

## Combined effect of hysteresis and heterogeneity on the stability of an embankment under transient seepage

Liu, Kang; Vardon, Phil; Hicks, Michael; Arnold, Patrick

**DOI**

[10.1016/j.enggeo.2016.11.011](https://doi.org/10.1016/j.enggeo.2016.11.011)

**Publication date**

2017

**Document Version**

Accepted author manuscript

**Published in**

Engineering Geology

**Citation (APA)**

Liu, K., Vardon, P., Hicks, M., & Arnold, P. (2017). Combined effect of hysteresis and heterogeneity on the stability of an embankment under transient seepage. *Engineering Geology*, 219, 140-150.  
<https://doi.org/10.1016/j.enggeo.2016.11.011>

**Important note**

To cite this publication, please use the final published version (if applicable).  
Please check the document version above.

**Copyright**

Other than for strictly personal use, it is not permitted to download, forward or distribute the text or part of it, without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license such as Creative Commons.

**Takedown policy**

Please contact us and provide details if you believe this document breaches copyrights.  
We will remove access to the work immediately and investigate your claim.

---

# Combined effect of hysteresis and heterogeneity on the stability of an embankment under transient seepage

Liu, K., Vardon, P.J., Hicks, M.A., Arnold, P.

*Geo-Engineering Section, Faculty of Civil Engineering and Geosciences, Delft*

*University of Technology, The Netherlands*

**Abstract:** The stability of most earth embankments is strongly influenced by the water content of the soil. The water content directly influences the suction or pore pressure in the soil, as well as the mass of material, thereby affecting the stress state and strength, and leading to changes in the stability. These aspects are coupled by the so-called soil water retention behaviour, which is observed to be a hysteretic phenomenon. Moreover, soils are known to be spatially variable or heterogeneous in nature, which can lead to preferential flow paths and stronger or weaker zones. In this paper the behaviour of a heterogeneous earth embankment subjected to cyclic water level fluctuation, including the impact of hysteresis, is investigated. The soil property values governing the unsaturated hydraulic response of the embankment are considered as spatially random variables, with the mechanical property values considered deterministic in order to isolate the impact of the hydraulic behaviour. The Monte Carlo Method (MCM) is used to conduct probabilistic analyses and an assessment of the relative influence of material properties illustrates that the saturated hydraulic conductivity,  $k_{sat}$ , plays a dominant role in the slope stability. Moreover, in the initially drying condition, the average factor of safety (FOS) and the

---

95th percentile FOS of the slope considering hysteresis are smaller than those without considering hysteresis, at all times, while the variability of the FOS considering hysteresis is larger than that when not considering hysteresis. In practice, this means that slopes under seepage conditions, which are assessed to have a low FOS, should be assessed including the hysteretic behaviour to ensure stability.

**Keywords:** Embankment, hysteresis, reliability, slope stability, spatial variability, transient seepage

## 1. Introduction

Slopes under seepage conditions, with both saturated and unsaturated zones, are common and of great concern in geotechnical engineering (Chen and Zhang, 2006; Rahardjo et al., 2010). The saturated–unsaturated seepage in the slope has a significant impact on the slope stability, via changes in shear strength and volumetric weights, and is strongly related to the water retention behaviour of the unsaturated soil (Gui et al., 2000; Le et al., 2012; Zhu et al., 2013; Zhang et al., 2015; Zhang et al., 2016).

The soil water retention curve (SWRC) describes the relationship between the suction head,  $h_s$ , and a measure of the water content, in this paper the volumetric water content (VWC),  $\theta$ , and in addition impacts the hydraulic conductivity,  $k$ , which further affects the distribution of pore water pressure (PWP) in the soil (Lam et al., 1987; Yang et al., 2012). Hysteresis in the water retention behaviour of unsaturated soils describes a non-unique relationship between  $h_s$  and  $\theta$ , and thus

---

also between  $h_s$  and  $k$  (Jaynes, 1984; Pham et al., 2005; Wu et al., 2012). Moreover, due to the existence of hysteresis, the VWC in the soil under cyclic drying and wetting processes may exhibit a significantly different response as compared to the non-hysteretic case (Ma et al., 2011). Indeed, the differences in the PWP and VWC induced by the hysteresis in the SWRC contribute to a hysteretic shear strength response which affects the stability and reliability of the slope (Bishop, 1959).

However, to simplify seepage analyses the effect of hysteresis is commonly ignored (e.g. Tsaparas et al., 2002; Le et al., 2012), even though it may generate inaccurate predictions of the distributions of PWP and VWC. Tami et al. (2004) investigated the variation in the suction profile in a soil column with a hysteretic SWRC. It was found that, due to the hysteresis, the suction at the newly reached steady state after a certain period of infiltration was significantly affected by the initial water content prior to the infiltration process. Yang et al. (2012) studied the variation of matric suction and VWC in a soil column under cyclic precipitation and evaporation. It was found that the computed results were closer to the experimental results when considering hysteresis.

Recently, several researchers have investigated the effect of hysteresis on the stability of soil slopes. Ebel et al. (2010) pointed out that simulations ignoring hysteresis could underestimate the potential for landslides. Ma et al. (2011) conducted an experimental and numerical study of a soil slope to assess the effect of hysteresis, both on the hydraulic response and the slope stability. It was found that the distribution of water content was influenced by hysteresis and that the calculated

---

FOS of the slope considering hysteresis recovered quickly after rainfall and was larger than that without considering hysteresis for any given time.

Most research that includes the effect of hysteresis focuses on homogeneous soils. Conversely, if the heterogeneity of soil property values is taken into account, the impact of hysteresis is typically not accounted for (Arnold and Hicks, 2010; 2011; Zhu et al., 2013). However, Nakagawa et al. (2012) highlighted the importance of considering both hysteresis and heterogeneity in the simulation of unsaturated flow by comparing numerical results with experimental data. Very few studies have incorporated hysteresis and heterogeneity in the assessment of slope stability. Yang et al. (2013) accounted for the effect of hysteresis and spatial variability of soil property values, i.e. of the saturated hydraulic conductivity and some SWRC fitting parameters, in a one-dimensional infiltration problem. It was shown that the combined effect of hysteresis and heterogeneity of soil property values increased the uncertainty in the estimation of the ability to prevent penetration in soil covers, compared to the non-hysteretic but heterogeneous case. Zhang (2007) incorporated both hysteresis and heterogeneity into the stability analysis of a 2D slope under cyclic precipitation and evaporation, with the analysis starting on the wetting SWRC. The results suggested that simulations without considering the effect of hysteresis may underestimate the slope reliability.

This paper investigates the slope stability of an embankment under transient seepage, i.e. due to a cyclic external water level. The effects of both hysteresis and heterogeneity of the soil property values on the seepage response are considered.

---

First, the mechanical and stochastic model framework for slope stability under saturated–unsaturated seepage is briefly introduced. Next, the numerical implementation of the framework is explained, and a specific example then utilised to investigate the impact of considering hysteresis for a homogeneous embankment. Finally, the effect of spatial variability of the soil property values is considered, by conducting a probabilistic analysis of the slope stability and comparing the results of the hysteretic and non-hysteretic cases.

## 2. Formulation

### 2.1 Governing flow equation

The governing equation of 2D transient unsaturated–saturated flow is based on mass conservation. In the flow analysis, the soil skeleton is considered to be rigid, which means that any volume change during the seepage process is not accounted for. Therefore, the governing flow equation, in incremental form, is (e.g. Celia et al., 1990)

$$\frac{\partial}{\partial x} \left( k_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial h}{\partial z} + k_z \right) = C(h) \frac{\partial h}{\partial t} \quad (1)$$

where  $k_x$  and  $k_z$  are the hydraulic conductivities in the  $x$  and  $z$  directions, respectively,  $h$  is the PWP head,  $t$  is time and  $C(h) = \partial\theta/\partial h$  is the specific moisture capacity function with  $\theta$  being the VWC. The same description of mass conservation can be used for heterogeneous porous media (e.g. Gui et al., 2000) with an appropriate selection of hydraulic conductivities at each location.

### 2.2 Water retention behaviour

The SWRC is a function relating the suction head,  $h_s$ , with  $\theta$ . The suction head is

---

defined as the negative component of  $h$  and is represented by

$$h_s = -h = s/\gamma_w = (u_a - u_w)/\gamma_w \quad (2)$$

where  $s$  is the matric suction,  $u_a$  is the pore air pressure, which is assumed to be atmospheric in this paper,  $u_w$  is the PWP and  $\gamma_w$  is the unit weight of water.

The van Genuchten (1980) model is frequently used to describe the SWRC (e.g. Gui et al., 2000; Sivakumar Babu and Murthy, 2005; Phoon et al., 2010) as it can give a good approximation of the experimental results of many soil types (e.g. Han et al., 2010). It is given by

$$S = \frac{\theta - \theta_r}{\theta_s - \theta_r} = \frac{1}{\left[1 + (\alpha |h|^n)\right]^m} \quad h < 0$$
$$S = 1 \quad h \geq 0 \quad (3)$$

where  $S$  is effective degree of saturation,  $\alpha$  is a parameter which is approximately the inverse of the air-entry suction head,  $h_{s,ae}$ , and  $\theta_s$  and  $\theta_r$  are the saturated and residual VWC, respectively (see Figure 1 (a)). Due to the hysteretic behaviour described by the water retention curve, the main wetting and drying curves have different values of  $\alpha$ , i.e.  $\alpha_w \approx \frac{1}{h_{s,ae,w}} > \alpha_d \approx \frac{1}{h_{s,ae,d}}$ . The model parameter  $n$  defines the slope of the water retention curve, which is here assumed to be identical for the main wetting and drying responses, and  $m$  is a curve fitting parameter which is here approximated (Mualem, 1976) by

$$m = 1 - 1/n \quad (4)$$

The hydraulic conductivity of the unsaturated soil is typically a function of the effective degree of saturation (Figure 1 (b)). It can be derived from the SWRC by using the van Genuchten (1980) model, for which one common expression is

---


$$k = k_{sat} \sqrt{S} [1 - (1 - S^{(n/n-1)})^m]^2 \quad (5)$$

where  $k_{sat}$  is the saturated hydraulic conductivity of the soil. It can be seen from Figure 1 that, due to the hysteresis in the SWRC, there will be also hysteresis in the hydraulic conductivity function with respect to the suction head.

Jaynes (1984) compared four methods of modelling hysteresis in the water retention behaviour, i.e. in approximating the curves between the main wetting and drying curves, referred to as scanning curves, (see Figure 1 (a)), and found that the performance of the four methods were equally good in the numerical simulation of hysteretic flow. Furthermore, the author pointed out that, due to simplicity, linear scanning curves were often used in numerical analyses. Therefore, as this paper is not considering a specific soil, this approach has been adopted here to describe the transition from wetting to drying and vice versa (Figure 1 (a)), utilising parameter  $\kappa$  to define the gradient of the curves.

### 2.3 Slope stability assessment

Slope stability equilibrium is solved using a linear elastic, perfectly plastic material model. Bishop's effective stress combined with the extended Mohr–Coulomb failure criterion has been used to model the shear strength (Bishop, 1959):

$$\tau = c' + \sigma_t \tan \varphi' + \chi s \tan \varphi' \quad (6)$$

where  $\tau$  is the shear strength,  $c'$  and  $\varphi'$  are the effective cohesion and friction angle,  $\chi$  is a scalar parameter defining the suction-induced effective stress,  $s$  is the matric suction and  $\sigma_t$  is the total stress normal to the sliding plane (Cai and Ugai, 2004). In this paper, the suction stress,  $\chi s$ , is defined as



---


$$\begin{aligned} \chi s = Ss &= \frac{\theta - \theta_r}{\theta_s - \theta_r} s & s > 0 \\ \chi s = Ss &= s & s \leq 0 \end{aligned} \quad (7)$$

where the suction stress is assumed to equal the effective degree of saturation (Vanapalli et al., 1996). When the suction is negative, i.e. there is a positive PWP, the conventional effective stress is used.

The unit weight,  $\gamma$ , of the unsaturated soil is a function of  $\theta$  (Tsai and Chen, 2010) and can be expressed as

$$\gamma = [(1 - \theta_s)\rho_w G_s + \rho_w \theta]g \quad (8)$$

where  $\rho_w$  is the density of water,  $G_s$  is the specific gravity of the soil particles and  $g$  is the gravitational acceleration.

## 2.4 Spatial variability of soil properties

The heterogeneity of the soil is considered by some soil property values being spatially random variables following normal or lognormal distributions. The point statistics of the variables are described by the distribution type, mean, variance and cross-correlation between different variables. The coefficient of variation ( $COV$ ) is a normalised measure of the variability, defined as

$$COV = \frac{\sigma}{\mu} \quad (9)$$

where  $\mu$  and  $\sigma$  are the mean and standard deviation, respectively.

The soil property values are also spatially correlated due to the deposition process. The scale of fluctuation,  $l$ , is the distance over which parameter values are significantly correlated (Fenton and Vanmarcke, 1990). In addition, the deposition process causes different scales of fluctuation in the horizontal and vertical directions. Hence, the ratio of the horizontal scale of fluctuation to the vertical scale of

---

fluctuation is referred to as the degree of anisotropy of the heterogeneity,  $\xi$  (Hicks and Samy, 2002; 2004), i.e.

$$\xi = \frac{l_h}{l_v} \quad (10)$$

where  $l_h$  and  $l_v$  are the horizontal and vertical scales of fluctuation, respectively.

For more information about the statistics of soil property values, the reader may refer to Phoon and Kulhawy (1999).

### **3 Numerical implementation**

#### **3.1 Slope stability under transient seepage**

The flow and slope stability analyses both use finite element (FE) programs that have been developed based on Smith et al. (2013); that is, using 4-node quadrilateral elements for the flow analysis and 8-node quadrilateral elements for the slope stability analysis. First, a flow analysis is performed and the variation of suction stress and VWC with time, i.e.  $\chi s(t)$  and  $\theta(t)$ , are computed. Next, the suction stress and VWC from the flow analysis are imported into the slope stability FE program and mapped onto the Gauss points for computing the effective stresses due to gravitational (self-weight) loading.

The governing flow equation has been solved by using the modified Picard iteration method (Celia et al., 1990; Lehmann and Ackerer, 1998). In the flow analysis program, the VWC for the main drying and wetting curves is computed via Equation 3. The water retention behaviour along the linear scanning curve is computed via an algorithm proposed by Yang et al. (2011), in which the VWC at the current time,  $t + 1$ , is determined as a function of the current change in suction head and the

---

previous VWC, i.e.  $\theta^{t+1} = f(\Delta h_s^{t+1}, \theta^t)$ .

If the PWP at the Gauss point is negative, the soil unit weight is updated by Equation 8 in the FE slope stability analysis; otherwise the unit weight is equal to the saturated unit weight. The strength reduction method (Griffiths and Lane, 1999) has then been applied to compute the FOS of the slope. In this method, the FOS is defined as the factor by which the original shear strength is reduced in order to cause the slope to fail. Hence,

$$\begin{aligned} c'_f &= c' / \text{FOS} \\ \phi'_f &= \arctan\left(\frac{\tan \phi'}{\text{FOS}}\right) \end{aligned} \quad (11)$$

where  $c'_f$  and  $\phi'_f$  are the factored shear strength parameters at slope failure.

### 3.2 Probabilistic simulation

For modelling soil heterogeneity, the Local Average Subdivision (LAS) method (Fenton and Vanmarcke, 1990) has been used to generate stationary, spatially correlated, random fields of soil parameter values. In this paper, the exponential Markov correlation function has been used to model the correlation between the parameter values at different locations:

$$\rho(\tau) = \exp\left(-\frac{2}{l}\tau\right) \quad (12)$$

where  $\tau$  is the lag distance between two points at different locations in space within the random field, and  $l$  is the scale of fluctuation.

The Monte Carlo Method (MCM) has been used to investigate the characteristics of the slope stability under stochastic transient seepage. Each Monte Carlo simulation involves multiple realisations of the problem, in which each random

---

field is based on the same set of statistics, but yields a unique representation of the spatial variation in a material property. Individual random fields are generated for each soil parameter in standard normal space, and then transformed into physical space. The random field cell values of the parameters, generated by LAS, are mapped onto the finite element mesh at the Gauss point level.

In this investigation, for the purpose of comparing non-hysteretic and hysteretic responses, 1000 realisations was found to be sufficient, as well as being consistent with other studies, e.g. Griffiths and Fenton (2004), Hicks and Spencer (2010) and Santoso et al. (2011).

#### **4. Slope stability example**

A heterogeneous embankment subjected to cyclic external water level fluctuation has been taken as an example to demonstrate the influence of the combined effect of hysteresis and heterogeneity on slope stability. For comparative purposes, the investigation has also included non-hysteretic and homogeneous analyses.

The geometry of the embankment is shown in Figure 2. Its height is 12 m, and the width of its crest and base are 4 m and 52 m, respectively. The embankment experiences a water level fluctuation on its upstream side. WL1 and WL2 are the highest and lowest water levels, whereas line A-A (at  $x = 34$  m) and point B (at  $x = 28$  m) denote the observation cross-section and point where results are recorded. The downstream water level remains at foundation level ( $z = 0$  m), and the bottom boundary of the embankment is impermeable and fixed.

The water level fluctuation with time has been simulated by the summation of

two sinusoidal curves, with two different frequencies (denoted by functions  $C_1$  and  $C_2$  in Figure 3). For both curves, the mean and amplitude are 3.5 m and 3 m respectively, whereas the time period of  $C_1$ , termed  $T_1$  (and equal to 10 days), is three times that of  $C_2$ . The resulting water level fluctuation, shown by the green line in Figure 3, has a maximum water level of 10 m (WL1) and a minimum water level of 4 m (WL2).

The parameter values of the deterministic homogenous case, in which the spatial variability of parameters is not included, are listed in Table 1, with the SWRC properties following Yang et al. (2011) and the mechanical properties following Hicks and Spencer (2010). The specific gravity is typical of an organic soil, i.e.  $\sim 2$ , as is the hydraulic conductivity, at  $1 \times 10^{-6}$  m/s. The SWRC is given in Figure 4. In the non-hysteretic case, the drying property values are used (as explained in Section 5.1). For the heterogeneous case, in which several hydraulic parameters are spatially random, the distributions of these parameters are listed in Table 2. Nielsen et al. (1973), Freeze (1975) and ASCE (2008) have shown that  $k_{sat}$  can be assumed to be log-normally distributed, and a  $COV_{k_{sat}}$  of 0.9 to 1.0 was reported in Nielsen et al. (1973).  $\alpha_d$  has also been described as log-normally distributed (Carsel and Parris, 1988; Russo and Bouton, 1992; de Rooij et al., 2004) and, in Carsel and Parris (1988),  $COV_{\alpha_d}$  is from 0.203 to 1.603. Russo and Bouton (1992) suggested a normal distribution for  $n$ , while Carsel and Parris (1988) indicated that  $COV_n$  is from 0.033 to 0.203. De Rooij et al. (2004) showed that the distribution of  $\theta_s$  is log-normal, while Carsel and Parris (1988) reported that  $COV_{\theta_s}$  is from 0.15 to 0.355. In de Rooij

---

et al. (2004),  $COV_{\theta_r}$  is equal to 0.031 and the distribution is assumed to be log-normal.

## 5 Results

In the following numerical investigation, the effects of hysteresis, parameter variation and heterogeneity are systematically investigated. In Section 5.1, hysteresis in isolation, i.e. a homogenous embankment, has been studied, in Section 5.2 the influence of the variable parameters is investigated and in Section 5.3 the impact of hysteresis on a spatially variable embankment has been presented. Table 3, summarises the items investigated in each sub-section.

### 5.1 Influence of hysteresis on the seepage and stability of a homogeneous embankment

Several analyses were initially undertaken to find an optimal mesh size and time step which would ensure both accuracy and efficiency. A standard finite element size of 0.5 m (vertical) by 1.0 m (horizontal) was selected and the time step was chosen to be 0.05 d.

In Liu et al. (2015), the current authors illustrated that hysteretic water retention behaviour significantly influenced the water flow and soil suction distributions in the embankment, and thereby the slope stability. Figure 5 shows the variation of the FOS with time. It increases from the initial conditions, illustrating the benefit of undertaking transient analysis. For both the hysteretic and non-hysteretic cases, the FOS is seen to react to the change in water level, where, if the water level is high, the FOS is low and vice versa. Figure 5 shows that, at any instant in time, the FOS of the

---

hysteretic case is always smaller than that of the non-hysteretic case, due to the selection of the main drying curve as the starting point of the transient seepage analysis for both cases. The reason for choosing the main drying curve is that Ebel (2010) reported it to be the most easily measured in the laboratory and therefore the most frequently used. In addition, in the numerical example, the embankment first experiences water level drawdown, i.e. a drying process. If the non-hysteretic SWRC is instead taken to be the wetting curve, a difference in the FOS between the non-hysteretic and hysteretic cases would still exist, although the FOS of the hysteretic case would then be bigger than that of the non-hysteretic case.

In Figure 5, the hysteretic case reacts quicker to changes in water level due to the variation of VWC with suction moving along the scanning curve, and this, in general, leads to a more significant and faster reduction in the FOS when the water level rises. The largest difference coincides with the highest water level and lowest FOS (ignoring the first part which is affected by the initial conditions). It is emphasised that in this analysis, all properties are constant throughout the domain, and no heterogeneity is considered. Section 5.3 provides a demonstration of the impact of considering heterogeneity alongside hysteresis.

Figure 6 compares the PWP head and VWC variation with time between the non-hysteretic and hysteretic cases, at three depths along cross-section A-A (see Figure 2). It can be seen that the VWC of the hysteretic case is usually larger or equal to that of the non-hysteretic case, and that the PWP head of the hysteretic case is also usually larger or equal to the non-hysteretic case. The larger the VWC, the larger

---

the overturning moment due to the greater soil weight; in addition, the smaller suction head results in a smaller shear resistance to sliding. Therefore, the combined effects lead to the FOS in the hysteretic case being smaller than in the non-hysteretic case (Figure 5) .

In Figure 6, it is seen that the PWP head in the hysteretic case can change more rapidly than that in the non-hysteretic case, due to the suction head moving along the scanning curves. In addition, it can be seen that the pore pressures are more sensitive to boundary condition (i.e. external water level) changes in the hysteretic case, with a similar change for different depths as seen from the blue curves in the left plots of Figure 6. In the non-hysteretic case, the red curve describing the highest point (Figure 6 (a)) does not change significantly with a change of boundary condition, whereas that describing the lowest point is almost as sensitive as in the hysteretic case. The response delay of the non-hysteretic case during transient seepage can be explained by referring to the wetting and drying process in a soil column (Yang et al., 2011; 2012).

The state at any point in the slope domain may be categorised into three types: always unsaturated, saturated–unsaturated and always saturated. The suction variations at typical points representing these three cases are illustrated in Figure 7, which shows the variation of the VWC with suction for points 1–3 shown in Figure 2. Figures 7(a) and (b) denote point 1 which is always in the unsaturated condition; Figures 7(c) and (d) denote point 2 which changes between saturated and unsaturated conditions; and Figures 7(e) and (f) denote point 3 which is always in the



---

saturated condition. In Figure 7, it can be noted that the suction in the hysteretic case shows larger variation than that in the non-hysteretic case, due to the suction head varying in the area enclosed by the main drying and wetting curves.

## 5.2 Relative importance of hydraulic parameters

To investigate the hydraulic parameters that are here represented by statistical distributions, i.e.  $\mathbf{X} = [k_{sat}, \alpha_d, n, \theta_s, \theta_r]$ , a sensitivity analysis has been undertaken to assess their relative importance on the embankment response under both non-hysteretic and hysteretic conditions. Gardner et al. (1981) suggested the use of the correlation coefficient derived from Monte Carlo simulation to evaluate the relative importance of the input parameters on the output. Hence, one thousand values from the distribution of each parameter (Table 2) have been randomly sampled, and assembled into a  $1000 \times 5$  input matrix:

$$\begin{bmatrix} \mathbf{X}_1 \\ \vdots \\ \mathbf{X}_{1000} \end{bmatrix} = \begin{bmatrix} k_{sat,1} & \alpha_{d,1} & n_1 & \theta_{s,1} & \theta_{r,1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ k_{sat,1000} & \alpha_{d,1000} & n_{1000} & \theta_{s,1000} & \theta_{r,1000} \end{bmatrix} \quad (12)$$

No correlation is assumed between the parameters, although physically likely due to the dependence on grain size, minerology and density, as the purpose of this investigation is to assess which of the parameters is important in controlling the system response.

Each combination of the random values of the five parameters (one row in the matrix) were utilised in a transient seepage analysis (with the domain being taken as homogeneous). The Pearson correlation coefficient was then utilised to give a measure of the correlation between two variables, and is defined as

---


$$\rho_{x_i, \text{FOS}} = \frac{\text{COV}(x_i, \text{FOS})}{\sigma_{x_i} \sigma_{\text{FOS}}} = \frac{E[(x_i - \mu_{x_i})(\text{FOS} - \mu_{\text{FOS}})]}{\sigma_{x_i} \sigma_{\text{FOS}}} \quad (13)$$

where  $x_i$  is the  $i$ th random variable in  $\mathbf{X}$ ,  $\rho_{x_i, \text{FOS}}$  is the Pearson correlation coefficient,  $\text{COV}(x_i, \text{FOS})$  is the covariance of the two variables, and  $\sigma_{x_i}$  and  $\sigma_{\text{FOS}}$  are the standard deviation of  $x_i$  and FOS, respectively.

Figure 8 shows the Pearson correlation coefficients between the FOS and different hydraulic parameters for the non-hysteretic case at time  $t = 2T_2$ . The black values are for the entire data set and the red values are for FOS below 1.25. It can be seen that the FOS increases with an increase in  $k_{sat}$ , but decreases with increasing  $\alpha_d$ ,  $n$  and  $\theta_s$ . The positive relationship between  $k_{sat}$  and FOS is because, when  $k_{sat}$  is large, the outflow of water is faster, leading to a higher FOS. The negative correlations between FOS and  $\alpha_d$ ,  $n$  and  $\theta_s$  may be explained with the aid of Figure 9, which illustrates the sensitivity of the SWRC equation to the different parameters, and by recalling that a lower FOS is generally obtained for higher VWC and lower suction. With reference to Figure 9, for curves ① and ②, the only difference is in the value of parameter  $\theta_s$ , where  $\theta_{s1} > \theta_{s2}$ . An increase in  $\theta_s$  means that the detained water in the embankment increases, which induces a lower FOS. For curves ② and ④, the only difference is parameter  $\alpha_d$ , where  $\alpha_{d2} < \alpha_{d4}$ . For the same VWC, the suction is larger for  $\alpha_{d2}$  and this causes a higher FOS. For curves ② and ③, the only difference is parameter  $n$ , where  $n_2 < n_3$ . For the same VWC, the suction is larger for  $n_2$  and this causes a higher FOS. Figure 8 shows that  $\theta_r$  has almost no influence on the FOS, because the suction cannot reach the range of values where the value of  $\theta_r$  has a large influence. The values of the

---

Pearson correlation coefficients for  $FOS < 1.25$  were calculated to investigate whether FOS closer to failure had similar correlations to those of the whole dataset. All trends are similar, in that positive correlations remain positive and vice versa, although the correlations are lower due to the removal of higher calculated FOS. However, there is still a strong positive correlation between FOS and  $k_{sat}$ .

Figure 10 shows the correlation coefficients as a function of time. The left vertical axis applies to the solid lines and the right axis applies to the dashed lines. In Figure 10, the correlation coefficients significantly decrease for three hydraulic parameters, i.e. for  $k_{sat}$ ,  $\alpha_d$  and  $n$ , whereas the correlation coefficients for  $\theta_s$  and  $\theta_r$  remain fairly stable. This is due to the impact of the propagation of water in the embankment, induced by the upstream water level fluctuation, towards the downstream. At the end of the first main cycle ( $T_1 = 10$  d) the water level has returned to its highest position, so that the unsaturated zone in the sliding area has become smaller. In addition, the water flowing into the embankment due to the increasing external water level has not had time to drain out from the downstream side. These factors result in the FOS reducing, even when  $k_{sat}$  is relatively high and  $\alpha_d$  and  $n$  are relatively small. Hence, when the data of the three parameters versus FOS, for  $t = T_1 = 3T_2$ , are plotted in the same way as in the first three plots in Figure 8 (though not shown here), the dots become more scattered and the correlation coefficients decrease.

As was seen in Figure 8, Figure 10 shows that the saturated hydraulic conductivity has the largest influence on the FOS, while the residual VWC has almost

---

no influence. Indeed, the correlation coefficient between the FOS and  $k_{sat}$  remains high in comparison with the other parameters, even at its lowest point, which indicates that  $k_{sat}$  plays a dominant role in the final computation of the FOS. Therefore, only the heterogeneity of  $k_{sat}$  has been incorporated into the transient seepage analysis in the following section. This conclusion is also supported by other studies reported in literature. Rahardjo et al. (2007) pointed out that the saturated hydraulic conductivity played a dominant role in rainfall infiltration compared to other hydraulic parameters. Avaniadou and Paleologos (2002) and Chen et al. (1994a, 1994b) also suggested that the saturated hydraulic conductivity was the most important parameter in unsaturated heterogeneous soils. Zhang et al. (2005) studied rainfall-induced slope failure in a heterogeneous slope. In the study, only the saturated hydraulic conductivity, saturated volumetric water content, the parameter related to the air entry value and the slope of the water retention curve were considered to be variable, because these four parameters were considered to be important and influence the computation of slope stability. The residual volumetric water content was considered to be not important.

### **5.3 Influence of hysteresis on the seepage and stability of a heterogeneous embankment**

In this section, a spatially variable saturated hydraulic conductivity is used and both hysteretic and non-hysteretic SWRCs are considered; all other parameters are constant. As an indicative example, the scale of fluctuation,  $l_{lnk_{sat}}$ , has been taken to be 1.0 m in both the vertical and horizontal directions, i.e. giving isotropic

---

variability, and 1000 realisations have been performed. Note that, for more comprehensive conclusions on the influence of spatial variability, further studies are needed.

### 5.3.1 Influence of hysteresis on stochastic seepage

In Figure 11, the VWCs at Point B from Figure 2 at  $t = T_1 = 10$  d are compared (one dot from each realisation), with the blue dots representing the non-hysteretic case and the red dots representing the hysteretic case. It can be seen from Figure 11 that the results of the hysteretic case show larger variation. The variation is also not limited along a single line in  $\theta-h_s$  space, as in the non-hysteretic case, but varies in a wider area. Note that, because  $k_{sat}$  was the only randomized parameter, the main drying curve in the SWRC is the same for both the non-hysteretic and hysteretic cases. This means that, in the non-hysteretic case, the spatial variability can only cause the scattering of blue dots located along the main drying curve in the SWRC. However, in the hysteretic case, the hysteretic behaviour allows the suction to vary along the scanning curve. When the spatial variability is added into the hysteretic effect, this causes the suction to vary in the area enclosed by the main drying and wetting curves.

### 5.3.2 Influence of hysteresis on the stability of the heterogeneous slope

The factor of safety has been calculated for each realisation and a log-normal distribution fitted to the resulting ensemble distribution of FOS. Table 4 gives the mean  $\mu_{FOS}$  and standard deviation  $\sigma_{FOS}$  of the FOS for both the non-hysteretic and hysteretic cases. The mean of the FOS at all times was smaller for the hysteretic

---

case than for the non-hysteretic case. In addition, the standard deviation of the FOS was usually higher for the hysteretic case. The reason is that, in the hysteretic case, the hysteresis in the water retention behaviour induced much more variation in the PWP and VWC, because the PWP and VWC could vary in the area enclosed by the main drying and wetting curves. Therefore, the uncertainty in the FOS is larger for the hysteretic case.

Figure 12 shows the cumulative distribution functions (CDF) of FOS for both the non-hysteretic (dashed line) and hysteretic cases (solid line) at different times. The limit value of the FOS (dash-dotted line) represents the FOS calculated in a homogeneous non-hysteretic analysis based on the mean value of  $k_{sat}$ . From the results in Figure 12, it is seen that if a deterministic method is used to analyse the safety of the embankment slope, there is a high probability of overestimating the FOS (in this case, by up to 20% compared to a 95% reliability). Moreover, based on the comparison between the solid and dashed lines, it can be concluded that, if only the heterogeneity is considered while the hysteretic effect in the SWRC is ignored, the computed probability of slope failure would be lower. This proves that, although the contribution to the variation in results due to the uncertainties in the material properties and the hysteresis can be different (the former factor could be larger if there are strong uncertainties in the parameters), both factors play an important role and should be considered in the analysis.

## 6 Conclusion

The combined effect of hysteresis in the water retention behaviour and

---

heterogeneity of an unsaturated soil on the stability of an embankment under transient seepage has been investigated. The stability and reliability of the embankment shows significant differences with analyses which ignore either or both of these factors.

Under the influence of both hysteresis and heterogeneity, the PWP and VWC in the transient seepage process have a larger variation than would otherwise be the case. Considering slope stability, in the initially drying condition, the mean of the FOS for the hysteretic case is smaller than that of the non-hysteretic case. Moreover, the standard deviation of the FOS is usually larger. It has been found that, due to hysteresis in the water retention behaviour, the influence of the heterogeneity of soil property values on slope stability could be amplified. Furthermore, in the sensitivity analysis of hydraulic parameters, it was found the saturated hydraulic conductivity,  $k_{sat}$ , plays a dominant role in slope stability compared to other hydraulic parameters. Further studies on the impact of spatial variability on these processes are needed to provide comprehensive conclusions.

#### **Acknowledgements**

The authors wish to acknowledge the support of the China Scholarship Council coupled with the Geo-Engineering Section of Delft University of Technology for financial support of the for the first author and a Marie Curie Career Integration Grant, No. 333177, for the second author.

$c'$	effective cohesion
$c'_f$	factored effective cohesion at slope failure
$COV$	coefficient of variation
FOS	factor of safety
$g$	gravitational acceleration
$G_s$	specific gravity of the soil particles
$h$	pore water pressure head
$h_s$	suction head
$h_{s,ae}$	air-entry suction head
$h_{s,ae,d}$	air-entry suction head for the main drying curve
$h_{s,ae,w}$	air-entry suction head for the main wetting curve
$k$	hydraulic conductivity
$k_{sat}$	saturated hydraulic conductivity
$k_x$	hydraulic conductivity in the $x$ direction
$k_z$	hydraulic conductivity in the $z$ direction
$l$	scale of fluctuation
$l_h$	scale of fluctuation in the horizontal direction
$l_v$	scale of fluctuation in the vertical direction
$m$	fitting parameter for the soil water retention curve
MCM	Monte Carlo method
$n$	fitting parameter for the soil water retention curve



---

$N$	total number of realisations
$N_f$	number of realisations in which the slope fails
PWP	pore water pressure
$s$	matric suction
$S$	effective degree of saturation
SWRC	soil water retention curve
$t$	time
$T_1$	period of the first sinusoid
$T_2$	period of the second sinusoid
$u_a$	pore air pressure
$u_w$	pore water pressure
VGM	van Genuchten–Mualem model
VWC	volumetric water content
WL	water level
$x$	coordinate in the horizontal direction relative to upstream toe of embankment
$z$	coordinate in the vertical direction relative to upstream toe of embankment
$\alpha_d$	approximately the inverse of the air-entry suction head for main drying curve
$\alpha_w$	approximately the inverse of the air-entry suction head for main wetting curve
$\gamma$	unit weight of soil

---

$\gamma_w$	unit weight of water
$\theta$	volumetric water content
$\theta_s$	saturated volumetric water content
$\theta_r$	residual volumetric water content
$\kappa$	slope of the scanning curve
$\mu$	mean
$\xi$	degree of anisotropy of the heterogeneity
$\rho$	correlation coefficient between two points
$\rho_w$	density of water
$\sigma$	standard deviation
$\sigma_t$	total stress normal to the sliding plane
$\tau$	shear stress
$\varphi'$	effective friction angle
$\varphi'_f$	factored effective friction angle at slope failure
$\chi$	scalar defining the suction-induced effective stress

## References

- Arnold, P. and Hicks, M. A. (2010). Stochastic modelling of unsaturated slope stability. *Proceedings of 5<sup>th</sup> International Conference on Unsaturated Soils (UNSAT)*, Barcelona, 1237-1242.
- Arnold, P. and Hicks, M. A. (2011). A stochastic approach to rainfall-induced slope failure. *Proceedings of 3rd International Symposium on Geotechnical Safety and Risk*, Munich, 107-115.
- ASCE (2008). *Standard Guideline for Fitting Saturated Hydraulic Conductivity Using Probability Density Functions*, ASCE Standard ASCE/EWRI 50-08, ASCE, Reston.

---

488 Avaniidou, T. and Paleologos, E. K. (2002). Infiltration in stratified, heterogeneous soils: Relative  
 489 importance of parameters and model variations. *Water Resources Research*, 38(11),  
 490 1232-1236.

491 Bishop, A. W. (1959). The principle of effective stress. *Teknisk Ukeblad*, 106(39), 859–863.

492 Cai, F. and Ugai, K. (2004). Numerical analysis of rainfall effects on slope stability. *International Journal*  
 493 *of Geomechanics*, 4(2), 69-78.

494 Carsel, R. F. and Parrish, R. S. (1988). Developing joint probability distributions of soil water retention  
 495 characteristics. *Water Resources Research*, 24(5), 755-769.

496 Celia, M. A., Bouloutas, E. T. and Zarba, R. L. (1990). A general mass-conservative numerical solution  
 497 for the unsaturated flow equation. *Water Resources Research*, 26(7), 1483-1496.

498 Chen, Q. and Zhang, L. M. (2006). Three-dimensional analysis of water infiltration into the Gouhou  
 499 rockfill dam using saturated unsaturated seepage theory. *Canadian Geotechnical Journal*,  
 500 43(5), 449-461.

501 Chen, Z., Govindaraju, R. S. and Kavvas, M. L. (1994a). Spatial averaging of unsaturated flow equations  
 502 under infiltration conditions over areally heterogeneous fields, 1, Development of models.  
 503 *Water Resources Research*, 30(2), 523– 533.

504 Chen, Z., Govindaraju, R. S. and Kavvas, M. L. (1994b). Spatial averaging of unsaturated flow equations  
 505 under infiltration conditions over areally heterogeneous fields, 2, Numerical simulations.  
 506 *Water Resources Research*, 30(2), 535– 548.

507 De Rooij, G. H., Kasteel, R. T., Papritz, A. and Flüßler, H. (2004). Joint distributions of the unsaturated  
 508 soil hydraulic parameters and their effect on other variates. *Vadose Zone Journal*, 3(3),  
 509 947-955.

---

510 Ebel, B. A., Loague, K. and Borja, R. I. (2010). The impacts of hysteresis on variably saturated  
511 hydrologic response and slope failure. *Environmental Earth Sciences*, 61(6), 1215-1225.

512 Fenton, G. A. and Vanmarcke, E. H. (1990). Simulation of random fields via local average subdivision.  
513 *Journal of Engineering Mechanics*, 116(8), 1733-1749.

514 Freeze, R. A. (1975). A stochastic-conceptual analysis of one-dimensional groundwater flow in  
515 nonuniform homogeneous media. *Water Resources Research*, 11(5), 725-741.

516 Gardner, R. H., O'Neill, R. V., Mankin, J. B. and Carney, J. H. (1981). A comparison of sensitivity analysis  
517 and error analysis based on a stream ecosystem model. *Ecological Modelling*, 12(3), 173-190.

518 Griffiths, D. V. and Lane, P. A. (1999). Slope stability analysis by finite elements. *Géotechnique*, 49(3),  
519 387-403.

520 Griffiths, D. V. and Fenton, G. A. (2004). Probabilistic slope stability analysis by finite elements. *Journal*  
521 *of Geotechnical and Geoenvironmental Engineering*, 130(5), 507-518.

522 Gui, S., Zhang, R., Turner, J. P. and Xue, X. (2000). Probabilistic slope stability analysis with stochastic  
523 soil hydraulic conductivity. *Journal of Geotechnical and Geoenvironmental Engineering*,  
524 126(1), 1-9.

525 Han, X.-W., Shao, M.-A. and Horton, R. (2010). Estimating van Genuchten model parameters of  
526 undisturbed soils using an integral method. *Pedosphere*, 20(1), 55-62.

527 Hicks, M. A. and Samy, K. (2002). Influence of heterogeneity on undrained clay slope stability.  
528 *Quarterly Journal of Engineering Geology and Hydrogeology*, 35(1), 41-49.

529 Hicks, M. A. and Samy, K. (2004). Stochastic evaluation of heterogeneous slope stability. *Italian*  
530 *Geotechnical Journal*, 38(2), 54-66.

531 Hicks, M. A. and Spencer, W. A. (2010). Influence of heterogeneity on the reliability and failure of a

---

532 long 3D slope. *Computers and Geotechnics*, 37(7), 948-955.

533 Jaynes, D. B. (1984). Comparison of soil–water hysteresis models. *Journal of Hydrology*, 75(1),

534 287-299.

535 Lam, L., Fredlund, D. G. and Barbour, S. L. (1987). Transient seepage model for saturated–unsaturated

536 soil systems: a geotechnical engineering approach. *Canadian Geotechnical Journal*, 24(4),

537 565-580.

538 Le, T. M. H., Gallipoli, D., Sanchez, M. and Wheeler, S. J. (2012). Stochastic analysis of unsaturated

539 seepage through randomly heterogeneous earth embankments. *International Journal for*

540 *Numerical and Analytical Methods in Geomechanics*, 36(8), 1056-1076.

541 Lehmann, F. and Ackerer, P. H. (1998). Comparison of iterative methods for improved solutions of the

542 fluid flow equation in partially saturated porous media. *Transport in Porous Media*, 31(3),

543 275-292.

544 Liu, K., Vardon, P. J., Arnold, P. and Hicks, M. A. (2015). Effect of hysteresis on the stability of an

545 embankment under transient seepage. *IOP Conference Series: Earth and Environmental*

546 *Science*, 26(1), 012013.

547 Ma, K. C., Tan, Y. C. and Chen, C. H. (2011). The influence of water retention curve hysteresis on the

548 stability of unsaturated soil slopes. *Hydrological Processes*, 25(23), 3563-3574.

549 Mualem, Y. (1976). A new model for predicting the hydraulic conductivity of unsaturated porous

550 media. *Water Resources Research*, 12(3), 513-522.

551 Nakagawa, K., Saito, M. and Berndtsson, R. (2012). On the importance of hysteresis and heterogeneity

552 in the numerical simulation of unsaturated flow. *Hydrological Research Letters*, 6, 59-64.

553 Nielsen, D. R., Biggar, J. W. and Erh, K. T. (1973). Spatial variability of field-measured soil–water

---

554 properties. *Hilgardia*, 42, 215-259.

555 Pham, H. Q., Fredlund, D. G. and Barbour, S. L. (2005). A study of hysteresis models for soil–water

556 characteristic curves. *Canadian Geotechnical Journal*, 42(6), 1548-1568.

557 Phoon, K.-K. and Kulhawy, F. H. (1999). Characterization of geotechnical variability. *Canadian*

558 *Geotechnical Journal*, 36(4), 612-624.

559 Phoon, K.-K., Santoso, A. and Quek, S.-T. (2010). Probabilistic analysis of soil–water characteristic

560 curves. *Journal of Geotechnical and Geoenvironmental Engineering*, 136(3), 445-455.

561 Rahardjo, H., Ong, T., Rezaury, R., and Leong, E. C. (2007). Factors controlling instability of

562 homogeneous soil slopes under rainfall. *Journal of Geotechnical and Geoenvironmental*

563 *Engineering*, 133(12), 1532-1543.

564 Rahardjo, H., Nio, A. S., Leong, E. C. and Song, N. Y. (2010). Effects of groundwater table position and

565 soil properties on stability of slope during rainfall. *Journal of Geotechnical and*

566 *Geoenvironmental Engineering*, 136(11), 1555-1564.

567 Russo, D. and Bouton, M. (1992). Statistical analysis of spatial variability in unsaturated flow

568 parameters. *Water Resources Research*, 28(7), 1911-1925.

569 Santoso, A. M., Phoon, K.-K. and Quek, S.-T. (2011). Effects of soil spatial variability on rainfall-induced

570 landslides. *Computers and Structures*, 89(11), 893-900.

571 Sivakumar Babu, G. L. and Murthy, D. S. (2005). Reliability analysis of unsaturated soil slopes. *Journal*

572 *of Geotechnical and Geoenvironmental Engineering*, 131(11), 1423-1428.

573 Smith, I. M., Griffiths, D. V. and Margetts, L. (2013). *Programming the finite element method*. John

574 Wiley and Sons, Chichester, UK, 5th edition.

575 Tami, D., Rahardjo, H. and Leong, E.-C. (2004). Effects of hysteresis on steady-state infiltration in

---

576           unsaturated slopes. *Journal of Geotechnical and Geoenvironmental Engineering*, 130(9),  
577           956-967.

578   Tsai, T.-L. and Chen, H.-F. (2010). Effects of degree of saturation on shallow landslides triggered by  
579           rainfall. *Environmental Earth Sciences*, 59(6), 1285-1295.

580   Tsaparas, I., Rahardjo, H., Toll, D. G. and Leong, E. C. (2002). Controlling parameters for rainfall-induced  
581           landslides. *Computers and Geotechnics*, 29(1), 1-27.

582   Van Genuchten, M. T. (1980). A closed-form equation for predicting the hydraulic conductivity of  
583           unsaturated soils. *Soil Science Society of America Journal*, 44(5), 892-898.

584   Vanapalli, S. K., Fredlund, D. G., Pufahl, D. E. and Clifton, A. W. (1996). Model for the prediction of  
585           shear strength with respect to soil suction. *Canadian Geotechnical Journal*, 33(3), 379-392.

586   Wu, L. Z., Huang, R. Q. and Xu, Q. (2012). Incorporating hysteresis in one-dimensional seepage  
587           modeling in unsaturated soils. *KSCE Journal of Civil Engineering*, 16(1), 69-77.

588   Yang, C., Sheng, D. and Carter, J. P. (2011). Hysteretic seepage analysis in unsaturated soil covers.  
589           *Proceedings of 5th Asia-Pacific Conference on Unsaturated Soils*, Pattaya, 541-546.

590   Yang, C., Sheng, D. and Carter, J. P. (2012). Effect of hydraulic hysteresis on seepage analysis for  
591           unsaturated soils. *Computers and Geotechnics*, 41, 36-56.

592   Yang, C., Sheng, D., Carter, J. P. and Huang, J. (2013). Stochastic evaluation of hydraulic hysteresis in  
593           unsaturated soils. *Journal of Geotechnical and Geoenvironmental Engineering*, 139(7),  
594           1211-1214.

595   Zhang, L. L. (2007). Effects of soil hydraulic hysteresis on slope reliability. *Proceedings of 1<sup>st</sup>*  
596           *International Symposium on Geotechnical Safety and Risk (ISGSR 2007)*, Shanghai, 399-405.

597   Zhang, L. L., Fredlund, M. D., Fredlund, D. G., Lu, H. and Wilson, G. W. (2015). The influence of the

---

598           unsaturated soil zone on 2-D and 3-D slope stability analyses. *Engineering Geology*, 193,  
599           374-383.

600   Zhang L., Li, J., Li, X., Zhang, J. and Zhu, H. (2016). *Rainfall-induced soil slope failure*. CRC Press, Boca  
601           Raton.

602   Zhang, L., Zhang, L. and Tang, W. H. (2005). Rainfall-induced slope failure considering variability of soil  
603           properties. *Géotechnique*, 55(2), 183-188.

604   Zhu, H., Zhang, L. M., Zhang, L. L. and Zhou, C. B. (2013). Two-dimensional probabilistic infiltration  
605           analysis with a spatially varying permeability function. *Computers and Geotechnics*, 48,  
606           249-259.



Table 1 Parameter values for the homogenous case.

Parameter	Symbol	Units	Value
Saturated hydraulic conductivity	$k_{sat}$	m/d	0.0864
VGM parameter for the main drying (and non-hysteretic) curve	$\alpha_d$	$m^{-1}$	0.1
VGM parameter for the main wetting curve	$\alpha_w$	$m^{-1}$	0.2
Fitting parameter for VGM model	$n$	-	1.226
Saturated VWC	$\theta_s$	-	0.38
Residual VWC	$\theta_r$	-	0.0038
Slope of the scanning curve	$\kappa$	$m^{-1}$	0.00006
Stiffness	$E$	kPa	$1.0 \times 10^5$
Poisson's ratio	$\nu$	-	0.3
Effective cohesion	$c'$	kPa	15
Effective friction angle	$\varphi'$	$^\circ$	20
Specific gravity	$G_s$	-	2.02

Note: VGM represents the van Genuchten–Mualem model described in Section 2.2.

Table 2 Statistical distributions of the hydraulic parameters

Parameter	$k_{sat}$ [m/d]	$ \alpha_d  [m^{-1}]$	$n$ [-]	$\theta_s$ [-]	$\theta_r$ [-]
Distribution	Lognormal	Lognormal	Normal	Lognormal	Lognormal
$\mu$	0.0864	0.1	1.226	0.38	0.0038
$\sigma$	0.0864	0.05	0.08	0.06	0.0002
$COV$	1.0	0.5	0.065	0.16	0.053

Table 3 Outline of numerical investigation

Sub-section	Hysteresis	Parameter sensitivity	Spatial variability
5.1	✓		
5.2	✓	✓	
5.3	✓		✓

Table 4 Statistical information of the FOS based on 1000 realisations

Case	Time $t$	$\mu_{\text{FOS}}$	$\sigma_{\text{FOS}}$	Case	Time $t$	$\mu_{\text{FOS}}$	$\sigma_{\text{FOS}}$
Hysteretic	$1T_2$	1.199	0.0374	Non-hysteretic	$1T_2$	1.221	0.0364
	$2T_2$	1.233	0.046		$2T_2$	1.278	0.045
	$T_1 = 3T_2$	1.182	0.0467		$T_1 = 3T_2$	1.245	0.0488
	$4T_2$	1.232	0.049		$4T_2$	1.274	0.0475
	$5T_2$	1.253	0.0535		$5T_2$	1.311	0.0529

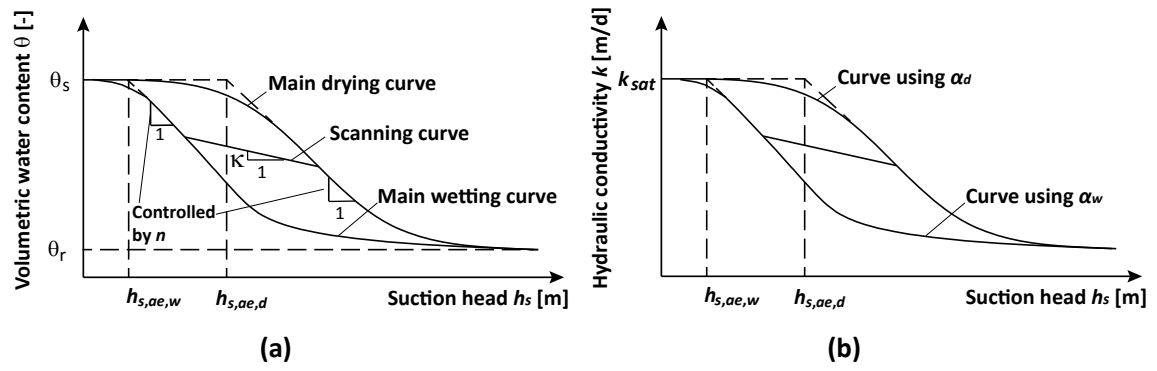


Figure 1. The relationships between suction head and: (a) VWC, and (b) hydraulic conductivity.

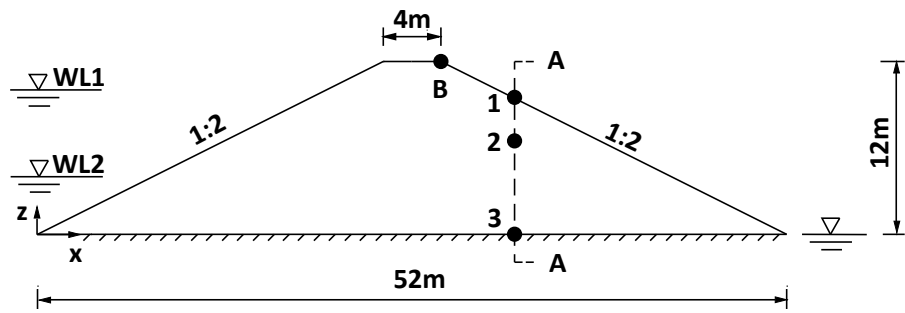


Figure 2. Geometry of the embankment.

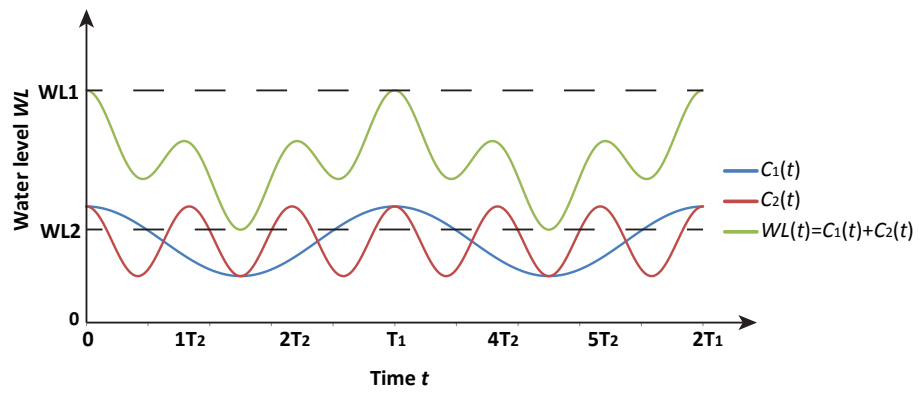


Figure 3. Water level fluctuation simulated by the sum of two sinusoidal curves.

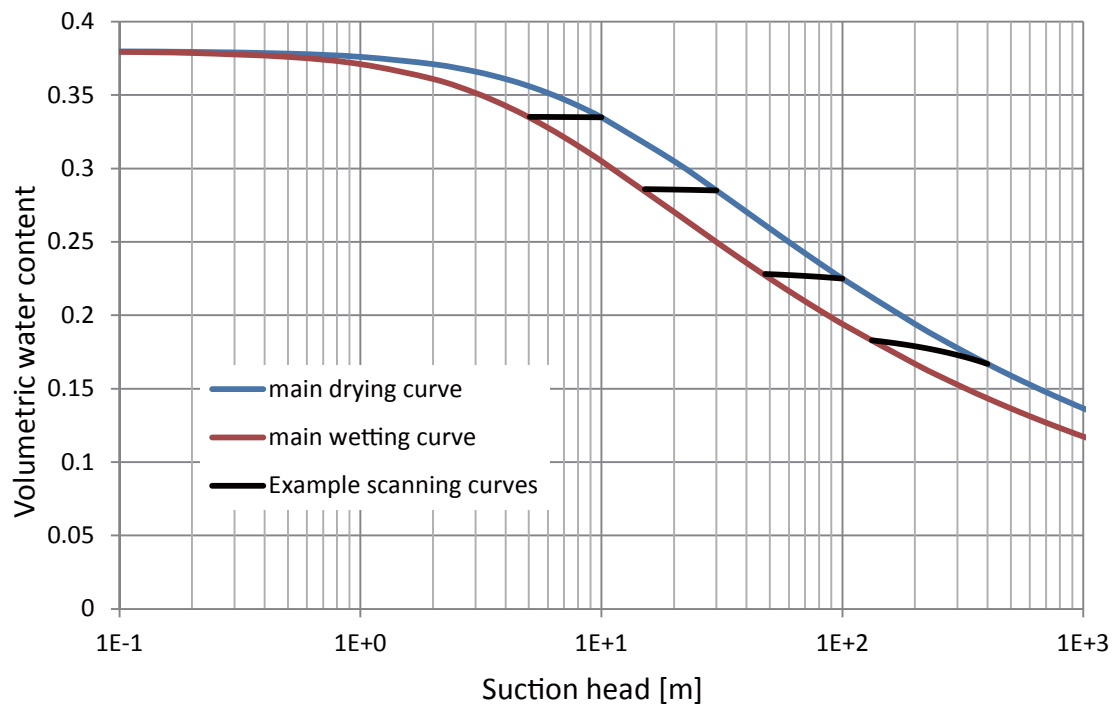


Figure 4. SWRC used in the analyses



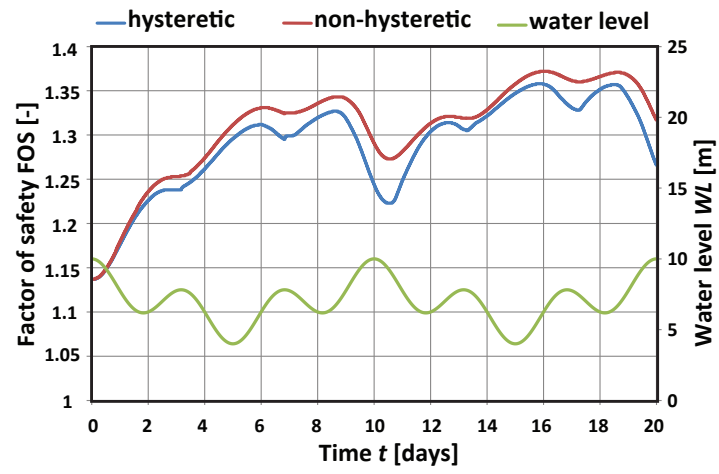


Figure 5. FOS variation with time.

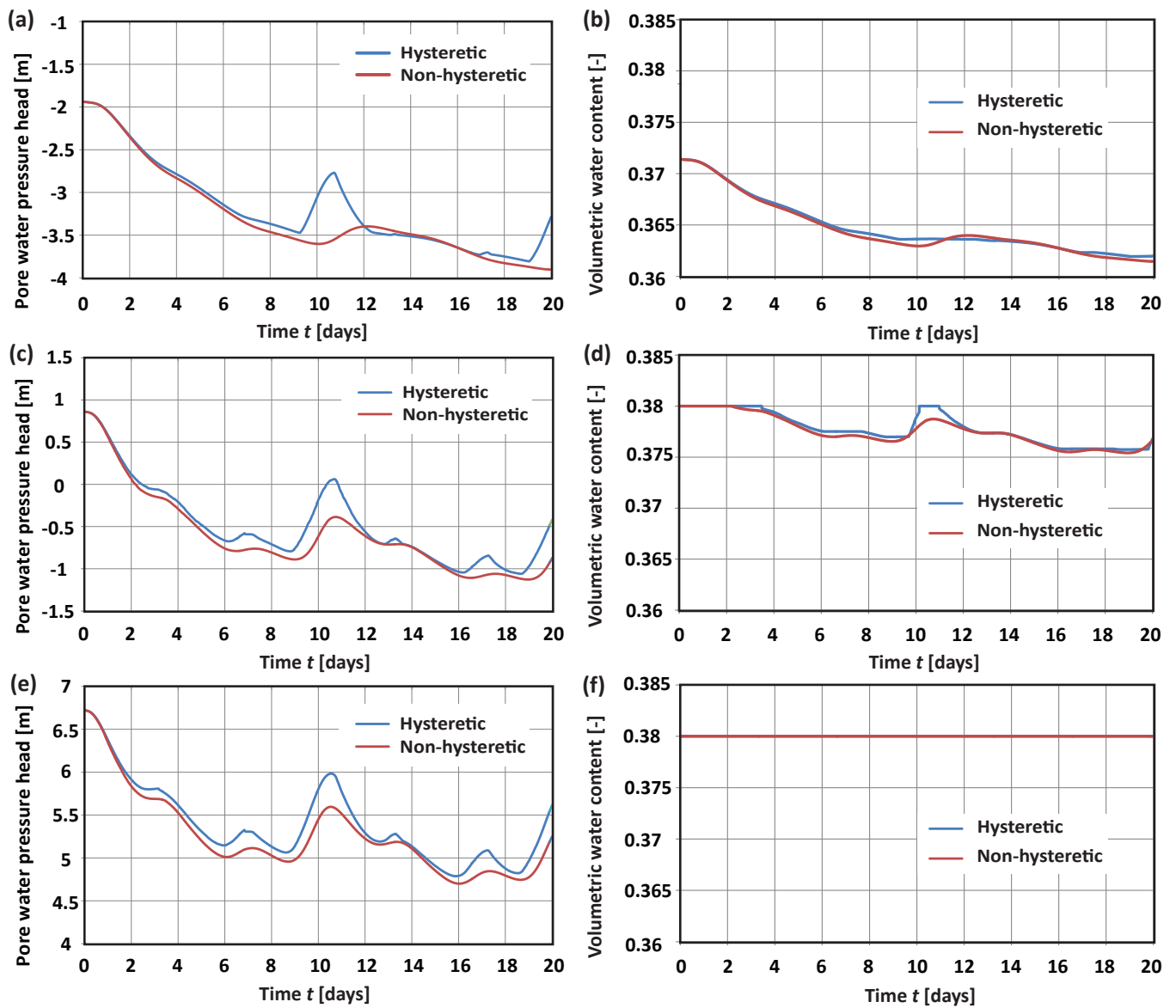


Figure 6. PWP head and VWC versus time at different depths (points 1-3 in Figure 2): (a) and (b)  $z_1 = 9$  m; (c) and (d)  $z_2 = 6$  m; and (e) and (f)  $z_3 = 0$  m.

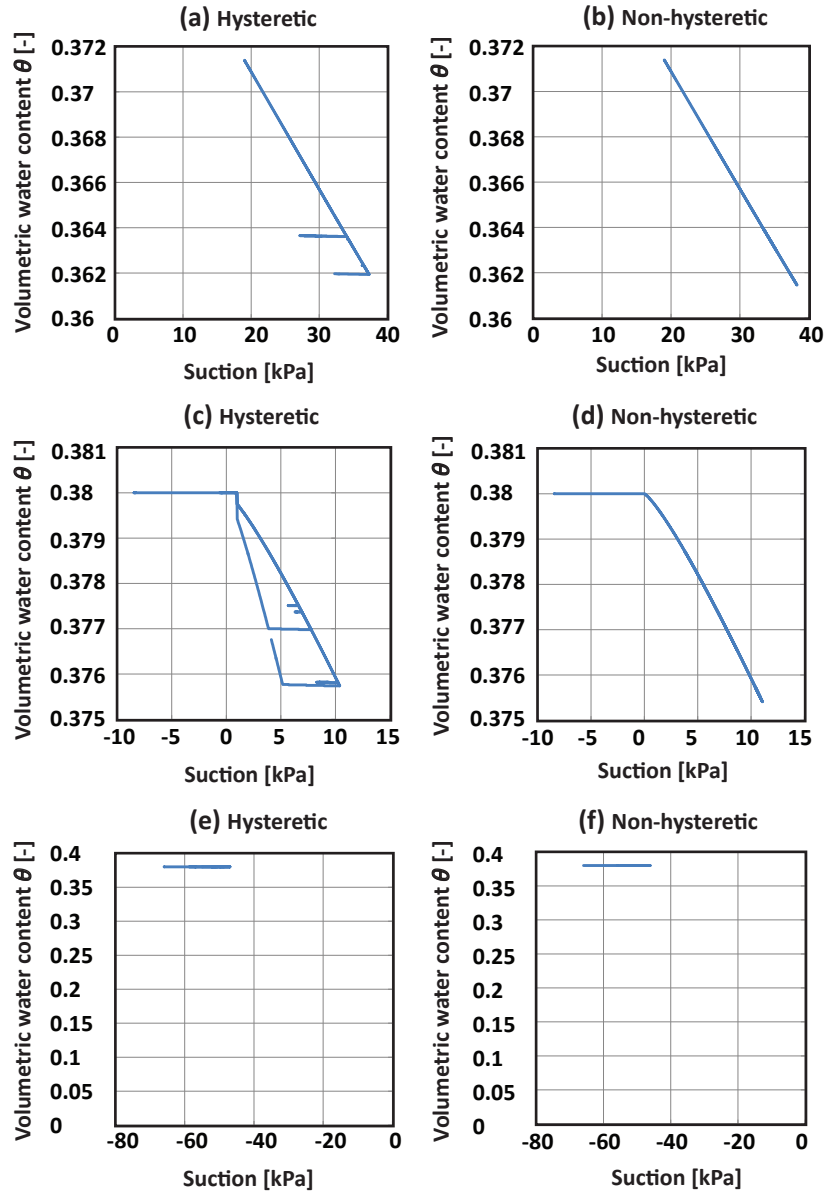


Figure 7. VWC versus suction at points along profile A–A at different depths: (a) and (b)  $z_1 = 9$  m; (c) and (d)  $z_2 = 6$  m; and (e) and (f)  $z_3 = 0$  m.

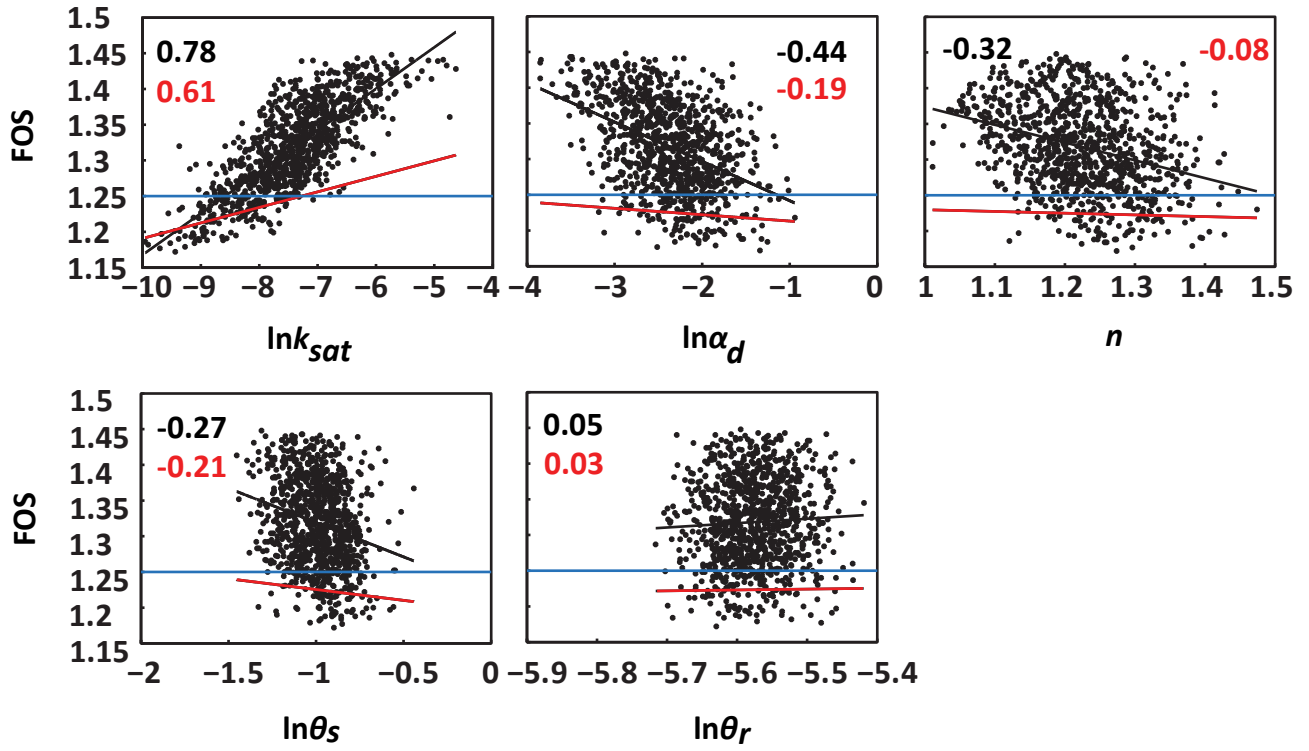


Figure 8. Correlation between FOS and different hydraulic parameters for non-hysteretic case at  $t = 2T_2$ . (The blue line represents  $FOS = 1.25$ , the black line and black number represent the correlation based on all the FOS values, and the red line and red number represent the correlation based on FOS values lower than  $FOS = 1.25$ ).

Parameter	Curves	Relation
$\theta_s$	1, 2	$\theta_{s1} > \theta_{s2}$
$\alpha_d$	2, 4	$\alpha_{d2} < \alpha_{d4}$
$n$	2, 3	$n_2 < n_3$

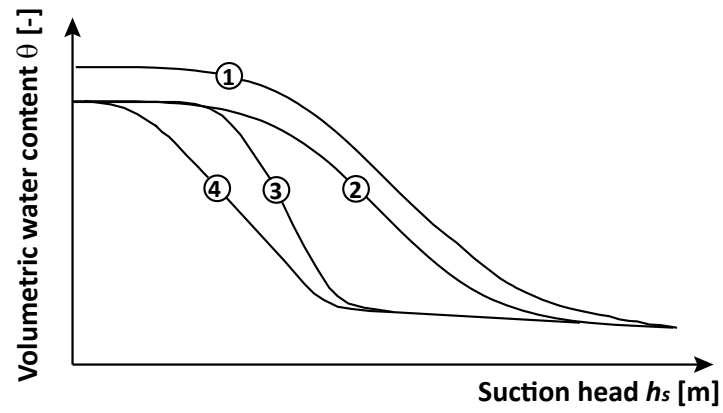


Figure 9. Sensitivity of the SWRC equation to model parameters.

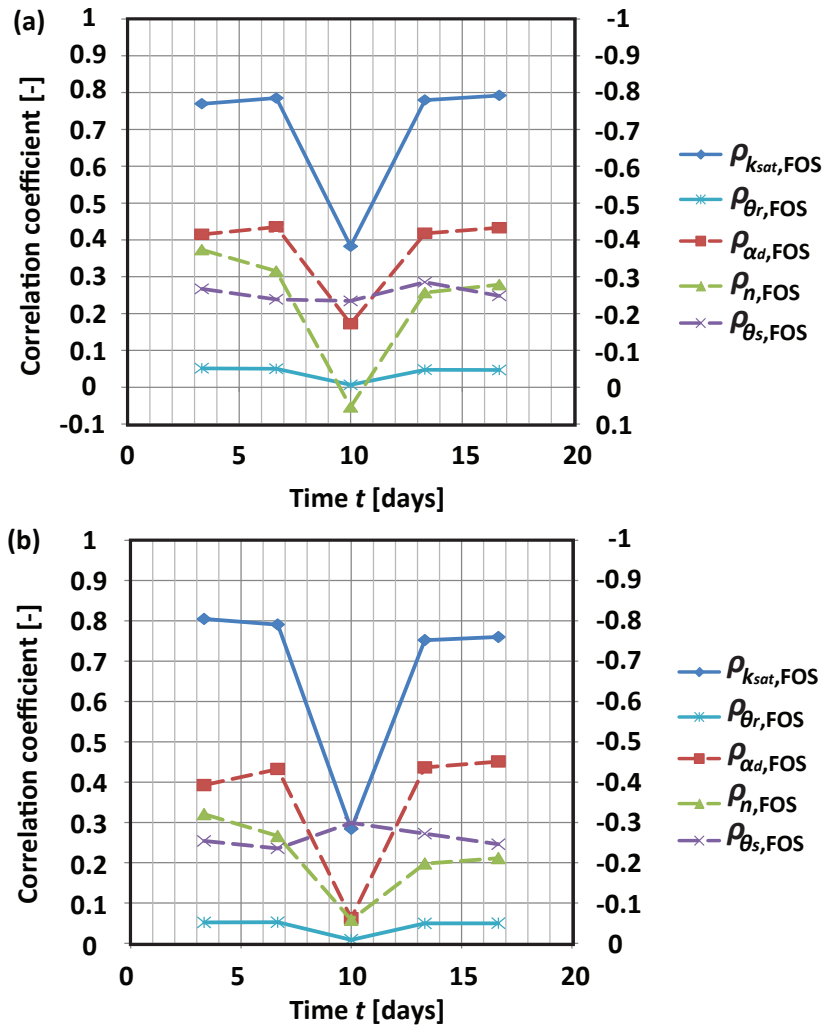


Figure 10. Correlation coefficients between FOS and different hydraulic parameters at several specific times: (a) non-hysteretic case and (b) hysteretic case.

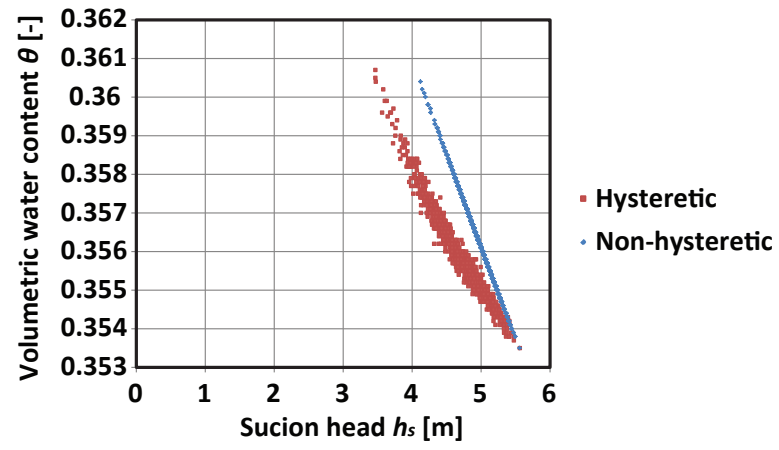


Figure 11. VWC versus suction head at Point B at  $t = T_1 = 10$  d based on 1000 realisations.

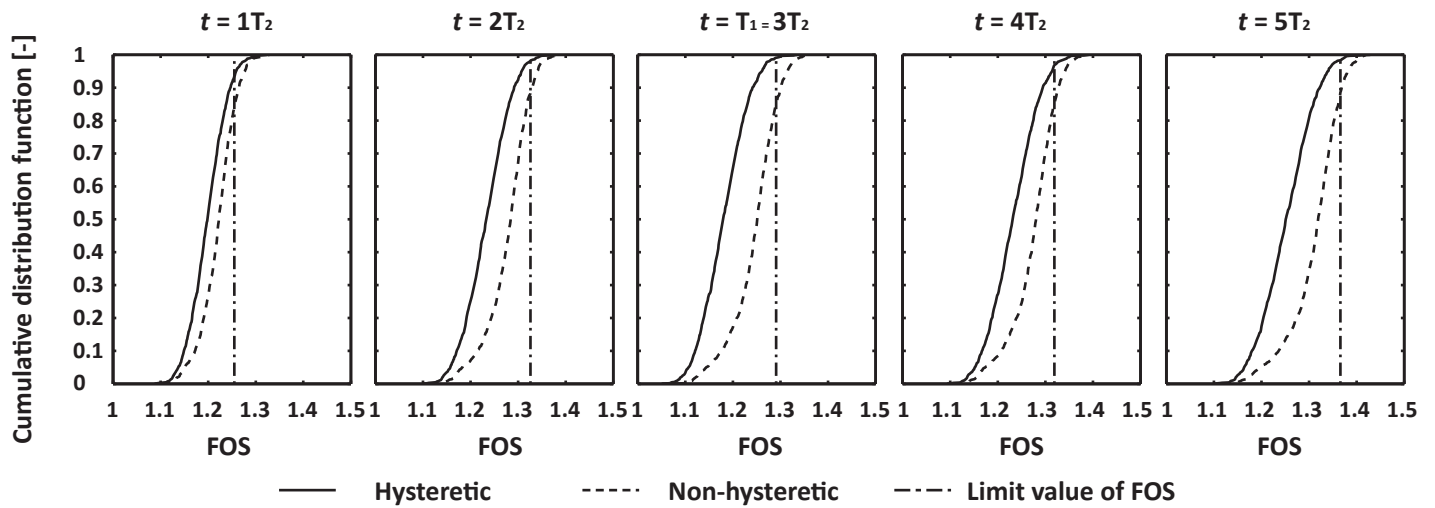


Figure 12. Cumulative distribution functions (CDFs) of FOS for both non-hysteretic and hysteretic cases at different times. The solid line is the CDF for the hysteretic case and the dotted line is for the non-hysteretic case. The dash-dotted line shows the factor of safety calculated without considering heterogeneity or hysteresis.