in Figure 22, highlights the goal deformation of our method: ensuring the porous structure remains closely aligned to it.

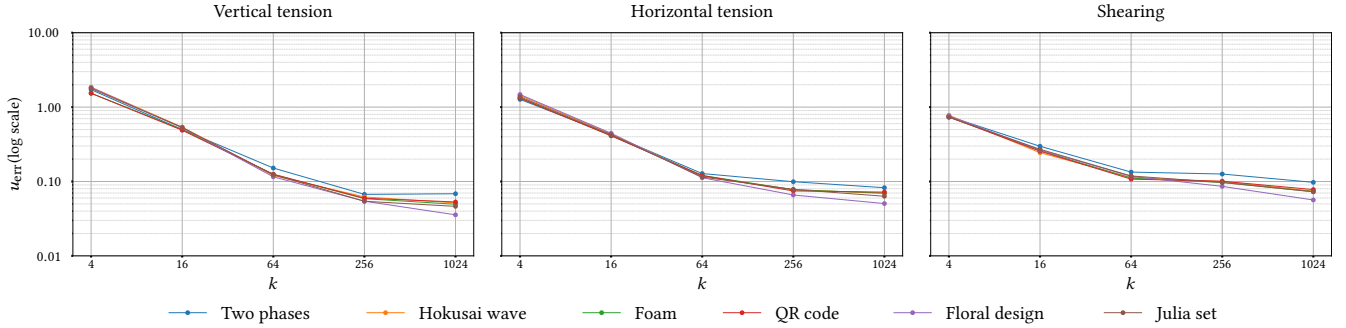


Fig. 18. Analysis of the relative displacement error u_{err} for increasing number of points k , for each load case of Figure 17, and considering the different halftoning results shown in Figure 22

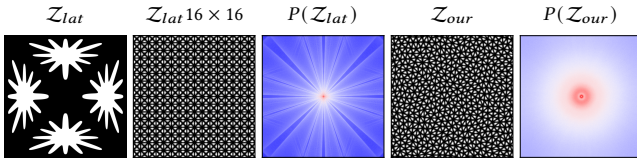


Fig. 19. Comparison between a triangulation given by some point lattice Z_{lat} and our method Z_{our} . The power spectrum is $P(Z) = |F(Z)|^2$, where F is the Fourier transform. Since F is periodic, we consider a unit tile Z_{lat} , tiled to visually match the number of points in Z_{our} . The colormap is normalized and plotted on a logarithmic scale, $(\log(1 + P(Z)))$, with blue representing low values and red the high ones. We set the DC component of $F(Z)$ to zero to improve the contrast in the spectral plot. Z_{lat} exhibits a clear power spectrum anisotropy, whereas our method yields a nearly isotropic one.

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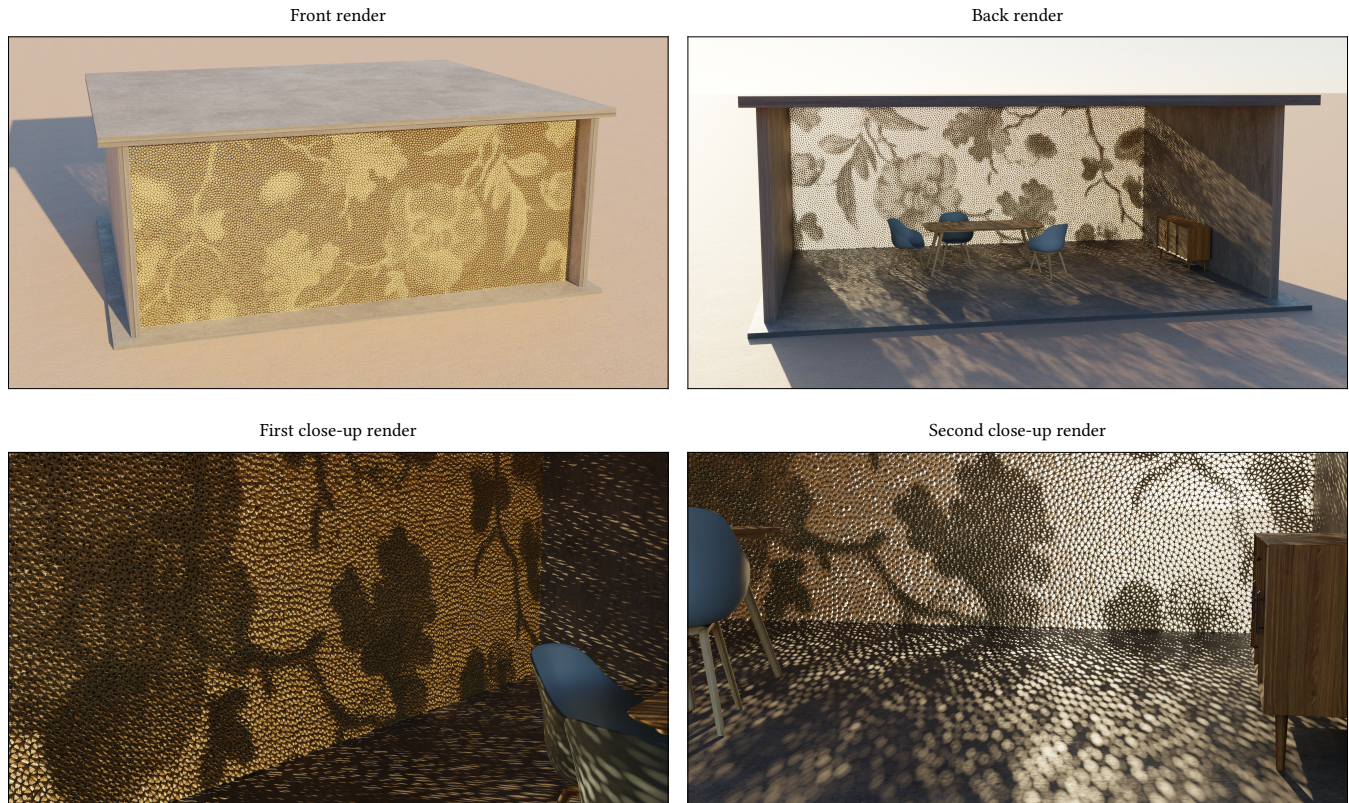


Fig. 20. Different renderings of an architectural pavilion with a floral motif on the front, generated using our method and based on the same pattern as fig. 1. The porous panel behaves homogeneously and isotropically while encoding a floral gradient, creating an appealing visual effect. See the accompanying video for an additional time-lapse rendering.

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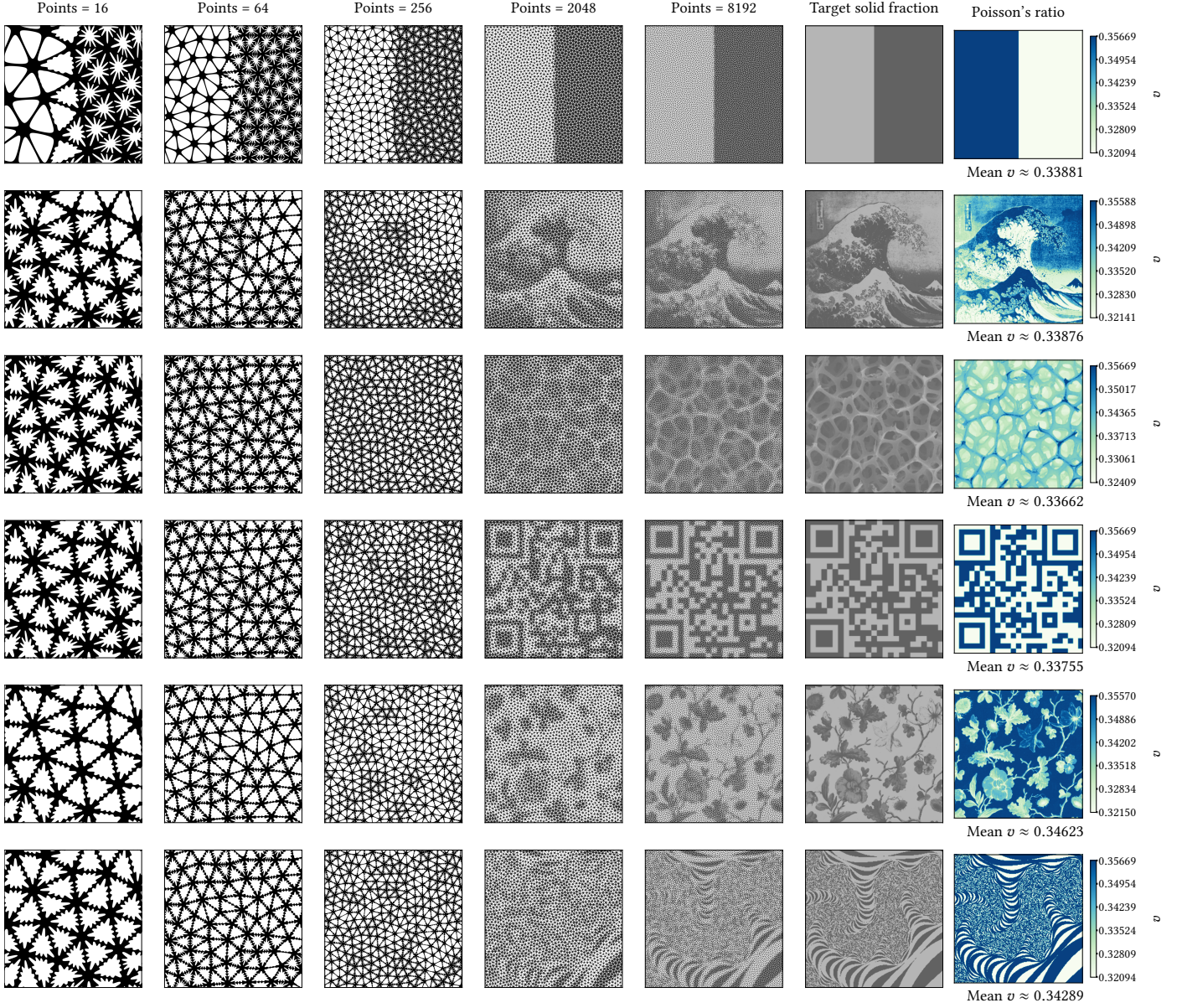


Fig. 21. Halftoning results of porous materials with target Young's modulus $E_t = 0.1$. From left to right, more points improve halftoning accuracy. The rightmost image corresponds to the input grayscale field. First row: **Two phases**, an abrupt transition between the two extremes of achievable grayscale values. Second row: **Hokusai wave**, *The Great Wave of Kanagawa* (1938) by Katsushika Hokusai. Third row: **Foam**, image of a polyurethane foam from [Haugen et al. 2004]. Fourth row: **QR code**, a QR code encoding the link <https://example.com/>. Fifth row: **Floral design**, a photo of a floral design for printed textile (1800–1818) by Louis-Albert DuBois. Sixth row: **Julia set**, halftoning of a binarized Julia set.

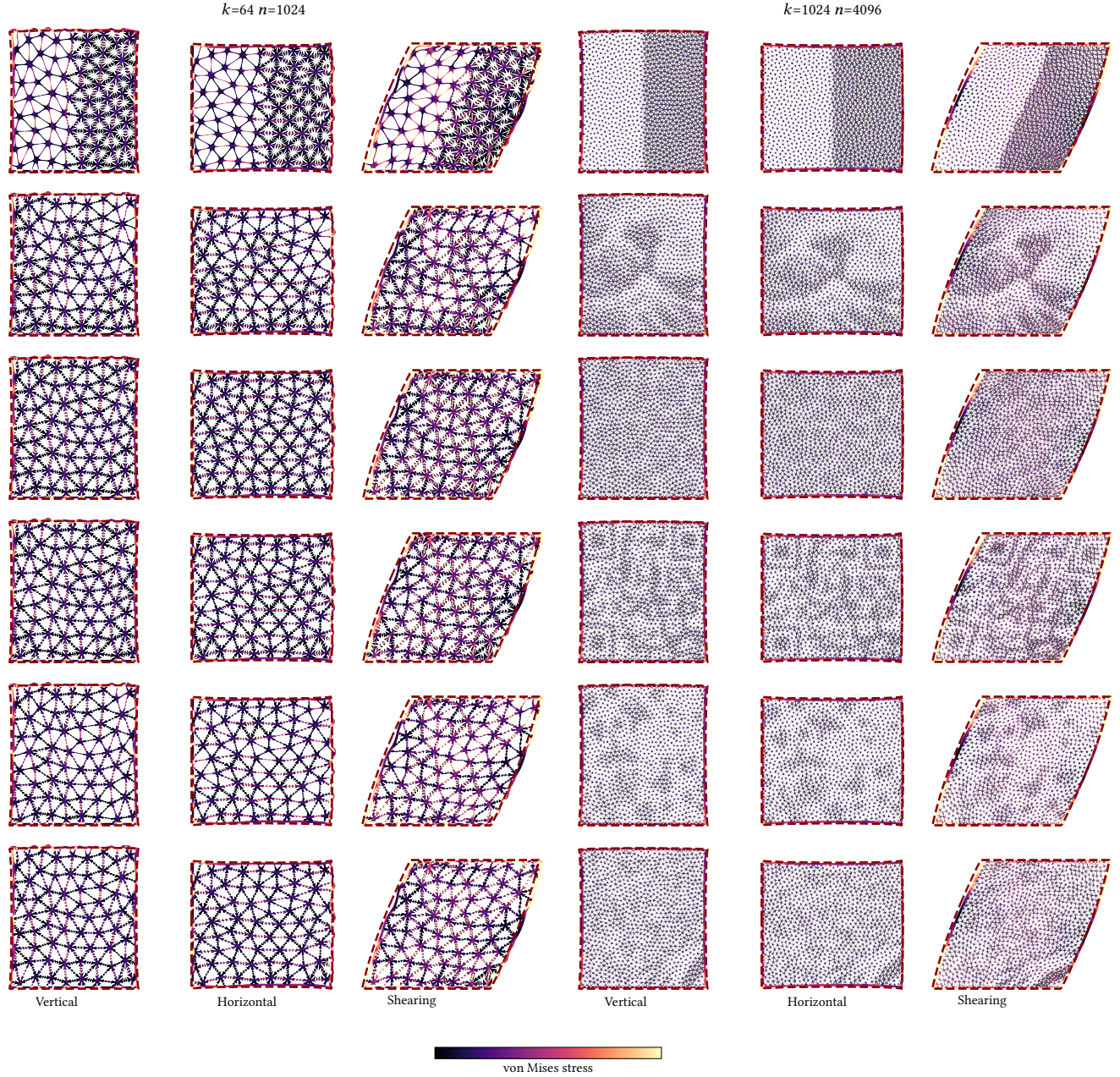


Fig. 22. Finite element analysis of porous materials with target Young's modulus $E_t = 0.1$. For each load case in Figure 22, we consider an increasing number of points with fixed $n_k = 2^{14}$. The red dashed line shows an isotropic material with varying target Young's modulus E_t and constant Poisson's ratio $\nu_s = 0.3$. The color scale indicates the von Mises stresses of the structure, normalized for each load case. This figure shows a fine-grained simulation of porous structures, accurately capturing their behavior at a specific scale under given boundary conditions.