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Title: **Performance analysis of a hybrid direct sequence / slow frequency hopping CDMA system in an indoor wireless slow Rician fading environment.**

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Type : graduation thesis
Size : 123 + ix
Date : August 31, 1993

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Period : January 1993 - July 1993

Abstract

The bit error rate of a hybrid direct sequence/slow frequency hopping system for DPSK modulation in a Rician fading radio channel in an indoor environment has been investigated with help of a theoretical model. We have considered the effect of selection diversity and maximal ratio combining on the performance. The effect of FEC coding on the performance was also part of the investigation. A comparison of the hybrid system with pure direct sequence has been made as well as a comparison with another model from literature.

The bit error rate performance of hybrid DS/SFH systems, employing BPSK and QPSK as modulation techniques, is also discussed. In this way a comparison could be made of three systems, using the three mentioned modulation schemes.

Finally, a throughput and delay analysis of a CDMA network using hybrid/ DS/SFH transceivers with DPSK modulation is presented in this report. A comparison of the throughput of the hybrid system has been compared with the throughput of a system based on direct sequence.

Indexing terms:

CDMA, Direct Sequence, Delay, Indoor wireless communications, Slow Frequency Hopping, Throughput.

SUMMARY

Wireless communication systems have regained a lot of interest recently. Especially the application in an indoor environment gives the advantages of flexibility and costs saving with respect to cabling. Users can get access to the radio channel by using multiple access techniques, such as Code Division Multiple Access (CDMA) of which Direct Sequence (DS) and Slow Frequency Hopping (SFH) are two implements. From previous studies however, it was observed that direct sequence (DS) suffers severely from the near-far effect. A hybrid direct sequence/slow frequency hopping (DS/SFH) system is able to overcome this problem.

In this report we have investigated the performance of a hybrid DS/SFH system in a Rician fading environment for two types of diversity, viz. selection diversity and maximal ratio combining with help of a theoretical model. The performance of the system is expressed in terms of average bit error probability or outage probability. Next, a comparison of DPSK with BPSK and QPSK is presented and the model under consideration with DPSK modulation is compared with a model described in literature. Second, the throughput and delay of a slotted CDMA network based on a hybrid DS/SFH is assessed with a theoretical model.

The models, used to describe the performance of the hybrid DS/SFH technique and the CDMA network based on this technique, are only valid when several key requirements are met. However, they give a good first impression of the behaviour of the systems. From the numerical results we have concluded that:

- The model in this report yields a higer bit error probability than the model presented in literature, because of the difference in approximations made in both models;
- Given a fixed bandwidth, a hybrid DS/SFH system with FEC coding and with large spreading codes and a relative low number of frequencies, yields a better result than a system with a large number of frequencies and short spreading codes;
- For the same bandwidth, DS operates better than hybrid DS/SFH in case of a large. For a relatively low number of resolvable paths the performance is in favour of the hybrid system;
- Comparing the performance of DPSK, BPSK and QPSK at fixed bandwidth and data rate shows that DPSK yields the poorest performance, next BPSK and the best is QPSK;
- A resonable throughput and accordingly a small system delay can be obtained by using small packet sizes and a sufficient order of diversity or in case of large packet sizes, the use of FEC coding in combination with a high order of diversity.

PREFACE

The performance analysis presented in this report has required quite some computations. Especially the bit error calculations have demanded a lot of programming. Fortunately, some predecessors have been working in the field of CDMA and in particular in the field of direct sequence. The software for the calculations of the bit error probability in case of direct sequence was already written.

In that respect I would like to thank Howard Misser and Caspar Wijffels for putting their software at my disposal. Further I would like to thank Hami Çamkerten and Alexander Zigic for the many discussions we had.

René Rooimans,
9 augustus 1993

LIST OF SYMBOLS

$\alpha(t)$	attenuation factor
β	path gain
β_{max}	maximal path gain
γ_b	bit signal-to-noise ratio
ξ	decision variable
η	real part of the noise at the matched filter output
ν	imaginary part of the noise at the matched filter output
μ	conditional covariance of the complex envelope at the current and the previous sampling instant
μ_o	conditional variance of the complex envelope at the current sampling instant
μ_{-1}	conditional variance of the the complex envelope at the previous sampling instant
σ_r^2	received power of the scattered signals
σ_n^2	variance of the noise samples
τ	path delay
$(\Delta f)_c$	coherence bandwidth
$(\Delta t)_c$	coherence time
$a_k(t)$	direct sequence code waveform of user k
$b_k(t)$	data waveform of user k
b_k^o	current data bit of user k
b_k^{-1}	previous data bit of user k
B_d	Doppler spread
B_T	Transmission bandwidth
C	CDMA threshold (maximum number of simultaneous transmitting users in the network)
C_{1k}	discrete aperiodic correlation function
D	system delay
d_1	random variable corresponding with the event of two frequencies being the same or not respectively associated with the the previous bit;
d_2	random variable corresponding with the event of two frequencies being the same or not respectively associated with the current bit

E_b	bit energy
G	offered traffic
G_p	processing gain
K	number of simultaneously transmitting users
L	number of resolvable paths
m	mean of the matched filter outputs
M	order of diversity
N	period of the spread spectrum codes
N_b	number of bits per hop
N_p	number of bits in a packet
N_o	single sided spectral density of the white Gaussian noise
P	transmitted signal power per user
q	number of frequencies in the hopping pattern
r	distance between transmitter and receiver
$r(t)$	received signal at the dehopper input
$r_d(t)$	signal at the dehopper output
R	Rice factor
R_c	channel bitrate
R_{1k}	partial correlation for user k
S	peak value of the envelope of the LOS component
S_n	normalized throughput
T	data bit duration
T_a	duration of an acknowledgement packet
T_c	chip time
T_h	hop duration
T_m	time delay spread
T_p	duration of a packet
T_r	retransmission delay
W	Message bandwidth
X_o	real part of the complex envelope at the current sampling instant
X_{-1}	real part of the complex envelope at the previous sampling instant
Y_o	imaginary part of the complex envelope at the current sampling instant
Y_{-1}	imaginary part of the complex envelope at the previous sampling instant

LIST OF ABBREVIATIONS

AWGN	Additive White Gaussian Noise
BPSK	Binary Phase Shift Keying
CDF	Cumulative Density Function
CDMA	Code Division Multiple Access
DPSK	Differential Phase Shift Keying
DS	Direct Sequence
FDMA	Frequency Division Multiple Access
FEC	Forward Error Correction
FFH	Fast Frequency Hopping
LOS	Line Of Sight
MRC	Maximal Ratio Combining
PDF	Probability Density Function
PN	Pseudo Noise
QPSK	Quartenary Phase Shift Keying
RF	Radio Frequency
SFH	Slow Frequency Hopping
SD	Selection Diversity
SS	Spread Spectrum
TDMA	Time Division Multiple Access
TH	Time Hopping

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1. INTRODUCTION

Wireless communication is a part of telecommunications that has regained a lot of interest recently. This appears from the increasing demand for wireless mobile services such as vehicular telephony, radio paging systems and cordless telephones.

Beside applications of these wireless services in an outdoor environment, there is an increasing development of wireless systems in the indoor environment. Especially in the office environment the application of these systems, such as wireless computer networks and cordless telephones, offers the advantages of flexibility and saving of costs compared to the cable systems.

In order to provide the possibility for users to transmit simultaneously and to overcome the severe fading effects of the indoor radio channel, spread spectrum multiple access techniques have to be used. A very promising technique is Code Division Multiple Access (CDMA), of which Direct Sequence (DS) and Slow Frequency Hopping (SFH) are two implements.

In the last few years, a lot of attention has been paid to direct sequence spread spectrum (DS/SS) communication systems over fading channels in presence of multi-user and multipath interference [3],[7],[10]. However, it has been observed that DS/SS suffers severely from the near-far effect.

For a low spectral sidelobe level of the transmitted signal, frequency hopping can reduce the near-far problem by reducing the probability that two or more users have the same instantaneous carrier frequency. A disadvantage of frequency hopping is the larger vulnerability of multipath interference in comparison with direct sequence. With a hybrid form of direct sequence and slow frequency hopping we can combine the advantages of both techniques.

A theoretical model for the performance of hybrid DS/SFH with DPSK modulation in terms of average bit error probability has been presented in [1],[2] and [9]. In this report however, we present an improved version of the forementioned model in order to assess the performance of the hybrid DS/SFH system with Differential Phase Shift Keying (DPSK) as modulation technique. The performance of the system is expressed in the following four terms:

- The Bit Error Rate (BER);
- The outage probability;
- The throughput;
- The delay.

In this performance analysis, we consider selection diversity and maximal ratio combining as diversity techniques. Furthermore, the influence of forward error correcting (FEC) codes on the performance is investigated. Finally, we are able to make the following comparisons:

- The theoretical model presented in this report with the model presented in literature;
- The performance of the hybrid system using DPSK with the performance of the hybrid system using BPSK and QPSK as modulation techniques presented in [4];
- The performance of DS with the performance of hybrid DS/SFH.
- The throughput of a CDMA system based on pure DS with that of a CDMA system based on hybrid DS/SFH;

The system we will use for the performance analysis is a starconnected CDMA network with K users as shown in figure 1.1

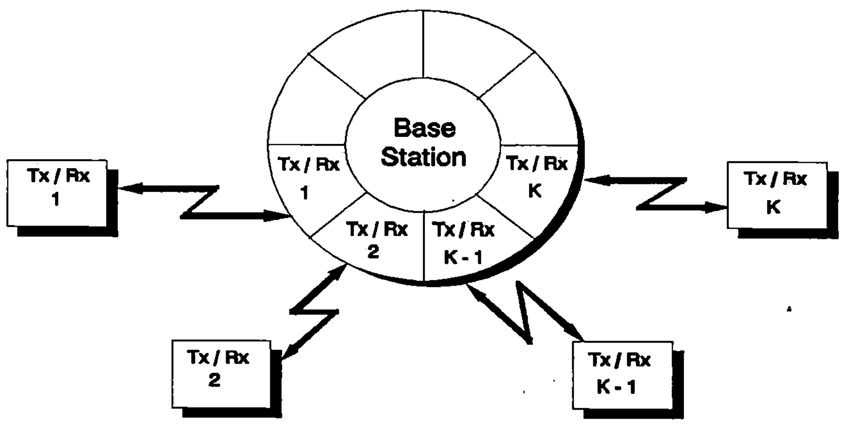

Figure 1.1: *Centralized wireless local area network with CDMA*

The base station consists of a bank of spread-spectrum transmitters/receivers, one for each active user. We assume that each user has an unique spread-spectrum code sequence and a unique hopping pattern .

In chapter Two there will be a discussion about code division multiple access techniques. Chapter Three will consider the propagation aspects of indoor wireless communications, such as pathloss and multipath interference. In chapter Four a description of the theoretical model is presented. Chapter Five is an analysis of the performance of the hybrid model. In chapter Six we present the numerical results of the performance described in chapter Five. Beside that, the comparison with the other systems as well as the discussions of the results will be given. In chapter Seven we present the throughput and delay analysis. The numerical results as well as the discussion of these results are presented in chapter Eight. Chapter Nine gives the conclusions and chapter Ten the recommendations for further research.

2. CODE DIVISION MULTIPLE ACCESS TECHNIQUES

In a wireless communication system all users should be offered the possibility to have communication simultaneously, which means that they must have multiple access capability. There are three main multiple access techniques: frequency division multiple access (FDMA), time division multiple access (TDMA), and code division multiple access (CDMA).

In FDMA all users are able to transmit simultaneously but use disjoint frequency bands. Every single user has his own channel in the frequency domain, across which he can transmit his information.

In TDMA every single user has the possibility to transmit his information during a short period of time, referred to as a time slot. So all users occupy the same bandwidth, but sequentially in time.

In CDMA, channels are created by using mutually different code sequences. Such a code sequence serves as a selection means and as a carrier of information. By using CDMA, the transmitted signal spectrum will be spread over a frequency range much greater than the message bandwidth. That is why this technique is often referred to as a spread-spectrum technique. In figure 2.1 the principles of the three multiple access methods discussed here are shown schematically.

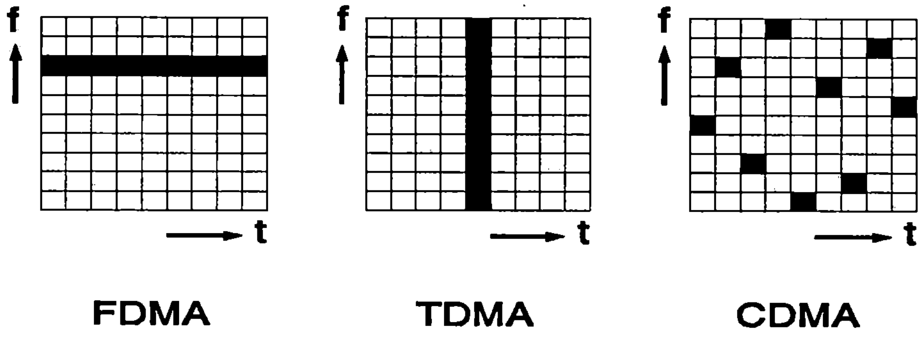

Figure 2.1: *Schematic representation of three multiple access techniques*

Since CDMA is the most promising of the three techniques, this one will be considered from now on. The two most important ways to implement CDMA, viz. direct sequence (DS) and frequency hopping (FH) will be discussed in the next two sections.

2.1 Direct Sequence

The principle of direct sequence is the modulation of the data signal with pseudo random code sequences, which consist of wideband digital signals. These three signals are shown in figure 2.2.

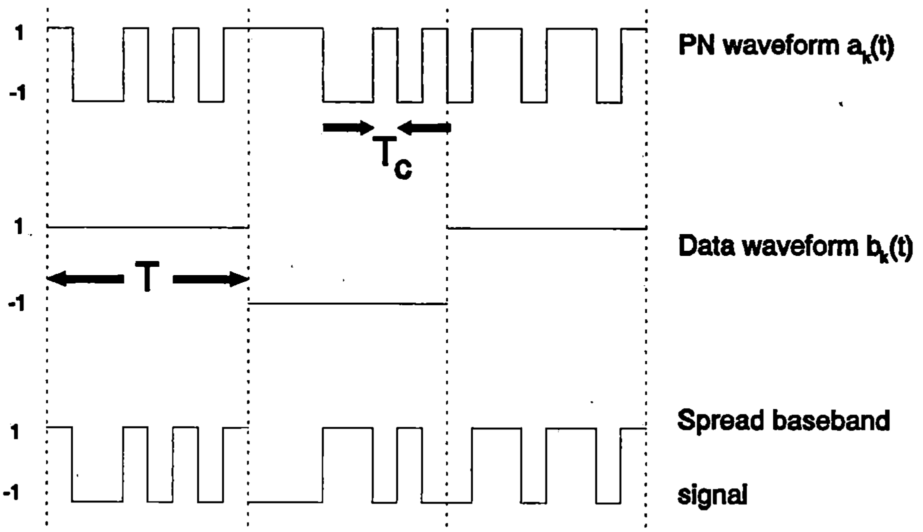

Figure 2.2: *The principle of direct sequence*

Each user obtains such a sequence that distinguishes itself sufficiently from the other code sequences. Therefore, it is necessary to have enough sequences of a certain length with mutually low crosscorrelation coefficients. Interesting codes in this respect are the Gold codes [20].

These Gold codes are composed of a combination of maximum-length sequences. The resulting sequences don't have this maximum-length property, however they have a very suitable correlation behaviour.

A measure for the mutual interference of the sequences is given by the ratio of the maximum cross-correlation coefficient *CC* and the auto correlation coefficient *AC*. In here *AC* is equal to $2^n - 1$ (i.e., the number of Gold sequences). Gold proved [20] that *CC* is given by:

$$CC = \begin{cases} 2^{(n+1)/2} + 1 & \text{for } n \text{ odd} \\ 2^{(n+2)/2} + 1 & \text{for } n \text{ even} \end{cases} \tag{2.1}$$

For $n=5$ for instance, we find that the ratio *CC/AC* equals 0,29. This means that the cross-correlation peaks have a normalized height of 0,29 , while the autocorrelation peaks have a height equal to 1. Fig 2.3 shows the cross and autocorrelation functions of this example. The sequences are generated by a Gold code generator, described in [17]

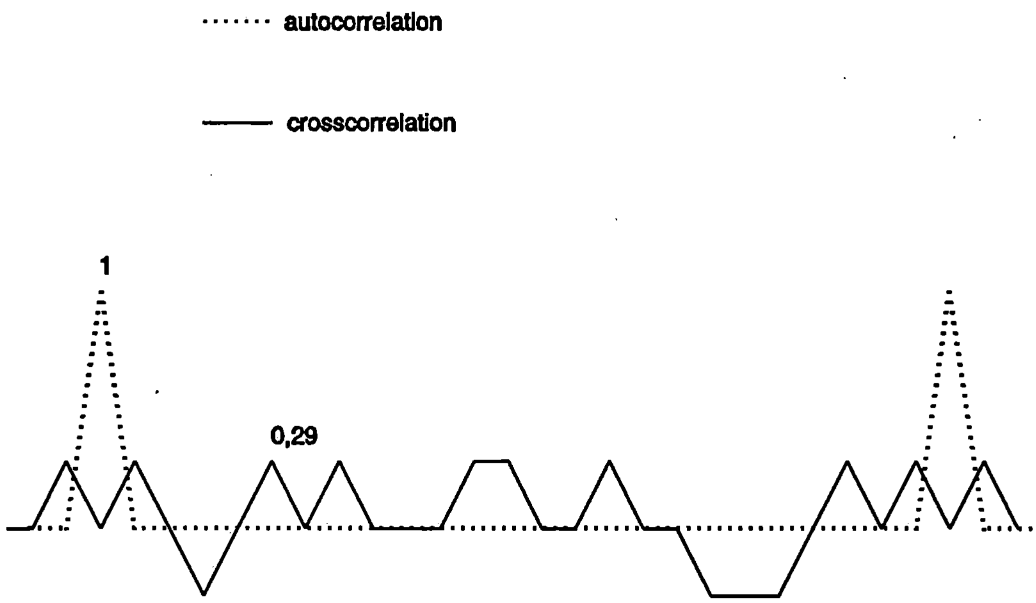

Figure 2.3: *Auto- and crosscorrelation of a 31-bit Gold-sequence*

The generation process of these sequences is often referred to as *pseudo-noise (PN) process,* due to the correlation properties, which are noise-like. Such a process looks random, but can be replicated by authorized users.

DS involves spread spectrum and as the name suggests, the transmitted signal spectrum is much greater than the message bandwidth. The ratio between the transmission bandwidth B_T and the message bandwidth W is called *processing gain.*

$$G_p = \frac{B_T}{W} \tag{2.2}$$

DS systems have several advantages and disadvantages.

Advantages:

- best noise and anti-jam performance;
- most difficult to detect
- best discrimination against multi-path fading.

Disadvantages:

- long acquisition time;
- fast code generator needed;
- near-far problem [1], [2];

2.2 Frequency hopping

In a frequency hopping system, the carrier frequency hops "randomly" from one value to another under control of an PN-process. Since each user has his own hopping pattern, they are able to transmit their information simultaneously. Frequency hopping systems can be divided in two categories: *fast frequency hopping* (FFH) and *slow frequency hopping* (SFH). The principles of these two ways of hopping are shown in figure 2.4.

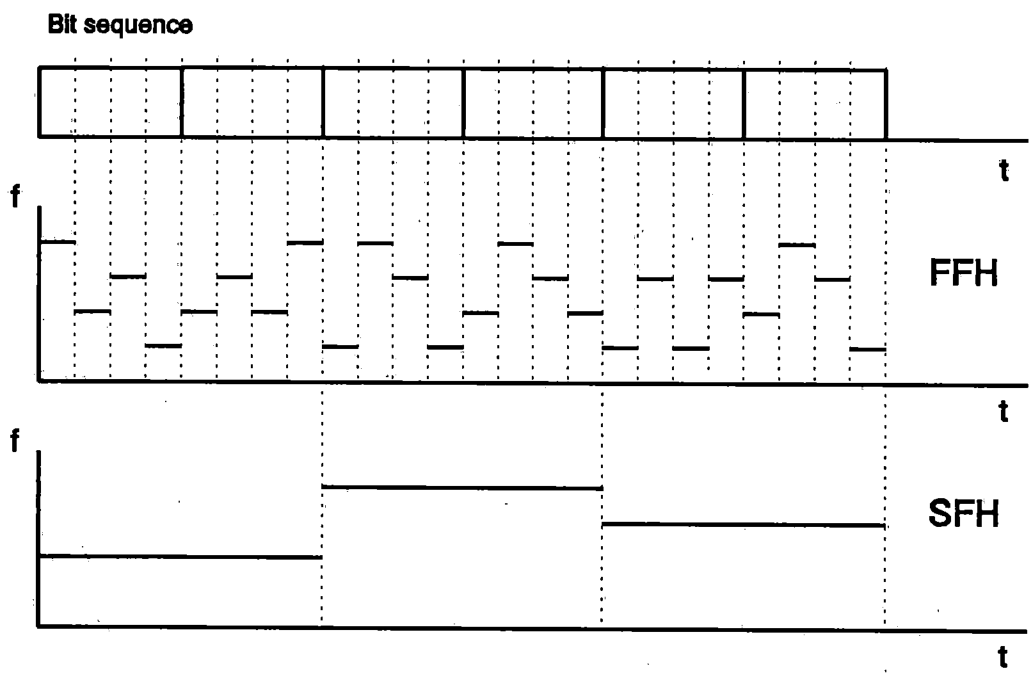

Figure 2.4: *Principle of fast and slow frequency hopping*

In a fast frequency hopping system, a data bit is transmitted during multiple hops.
If we consider a "snapshot" of the spectrum of this transmitted signal, we are not able to make a distinction between the different hops, because of the high hopping speed. The frequency band, allocated for this purpose will be fully occupied and spread spectrum is also involved.

A slow frequency hopping system transmits multiple data bits per hop and here no direct spreading of the spectrum is involved, because of the slow hopping speed. If we consider a "snapshot" of the spectrum again, we only see the message bandwidth, slowly hopping from one carrier frequency to another.

When several frequency hopping signals occupy a common RF channel, it might occur that these signals interfere. Events of this type are called *hits*. These hits become more and more of a problem when the number users hopping over a fixed bandwidth increases.

Frequency hopping systems have several advantages and disadvantages over other CDMA techniques.

Advantages:

- FFH yields the greatest amount of spreading;
- programmable hopping pattern so portions of the spectrum can be avoided;
- short acquisition time;
- less affected by the near-far problem.

Disadvantages:

- complex frequency synthesizer required;
- error correction needed.

3. PROPAGATION ASPECTS IN AN INDOOR ENVIRONMENT

In order to design an indoor wireless communication system we need to have some knowledge of the indoor radio propagation characteristics. Two phenomena, namely path loss and multipath fading, determine the characteristics of the radio channel in a large measure. In the following sections we will give a characterization of these phenomena.

3.1 The path loss

In general the spatially averaged value of the multipath power gain in a point at a distance r from the transmitter is represented by a distance-power law of the following form:

$$P \propto r^{-\alpha} \tag{3.1}$$

In free space $\alpha = 2$, but results from literature [11,16] show that whitin buildings α can be smaller or larger than 2, dependent on the position of the transmitter with regard to the receiver and the environment.

If the transmitter and receiver are placed in the same hallway of an office building for instance, α usually is smaller than 2. This gain over the free space situation is likely due to the waveguiding effects in the hallway.

In rooms which are located off the hallway, α can exceed values of 3,4 or even 5, dependent on the position of the room with regard to the transmitter.

3.2 Description of the multipath fading

Multipath fading is caused by multiple reflections of the transmitted signal from the building structure and surrounding inventory. The fading phenomenon is primarily a result of the time variation in the phases $\theta_n(t)$ of the different signal components. As a result of this the signal vectors might add destructively at times, which results in a very small received signal. When the signal vectors add constructively, the received signal will be large.

Since the channel parameters play an important role with respect to multipath fading, they will be discussed in the next sections.

3.3 Parameters of the radio channel

Starting point for the investigation of the properties of a channel is the impulse response of the channel. From this impulse response two parameters, viz. coherence bandwidth and coherence time, can be derived. The time-variant impulse response of the equivalent low-pass channel, which contains discrete multipath components, is described by [18]:

$$c(\tau;t) = \sum_n \alpha_n(t)\, e^{-j2\pi f_c \tau_n(t)}\, \delta(\tau - \tau_n(t)) \tag{3.2}$$

Here, $\alpha_n(t)$ is the attenuation factor for the received signal on the *nth* path and $\tau_n(t)$ is the propagation delay for the *nth* path. From such an impulse response several separated responses can be distinguished, due to the fact that the transmitted impulse reaches the receiver along different paths of different length. The *coherence bandwidth* and the *coherence time*, two important parameters of the channel, will be discussed in the next two sections.

3.3.1 Coherence bandwidth of the channel

With help of the impulse response of the channel, we are able to obtain some useful information of the radio channel. Due to reflections, copies of the signal will arrive at the receiver at different times. The average output power of the channel (denoted by: $\phi_c(\tau)$), corresponding to the power of the different copies of the signal, can be given as a function of the corresponding delays. We then obtain a delay-density function [12], where the delay is weighted by the signal level at that delay. The root mean square of this distribution is called the *multipath spread* or *rms delay spread* of the channel and is denoted by T_m [18].

The Fourier transform of $\phi_c(\tau)$ yields an autocorrelation function in the frequency variable, which is denoted by $\phi_c(\Delta f)$. This latter quantity is a measure of the frequency coherence of the channel. In figure 3.1 the relationship between these latter two quantities is shown.

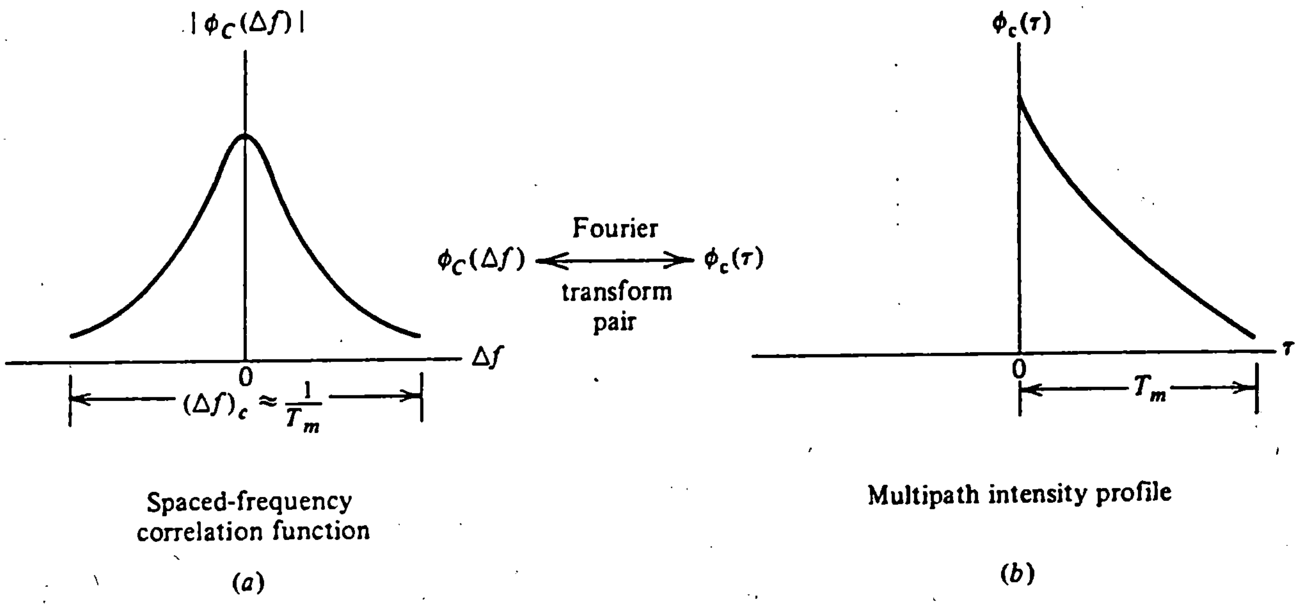

***Figure 3.1:** Relationship between the spaced-frequency correlation function and the multipath intensity profile*

As a result of the Fourier transformation relationship between $\phi_c(\tau)$ and $\phi_c(\Delta f)$, the reciprocal of the multipath delay spread is called the *coherence bandwidth* of the channel. That is:

$$(\Delta f)_c \approx \frac{1}{T_m} \tag{3.3}$$

Where $(\Delta f)_c$ denotes the coherence bandwidth. The channel is said to be frequency-selective when the signal bandwidth is larger than the coherence bandwidth. In that case the signal will undergo different gains and phase shifts across the band and is severely distorted by the channel.

On the other hand if the signal bandwidth is much smaller than the coherence bandwidth the channel is said to be frequency-nonselective. All frequency components in the transmitted signal undergo the same attenuation and phase shift.

3.3.2 Coherence time of the channel

The signal intensity as a function of the Doppler frequency f is given by the Doppler power spectrum of the channel, denoted by $S_c(f)$. The Doppler spread B_d is the range of values over which the Doppler power spectrum is nonzero. The Fourier transform of $S_c(f)$ yields an autocorrelation function in the time variable, denoted by $\phi_c(\Delta t)$. This latter quantity is a measure for the time coherence of the channel. This is shown in figure 3.2.

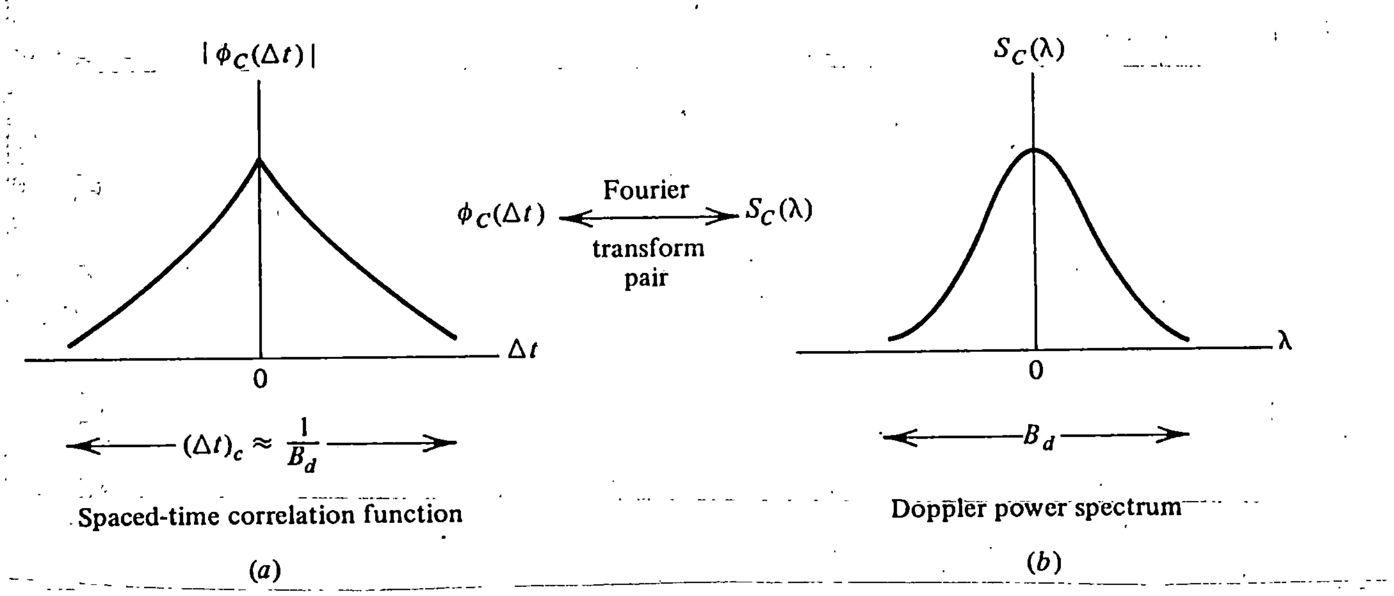

***Figure 3.2:** Relationship between the spaced-time correlation function and the Doppler power spectrum*

Because of the relationship between $S_c(f)$ and $\phi_c(\Delta t)$, the reciprocal of B_d is called the *coherence time* of the channel. That is:

$$(\Delta t)_c \approx \frac{1}{B_d} \tag{3.4}$$

Where $(\Delta t)_c$ denotes the coherence time. It is obvious that a slowly changing channel has a large coherence time or, equivalently a small Doppler spread.

The rapidity of the fading in a frequency nonselective channel is determined either from the correlation function $\phi_c(\Delta t)$ or from the Doppler power spectrum $S_c(f)$. Suppose that the period T of the transmitted signal is smaller than the coherence time of the channel. Then the channel attenuation and phase shift do not change considerably for the duration of at least one signalling interval. When this condition holds, the channel is said to be a *slow fading* channel.

In the next section we will discuss the influence of the channel parameters on the transmitted signal.

3.4 Influence of the channel parameters on the radio signal

As we have discussed, many reflections of the signal arrive at the receiver at different times. All these reflections of the signal add up and cause signal peaks and dips, dependent on the phases of the different reflections.

The reflections can be grouped in clusters. These clusters contain the signals with a time difference between the arrival of the first and the last ray, which is less than the chip time T_c of the user code. These clusters can be resolved by a receiver and are therefore called *resolvable paths*. The principle in shown in figure 3.3.

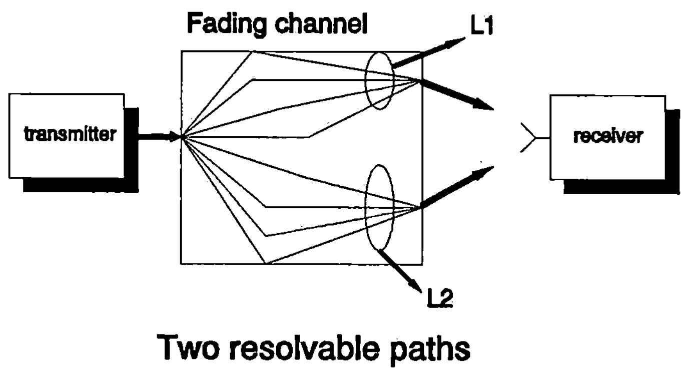

Figure 3.3: *Example of multiple resolvable paths*

The criterium for the existence of multiple resolvable paths is that the signal bandwidth W is much larger than the coherence bandwidth $(\Delta f)_c$.

In case of direct sequence, each data bit is multiplied by a unique user code. Such a code consists of N pulses (chips) of duration $T_c = T/N$, where T is the duration of one data bit. The maximum number of multiple resolvable paths L in this case is given by [13] as:

$$L = \left\lfloor \frac{T_m}{T_c} \right\rfloor + 1 = \left\lfloor T_m\, R_c\, N \right\rfloor + 1 \tag{3.5}$$

where T_m denotes the rms delay spread of the channel. Equation (3.5) is based on the time resolution of DS/SS signals, which is T_c seconds. So spread spectrum provides independently fading multiple signal paths, because the transmission bandwidth is much larger than the coherence bandwidth of the channel.

3.5 Statistical description of fading

The time variations of the channel seem to be unpredictable to the user of the channel. Therefore it is reasonable to characterize the time-variant multipath channel statistically. In the following sections we will discuss statistical characterization of the envelope of signals submitted to signal fading.

3.5.1 Rician fading envelope

The Rician PDF describes the envelope of the sum of two orthogonal statistically independent nonzero mean Gaussian random variables with identical variances σ_r^2. This distribution is applicable when a significant part of the received signal envelope is due to a constant path (such as a line-of-sight component or fixed signal reflectors). Since in an indoor environment there are a lot of fixed signal reflectors, which provide a LOS path, the Rician PDF would be appropriate [11].

The Rice probability density function is defined as:

$$p(r) = \frac{r}{\sigma_r^2} \exp\left[-\frac{r^2+S^2}{2\sigma_r^2}\right] I_o\left[\frac{Sr}{\sigma_r^2}\right] \qquad r \geq 0, \quad S \geq 0 \tag{3.6}$$

where $I_0(\ )$ is the modified Besselfunction of the first kind and zero order. The second central moment of the Rice distribution is given by:

$$E[r^2] = S^2 + 2\sigma_r^2 \tag{3.7}$$

The average power of a bandpass signal with an Rician distributed envelope is given by:

$$P = \frac{1}{2}E\left[r^2\right] = \frac{1}{2}S^2 + \sigma_r^2 \tag{3.8}$$

where ½S^2 represents the power of the dominant component and σ_r^2 the power of the scattered signals. The ratio of the dominant received power to the scattered power is called the Rice factor, denoted as *R*. So, we can write:

$$R = \frac{S^2}{2\sigma_r^2} \tag{3.9}$$

From propagation measurements done in an indoor environment [11] some values of R are determined. We know that typically R = 6,8 dB corresponds to a 30-year-old brick building with reinforced concrete and plaster, as well as some ceramic block interior partitions. R=11 dB corresponds to a building that has the same construction, but has an open-office interior floor plan and non-metallic ceiling tiles.

3.5.2 Rayleigh fading envelope

The Rayleigh PDF describes the envelope of the sum of two orthogonal statistically independent zeromean Gaussian random variables with equal variance σ_r^2. In case there is no dominant component in the received signal the envelope can be described by a Rayleigh distribution [11], [12] which is given by:

$$p(r) = \frac{r}{\sigma_r^2}\exp\left[-\frac{r^2}{2\sigma_r^2}\right] \qquad r \geq 0 \tag{3.10}$$

The first and the second moment of this distribution are given by:

$$E[r] = \sqrt{\frac{\pi}{2}\sigma_r^2} \tag{3.11}$$

$$E\left[r^2\right] = 2\sigma_r^{\,2} \tag{3.12}$$

The average power of a bandpass signal with a Rayleigh fading envelope is given by:

$$P = \frac{1}{2}E\left[r^2\right] = \sigma_r^{\,2} \tag{3.13}$$

The Rayleigh distribution is a special case of the Rice distribution (with S=0).

3.6 Diversity techniques for fading multipath channels

The major problem involving the reception of radio signals is when the transmitted signal is in a deep fade. A solution of this problem is to use the replicas of the signal, transmitted over independently fading channels. This latter phenomenon is called *diversity*.

Direct sequence spread spectrum provides multiple resolvable paths if the delay spread is larger than the duration of a chip in a user code. This is called *inherent spread spectrum diversity*. In the next two sections we will discuss two ways receivers use diversity for signal reception, namely *selection diversity* and *maximal ratio combining*.

3.6.1 Selection diversity

Selection diversity is based on selecting the largest in a group of signals carrying the same information. The multiple resolvable paths can be used for selection diversity by selecting the path with the largest output of the matched filter in the receiver.

The order of diversity M that can be achieved with selection diversity is defined as the number of resolvable paths multiplied by the number of antennas. So, one could use multiple antennas in order to increase the order of diversity. They must be spaced sufficiently far apart so the multipath components in the signal have significantly different propagation delays at the antennas.

3.6.2 Maximal ratio combining

Maximal ratio combining is based on summing the demodulation results of a group of signals carrying the same information. The result of this summation is used as a decision variable in the data decision process.

In general the order of diversity M that can be achieved with maximal ratio combining is equal to the number of resolvable paths L. However the use of multiple antennas increases the order of diversity. This type of diversity is very attractive for spread spectrum with DPSK modulation, because it's very easy to implement and it gives a significant improvement in performance.

4. SYSTEM DESCRIPTION FOR DPSK, BPSK AND QPSK

The modulation scheme DPSK has been the starting point in this report. However, we will also consider two other modulation schemes, viz. (B)PSK and QPSK. The system description performance of the hybrid DS/SFH system with BPSK and QPSK modulation have been presented in [4]. However, in order to make a good comparison and discussion of the performance of the three modulation schemes, we will give a short system description of the hybrid system with BPSK and QPSK in this chapter as well.

The system is a star connected network with K users, where all of them are considered to be at the same distance from the base station. The performance analysis of the system is done assuming that the base station is in the receiving mode.

The model under consideration consists three parts, viz. the transmitter, the channel and the receiver. In the following sections the models of these three parts will be discussed for DPSK, BPSK and QPSK as modulation schemes.

4.1 Description of the transmitter model

In the following sections we will describe the transmitter models of the hybrid DS/SFH system with DPSK and BPSK/QPSK respectively.

4.1.1 Transmitted signal with DPSK modulation

In figure 4.1 the transmitter model of a hybrid DS/SFH system with DPSK modulation is shown.

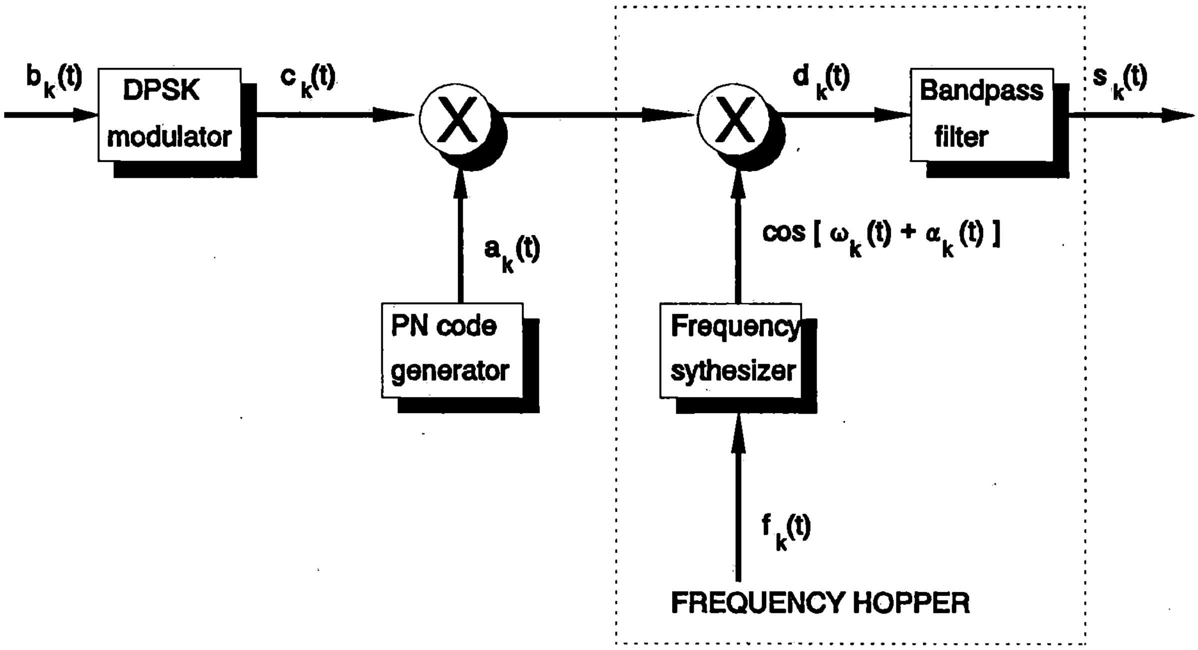

Figure 4.1: *Transmitter model of a hybrid DS/SFH system*

In case of channel coding, the channel encoder and the interleaver have to precede the DPSK modulator in the diagram of figure 4.1. Each user produces a data waveform given by:

$$b_k(t) = \sum_j b_k^j \, P_T\,(t-jT) \tag{4.1}$$

where b^j_k is the jth data bit of user k and belongs to the set {0,1}. Furthermore $P_T(t)$ is a rectangular NRZ pulse of unit height and duration T. The signal $b_k(t)$ is first DPSK modulated and can be written as:

$$c_k(t) = b_k(t)\cos(\omega_c t+\theta_k) \tag{4.2}$$

where θ_k is a random phase associated with user k. As a consequence of the DPSK modulation b^j_k now belongs to the set of {-1,1}.

This modulated signal is now multiplied by a spreading sequence $a_k(t)$, which accomplishes DS.

$$a_k(t) = \sum_j a_k^j \, P_{T_c}\!\left(t-jT_c\right) \tag{4.3}$$

where the j th pulse of $a_k(t)$ belongs to the set of {-1,1}. Here T_c is the chip duration and

$P_{Tc}(t)$ a rectangular NRZ pulse of unit height and duration T_c. This sequence is periodical with period T. We assume that there are *N* code pulses during each encoded bit ($T=NT_c$).

This spread signal is then frequency-hopped according to the hopping pattern $f_k(t)$ associated with user k. We assume random hopping patterns with q frequencies in this model. The number of data bits transmitted per hop $N_b = T_h/T$ is a positive integer. In case of slow frequency hopping, we have $N_b >> 1$. The frequency hopped signal is then given by:

$$d_k(t) = a_k(t)b_k(t)\cos\left(\omega_c t+\theta_k\right) \cos\left(\omega_k(t)t+\alpha_k(t)\right) \tag{4.4}$$

The frequency $f_k(t)$ and the phase $\alpha_k(t)$ are constant over a time interval of duration T_h. The bandpass filter in the frequency hopper, shown in figure 4.1, removes unwanted frequency components present at the output of the multiplier. So the transmitted signal becomes:

$$s_k(t) = \sqrt{2P}\, a_k(t)b_k(t)\cos\left(\omega_c t+\omega_k(t)t+\theta_k+\alpha_k(t)\right) \tag{4.5}$$

where P is the transmitted power. The quantity $\alpha_k(t)$ represents the phase shift introduced by the frequency hopper when it switches from one frequency to another. We assume $\alpha_k(t)$ to be constant during the time intervals that $f_k(t)$ is constant.

4.1.2 Transmitted signal with BPSK and QPSK modulation

The sequences with rate 2/T produces by user *k* is separated in 2 sequences, denoted by $b_{c,k}(t)$ and $b_{s,k}(t)$ respectively, each having a rate 1/T.

$$\begin{aligned} b_{c,k} &= \sum_j b^j_{c,k}\, P_T(t - jT) \\ b_{s,k} &= \sum_j b_{s,k}\, P_T(t - jT) \end{aligned} \tag{4.6}$$

where $b^j_{x,k}$ belongs to the set {-1,1} and $P_T(t)$ is a rectangular pulse of duration T. These signals are first multiplied by the spreading sequences $a_{c,k}(t)$ and $a_{s,k}(t)$, which are defined the same way as in equation (4.3). These sequences are periodical with period T. The signals $a_{c,k}(t)b_{c,k}(t)$ and $a_{s,k}(t)b_{s,k}(t)$ are PSK modulated on two carriers in quadrature.

This modulated signal is then frequency hopped, according to the hopping pattern associated with user k. After appropriate filtering, this hopped signal for QPSK becomes:

$$s_k^Q(t) = \sqrt{2P}\, a_{c,k}(t) b_{c,k}(t).\cos(\omega_c t + \omega_k(t)t + \alpha_k + \theta_k)$$
$$+ \sqrt{2p}\, a_{s,k}(t) b_{s,k}(t).\sin(\omega_c t + \omega_k(t)t + \alpha_k + \theta_k) \tag{4.7}$$

where the phases are the same as defined in section 4.1.1. The hopped signal for PSK can be obtained by setting $b_{s,k}(t)$ in equation (4.7) to 0

4.2 Description of the channel model

The link between the kth user and the base station is characterized by a lowpass equivalent transfer function given by:

$$h_k(t) = \sum_{l=1}^{L} \beta_{kl}\delta(t-\tau_{kl})\exp(j\gamma_{kl}) \tag{4.8}$$

where kl refers to path l of user k. We assume there are L paths associated to each user. The l*th* path of the k*th* user is characterized by three random variables: the gain β_{kl}, delay τ_{kl} and the phase γ_{kl}. In this study we make the following five assumptions concerning the channel:

- path gain β_{kl}, the phase γ_{kl} and delays τ_{kl} of the different paths are statistically independent for different values of k and l;
- the path gain β_{kl} is Rician distributed;
- the phase factor γ is uniformly distributed over $[0,2\pi]$ and the path delay τ_{kl} is uniformly distributed in [0,T];
- the rms delay spread T_m is less than the bit duration T in order to avoid intersymbol interference;
- the channel introduces additive white Gaussian noise n(t), with two-sided power spectral density $N_o/2$.

4.3 Description of the receiver model

In this model we assume that the DPSK modulated signal is detected noncoherently and the BPSK and QPSK modulated signals are detected coherently. Because of the stable LOS component, phase recovery at the receiver can be achieved, which makes coherent detection possible.

4.3.1 Description of the received signal in case of DPSK

In case of DPSK we consider a noncoherent receiver. This means that the receiver uses the phase information which is concealed in two consecutive bits. The receiver consists of a dehopper, a bandpass matched filter, a DPSK demodulator and a hard-decision device. Besides, when channel coding is used, a deinterleaver and a hard-decision decoder are added to the receiver. In figure 4.2 the receiver structure is shown:

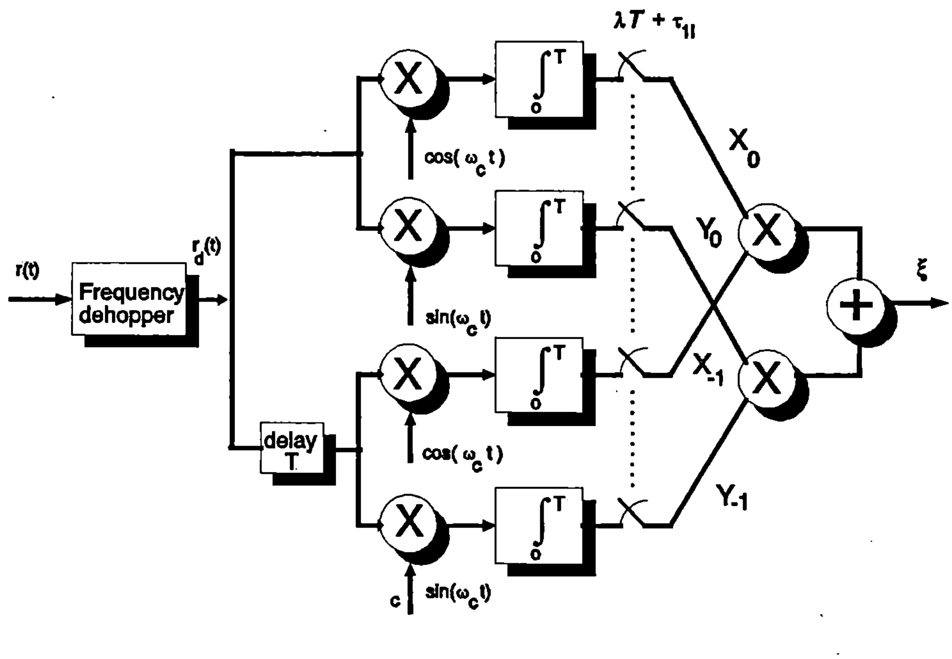

Figure 4.2: *Frequency dehopper and DPSK demodulator*

The received signal can be written as:

$$r(t) = \sqrt{2P} \sum_k \sum_l \beta_{kl}\, a_k(t-\tau_{kl}) b_k(t-\tau_{kl}) \cos\left(\omega_c t + \omega_k(t-\tau_{kl})t + \phi_{kl}\right) + n(t) \qquad (4.9)$$

with

$$\phi_{kl} = \alpha_{kl}(t-\tau_{kl}) + \theta_{kl} - \left[\omega_c + \omega_k(t-\tau_{kl})\right]\tau_{kl} + \gamma_{kl} \qquad (4.10)$$

and $n(t)$ is white Gaussian noise with two-sided power spectral density $N_o/2$ [W/Hz]. The received signal $r(t)$ enters a bandpass filter which removes out-of-band noise. The mixer of the dehopper performs the appropriate frequency translation.

We assume that the hopping pattern of the receiver is synchronized with the hopping pattern associated with the *jth* path of user 1 (which will be denoted as the reference path). The dehopper introduces a phase $\beta_1(t)$, which constant over the hopping interval T_h. A bandpass filter which follows the mixer removes high frequency terms. The resulting dehopper output signal is given by:

$$r_d(t) = \sqrt{P/2} \sum_k \sum_l \beta_{kl}\, a_k(t-\tau_{kl}) b_k(t-\tau_{kl}) \delta\left[f_1(t-\tau_{1j}), f_k(t-\tau_{kl})\right] \cos\left(\omega_c t + \phi_{kl} + \beta_1(t)\right) + n_d(t) \quad (4.11)$$

where $n_d(t)$ can be considered as white Gaussian noise with two-sided power spectral density $N_o/8$. The delta function in this expression is defined as follows

$$\delta(u,v) = \begin{matrix} 1 & \mathit{for} & u=v \\ 0 & \mathit{for} & u \neq v \end{matrix} \qquad (4.12)$$

in which both u and v are real. Note that the dehopper suppresses, at any instant t, all path signals whose frequency at instant t differs from $f_1(t-\tau_{1j})$. Evidently, the reference path signal is not suppressed; the other path signals from the reference user are suppressed only during a part of the first or last bit of a hop, depending on the relative delay of the considered path with respect to the reference path. Path signals from users different from the reference user contribute to the dehopper output only during those time intervals for which their frequency accidentally equals the frequency of the reference path signal.

We can express $r_d(t)$ by means of its quadrature components:

$$r_d(t) = x(t)\cos(\omega_c t) - y(t)\sin(\omega_c t) \tag{4.13}$$

with

$$x(t) = \sqrt{P/2}\sum_k\sum_l \beta_{kl}\, a_k(t-\tau_{kl})b_k(t-\tau_{kl})\delta\left[f_1(t-\tau_{1j}), f_k(t-\tau_{kl})\right]\cos\left(\phi_{kl}+\beta_1(t)\right) + n_c(t) \tag{4.14}$$

$$= \sqrt{P/2}\sum_k\sum_l \beta_{kl}\, a_k(t-\tau_{kl})b_k(t-\tau_{kl})\delta\left[f_1(t-\tau_{1j}), f_k(t-\tau_{kl})\right]\cos\left(\psi_{kl}\right) + n_c(t) \tag{4.15}$$

$$y(t) = \sqrt{P/2}\sum_k\sum_l \beta_{kl}\, a_k(t-\tau_{kl})b_k(t-\tau_{kl})\delta\left[f_1(t-\tau_{1j}), f_k(t-\tau_{kl})\right]\sin\left(\phi_{kl}+\beta_1(t)\right) + n_s(t) \tag{4.16}$$

$$= \sqrt{P/2}\sum_k\sum_l \beta_{kl}\, a_k(t-\tau_{kl})b_k(t-\tau_{kl})\delta\left[f_1(t-\tau_{1j}), f_k(t-\tau_{kl})\right]\sin\left(\psi_{kl}\right) + n_s(t) \tag{4.17}$$

The dehopper output signal $r_d(t)$ enters a DPSK demodulator, whose impulse response is matched to a T-seconds segment of $a_1(t)\cos(\omega_c t)$.

The bandpass signal at the matched filter output contains narrow peaks of width $2T_c$, at instants $\lambda T + \tau_{1j}$ ($\lambda = 0, \pm1, \pm2, \ldots$), which are caused by the reference path signal. The other L - 1 path signal from the reference user give rise to peaks at instants $\lambda T + \tau_{1l}$ ($l \neq j$). However, there is no peak during the first or last bit of a hop, because each of these L -1 path signals is partly suppressed by the dehopper. Path signals from the nonreference users also give rise to peaks, however usually smaller than the peaks caused by the reference user. These peaks of the nonreference user are caused by the fact that the spreading codes are not ideal in the sense that they are correlated to some extend.

The DPSK demodulator output is a lowpass signal, which has peaks at instants $\lambda T + \tau_{1l}$ as we already have seen. A positive peak indicates that the corresponding channel encoder output bit is likely to be a logical one and a logical zero when the peak is negative.

We assume that the delay differences between path signals from the same user are larger than $2T_c$, so these peaks do not overlap; this assumption is justified when the PN sequence period N is relatively large. The DPSK demodulator output signal is sampled at the instants $\lambda T + \tau_{1l}$, and this gives rise to the decision variables $\xi_l(\lambda)$.

4.3.2 Signal description of the DPSK demodulator output

In DPSK the message bits are coded with two consecutive code bits. If a binary one is to be sent during a bit interval, it is sent as a signal with the same phase as the previous bit. If a binary zero is sent, it is transmitted with the opposite phase of the previous bit. So the demodulator has to correlate two consecutive signal elements and create the decision variable ξ_l, which can be written as:

$$\xi = Re\left[V_o V_{-1}^*\right] \tag{4.18}$$

with

$$\begin{aligned} V_o &= X_o + jY_o \\ V_{-1} &= X_{-1} + jY_{-1} \end{aligned} \tag{4.19}$$

in which V_o denotes the complex envelope at the current sampling instant at the output of the demodulator and the same for V_{-1} at the previous sampling instant. The real and the imaginary parts of the complex envelope denote the output of the in-phase integrator and the output of the quadrature-branch integrator respectively. The output of the in-phase and the quadrature branches are given by:

$$X_o = \sqrt{P/8} \sum_k \sum_l \beta_{kl} \cos\left(\psi_{kl}\right) \int_{\lambda T}^{(\lambda+1)T} a_1(t) a_k(t-\tau_{kl}) b_k(t-\tau_{kl}) \delta\left[f_1(t-\tau_{1j}), f_k(t-\tau_{kl})\right] dt + \eta \tag{4.20}$$

$$Y_o = \sqrt{P/8} \sum_k \sum_l \beta_{kl} \sin\left(\psi_{kl}\right) \int_{\lambda T}^{(\lambda+1)T} a_1(t) a_k(t-\tau_{kl}) b_k(t-\tau_{kl}) \delta\left[f_1(t-\tau_{1j}), f_k(t-\tau_{kl})\right] dt + \nu \tag{4.21}$$

$$\eta = \int_{\lambda T}^{(\lambda+1)T} a_1(s) n_c ds \qquad \nu = \int_{\lambda T}^{(\lambda+1)T} a_1(s) n_s ds \tag{4.22}$$

where υ and η are zero-mean Gaussian random variables with equal variances given by $N_0T/16$. Bit under consideration is bit number $\lambda = jN_b + p$. Actually, the phase Ψ is time-dependent, but it has a fixed value for the time interval under consideration, because the δ function will only be equal to one if $f_k(t - \tau_{kl}) = f_1(t-\tau_{1j})$, which means a fixed value for f and α.

Because of the limited interval for the delays, two bits will be under consideration: bits 0 and -1 of the sequence k (or equivalently λ and λ-1 respectively) as shown in figure 4.3.

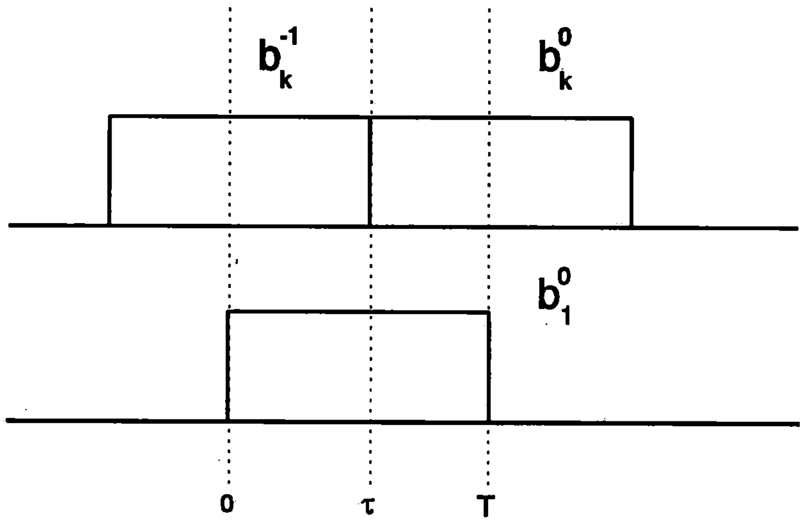

Figure 4.3: *Time interval with the bits under consideration*

So, in order to perform the cross correlations of the bits of the nonreference user with the bits of the reference user, we have to split up the time interval [0,T] in two parts, viz. $[0,\tau] \cup [\tau,T]$ as shown in figure 4.3. According to [10] we define:

$$R_{1k}(\tau_{kl}) = \int_0^{\tau_{kl}} a_1(t) a_k(t-\tau_{kl}) dt \tag{4.23}$$

$$\hat{R}_{1k}(\tau_{kl}) = \int_{\tau_{kl}}^{T} a_1(t) a_k(t-\tau_{kl}) dt \tag{4.24}$$

In these definitions, the periodicity of the spreading sequence has been taken into account. As a consequence, we can write:

$$X_o = \sqrt{P/8} \sum_k \sum_l \beta_{kl} \cos\left(\psi_{kl}\right) \left[d_1\left(b_k^{-1}\right) b_k^{-1} R_{1k}(\tau_{kl}) + d_2\left(b_k^0\right) b_k^0 \hat{R}_{1k}(\tau_{kl}) \right] + \nu \tag{4.25}$$

$$Y_o = \sqrt{P/8} \sum_k \sum_l \beta_{kl} \sin\left(\psi_{kl}\right) \left[d_1\left(b_k^{-1}\right) b_k^{-1} R_{1k}(\tau_{kl}) + d_2\left(b_k^0\right) b_k^0 \hat{R}_{1k}(\tau_{kl}) \right] + \eta \tag{4.26}$$

where b_k^{-1} and b_k^{o} are the bits with numbers $\lambda - 1$ and λ of the *k*th user. The functions d_1 and d_2 account for the possible hits between the current frequency of user 1 and the frequencies associated with the two bits under consideration. X_{-1} and Y_{-1} are given by equations similar to (4.25) and (4.26) with λ becoming λ-1 (i.e. 0 becomes -1 and -1 becomes -2).

We define:

$$X_1 = \sum_{l, l \neq j} \beta_{1l} \cos\left(\psi_{1l}\right) R_{11}(\tau_{1l}) \tag{4.27}$$

$$X_k = \sum_l \beta_{kl} \cos\left(\psi_{kl}\right) R_{1k}(\tau_{kl}) \tag{4.28}$$

$$\hat{X}_1 = \sum_{l, l \neq j} \beta_{1l} \cos\left(\psi_{1l}\right) \hat{R}_{11}(\tau_{1l}) \tag{4.29}$$

$$\hat{X}_k = \sum_l \beta_{kl} \cos\left(\psi_{kl}\right) \hat{R}_{1k}(\tau_{kl}) \tag{4.30}$$

and similar expressions for Y. Without loss of generality, we can assume that $\tau_{1j} = 0$, $\Psi_{1j} = 0$, because we have considered the *j*th path between transmitter 1 and receiver as the reference path. So, we can write:

$$X_o = \sqrt{P/8}\, \hat{R}_{1k}(0) \beta_{1j}\, b_1^{\lambda} + \sqrt{P/8} \sum_k \left[d_1\left(b_k^{-1}\right) b_k^{-1} X_k + d_2\left(b_k^0\right) b_k^0 \hat{X}_k \right] + \nu \tag{4.31}$$

$$Y_o = \sqrt{P/8} \sum_k \left[d_1\left(b_k^{-1}\right) b_k^{-1} Y_k + d_2\left(b_k^0\right) b_k^0 \hat{Y}_k \right] + \eta \tag{4.32}$$

in which we define *m* as:

$$m = \sqrt{P/8}\, \hat{R}_{11}(0)\beta_{1j}\, b_1^{\lambda} \tag{4.33}$$

The complex envelope, which is considered to be a random variable, consists of the following four contributions:

* A useful term, which is due to the *j*th path signal of the reference user;
* A complex Gaussian noise term, of which the real and imaginary parts are statistically independent and have the same variance, $N_o T/16$;
* A multipath interference term, which is due to the L -1 other path signals from the reference user. The contributions from the different paths are uncorrelated.
* A multiple access interference term, which is due to the path signals of the K - 1 nonreference users. Here also, the contributions of the different paths are uncorrelated.

The expressions derived in this section will be the starting point for the derivation of the expression for the bit error probability for both selection diversity and maximal ratio combining.

4.3.3 Description of the received signal in case of BPSK and QPSK

We assume that a received signal is composed of the contributions of the different users, their different paths and additive white Gaussian noise. Therefore, we can write the expression for the received signal for QPSK as:

$$\begin{aligned} r(t) = {} & \sqrt{2P}\sum_k \sum_l \beta_{kl} a_{a,c}(t - \tau_{kl}) b_{c,k}(t - \tau_{kl}) \cos\left(\omega_c t + \int_{-\infty}^{t-\tau_{kl}} \omega_k(\tau)d\tau + \alpha_k + \theta_k + \gamma_{kl}\right) \\ & + \sqrt{2P}\sum_k \sum_l \beta_{kl} a_{s,k}(t - \tau_{kl}) b_{s,k}(t - \tau_{kl}) \sin\left(\omega_c t + \int_{-\infty}^{t-\tau_{kl}} \omega_k(\tau)d\tau + \alpha_k + \theta_k + \gamma_{kl}\right) \\ & + n(t) \end{aligned} \tag{4.34}$$

where $n(t)$ is AWGN with single-sided power spectral density N_o [W/Hz]. Since we consider user 1 as the reference user, the dehopping sequence is the one associated with user 1. After dehopping each of the 2 components of the dehopped signal is multiplied by the DS code associated with user 1 and multiplied by the carrier signal and next integrated over the bit duration.

The bit under consideration is bit number λ. However, without loss of generality we can assume that $\lambda=0$ and that the jth path between transmitter 1 and the receiver is the reference. Because of the dominant and stable component, the receiver can be time-synchronous with this path. Besides, the receiver is able to acquire the phase of the stable component of this path, due to its tracking mechanism. Therefore, we say that $\tau_{1j}=0$ and $\Psi_{1j}=0$, in which Ψ is the overall phase shift. All delays are then defined as relative delays according to this reference path.

4.3.4 Signal description of the BPSK and QPSK demodulator output

Let us focus on the in-phase signal. The output of the in-phase integrator is given by:

$$
\begin{aligned}
z_{o,c} &= \sqrt{P/8}\sum_k \sum_l \beta_{kl}\cos(\psi_{kl}) \int_{\lambda T}^{(\lambda+1)T} a_{c,1}(t)a_{c,k}(t-\tau_{kl})b_{c,k}(t-\tau_{kl})\delta[f_1(t),f_k(t-\tau_{kl})]dt \\
&+ \sqrt{P/8}\sum_k \sum_l \beta_{kl}\sin(\psi_{kl}) \int_{\lambda T}^{(\lambda+1)T} a_{c,1}(t)a_{s,k}(t-\tau_{kl})b_{s,k}(t-\tau_{kl})\delta[f_1(t),f_k(t-\tau_{kl})]dt \qquad (4.35)\\
&+ \nu_c
\end{aligned}
$$

where ν_c is a zero-mean Gaussian random variable with variance $N_oT/16$. The δ-function accounts for the possible hits: a hit occurs when the frequency $f_1(t)$ used by the reference user and the frequency $f_k(t-\tau_{kl})$ used by nonreference users are the same.

Because of the limited interval of the delays, two bits will be under consideration, viz. bits 0 and -1 of sequence k. According to [10] we define the following correlation functions:

$$R^{xy}_{1k}(\tau_{kl}) = \int_0^{\tau_{kl}} a_{x,1}(t)a_{y,k}(t - \tau_{kl})dt \tag{4.36}$$

$$\hat{R}^{xy}_{1k}(\tau_{kl}) = \int_{\tau_{kl}}^{T} a_{x,1}(t)a_{y,k}(t - \tau_{kl})dt \tag{4.37}$$

in which x and y may be both c or s. In these definitions, the periodicity has been taken into account. We define the following parameters:

$$X^{xy}_{k} = \sum_{l} \beta_{kl}\, f\, [\psi_{kl}]\, R^{xy}_{1k}(\tau_{kl}) \tag{4.38}$$

$$\hat{X}^{xy}_{k} = \sum_{l} \beta_{kl}\, f\, [\psi_{kl}]\, \hat{R}^{xy}_{1k}(\tau_{kl}) \tag{4.39}$$

with x and y defined as previously. When $x = y$ then the function f is the cosine function else it is the sine function. If $k = 1$, then $l \neq j$, so we have:

$$\begin{aligned} z_{o,c} &= \sqrt{P/8}T\beta_{1j}b^{o}_{c,1} \\ &+ \sqrt{P/8}\sum_{k}\left[d_1(b^{-1}_{c,k})b^{-1}_{c,k}X^{cc}_{k} + d_2(b^{o}_{c,k})b^{o}_{c,k}\hat{X}^{cc}_{k} + d_1(b^{-1}_{s,k})b^{-1}_{s,k}X^{cs}_{k} + d_2(b^{o}_{s,k})b^{o}_{s,k}\hat{X}^{cs}_{k}\right] \\ &+ \nu \end{aligned} \tag{4.40}$$

where $b^{-1}_{c,k}$ (resp. $b^{-1}_{s,k}$) and $b^{o}_{c,k}$ (resp. $b^{o}_{s,k}$) are the current bit and the previous bit of in-phase (resp. quadrature) data stream of the kth user. The functions d_1 and d_2 account for the possible hit between the current frequency of user 1 and the frequencies of the associated with the two bits under consideration.

We see that there is cross-rail interference, in the sense that the bits associated with the in-phase (quadrature) component interfere with the quadrature (in-phase) component. In case of BPSK we have:

$$z_o = \sqrt{P/8}T\beta_{1j}\, b^{o}_{1} + \sqrt{P/8}\sum_{k}\left[d_1(b^{-1}_{k})b^{-1}_{k}X_k + d_2(b^{o}_{k})b^{o}_{k}\hat{X}_k\right] + \nu \tag{4.41}$$

in which X_k and $\hat{X}_k$ are defined as follows:

$$X_k = \sum_l \beta_{kl} \cos(\psi_{kl}) \, R_{1k}(\tau_{kl}) \tag{4.42}$$

$$\hat{X}_k = \sum_l \beta_{kl} \cos(\psi_{kl}) \, \hat{R}_{1k}(\tau_{kl}) \tag{4.43}$$

The same as in case of QPSK, we have that $l \neq j$ if $k = 1$, according eqs. (4.27) and (4.29). We see that the cross-rail term has disappeared in equation (4.41).

With the help of all these expressions we are able to derive the expressions for the average bit error probability for both selection diversity and maximal ratio combining. This will be done in the next chapter.

5. PERFORMANCE OF THE HYBRID DS/SFH SYSTEM

The performance of the hybrid DS/SFH system can be assessed in different ways. The average bit error probability and the outage probability are measures to get an idea about the system performance.

In the following chapters we will investigate the average bit error probability, the outage probability and the influence of FEC coding of the hybrid DS/SFH with DPSK modulation. Next we will discuss the average probability of the hybrid system using BPSK and QPSK as modulation techniques.

5.1 Study of the bit error probability

There are three different main phenomena which contribute to errors in the system under consideration.

First, even in the absence of noise and fading, errors may occur when a signal is hopped to a frequency slot that is occupied by another signal. Whenever two different signals occupy one frequency slot simultaneously, we say a that *hit* occurs.

Second, the different spread spectrum codes assigned to the different users, show mutually cross-correlation effects. This, because the codes are not ideal and of finite length. These cross-correlation effects result in side pulses in the detector, which might give rise to errors in the detection of the bits.

Third, even in the absence of the multiple access interference effects described above, errors may occur due to fading and additive white noise.

According to [1],[2] and[9], we first decouple the effects of hits from other users due to frequency hopping from the multiple access interference due to the direct sequence spread spectrum signals. In [1],[2] and [9] this is done by first evaluating the conditional error probability, given the number of hits, and then averaging with respect to the distribution of the hits.

In our model we evaluate the conditional probability not only given a number of hits, but also given the path gain β and the delay τ. Then we first average with respect to the number of hits and next with respect to the path gains and the delays. We can actually say that given a number of hits from other users has occurred, the hybrid system is almost equivalent to a DS/SSMA system with noncoherent reception.

The conditional bit error probability, which is computed as the mean of several situations, corresponding to the possible hit situations produced by the multiple access interferers, is then given by:

$$P_e(\ |\beta,\tau_{kl}) = \sum_{n_i=0}^{K-1} P_e(\ |n_i,\beta,\tau_{kl})P(n_i) \tag{5.1}$$

Where $P_e(\ |n_i,\beta,\tau_{kl})$ is the conditional bit error probability in the absence of coding, assuming there are n_i active interferers out of K - 1 users. An expression for this probability will be derived in the next section. $P(n_i)$ is the probability of having n_i active interferers out of K - 1 users. For random hopping patterns with a number of frequencies equal to q, the probability that any two users use the same frequency is given by 1/q. So, $P(n_i)$ is given by:

$$P(n_i) = \binom{K-1}{n_i}\left(\frac{1}{q}\right)^{n_i}\left(1-\frac{1}{q}\right)^{K-1-n_i} \tag{5.2}$$

which is a binomial distribution.

In order to use this approximation we have to meet the following requirements:

- All users yield the same average power at the receiver (a symmetrical system).
- All K - 1 nonreference users have the same path power, so that the multi-access interference power only depends on the number of hits.
- All path signals of the reference user have the same path power.

When these key requirements are met and the PN sequence period N is sufficiently large, the multipath and multiple access interference terms consist of a large number of statistically independent contributions with the same distribution. This means that each interference term can be well approximated by a Gaussian random variable.

The influence of the multipath term is actually dependent on the bit position in the hopping interval. For the first bit in a block, there is no hit with certitude with the previous bit, because they do not necessarily have the same frequency. For all other bits in the blocks, there will be a hit with certitude with the previous and current bit conveyed by interfering paths. We neglect the effect of this first bit, which is a reasonable assumption, because N_b is relatively large in case of slow frequency hopping.

5.2 Study of the bit error probability for DPSK modulation in case of selection diversity

In case of selection diversity of order M, the decision variable $\xi_{SD}(\lambda)$ is the maximum of M random variables $\xi_l(\lambda)$. Since the data bits are equiprobable, the bit error probability can be written as:

$$P\left[\xi_{\max}^{M}<0 \mid b_1^o b_1^{-1}=1\right] \tag{5.3}$$

If we assume multichannel reception in a time-invariant Rician fading channel and DPSK modulation for fixed delays, phase angles, bits and assuming the maximum of the envelope has been found (all other signals are seen as noise), we can use the bit error probability given in [18]:

$$P_e(\ |n_i,\beta_{\max},\tau_{kl}) = Q(a,b) - \frac{1}{2}\left(1+\frac{\mu}{\sqrt{\mu_o\mu_{-1}}}\right).\exp\left(-\frac{a^2+b^2}{2}\right).I_o(ab) \tag{5.4}$$

where Q is the Marcum-Q function, which is defined as:

$$Q(a,b) = \int_b^{\infty} x.\exp\left(-\frac{a^2+x^2}{2}\right).I_o(ax) \tag{5.5}$$

and where I_0 is the modified Bessel function of the first kind and zero order, which is defined as:

$$I_o(ab) = \frac{1}{2\pi}\int_o^{2\pi}\exp(\ a\ b\ \cos\theta\ )d\theta \tag{5.6}$$

The parameters a, b ,m ,μ ,μ_{-1} and μ_0 are given below.

$$a = \frac{m}{\sqrt{2}} \left| \frac{1}{\sqrt{\mu_o}} - \frac{1}{\sqrt{\mu_{-1}}} \right| \tag{5.7}$$

$$b = \frac{m}{\sqrt{2}} \left| \frac{1}{\sqrt{\mu_o}} + \frac{1}{\sqrt{\mu_{-1}}} \right| \tag{5.8}$$

$$\mu_o = VAR\left(V_o \,|\, L,\ \tau_{kl},\ n_i,\ b\right) \tag{5.9}$$

$$\mu_{-1} = VAR\left(V_{-1} \,|\, L,\ \tau_{kl},\ n_i,\ b\right) \tag{5.10}$$

$$\mu = COV\left(V_o, V_{-1} \,|\, L,\ \tau_{kl},\ n_i,\ b\right) \tag{5.11}$$

The derivation of expressions for these parameters are given in appendix A. After this, the bit error probability given in equation (5.1) has to be averaged for all possible values of β_{max} and on the delays.

5.2.1 Influence of the Rician fading statistics

We now have to weight this result by considering the Rician fading statistics, i.e. the probability density function of the maximal path gain β_{max}. The derivation of this PDF is given in appendix C. For an order of diversity equal to M and assuming of course, that all path gains are equally distributed independent Rician variables, this PDF becomes:

$$f_{\beta_{max}}(\beta_{max}) = M \left[\int_0^{\beta_{max}} \frac{z}{\sigma_r^2} \exp\left(-\frac{S^2+z^2}{2\sigma_r^2} \right) I_o\left(\frac{Sz}{\sigma_r^2} \right) dz \right]^{(M-1)} \frac{\beta_{max}}{\sigma_r^2} \exp\left(-\frac{S^2+\beta_{max}^2}{2\sigma_r^2} \right) I_o\left(\frac{S\beta_{max}}{\sigma_r^2} \right) \tag{5.12}$$

It is through this PDF that the influence of the order of diversity can be understood. In figure 5.1 the influence of the order of diversity M on the PDF of the maximal path gain is shown.

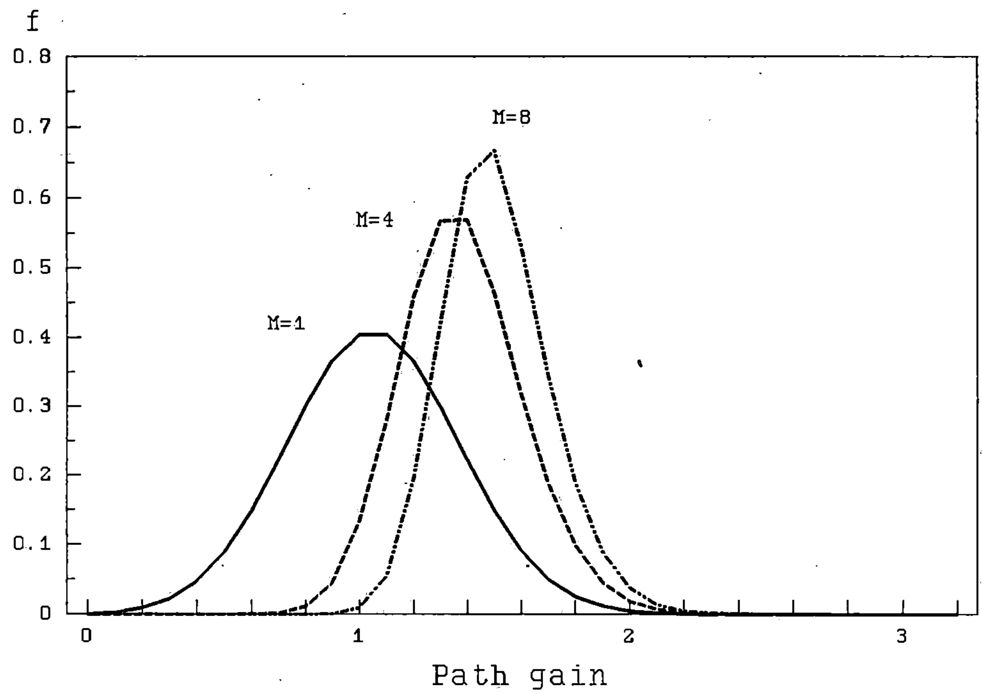

Figure 5.1: *Influence of the order of diversity on the PDF of the maximal path gain*

The increase of the order of diversity M results in a narrower PDF, which results in a higher probability of determination and realization of the maximal path gain.

5.2.2 Gaussian approximation on the delays

The bit error probability only conditioned on the delays τ_{kl} with selection diversity is given by:

$$P_e = \int_{\beta_{max}} \int_{\tau_{kl}} P_e(\mid \beta_{max}, \tau_{kl}) \, f_{\beta_{max}}(\beta_{max}) \, f_{\tau_k}(\tau_k) \, d\beta_{max} \, d\tau_{kl} \tag{5.13}$$

It has been mentioned by many authors that integration over all path delays τ_{kl} is very time consuming. Therefore an approximation has to be made.

We know that if the number of users K and the number of paths L is sufficiently large, the central limit theorem can be used. As a result of this, the conditional variances μ_o ,

μ_{-1} and μ are assumed to have a Gaussian distribution. The parameters belonging to the Gaussian distribution are the mean and the variance of the stochastic variable under consideration. The means and the variances with respect to τ of the three μ-parameters are derived in appendix B. The conditioning on the delays can now be removed by weighting the conditional bit error probability by the appropriate Gaussian probability density functions. Therefore, we finally get an expression for the average bit error probability in:

$$P_e = \int_{\mu}\int_{\mu_{-1}}\int_{\mu_o}\int_{\beta_{max}} P_e(\,|\,\beta_{max},\mu_o,\mu_{-1},\mu) f_{\beta_{max}}(\beta_{max}) f_{\mu_{-1}} f(\mu_{-1})_{\mu_o}(\mu_o) f_{\mu}(\mu) d\beta_{max} d\mu_o d\mu_{-1} d\mu \tag{5.14}$$

in which $f_{\beta_{max}}(\beta_{max})$ is the probability density function given in equation (5.12) and $f_{\mu_o}(\mu_o)$ is the Gaussian probability density function.This integral can be simplified by considering that μ appears explicitly in the function P_e (see equation (5.4)). We also know that:

$$\int_{-\infty}^{\infty} \mu\, f(\mu) d\mu \doteq E(\mu) \quad . \tag{5.15}$$

In the next section we will discuss the performance of the hybrid DS/SFH system with maximal ratio combining.

5.3 Study of the bit error probability for DPSK modulation in case of maximal ratio combining

As we noted earlier, the decision variable in maximal ratio combining of order M is given by $\xi_{mrc}(\lambda)$, which is the average of M random variables ξ_l. In order to be able to give an expression for the bit error probability in case of maximal ratio combining, we have to assume that the multiple access and the multipath interference is Gaussian. This means that the term

$$\sqrt{P/8}\left[\sum_k \left(d_1(b_k^{-1})b_k^{-1}X_k + d_2(b_k^o)b_k^o\hat{X}_k\right) + j\left(d_1(b_k^{-1})b_k^{-1}Y_k + d_2(b_k^o)b_k^o\hat{Y}_k\right)\right] \tag{5.16}$$

in V_o is complex Gaussian and also for the similar parts of V_{-1} (See equations (4.31) and (4.32)).

We have to keep in mind that this assumption is a simplification of the mathematical description of the bit error probability in case of maximal ratio combining. However, this assumption enables us to derive some closed expressions for the bit error probability and to get a first idea of the behaviour of the system.

5.3.1 Definition of the decision variable in case of MRC

We consider channels in which the path gains have identical average power and in which the path delays are uniformly distributed in [0,T]. The decision variable, which is given in [8, 7 App. 4 B] is identical to ξ_{mrc}, namely:

$$\xi_{mrc} = \frac{1}{2} \sum_{i=1}^{M} \left(V_{o,i} V^{*}_{-1,i} + V^{*}_{o,i} V_{-1,i} \right) \tag{5.17}$$

If the interference is assumed Gaussian, the sum of the interference and the AWGN is also Gaussian. These total noise terms will be denoted N_1 for V_0 and N_2 for V_{-1} respectively. So the decision variable then becomes:

$$\xi_{mrc} = Re\left[\sum_{i=1}^{M} \left(AT\beta_i b_1^o + N_{1i} \right) \cdot \left(AT\beta_i b_1^{-1} + N_{2i} \right)^{*} \right] \tag{5.18}$$

where β_i denotes the gain and N_{1i} and N_{2i} the Gaussian random variables, associated with the *ith* path. We now have to make two assumptions about the noise variables.

First, we assume that N_{1i} and N_{2i} are independent for each *i*. In our case this is justified, because calculations have shown that μ is small in comparison with μ_0 and μ_{-1}. This implies that the COV $[N_{1i}, N^{*}_{2i}]$ is negligible compared with VAR $[N_{1i}]$ and VAR $[N_{2i}]$.

Second, we will assume that the pair (N_{1i}, N_{2i}) is independent of the pair (N_{1j}, N_{2j}) for $i \neq j$. Mathematically, this is not correct, because the delays $\{\tau_{kl}\}_i$ and $\{\tau_{kl}\}_j$ are not independent. However, there are two reasons to make the assumption physically reasonable.

First, each set of delays is taken with reference to a different time origin (corresponding to the arrival time of the signal on the corresponding combined path).

Second, we know that any two resolved paths (i,j) are separated by at least one chip time period.

The computation of the total noise term can be obtained from the computation of μ_o by taking the expectation over the path delays. We denote this expectation by $E_\tau(\mu_o)$ and the expression is given by equation (B.8) in appendix B.

For path k, we have a signal power given by: $P\beta_k^2$ and a total noise power given by $N_T = E_\tau(\mu_o)$. Therefore, the signal to noise ratio of resoluted path k is given by:

$$\frac{P\beta_k^2}{E_\tau(\mu_o)} \tag{5.19}$$

With this information we are able to give an expression for the bit error probability in case of MRC. This will be done in the next section.

5.3.2 Definition of the bit error probability in case of MRC

If we assume that the values of the path gains are known, we can use the bit error probability given in [18] for maximal ratio combining and diversity of order M:

$$P_{e,mrc}(\,|\,\gamma_b, n_i) = \frac{1}{2^{(2M-1)}}\exp(-\gamma_b)\sum_{k=0}^{M-1} p_k\gamma_b^k \tag{5.20}$$

with

$$p_k = \frac{1}{k!}\sum_{n=0}^{M-1-k}\binom{2M-1}{n} \tag{5.21}$$

$$\gamma_b = \frac{E}{N_T}\sum_{k=1}^{M}\beta_k^2 \tag{5.22}$$

in which N_T represents the total noise power. In order to remove the conditioning on the interferers we have to substitute equation (5.20) in equation (5.1). We then obtain the probability which is only conditioned on γ_b. To find the average bit error probability, we only have to weight this conditional probability with the probability density function of γ_b, which is given in [18] as:

$$f_{\gamma_b}(\gamma_b) = \frac{1}{2E/N_T}\left[\frac{\gamma_b}{(E/N_T)S_M^2}\right]^{\frac{(M-1)}{2}} \exp\left[-\frac{\left(S_M^2 + \gamma_b N_T/E\right)}{2}\right] I_{M-1}\cdot\left[\frac{S_M}{\sigma^2}\sqrt{\frac{\gamma_b}{E/N_T}}\right] \tag{5.23}$$

with $S_M^2 = MS^2$ and I_{M-1} is the (M-1)th-order modified Bessel function. We finally compute the bit error probability by:

$$P_{e,mrc} = \int_{\gamma_b} P_{e,mrc}(\,|\,\gamma_b)\, f_{\gamma_b}(\gamma_b)d\gamma_b \tag{5.24}$$

Another measure for the performance is the outage probability. In the next section we will discuss the outage probability for the hybrid DS/SFH system with DPSK modulation.

5.4 The outage probability performance

The outage probability is defined as the probability that the instantaneous bit error probability exceeds a preset threshold. We denote the threshold value as ber_o. The instantaneous value of the bit error probability can be obtained by using equation (5.14). The averaging over β_{max} should be removed and a fixed value for β_{max}, β, should be substituted. Equation (5.14) then alters in

$$P_e(\beta) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P_e(\beta\,|\,\mu_o,\mu_{-1})f_{\mu_o}(\mu_o)f_{\mu_{-1}}(\mu_{-1})d\mu_o d\mu_{-1} = ber(\beta) \tag{5.25}$$

The outage probability (in case of selection diversity) can then be calculated as follows:

$$P_{out} = P(0 \le \beta_{max} \le \beta_o) = P(ber(\beta) \ge ber_o) = \int_0^{\beta_o} f_{\beta_{max}}(\beta_{max})d\beta_{max} \tag{5.26}$$

in which β_o is the value of β at which the instantaneous bit error probability is equal to ber_o. The integrand is just the PDF of β_{max} given in equation (5.14).

5.5 Effect of FEC coding on the performance

Error correcting codes can improve the performance of the system in some cases. In this section two specific codes are considered viz. the (7,4) Hamming code and the (23,12) Golay code. The figures between brackets (n,k) means that k source bits are transformed into a block of n channel bits by coding.

From coding theory we know that a code with a Hamming distance d_{min} is able to correct at least $t=(d_{min} - 1)/2$ errors [18]. The Hamming distance for the two types of codes are:

- Hamming code: $d_{min} = 3$;
- Golay code: $d_{min} = 7$.

This means that the codes can correct one and three errors respectively. The probability of having m errors in a block of n bits is:

$$P(m,n) = \binom{n}{m} P_e^m \left(1-P_e\right)^{n-m} \tag{5.27}$$

The probability of having more than t errors in a code block of n bits is:

$$P_{ec} = \sum_{m=t+1}^{n} P(m,n) \tag{5.28}$$

An approximation for the bit error probability after decoding is given in [1] as:

$$P_{ec1} = \frac{1}{n} \sum_{m=t+1}^{n} m.\binom{n}{m} P_e^m \left(1-P_e\right)^{n-m} \tag{5.29}$$

in which P_e is the conditional bit error probability at the hard decision output. This means the bit error probability obtained with equation (5.29) must be substituted in equation (5.1) in order to get the average bit error probability.

5.6 Study of the bit error probability for BPSK and QPSK modulation

The methodology which is followed is similar to the one presented [2] and section 5.1.
The multipath term of the in-phase signal of the QPSK receiver, denoted as $I_{c,mp}$ is given by:

$$I_{c,mp} = \sqrt{P/8}\left[b_{c,1}^{-1}X_1^{cc} + b_{c,1}^{o}\hat{X}_1^{cc} + b_{s,1}^{-1}X_1^{cs} + b_{s,1}\hat{X}_1^{cs}\right] \tag{5.30}$$

For BPSK the multipath term, denoted as I_{mp} is given by:

$$I_{mp} = \sqrt{P/8}\left[b_1^{-1}X_1 + b_1^{o}\hat{X}_1\right] \tag{5.31}$$

The multiple access noise, denoted as $I_{c,ma}$ corresponding to the assumption of n_i active interferers for QPSK is given by:

$$I_{c,ma} = \sqrt{P/8}\sum_{k=2}^{n_i+1}\left[b_{c,k}^{-1}X_k^{cc} + b_{c,k}^{o}\hat{X}_k^{cc} + b_{s,k}^{-1}X_k^{cs} + b_{s,k}\hat{X}_k^{cs}\right] \tag{5.32}$$

and for BPSK this term is given by:

$$I_{ma} = \sqrt{P/8}\sum_{k=2}^{n_i+1}\left[b_1^{-1}X_1 + b_1^{o}\hat{X}_1\right] \tag{5.33}$$

The bit error probability, assuming n_i interferers, is defined as the probability that the decision variable is lower than 0 assuming a "1" was transmitted (or conversely). In case of QPSK we then get:

$$P_e^{Q} = P\left[z_{o,c} < 0 \mid b_{c,1}^{o} = 1\right] \tag{5.34}$$

and for BPSK we have:

$$P_e^{B} = P\left[z_o < 0 \mid b_1^{o} = 1\right] \tag{5.35}$$

Here $z_{o,c}$ and z_o are given by equations (4.40) and (4.41) respectively. In order to compute the power of the multipath and multiple access terms for BPSK and QPSK exactly, we have to make some comments.

We assume that we deal with a given channel, which means that the absolute value of the delay range is known. The bit duration of BPSK was assumed to be T and the delay was assumed to be uniformly distributed over this bit duration. Accordingly, in case of QPSK the bit duration is twice the one of BPSK. Therefore, the delay for QPSK has to be considered as being distributed uniformly over half of the QPSK bit duration. Considering this and according to an approach proposed in [9], the power of the multipath and multiple access are very well approximated by:

$$\frac{P}{8}(L-1)\left(\sigma_r^2+\frac{S^2}{2}\right)\frac{2T^2}{3N} \tag{5.36}$$

and

$$\frac{P}{8}\, n_i\, L\left(\sigma_r^2+\frac{S^2}{2}\right)\frac{2T^2}{3N} \tag{5.37}$$

respectively, for both QPSK and BPSK.

The bit error probability can be computed as a function of the bit energy to total noise power spectral density assuming the value of β_{1j}. The bit error probability has to be averaged over the distribution of β_{1j}. If we assume that the total noise is Gaussian with zero-mean, we can use the bit error probability given in [18]:

$$P_e = \frac{1}{2} erfc\left(\sqrt{\frac{E}{N_T}}\right) \tag{5.38}$$

We define the following normalized variable:

$$\alpha_{1j} = \frac{\beta_{1j}}{\left(2\sigma_r^2+S^2\right)^{0,5}} \tag{5.39}$$

We finally can write the bit error probability, given a number of active interferers, as:

$$P_e(n_i) = \frac{1}{2} erfc \left\{ \left[\left[\frac{E_b(2\sigma_r^2 + S^2)}{N_o} \right]^{-1} \frac{1}{\alpha_{1j}^2} + \frac{2L}{3N\alpha_{1j}^2}(1 + n_i) \right]^{-0,5} \right\} \tag{5.40}$$

The bit error probabilities are given as functions of the mean received bit energy to noise ratio. It is assumed, that the multipath interference associated with the reference user is due to L paths instead of L - 1.

5.6.1 Study of the system with selection diversity

Selection diversity means that the largest of a group of M signals, carrying the same information, is selected. As mentioned in chapter 3, the order of diversity is equal to the number of resolvable paths times the number of antennas. The decision variable is denoted by $z^M_{o\,max}$, which is the largest among the set of M values. In order to obtain the average bit error probability, we have to average the probability in equation (5.40) with the PDF of the maximum path gain β_{max}, instead of β. A change of variables gives:

$$\alpha = \frac{\beta}{\left(S^2 + 2\sigma_r^2 \right)^{0,5}} \tag{5.41}$$

and

$$y = \frac{z}{\left(S^2 + 2\sigma_r^2 \right)^{0,5}} \tag{5.42}$$

For the PDF of the maximum path gain we then find:

$$\begin{aligned} f_{\alpha_{o\,max}}(\alpha) = M & \left[\int_0^{\alpha} 2\, y\, (1 + R) \exp\left(-R - (1 + R)\, y^2 \right) I_o \left(2\sqrt{R(1 + R)}\, y \right) dy \right]^{(M-1)} \\ & \times 2\, \alpha\, (1 + R) \exp\left(-\alpha^2 (1 + R) - R \right) I_o \left(2\sqrt{R(1 + R)}\, \alpha \right) \end{aligned} \tag{5.43}$$

The average bit error probability is given as a function of the mean bit energy to white noise ratio at the receiver.

5.6.2 Study of the system with maximal ratio combining

In case of maximal ratio combining, the contributions of several resolved paths are added together. The combiner that achieves the best performance is the one in which each matched filter output is multiplied by the corresponding complex-valued channel gain [18]. This complex gain compensates for the phase shift and introduces a weighting associated with the signal strength. The realization of such a receiver is based on the assumption that the channel attenuations and phase shifts are perfectly known.

Considering the decision variable z_o in case of MRC, we have:

$$z_o = Re\left[\sqrt{P/8}\ T \sum_{i=1}^{M} \beta_{1i}^2 + \sum_{i=1}^{M} \beta_{1i} N_{1i}\right] \tag{5.44}$$

Each term involved in the combination process is corrupted by AWGN and the multi-user and the multipath interference. We make the assumption that the noise due tothe multi-user and the multipath interference is also Gaussian and that all noise terms can be added together. We also assume that the noise terms affecting 2 different paths, are independent. On close examination this is not correct, however for the same reason as discussed in section 5.3.1, this assumption can be used.

Using the bit error probability given in [18], we have:

$$P_e = \frac{1}{2}\, erfc\left(\sqrt{\gamma_b}\right) \tag{5.45}$$

where the signal to noise ratio per bit is given by:

$$\gamma_b = \frac{T^2 E_b}{16 N_T} \sum_{m=1}^{M} \beta_{1m}^2 \tag{5.46}$$

We the need to average on the PDF of $t = \sum_{m=1}^{M} \beta_{1m}^2$. The average of this random variable is given by $M\ (2\sigma_r^2 + S^2)$, which also represents the average gain at the receiver. We will perform a normalization of t with respect to the average value. The normalized variable v is the given by:

$$\nu = \sum_{m=1}^{M} \frac{\beta_{1m}^2}{M\left(2\sigma_r^2 + S^2\right)} \tag{5.47}$$

The PDF of ν is then given by:

$$\begin{aligned} f(\nu) = M\left[\, 1 + R \,\right] \left[\nu\left(1 + \frac{1}{R}\right)\right]^{(M-1)/2} \exp\left(-MR - M\nu(1 + R)\right) \\ \times I_{m-1}\left(2M \sqrt{(\nu R(1 + R)} \right) \end{aligned} \tag{5.48}$$

The following bit error probability, averaged over this PDF provides the average bit error probability as a function of the mean received bit energy to noise ratio:

$$P_e(n_i) = \frac{1}{2}\, erfc \left\{ \left[\left(\frac{E_b(2\sigma_r^2 + S^2)}{N_o} \right)^{-1} \frac{1}{M\nu} + \frac{2L}{3NM\nu}\,(1 + n_i) \right]^{-0,5} \right\} \tag{5.49}$$

6. NUMERICAL RESULTS FOR THE PERFORMANCE

In this chapter we present the numerical results of the average BER and the outage probability of the hybrid DS/SFH system with diversity and FEC coding.

These results will be given as a function of the ratio of energy per bit E_b and the spectral density N_o of the AWGN. We know that E_b is equal to $P.T$, so in order be able to compare the performance for different bit rates for the same signal to noise ratio, we take care that the bit energy E_b remains the same for different bit rates. This means that if the bit duration T changes due to a changes in bit rate, the transmitted power P also changes so that E_b remains constant.

In the first section we will present the results of the BER without coding for different parameter settings. The second section deals about the outage probability also for different parameters. In the third section we will consider the effect of FEC coding on the BER. A comparison with the performance, obtained with another model and with other modulation schemes will be given in the fourth section

6.1 BER performance without coding

We will consider a number of users of K=15, unless stated otherwise. In table 6.1 and 6.2 the relation between the rms delay spread T_m, the various bit rates, different code lengths the maximum number of resolvable paths L is given. These results are obtained with equation (3.5).

Table 6.1: *Delay spreads (in ns) corresponding with three bit rates and the maximum number of resolvable paths for N=127.*

	32 kbit/s	64 kbit/s	144 kbit/s
L = 1	$T_m < 250$	$T_m < 123$	$T_m < 55$
L = 2	$250 \le T_m < 490$	$123 \le T_m < 246$	$55 \le T_m < 110$
L = 3	-	$246 \le T_m < 370$	$110 \le T_m < 165$
L = 4	-	-	$165 \le T_m < 220$
L = 5	-	-	$220 \le T_m < 275$

Table 6.2: *Delay spread (in ns) corresponding with three bit rates and the maximum number of resolvable paths for N=255.*

	32 kbit/s	64 kbit/s	144 kbit/s
L = 1	$T_m < 123$	$T_m < 61$	$T_m < 54$
L = 2	$123 \le T_m < 246$	$61 \le T_m < 122$	$27 \le T_m < 54$
L = 3	$246 \le T_m < 369$	$122 \le T_m < 183$	$54 \le T_m < 81$
L = 4	-	$183 \le T_m < 244$	$81 \le T_m < 108$
L = 5	-	$244 \le T_m < 305$	$108 \le T_m < 135$
L = 6	-	-	$135 \le T_m < 162$
L = 7	-	-	$162 \le T_m < 189$
L = 8	-	-	$189 \le T_m < 216$
L = 9	-	-	$216 \le T_m < 243$
L = 10	-	-	$243 \le T_m < 270$

In figure 6.1 the average BER is presented for selection diversity (SD) and maximal ratio combining (MRC) with the order of diversity M as parameter.

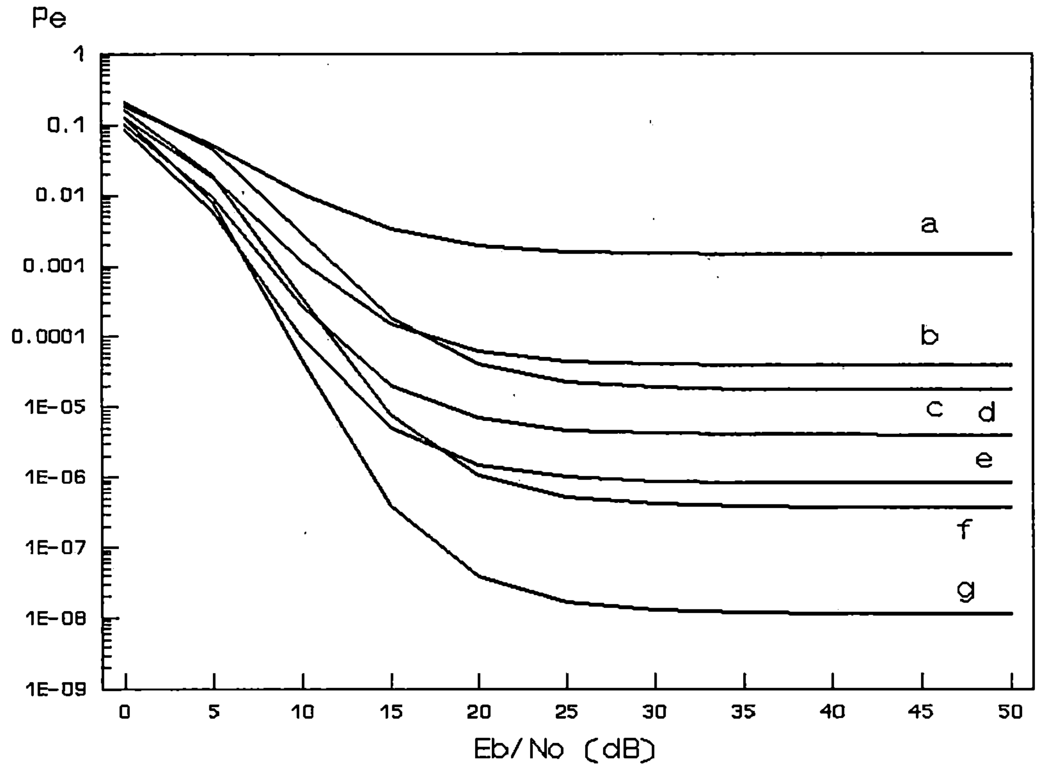

Figure 6.1: *BER for DS/SFH with SD and MRC with M as parameter; q=10; L=5; R_c= 64 kbit/s; K=15; N=255; R=6,8 dB*

a: nondiversity;
b: M=2 with SD;
c: M=2 with MRC;
d: M=3 with SD;
e: M=4 with SD;
f: M=3 with MRC;
g: M=4 with MRC;

From the plot is it seen that the performance of MRC is better than the performance of SD, however for a low order of diversity the difference in performance is less than for higher orders of diversity. The asymptotes of the curves are due to the multi-user interference; this limitation would disappear if the multipath interference would be zero, the signal-to-noise ratio were infinite and if the number of frequencies in the hopping pattern were infinite.

In figure 6.2 the effect of different parameter settings on the average BER in case of MRC is presented with the constraint of a fixed bandwidth. This means that $N.q.R_c$ is kept constant.

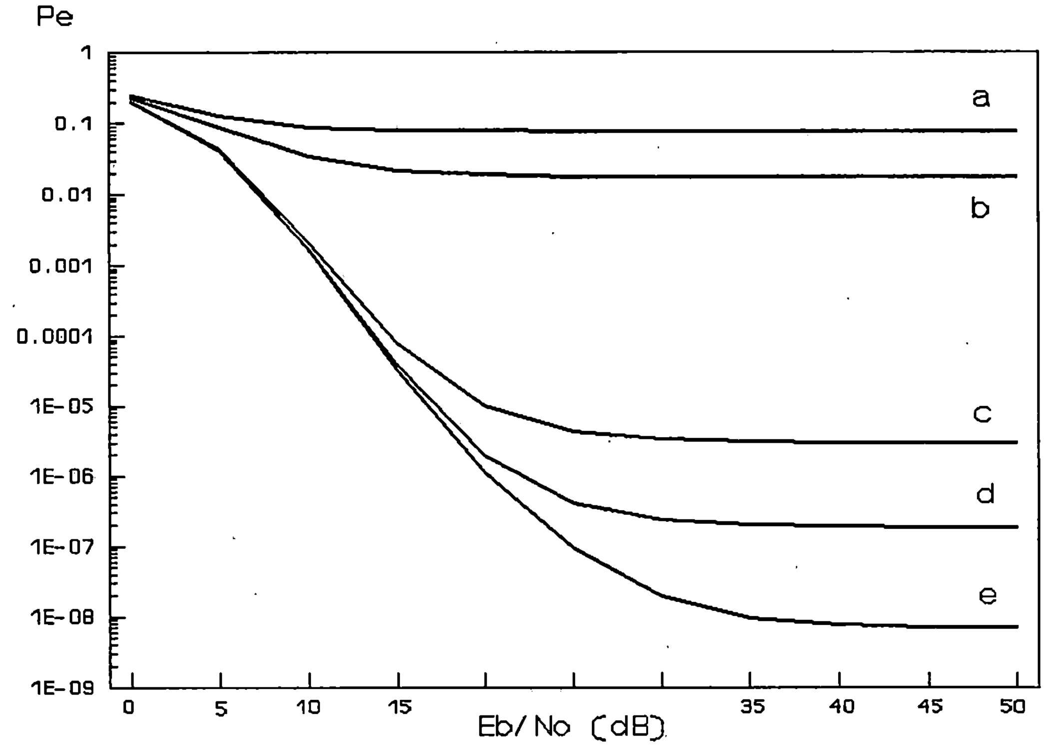

Figure 6.2: *BER of DS/SFH with MRC for different parameter settings given a fixed bandwidth; $M=2$; $T_m=250$ ns; $R=6,8$ dB.*

a: $N=127$; $Q=21$; $R_c=144$ kbit/s; $L=5$
b: $N=255$; $Q=10$; $R_c=144$ kbit/s; $L=10$
c: $N=255$; $Q=24$; $R_c=64$ kbit/s; $L=5$.
d: $N=127$; $Q=98$; $R_c=32$ kbit/s; $L=2$
e: $N=255$; $Q=49$; $R_c=32$ kbit/s; $L=3$

It is seen that a bit rate of 144 kbit/s yield a relatively poor performance. It is also obvious that code period of $N=127$ yields a worse performance than with the code period of $N=255$. We can explain these results by considering equation (3.5) and table (6.1) and (6.2).

First, according to equation (3.5) the number of resolvable paths is influenced by the rms delay spread T_m, the bit rate R_c and the period of the spreading codes N. An increase of the data rate increases the number of resolvable paths for the same spreading code period. The increase of the number of resolvable paths yields a higher level of multipath interference and this causes a relative high level of multipath interference.

Secondly, from the different combinations of constant N and q at fixed bit rate, it is seen that a doubling of the spreading code period yields a better performance than a doubling in the number of frequencies.

Third, the value of the maximum delay spread is of importance as well. A decrease of this parameter will decrease the number of resolvable paths. Accordingly, this will result in an increase in performance, because of the diminishing level of multipath interference.

6.2 Comparison of the BER of hybrid DS/SFH and pure DS

In table 6.3 we present a comparison of hybrid DS/SFH and DS in case of selection diversity (SD) for two bit rates and for two values of the order of diversity M. The transmission bandwidth B_T, which is proportional to $R_c.N.q$, is taken as parameter. The results for DS are obtained from [7].

Table 6.3: *BER comparison of DS and hybrid DS/SFH with SD for two bit rates, two values of M and the bandwidth as parameter; E_b/N_o=40 dB; T_m=250 ns; R=6,8 dB; K=15.*

M	R_c[kbit/s]	DS N=255	Hybr. DS/SFH q=2 N=127	Hybr. DS/SFH q=10 N=127	Hybr. DS/SFH q=5 N=255
4	32	$3{,}2.10^{-5}$	$2{,}0.10^{-5}$	$4{,}8.10^{-8}$	$2{,}9.10^{-11}$
4	64	$6{,}0.10^{-4}$	$3{,}1.10^{-2}$	$1{,}0.10^{-3}$	$1{,}1.10^{-5}$
8	32	$6{,}0.10^{-6}$	$9{,}0.10^{-7}$	$2{,}2.10^{-10}$	$1{,}6.10^{-12}$
8	64	$3{,}0.10^{-5}$	$1{,}8.10^{-2}$	$3{,}4.10^{-4}$	$8{,}2.10^{-7}$

It is seen that under the constraint of the same transmission bandwidth the performance of hybrid DS/SFH is slightly better than DS for the bit rate of R_c=32 kbit/s and for both orders of diversity. For the bit rate of R_c=64 kbit/s and under the same constraint, the performance is in favour of DS, for both orders of diversity. A higher bit rate means a higher number of resolvable paths, according to the tables (6.1) and (6.2) and this means a higher level of multipath interference. Apparently, for a low number of resolvable paths, which is the case for the lowest bit rate, hybrid DS/SFH with a number of frequencies $q=2$ is able to deal with the worse correlation properties of spreading codes with period $N=127$. For a higher number of resolvable paths, the number of frequencies of $q=2$ is insufficient to overcome the effect of increased multipath interference and the effect of the low spreading code period, for both orders of diversity.

The situation for which the transmission bandwidth is increased by a factor 5 for the hybrid system, through the combinations $N=127$ and $q=10$ or $N=255$ and $q=5$, shows something different. In case of $N=127$ and $q=10$ the performance of the hybrid system for the highest bit rate is better than DS and for the highest bit rate it is worse than DS for both orders of diversity. It is obvious that even a number of frequencies as high as $q=10$ is not able to overcome the effect of the spreading codes and the multipath effects, due to the increased number of resolvable paths in case of the highest bit rate.

The combination of $N=255$ and $q=5$ shows that the performance for both bit rates is in favour of the hybrid system for both orders of diversity. Besides, this combination yields a better performance than the previous combination of N and q, due to the better correlation properties of the spreading code with period $N=255$.

6.3 The outage probability performance

We consider the outage probability as a function of the signal to white noise ratio (ratio of the energy per bit and the density of the AWGN). The threshold value is taken to be 0.01 in our case. In figure 6.3 the effect of the order of diversity on the outage probability in case of SD is presented.

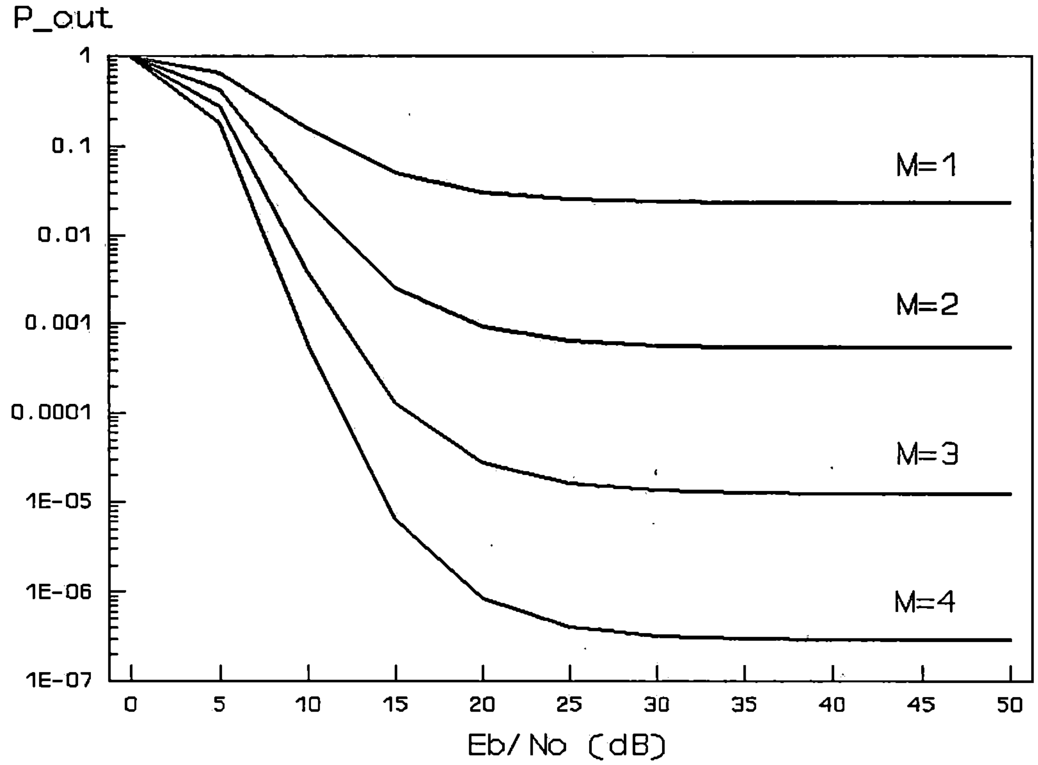

Figure 6.3: *Outage probability in case of SD for a fixed bandwidth with M as parameter; $N=255$; $Q=10$; $R=6,8$ dB; $L=5$; $R_c=64$ kbit/s; $T_m=250$ ns; $K=15$.*

a: $M=1$; b: $M=2$; c: $M=3$; d: $M=4$.

It is obvious that an increase in the order of diversity decreases the outage probability considerably. This is due to the fact if we consider figure 5.1, in which it shown that a higher order of diversity yields less spread in the PDF of the maximum path gain β_{max}.

In figure 6.4 we present the effect of different parameter settings on the outage probability, with the constraint of a fixed bandwidth.

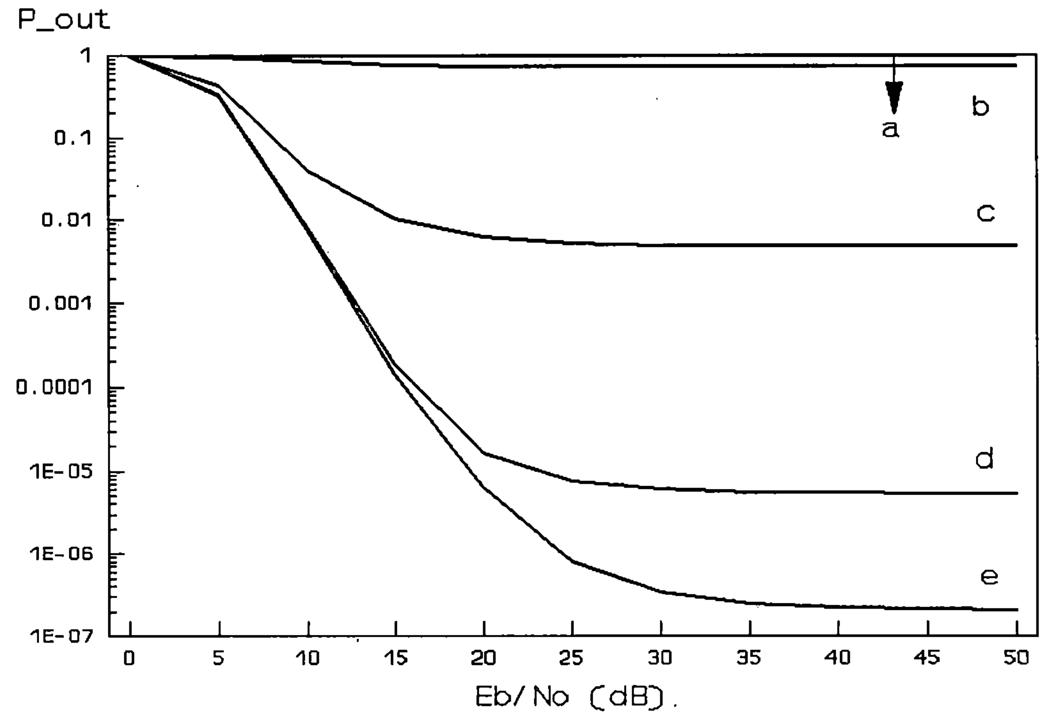

Figure 6.4: *Outage probability in case of SD with different parameter settings given a fixed bandwidth; M=2; R=6,8 dB*

a: N=127; Q=21; R_c*=144 kbit/s; L=5*
b: N=255; Q=10; R_c*=144 kbit/s; L=10*
c: N=255; Q=24; R_c*=64 kbit/s; L=5*
d: N=127; Q=98; R_c*=32 kbit/s; L=2*
e: N=255; Q=49; R_c*=32 kbit/s; L=3*

It is seen from these plots, that relative high bit rates yield a very poor performance in terms of outage probability. The same as with the bit error probability, this is due either to the worse correlation properties of codes with N=127 or to the relatively large number of resolvable paths at high bit rates. As discussed before, an increase of the number of resolvable paths causes an increase in the multipath interference level.

Under the constraint of a fixed bandwidth only the combination of relatively low bit rates and a sufficiently large spreading code period yield a reasonable performance. If the constraint of the fixed bandwidth is left, the performance can be enhanced by using a sufficiently large number of frequencies in combination with large spreading code periods.

6.4 Effect of FEC codes on the BER performance

We know that with DPSK demodulation, the errors at the hard decision device tend to occur in bursts of length two [1]. In order to avoid such a burst affecting two bits of a single code word, interleaving and deinterleaving of order two is applied. This will make the errors within a single codeword statistically independent, because these errors belong to different error events.

Another important observation is, that if the information bit rate should remain unchanged when FEC codes are used, the signalling rate should be increased [22]. This increases the chip rate, which causes an increase in the number of resolvable paths L.

Besides, the BER in case of no coding is given as a function of the ratio of the source bit energy E_b and the spectral density of the AWGN N_o. In case of coding we have a bit energy to noise ratio of E_i/N_o, which is given by:

$$\frac{E_i}{N_o} = \frac{E_b}{N_o} r_c \tag{6.1}$$

,where r_c is the code rate which equals k/n (see section 5.5). In case of the (7,4) Hamming and the (23,12) Golay codes, r_c is approximately equal to 0.5. This means that E_i/N_o is 3 dB lower than the ratio of the source bit energy E_b and N_o in case of FEC coding.

In figure 6.5 the effect of FEC coding on the performance of the hybrid system is presented, again with the constraint of a fixed bandwidth and as a function of the ratio of source bit energy and the spectral density of the AWGN. The code, which is used, is the (23,12) Golay code, which transforms a block of 12 bits into a block of 23 bits. This code has a code rate r_c equal to 12/23 (=0,52). This means that for the constraint of a fixed bandwidth, we can compare the following two cases:

A: - no coding, N=255 and q=10;
- coding, N=127 and q=10;

B: - no coding, N=255 and q=20;
- coding, N=255 and q=20

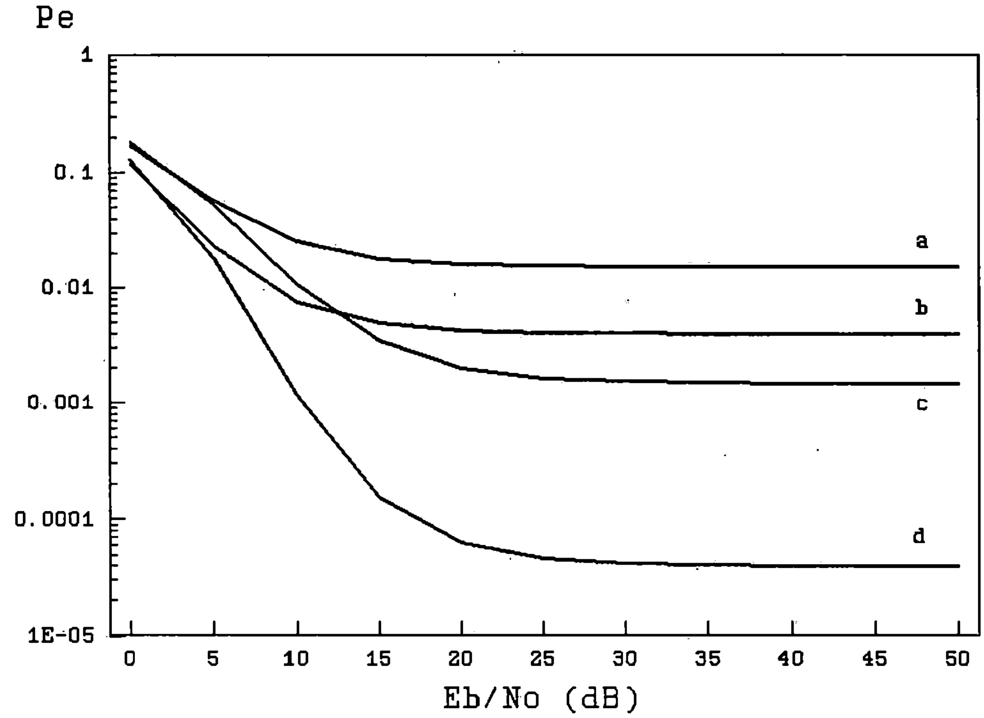

Figure 6.5: *BER of hybrid DS/SFH in case of SD with and without FEC coding at fixed bandwidth; $q=10$, $R=6{,}8$ dB; $K=15$; $T_m=250$ ns; $R_c=64$ kbit/s.*

a: $M=1$, Golay code, $N=127$, $L=6$	*c: $M=1$, no coding, $N=255$, $L=5$*
b: $M=2$, Golay code, $N=127$, $L=6$	*d: $M=2$, no coding, $N=255$, $L=5$*

It is seen that the performance with coding worsens the performance. This is due to the value of the spreading code period of $N=127$, which has worse correlation properties than $N=255$, together with a larger number of resolvable paths, which increases the multipath interference, too many errors occur. And since the error correction code is able to correct a limited number of errors, the performance is not enhanced.

Even selection diversity of order 2 together with FEC coding does not give a better performance than no diversity without coding.

In figure 6.6 we consider the bandwidth fixed, but now we change the number of frequencies in the hopping pattern instead of the spreading code sequence.

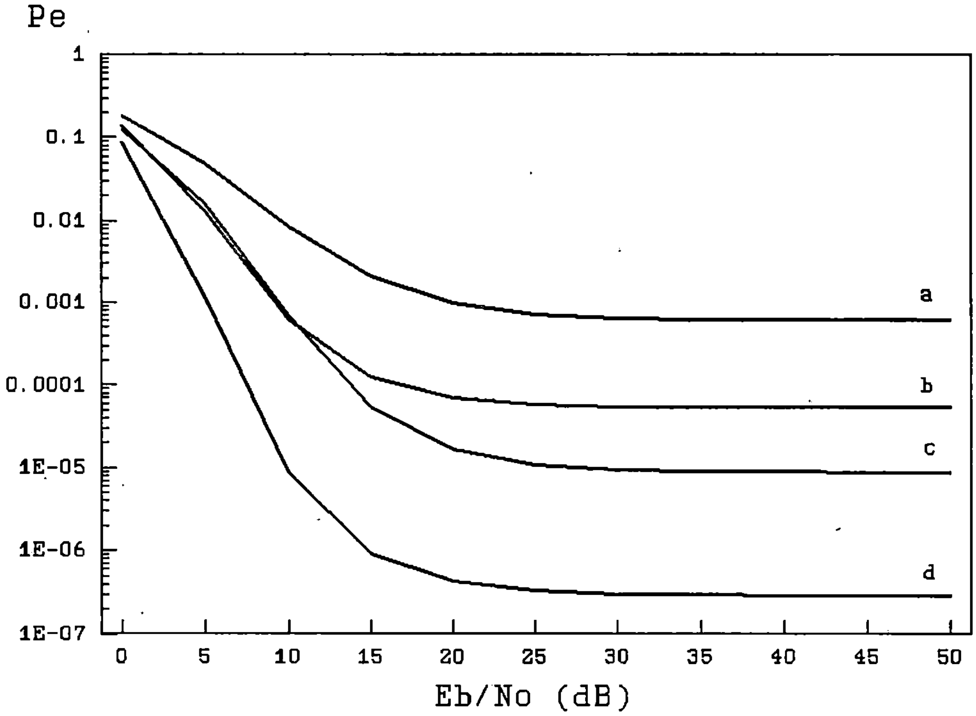

***Figure 6.6:** BER of hybrid DS/SFH in case of SD with and without FEC coding at fixed bandwidth; $N=255$, $R=6{,}8$ dB; $K=15$; $T_m=250$ ns and $R_c=64$ kbit/s.*

a: $M=1$, no coding, $q=20$, $L=5$;	*c: $M=2$; no coding, $q=20$, $L=5$*
b: $M=1$, Golay code, $q=10$, $L=10$	*d: $M=2$, Golay code, $q=10$, $L=10$*

From the figure it is seen that the use of FEC coding enhances the performance. Although the bandwidth is twice the one of figure 6.5, it is obvious that with a constant spreading code period $N=255$, a number of frequencies $q=10$ and Golay coding the performance is better than the situation without coding.

Despite the relative high number of resolvable paths in the case of coding, the sufficiently long spreading code, the large number of frequencies and the error capabilities of the FEC code in combination makes it possible to enhance the performance.

6.5 Comparison of the performance of the hybrid DS/SFH systems with QPSK, BPSK and DPSK modulation.

In the previous sections we have presented the numerical results of the performance of the hybrid DS/SFH system with DPSK as modulation scheme. In order to get a good idea about the performance of DPSK compared to the performance of other modulation schemes, we will compare the BER of DPSK with the BER of BPSK and QPSK. The results of the performance of the hybrid system, in terms of average BER have been presented in [4].

As far as the comparison of QPSK and BPSK is concerned, some constraints have to be taken into account.

If we require that the bit rate in both systems should be the same, we will have $T_q = 2T_b$, where T_q and T_b are the bit durations for QPSK and BPSK respectively. In addition, we require that the bandwidths should be the same, which means that the ratio Nq/T should be the same for both systems. This leads to the following requirement:

$$\frac{N_b\, q_b}{T_b} = \frac{N_q\, q_q}{T_q} \tag{6.1}$$

Considering the bit rate constraints, we have: $N_q\, q_q = 2N_b\, q_b$. If we finally assume the same number of frequencies for both systems, we get: $N_q = 2N_b$.

This has consequences for the multipath and multiple access noise. Considering the expressions for these types of noise, given in equations (5.36) and (5.37), we see that the multi-access and multipath noise for BPSK is twice of that of QPSK. This means that, under the condition of constant bandwidth and bit rate, the performance of a QPSK system with a code length of 128 chips is equivalent to the performance of a BPSK system with a code length of 256.

In this comparison, we have taken N_q=255 for QPSK, N_b=127 for BPSK and N=255 for DPSK.

In table 6.4 the BER, according to the three modulation techniques, has been presented. We have compared the performance in case of q=10 and q=50, M=1 and M=3 under the constraints of a fixed bandwidth and bit rate. Both SD and MRC have been considered.

Table 6.4: BER comparison of three modulation schemes; E_b/N_o=25 dB; L=8; K=15.

Diversity	M	Q	QPSK	BPSK	DPSK
SD	1	10	2.10^{-3}	6.10^{-3}	$4{,}3.10^{-2}$
SD	3	10	7.10^{-6}	2.10^{-4}	$1{,}2.10^{-2}$
SD	1	50	1.10^{-3}	3.10^{-3}	$6{,}7.10^{-3}$
SD	3	50	3.10^{-7}	8.10^{-6}	$7{,}5.10^{-4}$
MRC	1	10	2.10^{-3}	6.10^{-3}	$4{,}3.10^{-2}$
MRC	3	10	2.10^{-7}	6.10^{-6}	$4{,}0.10^{-3}$
MRC	1	50	1.10^{-3}	3.10^{-3}	$6{,}9.10^{-4}$
MRC	3	50	5.10^{-9}	3.10^{-7}	$1{,}7.10^{-4}$

From this table it is seen that systems using DPSK, perform poorer than systems using BPSK or QPSK. This is due to the fact that DPSK is assumed to be detected noncoherently at the receiver, as opposed to BPSK and QPSK, which are assumed to be detected coherently at the receiver. In the DPSK demodulator the noise variance is twice as large as in case of BPSK or QPSK. Besides, under the constraint of fixed bandwidth and data rate the system with QPSK yields a better performance than the one with BPSK.

This is due to the fact that the multi-user and multipath interference of the BPSK system is twice the interference of the QPSK system, according to equations (5.36) and (5.37). This is the consequence of the condition of the signature sequences: $N_q = 2N_b$, in order to have the same bandwidth.

Without the bandwidth and bit rate constraint, BPSK and QPSK should give approximately the same performance [18].

6.6 Comparison of two models of hybrid DS/SFH

Also the comparison of the model used in this performance analysis with other models described in literature is valuable. Therefore we will give a comparison of the performance of our model, denoted as model 1, with the performance of another model described in literature [2], denoted as model 2.

The model, given in [2] also describes the performance of a hybrid DS/SFH system with DPSK modulation over Rician fading channels. However, in order to simplify the calculations in model 2, random spreading code sequences were used. This means that for each user k, the set of spreading codes $(a_l^{(k)})$ consists of a sequence of mutually independent random variables, taking values {+1, -1} with equal probability. This means that the signature sequences, assigned to the different users, are mutually independent [9]. Especially for the necessary averaging over the path delays, this provides some extreme simplifications.

The model presented in this report takes model 2 as a starting point, but does not use random signature sequences. Instead, deterministic spreading sequences are used, which requires some extended calculations with regard to the averaging over the path delays.

In table 6.5 the BER of the two models are presented. The only difference in the comparison is that model 1 uses a spreading code period of 127 and model 2 uses a code period of 63.

Table 6.5: *Comparison of the BER of two models of hybrid DS/SFH with MRC: $L=4$; $K=10$; $Q=30$; $R=3$ dB; $E_b/N_o= 30$ dB.*

Order of diversity	model 1 with N=127	model 2 with N=63
M=1	$1{,}1.10^{-2}$	1.10^{-2}
M=2	$1{,}4.10^{-3}$	8.10^{-4}
M=3	$2{,}3.10^{-4}$	6.10^{-5}
M=4	$4{,}4.10^{-5}$	6.10^{-6}

From the table it is seen that model 2 yields a lower BER than model 1, despite the shorter signature sequence. The difference between the bit error rates becomes smaller for higher orders of diversity.

On first sight we would expect that the system with code length N=127 would yield a better result than the system with code length N=63. However, the lower BER obtained with model 2 is due the use of random spreading code sequences. The expectation of the correlation functions of these random sequences with respect to the delay τ, yields a different result than the expectation of the correlation functions of the deterministic signature sequences. As shown in section B.4 of appendix B, the expectations of the correlation functions of the deterministic signature sequences are quite extensive.

7. THROUGHPUT AND DELAY ANALYSIS OF A HYBRID DS/SFH SYSTEM WITH DPSK

In this chapter we discuss the performance of a slotted CDMA network, employing hybrid DS/SFH, in terms of throughput and delay. In order to give a good description of the network features of this CDMA communication system, a lot of phenomena has to be taken into account. From a mathematical viewpoint the modelling of such systems is quite complex.

Our purpose, however, is to get a first impression about the system behaviour. Therefore we will use a relatively simple model, which can be set up only when several requirements are met and some assumptions are made.

We will start the discussion with a description of the network model, with the assumptions and requirements. Second, the packet flow and arrival model is described. After that, derivations of the expressions for the packet success probability, throughput and the delay will be given.

7.1 Description of the network model

For our analysis we consider a centralized network, in which the various users communicate with each other by a coordinating base station. The base station consists of a bank of spread spectrum transceivers; each transceiver at the base station can communicate with one specific user (as shown in figure 1.1). Each user has his own spread spectrum code and hopping pattern. We consider a fixed frequency band, which is determined by the spreading code length and the number of frequencies in the hopping pattern. This network has the following characteristics:

- Data is transmitted in packets of N_p bits;
- The system is slotted;
- The time slot is equal to packet duration;
- The system has a positive acknowledgement scheme;

The following key requirements have to be met in order to be able to derive the appropriate expressions:

- All users are identical, i.e. each user face the same probabilistic circumstances in the channel; the same holds for the transceivers at the base station (symmetry condition);
- The average received power at the base station is equal for all users;
- The acknowledgements are almost free from a capacity viewpoint, fully reliable and instantaneous;
- The channel is memoryless, i.e. the transmitted packet experiences a channel in which the number of active users is uncorrelated with the users in the prior attempt.
- The system is in a stable state.

In CDMA systems based on DS and SFH there is always some interference among simultaneously transmitting users, which is mainly due to two causes:

First, practical codes don't have ideal cross- and/or autocorrelation properties. This is called cross correlation multiple access interference.

Second, two users might accidentally use the same carrier frequency, which causes errors in the transmitted bits. Therefore in the CDMA system there is a pronounced threshold, where the system performance degradation as a function of the number of users, ceases to be gradual and performance degrades rapidly. The point of destructive interference or contention is called the CDMA threshold. This threshold is therefore defined as the number of users that can transmit simultaneously within a shared frequency band without destructive interference occurring. We assume the system degradation to be a step function located at the system capacity of C users.

We assume the number of users transmitting data packets simultaneously within a specific time slot in slotted CDMA is k. In a CDMA system fatal collisions occur and all packets are destroyed as soon as the number of active users rises above the threshold capacity C. The packets are lost due to excessive bit errors when the number of simultaneous transmitting users exceeds C.

7.2 Description of the packet flow and arrival model

The arrival model describes the packet traffic resulting from C users which attempt to transmit during a time slot. The model is shown in figure 7.1.

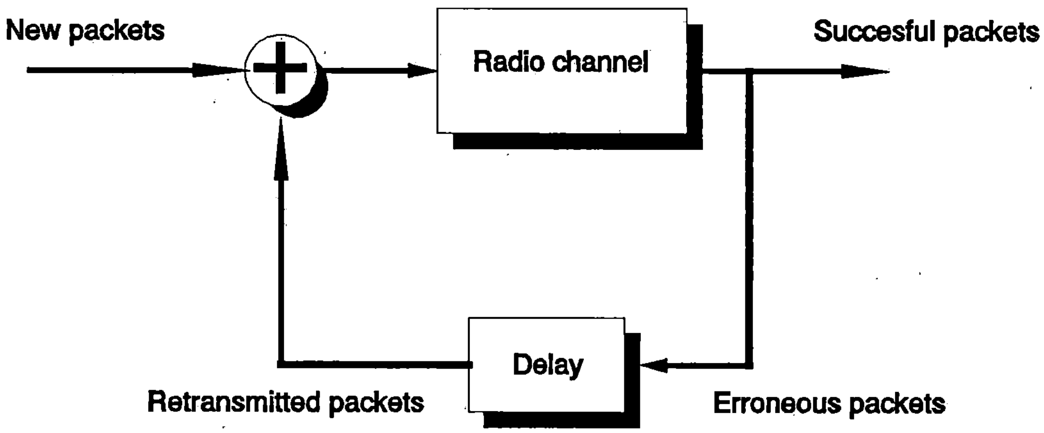

Figure 7.1: *The packet flow model*

The number of new transmission attempts is called the channel input. If a transmission attempt is not successful (no acknowledgement received) a retransmission is attempted after a random delay. The total number of transmission attempts (number of active users), denoted as k therefore consists of new and retransmission attempts. Under the condition of a finite number, identical and statistically independent users, we can describe the distribution of k by a binomial distribution. The probability that a user has a packet (new or retransmission packet) is denoted as P_{nt}. The probability P that k packets are generated during a time slot is then given by [19] as:

$$P_k = \binom{C}{k} \left[P_{nt}\right]^k \left[1-P_{nt}\right]^{C-k} \tag{7.1}$$

The offered traffic G, which is defined as the average number of transmissions (new packets plus retransmitted packets) per time slot by k users, is given by:

$$G = E(k) = C.P_{nt} \tag{7.2}$$

So we can write P_{nt} as:

$$P_{nt} = \frac{G}{C} \tag{7.3}$$

The basic limitation of this arrival model is the demand for a stable system, i.e. the time between generation of packets by a user (inter-arrival time) is larger the time needed for the successful transmission of a packet (average packet delay).

7.3 Derivation of the packet success probability

The event of a "packet success" can be interpreted in different ways [5]. We can regard the success probabilities from the transmitter viewpoint as well as from the receiver viewpoint and in general, these probabilities won't be the same. Besides, the success probabilities of the different users are in general not the same as well. All of these complications make the mathematical description of the packet success probability quite complex. In order to get a first impression of the system behaviour and to be able to derive a simplified expression for the packet success probability, we make the following three assumptions:

- The different transmitter-receiver pairs are assumed to be statistically independent;
- The transmitter success is equal to the receiver success, which is valid under the forementioned symmetry condition (page 64);
- Each transmitter has the same probability of success.

The probability of success from a particular transmitter's viewpoint, given another (k - 1) other packets in the channel is denoted as $P_k(k)$. We can interpret the success probability as the probability that the packet is received without any bit errors. Under the conditions mentioned above, we can consider the number of channel successes S, given k packets in a time slot, to be binomially distributed. The expression for this distribution then becomes:

$$P(S{=}s\,|\,k) = \binom{k}{s} \left[P_k(k)\right]^s \left[1 - P_k(k)\right]^{k-s} \qquad 0 < s < k \tag{7.4}$$

The conditioning on k can be removed with help of Bayes' theorem, which yield the expression for the probability of s successful packets.

$$P(S{=}s) = \sum_{k=0}^{C} P(S{=}s\,|\,k)P_k \tag{7.5}$$

It should be kept in mind that so far we didn't take the channel statistics into account. The effect of the channel will be discussed in the next sections.

7.3.1 Packet success probability in case of a slow Rician fading channel

With the bit error calculations, we have assumed the channel to be of the slow Rician fading type. With this channel model we assume that the channel parameters do not change considerably for the duration of one packet. This implies that all bits in a packet are received with almost the same average power.

In order to incorporate the channel statistics we have to reconsider the probability of success of one packet given $(k - 1)$ other packets $P_k(k)$. It is necessary now to condition this probability on the random parameters of the channel, i.e. β_{max}, μ_0 and μ_{-1}.

$$P_k\left(k\,|\,\beta_{max}, \mu_0, \mu_{-1}, \mu\right) = \left[1 - P_e\left(\,|\,\beta_{max}, \mu_0, \mu_{-1}, \mu, k\right)\right]^{N_b} \tag{7.6}$$

where the error probability is given by equation (5.4) with n_i replaced by k and τ_{kl} by the μ-parameters. The conditioning on the channel statistics can be removed the same way as with the determination of the average bit error probability given in equation (5.14). So, the expression for the probability of one successful packet given k simultaneous transmitting users $P_k'(k)$, is given as:

$$P_k^{/}(k)=\int\limits_{\mu_{-1}}\int\limits_{\mu_0}\int\limits_{\mu}\int\limits_{\beta_{max}}\left[1-P_e\left(\ |\beta_{max},\mu_0,\mu_{-1},\mu\ ,k\right)\right]^{N_b} f(\beta_{max})f(\mu)f(\mu_o)f(\mu_{-1})d\beta_{max}d\mu d\mu_0 d\mu_{-1} \quad (7.7)$$

where f() represents the PDF of the random variables. The packet success probability for a fast Rician fading channel will be discussed in the next section.

7.3.2 Packet success probability in case of a fast Rician fading channel

In case of fast fading, the channel variations are fast relative to the signalling interval. Thus each signalling symbol is affected by fading independently from the other symbols. Therefore, if we send data in packets over a fast fading channel, all data bits undergo the fading independently.

This means that the packet success probability doesn't have to be averaged over the path gain any more; the probability is just given by:

$$P_k^{//} = \left[1 - P_e(k)\right]^{N_b} \quad (7.8)$$

in which $P_e(k)$ is the average bit error probability given by equation (5.14). After the determination of the packet success probability, we now are able to derive an expression for the throughput, which will be done in the next section.

7.4 Derivation of the throughput and delay

We define the throughput as the number of successfully received packets per time slot. In this analysis we will normalize the throughput on the system capacity C, in order to be able to compare the results obtained here, with results from literature [19]. Assume k transmission attempts during a time slot, then the normalized throughput is given as the expected number of successes per time slot:

$$S_n = \frac{1}{C} E[S] = \frac{1}{C} \sum_{k=1}^{C} s.P(S=s) \quad (7.9)$$

Substitution of equation (7.5) and (7.7) in (7.9) then yields:

$$S_n = \frac{1}{C} \sum_{s=1}^{C} s \sum_{k=1}^{C} P(S{=}s\,|\,k)P_k \tag{7.10}$$

When we exchange the summation and rearrange the expression, we get:

$$S_n = \frac{1}{C} \sum_{k=1}^{C} P_k \sum_{s=1}^{C} s.P(S{=}s\,|\,k) \tag{7.11}$$

The second summation yields the conditional expectation of s given k and equals the conditional expectation belonging to the binomial distribution of equation (7.4), which is given by $E[s\,|\,k] = k\ P_k'(k)$. In which $P_k'(k)$ is given by equation (7.7) in case of slow fading. So, we finally get the expression for the normalized throughput for a slow Rician fading channel:

$$S_n = \frac{1}{C} \sum_{k=1}^{C} kP_kP_k'(k) \qquad [\,packets\ /\ time\ slot\,] \tag{7.12}$$

In should be mentioned that in //case of fast Rician fading the packet success probability in equation (7.12) must be replaced by equation (7.8).

The average delay D of the system is defined as the average number of packet slots between the generation and successful reception of a packet.

A new packet will be generated at a random time within a time slot. The average time between the generation of a packet and the start of the next time slot is $\frac{1}{2}T_p$, with T_p being the duration of a packet. The packet is transmitted in the first time slot after the slot in which it was generated. When the packet is received successfully, the total delay between generation and successful reception is equal to $1{,}5T_p$, in which we have neglected the propagation delay. This is reasonable in an indoor environment. The receiver becomes aware of an unsuccessful transmission after waiting T_a seconds, in which T_a is the duration of an acknowledgement packet. The transmitter will wait T_r seconds until the start of the next time slot and will transmit the packet in a second attempt. T_r is called the retransmission delay. The delay is shown in a timing diagram, which is given in figure 7.2.

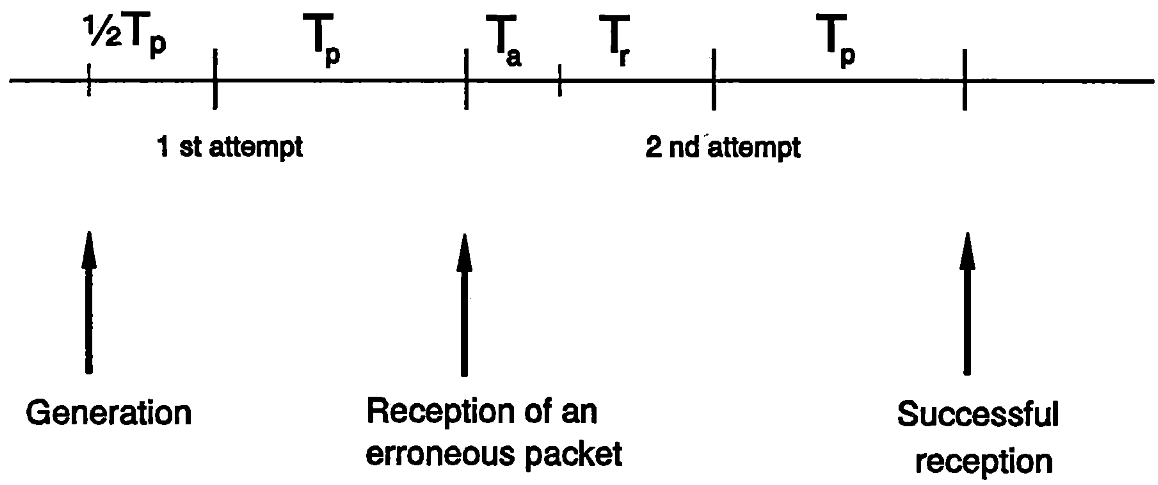

Figure 7.2: *Timing diagram illustrating the packet delay*

We can define the probability of success per transmitted packet as:

$$P(suc.\ per\ packet) = \frac{C.S_n}{G} \tag{7.13}$$

Then we can give the average delay as:

$$D = 1{,}5T_p + \left[\frac{G}{C.S_n} - 1\right]\left(T_p + T_a + T_r\right) \tag{7.14}$$

in which $(G/CS_n - 1)$ is the average number of retransmissions needed for a packet to be received successfully.

8. NUMERICAL RESULTS FOR THE PERFORMANCE

In this chapter we present the numerical results of the throughput and delay analysis of the CDMA network based on hybrid DS/SFH with selection diversity. The normalized throughput will be given as a function of the offered traffic G. We have investigated the performance at a signal to white noise ratio of 20 dB. Further, it is assumed that the CDMA threshold is 30 active users. The indoor radio channel is assumed to be of the slow Rician fading type.

In the first section we will consider the throughput for different parameter settings without FEC coding. The second section gives the effect of FEC coding on the throughput. The third and fourth section give the delay results for a system without and a system with FEC coding respectively.

8.1 Throughput performance without coding

In figure 8.1 the influence of the order of diversity M for a packet size of 128 bits is presented. It is seen that the throughput of the system increases with an increase of the order of diversity. This is due to the fact that with an increase of M the BER decreases. This results in a larger packet success probability, according to equation (7.6) and (7.7).

Further, it is obvious that the maximum of the throughput shifts to the right as the order of diversity increases.

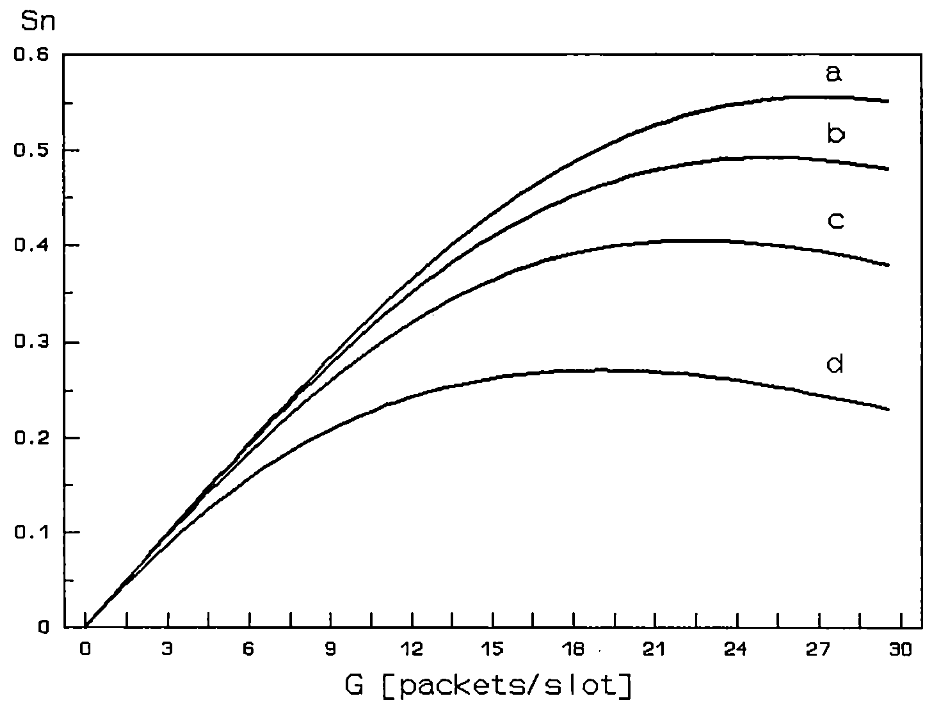

Figure 8.1: *Effect of the order of diversity on the throughput with a packet size of 128 in case of SD; Q=10; N=255; T_m=100 ns; L=4; R=6,8 dB*

a: M=4; *b: M=3;* *c: M=2;* *d: M=1*

The effect of the order of diversity on the throughput with a packet size of 1024 bits is given in figure 8.2. It is seen that the throughput still increases with an increase of the order of diversity, however the maximal throughput in case of M=4 is almost half the maximal throughput in case of a packet size of 128 bits. This is due to the fact that the increased packet size decreases the packet success probability, according to equation (7.6).

However, an increase of the order of diversity yield a smaller BER. So, in order to obtain a reasonable throughput of large packets, one should use a receiver with a sufficient large order of diversity.

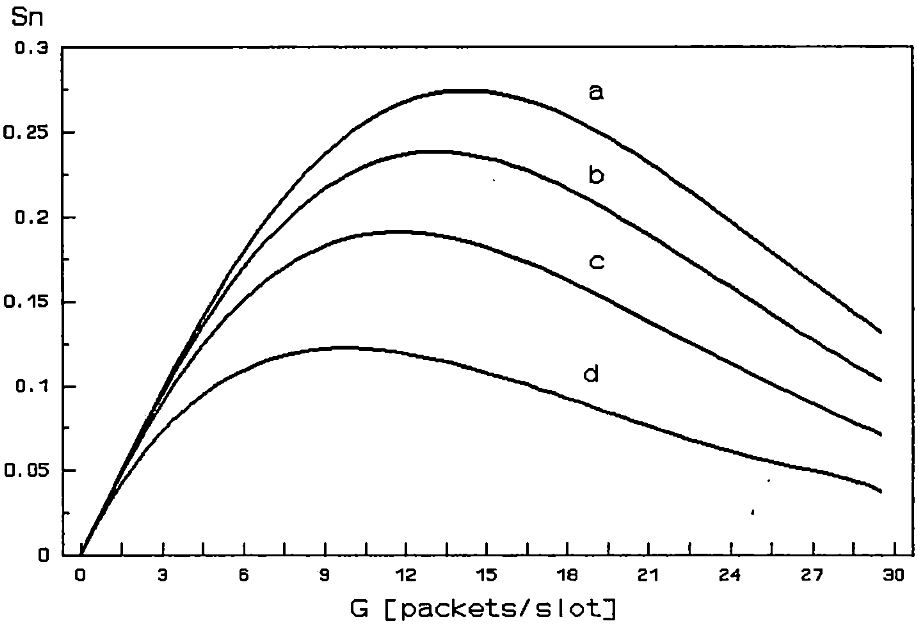

Figure 8.2: *Effect of the order of diversity on the throughput with a packet size 1024 in case of SD;* E_b/N_o*=20 dB; Q=10; L=4;* T_m*=100 ns; N=255; R=6,8 dB*

a: M=4; *b: M=3;* *c: M=2;* *d: M=1.*

In figure 8.3 the effect of the packet size on the throughput is presented. We have taken the order of diversity M equal to two. The plots show that the throughput decreases with an increase of the packet size. Only with packet sizes of 64 bits and 128 bits a reasonable value of the maximum throughput is achieved.

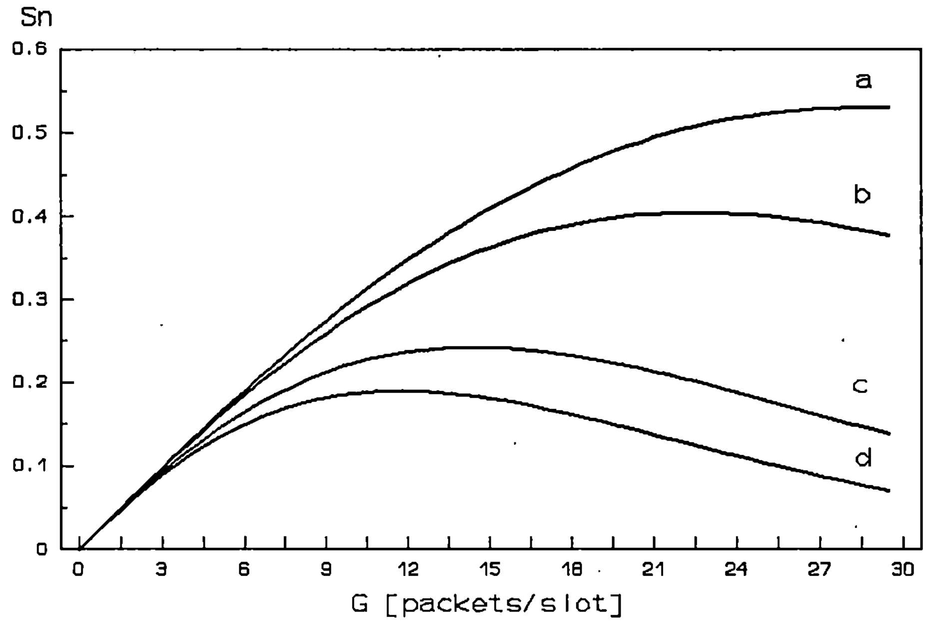

Figure 8.3: *Influence of the packet size on the throughput in case of SD; $M=2$; $Q=10$; $N=255$; $L=4$; $T_m=100$ ns; a: $N_p=64$; b: $N_p=128$; c: $N_p=512$; d: $N_p=1024$.*

In figure 8.4 we have considered the throughput performance for different parameter settings with the constraint of a fixed bandwidth and a fixed data rate. The plots show a large range in which the maximum throughput varies. It is seen that in general the throughput decreases with an increase of the number of resolvable paths in combination with a small spreading code period *N*. Even the relatively small packet size of 42 bits does not give a reasonable value of the throughput when L is relatively large.

The cause for the poor performance is the increased level of multipath interference when *L* increases. This causes the BER to increase, which yield a smaller packet success probability. When the period of the spreading codes decreases, the multi-user interference increases, which also causes a higher BER.

The increase of the number of frequencies in the hopping pattern, is not capable to diminish the effect of increasing number of resolvable paths and decreasing the spreading code period. It is obvious that an appropriate choice of the parameters, a reasonable value of the throughput can be obtained.

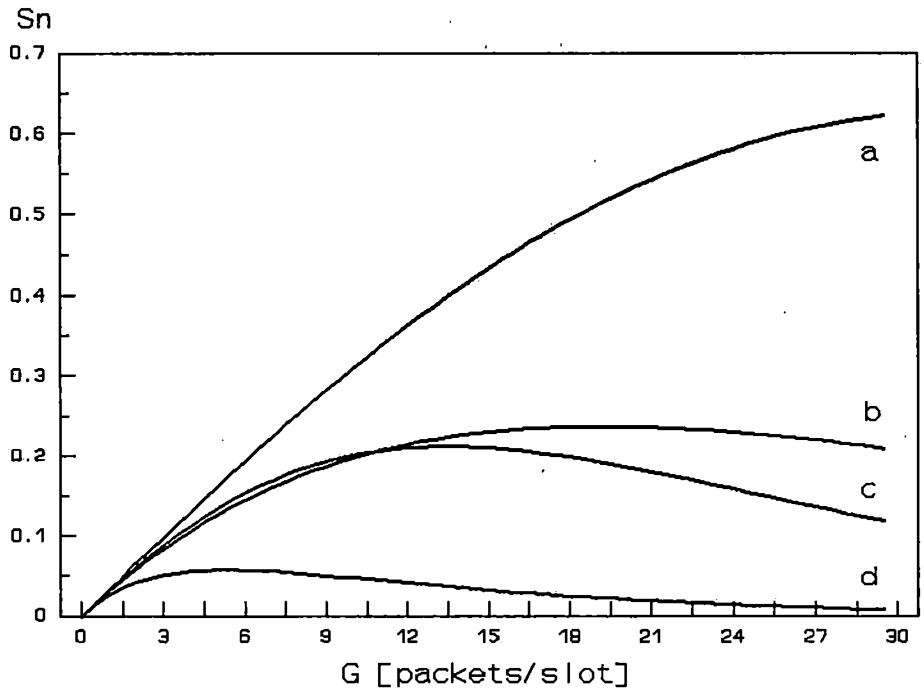

figure 8.4: *Effect of the different parameter settings on the throughput for a packet size of 42 bits given a fixed bandwidth; $M=2$; $R=6{,}8$ dB;*

a: $N=255$; $Q=10$; $L=4$; $T_m=100$ ns c: $N=255$; $Q=10$; $L=10$; $T_m=250$ ns
b: $N=127$; $Q=20$; $L=2$; $T_m=100$ ns d: $N=127$; $Q=20$; $L=5$; $T_m=250$ ns.

8.2 The influence of FEC coding on the throughput

Figure 8.5 shows the comparison of the throughput with a packet size of 128 bits for systems with and without coding. It is seen that nondiversity without coding yields the poorest result, with respect to the throughput. The best result is obtained with high diversity order and Golay coding.

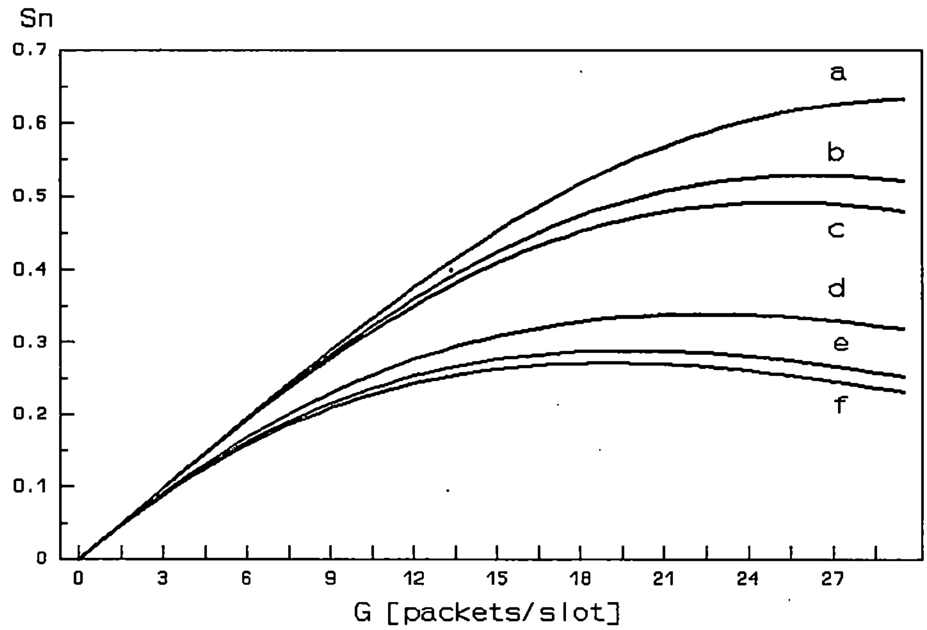

Figure 8.5: *Comparison of the throughput with and without FEC coding for a packet size of 128 bits; T_m=100 ns; N=255; Q=10; R=6,8 dB;*

a: M=3, Golay code, L=8

b: M=3, Hamming code, L=8

c: M=3, no coding, L=4

d: M=1, Golay code, L=8

e: M=1, Hamming code, L=8

f: M=1 no coding, L=4

Figure 8.6 shows the effect of FEC coding on the throughput in case of a packet size of 1024 bits. It is seen that nondiversity with and without coding yield a relatively poor result. Only diversity of order 3 with Golay coding yields a reasonable throughput.

In general, we can say that in order to obtain an acceptable performance for large packet sizes, we have to use a sufficient large order of diversity and FEC coding.

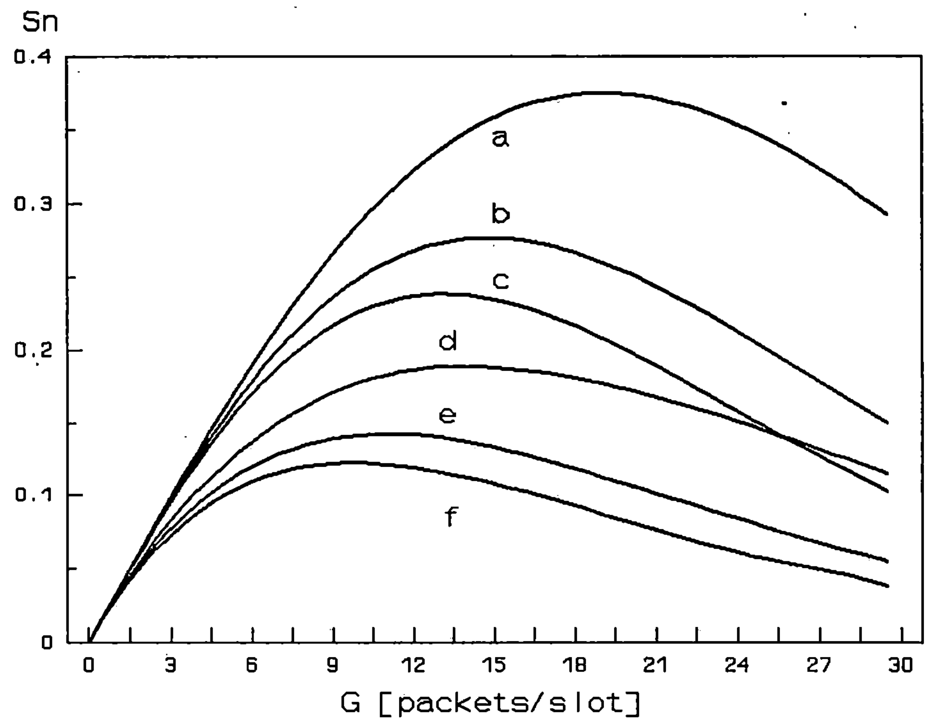

Figure 8.6: *Comparison of the throughput with and without FEC coding for a packet size of 1024 bits; E_b/N_0=20 dB; N=255; Q=10; R=6,8 dB*

a: M=3, Golay code, L=8	*d: M=1, Golay code, L=8*
b: M=3, Hamming code, L=8	*e: M=1, Hamming code, L=8*
c: M=3 no coding, L=4	*f: M=1, no coding, L=4*

8.3 The delay performance without coding

We have considered the delay as a function of the offered traffic. Further, we have taken the retransmission delay T_r equal to one. We will assume the same parameter settings as with the corresponding plots of the throughput.

Figure 8.7 presents the effect of the order of diversity on the delay for a packet size with the same parameters as in figure 8.1. It is seen that a high order of diversity causes a relative low delay in the system. Systems without diversity (M=1), however, yield relative large delay in the system.

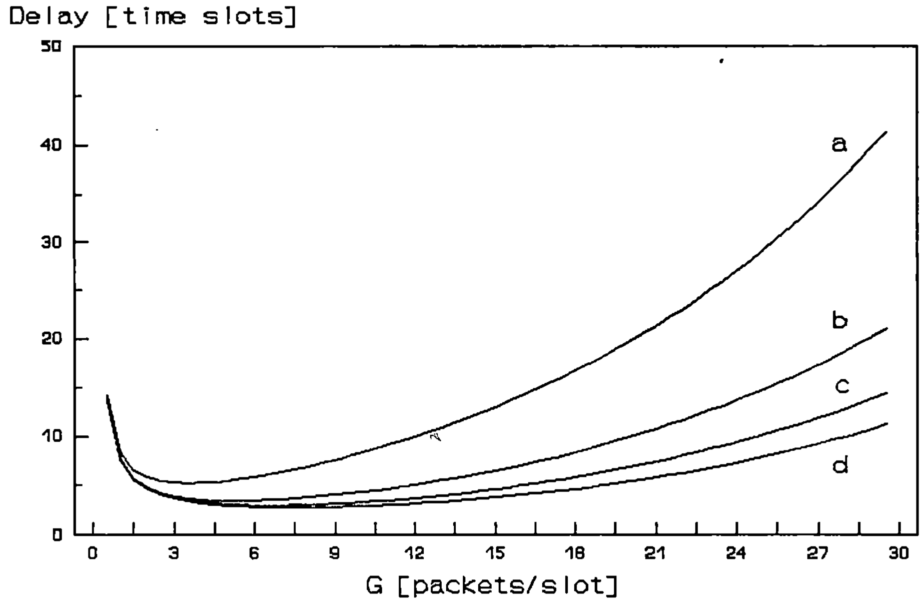

Figure 8.7: *Effect of the order of diversity on the delay with a packet size of 128 bits;*
a: M=1; b: M=2; c: M=3; d: M=4

In figure 8.8 the effect of the order of diversity on the delay is shown, but now for a packet size equal to 1024 bits. The parameter setting is equal to the one in figure 8.2. It is obvious that large packet sizes in combination with a low order of diversity yield a considerable delay in the system. Only a high order of diversity gives a reasonable value of the delay.

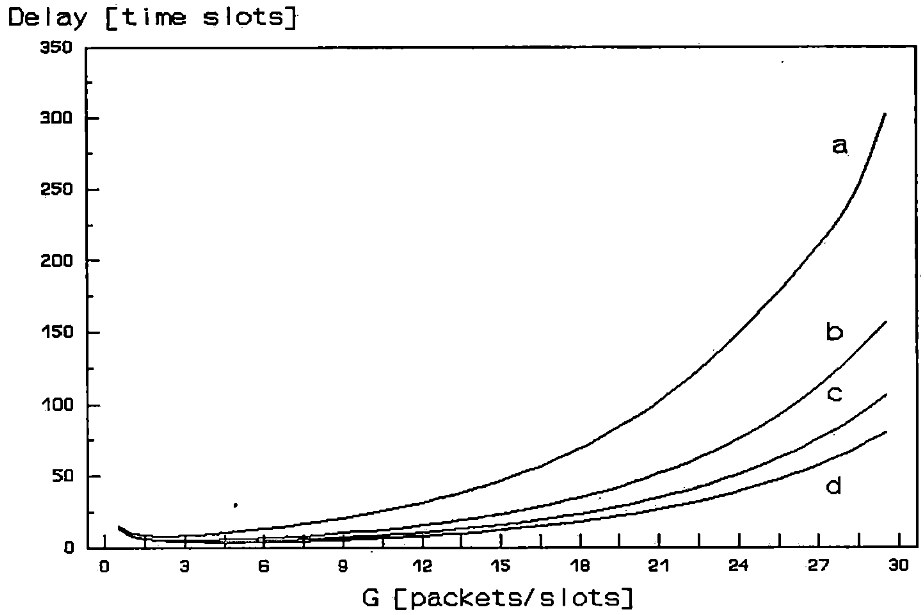

Figure 8.8: *Effect of the order of diversity on the delay with a packet size of 1024 bits;*
a: M=1; b: M=2; c: M=3; d: M=4

Figure 8.9 shows the effect of the packet size on the delay with the same parameters as in figure 8.3. The plots show that large packet sizes cause considerable delays in the system. Only for packet sizes equal to 128 bits or 64 bits we obtain reasonable values of the delay.

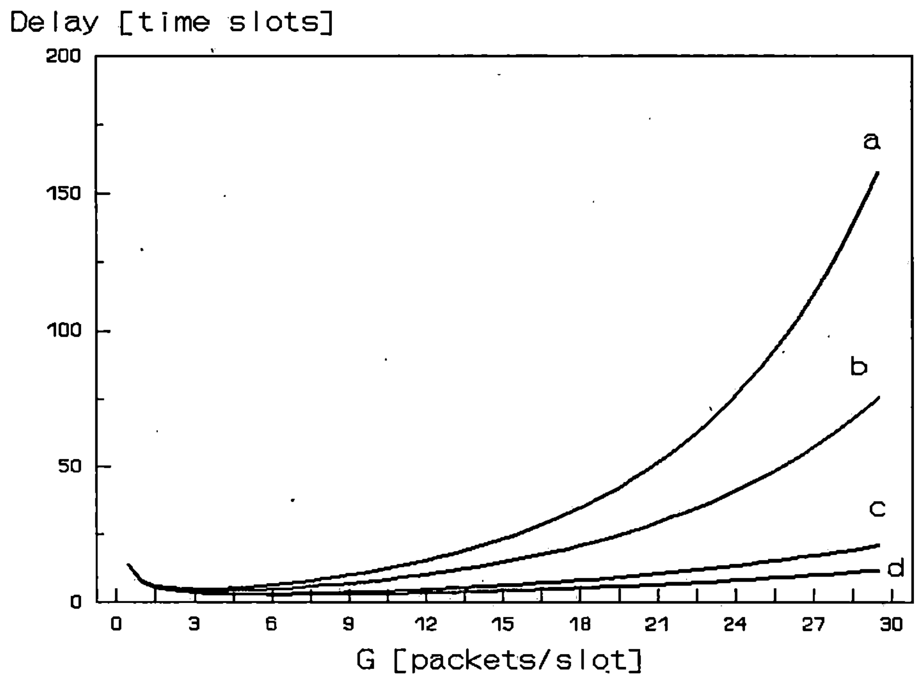

Figure 8.9: *Effect of the packet size on the delay;*
a: N_b=1024; b: N_b=512; c: N_b=128; d: N_b=64.

8.4 Effect of FEC coding on the delay performance

Figure 8.10 presents the comparison of the delay of systems using FEC coding and the delay of systems without coding for a packet size of 128 bits. The parameters are the same as those of figure 8.5. It is seen that FEC coding decreases the delay in the system considerably.

Golay coding in combination with an order of diversity of three, yields the lowest delay for this packet size. Nondiversity without coding yields the poorest result for this relatively small packet size.

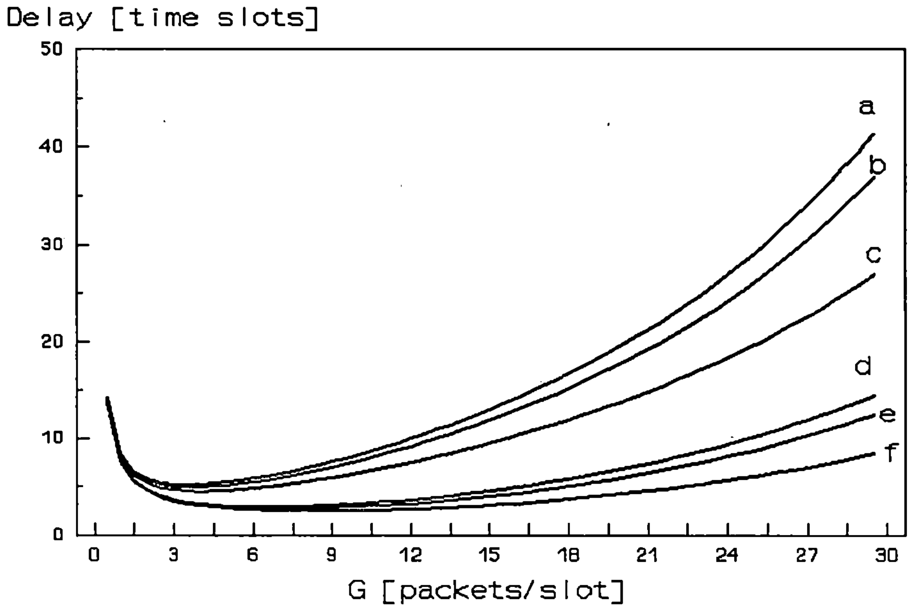

Figure 8.10: *Effect of FEC coding on the delay for a packet size of 128 bits;*

a: M=1 no coding;	*d: M=3 no coding;*
b: M=1 Hamming code;	*e: M=3 Hamming code;*
c: M=1 Golay code;	*f: M=3 Golay code.*

Figure 8.11 presents the effect of FEC coding for a packet size of 1024 bits, with the same parameter setting as figure 8.6. It is seen that FEC coding with diversity yields a relatively lower delay than FEC coding without diversity. However, the Golay code with M=1 almost yields the same delay as a system with M=3 without coding.

In general we see that in case of large packet sizes FEC coding and diversity are required to obtain a sufficiently low delay in the system

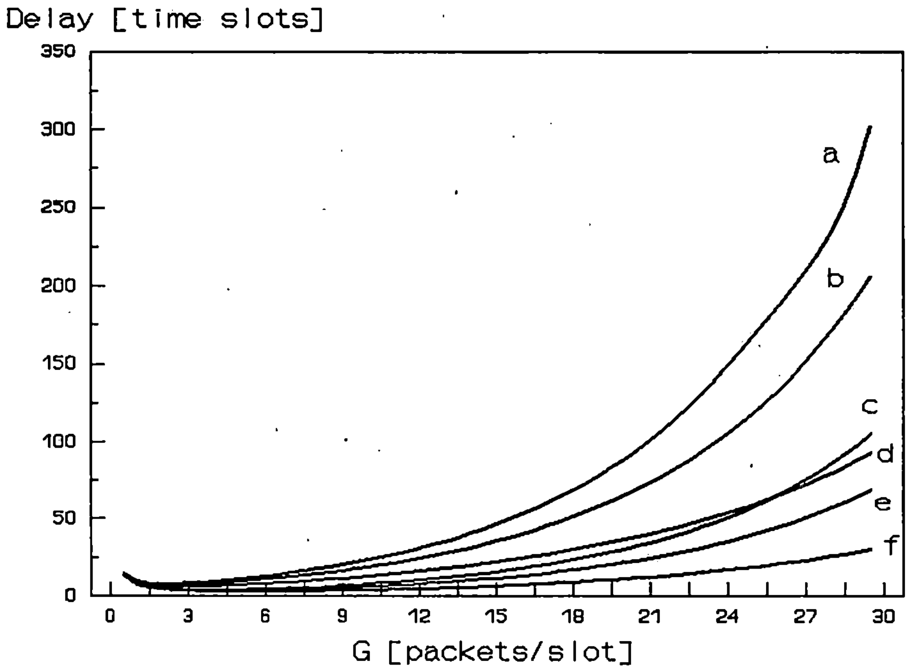

Figure 8.11: *Effect of FEC coding on the delay for a packet size of 1024;*

a: M=1 no coding;	*d: M=3 no coding;*
b: M=1 Hamming code;	*e: M=3 Hamming code;*
c: M=1 Golay code;	*f: M=3 Golay code.*

8.5 Comparison of the throughput of hybrid DS/SFH and pure DS over a fast Rician fading radio channel

In [19] the results of the throughput of a slotted CDMA network using DPSK modulation in a micro-cellular mobile radio environment have been presented. The network consists of transmitter-receiver combinations, which operate with direct sequence as spread spectrum technique.

In case of micro-cells, which have a size of 0,4 - 2 km diameter, the radio channel can be very well characterized by a Rician fading channel [21]. The larger size of micro-cells in comparison with pico-cells, result in a larger value for the rms delay spread (e.g. 2 μs for micro-cells). As a consequence, this will yield a larger number of resolvable paths.

In table 8.1 we will present a comparison of the throughput of a CDMA system based on pure DS and a system based on hybrid DS/SFH, using selection diversity. We have done the comparison in a single micro-cell environment; the single cell means, that we didn't consider the interference of adjacent cells. The results for pure DS are obtained from [19].

The radio channel is considered to be of the fast Rician fading type, with a Rice factor of R=12 dB. This relatively high value of the Rice factor means a relative strong LOS component. Further, we consider the rms delay spread and the data rate to be constant for both systems, which means that the number of resolvable paths is only determined by the spreading code period N, according to equation (3.5). The bandwidth, which is proportional with $R_c.N.q$, is used as parameter.

Table 8.1: *Comparison of the throughput of a CDMA system with selection diversity based on pure DS and on hybrid DS/SFH with the bandwidth as parameter;* $M=2$, $R=12$ *dB,* $N_p=42$, $C=30$, $R_c=32$ *kbit/s,* $T_m=1961$ *ns.*

	G=5	G=10	G=15	G=20	G=25	$S_{n(max)}$	$G(S_{n(max)})$
DS N=255 L=16	0,16	0,23	0,11	0,025	0,005	0,24	9
Hybr. DS/SFH N=127 q=2 L=8	0,17	0,27	0,14	0,030	0,020	0,27	10
Hybr. DS/SFH N=127 q=10 L=8	0,15	0,29	0,43	0,58	0,74	0,74	25
Hybr. DS/SFH N=255 q=2 L=16	0,17	0,32	0,31	0,20	0,17	0,34	12
Hybr. DS/SFH N=255 q=5 L=16	0,17	0,33	0,49	0,65	0,81	0,81	25

For the combination N=255 for DS and N=127 with q=2 for hybrid DS/SFH, which means the same bandwidth, we see that the performance of the hybrid system is slightly better than the one for pure DS.

The number of resolvable paths in case of DS is twice the number in case of hybrid DS/SFH, which means that the multi-user interference in case of DS is higher than case the hybrid case. Despite the larger code length in case of DS, the relatively low number of frequencies in the hybrid system is able to overcome the poorer correlation properties of N=127.

The combination N=255 with q=2 for the hybrid system gives the performance for twice the bandwidth of pure DS. It is seen from table 8.1 that the maximum throughput $S_{n(max)}$ for the hybrid DS/SFH system is less than twice the maximum throughput of the DS system. An increase in bandwidth by a factor 5, through the combination N=255 with q=5 or N=127 with q=10, only gives an improvement in maximum throughput by a factor three. The first combination yields the best performance.

The throughput in fast fading channels is lower than in slow fading channels. However, a relative large Rice factor improves the throughput performance in these fast fading channels [19]. In order to obtain a reasonable throughput performance (e.g. $S_{n(max)}$ > 0,40), a hybrid DS/SFH with a sufficient number of frequencies is preferred over a pure DS system. However, this requires a larger bandwidth for the hybrid system than pure DS.

9. CONCLUSIONS

We have theoretically and numerically assessed the performance of a hybrid DS\SFH communication system with DPSK modulation for selection diversity and maximal ratio combining and a CDMA network based on hybrid DS/SFH transceivers with DPSK modulation. The numerical results have been obtained with help of a theoretical mathematical models.

With the BER results for the hybrid DS/SFH system with DPSK, we have made a comparison with the performance of a hybrid system based on BPSK and QPSK modulation. Besides, we have made a comparison of the BER performance hybrid model with DPSK presented in this report, and a model described in literature.

Furthermore, the throughput of a CDMA network based on hybrid DS/SFH has been compared with the throughput of a CDMA system based on pure DS, in a fast Rician fading radio channel. From these results we draw the following conclusions.

1. The modelling of multipath and multi-user interference is very complicated and an approximation by random Gaussian variables is allowed only when some key requirements are met.

2. A comparison of two models of the hybrid DS/SFH system shows that the model described in literature yiels a lower bit error probability than the model described in this report. This is due to the use of random signature sequences in the model described in literature, in contrast with the model of this report, which uses deterministic signature sequences.

3. The average BER performance of systems employing MRC is better than the BER performance of employing SD for the same order of diversity. The advantage of MRC over SD increases with an increasing order of diversity.

4. Under the constraint of a fixed bandwidth ($q.N$ is constant) and a given bit rate for the hybrid DS/SFH with DPSK and without coding, the combination of a relatively large signature sequence N and a certain number of frequencies q yields in general a better

performance, both for the BER and for the outage probability, than a small value of N and a large value of q. It is also seen that the performance degrades with an increase in bit rate, due to the high level of multipath interference.

5. A comparison of the BER of the hybrid DS/SFH system with the DS system for the same transmission bandwidths shows that for low bit rates the hybrid system yields a slightly better performance. For higher bit rates the performance is in favour of DS. An increase in bandwidth by a factor 5 for the hybrid system, the performance at low bit rates is in favour of the hybrid system; however, at high bit rates the performance of the hybrid system, in case of low code length N and relative large q, is worse than the performance of pure DS. The combination of large N with a small q for the same higher bit rate, yield a better performance for the hybrid system.

6. The use of FEC coding, under the constraint of a fixed bandwidth, only improves the performance for the combination of large signature sequences in combination with a relatively small number of frequencies, especially for $M > 1$.

7. Comparison of the modulation schemes DPSK, BPSK and QPSK has shown that DPSK yields in general the poorest result, due to the poorer noise resistance at the DPSK demodulator. Under the condition of a fixed bandwidth and data rate, QPSK yields a better performance than BPSK, due to the reduction of the multi-user and multipath interference in case of QPSK.

   Since the indoor channel provides a relatively strong LOS component, which can be used for synchronization at coherent receivers, QPSK or BPSK is preferred over DPSK when relatively low bit error probabilities are necessary.

8. The model which describes the throughput and the delay in this report is only valid under special conditions, concerning the statistical properties of the different users and the receivers at the base station. The model is a simplification of the very complex behaviour of the CDMA network, but it provides a first impression of the throughput and delay performance of the CDMA system.

9. The throughput of a slotted CDMA network based on hybrid DS/SFH communication systems increases for an increasing order of diversity. Besides, the corresponding delay in the system then decreases. The higher order of diversity provides a lower bit error probability and, accordingly, a higher packet success probability.

10. Relatively small packet sizes yield a high throughput and accordingly a low delay in the system. In order to obtain a reasonable value for the throughput and delay in case of large packet sizes, the combination of the Golay code and a sufficiently high order of diversity is necessary.

11. The throughput decreases with an increase in the number of resolvable paths given a spreading code period N under the condition of a fixed bandwidth. So the performance gets worse when either the delay spread or the data rate is increased.

12. A comparison of the throughput of a hybrid DS/SFH system with a CDMA system based on DS for a fast Rician fading channel, shows that the hybrid system yields a slightly higher maximum throughput than pure DS under the condition of a fixed bandwidth. The maximal throughput increases when the transmission bandwidth of the hybrid system is increased, however the throughput doesn't increase proportionally. Further, at larger bandwidths a higher offered traffic can be put through the channel.

13. Although a fast fading channel provides in general a worse throughput than slow fading channels [19], the reasonable throughput performance over the fast fading channel is mainly due to the relatively high Rice factor.

10. RECOMMENDATIONS FOR FURTHER RESEARCH

An important issue at this stage of the investigation of CDMA communication systems is verification of the models that have been used up till now. Computer simulations of these systems will provide an impression of the validity of the models. Together with measurement results, one can get a good idea about the models.

As far as the hybrid DS/SFH system is concerned, the following parts of the model should be verified with help of computer simulations:

- The Gaussian approximation used to describe the multi-user interference;

- The assumption of the uniform delay profile;

- The hit model, which is now modelled as a first-order stationary Markov process;

If the simulations provide total differently results than the results obtained with the models described in literature or in this report, the forementioned parts of the model have to be revised. This will give rise to extra complexities in the derivation of the analytical expressions.

The model which describes the throughput and delay analysis of a CDMA network, employing the hybrid DS/SFH technique given in this report, should also be checked with help of simulations.

If the simulation results show a large difference with the results obtained with the model under consideration, the model should be changed in the following way.

- Consider the event of "success" from the transmitter's viewpoint different from the receiver's point of view, which results in different packet success probabilities;

- Take different probabilities for the packet successes for the different users;

- As a consequence the packet arrival model has to be changed from a binomial model into a higher-order stationary Markov process.

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APPENDIX A: DERIVATION OF THE STATISTICAL MOMENTS

Let V_o and V_{-1} denote the complex envelope of the matched filter output at the current sampling instant and the previous sampling instant respectively.

$$V_o = X_o + jY_o \tag{A.1}$$

$$V_{-1} = X_{-1} + jY_{-1} \tag{A.2}$$

The output of the matched filter calculates the decision variable.

$$\xi = Re\left[V_o V_{-1}^*\right] = X_o X_{-1} + Y_o Y_{-1} \tag{A.3}$$

A.1 COMPUTATION OF μ_o

The definition of μ_o is given here below.

$$\mu_o = VAR\left(V_o \,|\, L,\, \tau_{kl},\, n_i,\, b\right) = E\left[(V_o - m)(V_o - m)^* \,|\, L,\, \tau_{kl},\, n_i,\, b\right] \tag{A.4}$$

If we consider $X_{o,c}$ and $Y_{o,c}$ as the centered versions of X_o and Y_o where the noise terms have been dropped, we can write:

$$\mu_o = E\left[X_{o,c}^2 + Y_{o,c}^2\right] \tag{A.5}$$

Let us first consider the mean of the first term. If we forget the P/8 factor fot the moment, we have:

$$X_{o,c}^2 = \sum_k \left[d_1(b_k^{-1})b_k^{-1}\, X_k + d_2(b_k^o)b_k^o\, \hat{X}_k\right] \cdot \sum_{k'} \left[d_1(b_{k'}^{-1})b_{k'}^{-1}\, X_{k'} + d_2(b_{k'}^o)b_{k'}^o\, \hat{X}_{k'}\right] \tag{A.6}$$

We know that d_1 and d_2 are equal to 1 with certitude for n interfering users and equal to 0 for the K-1-n_i other users. Equation (A.6) can be written as:

$$X_{o,c}^2 = \sum_k \left[b_k^{-1} X_k + b_k^o \hat{X}_k\right] \cdot \sum_{k'} \left[b_{k'}^{-1} X_{k'} + b_{k'}^o \hat{X}_{k'}\right] \tag{A.7}$$

$$= \sum_k \left[b_k^{-1} X_k + b_k^o \hat{X}_k\right]^2 + \sum_k \sum_{k' \neq k} \left[b_k^{-1} X_k + b_k^o \hat{X}_k\right] \cdot \left[b_{k'}^{-1} X_{k'} + b_{k'}^o \hat{X}_{k'}\right] \tag{A.8}$$

$$= \sum_k \left[\left(b_k^{-1}\right)^2 X_k^2 + \left(b_k^o\right)^2 \hat{X}_k^2 + 2 b_k^{-1} b_k^o X_k \hat{X}_k\right]$$

$$\tag{A.9}$$

$$+ \sum_k \sum_{k' \neq k} \left[b_k^{-1} b_{k'}^{-1} X_k X_{k'} + b_k^{-1} b_{k'}^o X_k \hat{X}_{k'} + b_k^o b_{k'}^{-1} \hat{X}_k X_{k'} + b_k^o b_{k'}^o \hat{X}_k \hat{X}_{k'}\right]$$

The bit error probability is defined as:

$$P\left[\xi < 0 \mid b_1^o\, b_1^{-1} = 1\right] \tag{A.10}$$

So we make the assumption that:

$$E\left[b_k^i\, b_k^j\right] = 0 \tag{A.11}$$

for all k, i and j except in case of k=1, i=-1 and j=0, where this expectation is equal to 1. So we have:

$$E\left[X_{o,c}^2\right] = \sum_k \left[E\left(b_k^{-1}\right)^2 E\left(X_k^2\right) + E\left(b_k^o\right)^2 E\left(\hat{X}_k^2\right) + 2E\left(b_k^{-1} b_k^o\right) E\left(X_k \hat{X}_k\right)\right]$$

$$+ \sum_k \sum_{k' \neq k} [\, E\left(b_k^{-1} b_{k'}^{-1}\right) E\left(X_k X_{k'}\right) + E\left(b_k^{-1} b_{k'}^o\right) E\left(X_k \hat{X}_{k'}\right) \tag{A.12}$$

$$+ E\left(b_k^o b_{k'}^{-1}\right) E\left(\hat{X}_k X_{k'}\right) + E\left(b_k^o b_{k'}^o\right) E\left(\hat{X}_k \hat{X}_{k'}\right)\,]$$

We know that the mean of the bit products of different users is zero, so the double summation won't give a contribution. We have:

$$E\left[X_{o,c}^2\right] = \sum_k \left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + 2E\left(X_1\hat{X}_1\right) \tag{A.13}$$

We now consider the mean of the second term in equation (A.5). This term has the same structure as the first term.

$$Y_{o,c}^2 = \sum_k \left[d_1(b_k^{-1})b_k^{-1}\, Y_k + d_2(b_k^o)b_k^o\, \hat{Y}_k\right] \sum_{k'} \left[d_1(b_{k'}^{-1})b_{k'}^{-1}\, Y_{k'} + d_2(b_{k'}^o)b_{k'}^o\, \hat{Y}_{k'}\right] \tag{A.14}$$

The contribution of this term will be the same as the contribution of the first term.

$$E\left[Y_{o,c}^2\right] = \sum_k \left[E\left(Y_k^2\right) + E\left(\hat{Y}_k^2\right)\right] + 2E\left(Y_1\hat{Y}_1\right) \tag{A.15}$$

We finally obtain the expression for μ_o with the AWGN included.

$$\mu_o = \frac{P}{8}\sum_{k=1}^{n_i} \left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right) + E\left(Y_k^2\right) + E\left(\hat{Y}_k^2\right)\right] + \frac{P}{4}\left[E\left(X_1\hat{X}_1\right) + E\left(Y_1\hat{Y}_1\right)\right] + 2\sigma_N^2 \tag{A.16}$$

A.2 COMPUTATION OF μ_{-1}

The definition of μ_{-1} is given here below:

$$\mu_{-1} = VAR\left(V_{-1} \mid L, \tau_{kl}, n_i, b\right) = E\left[(V_{-1} - m)(V_{-1} - m)^* \mid L, \tau_{kl}, n_i, b\right] \tag{A.17}$$

If we consider $X_{-1,c}$ and $Y_{-1,c}$ as the centered versions of X_{-1} and Y_{-1} where the noise terms have been dropped, we can write:

$$\mu_{-1} = E\left[X_{-1,c}^2 + Y_{-1,c}^2\right] \tag{A.18}$$

Let us first consider the mean of the first term. Again we forget the P/8 factor for this moment, so we have:

$$X^2_{-1,c} = \sum_k \left[d_1(b_k^{-2})b_k^{-2}\, X_k + d_2(b_k^{-1})b_k^{-1}\, \hat{X}_k\right] \cdot \sum_{k'} \left[d_1(b_{k'}^{-2})b_{k'}^{-2}\, X_{k'} + d_2(b_{k'}^{-1})b_{k'}^{-1}\, \hat{X}_{k'}\right] \tag{A.19}$$

We know that d_1 and d_2 are equal to 1 with certitude for n interfering users and equal to 0 for the K-1-n_i other users. Equation (A.19) can be written as:

$$X^2_{-1,c} = \sum_k \left[b_k^{-2}\, X_k + b_k^{-1}\, \hat{X}_k\right] \cdot \sum_{k'} \left[b_{k'}^{-2}\, X_{k'} + b_{k'}^{-1}\, \hat{X}_{k'}\right] \tag{A.20}$$

$$= \sum_k \left[b_k^{-2}\, X_k + b_k^{-1}\, \hat{X}_k\right]^2 + \sum_k \sum_{k' \neq k} \left[b_k^{-2}\, X_k + b_k^{-1}\, \hat{X}_k\right] \cdot \left[b_{k'}^{-2}\, X_{k'} + b_{k'}^{-1}\, \hat{X}_{k'}\right] \tag{A.21}$$

$$= \sum_k \left[\left(b_k^{-2}\right)^2 X_k^2 + \left(b_k^{-1}\right)^2 \hat{X}_k^2 + 2b_k^{-2}b_k^{-1}\, X_k\hat{X}_k\right]$$

$$+ \sum_k \sum_{k' \neq k} \left[b_k^{-2}b_{k'}^{-2}\, X_kX_{k'} + b_k^{-2}b_{k'}^{-1}\, X_k\hat{X}_{k'} + b_k^{-1}b_{k'}^{-2}\, \hat{X}_kX_{k'} + b_k^{-1}b_{k'}^{-1}\, \hat{X}_k\hat{X}_{k'}\right] \tag{A.22}$$

When we take the mean of this expression we get:

$$E\left[X^2_{-1,c}\right] = \sum_k \left[E\left(b_k^{-2}\right)^2 E\left(X_k^2\right) + E\left(b_k^{-1}\right)^2 E\left(\hat{X}_k^2\right) + 2E\left(b_k^{-2}b_k^{-1}\right) E\left(X_k\hat{X}_k\right)\right]$$

$$+ \sum_k \sum_{k' \neq k} [\, E\left(b_k^{-2}b_{k'}^{-2}\right) E\left(X_kX_{k'}\right) + E\left(b_k^{-2}b_{k'}^{-1}\right) E\left(X_k\hat{X}_{k'}\right) \tag{A.23}$$

$$+ E\left(b_k^{-1}b_{k'}^{-2}\right) E\left(\hat{X}_kX_{k'}\right) + E\left(b_k^{-1}b_{k'}^{-1}\right) E\left(\hat{X}_k\hat{X}_{k'}\right)\,]$$

We know that the mean of the bit products of different users is zero, so the double summation won't give a contribution. We have:

$$E\left[X_{-1,c}^2\right] = \sum_k \left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] \tag{A.24}$$

We now consider the mean of the second term of (A.18). This term has the same structure as the first term.

$$Y_{-1,c}^2 = \sum_k \left[d_1(b_k^{-2})b_k^{-2}\, Y_k + d_2(b_k^{-1})b_k^{-1}\, \hat{Y}_k\right] \cdot \sum_{k'} \left[d_1(b_{k'}^{-2})b_{k'}^{-2}\, Y_{k'} + d_2(b_{k'}^{-1})b_{k'}^{-1}\, \hat{Y}_{k'}\right] \tag{A.25}$$

The contribution of this term will be the same as the contribution of the first term.

$$E\left[Y_{-1,c}^2\right] = \sum_k \left[E\left(Y_k^2\right) + E\left(\hat{Y}_k^2\right)\right] \tag{A.26}$$

We finally obtain the expression for μ_{-1} with the AWGN included:

$$\mu_{-1} = \frac{P}{8} \sum_{k=1}^{n_i} \left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right) + E\left(Y_k^2\right) + E\left(\hat{Y}_k^2\right)\right] + 2\sigma_N^2 \tag{A.27}$$

A.3 COMPUTATION OF μ

The definition of μ is given here below:

$$\mu = COV\left(V_o, V_{-1} \,|\, L,\, \tau_{kl},\, n_i,\, b\right) = E\left[(V_o - m)(V_{-1} - m)^* \,|\, L,\, \tau_{kl},\, n_i,\, b\right] \tag{A.28}$$

If we consider $X_{o,c}$ and $Y_{o,c}$ as the centered versions of X_o and Y_o (and the same for X and Y with -1 as subscript), we can write:

$$\mu = E\left[X_{o,c}\, X_{-1,c} + Y_{o,c}\, Y_{-1,c}\right] + jE\left[Y_{o,c}\, X_{-1,c} - X_{o,c}\, Y_{-1,c}\right] \tag{A.29}$$

The noise terms will disappear in the calculations. Let us first consider:

$$E\left[X_{o,c}\,X_{-1,c}\right] \tag{A.30}$$

If we forget the P/8 factor we have:

$$X_{o,c}\,X_{-1,c} = \sum_k \left[d_1(b_k^{-1})b_k^{-1}\,X_k + d_2(b_k^o)b_k^o\,\hat{X}_k\right] \cdot \sum_{k'} \left[d_1(b_{k'}^{-2})b_{k'}^{-2}\,X_{k'} + d_2(b_{k'}^{-1})b_{k'}^{-1}\,\hat{X}_{k'}\right] \tag{A.31}$$

Again we apply the assumption that d_1 and d_2 are equal to 1 with certitude for n interfering users and equal to 0 with certitude for the K-1-n_i other users. So we get:

$$X_{o,c}\,X_{-1,c} = \sum_k \left[b_k^{-1}\,X_k + b_k^o\,\hat{X}_k\right] \cdot \sum_{k'} \left[b_{k'}^{-2}\,X_{k'} + b_{k'}^{-1}\,\hat{X}_{k'}\right] \tag{A.32}$$

$$= \sum_k \left[b_k^{-1}b_k^{-2}\,X_k^2 + \left(b_k^{-1}\right)^2 X_k\,\hat{X}_k + b_k^o b_k^{-2}\,\hat{X}_k X_k + b_k^o b_k^{-1}\,\hat{X}_k^2\right]$$

$$+ \sum_k \sum_{k' \neq k} \left[b_k^{-1}b_{k'}^{-2}\,X_k X_{k'} + b_k^{-1}b_{k'}^{-1}\,X_k\hat{X}_{k'} + b_k^o b_{k'}^{-2}\,\hat{X}_k X_{k'} + b_k^o b_{k'}^{-1}\,\hat{X}_k\hat{X}_{k'}\right] \tag{A.33}$$

When we take the mean of this expression, we get:

$$E\left[X_{o,c}\,X_{-1,c}\right] = \sum_k [\, E\left(b_k^{-1}\,b_k^{-2}\right) E\left(X_k^2\right) + E\left(b_k^{-1}\right)^2 E\left(X_k\hat{X}_k\right)$$

$$+ E\left(b_k^o b_k^{-2}\right) E\left(\hat{X}_k X_k\right) + E\left(b_k^o b_k^{-1}\right) E\left(\hat{X}_k^2\right)\,] \tag{A.34}$$

We know that the mean of the bit products of different users is zero, so the double summation won't give a contribution. We have:

$$E\left[X_{o,c}\,X_{-1,c}\right] = \sum_k \left[E\left(X_k\hat{X}_k\right)\right] + E\left(\hat{X}_1^2\right) \tag{A.35}$$

The derivation of the mean of $Y_{o,c}Y_{-1,c}$ is exactly equivalent with the one for X_oX_{-1}, so we have:

$$E\left[Y_{o,c}Y_{-1,c}\right] = \sum_k \left[E\left(Y_k \hat{Y}_k\right)\right] + E\left(\hat{Y}_1^2\right) \tag{A.36}$$

It can be shown very easily that:

$$E\left[Y_{o,c}\;X_{-1,c}\right] = E\left[X_{o,c}\;Y_{-1,c}\right] \tag{A.37}$$

Therefore the imaginary part in equation (A.29) will vanish. So the expression for μ becomes:

$$\mu = \frac{P}{8}\sum_{k=1}^{n_i}\left[E\left(X_k\hat{X}_k\right) + E\left(Y_k\hat{Y}_k\right)\right] + \frac{P}{8}\left[E\left(\hat{X}_1^2\right) + E\left(\hat{Y}_1^2\right)\right] \tag{A.38}$$

A.4 OVERVIEW OF THE CONDITIONAL EXPECTATIONS

In the previous part we derived expressions for the μ variables. In this part expressions for the expectations will be given. With the definitions of equations (4.27) to (4.30) the following expectations can be derived:

$$E\left[X_1^2\right] = E\left[Y_1^2\right] = \sum_{\substack{l=1\\ l\neq j}}^{L} R_{11}^2\left(\tau_{1l}\right)\cdot\left[\sigma_r^2 + \frac{S^2}{2}\right] + \sum_{\substack{l=1\\ l\neq j}}^{L}\sum_{\substack{p\neq l\\ p\neq j}}^{L} R_{11}\left(\tau_{1l}\right).R_{11}\left(\tau_{1p}\right)\cdot\frac{S^2}{2} \tag{A.39}$$

$$E\left[\hat{X}_1^2\right] = E\left[\hat{Y}_1^2\right] = \sum_{\substack{l=1\\ l\neq j}}^{L} \hat{R}_{11}^2\left(\tau_{1l}\right)\cdot\left[\sigma_r^2 + \frac{S^2}{2}\right] + \sum_{\substack{l=1\\ l\neq j}}^{L}\sum_{\substack{p\neq l\\ p\neq j}}^{L} \hat{R}_{11}\left(\tau_{1l}\right).\hat{R}_{11}\left(\tau_{1p}\right)\cdot\frac{S^2}{2} \tag{A.40}$$

$$E\left[X_1\hat{X}_1\right] = E\left[Y_1\hat{Y}_1\right] = \sum_{\substack{l=1\\ l\neq j}}^{L} R_{11}\left(\tau_{1j}\right).\hat{R}_{11}\left(\tau_{1l}\right)\cdot\left[\sigma_r^2 + \frac{S^2}{2}\right] + \sum_{\substack{l=1\\ l\neq j}}^{L}\sum_{\substack{p\neq l\\ p\neq j}}^{L} R_{11}\left(\tau_{1l}\right).\hat{R}_{11}\left(\tau_{1p}\right)\cdot\frac{S^2}{2} \tag{A.41}$$

$$E\left[X_k^2\right] = E\left[Y_k^2\right] = \sum_{l=1}^{L} R_{1k}^2(\tau_{kl}) \cdot \left(\sigma_r^2 + \frac{S^2}{2}\right) + \sum_{l=1}^{L}\sum_{p \neq l}^{L} R_{1k}(\tau_{lk}) . R_{1k}(\tau_{pk}) \cdot \frac{S^2}{2} \tag{A.42}$$

$$E\left[\hat{X}_k^2\right] = E\left[\hat{Y}_k^2\right] = \sum_{l=1}^{L} \hat{R}_{1k}^2(\tau_{kl}) \cdot \left(\sigma_r^2 + \frac{S^2}{2}\right) + \sum_{l=1}^{L}\sum_{p \neq l}^{L} \hat{R}_{1k}(\tau_{kl}) . \hat{R}_{1k}(\tau_{kp}) \cdot \frac{S^2}{2} \tag{A.43}$$

$$E\left[X_k \hat{X}_k\right] = E\left[Y_k \hat{Y}_k\right] = \sum_{l=1}^{L} R_1 k(\tau_{kl}) . \hat{R}_{1k}(\tau_{kl}) \cdot \left(\sigma_r^2 + \frac{S^2}{2}\right) + \sum_{l=1}^{L}\sum_{p \neq l}^{L} R_{1k}(\tau_{kl}) . \hat{R}_{1k}(\tau_{kp}) \cdot \frac{S^2}{2} \tag{A.44}$$

The calculations of the Gaussian approximation, given in appendix B, require the square of some of the expectations given above. The calculation of these squares can be done in a very straight forward way.

APPENDIX B: CALCULATIONS OF THE MOMENTS OF THE GAUSSIAN RANDOM VARIABLES

The parameters belonging to the Gaussian distribution are the mean and the variance of the stochastic varible under consideration. These moments of the μ-variables will be determined in the following sections.

B.1 COMPUTATION OF THE VARIANCE OF μ_o

We have derived an expression for μ_o. The equation is given here below:

$$\mu_o = \frac{P}{8}\sum_{k=1}^{n_i}\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right) + E\left(Y_k^2\right) + E\left(\hat{Y}_k^2\right)\right] + \frac{P}{4}\left[E\left(X_1\hat{X}_1\right) + E\left(Y_1\hat{Y}_1\right)\right] + 2\sigma_N^2 \tag{B.1}$$

Since the contributions of $E[X_{o,c}^2]$ and $E[Y_{o,c}^2]$ are the same, we can write μ_o as:

$$\mu_o = \frac{P}{4}\sum_{k=1}^{n_i}\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + \frac{P}{2}E\left(X_1\hat{X}_1\right) + 2\sigma_N^2 \tag{B.2}$$

In order to calculate the variance of μ_o, we have to calculate the mean of μ_o squared and the square of the mean of μ_o. The variance can then be determined as follows:

$$VAR\left(\mu_o\right) = E_T\left(\mu_o^2\right) - E_T^2\left(\mu_o\right) \tag{B.3}$$

The derivation of the square of μ_o is given here below:

$$\mu_o^2 = \left\{\frac{P}{4}\sum_k\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + \frac{P}{2}E\left(X_1\hat{X}_1\right) + 2\sigma_N^2\right\}^2 \tag{B.4}$$

$$= \frac{P^2}{16}\left\{\sum_k \left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right]\right\}^2 + \frac{P^2}{4} E\left(X_1\hat{X}_1\right) \sum_k \left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right]$$

$$+ P\sigma_N^2 \sum_k \left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + \frac{P^2}{4} E^2\left(X_1\hat{X}_1\right) + 2\sigma_N^2 PE\left(X_1\hat{X}_1\right) + 4\sigma_N^4 \qquad \text{(B.5)}$$

$$= \frac{P^2}{16}\sum_k \left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right]^2 + \frac{P^2}{16}\sum_k \sum_{k' \neq k} \left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right]\left[E\left(X_{k'}^2\right) + E\left(\hat{X}_{k'}^2\right)\right]$$

$$+ \frac{P^2}{4} E\left(X_1\hat{X}_1\right) \sum_k \left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + P\sigma_N^2 \sum_k \left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] \qquad \text{(B.6)}$$

$$+ \frac{P^2}{4}E^2\left(X_1\hat{X}_1\right) + 2\sigma_N^2 PE\left(X_1\hat{X}_1\right) + 4\sigma_N^4$$

When we take the mean of this expression with respect to τ we get:

$$E_\tau\left[\mu_o^2\right] = \frac{P^2}{16}\sum_k \{ E_\tau\left[E\left(X_k^2\right)\right]^2 + 2E_\tau\left[E\left(X_k^2\right)E\left(\hat{X}_k^2\right)\right] + E_\tau\left[E\left(\hat{X}_k^2\right)\right]^2 \}$$

$$+ \frac{P^2}{16}\sum_k \sum_{k'\neq k} E_\tau\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] E_\tau\left[E\left(X_{k'}^2\right) + E\left(\hat{X}_{k'}^2\right)\right]$$

$$+ \frac{P^2}{4} E_\tau\left[E\left(X_1\hat{X}_1\right)\right] \sum_{k\neq 1} E_\tau\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] \tag{B.7}$$

$$+ \frac{P^2}{4} E_\tau\left[E\left(X_1\hat{X}_1\right)E\left(X_1^2\right)\right] + \frac{P^2}{4} E_\tau\left[E\left(X_1\hat{X}_1\right)E\left(\hat{X}_1^2\right)\right]$$

$$+ P\sigma_N^2 \sum_k E_\tau\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + \frac{P^2}{4} E_\tau\left[E\left(X_1\hat{X}_1\right)\right]^2$$

$$+ 2\sigma_N^2 P E_\tau\left[E\left(X_1\hat{X}_1\right)\right] + 4\sigma_N^4$$

When we take the mean of μ_o with respect to tau we get:

$$E_\tau\left[\mu_o\right] = \frac{P}{4}\sum_{k=1}^{n_i} E_\tau\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + \frac{P}{2} E_\tau\left[E\left(X_1\hat{X}_1\right)\right] + 2\sigma_N^2 \tag{B.8}$$

This expression squared yields:

$$E_\tau^2\left[\mu_o\right] = \frac{P^2}{16}\left\{\sum_k E_\tau\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right]\right\}^2$$

$$+ \frac{P^2}{4} E_\tau\left[E\left(X_1\hat{X}_1\right)\right]\sum_k E_\tau\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] \tag{B.9}$$

$$+ P\sigma_N^2 \sum_k E_\tau\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + \frac{P^2}{4} E_\tau^2\left[E\left(X_1\hat{X}_1\right)\right]$$

$$+ 2\sigma_N^2 PE_\tau\left[E\left(X_1\hat{X}_1\right)\right] + 4\sigma_N^4$$

$$E_\tau^2\left[\mu_o\right] = \frac{P^2}{16}\sum_k \{ E_\tau^2\left[E\left(X_k^2\right)\right] + 2E_\tau\left[E\left(X_k^2\right)\right]E_\tau\left[E\left(\hat{X}_k^2\right)\right] + E_\tau^2\left[E\left(\hat{X}_k^2\right)\right]\}$$

$$+ \frac{P^2}{16}\sum_k\sum_{k'\neq k} E_\tau\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right]E_\tau\left[E\left(X_{k'}^2\right) + E\left(\hat{X}_{k'}^2\right)\right]$$

$$+ \frac{P^2}{4} E_\tau\left[E\left(X_1\hat{X}_1\right)\right]\sum_{k\neq 1} E_\tau\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] \tag{B.10}$$

$$+ \frac{P^2}{4} E_\tau\left[E\left(X_1\hat{X}_1\right)\right]E_\tau\left[E\left(X_1^2\right)\right] + \frac{P^2}{4} E_\tau\left[E\left(X_1\hat{X}_1\right)\right]E_\tau\left[E\left(\hat{X}_1^2\right)\right]$$

$$+ P\sigma_N^2 \sum_k E_\tau\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + \frac{P^2}{4} E_\tau^2\left[E\left(X_1\hat{X}_1\right)\right]$$

$$+ 2\sigma_N^2 PE_\tau\left[E\left(X_1\hat{X}_1\right)\right] + 4\sigma_N^4$$

Substracting equation (B.10) from equation (B.7) yields the variance of μ_o:

$$
\begin{aligned}
VAR\left(\mu_o\right) = \frac{P^2}{16}\sum_{k=1}^{n_i}\Bigg\{ & E_T\left[E\left(X_k^2\right)\right]^2 + 2E_T\left[E\left(X_k^2\right)E\left(\hat{X}_k^2\right)\right] + E_T\left[E\left(\hat{X}_k^2\right)\right]^2\Bigg\} \\
& + \frac{P^2}{4}E_T\left[E\left(X_1\hat{X}_1\right)E\left(X_1^2\right)\right] + \frac{P^2}{4}E_T\left[E\left(X_1\hat{X}_1\right)E\left(\hat{X}_1^2\right)\right] \\
& + \frac{P^2}{4}E_T\left[E\left(X_1\hat{X}_1\right)\right]^2 \\
& - \left\{\frac{P^2}{16}\sum_k E_T^2\left[E\left(X_k^2\right)\right] + 2E_T\left[E\left(X_k^2\right)\right]E_T\left[E\left(\hat{X}_k^2\right)\right] + E_T^2\left[E\left(\hat{X}_k^2\right)\right]\right\} \\
& - \frac{P^2}{4}E_T\left[E\left(X_1\hat{X}_1\right)\right]E_T\left[E\left(X_1^2\right)\right] - \frac{P^2}{4}E_T\left[E\left(X_1\hat{X}_1\right)\right]E_T\left[E\left(\hat{X}_1^2\right)\right] \qquad \text{(B.11)} \\
& - \frac{P^2}{4}E_T^2\left[E\left(X_1\hat{X}_1\right)\right]
\end{aligned}
$$

B.2 COMPUTATION OF THE VARIANCE OF μ_{-1}

We have derived an expression for μ_{-1}. The equation is given here below:

$$
\mu_{-1} = \frac{P}{8}\sum_{k=1}^{n_i}\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right) + E\left(Y_k^2\right) + E\left(\hat{Y}_k^2\right)\right] + 2\sigma_N^2 \qquad \text{(B.12)}
$$

Since the contributions of $E[X^2_{-1,c}]$ and $E[Y^2_{-1,c}]$ are the same we can write μ_{-1} as:

$$\mu_{-1} = \frac{P}{4}\sum_{k=1}^{n_i}\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + 2\sigma_N^2 \tag{B.13}$$

In order to calculate the variance of μ_{-1} we have to calculate the mean of μ_{-1} squared and the square of the mean of μ_{-1}. The variance is then determined in an equivalent way as the variance of μ_o. The derivation of the square of μ_{-1} is given here below:

$$\mu_{-1}^2 = \left\{\frac{P}{4}\sum_{k=1}^{n_i}\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + 2\sigma_N^2\right\}^2 \tag{B.14}$$

$$= \frac{P^2}{16}\left\{\sum_k\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right]\right\}^2 + P\sigma_N^2\sum_k\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + 4\sigma_N^4 \tag{B.15}$$

$$= \frac{P^2}{16}\sum_k\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right]^2 + \frac{P^2}{16}\sum_k\sum_{k'\neq k}\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right]\left[E\left(X_{k'}^2\right) + E\left(\hat{X}_{k'}^2\right)\right]$$

$$+ P\sigma_N^2\sum_k\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + 4\sigma_N^4 \tag{B.16}$$

When we take the mean of this expression with respect to τ we get:

$$E_\tau\left[\mu_{-1}^2\right] = \frac{P^2}{16}\sum_{k=1}^{n_i}\left\{E_\tau\left[E\left(X_k^2\right)\right]^2 + 2E_\tau\left[E\left(X_k^2\right)E\left(\hat{X}_k^2\right)\right] + E_\tau\left[E\left(\hat{X}_k^2\right)\right]^2\right\}$$

$$+ \frac{P^2}{16}\sum_k\sum_{k'\neq k}E_\tau\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right]E_\tau\left[E\left(X_{k'}^2\right) + E\left(\hat{X}_{k'}^2\right)\right] \tag{B.17}$$

$$+ P\sigma_N^2\sum_k E_\tau\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + 4\sigma_N^4$$

When we take the mean of μ_{-1} with respect to tau we get:

$$E_T[\mu_{-1}] = \frac{P}{4}\sum_{k=1}^{n_i}\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + 2\sigma_N^2 \tag{B.18}$$

This expression squared yields:

$$E_T^2\left[\mu_{-1}\right] = \frac{P^2}{16}\left\{\sum_k E_T\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right]\right\}^2 + P\sigma_N^2\sum_k E_T\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + 4\sigma_N^4 \tag{B.19}$$

Further calculation leads to:

$$E_T^2\left[\mu_{-1}\right] = \frac{P^2}{16}\sum_k E_T^2\left[E\left(X_k^2\right)\right] + 2E_T\left[E\left(X_k^2\right)\right]E_T\left[E\left(\hat{X}_k^2\right)\right] + E_T^2\left[E\left(\hat{X}_k^2\right)\right]$$

$$+ \frac{P^2}{16}\sum_k\sum_{k'\neq k} E_T\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right]E_T\left[E\left(X_{k'}^2\right) + E\left(\hat{X}_{k'}^2\right)\right] \tag{B.20}$$

$$+ P\sigma_N^2\sum_k E_T\left[E\left(X_k^2\right) + E\left(\hat{X}_k^2\right)\right] + 4\sigma_N^4$$

Substracting equation (B.20) from equation (B.17) yields the variance of μ_{-1}:

$$VAR\left(\mu_{-1}\right) = \frac{P^2}{16}\sum_{k=1}^{n_i}\left\{E_T\left[E\left(X_k^2\right)\right]2 + 2E_T\left[E\left(X_k^2\right)E\left(\hat{X}_k^2\right)\right] + E_T\left[E\left(\hat{X}_k^2\right)\right]^2\right\}$$

$$-\left\{\frac{P^2}{16}\sum_k E_T^2\left[E\left(X_k^2\right)\right] + 2E_T\left[E\left(X_k^2\right)\right]E_T\left[E\left(\hat{X}_k^2\right)\right] + E_T^2\left[E\left(\hat{X}_k^2\right)\right]\right\} \tag{B.21}$$

B.3 COMPUTATION OF THE VARIANCE OF μ

We have derived an expression for μ. The equation is given here below:

$$\mu = \frac{P}{8}\sum_{k=1}^{n_i}\left[E\left(X_k\hat{X}_k\right) + E\left(Y_k\hat{Y}_k\right)\right] + \frac{P}{8}\left[E\left(\hat{X}_1^2\right) + E\left(\hat{Y}_1^2\right)\right] \tag{B.22}$$

Since the contributions of $E[X_{0,c}X_{-1,c}]$ and $E[Y_{0,c}Y_{-1,c}]$ are the same, we can write μ as:

$$\mu = \frac{P}{4}\sum_{k=1}^{n_i}E\left(X_k\hat{X}_k\right) + \frac{P}{4}E\left(\hat{X}_1^2\right) \tag{B.23}$$

First we have to determine the square of μ

$$\mu^2 = \frac{P^2}{16}\sum_k E^2\left(X_k\hat{X}_k\right) + \frac{P^2}{16}\sum_k\sum_{k'\neq k}E\left(X_k\hat{X}_k\right)E\left(X_{k'}\hat{X}_{k'}\right)$$
$$+ \frac{P^2}{8}E\left(\hat{X}_1\right)\sum_k E\left(X_k\hat{X}_k\right) + \frac{P^2}{16}E^2\left(\hat{X}_1^2\right) \tag{B.24}$$

When we take the mean of this expression with respect to tau, we get:

$$E_\tau\left[\mu^2\right] = \frac{P^2}{16}\sum_k E_\tau\left[E\left(X_k\hat{X}_k\right)\right]^2 + \frac{P^2}{16}\sum_k\sum_{k'\neq k}E_\tau\left[E\left(X_k\hat{X}_k\right)\right]E_\tau\left[E\left(X_{k'}\hat{X}_{k'}\right)\right]$$
$$+ \frac{P^2}{8}E_\tau\left[E\left(X_1^2\right)E\left(X_1\hat{X}_1\right)\right] + \frac{P^2}{8}E_\tau\left[E\left(\hat{X}_1^2\right)\right]\sum_{k\neq 1}E_\tau\left[E\left(X_k\hat{X}_k\right)\right] \tag{B.25}$$
$$+ \frac{P^2}{16}E_\tau\left[E\left(\hat{X}_1^2\right)\right]^2$$

When we take the mean of μ with respect to tau, we get:

$$E_\tau[\mu] = \frac{P}{4}\sum_{k=1}^{n_i}E_\tau\left[E\left(X_k\hat{X}_k\right)\right] + \frac{P}{4}E_\tau\left[E\left(\hat{X}_1^2\right)\right] \tag{B.26}$$

This expression squared yields:

$$E_\tau^2[\mu] = \frac{P^2}{16}\sum_k E_\tau^2\left[E\left(X_k\hat{X}_k\right)\right] + \frac{P^2}{16}\sum_k\sum_{k'\neq k} E_\tau\left[E\left(X_k\hat{X}_k\right)\right]E_\tau\left[E\left(X_{k'}\hat{X}_{k'}\right)\right]$$

$$+ \frac{P^2}{8}E_\tau\left[E\left(X_1^2\right)\right]E_\tau\left[E\left(X_1\hat{X}_1\right)\right] + \frac{P^2}{8}E_\tau\left[E\left(\hat{X}_1^2\right)\right]\sum_{k\neq 1}E_\tau\left[E\left(X_k\hat{X}_k\right)\right] \qquad \text{(B.27)}$$

$$+ \frac{P^2}{16}E_\tau^2\left[E\left(\hat{X}_1^2\right)\right]$$

The variance of μ can be calculated by substracting equation (B.27) from equation (B.25):

$$VAR(\mu) = \frac{P^2}{16}\sum_k E_\tau\left[E\left(X_k\hat{X}_k\right)\right]^2 + \frac{P^2}{8}E_\tau\left[E\left(X_1^2\right)E\left(X_1\hat{X}_1\right)\right] + \frac{P^2}{16}E_\tau\left[E\left(\hat{X}_1^2\right)\right]^2$$

$$- \frac{P^2}{16}\sum_k E_\tau^2\left[E\left(X_k\hat{X}_k\right)\right] - \frac{P^2}{8}E_\tau\left[E\left(X_1^2\right)\right]E_\tau\left[E\left(X_1\hat{X}_1\right)\right] - \frac{P^2}{16}E_\tau^2\left[E\left(\hat{X}_1^2\right)\right] \qquad \text{(B.28)}$$

In order to give the expressions for the expectations with respect to τ, we need to determine the expectations of the partial cross correlation functions. This will be done in the next section.

B.4 REVIEW OF THE EXPECTATIONS OF THE PARTIAL CROSS CORRELATION FUNCTIONS

When we consider the expectations with respect to the path delays, various expressions are encountered. For the computation of various expectations of partial cross correlation functions, we can partly refer to [3]. In this section the only the results will be presented

For the delay τ such that we have $0 \leq nT_c \leq \tau \leq (n+1)T_c \leq T$, we define the following terms:

$$R_{1k}(\tau) = A_{n1k}T_c + B_{n1k}(\tau - nT_c) \tag{B.29}$$

$$\hat{R}_{1k}(\tau) = \hat{A}_{n1k}T_c + \hat{B}_{n1k}(\tau - nT_c) \tag{B.30}$$

with

$$A_{n1k} = C_{1k}(n-N) \tag{B.31}$$

$$B_{n1k} = C_{1k}(n+1-N) - C_{1k}(n-N) \tag{B.32}$$

$$B_{n1k} = C_{1k}(n+1-N) - C_{1k}(n-N) \tag{B.33}$$

$$\hat{A}_{n1k} = C_{1k}(n) \tag{B.34}$$

$$\hat{B}_{n1k} = C_{1k}(n+1) - C_{1k}(n) \tag{B.35}$$

$$C_{1k}(n) = \begin{cases} \sum_{j=0}^{N-1-n} d_k^j \, d_1^{j+n} & \text{for } 0 \leq n \leq N-1 \\ \sum_{j=0}^{N-1+n} d_k^{j-n} d_1^j & \text{for } 1-N \leq n \leq 0 \end{cases} \tag{B.36}$$

We then get the following expectations:

$$\eta_{k1} = E_\tau\left[R_{1k}(\tau_{k1})\right] = \frac{T_c^2}{T} \sum_{n=0}^{N-1} \left[A_{n1k} + \frac{1}{2}B_{n1k}\right] \tag{B.37}$$

$$\eta_{k2} = E_\tau\left[R_{1k}^2(\tau_{k1})\right] = \frac{T_c^3}{T} \sum_{n=0}^{N-1} \left[A_{n1k}^2 + A_{n1k}B_{n1k} + \frac{1}{3}B_{n1k}^2\right] \tag{B.38}$$

$$\eta_{k3} = E_\tau\left[R_{1k}^3(\tau_{kl})\right] = \frac{T_c^4}{T}\sum_{n=0}^{N-1}\left[A_{nlk}^3 + \frac{3}{2}A_{nlk}^2 B_{nlk} + A_{nlk}B_{nlk}^2 + \frac{1}{4}B_{nlk}^3\right] \tag{B.39}$$

$$\eta_{k4} = E_\tau\left[R_{1k}^4(\tau_{kl})\right] = \frac{T_c^5}{T}\sum_{n=0}^{N-1}\left[A_{nlk}^4 + 2A_{nlk}^3 B_{nlk} + 2A_{nlk}^2 B_{nlk}^2 + A_{nlk}B_{nlk}^3 + \frac{1}{5}B_{nlk}^4\right] \tag{B.40}$$

$$\nu_{k1} = E_\tau\left[\hat{R}_{1k}(\tau_{kl})\right] = \frac{T_c^2}{T}\sum_{n=0}^{N-1}\left[\hat{A}_{nlk} + \frac{1}{2}\hat{B}_{nlk}\right] \tag{B.41}$$

$$\nu_{k2} = E_\tau\left[\hat{R}_{1k}^2(\tau_{kl})\right] = \frac{T_c^3}{T}\sum_{n=0}^{N-1}\left[\hat{A}_{nlk}^2 + \hat{A}_{nlk}\hat{B}_{nlk} + \frac{1}{3}\hat{B}_{nlk}^2\right] \tag{B.42}$$

$$\nu_{k3} = E_\tau\left[\hat{R}_{1k}^3(\tau_{lk})\right] = \frac{T_c^4}{T}\sum_{n=0}^{N-1}\left[\hat{A}_{nlk}^3 + \frac{3}{2}\hat{A}_{nlk}^2 B_{nlk} + \hat{A}_{nlk}\hat{B}_{nlk}^2 + \frac{1}{4}\hat{B}_{nlk}^3\right] \tag{B.43}$$

$$\nu_{k4} = E_\tau\left[\hat{R}_{1k}^4(\tau_{kl})\right] = \frac{T_c^5}{T}\sum_{n=0}^{N-1}\left[\hat{A}_{nlk}^4 + 2\hat{A}_{nlk}^3 \hat{B}_{nlk} + 2\hat{A}_{nlk}^2 \hat{B}_{nlk}^2 + \hat{A}_{nlk}\hat{B}_{nlk}^3 + \frac{1}{5}\hat{B}_{nlk}^4\right] \tag{B.44}$$

$$\begin{aligned}\alpha_{k11} = E_\tau\left[R_{1k}(\tau_{kl})\hat{R}_{1k}(\tau_{kl})\right] = \frac{T_c^3}{T}\sum_{n=0}^{N-1}[\, & A_{nlk}\hat{A}_{nlk} + \frac{1}{2}A_{nlk}\hat{B}_{nlk} + \frac{1}{2}\hat{A}_{nlk}B_{nlk} \\ & + \frac{1}{3}B_{nlk}\hat{B}_{nlk}]\end{aligned} \tag{B.45}$$

$$\begin{aligned}\alpha_{k22} = E_\tau\left[R_{1k}^2(\tau_{kl})\hat{R}_{1k}^2(\tau_{kl})\right] = \frac{T_c^5}{T}\sum_{n=0}^{N-1}[\, & A_{nlk}^2\hat{A}_{nlk}^2 + A_{nlk}^2\hat{A}_{nlk}\hat{B}_{nlk} + \frac{1}{3}A_{nlk}^2\hat{B}_{nlk}^2 \\ + A_{nlk}\hat{A}_{nlk}^2 B_{nlk} + \frac{4}{3}A_{nlk}\hat{A}_{nlk}B_{nlk}\hat{B}_{nlk} & + \frac{2}{3}A_{nlk}B_{nlk}\hat{B}_{nlk}^2 \\ + \frac{1}{2}\hat{A}_{nlk}B_{nlk}^2\hat{B}_{nlk} + \frac{1}{3}\hat{A}_{nlk}^2 B_{nlk}^2 & + \frac{1}{5}B_{nlk}^2\hat{B}_{nlk}^2]\end{aligned} \tag{B.46}$$

$$\alpha_{k21} = E_\tau\left[R_{1k}^2(\tau_{kl})\hat{R}_{1k}(\tau_{kl})\right] = \frac{T_c^4}{T}\sum_{n=0}^{N-1}[\, A_{nlk}^2\hat{A}_{nlk} + \frac{1}{2}A_{nlk}^2\hat{B}_{nlk} + \frac{1}{3}\hat{A}_{nlk}B_{nlk}^2 + \frac{1}{4}B_{nlk}^2\hat{B}_{nlk} + A_{nlk}\hat{A}_{nlk}B_{nlk} + \frac{2}{3}A_{nlk}B_{nlk}\hat{B}_{nlk}] \tag{B.47}$$

$$\alpha_{k12} = E_\tau\left[R_{1k}(\tau_{kl})\hat{R}_{1k}^2(\tau_{kl})\right] = \frac{T_c^4}{T}\sum_{n=0}^{N-1}[\, \hat{A}_{nlk}^2 A_{nlk} + \frac{1}{2}\hat{A}_{nlk}^2 B_{nlk} + \frac{1}{3}A_{nlk}\hat{B}_{nlk}^2 + \frac{1}{4}\hat{B}_{nlk}^2 B_{nlk} + A_{nlk}\hat{A}_{nlk}\hat{B}_{nlk} + \frac{2}{3}\hat{A}_{nlk}\hat{B}_{nlk}B_{nlk}] \tag{B.48}$$

$$\alpha_{k31} = E_\tau\left[R_{1k}^3(\tau_{kl})\hat{R}_{1k}(\tau_{kl})\right] = \frac{T_c^5}{T}\sum_{n=0}^{N-1}[\, A_{nlk}^3\hat{A}_{nlk} + \frac{1}{2}A_{nlk}^3\hat{B}_{nlk} + \frac{3}{2}A_{nlk}^2\hat{A}_{nlk}B_{nlk} + A_{nlk}^2 B_{nlk}\hat{B}_{nlk} + A_{nlk}\hat{A}_{nlk}B_{nlk}^2 + \frac{3}{4}A_{nlk}B_{nlk}^2\hat{B}_{nlk} + \frac{1}{4}\hat{A}_{nlk}B_{nlk}^3 + \frac{1}{5}B_{nlk}^3\hat{B}_{nlk}] \tag{B.49}$$

$$\alpha_{k13} = E_\tau\left[R_{1k}(\tau_{kl})\hat{R}_{1k}^3(\tau_{kl})\right] = \frac{T_c^5}{T}\sum_{n=0}^{N-1}[\, \hat{A}_{nlk}^3 A_{nlk} + \frac{1}{2}\hat{A}_{nlk}^3 B_{nlk} + \frac{3}{2}\hat{A}_{nlk}^2 A_{nlk}\hat{B}_{nlk} + \hat{A}_{nlk}^2 A_{nlk}\hat{B}_{nlk} + \hat{A}_{nlk}^2\hat{B}_{nlk}B_{nlk} + \hat{A}_{nlk}A_{nlk}\hat{B}_{nlk}^2 + \frac{3}{4}\hat{A}_{nlk}\hat{B}_{nlk}^2 B_{nlk} + \frac{1}{4}A_{nlk}\hat{B}_{nlk}^3 + \frac{1}{5}\hat{B}_{nlk}^3 B_{nlk}] \tag{B.50}$$

B.5 REVIEW OF THE EXPECTATIONS WITH RESPECT TO THE PATH DELAYS

With help the equations (A.39) up to (A.44) we then find the following equations for the expectations with respect to the delays. It should be mentioned that when k=1, L should be replaced everywhere by L-1.

$$E_\tau\left[E\left(X_k^2\right)\right] = L(L-1)\frac{S^2}{2}\eta_{k1}^2 + L\left(\sigma_r^2 + \frac{S^2}{2}\right)\eta_{k2} \tag{B.51}$$

$$E_\tau\left[E\left(\hat{X}_k^2\right)\right] = L(L-1)\frac{S^2}{2}\nu_{k1}^2 + L\left(\sigma_r^2 + \frac{S^2}{2}\right)\nu_{k2} \tag{B.52}$$

$$
\begin{aligned}
E_r\left[E\left(X_k^2\right)\right]^2 &= \frac{S^4}{4}L(L-1)\left[2\eta_{k2}^2 + (L-2)4\eta_{k2}\eta_{k1}^2 + (L-2)(L-3)\eta_{k1}^4\right] \\
&+ \left(\sigma_r^2 + \frac{S^2}{2}\right)^2 L\left[\eta_{k4} + (L-1)\eta_{k2}^2\right] \\
&+ \frac{S^2}{2}\left(\sigma_r^2 + \frac{S^2}{2}\right)L(L-1)\left[4\eta_{k3}\eta_{k1} + 2(L-2)\eta_{k2}\eta_{k1}^2\right]
\end{aligned}
\tag{B.53}
$$

$$
\begin{aligned}
E_r\left[E\left(\hat{X}_k^2\right)\right]^2 &= \frac{S^4}{4}L(L-1)\left[2\nu_{k2}^2 + (L-2)4\nu_{k2}\nu_{k1}^2 + (L-2)(L-3)\nu_{k1}^4\right] \\
&+ \left(\sigma_r^2 + \frac{S^2}{2}\right)^2 L\left[\nu_{k4} + (L-1)\nu_{k2}^2\right] \\
&+ \frac{S^2}{2}\left(\sigma_r^2 + \frac{S^2}{2}\right)L(L-1)\left[4\nu_{k3}\nu_{k1} + 2(L-2)\nu_{k2}\nu_{k1}^2\right]
\end{aligned}
\tag{B.54}
$$

$$
\begin{aligned}
E_r\left[E\left(X_k^2\right)E\left(\hat{X}_k^2\right)\right] &= \frac{S^4}{4}L(L-1)\left[2\alpha_{k11}^2 + 4(L-2)\alpha_{k11}^2\nu_{k1}\eta_{k1} + (L-2)(L-3)\nu_{k1}^2\eta_{k1}^2\right] \\
&+ \left(\sigma_r^2 + S^2{}_2\right)^2 L\left[\alpha_{k22} + (L-1)\nu_{k2}\eta_{k2}\right] + \frac{S^2}{2}\left(\sigma_r^2 + \frac{S^2}{2}\right)L(L-1) \\
&\times\left[2\alpha_{k21}\eta_{k1} + 2\alpha_{k12}\nu_{k1} + (L-2)\nu_{k2}\eta_{k1}^2 + (L-2)\eta_{k2}\nu_{k1}^2\right]
\end{aligned}
\tag{B.55}
$$

$$
E_r\left[E\left(X_k\hat{X}_k\right)\right] = L\left(\sigma_r^2 + \frac{S^2}{2}\right)\alpha_{k11} + 2\frac{S^2}{2}L(L-1)\eta_{k1}\nu_{k1}
\tag{B.56}
$$

$$
\begin{aligned}
E_r\left[E\left(X_k^2\right)E\left(\hat{X}_k^2\right)\right] &= \frac{S^4}{4}L(L-1)\left[2\alpha_{k11}^2 + 4(L-2)\alpha_{k11}^2\nu_{k1}\eta_{k1} + (L-2)(L-3)\nu_{k1}^2\eta_{k1}^2\right] \\
&+ \left(\sigma_r^2 + \frac{S^2}{2}\right)^2 L\left[\alpha_{k22} + (L-1)\nu_{k2}\eta_{k2}\right] + \frac{S^2}{2}\left(\sigma_r^2 + \frac{S^2}{2}\right)L(L-1) \\
&\times\left[2\alpha_{k21}\eta_{k1} + 2\alpha_{k12}\nu_{k1} + (L-2)\nu_{k2}^2\eta_{k1} + (L-2)\eta_{k2}\nu_{k1}^2\right]
\end{aligned}
\tag{B.58}
$$

$$E_r\left[E\left(X_k\hat{X}_k\right)\right]^2 = \frac{S^4}{4}L(L-1)\left[\nu_{k2}\eta_{k2} + \alpha_{k11}^2\right]$$
$$+ \frac{S^4}{4}L(L-1)(L-2)\left[\nu_{k2}\eta_{kl}^2 + 2\nu_{kl}\alpha_{k11}\eta_{kl} + \nu_{kl}^2\eta_{k2}\right]$$
$$+ \frac{S^4}{4}L(L-1)(L-2)(L-3)\nu_{kl}^2\eta_{kl}^2 + \left(\sigma_r^2 + \frac{S^2}{2}\right)L\left[\alpha_{k22} + (L-1)\alpha_{k11}^2\right] \tag{B.57}$$
$$+ 2\,\frac{S^2}{2}\left(\sigma_r^2 + \frac{S^2}{2}\right)L(L-1)\left[\alpha_{k21}\eta_{kl} + \alpha_{k12}\nu_{kl} + (L-2)\nu_{kl}\eta_{kl}\alpha_{k11}\right]$$

$$E_r\left[E\left(X_1\hat{X}_1\right)E\left(X_1^2\right)\right] = \frac{S^4}{4}(L-1)(L-2)2\nu_{12}\alpha_{111}$$
$$+ \frac{S^4}{4}(L-1)(L-2)(L-3)\left[2\nu_{12}\eta_{11}\nu_{11} + 2\nu_{11}^2\alpha_{111}\right]$$
$$+ \frac{S^4}{4}(L-1)(L-2)(L-3)(L-4)\nu_{11}^3\eta_{11}$$
$$+ \frac{S^2}{2}\left(\sigma_r^2 + \frac{S^2}{2}\right)(L-1)(L-2)\left[\nu_{13}\eta_{11} + 3\nu_{11}\alpha_{121}\right] \tag{B.59}$$
$$+ \frac{S^2}{2}\left(\sigma_r^2 + \frac{S^2}{2}\right)(L-1)(L-2)(L-3)\left[\nu_{11}\nu_{12}\eta_{11} + \nu_{11}^2\alpha_{111}\right]$$
$$+ \left(\sigma_r^2 + \frac{S^2}{2}\right)^2(L-1)\left[\alpha_{131} + (L-2)\alpha_{111}\nu_{12}\right]$$

$$
\begin{aligned}
E_r\left[E\left(X_1\hat{X}_1\right)E\left(\hat{X}_1^2\right)\right] = {} & \frac{S^4}{4}(L-1)(L-2)2\eta_{12}\alpha_{111} \\
& + \frac{S^4}{4}(L-1)(L-2)(L-3)\left[\, 2\nu_{11}\eta_{12}\nu_{11} + 2\eta_{11}^2\alpha_{111}\right] \\
& + \frac{S^4}{4}(L-1)(L-2)(L-3)(L-4)\eta_{11}^3\nu_{11} \\
& + \frac{S^2}{2}\left(\sigma_r^2 + \frac{S^2}{2}\right)(L-1)(L-2)\left[\, 2\eta_{13}\nu_{11} + 2\eta_{11}\alpha_{112}\,\right] \\
& + \frac{S^2}{2}\left(\sigma_r^2 + \frac{S^2}{2}\right)(L-1)(L-2)(L-3)\left[\, \nu_{11}\eta_{12}\eta_{11} + \nu_{11}\eta_{11}\alpha_{111}\,\right] \\
& + \left(\sigma_r^2 + \frac{S^2}{2}\right)^2(L-1)\left[\, \alpha_{113} + (L-2)\alpha_{111}\eta_{12}\,\right]
\end{aligned}
\tag{B.60}
$$

APPENDIX C: DERIVATION OF THE PDF OF THE MAXIMAL PATH GAIN

Selection diversity is based on selecting the strongest signal from several statistically independent signals carrying the same data. These signals are usually the multiple resolvable paths due to inherent diversity of spread spectrum, but these signals can also arrive from different antennas, in order to increase the order of diversity.

Suppose we have M identically distributed random variables $\{\beta_1,...,\beta_M\}$. For the largest random variable β_{max}, the following inequality holds:

$$\beta_{max} > \beta_i \qquad i \in \{1,2,...,M\} \tag{C.1}$$

The probability that $\beta_{max} < y$, denoted as $P_{\beta_{max}}(y)$ is now given by:

$$P_{\beta_{max}}(y) = Pr\{\ \beta_1 < y\ \}.\ ....\ .Pr\{\ \beta_M < y\ \} = P_{\beta_1}(y).\ ...\ .P_{\beta_M} \tag{C.2}$$

This means that the CDF of β_{max} is the product of the CDFs of the M path gains.

Since β is Rician distributed, β^2 has a non-central chi-square probability distribution with two degrees of freedom. With the PDF given in equation (3.6) we can derive the CDF just by integrating the PDF.

$$P_{\beta_i}(\beta_i) = \int_0^{\beta_i} \frac{r}{\sigma_r^2} \exp\left[-\frac{S^2+r^2}{2\sigma_r^2}\right] .I_o\left[\frac{S.r}{\sigma_r^2}\right] \tag{C.3}$$

The CDF of β_{max} is now given by:

$$P_{\beta_{max}}(\beta_{max}) = \left[\ P_{\beta_i}(\beta_i)\ \right]^M \tag{C.4}$$

The PDF can now be obtained by calculating the derivative with respect to β_{max}.

$$f_{\beta_{\max}}(\beta_{\max}) = M.\left[P_{\beta_{\max}}(\beta_{\max}) \right]^{M-1}.\frac{dP_{\beta_{\max}}(\beta_{\max})}{d\beta_{\max}} \tag{C.5}$$

By calculating this derivative, we get the following PDF:

$$f_{\beta_{\max}}(\beta_{\max}) = M\left[\int_0^{\beta_{\max}} \frac{z}{\sigma_r^2}\exp\left(-\frac{s^2+z^2}{2\sigma_r^2}\right) I_o\left(\frac{sz}{\sigma_r^2}\right)dz \right]^{(M-1)}.\frac{\beta_{\max}}{\sigma_r^2}\exp\left(-\frac{s^2+\beta_{\max}}{2\sigma_r^2}\right) I_o\left(\frac{s\beta_{\max}}{\sigma_r^2}\right) \tag{C.6}$$

APPENDIX D: CONTENT OF THE SUBMITTED PAPER

Here we present the content of the paper "*Hybrid Slow Frequency Hopping/Direct Sequence Spread Specrtrum Communications Systems with B- and QPSK Modulation in an Indoor Wireless Environment*", which is accepted for the "Proceedings of the Fourth International Symposium on Personal, Indoor and Mobile Radio Communications", to be held on September 9-11, 1993 in Yokohama, Japan.

Hybrid Slow Frequency Hopping/Direct Sequence Spread Spectrum Communications Systems with B- and QPSK Modulation in an Indoor Wireless Environment

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Abstract

This paper analyses the performance of a spread-spectrum multiple access system based on hybrid direct sequence/slow frequency hopping techniques with QPSK modulation. The performance is analyzed considering multipath and multiple access interferences and an indoor channel having Rician distributed path gains. Two diversity techniques are investigated and compared, for given delay spread, bandwidth and bit rate. It is shown that the larger bit duration of QPSK is favorable to combat the effect of multipath propagation.

1 Introduction

Currently, a lot of attention is paid to CDMA systems for personal and mobile communications systems [1, 2, 3, 4]. CDMA systems are attractive not only for combating interferers, but also from the point of view of multiple access capabilities. DS/SS (direct sequence/spread-spectrum) systems provide a good answer to the problem of multiple access communication systems in fading environments. When DS/SS is combined with diversity techniques, the performances are really attractive. Direct sequence is a good mean to combat the multipath effect. Nevertheless, DS/SS is known to suffer from the near-far effect. On the other hand, slow frequency hopping (SFH) is a good solution to cope with the multiple access interference. Besides, it is not sensitive to the near-far effect. As a consequence, the combination of direct sequence with slow frequency hopping in a hybrid system is an attractive solution for combating the multipath effect, the multiple access interference and solving the near-far problem [4]. *The aim of the paper is to analytically determine the performance of a hybrid SFH-DS/SS system with QPSK modulation in a Rician fading environment and compare this performance with the one achieved by BPSK modulation under some constraints.* In [4], PSK and maximal ratio combining has been studied. In this paper, QPSK is used and selection diversity is compared with maximal ratio combining. The multipath effect and the multiple access interference are taken into account.

*L. Vandendorpe would like to thank the Belgian NSF for its financial support

2 System description

The system is a star connected network with K users. All of them are at the same distance from the base-station. We study the performance of the system when the base station is in receiving mode.

2.1 Transmitted signal

The sequence of bits with rate $2/T$ produced by user k is separated in 2 sequences, denoted by $b_{c,k}(t)$ and $b_{s,k}(t)$ respectively and having a rate $1/T$: $b_{c,k}(t) = \sum_j b^j_{c,k} P_T(t-jT)$, $b_{s,k}(t) = \sum_j b^j_{s,k} P_T(t-jT)$ where $b^j_{x,k}$ belongs to the set $\{-1,1\}$ and $P_S(t)$ is a rectangular pulse of duration S. These signals are first multiplied by a spreading sequence $a_{c,k}(t)$ and $a_{s,k}(t)$ with $a_{c,k}(t) = \sum_j a^j_{c,k} P_{T_c}(t-jT_c)$, $a_{s,k}(t) = \sum_j a^j_{s,k} P_{T_c}(t-jT_c)$ where T_c is the chip duration. These sequences are periodical with period T. The signals $a_{c,k}(t)b_{c,k}(t)$ and $a_{s,k}(t)b_{s,k}(t)$ PSK modulate 2 carriers in quadrature. This modulated signal is then frequency hopped according to the hopping pattern associated with user k. After appropriate filtering, this hopped signal becomes :

$$\begin{aligned} s^Q_k(t) &= \sqrt{2P}a_{c,k}(t)\,b_{c,k}(t) \\ &\times \cos[\omega_c t + \int_{-\infty}^{t} \omega_k(\tau)\mathrm{d}\tau + \alpha_k + \theta_k] \\ &+ \sqrt{2P}a_{s,k}(t)\,b_{s,k}(t) \\ &\times \sin[\omega_c t + \int_{-\infty}^{t} \omega_k(\tau)\mathrm{d}\tau + \alpha_k + \theta_k] \quad (1) \end{aligned}$$

where θ_k is a random phase associated with user k. and α_k is a phase factor introduced by the hopper. The frequency $f_k(t)$ is constant over a time interval of duration $T_h = N_b T$ where N_b is the number of bits in a hopping interval. The asynchronous hopping patterns are assumed to be generated by first order stationary Markov processes. The number of available frequencies is q. They are assumed to have a minimum spacing $2/T_c$, so that the overlap of DS signals hopped to adjacent frequencies is negligible. The results of the previous section can be particularized to the situation of BPSK modulation by setting $b_{s,k}(t) = 0$.

2.2 Channel description

The link between the kth user and the base station is characterized by a lowpass equivalent transfer function given by $h_k(t) = \sum_{l=1}^{L} \beta_{kl}\,\delta(t-\tau_{kl})\exp(j\gamma_{kl})$ where kl refers to path l of user k. We assume that the path gains β_{kl} are Rician distributed. The delays τ are assumed to be uniformly distributed random variables over $[0,T]$. The phase factor γ is uniformly distributed over $[0, 2\pi]$. The random variables β, γ and τ are independent for different values of k and l. The fading is assumed to be slow and the different random variables are constant over the duration of one bit. The Rice probability function is given by :

$$p_\beta(r) = \frac{r}{\sigma_r^2}\exp^{\left[-\frac{r^2+S^2}{2\sigma_r^2}\right]} I_0\left(\frac{rS}{\sigma_r^2}\right) \qquad (2)$$

where $r \geq 0, S \geq 0$. An important parameter is R, known as the Rician parameter. Its value is $S^2/(2\sigma_r^2)$ and represents the ratio of the power associated with the direct component and the scattered components.

2.3 Receiver description

We assume that a received signal is composed of the contributions of the different users, their different paths and additive white Gaussian noise. Therefore, we have for QPSK, a received signal which is given by :

$$\begin{aligned} r(t) &= \sqrt{2P}\sum_k\sum_l \beta_{kl}\,a_{c,k}(t-\tau_{kl})\,b_{c,k}(t-\tau_{kl}) \\ &\times \cos\left[\omega_c t + \int_{-\infty}^{t-\tau_{kl}} \omega_k(\tau)d\tau + \alpha_k + \theta_k + \gamma_{kl}\right] \\ &+ \sqrt{2P}\sum_k\sum_l \beta_{kl}\,a_{s,k}(t-\tau_{kl})\,b_{s,k}(t-\tau_{kl}) \\ &\times \sin\left[\omega_c t + \int_{-\infty}^{t-\tau_{kl}} \omega_k(\tau)d\tau + \alpha_k + \theta_k + \gamma_{kl}\right] \\ &+ n(t) \end{aligned} \qquad (3)$$

and $n(t)$ is a white Gaussian noise with single-sided power spectral density N_0 [W/Hz]. We consider user 1 as reference user. Therefore, the dehopping sequence is the one associated with user 1. Each one of the 2 components is dehopped, multiplied by the DS code associated with user 1, the carrier and integrated over the bit duration. Let us focus on the in-phase signal. The output of the in-phase integrator is given by :

$$\begin{aligned} z_{0,c} &= \sqrt{P/8}\sum_k\sum_l \beta_{kl}\cos[\Psi_{kl}]\int_{\lambda T}^{(\lambda+1)T} a_{c,1}(t) \\ &\quad a_{c,k}(t-\tau_{kl})\,b_{c,k}(t-\tau_{kl})\,\delta[f_1(t), f_k(t-\tau_{kl})]\,dt \\ &+ \sqrt{P/8}\sum_k\sum_l \beta_{kl}\sin[\Psi_{kl}]\int_{\lambda T}^{(\lambda+1)T} a_{c,1}(t) \\ &\quad a_{s,k}(t-\tau_{kl})\,b_{s,k}(t-\tau_{kl})\,\delta[f_1(t), f_k(t-\tau_{kl})]\,dt \\ &+ \nu_c \end{aligned} \qquad (4)$$

where ν_c is a zero-mean Gaussian random variable with a variance given by $N_0T/16$. The δ function accounts for the possible hits : a hit occurs when the frequency $f_1(t)$ used by reference user and the frequency $f_k(t-\tau_{kl})$ used in the delayed version of signals k are the same. The phase Ψ is the overall phase shift. The bit under consideration is bit number λ. Let us assume without loss of generality that $\lambda = 0$. We can also assume without loss of generality that the jth path between transmitter 1 and the receiver is the reference path. Because of the dominant and stable component, the receiver can be time-synchronous with this path and besides, it can acquire knowledge of the phase of the stable component of this path, thanks to its tracking mechanism. Therefore, we say $\tau_{1j} = 0$, $\Psi_{1j} = 0$. All delays are then defined as relative delays according to this reference path. We further assume that path j is the shortest path between emitter and receiver. The relative delays are then all positive valued. Because of the limited interval for the delays, two bits will be under consideration : bits 0 and -1 of sequence k. We define [2] $R_{1k}^{xy}(\tau_{kl}) = \int_0^{\tau_{kl}} a_{x,1}(t)\,a_{y,k}(t-\tau_{kl})\,dt$ and $\hat{R}_{1k}^{xy}(\tau_{kl}) = \int_{\tau_{kl}}^{T} a_{x,1}(t)\,a_{y,k}(t-\tau_{kl})\,dt$ where x and y may both be c or s. In these definitions, the periodicity of the spreading sequence has been taken into account. We define the following parameters : $X_k^{xy} = \sum_l \beta_{kl} f[\Psi_{kl}] R_{1k}^{xy}(\tau_{kl})$, and $\hat{X}_k^{xy} = \sum_l \beta_{kl} f[\Psi_{kl}] \hat{R}_{1k}^{xy}(\tau_{kl})$, where x and y may both be c or s; when $x = y$ (for instance ss), the function f is cos otherwise, it is sin. When $k = 1$, the summation over l is for $l \neq j$. Then,

$$\begin{aligned} z_{0,c} &= \sqrt{P/8}\,T\,\beta_{1j}\,b_{c,1}^0 + \nu \\ &+ \sqrt{P/8}\sum_k \left[d_1(b_{c,k}^{-1})\,b_{c,k}^{-1}\,X_k^{cc} + d_2(b_{c,k}^0)\,b_{c,k}^0\,\hat{X}_k^{cc}\right. \end{aligned}$$

$$+ \quad d_1(b_{s,k}^{-1})\, b_{s,k}^{-1}\, X_k^{cs} + d_2(b_{s,k}^{0})\, b_{s,k}^{0}\, \hat{X}_k^{cs}\Big]$$

where $b_{c,k}^{-1}$ (resp. $b_{s,k}^{-1}$) and $b_{c,k}^{0}$ (resp. $b_{s,k}^{0}$) are the current bit and the previous bit of the kth user in-phase (resp. in-quadrature) data stream. The functions d_1 and d_2 account for the possible hit between the current frequency of user 1 and the frequencies associated with the two bits under consideration. We see that there is cross-rail interference, in the sense that the bits associated with the in-phase (in-quadrature) component interfere with the (in-phase) in-quadrature component. For PSK only, we have :

$$\begin{aligned} z_0 &= \sqrt{P/8}\,T\,\beta_{1j}\,b_1^0 + \nu \\ &+ \sqrt{P/8}\sum_k \left[d_1(b_k^{-1})\,b_k^{-1}\,X_k + d_2(b_k^0)\,b_k^0\,\hat{X}_k\right] \quad (5) \end{aligned}$$

with $X_k = \sum_l \beta_{kl}\cos[\Psi_{kl}]R_{1k}(\tau_{kl})$, and $\hat{X}_k = \sum_l \beta_{kl}\cos[\Psi_{kl}]\hat{R}_{1k}(\tau_{kl})$. When $k = 1$, we have $l \neq j$. In comparison with the QPSK situation, the cross-rail term has disappeared.

3 Study of the bit error probability

The methodology which is followed is similar to the one presented in [4]. The bit error probability is computed as the mean of several situations, corresponding to the possible hits situations produced by the multiple access interferers. Therefore, $P_e = \sum_{n_i=0}^{K-1} P_e(n_i)\,P(n_i)$ where $P_e(n_i)$ is the probability of error assuming there are n_i active interferers out of $K-1$, and $P(n_i)$ is the probability of having n_i active interferers out of $K-1$. For hopping patterns which are first-order stationary Markov and a number of available frequencies q, $P(n_i) = \binom{K-1}{n_i}\left(\frac{1}{q}\right)^{n_i}\left(1-\frac{1}{q}\right)^{K-1-n_i}$ which corresponds to a binomial distribution. This result applied for the multi-access interference assumes that all paths signals of a hitting user cause full hits and neglects the effects due to a partial hit [4]. However the effects associated with partial hits have been shown to be negligible in the case of slow frequency hopping [4]. The multipath interference and the multiple access interference are independent. We assume both are Gaussian distributed. The influence of the multipath term is actually dependent on the bit position in the hopping interval. For the first bit in the block, there is no hit with certitude with the previous bit because they do not necessarily have the same frequency. For all other bits in the blocks, there will be a hit with certitude with the previous and current bit conveyed by interfering paths. We neglect the effect of this first bit. In addition, maximal ratio combining cannot be used for the first and last bits of a hop, because some of the non-reference path signals are partially destroyed by the dehopper. We forget this problem. We also neglect the bits required to solve the phase ambiguity and making the number of useful bits smaller. This approximation is valid when N_b is large, which should be the case in slow frequency hopping. The multipath term on the in-phase signal of the QPSK receiver is given by : $I_{c,mp} = \sqrt{P/8}\left[b_{c,1}^{-1}X_1^{cc} + b_{c,1}^{0}\hat{X}_1^{cc} + b_{s,1}^{-1}X_1^{cs} + b_{s,1}^{0}\hat{X}_1^{cs}\right]$ respectively. For PSK, it is given by : $I_{mp} = \sqrt{P/8}\left[b_1^{-1}X_1 + b_1^0\hat{X}_1\right]$. The multiple access noise corresponding to the assumption of n_i active interferers is given, for QPSK, by $I_{c,ma} = \sqrt{P/8}\sum_{k=2}^{n_i+1}\left[b_{c,k}^{-1}X_k^{cc} + b_{c,k}^{0}\hat{X}_k^{cc} + b_{s,k}^{-1}X_k^{cs} + b_{s,k}^{0}\hat{X}_k^{cs}\right]$ and by $I_{ma} = \sqrt{P/8}\sum_{k=2}^{n_i+1}\left[b_k^{-1}X_k + b_k^0\hat{X}_k\right]$ for PSK. The bit error probability assuming n_i interferers is defined as the probability that the decision variable is lower than 0 assuming a '1' was transmitted (or conversely). It is the same for each of the 2 components when QPSK is considered. If we say $z_{0,c} = \sqrt{P/8}\,\beta_{1j}\,b_{c,1}^0\,T + I_{c,mp} + I_{c,ma} + \nu_c$, we have that $P_e^Q = P[z_{c,0} < 0|b_{c,1}^0 = 1]$. For PSK, we have $z_0 = \sqrt{P/8}\,\beta_{1j}\,b_1^0\,T + I_{mp} + I_{ma} + \nu$ and $P_e^P = P[z_0 < 0|b_1^0 = 1]$. In order to compute exactly the power of the multipath and the multiple access terms for B- and QPSK, some comments have to made. We assume to deal with a given channel, which means that the absolute value of the delay range is known. We denoted by T the bit duration in the BPSK system, and assumed a delay uniformly distributed over this bit duration. Accordingly, with QPSK, the bit duration is twice the one of BPSK. Therefore, *the delay for QPSK has to be considered as being uniformly distributed over half the QPSK bit duration.* Considering this, and according to an approach proposed in [1], the power of the multipath and multiple access terms are well approximated by $\frac{P}{8}(L-1)\left(\sigma_r^2 + \frac{S^2}{2}\right)\frac{2T^2}{3N}$ and $\frac{P}{8}n_i L\left(\sigma_r^2 + \frac{S^2}{2}\right)\frac{2T^2}{3N}$ for both QPSK and BPSK. The bit error probability can be computed as a function of the bit energy to total noise power spectral density assuming the value of β_{1j}. The bit error probability has to be averaged over the distribution of β_{1j}. If we further assume that the multipath interference associated with the reference user is due to L paths instead of $L-1$, using the normalization $\alpha_{1j} = \frac{\beta_{1j}}{(2\sigma_r^2+S^2)^{0.5}}$, we can write for

$$P_e(n_i) = \frac{1}{2}\mathrm{erfc}\left\{\left[\left(\frac{E_b(2\sigma_r^2+S^2)}{N_0}\right)^{-1}\frac{1}{\alpha_{1j}^2}\right.\right.$$

$$+ \quad \frac{2L}{3N\alpha_{1j}^2}(1+n_i)\Big]^{-0.5}\Bigg\} \qquad (7)$$

and the bit error probabilities are given as functions of the mean received bit energy to noise ratio.

4 Performance with diversity

4.1 Selection diversity

Selection diversity means that the largest one of a group of M signals carrying the same information is selected. The order of diversity is equal to the number of paths times the number of antennas. The decision variable is $z_{0,\max}^{M}$ which is the largest among the set of M values. We can take the result of the previous section, and average the bit error probability on the pdf of the maximum instead of that of β.

4.2 Maximal ratio combining

In the case of maximal ratio combining, the contribution of several resoluted paths are added together. The combiner which achieves the best performance is one in which each matched filter output is multiplied by the corresponding complex-valued channel gain. The phase shift is compensated for and a weighting associated with the signal strength is introduced. The realization of such a receiver is based on the assumption that the channel attenuations and the phase shifts are perfectly known. Let us consider again the decision variable $z_0 = \mathrm{Re}\left[\sqrt{\frac{P}{8}}T\sum_{i=1}^{M}\beta_{1i}^2 + \sum_{i=1}^{M}\beta_{1i}N_{1i}\right]$. Each term involved in the combination process is corrupted by AWGN, the multiuser interference and the multipath delayed versions. We make the assumption that the noise due to the multiuser interference and the residual multipath interference is also Gaussian and that all noise terms add together. We also assume that the noise terms affecting 2 different paths are independent. This is of course not correct. We need to average the bit error probability over the pdf of $t = \sum_{i=1}^{M}\beta_{1i}^2$. We define $v = t/[M\,(2\sigma_r^2 + S^2)]$. The following bit error probability, averaged over the pdf of v provides the bit error probability as a function of the mean received bit energy to noise ratio.

$$\begin{aligned} \mathrm{P}_e^P(n_i) &= \frac{1}{2}\mathrm{erfc}\left\{\left[\left(\frac{E_b\,(2\sigma_r^2+S^2)}{N_0}\right)^{-1}\frac{1}{Mv}\right.\right. \\ &+ \left.\left.\frac{2L}{3NMv}(1+n_i)\right]^{-0.5}\right\} \end{aligned} \qquad (8)$$

5 Computational results and discussion

This section is devoted to the presentation of some computational results. Equation 9 shows that a bit error curve derived for QPSK and BPSK are the same, assuming the same absolute delay range is taken into account. For the computations, the following parameters have been considered : number of resolvable paths $L = 8$, code length $N_b = 255$ or $N_b = 127$, number of users $K = 15$, Rician parameter $R = 6.8dB$. Figure 1 provides the results obtained by means of Selection Diversity (SC) for 10 frequencies ($q = 10$) and code lengths of $N_b = 127$ or $N_b = 255$, with the order of diversity as a parameter. Figure 2 shows the same results for Maximal ratio combining. As expected, MRC performs better than SC, and the performances of both systems are improved when the order of diversity increases. Besides, the effect of increasing the code length is also obvious. Figures 3 and 4 provide the same results as figures 1 and 2 but for $q = 50$. We also see that a higher number of frequencies improves the performances of the hybrid system. If the number of frequencies could be made infinite, the multiuser interference would be completely removed, but the multipath interference would still be present. As regards the comparison of QPSK with BPSK, some constraints have to be taken into account. If we require that the bit rate in both systems should be the same, we will have $T_q = 2T_b$ where T_q and T_b are the bit durations for QPSK and PSK respectively. In addition, we require the bandwidths be the same. It means that the ratio Nq/T should be the same for both systems. More accurately : $\frac{N_p q_p}{T_p} = \frac{N_q q_q}{T_q}$. Considering the bit rate constraint leads to $N_q q_q = 2N_p q_p$. If we assume the same number of frequencies $q_q = q_p$, we finally end with $N_q = 2N_p$. From equation 9, we see that with these choices, the multiple access noise and the multipath noise for QPSK are divided by 2 compared to BPSK. Stated with other words, *under the constraints of constant bandwidth and bit rate, for a given value of the bit energy over white noise psd ratio, we obtain with QPSK and a given code length, the performance we would obtain by means of BPSK and a code with 2 times the length of the QPSK code.* A same bit energy means identical product of power and bit duration. If P is the power used for BPSK, the power used with QPSK should be P/2 for each carrier, which means the same overall power. As an example, under the bandwidth and bit rate constraints, the performance of a QPSK system with a code length of 128 chips is equivalent to the one of a BPSK system with a code length of 256. Therefore, the curves giving the results for $N_b = 127$ and

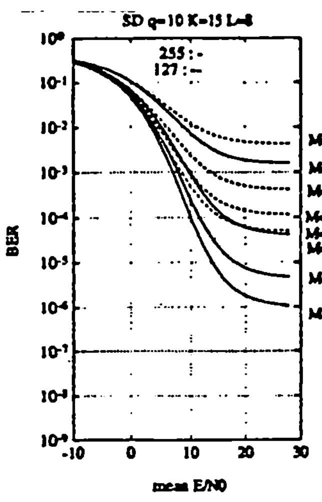

Figure 1: BER for DS/SFH with selection diversity with $M = 1,2,3,4$ and 2 code lengths. Number of frequencies $q = 10$

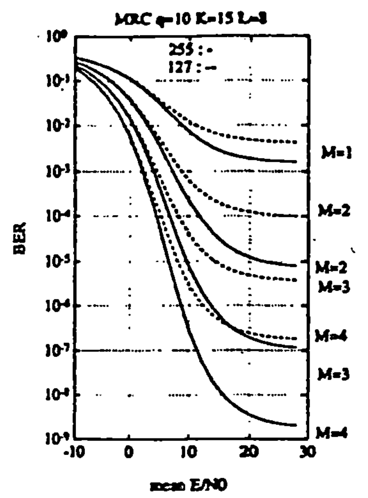

Figure 2: BER for DS/SFH with maximal ratio combining with $M = 1,2,3,4$ and 2 code lengths. Number of frequencies $q = 10$

$N_b = 256$ are representative of the performance of BPSK and QPSK under constraints of fixed bandwidth, bit rate, energy and delay range.

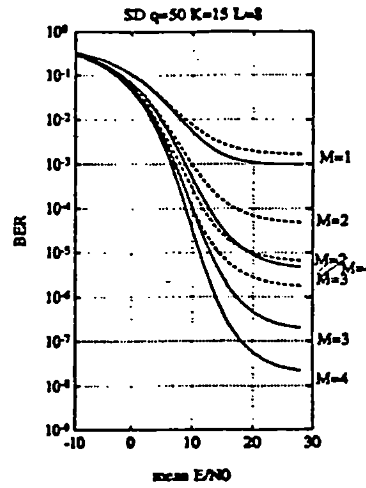

Figure 3: BER for DS/SFH with selection diversity with $M = 1,2,3,4$ and 2 code lengths. Number of frequencies $q = 50$

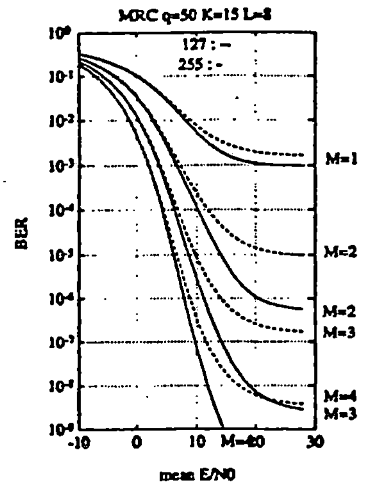

Figure 4: BER for DS/SFH with maximal ratio combining with $M = 1,2,3,4$ and 2 code lengths. Number of frequencies $q = 50$

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