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COMPUTATION OF PITCH AND HEAVE MOTIONS FOR ARBITRARY SHIP FORMS.

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Computation of Pitch and Heave Motions for Arbitrary Ship Forms

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Abstract

Analytical methods are used to determine the pitch and heave motions in headwaves for three ship forms of the destroyer type. A computational method using a multiple coefficient transformation for the ship cross-sectional shapes is used. Transformation method for arbitrarily shaped ship sections are discussed. The results from computation and experiment are compared. Agreement is found to depend significantly on the accuracy of the cross-section transformation. When the proper transformation is used, the influence of variation in hull shape on the motion can be accounted for. Agreement between motion computation and experiment is excellent. Computed longitudinal distributions of damping, added mass and exciting forces are discussed.

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Introduction

An analytical method for the computation of ship motions in a seaway has long been of major interest to both the ship designer and seakeeping researcher. The need for such a technique has been greatly increased by the appearance of many unusual hull shapes such as low resistance forms, bulbous bows, sonar dome, etc., for which an evaluation of the effects of hull shapes on the motion characteristics is vital.

For pitch and heave motions in head seas, the formulation of the problem is reasonably complete and may be described as that of obtaining the coefficients of an appropriate set of equations which relate for a particular ship's geometry the wave surface amplitude or some other measurable wave property to the resulting motion of a ship. The fundamental work over the past century has been primarily that of: (1) determining the appropriate form of the equations of motions; (2) obtaining a valid relationship between a particular hull geometry and the coefficients of the equations (so-called left-hand side); and (3) relating the free surface contours or wave shape to the resulting force on a specified hull form (so-called right-hand side). The motion equations are:

$$(a + \rho\nabla)\ddot{z} + b\dot{z} + cz - d\ddot{\theta} - e\dot{\theta} - g\theta = F_a \cos(\omega_e t + \epsilon_{F\zeta})$$

$$(A + \rho\nabla k_{yy}^2)\ddot{\theta} + B\dot{\theta} + C\theta - D\ddot{z} - E\dot{z} - Gz = M_a \cos(\omega_e t + \epsilon_{M\zeta})$$

The equations of motion consist of two coupled linear differential equations containing cross coupling terms proportional to acceleration, velocity and displacement. This representation, while originally developed by Korvin Kroukovsky [1] using a strip theory approach is, however, in no way related to strip theory and is completely general as far as the motion representation is concerned.

Numbers in brackets refer to the references at the end of this paper.

In fact, the only assumptions inherent in such a representation are those (a) of linearity and (b) that for long crested head waves the coupling of other modes of motion into pitch and heave is small and can be neglected. Further, they are easily extended to include all six coupled modes of motion as shown by Cummins [2]. The validity of such a representation was established experimentally by Gerritsma [3]. This same experiment also established the theory of super position, the equivalence of regular and random wave testing, and the frequency dependence of the equation of motion coefficients.

The coefficients and exciting forces were formulated via a so-called strip theory method as developed by Korvin Kroukovsky [1] and extended by Gerritsma [4], [5], [6]. This formulation permits the evaluation of the coefficients and exciting forces in terms of the damping and added mass associated with a particular hull form. It also contains velocity dependent terms which account for most of the forward speed effects in both the coefficients and the exciting forces. This has been experimentally demonstrated by Gerritsma [4], [5], [6], [11], [12].

There remained only the problem of computing for a specific geometry a two dimensional damping and added mass for each of the ship's sections. This can be accomplished by using the Ursell [7] two dimensional solution for a circular cylinder oscillating at the free surface, and conformally mapping this solution for the circle into a particular sectional shape. Such a mapping or transformation was originally accomplished by Tasai [8] in which a three coefficient or Lewis form transformation was used. This works quite well for many of the simpler ship forms whose shapes are closely approximated by the Lewis form family. It, however, gives poor results for sections not properly fitted by the Lewis form coefficients. Also, it can be shown that if this method is used to determine the effects of sectional shape variations on the motions, the difference between the computed values and those obtained experimentally is approximately equal to the differences being investigated.

Porter [9] experimentally verified the Ursell solution for the circular cylinder and extended the transformation expressions to include an arbitrarily large number of transformation coefficients. He also showed experimentally the accuracy of such a transform solution for a number of two dimensional ship-like sections. Porter did not, however, provide a method for determining the coefficients for a given section. Such a method has now been developed which permits the transformation of the unit circle into any simply connected sectional shape. With this and the modified form of strip theory, as developed by Gerritsma [5][6] a method for evaluating not only the motions but the influence of hull shape on the motions, is available.

The availability of such a program immediately presents many possibilities. At long last, we can do quick and inexpensive experiments on a computer. Further, it is possible to look in detail at the various terms of the equation of motion. This should provide new insight into the physical mechanisms involved. As additional computer experiments are performed and the limitations of the program are evaluated, this in itself should provide additional information concerning the physics of ship motion.

It has long been recognized that when experimentally investigating ship motions, the change of one hull dimension is extremely difficult. This is not so for a computer program, an individual design dimension can be artificially varied on a computer and its effects assessed. In addition, there is evidence that such a multi-coefficient or close fit program is required for even the simplest hull forms when computing relative motions, bending moments, bow immersions, etc.

Ship Models Used for Calculation and Experiment

In order to evaluate the capabilities of such a computation method three hull forms were selected. The forms chosen were: (a) a conventional frigate hull which had been previously tested by the Delft Shipbuilding Laboratory; (b) a similar form which had been tested at the Davidson Laboratory; and (c) a radically shaped destroyer which had been designed and tested at the Davidson Laboratory.

Each of the three forms are similar in total displacement and cross-sectional area. (See Table 1.) The first form selected, (Figure 1) a Friesland class frigate, is one for which the motion characteristics have been extensively investigated by this laboratory. The motions have been measured and compared at both full and model scale (Gerritsma and Smith [6] , Bledsoe, Bussemaker and Cummins [14] .) Further, the coefficients of the equations of motion have been determined from forced oscillation model experiment and, similarly, the wave exciting forces and moments have been measured (Smith [15]). This model, therefore, provides a standard for reference which not only demonstrates the accuracy of the motion computations for a conventional hull form but also provides a detailed standard for the various terms in the equations of motion. The second, (Figure 2) the DD 692 class, is a conventional destroyer hull for which the motion characteristics were determined experimentally at the Davidson Laboratory. (Breslin and Eng [10]). This particular ship, like the Friesland class frigate, is a form for which the motion computation program is known to work well. A comparison between the Davidson Laboratory experiment for this hull and computed values would, therefore, in effect be a comparison of motion responses obtainable from experiments in the two tanks. The third, (Figure 3) a Davidson type A destroyer, is a ship with a conventional afterbody, but with a strongly bulged forebody. The forebody sectional shapes are of unconventional design with a narrow water line but which widens with increasing draft. Sectional shapes of this type are ones which the Lewis form transformation either fits badly or, as in the case of sections 14 through 20, does not even exist as a simply connected shape. This ship, therefore, provides an excellent test of the program's multiple coefficient transformation capability. Also, a comparison between such a close fit computation and experiment should provide an indication as to whether a potential solution and modified strip theory can properly represent the hydrodynamics of a radically flared or bulbous section, or whether such non linear effects as eddy currents, flow separation, viscosity, etc. are sufficiently large to significantly affect the computation accuracy. It could further indicate conclusively whether or not the effect on the motion due to hull shape variation can be accounted for using such a theory. Accordingly, a set of close fit transformation coefficients were obtained for each of the ship forms and these in turn were used in the computation of pitch and heave motion responses for a range of wave length and ship speeds. The speeds considered were Fn = .15, .25, .35, .45, and .55. The wave lengths considered were for a range from $L/\lambda = .3$ to $L/\lambda = 2.5$.

For the Friesland class frigate computed results were compared with experimentally obtained motion responses and phase angles. The DD 692 and Davidson type A were compared with experimental results for three wave lengths as extracted from the Breslin and Eng report [10] .

Motion Tests (Friesland Class)

The Friesland class hull form was tested by the Delft Shipbuilding Laboratory at both model and full scale. Since the results from the model and full scale tests were virtually identical as far as motion responses are concerned, only the model test results are used for comparison. The model was tested in regular long crested head waves for pitch and heave motions. The model length was 2.81m and was operated with a radius of gyration of $.25L_{oa}$ or $.259L_{pp}$.

All testing was done in regular long crested head waves with a peak to peak height of approximately $L_{pp}/40$. Wave lengths were varied from $L_{pp}/\lambda = .5$ to $L_{pp}/\lambda = 2.0$. Testing was done for a range of Froude numbers from $F_n = .15$ to $F_n = .55$. Test conditions are summarized in Table 2.

Table 2
Model Test Conditions

Speed	$F_n =$	.15, .25, .35, .45, .55
Wave length ratio	$L/\lambda =$	.500, .555, .625, .714, .833, 1.000, 1.250, 1.670, 2.000
Wave height ratio	$2\zeta_a/L = 1/40$	

Motion Tests (DD 692 and Davidson Type A

The motion test results as extracted from the Breslin and Eng report [10] were performed in regular waves of .75, 1.0, and 1.25 times the model length over a range of Froude numbers from 0 to .60. The wave height (double amplitude) used in these tests was 1/40 model length.

A comparison of the pitch and heave motions made in this report between the Davidson type A and the DD 692 shows a remarkable reduction in pitch for all Froude numbers above $F_n = .15$.

Calculations

The calculations are based on a form of the strip theory originally developed by Korvin-Kroukovsky [1] and modified and extended by Gerritsma [5] , [6] . Briefly, the procedure is as follows:

(1) The ship is divided up into a number of sections and the individual sections are each represented by a set of (y, z) offset values. Depending on the severity of the sectional shape, an adequate representation is provided by 15 to 30 offset values evenly spaced around the periphery.

(2) Transformation coefficients are computed using the (y, z) offset values in a itterative process which is permitted to converge until the root mean square difference between the actual sections (offset values) and the transformed shape is as small as desired.

(3) The two dimensional added mass, damping, and the variation of added mass ($\frac{dm}{dx}$) longitudinally along the ship are computed for each of the sections by methods from [7, 9].

(4) A modified form of strip theory [5] is used to determine the coefficients of the equations of motion for the various frequencies and speeds of advance.

(5) Exciting forces are computed for each section using [6].

(6) The equations of motion are solved and the complex frequency response functions are computed for the speeds and frequencies desired.

The heave and pitch equations of motion assuming negligible coupling between the other four modes of motion are:

$$
\begin{aligned}
\rho \nabla \ddot{z} &= F \\
\rho \nabla k_{yy}^{2} \ddot{\theta} &= M
\end{aligned}
\qquad (1)
$$

In terms of the force and moment distributions along the ship force F and the moment M are:

$$
\begin{aligned}
F &= \int_L F' dx_b \\
M &= - \int_L F' x_b dx_b
\end{aligned}
\qquad (2)
$$

where:

z	- heave displacement
θ	- pitch displacement
F	- total vertical force on the ship
M	- total pitch moment on the ship
F'	- vertical force on a section
x_b	- longitudinal ship coordinate
k_{yy}	- radius of gyration in pitch

Dividing the ship into sections and employing a modified form of strip theory which includes forward speed effects, the sectional force is:

$$
\begin{aligned}
F' = & -2\rho g\, y_w \left(z_b - x_b\theta_b - \zeta^{*}\right) \\
& - N' \left(\dot{z}_b - x_b\dot{\theta}_b + V\theta_b - \dot{\zeta}^{*}\right) \\
& - \frac{d}{dt}\left[m' \left(\dot{z} - x_b\dot{\theta}_b + V\theta - \dot{\zeta}^{*}\right)\right]
\end{aligned}
\qquad (3)
$$

V - forward speed of the ship

y_w - half width of water line

m' - sectional added mass

N' - sectional damping

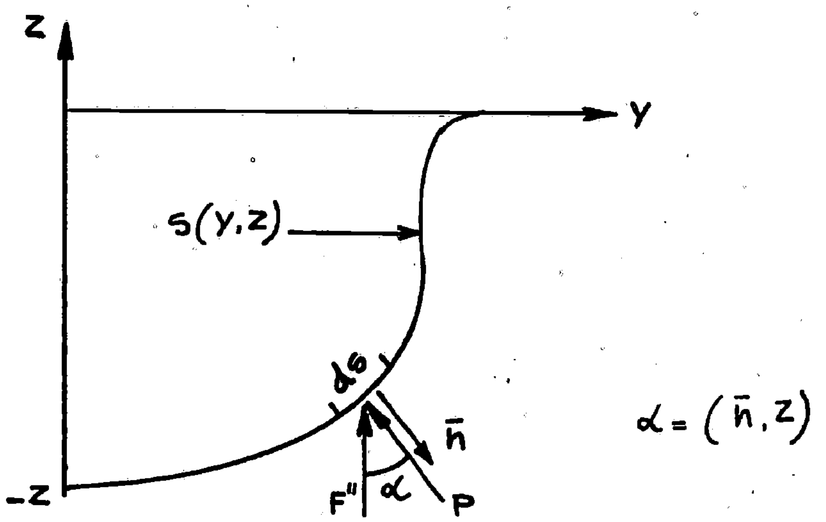
T - draft of a section

ζ - instantaneous wave elevation

$$\zeta^* = \zeta (1 - \frac{k}{y_w} \int_{-T}^{0} y_b \, e^{kz_b} \, dz_b)$$

For a particular section and considering only the hydrodynamic part of the force, the vertical component of the force per unit area on the section surface S (y, z) is:

$$F'' = -P\cos(n,z) \qquad (4)$$



or dividing into parts in phase with the acceleration and with the velocity:

$$F'' = m'' \ddot{z}_b + N'' \dot{z}_b \qquad (5)$$

F'' - vertical force per unit area on section

S (y, z) - surface of section

m'' - added mass per unit surface area

$\frac{dm''}{dx_b}$ - local rate of change of added mass in the x_b direction

P - pressure on the sectional surface

N'' - damping per unit surface area.

Therefore the sectional damping and added mass become:

$$m' = \int_S m'' \, ds \tag{6}$$

$$N' = \int_S N'' \, ds \tag{7}$$

$$\frac{dm'}{dx_b} = \int_S \frac{dm''}{dx_b} \, ds \tag{8}$$

Rearranging (3) and retaining on the right only terms representing wave forces, the equationsof motion become:

$$(a + \rho\nabla)\ddot{z} + b\dot{z} + cz - d\ddot{\theta} - e\dot{\theta} - g\theta = F_a \cos(\omega_e t + \varepsilon_{F\zeta}) \tag{9}$$

$$(A + \rho\nabla k_{yy}^2)\ddot{\theta} + B\dot{\theta} + C\theta - D\ddot{z} - E\dot{z} - Gz = M_a \cos(\omega_e t + \varepsilon_{M\zeta})$$

From references [1], [5] the coefficients are:

$$a = \int_L m' \, dx_b \tag{10}$$

$$b = \int_L N' \, dx_b - V \int_L \frac{dm'}{dx_b} dx_b$$

$$c = \rho g A_w$$

$$d = \int_L m' \, x_b \, dx_b$$

$$e = \int_L N' x_b \, dx_b - 2Va - V \int_L \frac{dm'}{dx_b} x_b \, dx_b \tag{10}$$

$$g = \rho g S_w - Vb$$

$$A = \int_L m' x_b^2 \, dx_b$$

$$B = \int_L N' x_b^2 \, dx_b - 2VD - V \int_L \frac{dm'}{dx_b} x_b^2 \, dx_b$$

$$C = \rho g I_w - VE$$

$$D = \int_L m' x_b \, dx_b$$

$$E = \int_L N' x_b \, dx_b - V \int_L \frac{dm'}{dx_b} x_b \, dx_b$$

$$G = \rho g S_w$$

From reference (6) the exciting forces and moments are:

$$\frac{F_a}{\varsigma_a} \cos \varepsilon_{F\varsigma} = 2\rho g \int_L y_w \cos (kx_b) \, dx_b \tag{11}$$

$$- 2\rho\omega^2 \int_L \int_{-T}^{0} y_b \, e^{kz_b} \cos (kx_b) \, dz_b \, dx_b$$

$$+ \omega V \int_L \int_S \frac{dm''}{dx_b} e^{kz_b} \sin (kx_b) \, ds \, dx_b$$

$$- \omega^2 \int_L \int_S m'' \, e^{kz_b} \cos (kx_b) \, ds \, dx_b$$

$$- \omega \int_L \int_S N'' \, e^{kz_b} \sin(kx_b) \, ds \, dx_b \qquad (11)$$

$$\frac{F_a}{\zeta_a} \sin \varepsilon_{F\zeta} = 2\rho g \int_L y_w \sin(kx_b) \, dx_b$$

$$- 2\rho\omega^2 \int_L \int_{-T}^{0} y_b \, e^{kz_b} \sin(kx_b) \, dz_b \, dx_b$$

$$- \omega V \int_L \int_S \frac{dm''}{dx_b} \, e^{kz_b} \cos(kx_b) \, ds \, dx_b$$

$$- \omega^2 \int_L \int_S m'' \, e^{kz_b} \sin(kx_b) \, ds \, dx_b$$

$$+ \omega \int_L \int_S N'' \, e^{kz_b} \cos(kx_b) \, ds \, dx_b$$

$$\frac{M_a}{\zeta_a} \cos \varepsilon_{M\zeta} = -2\rho g \int_L y_w \, x_b \cos(kx_b) \, dx_b$$

$$+ 2\rho\omega^2 \int_L \int_{-T}^{0} y_b \, x_b \, e^{kz_b} \cos(kx_b) \, dz_b \, dx_b$$

$$- \omega V \int_L \int_S \frac{dm''}{dx_b} \, x_b \, e^{kz_b} \sin(kx_b) \, ds \, dx_b$$

$$+ \omega^2 \int_L \int_S m'' \, x_b \, e^{kz_b} \cos(kx_b) \, ds \, dx_b$$

$$+ \omega \int_L \int_S N'' \, x_b \, e^{kz_b} \sin(kx_b) \, ds \, dx_b$$

$$\frac{M_a}{\zeta_a} \sin \varepsilon_{M\zeta} = -2\rho g \int_L y_w \, x_b \sin(kx_b) \, dx_b$$

$$+ 2\rho\omega^2 \int_L \int_{-T}^{0} y_b \, x_b \, e^{kz_b} \sin(kx_b) \, dz_b \, dx_b$$

$$+ \omega V \int_L \int_S \frac{dm''}{dx_b} x_b e^{kz_b} \cos (kx_b) \, ds \, dx_b \qquad (11)$$

$$+ \omega^2 \int_L \int_S m'' x_b e^{kz_b} \sin (kx_b) \, ds \, dx_b$$

$$- \omega \int_L \int_S N'' x_b e^{kz_b} \cos (kx_b) \, ds \, dx_b$$

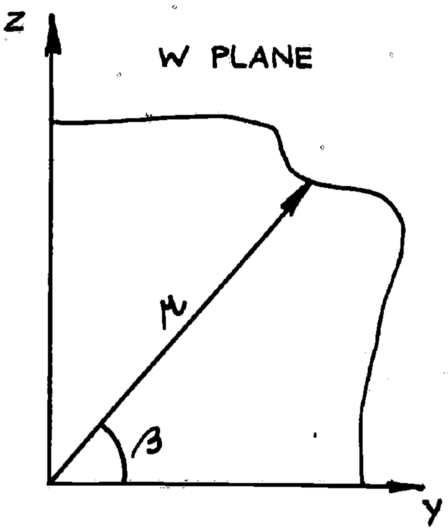
where:

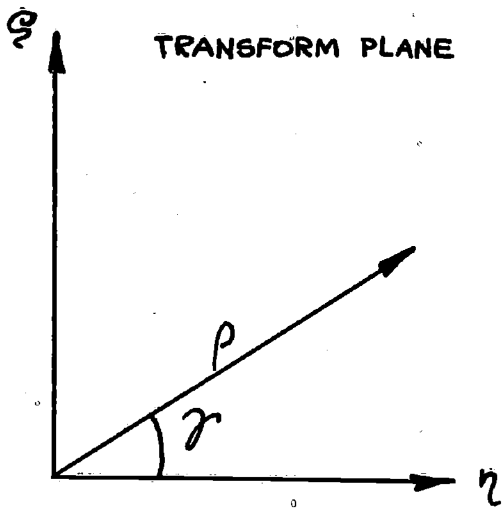
S -is the surface of the section.

K -is equal to ω^2/g.

Transformation Coefficients

For the transformation coefficients a numerical method is used to generate a set of coefficients which conformally maps the exterior of the unit circle $|\delta| \geq 1$ into the exterior of a given simply connected region. For this program the boundary of the region may be given analytically, or by a discreet set of (y, z) points, i.e. a table of offset values (y_i, z_i).





The mapping function is:

$$W = \sum_{n=-1}^{N} C_n \delta^{-n} = \sum_{n=-1}^{N} (A_n + i B_n) (\cos n\gamma - i \sin n\gamma) \qquad (12)$$

where:

$$W = y + iz = re^{i\beta}$$

and:

$$\delta = \rho e^{i\gamma}$$

The notation, which is somewhat different from that used by Tasai, Porter, etc. was selected to conform with the standard right-handed coordinate system normally used to describe ship motions (Figure 4).

From equation 12 for a particular set of offset values we have:

$$y_i = \sum_{n=-1}^{N} (A_n \cos n\gamma_i + B_n \sin n\gamma_i) \qquad (13)$$

$$z_i = \sum_{n=-1}^{N} (- A_n \sin n\gamma_i + B_n \cos n\gamma_i)$$

$$i = 1, 2, 3 \ldots, I$$

This system of equations (13), for equally spaced arguments, is characterized by an interesting property, it is easily inverted with respect to the coefficients A_n and B_n. This is a consequence of the property of orthogonality, which trigonometric functions of discreet arguments possess in the case of equally spaced points (Krylov [11]).

Inverting equation 13:

$$A_n = \frac{1}{I} \sum_{i=1}^{I} (y_i \cos n\gamma_i - z_i \sin n\gamma_i) \qquad (14)$$

$$B_n = \frac{1}{I} \sum_{i=1}^{I} (y_i \sin n\gamma_i + z_i \cos n\gamma_i)$$

$$n = -1, 0, 1, 2 \ldots, I-2$$

Equation 14 permits the coefficients A_n and B_n to be easily calculated by an itterative numerical process which can provide transformation coefficients which transforms a simply connected region with any reasonable preassigned accuracy (sectional fit).

The coefficient program is designed to handle any simply connected shape, symmetrical or assymetrical, with respect to the coordinate axis. Further, it can accomodate any shape capable of being transformed with a pre-selected accuracy by not more than 256 A_n and 256 B_n. Even though the program can accomodate completely assymetrical shapes, the sectional outlines usually encountered in shipbuilding are symmetrical with respect to both the y and z axis. This, of course, refers only to the portion of the hull below the mean free surface and for the y axis symmetry the upper two quandrants are considered to be mirror images of the submerged portion. This symmetry assumption insures that all of the B_n coefficients are zero and likewise, the A_n coefficients for even n are also zero. The resultant transformation equations are:

$$y_i = \sum_{n=-1}^{N} A_{2n+1} \cos (2n+1)\gamma_i \qquad (15)$$

$$z_i = \sum_{n=-1}^{N} -A_{2n+1} \sin (2n+1)\gamma_i$$

$$i = 1, 2, 3 \ldots, I$$

or in normalized form:

$$\frac{y_i}{A_{-1}} = \cos \gamma_i + \sum_{n=0}^{N} a_{2n+1} \cos (2n+1)\gamma_i \qquad (16)$$

$$\frac{z_i}{A_{-1}} = \sin \gamma_i - \sum_{n=0}^{N} a_{2n+1} \sin (2n+1)\gamma_i$$

where

$$A_{-1} = \frac{y_w}{1+\sum_{n=0}^{N} a_{2n+1}}$$

which may be treated as a scale factor.

For symmetrical shapes represented by equations (15) which include all of the sections considered in this paper, the computation time and the number of coefficients required are quite modest. For example, only five coefficients and 25 seconds of computer time were required to obtain a representation of the Davidson Type A midship section. The relatively radical section 19 of the same ship form required six minutes and 16 coefficients.

A variety of sectional shapes have been mapped with this program, including such extremes as rectangles, triangles, sections with bilge keels, and sections with anti-pitch fins. In every case an extremely close fit was obtained.

Discussion

The computation method for the three ships was as follows: Each of the ships was represented by 21 cross sections which, as is the practice in naval architecture, were evenly spaced along the ship with the first cross section located at the aft perpendicular and the 21st cross section at the forward perpendicular. Each of the cross sections was represented by a table of 20 (y, z) offset values. For the Friesland appropriate offset values for each section were obtained from a master table of offsets provided by the ship's designers. The required values for the DD 692 and Davidson type A were taken from body plan diagrams provided in the Breslin, Eng report [10]. The offset values for each cross section were selected so that they were approximately evenly spaced around the periphery of the half section lying between the load water line, $\beta = 0$, and the keel, $\beta = \pi/2$. It should be emphasized that, while this is contrary to the normal ship designers practice of using evenly spaced water lines, the equal spacing around the periphery is very necessary to insure a proper fit by the transformation coefficients.

The offset values for the 21 sections were used as input to the transformation coefficient program. For the ships considered here, an itterative fitting process was allowed to continue for each section until the sum of the square of the difference between the 20 new or transformed values and the actual or original offset values was less than .01 percent of the mean beam A_x/T_x. The transformed shapes so obtained were compared with the original cross sections and in every case, including the rather radical shapes of the Davidson type A forebody, the two were virtually identical. The convergence criteria of 1.0 percent A_x/Tx has been found to be sufficient for all normal computations. The normalized coefficient values obtained for the three ships are given in Tables 3, 4 and 5.

The 21 sets of transformation coefficients obtained for each ship were then used to calculate the pitch and heave motion responses.

During the motion computations intermediate values such as sectional added mass and damping, coefficients of the equation of motion, exciting forces and moments were obtained. This, therefore, permits a comparison and evaluation of these intermediate values as well as the motion characteristics. The motions and intermediate values were computed for a number of wave lengths and ship speeds.

(1) Friesland Class

The motion comparison between computation and experiment for the Friesland was quite good, with virtually perfect agreement for all conditions except Froude number .55. In this case, the computer values for the pitch amplitude are slightly higher than experiment. It should also be noted that the experimental values shown for this ship have also been compared with full scale measurements, Gerritsma, Smith [6] where the agreement again was almost perfect. In the case of the full scale comparison paper a Lewis form (three coefficient) transformation was used. The Lewis form computer results showed small differences at the higher frequencies, even though for this ship the Lewis form fit is a good one. The close fit program has produced even better agreement. The differences between the two computation methods are insignificant when considering the design aspects of ship motions, but are in themselves interesting since they demonstrate that a close fit computation is capable of accounting for small differences in hull shape. Also, it provides an excellent check on the correctness of programming and numerical analysis aspects of the close fit program.

(2) DD 692

The comparison between computation and experiment for the DD 692 is a comparison between close fit computer results and Davidson laboratory experimental results extracted from the Breslin, Eng report [10]. The motion amplitude comparison generally gave only a fair agreement, with the pitch motion amplitudes agreeing better than the heave. The experimental values are generally higher than those from computation, with the largest differences occuring at the lower frequencies. Also, it should be pointed out that this is only a limited comparison, since experimental data is available for only three wave frequencies. As this ship is one of a class or type, for which both the Lewis form and close fit computations have always shown good agreement with experiments, such a comparison of computation and experiment is, in effect, a comparison between motion responses obtainable from experiments in the two tanks. There is apparently a rather large difference between the experiments in the two tanks, especially in the heave amplitudes, and is thought to be of sufficient significance to warrant additional investigation.

(3) Davidson Type A

The Davidson type A results are also a comparison between close fit computation and Davidson laboratory experiments. The Davidson experiments for this ship show a remarkable reduction in pitch amplitudes at high speed when compared with more conventional ships. It was felt that such an unusual form would be an excellent example for the investigation of the accuracy limitations inherent in the close fit multiple transform computation method. Of greater interest is the fact that a specific change in a hull design has produced such a large and clearly definable variation in the motion. Here, then, is an ideal situation for investigating the equation of motion terms which are responsible for this change and their relationship to the shape of the hull. With this objective in mind, the computed values of all equation of motion terms for the Davidson type A and the Friesland were compared. Also, the distribution of added mass, damping and exciting forces along these ships was investigated.

When comparing the computed and measured motions for the Davidson type A, the results are remarkably good. Of foremost interest is the nearly perfect agreement between Davidson experiment and computed pitch motions at all speeds. The large reductions in pitch amplitude as shown in the experiments are also clearly shown in the computation. This in itself provides convincing proof as to the validity of the modified strip theory for even radically shaped hull forms. The computed heave motions do not show as good agreement for Fn .15 and .25. In these instances the computed heave motion amplitude is overestimated near resonance. The general agreement is good for the limited amount of experimental data available; however, a more detailed experiment over the entire frequency range of comparison will be necessary for a completely conclusive evaluation.

The dynamic coefficients of the motion equations (a, b, d, e, A, B, D, E) are given in Figures 13 and 14. Computed values only are given for the Davidson type A and computed and experimental values for the Friesland. Results are given for Fn .15 and .45. The forward speed effects normally associated with the static restoring coefficients (C, g), equation 10, have been included in the added mass coefficients (A, d). This change in the static coefficients was made arbitrarily to facilitate comparison with experimental data. The modified coefficients are:

$$\bar{d} = d + \frac{Vb}{\omega^2} = \int_L m' \, x_b \, dx_b + \frac{Vb}{\omega^2}$$

$$\bar{g} = g + Vb = \rho g S_w$$

$$\bar{A} = A + \frac{VE}{\omega^2} = \int_L m' \, x_b^2 \, dx_b + \frac{VE}{\omega^2}$$

$$\bar{C} = C + VE = \rho g I_w$$

The experimental coefficients for the Friesland are from forced oscillation experiments (Smith [15]). As shown in the figures testing was for a number of oscillator amplitudes and frequencies. The Friesland computations and experiments show good agreement at all speeds and frequencies and again demonstrate the ability of modified strip theory to account for forward speed effects.

A comparison between the Davidson type A and Friesland coefficients shows remarkably little difference in the main added mass and damping terms (a, b, A, B), with the greatest difference less than ten percent. When the cross coupling terms (e, e, g, D, E, G) are compared, however, the situation is quite different, with the damping cross coupling terms differing by as much as 400 percent. This demonstrates the importance, in motion computation, of the cross coupling terms. Further, it indicates that differences in the motions due to hull shape variation are primarily a result of changes in the longitudinal dynamic symmetry and the resultant change in the cross coupling terms. As a demonstration of this effect, the d and e terms in the motion computation for the Davidson type A were set equal to zero. The motion computation then demonstrates the large effects of coupling (Figure 5).

The added mass, damping and wave exciting force distribution along the ship are compared. The results are given in non dimensional form.

The sectional damping is:

$$b' = N' - V \frac{dm'}{dx_b}$$

or in non dimensional form:

$$\frac{b'}{\rho \nabla} \sqrt{\frac{L_{pp}^3}{g}}$$

the non dimensional sectional added mass:

$$\frac{m' \; L_{pp}}{\rho \nabla}$$

The sectional exciting force is:

$$\frac{F' \; L_{pp}}{c}$$

The damping distribution for the forward section of the Davidson type A is unusual in that, even when forward speed effects are included, several of the sections exhibit virtually zero damping for a limited range of frequencies. Also, the same sections show nearly zero exciting forces. This, then, would appear to be a major reason for the extreme difference in the motion characteristics of the two ships, and apparently offers considerable promise as a device for tuning a ship and thus optimizing the motions. This factor in itself would seem to be of sufficient interest to warrant future investigation.

The distribution of added mass for the two ships is very similar, with only significant differences occuring in the forward part and at the higher frequencies. While the total added mass is virtually identical for both ships, the slope of the added mass distribution curve for the Davidson type A is much greater in the bow, thus indicating larger values for the speed correction term $\frac{dm}{dx}$. The damping distribution for the Davidson type A, however, is quite different, with large modifications in the two dimensional damping N' by the speed correction term. Of particular is section 20, the forward-most section, which shows a large damping at high speed even though the added mass and sectional area are zero. This is entirely forward speed effect. The exciting force distributions behave similarly to the damping term and clearly show the strong relationship between exciting forces and damping. As previously mentioned, while the distribution of added mass, damping and exciting forces for the two ships is quite different, the total or integrated value for the whole ship in each case is practically the same. This also accounts for the large differences in dynamic cross coupling coefficients. To demonstrate the large effect of the cross coupling term, the motions for Froude number .15 were computed with the d and e terms zero. The resulting motion amplitude is shown in Figure 5.

Conclusions

(1) The use of modified strip theory and a multiple coefficient transformation computation for pitch and heave motions is confirmed and extended by this comparison.

(2) The influence of variations in hull shape can be accounted for using close fit transformation methods.

(3) The large variation in dynamic symmetry or fore and aft distribution of exciting forces, moments, added mass and damping producable by hull shape variations strongly indicates that such variations can be used to optimize the motions.

(4) A close fit program which can account for the fore and aft dynamic distributions is mandatory when computing bending moments, relative motion, etc.

(5) The dynamic cross coupling terms in the equations of motion are of paramount importance when optimizing the motions.

(6) An efficient program which can generate conformal transformation coefficients for an arbitrary simply connected shape is demonstrated.

Acknowledgement.

I wish to acknowledge the continued encouragement and assistance offered by Prof. Ir J. Gerritsma and particularly to express my thanks for the opportunity to study and work under his direction.
I am also greatly indebted to the staff of the Delft Computer Laboratory for their cheerful and continuous assistance in all phases of the computation work. Of particular note is the extremely quick computation service rendered. Without such service this project would have been impossible.
The completion of this project was greatly aided by the enthusiastic assistance of the Shipbuilding Laboratory Staff. I am further indebted to Mr Ralph D. Cooper, of the Office of Naval Research, for continued advice and assistance.

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Nomenclature.

a b c d e g A B C D E G	- Coefficients of the equations of motion for heave and pitch.
A_n	- Transformation coefficient.
A_w	- Area of waterplane.
a_n	- Normalized transformation coefficient.
A_x	- Area of cross-section.
b'	- Section damping coefficient.
B_M	- Midship beam.
B_n	- Transformation coefficient.
C_B	- Block coefficient.
F	- Total vertical force on ship.
F'	- Vertical force on a section.
F''	- Vertical hydrodynamic force per unit area on section.
F_a	- Wave force amplitude on restrained ship.
$F_n = \frac{V}{\sqrt{gL_{pp}}}$	- Froude number.
g	- Acceleration due to gravity.
I_w	- Longitudinal moment of inertia of waterplane area with respect to the y_b axis.
I_{yy}	- Real moment of inertia of ship.
$k = \frac{2\pi}{\lambda}$	- Wave number.
k_{yy}	- Radius of gyration in pitch.
L_{oa}	- Length over all.
L_{pp}	- Length between perpendiculars.
M	- Total moment on ship.
M_a	- Wave moment amplitude on restrained ship.
m	- Total added mass for heave.
m'	- Sectional added mass.
m''	- Added mass per unit surface area

N' - Sectional damping (without speed effect).

N'' - Damping per unit surface area.

P - Pressure on sectional surface.

$S(y,z)$ - Section surface.

S_w - Statical moment of waterplane area.

t - Time.

T - Draft of ship.

T_x - Draft of cross-section.

V - Speed of ship.

x_b, y_b, z_b - Right-handed body axis system.

$y_w(x)$ - Half width of waterline.

z - Heave displacement.

z_a - Heave amplitude.

ε - Phase angle between the motions (forces, moments) and the waves.

ς - Instantaneous wave elevation.

ς_a - Wave amplitude.

θ - Pitch angle.

θ_a - Pitch amplitude.

γ - Transform plane angle.

λ - Wave length.

β - Physical plane angle.

ρ - Density of water.

∇ - Displacement volume.

ω - Circular frequency.

ω_e - Circular frequency of encounter.

$\frac{dm'}{dx_b}$ - Rate of change of added mass in the x_b direction.

$\frac{dm''}{dx_b}$ - Local rate of change of added mass in the x_b direction

Δ - Displaced weight.

Table 1.

Model characteristics.

	Friesland	DD692	Davidson A
Scale ratio	40	67.09	-
Length L_{pp} M.	2.810	1.741	1.741
Beam M.	.2935	.187	.185
Draft (DWL) M.	.0975	.0635	.0635
Displacement KG	44.55	10.90	10.98
Block coefficient	.554	.524	.536
Midship area coefficient	.815	.824	.778
Prismatic coefficient	.679	.636	.689
Waterplane area coefficient	.798	.762	.739
Longitudinal center of mass M $\frac{L_{pp}}{2}$	.0293 AFT	.0345 AFT	.0280 FWD
Radius of gyration pitch	.259 L_{pp}	.25 L_{pp}	.25 L_{pp}

Table 3.

Friesland Class Transformation Coefficients

Normalized Form.

Coeff.	Section 0	1	2	3	4	5	6	7	8	9	10
$Y_w(X)$	+. 0755	+. 0985	+. 11320	+. 12450	+. 13220	+. 13770	+. 1420	+. 14480	+. 14675	+. 14675	+. 14675
a_1	+.756566	+.469845	+.320917	+.207701	+.207195	+.207667	+.211572	+.213333	+.216503	+.218439	+.216715
a_3	-.011692	+.052863	+.075972	+.106067	+.061324	+.029689	+.004282	+.012338	-.020112	-.025715	-.027852
a_5	+.001154	-.019783	-.038869	-.066257	-.042457	-.029765	-.022487	-.018722	-.018272	-.018842	-.017610
a_7	+.000056	+.013053	+.029925	+.035318	+.017287	+.008908	+.006029	+.003570	+.002268	-.000020	-.000264
a_9	+.000643	-.000377	-.005396	-.011768	-.006230	-.004031	-.002504	-.001663	-.000439	-.003263	-.002677
a_{11}	+.000012	+.005580	+.007198	+.009417	+.004646	+.002923	+.001846	+.001182	+.001419	-.001406	-.000586
a_{13}	-.001163	-.003778	-.001389	+.000106	+.000776	+.001235	-.000361	-.001411	-.000603	-.001456	-.001643
a_{15}	+.0	+.0	+.0	-.000325	+.0	+.0	+.0	+ 0	+ 0	+ 0	+ 0

Coeff.	Section 11	12	13	14	15	16	17	18	19	20
$Y_w(X)$	+. 14580	+. 1440	+. 13930	+. 13090	+. 11960	+. 1047	+. 0860	+. 0623	+. 0335	+. 00210
a_1	+.212799	+.202529	+.181780	+.149260	+.102148	+.032946	-.076855	-.251106	-.543066	-.771895
a_3	-.024269	-.018025	-.009248	-.001207	+.012882	+.031142	+.048723	+.067091	+.072455	-.011656
a_5	-.017266	-.013083	-.007731	-.005303	-.001754	+.000396	+.005114	+.008321	+.006943	+.008758
a_7	+.000713	+.002804	+.004184	+.006101	+.006435	+.006409	+.009881	+.009783	+.010588	-.007105
a_9	-.002138	-.000938	-.000392	+.001176	+.000622	+.000692	+.002496	+.002557	+.001713	+.012262
a_{11}	-.000237	+.000293	+.000411	+.000798	+.001650	+.002305	+.003241	+.003026	+.005503	-.005027
a_{13}	-.000659	-.000219	-.000114	-.000377	+.000754	+.000734	+.000835	+.000436	-.000442	-.000376
a_{15}	+ 0	+ 0	+ 0	+ 0	+ 0	+ 0	+ 0	+ 0	+ 0	+ 0

Table 4.

DD 692 Transformation Coefficients

Normalized Form.

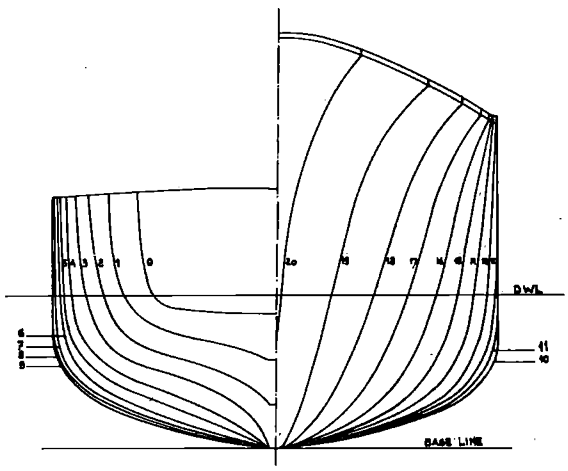
Coeff.	Section.										
	0	1	2	3	4	5	6	7	8	9	10
$Y_w(X)$		+. 0640	+. 0732	+. 0802	+. 0851	+. 0887	+. 0913	+. 0926	+. 0935	+. 0935	+. 0935
a_1	0	+.503369	+.417424	+.345471	+.276803	+.223354	+.188119	+.177393	+.177750	+.175243	+.170747
a_3	0	+.004956	+.018417	+.017416	+.013005	+.005442	-.004492	-.020053	-.028284	-.036155	-.032108
a_5	0	-.043042	-.024534	-.014502	-.010177	-.005611	-.011570	-.003672	-.000034	+.001081	+.001151
a_7	0	+.019135	-.001778	+.003559	+.001562	+.006130	+.010788	+.004691	+.002839	+.001459	+.000873
a_9	0	0	0	0	0	0	0	0	0	0	0
a_{11}	0	0	0	0	0	0	0	0	0	0	0

Coeff.	Section									
	11	12	13	14	15	16	17	18	19	20
$Y_w(X)$	+. 0914	+. 0876	+. 0822	+. 0747	+. 0654	+. 0545	+. 0420	+. 0283	+. 0139	0
a_1	+.163304	+.145807	+.116446	+.068515	-.000753	-.092987	-.227172	-.409429	-.628993	0
a_3	-.027961	-.015651	-.003523	+.011781	+.025146	+.033940	+.040215	+.037197	-.019362	0
a_5	+.002512	+.001560	+.001432	+.002266	+.002546	-.000376	-.002547	-.001702	-.026939	0
a_7	+.003814	+.005334	+.005635	+.009073	+.009086	+.011585	+.010463	+.004567	+.025548	0
a_9	0	0	0	0	0	0	0	0	+.010061	0
a_{11}	0	0	0	0	0	0	0	0	-.012814	0

Table 5. DAVIDSON TYPE A TRANSFORMATION COEFFICIENTS, NORMALIZED FORM.

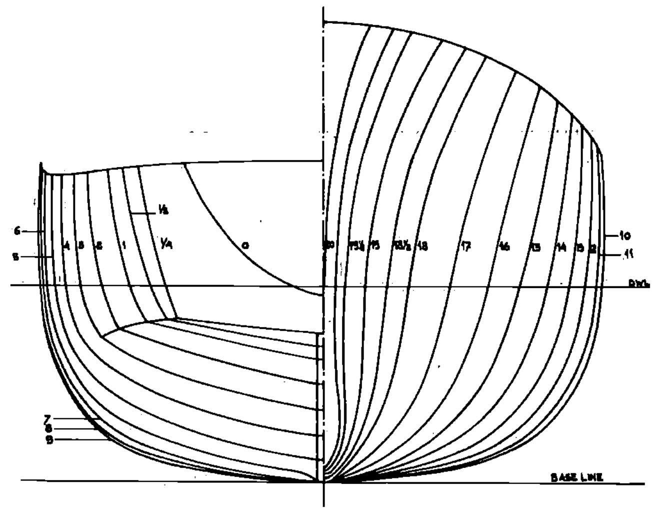
	Section 0	1	2	3	4	5	6	7	8	9	10
$Y_w(x)$	+0.0548	+0.066100	+0.075400	+0.081800	+0.085300	+0.087800	+0.089400	+0.091000	+0.092400	+0.092700	+0.092700
a_1	+0.634763	+0.538712	+0.475447	+0.405580	+0.337851	+0.276044	+0.227559	+0.204095	+0.196954	+0.194498	+0.193795
a_3	+0.008824	+0.008272	+0.016456	+0.033001	+0.040889	+0.049993	+0.061934	+0.048229	+0.030584	+0.016564	+0.004721
a_5	-0.018587	-0.032341	-0.029574	-0.028604	-0.029224	-0.031944	-0.031540	-0.016625	-0.006254	-0.003277	-0.002115
a_7	-0.014569	+0.000787	+0.006433	+0.004568	+0.009562	+0.012645	+0.008042	-0.001761	-0.002119	-0.004268	-0.004379
a_9	0	0	0	0	0	0	-0.004427	0	0	0	0

	Section 11	12	13	14	15	16	17	18	19	20
$Y_w(x)$	+0.091300	+0.085900	+0.077500	+0.066600	+0.056000	+0.043500	+0.031500	+0.021100	+0.012000	+0.000100
a_1	+0.182833	+0.159812	+0.129027	+0.087717	+0.044207	-0.008312	-0.069047	-0.156982	-0.321085	0
a_3	-0.000003	-0.013634	-0.037071	-0.063870	-0.092086	-0.115450	-0.133027	-0.153040	-0.167356	+0.849541
a_5	-0.002151	-0.006390	-0.023765	-0.038991	-0.053038	-0.062931	-0.065924	-0.077415	-0.079463	0
a_7	-0.002419	-0.004420	-0.015333	-0.024243	-0.029453	-0.039177	-0.044839	-0.051034	-0.053505	0
a_9	0	0	0	-0.016527	-0.017523	-0.029847	-0.035147	-0.036673	-0.036656	0
a_{11}	0	0	0	-0.007892	-0.011381	-0.021379	-0.024859	-0.027850	-0.028859	0
a_{13}	0	0	0	0	-0.011643	-0.018640	-0.021567	-0.023534	-0.022944	0
a_{15}	0	0	0	0	-0.008125	-0.012810	-0.014849	-0.016615	-0.016122	0
a_{17}	0	0	0	0	-0.006435	-0.011540	-0.014018	-0.016084	-0.016587	0
a_{19}	0	0	0	0	0	-0.008154	-0.010711	-0.011900	-0.009329	0
a_{21}	0	0	0	0	0	0	-0.008382	-0.009449	0	0
a_{23}	0	0	0	0	0	0	-0.008175	-0.008663	0	0
a_{25}	0	0	0	0	0	0	-0.007344	-0.007466	0	0
a_{27}	0	0	0	0	0	0	-0.004600	-0.005942	0	0
a_{29}	0	0	0	0	0	0	-0.004671	-0.005488	0	0
a_{31}	0	0	0	0	0	0	-0.004484	-0.004837	0	0



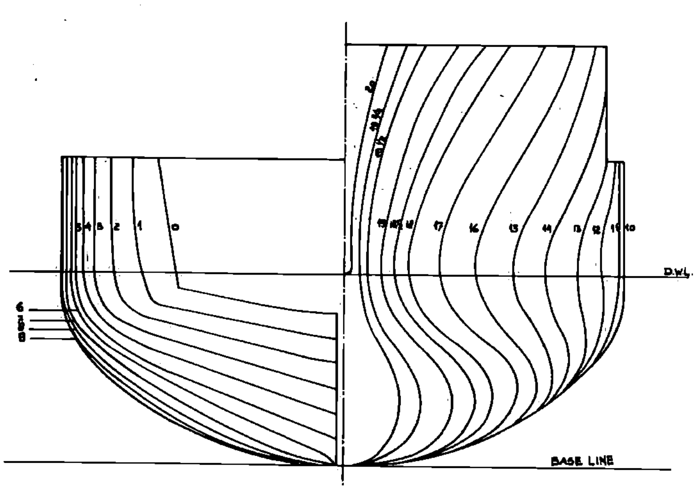
FRIESLAND CLASS FRIGATE

FIGURE 1



DD-692 DESTROYER

FIGURE 2



DAVIDSON TYPE A DESTROYER

FIGURE 3

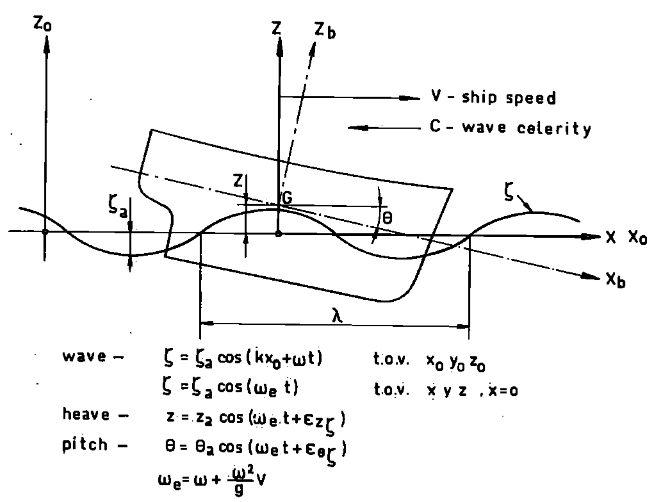
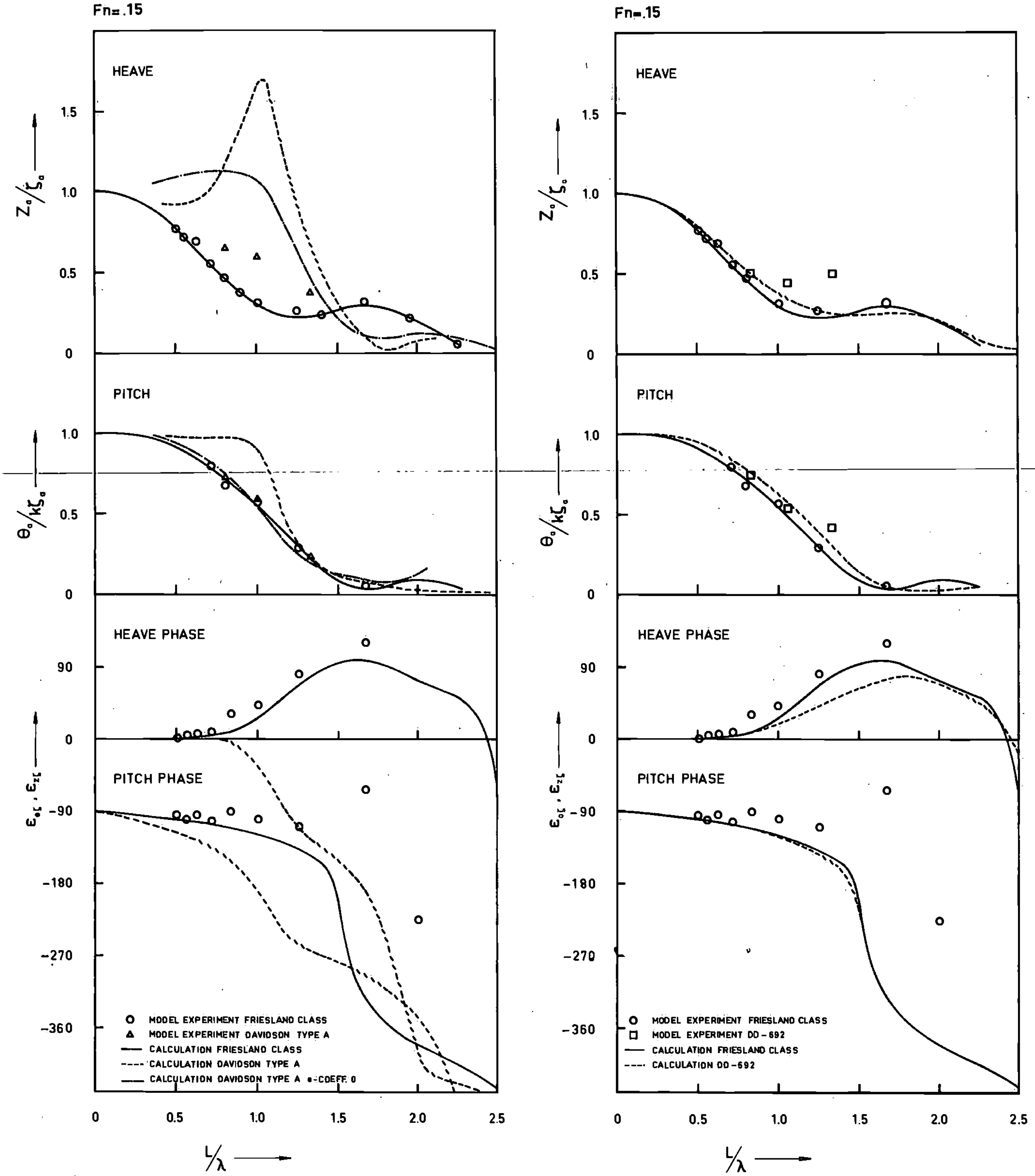
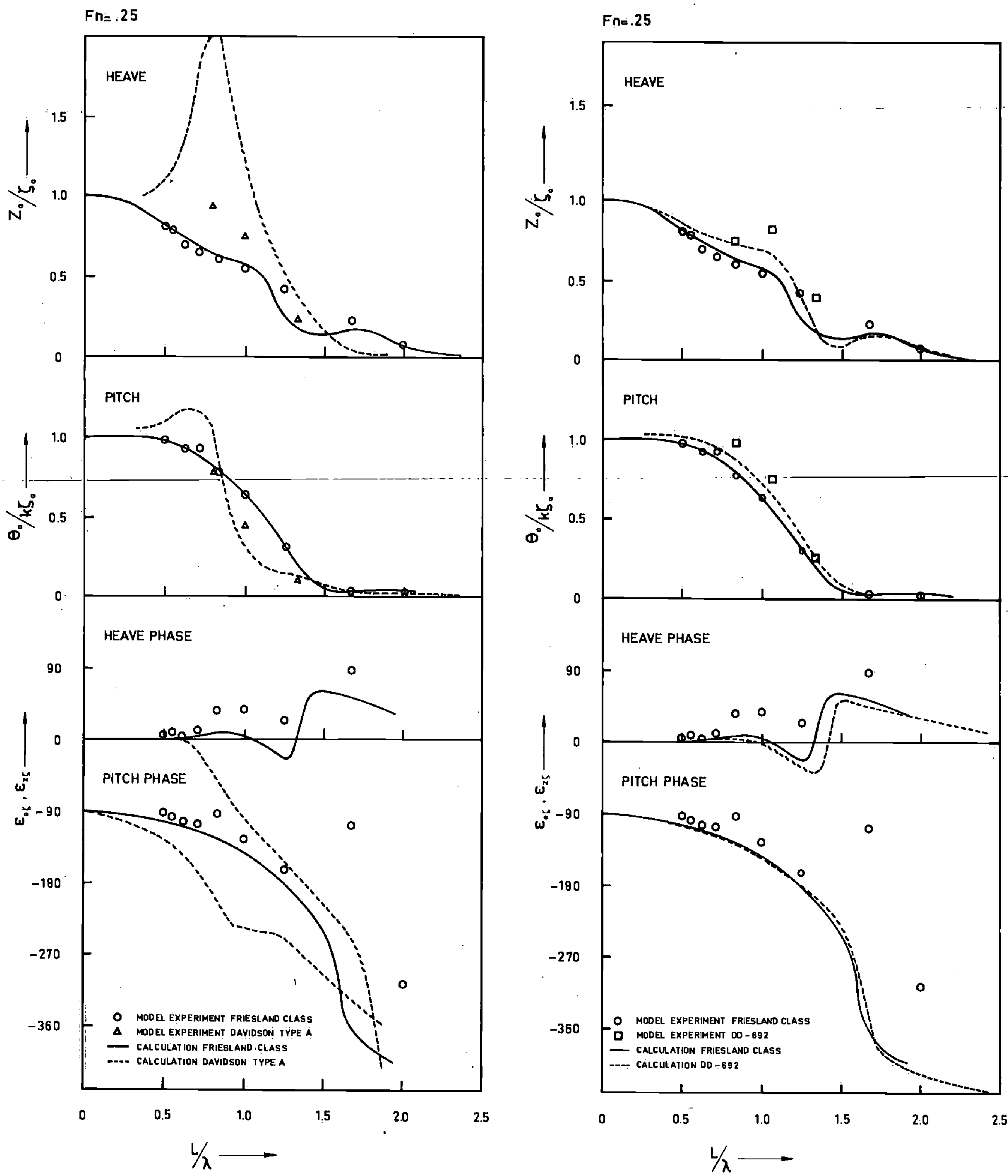


Figure 4. Definition of wave and motions



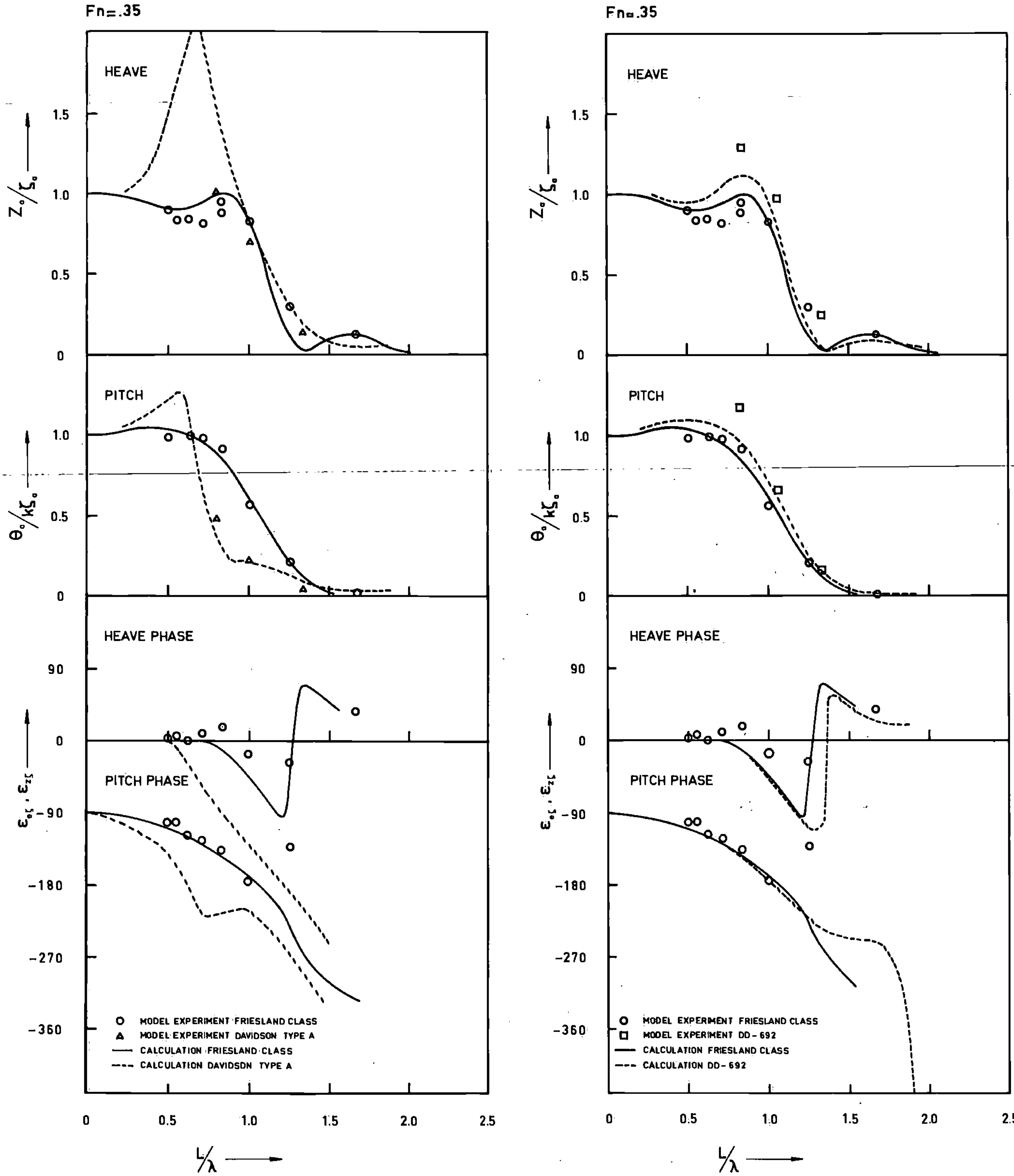
COMPARISON OF CALCULATION AND MODEL EXPERIMENT Fn=.15

FIGURE 5



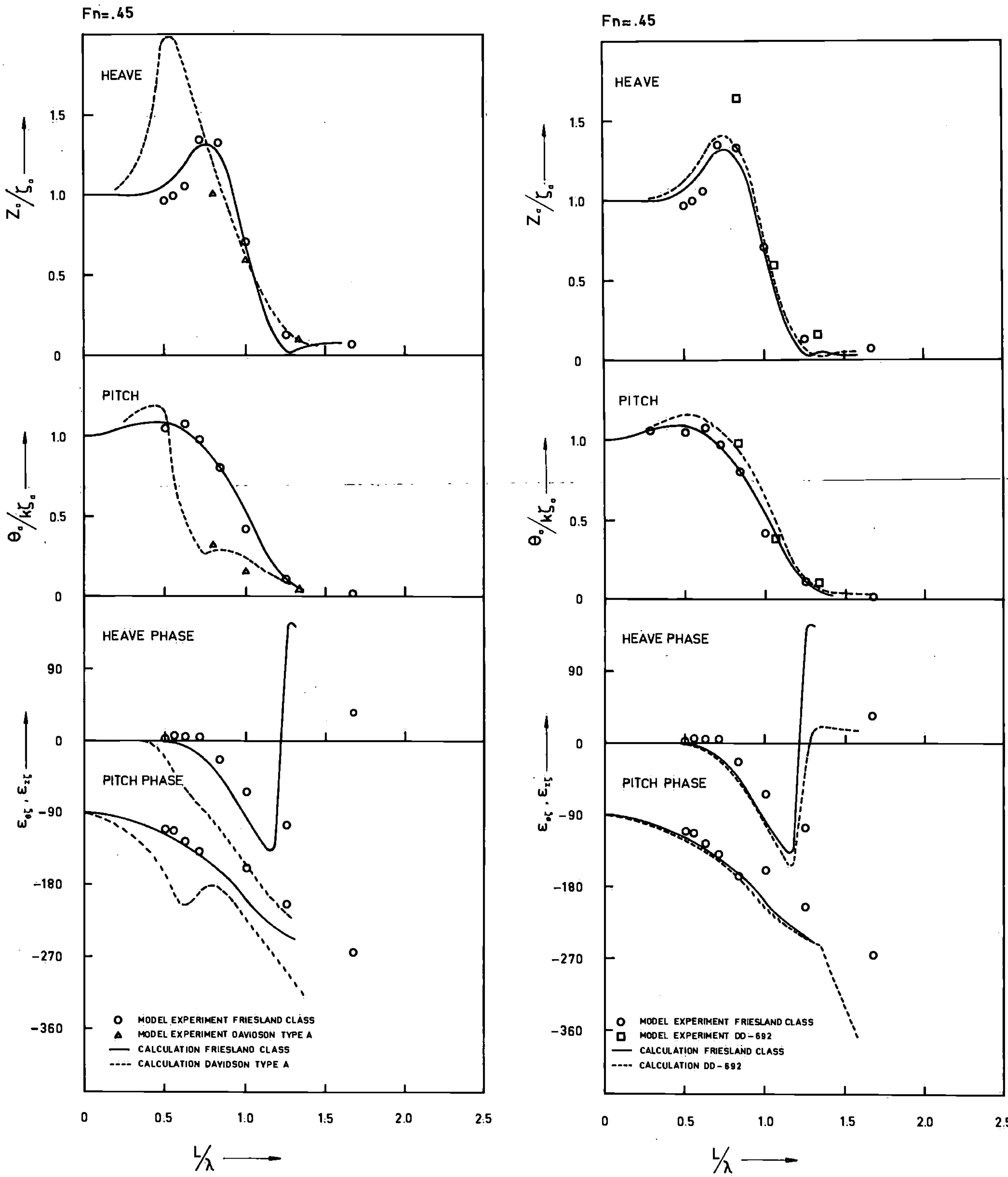
COMPARISON OF CALCULATION AND MODEL EXPERIMENT Fn=.25

FIGURE 6



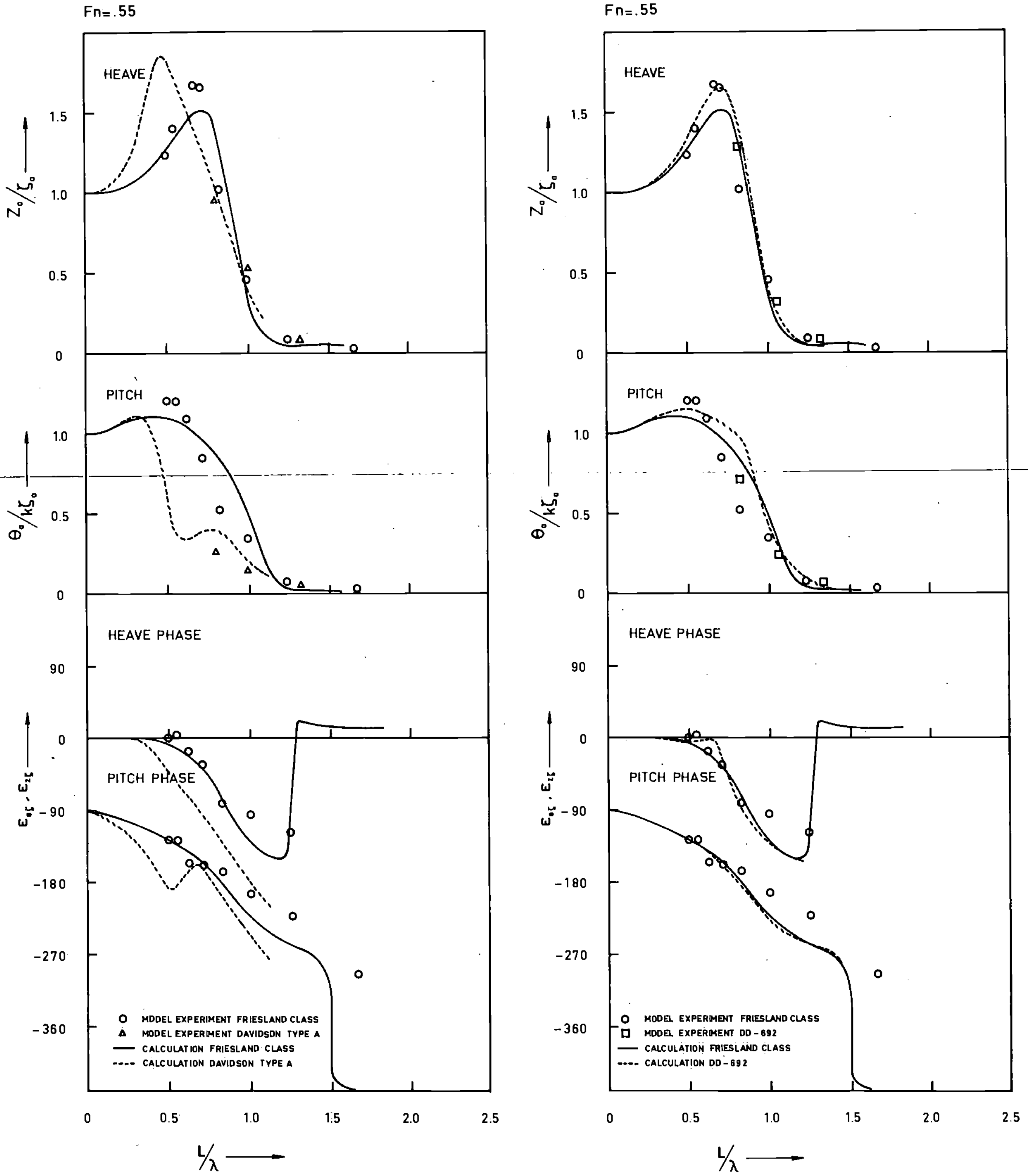
COMPARISON OF CALCULATION AND MODEL EXPERIMENT Fn=.35

FIGURE 7



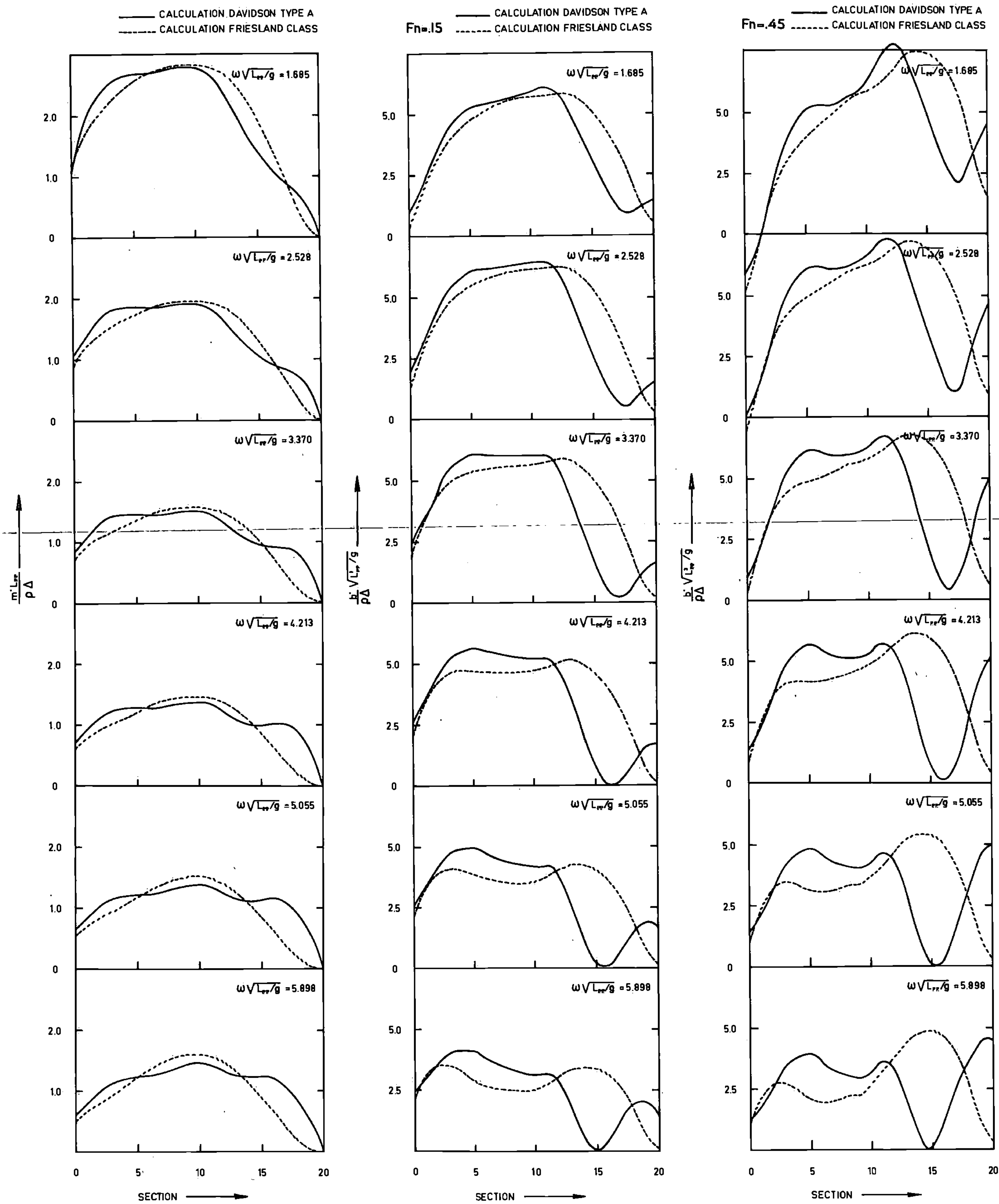
COMPARISON OF CALCULATION AND MODEL EXPERIMENT Fn=.45

FIGURE 8



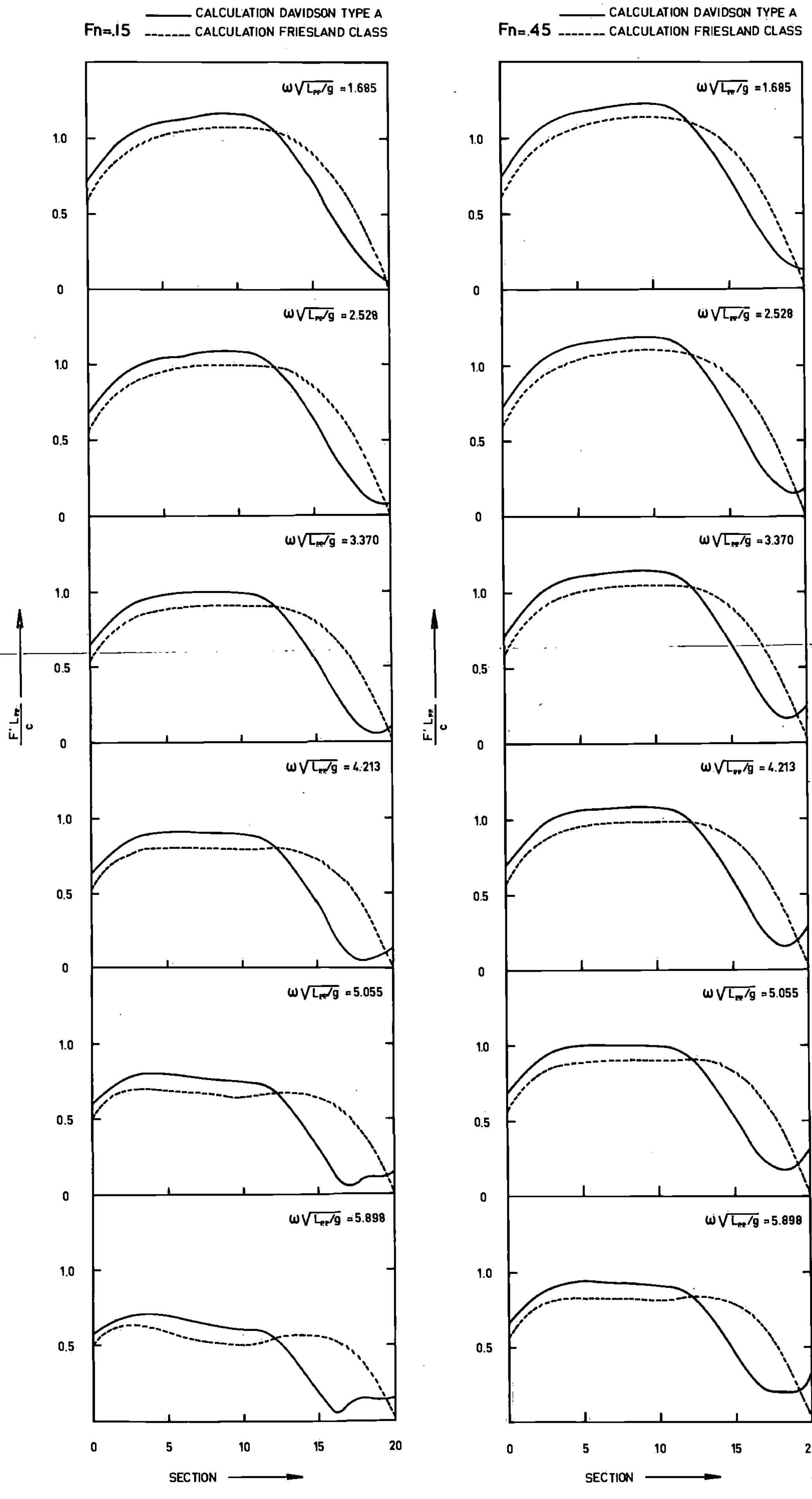
COMPARISON OF CALCULATION AND MODEL EXPERIMENT Fn=.55

FIGURE 9



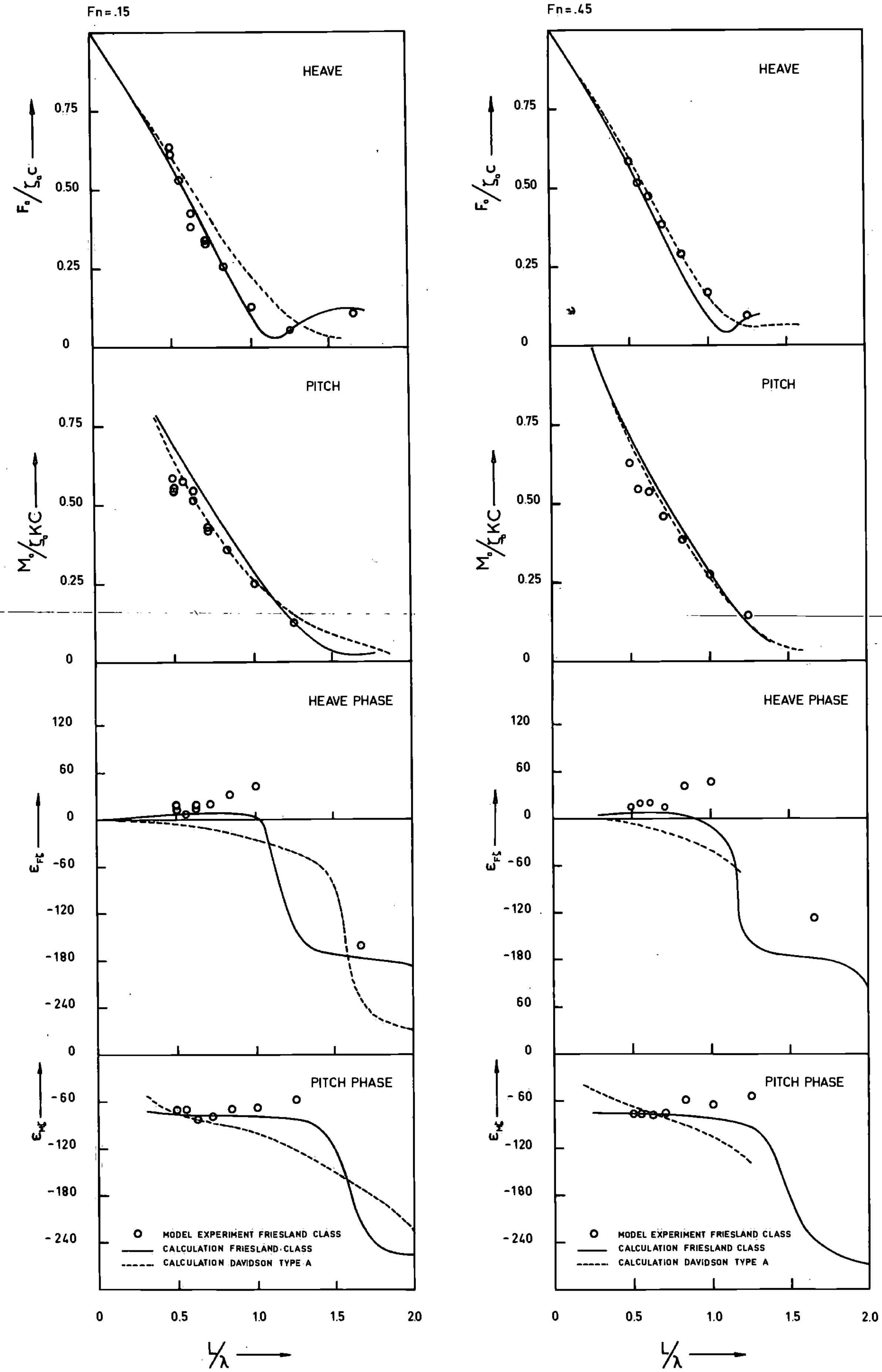
ADDED MASS AND DAMPING DISTRIBUTION Fn=.15 AND Fn=.45

FIGURE 10



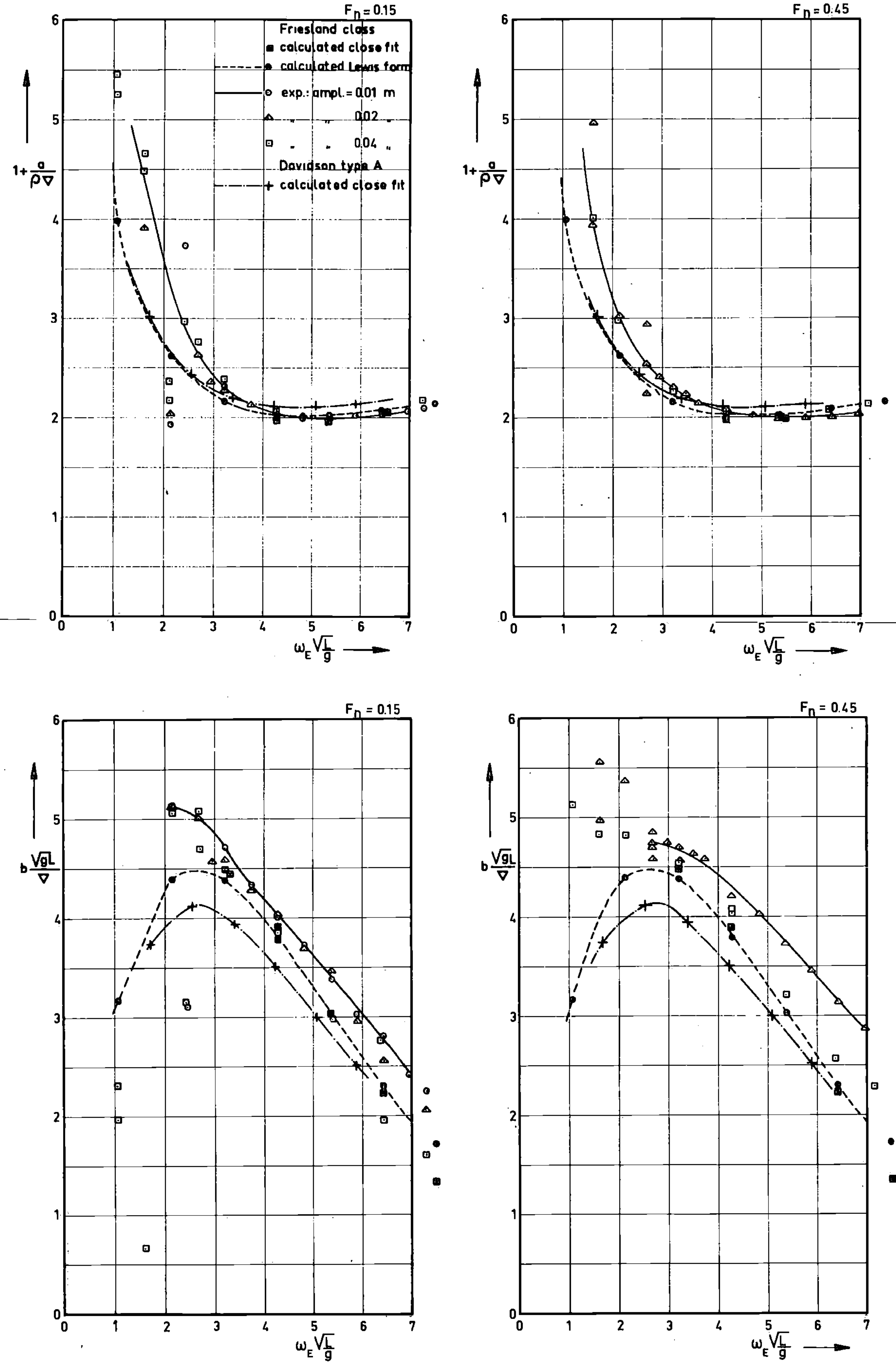
EXCITING FORCE DISTRIBUTION Fn=.15 AND Fn=.45

FIGURE 11



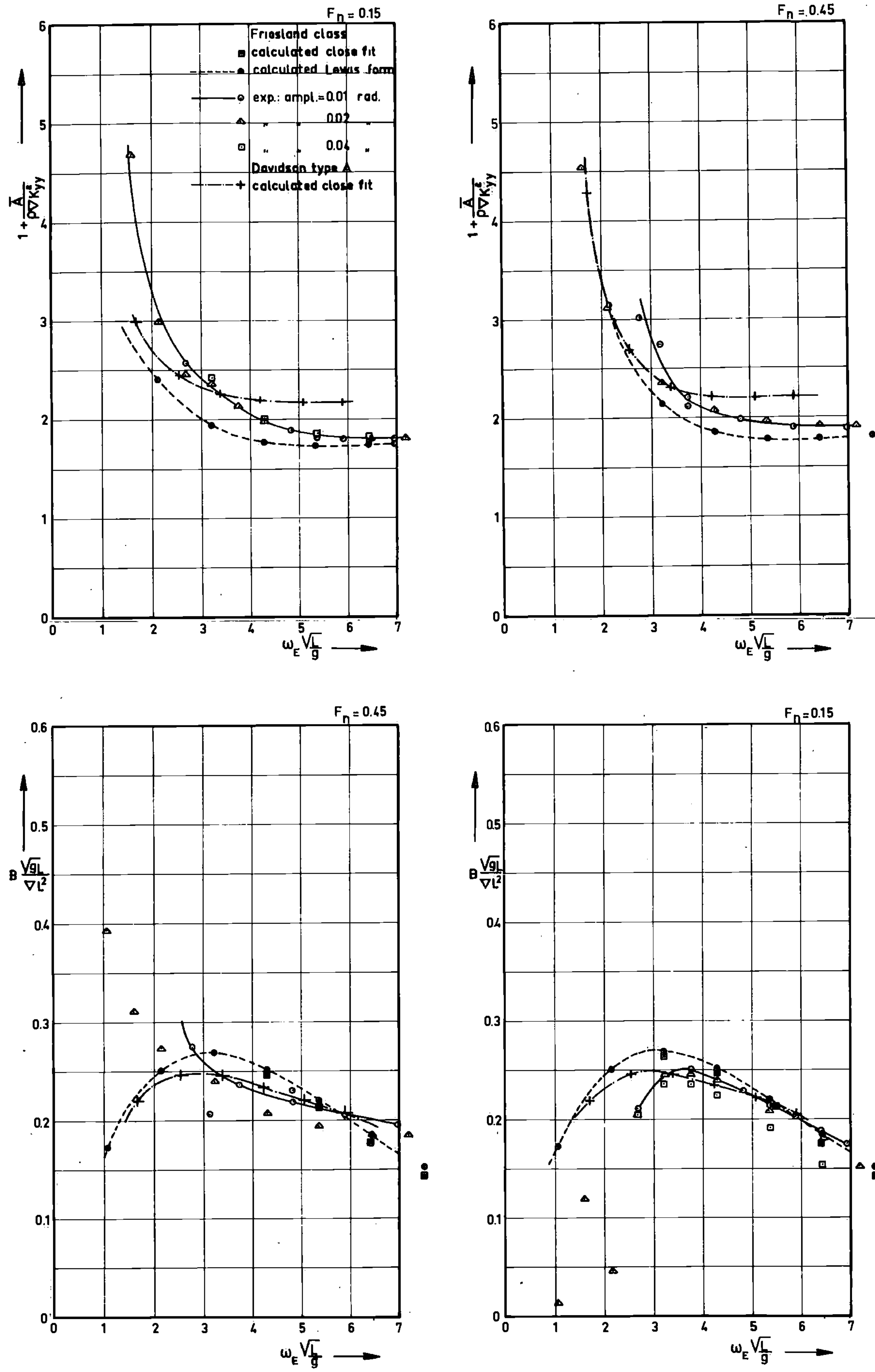
TOTAL EXCITING FORCES CALCULATION AND EXPERIMENT Fn=.15 AND Fn=.45

FIGURE 12



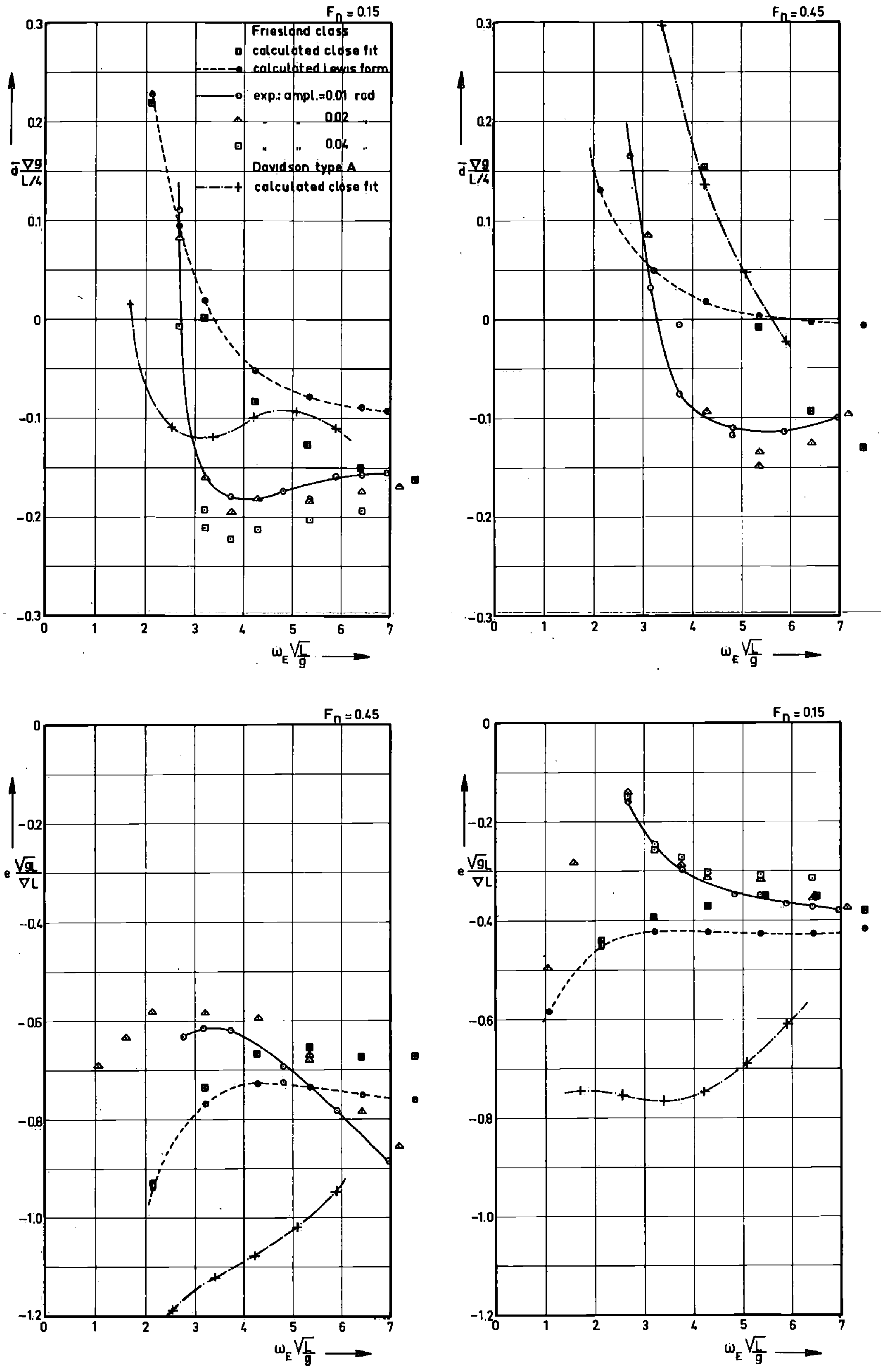
EQUATION OF MOTION COEFFICIENTS a AND b

FIGURE 13



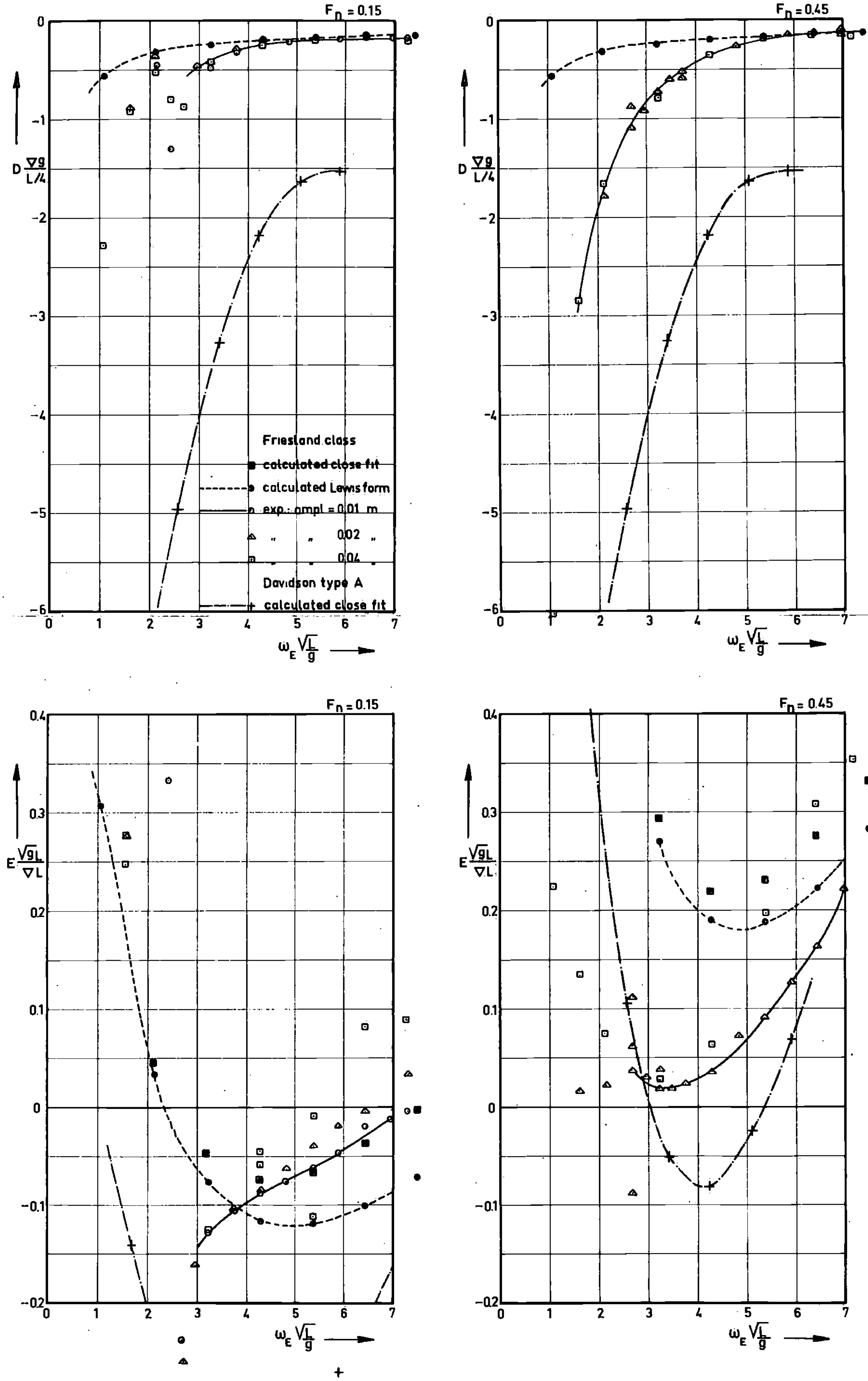
EQUATION OF MOTION COEFFICIENTS A AND B

FIGURE 14



EQUATION OF MOTION COEFFICIENTS d AND e

FIGURE 15



EQUATION OF MOTION COEFFICIENTS D AND E

FIGURE 16

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April 1967

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NETHERLANDS SHIP RESEARCH CENTRE TNO
SHIPBUILDING DEPARTMENT LEEGHWATERSTRAAT 5, DELFT

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COMPUTATION OF PITCH AND HEAVE MOTIONS FOR ARBITRARY SHIP FORMS

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VOORWOORD

Het belang, dat gehecht moet worden aan onderzoek op het gebied van de bewegingen van het schip in zeegang kan nauwelijks overschat worden, als men denkt aan het met deze scheepsbewegingen verband houdende mogelijke snelheidsverlies, de veiligheid van de lading en het ongemak voor passagiers en bemanning.

Tot voor kort konden voor het oplossen van de bewegingsvergelijkingen voor dompen en stampen de constanten van de gekoppelde lineaire bewegingsvergelijkingen slechts bepaald worden aan de hand van in langsscheepse golven uitgevoerde modelproeven, waarna een voorspelling van het gedrag in zeegang kon volgen.

Verschillende auteurs hebben zich bezig gehouden met de theoretische berekening van de scheepsbewegingen met methoden, die gebaseerd waren op de potentiaaltheorie voor twee-dimensionale vormen, gelijkend op een scheepsdoorsnede of voor vereenvoudigde scheepsvormen. Nu het verband van de coëfficiënten en de uitwendige krachten in de termen van demping en toegevoegde massa is vastgelegd, blijft alleen het vraagstuk over om een twee-dimensionale demping en een toegevoegde massa voor elke scheepsdoorsnede te berekenen.

Hier is door het transformeren van een cirkelvormige doorsnede naar een scheepsvorm in combinatie met de gemodificeerde striptheorie, een methode ontwikkeld, die niet alleen de bewegingen geeft, maar ook de invloed kan laten zien van de scheepsvorm op de bewegingen.

De uitkomsten van de berekeningen zijn vergeleken met de resultaten van uitgebreide modelproeven. Voor dit onderzoek werden drie typen jagers bezien, waarvan toch reeds zeer veel modelexperimentele en ware-grootte-informatie beschikbaar was. Hierbij werden niet alleen de bewegingen zelf vergeleken, maar ook de coëfficiënten van de bewegingsvergelijkingen afzonderlijk. De coëfficiënten zoals berekend en zoals verkregen uit modelproeven, geven somtijds verschillen te zien, maar voor de bewegingen zelf is de overeenstemming goed.

De ontwikkelde methode opent de mogelijkheid om de invloed van een wijziging in de scheepsvorm op de scheepsbewegingen te voorspellen. De berekening kan door middel van een computer uitgevoerd worden en is in principe toepasbaar voor elk type schip.

HET NEDERLANDS SCHEEPSSTUDIECENTRUM TNO

PREFACE

It is hardly possible to over-estimate the importance of the research on ship motions in seaway, keeping in mind the loss of speed, the safety of the cargo and the comfort of passengers and crew.

Before some time the only way to get a solution of the coupled linear differential motion equations for heave and pitch was the determination of the coefficients of these equations by measuring the exciting forces on a model during forced oscillation tests and on a restrained model in waves. Subsequently the behaviour of the ship in longitudinal waves could be predicted.

Several authors have been occupied with the theoretical calculation of ship motions based on potential theory for two-dimensional shiplike sections or simplified ship forms. Since the evaluation of the coefficients and exciting forces in terms of damping and added mass is completed, there remains only the problem of computing a two-dimensional damping and added mass for each of the ship's sections.

Here, by transforming the unit circle into a sectional shipshape, in combination with the modified striptheory, a method became available for evaluating both the motions and the influence of the shape of the hull on the motions.

The results of the computations are verified with the outcome of extensive model experiments. For this investigation three types of destroyers have been examined, of which a great amount of information was available already, both on model-scale and on board. Not only the motions itselves were compared, but also the coefficients of the motion equations separate. The coefficients computed and those determined from model experiments differ sometimes slightly, but for the motions the results are good in keeping with each other.

The method developed here gives the opportunity to predict the influence of a change in the shipform. The computation can be carried out by using a computer and can be applied to all types of ships.

THE NETHERLANDS SHIP RESEARCH CENTRE TNO

CONTENTS

LIST OF SYMBOLS

$a\ b\ c\ d\ e\ g$ $A\ B\ C\ D\ E\ G$	Coefficients of the equations of motion for heave and pitch
A_n	Transformation coefficient
A_w	Area of waterplane
a_n	Normalized transformation coefficient
A_x	Area of cross-section
b'	Section damping coefficient
B_M	Midship beam
B_n	Transformation coefficient
C_B	Block coefficient
F	Total vertical force on ship
F'	Vertical force on a section
F''	Vertical hydrodynamic force per unit area on section
F_a	Wave force amplitude on restrained ship
$\mathrm{Fn} = \dfrac{V}{\sqrt{g L_{pp}}}$	Froude number
g	Acceleration due to gravity
I_w	Longitudinal moment of inertia of waterplane area with respect to the y_b axis
I_{yy}	Real moment of inertia of ship
$k = \dfrac{2\pi}{\lambda}$	Wave number
k_{yy}	Radius of gyration in pitch
L_{oa}	Length over all
L_{pp}	Length between perpendiculars
M	Total moment on ship
M_a	Wave moment amplitude on restrained ship
m	Total added mass for heave
m'	Sectional added mass
m''	Added mass per unit surface area
N'	Sectional damping (without speed effect)
N''	Damping per unit surface area
P	Pressure on sectional surface
$S(y, z)$	Section surface
S_w	Statical moment of waterplane area
t	Time
T	Draft of ship
T_x	Draft of cross-section
V	Speed of ship
x_b, y_b, z_b	Right-handed body axis system
$y_w(x)$	Half width of waterline
z	Heave displacement
z_a	Heave amplitude
ε	Phase angle between the motions (forces, moments) and the waves
ζ	Instantaneous wave elevation
ζ_a	Wave amplitude
θ	Pitch angle
θ_a	Pitch amplitude
γ	Transform plane angle
λ	Wave length
β	Physical plane angle
ϱ	Density of water
∇	Displacement volume
ω	Circular frequency
ω_e	Circular frequency of encounter
$\dfrac{\mathrm{d}m'}{\mathrm{d}x_b}$	Rate of change of added mass in the x_b direction
$\dfrac{\mathrm{d}m''}{\mathrm{d}x_b}$	Local rate of change of added mass in the x_b direction
Δ	Displaced weight

COMPUTATION OF PITCH AND HEAVE MOTIONS FOR ARBITRARY SHIP FORMS *)

by

W. E. SMITH **)

Summary

Analytical methods are used to determine the pitch and heave motions in headwaves for three ship forms of the destroyer type. A computational method using a multiple coefficient transformation for the ship cross-sectional shapes is used. Transformation methods for arbitrarily shaped ship sections are discussed. The results from computation and experiment are compared. Agreement is found to depend significantly on the accuracy of the cross-section transformation. When the proper transformation is used, the influence of variation in hull shape on the motion can be accounted for.
Agreement between motion computation and experiment is excellent. Computed longitudinal distributions of damping, added mass and exciting forces are discussed.

1 Introduction

An analytical method for the computation of ship motions in a seaway has long been of major interest to both the ship designer and seakeeping researcher. The need for such a technique has been greatly increased by the appearance of many unusual hull shapes such as low resistance forms, bulbous bows, sonar domes, etc., for which an evaluation of the effects of hull shapes on the motion characteristics is vital.

For pitch and heave motions in head seas, the formulation of the problem is reasonably complete and may be described as that of obtaining the coefficients of an appropriate set of equations which relate for a particular ship's geometry the wave surface amplitude or some other measurable wave property to the resulting motion of a ship. The fundamental work over the past century has been primarily that of: (a) determining the appropriate form of the equations of motions; (b) obtaining a valid relationship between a particular hull geometry and the coefficients of the equations (so-called left-hand side); and (c) relating the free surface contours or wave shape to the resulting force on a specified hull form (so-called right-hand side). The motion equations are:

Heaving:

$$(a+\varrho\nabla)\ddot{z}+b\dot{z}+cz-d\ddot{\theta}-e\dot{\theta}-g\theta = F_a\cos(\omega_e t+\varepsilon_{F\zeta})$$

Pitching:

$$(A+\varrho\nabla k_{yy}^2)\ddot{\theta}+B\dot{\theta}+C\theta-D\ddot{z}-E\dot{z}-Gz = M_a\cos(\omega_e t+\varepsilon_{M\zeta})$$

The equations of motion consist of two coupled linear differential equations containing cross coupling terms proportional to acceleration, velocity and displacement. This representation, while originally developed by KORVIN KROUKOVSKY [1] using a strip theory approach is, however, in no way related to strip theory and is completely general as far as the motion representation is concerned. In fact, the only assumptions inherent in such a representation are those (a) of linearity and (b) that for long crested head waves the coupling of other modes of motion into pitch and heave is small and can be neglected. Further, they are easily extended to include all six coupled modes of motion as shown by CUMMINS [2]. The validity of such a representation was established experimentally by GERRITSMA [3]. This same experiment also established the theory of superposition, the equivalence of regular and random wave testing, and the frequency dependence of the equation of motion coefficients.

The coefficients and exciting forces were formulated via a so-called strip theory method as developed by KORVIN KROUKOVSKY [1] and extended by GERRITSMA [4, 5, 6]. This formulation permits the evaluation of the coefficients and exciting forces in terms of the damping and added mass associated with a particular hull form. It also contains velocity dependent terms which account for most of the forward speed effects in both the coefficients and the exciting forces. This has been experimentally demonstrated by GERRITSMA [4, 5, 6, 11, 12].

There remained only the problem of computing for a specific geometry a two dimensional damping and added mass for each of the ship's sections. This can be accomplished by using the URSELL [7] two dimensional solution for a circular cylinder oscillating at the free surface, and conformally mapping

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this solution for the circle into a particular sectional shape. Such a mapping or transformation was originally accomplished by Tasai [8] in which a three coefficient or Lewis form transformation was used. This works quite well for many of the simpler ship forms whose shapes are closely approximated by the Lewis form family. It, however, gives poor results for sections not properly fitted by the Lewis form coefficients. Also, it can be shown that if this method is used to determine the effects of sectional shape variations on the motions, the difference between the computed values and those obtained experimentally is approximately equal to the differences being investigated.

Porter [9] experimentally verified the Ursell solution for the circular cylinder and extended the transformation expressions to include an arbitrarily large number of transformation coefficients. He also showed experimentally the accuracy of such a transform solution for a number of two dimensional ship-like sections. Porter did not, however, provide a method for determining the coefficients for a given section. Such a method has now been developed which permits the transformation of the unit circle into any simply connected sectional shape. With this and the modified form of strip theory, as developed by Gerritsma [5, 6] a method for evaluating not only the motions but the influence of hull shape on the motions, is available.

The availability of such a program immediately presents many possibilities. At long last, we can do quick and inexpensive experiments on a computer. Further, it is possible to look in detail at the various terms of the equation of motion. This should provide new insight into the physical mechanisms involved. As additional computer experiments are performed and the limitations of the program are evaluated, this in itself should provide additional information concerning the physics of ship motion.

It has long been recognized that when experimentally investigating ship motions, the change of one hull dimension is extremely difficult. This is not so for a computer program, an individual design dimension can be artificially varied on a computer and its effects assessed. In addition, there is evidence that such a multi-coefficient or close fit program is

Table I Model characteristics

	Friesland	DD 692	Davidson A
Scale ratio	40	67.09	–
Length L_{pp} in m	2.810	1.741	1.741
Beam in m	0.2935	0.187	0.185
Draft (DWL) in m	0.0975	0.0635	0.0635
Displacement in kg	44.55	10.90	10.98
Block coefficient	0.554	0.524	0.536
Midship area coefficient	0.815	0.824	0.778
Prismatic coefficient	0.679	0.636	0.689
Waterplane area coefficient	0.798	0.762	0.739
Longitudinal center of mass aft or forward $L_{pp}/2$ in m	0.0293 aft	0.0345 aft	0.0280 fwd
Radius of gyration pitch	0.259 L_{pp}	0.25 L_{pp}	0.25 L_{pp}

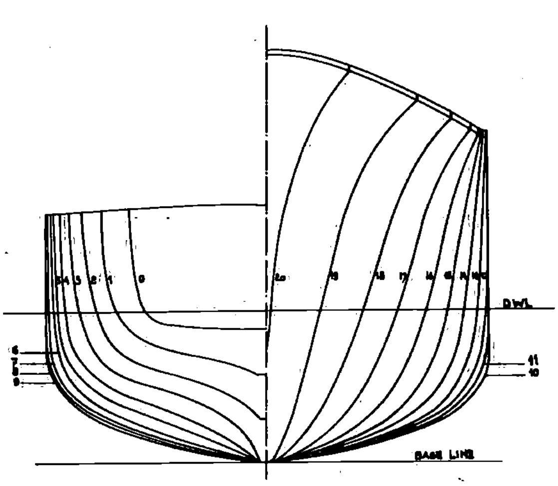


Fig. 1 Friesland class frigate

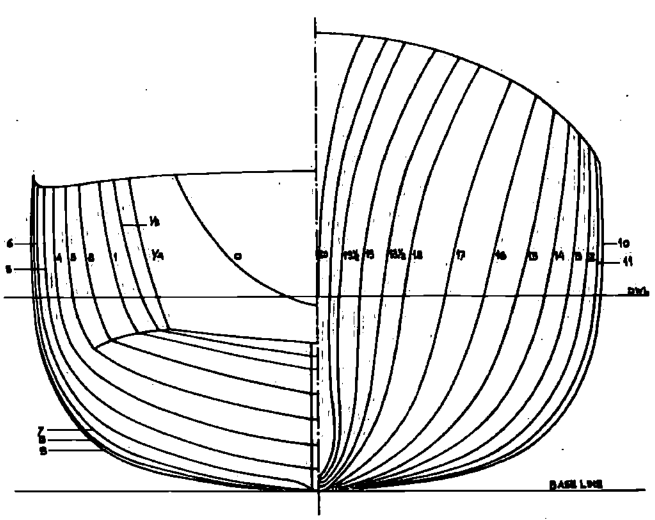


Fig. 2 DD 692 destroyer

required for even the simplest hull forms when computing relative motions, bending moments, bow immersions, etc.

2 Ship models used for calculation and experiment

In order to evaluate the capabilities of such a computation method three hull forms were selected. The forms chosen were: (a) a conventional frigate hull which had been previously tested by the Delft Shipbuilding Laboratory; (b) a similar form which had been tested at the Davidson Laboratory; and (c) a radically shaped destroyer which had been designed and tested at the Davidson Laboratory.

Each of the three forms are similar in total displacement and cross-sectional area (see Table I). The first form selected (figure 1), a Friesland class frigate, is one for which the motion characteristics have been extensively investigated by the Delft Shipbuilding Laboratory.

The motions have been measured and compared at both full and model scale (Gerritsma and Smith [6], Bledsoe, Bussemaker and Cummins [14]). Further, the coefficients of the equations of motion have been determined from forced oscillation model experiments and, similarly, the wave exciting forces and moments have been measured (Smith [15]). This model, therefore, provides a standard for reference which not only demonstrates the accuracy of the motion computations for a conventional hull form but also provides a detailed standard for the various terms in the equations of motion. The second (figure 2), the DD 692 class, is a conventional destroyer hull for which the motion characteristics were determined experimentally at the Davidson Laboratory (Breslin and Eng [10]). This particular ship, like the Friesland class frigate, is a form for which the motion computation program is known to work well. A comparison between the Davidson Laboratory experiment for this hull and computed values would, therefore, in effect be a comparison of motion responses obtainable from experiments in the two tanks. The third (figure 3), a Davidson Type A destroyer, is a ship with a conventional afterbody, but with a strongly bulged forebody. The forebody sectional shapes are of unconventional design with a narrow water line but which widens with increasing draft. Sectional shapes of this type are ones which the Lewis form transformation either fits badly or, as in the case of sections 14 through 20, does not even exist as a simply connected shape. This ship, therefore, provides an excellent test of the program's multiple coefficient transformation capability. Also, a comparison between such a close fit computation and experiment should provide an indication as to whether a potential solution and modified strip theory can properly represent the hydrodynamics of a radically flared or bulbous section, or whether such non linear effects as eddy currents, flow separation, viscosity, etc. are sufficiently large to significantly affect the computation accuracy. It could further indicate conclusively whether or not the effect on the motion due to hull shape variation can be accounted for using such a theory. Accordingly, a set of close fit transformation coefficients were obtained for each of the ship forms and these in turn were used in the computation of pitch and heave motion responses for a range of wave lengths and ship speeds. The speeds considered were in a Froude number range from

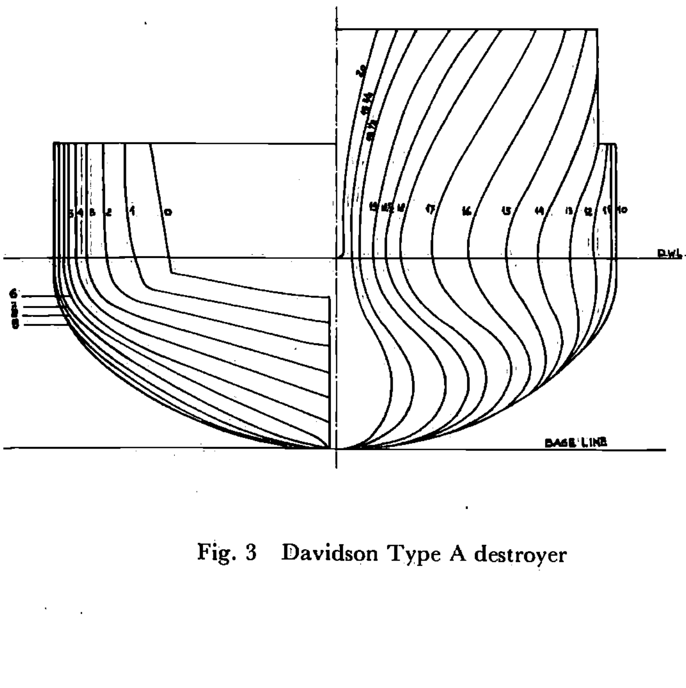


Fig. 3 Davidson Type A destroyer

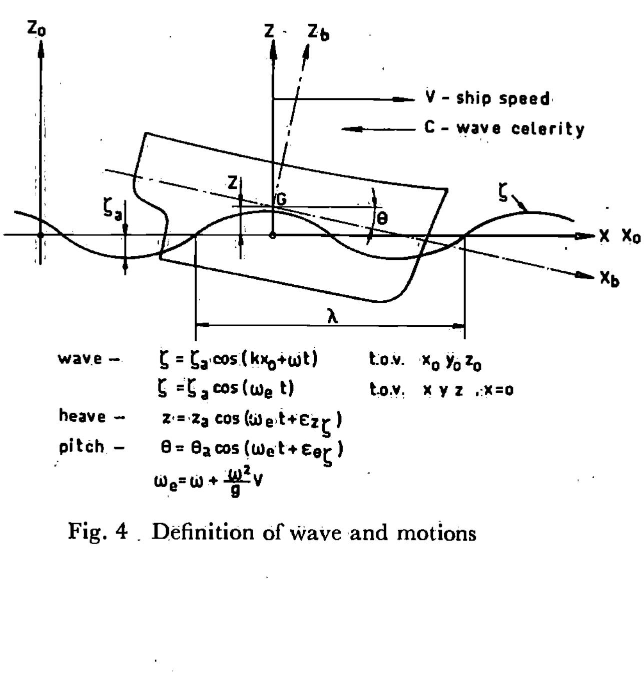


Fig. 4 Definition of wave and motions

Fn = 0.15 to Fn = 0.55. The wave lengths considered were for a range from $L_{pp}/\lambda = 0.3$ to $L_{pp}/\lambda = 2.5$.

The definition of the wave and ship motions concerned is given in figure 4.

For the Friesland class frigate computed results were compared with experimentally obtained motion responses and phase angles. The DD 692 and Davidson Type A were compared with experimental results for three wave lengths as extracted from the BRESLIN and ENG report [10].

3 Motion tests

3.1 *Friesland class*

The Friesland class hull form was tested by the Delft Shipbuilding Laboratory at both model and full scale. Since the results from the model and full scale tests were virtually identical as far as motion responses are concerned, only the model test results are used for comparison. The model was tested in regular long crested head waves for pitch and heave motions. The model length was 2.81 m and was operated with a radius of gyration of 0.25 L_{oa} or 0.259 L_{pp}.

All testing was done in regular long crested head waves with a peak to peak height $2\zeta_a = L_{pp}/40$. Wave lengths were varied from $L_{pp}/\lambda = 0.5$ to $L_{pp}/\lambda = 2.0$. Testing was done for a range of Froude numbers from Fn = 0.15 to Fn = 0.55. Test conditions are summarized in Table II.

3.2 *DD 692 and Davidson Type A*

The motion test results as extracted from the BRESLIN and ENG report [10] were performed in regular waves of $L_{pp}/\lambda = 0.75$, 1.0, and 1.25 over a range of Froude numbers from Fn = 0 to 0.60. The wave height (double amplitude) used in these tests was again $2\zeta_a = L_{pp}/40$.

A comparison of the pitch and heave motions made in this report between the Davidson Type A and the DD 692 shows a remarkable reduction in pitch for all Froude numbers above Fn = 0.15 for the first mentioned type (see figures 5 through 9).

4 Calculations

The calculations are based on a form of the strip theory originally developed by KORVIN KROUKOVSKY [1] and modified and extended by GERRITSMA [5, 6]. Briefly, the procedure is as follows:

(a) The ship is divided up into a number of sections and the individual sections are each represented by a set of (y, z) offset values. Depending on the severity of the sectional shape, an adequate representation is provided by 15 to 30 offset values evenly spaced around the periphery.

(b) Transformation coefficients are computed using the (y, z) offset values in an iterative process which is permitted to converge until the root mean square difference between the actual sections (offset values) and the transformed shape is as small as desired.

(c) The two dimensional added mass, damping, and the variation of added mass (dm/dx) longitudinally along the ship are computed for each of the sections by methods from [7, 9].

(d) A modified form of strip theory [5] is used to determine the coefficients of the equations of motion for the various frequencies and speeds of advance.

(e) Exciting forces are computed for each section using [6].

(f) The equations of motion are solved and the complex frequency response functions are computed for the speeds and frequencies desired.

The heave and pitch equations of motion assuming negligible coupling between the other four modes of motion are:

$$\begin{aligned} &\text{Heaving:} \quad \varrho\nabla\ddot{z} = F \\ &\text{Pitching:} \quad \varrho\nabla k_{yy}^2\ddot{\theta} = M \end{aligned} \qquad \text{(eq. 1)}$$

In terms of the force and moment distributions along the ship force F and the moment M are:

$$\begin{aligned} &\text{Heaving:} \quad F = \int_L F' \mathrm{d}x_b \\ &\text{Pitching:} \quad M = -\int_L F' x_b \mathrm{d}x_b \end{aligned} \qquad \text{(eq. 2)}$$

where:

z – heave displacement
θ – pitch displacement
F – total vertical force on the ship
M – total pitch moment on the ship
F' – vertical force on a section
x_b – longitudinal ship coordinate
k_{yy} – radius of gyration in pitch.

Table II Model test conditions

Speed	Fn =	0.15, 0.25, 0.35, 0.45, 0.55
Wave length ratio	L_{pp}/λ =	0.500, 0.555, 0.625, 0.714, 0.833, 1.000, 1.250, 1.670, 2.000
Wave height ratio	$2\zeta_a/L$ =	1/40

Dividing the ship into sections and employing a modified form of strip theory which includes forward speed effects, the sectional force is the sum of three parts: the hydrostatic force, the damping force and the inertia force.

$$F' = -2\varrho g y_w(z_b - x_b\theta_b - \zeta^*) + - N'(\dot{z}_b - x_b\dot{\theta}_b + V\theta_b - \dot{\zeta}^*) + - \frac{\mathrm{d}}{\mathrm{d}t}[m'(\dot{z} - x_b\dot{\theta}_b + V\theta - \dot{\zeta}^*)] \quad \text{(eq. 3)}$$

in which:

V – forward speed of the ship
y_W – half width of water line
m' – sectional added mass
N' – sectional damping
T – draft of a section
ζ – instantaneous wave elevation

and $\zeta^* = \zeta(1 - \frac{k}{y_w}\int_{-T}^{0} y_b\, e^{kz_b}\, \mathrm{d}z_b)$

The expression for ζ^* may be written in the following form:

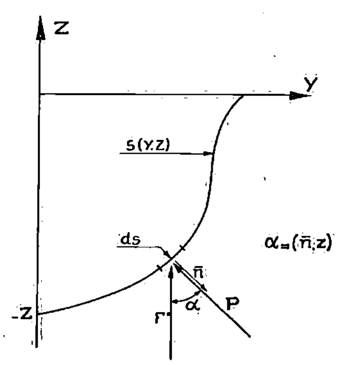
$$\zeta^* = \zeta \cdot e^{-kT^*}$$

where T^* is determined from:

$$T^* = -\frac{1}{k}\lg\,(1 - \frac{k}{y_w}\int_{-T}^{0} y_b \cdot e^{kz_b} \cdot \mathrm{d}z_b)$$

Here T^* can be considered as the distance under the wave surface of the point of application of the pressure forces, which are caused by the wave.

For a particular section and considering only the hydrodynamic part of the force, the vertical component of the force per unit area on the section surface $S(y, z)$ is:



$$F'' = -P\cos(\bar{n}, z) \quad \text{(eq. 4)}$$

or dividing into parts in phase with the acceleration and with the velocity:

$$F'' = m''\ddot{z}_b + N''\dot{z}_b, \quad \text{(eq. 5)}$$

where:

F'' – vertical force per unit area on section
$S(y, z)$ – surface of section
m'' – added mass per unit surface area
P – pressure on the sectional surface
N'' – damping per unit surface area.

Therefore the sectional damping and added mass become:

$$N' = \int_S N''\,\mathrm{d}s \quad \text{(eq. 6)}$$

$$m' = \int_S m''\,\mathrm{d}s \quad \text{(eq. 7)}$$

Substituting eq. (2) and (3) in eq. (1), with the application of eq. (5) through (7) and retaining on the right only terms representing wave forces, the equations of motion become:

Heaving:

$$(a+\varrho\nabla)\ddot{z} + b\dot{z} + cz - d\ddot{\theta} - e\dot{\theta} - g\theta = F_a\cos(\omega_e t + \varepsilon_{F\zeta})$$

Pitching:

$$(A+\varrho\nabla k_{yy}^2)\ddot{\theta} + B\dot{\theta} + C\theta - D\ddot{z} - E\dot{z} - Gz = M_a\cos(\omega_e t + \varepsilon_{M\zeta}) \quad \text{(eq. 8)}$$

From references [1, 5] the coefficients are:

$$a = \int_L m'\,\mathrm{d}x_b$$

$$b = \int_L N'\,\mathrm{d}x_b - V\int_L \frac{\mathrm{d}m'}{\mathrm{d}x_b}\,\mathrm{d}x_b$$

$$c = \varrho g A_w$$

$$d = \int_L m' x_b\,\mathrm{d}x_b$$

$$e = \int_L N' x_b\,\mathrm{d}x_b - 2Va - V\int_L \frac{\mathrm{d}m'}{\mathrm{d}x_b} x_b\,\mathrm{d}x_b$$

$$g = \varrho g S_w - Vb$$

$$A = \int_L m' x_b^2\,\mathrm{d}x_b$$

$$B = \int_L N' x_b^2\,\mathrm{d}x_b - 2VD - V\int_L \frac{\mathrm{d}m'}{\mathrm{d}x_b} x_b^2\,\mathrm{d}x_b$$

$$C = \varrho g I_w - VE$$

$$D = \int_L m' x_b\,\mathrm{d}x_b$$

$$E = \int_L N' x_b\,\mathrm{d}x_b - V\int_L \frac{\mathrm{d}m_b}{\mathrm{d}x_b} x_b\,\mathrm{d}x_b$$

$$G = \varrho g S_w \quad \text{(eq. 9)}$$

in which: $\frac{\mathrm{d}m'}{\mathrm{d}x_b} = \int_S \frac{\mathrm{d}m''}{\mathrm{d}x_b}\cdot\mathrm{d}s$ (eq. 10)

and: $\frac{\mathrm{d}m''}{\mathrm{d}x_b}$ – local rate of change of added mass in the x_b direction.

The representation of the terms of the coefficients is somewhat different from the usual one. The reason is the addition of the forward speed effect to the added mass coefficients. This facilitates the comparison with experimental data, as explained further in section 6.4, page 21.

From reference [6] but written in a different way, the exciting forces and moments are:

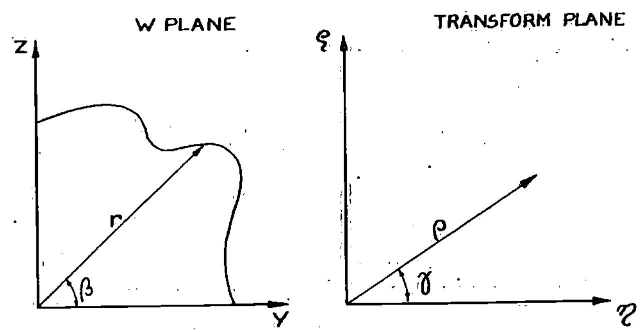
$$\left.\begin{aligned}
\frac{F_a}{\zeta_a}\cos\varepsilon_{F\zeta} &= 2\varrho g\int_L y_w \cos(kx_b)\,\mathrm{d}x_b + \\
&-2\varrho\omega^2\int_L\int_{-T}^{O} y_b\,\mathrm{e}^{kz_b}\cos(kx_b)\,\mathrm{d}z_b\,\mathrm{d}x_b + \\
&+\omega V\int_L\int_S \frac{\mathrm{d}m''}{\mathrm{d}x_b}\mathrm{e}^{kz_b}\sin(kx_b)\,\mathrm{d}s\,\mathrm{d}x_b + \\
&-\omega^2\int_L\int_S m''\,\mathrm{e}^{kz_b}\cos(kx_b)\,\mathrm{d}s\,\mathrm{d}x_b + \\
&-\omega\int_L\int_S N''\,\mathrm{e}^{kz_b}\sin(kx_b)\,\mathrm{d}s\,\mathrm{d}x_b \\
\frac{F_a}{\zeta_a}\sin\varepsilon_{F\zeta} &= 2\varrho g\int_L y_w \sin(kx_b)\,\mathrm{d}x_b + \\
&-2\varrho\omega^2\int_L\int_{-T}^{O} y_b\,\mathrm{e}^{kz_b}\sin(kx_b)\,\mathrm{d}z_b\,\mathrm{d}x_b + \\
&-\omega V\int_L\int_S \frac{\mathrm{d}m''}{\mathrm{d}x_b}\mathrm{e}^{kz_b}\cos(kx_b)\,\mathrm{d}s\,\mathrm{d}x_b + \\
&-\omega^2\int_L\int_S m''\,\mathrm{e}^{kz_b}\sin(kx_b)\,\mathrm{d}s\,\mathrm{d}x_b + \\
&+\omega\int_L\int_S N''\,\mathrm{e}^{kz_b}\cos(kx_b)\,\mathrm{d}s\,\mathrm{d}x_b \\
\frac{M_a}{\zeta_a}\cos\varepsilon_{M\zeta} &= -2\varrho g\int_L y_w x_b\cos(kx_b)\,\mathrm{d}x_b + \\
&+2\varrho\omega^2\int_L\int_{-T}^{O} y_b x_b\,\mathrm{e}^{kz_b}\cos(kx_b)\,\mathrm{d}z_b\,\mathrm{d}x_b + \\
&-\omega V\int_L\int_S \frac{\mathrm{d}m''}{\mathrm{d}x_b} x_b\,\mathrm{e}^{kz_b}\sin(kx_b)\,\mathrm{d}s\,\mathrm{d}x_b + \\
&+\omega^2\int_L\int_S m'' x_b\,\mathrm{e}^{kz_b}\cos(kx_b)\,\mathrm{d}s\,\mathrm{d}x_b + \\
&+\omega\int_L\int_S N'' x_b\,\mathrm{e}^{kz_b}\sin(kx_b)\,\mathrm{d}s\,\mathrm{d}x_b \\
\frac{M_a}{\zeta_a}\sin\varepsilon_{M\zeta} &= -2\varrho g\int_L y_w x_b\sin(kx_b)\,\mathrm{d}x_b + \\
&+2\varrho\omega^2\int_L\int_{-T}^{O} y_b x_b\,\mathrm{e}^{kz_b}\sin(kx_b)\,\mathrm{d}z_b\,\mathrm{d}x_b + \\
&+\omega V\int_L\int_S \frac{\mathrm{d}m''}{\mathrm{d}x_b} x_b\,\mathrm{e}^{kz_b}\cos(kx_b)\,\mathrm{d}s\,\mathrm{d}x_b + \\
&+\omega^2\int_L\int_S m'' x_b\,\mathrm{e}^{kz_b}\sin(kx_b)\,\mathrm{d}s\,\mathrm{d}x_b + \\
&-\omega\int_L\int_S N'' x_b\,\mathrm{e}^{kz_b}\cos(kx_b)\,\mathrm{d}s\,\mathrm{d}x_b
\end{aligned}\right\}\text{(eq. 11)}$$

where:

S – is the surface of the section.
k – is equal to ω^2/g.

5 Transformation coefficients

For the transformation coefficients a numerical method is used to generate a set of coefficients which conformally maps the exterior of the unit circle $|\delta| \geqslant 1$ into the exterior of a given simply connected region. For this program the boundary of the region may be given analytically, or by a discreet set of (y, z) points, i.e. a table of offset values (y_i, z_i).



The mapping function is:

$$W = \sum_{n=-1}^{N} C_n\delta^{-n} = \sum_{n=-1}^{N}(A_n+iB_n)(\cos n\gamma - i\sin n\gamma) \qquad \text{(eq. 12)}$$

where:

$W = y+iz = r\mathrm{e}^{i\beta}$

and:

$\delta = \varrho\mathrm{e}^{i\gamma}$

The notation, which is somewhat different from that used by Tasai, Porter, etc. was selected to conform with the standard right-handed coordinate system normally used to describe ship motions (figure 4).

From equation (12) for a particular set of offset values we have:

$$y_i = \sum_{n=-1}^{N} (A_n \cos n\gamma_i + B_n \sin n\gamma_i)$$
$$z_i = \sum_{n=-1}^{N} (-A_n \sin n\gamma_i + B_n \cos n\gamma_i) \quad \text{(eq. 13)}$$
$$i = 1, 2, 3 \ldots I$$

This system of equations (13), for equally spaced arguments, is characterized by an interesting property, it is easily inverted with respect to the coefficients A_n and B_n. This is a consequence of the property of orthogonality, which trigonometric functions of discreet arguments possess in the case of equally spaced points (KRYLOV [13]).

Inverting equation (13):

$$A_n = \frac{1}{I} \sum_{i=1}^{I} (y_i \cos n\gamma_i - z_i \sin n\gamma_i)$$
$$B_n = \frac{1}{I} \sum_{i=1}^{I} (y_i \sin n\gamma_i + z_i \cos n\gamma_i) \quad \text{(eq. 14)}$$
$$n = -1, 0, 1, 2 \ldots I\text{–}2$$

Equation (14) permits the coefficients A_n and B_n to be easily calculated by an iterative numerical process which can provide transformation coefficients which transforms a simply connected region with any reasonable preassigned accuracy (sectional fit).

The coefficient program is designed to handle any simply connected shape, symmetrical or asymetrical, with respect to the coordinate axis. Further, it can accommodate any shape capable of being transformed with a pre-selected accuracy by not more than 256 A_n and 256 B_n. Even though the program can accommodate completely asymetrical shapes, the sectional outlines usually encountered in ship-building are symmetrical with respect to both the y and z axis. This, of course, refers only to the portion of the hull below the mean free surface and for the y axis symmetry the upper two quadrants are considered to be mirror images of the submerged portion. This symmetry assumption insures that all of the B_n coefficients are zero and likewise, the A_n coefficients for even n are also zero. The resultant transformation equations are:

$$y_i = \sum_{n=-1}^{N} A_{2n+1} \cos (2n+1) \gamma_i$$
$$z_i = \sum_{n=-1}^{N} -A_{2n+1} \sin (2n+1) \gamma_i \quad \text{(eq. 15)}$$
$$i = 1, 2, 3 \ldots I$$

or in normalized form:

$$\frac{y_i}{A_{-1}} = \cos \gamma_i + \sum_{n=0}^{N} a_{2n+1} \cos (2n+1) \gamma_i$$
$$\frac{z_i}{A_{-1}} = \sin \gamma_i - \sum_{n=0}^{N} a_{2n+1} \sin (2n+1) \gamma_i \quad \text{(eq. 16)}$$

where:

$$A_{-1} = \frac{y_w}{1 + \sum_{n=0}^{N} a_{2n+1}}$$

which may be treated as a scale factor.

For symmetrical shapes represented by equations (15) which include all of the sections considered in this paper, the computation time and the number of coefficients required are quite modest. For example, only five coefficients and 25 seconds of computer time were required to obtain a representation of the Davidson Type A midship section. The relatively radical section 19 of the same ship form (see figure 3) required six minutes and 16 coefficients.

A variety of sectional shapes have been mapped with this program, including such extremes as rectangles, triangles, sections with bilge keels, and sections with anti-pitch fins. In every case an extremely close fit was obtained.

6 Discussion

The computation method for the three ships was as follows: Each of the ships was represented by 21 cross-sections which, as is the practice in naval architecture, were evenly spaced along the ship with the first cross-section located at the aft perpendicular and the 21st cross-section at the forward perpendicular. Each of the cross-sections was represented by a table of 20 (y, z) offset values. For the Friesland appropriate offset values for each section were obtained from a master table of offsets provided by the ship's designers. The required values for the DD 692 and Davidson Type A were taken from body plan diagrams provided in the BRESLIN, ENG report [10]. The offset values for each cross-section were selected so that they were approximately evenly spaced around the periphery of the half section lying between the load water line, $\beta = 0$, and the keel, $\beta = \pi/2$. It should be emphasized that, while this is contrary to the normal ship designers practice of using evenly spaced water lines, the equal spacing around the periphery is very necessary to insure a proper fit by the transformation coefficients.

The offset values for the 21 sections were used as input to the transformation coefficient program. For the ships considered here, an iterative fitting process was allowed to continue for each section until the sum of the square of the difference between the 20 new or transformed values and the actual or original offset values was less than 0.01 percent of the mean beam A_x/T_x. The transformed shapes so obtained were compared with the original cross-sections and in every case, including the rather radical shapes of the Davidson Type A forebody, the two were virtually identical. The convergence criteria of 1.0 percent A_x/T_x has been found to be sufficient for all normal computations. The normalized coefficient values obtained for the three ships are given in Tables III, IV and V.

The 21 sets of transformation coefficients obtained for each ship were then used to calculate the pitch and heave motion responses.

During the motion computations intermediate values such as sectional added mass and damping, coefficients of the equation of motion, exciting

Table III Friesland class transformation coefficients, normalized form

Coeff.	Section 0	1	2	3	4	5	6	7	8	9
$y_w(x)$	+0.075500	+0.098500	+0.113200	+0.124500	+0.132200	+0.137700	+0.142000	+0.144800	+0.146750	+0.14675
a_1	+0.756566	+0.469845	+0.320917	+0.207701	+0.207195	+0.207667	+0.211572	+0.213333	+0.216503	+0.21843
a_3	−0.011692	+0.052863	+0.075972	+0.106067	+0.061324	+0.029689	+0.004282	−0.012338	−0.020112	−0.02571
a_5	+0.001154	−0.019783	−0.038869	−0.066257	−0.042457	−0.029765	−0.022487	−0.018722	−0.018272	−0.01884
a_7	+0.000056	+0.013053	+0.029925	+0.035318	+0.017287	+0.008908	+0.006029	+0.003570	+0.002268	−0.00002
a_9	+0.000643	−0.000377	−0.005396	−0.011768	−0.006230	−0.004031	−0.002504	−0.001663	−0.000439	−0.00326
a_{11}	+0.000012	+0.005580	+0.007198	+0.009417	+0.004646	+0.002923	+0.001846	+0.001182	+0.001419	−0.00140
a_{13}	−0.001163	−0.003778	−0.001389	+0.000106	+0.000776	+0.001235	−0.000361	−0.001411	−0.000603	−0.00145
a_{15}	0	0	0	−0.000325	0	0	0	0	0	0

Table IV DD 692 transformation coefficients, normalized form

Coeff.	Section 0	1	2	3	4	5	6	7	8	9
$y_w(x)$		+0.064000	+0.073200	+0.080200	+0.085100	+0.088700	+0.091300	+0.092600	+0.093500	+0.09350
a_1	0	+0.503369	+0.417424	+0.345471	+0.276803	+0.223354	+0.188119	+0.177393	+0.177750	+0.17524
a_3	0	+0.004956	+0.018417	+0.017416	+0.013005	+0.005442	−0.004492	−0.020053	−0.028284	−0.03615
a_5	0	−0.043042	−0.024534	−0.014502	−0.010177	−0.005611	−0.011570	−0.003672	−0.000034	+0.00108
a_7	0	+0.019135	−0.001778	+0.003559	+0.001562	+0.006130	+0.010788	+0.004691	+0.002839	+0.00145
a_9	0	0	0	0	0	0	0	0	0	0
a_{11}	0	0	0	0	0	0	0	0	0	0

Table V Davidson Type A transformation coefficients, normalized form

Coeff.	Section 0	1	2	3	4	5	6	7	8	9
$y_w(x)$	+0.054800	+0.066100	+0.075400	+0.081800	+0.085300	+0.087800	+0.089400	+0.091000	+0.092400	+0.09270
a_1	+0.634763	+0.538712	+0.475447	+0.405580	+0.337851	+0.276044	+0.227559	+0.204095	+0.196954	+0.19449
a_3	+0.008824	+0.008272	+0.016456	+0.033001	+0.040889	+0.049993	+0.061934	+0.048229	+0.030584	+0.01656
a_5	−0.018587	−0.032341	−0.029574	−0.028604	−0.029224	−0.031944	−0.031540	−0.016625	−0.006254	−0.00327
a_7	−0.014569	+0.000787	+0.006433	+0.004568	+0.009562	+0.012645	+0.008042	−0.001761	−0.002119	−0.00426
a_9	0	0	0	0	0	0	−0.004427	0	0	0
a_{11}	0	0	0	0	0	0	0	0	0	0
a_{13}	0	0	0	0	0	0	0	0	0	0
a_{15}	0	0	0	0	0	0	0	0	0	0
a_{17}	0	0	0	0	0	0	0	0	0	0
a_{19}	0	0	0	0	0	0	0	0	0	0
a_{21}	0	0	0	0	0	0	0	0	0	0
a_{23}	0	0	0	0	0	0	0	0	0	0
a_{25}	0	0	0	0	0	0	0	0	0	0
a_{27}	0	0	0	0	0	0	0	0	0	0
a_{29}	0	0	0	0	0	0	0	0	0	0
a_{31}	0	0	0	0	0	0	0	0	0	0

forces and moments were obtained. This, therefore, permits a comparison and evaluation of these intermediate values as well as the motion characteristics. The motions and intermediate values were computed for a number of wave lengths and ship speeds.

6.1 *Friesland class*

The motion comparison between computation and experiment for the Friesland was quite good, with virtually perfect agreement for all conditions except Froude number Fn = 0.55. In this case, the computer values for the pitch amplitude are slightly higher than experiment. It should also be noted that the experimental values shown for this ship have also been compared with full scale measurements, GERRITSMA, SMITH [6] where the agreement again was almost perfect. In the full scale comparison a Lewis form (three coefficient) transformation was used. The Lewis form computer results showed small differences at the higher frequencies, even though for this ship the Lewis form fit is a

Table III

Section										
10	11	12	13	14	15	16	17	18	19	20
0.146750	+0.145800	+0.144000	+0.139300	+0.130900	+0.119600	+0.104700	+0.086000	+0.062300	+0.033500	+0.002100
0.216715	+0.212799	+0.202529	+0.181780	+0.149260	+0.102148	+0.032946	−0.076855	−0.251106	−0.543066	−0.771895
0.027852	−0.024269	−0.018025	−0.009248	−0.001207	+0.012882	+0.031142	+0.048723	+0.067091	+0.072455	−0.011656
0.017610	−0.017266	−0.013083	−0.007731	−0.005303	−0.001754	+0.000396	+0.005114	+0.008321	+0.006943	+0.008758
0.000264	+0.000713	+0.002804	+0.004184	+0.006101	+0.006435	+0.006409	+0.009881	+0.009783	+0.010588	−0.007105
0.002677	−0.002138	−0.000938	−0.000392	+0.001176	+0.000622	+0.000692	+0.002496	+0.002557	+0.001713	+0.012262
0.000586	−0.000237	+0.000293	+0.000411	+0.000798	+0.001650	+0.002305	+0.003241	+0.003026	+0.005503	−0.005027
0.001643	−0.000659	−0.000219	−0.000114	−0.000377	+0.000754	+0.000734	+0.000835	+0.000436	−0.000442	−0.000376
0	0	0	0	0	0	0	0	0	0	0

Table IV

Section										
10	11	12	13	14	15	16	17	18	19	20
0.093500	+0.091400	+0.087600	+0.082200	+0.074700	+0.065400	+0.054500	+0.042000	+0.028300	+0.013900	0
0.170747	+0.163304	+0.145807	+0.116446	+0.068515	−0.000753	−0.092987	−0.227172	−0.409429	−0.628993	0
0.032108	−0.027961	−0.015651	−0.003523	+0.011781	+0.025146	+0.033940	+0.040215	+0.037197	−0.019362	0
0.001151	+0.002512	+0.001560	+0.001432	+0.002266	+0.002546	−0.000376	−0.002547	−0.001702	−0.026939	0
0.000873	+0.003814	+0.005334	+0.005635	+0.009073	+0.009086	+0.011585	+0.010463	+0.004567	+0.025548	0
0	0	0	0	0	0	0	0	0	+0.010061	0
0	0	0	0	0	0	0	0	0	−0.012814	0

Table V

Section										
10	11	12	13	14	15	16	17	18	19	20
0.092700	+0.091300	+0.085900	+0.077500	+0.066600	+0.056000	+0.043500	+0.031500	+0.021100	+0.012000	+0.000100
0.193795	+0.182833	+0.159812	+0.129027	+0.087717	+0.044207	−0.008312	−0.069047	−0.156982	−0.321085	0
0.004721	−0.000003	−0.013634	−0.037071	−0.063870	−0.092086	−0.115450	−0.133027	−0.153040	−0.167356	+0.849541
0.002115	−0.002151	−0.006390	−0.023765	−0.038991	−0.053038	−0.062931	−0.065924	−0.077415	−0.079463	0
0.004379	−0.002419	−0.004420	−0.015333	−0.024243	−0.029453	−0.039177	−0.044839	−0.051034	−0.053505	0
0	0	0	0	−0.016527	−0.017523	−0.029847	−0.035147	−0.036673	−0.036656	0
0	0	0	0	−0.007892	−0.011381	−0.021379	−0.024859	−0.027850	−0.028859	0
0	0	0	0	0	−0.011643	−0.018640	−0.021567	−0.023534	−0.022944	0
0	0	0	0	0	−0.008125	−0.012810	−0.014849	−0.016615	−0.016122	0
0	0	0	0	0	−0.006435	−0.011540	−0.014018	−0.016084	−0.016587	0
0	0	0	0	0	0	−0.008154	−0.010711	−0.011900	−0.009329	0
0	0	0	0	0	0	0	−0.008382	−0.009449	0	0
0	0	0	0	0	0	0	−0.008175	−0.008663	0	0
0	0	0	0	0	0	0	−0.007344	−0.007466	0	0
0	0	0	0	0	0	0	−0.004600	−0.005942	0	0
0	0	0	0	0	0	0	−0.004671	−0.005488	0	0
0	0	0	0	0	0	0	−0.004484	−0.004837	0	0

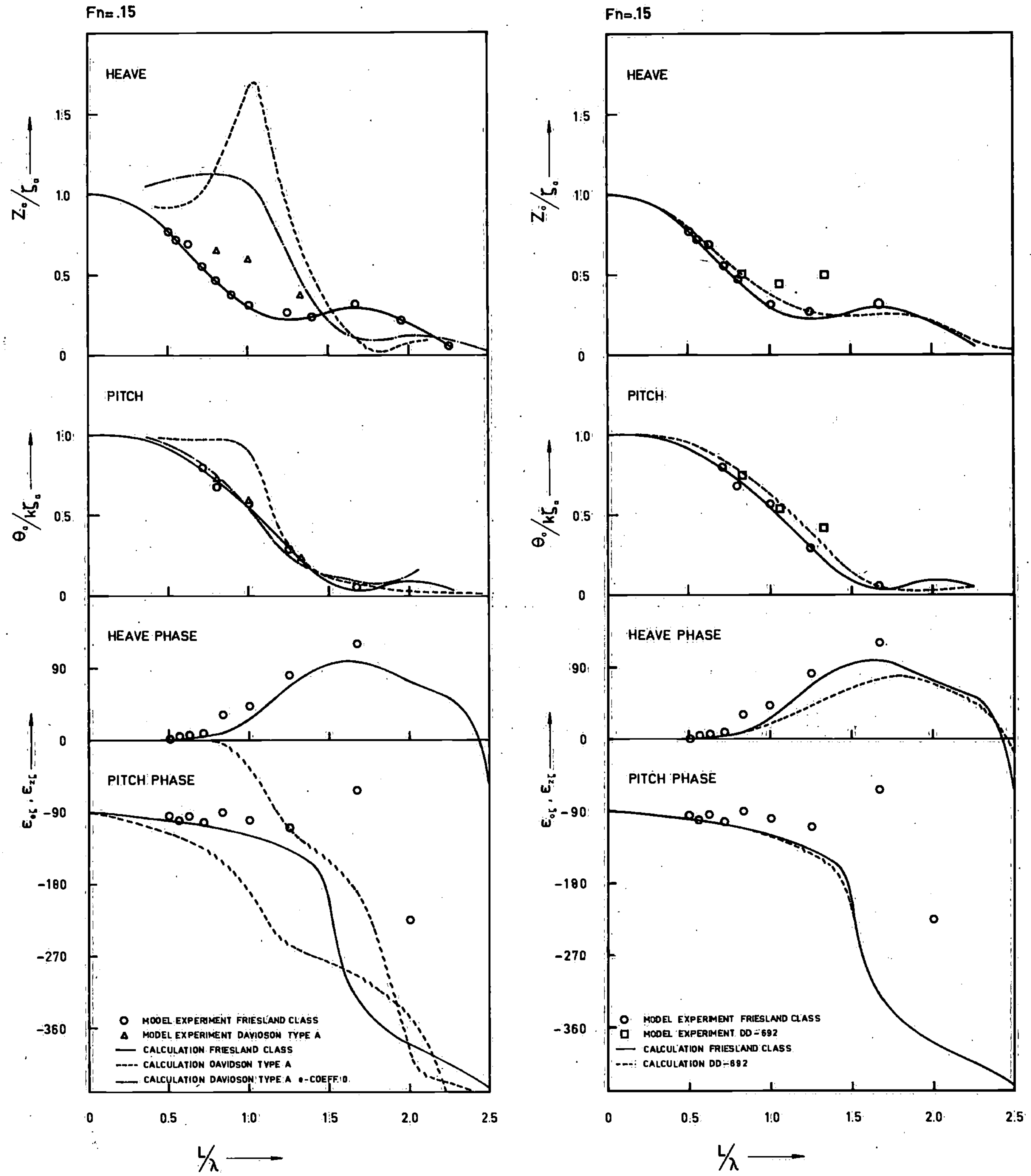


Fig. 5 Comparison of calculation and model experiment Fn = 0.15

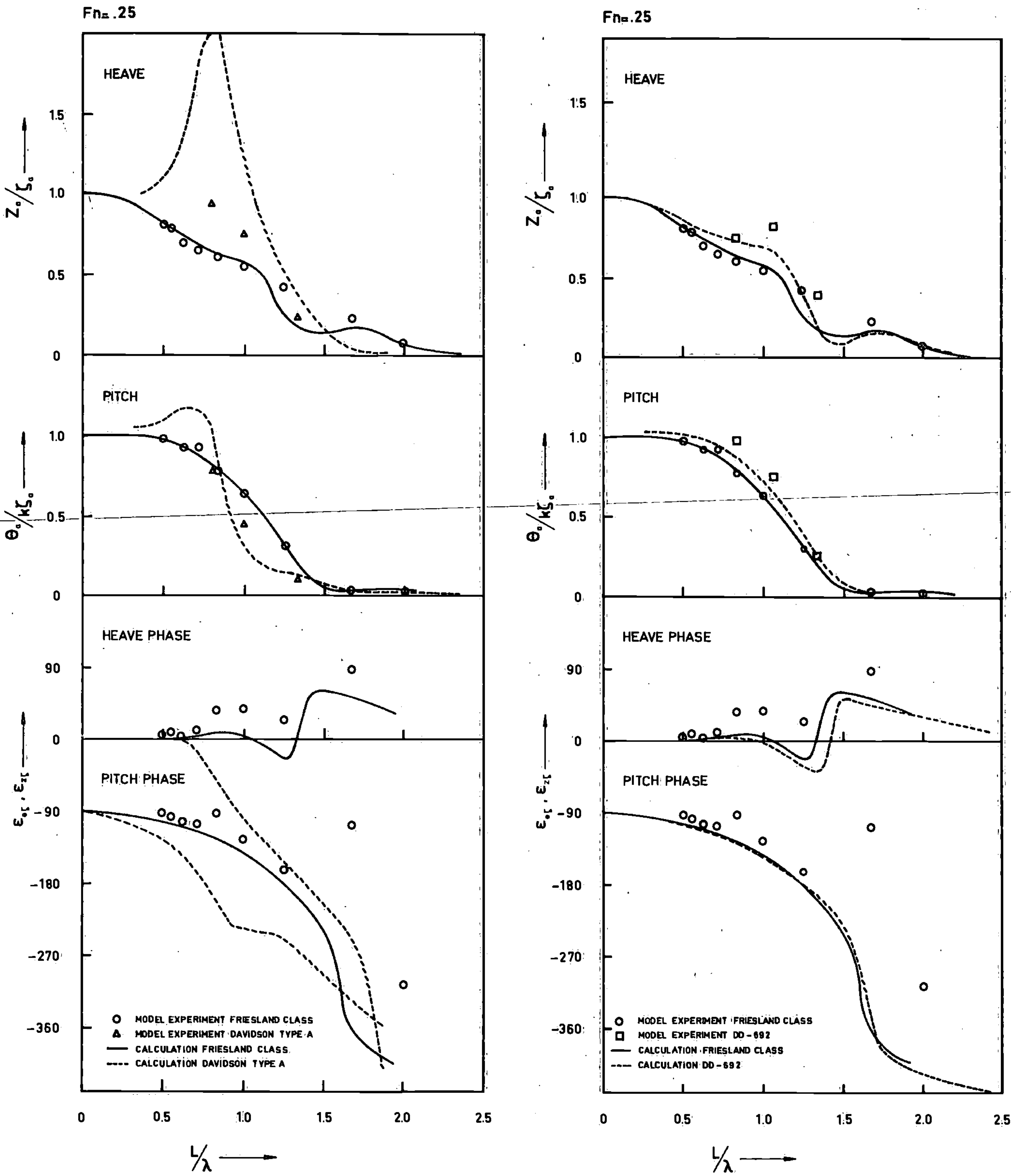


Fig. 6 Comparison of calculation and model experiment Fn = 0.25

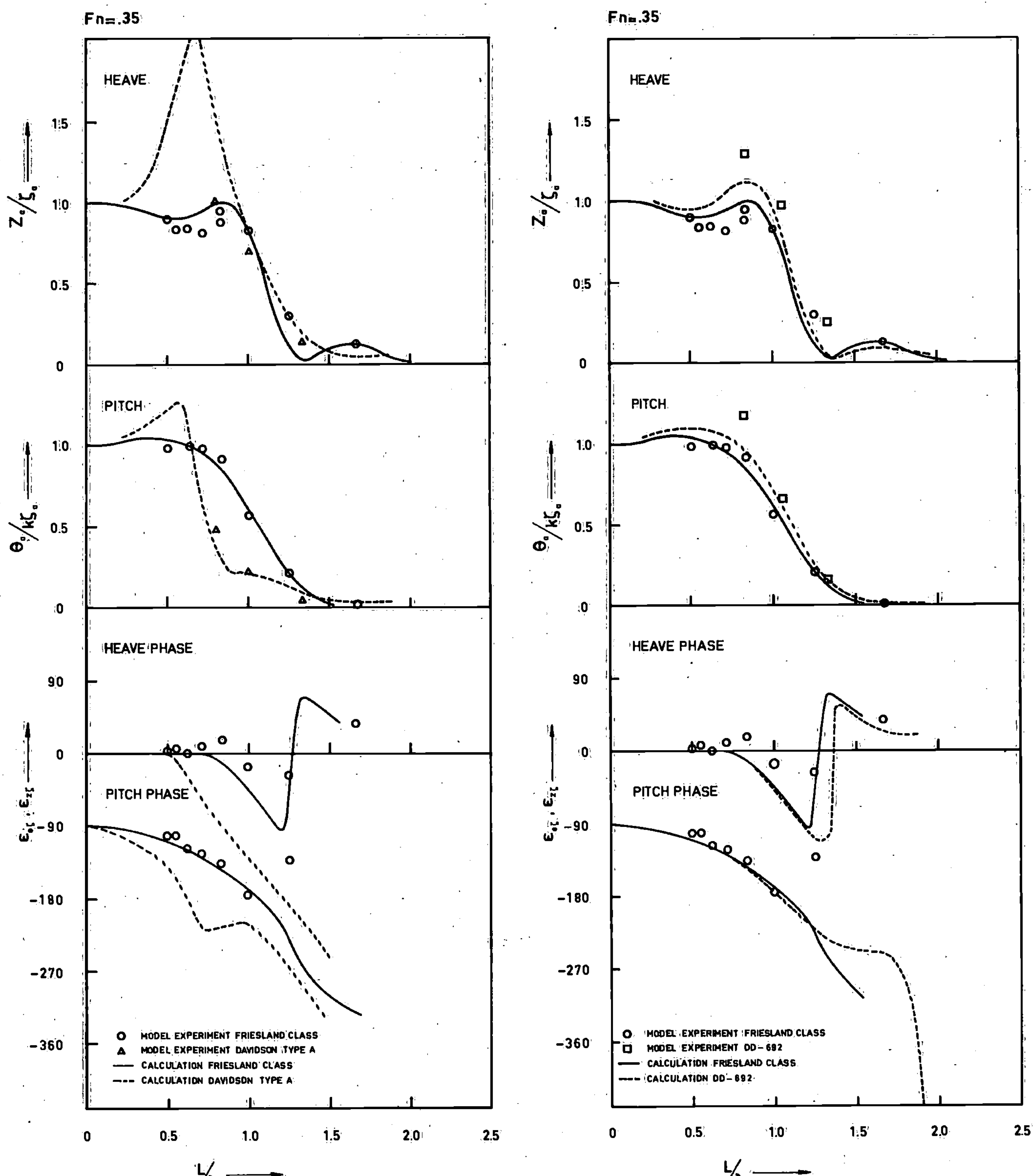


Fig. 7 Comparison of calculation and model experiment Fn = 0.35

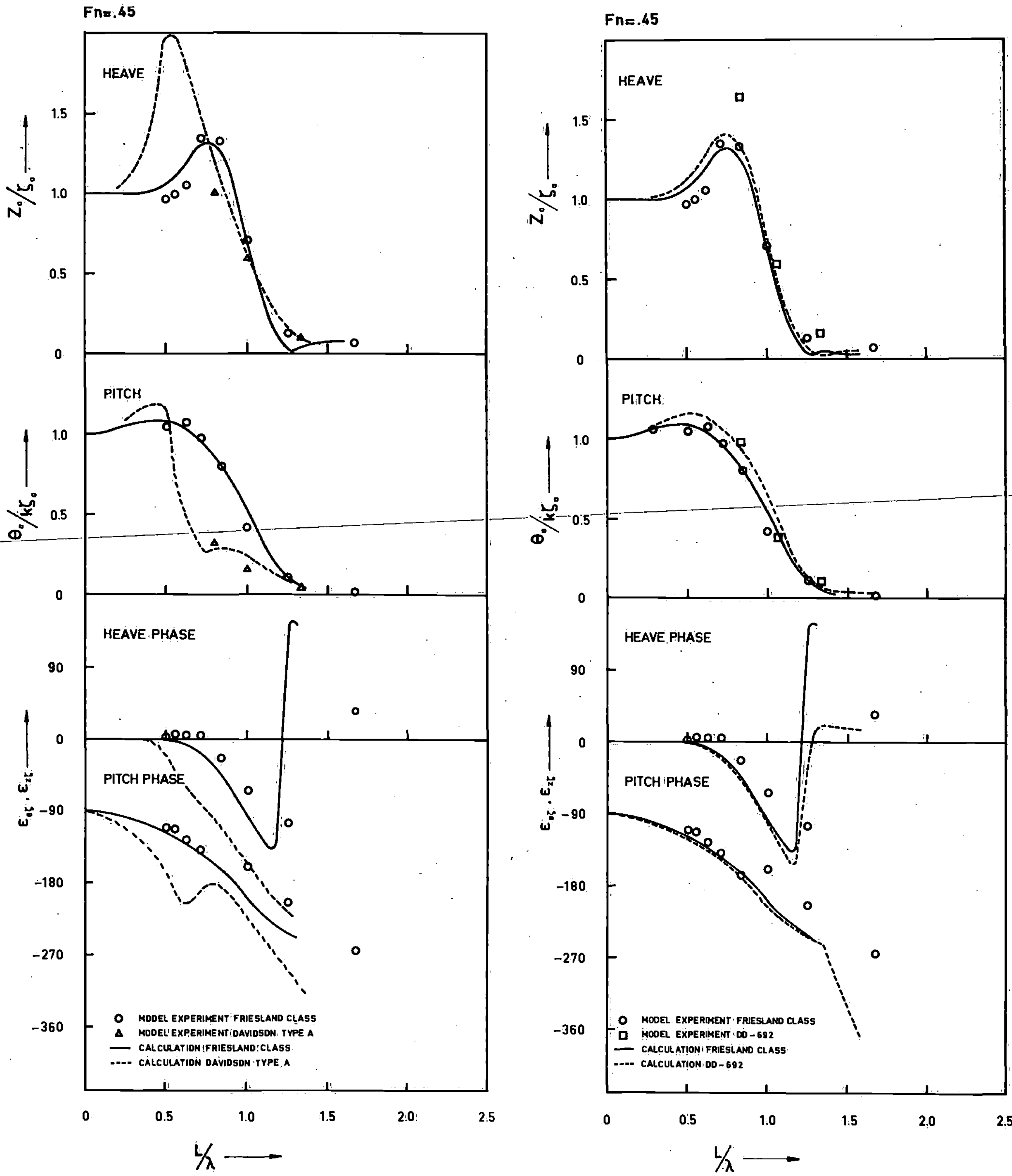


Fig. 8 Comparison of calculation and model experiment Fn = 0.45

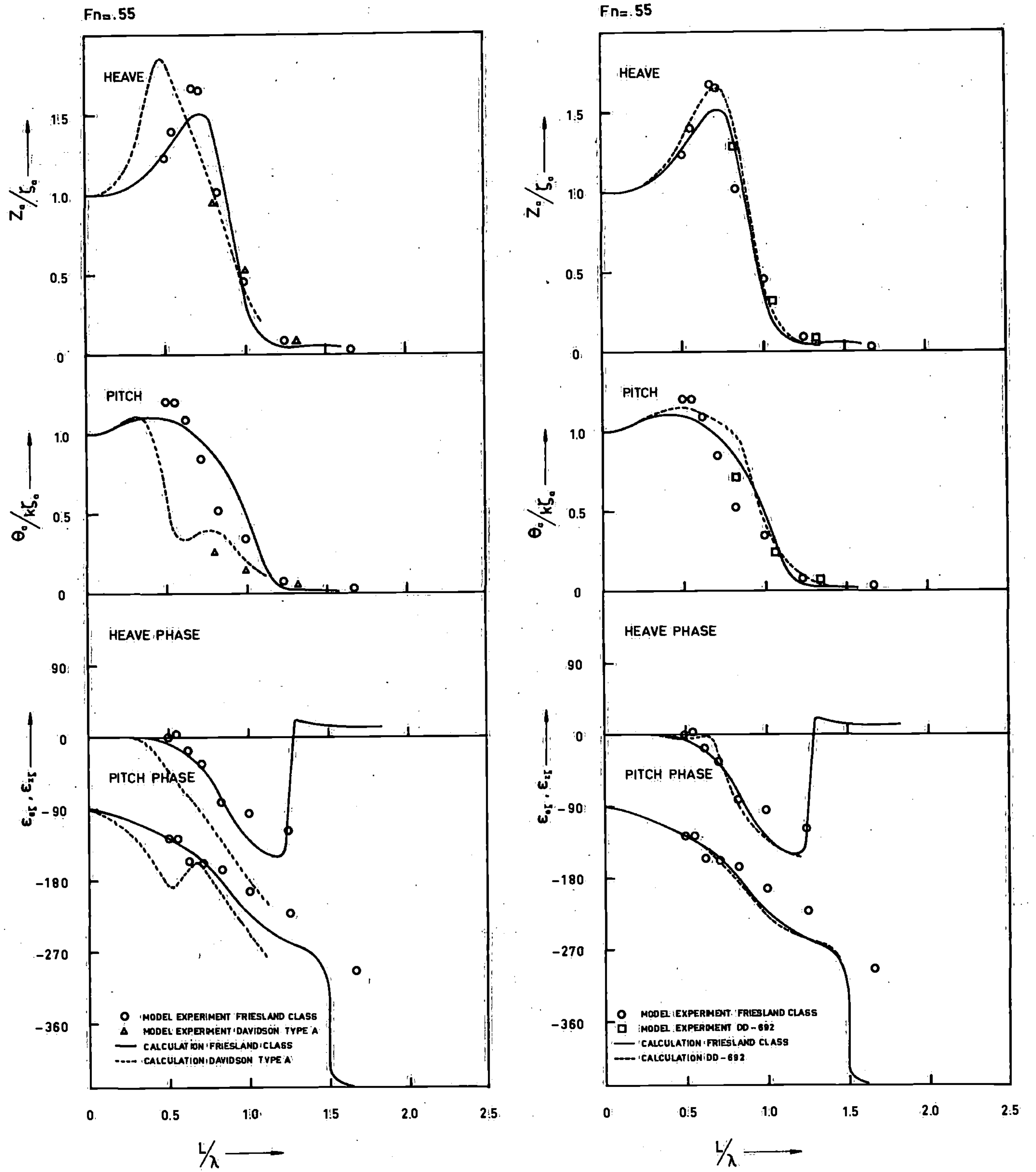


Fig. 9 Comparison of calculation and model experiment Fn = 0.55

good one. The close fit program has produced even better agreement. The differences between the two computation methods are insignificant when considering the design aspects of ship motions, but are in themselves interesting since they demonstrate that a close fit computation is capable of accounting for small differences in hull shape. Also, it provides an excellent check on the correctness of programming and numerical analysis aspects of the close fit program.

6.2 *DD 692*

The comparison between computation and experiment for the DD 692 is a comparison between close fit computer results and Davidson laboratory experimental results extracted from the BRESLIN, ENG report [10]. The motion amplitude comparison generally gave only a fair agreement, with the pitch motion amplitudes agreeing better than the heave. The experimental values are generally higher than those from computation, with the largest differences occurring at the lower frequencies. Also, it should be pointed out that this is only a limited comparison, since experimental data is available for only three wave frequencies. As this ship is one of a class or type, for which both the Lewis form and close fit computations have always shown good agreement with experiments, such a comparison of computation and experiment is, in effect, a comparison between motion responses obtainable from experiments in the two tanks. There is apparently a rather large difference between the experiments in the two tanks, especially in the heave amplitudes, and is thought to be of sufficient significance to warrant additional investigation.

6.3 *Davidson Type A*

The Davidson Type A results are also a comparison between close fit computation and Davidson laboratory experiments. The Davidson laboratory experiments for this ship show a remarkable reduction in pitch amplitudes at high speed when compared with more conventional ships.

When comparing the computed and measured motions for the Davidson Type A, the results are remarkably good. Of foremost interest is the nearly perfect agreement between Davidson experiment and computed pitch motions at all speeds. The large reductions in pitch amplitude as shown in the experiments are also clearly shown in the computation. This in itself provides convincing proof as to the validity of the modified strip theory for even radically shaped hull forms. The computed heave motions do not show as good agreement for Fn = 0.15 and 0.25. In these instances the computed heave motion amplitude is overestimated near resonance. The general agreement is good for the limited amount of experimental data available, however, a more detailed experiment over the entire frequency range of comparison will be necessary for a completely conclusive evaluation.

6.4 *Comparison of the results*

It was felt that such an unusual form as the Davidson Type A would be an excellent example for the investigation of the accuracy limitations inherent in the close fit multiple transform computation method. Of greater interest is the fact that a specific change in a hull design has produced such a large and clearly definable variation in the motion. Here, then, is an ideal situation for investigating the equation of motion terms which are responsible for this change and their relationship to the shape of the hull. With this objective in mind, the computed values of all equation of motion terms for the Davidson Type A and the Friesland were compared. Also, the distribution of added mass, damping and exciting forces along these ships was investigated.

The dynamic coefficients of the motion equations (a, b, d, e, A, B, D, E) are given in figures 10, 11, 12 and 13. Computed values only are given for the Davidson Type A and computed and experimental values for the Friesland. Results are given for Fn = 0.15 and 0.45. The forward speed effects normally associated with the static restoring coefficients (C, g), equation (9), have been included in the added mass coefficients (A, d). This change in the static coefficients was made arbitrarily to facilitate comparison with experimental data. The modified coefficients are:

$$\bar{d} = d + \frac{Vb}{\omega^2} = \int_L m' x_b \, dx_b + \frac{Vb}{\omega^2}$$

$$\bar{g} = g + Vb = \varrho g S_w$$

$$\bar{A} = A + \frac{VE}{\omega^2} = \int_L m' x_b^2 \, dx_b + \frac{VE}{\omega^2}$$

$$\bar{C} = C + VE = \varrho g I_w$$

The experimental coefficients for the Friesland are from forced oscillation experiments (SMITH [15]). As shown in the figures testing was done for a number of oscillator amplitudes and frequencies. The Friesland computations and experiments show reasonable good agreement at all speeds and frequencies

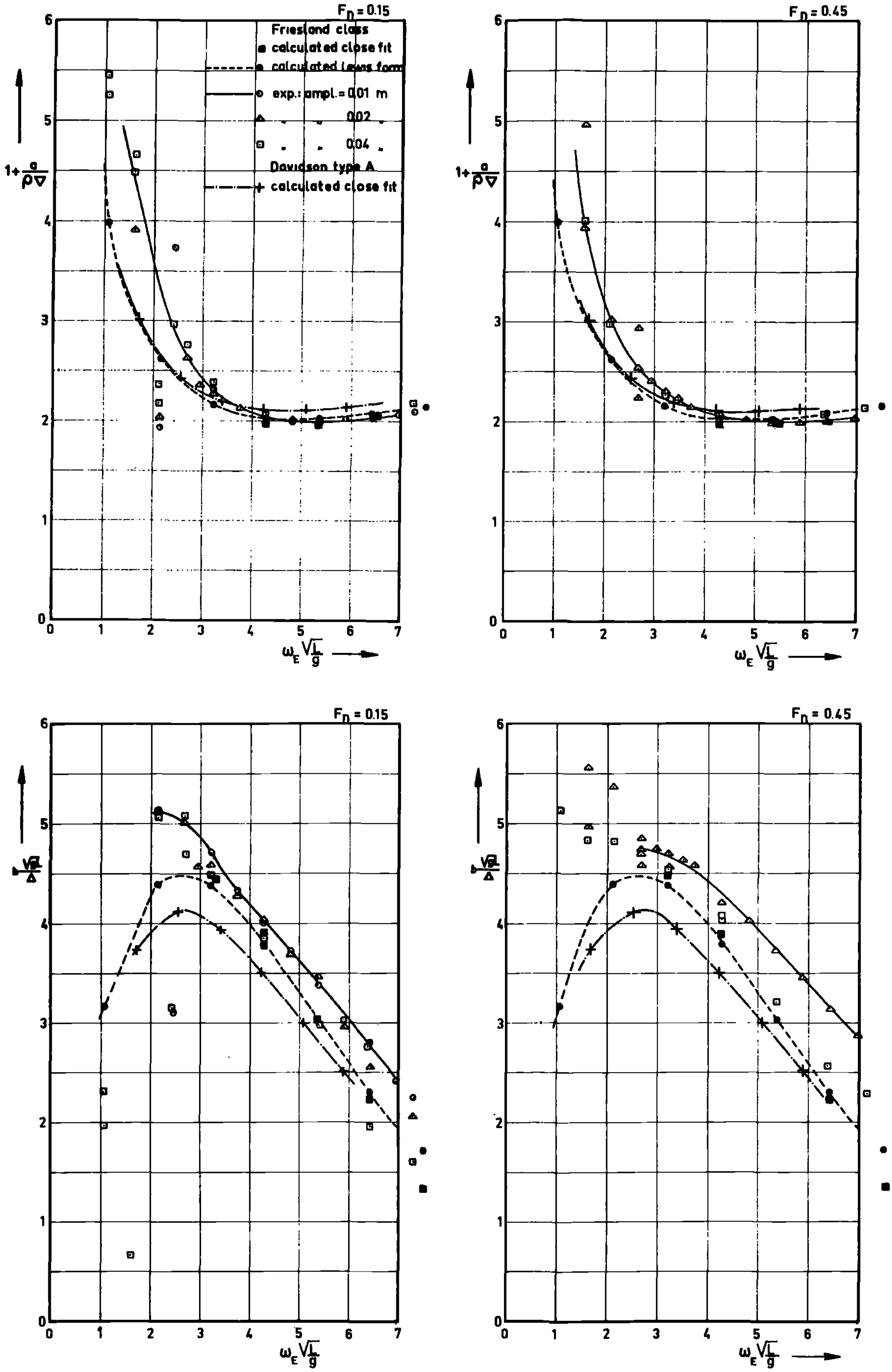


Fig. 10 Equation of motion coefficients *a* and *b*

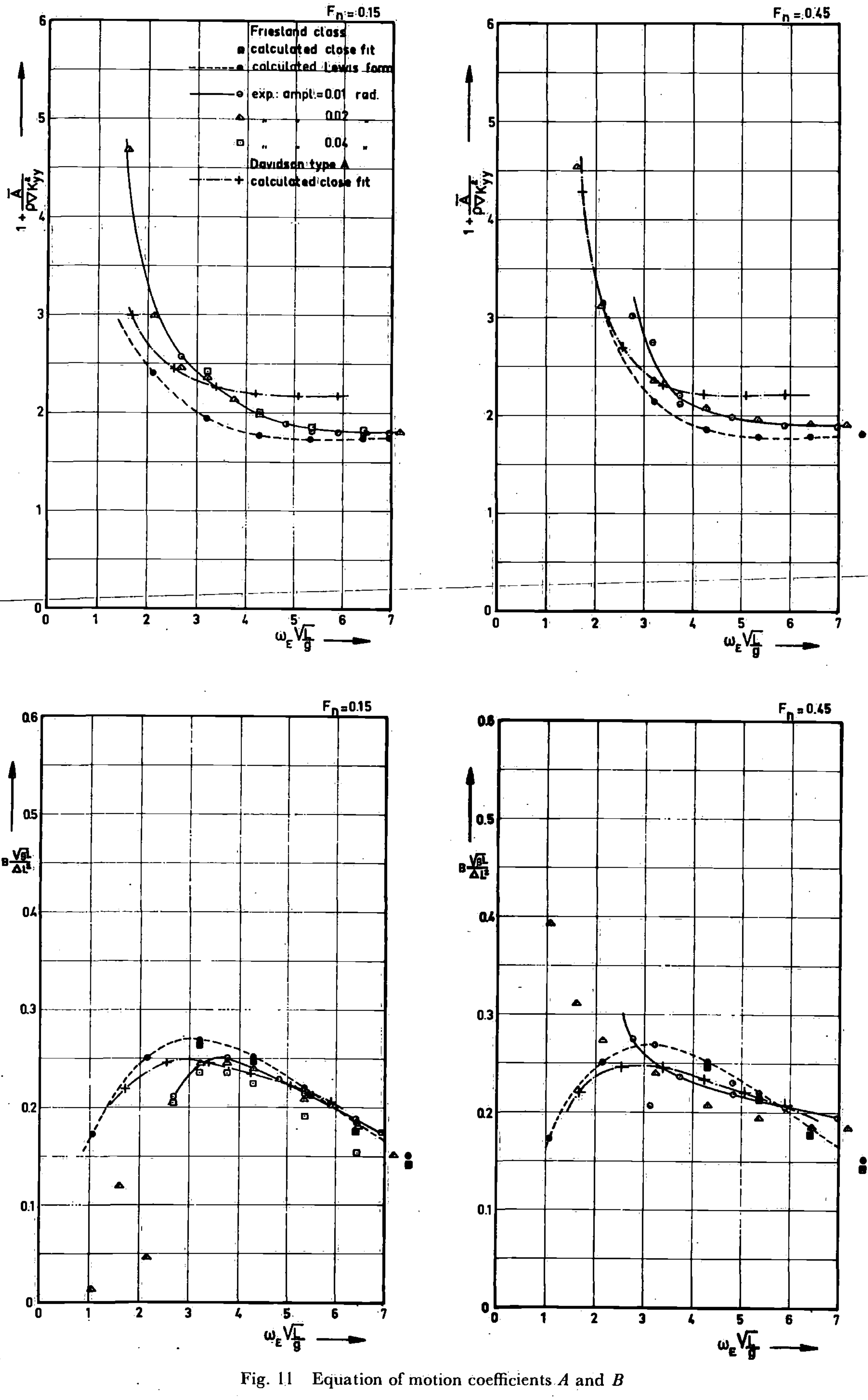


Fig. 11 Equation of motion coefficients A and B

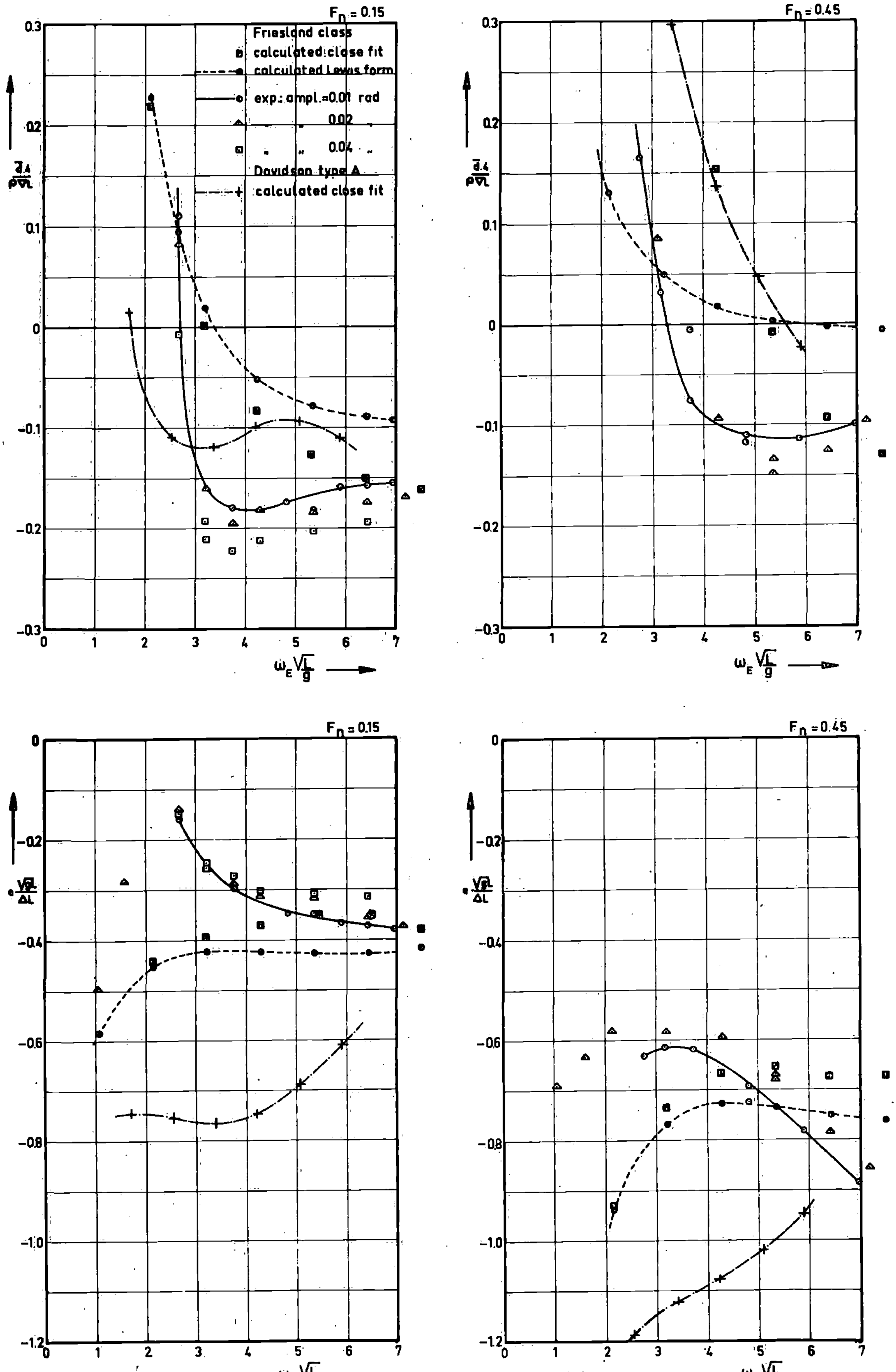


Fig. 12 Equation of motion coefficients *d* and *e*

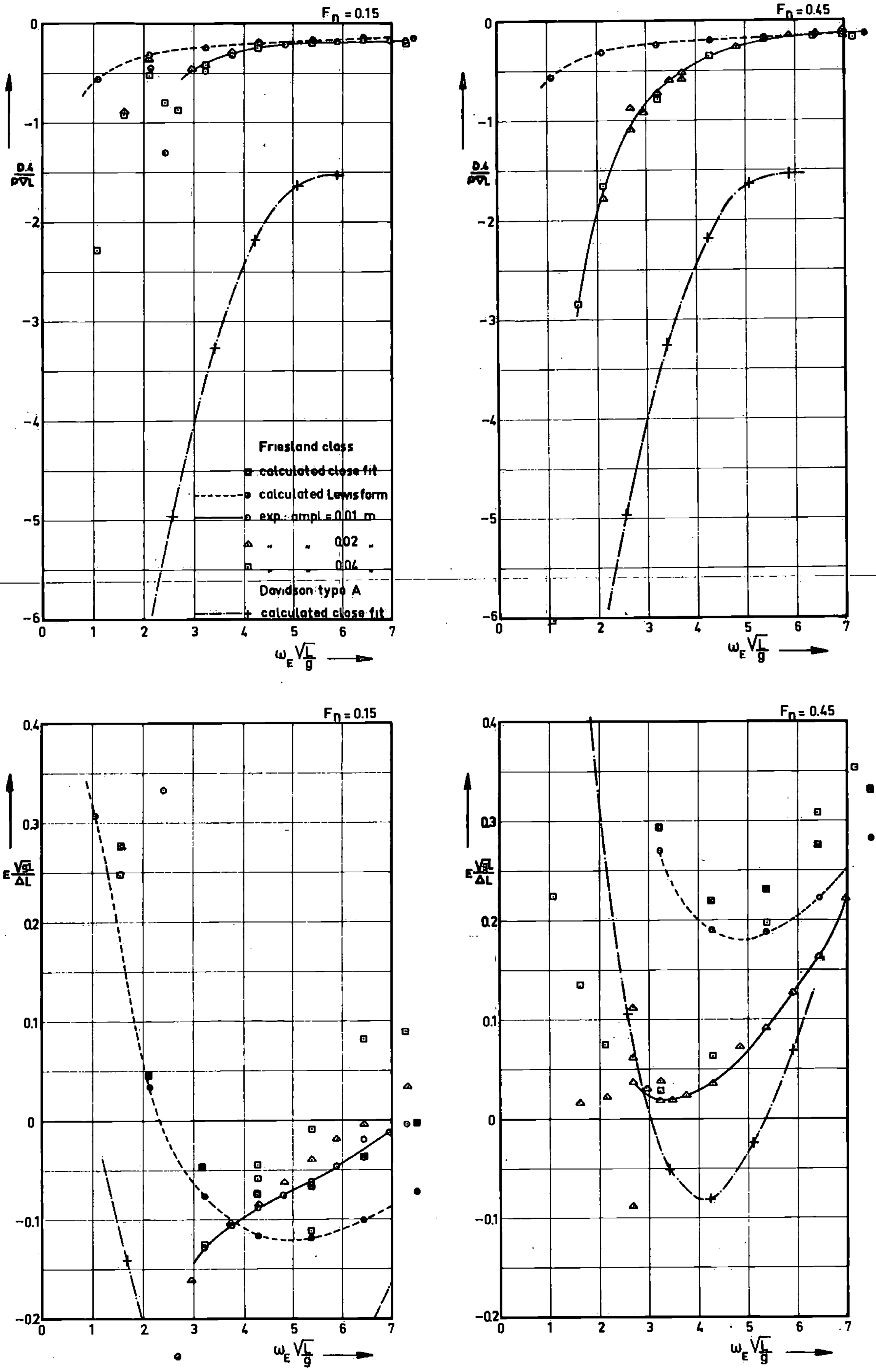


Fig. 13 Equation of motion coefficients D and E

and again demonstrate the ability of modified strip theory to account for forward speed effects.

A comparison between the Davidson Type A and Friesland coefficients shows remarkably little difference in the main added mass and damping terms (a, b, A, B), with the greatest difference less than ten percent. When the cross coupling terms (d, e, g, D, E, G) are compared, however, the situation is quite different, with the damping cross coupling terms differing by as much as 400 percent. This demonstrates the importance, in motion computation, of the cross coupling terms. Further, it indicates that differences in the motions due to hull shape variation are primarily a result of changes in the longitudinal dynamic symmetry and the resultant change in the cross coupling terms. As a demonstration of this effect, the d and e terms in the motion computation for the Davidson Type A were set equal to zero. The motion computation then demonstrates the large effects of coupling as illustrated in figure 5.

The added mass, damping and wave exciting force distribution along the ship are compared in figures 14, 15 and 16. The results are given in non-dimensional form.

The sectional damping is:

$$b' = N' - V\,\frac{\mathrm{d}m'}{\mathrm{d}x_b}$$

The sectional damping in non-dimensional form:

$$\frac{b'}{\varrho\nabla}\sqrt{\frac{L_{pp}^3}{g}}$$

The non-dimensional sectional added mass:

$$\frac{m' L_{pp}}{\varrho\nabla}$$

The sectional exciting force is:

$$\frac{F' L_{pp}}{c}$$

The damping distribution for the forward section of the Davidson Type A is unusual in that, even when forward speed effects are included, several of the sections exhibit virtually zero damping for a limited range of frequencies. Also, the same sections show nearly zero exciting forces. This, then, would appear to be a major reason for the extreme difference in the motion characteristics of the two ships, and apparently offers considerable promise as a device for tuning a ship and thus optimizing the motions. This factor in itself would seem to be of sufficient interest to warrant future investigation.

The distribution of added mass for the two ships is very similar, with only significant differences occurring in the forward part and at the higher frequencies. While the total added mass is virtually identical for both ships, the slope of the added mass distribution curve for the Davidson Type A is much greater in the bow, thus indicating larger values for the speed correction term $\mathrm{d}m/\mathrm{d}x$. The damping distribution for the Davidson Type A, however, is quite different, with large modifications in the two dimensional damping N' by the speed correction term. Of particular interest is section 20, the forward-most section, which shows a large damping at high speed even though the added mass and sectional area are zero. This is entirely forward speed effect. The exciting force distributions (see figure 16) behave similarly to the damping term and clearly show the strong relationship between exciting forces and damping. As previously mentioned, while the distribution of added mass, damping and exciting forces for the two ships is quite different, the total or integrated value for the whole ship in each case is practically the same. This also accounts for the large differences in dynamic cross coupling coefficients, which is demonstrated before, in fig. 5, where for Fn = 0.15 the resulting calculated motion amplitude can be compared with the other experiments and calculations.

7 Conclusions

(a) The use of modified strip theory and a multiple coefficient transformation computation for pitch and heave motions is confirmed and extended by this comparison.

(b) The influence of variations in hull shape can be accounted for using close fit transformation methods.

(c) The large variation in dynamic symmetry or fore and aft distribution of exciting forces, moments, added mass and damping producable by hull shape variations strongly indicates that such variations can be used to optimize the motions.

(d) A close fit program which can account for the fore and aft dynamic distributions is mandatory when computing bending moments, relative motion, etc.

(e) The dynamic cross coupling terms in the equations of motion are of paramount importance when optimizing the motions.

(f) An efficient program which can generate conformal transformation coefficients for an arbitrary simply connected shape is demonstrated.

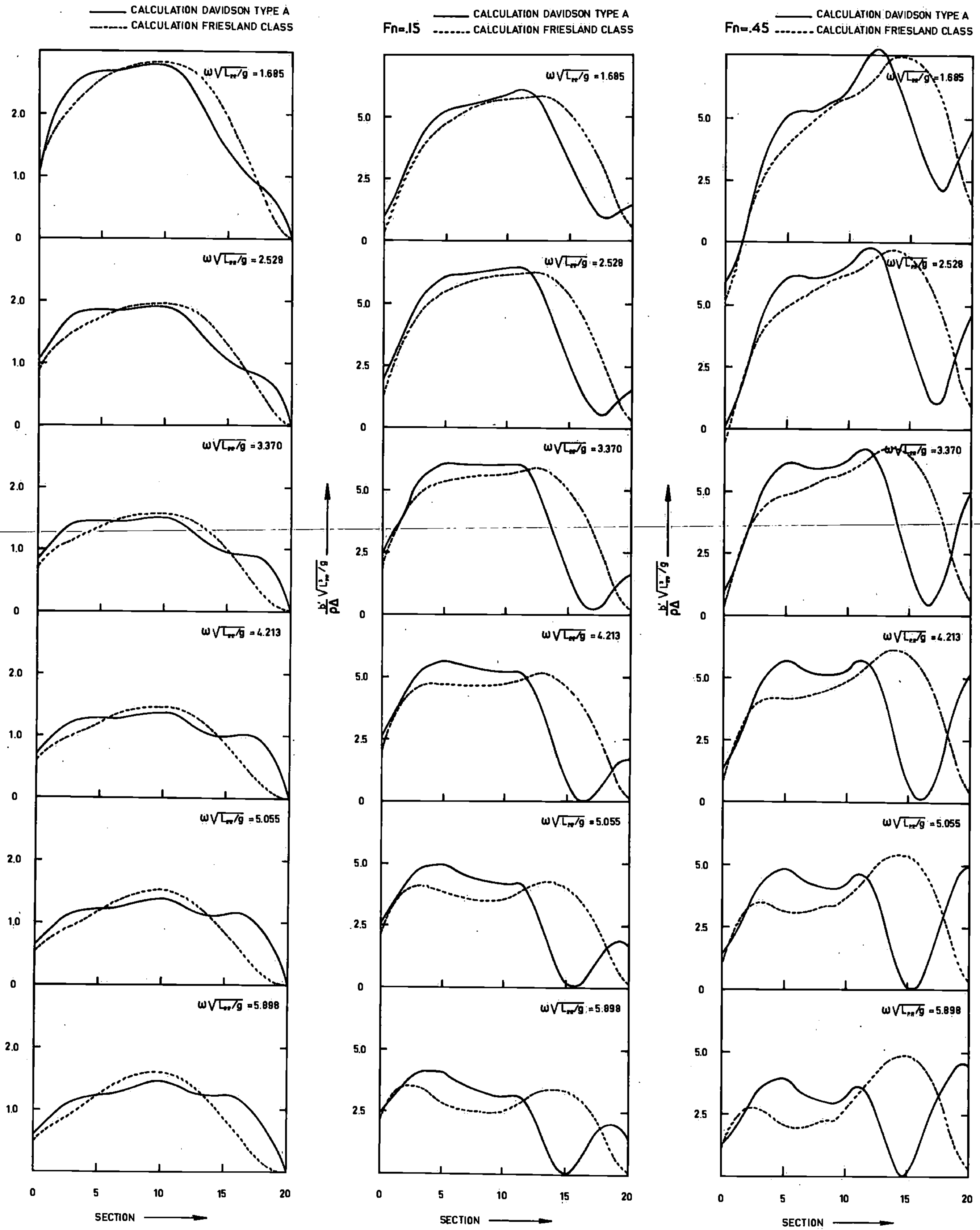


Fig. 14 Added mass and damping distribution Fn = 0.15 and Fn = 0.45

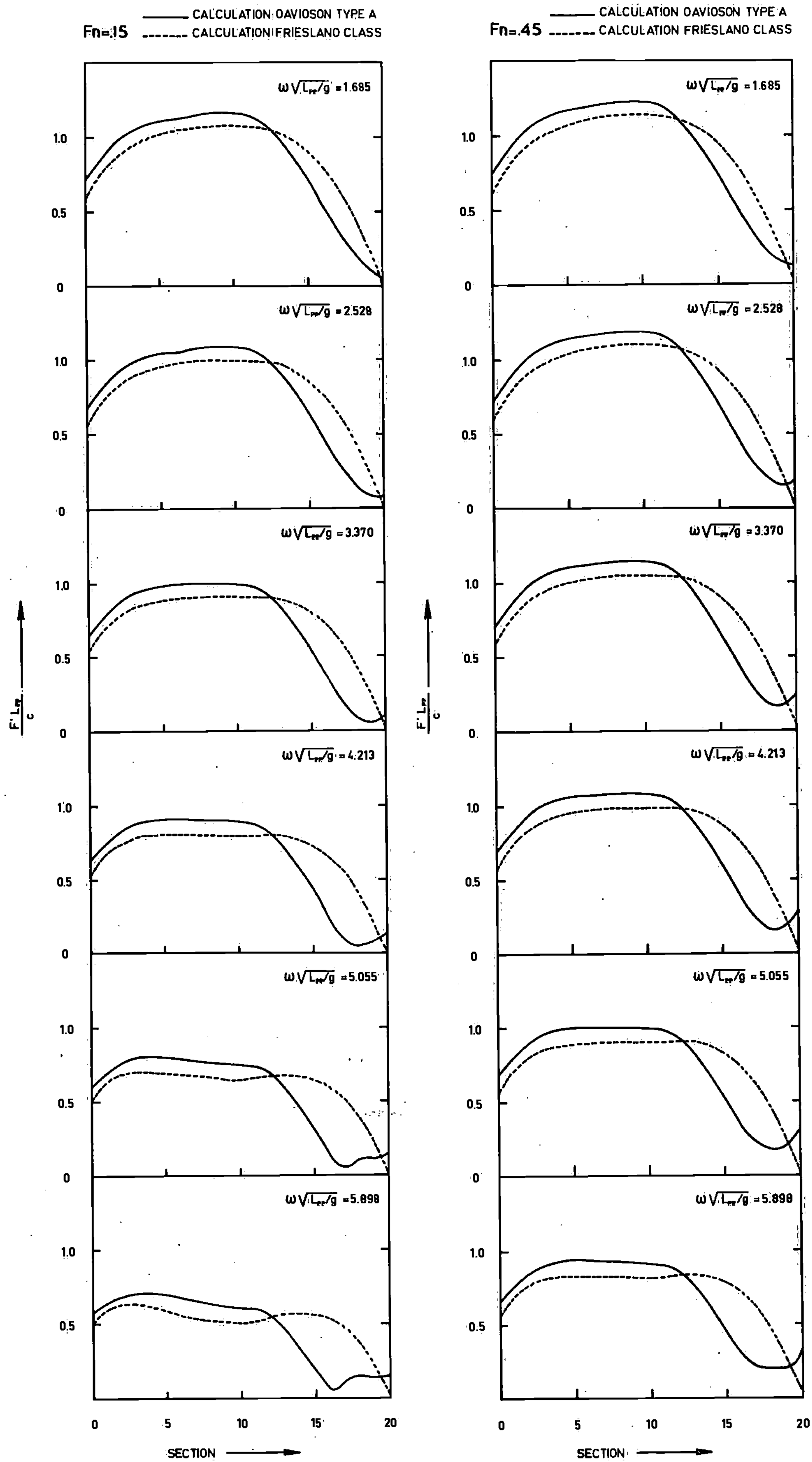


Fig. 15 Exciting force distribution Fn = 0.15 and Fn = 0.45

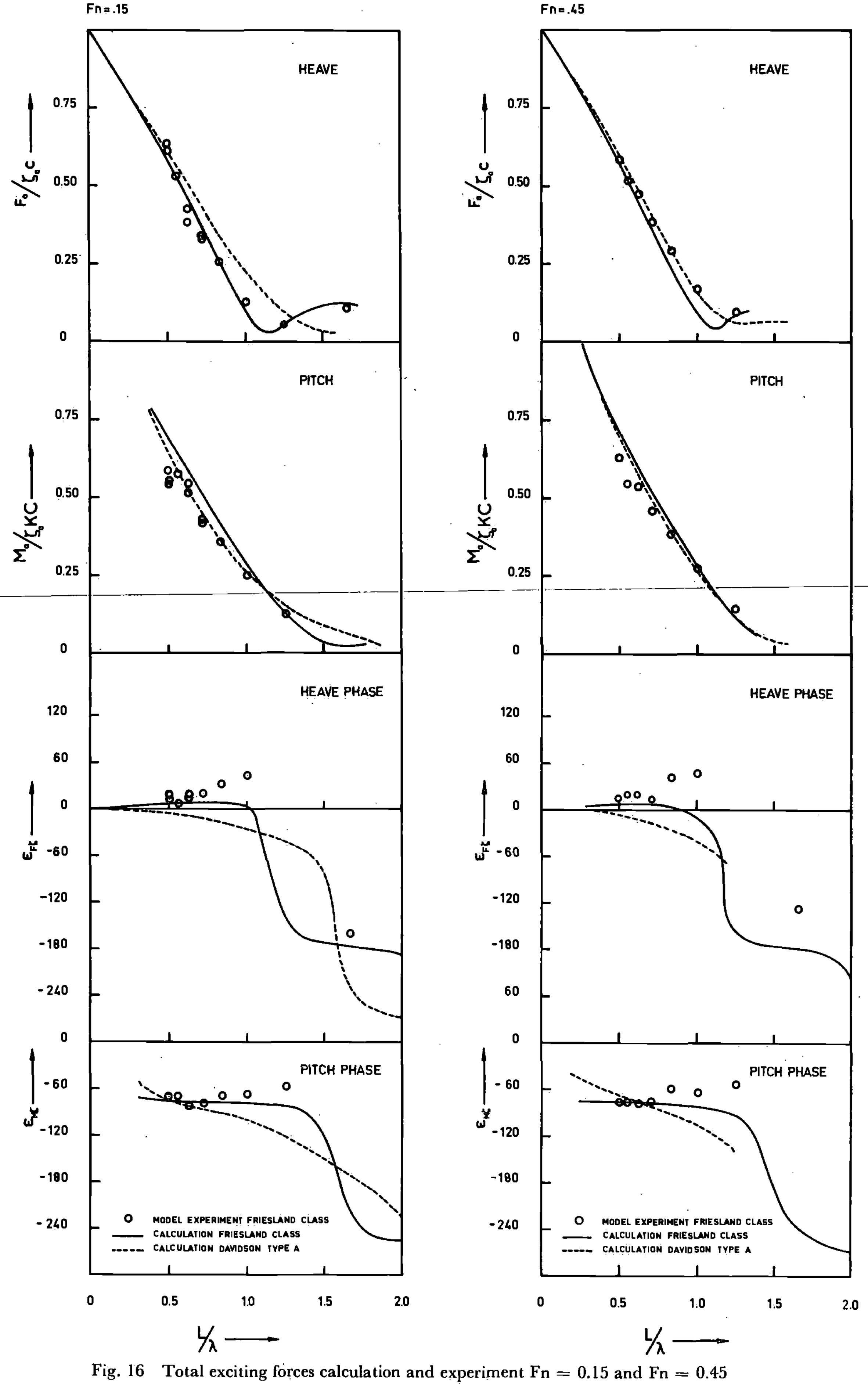


Fig. 16 Total exciting forces calculation and experiment Fn = 0.15 and Fn = 0.45

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