

Document Version

Final published version

Citation (APA)

Van der Werf, I. (2026). *Channel-Aware OTFS Modulation for Underwater Acoustic Communications: From Theoretical Analysis to Experimental Validation*. [Dissertation (TU Delft), Delft University of Technology].
<https://doi.org/10.4233/uuid:fae4dda8-23b9-4b1e-aef8-08e19ce81094>

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Channel-Aware OTFS Modulation

Ids van der Werf

for
Underwater
Acoustic Communications

From Theoretical Analysis to Experimental Validation



CHANNEL-AWARE OTFS MODULATION FOR UNDERWATER ACOUSTIC COMMUNICATIONS

**FROM THEORETICAL ANALYSIS TO EXPERIMENTAL
VALIDATION**

CHANNEL-AWARE OTFS MODULATION FOR UNDERWATER ACOUSTIC COMMUNICATIONS

**FROM THEORETICAL ANALYSIS TO EXPERIMENTAL
VALIDATION**

Dissertation

for the purpose of obtaining the degree of doctor
at Delft University of Technology
by the authority of the Rector Magnificus,
Prof. dr. ir. H. Bijl,
chair of the Board for Doctorates
to be defended publicly on
Tuesday 27 October 2026 at 17:30

by

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This research was financially supported by the Netherlands Organisation for Applied Scientific Research (TNO) and the Netherlands Defence Academy (NLDA), reference no. TNO-10026587.



Ministerie van Defensie

Keywords: Underwater acoustic communications, doubly selective channels, pilot design, modulation, V-OFDM, OTFS, OSDM

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ISBN 978-94-6518-431-9

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Summary

Underwater wireless communication is a cornerstone of modern marine exploration, infrastructure maintenance, and naval operations. While terrestrial systems rely on electromagnetic radio-frequency (RF) waves, these are impractical underwater due to rapid attenuation. Consequently, underwater communication typically utilizes acoustic pressure waves, which can propagate over tens of kilometers. However, underwater acoustic (UWA) communication is characterized by limited bandwidth, high latency due to slow propagation, and severe distortions from multipath-induced delay and motion-induced Doppler spreads. This dissertation investigates the design of an UWA system that is both analytically tractable and sufficiently flexible to adapt to these volatile conditions in real time.

The research begins by examining state-of-the-art communication schemes in both UWA and terrestrial domains. A key theoretical contribution is the demonstration that two prominent schemes, developed independently in their respective fields, are mathematically equivalent. Further analysis of the time-frequency signal structure reveals that the benefit of these schemes stems from the fact that information is spread across both time and frequency.

A central challenge addressed is the trade-off between training and data transmission. By proposing a novel training design, this dissertation demonstrates a 50% reduction in training overhead compared to conventional designs without sacrificing estimation accuracy. Finally, to bridge the gap between theory and application, a channel-aware adaptation protocol is introduced. This allows the system to optimize parameters in-situ, ensuring a robust link. The approach was validated through synthetic simulations, channel replay, and real-world experiments.

Samenvatting

Draadloze onderwatercommunicatie speelt een belangrijke rol in moderne maritieme exploratie, onderhoud van infrastructuur en marineoperaties. Waar systemen op het land gebruik maken van elektromagnetische radiofrequentiegolven, zijn deze onder water onpraktisch vanwege de sterke verzwakking (attenuatie). Onderwatercommunicatie maakt daarom doorgaans gebruik van akoestische drukgolven, die zich over tientallen kilometers kunnen voortplanten. Akoestische onderwatercommunicatie wordt echter gekenmerkt door een beperkte bandbreedte, hoge latentie door de lage voortplantings-snelheid, en ernstige vervormingen veroorzaakt door echo's en frequentieverschuiving door beweging. Dit proefschrift onderzoekt het ontwerp van een akoestisch onderwater-communicatiesysteem dat zowel analytisch hanteerbaar als voldoende flexibel is om zich aan deze veranderlijke omstandigheden aan te passen.

Het onderzoek begint met het bestuderen van de huidige stand van de techniek op het gebied van zowel akoestische onderwater- als terrestrische communicatieschema's. Een belangrijke theoretische bijdrage is het bewijs dat twee prominente schema's, die onafhankelijk van elkaar in hun respectievelijke vakgebieden zijn ontwikkeld, wiskundig equivalent zijn. Verdere analyse van de signaalstructuur in het tijd-frequentiedomein laat zien dat het voordeel van deze schema's voortkomt uit het feit dat informatie over zowel tijd als frequentie wordt gespreid.

Een centrale uitdaging die in dit proefschrift wordt behandeld, is de afweging tussen training en datatransmissie. Door een nieuw trainingsontwerp voor te stellen, toont dit proefschrift een reductie van 50% in trainingsoverhead aan in vergelijking met conventionele ontwerpen, zonder daarbij in te leveren op de nauwkeurigheid van de kanaalschatting. Tot slot wordt, om de kloof tussen theorie en praktijk te overbruggen, een kanaalbewust adaptatieprotocol geïntroduceerd. Hiermee kan het systeem de parameters in-situ optimaliseren, wat in een robuuste communicatieverbinding resulteert. De haalbaarheid van deze aanpak is gevalideerd door middel van synthetische simulaties, channel replay-simulaties en experimenten in de praktijk.



1

Introduction

This chapter establishes the context of the dissertation. It outlines the principal challenges in underwater communication and examines the role of signal processing in addressing these. The chapter presents the current research limitations and objectives considered in this work. It concludes with an outline of the dissertation structure.

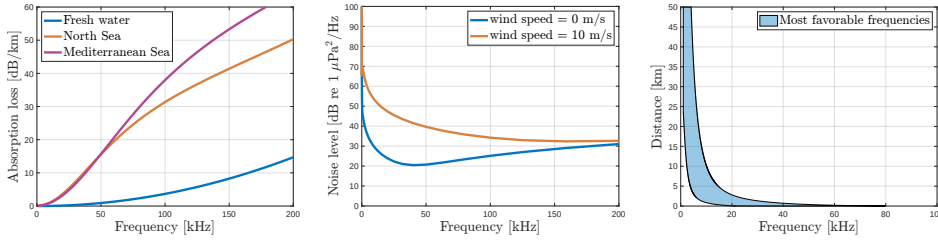
Communication is a vital part of life on Earth and comes in many forms. Trees, for instance, appear to communicate underground through complex fungal networks that connect their roots [1]; peacocks communicate visually by displaying their vibrant feathers; and many animals, mammals in particular, use sound to coordinate, navigate, or signal intent. Humans have historically made use of the same fundamental communication modalities found in nature: chemical, visual, and acoustic. Over the past century, however, we have also started to communicate through engineered systems, most notably through the invention of radio-frequency technology, as used by mobile phones. Another form of human-engineered communication, although less noticeable in everyday life, is wireless underwater communication. Its origin lies in the need for ships to navigate safely or to be warned of fixed hazards. At the start of the 20th century, a fog horn and underwater bell would be sounded simultaneously from the coast. By measuring the difference in time-of-arrival of the two signals, a ship could determine its offshore distance [2]. The development of underwater communication surged during the two World Wars, eventually leading to the invention of the underwater telephone in 1945 [3], which made it possible to communicate with submerged submarines. Since then, underwater communication drew relatively little public attention until the sabotage of the Nord Stream pipeline in the Baltic Sea in September 2022. Despite this low visibility, substantial scientific progress has been made over the decades. Today, underwater communication is increasingly used in commercial applications such as maintenance of offshore structures, sea-life monitoring, and inspection of pipelines and power lines, alongside its traditional role in naval warfare.

Underwater communication is typically carried out using acoustic pressure waves. Although electromagnetic waves, including both radio-frequency signals and optical waves (i.e., light), would be desirable due to their higher bandwidth and faster propagation speed, they are strongly attenuated in (sea)water. Therefore, electromagnetic waves are unsuitable for long-range underwater communication. Optical waves provide a familiar example: light is rapidly scattered and absorbed, especially in turbid water. Acoustic waves, on the other hand, can propagate over much greater distances but introduce significant challenges for the design of communication systems, as will be discussed below.

1.1. Challenges of acoustic communications

Communication systems are designed to maximize the data rate – the amount of information transmitted per unit time. Higher data rates enable more sophisticated applications, such as video streaming, rather than just simple text messages. However, data rates achievable with underwater acoustic communication systems are several orders of magnitude lower than those for (terrestrial) radio-frequency systems. Typical acoustic data rates are on the order of kbit/s, whereas radio-frequency links commonly operate on the order of MB/s.¹ As a result, sending a message through an acoustic link can easily take 8000 times longer. This enormous gap has several underlying causes.

¹Note that one byte corresponds to eight bits; hence $1 \text{ MB/s} = 8000 \text{ kbit/s}$.



(a) Absorption loss for sea and fresh waters (model adapted from [4]). (b) Noise level for different wind speeds (model from [5]). (c) Most favorable frequencies (adapted from [6]).

Figure 1.1.: Due to the frequency-dependent absorption loss (a) and noise level (b), the usable bandwidth for underwater acoustic communications is limited (c).

1.1.1. Limited usable bandwidth

The usable bandwidth, meaning the range of frequencies that can effectively carry information, in underwater acoustic communication systems is severely limited. This arises from the interplay of propagation loss and ambient noise. The limited usable bandwidth constrains the maximum achievable data rate, much like a narrow pipe restricts the flow of water.

Propagation loss

As an acoustic wave propagates through water, its energy, which is directly related to its pressure level, decreases. This propagation loss arises from two mechanisms: spreading loss and absorption loss. As the wave travels, its energy spreads geometrically over a larger volume, reducing its pressure level (spreading loss). In parallel, part of the wave's energy is absorbed by the medium (absorption loss), e.g. converted into heat. Absorption loss depends on environmental conditions such as temperature and salinity, but most strongly on the frequency of the wave: higher frequencies suffer much greater absorption, limiting their usable range. This effect is illustrated in Figure 1.1a, which shows the frequency-dependent absorption loss for sea and fresh water. Consequently, only a relatively small portion of the low-frequency spectrum remains usable for communication over practical distances.

Ambient noise

The usable bandwidth is further constrained by noise, the background sound naturally present in water. This noise has several origins. Marine life and nearby ships generate sound, and the environment itself contributes through wind, rainfall, breaking waves, and swell generated by distant storms. Because the noise level varies across the spectrum, the extent to which noise affects a communication link depends strongly on the frequencies used by the communication system. However, also other factors such as operating depth and local bathymetry play an important role. Figure 1.1b illustrates the noise level as a function of frequency for different wind speeds.

To summarize, the usable bandwidth is squeezed from both sides: high frequencies are absorbed before they can travel useful distances (Fig. 1.1a), while low frequencies are masked by ambient noise (Fig. 1.1b). The blue region in Fig. 1.1c captures this directly. Its right edge is pushed left by absorption, and its left edge is pushed right by noise, leaving only a narrow window of favorable frequencies in between. For this reason, underwater acoustic systems typically operate between approximately 2 kHz and 40 kHz. Within this usable range, the most favorable frequencies depend on the communication distance. Lower frequencies support longer ranges and higher frequencies can be used for shorter links.

1.1.2. Large propagation delays

Another challenge in underwater acoustic communication is the propagation delay – the time it takes for a message to travel from the sender to the receiver – which is relatively large compared to radio-frequency systems. This is because acoustic waves travel through water at only about 1500 m/s, which is extremely slow compared to electromagnetic waves that travel at the speed of light, roughly 3×10^8 m/s.

The propagation delay does not directly limit the data rate itself. This is similar to turning on a tap: there is a short wait before the water arrives, but the flow rate itself is unchanged once it does. However, it has a strong indirect impact because the resulting latency affects the communication protocols. For instance, if a message is received incorrectly and a retransmission is required, the request for retransmission takes a long time to reach the sender, and the repeated message takes equally long to return. These long round-trip times reduce the throughput of messages and hence the effective data rate.

1.1.3. Channel distortions

When a message travels through the ‘channel’, i.e., as it propagates from the sender to the receiver, it can become distorted. These channel distortions have various origins.

Delay spread

Firstly, in the underwater acoustic channel, multiple propagation paths typically exist between the sender and the receiver, for example a direct path and reflections from the water surface or seabed, as visualized in Fig. 1.2. As a result, the receiver often observes delayed replicas of the same message, which degrade correct reception (much like echoes in a church can make speech difficult to understand). The delay spread, the time difference between the first and last significant arrival, is often several tens of milliseconds.

Doppler spread

Secondly, motion of the water surface and relative movement between sender and receiver cause frequency shifts, known as Doppler shifts (similar to how an ambulance siren changes pitch as it passes). Because the relative motion can differ across propagation paths, each path may experience a different frequency shift. The Doppler

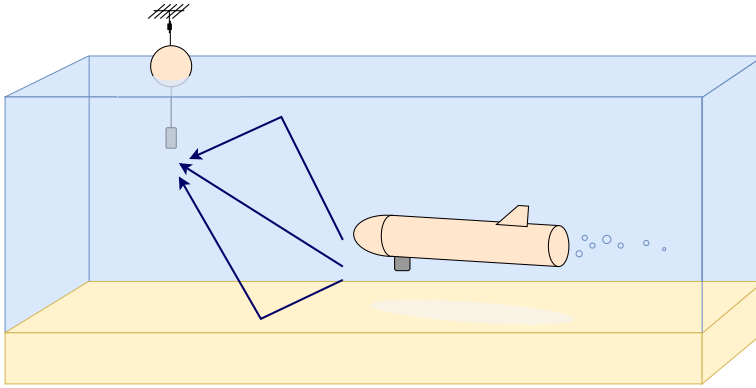


Figure 1.2.: Example of an underwater acoustic communication scenario: an autonomous vehicle sends a message to a gateway buoy, which can relay the message using radio-frequency communications. Due to reflections from the water surface and seabed, the buoy receives multiple delayed replicas of the same message.

spread is the frequency-domain equivalent of delay spread and refers to the difference between the largest positive and largest negative significant frequency shift observed across all paths. The magnitude of the shift depends on the ratio v/c , where v is the relative velocity along the path and c is the propagation speed of the wave. Hence, the more surface waves or the higher the relative velocity between the sender and receiver, the more the message is distorted.

Non-homogeneous propagation speed

The temperature, salinity and pressure levels in the water column are not uniform, which leads to spatial variations in the sound speed. This causes acoustic waves to bend rather than travel in straight lines, as waves always refract toward regions of lower propagation speed. In deep waters, such as oceans, temperature, salinity and pressure gradients are common and bending often introduces additional reflections. This increases the extent of multipath propagation and can enhance Doppler effects, further distorting the received signal.

Underwater acoustic vs. radio-frequency communication

While the challenges for underwater acoustic communication are often analogous to those encountered in radio-frequency systems, they are far more pronounced due to the relatively low propagation speed in water. For instance, delay spread causes much stronger distortion in acoustic channels because the propagation speed is low, making path delays comparable to typical message durations, whereas in radio-frequency channels the high speed of electromagnetic waves keeps delays small relative to the signal length. Similarly, Doppler effects are amplified: since the magnitude of the frequency shift depends on the ratio v/c , the same relative velocity produces substantially larger Doppler shifts in acoustic channels, resulting in a larger Doppler spread. Consequently,

underwater acoustic communication requires solutions specifically tailored to these conditions. Although concepts and signal processing techniques from radio-frequency systems can provide inspiration, direct application is generally not possible.

1.2. The role of signal processing

As discussed in the previous section, underwater acoustic communication faces several significant challenges. The field of signal processing plays a crucial role in addressing these challenges by developing models that describe the channel distortions, algorithms that estimate the resulting effects, and methods that compensate for these distortions during reception.

Channel modeling

Although the physical origins of underwater channel distortions are known, accurately modeling the channel remains difficult. The received signal is shaped by a complicated mix of reflections, refractions, and motion-induced Doppler shifts. In addition, many influential factors, such as the temperature profile, surface-wave motion, seabed properties, and platform movement are unknown or only partially measurable. For this reason, underwater channels are typically described using simplified physical or statistical models that capture their overall behavior rather than their exact details. These models provide the foundation on which a communication system is built.

Channel estimation and equalization

Over the past decade, much research has focused on *channel estimation*, techniques that determine how the channel distorts the transmitted signal, and on *equalization*, algorithms that reverse these effects to recover the original message. In practice, the performance of the equalization depends critically on the accuracy of the channel estimation. In other words, one first has to estimate how the transmitted signal is distorted before attempting to undo those effects. The overall performance and achievable data rate of a communication system therefore rely on how effectively these two components work together.

While these algorithms operate at the receiver, their exact implementation and performance are fundamentally influenced by the design of the transmitted signal itself. The structure of the transmitted waveform determines how, and how well the receiver can estimate the channel and how efficiently it can recover the data.

Design of the transmitted signal

Every transmitted message generally consists of two parts: a training portion, which is known to the receiver, and a data portion, which carries the information to be decoded.² Throughout this work, the terms “training” and “pilot” are used interchangeably to describe the known signal portion. Even with this basic division, there is considerable freedom in how the transmitted signal is constructed.

²Blind equalization techniques, which operate without explicit training, are considered outside the scope of this dissertation.

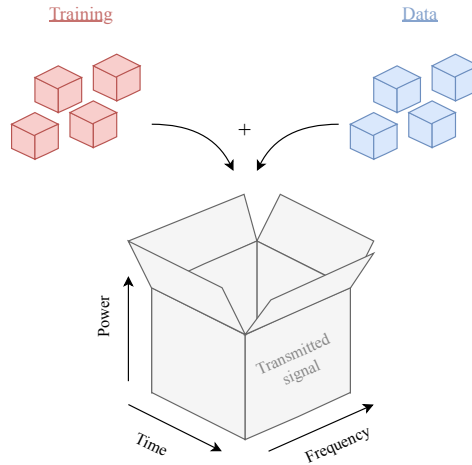


Figure 1.3.: Visualization of the transmit signal design problem. Training and data parts of the signal have to be ordered in time and frequency. Placing one cube on top of another indicates more power is to be transmitted at a specific time and frequency.

An intuitive way to understand this design space is to imagine that power can be placed in two domains: time and frequency, see Fig. 1.3. The transmitter must decide when (time) to send training and at which frequencies (frequency). These choices lead to fundamental trade-offs. More training improves channel estimation and makes equalization easier, but leaves less room for data. Too little training increases the data rate, but risks poor channel estimates and hence degraded equalization. Transmit signal design therefore requires balancing these competing objectives.

1.2.1. Limitations of existing research

The challenges described above are highly dependent on the environment, e.g., temperature, wind conditions, water depth, distance, and seabed characteristics. As a result, different locations can exhibit very different channel conditions. For example, a harbor is typically a relatively static environment with minimal Doppler effects, but the presence of hard quay walls can lead to strong and complex reflections. In contrast, the open sea is far more dynamic: surface waves and underwater currents introduce significant motion, resulting in larger Doppler shifts. Moreover, different seas or ocean regions can behave very differently due to variations in temperature profiles, salinity, or even surface ice. These strong location-dependent variations make it difficult to design a single, universally effective communication system. Signal processing algorithms and parameters that work well in one environment may perform poorly in another, meaning that models, channel estimators, and equalizers must either be carefully tuned to local conditions or designed to adapt automatically.

Lack of analytical tractability

Most underwater communication systems rely on a highly nonlinear receiver structure. While such systems can perform well in practice (when tuned well), they typically lack a closed-form analytical description. This makes it difficult to understand their behavior theoretically, predict performance under new channel conditions, or optimize them systematically. As a consequence, system tuning usually relies on large-scale parameter searches, which are computationally demanding and risk overfitting to specific measurement conditions.

Neglected training/data trade-off

To cope with environmental variability, some research (e.g., [7], [8], [9]) has proposed running multiple receiver configurations in parallel, each with different (hyper)parameters, and selecting the one that performs best. However, such approaches focus solely on the receiver and neglect the design of the transmitted signal. In reality, the balance between training and data, and the allocation of power across time and frequency, strongly influences both channel estimation and equalization performance. Ignoring these trade-offs limits the overall adaptability of the system.

Insufficiently flexible adaptive frameworks

Another line of work proposes using a small set of predefined system configurations, each combining a particular transmit signal and receiver algorithm, and learning when to switch between them. Although this provides some adaptability, the approach has two fundamental limitations: i) a small, fixed collection of configurations cannot capture the wide variety of channel behaviors encountered in practice, and ii) switching decisions are driven by communication system performance rather than by underlying channel conditions. Moreover, the communication performance metrics often do not depend on the system parameters. Consequently, such methods offer only coarse, reactive rather than condition-aware adaptation.

1.3. Research objective

The limitations identified above clearly highlight the need for underwater acoustic communication systems that are both analytically tractable and sufficiently flexible to cope with highly time-varying environments. This motivates the development of a simple, interpretable system architecture that can be optimized to fit diverse channel conditions and adapted to changing environments in real time. The work focuses specifically on short-range uplink scenarios, typically up to a few kilometers, where platform motion induces significant Doppler spreading, making the channel highly time-varying. The overarching aim of this dissertation is therefore:

To design an underwater acoustic system that can adapt to the channel conditions in real time.

To address this main goal, the dissertation investigates three core aspects: (i) understanding the behavior of simple modulation schemes under time-varying conditions, (ii) optimizing transmit-signal design when channel statistics are available, and (iii) enabling real-time system adaptation when statistics are unknown. To this end, the following key questions are posed.

RQ1: *How do state-of-the-art modulation schemes perform under time-varying channels?*

RQ2: *Given channel statistics, how can the transmitted signal be optimized to maximize throughput?*

RQ2.1 *Which pilot allocation strategies minimize overhead while ensuring accurate channel estimation performance?*

RQ2.2 *How should power be distributed between pilot and data signals to maximize throughput?*

RQ3: *How can the communication system be adaptively optimized in real time to maximize throughput under changing channel conditions?*

1.4. Dissertation outline

The remainder of this dissertation is organized as follows. Figure 1.4 provides a visual roadmap of the chapter progression and illustrates how each relates to the core research questions. The core technical chapters, Chapter 3 through Chapter 7, are arranged along a spectrum ranging from theoretical foundations to experimental validation.

To assist readers in navigating this work based on their specific interests:

- For a general overview: Readers seeking a high-level summary of the contributions are advised to focus on Chapters 1 and 8.
- For theoretical depth: Those interested in the underlying framework and design methodology should refer to Chapters 3, 4 and 6.
- For implementation and validation: Readers focused on practical results and experimental setups are directed to Chapters 5 to 7.

The following paragraphs provide a brief overview of the specific contributions of each chapter.

Chapter 2: Background provides the necessary theoretical background for the contributions presented later in this thesis. It introduces key concepts in (underwater acoustic) communication theory, aiming to bridge the gap for researchers in signal processing who may lack specific expertise in these areas.

Chapter 3: Modulation Techniques addresses the first research question by investigating how various modulation schemes influence the transmitted signal. This chapter provides a comparative analysis of several techniques, evaluating their structural similarities and their resilience to specific channel distortions.

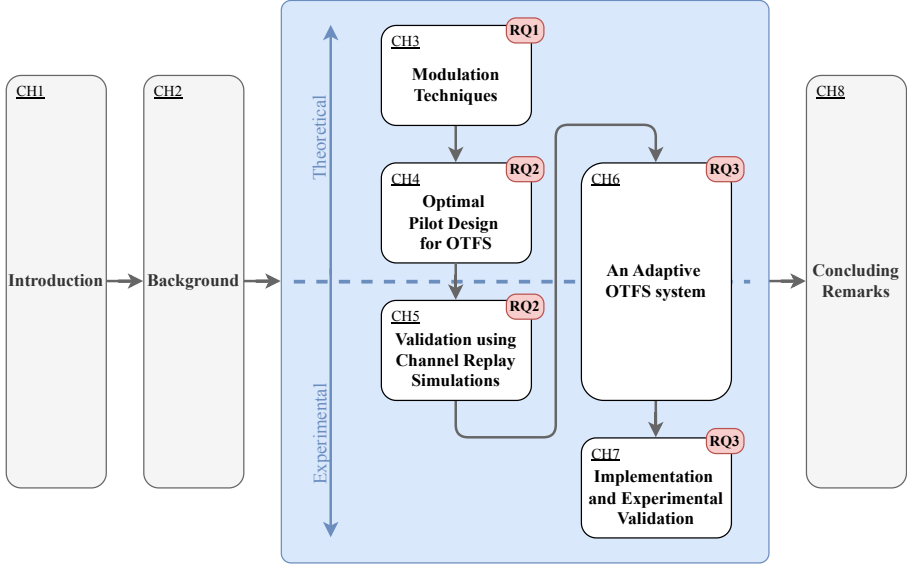


Figure 1.4.: Schematic representation of the dissertation structure. The core research chapters (CH3–CH7) are categorized by their theoretical or experimental focus and are mapped to the primary research questions (RQ1–RQ3).

Chapter 4: Optimal Pilot Design for OTFS focuses on the optimization of pilot signals to address the second research question. It evaluates various pilot designs across multiple modulation schemes, comparing them in terms of overhead and channel estimation performance. Moreover, this chapter proposes a novel pilot design specifically tailored for orthogonal time frequency space (OTFS) modulation.

Chapter 5: Validation using Channel Replay Simulations provides empirical validation of the proposed pilot design through channel replay simulations. By comparing the proposed framework against current state-of-the-art approaches, this chapter further addresses the second research question and demonstrates the performance gains of the new design in controlled but realistic environments.

Chapter 6: An Adaptive OTFS System further bridges the gap between theory and practice, extending the analysis of Chapter 4 to the more complex, real-world underwater acoustic (UWA) channels. It details the integration of the proposed framework into a communication system, thereby addressing the third research question.

Chapter 7: Implementation and Experimental Validation considers the practical implementation of the methods introduced by the preceding chapters. It describes the system architecture and reports on the communication performance during real-world experiments.

Chapter 8: Concluding Remarks concludes the dissertation by revisiting the three primary research questions. It synthesizes the theoretical and experimental findings to provide comprehensive answers. Additionally, it identifies remaining technical challenges, and discusses promising directions for future research in UWA communications.

1.5. List of publications and contributions

Included publications and contributions

The research contributions included in this thesis are the following:

Journal articles

- J1 **I. van der Werf**, K.C.H. Blom, G. Leus, R. C. Hendriks, and R. Heusdens, “An Adaptive OTFS System for Underwater Acoustic Communications,” *submitted to IEEE Journal on Oceanic Engineering*, Feb. 2026.
- J2 **I. van der Werf**, R. Heusdens, R. C. Hendriks and G. Leus, “Optimal Pilot Design for OTFS in Linear Time-Varying Channels,” *Signal Processing*, vol. 239, p. 110223, Feb. 2026.
- J3 **I. van der Werf**, H. Dol, K. Blom, R. Heusdens, R. C. Hendriks, and G. Leus, “On the equivalence of OSDM and OTFS,” *Signal Processing*, vol. 214, p. 109254, Jan. 2024.

Conference proceedings

- C1 **I. van der Werf**, K. Pelekanakis, R. C. Hendriks, R. Heusdens and G. Leus, “Optimized Pilot Design for OTFS Modulation in Underwater Acoustic Communications,” *7th Underwater Communications and Networking Conference (UComms)*, Sestri Levante, Italy, Sept. 2024, pp. 1-5.

Other achievements

- A1 Finalist in the three minute thesis (3MT) presentation contest at EUSIPCO 2023, Helsinki. Published as:
I. van der Werf, Richard Heusdens, Richard C. Hendriks, “Doppler-tolerant underwater communications,” *Science Talks*, Volume 9, Mar. 2024.

Excluded publications and contributions

The research contributions excluded from this thesis are the following:

Journal articles

- J4 **I. van der Werf**, R. Rajamäki and G. Leus, “Jointly Optimal Array Geometries and Waveforms in Active Sensing,” in *IEEE Transactions on Signal Processing*, (Early Access), Jun. 2025.

Conference proceedings

- C2 **I. van der Werf** and G. Leus, “On Jointly CRB-optimal Arrays and Waveforms for Multi-Target Active Sensing,” *59th Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, USA, Oct. 2025, pp. 1724-1728.
- C3 **I. van der Werf**, G. Leus, R. Rajamäki, “Jointly Optimal Array Geometries and Waveforms in Active Sensing: New Insights Into Array Design via the Cramér-Rao

- Bound,” *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Hyderabad, India, Apr. 2025, pp. 1-5.
- C4 **I. van der Werf**, S. P. Chepuri and G. Leus, “Sparse MIMO Array Design Using Uplink-Downlink Duality and Hole-free Zone Co-array Sensing,” *58th Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, USA, Oct. 2024, pp. 396-400.
- C5 **I. van der Werf**, C. A. Kokke, R. Heusdens, R.C. Hendriks, G. Leus and M. Coutino, “Transmit and Receive Sensor Selection Using the Multiplicity in the Virtual Array,” *32nd European Signal Processing Conference (EUSIPCO)*, Lyon, France, Aug. 2024, pp. 2272-2276.
- C6 **I. van der Werf**, G. Leus and S. P. Chepuri, “Receiver Antenna Allocation for Joint Sensing and Communications,” *IEEE Sensor Array and Multichannel Signal Processing Workshop (SAM)*, Corvallis, USA, July 2024, pp. 1-5.
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- C8 **I. van der Werf**, R. C. Hendriks and R. Heusdens, “Channel Parameter Estimation Using a Wideband LFM Preamble: Comparison of the Fractional Fourier Transform and Matched Filtering,” *31st European Signal Processing Conference (EUSIPCO)*, Helsinki, Finland, Sept. 2023, pp. 1430-1434.
- C9 **I. van der Werf**, P. Martínez-Nuevo, M. Møller, R. Hendriks and J. Martinez, “Towards Gridless Sound Field Reconstruction,” *2022 30th European Signal Processing Conference (EUSIPCO)*, Belgrade, Serbia, Aug. 2022, pp. 862-866.

Other achievements

- A2 Best student presentation award for the presentation:
I. van der Werf, “Towards Gridless Sound Field Reconstruction,” *IEEE-SPS/EURASIP Summer School on “Data and Graph Driven Learning for Communications and Signal Processing”*, Banja Luka, Bosnia and Herzegovina, Sept. 2022.

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2

Background

The previous chapter established the context and key challenges of underwater communication and posed the research objective of this dissertation.

This chapter serves as an introduction to fundamental signal processing techniques for (underwater acoustic) communications. It begins with a mathematical model of a general communication pipeline in Section 2.1, followed by a description of common practices in underwater acoustic communications in Section 2.2.

2.1. The general communication pipeline

This section discusses the communication pipeline step by step. First, in Section 2.1.1, the channel model is introduced, after which Sections 2.1.2 and 2.1.3 consider the pipeline at baseband and normalized baseband, respectively.

2.1.1. The physical channel model: the linear time-varying impulse response

As discussed in the previous chapter, underwater communication relies on the transmission of acoustic pressure waves. Let the transmitted signal $x(t)$ denote the acoustic pressure emitted by the source as a function of time. In principle, the received signal at a given location could be obtained by solving the underlying (acoustic) wave equations for the propagation environment. Such an approach, however, becomes intractable in realistic scenarios involving multiple propagation paths, relative motion between the transmitter, receiver, and surrounding scatterers, or non-homogeneous propagation speeds.

Fortunately, deriving an explicit physical solution for every propagation scenario is not required, as these effects can be accurately captured by modeling the underwater acoustic channel as a linear time-variant (LTV) system [1]. The (noiseless) received signal is expressed as

$$r(t) = \int_{-\infty}^{+\infty} h(t, \tau) x(t - \tau) d\tau, \quad (2.1)$$

where $h(t, \tau)$ denotes the time-varying channel impulse response (CIR) in the *time-delay* domain. A visualization of the communication pipeline including only the physical channel is provided in Fig. 2.1.

Example 1 (Linear time-invariant (LTI) channel). *Suppose that a single line-of-sight propagation path of length d exists between the transmitter and the receiver. The corresponding CIR is then $h(t, \tau) = \alpha \delta(\tau - \frac{d}{c})$ where c denotes the speed of propagation and $\alpha \in \mathbb{R}$ models the path loss. The received signal is consequently given by $r(t) = \alpha x(t - \frac{d}{c})$.*

Since the channel impulse response does not depend on the observation time t , the channel is time-invariant. Thus, the general linear time-varying channel model in (2.1) subsumes the LTI channel as the special case.

Domains of the channel impulse response (CIR)

Apart from the time-delay domain of the CIR characterized by $h(t, \tau)$, it can be useful to characterize the underwater acoustic channel in other domains. We briefly discuss commonly used domains here.

Time-delay domain The LTV impulse response $h(t, \tau)$ provides a general description of the channel, explicitly capturing how propagation delays and path losses evolve over time.

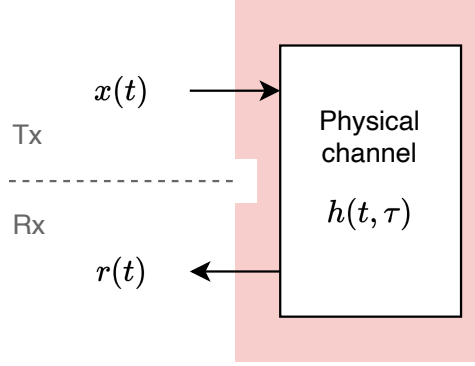


Figure 2.1.: Visualization of the communication pipeline including only the physical channel.

Time-frequency domain The relation from the time-delay domain to the time-frequency response of the channel is

$$h(t, \tau) = \int_{-\infty}^{+\infty} H_{TF}(t, f) e^{j2\pi f\tau} df, \quad (2.2)$$

where f denotes frequency and $H_{TF}(t, f)$ is the CIR in the time-frequency domain. The received signal can be expressed as

$$r(t) = \int_{-\infty}^{+\infty} H_{TF}(t, f) \underbrace{\int_{-\infty}^{+\infty} x(t-\tau) e^{j2\pi f\tau} d\tau}_{X(f) e^{j2\pi ft}} df \quad (2.3)$$

$$= \int_{-\infty}^{+\infty} H_{TF}(t, f) X(f) e^{j2\pi ft} df. \quad (2.4)$$

Here $X(f) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi ft} dt$.

Delay-Doppler domain An alternative and often insightful representation of the underwater acoustic channel is obtained in the delay-Doppler domain. The relation from the time-delay domain to the delay-Doppler response of the channel is

$$h(t, \tau) = \int_{-\infty}^{+\infty} H_{DD}(\nu, \tau) e^{j2\pi \nu t} d\nu, \quad (2.5)$$

where ν denotes the Doppler shift and $H_{DD}(\nu, \tau)$ is the CIR in the delay-Doppler domain. The corresponding input-output relation is given by

$$r(t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} H_{DD}(\nu, \tau) x(t-\tau) e^{j2\pi \nu t} d\tau d\nu. \quad (2.6)$$

Physically, $H_{DD}(\nu, \tau)$ describes how the channel redistributes transmitted signal energy across time delay and Doppler shift. In underwater acoustic propagation environments,

the function $H_{\text{DD}}(\nu, \tau)$ is often sparse or approximately sparse, consisting of a small number of dominant components corresponding to distinct propagation paths.¹

Lossless medium

In a lossless medium, propagation loss is governed entirely by geometric spreading; absorption losses are absent due to the lossless medium assumption. The intensity decay of an acoustic wave due to the geometric spreading loss can be characterized by the ratio $A(d) = \left(\frac{d}{d_r}\right)^\kappa$, where d and d_r denote the transmission and reference distances. The exponent κ accounts for the dimensionality of the medium (for cylindrical spreading $\kappa = 1$, and for spherical spreading $\kappa = 2$) [2].

Let us consider a propagation environment consisting of P distinct paths in a lossless medium. Each path is characterized by a time-varying propagation delay $\tau_p(t)$, with a corresponding path length $d_p(t) = c \tau_p(t)$, where c denotes the propagation speed, and a (cumulative) reflection coefficient γ_p . The corresponding CIR in the time–frequency domain is

$$H_{\text{TF}}(t, f) = \sum_{p=0}^{P-1} \frac{\gamma_p}{\sqrt{A(d_p(t))}} e^{-j2\pi f \tau_p(t)} = \sum_{p=0}^{P-1} \gamma_p \left(\frac{d_r}{d_p(t)} \right)^{\kappa/2} e^{-j2\pi f \tau_p(t)}. \quad (2.7)$$

The resulting CIR in time–delay domain is

$$h(t, \tau) = \sum_{p=0}^{P-1} h_p(t) \delta(\tau - \tau_p(t)). \quad (2.8)$$

where $h_p(t) = \gamma_p \left(\frac{d_r}{d_p(t)} \right)^{\kappa/2}$ captures the time-varying attenuation, i.e, the combined effect of the reflection coefficient and the spreading loss, along the p 'th path.

Example 2 (Underwater uplink communication). *Suppose there exist two propagation paths between an autonomous underwater vehicle (AUV) (transmitter) and buoy (receiver): a direct path and a bottom reflection. To simulate the motion, the buoy is moving up and down in the vertical dimension (as if on a wave), while the AUV is passing by at constant depth at 1.5 m s^{-1} . Hence, the positions of the buoy and AUV can be described by*

$$\begin{bmatrix} x_{\text{buoy}}(t) \\ z_{\text{buoy}}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ \sin(\frac{\pi}{3} t) \end{bmatrix}, \quad \begin{bmatrix} x_{\text{AUV}}(t) \\ z_{\text{AUV}}(t) \end{bmatrix} = \begin{bmatrix} -10 + 1.5t \\ -15 \end{bmatrix}. \quad (2.9)$$

Fig. 2.2a shows the described scenario at $t = 0 \text{ s}$. Furthermore, the propagation speed is assumed constant $c = 1500 \text{ m s}^{-1}$, such that $\tau_p(t) = \tau_0 + \frac{d_p(t)}{c}$, where $d_p(t)$ is the path length in meters. Figs. 2.2b and 2.2c show the path loss and propagation delay of the two paths as a function of time, respectively.

¹Fig. 2.11b in Section 2.2.3 shows a measured CIR in the delay–Doppler domain.

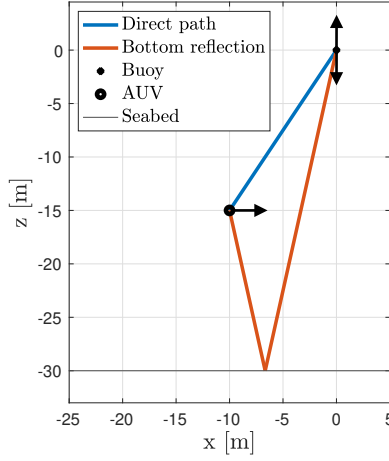
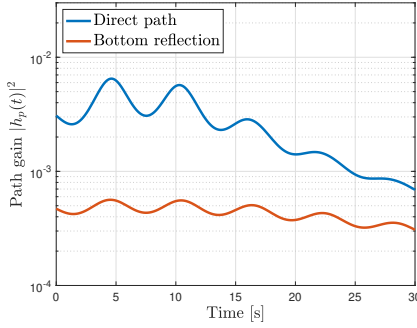
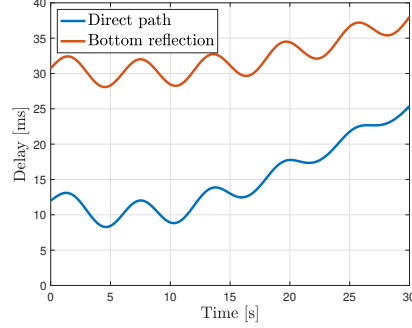
(a) The scenario at $t = 0$ s.(b) Path loss over time, $h_p(t) = \gamma_p (d_r/d_p(t))^\kappa$.(c) Path delay over time, $\tau_p(t)$.

Figure 2.2.: Simplified example scenario of changing path lengths between an AUV and buoy in a lossless medium. The buoy is moving in the vertical dimension, $z_{\text{buoy}}(t) = \sin(\frac{\pi}{3}t)$, while the AUV is passing by at constant depth $z_{\text{buoy}}(t) = -15$ at a speed of 1.5 m s^{-1} , i.e., $x_{\text{AUV}}(t) = -10 + 1.5t$. The reflection coefficient for the direct path is $\gamma_0 = 1$. Assuming a lossless reflection, the reflection coefficient for the bottom reflection is set to $|\gamma_1| = 1$. Furthermore, $\kappa = 2$ and $d_r = 1$.

Lossy medium

In a lossy medium, propagation also incurs frequency-dependent absorption. The propagation loss can in this case be described by $A(d, f) = \left(\frac{d}{d_r}\right)^\kappa \alpha(f)^{d_p(t) - d_r}$, where $\alpha(f)$ denotes the frequency dependent absorption coefficient [2].² The propagation loss

²While $\alpha(f)$ here acts as a dimensionless base per meter, the quantity $1000 \cdot 10\log_{10}(\alpha(f))$ has units dB km^{-1} and is typically found in literature and plots.

$A(d, f)$ can be incorporated directly into the CIR in the time–frequency domain as³

$$H_{\text{TF}}(t, f) = \sum_{p=0}^{P-1} \gamma_p \left(\frac{d_r}{d_p(t)} \right)^{\kappa/2} \alpha(f)^{-\frac{1}{2}(d_p(t)-d_r)} e^{-j2\pi f \tau_p(t)}. \quad (2.10)$$

Note that different models for $\alpha(f)$ were visualized in Fig. 1.1a. Discussion of different absorption models is, however, beyond the scope of this work; the interested reader is referred to [2], [3], [4] for a detailed analysis of underwater acoustic absorption losses. Here a simplified absorption model is assumed; $\alpha(f) = e^{\beta f^2}$ where β is a positive coefficient.

The corresponding CIR in the time-delay domain is

$$h(t, \tau) = \int_{-\infty}^{+\infty} H_{\text{TF},p}(t, f) e^{j2\pi f \tau} df \quad (2.11)$$

$$= \sum_{p=0}^{P-1} \gamma_p \left(\frac{d_r}{d_p(t)} \right)^{\kappa/2} \int_{-\infty}^{+\infty} e^{-j2\pi f \tau_p(t)} e^{-\frac{1}{2}\beta f^2 (d_p(t)-d_r)} e^{j2\pi f \tau} df \quad (2.12)$$

$$= \sum_{p=0}^{P-1} \gamma_p \left(\frac{d_r}{d_p(t)} \right)^{\kappa/2} \sqrt{\frac{\pi}{\frac{1}{2}\beta(d_p(t)-d_r)}} e^{-\frac{\pi^2 (\tau - \tau_p(t))^2}{\frac{1}{2}\beta(d_p(t)-d_r)}} \quad (2.13)$$

$$= \sum_{p=0}^{P-1} h_p(t) \xi_p(\tau - \tau_p(t)), \quad (2.14)$$

where $h_p(t)$ is defined as before and $\xi_p(\tau) = \sqrt{\frac{\pi}{\frac{1}{2}\beta(d_p(t)-d_r)}} e^{-\frac{\pi^2 \tau^2}{\frac{1}{2}\beta(d_p(t)-d_r)}}$. Compared to the lossless case in (2.8), the Dirac delta function is replaced by a Gaussian pulse $\xi_p(\tau)$, which acts as a distance-dependent low-pass filter. As $d_p(t)$ increases, the pulse widens and its amplitude decreases, characterizing the greater attenuation of high-frequency components. Fig. 2.3 shows the Gaussian pulse for different distances.

Having developed a clearer understanding of the CIR, the next step is to examine how the transmitted signal interacts with the channel at baseband.

2.1.2. Transmission and reception in baseband

In practical systems, transmission takes place over a band of frequencies centered around a carrier frequency f_c , occupying the passband $[f_c - B/2, f_c + B/2]$. However, for signal processing, analysis, and algorithm design, it is more convenient to work with an equivalent *baseband* representation, where the signal spectrum is confined to $[-B/2, B/2]$. This approach decouples the signal description from the carrier frequency, offering two main advantages. First, signal processing can be performed at sampling rates proportional to the signal bandwidth rather than the carrier frequency. Since

³For simplicity of exposition, the frequency-dependent phase response associated with frequency-dependent absorption is neglected here. In a causal medium, attenuation and phase velocity are related through the Kramers–Kronig relations. For typical underwater acoustic communication frequencies and ranges, however, intrinsic dispersion is generally negligible compared with other propagation effects, and a frequency-independent propagation delay is therefore assumed.

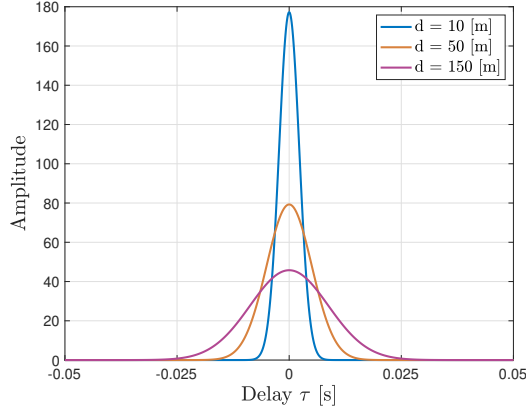


Figure 2.3.: Visualization of the Gaussian pulse $\xi(\tau) = \sqrt{\frac{\pi}{\beta(d-d_r)}} e^{-\frac{\pi^2 \tau^2}{\beta(d-d_r)}}$, for three distances. The reference distance is $d_r = 1$ m, the coefficient is set to $\beta = 2 \cdot 10^{-5}$.

typically $B < f_c$, this reduces computational complexity and storage requirements. Second, baseband representations provide a carrier-independent description of the communication pipeline, enabling transmitter and receiver designs that are independent of the carrier frequencies.

Let the (complex-valued) baseband signal be denoted by $x_{bb}(t)$. It is then upconverted to the passband for transmission,

$$x_{pb}(t) = \text{Re} \left\{ x_{bb}(t) e^{j2\pi f_c t} \right\} = \frac{1}{2} \left(x_{bb}(t) e^{j2\pi f_c t} + x_{bb}^*(t) e^{-j2\pi f_c t} \right). \quad (2.15)$$

The frequency spectrum of the passband signal $x_{pb}(t)$ is given by $X_{pb}(f) = \frac{1}{2} X_{bb}(f - f_c) + \frac{1}{2} X_{bb}^*(-f - f_c)$, as visualized in Fig. 2.4. Note that since the passband signal is real valued, its spectrum is symmetric.

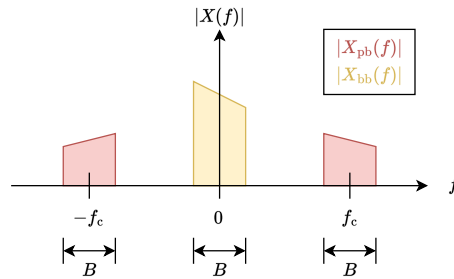


Figure 2.4.: Frequency spectrum of the baseband signal (yellow) and corresponding passband signal (red).

Upon reception, the signal is first downconverted to baseband and subsequently passed through a low-pass filter (LPF) with bandwidth B , corresponding to the bandwidth of the transmitted baseband signal $x_{\text{bb}}(t)$, in order to suppress the spectral images.⁴ The received signal in baseband is

$$\begin{aligned}
 r_{\text{bb}}(t) &= 2 \cdot \text{LPF}_B \{ r_{\text{pb}}(t) e^{-j2\pi f_c t} \} \\
 &= 2 \cdot \text{LPF}_B \left\{ \int_{-\infty}^{+\infty} h(t, \tau) x_{\text{pb}}(t - \tau) e^{-j2\pi f_c t} d\tau \right\} + \underbrace{2 \cdot \text{LPF}_B \{ n(t) e^{-j2\pi f_c t} \}}_{w(t)} \\
 &= \text{LPF}_B \left\{ \int_{-\infty}^{+\infty} \underbrace{h(t, \tau) e^{-j2\pi f_c \tau}}_{h_{\text{bb}}(t, \tau)} x_{\text{bb}}(t - \tau) d\tau \right\} \\
 &\quad + \text{LPF}_B \left\{ \int_{-\infty}^{+\infty} h(t, \tau) x_{\text{bb}}^*(t - \tau) e^{-j2\pi f_c (2t - \tau)} d\tau \right\} + w(t) \\
 &\stackrel{(a)}{=} \int_{-\infty}^{+\infty} h_{\text{bb}}(t, \tau) x_{\text{bb}}(t - \tau) d\tau + 0 + w(t), \tag{2.16}
 \end{aligned}$$

where $w(t) = 2 \cdot \text{LPF}_B \{ n(t) e^{-j2\pi f_c t} \}$ denotes the additive noise in baseband and

$$h_{\text{bb}}(t, \tau) = h(t, \tau) e^{-j2\pi f_c \tau}, \tag{2.17}$$

is the baseband-equivalent CIR. In (a) we have used that, in the frequency domain, $x_{\text{bb}}(t)$ and $h_{\text{bb}}(t, \tau)$ are bandlimited, i.e., $X_{\text{bb}}(f) = 0$ and $H_{\text{TF,bb}}(t, f) = 0$ for $f \notin [-B/2, B/2]$. Figure 2.5 shows the frequency spectrum of the received signal in passband, the frequency shifted signal and the (ideal) LPF. Figure 2.6 shows the communication pipeline at baseband.

Baseband models, while independent of the carrier frequency, are inherently bandwidth-specific, which leads to the concept of *normalized baseband*, introduced in the following subsection.

2.1.3. Transmission and reception in normalized baseband

The baseband waveform $x_{\text{bb}}(t)$ is generated from a sequence of information symbols $\{x[i]\}$ through pulse shaping,

$$x_{\text{bb}}(t) = \sum_{i \in \mathbb{Z}} x[i] g(t - iT_{\text{symp}}), \tag{2.18}$$

where $g(t)$ is the transmit pulse and T_{symp} denotes the symbol period. This construction ensures that $x_{\text{bb}}(t)$ is bandlimited to $B = 1/T_{\text{symp}}$, provided that $g(t)$ is bandlimited to

⁴In the presence of relative motion between transmitter, receiver and/or scattering objects, Doppler effects may have shifted the signal spectrum away from the nominal carrier frequency f_c . In such cases, the low-pass filter bandwidth must be chosen sufficiently large to accommodate this frequency offset.

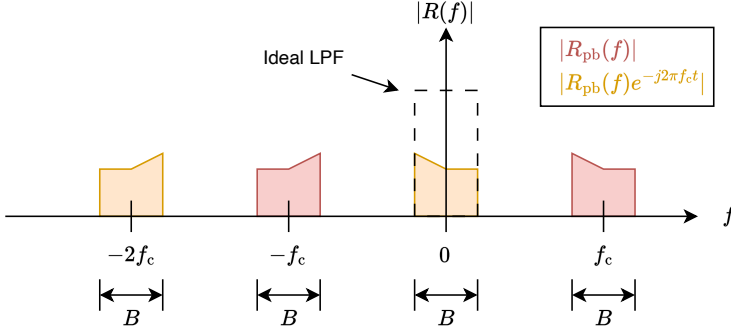


Figure 2.5.: Frequency spectrum of the received signal in passband (red) and the corresponding downconverted signal (orange). The baseband signal is obtained after applying the LPF to the downconverted signal.

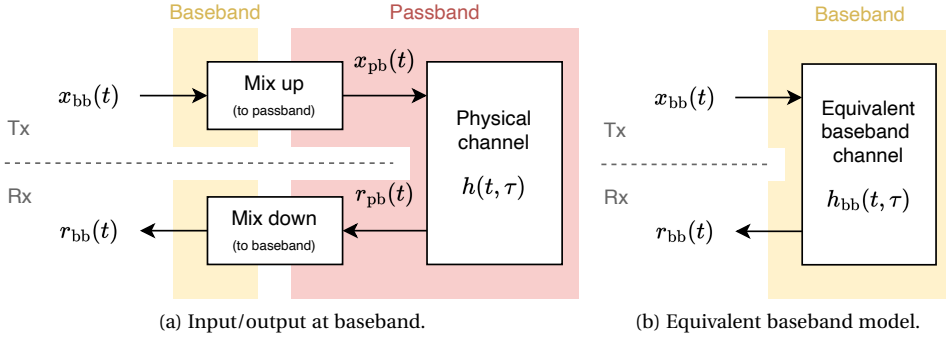


Figure 2.6.: Visualization of the communication pipeline at baseband.

$B = 1/T_{\text{symp}}$.⁵ Substituting (2.18) into the received baseband signal, given by (2.16), yields

$$r_{bb}(t) = \sum_{i \in \mathbb{Z}} x[i] \int_{-\infty}^{+\infty} h_{bb}(t, \tau) g(t - \tau - iT_{\text{symp}}) d\tau + w(t). \quad (2.19)$$

At the receiver, the baseband signal is passed through a receive filter matched to the transmit pulse and sampled at the symbol rate,

$$r[k] = \int_{-\infty}^{+\infty} r_{bb}(t) g^*(t - kT_{\text{symp}}) dt. \quad (2.20)$$

⁵Physical signals are often not perfectly bandlimited. The pulse $g(t)$ is typically designed to concentrate its energy within a specific range. In practice, the occupied bandwidth is defined as $B = (1 + \beta)/T_{\text{symp}}$, where $\beta \in [0, 1]$ is the roll-off factor. Our assumption of $B = 1/T_{\text{symp}}$ corresponds to the ideal Nyquist case with $\beta = 0$.

Substituting the expression for $r_{\text{bb}}(t)$ in (2.19) into (2.20) gives

$$r[k] = \sum_{i \in \mathbb{Z}} x[i] \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h_{\text{bb}}(t, \tau) g(t - \tau - i T_{\text{symp}}) g^*(t - k T_{\text{symp}}) d\tau dt + w[k], \quad (2.21)$$

where

$$w[k] \triangleq \int_{-\infty}^{+\infty} w(t) g^*(t - k T_{\text{symp}}) dt. \quad (2.22)$$

Changing the summation index via $i = k - l$, the input–output relation at *normalized baseband* can be written compactly as

$$r[k] = \sum_{l \in \mathbb{Z}} h_{\text{nbb}}[k, l] x[k - l] + w[k], \quad (2.23)$$

where the discrete-time normalized baseband channel taps are defined as

$$\begin{aligned} h_{\text{nbb}}[k, l] &\triangleq \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h_{\text{bb}}(t, \tau) g(t - \tau - (k - l) T_{\text{symp}}) g^*(t - k T_{\text{symp}}) d\tau dt \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(t, \tau) e^{-j2\pi f_c \tau} g(t - \tau - (k - l) T_{\text{symp}}) g^*(t - k T_{\text{symp}}) d\tau dt \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} h(t + k T_{\text{symp}}, \tau + l T_{\text{symp}}) e^{-j2\pi f_c (\tau + l T_{\text{symp}})} g(t - \tau) g^*(t) d\tau dt. \end{aligned} \quad (2.24)$$

Note that $h_{\text{nbb}}[k, l]$ is a filtered response of $h(t, \tau)$ at the point $(k T_{\text{symp}}, l T_{\text{symp}})$. Moreover, the discrete-time model in (2.23) shows that the received symbols are a *time-varying* convolution of the transmitted symbols. Fig. 2.7 shows the communication pipeline at normalized baseband.

Remark 1. (*Finite delay support*) A common assumption is that the discrete-time time-varying channel impulse response $h_{\text{nbb}}[k, l]$ has finite delay support,

$$h_{\text{nbb}}[k, l] = 0, \quad l \in \mathbb{Z} \setminus \{0, \dots, L\}. \quad (2.25)$$

The restriction to nonnegative delays reflects the causality of the physical propagation channel, since non-zero taps for $l < 0$ would imply a dependence of the received signal $r[k]$ on future transmitted symbols. Moreover, the finite upper bound L models the fact that multipath components experience bounded propagation delays, so that contributions beyond a maximum delay can be neglected.

Example 3 (Two-path channel). Assume that the channel consists of two paths in a lossless medium. The first path is static and the second path lengthens at a constant rate. The CIR can be described by⁶

$$h(t, \tau) = \delta(t - 4 T_{\text{symp}}) + \frac{1}{2} \delta\left(t - (6 T_{\text{symp}} + \frac{3}{1500} t)\right). \quad (2.26)$$

⁶For simplicity, the attenuation of the second path is assumed constant, and its dependence on the time-varying path length is neglected.

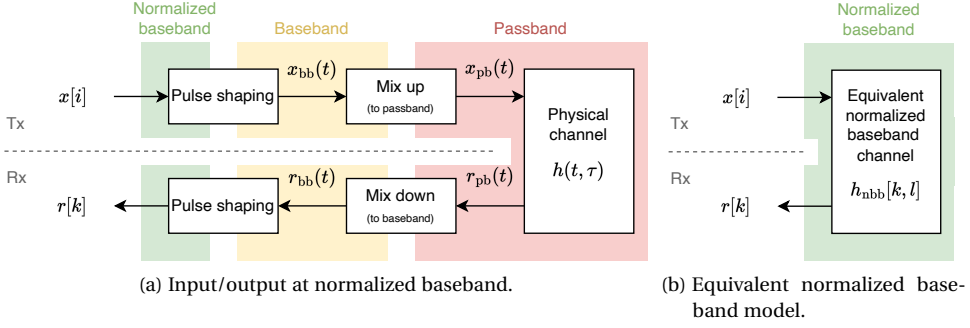


Figure 2.7.: Visualization of the communication pipeline at normalized baseband.

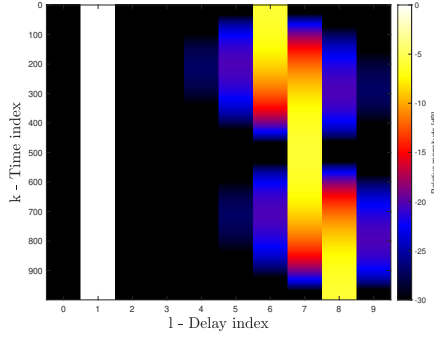


Figure 2.8.: Example of the normalized baseband channel coefficients $h_{nbb}[k, l]$ for a two-path channel. The symbol period is $T_s = 1/13000$ s, and $K = 1000$ symbol intervals are shown. The channel consists of two propagation paths described in (2.26).

The resulting discrete-time channel coefficients $h_{nbb}[k, l]$ are shown in Fig. 2.8. If the delay of a path is static and an integer multiple of the sampling period (e.g., the first path), then the corresponding physical path contributes to a single discrete-time channel tap. In contrast, when the delay is not an integer multiple of the sampling period or when the path length is changing (e.g., the second path), a single physical path spreads its energy across multiple discrete-time channel taps.

Matrix vector notation

In practice, multiple received symbols are processed jointly. This motivates a matrix-vector representation of the equivalent normalized baseband input-output relation.

By stacking K received symbols into the vector $\mathbf{r} = [r[0], \dots, r[K-1]]^T$, the input-output

relation can be written as

$$\mathbf{r} = \begin{bmatrix} h[0,L] & \cdots & h[0,0] & 0 & \cdots & 0 \\ 0 & h[1,L] & \cdots & h[1,0] & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & h[K-1,L] & \cdots & h[K-1,0] \end{bmatrix} \begin{bmatrix} x[-L] \\ \vdots \\ x[0] \\ x[1] \\ \vdots \\ x[K-1] \end{bmatrix} + \begin{bmatrix} w[0] \\ \vdots \\ w[K-1] \end{bmatrix}.$$

For each time index k , the received symbol depends on $L+1$ channel coefficients. Consequently, the total number of unknown channel parameters scales with $K(L+1)$, which poses a fundamental challenge for channel estimation. Indeed, even in the idealized scenario where all $K+L$ transmitted symbols are perfectly known at the receiver, the resulting system of equations is underdetermined, since the number of observations K is insufficient to uniquely identify the $K(L+1)$ channel coefficients.

To render the channel estimation problem well posed, we impose a parametric model on the temporal evolution of the channel coefficients. Specifically, we adopt a low-dimensional basis expansion model (BEM).

A BEM represents the temporal variation of $h_{\text{nb}}[k, l]$ using a set of $Q+1$ basis functions $\{b_q[k]\}_{q=-Q/2, \dots, Q/2}$, that is,

$$h_{\text{nb}}[k, l] = \sum_{q=-Q/2}^{Q/2} c_{l,q} b_q[k]. \quad (2.27)$$

Note that if $Q+1 = K$ and if the set of basis vectors spans $\mathbb{C}^K$, the decomposition in (2.27) is unique. To ensure that the channel estimation problem is well posed, we assume that the channel can be accurately represented using a lower-order basis, i.e., $Q+1 < K$.

We now incorporate the BEM representation into the matrix-vector input-output relation derived previously. Let $\mathbf{b}_q = [b_q[0], \dots, b_q[K-1]]^\top$ denote the sampled basis functions and define the diagonal matrix $\Lambda_K^{(q)} \triangleq \text{diag}(\mathbf{b}_q)$. Moreover, let $\mathbf{P}_{K \times (K+L)}^{(l)} = [\mathbf{0}_{K \times (L-l)}, \mathbf{I}_K, \mathbf{0}_l]$ denote the delay operator which extracts the contribution of the l th channel tap.

Using the BEM representation in (2.27), the received signal vector can be written as

$$\mathbf{r} = \left(\sum_{l=0}^L \sum_{q=-Q/2}^{Q/2} c_{l,q} \Lambda_K^{(q)} \mathbf{P}_{K \times (K+L)}^{(l)} \right) \mathbf{x}_{\text{ext}} + \mathbf{w}, \quad (2.28)$$

where $\mathbf{x}_{\text{ext}} = [x[-L], \dots, x[K-1]]^\top$ collects the transmitted symbols and $\mathbf{w} = [w[0], \dots, w[K-1]]^\top$ collects the additive noise affecting the received block.

Note that the received symbol vector $\mathbf{r}$ depends on $K+L$ transmitted symbols. Consequently, when transmitting consecutive symbol blocks of length K , the received block would generally depend on multiple transmitted blocks. To avoid this so-called *block interference*, a cyclic prefix (CP) or zero-padding (ZP) is used.

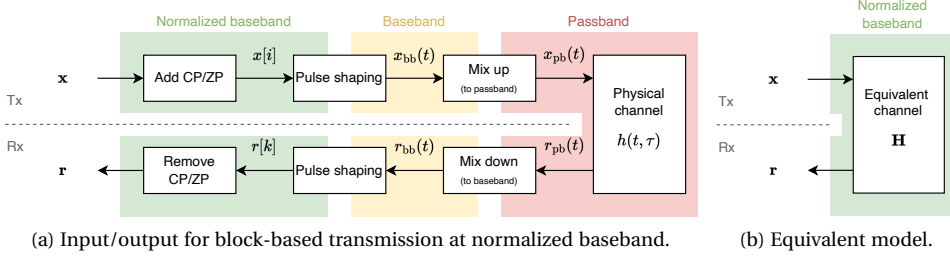


Figure 2.9.: Visualization of the communication pipeline for block-based transmission at normalized baseband.

Cyclic prefix (CP) and zero padding (ZP)

Let $\mathbf{x} = [x[0], \dots, x[K-1]]^T$ denote the transmitted symbol vector. A CP or ZP appends L symbols in front of $\mathbf{x}$. For CP transmission, these symbols equal the last L entries of $\mathbf{x}$, whereas for ZP transmission they are set to zero. These operations can be represented by the linear mappings

$$\mathbf{T}_{\text{CP}} = \begin{bmatrix} \mathbf{0}_{L \times (K-L)} & \mathbf{I}_L \\ & \mathbf{I}_K \end{bmatrix} \in \{0, 1\}^{(K+L) \times K}, \quad (2.29)$$

$$\mathbf{T}_{\text{ZP}} = \begin{bmatrix} \mathbf{0}_{L \times K} \\ \mathbf{I}_K \end{bmatrix} \in \{0, 1\}^{(K+L) \times K}. \quad (2.30)$$

Substituting $\mathbf{x}_{\text{ext}} = \mathbf{T}\mathbf{x}$ into (2.28), where $\mathbf{T} \in \{\mathbf{T}_{\text{CP}}, \mathbf{T}_{\text{ZP}}\}$, yields

$$\mathbf{r} = \left(\sum_{l=0}^L \sum_{q=-Q/2}^{Q/2} c_{l,q} \Lambda_K^{(q)} \mathbf{P}_K^{(l)} \right) \mathbf{x} + \mathbf{w} = \mathbf{H}\mathbf{x} + \mathbf{w}, \quad (2.31)$$

where $\mathbf{P}_K^{(l)} = \mathbf{P}_{K \times (K+L)}^{(l)} \mathbf{T}$. For CP transmission, $\mathbf{P}_K^{(l)}$ is a circulant (cyclic shift) matrix given by the l 'th power of

$$\mathbf{P}_K = \begin{bmatrix} 0 & 0 & \dots & 0 & 1 \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & 0 & & \vdots \\ \vdots & \ddots & \ddots & \ddots & \\ 0 & \dots & 0 & 1 & 0 \end{bmatrix}, \quad (2.32)$$

whereas for ZP transmission $\mathbf{P}_K^{(l)}$ is a lower-triangular Toeplitz (linear shift) matrix, i.e., a matrix with ones on the l 'th subdiagonal and zeros elsewhere. Fig. 2.9 shows the communication pipeline for block-based transmission at normalized baseband.

Modulation and demodulation

Having established a block-based input-output relation, we now introduce a linear modulation framework. Let $\mathbf{s} \in \mathbb{C}^K$ denote a vector of information symbols. The

corresponding transmitted time-domain symbol vector is obtained via

$$\mathbf{x} = \mathbf{M}^H \mathbf{s}, \quad (2.33)$$

where $\mathbf{M} \in \mathbb{C}^{K \times K}$ denotes the modulation matrix.

The choice of $\mathbf{M}$ determines how the information symbols in $\mathbf{s}$ are mapped onto the time-frequency resources of the transmission block. Several common modulation schemes can be expressed within this framework. For example, orthogonal frequency division multiplexing (OFDM) corresponds to $\mathbf{M} = \mathbf{F}_K$, where $\mathbf{F}_K$ denotes the K -point discrete Fourier transform (DFT) matrix, whereas single-carrier modulation (SCM) is obtained by choosing $\mathbf{M} = \mathbf{I}_K$. For orthogonal time frequency space (OTFS) modulation, the modulation matrix takes the form $\mathbf{M} = \mathbf{F}_N \otimes \mathbf{I}_M$, with $K = NM$. Chapter 3 will treat these modulation techniques in more detail.

At the receiver, demodulation is performed by applying the corresponding linear transform,

$$\mathbf{y} = \mathbf{M}\mathbf{r} = \mathbf{M}\mathbf{M}^H \mathbf{s} + \mathbf{M}\mathbf{w}. \quad (2.34)$$

The information symbols are recovered from the received signal vector using channel equalization.

From bits to symbols

Finally, we describe the mapping from a binary information sequence to the symbol vector $\mathbf{s}$. This operation, referred to as *symbol mapping*, assigns groups of bits to complex-valued constellation points.

In general, symbol mapping is described by the function

$$f : \{0, 1\}^{\log_2 m} \rightarrow \mathbb{C}, \quad (2.35)$$

where m denotes the constellation size. Each block of $\log_2 m$ bits is mapped to one symbol from a finite constellation in the complex plane.

Fig. 2.10 illustrates several commonly used constellations, including binary phase-shift keying (BPSK), quadrature phase-shift keying (QPSK), and quadrature amplitude modulation (QAM). In these schemes, information is conveyed through the phase of the transmitted symbols and, in the case of QAM, additionally through their amplitude.

2.2. Common practices for underwater acoustic (UWA) communications

Building on the general communication system pipeline introduced in the previous section, this section details the specifics of underwater acoustic (UWA) communications.

2.2.1. Channel model

As discussed in Chapter 1, the signals propagating through an UWA channel typically suffer from frequency-dependent attenuation and noise. Additionally, due to multiple

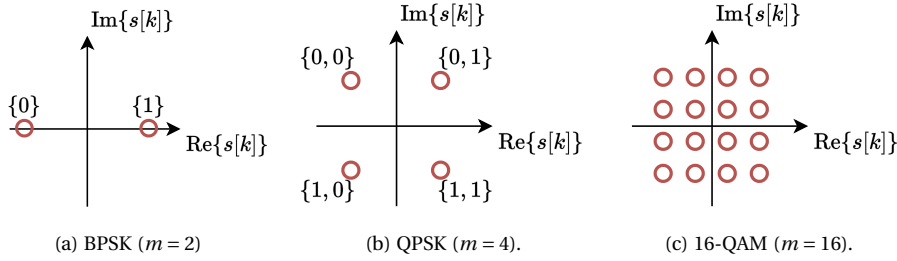


Figure 2.10.: Illustration of digital modulation constellations, where information is encoded (a),(b) in the phase (BPSK, QPSK) and (c) additionally in the amplitude (QAM).

propagation paths, with time-varying path lengths, the received signal is both spread in time (delay spread) and in frequency (Doppler spread). In addition, the inhomogeneous speed of sound can cause waves to bend.

Although mathematical models exist for these phenomena, transmitter and receiver structures are typically designed on a simplified channel model, as this eases the design and analysis. Typically, the UWA channel is modeled as a composition of P propagation paths with time-varying delays, where the path loss is assumed to be frequency-independent (i.e., absorption losses are neglected). The corresponding time-varying CIR can be written as

$$h(t, \tau) = \sum_{p=0}^{P-1} h_p(t) \delta(\tau - \tau_p(t)). \quad (2.36)$$

Additionally, it is often assumed that, within a single communication packet⁷, the path losses vary slowly and can therefore be treated as constant, that is, $h_p(t) \approx h_p$, and that the propagation delays can be approximated as linearly varying over time, i.e., $\tau_p(t) \approx \tau_{0,p} + a_p t$ [5], [6], [7]. Here the so-called Doppler scale a_p is related to the path's radial velocity v_p by $a_p = \frac{v_p}{c}$, where c is the propagation speed.⁸ Let the transmitted signal in passband again be denoted by $x_{\text{pb}}(t)$. Including the discussed assumptions, the received signal is modeled as

$$r_{\text{pb}}(t) = \sum_{p=0}^{P-1} h_p x_{\text{pb}}((1 - a_p)t - \tau_{0,p}) + n(t), \quad (2.37)$$

where $n(t)$ denotes the additive noise.

⁷In communications, information is typically sent piece by piece, in so-called “packets.” This helps make communication systems reliable and scalable.

⁸The velocity v_p generally results from the combined motion of the transmitter, receiver and/or scattering objects.

Noise

While many system designs in UWA communications rely on the simplifying assumption of additive white Gaussian noise (AWGN), it is well-established that the underwater acoustic environment is characterized by frequency-dependent (colored) noise, as discussed in Section 1.1.1. Beyond the non-flat power spectral density, certain environments exhibit non-Gaussian characteristics. For instance, in tropical regions such as the waters around Singapore, so-called snapping shrimps generate high-intensity, impulsive noise. This phenomenon has motivated extensive research into communication performance under such noise [8] and the statistical characterization of impulsive distributions in the underwater channel [9, Chapter 3]. Moreover, in a channel replay simulation tool that is currently being developed, measured noise can be replayed [10, Chapter 4].

Narrowband assumption

Early works on underwater acoustic communications (e.g., [11]) often assume that the signal bandwidth is much smaller than the carrier frequency, i.e., $B \ll f_c$. This narrowband assumption is also commonly adopted in terrestrial wireless communications.

Under this assumption, each propagation path can be characterized by a fixed delay τ_p and a Doppler shift ν_p , resulting in the following representation of the channel impulse response:

$$h(t, \tau) = \sum_{p=0}^{P-1} h_p e^{j2\pi\nu_p t} \delta(\tau - \tau_p). \quad (2.38)$$

Such a channel model is commonly referred to as a *delay-Doppler* channel, since it admits a sparse representation in the delay-Doppler domain, namely

$$H_{DD}(\nu, \tau) = \sum_{p=0}^{P-1} \delta(\nu - \nu_p) \delta(\tau - \tau_p). \quad (2.39)$$

The corresponding received passband signal can then be expressed as

$$r_{pb}(t) = \sum_{p=0}^{P-1} h_p e^{j2\pi\nu_p t} x_{pb}(t - \tau_p) + n(t). \quad (2.40)$$

2.2.2. Detection and synchronization using a preamble

In UWA communication systems, each packet is typically preceded by a *preamble*. The preamble is a known signal that enables packet detection and facilitates synchronization in both time and frequency.

A commonly used preamble is a SCM waveform with BPSK symbols mapped from a maximum length sequence (m-sequence) [12], [13] [14, Chapter 13]. Such sequences have desirable autocorrelation properties. Let L_{seq} denote the length of the m-sequence. The

corresponding passband preamble signal is

$$x_{\text{preamble}}(t) = x_{\text{m-seq}}(t) = \text{Re} \left\{ \sum_{i=0}^{L_{\text{seq}}-1} c_i g(t - iT_{\text{symb}}) e^{j2\pi f_c t} \right\}, \quad (2.41)$$

where $c_m \in \{\pm 1\}$ denotes the m 'th element of the m-sequence, $g(t)$ is the transmit pulse, T_{symb} is the symbol period, and f_c is the carrier frequency.

Due to the impulse-like autocorrelation of the m-sequence, correlating the received signal with a locally generated replica of the preamble yields a sharp peak at the packet arrival time. This property enables robust packet detection and coarse synchronization, even in the presence of multipath propagation and noise.

The propagation delay and Doppler scale can be jointly estimated by solving

$$(\hat{\tau}, \hat{a}) = \arg \max_{\tau, a} \int r_{\text{preamble}}(t) x_{\text{preamble}}^*((1-a)t - \tau) dt. \quad (2.42)$$

Note that the estimated propagation delay $\hat{\tau}$ and Doppler scale $\hat{a}$ correspond to the strongest path. The working assumption is that this strongest path is the direct path, and hence the Doppler scale of the direct path resembles the average of all the a_p 's.

Using these estimates, the received communication packet can be synchronized in both time and frequency. Frequency synchronization is achieved by compensating for the Doppler-induced time scaling,

$$\tilde{r}_{\text{pb}}(t) = r_{\text{pb}} \left(\frac{1}{1-\hat{a}} t \right). \quad (2.43)$$

In the discrete-time domain, this operation corresponds to resampling the received signal.

2.2.3. Channel replay

In the previous subsections, the different components of the communication pipeline were discussed. The design and evaluation of communication systems, however, still require an accurate description of the propagation channel $h(t, \tau)$. In many practical scenarios, and in particular in UWA communications, a mathematical description is either unavailable or difficult to obtain with sufficient accuracy. Simplified physical models, such as ray-based approaches or statistical models, often fail to capture the full complexity and time-variability of the channel.

To address this challenge, it is common practice in UWA communications to rely on *channel replay*. In this approach, the channel is first measured *in situ*, yielding an estimate of the time-varying CIR $h(t, \tau)$. Subsequently, *offline*, the measured channel is used to simulate the propagation effects by convolving new transmitted signals with the measured CIR. In this way, the resulting received signal closely approximates what would have been observed had the new waveform been transmitted through the original in-situ channel. Channel replay has two main benefits: i) it is more realistic than simplified physical models and, ii) its results are reproducible.

The Watermark framework [15] is among the most commonly used channel replay frameworks in UWA communications. The following subsections outline both the procedure for channel measurement and the subsequent channel replay process.

Measuring a channel

The channel is measured using a so-called “probe” signal. The probe signal is a signal in the passband of interest, $[f_c - B/2, f_c + B/2]$, and typically consists of repeated traces of the same signal,

$$x_{\text{probe}}(t) = \text{Re} \left\{ \left(\sum_{n=0}^{N-1} x_{\text{trace}}(t - nT) \right) e^{j2\pi f_c t} \right\}. \quad (2.44)$$

Here $x_{\text{trace}}(t)$ is a signal with non-zero support for $t \in [0, T)$. For example, a linear frequency modulated (LFM) signal can be used as a trace

$$x_{\text{trace}}(t) = \begin{cases} e^{-j\pi/2 + j2\pi(-\frac{B}{2}t + \frac{B}{2}\frac{1}{T}t^2)}, & t \in [0, T), \\ 0 & \text{else.} \end{cases} \quad (2.45)$$

Remark 2 (Probe vs. pilot signals). *While both probe and pilot signals are known sequences used for channel estimation, they differ in their operational use cases. A probe signal is typically a standalone transmission, whereas a pilot (or training) signal is embedded within the data packet and facilitates equalization.*

Remark 3 (Channel identifiability). *For a channel to be identifiable, the repetition interval T has to be larger than the maximum delay $\tau_{\max}$ in the channel, i.e., $T \geq \tau_{\max}$. At the same time, the repetition interval needs to be short enough compared to the maximum Doppler frequency $\nu_{\max}$ to track the channel, i.e., $T \leq 1/(2\nu_{\max})$. Thus, the channel is identifiable if [16, Ch. 8]*

$$2\tau_{\max}\nu_{\max} \leq 1. \quad (2.46)$$

Channels fulfilling this property are called underspread.

The received probe signal is given by

$$r_{\text{probe}}(t) = \int_{-\infty}^{+\infty} h(t, \tau) x_{\text{probe}}(t - \tau) d\tau + n(t). \quad (2.47)$$

Upon reception, (after initial detection and synchronization in time) the probe signal is synchronized in frequency by removing the estimated “mean” Doppler scale

$$\tilde{r}_{\text{probe}}(t) = r_{\text{pb}} \left(\frac{1}{1 - \hat{a}} t \right). \quad (2.48)$$

The signal is then brought to baseband.

$$r_{\text{bb}}(t) = 2 \cdot \text{LPF}_{1.5B} \left\{ \tilde{r}_{\text{pb}}(t) e^{-j2\pi f_c t} \right\}. \quad (2.49)$$

Note that the Watermark framework applies a LPF with a bandwidth of $1.5B$ to accommodate Doppler-induced frequency offsets. To obtain an estimate of the baseband-equivalent CIR $h_{\text{bb}}(t, \tau)$, the signal $r_{\text{bb}}(t)$ is matched filtered with the signal $x_{\text{trace}}(t)$, that is,

$$\tilde{h}_{\text{bb}}(nT, lT_s) = \int_{-\infty}^{+\infty} r_{\text{bb}}(t) x_{\text{trace}}^*(t - lT_s - nT) dt. \quad (2.50)$$

Here T_s denotes the sampling period, which is typically a fraction of the symbol period. Fig. 2.11 shows an example of a measured CIR.

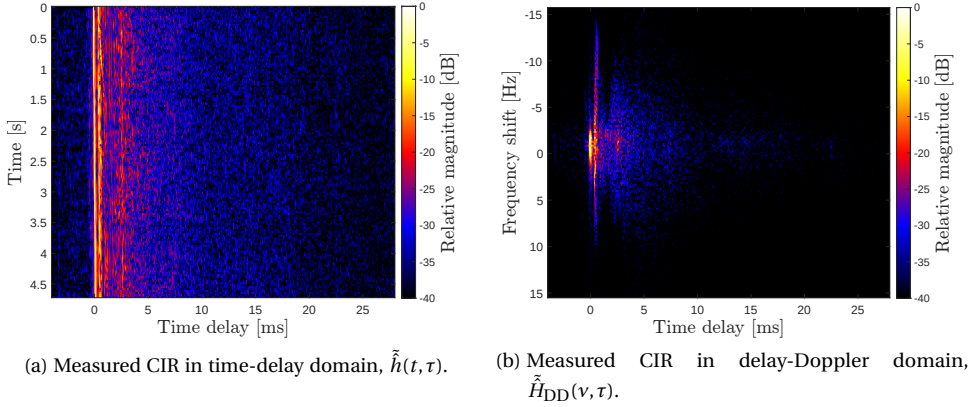


Figure 2.11.: Measured channel at the Haringvliet estuary on March 19, 2025. The transmitter and receiver were suspended from a surface vessel and jetty, respectively. The estimated “mean” Doppler scale was measured to be $\hat{a} = -8.24 \cdot 10^{-5}$.

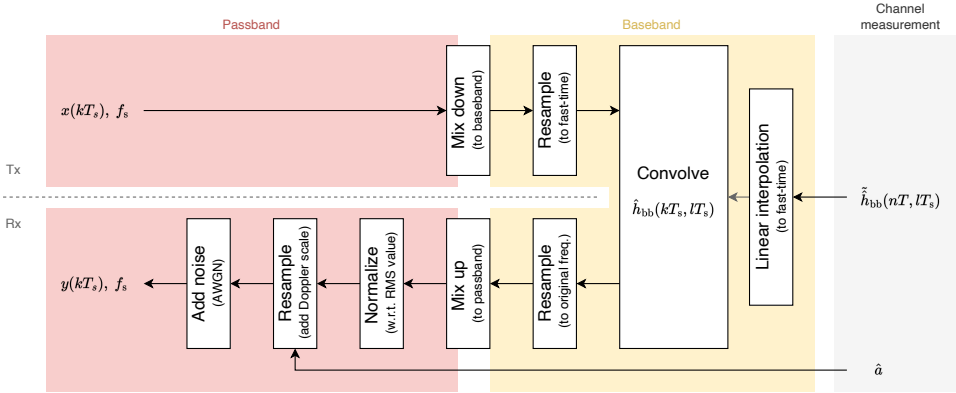


Figure 2.12.: Overview of the general pipeline of the channel replay operation by the Watermark framework [15], [17]. The transmitted signal $x(kT_s)$ is convolved with a measured channel impulse response in baseband. The resample operation before and after the ‘convolve’ operation ensures that the input signal is sampled at the same sampling frequency as the measured channel. These resampling operations have been omitted in the mathematical overview in this subsection to maintain clarity.

Replaying a measured channel

This subsection provides a brief overview of the main steps for channel replay, using the Watermark framework [15]. An overview of the general pipeline is visualized in Fig. 2.12. For further details, the reader is referred to [17].

The replay procedure operates on discretely sampled signals. Let T_s denote the sampling period, which is chosen sufficiently small to capture the passband signal bandwidth and is typically much smaller than the symbol period. All signals are represented in discrete time at this sampling rate.

First, the transmitted passband signal in discrete-time,

$$x_{\text{pb}}(kT_s) = \text{Re} \left\{ x_{\text{bb}}(kT_s) e^{j2\pi f_c kT_s} \right\}, \quad (2.51)$$

is translated to “baseband” by multiplication with $e^{-j2\pi f_c kT_s}$, that is

$$\begin{aligned} \tilde{x}_{\text{bb}}(kT_s) &= x_{\text{pb}}(kT_s) e^{-j2\pi f_c kT_s} \\ &= \text{Re} \left\{ x_{\text{bb}}(kT_s) e^{j2\pi f_c kT_s} \right\} e^{-j2\pi f_c kT_s} \\ &= \left(\frac{1}{2} x_{\text{bb}}(kT_s) e^{j2\pi f_c kT_s} + \frac{1}{2} x_{\text{bb}}^*(kT_s) e^{-j2\pi f_c kT_s} \right) e^{-j2\pi f_c kT_s} \\ &= \frac{1}{2} x_{\text{bb}}(kT_s) + \frac{1}{2} x_{\text{bb}}^*(kT_s) e^{-j2\pi 2f_c kT_s}. \end{aligned} \quad (2.52)$$

Note that the signal $\tilde{x}_{\text{bb}}(kT_s)$ is not truly baseband; it contains an image of the signal centered at $-2f_c$. In contrast to conventional receiver processing, the Watermark framework does not apply a LPF at this stage. This is because the subsequent convolution with the baseband CIR acts as an implicit low-pass filter, suppressing the high-frequency image centered at $-2f_c$. We use a tilde to make a distinction between $\tilde{x}_{\text{bb}}(kT_s)$ and the true baseband signal $x_{\text{bb}}(kT_s)$.

The signal $\tilde{x}_{\text{bb}}(kT_s)$ is then convolved with the measured CIR. However, the channel was measured in time with period T , an integer multiple of the time delay resolution T_s . Before convolution, the sampling interval of the time axis is matched with the time delay axis; the time axis is resampled using linear interpolation. Let $T = LT_s$, then the final channel estimate is

$$\hat{h}_{\text{bb}}(kT_s, lT_s) = (1 - \theta) \cdot \tilde{h}_{\text{bb}}(\lfloor k/L \rfloor T, lT_s) + \theta \cdot \tilde{h}_{\text{bb}}((\lfloor k/L \rfloor + 1)T, lT_s), \quad (2.53)$$

where $\theta = k/L - \lfloor k/L \rfloor$. Following the interpolation, the convolution is given by

$$\begin{aligned} r_{\text{bb}}(kT_s) &= \sum_{l=0}^L \hat{h}_{\text{bb}}(kT_s, lT_s) \tilde{x}_{\text{bb}}((k-l)T_s) \\ &= \sum_{l=0}^L \hat{h}_{\text{bb}}(kT_s, lT_s) \left(\frac{1}{2} x_{\text{bb}}((k-l)T_s) + \frac{1}{2} x_{\text{bb}}^*((k-l)T_s) e^{-j2\pi 2f_c (k-l)T_s} \right) \\ &= \sum_{l=0}^L \frac{1}{2} \hat{h}_{\text{bb}}(kT_s, lT_s) x_{\text{bb}}((k-l)T_s). \end{aligned} \quad (2.54)$$

Here the convolution with the high-frequency image is assumed to be zero due to the implicit low-pass behavior of $\hat{h}_{\text{bb}}[k, l]$. The signal is then brought back to passband,

$$\begin{aligned} \tilde{r}_{\text{pb}}(kT_s) &= \text{Re} \{ r_{\text{bb}}(kT_s) e^{j2\pi f_c kT_s} \} \\ &= \frac{1}{2} \text{Re} \left\{ \hat{h}_{\text{bb}}[k, l] x_{\text{bb}}((k-l)T_s) e^{j2\pi f_c kT_s} \right\}. \end{aligned} \quad (2.55)$$

Finally, the passband signal is resampled to reinstate the estimated “mean” Doppler scale, and noise is added,

$$r_{\text{pb}}(kT_s) = \tilde{r}_{\text{pb}}((1 - \hat{a})kT_s) + n(kT_s). \quad (2.56)$$

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3

Modulation Techniques

The previous chapters presented the context of underwater acoustic communication and the fundamental signal processing techniques employed in this domain.

This chapter focuses on the modulation technique, a core component of the communication pipeline. It examines several modulation techniques at normalized baseband, discusses their robustness to specific channel distortions, and highlights the relationships among them.

This chapter is based on the journal article published as:

I. van der Werf, H. Dol, K. Blom, R. Heusdens, R. C. Hendriks, and G. Leus, "On the equivalence of OSDMM and OTFS," *Signal Processing*, vol. 214, 2024.

3.1. Introduction

In this chapter, we investigate the role of the modulation technique in the communication pipeline. We identify two main fields of research in which the design of modulation techniques is addressed: underwater acoustic (UWA) communications and radio-frequency (RF) communications. Although the two fields differ in some aspects, the problems and proposed solutions in terms of modulation techniques are very similar. In RF communications, high-mobility scenarios and broader frequency bands bring new challenges, including significant delay and Doppler spread. Similarly, in UWA communications, the channel is often deteriorated by significant delay and Doppler spread as the environmental variability is overlapping with the temporal and spatial scales of the acoustic signals. Causes of variability typically are variations in the sound speed profile, the movement of surface waves and the relative movement of transmitter and receiver.

Orthogonal sequence division multiplexing (OSDM) and orthogonal time frequency space (OTFS) are two modulation schemes that are used in UWA communication and RF communication, respectively. OTFS involves initially mapping symbols from the delay-Doppler domain to the time-frequency domain and subsequently to the time domain using the Heisenberg transform.

3.1.1. Contributions of this chapter

Although the literature suggests a link between OSDM and OTFS by connecting them to related modulation schemes like vector orthogonal frequency division multiplexing (OFDM) (V-OFDM) and asymmetric OFDM (A-OFDM), a direct mathematical equivalence has not yet been established. The main novelty of this chapter is therefore to show the mathematical equivalence of the two schemes. The key findings of this chapter can be summarized as follows:

- We show that OSDM and OTFS are mathematically equivalent, by deriving equivalent input-output relations of both techniques from first principles presented in [1] and [2], respectively.
- In addition, by combining the knowledge of acoustic and radio-frequency communications, we give insight into the performance of OSDM/OTFS in terms of intersymbol interference (ISI) and intercarrier interference (ICI) by analyzing its signal structure.

3.2. Related work on modulation techniques

In RF communication, the conventional modulation technique OFDM is not suitable for environments with large Doppler spread, as it will suffer from ICI [3]. OTFS modulation [2] has therefore become the new state-of-the-art modulation technique being more robust in channels with large Doppler spread.

OTFS modulation defines symbols in the delay-Doppler (DD) domain, after which they are transformed to a time domain signal before transmission. By defining symbols in the DD domain, full channel diversity over time and frequency can be obtained. In general,

OTFS is favorable in terms of peak-to-average power ratio (PAPR) [4] and is said to have less ICI in systems with high Doppler and short symbol periods, compared to OFDM [2], [5], [6]. For that reason, the use of OTFS in 5G networks is currently studied [7], [8], and it is considered as one of the contending modulation schemes for 6G networks [9]. Before its first introduction in [2], the OTFS method was patented in the United States by Hadani and Rakib [10].

Generalized frequency division multiplexing (GFDM) [11] was introduced to exploit white spaces in case of (strong) spectrum fragmentation. Compared to OFDM, GFDM has lower PAPR. Nimr *et al.* [12] extended the GFDM framework and noted that there is a close relationship between GFDM and OTFS. To be more specific, the modulated symbols of GFDM and OTFS are equal up to a permutation.

A-OFDM was proposed to bridge single-carrier modulation (SCM) and OFDM, arguing that flexibility between the two increases the overall performance [13]. A-OFDM is based on the divide-and-conquer algorithm, used for the fast Fourier transform (FFT) algorithm. Raviteja *et al.* [14] showed that A-OFDM is equivalent to OTFS.

In coherent underwater communications, initially, modulation techniques such as SCM [15] and OFDM [16] were used. However, two decades ago, OSDM was introduced as an alternative, which performs much better in terms of bit error ratio (BER) and has a lower PAPR [1], [17], [18], [19], [20].

Xia [21] proposed V-OFDM as a special case of precoded OFDM. Compared to conventional OFDM, the overhead due to the cyclic prefix (CP) is reduced by the use of precoding. In [22] and [23] the “compatibility” between V-OFDM and OSDM was noted. In [24], the mathematical equivalence between V-OFDM and A-OFDM was remarked. Although the two are equivalent, A-OFDM and V-OFDM seem to be inspired by different principles. Recently, Xia argued that the transmitted signals of V-OFDM and OTFS are the same [25]. The timeline of the related literature is visualized in Fig. 3.1.

As the literature suggests, OSDM and OTFS are related by connecting them with V-OFDM. However, a direct mathematical comparison is still missing. Independently from this work, Dong *et al.* [26], recently mentioned the relation between OSDM and OTFS, however, without any mathematical proof or background. Therefore, in this chapter, we show the mathematical equivalence of the two schemes, and show their relation to V-OFDM.

3.3. Different modulation techniques

In this section, we will discuss V-OFDM, OSDM and OTFS. By $\{s_k\}_{k=0}^{K-1}$ we denote the sequence of symbols we want to modulate and transmit over the channel. The output of the modulation will be denoted by $\{x_k\}_{k=0}^{K-1}$. To make a fair comparison, we will compare the modulation techniques without the source coding, transmit pulse and carrier frequency.

3.3.1. Vector orthogonal frequency division multiplexing (V-OFDM)

V-OFDM divides the sequence of symbols into N subblocks of length M , thus $K = NM$. In matrix notation, $\mathbf{S} = [\mathbf{s}_0, \dots, \mathbf{s}_{N-1}]$, where $\mathbf{s}_n = [s_{nM+0}, s_{nM+1}, \dots, s_{nM+M-1}]^T$. Then, an

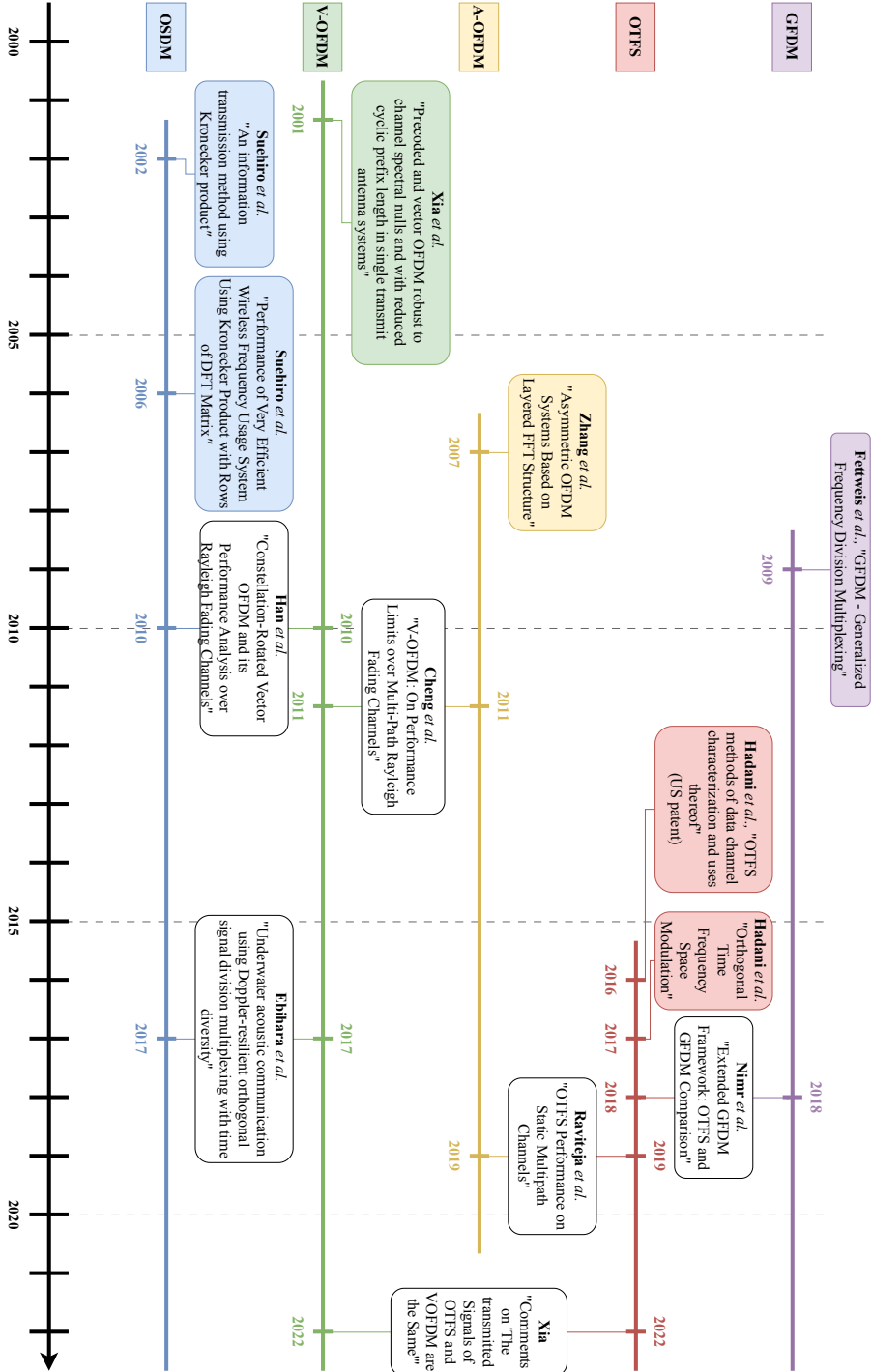


Figure 3.1.: Timeline of literature on modulation techniques related to OSDM and OTFS.

N -point inverse discrete Fourier transform (IDFT) is performed on the rows of $\mathbf{S}$. The modulated symbols become,¹

$$\mathbf{x} = \text{vec}(\mathbf{S}\mathbf{F}_N^H) = (\mathbf{F}_N^H)^T \otimes \mathbf{I}_M \text{vec}(\mathbf{S}) = (\mathbf{F}_N^H \otimes \mathbf{I}_M) \text{vec}(\mathbf{S}) = (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s}. \quad (3.1)$$

Here $\otimes$ denotes the Kronecker product. By performing an IDFT on the rows of $\mathbf{S}$ and vectorizing columns of length M , it is as if we zero-pad each sample with $M - 1$ zeros before we compute the IDFT. As a result, we get M (block) repetitions in the frequency domain. Similarly we get repetitions in the time domain. The IDFT matrix $\mathbf{F}_N^H$ has N rows, so each element in $\mathbf{s}$ will appear N times in the output vector $\mathbf{x}$, resulting in N (block) repetitions in time. Fig. 3.2c shows the structure of the transmitted signal for V-OFDM. From Fig. 3.2c, one can see that if $(M \neq 1 \wedge N \neq 1)$, the structure of the transmitted signal of V-OFDM is fundamentally different from SCM, shown in Fig. 3.2a, and OFDM, shown in Fig. 3.2b. Each symbol now appears on an $M \times N$ grid.

From (3.1) it is clear that V-OFDM can make a compromise between SCM and OFDM. When $M = 1$, (3.1) reduces to $\mathbf{x} = \mathbf{F}_N^H \mathbf{s}$, and thus, V-OFDM is equal to OFDM. Similarly, when $N = 1$, (3.1) reduces to $\mathbf{x} = \mathbf{I}_M \mathbf{s}$, and V-OFDM equals SCM. Due to the flexibility, one would say that V-OFDM is generally more robust against Doppler spread than OFDM and more robust against ISI than SCM. In Section 3.4 we will discuss its exact properties.

3.3.2. Orthogonal sequence division multiplexing (OSDM)

Let $K = NM$. OSDM then splits the symbol sequence $\{s_k\}_{k=0}^{K-1}$ into N sequences of M symbols. Thus, again, $\mathbf{s}_n = [s_{nM+0}, s_{nM+1}, \dots, s_{nM+M-1}]^T$ is the n 'th sequence of M symbols, corresponding to the n 'th column of $\mathbf{S}$. Then, the modulated $NM \times 1$ symbol vector $\mathbf{x}$ is the summation over all the columns, where to each column $\mathbf{s}_n$ the Kronecker product with $\mathbf{f}_n^*$, the $(n + 1)$ 'th column of $\mathbf{F}_N^H$, is applied.

$$\mathbf{x} = \sum_{n=0}^{N-1} \mathbf{f}_n^* \otimes \mathbf{s}_n. \quad (3.2)$$

Note that, using the mixed-product property, the periodic cross-correlation between the sequences $(\mathbf{f}_{n_1}^* \otimes \mathbf{s}_{n_1})$ and $(\mathbf{f}_{n_2}^* \otimes \mathbf{s}_{n_2})$ is zero at every term for $n_1 \neq n_2$. The derivation of this property is given in Appendix A. The zero periodic cross-correlation property is very useful as will become clear in Section 3.4.

However, (3.2) can be rewritten as follows,

$$\begin{aligned} \mathbf{x} &= \sum_{n=0}^{N-1} \mathbf{f}_n^* \otimes \mathbf{s}_n = (\mathbf{F}_N^H \odot \mathbf{S}) \mathbf{1}_{N \times 1} = \text{vec}(\mathbf{S} \mathbf{I}_N \mathbf{F}_N^H) \\ &= \text{vec}(\mathbf{I}_M \mathbf{S} \mathbf{F}_N^H) = (\mathbf{F}_N^H \otimes \mathbf{I}_M) \text{vec}(\mathbf{S}) = (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s}. \end{aligned} \quad (3.3)$$

Here $\odot$ denotes Khatri-Rao (columnwise Kronecker) product. From (3.3) it is clear that OSDM coincides with V-OFDM, as was remarked in [22], [23]. The structure of the transmitted signal of OSDM can therefore also be depicted by Fig. 3.2c. In [20] it was shown that OSDM (and thus also V-OFDM) has lower PAPR compared to OFDM.

¹Note that for the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, $\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A}) \text{vec}(\mathbf{B})$, and that since $\mathbf{F}_N^H$ is square, $(\mathbf{F}_N^H)^T = \mathbf{F}_N^H$.

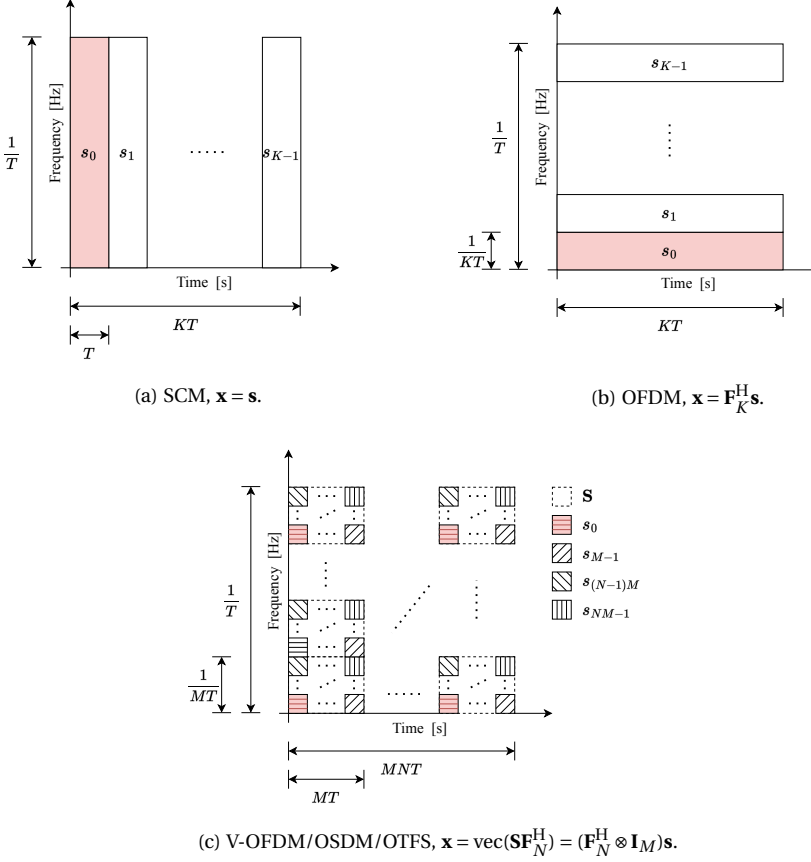


Figure 3.2.: Structure of transmitted signal in the time-frequency domain for (a)SCM, (b)OFDM and (c)V-OFDM/OSDM/OTFS. One symbol is spread in frequency (SCM), in time (OFDM) and in both frequency and time (V-OFDM/OSDM/OTFS).

We want to emphasize here that due to the mentioned zero periodic cross-correlation, the ISI (thus the interference between the block repetitions of $\mathbf{S}$ in Fig. 3.2c) is eliminated. Hence, V-OFDM and OSDM can do with only one CP per N repetitions of $\mathbf{S}$ in time[20].

3.3.3. Orthogonal time frequency space (OTFS)

OTFS defines its symbols in the DD domain, $s_{\text{DD}}[k, l]$, for $k = 0, \dots, M-1$ and $l = 0, \dots, N-1$. The symbols are first mapped to the time-frequency (TF) domain using a combination of the inverse discrete symplectic² Fourier transform (DSFT) and

²The term symplectic indicates that the transform applies a DFT in one domain variable and an IDFT in the other, in contrast to the standard 2D DFT which applies the same operation to both.

windowing [2], for $m = 0, \dots, M-1$ and $n = 0, \dots, N-1$,

$$s_{\text{TF}}[m, n] = w_{\text{tx}}[m, n] \frac{1}{\sqrt{NM}} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} s_{\text{DD}}[k, l] e^{-2\pi j m \frac{k}{M}} e^{2\pi j n \frac{l}{N}}, \quad (3.4)$$

where $w_{\text{tx}}[m, n]$ is the weight of the window. In the literature, these weights are predominantly left out or set to one [14], [27], [28], [29], [30]. The use of weighting functions different from all ones was investigated in [31]. A performance improvement was found only when the estimated channel state information (CSI) is present at both the receiver and transmitter, which is a scenario that is never satisfied in practice. Additionally, the performance improvement is strongly dependent on the type of detection. Thus, choosing a different weighting function is in general not beneficial. Therefore, in this chapter, we will use a weighting function of all ones.

Note that the DSFT is equivalent to consecutively performing an N -point IDFT and an M -point DFT on $s_{\text{DD}}[k, l]$. In matrix-vector notation one could write, $\mathbf{S}_{\text{TF}} = \mathbf{W}_{\text{tx}} \circ (\mathbf{F}_M \mathbf{S}_{\text{DD}} \mathbf{F}_N^H)$, where $\mathbf{S}_{\text{TF}}$, $\mathbf{W}_{\text{tx}}$ and $\mathbf{S}_{\text{DD}}$ are matrices of size $M \times N$ with elements $s_{\text{TF}}[m, n]$, $w_{\text{tx}}[m, n]$ and $s_{\text{DD}}[k, l]$, respectively. Here $\circ$ denotes the Hadamard (elementwise) product. The symbols in the TF domain are then converted to the time domain by the Heisenberg transform [2],

$$x(t) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \frac{s_{\text{TF}}[m, n]}{\sqrt{M}} g_{\text{tx}}(t - nMT) e^{2\pi j m \frac{\Delta f}{M} (t - nMT)}, \quad (3.5)$$

where $g_{\text{tx}}(t)$ is the transmit pulse, which is assumed zero outside the interval $[0, MT)$, and where $\Delta f = \frac{1}{T}$. From (3.5), one would not directly see the link between OTFS and V-OFDM and OSDM. However, by adopting the translation $t \rightarrow m'T + n'MT$ in (3.5) we can make the connection. To do so, we write out the translation in (3.6),

$$\begin{aligned} & x(m'T + n'MT) \\ &= \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} s_{\text{TF}}[m, n] g_{\text{tx}}(m'T + n'MT - nMT) e^{2\pi j m \frac{\Delta f}{M} (m'T + n'MT - nMT)} \\ &\stackrel{(a)}{=} \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} s_{\text{TF}}[m, n'] g_{\text{tx}}(m'T) e^{2\pi j m \frac{\Delta f}{M} (m'T)} \\ &= g_{\text{tx}}(m'T) \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} s_{\text{TF}}[m, n'] e^{2\pi j m \frac{m'}{M}} \\ &= g_{\text{tx}}(m'T) \frac{1}{\sqrt{M}} \sum_{m=0}^{M-1} \left[w_{\text{tx}}[m, n'] \frac{1}{\sqrt{NM}} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} s_{\text{DD}}[k, l] e^{-2\pi j m \frac{k}{M}} e^{2\pi j n' \frac{l}{N}} \right] e^{2\pi j m \frac{m'}{M}} \quad (3.6) \\ &\stackrel{(b)}{=} g_{\text{tx}}(m'T) \frac{1}{M\sqrt{N}} \sum_{m=0}^{M-1} \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} s_{\text{DD}}[k, l] e^{-2\pi j m \frac{k-m'}{M}} e^{2\pi j n' \frac{l}{N}} \\ &= g_{\text{tx}}(m'T) \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} \sum_{k=0}^{M-1} s_{\text{DD}}[k, l] \left[\frac{1}{M} \sum_{m=0}^{M-1} e^{-2\pi j m \frac{k-m'}{M}} \right] e^{2\pi j n' \frac{l}{N}} \\ &\stackrel{(c)}{=} g_{\text{tx}}(m'T) \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} s_{\text{DD}}[m', l] e^{2\pi j n' \frac{l}{N}}, \end{aligned}$$

where for (a) we have used that $g_{\text{tx}}(t)$ is zero outside the interval $[0, MT)$, for (b) we have used that $w_{\text{tx}}[m, n] = 1$ and for (c) we have used that,

$$\frac{1}{M} \sum_{m=0}^{M-1} e^{-2\pi j m \frac{k-m'}{M}} = \begin{cases} 1 & \text{if } k = m' \\ 0 & \text{otherwise.} \end{cases} \quad (3.7)$$

Now we stack M samples of $x(t)$, namely $x(m'T + n'MT)$ for $m' = 0, 1, \dots, M$, in one column. Using the result in (3.6) we can write,

$$\begin{aligned} & \begin{bmatrix} x(0 \cdot T + n'MT) \\ x(1 \cdot T + n'MT) \\ \vdots \\ x((M-1)T + n'MT) \end{bmatrix} = \\ &= \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} e^{2\pi j \frac{n'l}{N}} \begin{bmatrix} g_{\text{tx}}(0 \cdot T) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & g_{\text{tx}}((M-1) \cdot T) \end{bmatrix} \begin{bmatrix} s_{\text{DD}}[0, l] \\ s_{\text{DD}}[1, l] \\ \vdots \\ s_{\text{DD}}[M-1, l] \end{bmatrix} \\ &= \frac{1}{\sqrt{N}} \mathbf{G}_{\text{tx}} \begin{bmatrix} s_{\text{DD}}[0, 0] & \cdots & s_{\text{DD}}[0, N-1] \\ \vdots & \ddots & \vdots \\ s_{\text{DD}}[M-1, 0] & \cdots & s_{\text{DD}}[M-1, N-1] \end{bmatrix} \begin{bmatrix} 1 \\ e^{2\pi j \frac{n'}{N}} \\ \vdots \\ e^{2\pi j \frac{n'(N-1)}{N}} \end{bmatrix} \\ &= \frac{1}{\sqrt{N}} \mathbf{G}_{\text{tx}} \mathbf{S}_{\text{DD}} \begin{bmatrix} 1 \\ e^{2\pi j \frac{n'}{N}} \\ \vdots \\ e^{2\pi j \frac{n'(N-1)}{N}} \end{bmatrix}. \end{aligned} \quad (3.8)$$

Here $\mathbf{G}_{\text{tx}} = \text{diag}(g_{\text{tx}}(0 \cdot T), g_{\text{tx}}(1 \cdot T), \dots, g_{\text{tx}}((M-1) \cdot T))$ and $[\mathbf{S}_{\text{DD}}]_{m,n} = s_{\text{DD}}[m, n]$. If we then take N columns of M samples, we can write $\mathbf{X} = \mathbf{G}_{\text{tx}} \mathbf{S}_{\text{DD}} \mathbf{F}_N^H$, where $[\mathbf{X}]_{m,n} = x(mT + nMT)$. The modulated symbols including a transmit pulse are,

$$\mathbf{x} = \text{vec}(\mathbf{X}) = \text{vec}(\mathbf{G}_{\text{tx}} \mathbf{S}_{\text{DD}} \mathbf{F}_N^H) = (\mathbf{F}_N^H \otimes \mathbf{G}_{\text{tx}}) \mathbf{s} = (\mathbf{I}_N \otimes \mathbf{G}_{\text{tx}}) (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s}. \quad (3.9)$$

where $\mathbf{s} = \text{vec}(\mathbf{S}_{\text{DD}})$. Note that, by not considering the transmit pulse for V-OFDM and OTFS, it is implicitly assumed that a rectangular (transmit) pulse is used. Therefore, to make a fair comparison with the other modulation schemes, we should use a rectangular pulse for OTFS as well, such that the transmitted symbols for OTFS become

$$\mathbf{x} = (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s}. \quad (3.10)$$

From (3.10) it is clear that OTFS coincides with V-OFDM and OSDM.

Before we continue, a final remark about the transmit pulse is in order.

Remark 4. In [2] it is assumed that the transmit and receive pulses $g_{tx}(t)$ and $g_{rx}(t)$ are bi-orthogonal with respect to a translation of an integer multiple of MT in time and $1/MT$ in frequency. Additionally, it is assumed that $g_{tx}(t)$ and $g_{rx}(t)$ are bi-orthogonal with respect to a translation of an integer multiple of T in time, within the support of the channel (maximum time delay), and to a translation of an integer multiple of $1/T$ in frequency, within the support of the channel (maximum frequency shift). This can actually never be satisfied by $g_{tx}(t)$ and $g_{rx}(t)$ in practice [29]. In current literature a rectangular pulse is predominantly used (see e.g., [14], [28], [29]), since the use of a rectangular function for the transmit pulse is optimal in terms of BER (see [30, Fig. 1]).

3.4. Influence of the channel

In this section, we will discuss the properties of V-OFDM/OSDM/OTFS with respect to different channel characteristics. We want to stress, however, that the equivalence between the modulation schemes, as shown in the previous section, does not depend on these channel characteristics.

In Section 3.3.2 we saw that for V-OFDM/OSDM/OTFS, the modulated symbol vector $\mathbf{x}$ is the summation over the sequences $(\mathbf{f}_n^* \otimes \mathbf{s}_n)$, which have zero periodic cross-correlation across n . In this section, we investigate if this property remains after propagating through the channel with delay and Doppler spread.

The communication channel can be approximated by a basis expansion model (BEM) [32], [33]. Let $\mathbf{P}_K$ be the cyclic permutation matrix of size K . In a BEM, the delays³ are modeled by a linear filter, which (assuming a CP is used) is described by a circulant matrix, $\mathbf{H}_q = \sum_{l=0}^L c_{l,q} \mathbf{P}_K^l \in \mathbb{C}^{K \times K}$, defined by the taps of the filter, $c_{l,q}$, where $l = 0, 1, \dots, L$ and $q = -Q/2, \dots, Q/2$ index the delay and Doppler shift component respectively. A Doppler shift is modeled by a diagonal matrix, $\mathbf{\Lambda}_q = \text{diag}(e^{2\pi j \frac{K}{K} 0}, e^{2\pi j \frac{1}{K} q}, \dots, e^{2\pi j \frac{K-1}{K} q})$. An effective channel with both delays and Doppler shifts is thus described by, $\mathbf{H} = \sum_{q=-Q/2}^{Q/2} \mathbf{\Lambda}_q \mathbf{H}_q$.

As was mentioned earlier (and shown in Appendix A), the sequences have zero periodic cross-correlation across n , that is,

$$(\mathbf{f}_{n_1}^* \otimes \mathbf{s}_{n_1})^H \mathbf{P}_K^k (\mathbf{f}_{n_2}^* \otimes \mathbf{s}_{n_2}) = 0, \quad n_1 \neq n_2, \quad \forall k \in \{0, 1, \dots, K-1\}. \quad (3.11)$$

We note that a circulant matrix is a summation of weighted cyclic permutation matrices, i.e., $\mathbf{H}_q = \sum_{l=0}^L c_{l,q} \mathbf{P}_K^l$. As a result, the sequences remain orthogonal after propagating

³Note that delay spread and Doppler spread encompass the collective effect of all the delays and Doppler shifts respectively. Hence, the effects of delay spread and Doppler spread are discussed in terms of multiple delays and Doppler shifts.

through a channel $\mathbf{H}_q$ without Doppler shifts with only (possibly multiple) delays,

$$\begin{aligned}
 (\mathbf{f}_{n_1}^* \otimes \mathbf{s}_{n_1})^H \mathbf{H}_q (\mathbf{f}_{n_2}^* \otimes \mathbf{s}_{n_2}) &= (\mathbf{f}_{n_1}^* \otimes \mathbf{s}_{n_1})^H \left(\sum_{l=0}^L c_{l,q} \mathbf{P}_K^l \right) (\mathbf{f}_{n_2}^* \otimes \mathbf{s}_{n_2}) \\
 &= \sum_{l=0}^L c_{l,q} (\mathbf{f}_{n_1}^* \otimes \mathbf{s}_{n_1})^H \mathbf{P}_K^l (\mathbf{f}_{n_2}^* \otimes \mathbf{s}_{n_2}) \\
 &= \sum_{l=0}^L c_{l,q} \cdot 0 = 0, \quad n_1 \neq n_2.
 \end{aligned} \tag{3.12}$$

This means that there is no ISI between the symbols of different sequences and as such the receiver can process the sequences independently. This can be further clarified when we look at the transfer function between transmitter and receiver where the modulation, the channel and the demodulation are aggregated. The receiver demodulates the received signal using $(\mathbf{F}_N \otimes \mathbf{I}_M)$. The transfer function for the symbols becomes,

$$\tilde{\mathbf{H}}_1 = (\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{H}_q (\mathbf{F}_N^H \otimes \mathbf{I}_M). \tag{3.13}$$

The matrix $\tilde{\mathbf{H}}_1$ is block diagonal (N blocks of size M) ([34]), due to the zero cross-correlation property in (3.12).

However, the sequences do not remain orthogonal if a delay and Doppler shift is considered, since $\Lambda_q \mathbf{H}_q$ is not circulant and thus not a summation of weighted permutation matrices anymore. There exists at least one pair⁴ $(n_1, n_2) \in \{1, 2, \dots, N\}^2$ such that $n_1 \neq n_2$, for which,

$$(\mathbf{f}_{n_1}^* \otimes \mathbf{s}_{n_1})^H \Lambda_q \mathbf{H}_q (\mathbf{f}_{n_2}^* \otimes \mathbf{s}_{n_2}) \neq 0. \tag{3.14}$$

Therefore, the receiver cannot process the sequences independently anymore. Moreover, since the sequences do not remain orthogonal, the transfer function,

$$\tilde{\mathbf{H}}_2 = (\mathbf{F}_N \otimes \mathbf{I}_M) \Lambda_q \mathbf{H}_q (\mathbf{F}_N^H \otimes \mathbf{I}_M), \tag{3.15}$$

is not block diagonal, which is another way of describing that the sequences cannot be processed independently.

Overall, we can conclude the following about the tolerance of V-OFDM/OSDM/OTFS against Doppler shifts. If the channel experiences only one Doppler shift, thus a channel of the form $\Lambda_{q_1} \mathbf{H}_{q_1}$, the receiver can still compensate for this one Doppler shift *before* demodulation and further processing, namely by multiplying with $\Lambda_{q_1}^{-1}$. Hence, interference between sequences can still be avoided. However, when the communication channel consists of multiple delays, of which at least two have different Doppler shifts, thus a channel of the form $\sum_i \Lambda_{q_i} \mathbf{H}_{q_i}$, this compensation before demodulation is not possible anymore. Consequently, interference between different sequences is inevitable.

Therefore, the claim that V-OFDM/OSDM/OTFS is more robust against Doppler shifts than OFDM (e.g. [2]) should be interpreted carefully. V-OFDM/OSDM/OTFS can only outperform OFDM if the right processing is performed at the receiver. Han *et al.* [34] described such a receiver that cancels the ICI between the blocks in $\tilde{\mathbf{H}}_2$. Alternatively, Raviteja *et al.* [30] describe a receiver in the delay-Doppler domain. Similar to OFDM, V-OFDM/OSDM/OTFS can also have guard bands to be (more) tolerant of large Doppler spread, as was proposed in [35] and was called Doppler resilient OSDM (D-OSDM).

⁴For example, all pairs that satisfy $q = n_1 - n_2 \neq 0$.

3.5. Conclusion

In this chapter, we have shown that OSDM and OTFS are mathematically equivalent by deriving the input-output relation of the modulation techniques from the first principles presented in [1] and [2], respectively. Additionally, we have given insight in the performance in terms of ISI and ICI by discussing its signal structure.

Due to the equivalence, all the results that have been presented in underwater communications on OSDM can be carried over and continued in RF communications on OTFS, and vice versa. We conclude that, while OTFS may be derived from different principles, the resulting modulation technique is equivalent to V-OFDM, A-OFDM (shown in [24] and [14]) and OSDM (shown in this chapter) which were all invented prior to OTFS.

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Pacific Ocean, Uvita, Costa Rica

4

Optimal Pilot Design for OTFS

The previous chapter examined various modulation schemes in detail and established that orthogonal time frequency space (OTFS) is equivalent to, among other schemes, the previously proposed orthogonal sequence division multiplexing (OSDM) for underwater acoustic communications.

This chapter further investigates OTFS by focusing on the design of pilot signals. It begins with a review of existing pilot designs for various modulation schemes, followed by the derivation of a minimal-overhead pilot structure tailored to OTFS. Finally, the power allocation between pilot and data signals is optimized based on a lower bound on the channel capacity.

This chapter is based on the journal article published as:

I. van der Werf, R. Heusdens, R. C. Hendriks and G. Leus, "Optimal pilot design for OTFS in linear time-varying channels," *Signal Processing*, vol. 239, 2026.

4.1. Introduction

As established in the previous chapter, the choice of a communication system's modulation scheme dictates how transmitted symbols are distributed across the time and frequency resources. In recent wireless radio-frequency (RF) communication standards, orthogonal frequency division multiplexing (OFDM) has been widely adopted as the preferred choice [1], and single-carrier modulation (SCM) is still a common choice for underwater acoustic (UWA) communications. Because OFDM and SCM often struggle with intercarrier interference (ICI) and intersymbol interference (ISI) in doubly selective channels, OTFS has been presented as a robust alternative (see e.g., [2], [3]). Throughout the remainder of this dissertation, we use OTFS as an umbrella term to encompass equivalent schemes such as vector OFDM (V-OFDM), asymmetric OFDM (A-OFDM), and OSDM, except where specific references to those individual modulations are required. While OTFS leverages diversity in both time and frequency to outperform OFDM in highly dynamic environments [4], [5], [6], it is not inherently immune to all channel impairments. As noted previously, its performance remains sensitive to the specific characteristics of delay and Doppler spread.

In this chapter, we focus on pilot design for OTFS. Pilot symbols serve as the “known” reference symbols within a transmitted packet; because the receiver knows their original value, it can compare the received version to the transmitted version to estimate the channel impulse response (CIR). Without accurate channel estimation facilitated by these pilots, equalization of the symbols would be impossible.

4.1.1. Contributions of this chapter

To get the most out of a communication system, careful design of the pilot symbols is required. Although many pilot designs have been proposed for OTFS (see, e.g., [7], [8], [9], [10]), a comparison and (mathematical) analysis of the optimality of these designs is lacking. This chapter addresses this gap by first providing a comprehensive overview of optimal pilot design for linear time-invariant (LTI) and linear time-variant (LTV) channels for OTFS and related modulation schemes. Following this foundation, we present a new pilot design for OTFS. The contributions of this chapter can be summarized as follows:

- We provide an overview of the work on (optimal) pilot design for LTI and LTV channels, and of the work on specific pilot designs for the most relevant modulation schemes.
- We propose two new pilot allocations for OTFS that reduce the pilot overhead by approximately 50%. These pilot allocations still achieve the minimum mean squared error (MMSE) on the estimation of the channel taps.
- We propose to choose the power distribution between the pilot and the data symbols such that it maximizes a lower bound on the channel capacity.

Our findings can be used as guidance for designing the OTFS parameters based on the delay and Doppler spread to increase the channel capacity.

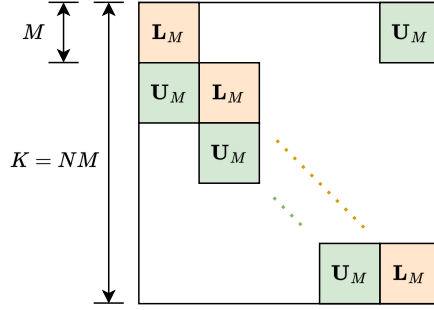


Figure 4.1.: Visualization of the decomposition $\mathbf{P}_K = \mathbf{I}_N \otimes \mathbf{L}_M + \mathbf{P}_N \otimes \mathbf{U}_M$.

Notation

Let K be a positive integer and let $\mathbf{P}_K$ denote a $K \times K$ cyclic permutation matrix. Further, let $\mathbf{P}_K^l$ denote the l 'th power of $\mathbf{P}_K$ for some integer l . Let N and M be positive integers. If $K = NM$, we can rewrite the cyclic permutation matrix as $\mathbf{P}_K = \mathbf{I}_N \otimes \mathbf{L}_M + \mathbf{P}_N \otimes \mathbf{U}_M$, where $\mathbf{L}_M$ is a cyclic permutation matrix of size M except for the top right element, which is zero, and $\mathbf{U}_M$ is a zero matrix of size M except for the top right element, which is one (thus $\mathbf{P}_M = \mathbf{U}_M + \mathbf{L}_M$). Fig. 4.1 visualizes the decomposition of $\mathbf{P}_K$. Similarly, provided $l < M$, we can write $\mathbf{P}_K^l = \mathbf{I}_N \otimes \mathbf{L}_M^{(l)} + \mathbf{P}_N \otimes \mathbf{U}_M^{(l)}$, where the matrices $\mathbf{L}_M^{(l)}$ and $\mathbf{U}_M^{(l)}$ denote the lower and upper part of $\mathbf{P}_M^{(l)}$, respectively. Lastly, we define the matrix $\Lambda_K = \text{diag}(e^{j2\pi 0/K}, \dots, e^{j2\pi (K-1)/K})$. Before we elaborate on the related work, we start by a brief discussion of the signal model in the next section.

4.2. Signal model

In this section, we discuss the OTFS modulation scheme and the LTV channel model at normalized baseband.

4.2.1. OTFS transmitter

For OTFS, the transmitter characterizes $K = NM$ symbols in the delay-Doppler (DD) domain (M by N symbols in the delay and Doppler dimension respectively), collected in the matrix $\mathbf{S}_{\text{DD}} \in \mathbb{C}^{M \times N}$, after which the symbols are consecutively transformed to the time-frequency and the time domain by the inverse symplectic finite Fourier transform (ISFFT) and the Heisenberg transform [3], respectively. Assuming a rectangular transmit pulse is used [11], the transmitted signal in discrete-time baseband is given by (see Chapter 3 or [12]),

$$\mathbf{x} = \text{vec}(\mathbf{S}_{\text{DD}} \mathbf{F}_N^H) = (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s}, \quad (4.1)$$

where $\mathbf{F}_N^H$ denotes the inverse discrete Fourier transform matrix, $\mathbf{I}_M$ is an identity matrix of size M and $\mathbf{s} = \text{vec}(\mathbf{S}_{\text{DD}})$. In Fig. 4.2, a visual representation of the operation at the transmitter, i.e., (4.1), is shown.

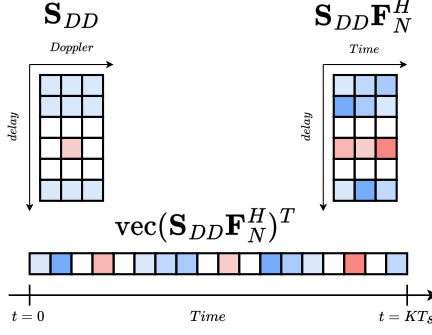


Figure 4.2.: Visualization of the OTFS transmitted signal, $\{K, N, M\} = \{18, 3, 6\}$.

4.2.2. Channel model

Let $r[k]$, $h_{\text{nbb}}[k, l]$, $x[k]$, and $n[k]$ denote the discrete-time received signal, channel impulse response, transmitted signal, and additive channel noise, respectively, in normalized baseband (as discussed in Section 2.1.3). We assume Nyquist-rate sampling, such that the signals are observed at time instances $t_k = kT_{\text{symp}}$ and time delays $\tau_l = lT_{\text{symp}}$, with T_{symp} denoting the symbol period. The received signal is then described by

$$r[k] = \sum_{l=0}^L h_{\text{nbb}}[k, l] x[k-l] + n[k].$$

Note that an LTI channel is subsumed by this model; if $h_{\text{nbb}}[k, l] = h_{\text{nbb}}[l]$, the model is time-invariant. Suppose we collect a total of K samples over time, then estimating all the $K(L+1)$ channel coefficients is an ill-posed problem, as in general $K < K(L+1)$. In order to decrease the number of coefficients that need to be estimated, the basis expansion model (BEM) was introduced [13].

The BEM is a model that represents how the channel changes over time. It approximates the varying channel taps by expressing them using a lower order basis of size $Q+1$ (with $Q+1 < K$),

$$h_{\text{nbb}}[k, l] = \sum_{q=-Q/2}^{Q/2} c_{l,q} b_q[k]. \quad (4.2)$$

Here $b_q[k] \in \mathbb{C}$ is the function representing the basis and $c_{l,q} \in \mathbb{C}$, for $l = 0, \dots, L$ and $q = -Q/2, \dots, Q/2$, are the BEM channel coefficients. In general $Q+1 \ll K$, and thus the total number of coefficients to be estimated is now lower, casting the problem well posed if $K \geq (Q+1)(L+1)$.

The BEM can be used with different bases and thus functions $b_q[k]$. Over the past decades, many various bases have been proposed. The most well-known is the complex exponential BEM (CE-BEM) [13], for which $b_q[k] = e^{j\omega_q k}$, $\omega_q = 2\pi q/K$. Note that the CE-BEM is quite comprehensible; the coefficients of the CE-BEM each represent a unique pair of Doppler shift ω_q and time delay τ_l . Some other popular BEMs are the generalized CE-BEM (GCE-BEM) [14], for which $b_q[k] = e^{j\omega_q k}$, $\omega_q = 2\pi q/(KR)$, $R \geq 1$, the

polynomial BEM (P-BEM) [15], the discrete Karhunen–Loève BEM (DKL-BEM) [16] and the discrete prolate spheroidal BEM (DPS-BEM) [17]. Of course, the modeling accuracy of each choice differs per application.

Remark 5. *In the OTFS literature, the so-called delay-Doppler channel is often used instead of a BEM. However, assuming that the time delays and Doppler shifts fall on the Nyquist grid, the delay-Doppler channel coincides with the (conventional) CE-BEM.*

In this work, we adopt the CE-BEM due to its intuitive structure and widespread use in the literature. Assuming that we use a cyclic prefix (CP) and that we model the channel with additive Gaussian noise with zero mean and covariance $\mathbf{R}_n$, the received signal is given by

$$\begin{aligned} \mathbf{r} &= \mathbf{H}\mathbf{x} + \mathbf{n} = \left(\sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L c_{l,q} \text{diag}(b_q[0], \dots, b_q[K-1]) \mathbf{P}_K^l \right) \mathbf{x} + \mathbf{n} \\ &= \left(\sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L c_{l,q} \mathbf{\Lambda}_K^q \mathbf{P}_K^l \right) \mathbf{x} + \mathbf{n}, \end{aligned}$$

where the vectors $\mathbf{r}$, $\mathbf{x}$ and $\mathbf{n}$ collect the samples of $r[k]$, $x[k]$ and $n[k]$, respectively, for $k = 0, 1, \dots, K-1$. Note that when $Q = 0$, the LTV channel becomes time-invariant and thus coincides with an LTI channel.

Remark 6. *The parameters L and Q denote the maximum time delay and Doppler shift in the channel, i.e. $\tau_{\max} = LT_{\text{symb}}$ and $f_{\max} = Q/(2KT_{\text{symb}})$, respectively. Although specific channel taps vary, these parameters remain constant over a long time. Upper bounds for these values are typically well-known for particular environments, allowing L and Q to be set to these bounds before communication.*

4.2.3. OTFS receiver

At the receiver, the demodulated signal is given by,

$$\mathbf{y} = (\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{r}.$$

To obtain the signal received in the DD domain, the vector $\mathbf{y}$ is reshaped into an $M \times N$ matrix, i.e., $\mathbf{Y} = \text{vec}^{-1}(\mathbf{y})$. Typically, we choose $N \geq Q+1$ and $M \geq L+1$ to ensure that the channel can be properly resolved in both delay and Doppler dimensions, as will become clear shortly. The literature on OTFS receivers is rich; see, e.g. [6], [18], [19], [20]. However, the most straightforward way is to apply a linear MMSE estimator to estimate the transmitted signal.

4.3. Related work on pilot design

In this section, we examine the existing literature on the development of (optimal) pilot designs.

Designing a pilot signal can be divided into two main components: pilot allocation and power distribution, which we will treat separately. Due to its practical relevance, we also

briefly discuss pilot design in relation to peak-to-average power ratio (PAPR). We will focus exclusively on designs where pilot and data symbols do not overlap, an assumption that underpins the remainder of this chapter. For studies on superimposed pilot designs, we refer readers to [21] and references therein.

Throughout the first two subsections, we discuss various pilot allocations for different modulation schemes and different channel models. A comparison of the pilot overhead of the discussed allocations is provided in Table 4.1.

4.3.1. Pilot allocation for SCM and OFDM

In this subsection, we discuss the existing pilot allocation strategies for SCM and OFDM. First, we examine literature on LTI channels, followed by a discussion on LTV channels. Note that OTFS reduces to SCM when $N = 1$ and $M = K$, and to OFDM when $N = K$ and $M = 1$.

Research on pilot allocation dates back to the late 1990s, considering metrics such as mean squared error (MSE), channel capacity, and the Cramér–Rao bound (CRB), which often led to similar optimal designs. For LTI channels (i.e., $Q = 0$), a BEM is unnecessary, and typically, the system is overdetermined ($K > L + 1$). In the SCM setting, [22] showed that clustering at least $K_p = 2L + 1$ pilot symbols, zeroing the leading and trailing L symbols, yields capacity-optimal results. This structure was also found optimal under the CRB on the channel tap estimator in [23], reflecting the equivalence of MSE and CRB under additive white Gaussian noise.

For OFDM, optimal pilot placement involves only $K_p = L + 1$ equispaced tones. This was shown to minimize the MSE on the channel taps [24] and maximize a capacity lower bound [22].

Unlike the LTI case, studies on LTV channels generally avoid assuming a specific modulation scheme. However, pilot and data symbols are often separated in either time or frequency, naturally aligning with SCM and OFDM, respectively. Similar to this work, existing works on pilot design typically model LTV channels using a CE-BEM.

In [26], assuming time-separated pilots and data, equispaced and equipowered pilot clusters of length $2L + 1$ (with L leading and trailing zeros) are shown to maximize a lower bound on average capacity. A total of $Q + 1$ such clusters are placed equidistantly in time, reflecting the Nyquist criterion for Doppler sampling (sampling at twice the maximum Doppler frequency, $Q/2$ in normalized baseband). This leads to a total pilot overhead of $K_p = (2L + 1)(Q + 1)$. This pilot design effectively probes the channel $Q + 1$ times using impulse-like signals, as visualized in Fig. 4.3a. Notably, for $Q = 0$, this reduces to the optimal SCM-based design for LTI channels.

Independently, [27] proposed three MSE-optimal pilot designs. The first mirrors [26] (time-separated). The second uses frequency separation, inserting guard bands to prevent pilot-data overlap post-channel. As visualized in Fig. 4.3b, it probes the channel $L + 1$ times in frequency, using equispaced, equipowered pilots surrounded by Q zeros, generalizing the OFDM-based design when $Q = 0$. This second design has a pilot overhead of $K_p = (L + 1)(2Q + 1)$. The third, based on linear chirps [27], [28], has lower spectral efficiency and is therefore not considered further here.

Interestingly, frequency separation outperforms time separation when $L > Q$, and vice

Table 4.1.: Comparison of pilot overhead across different modulation schemes and channel models. The minimum pilot overhead values (for all L and Q) are indicated in bold.

Channel	Paper	Modulation	MMSE optimal	Pilot overhead K_p
LTI	[22], [23]	SCM	yes	$2L + 1$
	[24], [25]	OFDM	yes	$L + 1$
	[4]	OSDM	yes	M
LTV	[26], [27], [28]	SCM	yes	$(2L + 1)(Q + 1)$
	[27], [28]	OFDM	yes	$(L + 1)(2Q + 1)$
	[7]	OTFS	yes	$(2L + 1)(2Q + 1)$
	[8]	OTFS	yes	$(2L + 1)N$
	[10]	OTFS	no	$(2L + 1)N$
	[29]	OSDM	no	$M(2Q + 1)$
	This work	OTFS	yes	$\min\{(L + 1)(2Q + 1), (2L + 1)(Q + 1)\}$

versa when $L < Q$ in terms of pilot overhead K_p [27], [28].¹ Moreover, for both cases, it suffices that pilot clusters are equispaced; their absolute positions in time or frequency are irrelevant for MSE performance.

Finally, [30] and [31] address scenarios where the Doppler spread is overestimated. In such cases, changing zero-valued pilot symbols into nonzero-valued pilot symbols can improve estimation, though this yields sub-optimal designs because the pilot/data structure is not fully adapted to the actual channel.

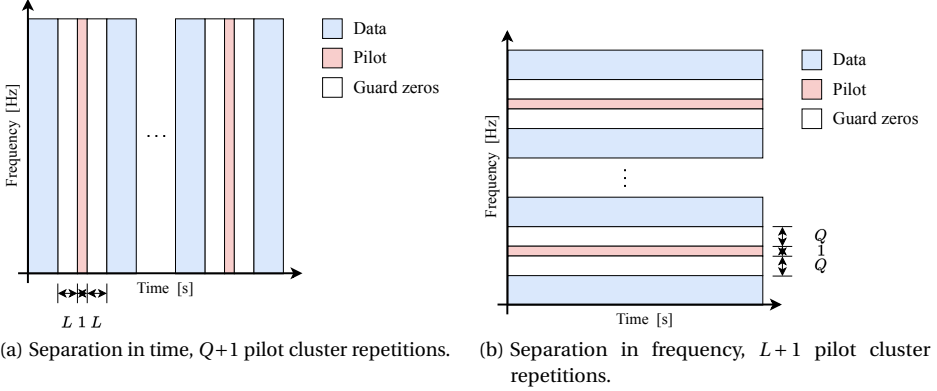


Figure 4.3.: Two methods, as proposed by (a)[26], [27], [28] and (b)[27], [28].

4.3.2. Pilot allocation for OSDM and OTFS

In this section, we discuss the most relevant existing pilot allocations for OSDM and OTFS. Note that since the two modulation schemes are equivalent, as shown in the previous chapter, pilot designs proposed for one scheme can be directly applied to the other.

In [4], the authors approach the (OSDM) modulation from a sequence-based perspective. They propose allocating one out of the N available sequences – each of length M – entirely to pilot symbols, employing a shift-orthogonal sequence. When applied to an LTI channel, no guard intervals are inserted between pilot and data symbols in this design. This idea was later extended to a modulation scheme termed Doppler-resilient OSDM (D-OSDM) [29], which introduces guard intervals to enhance robustness in LTV channels. A visual representation of this latter scheme is provided in Fig. 4.4.

In [7], [8], two pilot allocations are proposed for OTFS. The first approach places a single pilot in the DD domain, surrounded by zeros to prevent interference with data symbols at the receiver. The pilot overhead is thus $K_p = (2Q+1)(2L+1)$, as visualized in Fig. 4.5a. The second scheme accounts for fractional Doppler shifts, which occur when the actual Doppler shifts do not align perfectly with the grid. In this case, all neighboring Doppler bins are set to zero to mitigate interference. The pilot overhead of this second

¹Note that when $L > Q$ we have $(2L+1)(Q+1) > (L+1)(2Q+1)$ and that when $L < Q$ we have $(2L+1)(Q+1) < (L+1)(2Q+1)$.

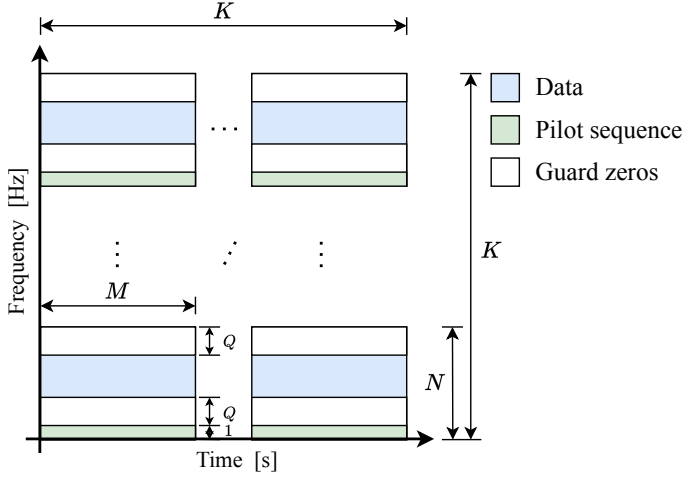


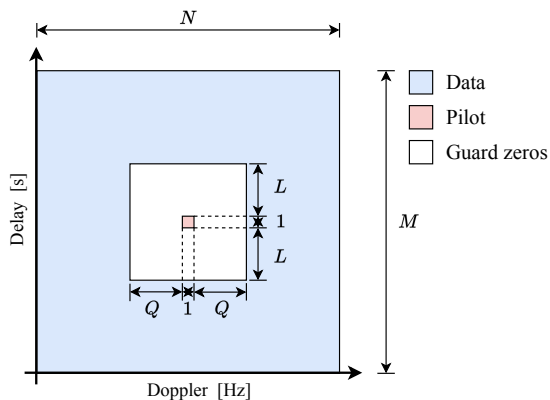
Figure 4.4.: Time-frequency plot for D-OSDM [29].

approach is therefore $K_p = N(2L + 1)$, as illustrated in Fig. 4.5b. Note that both pilot allocations have a higher overhead than the allocations proposed for SCM and OFDM by [26], [27], [28]. To reduce the high pilot overhead of the fractional Doppler scheme in [7], a different method is introduced in [20]. The work adopts the GCE-BEM and proposes a two-step estimation process. In the first step, the BEM order Q is fixed and $R = 1$ is used. In the second step, Q is increased by adjusting R from 1 to 2. Both the channel coefficients and data symbols are iteratively refined, incorporating initial demodulation results. By starting with a reduced BEM order, pilot overhead is significantly lowered compared to the one-step method in [7], while improving estimation accuracy through model refinement. However, besides the higher pilot overhead, the pilot designs for OTFS by [7], [8], [12], [20] did not provide insight into the design by discussing the pilot signal in the time-frequency domain, in contrast to the works on SCM and OFDM.

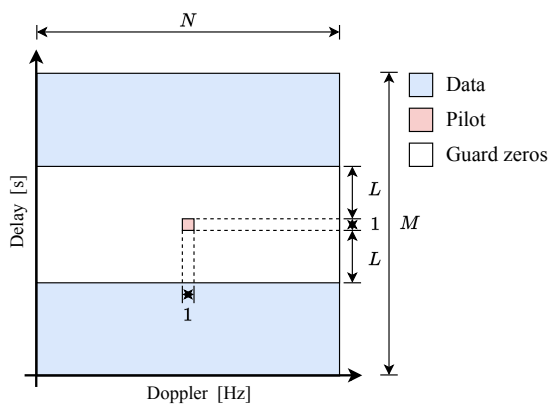
Interestingly, it is possible to characterize pilot designs for OTFS in the time-frequency domain. In Chapter 3, we identified the equivalence of OTFS and OSDM, showing that symbols in the DD domain appear as repeated structures in the time-frequency domain. More specifically, *each OTFS symbol in the DD domain is repeated N times in time and M times in frequency*.

This phenomenon, also appearing in Fig. 4.2 (where one red symbol in the DD domain is repeated $N = 3$ times in time), is further visualized in Fig. 4.5c, which shows the time-frequency plot of the delay-Doppler signal of Fig. 4.5b. Note that Fig. 4.5c, where the signal exhibits N repeated pilot clusters in time, each with L leading and trailing symbols set to zero, closely resembles the time-separated pilot design in [26], [27], [28], shown in Fig. 4.3a, where $Q + 1$ repeated pilot clusters also have L leading and trailing symbols equal to zero. This raises the question: how many repetitions are optimal?

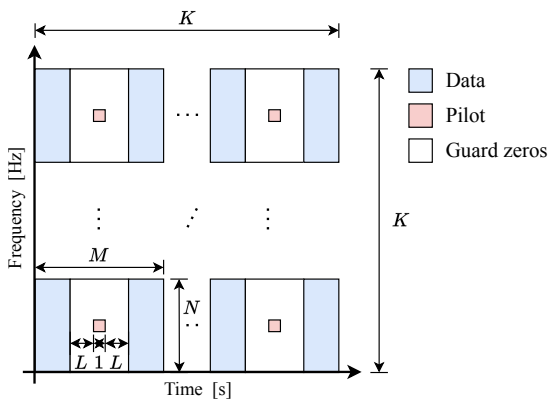
Given $L + 1$ delay taps and $Q + 1$ Doppler taps, we require at least $Q + 1$ time samples and $L + 1$ frequency samples of the pilot signal. Since OTFS exhibits N repetitions in time and M in frequency, this requirement can be met by imposing $N \geq Q + 1$ and $M \geq L + 1$,



(a) Integer Doppler case [7], [8].



(b) Fractional Doppler case [7], [8].



(c) Time-frequency plot of (b).

Figure 4.5.: Pilot allocations in the delay-Doppler, (a) and (b), and in the time-frequency domain, (c).

and using one pilot symbol in the DD domain. Alternatively, it is also possible to meet the sampling requirement for $N < Q + 1$ and/or $M < L + 1$, however, this requires multiple equispaced pilots in the delay and/or Doppler dimension, respectively, each with appropriate guard bands. This, however, complicates the pilot design, does not reduce the pilot overhead, which is why we will not further investigate this option. With $K = NM$, we can flexibly choose either N or M (as long as K/N or K/M remains an integer). In this work, we will exploit this flexibility to reduce the pilot overhead by approximately 50%, from $K_p = (2L + 1)(2Q + 1)$ to $K_p = \min\{(L + 1)(2Q + 1), (2L + 1)(Q + 1)\}$.

Remark 7. *A notable difference between SCM and OTFS is that the repetitions of the OTFS pilot impulses in time have different phases. As a result, the pilots are not active on all frequencies, whereas the frequency characteristics of pilot clusters in [26], [27], [28] were not explicitly analyzed. The authors of [26] did, however, observe that periodic repetition in time implicitly induces repetition in frequency².*

Furthermore, while [26], [27], [28] do not specify *how* data symbols are allocated, OTFS inherently provides a form of precoding for these symbols. In fact, [26] does not consider the structure or coding of data symbols at all.

4.3.3. Power distribution

For distributing power, one can consider how to allocate power across multiple pilot symbols (if one has multiple) and/or how to divide the total available transmitter power between the pilot signal and the data signal.

For LTI channels, employing OFDM, [25] proved that $L + 1$ pilot tones with equal energy are optimal with respect to the MSE on the channel taps. The optimal power distribution between pilot and data signals was derived in [22], [25]. These results were extended to stochastic channels in [32]. A similar derivation for LTV channels was provided by [22], [25], yielding significant gains.

In most OTFS pilot designs, only a single pilot symbol is specified in the delay-Doppler domain, which focuses the concern on the power allocation between the pilot and the data signal. While it is recognized that assigning different power levels to pilot and data signals impacts the performance ([7], [8], [12]), no optimality analysis has been provided in these works. Additionally, an optimal power distribution has (also) not been derived in the existing literature on OSDM.

This chapter aims to fill this gap and provides a closed-form expression to identify the optimal power allocation.

4.3.4. Pilot designs mitigating PAPR for OTFS

A PAPR analysis of OTFS is conducted in [33], revealing that increasing pilot power significantly raises the PAPR. This contradicts the claim in [7] that the “spread-spectrum

²Quote: “We wish to show in this subsection that our optimal PSAM (...) enables 2-D sampling and estimation of our time-frequency selective channel. Intuitively thinking, the Kronecker deltas (...) surrounded by zero-guards implement time-domain sampling with pilot symbols; furthermore, the fact that these deltas are periodically inserted implies that they are also equivalent to Kronecker deltas in the frequency-domain and thus serve as pilot tones as well.”

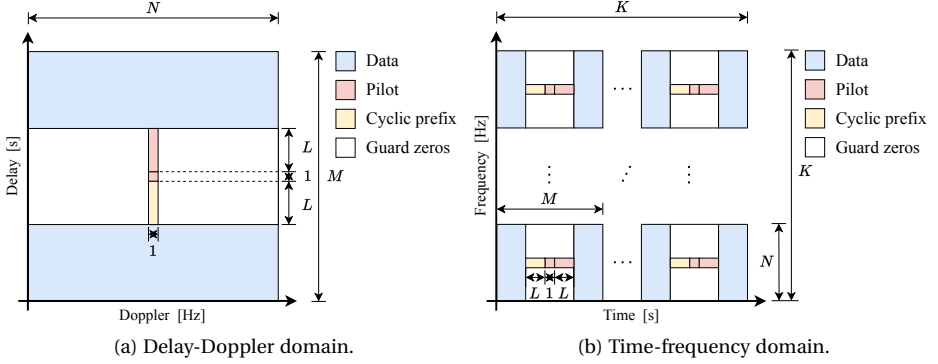


Figure 4.6.: Pilot allocation as proposed by [10].

nature of OTFS” allows for higher pilot power without PAPR increase. Unfortunately, the pilot structure details (e.g., guard size) are not fully disclosed in [33]. Moreover, while the optimal power distribution is found empirically, the lack of theoretical analysis limits the generalizability of the findings.

An alternative scheme aimed at PAPR reduction is proposed in [10]. Here, the pilot from [7] is modified by extending it along the delay axis using a Zadoff-Chu (ZC) sequence and inserting a CP. This configuration, shown in Fig. 4.6, significantly reduces PAPR, albeit at the cost of increased bit error ratio (BER).

It is worth noting that both [29] and [10] make use of shift-orthogonal sequences, yet they differ in their pilot-data separation strategy: [29] achieves separation in the frequency domain, while [10] does so in the time domain. This suggests that D-OSDM may exhibit improved PAPR performance as well, compared to the impulse pilot scheme for OTFS [8], although this comes at the cost of reduced BER performance.

Another approach is presented in [9], where multiple pilots are placed at the corners of the DD domain, each with surrounding guard symbols. However, the rationale for this placement is unclear, and although a comparison with [7] is included, the use of different estimators per pilot design renders the comparison inconclusive.

4.4. OTFS modulation through the LTV channel

In this section, we explore the interaction of the OTFS modulation scheme with a CE-BEM channel with L temporal delays and Q Doppler shifts. It will be observed that the modulation scheme transforms the time-varying channel into a time-invariant channel in the DD domain. Furthermore, the received signal, in the DD domain, can be represented as a circularly shifted form of the transmitted signal (in the same domain).

If we receive K samples over time, where K is factored as $K = NM$, the received signal

can be expressed as,

$$\begin{aligned}
 \mathbf{r} &= \mathbf{H}(\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s} + \mathbf{n} \\
 &= \left(\sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L c_{l,q} \Lambda_K^q \mathbf{P}_K^l \right) (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s} + \mathbf{n} \\
 &= \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L c_{l,q} (\Lambda_N^q \otimes \Lambda_M^{q/N}) \overbrace{\left[(\mathbf{I}_N \otimes \mathbf{L}_M^{(l)}) + (\mathbf{P}_N \otimes \mathbf{U}_M^{(l)}) \right]}^{\mathbf{P}_K^l} (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s} + \mathbf{n} \\
 &= \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L c_{l,q} \left[(\Lambda_N^q \mathbf{I}_N \otimes \Lambda_M^{q/N} \mathbf{L}_M^{(l)}) + (\Lambda_N^q \mathbf{P}_N \otimes \Lambda_M^{q/N} \mathbf{U}_M^{(l)}) \right] (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s} + \mathbf{n} \\
 &= \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L c_{l,q} \left[(\Lambda_N^q \mathbf{F}_N^H \otimes \Lambda_M^{q/N} \mathbf{L}_M^{(l)}) + (\Lambda_N^q \mathbf{P}_N \mathbf{F}_N^H \otimes \Lambda_M^{q/N} \mathbf{U}_M^{(l)}) \right] \mathbf{s} + \mathbf{n}. \tag{4.3}
 \end{aligned}$$

At the receiver side we first demodulate the signal,

$$\begin{aligned}
 \mathbf{y} &= (\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{r} \\
 &= \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L c_{l,q} \left[\underbrace{(\mathbf{F}_N \Lambda_N^q \mathbf{F}_N^H \otimes \Lambda_M^{q/N} \mathbf{L}_M^{(l)})}_{\mathbf{P}_N^q} + \underbrace{(\mathbf{F}_N \Lambda_N^q \mathbf{P}_N \mathbf{F}_N^H \otimes \Lambda_M^{q/N} \mathbf{U}_M^{(l)})}_{\mathbf{P}_N^q \Lambda_N^{-1}} \right] \mathbf{s} \\
 &\quad + (\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{n}. \tag{4.4}
 \end{aligned}$$

where we used $\mathbf{F}_N \Lambda_N^q \mathbf{F}_N^H = \mathbf{P}_N^q$ and $\mathbf{F}_N \mathbf{P}_N \mathbf{F}_N^H = \Lambda_N^{-1}$. Then, using $\text{vec}(\mathbf{B} \mathbf{S} \mathbf{A}^\top) = (\mathbf{A} \otimes \mathbf{B}) \text{vec}(\mathbf{S})$, the vector $\mathbf{y}$ reshaped into an $M \times N$ matrix reads

$$\begin{aligned}
 \mathbf{Y} &= \text{vec}^{-1}(\mathbf{y}) \\
 &= \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L c_{l,q} \Lambda_M^{q/N} \left[\mathbf{L}_M^{(l)} \mathbf{S} (\mathbf{P}_N^q)^\top + \mathbf{U}_M^{(l)} \mathbf{S} (\mathbf{P}_N^q \Lambda_N^{-1})^\top \right] + \mathbf{N} \mathbf{F}_N \\
 &= \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L c_{l,q} \Lambda_M^{q/N} \times \\
 &\quad \left[\mathbf{L}_M^{(l)} \mathbf{S} \mathbf{P}_N^{-q} + \mathbf{U}_M^{(l)} \mathbf{S} \mathbf{P}_N^{-q} \text{diag}\left(e^{-j2\pi(0-q)/N}, \dots, e^{-j2\pi((N-1)-q)/N}\right) \right] + \mathbf{N} \mathbf{F}_N \\
 &= \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L c_{l,q} \mathbf{W}_{l,q} \circ \left(\mathbf{P}_M^l \mathbf{S} \mathbf{P}_N^{-q} \right) + \mathbf{N} \mathbf{F}_N, \tag{4.5}
 \end{aligned}$$

where the last equality follows since $\mathbf{L}_M^{(l)}$ and $\mathbf{U}_M^{(l)}$ have disjoint row supports and $\mathbf{P}_M^l = \mathbf{L}_M^{(l)} + \mathbf{U}_M^{(l)}$, so that the bracketed term equals $\mathbf{P}_M^l \mathbf{S} \mathbf{P}_N^{-q}$ with a row-dependent column scaling, i.e. an element-wise mask,

$$[\mathbf{W}_{l,q}]_{m,n} = \begin{cases} e^{j2\pi qm/N}, & \text{if } m \geq l, \\ e^{j2\pi qm/N} e^{-j2\pi(n-q)/N}, & \text{if } m < l, \end{cases} \tag{4.6}$$

for $m = 0, 1, \dots, M-1$, $n = 0, 1, \dots, N-1$.

From (4.5), we learn that the delay tap l (circularly) shifts the rows of $\mathbf{S}$. Similarly, the Doppler tap q (circularly) shifts the columns in $\mathbf{S}$. Therefore, the first and second dimensions of $\mathbf{S}$ are often called the delay and Doppler dimensions, respectively. The channel coefficient $c_{l,q}$ and the matrix $\mathbf{W}_{l,q}$ apply a change in amplitude and phase.

4.5. Optimal pilot design for OTFS

In this section, we will propose a new pilot design for OTFS modulation, taking into account the relationship between the channel and the transmitted signal, as derived in the previous section. The following steps will be followed:

1. We begin by establishing a criterion that ensures no overlap between pilot and data symbols at the receiver (and consequently at the transmitter), and analyze the pilot allocation schemes that satisfy this condition. (Section 4.5.1)
2. Then, we derive the allocations with the minimum number of pilot overhead. (Section 4.5.2)
3. We show that for fixed pilot power, these allocations achieve a channel estimate that minimizes the MSE on channel taps. (Section 4.5.3)
4. Finally, we optimize the power distribution between the pilot and the data symbols with respect to the channel capacity. (Section 4.5.4)

4.5.1. Step 1: Analyzing possible pilot allocations

We will assign the available symbols to pilot and data symbols. Let K_d and K_p denote the number of data and pilot symbols, such that $K = K_d + K_p$. Suppose we have a total power budget P , which can be distributed between the pilot and data symbols. Let $P_d = \alpha P$ and $P_p = (1 - \alpha)P$, where $\alpha \in [0, 1]$ controls the power allocation between data and pilot symbols, while the total power constraint $\|\mathbf{s}\|_2^2 = P_d + P_p = P$ is satisfied. In matrix vector notation, this division can be written as

$$\mathbf{s} = \sqrt{P_d} \Phi_d \mathbf{s}_d + \sqrt{P_p} \Phi_p \mathbf{s}_p. \quad (4.7)$$

Here, $\Phi_d \in \{0, 1\}^{K \times K_d}$ and $\Phi_p \in \{0, 1\}^{K \times K_p}$ are selection matrices containing only K_d and K_p columns of $\mathbf{I}_K$, indexed by $\mathbf{p}_d \in \{0, 1\}^{K \times 1}$ and $\mathbf{p}_p \in \{0, 1\}^{K \times 1}$. We have $\mathbf{1}_{K \times 1}^T \mathbf{p}_d = K_d$ and $\mathbf{1}_{K \times 1}^T \mathbf{p}_p = K_p$, and a symbol is either used for the pilot or communication, hence $\mathbf{p}_d + \mathbf{p}_p = \mathbf{1}_{K \times 1}$. Note that we require $\|\Phi_d \mathbf{s}_d\|_2^2 = \|\Phi_p \mathbf{s}_p\|_2^2 = 1$.

After demodulation we can write our received signal as,

$$\begin{aligned} \mathbf{y} &= (\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{H} (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s} + (\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{n} \\ &= \sqrt{P_d} \mathbf{H}_{DD} \Phi_d \mathbf{s}_d + \sqrt{P_p} \mathbf{H}_{DD} \Phi_p \mathbf{s}_p + \mathbf{w}. \end{aligned}$$

Here $\mathbf{H}_{DD} = (\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{H} (\mathbf{F}_N^H \otimes \mathbf{I}_M)$ and $\mathbf{w} = (\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{n}$. At the receiver, we can divide the received signal into a data part and a pilot part, where we denote the data part by

$\mathbf{y}_d = \Psi_d^H \mathbf{y}$ and the pilot part by $\mathbf{y}_p = \Psi_p^H \mathbf{y}$. Here, the matrices $\Psi_d \in \mathbb{C}^{K \times R_d}$ and $\Psi_p \in \mathbb{C}^{K \times R_p}$ denote selection matrices, similar to the definition of Φ_d and Φ_p , and contain R_d and R_p columns of $\mathbf{I}_K$ indexed by $\tilde{\mathbf{p}}_d \in \{0, 1\}^{K \times 1}$ and $\tilde{\mathbf{p}}_p \in \{0, 1\}^{K \times 1}$, respectively. It is important to mention that the Ψ matrices are selecting all the received symbols that include a data symbol (for Ψ_d) or a pilot symbol (for Ψ_p). Thus, if Φ is designed, Ψ follows automatically. Besides, note that the selection of Ψ_p is larger than that of Φ_p , i.e. $R_d \geq K_d$, because the transmitted data symbols are spread out by the channel. A block diagram of the pipeline is shown in Fig. 4.7.

In order to facilitate a low-complexity receiver, we design the pilot and data signals such that they do not interfere at the receiver. This configuration permits independent processing of channel estimation and symbol estimation. Hence, we have the following criterion:

C1 There is no overlap between pilot and data symbols at the receiver side.

It follows from **C1** that we should have $\Psi_d^H \mathbf{H}_{DD} \Phi_p \mathbf{s}_p = \mathbf{0}$ and $\Psi_p^H \mathbf{H}_{DD} \Phi_d \mathbf{s}_d = \mathbf{0}$. From the interaction between the channel and the modulation, as derived in (4.5), we know that this is possible as long as the pilot and data symbols are guarded by zeros accordingly, so that the shift operations of the channel do not mix them.

Using these properties, the data part is given by

$$\begin{aligned} \mathbf{y}_d &= \Psi_d^H \mathbf{y} = \Psi_d^H \left[\sqrt{P_d} \mathbf{H}_{DD} \Phi_d \mathbf{s}_d + \sqrt{P_p} \mathbf{H}_{DD} \Phi_p \mathbf{s}_p + \mathbf{w} \right] \\ &= \sqrt{P_d} \Psi_d^H \mathbf{H}_{DD} \Phi_d \mathbf{s}_d + \mathbf{w}_d, \end{aligned} \quad (4.8)$$

where $\mathbf{w}_d = \Psi_d^H \mathbf{w}$ and the pilot part is given by

$$\begin{aligned} \mathbf{y}_p &= \Psi_p^H \mathbf{y} = \Psi_p^H \left[\sqrt{P_d} \mathbf{H}_{DD} \Phi_d \mathbf{s}_d + \sqrt{P_p} \mathbf{H}_{DD} \Phi_p \mathbf{s}_p + \mathbf{w} \right] \\ &= \sqrt{P_p} \Psi_p^H \mathbf{H}_{DD} \Phi_p \mathbf{s}_p + \mathbf{w}_p, \end{aligned} \quad (4.9)$$

where $\mathbf{w}_p = \Psi_p^H \mathbf{w}$.

Recognizing that the LTV channel (circularly) shifts the rows and columns of the transmitted symbols $\mathbf{S}_{DD}$, we can deduce which type of pilot allocation respects **C1**. We identify that every pilot allocation must consist of one of the three “basic” cases. Strictly speaking, a combination of multiple such cases is also possible. This requires multiple equispaced pilots in the delay and/or Doppler dimension, each with appropriate guard bands. This, however, complicates the pilot design, does not reduce the pilot overhead and does not improve the channel estimation, which is why we will not further investigate this option. The “basic” cases are:

1. “Island case” - in case the pilot symbol “area” is embedded both in the delay and Doppler direction by data symbols, in order to have nonzero pilot symbols, the “area” should be at least $(2Q + 1) \times (2L + 1)$, thus $K_p \geq (2Q + 1)(2L + 1)$. See Fig. 4.8a for a visualization. Note that this pilot allocation requires $N \geq 2Q + 1$ and $M \geq 2L + 1$.

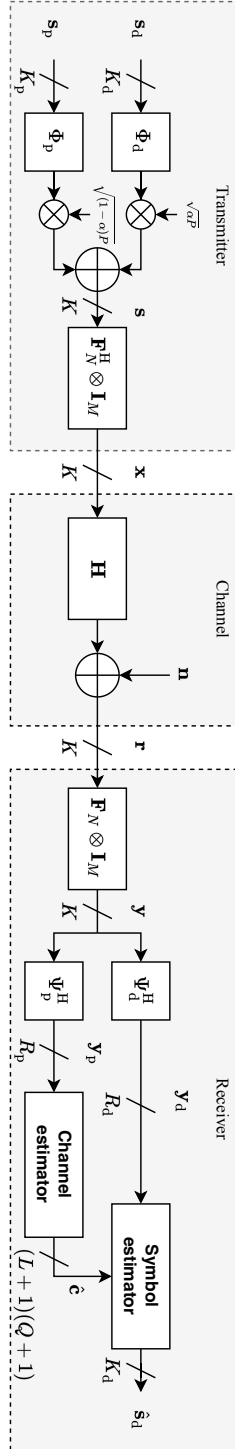


Figure 4.7.: Block diagram of the OTFS communication pipeline. Insertion and deletion of the CP is omitted.

2. “Doppler slab” - in case the pilot symbol “area” is embedded only in the delay direction but not in the Doppler direction, the area should be at least $N \times (2L + 1)$, thus $K_p \geq N(2L + 1)$. See Fig. 4.8b for a visualization. Note that this pilot allocation requires $N \geq Q + 1$ and $M \geq 2L + 1$.
3. “Delay slab” - in case the pilot symbol “area” is embedded only in the Doppler direction but not in the delay direction, the area should be at least $(2Q + 1) \times M$, thus $K_p \geq (2Q + 1)M$. See Fig. 4.8c for a visualization. Note that this pilot allocation requires $N \geq 2Q + 1$ and $M \geq L + 1$.

Having determined the possibilities for the pilot design, we are interested in its symbol overhead and the estimation performance.

4.5.2. Step 2: Determining pilot allocations with the lowest overhead

The minimum number of pilot symbols for each case is given by

1. “Island case” - $K_p = (2Q + 1)(2L + 1)$, consequently only one pilot symbol is nonzero.
2. “Doppler slab” - $K_p = (Q + 1)(2L + 1)$, thus choosing $N = Q + 1$, consequently only one row containing $Q + 1$ pilot symbols is nonzero.
3. “Delay slab” - $K_p = (2Q + 1)(L + 1)$, thus choosing $M = L + 1$, consequently only one column containing $L + 1$ pilot symbols is nonzero.

It is clear that case 2) and case 3) will require less pilot overhead compared to case 1), for every choice of L and Q . Therefore, it is crucial to examine whether there are any performance distinctions among these pilot designs. In the subsequent section, we will demonstrate that all these designs (1, 2, and 3) achieve the same minimum MSE.

4.5.3. Step 3: Showing MMSE optimality

The pilot part at the receiver side can be rewritten as

$$\mathbf{y}_p = \sqrt{P_p} \mathbf{\Psi}_p^H \mathbf{H}_{DD} \mathbf{\Phi}_p \mathbf{s}_p + \mathbf{w}_p = \mathbf{Z} \mathbf{c} + \mathbf{w}_p, \quad (4.10)$$

where $\mathbf{c} \in \mathbb{C}^{(L+1)(Q+1) \times 1}$ contains the coefficients $c_{l,q}$ of the BEM and where the columns of $\mathbf{Z} \in \mathbb{C}^{R_p \times (L+1)(Q+1)}$ are given by,

$$\mathbf{z}_{l+q(L+1)} = \sqrt{P_p} \mathbf{\Psi}_p^H (\mathbf{F}_N \otimes \mathbf{I}_M) \left(\mathbf{\Lambda}_K^q \mathbf{P}_K^l \right) (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{\Phi}_p \mathbf{s}_p. \quad (4.11)$$

Let $\hat{\mathbf{c}}$ represent a channel estimator. Our objective is to minimize the MSE of this estimator, that is, $\mathbb{E}[(\mathbf{c} - \hat{\mathbf{c}})^H (\mathbf{c} - \hat{\mathbf{c}})]$. Note that we have a linear model, and consequently the MSE can be minimized by the linear minimum MSE (LMMSE) estimator. Assume the channel taps are independently distributed following a Gaussian distribution with a mean of zero and potentially varying variances, such that $\mathbb{E}[\mathbf{c} \mathbf{c}^H] = \text{diag}([\sigma_{c_{0,0}}^2, \dots, \sigma_{c_{L,Q}}^2]) = \mathbf{R}_c$ and let $\mathbb{E}[\mathbf{w}_p \mathbf{w}_p^H] = \mathbf{R}_{w_p}$, then, the expression for the LMMSE estimator is

$$\hat{\mathbf{c}} = \left(\mathbf{R}_{w_p}^{-1} \mathbf{Z} (\mathbf{R}_c^{-1} + \mathbf{Z}^H \mathbf{R}_{w_p}^{-1} \mathbf{Z})^{-1} \right)^H \mathbf{y}_p. \quad (4.12)$$

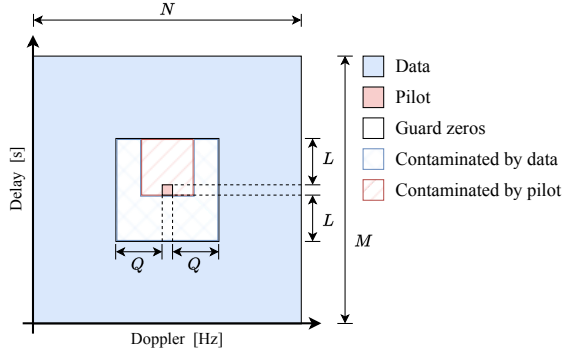
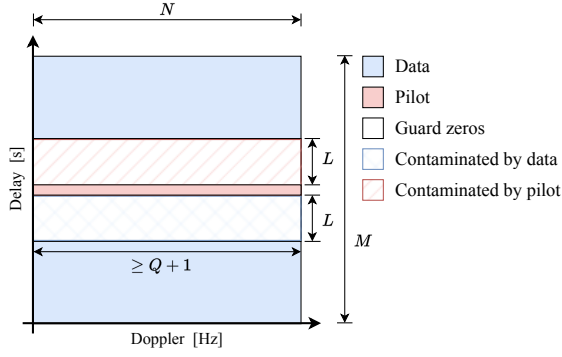
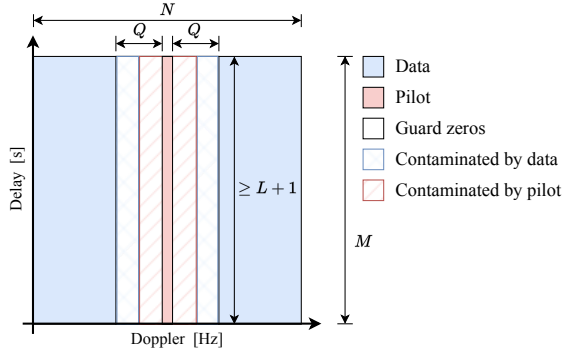
(a) $M \geq 2L + 1$ and $N \geq 2Q + 1$.(b) $M \geq 2L + 1$ and $N \geq Q + 1$.(c) $M \geq L + 1$ and $N \geq 2Q + 1$.

Figure 4.8.: DD configurations satisfying C1.

If we assume that the noise is white, i.e. $\mathbf{R}_n = \sigma_n^2 \mathbf{I}_K$, then $\mathbf{R}_w = \mathbb{E}[(\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{n} \mathbf{n}^H (\mathbf{F}_N \otimes \mathbf{I}_M)] = \sigma_n^2 \mathbf{I}_K$, and consequently,

$$\mathbf{R}_{w_p} = \mathbb{E}[\Psi_p^H \mathbf{w} \mathbf{w}^H \Psi_p] = \Psi_p^H \mathbf{R}_w \Psi_p = \sigma_n^2 \mathbf{I}_{R_p}. \quad (4.13)$$

As a result, we can rewrite the LMMSE estimator as

$$\hat{\mathbf{c}} = (\sigma_n^2 \mathbf{R}_c^{-1} + \mathbf{Z}^H \mathbf{Z})^{-1} \mathbf{Z}^H (\mathbf{Z} \mathbf{c} + \mathbf{w}_p). \quad (4.14)$$

The MSE of this estimator is given by,

$$\mathbb{E}[(\mathbf{c} - \hat{\mathbf{c}})^H (\mathbf{c} - \hat{\mathbf{c}})] = \mathbb{E}[\text{tr}((\mathbf{c} - \hat{\mathbf{c}})(\mathbf{c} - \hat{\mathbf{c}})^H)] = \text{tr}\left(\left(\mathbf{R}_c^{-1} + \frac{1}{\sigma_n^2} \mathbf{Z}^H \mathbf{Z}\right)^{-1}\right). \quad (4.15)$$

To reach the minimum MSE, the pilot symbols should be placed such that $\mathbf{Z}^H \mathbf{Z}$ is diagonal ([32], Lemma 1). Note that the columns of $\mathbf{Z}$ are given by

$$\begin{aligned} \mathbf{z}_{l+q(L+1)} &= \sqrt{P_p} \Psi_p^H (\mathbf{F}_N \otimes \mathbf{I}_M) \left(\Lambda_K^q \mathbf{P}_K^l \right) (\mathbf{F}_N^H \otimes \mathbf{I}_M) \Phi_p \mathbf{s}_p \\ &= \sqrt{P_p} \Psi_p^H \text{vec} \left(\mathbf{W}_{l,q} \circ \left(\mathbf{P}_M^l \text{vec}^{-1}(\Phi_p \mathbf{s}_p) \mathbf{P}_N^{-q} \right) \right). \end{aligned} \quad (4.16)$$

Let $\mathbf{S}_p = \text{vec}^{-1}(\Phi_p \mathbf{s}_p)$. The elements of the matrix $\mathbf{Z}^H \mathbf{Z}$ can be rewritten³ as

$$\begin{aligned} &\mathbf{z}_{l_1+q_1(L+1)}^H \mathbf{z}_{l_2+q_2(L+1)} \\ &= P_p \text{vec}^H \left(\mathbf{W}_{l_1,q_1} \circ \left(\mathbf{P}_M^{l_1} \mathbf{S}_p \mathbf{P}_N^{-q_1} \right) \right) \Psi_p \Psi_p^H \text{vec} \left(\mathbf{W}_{l_2,q_2} \circ \left(\mathbf{P}_M^{l_2} \mathbf{S}_p \mathbf{P}_N^{-q_2} \right) \right) \\ &\stackrel{(a)}{=} P_p \left[\text{vec}^* \left(\mathbf{W}_{l_1,q_1} \circ \left(\mathbf{P}_M^{l_1} \mathbf{S}_p \mathbf{P}_N^{-q_1} \right) \right) \circ \text{vec} \left(\mathbf{W}_{l_2,q_2} \circ \left(\mathbf{P}_M^{l_2} \mathbf{S}_p \mathbf{P}_N^{-q_2} \right) \right) \right]^T \text{diag}(\Psi_p \Psi_p^H) \\ &= P_p \text{vec}^T \left(\mathbf{W}_{l_1,q_1}^* \circ \mathbf{W}_{l_2,q_2} \circ \left(\mathbf{P}_M^{l_1} \mathbf{S}_p^* \mathbf{P}_N^{-q_1} \right) \circ \left(\mathbf{P}_M^{l_2} \mathbf{S}_p \mathbf{P}_N^{-q_2} \right) \right) \text{diag}(\Psi_p \Psi_p^H). \end{aligned} \quad (4.17)$$

For the diagonal elements, i.e. $(l_1, q_1) = (l_2, q_2) \in \{0, 1, \dots, L\} \times \{0, 1, \dots, Q\}$, we have,

$$\begin{aligned} \mathbf{z}_i^H \mathbf{z}_i &= P_p \text{vec}^T \left(\mathbf{W}_{l,q}^* \circ \mathbf{W}_{l,q} \circ \left(\mathbf{P}_M^l (\mathbf{S}_p^* \circ \mathbf{S}_p) \mathbf{P}_N^{-q} \right) \right) \text{diag}(\Psi_p \Psi_p^H) \\ &= P_p \text{vec}^T \left(\left(\mathbf{P}_M^l (\mathbf{S}_p^* \circ \mathbf{S}_p) \mathbf{P}_N^{-q} \right) \right) \text{diag}(\Psi_p \Psi_p^H) = P_p \text{vec}^T \left(\left(\mathbf{P}_M^l (\mathbf{S}_p^* \circ \mathbf{S}_p) \mathbf{P}_N^{-q} \right) \right) \tilde{\mathbf{p}}_p \\ &\stackrel{(b)}{=} P_p \left(\mathbf{s}_p^* \circ \mathbf{s}_p \right)^T \mathbf{1}_{K_p \times 1} = P_p \mathbf{s}_p^H \mathbf{s}_p = P_p, \end{aligned} \quad (4.18)$$

where in (b) we have used the fact that $\tilde{\mathbf{p}}_p$ is selecting all the symbols that include a pilot symbol after passing through the channel. Thus, the diagonal of $\mathbf{Z}^H \mathbf{Z}$ only contains the total power P_p of the pilot symbols. Note that the diagonal elements do not depend on K_p . If $\mathbf{Z}^H \mathbf{Z}$ must be diagonal, then the off-diagonal elements of $\mathbf{Z}^H \mathbf{Z}$ must be zero, that is,

$$\mathbf{z}_{l_1+q_1(L+1)}^H \mathbf{z}_{l_2+q_2(L+1)} = 0, \quad (l_1, q_1) \neq (l_2, q_2). \quad (4.19)$$

³Where (a) uses the fact that if $\mathbf{a}_1$ and $\mathbf{a}_2$ are two column vectors and $\mathbf{X}$ is a diagonal matrix, then $\mathbf{a}_1^H \mathbf{X} \mathbf{a}_2 = \text{vec}(\mathbf{a}_2^T \circ \mathbf{a}_1^H) \text{diag}(\mathbf{X}) = (\mathbf{a}_1^* \circ \mathbf{a}_2)^T \text{diag}(\mathbf{X})$.

By inspecting (4.17) for $(l_1, q_1) \neq (l_2, q_2)$, we see that, left of the element-wise product, the pilot symbols are shifted by l_1 in the delay direction and q_1 in the Doppler direction, while on the right of the element-wise product, the pilot symbols are shifted by l_2 in the delay direction and q_2 in the Doppler direction. From this observation we can deduce that, to have the outcome equal to zero, *the pilot symbols in the delay-Doppler domain, that is the matrix $\text{vec}^{-1}(\Phi_p \mathbf{s}_p)$, should have shift orthogonal rows and columns.*

We can draw some conclusions:

1. “Island case” - For a fixed power P_p , this pilot allocation, with $K_p = (2Q + 1)(2L + 1)$ and with only one nonzero pilot symbol in the middle, achieves the minimum MSE. However, it uses more pilot symbols compared to the “Doppler slab” and “Delay slab” case.
2. “Doppler slab” - For a fixed power P_p , this pilot allocation, with $K_p = (Q + 1)(2L + 1)$ and with $Q + 1$ possibly nonzero pilot symbols, achieves the minimum MSE if and only if the nonzero pilots are shift orthogonal in both delay and Doppler direction at the same time. The only option adhering to this orthogonality is to have only one nonzero pilot symbol. This is visualized in Fig. 4.9a. Although in the figure the pilot symbol is placed in the middle, the symbol could be placed anywhere along the Doppler direction (the delay position is fixed).
3. “Delay slab” - For a fixed power P_p , this pilot allocation, with $K_p = (2Q + 1)(L + 1)$ and with $L + 1$ possibly nonzero pilot symbols, achieves the minimum MSE if and only if the nonzero pilots are shift orthogonal in both delay and Doppler direction at the same time. The only option adhering to this orthogonality is to have only one nonzero pilot symbol. This is visualized in Fig. 4.9b. Although in the figure the pilot symbol is placed in the middle, the symbol could be placed anywhere along the delay direction (the Doppler position is fixed).

To recap, given **C1**, the pilot allocations with the lowest pilot overhead are (also) allocations that attain the minimum MSE. The latter result is supported by simulation experiments, as shown in Fig. 4.10, where the average MSE of the channel taps is computed over 100 Monte Carlo runs. In each run, a new channel and noise realization is drawn from their respective distributions, with $\sigma_{c_{l,q}}^2 = 1/((L + 1)(Q + 1))$ for all l and q .

Note that the pilot allocations using the ZC sequence do not achieve the MMSE performance. This is because the pilot symbols in the DD domain do not produce shift-orthogonal rows and columns. For the current parameters, $\{K, L, Q\} = \{441, 8, 8\}$, the resulting performance gap is approximately 3 dB, although this difference depends on the specific simulation parameters.

Moreover, the three cases, the island case, Doppler slab, and delay slab, do achieve the same MMSE; however, our proposed Doppler slab and delay slab do so with less pilot overhead. As a result, the proposed allocations can fit more data symbols. When the three allocations would operate at the same data rate, the proposed allocations will have a better BER than the island case, since in practice, the latter will have to use higher-order constellations, or less error correction coding. Therefore, our proposed Doppler slab and delay slab allocations are the preferred options. In order to use the least overhead at all times, we must choose the Doppler slab (set $N = Q + 1$) in case $Q < L$, and we must

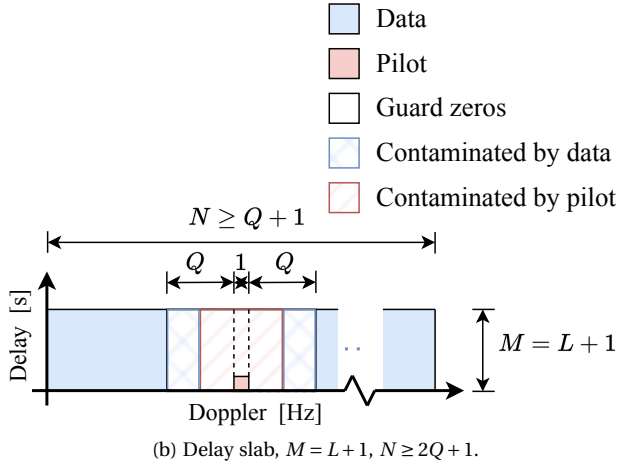
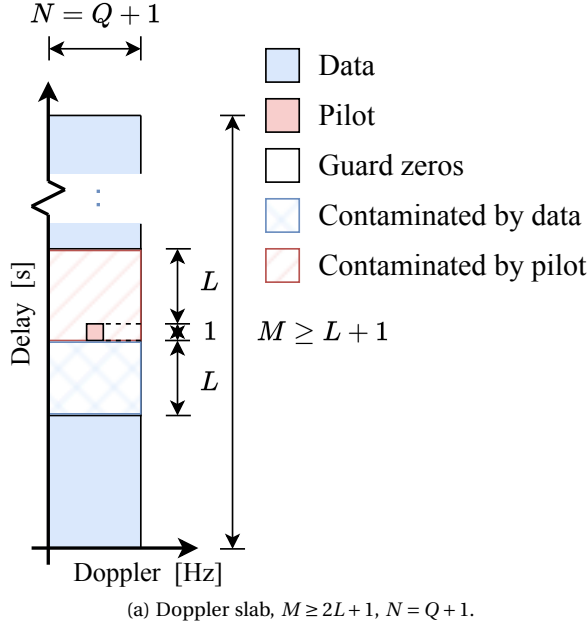


Figure 4.9.: (a) and (b): Pilot allocations satisfying i) **C1**, ii) achieving the MMSE ($\mathbf{Z}^H \mathbf{Z}$ diagonal), and iii) having the lowest overhead.

choose the delay slab (set $M = L + 1$) in case $Q > L$. With this decision rule, we can attain a pilot overhead of $K_p = \min\{(L + 1)(2Q + 1), (2L + 1)(Q + 1)\}$.

As the optimal pilot allocation is now determined, the next step is to optimize the power distribution between the pilot and the data symbols.

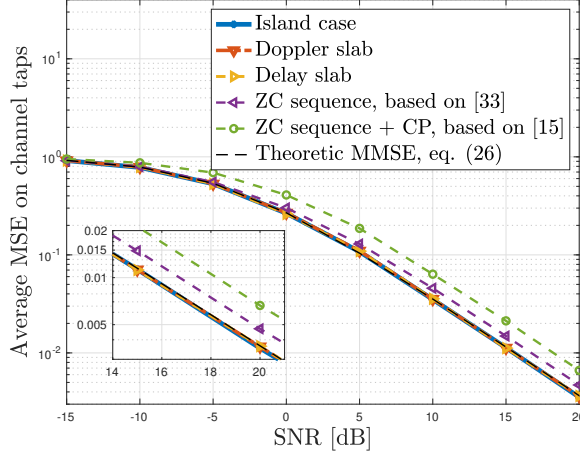


Figure 4.10.: Channel estimation performance of proposed pilot allocations compared to existing ones, for $\{K, L, Q\} = \{441, 8, 8\}$.

4.5.4. Step 4: Optimizing the power balance

Since pilot overhead is critical in time-varying communication systems, we first (step 2 of our approach) focused on reducing this overhead as much as possible. This naturally also maximizes the time-frequency occupancy of the communications symbols, and hence implicitly focuses on capacity⁴. Therefore, in this step 4, it makes sense to also optimize the power distribution in terms of the channel capacity. Moreover, channel capacity is the primary performance metric in communication systems. Unlike the BER, which depends on modulation order and channel coding, channel capacity provides a modulation- and coding-independent measure of link performance. It also has a closed-form expression, making it suitable for system analysis and optimization. Finally, systems that transmit large amounts of data and employ adaptive modulation and coding can approach the channel capacity in practice.

The received data part is given by,

$$\mathbf{y}_d = \sqrt{P_d} \mathbf{\Psi}_d^H \mathbf{H}_{DD} \mathbf{\Phi}_d \mathbf{s}_d + \mathbf{w}_d = \mathbf{H}_d \mathbf{s}_d + \mathbf{w}_d. \quad (4.20)$$

where $\mathbf{H}_d = \sqrt{P_d} \mathbf{\Psi}_d^H \mathbf{H}_{DD} \mathbf{\Phi}_d$. The capacity of the channel, averaged over the random channel $\mathbf{H}$, induced by the coefficients in $\mathbf{c}$, is given by (see [26], [34]),

$$C = \frac{1}{K+L} \mathbb{E}_{\mathbf{H}} \left[\max_{p_{\mathbf{s}_d}(\cdot), \mathbb{E}[\mathbf{s}_d^H \mathbf{s}_d] = 1} \mathcal{J}(\mathbf{y}_d; \mathbf{s}_d | \hat{\mathbf{c}}) \right] \quad \text{bit/s/Hz.} \quad (4.21)$$

Here, $\mathcal{J}(\mathbf{y}_d; \mathbf{s}_d | \hat{\mathbf{c}})$ is the conditional mutual information between the received signal $\mathbf{y}_d$ and the transmitted symbols $\mathbf{s}_d$, given the channel coefficient estimate $\hat{\mathbf{c}}$, $p_{\mathbf{s}_d}(\cdot)$ is the probability distribution of $\mathbf{s}_d$ with fixed energy $\|\mathbf{s}_d\|_2^2 = 1$. Note that we divide by $K+L$ to account for the CP of length L .

⁴Interestingly, a similar connection between channel capacity and channel MMSE has been noticed in [26].

Let the channel estimate be given by $\hat{\mathbf{c}}$, and let the *estimated* channel matrix be denoted by $\hat{\mathbf{H}}_d$ and the channel estimation error matrix by $\tilde{\mathbf{H}}_d = (\mathbf{H}_d - \hat{\mathbf{H}}_d)$ then the received data part can be rewritten as,

$$\mathbf{y}_d = \hat{\mathbf{H}}_d \mathbf{s}_d + (\mathbf{H}_d - \hat{\mathbf{H}}_d) \mathbf{s}_d + \mathbf{w}_d = \hat{\mathbf{H}}_d \mathbf{s}_d + \mathbf{v}, \quad (4.22)$$

where $\mathbf{v} = (\mathbf{H}_d - \hat{\mathbf{H}}_d) \mathbf{s}_d + \mathbf{w}_d$. Now, since no knowledge is available at the transmitter about the channel, it is reasonable to assign equal energy to all data symbols, i.e. $\mathbf{R}_{s_d} = \frac{1}{K_d} \mathbf{I}_{K_d}$. With a fixed (i.e. equal) data symbol power, a lower bound on the channel capacity is given by ([26], [34]),

$$C \geq \frac{1}{K+L} \mathbb{E}_{\mathbf{H}} \left[\log_2 \det \left(\mathbf{I}_{R_d \times R_d} + \frac{1}{K_d} \mathbf{R}_{\mathbf{v}}^{-1} \hat{\mathbf{H}}_d \hat{\mathbf{H}}_d^H \right) \right]. \quad (4.23)$$

Here,

$$\mathbf{R}_{\mathbf{v}} = \mathbb{E}[\mathbf{v}\mathbf{v}^H] = \frac{1}{K_d} \mathbb{E}[(\mathbf{H}_d - \hat{\mathbf{H}}_d)(\mathbf{H}_d - \hat{\mathbf{H}}_d)^H] + \sigma_{\mathbf{n}}^2 \mathbf{I}_{R_d \times R_d}. \quad (4.24)$$

The goal is to determine the appropriate power distribution, that is, select α , to maximize the lower bound on the channel capacity C . To express the lower bound in terms of the power distribution parameter α , we proceed with some derivations. First of all, we can write (see Appendix B for a detailed derivation),

$$\mathbb{E}[(\mathbf{H}_d - \hat{\mathbf{H}}_d)(\mathbf{H}_d - \hat{\mathbf{H}}_d)^H] \leq P_d \mathbb{E}[\text{tr}((\mathbf{c} - \hat{\mathbf{c}})(\mathbf{c} - \hat{\mathbf{c}})^H)] \mathbf{I}_{R_d}. \quad (4.25)$$

Secondly, because the pilot allocation makes $\mathbf{Z}^H \mathbf{Z}$ diagonal, we can write

$$\begin{aligned} \left(\mathbf{R}_{\mathbf{c}}^{-1} + \frac{1}{\sigma_{\mathbf{n}}^2} \mathbf{Z}^H \mathbf{Z} \right)^{-1} &= \left(\text{diag}([\sigma_{c_{0,0}}^2, \dots, \sigma_{c_{L,Q}}^2])^{-1} + \frac{1}{\sigma_{\mathbf{n}}^2} P_p \mathbf{I}_{(L+1)(Q+1)} \right)^{-1} \\ &= \text{diag} \left(\frac{\sigma_{c_{0,0}}^2 \sigma_{\mathbf{n}}^2}{\sigma_{\mathbf{n}}^2 + \sigma_{c_{0,0}}^2 P_p}, \dots, \frac{\sigma_{c_{L,Q}}^2 \sigma_{\mathbf{n}}^2}{\sigma_{\mathbf{n}}^2 + \sigma_{c_{L,Q}}^2 P_p} \right), \end{aligned} \quad (4.26)$$

so that, the channel MSE (4.15) is given by,

$$\mathbb{E}[\text{tr}(\mathbf{c} - \hat{\mathbf{c}})(\mathbf{c} - \hat{\mathbf{c}})^H] = \sum_{l=0}^L \sum_{q=0}^Q \frac{\sigma_{c_{l,q}}^2 \sigma_{\mathbf{n}}^2}{\sigma_{\mathbf{n}}^2 + \sigma_{c_{l,q}}^2 P_p} = \sigma_{\mathbf{H}}^2. \quad (4.27)$$

We can then combine (4.25) and (4.27) to rewrite $\mathbf{R}_{\mathbf{v}}$ as

$$\begin{aligned} \mathbf{R}_{\mathbf{v}} &\leq \frac{P_d}{K_d} \mathbb{E}[\text{tr}(\mathbf{c} - \hat{\mathbf{c}})(\mathbf{c} - \hat{\mathbf{c}})^H] \mathbf{I}_{R_d} + \sigma_{\mathbf{n}}^2 \mathbf{I}_{R_d} \\ &= \left[\frac{P_d}{K_d} \sum_{l=0}^L \sum_{q=0}^Q \frac{\sigma_{c_{l,q}}^2 \sigma_{\mathbf{n}}^2}{\sigma_{\mathbf{n}}^2 + \sigma_{c_{l,q}}^2 P_p} + \sigma_{\mathbf{n}}^2 \right] \mathbf{I}_{R_d} = \left[\frac{P_d}{K_d} \sigma_{\mathbf{H}}^2 + \sigma_{\mathbf{n}}^2 \right] \mathbf{I}_{R_d}. \end{aligned} \quad (4.28)$$

Finally, we can derive a looser lower bound on the capacity from (4.23) as

$$\begin{aligned}
 C &\geq \frac{1}{K+L} \mathbb{E}_{\mathbf{H}} \left[\log_2 \det \left(\mathbf{I}_{R_d} + \frac{1}{K_d} \mathbf{R}_v^{-1} \hat{\mathbf{H}}_d \hat{\mathbf{H}}_d^H \right) \right] \\
 &\geq \frac{1}{K+L} \mathbb{E}_{\mathbf{H}} \left[\log_2 \det \left(\mathbf{I}_{R_d} + \frac{1}{K_d} \left[\frac{P_d}{K_d} \sigma_{\hat{\mathbf{H}}}^2 + \sigma_{\mathbf{n}}^2 \right]^{-1} \hat{\mathbf{H}}_d \hat{\mathbf{H}}_d^H \right) \right] \\
 &= \frac{1}{K+L} \mathbb{E}_{\mathbf{H}} \left[\log_2 \det \left(\mathbf{I}_{K_d} + \frac{1}{K_d} \left[\frac{P_d}{K_d} \sigma_{\hat{\mathbf{H}}}^2 + \sigma_{\mathbf{n}}^2 \right]^{-1} \hat{\mathbf{H}}_d^H \hat{\mathbf{H}}_d \right) \right] = \underline{C}. \tag{4.29}
 \end{aligned}$$

Now lastly, we inspect $\hat{\mathbf{H}}_d^H \hat{\mathbf{H}}_d$. We can show that (refer to Appendix C for the derivation),

$$\mathbb{E} \left[\|\hat{\mathbf{H}}_d\|_F^2 \right] = P_d K_d \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L \frac{P_p \sigma_{c_{l,q}}^4}{\sigma_{\mathbf{n}}^2 + P_p \sigma_{c_{l,q}}^2} = P_d \sigma_{\hat{\mathbf{H}}}^2. \tag{4.30}$$

Hence we can rewrite $\underline{C}$ in (4.29) as

$$\begin{aligned}
 \underline{C} &= \frac{1}{K+L} \mathbb{E}_{\mathbf{H}} \left[\log_2 \det \left(\mathbf{I}_{K_d} + \frac{1}{K_d} \left[\frac{P_d}{K_d} \sigma_{\hat{\mathbf{H}}}^2 + \sigma_{\mathbf{n}}^2 \right]^{-1} P_d \sigma_{\hat{\mathbf{H}}}^2 \frac{\hat{\mathbf{H}}_d^H \hat{\mathbf{H}}_d}{\mathbb{E} \left[\|\hat{\mathbf{H}}_d\|_F^2 \right]} \right) \right] \\
 &= \frac{1}{K+L} \mathbb{E}_{\mathbf{H}} \left[\log_2 \det \left(\mathbf{I}_{K_d} + \rho \frac{\hat{\mathbf{H}}_d^H \hat{\mathbf{H}}_d}{\mathbb{E} \left[\|\hat{\mathbf{H}}_d\|_F^2 \right]} \right) \right], \tag{4.31}
 \end{aligned}$$

where,

$$\rho = \frac{\frac{P_d}{K_d} \sigma_{\hat{\mathbf{H}}}^2}{\frac{P_d}{K_d} \sigma_{\hat{\mathbf{H}}}^2 + \sigma_{\mathbf{n}}^2}, \tag{4.32}$$

with

$$\sigma_{\hat{\mathbf{H}}}^2 = K_d \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L \frac{P_p \sigma_{c_{l,q}}^4}{\sigma_{\mathbf{n}}^2 + P_p \sigma_{c_{l,q}}^2}, \tag{4.33}$$

$$\sigma_{\hat{\mathbf{H}}}^2 = \sum_{l=0}^L \sum_{q=0}^Q \frac{\sigma_{c_{l,q}}^2 \sigma_{\mathbf{n}}^2}{\sigma_{\mathbf{n}}^2 + \sigma_{c_{l,q}}^2 P_p}. \tag{4.34}$$

Remark 8. Note that ρ is essentially a signal-to-interference-plus-noise ratio (SINR). The numerator contains the energy related to the (target) signal, i.e. $\hat{\mathbf{H}}_d \mathbf{s}_d$, while the denominator contains the interference energy due to the channel estimation error, related to $(\mathbf{H}_d - \hat{\mathbf{H}}_d) \mathbf{s}_d$, and the noise energy.

Observe that ρ is a function of α through P_d and P_p . We can hence optimize $\underline{C}$ in (4.31) by maximizing ρ with respect to α . The optimal power distribution is

$$\alpha^* = \arg \max_{\alpha} \rho. \tag{4.35}$$

Before proceeding to the simulations in the next section, we make some remarks with respect to the complexity and implementation of our proposed pilot design.

Note that using a different pilot *allocation* does not increase or decrease complexity of the communication system. Determining the optimal *power distribution* requires maximizing a one-dimensional (concave) function, which has negligible complexity compared to the channel or symbol estimation steps, which typically involve matrix inversions or iterative algorithms. The complexity of our pilot design is therefore comparable to that of existing designs.

Implementation of our pilot design in practical systems is not significantly different from existing approaches. All discussed pilot designs assume that L and Q are known, which is reasonable in practice as noted in Remark 6, and that the noise level $\sigma_{\mathbf{n}}^2$ is available. In modern communication systems, the noise level is typically tracked by the receiver, since both channel and symbol estimators depend on it.

The only extra requirement of our method is knowledge of the channel tap variance. Specifically, under the working assumption that $c_{l,q} \sim \mathcal{N}(0, \sigma_{c_{l,q}}^2)$, our method requires knowledge of $\sigma_{c_{l,q}}^2$. These variables can be estimated using a probe signal, or a preamble, and in practice, this distribution tends to remain stable over multiple communication packets as well.

4.6. Simulations

In this section, we validate the theoretical findings and compare our framework with related work.

4.6.1. Numerical validation

We simulate three baseband channels (i.e., frequency shifts between 0 and 1 and time delays of multiples of the symbol period) with different parameters, which are specified in Table 4.2. We call a channel ‘Doppler dominant’ if $Q > L$ and ‘delay dominant’ if $Q < L$. Thus, Channel 2 and Channel 3 are Doppler and delay dominant, respectively. We define the signal-to-noise ratio with respect to the transmitted signal $\mathbf{x}$ as,

$$\text{SNR}_{\text{tx}} = \frac{\mathbb{E}[\mathbf{x}^H \mathbf{x}]}{\mathbb{E}[\mathbf{n}^H \mathbf{n}]} = \frac{P}{K \sigma_{\mathbf{n}}^2}.$$

We set $P = 1$, and change the value of $\sigma_{\mathbf{n}}^2$ according to the desired SNR_{tx} . In all simulations, the (pilot and communication) symbols are uncoded QPSK symbols. The channel coefficients $c_{l,q}$ are realizations of an (independent) complex Gaussian process with zero mean and variance $\sigma_{c_{l,q}}^2 = 1/((Q+1)(L+1))$. For all three channels we draw ten noise and channel realizations, and calculate the lower bound on the average capacity, i.e., (4.31). The results are shown in Fig. 4.11. We can draw the following conclusions. First of all, the Doppler slab and delay slab, which alter the modulation parameters N and M according to the channel, exhibit higher capacity in all three channels, compared to the ‘island case’ proposed in [7], [8]. Moreover, we note that either the Doppler slab or delay slab performs the best, according to whether the channel is Doppler or delay dominant. This is to be expected, since the Doppler (delay) slab has the lowest pilot overhead for a

Table 4.2.: Parameters for the simulations in Fig. 4.11.

Parameters	Channel 1	Channel 2	Channel 3
K	441	441	441
Q	6	8	2
L	6	2	8
SNR_{Tx}	20 dB	20 dB	20 dB
$\sigma_{c_{l,q}}^2$	$\frac{1}{(Q+1)(L+1)}, V\{l, q\}$	$\frac{1}{(Q+1)(L+1)}, V\{l, q\}$	$\frac{1}{(Q+1)(L+1)}, V\{l, q\}$
Island case - $\{N, M, \alpha_{\text{opt}}\}$	$\{21, 21, 0.7015\}$	$\{21, 21, 0.7834\}$	$\{21, 21, 0.7834\}$
Doppler slab - $\{N, M, \alpha_{\text{opt}}\}$	$\{7, 63, 0.7270\}$	$\{9, 49, 0.7922\}$	$\{3, 147, 0.7910\}$
Delay slab - $\{N, M, \alpha_{\text{opt}}\}$	$\{63, 7, 0.7270\}$	$\{147, 3, 0.7910\}$	$\{49, 9, 0.7922\}$

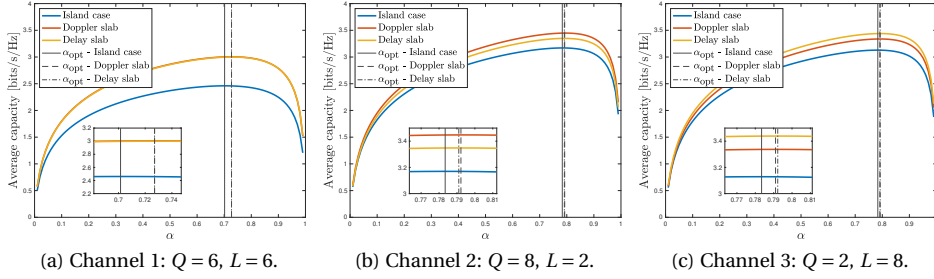


Figure 4.11.: Performance of the pilot allocation schemes with respect to the power distribution for three different channels.

Doppler (delay) dominant channel. Finally, we can see that all three pilot allocations benefit from the optimal power allocation. We see that, indeed, the maximum is reached at the power distribution α_{opt} . In the simulation for Fig. 4.11, we considered only those pilot allocations for OTFS that achieve the channel MMSE. For allocations that do not attain the MMSE, a general expression for the channel estimation error is not available, making it nontrivial to derive a meaningful lower bound on the channel capacity.

In Fig. 4.12, we illustrate (with lines without markers) how the capacity optimal power distribution, obtained from (4.35), evolves with different noise levels. We set $\{K, Q, L\} = \{441, 2, 6\}$ and consider two different channels: in Fig. 4.12a, we use channel taps with identical distributions, while in Fig. 4.12b, we let the variance of the channel taps decay exponentially. The capacity-optimal power distribution evolves differently per channel, but both curves saturate to $\alpha = 0.5$ for high noise levels. A similar trend was found for SCM in [26].

Additionally, we plot (with lines with markers) the power distribution that is optimal in terms of BER. The minimum BER over 50 Monte Carlo runs has been found by simulating every combination of noise level and power distribution. Results at extremely high and low noise levels are omitted, as the BER saturates to 0 (low noise level) and 0.5 (high noise level) for all power distributions, so no meaningful optimum can be identified in those cases.

Fig. 4.12 shows that the BER optimal power distribution closely follows the capacity-optimal power distribution, even for D-OSDM, which employs a pilot design not suited for our capacity optimization. This suggests that (4.35) can be used not only to optimize channel capacity but also to improve the BER.

4.6.2. Comparison with related work

So far, we have compared our proposed pilot designs to the ‘island case’ pilot allocation from [7], [8] using our capacity optimal power distribution for all cases. Here, we explicitly compare the capacity of our power distribution with the one originally used in [8]. To ensure fairness, we align the parameters used in [8] with those discussed in this chapter.

Let s_p denote a pilot symbol and let s_d denote a data symbol, then in [8] the pilot and

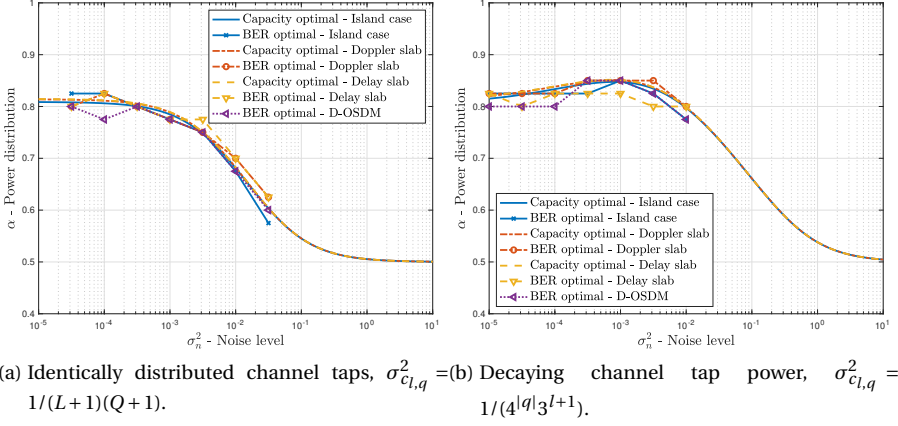


Figure 4.12.: Capacity optimal power distribution (i.e., (4.35)), versus BER optimal power distribution.

data signal-to-noise ratio (SNR) were defined per *symbol*, that is,

$$\text{SNR}_p = \frac{|s_p|^2}{\sigma_n^2}, \quad \text{SNR}_d = \frac{\mathbb{E}[|s_d|^2]}{\sigma_n^2}.$$

Note that the relation to our SNR of the transmitted signal (thus pilot and data signal together) is given by $\text{SNR}_{\text{tx}} = \frac{1}{K} \text{SNR}_p + \frac{K_d}{K} \text{SNR}_d$. We can relate the notion of SNR per *symbol* of [8] to our power distribution parameter,

$$\alpha = \frac{K_d \text{SNR}_d}{K_d \text{SNR}_d + \text{SNR}_p}.$$

Then, in Table 4.3 we list: the parameters that were used in [8] (first four columns), “our” parameters that follow from that (fifth to ninth column), and the parameters for the optimal pilot design (last four columns). We can see that the α that follows from [8] differs a lot from the optimal distribution α^* . In fact, we see that modifying the power distribution can lead to a substantial improvement in performance (compare seventh and ninth column). Furthermore, selecting the appropriate values for M and N can further enhance performance as we can see in the eleventh column. Note that an increase in SNR_{tx} from 21.6 dB to 27.7 dB and from 25.5 dB to 29.0 dB is more than double the amount of power. It is important to highlight that this increase in power leads to only a minor BER improvement in [8] as well as a minor capacity improvement as seen in the seventh column. By contrasting the optimal power allocation with the actual power distribution, the slight increase in BER becomes more understandable; the power increase is counteracted by the bad power distribution.

Table 4.3.: Results of the comparison with [8] for a simulated channel with $Q = 2$, $L = 6$. The SNR and capacity values are listed in dB and bit/s/Hz, respectively.

Island case $M = 128, N = 16$					Doppler slab $M = 686, N = 3$		Delay slab $M = 7, N = 294$		
Parameters from [8], cf. Fig. 14			Resulting parameters [8]		Sub-optimal:		Optimal choice:		
SNR_p	SNR_d	σ^2	BER	SNR_{tx}	α	α^*	$\underline{C(\alpha^*)}$	α^*	$\underline{C(\alpha^*)}$
50	20	1	$\approx 1 \cdot 10^{-2}$	21.6	0.66	0.91	4.14	0.91	4.17
60	20	1	$\approx 8.5 \cdot 10^{-3}$	27.7	0.17	0.91	5.50	0.91	5.55
50	25	1	$\approx 2 \cdot 10^{-3}$	25.5	0.86	0.91	5.05	0.91	5.09
60	25	1	$\approx 1.5 \cdot 10^{-3}$	29.0	0.39	0.91	5.75	0.91	5.81

4.7. Conclusion

In this chapter, we aimed to contribute to the understanding of how to design pilot signals for OTFS. We conducted an investigation into the literature on pilot design and established connections between the work on LTI and LTV channels, SCM, OFDM and OTFS modulation.

We have proposed two new pilot allocations for OTFS, that adjust M or N according to the channel parameters L and Q . Our pilot allocations save approximately 50% on pilot overhead compared to the conventional allocation, while still achieving the MMSE for channel estimation. We have also addressed the aspect of optimizing the power distribution between the pilot and data signal with respect to the channel capacity. Our results indicate that selecting an appropriate power distribution significantly enhances a (lower bound on the) channel capacity of the communication system. Moreover, we empirically show that, across different channel settings, our proposed power distribution, although optimized for capacity, also minimizes the BER.

In summary, this chapter demonstrated that the careful selection of OTFS parameters, together with pilot design (including the allocation and power distribution) can lead to a significant improvement in the performance of an OTFS based communication system.

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5

Validation using Channel Replay Simulations

The previous chapter proposed a pilot design for the orthogonal time frequency space (OTFS) modulation scheme that depends on the delay and Doppler spreads of the channel.

This chapter first validates the expected behavior of the pilot power distribution using channel-replay simulations. It then compares the proposed pilot design with a conventional approach, also using channel-replay simulations.

This chapter is based on the conference proceeding published as:

I. van der Werf, K. Pelekanakis, R. C. Hendriks, R. Heusdens and G. Leus, "Optimized Pilot Design for OTFS Modulation in Underwater Acoustic Communications," 2024 Seventh Underwater Communications and Networking Conference (UComms), Sestri Levante, Italy, 2024.

5.1. Introduction

The previous chapter introduced a novel approach for selecting OTFS system parameters based on the channel conditions, via a pilot allocation and power distribution strategy. This method combines an minimum mean squared error (MMSE)-optimal channel estimator with a capacity-optimal power distribution. While those results were verified on synthetic channels with complex Gaussian distributed taps, this assumption does not always hold in practical underwater acoustic (UWA) environments. Consequently, this chapter corroborates our theoretical framework using the Watermark [1] channel replay simulator. In addition, the OTFS performance is compared against that of a decision feedback equalizer (DFE) based system, showing promising results.

5.1.1. Contributions of this chapter

The contributions of this chapter can be summarized as follows:

- We assess the performance of the pilot design as proposed in Chapter 4 using channel replay simulations. The channel replay simulations show close agreement with the theoretically optimal power distribution.
- We demonstrate that the proposed OTFS system outperforms a conventional DFE (from [2]) in environments with severe multipath and Doppler spread, particularly at lower data rates.

5.2. Related work

Sound propagation in horizontal underwater channels is typically characterized by extensive time-varying multipath [3]. As a result, classical modulation techniques such as orthogonal frequency division multiplexing (OFDM) [4], [5] perform well in relatively stable channels but face challenges with significant Doppler spreads [6], [7]. Since sound propagates slowly, motion distorts the time scale of the waveform rather than merely shifting its carrier frequency. Due to this issue, various improvements have been proposed, ranging from front-end resampling and Doppler-scale or frequency-offset estimation [4], [8], [9], [10], [11] to detection and channel estimation schemes designed to operate under the residual Doppler distortion [12], [13], [14], [15]. Such front-end processing acts on the received waveform and is largely independent of the modulation format, so it remains available whichever scheme is used. For such dynamic channels, alternative modulation techniques such as single-carrier modulation (SCM) [16] or multiband modulation combined with a DFE and a phase-locked-loop (PLL) may become effective [17], [18]; however, they require careful tuning of filter update parameters to adapt to channel variability.

Recently, the seemingly novel modulation scheme OTFS was introduced in the communication society for terrestrial communication purposes [19], and was later adopted by the UWA community [20], [21], [22], [23]. In Chapter 3, we have shown that OTFS is equivalent to orthogonal sequence division multiplexing (OSDM), which was proposed for underwater acoustic communications over a decade ago. However, OTFS is derived from different principles, which leads to new insights for OSDM as well.

The main benefit of OTFS is that it transforms a time-varying multipath channel into a time-invariant one within the delay-Doppler (DD) domain, offering a novel way to exploit the channel time-frequency diversity.

The literature on OTFS and OSDM is extensive, especially on the receiver structure. Nonlinear equalizers based on the message passing algorithm [24] or detection through Markov chain Monte Carlo sampling [25] offer the best performance in general, at the cost of computational complexity. Others have proposed to iteratively adjust the order of the channel model [26]. In [27], a receiver based on the MMSE estimator was used; however, the pilot scheme was not optimized, and as such their pilot is not MMSE optimal (see Chapter 4). Iterative approaches have been suggested for OSDM, as demonstrated in [28], where the receiver iterates between detection, channel estimation, phase estimation, equalization, and decoding. Moreover, [28] employed the same suboptimal pilot as [27].

Despite the diverse approaches, a common limitation across these studies is the fact that there is no optimization over the parameters of OTFS, the pilot allocation, or the power distribution between the pilot and the data symbols. The studies discussed previously suggest a *fixed* pilot design and receiver setup intended to accommodate all channels. However, it is evident from the literature that an adaptive pilot scheme, tailored to specific channel characteristics, would improve performance.

5.3. Channel model

Given Nyquist-rate sampling, we denote the discrete-time versions of the received signal, the time-varying channel impulse response (CIR) (in normalized baseband), the transmitted signal, and the additive channel noise as $r[k]$, $h_{\text{nb}}[k, l]$, $x[k]$, and $n[k]$, respectively. Note that these quantities are sampled at symbol rate with period T_{symp} . The received signal can be expressed as follows:

$$r[k] = \sum_{l=0}^L h_{\text{nb}}[k, l] x[k-l] + n[k]. \quad (5.1)$$

Here L denotes the maximum time delay. Note that if $h_{\text{nb}}[k, l] = h_{\text{nb}}[l]$, the CIR is time-invariant.

To make the problem of estimating the channel taps $\{h_{\text{nb}}[k, l]\}_{k=0, l=0}^{K, L}$ well posed, we model the CIR with the complex exponential basis expansion model (BEM) (CE-BEM) [29], which leads to

$$h_{\text{nb}}[k, l] = \sum_{q=-Q/2}^{Q/2} c_{l,q} b_q[k]. \quad (5.2)$$

Here $b_q[k] = e^{j\omega_q k}$, with $\omega_q = 2\pi q/K$, is the basis function. Note that the CE-BEM is quite comprehensible; each coefficient of the CE-BEM represents a unique pair of Doppler shift ω_q and time delay τ_l .

Modeling the channel with a CE-BEM and considering additive Gaussian noise with zero mean and covariance $\mathbf{R}_n$, the received signal is given by

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} = \left(\sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L c_{l,q} \mathbf{\Lambda}_K^{(q)} \mathbf{P}_K^l \right) \mathbf{x} + \mathbf{n}, \quad (5.3)$$

where the vectors $\mathbf{r}$, $\mathbf{x}$ and $\mathbf{n}$ collect the samples of $r[k]$, $x[k]$ and $n[k]$, respectively, for $k = 0, 1, \dots, K-1$. We further have $\mathbf{\Lambda}_K^{(q)} = \text{diag}(e^{j2\pi q 0/K}, \dots, e^{j2\pi q (K-1)/K})$ and the matrix $\mathbf{P}_K$ is the cyclic shift matrix.

5.4. OTFS modulation in baseband

In this section, we introduce the OTFS modulation and demodulation.

The channel symbols are arranged in a two-dimensional matrix $\mathbf{S} \in \mathbb{C}^{M \times N}$, where the first and second dimensions are related to the CIR delay and Doppler dimension, respectively. Vectorizing the symbols to a vector of length $K = NM$, i.e., $\mathbf{s} = \text{vec}(\mathbf{S})$ and denoting the discrete Fourier transform matrix of size N as $\mathbf{F}_N$, the transmitted vector can be written as (see Chapter 3 or [30]),

$$\mathbf{x} = (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s}, \quad (5.4)$$

where $\otimes$ denotes the Kronecker product. Note that this type of modulation coincides with OFDM and SCM when $M = 1$ and $N = 1$, respectively.

After passing through the channel, the received signal is demodulated by,

$$\mathbf{y} = (\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{r}. \quad (5.5)$$

To obtain the signal received in the DD domain, the vector $\mathbf{y}$ is reshaped again into a matrix of size $M \times N$, i.e., $\mathbf{Y} = \text{vec}^{-1}(\mathbf{y})$.

In our implementation, we use the MMSE channel estimator on the demodulated signal [27]. This estimator allows us to perform a systematic theoretical analysis of the pilot allocation and power distribution, as will be described in the next section.

5.5. Pilot allocation and power distribution

In this section, we provide a brief overview of the traditional pilot design [31], [32], and summarize the novel pilot design approach provided in Chapter 4.

5.5.1. Pilot allocation

Island case

Traditional pilot design for OTFS involves positioning a single pilot symbol surrounded entirely by zeros, ensuring that there is no overlap between pilot and data symbols on the receiver side [31], [32]. For the implementation of this pilot allocation in the symbol matrix $\mathbf{S}$, a block of size $(2L+1) \times (2Q+1)$ must be reserved only for pilots. Due to the relatively large resulting overhead, different approaches were proposed to reduce the pilot overhead [26], [33]. Furthermore, despite the unequal power levels between pilot and data symbols in [31] and [32], an analysis of the optimal power distribution is lacking. We will refer to this pilot allocation as the “island case”.

New pilot design strategy

In the DD domain, the channel performs a circular shift of the symbols in $\mathbf{S}$ along the delay and Doppler dimensions, with l and q respectively. Additionally, it should be emphasized that the values of N and M can be chosen based on our preference. This adaptability is not utilized in the island case, where the sole conditions are that $M \geq 2L + 1$ and $N \geq 2Q + 1$. Utilizing this adaptability results in the following pilot allocations.

- Delay slab - here we set $M = L + 1$ and $N = K/M$. As a result, the pilot overhead can be reduced to a block of size $K_p = (L + 1) \times (2Q + 1)$.
- Doppler slab - here we set $N = Q + 1$ and $M = K/N$. As a result, the pilot overhead can be reduced to a block of size $K_p = (2L + 1) \times (Q + 1)$.

In the previous chapter, it was shown that the island case, delay slab and Doppler slab all minimize the error of the MMSE channel estimator. Therefore, the delay slab and Doppler slab are favored due to their reduced pilot overhead. Moreover, note that if $L > Q$ ($L < Q$) the delay slab (Doppler slab) is the allocation with the lowest overhead. In practical channels, usually one has $L > Q$. Finally, let R_s and m denote the symbol rate and constellation order, respectively, then the effective data rate of the OTFS system is given by,

$$R_d = \log_2(m) R_s \frac{K - K_p}{K + L}. \quad (5.6)$$

5.5.2. Power distribution

Having set the pilot allocation, we can still choose the power distribution between the pilot and data symbols. Let P denote the total power available at the transmitter and suppose that $P_p = (1 - \alpha)P$ and $P_d = \alpha P$ power is used for the pilot and data symbols, respectively. That is,

$$\|\mathbf{x}_d\|_2^2 + \|\mathbf{x}_p\|_2^2 = P_d + P_p = P, \quad (5.7)$$

where $\mathbf{x}_d$ and $\mathbf{x}_p$ denote the data and pilot signal respectively. Based on a lower bound of the capacity, the optimal power distribution is given by,

$$\alpha^* = \arg \max_{\alpha} \rho(\alpha), \quad (5.8)$$

where $\rho(\alpha)$ is given by (see (4.32), (4.33) and (4.34))

$$\rho(\alpha) = \frac{\frac{P_d}{K_d} \sigma_{\mathbf{H}}^2}{\frac{P_d}{K_d} \sigma_{\mathbf{H}}^2 + \sigma_{\mathbf{n}}^2}, \quad (5.9)$$

with

$$\sigma_{\mathbf{H}}^2 = K_d \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L \frac{P_p \sigma_{c_{l,q}}^4}{\sigma_{\mathbf{n}}^2 + P_p \sigma_{c_{l,q}}^2}, \quad (5.10)$$

$$\sigma_{\mathbf{H}}^2 = \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L \frac{\sigma_{c_{l,q}}^2 \sigma_{\mathbf{n}}^2}{\sigma_{\mathbf{n}}^2 + \sigma_{c_{l,q}}^2 P_p}. \quad (5.11)$$

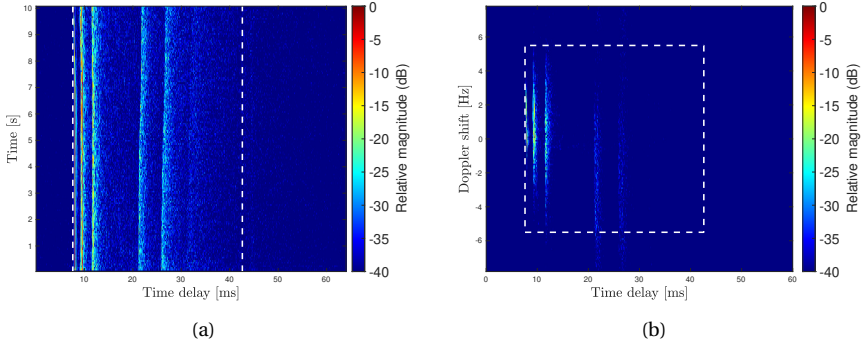


Figure 5.1.: (a) Measured CIR. (b) Measured CIR in the delay-Doppler domain.

Here, $K_d = K - K_p$ denotes the number of data symbols. Note that the optimal power distribution in (5.8) depends, among others, on the noise level σ_n^2 and the variance of the channel taps $\sigma_{c_{l,q}}^2$. These parameters should be estimated up front, which generally does not pose a significant issue since they are approximately constant over a longer period of time.

5.6. Simulations

5.6.1. The Watermark channel replay

The proposed OTFS pilot design is tested via the Watermark channel replay simulator [1]. The simulated channel is based on a 240m acoustic link between an autonomous underwater vehicle and a surface ship that was established during the NATO REPMUS exercise on September 18, 2021, off the coast of Sesimbra, Portugal. The real channel was estimated by successive transmissions of a linear frequency modulated (LFM) occupying the 21 – 37 kHz band. Fig. 5.1a shows the amplitudes of the estimated baseband CIRs, with the y-axis representing the relative time of the LFM arrival and the x-axis representing the multipath delay. Fig. 5.1b illustrates the CIR in the delay-Doppler domain, computed by applying the Fourier transform along the y-axis of Fig. 5.1a. Note that the CIR is characterized by multiple arrivals mainly within 24 ms delay spread, and Doppler spreads up to about 12 Hz. The Watermark channel replay [1] applies the estimated CIR of Fig. 5.1a to any waveform that lies within the duration and bandwidth of the channel.

Given that Watermark processes signals in passband, while previous discussions have been centered on baseband, an OTFS communication pipeline has been developed around Watermark, as depicted in Fig. 5.2. Here, we do not consider coding, and timing and Doppler synchronization (i.e. resampling) are assumed to be ideal for both the OTFS and the SCM system, so that the comparison reflects the channel estimation and equalization stage rather than the receiver front end¹. Integration of a detection

¹OTFS exhibits robustness to synchronization errors, provided that the channel energy remains within the bounds of the estimated channel model.

preamble is considered in subsequent chapters.

5.6.2. OTFS scheme

The quadrature phase-shift keying (QPSK) symbols are pulse-shaped based on a root-raised cosine filter with a roll-off factor $\beta = 0.2$. The communication band is 21-37 kHz, leading to a symbol rate of $R_s = 13300$ symbols per second (sps). For the OTFS modulation, we use the pilot design as discussed in Section 5.5, with an MMSE channel estimator and an MMSE symbol detector.

The first step is to select a suitable set of parameters $\{L, Q\}$ for the channel we intend to use. Consequently, based on the coherence time of the delay-Doppler channel, the parameters K , N , and M are derived. Note that, in complex baseband, the resolution of the time delay corresponds to the sampling interval, which is the reciprocal of the sampling frequency or the symbol rate, denoted as $L = \lfloor \tau_{\max}/T_{\text{symp}} \rfloor$. Similarly, the resolution of the Doppler shift is determined by the total duration of the sampling, expressed as $Q = \lfloor 2f_{\max}KT_{\text{symp}} \rfloor$. Thus, selecting the set $\{L, Q\}$ consists of determining the maximum time delay and Doppler shift of the channel. It is assumed that we have a coarse estimate of both the delay-Doppler plane and the noise intensity, which could typically be computed from the preamble. For the simulations in this section, the delay-Doppler plot shown in Fig. 5.1b was used. For example, if the maximum time delay is 35 ms and the maximum Doppler shift 5.5 Hz, visualized with the white dashed square in Fig. 5.1b, all the channel energy within that square is modeled and estimated, while any energy outside the box becomes self-interference. In this scenario, a balance must be struck between bit error ratio (BER) and data rate; selecting greater maximum delays and Doppler shifts enhances channel estimation, thereby reducing BER, whereas selecting smaller maximum delays and Doppler shifts reduces pilot overhead, consequently increasing the data rate. The considered rates are summarized in Table 5.1. Note that both the OTFS and the SCM signals are each repeated several times (seven in this case) to match the duration of the measured Watermark channel (10 s in this case). This enables averaging the performance of both methods over multiple repetitions/packets for each noise realization.

Table 5.1.: Settings for the different rates of OTFS.

Effective bit rate R_d [bits s^{-1}]	Packet length T [s]	Delay spread $\tau_{\max}$ [ms]	Doppler spread $2f_{\max}$ [Hz]
4835	0.7847	35.0	11.0
8864	0.7874	37.5	8.0
9670	0.7853	35.7	9.0

5.6.3. SCM scheme

Here, we describe the SCM scheme compared against OTFS. The QPSK symbols are pulse-shaped with a raised cosine filter (roll-off factor $\beta = 0.2$) and a symbol rate of $R_s = 13000$ sps. Similar to OTFS, the SCM signal passes through Watermark seven times

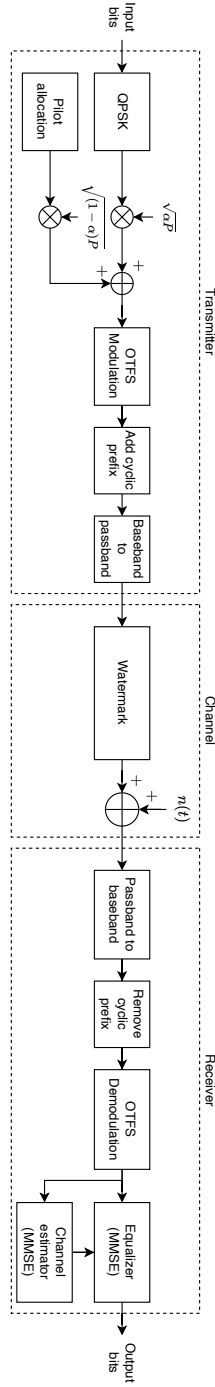


Figure 5.2.: Communication pipeline including the watermark framework for the OTFS system.

for a channel duration of 7 s. A subset of transmitted QPSK symbols, K_d out of K , is data, while the rest are pilots. The effective bit rate is $R_d = \log_2(m) R_s \frac{K_d}{K}$.

The SCM receiver front-end uses a DFE as in [2], performing Doppler compensation via a first-order PLL, inter-symbol interference cancellation via channel estimation, and linear equalization. The channel filter length is 26 ms and is updated via the improved-proportionate normalized least mean squares (IPNLMS) algorithm. The linear equalization filter length is 6 ms and is updated via the recursive least squares (RLS) algorithm. The first 750 symbols of the signal are used for DFE convergence. During this phase, the IPNLMS and RLS are jointly tuned to maximize the output signal-to-noise ratio (SNR). After training, the DFE parameters are frozen, and data and pilot symbols are interleaved to strike a balance between BER and data rate. Table 5.2 summarizes the effective bit rates and data-pilot patterns used after initial training.

Table 5.2.: Settings for the different rates of SCM.

Effective bit rate R_d [bits s^{-1}]	Packet length T [s]	data-pilot pattern	#data symbols K_d
4810	0.77 s	1 – 4	1850
8018	0.77 s	1 – 2	3084
9620	0.77 s	2 – 3	3700

5.6.4. Results

The OTFS communication pipeline in Fig. 5.2 is simulated for a range of SNR and α values. Fig. 5.3 shows the BER, as a function of the SNR and α for the rate of 4835 bps. The solid blue line indicates the power distribution that leads to the maximum output SNR, while the dashed blue line indicates the theoretical optimal power distribution, obtained from (5.8). The practical optimal power distribution changes similarly to the theoretical optimal power distribution. The slight deviation from the theoretical optimum can be attributed to the fact that the CE-BEM used by the OTFS system does not perfectly align with the true channel. This discrepancy becomes evident at high SNR values, as the optimal power distribution is less influenced by the noise level. An additional discrepancy is the assumed Gaussian distribution of channel taps and data symbols.

Fig. 5.4 illustrates the BER performance as a function of the energy per bit to noise power spectral density ratio. Note that the theoretical and practical optimum OTFS performances are nearly identical, and hence the theoretical model provides an accurate performance forecast of the practical optimal power distribution. Additionally, Fig. 5.4 shows the comparison between the OTFS and SCM schemes on the measured channel, for different data rates. The OTFS system shows slightly better performance for the higher data rates but for the lowest rate the improvement is significant: more than 5 dB at BER = 10^{-2} . This can be attributed to the OTFS system having sufficient pilot overhead, allowing for precise channel estimation and equalization.

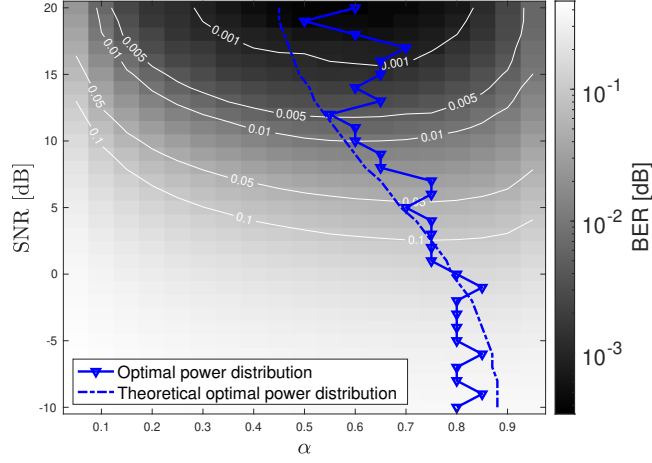


Figure 5.3.: BER of the OTFS system for various SNR and power distribution values.

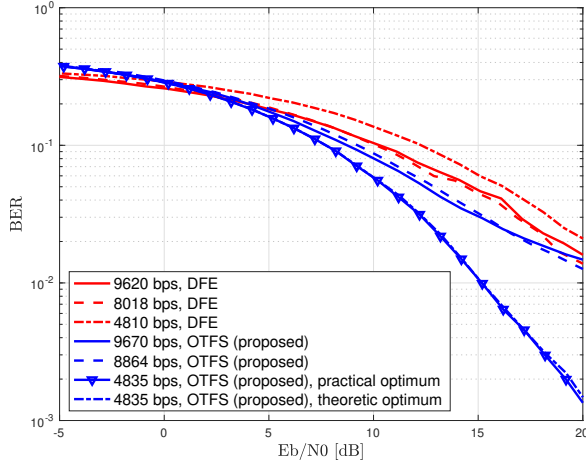


Figure 5.4.: BER performance comparison for various bit rates.

5.7. Conclusion

This chapter applied the novel pilot design scheme for OTFS, proposed in the previous chapter, in a specific UWA link. The scheme reduced the pilot overhead and optimized the power distribution by choosing appropriate OTFS parameters based on the channel delay and Doppler spread. This was a key advantage relative to the DFE, whose selection of the equalization parameter can become cumbersome. Using the Watermark channel

replay simulator, the pilot design scheme was validated demonstrating close agreement with the theoretically optimal power distribution, based on the CE-BEM channel model. Furthermore, the proposed OTFS system was compared against a conventional DFE for different data rates. The superiority of OTFS at lower data rates was clearly shown, largely due to its ability to perform more accurate channel estimation in the presence of severe multipath and Doppler spread.

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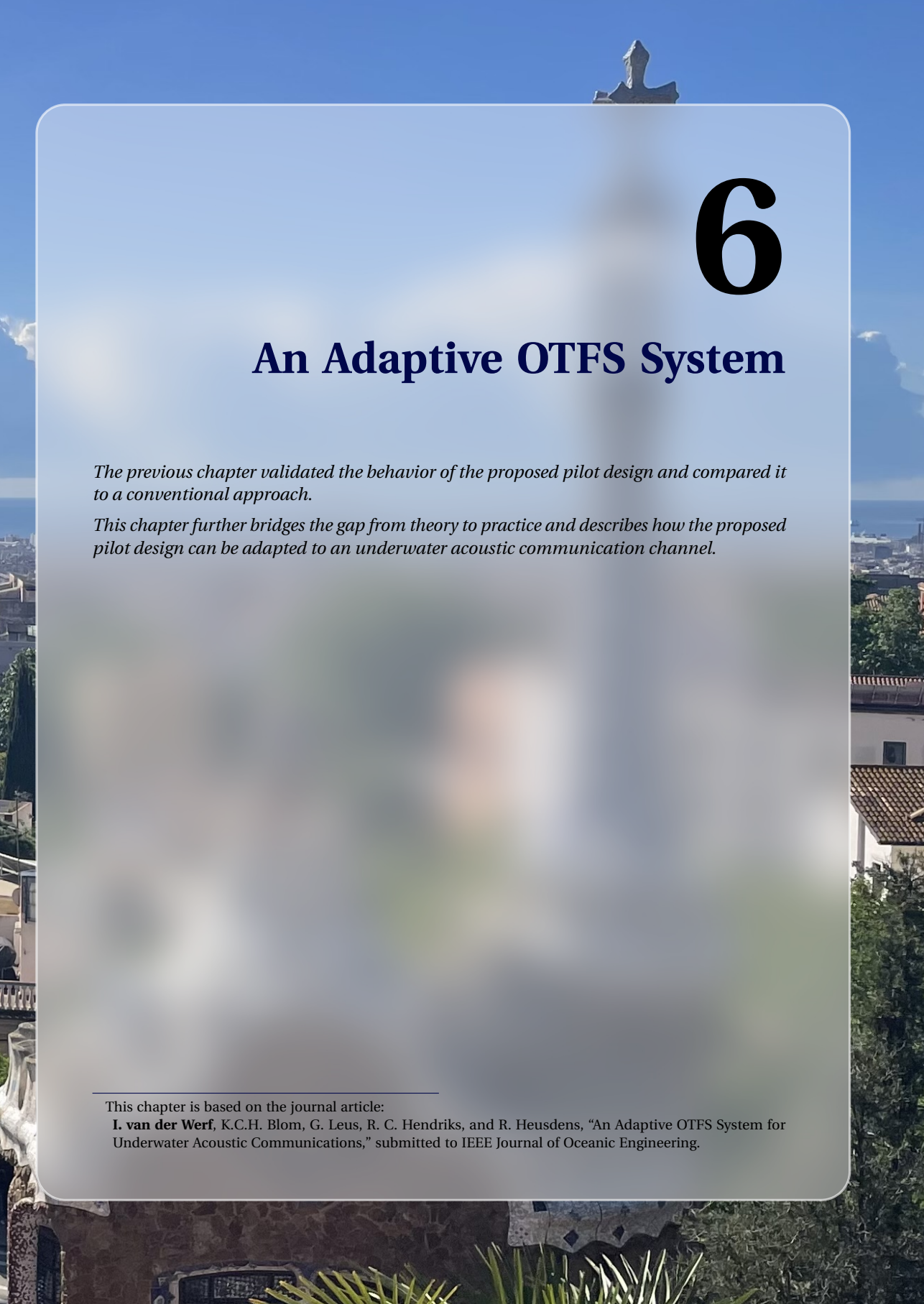
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Mediterranean Sea, Barcelona, Spain



6

An Adaptive OTFS System

The previous chapter validated the behavior of the proposed pilot design and compared it to a conventional approach.

This chapter further bridges the gap from theory to practice and describes how the proposed pilot design can be adapted to an underwater acoustic communication channel.

This chapter is based on the journal article:

I. van der Werf, K.C.H. Blom, G. Leus, R. C. Hendriks, and R. Heusdens, "An Adaptive OTFS System for Underwater Acoustic Communications," submitted to IEEE Journal of Oceanic Engineering.

6.1. Introduction

The highly time-varying nature of underwater acoustic (UWA) channels, driven by dynamic environmental factors, necessitates the use of adaptive communication frameworks. While the preceding chapters established a theoretical foundation for optimizing orthogonal time frequency space (OTFS) system parameters based on instantaneous channel conditions, translating these insights into a practical UWA implementation requires addressing additional challenges.

First, the analysis in Chapter 4 relies on the assumption of finite delay and Doppler support. In realistic UWA scenarios, channels frequently exhibit extended delay-Doppler profiles characterized by slowly decaying tails. These effectively “infinite” responses introduce residual interference that complicates the direct application of the pilot design in Chapter 4. Second, the channel state information (CSI) required for parameter selection (specifically delay-Doppler spreads, second-order statistics, and noise levels) is not known a priori. In a deployed system, these parameters must be estimated at the receiver and communicated to the transmitter via a feedback link. Lastly, the proposed approach was designed for normalized baseband, hence, a full communication pipeline including preamble and synchronization needs to be added.

This chapter further bridges the gap between theory and implementation by extending the framework from Chapter 4 to accommodate channels with extended delay-Doppler profiles. We explicitly account for channel estimation inaccuracies and unmodeled components to develop a predictive adaptation framework centered on an analytically tractable effective signal-to-interference-plus-noise ratio (SINR) metric. Unlike idealized metrics, the proposed formulation incorporates partial channel knowledge and noise, allowing for evaluation using practically obtainable second-order statistics. Ultimately, this provides a principled methodology for the selection of OTFS parameters in real-world UWA environments.

6.1.1. Contributions of this chapter

The contributions of this chapter can be summarized as follows:

- We explain how the proposed framework, designed for normalized baseband, fits within a full communication pipeline (i.e., including detection and synchronization).
- We extend the analysis of Chapter 4 and derive a closed-form expression for the *effective* SINR, ρ_{eff} , that has as parameters both the delay and Doppler spread that the receiver is able to equalize. It also models the neglected part of the channel as interference.
- We show that ρ_{eff} serves as an accurate predictor of the output signal-to-noise ratio (SNR), validated on both synthetic and measured channels.
- We propose to use the derived *effective* SINR term for OTFS system optimization. We introduce a simple protocol that can be used to configure the OTFS system in-situ, using only the information available in practice.

- We compare the potential of the proposed OTFS system with the four data rates of frequency-repetition spread spectrum (FRSS) [1], [2] and conclude that the proposed system allows a higher throughput.

By demonstrating its effectiveness on real-data channel replay, we show that the OTFS system performance can be accurately optimized using practically obtainable channel estimates only, without relying on information unavailable in deployment.

6.2. Related work

Severe multi-path propagation and motion-induced Doppler effects make UWA communication particularly challenging. These channel distortions depend strongly on environmental conditions, such as temperature, wind conditions, water depth, and seabed characteristics [3], [4]. Consequently, channels can vary drastically over time and location, and a communication system tuned for one environment may perform poorly in another unless its channel model, estimator, and equalizer are adapted to the prevailing conditions.

At the physical layer, poor communication performance typically has two root causes: (i) insufficient input SNR, or (ii) insufficient robustness to channel distortion. The appropriate remedy depends on which limitation dominates: constellation size and/or coding rate adaptation addresses (i), whereas redesign of the pilot structure and equalizer parameters is needed for (ii). In practice, both must be handled jointly. However, first of all, mapping input SNR to post-equalization performance (e.g. output SNR or bit error ratio (BER)) is typically highly nonlinear and architecture-dependent. Second, determining optimal pilot/equalizer parameters for given channel conditions can be nontrivial. Nevertheless, various strategies have been proposed, which we discuss in the following.

6.2.1. Existing adaptive methods

To cope with environmental variability, prior works have explored several forms of adaptation. A structured overview is provided in Table 6.1.

Receiver-side adaptation

One approach involves running multiple receiver instances in parallel, each with different parameters, and selecting the best-performing configuration [5] (this approach was also used for the decision feedback equalizer (DFE)-based system in Chapter 5). While a subsequent sequential adaptation method reduces the computational cost by tracking parameters across packets rather than running all instances simultaneously [6], both approaches focus exclusively on the receiver. They neglect the impact of transmit-signal design, including the training-data balance and energy allocation across time and frequency, which fundamentally affects channel estimation and equalization performance.

Transmit-side adaptation

Another common approach is to adaptively adjust the constellation size and coding rate.¹ For example, [7] adapts these two based on output SNR and achievable information rate, but relies on empirically determined thresholds from simulation and in-situ data. Similarly, [8] demonstrates gains from constellation adaptation, though only in an additive white Gaussian noise (AWGN) channel with perfect CSI, making generalization to realistic UWA channels unclear. The work in [9] jointly adjusts constellation size and subcarrier power for OFDM, but maintains a fixed pilot structure.

Joint transmitter–receiver adaptation

Besides adjusting the system on the transmit or receive side, a further line of work uses a small set of predefined system configurations, each combining a specific transmit waveform and receiver algorithm, and switches among them based on observed performance or heuristics. For instance, [10], [11] switch between coherent and noncoherent schemes and adjust their symbol rates. Although [10] uses physical channel properties (e.g., estimated delay and Doppler spread) to guide switching, the decision rules are derived empirically and are based on simplified channel models. More recently, the SALS project [12] investigated switching between four configurations of FRSS [13, Section III.B]. These configurations differ in preamble length, number of sub-bands, and equalizer filter lengths, with switching decisions made according to output SNR [1].

6.2.2. Limitations of existing adaptive methods

Beyond method-specific drawbacks, two fundamental limitations recur across the discussed approaches:

- Most systems adapt only the data rate, via constellation size or coding rate, while leaving the pilot design and/or equalizer structure unchanged (e.g., [7], [8], [9], [10], [14], [15], [18]).
- Adaptation decisions rely on *posterior* metrics (e.g., output SNR or error statistics), but do not take into account *a priori* information about the channel or the equalization capability (e.g., [7], [8], [11], [19]).

These limitations often result in suboptimal adaptation decisions. For example, a system may reduce its data rate even when the true bottleneck lies in the pilot or equalizer configuration. At their core, these issues stem from the lack of a performance metric that accounts for the channel conditions and the equalizer's tolerance. This observation motivates the development of predictive performance metrics that capture these effects.

Several metrics have been proposed for predicting performance prior to full decoding. Input SNR [16] is simple but ignores channel conditions and receiver behavior. Pilot-symbol SNR [20] captures some channel effects but still neglects equalization.

¹This is often termed adaptive modulation and coding (AMC), although the term “modulation” in this context refers only to the constellation size. To avoid ambiguity, we use “modulation” to denote the waveform type, e.g., single-carrier modulation (SCM) or orthogonal frequency division multiplexing (OFDM).

Table 6.1.: Brief overview of adaptive underwater acoustic communication literature.

Category	Author	Ref.	Modulation	Adaptivity	Decision metric	Validation	Feedback
Rx-side	Melainine	[5]	SCM	DFE filter lengths, step sizes	Symbol detection MSE	Simulation	n/a
	Melainine	[6]	SCM	DFE filter lengths	Symbol detection MSE	Simulation	n/a
Tx-side	Mani	[7]	SCM	Constellation, coding, no. of Tx elements	Output SNR, achievable information rate	Simulation, in-situ	no
	Tomasi	[8]	SCM	Constellation	SNR	Simulation	Instant, delayed
	Radosevic	[9]	OFDM	Constellation, power allocation	channel tap estimates	Replay, in-situ	Instant, delayed
	Wan	[14]	OFDM	Constellation, coding	Output SNR	Simulation, in-situ	Instant
	Pelekanakis	[15]	SCM	Constellation, bandwidth	delay/Doppler spread, input SNR	In-situ	Instant
	Barua	[16]	OFDM	Constellation	input SNR	Simulation	Instant
	Zhang	[17]	OFDM	Constellation, coding, power allocation	Second-order stats, SINR	Simulation	Instant
Tx/Rx-side	Yildirim	[18]	SCM	Constellation, pilot pattern	PAPR, input/output SNR, spreading factor	in-situ	delayed
	Benson	[10]	FSK, SCM	Modulation, symbol rate, filter lengths	delay and Doppler spread	Simulation	no
	Wu	[11]	FSK, DSSS	Modulation, symbol rate	BER	Simulation, in-situ	Instant
	Blom	[1]	FRSS	no. sub-bands, filter lengths	output SNR	Simulation, in-situ	Delayed

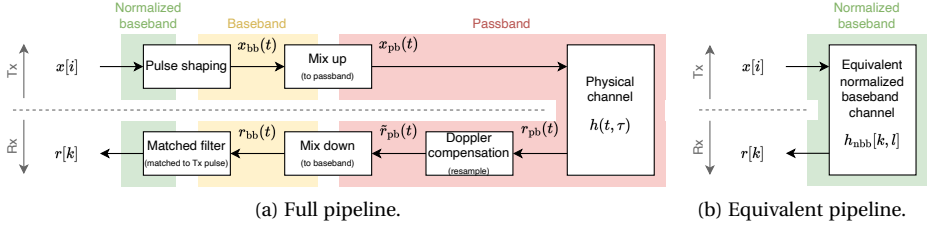


Figure 6.1.: Visualization of the communication pipeline at normalized baseband.

The “effective SNR” metric in [14] is, in practice, a measurement of output SNR and thus does not support predictive configuration. A more insightful approach is the delay–Doppler domain based SINR metric introduced in [2], which partitions the measured delay–Doppler power into regions accessible and inaccessible to the FRSS equalizer. While this metric correlates well with output SNR, it has two shortcomings: Doppler tolerance of the FRSS equalizer is difficult to quantify, leading to poor performance in dynamic channels. Secondly, the adaptive-filter behavior and decision errors are not captured.

To summarize, existing prediction metrics enable performance assessment but do not permit principled system optimization. To address this gap, we develop a predictive and analytically tractable metric that is a function of the OTFS system parameters, and can hence be used to optimize the system.

Outline

The remainder of the chapter is organized as follows. In Section 6.3, we develop the channel model and discuss the pilot–data trade-off. This forms the basis for the OTFS system described in Section 6.4. The proposed adaptation method and its use within a simple protocol are presented in Section 6.5. Finally, in Section 6.6, the adaptive system is validated using synthetic baseband channels and measured channel replay data, and compared with FRSS.

6.3. Signal model

This section presents the communication signal model at (normalized) baseband and its interaction with the physical channel at passband. An overview of the considered transmit–receive processing chain is provided in Fig. 6.1. Channel coding, constellation design, and OTFS modulation/demodulation and equalization are not considered here and are instead addressed in Section 6.4.

6.3.1. Transmitter

The baseband waveform $x_{\text{bb}}(t)$ is generated from a sequence of K complex information symbols $\{x[i]\}_{i=0}^{K-1}$ through pulse shaping,

$$x_{\text{bb}}(t) = \sum_{i=0}^{K-1} x[i]g(t - iT_{\text{symp}}), \quad (6.1)$$

where $g(t)$ is the transmit pulse and T_{symp} denotes the symbol period. Assuming the transmit pulse satisfies the Nyquist pulse-shaping criterion [21, Chapter 9], $x_{\text{bb}}(t)$ is bandlimited to $B = 1/T_{\text{symp}}$.² The corresponding transmitted passband signal is

$$x_{\text{pb}}(t) = \text{Re} \left\{ x_{\text{bb}}(t) e^{j2\pi f_c t} \right\}, \quad (6.2)$$

where f_c denotes the carrier frequency.

6.3.2. Time-varying baseband channel impulse response

We assume the UWA channel can be modeled as a linear time-variant (LTV) channel, i.e., with a time-varying channel impulse response (CIR) $h(t, \tau)$. Hence, the received signal is

$$r_{\text{pb}}(t) = \int_{-\infty}^{+\infty} h(t, \tau) x_{\text{pb}}(t - \tau) d\tau + n(t), \quad (6.3)$$

where $n(t)$ denotes the additive noise.

We model the time-varying CIR as

$$h(t, \tau) = \sum_{p=0}^{P-1} h_p(t) \delta(\tau - \tau_p(t)), \quad (6.4)$$

where $h_p(t)$ and $\tau_p(t)$ denote the (time-varying) path gain and propagation delay of the p 'th path, respectively. We assume that, within a single communication packet, the propagation delays can be approximated as linearly varying over time, i.e., $\tau_p(t) \approx \tau_{0,p} + a_p t$, and that the path gains vary slowly and can therefore be treated as constant, $h_p(t) \approx h_p$. The Doppler scale a_p is related to the path's radial velocity v_p by $a_p = \frac{v_p}{c}$, where c is the propagation speed.³

6.3.3. Receiver

Substituting (6.4) in (6.3) and using these two approximations, the received passband signal is

$$r_{\text{pb}}(t) = \sum_{p=0}^{P-1} h_p x_{\text{pb}}((1 - a_p)t - \tau_{0,p}) + n(t). \quad (6.5)$$

²Here we are referring to the -3 dB bandwidth. A discussion on the type and parameters of the (transmit) pulse is considered out of scope.

³The velocity v_p generally results from the combined motion of the transmitter, receiver and/or scattering objects.

Upon reception, the signal is first compensated for the motion-induced Doppler scalings a_p ; the time axis of the received signal is scaled by a factor $1 - a$, where a is typically an estimate of the Doppler scaling factor of the strongest path. The working assumption is that this strongest path is the direct path, and hence the Doppler scale of the direct path resembles the average of all the a_p 's. Transmitted signals can be prefixed with a detection preamble from which this estimate can be obtained. The received signal after Doppler compensation is given by

$$\begin{aligned}\tilde{r}_{\text{pb}}(t) &= r_{\text{pb}}\left(\frac{1}{1-a}t\right) \\ &= \sum_{p=0}^{P-1} h_p x_{\text{pb}}\left(\frac{(1-a_p)}{1-a}t - \tau_{0,p}\right) + \tilde{n}(t) \\ &= \sum_{p=0}^{P-1} h_p \text{Re}\left\{x_{\text{bb}}\left(\frac{(1-a_p)}{1-a}t - \tau_{0,p}\right)e^{j2\pi f_c\left(\frac{(1-a_p)}{1-a}t - \tau_{0,p}\right)}\right\} + \tilde{n}(t),\end{aligned}\quad (6.6)$$

where $\tilde{n}(t) = n(\frac{1}{1-a}t)$.

After this (initial) Doppler compensation, the received signal is shifted to baseband and passed through a low-pass filter (LPF) with bandwidth B , corresponding to the bandwidth of the signal $x_{\text{bb}}(t)$. The received signal in baseband is

$$\begin{aligned}r_{\text{bb}}(t) &= 2 \cdot \text{LPF}_B\{\tilde{r}_{\text{pb}}(t)e^{-j2\pi f_c t}\} \\ &= 2 \cdot \text{LPF}_B\left\{\sum_{p=0}^{P-1} h_p \text{Re}\left\{x_{\text{bb}}\left(\frac{(1-a_p)}{1-a}t - \tau_{0,p}\right)e^{j2\pi f_c\left(\frac{(1-a_p)}{1-a}t - \tau_{0,p}\right)}\right\}e^{-j2\pi f_c t}\right\} + w(t) \\ &= \sum_{p=0}^{P-1} h_p x_{\text{bb}}\left(\frac{(1-a_p)}{1-a}t - \tau_{0,p}\right)e^{j2\pi f_c\left(\frac{a-a_p}{1-a}t - \tau_{0,p}\right)} + w(t) \\ &= \sum_{p=0}^{P-1} h_p e^{-j2\pi f_c \tau_{0,p}} \sum_{i=0}^{K-1} x[i]g\left(\frac{(1-a_p)}{1-a}t - \tau_{0,p} - iT_{\text{symp}}\right)e^{j2\pi \nu_p t} + w(t),\end{aligned}\quad (6.7)$$

where $w(t) = 2 \cdot \text{LPF}_B\{\tilde{n}(t)e^{-j2\pi f_c t}\}$ denotes the additive noise in baseband and where $\nu_p = \frac{a-a_p}{1-a}f_c$. Without loss of generality, in the following, we absorb the constant phase term $e^{-j2\pi f_c \tau_{0,p}}$ in the complex path gain h_p .

The baseband signal is then passed through a receive filter matched to the transmit pulse $g(t)$ and sampled at the symbol rate, yielding

$$r[k] = \int_{-\infty}^{+\infty} r_{\text{bb}}(t)g^*(t - kT_{\text{symp}})dt. \quad (6.8)$$

Substituting the expression for $r_{\text{bb}}(t)$ into (6.8) and changing the summation index from i to l via $i = k - l$, we obtain

$$r[k] = \sum_l h_{\text{nbb}}[k, l]x[k - l] + w[k], \quad (6.9)$$

where $w[k] \triangleq \int_{-\infty}^{+\infty} w(t)g^*(t - kT_{\text{symp}})dt$ and where the discrete-time *normalized*

baseband channel coefficients $h_{\text{nb}}[k, l]$ are given by

$$h_{\text{nb}}[k, l] \triangleq \sum_{p=0}^{P-1} h_p \times \int_{-\infty}^{+\infty} g\left(\frac{1-a_p}{1-a}t - kT_{\text{sy}} + lT_{\text{sy}} - \tau_{0,p}\right) g^*(t - kT_{\text{sy}}) e^{j2\pi\nu_p t} dt. \quad (6.10)$$

Note that the integral in (6.10) can be interpreted as a cross-ambiguity function between two time- and Doppler-shifted versions of the pulse $g(t)$. As a consequence, the discrete-time channel coefficient $h_{\text{nb}}[k, l]$ attains significant magnitude primarily for delay indices l when lT_{sy} is close to the physical path delay $\tau_{0,p}$. This observation allows the summation in (6.9) to be truncated to a finite number of delay taps, e.g., $l = 0, \dots, L_{\text{max}}$, provided that $0 \leq \tau_{0,p} \leq L_{\text{max}}T_{\text{sy}}$ for all P paths.

An example of the resulting discrete-time channel coefficients $h_{\text{nb}}[k, l]$ for a two-path channel was previously provided in Chapter 2, but is reproduced here in Fig. 6.2 due to its strong relevance.

If the delay of a path is an integer multiple of the sampling period, i.e., $\tau_{0,p} = l_p T_{\text{sy}}$, and the motion-induced Doppler scaling is fully compensated ($a - a_p = 0$), then the corresponding physical path contributes to a single discrete-time channel tap. In contrast, when the delay is not an integer multiple of the sampling period or when $a - a_p \neq 0$, a single physical path generally spreads its energy across multiple discrete-time channel taps.

Although in principle estimating $\{h_p, a_p, \tau_{0,p}\}_{p=1}^P$ is sufficient, this is challenging in practice because the number of paths is unknown and the channel taps $h_{\text{nb}}[k, l]$ depend nonlinearly on the time delay and Doppler shift. Alternatively, estimating the channel taps $h_{\text{nb}}[k, l]$ from $r[k]$ (and the pilot part of $x[i]$) is typically an under-determined problem. To tackle this problem, we turn to the basis expansion model (BEM).

6.3.4. Basis expansion model (BEM)

A BEM represents the temporal variation of $h_{\text{nb}}[k, l]$ using a set of $Q_{\text{max}} + 1$ basis functions $\{b_q[k]\}_{q=-Q_{\text{max}}/2, \dots, Q_{\text{max}}/2}$, that is,

$$h_{\text{nb}}[k, l] = \sum_{q=-Q_{\text{max}}/2}^{Q_{\text{max}}/2} c_{l,q} b_q[k]. \quad (6.11)$$

Note that if $Q_{\text{max}} + 1 = K$, where K denotes the number of discrete time samples over which the channel is observed, and if the set of basis vectors $\{b_q[k]\}_{q=-Q_{\text{max}}/2, \dots, Q_{\text{max}}/2}$ spans $\mathbb{C}^K$, the decomposition in (6.11) is unique. To ensure that the channel estimation problem is well posed, we assume that the channel can be accurately represented using a lower-order basis, i.e., $Q_{\text{max}} + 1 < K$.

While various bases have been proposed [22], [23], [24], we will work with a complex exponential BEM (CE-BEM) [25], where $b_q[k] = e^{j\omega_q k}$, with $\omega_q = 2\pi q/K$. Fig. 6.3 shows the CE-BEM coefficients for the channel used in Fig. 6.2. Indeed, only $Q_{\text{max}} + 1 = 21$ basis functions are sufficient to accurately model the time-varying CIR $h_{\text{nb}}[k, l]$ over $K = 1000$ symbol intervals.

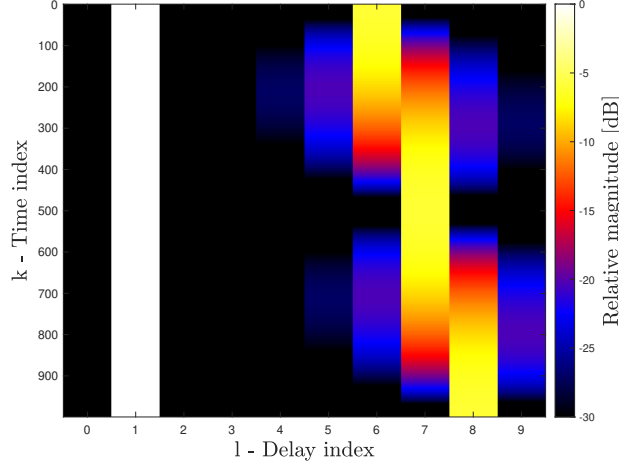


Figure 6.2.: Example of the normalized baseband channel coefficients $h_{\text{nbb}}[k, l]$ for a two-path channel. The symbol period is $T_{\text{symp}} = 1/13000\text{s}$, and $K = 1000$ symbol intervals are shown. The channel consists of two propagation paths with delays $\tau_{0,0} = T_{\text{symp}}$ and $\tau_{0,1} = 6T_{\text{symp}}$, and Doppler scaling factors $a_0 = 0$ and $a_1 = 3/1500$, respectively. No Doppler scaling compensation is applied ($a = 0$). The figure illustrates how integer-multiple delays lead to concentrated energy on a single tap, whereas Doppler scaling causes energy spreading across multiple delay taps.

By estimating the coefficients $\{c_{l,q}\}_{l=0, q=-Q_{\text{max}}/2}^{L_{\text{max}}, Q_{\text{max}}/2}$, we effectively estimate which combinations of Doppler shifts and time delays are present.

Remark 9. DFE based systems do not make any assumption on how the channel taps vary over time, only that they vary slowly with respect to the symbol duration, i.e. $h_{\text{nbb}}[k, l] \approx h_{\text{nbb}}[k+1, l]$, such that tracking is possible. A phase-locked-loop (PLL) is often included to account for the time-varying phase introduced by motion or oscillator offsets.

Stacking the samples $r[k]$, $x[k]$ and $w[k]$ for $k = 0, \dots, K-1$, in the vectors $\mathbf{r}$, $\mathbf{x}$ and $\mathbf{n}$, the received signal including the CE-BEM representation is given by

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n} = \left(\sum_{q=-Q_{\text{max}}/2}^{Q_{\text{max}}/2} \sum_{l=0}^{L_{\text{max}}} c_{l,q} \mathbf{\Lambda}^q \mathbf{P}^l \right) \mathbf{x} + \mathbf{n}, \quad (6.12)$$

where $\mathbf{\Lambda} = \text{diag}(e^{j2\pi 0/K}, \dots, e^{j2\pi(K-1)/K})$ applies the Doppler shift, and $\mathbf{P}$, denoting the cyclic shift matrix, applies the time delay, assuming a cyclic prefix precedes $\mathbf{x}$. Hence, this representation decomposes the channel into a sum of delay-Doppler shifts each weighted by a complex coefficient $c_{l,q}$.

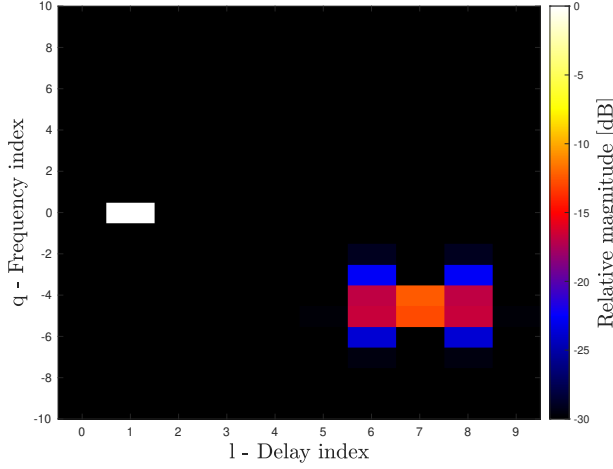


Figure 6.3.: Example of CE-BEM coefficients $c_{l,q}$ for the normalized baseband channel $h_{\text{nb}}[k, l]$ shown in Fig. 6.2. The channel is observed over $K = 1000$ symbol intervals and the first $Q_{\text{max}} + 1 = 21$ basis functions are shown. The temporal variation of the channel can be captured by a low-order CE-BEM.

6.3.5. Partial channel estimation: a pilot/data trade-off

Block-based systems, such as OFDM and OTFS, typically require a channel estimate for symbol equalization, in contrast to SCM-DFE systems. Estimating all $(L_{\text{max}} + 1)(Q_{\text{max}} + 1)$ channel taps, however, can lead to a substantial pilot overhead. To reduce this burden, the (transmit (Tx) and) receive (Rx) may estimate only a subset of the channel coefficients. To formalize this, we partition the set of all channel tap indices

$$\mathcal{C} = \left\{ (l, q) \mid l \in \{0, \dots, L_{\text{max}}\}, q \in \left\{ -\frac{Q_{\text{max}}}{2}, \dots, \frac{Q_{\text{max}}}{2} \right\} \right\},$$

into two disjoint subsets: $\mathcal{L}$, representing the estimated taps, and its complement $\mathcal{C} \setminus \mathcal{L}$. Specifically,

$$\mathcal{L} = \{0, \dots, L\} \times \{-Q/2, \dots, Q/2\}, \quad L \leq L_{\text{max}}, \quad Q \leq Q_{\text{max}}.$$

The channel matrix then decomposes into estimated ($\mathbf{H}_1$) and non-estimated ($\mathbf{H}_2$) parts:

$$\mathbf{H} = \sum_{(l,q) \in \mathcal{L}} c_{l,q} \mathbf{\Lambda}^q \mathbf{P}^l + \sum_{(l,q) \notin \mathcal{L}} c_{l,q} \mathbf{\Lambda}^q \mathbf{P}^l = \mathbf{H}_1 + \mathbf{H}_2. \quad (6.13)$$

Accordingly, the received signal in (6.12) becomes

$$\mathbf{r} = \mathbf{H}_1 \mathbf{x} + \mathbf{H}_2 \mathbf{x} + \mathbf{n}, \quad (6.14)$$

where $\mathbf{H}_2 \mathbf{x}$ acts as an interference term at the receiver.

Remark 10 (Practical viewpoint). *Although the delay–Doppler response of the channel may extend over a large support (i.e., $L_{\max}$ and $Q_{\max}$ can be large), the effective delay and Doppler spreads (i.e., L and Q) are typically much smaller due to energy concentration in a limited region of the delay–Doppler plane. In practice, these are often determined by applying a power threshold in decibels.*

Assuming successive symbols in $\mathbf{x}$ are uncorrelated, the SINR at the input of the receiver is

$$\text{SINR}_{\text{in}}(\mathcal{L}) = \frac{\mathbb{E}[\|\mathbf{H}_1 \mathbf{x}\|_2^2]}{\mathbb{E}[\|\mathbf{H}_2 \mathbf{x} + \mathbf{n}\|_2^2]} = \frac{\sum_{(l,q) \in \mathcal{L}} |c_{l,q}|^2 P}{\sum_{(l,q) \notin \mathcal{L}} |c_{l,q}|^2 P + \sigma_{\mathbf{n}}^2}, \quad (6.15)$$

where $\|\mathbf{x}\|_2^2 = P$ denotes the transmitted power (in normalized baseband) and where we have assumed that the noise is zero-mean white Gaussian, with variance $\sigma_{\mathbf{n}}^2$. Increasing L and Q improves the $\text{SINR}_{\text{in}}(\mathcal{L})$ at the cost of increased pilot overhead and reduced effective data rate.

To summarize, the effective delay and Doppler spreads, L and Q must be selected carefully; this process will be detailed in Section 6.5.

6.3.6. Channel tap distribution

Several works have investigated the statistical characterization of underwater acoustic channels; see, e.g., [4], [26]. While a variety of sophisticated channel models have been proposed, we adopt in this chapter a simple statistical model in which the delay–Doppler channel taps $c_{l,q}$ are assumed to be independent, zero-mean complex Gaussian random variables with variances $\sigma_{c_{l,q}}^2$. Under this assumption, a new channel realization is drawn for each channel use.

Based on this model, we design the proposed communication system in the next section. The impact of this assumption is assessed in Section 6.6, where system performance is evaluated using synthetic channels generated according to the assumed model and measured channel replay data, for which the assumption is not enforced.

6.4. OTFS system

This section describes the OTFS communication system. The theoretical framework of the pilot design was described in detail in Chapter 4, but assumed that $\mathbf{H}_2 = \mathbf{0}$. Here we generalize the analysis to any $\mathbf{H}_2$. In the following, we will treat the different parts of the communication pipeline. An overview of the pipeline is shown in Fig. 6.4.

6.4.1. Modulation and demodulation

For OTFS modulation the symbols are defined in the delay–Doppler domain, $\mathbf{S}_{\text{DD}} \in \mathbb{C}^{M \times N}$. The transmitted signal (in discrete time) is given by

$$\mathbf{x} = (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s}, \quad (6.16)$$

where $\mathbf{F}_N$ is the normalized discrete Fourier transform (DFT) matrix with elements $[\mathbf{F}_N]_{k,n} = \frac{1}{\sqrt{N}} e^{-j2\pi kn/N}$ and where the vector $\mathbf{s} = \text{vec}(\mathbf{S}_{\text{DD}})$ contains the transmitted

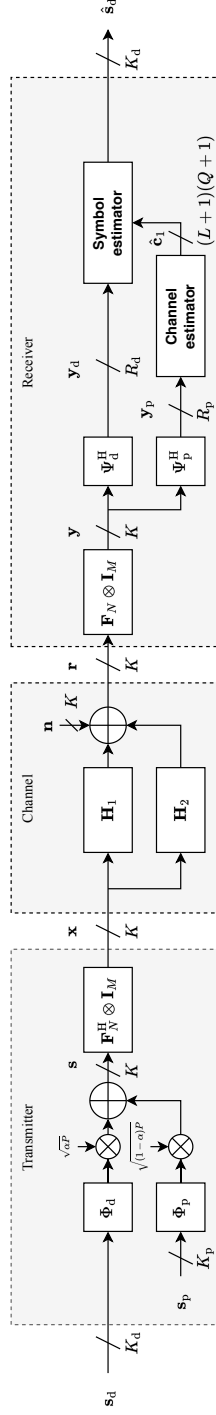


Figure 6.4.: Pipeline of the communication system at normalized baseband, including (de)modulation, inserted pilots, channel estimation and symbol estimation. Note that the division between $\mathbf{H}_1$ and $\mathbf{H}_2$ depends on L and Q , which are chosen by the transmitter and receiver (see (6.13)).

symbols. The parameters N and M can be chosen such that $K = NM$. At the receiver side, after the signal has passed through the channel, the signal is demodulated,

$$\begin{aligned} \mathbf{y} &= (\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{r} \\ &= (\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{H} (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s} + (\mathbf{F}_N \otimes \mathbf{I}_M) \mathbf{n} \\ &= \mathbf{H}_{DD} \mathbf{s} + \mathbf{w}. \end{aligned} \quad (6.17)$$

To obtain the signal received in the two-dimensional delay-Doppler (DD) domain, the vector $\mathbf{y}$ is reshaped into a matrix, $\mathbf{Y} = \text{vec}^{-1}(\mathbf{y}) \in \mathbb{C}^{M \times N}$.

6.4.2. Transmitted symbol vector

The transmitted symbol vector $\mathbf{s}$ consists of data and pilot symbols, $\mathbf{s}_d \in \mathbb{C}^{K_d}$ and $\mathbf{s}_p \in \mathbb{C}^{K_p}$, respectively, with $K = K_d + K_p$, such that

$$\mathbf{s} = \sqrt{\alpha P} \mathbf{\Phi}_d \mathbf{s}_d + \sqrt{(1 - \alpha)P} \mathbf{\Phi}_p \mathbf{s}_p. \quad (6.18)$$

Here the selection matrices $\mathbf{\Phi}_d \in \{0, 1\}^{K \times K_d}$ and $\mathbf{\Phi}_p \in \{0, 1\}^{K \times K_p}$ determine how the data and pilot symbols are allocated. In the following subsection we further detail the construction of the selection matrices and the design of the pilot symbol vector $\mathbf{s}_p$. The data symbol vector $\mathbf{s}_d$ contains symbols from a constellation of size m . The available power P at the transmitter is distributed between the data and pilot symbols using $\alpha \in [0, 1]$, as we set $\|\mathbf{\Phi}_d \mathbf{s}_d\|_2^2 = \|\mathbf{\Phi}_p \mathbf{s}_p\|_2^2 = 1$. In the following, we will use $P_d = \alpha P$ and $P_p = (1 - \alpha)P$ to denote the data and pilot power, respectively.

6.4.3. Pilot allocation

In Chapter 4, we have shown that the flexibility of choosing M and N can be exploited to reduce the pilot overhead by up to 50% compared to existing designs. The pilot allocation is described by two cases:

- *Delay slab*: we set $M = L + 1$ and $N = K/M$. The pilot overhead is $K_p = (L + 1) \times (2Q + 1)$.
- *Doppler slab*: we set $N = Q + 1$ and $M = K/N$. The pilot overhead is $K_p = (2L + 1) \times (Q + 1)$.

We use a single non-zero pilot symbol and fill the remaining pilot symbols with guard zeros, thereby eliminating interference between the data symbols and the (non-zero) pilot symbols. The two allocations are visualized in Fig. 6.5. The selection matrices are designed such that the (non-zero) pilot symbol does not interfere with the data symbols after propagation through $\mathbf{H}_1$ and vice versa. For this specific pilot symbol vector, we have

$$\mathbf{\Psi}_d \mathbf{M} \mathbf{H}_1 \mathbf{M}^H \mathbf{\Phi}_p \mathbf{s}_p = \mathbf{0}, \quad (6.19)$$

and for any data symbol vector, we have

$$\mathbf{\Psi}_p \mathbf{M} \mathbf{H}_1 \mathbf{M}^H \mathbf{\Phi}_d \mathbf{s}_d = \mathbf{0}. \quad (6.20)$$

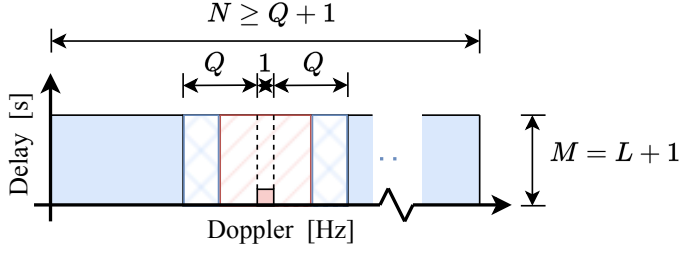
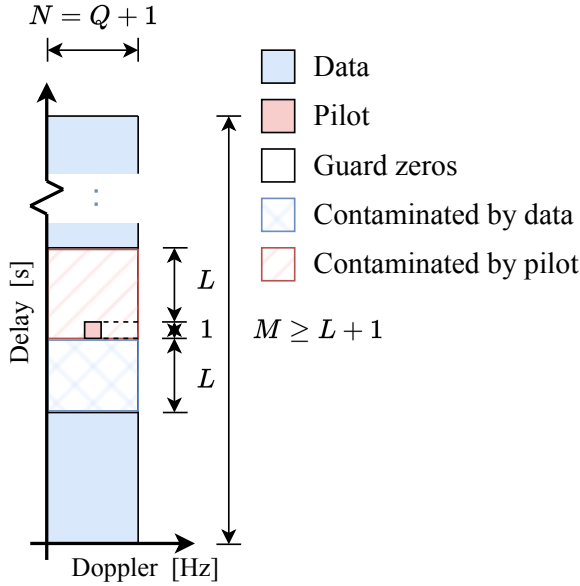
(a) Delay slab, $M = L + 1$, $N \geq Q + 1$.(b) Doppler slab, $M \geq L + 1$, $N = Q + 1$

Figure 6.5.: Pilot allocations for OTFS.

The corresponding selection matrices $\Phi_{(\cdot)}$ and $\Psi_{(\cdot)}$ are given in Appendix D.

Note that the two allocations can be combined. The allocation with the lowest pilot overhead depends on the relative spreads: if $L > Q$, the delay slab is optimal, whereas if $L < Q$, the Doppler slab is preferred. Accordingly, *the OTFS parameters (M, N) should be chosen based on the delay and Doppler spread:*

$$(M, N) = \begin{cases} (L + 1, K/(L + 1)), & \text{if } L > Q, \\ (K/(N + 1), N + 1), & \text{if } L < Q. \end{cases} \quad (6.21)$$

Following the strategy in (6.21), the pilot overhead of the allocation is given by,

$$K_p = \min \{(L+1)(2Q+1), (2L+1)(Q+1)\}. \quad (6.22)$$

6.4.4. Channel estimation

After demodulation, the received signal in the delay-Doppler domain $\mathbf{y}$, is split into a pilot part used for channel estimation $\mathbf{y}_p$ and a data part used for symbol estimation $\mathbf{y}_d$, using the selection matrices $\mathbf{\Psi}_p \in \{0, 1\}^{K \times R_p}$ and $\mathbf{\Psi}_d \in \{0, 1\}^{K \times R_d}$, with $K = R_p + R_d$. For brevity of notation, let $\mathbf{M} = \mathbf{F}_N \otimes \mathbf{I}_M$. Using (6.20), the part for channel estimation is

$$\begin{aligned} \mathbf{y}_p &= \sqrt{P_p} \mathbf{\Psi}_p^T \mathbf{M} \mathbf{H}_1 \mathbf{M}^H \Phi_p \mathbf{s}_p + \mathbf{\Psi}_p^T \mathbf{M} \mathbf{H}_2 \mathbf{M}^H \mathbf{s} + \mathbf{\Psi}_p^T \mathbf{M} \mathbf{n}. \\ &= \mathbf{Z}_p \mathbf{c}_1 + \mathbf{v}_p, \end{aligned} \quad (6.23)$$

where $\mathbf{Z}_p = [\dots, \sqrt{P_p} \mathbf{\Psi}_p^T \mathbf{M} \Lambda_K^q \mathbf{P}_K^l \mathbf{M}^H \Phi_p \mathbf{s}_p, \dots]$ is the sensing matrix for the channel taps $\mathbf{c}_1 = [\dots, c_{l,q}, \dots]$ indexed by $(l, q) \in \mathcal{L}$, and $\mathbf{v}_p = \mathbf{\Psi}_p^T \mathbf{M} \mathbf{H}_2 \mathbf{M}^H \mathbf{s} + \mathbf{\Psi}_p^T \mathbf{M} \mathbf{n}$. We use a classical linear minimum mean squared error (MMSE) channel estimator:

$$\hat{\mathbf{c}}_1 = \mathbf{W}_{\text{MMSE}}^H \mathbf{y}_p = \left(\mathbf{R}_{\mathbf{c}_1}^{-1} + \mathbf{Z}_p^H \mathbf{R}_{\mathbf{v}_p}^{-1} \mathbf{Z}_p \right)^{-1} \mathbf{Z}_p^H \mathbf{R}_{\mathbf{v}_p}^{-1} \mathbf{y}_p, \quad (6.24)$$

where $\mathbf{R}_{\mathbf{c}_1} = \mathbb{E}[\mathbf{c}_1 \mathbf{c}_1^H] = \text{diag}(\dots, \sigma_{c_{l,q}}^2, \dots)$ and $\mathbf{R}_{\mathbf{v}_p} = \mathbb{E}[\mathbf{v}_p \mathbf{v}_p^H]$.

6.4.5. Symbol estimation

Using (6.19), the part for symbol estimation is

$$\begin{aligned} \mathbf{y}_d &= \sqrt{P_d} \mathbf{\Psi}_d^T \mathbf{M} \mathbf{H}_1 \mathbf{M}^H \Phi_d \mathbf{s}_d + \mathbf{\Psi}_d^T \mathbf{M} \mathbf{H}_2 \mathbf{M}^H \mathbf{s} + \mathbf{\Psi}_d^T \mathbf{M} \mathbf{n}. \\ &= \hat{\mathbf{H}}_{d,1} \mathbf{s}_d + \mathbf{v}_d, \end{aligned} \quad (6.25)$$

where $\hat{\mathbf{H}}_{d,1} = \sqrt{P_d} \mathbf{\Psi}_d^T \mathbf{M} \hat{\mathbf{H}}_1 \mathbf{M}^H \Phi_d$ is the effective channel matrix and $\mathbf{v}_d = \hat{\mathbf{H}}_{d,1} \mathbf{s}_d + \mathbf{\Psi}_d^T \mathbf{M} \mathbf{H}_2 \mathbf{M}^H \mathbf{s} + \mathbf{\Psi}_d^T \mathbf{M} \mathbf{n}$ is the interference-plus-noise term where $\hat{\mathbf{H}}_{d,1} = \sqrt{P_d} \mathbf{\Psi}_d^T \mathbf{M} (\mathbf{H}_1 - \hat{\mathbf{H}}_1) \mathbf{M}^H \Phi_d$ denotes the channel estimation error matrix. For simplicity, the estimated symbols are obtained using a zero-forcing (ZF) estimator

$$\begin{aligned} \hat{\mathbf{s}}_d &= \mathbf{W}_{d,\text{ZF}}^H \mathbf{y}_d = \hat{\mathbf{H}}_{d,1}^\dagger \mathbf{y}_d \\ &= \left(\sum_{(l,q) \in \mathcal{L}} \hat{c}_{l,q} \sqrt{P_d} \mathbf{\Psi}_d^T \mathbf{M} \Lambda_K^q \mathbf{P}_K^l \mathbf{M}^H \Phi_d \right)^\dagger \mathbf{y}_d, \end{aligned} \quad (6.26)$$

where $(.)^\dagger$ denotes the pseudo-inverse.

6.4.6. Effective data rate

The number of communication symbols is given by $K_d = K - K_p$. Assuming a cyclic prefix (CP) of length L , the effective data rate of the OTFS system is

$$R = \log_2(m) R_b \frac{K_d}{K + L}, \quad (6.27)$$

where R_b denotes the baud rate. The effective rate of the OTFS system thus depends on the (estimated/modeled) delay and Doppler spread of the channel.

6.4.7. OTFS system configuration

The OTFS system configuration consists of the tuple (N, M, L, Q, α, m) . The estimator makes use of the second order channel statistics $(\{\sigma_{c_{l,q}}^2\}_{(l,q) \in \mathcal{C}}, \sigma_n^2)$. In the next section, we propose a method that determines the optimal OTFS system parameters from these channel statistics.

6.5. Proposed adaptation method

We require a method that, based on the second order channel statistics $(\{\sigma_{c_{l,q}}^2\}_{(l,q) \in \mathcal{C}}, \sigma_n^2)$ determines a configuration (L, Q, α, m) . The parameters N and M follow from the delay and Doppler spread L and Q respectively, through (6.21). The proposed method is based on an approximate lower bound on the channel capacity.

6.5.1. Approximate lower bound on the channel capacity

The channel capacity, averaged over the channel $\mathbf{H}$, is

$$C = \mathbb{E}_{\mathbf{H}} \left[\max_{p_{s_d}(\cdot), \mathbb{E}[\|\mathbf{s}_d\|_2^2]=1} \mathcal{J}(\mathbf{y}_d; \mathbf{s}_d | \hat{\mathbf{c}}_1) \right] \text{ bits/use}, \quad (6.28)$$

where $p_{s_d}(\cdot)$ denotes the probability density function of $\mathbf{s}_d$. Since a closed-form expression for this capacity is not available, we derive an approximate lower bound on the channel capacity (see Appendix E for the full derivation),

$$C \geq \underline{C} = \mathbb{E}_{\mathbf{H}} \left[\log_2 \left(\det \left(\mathbf{I}_{K_d} + \rho_{\text{eff}} \frac{\hat{\mathbf{H}}_{d,1}^H \hat{\mathbf{H}}_{d,1}}{\mathbb{E}[\|\hat{\mathbf{H}}_{d,1}\|_F^2]} \right) \right) \right]. \quad (6.29)$$

Here ρ_{eff} is the *effective* SINR given by

$$\rho_{\text{eff}} = \frac{\frac{P_d}{K_d} \sigma_{\hat{\mathbf{H}}}^2}{\frac{P_d}{K_d} \sigma_{\hat{\mathbf{H}}}^2 + \max \left\{ \frac{P_d}{K_d}, P_p \right\} \sigma_{\mathbf{H}_2}^2 + \sigma_{\mathbf{n}}^2}. \quad (6.30)$$

Here $\sigma_{\hat{\mathbf{H}}}^2$, $\sigma_{\mathbf{H}}^2$, $\sigma_{\mathbf{H}_2}^2$ and $\sigma_{\mathbf{n}}^2$ denote the useful (estimated) channel power, the channel estimation error power, the channel interference power and the noise power, respectively. These are defined as

$$\sigma_{\hat{\mathbf{H}}}^2 = K_d \sum_{(l,q) \in \mathcal{L}} \frac{\sigma_{c_{l,q}}^4 P_p}{\sigma_{c_{l,q}}^2 P_p + \max \left\{ \frac{P_d}{K_d}, P_p \right\} \sigma_{\mathbf{H}_2}^2 + \sigma_{\mathbf{n}}^2}, \quad (6.31)$$

$$\sigma_{\mathbf{H}}^2 = \sum_{(l,q) \in \mathcal{L}} \frac{\sigma_{c_{l,q}}^2 \left(\max \left\{ \frac{P_d}{K_d}, P_p \right\} \sigma_{\mathbf{H}_2}^2 + \sigma_{\mathbf{n}}^2 \right)}{\sigma_{c_{l,q}}^2 P_p + \max \left\{ \frac{P_d}{K_d}, P_p \right\} \sigma_{\mathbf{H}_2}^2 + \sigma_{\mathbf{n}}^2}, \quad (6.32)$$

$$\sigma_{\mathbf{H}_2}^2 = \sum_{(l,q) \notin \mathcal{L}} \sigma_{c_{l,q}}^2. \quad (6.33)$$

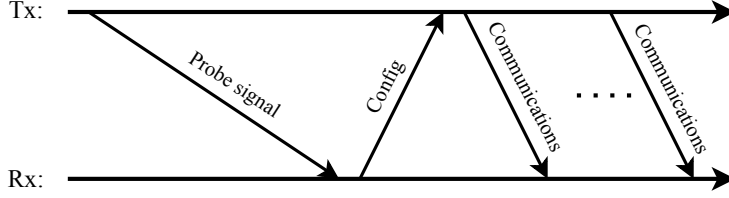


Figure 6.6.: Diagram of the simple protocol to configure the OTFS system.

Observe that if $\mathbf{H}_2 = 0$, then $\sigma_{\mathbf{H}_2}^2 = 0$ and hence ρ_{eff} in (6.30) simplifies to the expression in (4.32). Moreover, interestingly, that term is equal to the term [27, eq. (42)] derived for SCM and OFDM.

In Section 6.6, we show that ρ_{eff} is a good predictor of the output SNR.⁴ This observation motivates the decision metric proposed next.

6.5.2. Decision metric

Note that ρ_{eff} depends on the channel statistics $\{\sigma_{c_{l,q}}^2\}_{(l,q) \in \mathcal{C}}$ and $\sigma_{\mathbf{n}}^2$, as well as the OTFS system parameters L, Q, α . Consequently, for given channel statistics, the OTFS system parameters can be selected that maximize the data rate R , subject to a minimum output SNR requirement:

$$(L^*, Q^*, \alpha^*, m^*) = \arg \max_{L, Q, \alpha, m} \log_2(m) R_b \frac{K_d}{K + L} \quad (6.34)$$

$$\text{s.t. } \rho_{\text{eff}} \geq \rho_{\min}(m),$$

where $\rho_{\min}(m)$ denotes the minimum SNR_{out} required for the constellation order m . The parameters (M^*, N^*) follow through (6.21).

6.5.3. The OTFS protocol

We briefly outline a simple protocol that determines the OTFS configuration based on the current channel conditions. An overview of the protocol is shown in Fig. 6.6.

Suppose the Tx wants to initiate a communication link with the Rx. It begins by sending a probe signal that serves two purposes: it acts as a wake-up signal for the Rx and simultaneously enables channel estimation. Based on this probe, a single-snapshot estimate of the channel tap power is obtained as $\hat{\sigma}_{c_{l,q}}^2 = |\hat{c}_{l,q}|^2$, for $l = 0, \dots, L_{\max}$ and $q = -Q_{\max}/2, \dots, Q_{\max}/2$. In addition, an estimate of the noise power, denoted by $\hat{\sigma}_{\mathbf{n}}^2$, is obtained via a moving-average operation at the detector. More sophisticated estimation methods exist but are out of scope for this dissertation.

⁴We will formally introduce the output SNR in Section 6.6.

Given these estimates, the Rx selects an optimal configuration using (6.34). This configuration is then conveyed to the Tx via a reliable, low-data-rate transmission. Following this initial handshake, the Tx begins streaming OTFS packets using the agreed-upon configuration.

The selected configuration remains valid as long as the second-order channel statistics are approximately stationary. When significant changes in the channel statistics are detected, the protocol can be re-initiated by transmitting a new probe signal to update the configuration.

6.6. Validation

In this section, we validate the proposed framework through statistical and channel replay simulations.

6.6.1. Statistical simulations

We begin by simulating the normalized baseband channel, with the channel tap distribution described in Section 6.3.6. For these normalized baseband simulations, the channel statistics are assumed known at the receiver, whereas the specific channel realizations are unknown.

Figures 6.7a and 6.7b illustrate the correlation between $\rho_{\text{eff}}(L, Q)$ and the output SNR for all possible (L, Q) combinations, assuming a fixed power allocation and constellation size. The output SNR is defined as [2], [28]

$$\text{SNR}_{\text{out}} = \frac{K_d - 1}{K_d} \frac{\sum_{k=1}^{K_d} |s_k|^2}{\sum_{k=1}^{K_d} |s_k - \hat{s}_k / \gamma|^2}, \quad (6.35)$$

with $\gamma = \frac{1}{K_d} \sum_{k=1}^{K_d} \hat{s}_k / s_k$.

Results are shown for two different channel distributions; exponential (Fig. 6.7a) and polynomial (Fig. 6.7b) decay. In both cases, the average and median over 100 independent channel realizations are included. The results demonstrate that ρ_{eff} provides an accurate prediction of the output SNR. Figures 6.7c and 6.7d further illustrate how the output SNR decreases as the bandwidth efficiency, defined as $K_d / (K + L)$ and thus proportional to the effective data rate, increases. This behavior arises because smaller values of L and Q lead to reduced output SNR while simultaneously enabling higher effective data rates, thereby clearly highlighting the trade-off between data and pilot allocation discussed in Section 6.3.5.

6.6.2. Channel soundings

Results in Sections 6.6.3 and 6.6.4 are based on Watermark [29] channel replay simulations. This section briefly elaborates on the soundings used to generate the replay channels. One *set of soundings* was performed with a static and short-range geometry (38.9 m) on May 19, 2025, in the Haringvliet estuary, the Netherlands. The other set of soundings is based on a dynamic geometry; an acoustic link between an autonomous underwater vehicle (AUV) and a surface ship established during the NATO

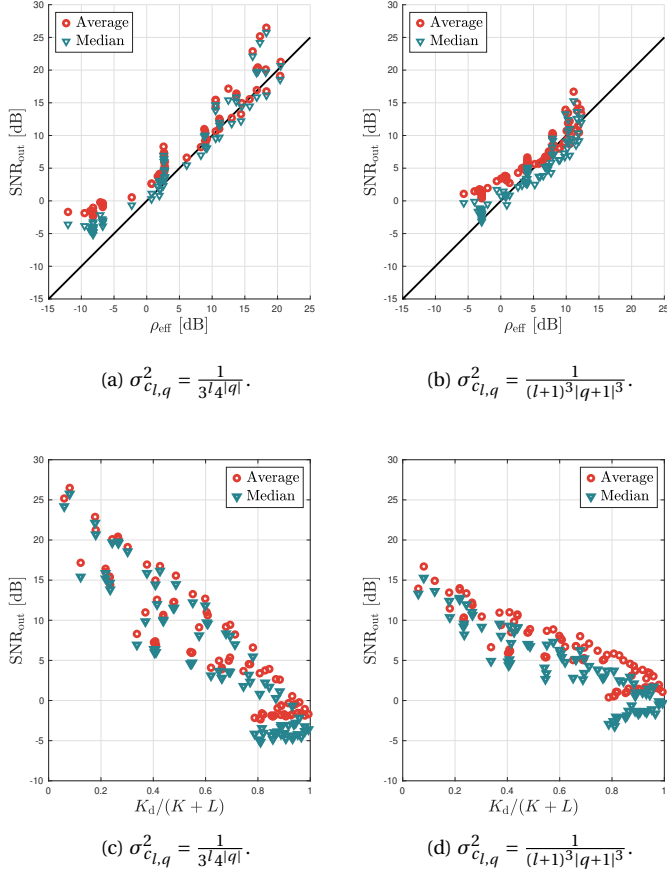


Figure 6.7.: SNR_{out} prediction and dependency between SNR_{out} and bandwidth efficiency for $K = 144$, $L_{\text{max}} = 16$, $Q_{\text{max}} = 16$, $\sigma_{\mathbf{n}}^2 = \frac{1}{K} \frac{P}{\text{SNR}_{\text{tx}}}$, $P = K$, $\alpha = 0.8$, $m = 4$, $\text{SNR}_{\text{tx}} = \frac{P}{K \sigma_{\mathbf{n}}^2} = 30 \text{ dB}$.

REPMUS exercise on September 18, 2021, off the coast of Sesimbra, Portugal. During this measurement, the surface ship was station-keeping and the AUV was sailing a lawn mower pattern; ranges varied from 240 m up to 760 m.

In the aforementioned sounding campaigns, channels were estimated by a train of linear frequency modulated (LFM) waveforms occupying (at least) the 21 kHz to 37 kHz band. The resulting (baseband) CIRs can be used for Watermark replay [29] of any waveform that lies within the duration and bandwidth of the channel. The time-varying CIR evolution for one of the benign Haringvliet channels is depicted in Fig. 6.8a. Figure 6.8b illustrates its CIR in the DD domain, computed by applying the discrete Fourier transform along the columns of Fig. 6.8a. The channel features a single path with

little Doppler spread.

The progressively more difficult CIRs of the NATO REPMUS channels in the DD domain are illustrated in Figures 6.8c and 6.8d. Figure 6.8c features multiple distinct arrivals with a Doppler spread that increases with delay. Next to a Doppler spread that increases with delay, Figure 6.8d also exhibits delay-domain smearing that gets more pronounced at larger delays.

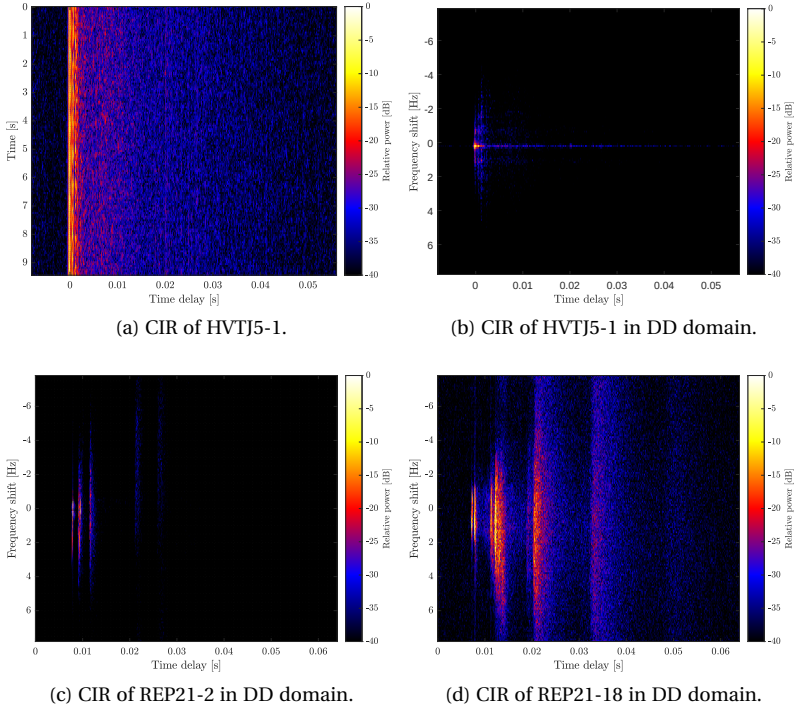


Figure 6.8.: Selection of channels used for Watermark replay simulation.

6.6.3. Channel replay simulations

All transmitted signals include a preamble containing a maximum length sequence, which is used for detection as well as time and frequency synchronization. The OTFS protocol outlined in Section 6.5.3 is executed, such that only practically available information is used at the receiver. The replayed channels consist of recordings with a duration of 10s. Following the protocol, the first (approximately two) seconds are occupied by the probe signal, a train of LFM signals. The validity of using LFM signals as a probe and its underlying assumptions are discussed in detail in [30]. Replacing the LFM-based probe with alternative waveforms, such as pseudonoise sequences that allow online control of the delay–Doppler resolution trade-off at the receiver, is left for future work.

Figure 6.9 illustrates the correlation between the predicted effective SNR $\rho(L, Q)$ and the measured output SNR of the communication packets for four different channel recordings. Several observations can be made. First, overall, the prediction correlates well with the measured output SNR. For some soundings, a slight offset is observed, which may have multiple causes; we attribute this primarily to the fact that, in practice, the channel tap distributions deviate from the assumed Gaussian model. Second, the variation in SNR_{out} across packets is small, indicating that the channel statistics estimated at the beginning of the sounding remain representative over the duration of the transmission. Third, the clusters of points in the lower-left region of the figures (most clearly visible in Fig. 6.9d) correspond to configurations with $Q = 0$, i.e., where the receiver assumes the absence of Doppler shifts. In this case, even small residual Doppler or frequency synchronization errors originating from the detection preamble propagate through the receiver and significantly degrade the equalization performance. Since this effect is not captured by the prediction metric, it results in poor correlation for these configurations.

6.6.4. Comparison with FRSS

Finally, we compare the performance of the proposed OTFS configurations with the four data rates of FRSS [1], which has recently been proposed as an extension to NATO standard STANAG 4748 [31]. The performance prediction methodology for FRSS has been studied extensively in [2]. For a fair comparison, both the FRSS and OTFS packets employ the same preamble structure and synchronization algorithm.

First, the four FRSS rates and all feasible OTFS configurations were replayed across all REP21 channel soundings. The FRSS payload length was fixed to 1024 information bits. This is the nearest available packet length that matches the mean of the OTFS packet lengths. For each sounding, the FRSS rate and the OTFS configuration yielding the lowest BER were selected. In cases where multiple rate settings and/or OTFS configurations achieved zero BER, the ones with the highest effective data rate were chosen. In Figure 6.10a, each vertical bar represents the mean packet delivery ratio (PDR) and standard deviation of a *rate or configuration*, averaged over the subset of soundings for which this rate/configuration was chosen; all points connected horizontally correspond to the full set of soundings. For OTFS, results were generated for constellation orders $m=2$ and $m=4$.

In terms of PDR, FRSS seems to outperform OTFS. Moreover, the highest effective bit rate of OTFS is only marginally larger than the bit rate of FRSS. However, the number of times a configuration is chosen should be taken into account. That is, FRSS with the highest effective data rate is used for only one sounding, whereas the highest effective rate of OTFS is utilized for 12 soundings. The difference becomes apparent in Fig. 6.10b where the total number of effective bits (i.e., the bits from error-free packets) transmitted per rate or configuration is shown. Clearly, in the REP21 channels, OTFS allows for error-free transmission of a larger net amount of effective bits than FRSS.

Currently, only reverb-limited replay simulations were performed. The effects of an increased peak-to-average power ratio (PAPR) and the potential reduction of the achievable SNR were not taken into account.

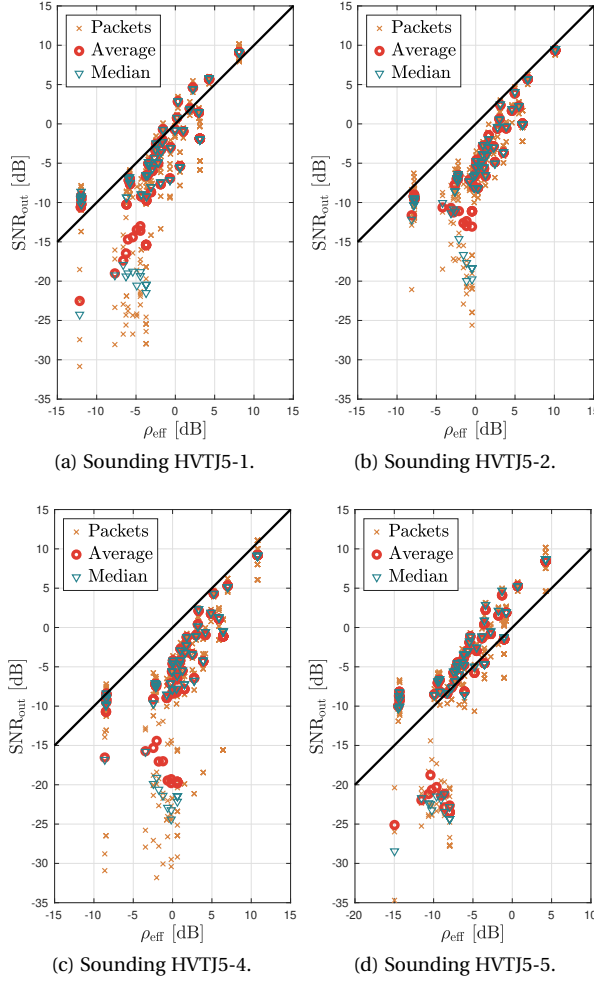


Figure 6.9.: Output SNR prediction for channel replay simulations. The third sounding has a frequency offset in its channel measurement (i.e. the probe), which might cause the offset.

6.7. Conclusion

This chapter presented an adaptive OTFS communication framework for underwater acoustic channels based on an analytically tractable effective SINR metric that explicitly accounts for the joint effects of multipath delay spread, Doppler spread, channel estimation error, and unmodeled channel components. Unlike conventional posterior performance metrics, the proposed effective SINR can be evaluated *a priori* from practically obtainable channel statistics and directly linked to system configuration parameters.

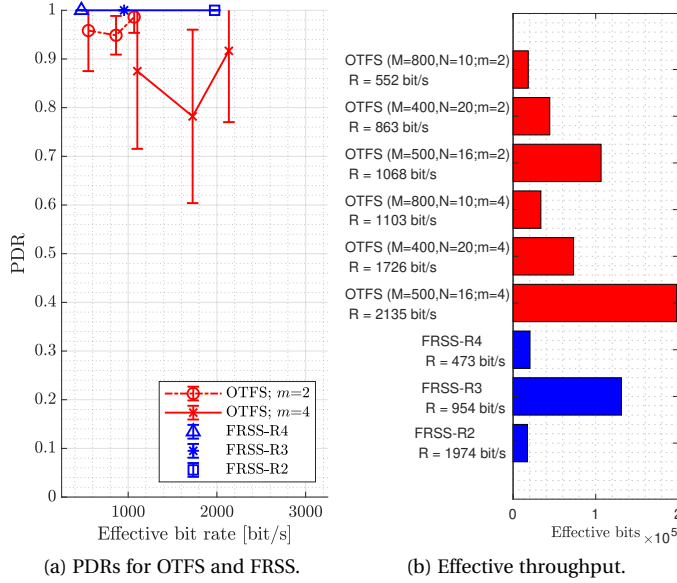


Figure 6.10.: Channel replay of OTFS and FRSS for the REP21 soundings.

We demonstrated, through both synthetic baseband simulations and measured channel replay experiments, that the effective SINR correlates strongly with the post-equalization output SNR. This validates its suitability as a predictive performance metric and enables principled optimization of OTFS parameters. Building on this insight, we proposed a simple yet practical adaptation protocol that configures the OTFS system in-situ using only information available at the receiver during normal operation. When compared to FRSS, the OTFS system achieves a higher effective throughput. These gains stem from both the increased configurability of OTFS and its superior robustness to Doppler effects.

Overall, this chapter demonstrates that predictive, model-based adaptation is feasible for practical underwater acoustic communications and that OTFS provides a flexible and powerful framework to exploit such adaptation.

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Haringvliet Estuary, Middelharnis, The Netherlands

7

Implementation and Experimental Validation

The previous chapters established the theoretical foundations of the proposed communication framework and evaluated its performance through synthetic and channel-replay simulations. This chapter shifts the focus to the practical implementation of the system and reports on its performance during real-world field experiments.

The author wishes to recognize the contributions of V.Y. Newsum (system integration), M. van Spellen and E. Brouns (field support), M. van Heteren (execution), K.C.H. Blom (system integration, supervision of the setup, execution and organization), H. Dol (organization and project management).

7.1. Introduction

While the previous chapters focused on the theoretical design and optimization of the underwater acoustic (UWA) communication pipeline, this chapter shifts attention to its practical realization. Specifically, it presents the integration into a hardware-based system, describing the experimental system architecture from both hardware and software perspectives rather than from an algorithmic or mathematical viewpoint. Furthermore, this chapter reports on a series of experiments conducted using this implementation, providing experimental validation of the developed approaches.

7.1.1. Contributions of this chapter

The main contributions of this chapter are twofold:

- We describe the experimental system architecture.
- We report on the experimental validation of the communication framework proposed in the preceding chapters.

7.2. System architecture

The experimental validation presented in this chapter is conducted using commercially available UWA communication hardware, augmented with custom software interfaces to enable full control of the experiment and data access.

The experiments employ Subnero acoustic modems from the M25M series [1], shown in Fig. 7.1, equipped with an embedded NVIDIA Jetson Orin Nano 4GB [2] co-processor. These modems operate as software-defined platforms, providing access to both the transmitted waveforms and the unprocessed received signals. This level of data transparency is essential for validating the algorithms discussed in the preceding chapters. Integration is facilitated by UnetStack, the software stack running on the modems. Using the Java application programming interface (API), all modem functionalities, including the physical layer parameters, can be controlled directly within MATLAB. Together with TNO, a small extension was built to support asynchronous buffering (with full control on the buffer size). Communication between the computer and the modem is established via an Ethernet connection over a local area network (LAN), enabling real-time interaction.

7.3. Basin experiments

In March 2025, a series of basin experiments was conducted to validate the proposed communication framework. This section provides an overview of the experimental setup and presents the results of these experiments.

7.3.1. Setup

The experiments were performed in the water basin at TNO¹, an octagonal tank with approximate dimensions of eight meters in both diameter and depth. The walls and

¹Location OWW, Oude Waalsdorperweg, The Hague, the Netherlands.



Figure 7.1.: Subnero modems from the Gen4X M25M series used in the experiments. Image courtesy of Subnero Pte Ltd, Singapore (reproduced with permission).

bottom are equipped with an anechoic lining designed to suppress acoustic reflections and approximate an infinite water column. This lining, shown in Fig. 7.2, consists of sound-absorbing pyramidal structures formed by stacked wedges made of cork elastomer, providing acoustic damping with a reflection loss of at least 10 dB over the frequency range 4 – 40 kHz.

During the experiment, two Subnero modems were deployed on opposite sides of the basin, at approximately 5 m distance. The setup was configured as a LAN, with both modems and their respective host computers connected via (cabled) Ethernet to a central switch. Each device was assigned a unique static internet protocol (IP) address within the subnet.

An overview of the experimental setup is illustrated in Fig. 7.3. Note that, while one modem is denoted as the transmit (Tx) and the other as the receive (Rx) modem, both are capable of transmission and reception.

The setup was used for two different experiments, which we will describe in the following.

7.3.2. Validating the effect of power distribution

In the first experiment, we evaluated the impact of the power distribution between data and pilot symbols, parameterized by $\alpha \in [0, 1]$, on the bit error ratio (BER).

The transmitted orthogonal time frequency space (OTFS) communication packets were transmitted at a carrier frequency $f_c = 29$ kHz and bandwidth $B = 16$ kHz. For the transmit pulse, we used a root-raised-cosine filter with a roll-off $\beta = 1/5$ resulting in a symbol rate $R_s = B/(1 + \beta) = 13.3$ kHz. At the start of the experiment, a channel sounding was performed using a train of linear frequency modulated (LFM) signals as probe. The received probe and estimated channel impulse response (CIR) in the delay-Doppler domain are shown in Fig. 7.4. Observe that the estimated CIR exhibits a non-zero Doppler spread, even though the measurements were conducted in a completely



Figure 7.2.: Section of the anechoic lining mounted on the walls and floor of the basin. The lining consists of sound-absorbing pyramidal structures formed by stacked cork elastomer wedges.

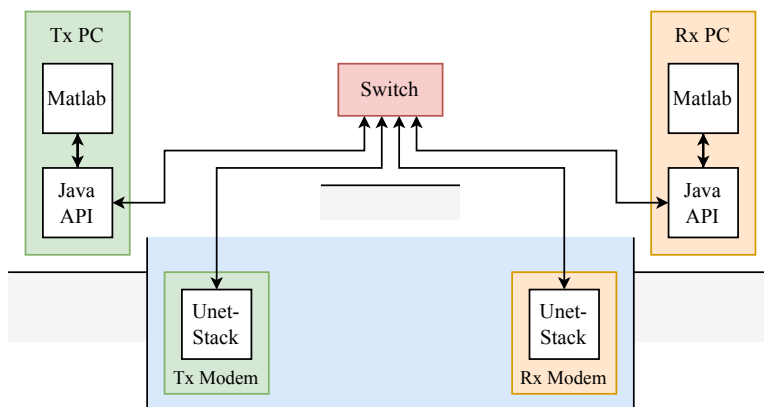


Figure 7.3.: Illustration of the setup for the basin experiments.

static environment. This widening of the Doppler spectrum is caused by a carrier frequency offset between the Tx and Rx modem, which is not fully compensated by the preamble-based synchronization.

Based on the estimated CIR, the OTFS parameters were set to $K = 6000$, $L = 74$

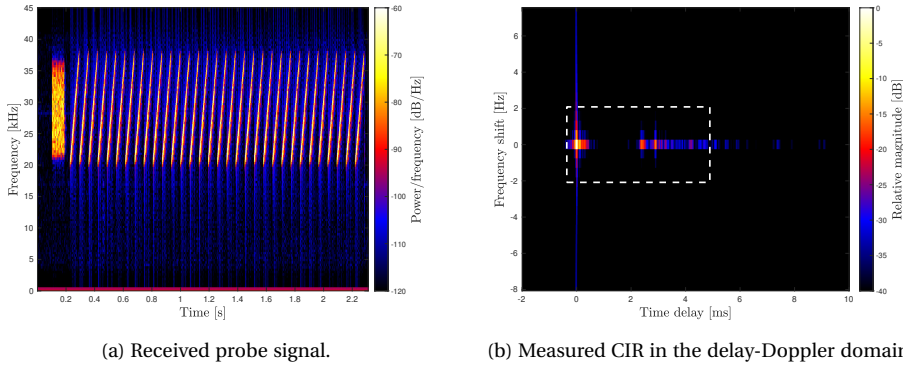


Figure 7.4.: Channel sounding prior to the power distribution experiment. (a) shows the received probe signal used to generate the CIR shown in (b). The horizontal and vertical dimension of the white dashed rectangle in (b) indicate the chosen values for L and Q .

and $Q = 2$. Thus, the duration of one communication packet, including cyclic prefix (CP) but excluding preamble, is about $T = (K + L)/R_s = 0.457$ s. The “delay slab” pilot allocation (cf. Chapter 4) was used, hence $M = L + 1 = 75$ and $N = K/M = 80$, resulting in $K_d = K - M(2Q + 1) = 5625$ data symbols. No coding was performed.

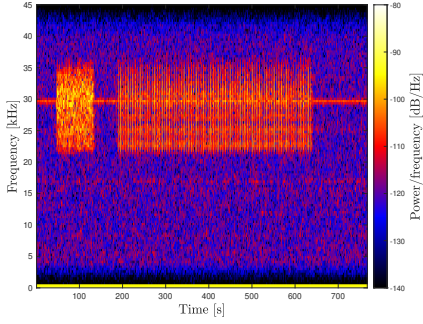
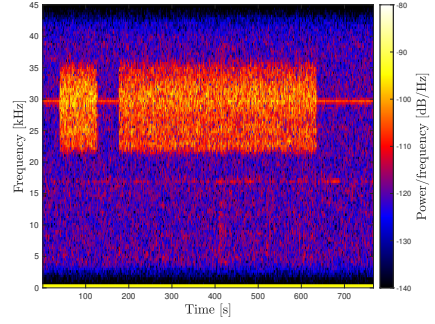
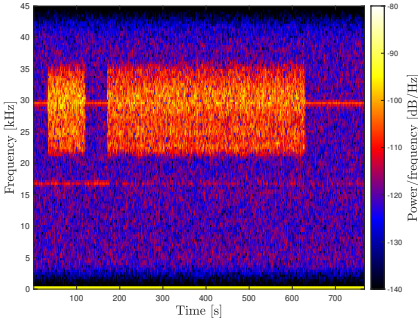
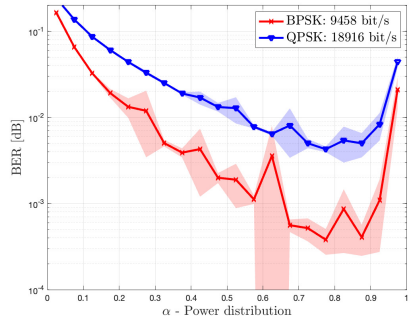
The communication packets were transmitted for $\alpha \in \{0.025, 0.075, \dots, 0.975\}$, and the procedure was repeated for a total of 85 packets and two different constellation sizes, $m = 2, 4$. The basin provides an ideal test-bed for this study, as its static environment ensures comparable conditions across the different α values.

Figs. 7.5a to 7.5c show three received OTFS packets corresponding to different power distributions. Note the difference between the received OTFS packet for $\alpha = 0.025$ in Fig. 7.5a and those for $\alpha = 0.775$ and $\alpha = 0.975$ in Figs. 7.5b and 7.5c, respectively; for smaller values of α , a larger fraction of the total energy is assigned to the pilot symbol, resulting in a waveform that looks like an impulse train. The resulting BER versus power distribution is shown in Fig. 7.5d. The graph shows a similar trend as for the synthetic channel simulations in Chapter 4 and the channel replay simulations in Chapter 5.

The tonal artifact at 30 kHz is an internal modem characteristic, likely originating from the power amplifier. It has a low power, independent of the transmission power; however, the artifact becomes prominent in these plots because the signal of interest was also transmitted at low power. This required the spectrogram’s color scale to be normalized to a lower range compared to Fig. 7.4a, making the artifact more visible.

7.3.3. Demonstration of the protocol

The primary goal of the second experiment was to implement a low-TRL (technology readiness level) demonstrator capable of facilitating reliable text-based data transmission between two modems. The protocol for this experiment closely followed the protocol

(a) Received OTFS packet with $\alpha = 0.025$.(b) Received OTFS packet with $\alpha = 0.775$.(c) Received OTFS packet with $\alpha = 0.975$.

(d) BER versus power distribution between data and pilot symbols.

Figure 7.5.: Results of the power distribution experiment. (a),(b) and (c) show received OTFS packets for different power distributions, (d) shows the sample mean and 95% confidence interval of the BER for 85 packets per power distribution value. The received SNR = 12.1 dB. The tonal around 30kHz in (a),(b) and (c) is an artifact from the modem. It becomes apparent since the signal of interest has low energy.

established in Section 6.5.3.

Protocol and signal description

In this setup, the transmitter initiates the communication link by emitting a probe signal. Upon reception and detection of the probe signal, the receiver performs channel estimation in the delay-Doppler domain to obtain $\hat{c}_{l,q}$. Using this estimate, the receiver determines the optimal modulation parameters for the current channel conditions. This configuration is then fed back to the transmitter using an OTFS packet with robust, low-data-rate settings. The transmitter then begins streaming OTFS packets using the optimized settings. The major difference from the protocol described in Section 6.5.3

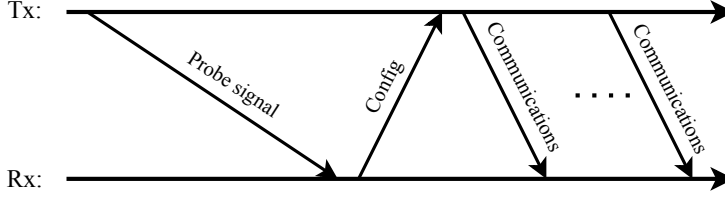


Figure 7.6.: Overview of the protocol.

lies in the criteria used for parameter selection, which is detailed in Section 7.3.3. The protocol is illustrated in Fig. 7.6, the specific logic for the Tx and Rx states is formalized using a finite state machine (FSM) in Chapter F.

All signals of the protocol operate within the same frequency band, with $f_c = 29\text{kHz}$, $B = 16\text{kHz}$, and are prefixed with a preamble to facilitate detection and synchronization. The preamble is a single-carrier modulation (SCM) waveform with binary phase-shift keying (BPSK) symbols generated from a maximum length sequence (m-sequence) of length $L_{\text{seq}} = 2^{10} - 1 = 1023$. To mitigate interference, the preamble and the subsequent signal are separated by a guard interval of $T_{\text{spacer}} = 40\text{ms}$. The configuration and communication signals use a root-raised-cosine filter with a roll-off factor of $\beta = 1/5$ as pulse shape, resulting in a symbol rate of $R_s = 13.3\text{kHz}$. The probe signal consists of a train of $N_{\text{lffm}} = 32$ LFM signals. One LFM signal has a duration of $T_{\text{lffm}} = 64\text{ms}$. The feedback/configuration signal is encapsulated in an OTFS packet configured with conservative, high-reliability parameters: $(N, M, L, Q, \alpha, m) = (10, 600, 599, 4, 0.6, 2)$. The “delay slab” pilot allocation was used, resulting in $K_d = K - M(2Q + 1) = 600$ data symbols. These data symbols carry the configuration settings encoded using JavaScript Object Notation (JSON). The data stream consists of OTFS packets. Some of the parameters are fixed, i.e., $K = 6000$, $\alpha = 0.6$ and $m = 4$, while the remaining OTFS parameters (N, M, L, Q) are chosen based on the configuration signal. As a result, the effective data rate and exact packet duration are flexible and depend on the measured channel. The data symbols of these packets contained text messages that were input by the user at the Tx modem.

Parameter selection heuristic

Since the experimental trials were conducted prior to the development of the analytical framework in Chapter 6, the parameter selection was done using an energy-based heuristic.

The selection of the OTFS parameters (N, M, L, Q) is driven by the estimated delay-Doppler channel taps, denoted by the set $\{\hat{c}_{l,q}\}$ for $l \in \{0, \dots, L_{\text{max}}\}$ and $q \in \{-Q_{\text{max}}/2, \dots, Q_{\text{max}}/2\}$. Note that L_{max} and Q_{max} are determined by the settings of the LFM signal in the probe signal. As established in Chapter 4, the parameters (N, M) can be derived as a function of the delay and Doppler spread (L, Q) . Consequently, the selection problem reduces to identifying an (L, Q) pair that sufficiently encapsulates the channel’s observed delay and Doppler spreads. The procedure for determining these values is

detailed below.

First, the channel tap energy is aggregated across the delay and Doppler dimensions, respectively:

$$P_{\text{delay}}(l) = \sum_{q=-Q_{\max}/2}^{Q_{\max}/2} |\hat{c}[l, q]|^2, \quad (7.1)$$

$$P_{\text{doppler}}(q) = \sum_{l=0}^{L_{\max}} |\hat{c}[l, q]|^2. \quad (7.2)$$

Then, we obtain L and Q by solving the following problems,

$$L = \min_{L_1, L_2} L_2 - L_1 \quad (7.3)$$

$$\text{s.t. } \frac{\sum_{l=L_1}^{L_2} P_{\text{delay}}(l)}{\sum_{l=0}^{L_{\max}} P_{\text{delay}}(l)} \geq \gamma, \quad (7.4)$$

$$L_1 < L_2 < L_{\max}, \quad (7.5)$$

and

$$Q = \min_{Q_1, Q_2} Q_2 - Q_1 \quad (7.6)$$

$$\text{s.t. } \frac{\sum_{q=Q_1}^{Q_2} P_{\text{doppler}}(q)}{\sum_{q=-Q_{\max}/2}^{Q_{\max}/2} P_{\text{doppler}}(q)} \geq \gamma, \quad (7.7)$$

$$-Q_{\max}/2 \leq Q_1 \leq Q_2 \leq Q_{\max}/2. \quad (7.8)$$

Here $\gamma \in (0, 1)$ denotes the portion of the total channel energy that one approximately captures with channel estimation, and is a hyperparameter.

Results

The adaptability of the proposed protocol was evaluated under two distinct experimental settings. In the first setting, both the transceivers and the environment remained stationary to establish a baseline. In the second setting, a dynamic scenario was created by physically swinging the Tx modem.

The estimated CIRs for both scenarios are illustrated in Fig. 7.7, where the horizontal and vertical dimensions of the white dashed rectangles represent the selected L and Q , respectively. The results demonstrate that the protocol effectively adapts to the instantaneous channel conditions: in the stationary case, a low Q is selected, whereas the dynamic scenario triggers an increase in Q to compensate for the broadened Doppler spread.

While the increased Q in the dynamic scenario inherently leads to a reduction in the effective data rate, the protocol achieved its primary objective in both cases. Specifically, all signals, the low-data-rate and the optimized OTFS packets, could be received error-free.

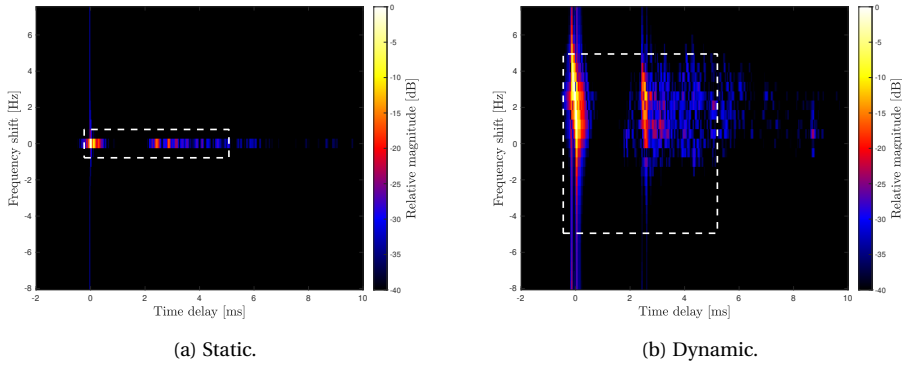


Figure 7.7.: CIRs in the delay-Doppler domain, estimated from probe signals during execution of the protocol. (a) measured while the Tx and Rx modem were stationary, (b) measured while the Tx modem was moving. The white dashed rectangles indicate the chosen values of L and Q (from which the configuration follows), using $\gamma = 0.95$.

7.4. Open-water experiments

In May 2025, a series of open-water trials was conducted to evaluate the performance of the proposed communication framework in a real-world environment. While the basin experiments provided a controlled baseline, the open-water trial aimed to test the robustness of the OTFS signaling and the link-adaptation protocol under representative field conditions.

7.4.1. Setup

The open-water trials were carried out in the Haringvliet estuary, a significant former inlet of the North Sea located south of Rotterdam, the Netherlands. The estuary spans approximately 28km in length with a width varying between 800m and 3km. The bathymetry of the estuary features an average depth of 9 m, with local trenches reaching a maximum depth of 39 m.

Similar to the basin trials, two Subnero modems were utilized: the Tx modem was towed by a small vessel, while the Rx modem was mounted on a jetty.² The distance between the modems varied between 50m and 500m. Unlike the setup in the basin, both modems were controlled by a single MATLAB instance on a central computer. To integrate the remote Tx modem into the LAN without a physical cable, two Silvus StreamCaster 4000 series radios were employed to provide a wireless Ethernet bridge. This configuration maintained the static IP addressing scheme across the wireless hop, allowing the central computer to interact with the mobile modem as if it were directly interfaced with the switch on the shore. The setup is illustrated in Fig. 7.8.

²Here, again, the Tx and Rx labels denote the role of the modems in the protocol. Both modems are capable of transmission and reception.

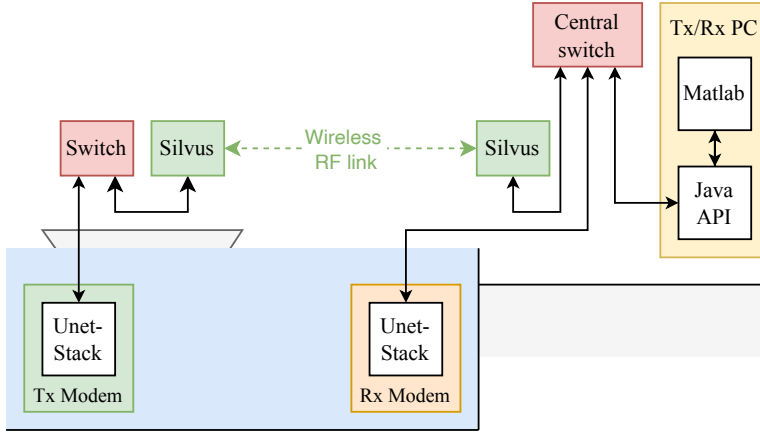


Figure 7.8.: Illustration of the setup for the open-water experiments.

The open-water trial encompassed multiple scenarios, including both stationary and dynamic configurations across varying ranges. A representative dynamic case is illustrated in Fig. 7.9, where the vessel towed the Tx modem at approximately 1.5 m s^{-1} at a distance of 200 m from the Rx modem. The distance between the Tx and Rx modems decreases during the first half of the vessel's trajectory and increases during the second half.

7.4.2. Performance of the protocol

The protocol, as visualized in Fig. 7.6 and detailed in the previous section, was evaluated across various configurations to assess its robustness in the estuary environment. This subsection briefly presents one execution of the protocol for the dynamic scenario depicted in Fig. 7.9. A discussion of the specific challenges encountered during the trials follows in the subsequent section.

The probe signal consists of a train of 32 LFM signals, each 32 ms in duration, resulting in a total signal length (including preamble) of approximately 1.1 s. The received waveform is shown in Fig. 7.10a. From the received probe signal, the CIR was estimated (Fig. 7.10b), leading to the selection of $L = 249$ and $Q = 12$. These parameters are indicated by the white dashed rectangle in Fig. 7.10b.

Both the configuration message and the subsequent data stream were transmitted as OTFS packets with $K = 8000$, $\alpha = 0.6$, and BPSK modulation ($m = 2$). Upon the successful decoding of the configuration message, the Tx modem initiated the data stream using the optimized parameters. Fig. 7.11 shows the received configuration signal and the first packet of the data stream.

Based on the selected L and Q , a “delay-slab” pilot allocation was implemented with the parameters $(N, M, L, Q) = (32, 250, 249, 12)$. This configuration resulted in $K_d = 1750$ available data symbols. With a code rate of $1/2$, each packet carried 875 information bits, yielding an effective throughput of 1155 bits s^{-1} (including preamble overhead).

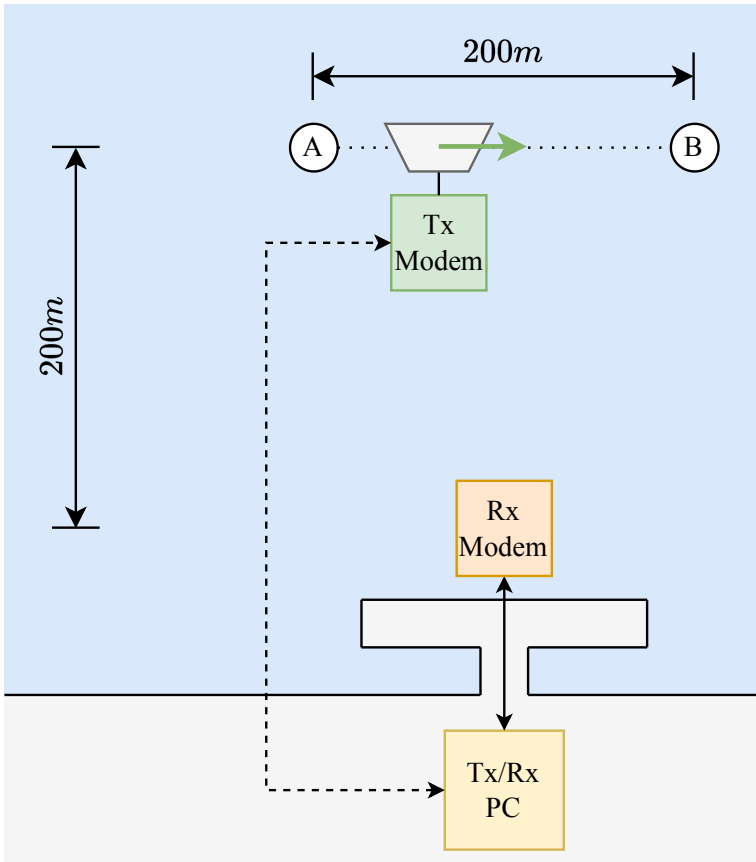


Figure 7.9.: Illustration (top view) of a dynamic scenario in which the Tx modem is towed by a vessel in a straight line from point A to point B, parallel to the shore and at a distance of 200 m from the Rx modem.

In the minutes following the initial probe and configuration signals, 17 OTFS packets were transmitted. The resulting BER for each packet is presented in Fig. 7.12. We observe an upward trend in bit errors as time progresses; this is attributed to the inherent non-stationarity of the acoustic channel, causing the initial configuration to become outdated. The channel coherence time, relative to the chosen modulation parameters, is limited, necessitating a restart of the protocol to maintain a reliable communication link.

7.5. Discussion of limitations and challenges

During the basin and open-water experiments, several challenges emerged.

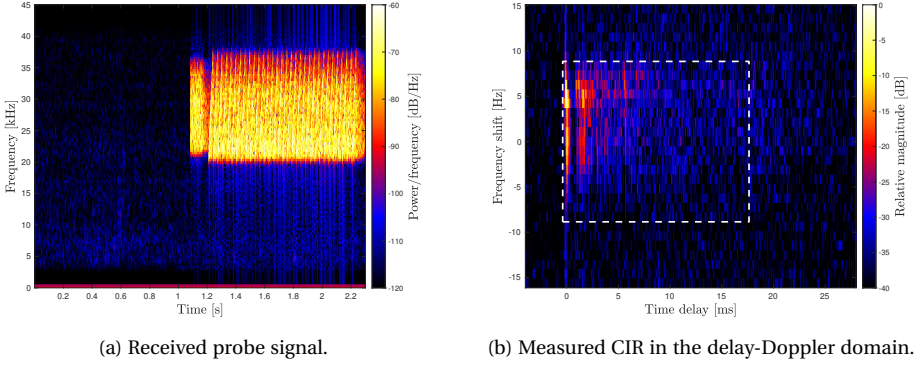


Figure 7.10.: Received probe signal and measured CIR during execution of the protocol in a dynamic scenario (as visualized in Fig. 7.9). The horizontal and vertical dimensions of the white dashed rectangle in (b) indicate the chosen values for L and Q .

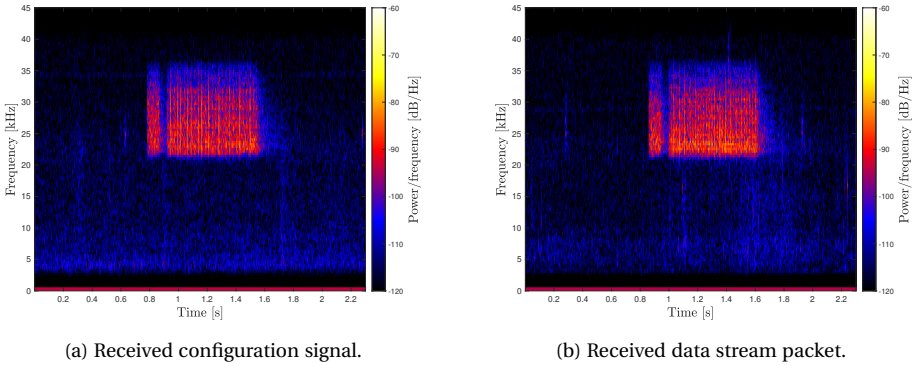


Figure 7.11.: Received communication signals during execution of the protocol at the Haringvliet estuary. (a) shows the received configuration message, (b) shows the first received data stream packet.

7.5.1. Overspread channels

A primary issue encountered during the open-water trials was the occurrence of *overspread* channels. A channel is formally considered overspread when the product of the maximum time delay $\tau_{\max}$ and Doppler shift $\nu_{\max}$ exceeds the limits of identifiability, such that:

$$2\tau_{\max}\nu_{\max} > 1. \quad (7.9)$$

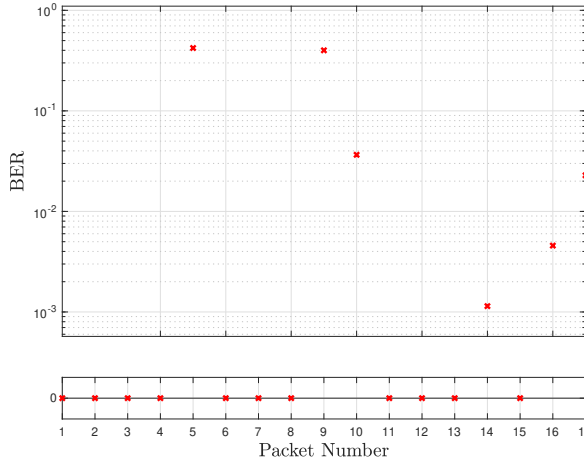


Figure 7.12.: BER of the data stream packets during the execution of the protocol at the Haringvliet estuary.

In these scenarios, it becomes theoretically impossible to uniquely resolve the channel taps, posing a fundamental limitation to system configuration via channel estimation.

More generally, the use of LFM signals for channel sounding involves an inherent resolution trade-off. Let T denote the duration of a single LFM signal in the train of LFM signals. To accurately capture the delay spread, the pulse duration must satisfy $T \geq \tau_{\max}$ to avoid interference between pulses. Conversely, to avoid aliasing in the Doppler dimension, T must be sufficiently small relative to the Doppler spread (typically $T \leq 1/(2v_{\max})$).

This introduces a paradox: to successfully estimate the channel and initialize the adaptive protocol, the operator must pre-configure T . However, the optimal value of T depends on the environmental delay and Doppler spreads, which are the very parameters the system aims to measure. Currently, this requires a-priori knowledge or manual tuning before the protocol begins. To mitigate this in future work, replacing the LFM train with a pseudo-random noise (PRN) sequence would be advantageous; such signals allow the receiver to perform correlation over flexible windows, providing online control of the delay-Doppler resolution trade-off during post-processing.

7.5.2. Peak-to-average power ratio (PAPR) of the transmitted signal

The optimization framework presented in this dissertation focuses on distributing energy between pilot and data symbols to maximize throughput. However, this energy-centric approach does not explicitly constrain the PAPR of the resulting waveform. Because the pilot signal is effectively an impulse train, assigning a significant fraction of the total power to the pilot symbol results in high-power peaks.

This characteristic introduced practical challenges during the open-water trials. To

avoid clipping, the transmit power had to be reduced. Consequently, the OTFS packets were transmitted with lower average energy than theoretically intended, leading to a significantly reduced signal-to-noise ratio (SNR). This mainly affects the performance at longer ranges. These findings suggest that future iterations of the optimization framework should include PAPR constraints or use peak-shaping techniques to better align the theoretical power distribution gains with the physical limitations of the hardware. A more detailed analysis of the PAPR and potential solutions for the proposed framework is provided in Chapter 8.

7.5.3. Parameter selection

At the time of the basin and open-water experiments, the comprehensive optimization framework detailed in Chapter 6 was not yet fully developed. Instead, the selection of the parameters L and Q relied on an energy-based heuristic, as described earlier, and the constellation size m remained fixed.

During the open-water trials, it became evident that this approach lacked generalization. Both the constellation order m and the heuristic hyperparameter γ required manual tuning to adapt to varying channel conditions. This dependence on empirical adjustment proved sub-optimal for a truly autonomous adaptive system. These observations directly motivated the development of the more robust and general framework presented in Chapter 6, which replaces manual heuristics with a rigorous analytical model for parameter optimization.

7.6. Conclusion

This chapter has detailed the transition of the proposed communication framework from theoretical design to practical hardware implementation. Through the development of a custom software interface for software-defined acoustic modems, the OTFS-based link-adaptation protocol was validated in both controlled basin environments and challenging open-water conditions in the Haringvliet estuary.

The experimental results confirmed that the proposed framework can effectively adapt to the time-varying UWA channel. Specifically, the basin experiments validated the theoretical relationship between pilot-data power distribution and bit-error performance in practice. Additionally, both the basin and the open-water trials demonstrated the feasibility of an adaptive OTFS system.

However, the field trials also highlighted critical practical constraints, most notably the challenges of overspread channel scenarios and the impact of high PAPR on effective transmit power. These real-world limitations provided the necessary empirical motivation for the more rigorous analytical optimization framework developed in Chapter 6. Ultimately, the successful error-free transmission of data in several different conditions confirms the potential of the developed framework for robust, high-performance UWA communication.

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8

Concluding Remarks

In Chapter 1, three main research questions were formulated to guide this dissertation. This chapter revisits these questions, collecting the theoretical and experimental findings presented throughout the dissertation to provide comprehensive answers. Furthermore, we identify some of the remaining challenges and discuss promising directions for future research.

8.1. Answers to research questions

In this section, we summarize the contributions of this dissertation by addressing the three main research questions. Collectively, these answers establish a comprehensive framework for maximizing throughput in underwater acoustic (UWA) channels through optimized pilot design and channel-aware adaptation.

First research question

The first research question we posed was:

RQ1 *How do state-of-the-art modulation schemes perform under time-varying channels?*

In Chapter 3, it was demonstrated that modulation schemes dictate how transmitted symbols are spread across time-frequency. A key finding is that, although derived from disparate principles, vector orthogonal frequency division multiplexing (OFDM) (V-OFDM), orthogonal sequence division multiplexing (OSDM), and orthogonal time frequency space (OTFS) are equivalent.

The primary distinction of OTFS to single-carrier modulation (SCM) and OFDM is that OTFS spreads symbols across both time and frequency, in contrast to SCM and OFDM, which localize symbols in time (spreading in frequency) and frequency (spreading in time), respectively.

Some works characterize OTFS as inherently delay- and Doppler-resistant [1], [2], which is based on the “bi-orthogonality” property of the transmit and receive pulses. In practice, this property cannot be strictly satisfied in doubly-selective channels [3]. Furthermore, we have shown that OTFS modulation is not inherently immune to the joint effects of delay and Doppler, specifically when the channel consists of multiple paths with distinct Doppler shifts. Under these conditions, the transmitted symbols inevitably experience interference.

Consequently, the claim that OTFS is more robust than OFDM against Doppler shifts must be interpreted with caution. The potential performance gains of OTFS are not automatic; they are conditional on the implementation of the right receiver processing to resolve the induced interference. This perspective is supported by recent findings [4] suggesting that the performance gap between OFDM and OTFS nearly vanishes when appropriate coding is applied to spread symbols across the time-frequency grid. This reinforces the conclusion that the primary advantage of OTFS stems from its inherent time-frequency spreading rather than a fundamental immunity to delay and Doppler spread.

Second research question

The second research question we posed was:

RQ2 *Given channel statistics, how can the transmitted signal be optimized to maximize throughput?*

As detailed in Chapter 1, the UWA communication channel introduces severe distortion to the transmitted signal. This necessitates equalization at the receiver, a process typically driven by a channel estimate. To facilitate this, the transmitted waveform must include

a known pilot sequence. Although blind equalization (i.e., channel estimation and equalization without pilot signals) presents an alternative methodology, it is outside the scope of this work. The design of the transmitted signal therefore depends on the allocation of pilot and data signals, as well as the power distribution between them. To this end, the second research question is partitioned into two sub-questions:

RQ2.1 *Which pilot allocation strategies minimize overhead while ensuring accurate channel estimation performance?*

RQ2.2 *How should power be distributed between pilot and data signals to maximize throughput?*

In Chapter 4, we address the minimization of pilot overhead. We restrict our analysis to designs where pilot and data symbols do not overlap. Our investigation revealed that the state-of-the-art pilot design [5], [6], referred to here as the “island case”, incurs a significantly higher overhead than the optimal allocations established for SCM and OFDM [7], [8], [9]. Since OTFS generalizes both SCM and OFDM, this discrepancy indicated that the island case is theoretically suboptimal. We then proposed two novel allocation strategies: the “delay slab” and “Doppler slab”. By adjusting the OTFS parameters M and N according to the channel delay and Doppler spreads, these designs reduce pilot overhead by approximately 50% compared to the conventional island case, while preserving minimum mean squared error (MMSE) channel estimation performance.

Regarding the second sub-question, Chapter 4 further addressed the optimal distribution of power between pilot and data signals. While increasing pilot power improves channel estimation accuracy, it simultaneously reduces the energy available for data transmission, creating a fundamental trade-off. We demonstrated that by selecting an appropriate power distribution, specifically by optimizing this distribution with respect to a lower bound on channel capacity, the overall system performance is significantly enhanced. The optimal power distribution depends on the channel through the (distribution of the) channel taps in the delay-Doppler domain and the noise level. Our results in Chapter 4 indicate that across various synthetic channels, this capacity-optimized power distribution not only increases the achievable rate but also empirically minimizes the bit error ratio (BER).

To bridge the gap between theory and practice, we further validated these findings in Chapter 5 using channel replay simulations. In these scenarios, the idealized stochastic assumptions regarding channel tap distributions often do not hold. Despite the presence of these non-idealities, the results in Chapter 5 demonstrate close agreement with the theoretically optimal power distribution.

Third research question

The insights gained from the second research question established that the design of the transmit signal is intrinsically related to the channel conditions. However, a significant limitation of the analysis in Chapter 4 is the assumption of finite delay and Doppler support. In practical UWA scenarios, channels often exhibit extended delay-Doppler profiles with slowly decaying tails, effectively “infinite” responses, that complicate direct

application of the results in Chapter 4. Consequently, we posed the final research question:

RQ3 *How can the communication system be adaptively optimized in real time to maximize throughput under changing channel conditions?*

To address this, in Chapter 6, we extended the analysis of Chapter 4 to accommodate channels with extended delay-Doppler profiles, explicitly accounting for channel estimation errors and unmodeled channel components. This led to an adaptive, channel-aware OTFS framework centered on an analytically tractable effective signal-to-interference-plus-noise ratio (SINR) metric. Crucially, unlike conventional metrics that evaluate performance *a posteriori*, this metric enables the *a priori* prediction of system performance using practically obtainable second-order channel statistics.

We validated this framework through both synthetic simulations and measured channel replay experiments, revealing a strong correlation between the predictive SINR and the actual post-equalization output signal-to-noise ratio (SNR). This facilitated the development of a practical, in-situ adaptation protocol that optimizes OTFS parameters in real time using only receiver-side information. Chapter 7 discussed the implementation details and corroborated that the proposed protocol is feasible in practice.

Consequently, we conclude that channel-aware adaptation is feasible for practical UWA communications and that OTFS provides a flexible and powerful framework to exploit such adaptation.

8.2. Future research directions

This dissertation has demonstrated the efficacy of OTFS modulation for UWA communication, particularly through novel pilot designs and an adaptive strategy. Although the proposed framework offers performance gains over traditional schemes, several practical challenges and opportunities remain for further research. This section outlines three key areas where future research could extend the applicability and efficiency of the system: i) the analysis and mitigation of the peak-to-average power ratio (PAPR), ii) the extension to multi-receiver architectures, and iii) the development of adaptive resampling techniques for high-mobility scenarios.

8.2.1. Analysis and mitigation of the peak-to-average power ratio (PAPR)

In practical wireless systems, the PAPR is a critical metric because it determines the efficiency of the power amplifier. When the PAPR is high, the power amplifier must be operated with a lower average output power to prevent clipping, which would otherwise lead to out-of-band emissions. The pilot design introduced in this dissertation can exhibit a high PAPR. Here, we briefly address this issue and outline potential remedies.

Let $x_{\text{pb}}(t)$ denote the transmitted passband signal over the time interval $t \in [0, T]$. The PAPR is defined as the ratio of the peak instantaneous power to the average power:

$$\text{PAPR}\{x_{\text{pb}}(t)\} = \frac{\max_{t \in [0, T]} |x_{\text{pb}}(t)|^2}{\frac{1}{T} \int_0^T |x_{\text{pb}}(t)|^2 dt}. \quad (8.1)$$

To evaluate the PAPR across different modulation schemes, we consider the relationship between the passband signal and its normalized baseband representation:

$$x_{\text{pb}}(t) = \text{Re} \left\{ x_{\text{bb}}(t) e^{j2\pi f_c t} \right\} = \text{Re} \left\{ \sum_{k=0}^{K-1} x[k] g(t - kT_{\text{symp}}) e^{j2\pi f_c t} \right\}, \quad (8.2)$$

where f_c is the carrier frequency, $g(t)$ is the pulse-shaping filter and T_{symp} is the symbol duration. By stacking K transmitted symbols into a vector $\mathbf{s} = [s[0], \dots, s[K-1]]^\top$, the normalized baseband symbol vector $\mathbf{x} = [x[0], \dots, x[K-1]]^\top$ for SCM, OFDM, and OTFS can be expressed as

$$\mathbf{x}_{\text{scm}} = \mathbf{I}_K \mathbf{s}, \quad (8.3)$$

$$\mathbf{x}_{\text{ofdm}} = \mathbf{F}_K^H \mathbf{s}, \quad (8.4)$$

$$\mathbf{x}_{\text{otfs}} = (\mathbf{F}_N^H \otimes \mathbf{I}_M) \mathbf{s}, \quad (8.5)$$

where $K = NM$.

The PAPR of an OTFS waveform has been shown to increase with the parameter N [10]. Since OTFS coincides with SCM when $N = 1$ and with OFDM when $N = K$, its PAPR performance inherently lies between these two extremes. This relationship is validated in Fig. 8.1, which illustrates the PAPR distribution, represented as a complementary cumulative distribution function (CCDF), obtained from 5000 Monte Carlo iterations. The results confirm that OTFS offers a more favorable PAPR profile compared to OFDM. It should be noted, however, that the results in Fig. 8.1 assume that all entries in $\mathbf{s}$ are equal-power quadrature phase-shift keying (QPSK) symbols. Specific pilot designs, such as the ones discussed in Chapter 4, typically increase the PAPR.

Recall that, in the proposed pilot design, the transmitted symbol vector $\mathbf{s}$ consists of data and pilot symbols, $\mathbf{s}_d \in \mathbb{C}^{K_d}$ and $\mathbf{s}_p \in \mathbb{C}^{K_p}$, respectively, with $K = K_d + K_p$, such that

$$\mathbf{s} = \sqrt{\alpha P} \Phi_d \mathbf{s}_d + \sqrt{(1-\alpha)P} \Phi_p \mathbf{s}_p, \quad (8.6)$$

where the selection matrices Φ_d and Φ_p define the allocation in the delay-Doppler domain. The power allocation parameter $\alpha \in [0, 1]$ influences the waveform's PAPR. Decreasing α shifts more power toward the pilot signal. Because the impulse-like pilot involves only a single non-zero entry in the delay-Doppler domain, this concentration of energy leads to a significant increase in the PAPR, as shown in Fig. 8.2.

A potential remedy is to use a pilot signal with constant-envelope properties, such as Zadoff-Chu (ZC) sequences. However, this introduces a trade-off: while such sequences lower the PAPR, they deviate from the impulsive pilot structure required to achieve the theoretical minimum mean squared error (MSE) in channel estimation. Future research could explore the optimization of these sequences to balance PAPR with channel estimation performance.

8.2.2. Extension to multiple receivers

In this dissertation, only single-input single-output (SISO) systems, utilizing a single transmit and receive antenna, were considered. However, the communication system

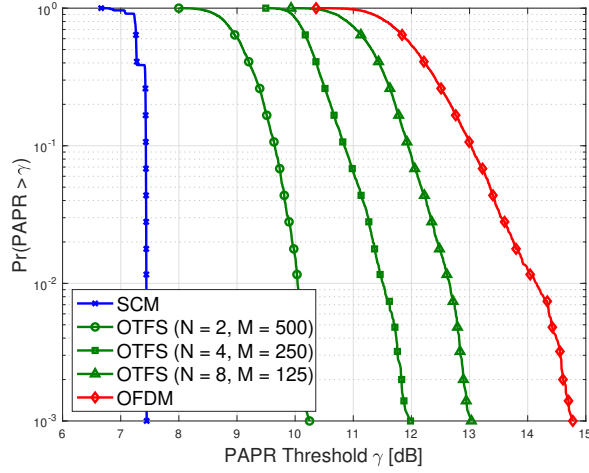


Figure 8.1.: Empirical CCDF of the PAPR for SCM, OFDM, and OTFS waveforms. Results are obtained for $K = 1000$ QPSK symbols across 5000 Monte Carlo iterations.

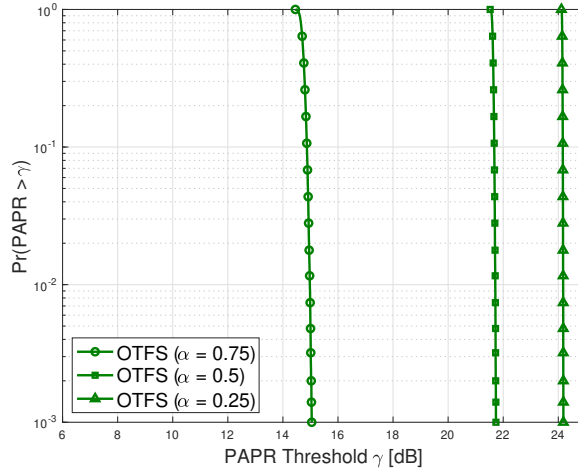


Figure 8.2.: Empirical CCDF of the PAPR for OTFS waveforms including pilot design. Results are obtained for $K = 1000$ across 5000 Monte Carlo iterations and three different power distributions.

could benefit significantly from the integration of an array of transmit and/or receive antennas. The primary advantages of such configurations include the ability to direct transmission energy along optimal propagation paths and to employ spatial filtering at the receiver, thereby increasing the effective SNR.

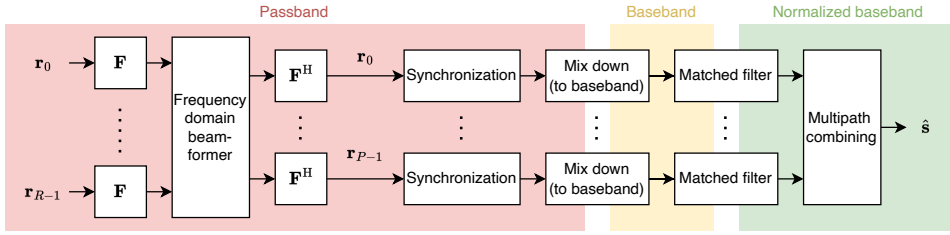


Figure 8.3.: SIMO-OTFS receiver architecture illustrating spatial filtering (in the frequency domain) and path combination.

In the context of UWA communications, both multi-transmitter (i.e., multiple-input single-output (MISO)) [11] and multi-receiver (i.e., single-input multiple-output (SIMO)) [12], [13] architectures have been proposed for SCM and OFDM systems. To fully exploit these gains, path angles (direction-of-arrival (DoA) and direction-of-departure (DoD)) must be known or estimated. In practice, receiver-side processing is often preferred, as estimating the DoA at the receiver is generally more feasible than acquiring precise DoD information at the transmitter.

A promising direction for future work is therefore the extension of the proposed OTFS framework to a multiple-receiver configuration. Consider a scenario with P propagation paths impinging on the receive array from P distinct angles. Given the wideband nature of the signal, these paths can be separated via spatial filtering implemented as a frequency-domain beamformer. A key advantage of this separation is that the delay and Doppler spreads of an isolated path are significantly smaller than those of the composite channel observed by a single receiver. Since the pilot design proposed in this dissertation relies directly on these spreads (i.e., through L and Q), a SIMO-OTFS architecture would enable a substantial reduction in pilot overhead. Fig. 8.3 illustrates the general architecture of the envisioned SIMO-OTFS system. The receiver has an array of R antennas. The P paths are separated using frequency-domain beamforming, after which the signals are synchronized on a per-path basis. Each individual “path signal” is then converted to normalized baseband, after which the discrete paths are combined for joint processing.

8.2.3. Adaptive resampling for block-based transmissions

The most straightforward method for compensating for motion-induced Doppler scaling is to resample the received signal at the packet level, an approach adopted within the framework of this dissertation. This method assumes that the (relative) platform velocity remains approximately constant over the duration of the packet. However, for long-duration packets or in high-dynamics scenarios, this assumption often fails to hold.

To address time-varying Doppler scales, several works in UWA communications have employed *adaptive* resampling techniques [14], [15], [16], [17], typically implemented via linear interpolation. These existing methods utilize SCM and rely on equalizer structures that operate on a symbol-by-symbol basis, such as the decision feedback equalizer (DFE). This symbol-level granularity allows the receiver to adapt the resampling rate

dynamically as the packet progresses.

Extending these adaptive resampling techniques to block-based transmissions, such as the OTFS framework proposed here, presents a significant challenge. Unlike SCM, where equalization is local in time, OTFS spreads information across the entire time-frequency grid, making it difficult to apply a time-varying resampling rate.

A direction for future work lies in exploiting the *cyclostationary* features inherent in communication signals [18], [19], [20], which arise from periodic operations such as pulse shaping. Doppler scaling manifests as a time-varying shift in the signal's cyclic frequencies (e.g., the symbol rate). By tracking the evolution of these cyclic frequencies over the duration of the packet, the receiver could potentially estimate the instantaneous Doppler scale. This estimate can then drive an adaptive resampling process for block-based transmissions.

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A

Orthogonality of sequences of V-OFDM/OSDM/OTFS

In this appendix we will show that the sequences of the orthogonal sequence division multiplexing (OSDM) signal have zero periodic cross-correlation. Let the sequences be defined by $(\mathbf{f}_n^* \otimes \mathbf{s}_n)$, and let $\mathbf{P}_K$ be a cyclic permutation matrix of size $K = NM$.

First, note that the columns of a (I)DFT matrix are orthogonal, $\mathbf{f}_{n_1}^{*H} \mathbf{f}_{n_2}^* = 0$, if $n_1 \neq n_2$, and remain orthogonal also after a cyclic permutation, thus we have,

$$\mathbf{f}_{n_1}^{*H} \mathbf{P}_N^k \mathbf{f}_{n_2}^* = 0, \quad n_1 \neq n_2, \forall k \in \{0, 1, \dots, K-1\}. \quad (\text{A.1})$$

Then, note that the sequences are orthogonal, that is,

$$\begin{aligned} (\mathbf{f}_{n_1}^* \otimes \mathbf{s}_{n_1})^H (\mathbf{f}_{n_2}^* \otimes \mathbf{s}_{n_2}) &= (\mathbf{f}_{n_1}^{*H} \mathbf{f}_{n_2}^*) \otimes (\mathbf{s}_{n_1}^H \mathbf{s}_{n_2}) \\ &= 0 \otimes (\mathbf{s}_{n_1}^H \mathbf{s}_{n_2}) = 0, \quad n_1 \neq n_2. \end{aligned} \quad (\text{A.2})$$

We can rewrite the cyclic permutation matrix, $\mathbf{P}_K = \mathbf{I}_N \otimes \mathbf{L}_M + \mathbf{P}_N \otimes \mathbf{U}_M$, where $\mathbf{L}_M$ is a cyclic permutation matrix of size M except for the top right element, which is zero, and $\mathbf{U}_M$ is a zero matrix of size M except for the top right element, which is one (thus $\mathbf{P}_M = \mathbf{L}_M + \mathbf{U}_M$). For the sequences we can then show that,

$$\begin{aligned} (\mathbf{f}_{n_1}^* \otimes \mathbf{s}_{n_1})^H \mathbf{P}_K (\mathbf{f}_{n_2}^* \otimes \mathbf{s}_{n_2}) &= (\mathbf{f}_{n_1}^* \otimes \mathbf{s}_{n_1})^H (\mathbf{I}_N \otimes \mathbf{L}_M + \mathbf{P}_N \otimes \mathbf{U}_M) (\mathbf{f}_{n_2}^* \otimes \mathbf{s}_{n_2}) \\ &= \left(\mathbf{f}_{n_1}^{*H} (\mathbf{I}_N + \mathbf{P}_N) \mathbf{f}_{n_2}^* \right) \otimes \left(\mathbf{s}_{n_1}^H (\mathbf{L}_M + \mathbf{U}_M) \mathbf{s}_{n_2} \right) \\ &= (0 + 0) \otimes \left(\mathbf{s}_{n_1}^H (\mathbf{L}_M + \mathbf{U}_M) \mathbf{s}_{n_2} \right) \\ &= 0 \quad n_1 \neq n_2. \end{aligned} \quad (\text{A.3})$$

Thus the sequences remain orthogonal after a cyclic permutation. To show that the sequences of OSDM have zero periodic cross-correlation, we have to show their orthogonality for (all) multiple cyclic permutations. For two cyclic permutations we can write,

$$\begin{aligned} \mathbf{P}_K^2 &= (\mathbf{I}_N \otimes \mathbf{L}_M + \mathbf{P}_N \otimes \mathbf{U}_M)^2 \\ &= (\mathbf{I}_N^2 \otimes \mathbf{L}_M^2) + (\mathbf{P}_N^2 \otimes \mathbf{U}_M^2) + (\mathbf{P}_N \otimes \mathbf{L}_M \mathbf{U}_M) + (\mathbf{P}_N \otimes \mathbf{U}_M \mathbf{L}_M). \end{aligned} \quad (\text{A.4})$$

Note that for any number of cyclic permutations, one can always write

$$\mathbf{P}_K^k = \sum_{i=1}^{2^k} f_i(\mathbf{I}_N, \mathbf{P}_N) \otimes g_i(\mathbf{L}_M, \mathbf{U}_M), \quad k \in \{0, 1, \dots, K-1\}, \quad (\text{A.5})$$

where $f_i(\mathbf{I}_N, \mathbf{P}_N)$ is a function which depends only on $\mathbf{I}_N$ and $\mathbf{P}_N$, and $g_i(\mathbf{L}_M, \mathbf{U}_M)$ is a function which depends only on $\mathbf{L}_M$ and $\mathbf{U}_M$. Now we can show that the sequences of OSDM have zero periodic cross-correlation,

$$\begin{aligned} & (\mathbf{f}_{n_1}^* \otimes \mathbf{s}_{n_1})^H \mathbf{P}_K^k (\mathbf{f}_{n_2}^* \otimes \mathbf{s}_{n_2}) = \\ & = (\mathbf{f}_{n_1}^* \otimes \mathbf{s}_{n_1})^H (\mathbf{I}_N \otimes \mathbf{L}_M + \mathbf{P}_N \otimes \mathbf{U}_M)^k (\mathbf{f}_{n_2}^* \otimes \mathbf{s}_{n_2}) \\ & = (\mathbf{f}_{n_1}^* \otimes \mathbf{s}_{n_1})^H \left[\sum_{i=1}^{2^k} f_i(\mathbf{I}_N, \mathbf{P}_N) \otimes g_i(\mathbf{L}_M, \mathbf{U}_M) \right] (\mathbf{f}_{n_2}^* \otimes \mathbf{s}_{n_2}) \\ & = \left(\mathbf{f}_{n_1}^* \right)^H \left[\sum_{i=1}^{2^k} f_i(\mathbf{I}_N, \mathbf{P}_N) \right] \mathbf{f}_{n_2}^* \otimes \left(\mathbf{s}_{n_1} \right)^H \left[\sum_{i=1}^{2^k} g_i(\mathbf{L}_M, \mathbf{U}_M) \right] \mathbf{s}_{n_2} \\ & = 0 \otimes \left(\mathbf{s}_{n_1} \right)^H \left[\sum_{i=1}^{2^k} g_i(\mathbf{L}_M, \mathbf{U}_M) \right] \mathbf{s}_{n_2} \\ & = 0 \quad n_1 \neq n_2, \forall k \in \{0, 1, \dots, K-1\}. \end{aligned} \quad (\text{A.6})$$

B

Upper bound on the expected channel estimation error matrix

We derive an upper bound as follows.

$$\begin{aligned}
 & \mathbb{E}[(\mathbf{H}_d - \hat{\mathbf{H}}_d)(\mathbf{H}_d - \hat{\mathbf{H}}_d)^H] \\
 &= \mathbb{E} \left[P_d (\boldsymbol{\Psi}_d^H (\mathbf{H}_{DD} - \hat{\mathbf{H}}_{DD}) \boldsymbol{\Phi}_d) (\boldsymbol{\Psi}_d^H (\mathbf{H}_{DD} - \hat{\mathbf{H}}_{DD}) \boldsymbol{\Phi}_d)^H \right] \\
 &= \mathbb{E} \left[P_d \left(\sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L (c_{l,q} - \hat{c}_{l,q}) \boldsymbol{\Psi}_d^H (\mathbf{F}_N \otimes \mathbf{I}_M) (\boldsymbol{\Lambda}_q \mathbf{P}^l) (\mathbf{F}_N^H \otimes \mathbf{I}_M) \boldsymbol{\Phi}_d \right) \times \right. \\
 & \quad \left. \left(\sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L (c_{l,q} - \hat{c}_{l,q}) \boldsymbol{\Psi}_d^H (\mathbf{F}_N \otimes \mathbf{I}_M) (\boldsymbol{\Lambda}_q \mathbf{P}^l) (\mathbf{F}_N^H \otimes \mathbf{I}_M) \boldsymbol{\Phi}_d \right)^H \right] \\
 &= \mathbb{E} \left[P_d \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L |c_{l,q} - \hat{c}_{l,q}|^2 \times \right. \\
 & \quad \left. \underbrace{\boldsymbol{\Psi}_d^H (\mathbf{F}_N \otimes \mathbf{I}_M) (\boldsymbol{\Lambda}_q \mathbf{P}^l) (\mathbf{F}_N^H \otimes \mathbf{I}_M) \boldsymbol{\Phi}_d \left(\boldsymbol{\Psi}_d^H (\mathbf{F}_N \otimes \mathbf{I}_M) (\boldsymbol{\Lambda}_q \mathbf{P}^l) (\mathbf{F}_N^H \otimes \mathbf{I}_M) \boldsymbol{\Phi}_d \right)^H}_{\text{Diagonal matrix with } K_d \text{ ones and } (R_d - K_d) \text{ zeros}} \right] \\
 &\leq P_d \mathbb{E} \left[\text{tr} \left((\mathbf{c} - \hat{\mathbf{c}})(\mathbf{c} - \hat{\mathbf{c}})^H \right) \right] \mathbf{I}_{R_d} \tag{B.1}
 \end{aligned}$$

C

Expected effective channel matrix

We can write,

$$\begin{aligned}
 \mathbb{E} [\hat{\mathbf{H}}_d^H \hat{\mathbf{H}}_d] &= \mathbb{E} \left[P_d (\Psi_d^H \hat{\mathbf{H}}_{DD} \Phi_d)^H (\Psi_d^H \hat{\mathbf{H}}_{DD} \Phi_d) \right] \\
 &= \mathbb{E} \left[P_d \left(\sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L \hat{c}_{l,q} \Psi_d^H (\mathbf{F}_N \otimes \mathbf{I}_M) (\Lambda_q \mathbf{P}^l) (\mathbf{F}_N^H \otimes \mathbf{I}_M) \Phi_d \right)^H \right. \\
 &\quad \left. \times \left(\sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L \hat{c}_{l,q} \Psi_d^H (\mathbf{F}_N \otimes \mathbf{I}_M) (\Lambda_q \mathbf{P}^l) (\mathbf{F}_N^H \otimes \mathbf{I}_M) \Phi_d \right) \right] \\
 &= P_d \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L \sigma_{\hat{c}_{l,q}}^2 \times \\
 &\quad \left(\Psi_d^H (\mathbf{F}_N \otimes \mathbf{I}_M) (\Lambda_q \mathbf{P}^l) (\mathbf{F}_N^H \otimes \mathbf{I}_M) \Phi_d \right)^H \left(\Psi_d^H (\mathbf{F}_N \otimes \mathbf{I}_M) (\Lambda_q \mathbf{P}^l) (\mathbf{F}_N^H \otimes \mathbf{I}_M) \Phi_d \right) \\
 &= P_d \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L \sigma_{\hat{c}_{l,q}}^2 \mathbf{I}_{K_d}. \tag{C.1}
 \end{aligned}$$

Note that (due to the K_d ones on the diagonal) we have

$$\mathbb{E} [\|\hat{\mathbf{H}}_d\|_F^2] = \text{tr} (\mathbb{E} [\hat{\mathbf{H}}_d^H \hat{\mathbf{H}}_d]) = P_d K_d \sum_{q=-Q/2}^{Q/2} \sum_{l=0}^L \sigma_{\hat{c}_{l,q}}^2 = P_d \sigma_{\hat{\mathbf{H}}}^2. \tag{C.2}$$

Note that $\sigma_{\hat{c}_{l,q}}^2$ is defined as

$$\begin{aligned}
 \mathbb{E}[\hat{\mathbf{c}}\hat{\mathbf{c}}^H] &= \mathbb{E} \left[\left(\sigma_{\mathbf{n}}^2 \mathbf{R}_{\mathbf{c}}^{-1} + \mathbf{Z}^H \mathbf{Z} \right)^{-1} \mathbf{Z}^H (\mathbf{Z}\mathbf{c} + \mathbf{w}_p) (\mathbf{Z}\mathbf{c} + \mathbf{w}_p)^H \mathbf{Z} (\sigma_{\mathbf{n}}^2 \mathbf{R}_{\mathbf{c}}^{-1} + \mathbf{Z}^H \mathbf{Z})^{-1} \right] \\
 &= \left(\sigma_{\mathbf{n}}^2 \mathbf{R}_{\mathbf{c}}^{-1} + \mathbf{Z}^H \mathbf{Z} \right)^{-1} \mathbb{E} \left[\mathbf{Z}^H \left(\mathbf{Z}\mathbf{c}\mathbf{c}^H \mathbf{Z}^H + \mathbf{w}_p \mathbf{w}_p^H \right) \mathbf{Z} \right] \left(\sigma_{\mathbf{n}}^2 \mathbf{R}_{\mathbf{c}}^{-1} + \mathbf{Z}^H \mathbf{Z} \right)^{-1} \\
 &= \left(\sigma_{\mathbf{n}}^2 \mathbf{R}_{\mathbf{c}}^{-1} + \mathbf{Z}^H \mathbf{Z} \right)^{-1} \left[\mathbf{Z}^H \mathbf{Z} \mathbf{R}_{\mathbf{c}} \mathbf{Z}^H \mathbf{Z} + \mathbf{Z}^H \mathbf{R}_{\mathbf{w}_p} \mathbf{Z} \right] \left(\sigma_{\mathbf{n}}^2 \mathbf{R}_{\mathbf{c}}^{-1} + \mathbf{Z}^H \mathbf{Z} \right)^{-1} \\
 &= \frac{P_p^2 \text{diag} \left([\sigma_{c_{0,0}}^2, \dots, \sigma_{c_{L,Q}}^2] \right) + P_p \sigma_{\mathbf{n}}^2 \mathbf{I}_{(L+1)(Q+1)}}{\left(\sigma_{\mathbf{n}}^2 \text{diag} \left([\sigma_{c_{0,0}}^2, \dots, \sigma_{c_{L,Q}}^2] \right)^{-1} + P_p \mathbf{I}_{(L+1)(Q+1)} \right)^2} \\
 &= \text{diag} \left(\left[\frac{P_p \sigma_{c_{0,0}}^4}{\sigma_{\mathbf{n}}^2 + \sigma_{c_{0,0}}^2 P_p}, \dots, \frac{P_p \sigma_{c_{L,Q}}^4}{\sigma_{\mathbf{n}}^2 + \sigma_{c_{L,Q}}^2 P_p} \right] \right) = \text{diag} \left(\left[\sigma_{\hat{c}_{0,0}}^2, \dots, \sigma_{\hat{c}_{L,Q}}^2 \right] \right). \quad (\text{C.3})
 \end{aligned}$$

D

Selection matrices for the OTFS system

This appendix details the construction of the selection matrices used in the orthogonal time frequency space (OTFS) system. Let $\mathbf{P}_{(\cdot)} \in \{0, 1\}^{M \times N}$ and $\mathbf{R}_{(\cdot)} \in \{0, 1\}^{M \times N}$ denote binary pilot and data allocation matrices in the delay–Doppler domain, respectively, where an entry equal to one indicates that the corresponding delay–Doppler index is assigned to a pilot or data symbol.

We define the corresponding selection vectors as $\mathbf{p}_{(\cdot)} = \text{vec}(\mathbf{P}_{(\cdot)})$, $\mathbf{r}_{(\cdot)} = \text{vec}(\mathbf{R}_{(\cdot)})$, both of length $K = MN$. The selection matrices are then constructed by retaining the columns of the $K \times K$ identity matrix corresponding to the nonzero entries of the selection vectors. That is, the pilot and data selection matrices are given by $\Phi_{(\cdot)} = \Phi_{(\cdot)}(\mathbf{p}_{(\cdot)})$, $\Psi_{(\cdot)} = \Psi_{(\cdot)}(\mathbf{r}_{(\cdot)})$.

For the ‘Doppler slab’ ($N = Q + 1$) configuration we hence have:

$$[\mathbf{P}_p]_{m,n} = \begin{cases} 1, & \text{if } m \leq (L+1) \vee m \geq M - (L+1) + 1 \\ 0, & \text{otherwise,} \end{cases} \quad (\text{D.1})$$

$$[\mathbf{P}_d]_{m,n} = 1 - [\mathbf{P}_p]_{m,n} = \begin{cases} 1, & \text{if } (L+1) + 1 \leq m \leq M - (L+1) \\ 0, & \text{otherwise,} \end{cases} \quad (\text{D.2})$$

$$[\mathbf{R}_p]_{m,n} = \begin{cases} 1, & \text{if } m \leq (L+1) \\ 0, & \text{otherwise,} \end{cases} \quad (\text{D.3})$$

$$[\mathbf{R}_d]_{m,n} = 1 - [\mathbf{R}_p]_{m,n} = \begin{cases} 1, & \text{if } m \geq (L+1) + 1 \\ 0, & \text{otherwise,} \end{cases} \quad (\text{D.4})$$

for $m = 1, \dots, M$, $n = 1, \dots, N$. For the ‘delay slab’ ($M = L + 1$) configuration we have:

$$[\mathbf{P}_p]_{m,n} = \begin{cases} 1, & \text{if } n \leq Q + Q/2 + 1 \vee n \geq N - Q/2 + 1 \\ 0, & \text{otherwise,} \end{cases} \quad (\text{D.5})$$

$$[\mathbf{P}_d]_{m,n} = 1 - [\mathbf{P}_p]_{m,n} = \begin{cases} 1, & \text{if } Q + Q/2 + 2 \leq n \leq N - Q/2 \\ 0, & \text{otherwise,} \end{cases} \quad (\text{D.6})$$

$$[\mathbf{R}_p]_{m,n} = \begin{cases} 1, & \text{if } n \leq (Q + 1) \\ 0, & \text{otherwise,} \end{cases} \quad (\text{D.7})$$

$$[\mathbf{R}_d]_{m,n} = 1 - [\mathbf{R}_p]_{m,n} = \begin{cases} 1, & \text{if } n \geq (Q + 1) + 1 \\ 0, & \text{otherwise.} \end{cases} \quad (\text{D.8})$$

for $m = 1, \dots, M$, $n = 1, \dots, N$.

E

Derivation of the approximate lower bound on the channel capacity

This appendix derives an approximate lower bound on the channel capacity, given in (6.28).

Let $\tilde{\mathbf{H}}_1 = \mathbf{H}_1 - \hat{\mathbf{H}}_1$. The received communication signal is

$$\begin{aligned} \mathbf{y}_d &= \sqrt{\alpha P} \Psi_d^\top \mathbf{M} \mathbf{H}_1 \mathbf{M}^H \Phi_d \mathbf{s}_d + \Psi_d^\top \mathbf{M} \mathbf{H}_2 \mathbf{M}^H \mathbf{s} + \Psi_d^\top \mathbf{M} \mathbf{n} \\ &= \sqrt{\alpha P} \Psi_d^\top \mathbf{M} \tilde{\mathbf{H}}_1 \mathbf{M}^H \Phi_d \mathbf{s}_d + \sqrt{\alpha P} \Psi_d^\top \mathbf{M} \hat{\mathbf{H}}_1 \mathbf{M}^H \Phi_d \mathbf{s}_d \\ &\quad + \Psi_d^\top \mathbf{M} \mathbf{H}_2 \mathbf{M}^H \mathbf{s} + \Psi_d^\top \mathbf{M} \mathbf{n}. \end{aligned} \tag{E.1}$$

The derivation of the approximate lower bound on the channel capacity is adapted from the analysis in [1] (and variants were used in [2], [3]) and relies on upper bounds on the interference-plus-noise terms together with a lower bound on the useful signal component. These will be derived in the following subsections.

An equivalent representation of the communication system, corresponding to the pipeline in Fig. 6.4, is shown in Fig. E.1. The analysis and resulting capacity lower bound in [1] assume that the signal $\mathbf{s}_d$, channel error $\sqrt{\alpha P} \Psi_d^\top \mathbf{M} (\tilde{\mathbf{H}}_1 + \mathbf{H}_2) \mathbf{M}^H \Phi_d$, and the noise term

$\sqrt{(1-\alpha)P} \Psi_d^\top \mathbf{M} \mathbf{H}_2 \mathbf{M}^H \Phi_p \mathbf{s}_p + \Psi_d^\top \mathbf{M} \mathbf{n}$ are mutually independent, and have zero mean. Since the latter two both depend on $\mathbf{H}_2$, this assumption is not met, which is why we are only able to obtain an “approximate” lower bound.

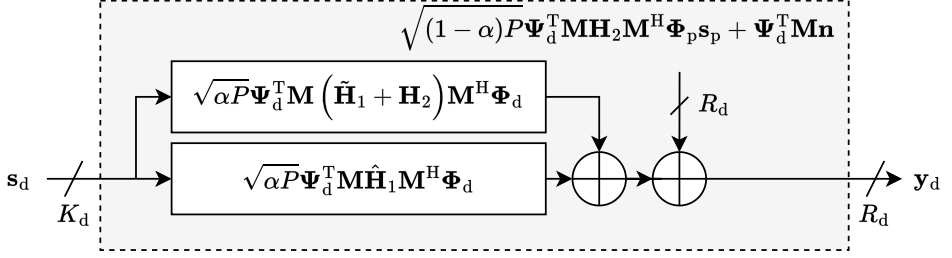


Figure E.1.: Equivalent pipeline of the communication system. The top white box denotes the influence of the channel estimation error and the data self-interference through $\mathbf{H}_2$. The bottom white box denotes the influence of the useful (estimated) channel.

Upper bound on channel estimation error

Let the channel estimator be given by (6.24) and let $P_d = \alpha P$ and $P_p = (1 - \alpha)P$. The covariance of the interference-plus-noise term $\mathbf{v}_p$ can be upper bounded:

$$\begin{aligned}
 \mathbf{R}_{\mathbf{v}_p} &= \mathbb{E} \left[\Psi_p^\top \mathbf{M} \mathbf{H}_2 \mathbf{M}^H \mathbf{s} \mathbf{s}^H \mathbf{M} \mathbf{H}_2^H \mathbf{M}^H \Psi_p \right] + \sigma_n^2 \mathbf{I}_{R_p} \\
 &\leq \max \left\{ \frac{P_d}{K_d}, P_p \right\} \mathbb{E} \left[\Psi_p^\top \mathbf{M} \mathbf{H}_2 \mathbf{H}_2^H \mathbf{M}^H \Psi_p \right] + \sigma_n^2 \mathbf{I}_{R_p} \\
 &= \max \left\{ \frac{P_d}{K_d}, P_p \right\} \Psi_p^\top \mathbf{M} \mathbb{E} [\mathbf{H}_2 \mathbf{H}_2^H] \mathbf{M}^H \Psi_p + \sigma_n^2 \mathbf{I}_{R_p} \\
 &= \max \left\{ \frac{P_d}{K_d}, P_p \right\} \Psi_p^\top \mathbf{M} \left(\sum_{l, q \notin \mathcal{L}} \sigma_{c_{l,q}}^2 \mathbf{I} \right) \mathbf{M}^H \Psi_p + \sigma_n^2 \mathbf{I}_{R_p} \\
 &= \max \left\{ \frac{P_d}{K_d}, P_p \right\} \left(\sum_{l, q \notin \mathcal{L}} \sigma_{c_{l,q}}^2 \right) \mathbf{I}_{R_p} + \sigma_n^2 \mathbf{I}_{R_p} \\
 &= \max \left\{ \frac{P_d}{K_d}, P_p \right\} \sigma_{\mathbf{H}_2}^2 \mathbf{I}_{R_p} + \sigma_n^2 \mathbf{I}_{R_p}, \tag{E.2}
 \end{aligned}$$

where $\sigma_{\mathbf{H}_2}^2 = \sum_{l, q \notin \mathcal{L}} \sigma_{c_{l,q}}^2$ and where we have used that $\mathbb{E}[\mathbf{s} \mathbf{s}^H] \leq \max \left\{ \frac{P_d}{K_d}, P_p \right\} \mathbf{I}_K$. Using the upper bound on $\mathbf{R}_{\mathbf{v}_p}$, the channel estimation error is bounded by

$$\begin{aligned}
 \mathbb{E}[(\mathbf{c}_1 - \hat{\mathbf{c}}_1)^H (\mathbf{c}_1 - \hat{\mathbf{c}}_1)] &= \text{tr} \left[\left(\mathbf{R}_{\mathbf{c}_1}^{-1} + \mathbf{Z}_p^H \mathbf{R}_{\mathbf{v}_p}^{-1} \mathbf{Z}_p \right)^{-1} \right] \\
 &\leq \sum_{(l,q) \in \mathcal{L}} \frac{\sigma_{c_{l,q}}^2 \left(\max \left\{ \frac{P_d}{K_d}, P_p \right\} \sigma_{\mathbf{H}_2}^2 + \sigma_n^2 \right)}{\sigma_{c_{l,q}}^2 P_p + \max \left\{ \frac{P_d}{K_d}, P_p \right\} \sigma_{\mathbf{H}_2}^2 + \sigma_n^2} = \sigma_{\hat{\mathbf{H}}}^2, \tag{E.3}
 \end{aligned}$$

where $\mathbf{R}_{\mathbf{c}_1} = \mathbb{E}[\mathbf{c}_1 \mathbf{c}_1^H]$.

Upper bound on the interference-plus-noise term in $\mathbf{y}_d$

The received communication signal is

$$\mathbf{y}_d = \hat{\mathbf{H}}_{d,1} \mathbf{s}_d + \mathbf{v}_d, \quad (\text{E.4})$$

where $\hat{\mathbf{H}}_{d,1} = \sqrt{P_d} \Psi_d^\top \mathbf{M} \hat{\mathbf{H}}_1 \mathbf{M}^H \Phi_d$ and where $\mathbf{v}_d = \sqrt{P_d} \Psi_d^\top \mathbf{M} \tilde{\mathbf{H}}_1 \mathbf{M}^H \Phi_d \mathbf{s}_d + \Psi_d^\top \mathbf{M} \mathbf{H}_2 \mathbf{M}^H \mathbf{s} + \Psi_d^\top \mathbf{M} \mathbf{n}$ collects the channel estimation error, the (unmodeled) channel interference and the noise terms.

The covariance of the vector $\mathbf{v}_d$ is then upper bounded as follows.

$$\begin{aligned} \mathbf{R}_{\mathbf{v}_d} &= \mathbb{E}[\mathbf{v}_d \mathbf{v}_d^H] \\ &\approx \mathbb{E} \left[P_d \Psi_d^\top \mathbf{M} \tilde{\mathbf{H}}_1 \mathbf{M}^H \Phi_d \mathbf{s}_d \mathbf{s}_d^H \Phi_d^\top \mathbf{M} \tilde{\mathbf{H}}_1^H \mathbf{M}^H \Psi_d \right] \\ &\quad + \mathbb{E} \left[\Psi_d^\top \mathbf{M} \mathbf{H}_2 \mathbf{M}^H \mathbf{s} \mathbf{s}^H \mathbf{M} \mathbf{H}_2 \mathbf{M}^H \Psi_d \right] + \sigma_{\mathbf{n}}^2 \mathbf{I}_{R_d} \end{aligned} \quad (\text{E.5})$$

$$\begin{aligned} &\leq \mathbb{E} \left[P_d \Psi_d^\top \mathbf{M} \tilde{\mathbf{H}}_1 \mathbf{M}^H \Phi_d \mathbf{s}_d \mathbf{s}_d^H \Phi_d^\top \mathbf{M} \tilde{\mathbf{H}}_1^H \mathbf{M}^H \Psi_d \right] + \max \left\{ \frac{P_d}{K_d}, P_p \right\} \sigma_{\mathbf{H}_2}^2 \mathbf{I}_{R_d} + \sigma_{\mathbf{n}}^2 \mathbf{I}_{R_d} \\ &\leq \frac{P_d}{K_d} \Psi_d^\top \mathbf{M} \mathbb{E} \left[\tilde{\mathbf{H}}_1 \tilde{\mathbf{H}}_1^H \right] \mathbf{M}^H \Psi_d + \max \left\{ \frac{P_d}{K_d}, P_p \right\} \sigma_{\mathbf{H}_2}^2 \mathbf{I}_{R_d} + \sigma_{\mathbf{n}}^2 \mathbf{I}_{R_d}. \end{aligned} \quad (\text{E.6})$$

In (E.5), we neglect the cross-covariance terms between the channel estimation error and the unmodeled interference, which are non-zero due to common dependence on $\mathbf{H}_2$, but are assumed small compared to the dominant diagonal terms. We further rewrite the term $\mathbb{E}[\tilde{\mathbf{H}}_1 \tilde{\mathbf{H}}_1^H]$

$$\begin{aligned} \mathbb{E}[\tilde{\mathbf{H}}_1 \tilde{\mathbf{H}}_1^H] &= \mathbb{E} \left[\left(\sum_{(l,q) \in \mathcal{L}} (c_{l,q} - \hat{c}_{l,q}) \mathbf{\Lambda}^q \mathbf{P}^l \right) \left(\sum_{(l,q) \in \mathcal{L}} (c_{l,q} - \hat{c}_{l,q}) \mathbf{\Lambda}^q \mathbf{P}^l \right)^H \right] \\ &= \mathbb{E} \left[(\hat{\mathbf{c}}_1 - \mathbf{c}_1)^H (\hat{\mathbf{c}}_1 - \mathbf{c}_1) \right] \mathbf{I}_K \\ &\quad + \mathbb{E} \left[\sum_{(l_1, q_1) \neq (l_2, q_2)} (c_{l_1, q_1} - \hat{c}_{l_1, q_1}) (c_{l_2, q_2} - \hat{c}_{l_2, q_2})^* \mathbf{\Lambda}^{q_1 - q_2} \mathbf{P}^{l_1 - l_2} \right] \\ &\approx \mathbb{E} \left[(\hat{\mathbf{c}}_1 - \mathbf{c}_1)^H (\hat{\mathbf{c}}_1 - \mathbf{c}_1) \right] \mathbf{I}_K \leq \sigma_{\tilde{\mathbf{H}}}^2 \mathbf{I}_K. \end{aligned} \quad (\text{E.7})$$

Note that $\sigma_{\tilde{\mathbf{H}}}^2$ was defined in (E.3). Here the approximation neglects the cross terms of the channel estimation error. These terms are non-zero because $\mathbb{E}[(\mathbf{c} - \hat{\mathbf{c}})(\mathbf{c} - \hat{\mathbf{c}})^H]$ is not diagonal due to interference through $\mathbf{H}_2$.

Substituting (E.7) into (E.6) we obtain

$$\mathbf{R}_{\mathbf{v}_d} \leq \left(\frac{P_d}{K_d} \sigma_{\tilde{\mathbf{H}}}^2 + \max \left\{ \frac{P_d}{K_d}, P_p \right\} \sigma_{\mathbf{H}_2}^2 + \sigma_{\mathbf{n}}^2 \right) \mathbf{I}_{R_d}. \quad (\text{E.8})$$

Lower bound on power in the effective channel matrix $\hat{\mathbf{H}}_{d,1}$

Next we bound the power in the effective channel matrix $\hat{\mathbf{H}}_{d,1}$. First note that

$$\begin{aligned}
 \mathbb{E} \left[\hat{\mathbf{H}}_{d,1}^H \hat{\mathbf{H}}_{d,1} \right] &= \mathbb{E} \left[P_d \Phi_d^\top \mathbf{M} \hat{\mathbf{H}}_1^H \mathbf{M}^H \Psi_d \Psi_d^\top \mathbf{M} \hat{\mathbf{H}}_1 \mathbf{M}^H \Phi_d \right] \\
 &= P_d \Phi_d^\top \mathbf{M} \mathbb{E} \left[\left(\sum_{(l,q) \in \mathcal{L}} \hat{c}_{l,q} \Lambda^q \mathbf{P}^l \right)^H \mathbf{M} \Psi_d \Psi_d^\top \mathbf{M} \left(\sum_{(l,q) \in \mathcal{L}} \hat{c}_{l,q} \Lambda^q \mathbf{P}^l \right) \right] \mathbf{M}^H \Phi_d \\
 &= P_d \sum_{(l,q) \in \mathcal{L}} \mathbb{E} \left[|\hat{c}_{l,q}|^2 \right] \mathbf{\Gamma}_{l,q}^H \mathbf{\Gamma}_{l,q} + P_d \sum_{l_1 \neq l_2, q_1 \neq q_2} \mathbb{E} \left[\hat{c}_{l_1, q_1} \hat{c}_{l_2, q_2}^* \right] \mathbf{\Gamma}_{l_1, q_1}^H \mathbf{\Gamma}_{l_2, q_2} \\
 &\approx P_d \sum_{(l,q) \in \mathcal{L}} \mathbb{E} \left[|\hat{c}_{l,q}|^2 \right] \mathbf{\Gamma}_{l,q}^H \mathbf{\Gamma}_{l,q} = P_d \mathbb{E} \left[\hat{\mathbf{c}}_1^H \hat{\mathbf{c}}_1 \right] \mathbf{I}_{K_d}, \tag{E.9}
 \end{aligned}$$

where $\mathbf{\Gamma}_{l,q} = \Psi_d^\top \mathbf{M} \Lambda^q \mathbf{P}^l \mathbf{M}^H \Phi_d$, and where we have used that $\mathbf{\Gamma}_{l,q}^H \mathbf{\Gamma}_{l,q} = \mathbf{I}_{K_d}$. The approximation neglects the cross terms of the channel tap estimates. These terms are non-zero due to the interference through $\mathbf{H}_2$.

The expected power in the effective channel matrix can hence be lower bounded by

$$\mathbb{E} [\| \hat{\mathbf{H}}_{d,1} \|_F^2] = \text{tr} \left(\mathbb{E} \left[\hat{\mathbf{H}}_{d,1}^H \hat{\mathbf{H}}_{d,1} \right] \right) \approx P_d K_d \text{tr} \left(\mathbb{E} \left[\hat{\mathbf{c}}_1^H \hat{\mathbf{c}}_1 \right] \right) \geq P_d \sigma_{\hat{\mathbf{H}}}^2, \tag{E.10}$$

where we have used that

$$\begin{aligned}
 \text{tr} \left(\mathbb{E} \left[\hat{\mathbf{c}} \hat{\mathbf{c}}^H \right] \right) &= \text{tr} \left(\left(\mathbf{R}_{c_1}^{-1} + \mathbf{Z}_p^H \mathbf{R}_{v_p}^{-1} \mathbf{Z}_p \right)^{-1} \mathbf{Z}_p^H \mathbf{R}_{v_p}^{-1} \mathbf{Z}_p \right) \\
 &\gtrsim \sum_{(l,q) \in \mathcal{L}} \frac{\sigma_{c_{l,q}}^4 P_p}{\sigma_{c_{l,q}}^2 P_p + \max \left\{ \frac{P_d}{K_d}, P_p \right\} \sigma_{\mathbf{H}_2}^2 + \sigma_{\mathbf{n}}^2}, \tag{E.11}
 \end{aligned}$$

and hence we define

$$\sigma_{\hat{\mathbf{H}}}^2 = K_d \sum_{(l,q) \in \mathcal{L}} \frac{\sigma_{c_{l,q}}^4 P_p}{\sigma_{c_{l,q}}^2 P_p + \max \left\{ \frac{P_d}{K_d}, P_p \right\} \sigma_{\mathbf{H}_2}^2 + \sigma_{\mathbf{n}}^2}. \tag{E.12}$$

Approximate lower bound

The channel capacity for this system can be approximately lower bounded by

$$\begin{aligned}
 C &= \mathbb{E}_{\mathbf{H}} \left[\max_{p_{\mathbf{s}_d}(\cdot), \mathbb{E}[\|\mathbf{s}_d\|_2^2]=1} \mathcal{J}(\mathbf{y}_d; \mathbf{s}_d | \hat{\mathbf{c}}_1) \right] \\
 &= \mathbb{E}_{\mathbf{H}} \left[\max_{p_{\mathbf{s}_d}(\cdot), \mathbb{E}[\|\mathbf{s}_d\|_2^2]=1} \log \det \left(\mathbf{I}_{R_d} + \mathbf{R}_{v_d}^{-1} \hat{\mathbf{H}}_{d,1} \mathbb{E} \left[\mathbf{s}_d \mathbf{s}_d^H \right] \hat{\mathbf{H}}_{d,1}^H \right) \right] \\
 &\geq \mathbb{E}_{\mathbf{H}} \left[\log_2 \det \left(\mathbf{I}_{R_d} + \mathbf{R}_{v_d}^{-1} \frac{1}{K_d} \hat{\mathbf{H}}_{d,1} \hat{\mathbf{H}}_{d,1}^H \right) \right] \\
 &\geq \mathbb{E}_{\mathbf{H}} \left[\log_2 \det \left(\mathbf{I}_{K_d} + \rho^{\text{eff}} \frac{\hat{\mathbf{H}}_{d,1}^H \hat{\mathbf{H}}_{d,1}}{\mathbb{E} \left[\|\hat{\mathbf{H}}_{d,1}\|_F^2 \right]} \right) \right] = \underline{C}, \tag{E.13}
 \end{aligned}$$

where

$$\rho_{\text{eff}} = \frac{\frac{P_d}{K_d} \sigma_{\mathbf{H}}^2}{\frac{P_d}{K_d} \sigma_{\mathbf{H}}^2 + \max\left\{\frac{P_d}{K_d}, P_p\right\} \sigma_{\mathbf{H}_2}^2 + \sigma_{\mathbf{n}}^2}. \quad (\text{E.14})$$

References

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F

Finite state machine (FSM) for the basin experiment

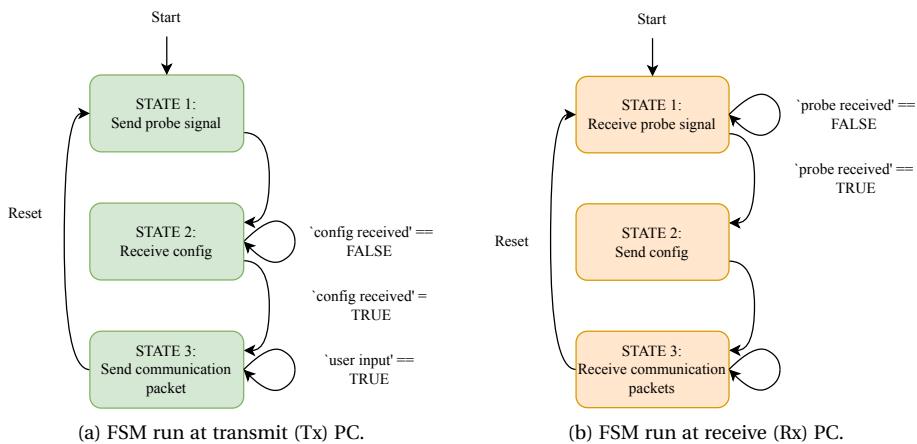


Figure F.1.: Diagrams of the FSMs used to execute the protocol during the basin experiments.

Glossary

Physical quantities

Constants

Symbol	Units	Description
c	m s^{-1}	Speed of sound

Variables

Symbol	Units	Description
f_c	s^{-1}	Carrier frequency
f_s	s^{-1}	Sampling frequency
T_{symb}	s	Symbol period

Mathematical notation

Sets

Notation	Description
$\mathbb{N}$	Natural numbers
$\mathbb{N}_+$	Positive natural numbers
$\mathbb{Z}$	Integer numbers
$\mathbb{R}$	Real numbers
$\mathbb{R}^N$	Real vectors of length N
$\mathbb{R}^{N \times M}$	Real matrices of size N
$\mathbb{C}$	Complex numbers
$\mathbb{C}^N$	Complex vectors of length N
$\mathbb{C}^{N \times M}$	Complex matrices of size N

Scalars, vectors and matrices

Notation	Description
a	Scalar (lowercase letter)
$\mathbf{a}$	Vector (bold lowercase letter)
$\mathbf{A}$	Matrix (bold uppercase letter)
$\mathbf{I}_N$	$N \times N$ identity matrix
$\mathbf{0}_N$	Zero vector of length N

$\mathbf{0}_{N \times M}$	Zero matrix of size $N \times M$
$\mathbf{1}_N$	Ones vector of length N
$\mathbf{1}_{N \times M}$	Ones matrix of size $N \times M$
$\mathbf{F}_N$	Normalized discrete Fourier transform (DFT) matrix of size $N \times N$ with elements $[\mathbf{F}_N]_{k,n} = \frac{1}{\sqrt{N}} e^{-2\pi j \frac{kn}{N}}$

Operators

Notation	Description
$\otimes$	Kronecker product
$\odot$	Khatri-Rao (columnwise Kronecker) product
$\circ$	Hadamard (elementwise) product
$(\cdot)^*$	Complex conjugate
$(\cdot)^\top$	Transpose
$(\cdot)^H$	Hermitian (conjugate transpose)

Functions

Notation	Description
$\det(\mathbf{A})$	Determinant of matrix $\mathbf{A}$
$\text{diag}(\mathbf{a})$	Diagonal matrix with $\mathbf{a}$ on the diagonal
$\text{diag}(\mathbf{A})$	(Column) vector with diagonal elements of $\mathbf{A}$
$\log(a)$	Natural logarithm
$\text{tr}(\mathbf{A})$	Trace of matrix $\mathbf{A}$
$\text{vec}(\mathbf{A})$	Vector of columns of $\mathbf{A}$
$\text{vec}^{-1}(\mathbf{a})$	Inverse operation of $\text{vec}(\mathbf{A})$
$\ \mathbf{a}\ _1$	ℓ_1 -norm of the vector $\mathbf{a}$
$\ \mathbf{a}\ _2$	ℓ_2 -norm of the vector $\mathbf{a}$ (Euclidean norm)

Random processes

Notation	Description
$\mathbb{E}[\cdot]$	Expected value of a random variable
$\mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance $\boldsymbol{\Sigma}$
$\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	Complex Gaussian distribution with (complex) mean $\boldsymbol{\mu}$ and (complex) covariance $\boldsymbol{\Sigma}$

Abbreviations

AUV	autonomous underwater vehicle
AWGN	additive white Gaussian noise

AMC	adaptive modulation and coding
API	application programming interface
BEM	basis expansion model
CCDF	complementary cumulative distribution function
CE-BEM	complex exponential basis expansion model
BER	bit error ratio
BPSK	binary phase-shift keying
CIR	channel impulse response
CP	cyclic prefix
CRB	Cramér–Rao bound
CSI	channel state information
DD	delay-Doppler
DFE	decision feedback equalizer
DFT	discrete Fourier transform
DLL	delay-locked-loop
DoA	direction-of-arrival
DoD	direction-of-departure
DSFT	discrete symplectic Fourier transform
FRSS	frequency-repetition spread spectrum
FSM	finite state machine
GFDM	generalized frequency division multiplexing
ICI	intercarrier interference
IDFT	inverse discrete Fourier transform
IP	internet protocol
ISI	intersymbol interference
LAN	local area network
LFM	linear frequency modulated
LMMSE	linear minimum mean squared error
LPF	low-pass filter
LTI	linear time-invariant
LTV	linear time-variant
MIMO	multiple-input multiple-output
MISO	multiple-input single-output
MMSE	minimum mean squared error
MSE	mean squared error
m-sequence	maximum length sequence
OFDM	orthogonal frequency division multiplexing
A-OFDM	asymmetric orthogonal frequency division multiplexing
V-OFDM	vector orthogonal frequency division multiplexing
OSDM	orthogonal sequence division multiplexing
D-OSDM	Doppler-resilient orthogonal sequence division multiplexing
OTFS	orthogonal time frequency space

PAPR	peak-to-average power ratio
PDR	packet delivery ratio
PLL	phase-locked-loop
PRN	pseudo-random noise
QAM	quadrature amplitude modulation
QPSK	quadrature phase-shift keying
RF	radio-frequency
RLS	recursive least squares
Rx	receive
SCM	single-carrier modulation
SIMO	single-input multiple-output
SINR	signal-to-interference-plus-noise ratio
SISO	single-input single-output
SNR	signal-to-noise ratio
TRL	technology readiness level
Tx	transmit
UWA	underwater acoustic
ZC	Zadoff–Chu
ZF	zero-forcing

Acknowledgements

When I started my PhD in August 2022, I had no idea what was ahead of me. A PhD has the reputation of being hard work and stressful, and I will not deny either. But now that I am finalizing this work, what stands out to me is something else: the freedom and the ownership it gave me, the beautiful places it took me and the people it introduced me to. It has been enormously rewarding, and I would do it all over again. A few of the people who made it so deserve a special mention.

I want to begin by sincerely thanking my promoters, Richard Hendriks and Richard Heusdens, who gave me the opportunity to start this PhD in the first place, and who then gave me all the freedom to do it my own way. Their doors were always open.

I would also like to thank Geert Leus. Over the past years I had the chance to work with him a great deal, in research as well as in teaching, and my professional development would not have been the same without him. I was also very lucky to go to conferences with him; Geert seems to know everyone (or everyone seems to know Geert), which has led me to meet many new people whom I can now call my friends and/or international colleagues.

One of them is Antonio G. Marques, who has become something of an academic uncle to me. I highly value our conversations, technical and philosophical, and owe him for the places he has shown me, and for the people he has introduced me to.

In the office I was in the good company of Ellen, Peiyuan and Yanbin. I thank them for listening to my daily complaints, for the many coffee breaks, and for our hotpot dinners.

I spent most of these years on the 17th floor of the EEMCS building at TU Delft. I would like to thank all my colleagues there, fellow PhD candidates, the professors and the support staff, for making it a place I was glad to return to every morning.

I am grateful to Costas, Henry, Koen and Mauries for our discussions, typically on Wednesdays, when I would travel to TNO in The Hague. I also want to thank Mark, Eef, Vincent and, in particular, again Koen for all their support during the measurements in the trough, the water basin and at the Haringvliet estuary, and for the documentation that followed. They are absolute professionals, who know how to execute a plan and how to deal with everything that is not in it.

To my parents, Ineke and Ype, and to my brother Tiis: thank you for supporting me, no matter what.

And finally, I want to thank Jeanne, for being by my side throughout all of this. She has been loving and endlessly patient with me, and she pushes me to follow my heart.

About the author

Ids van der Werf was born in Leiden, the Netherlands, in 1998. He received his B.Sc. (cum laude) and M.Sc. degrees in Electrical Engineering from Delft University of Technology in 2019 and 2022, respectively. During his M.Sc. thesis project, he collaborated with Bang & Olufsen on the topic of gridless sound field reconstruction.

Since late 2022, he has been working toward his Ph.D. degree within the Signal Processing Systems group at Delft University of Technology. His doctoral research is a collaborative effort with the Netherlands Defence Academy (NLDA) and the Netherlands Organisation for Applied Scientific Research (TNO). In 2025 and 2026, he was a visiting researcher at the Indian Institute of Science in Bangalore, India, and at the Rey Juan Carlos University in Madrid, Spain, respectively. Alongside his research, Ids has been involved in teaching the graduate-level course on Applied Convex Optimization and supervising students at both the B.Sc. and M.Sc. levels. His research interests include optimization techniques, modulation schemes, and sensor selection for (joint) sensing and communication systems.