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Optimizing multi-attribute pricing plans with time- and location-dependent rates for different carsharing user profiles

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ABSTRACT

One of the main challenges of one-way carsharing systems is to maximize profit by attracting potential customers and utilizing the fleet efficiently. Pricing plans are mid or long-term decisions that affect customers' decision to join a carsharing system and may also be used to influence their travel behavior to increase fleet utilization e.g., favoring rentals on off-peak hours. These plans contain different attributes, such as registration fee, travel distance fee, and rental time fee, to attract various customer segments, considering their travel habits. This paper aims to bridge a gap between business practice and state of the art, moving from unique single-tariff plan assumptions to a realistic market offer of multi-attribute plans. To fill this gap, we develop a mixed-integer linear programming model and a solving method to optimize the value of plans' attributes that maximize carsharing operators' profit. Customer preferences are incorporated into the model through a discrete choice model, and the Brooklyn taxi trip dataset is used to identify specific customer segments, validate the model's results, and deliver relevant managerial insights. The results show that developing customized plans with time- and location-dependent rates allows the operators to increase profit compared to fixed-rate plans. Sensitivity analysis reveals how key parameters impact customer choices, pricing plans, and overall profit.

1. Introduction

Carsharing is a type of shared-mobility system that enables users to have private and short-term access to a vehicle without the hassles of vehicle ownership, such as the costs of purchase and maintenance. The existing carsharing services fall primarily into the "round-trip" service in which the users are required to return the vehicle to the same location (station) where it was picked up, and the "one-way" service where users can deliver the car to a different location than where the trip started (Nourinejad et al., 2015; Jorge and Correia, 2013). The one-way services, which are the main focus of this study, are the most popular type of carsharing due to the flexibility they offer to users. However, one-way services come at the cost of an imbalanced distribution of cars, which may lead to vehicle shortages and customer dissatisfaction. Thus, one of the main challenges of one-way services is to execute cost-efficient strategies that can maintain a balanced supply–demand system to achieve proper utilization of vehicles. Consequently, a large body of literature has focused on relocation problems to move vehicles from a station/area with an oversupply to a station/area with a shortage of vehicles (Liu et al., 2022; Huang et al., 2020).

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The design and implementation of one-way carsharing systems pose various challenges, both practical and scientific. Hence, carsharing is being intensively studied in the academic literature. Nevertheless, there is still a gap between the literature and the business development of the market. In this context, we aim to consider the business practices and realistic requirements of carsharing systems to develop a model that could more easily be applied in practice. A recent business review by Golalikhani et al. (2021b) revealed that the pricing decisions of carsharing operators (CSOs) are among the most significant issues in the market. In the carsharing systems in general, pricing decisions can be used as a means of controlling the interactions between demand and supply to increase the profitability of CSOs (Turan et al., 2020). These pricing decisions can be divided into dynamic price incentives and "strategic pricing plans", that present key structural differences. The dynamic price incentives are short-term decisions that aim to encourage users to rent vehicles for specific trips and modify the origin/destination of the trips to achieve a better balance according to the dynamic distribution of the vehicles (Zhang et al., 2022; Giorgione et al., 2020, 2019). Even though dynamic pricing and relocation procedures can effectively balance the distribution of vehicles, carsharing systems can still benefit from applying more tactical and strategic decisions. In this context, the "strategic pricing plans" (hereafter referred to as "pricing plans") are mid/long-term decisions (e.g., yearly decisions) that play a crucial role in attracting potential customers to join a carsharing system as a user and mainly seek to adjust the demand to the supply in a long-term perspective for profit maximization (Golalikhani et al., 2021a). In practice, CSOs offer a few pricing plans to attract various segments of customers with different travel behaviors. These plans are often constructed from several attributes (e.g., registration fee, rental time fee, and travel distance fee) with different values considering spatial and temporal issues and customers' sensitivity to the value of these attributes. Nevertheless, to our knowledge, no study has been conducted to design realistic pricing plans. The main goal of this paper is to introduce a model designed to facilitate decision-making regarding realistic pricing plans that compete to capture those segments of users that yield maximum profitability. The model optimizes the values of pricing plans' attributes, considering different areas of the city and time periods according to their demand volumes. These pricing plans will be presented to potential customers in a user-friendly and transparent way, such as a price table. In summary, the main contributions of this paper are threefold:

1. **Multi-attribute pricing plans:** We tackle the problem of developing a combination of multi-attribute pricing plans for different segments of customers as a mid/long-term decision to be determined offline, considering the travel behavior of potential customers. The value of plans' attributes is based on the origin of the trips (e.g., high/low-demand areas), as well as the rental time (e.g., high/low-demand periods). We investigate how CSOs should develop pricing plans when customers are mainly concerned about the values of plans' attributes that are relevant to their usual travel behavior. To this end, the heterogeneity of travelers is mainly considered in terms of their travel patterns (e.g., average travel time and travel distance per trip), which are highly related to the attributes of plans (e.g., rental time fee and travel distance fee).
2. **Mathematical model:** We develop a mixed-integer nonlinear programming (MINLP) model to maximize profit for optimally setting the best combination of plans in one-way carsharing systems. A discrete choice model is incorporated into the optimization formulation to express customers' responses to the value of the plans' attributes. Given the nonlinearity of the model, a linear transformation method is applied to allow the computation of optimal plans. We also proposed a decomposition algorithm to produce high-quality results efficiently within a shorter timeframe.
3. **Managerial and practical insights:** We deliver valuable insights from the application of the model to a real case study in Brooklyn, New York. Our approach to identifying different types of areas and periods within the city provides useful insights for various shared-mobility systems that need to analyze temporal and spatial factors affecting demand. We identify the main factors affecting the system's profit and complexity, thereby providing insights for modeling carsharing pricing strategies. By comparing time- and location-dependent rates with fixed-rate plans and demonstrating the trade-offs between profit potential and the simplicity of plans, we offer practical guidance for future pricing decision efforts. Additionally, we conduct a detailed sensitivity analysis to investigate the impact of key parameters (e.g., the ratio between the number of vehicles and customers) on pricing plans and system profitability, offering further managerial insights into the planning and operations of carsharing systems.

This paper is organized as follows. First, Section 2 reviews the relevant literature and business practices on pricing decisions. Then, in Section 3, a MILP model for optimizing the pricing plans of one-way carsharing systems is formulated. Section 4 describes the case study and data analyses. In Section 5, computational results and sensitivity analysis are presented. The conclusions and future research directions are discussed in Section 6.

2. Literature review and business practices

This paper aims to fill a relevant gap between state of the art and business practices by optimizing multi-attribute plans in one-way systems. This section reviews the key carsharing pricing papers (Section 2.1) and presents some of the relevant pricing practices that are usually oversimplified in the literature (Section 2.2).

2.1. Scientific literature

Designing strategic pricing plans for different customer segments involves critical and complex decisions, as highlighted by Perboli et al. (2018), who emphasized that customized pricing plans are crucial for the success of CSOs. However, most existing pricing studies focus on dynamic pricing problems as a short-term decision. Dynamic pricing may be an effective way of balancing the system by modifying rental fees based on actual vehicle availability, as it offers users trips for lower prices at overstocked areas/periods and higher prices for requests in areas/periods with vehicle shortages. During the past years, several studies have focused on dynamic pricing to mitigate the effects of vehicle stock imbalance in one-way CSOs. For instance, Di Febbraro et al. (2012) proposed a price incentive mechanism that offers a destination-based variable discount to users. Giorgione et al. (2019) presented an origin-based dynamic pricing model in which price depends on vehicle availability in booking stations. More recently, Zhang et al. (2022) proposed a MINLP model to determine the optimal discounting scheme and request allocation plans for one-way carsharing systems.

Apart from the dynamic pricing studies addressed above, some scholars tackled the strategic pricing decisions of CSOs. Huang et al. (2022a) developed a carsharing demand function and a profit maximization model to evaluate the effect of fixed pricing policy and dynamic pricing policy of a carsharing system. Nevertheless, unlike this study, the fixed pricing policy consists of a single attribute (i.e., rental time fee per hour), which is the same during peak and off-peak hours and high and low-demand areas. Studies by Jorge et al. (2015) and Xu et al. (2018) contain optimization models to determine trip prices that maximize the profit of one-way carsharing systems. Unlike the model proposed by Jorge et al. (2015) with linear elastic demand concerning the trip price, the model developed by Xu et al. (2018) investigated trip pricing as a real-time mechanism under nonlinear elastic demand. However, in both models, the heterogeneity of customers was not considered, and the trip prices consisted of only a single attribute (i.e., rental time fee) equal for all users and only differed between the origin and destination pairs. Lu et al. (2021) and Nguyen et al. (2022) developed bi-level mathematical models to maximize the profits of one-way CSOs. Both studies focused on optimizing a single-attribute trip price at the upper level.

In summary, this study differs from previous works in several ways. First, while prior literature has focused on single-attribute strategic pricing policies, our study develops multi-attribute plans that align with market practices. Second, unlike previous studies that typically considered only one plan and assumed homogeneous customers, we design multiple plans to attract various customer segments with different needs by analyzing their travel behaviors. Finally, consistent with business practices, each plan includes varying rental time fees for different periods and areas based on the trip's origin and rental time (e.g., weekends) to increase fleet utilization. To our knowledge, this is the first attempt to develop a combination of multi-attribute pricing plans with time- and location-dependent rates for different customer segments, aiming to maximize the profit of CSOs.

2.2. Business practices

In order to develop a realistic and practical model, it is crucial to consider the relevant business practices and contexts of CSOs. More recently, Golalikhani et al. (2021b) conducted a business review and concluded that, due to the recency of this business, there is still a "gap of understanding" of the scientific community concerning the business contexts of carsharing, which leads to the developments of pricing models that are often oversimplified. Hence, this section presents some of the business practices regarding the pricing plans of CSOs in the market that are relevant for this paper.

The existing pricing plans in the market are moving from single-tariff models to increasingly complex plans. In this regard, CSOs use a combination of different attributes in the structure of plans which are often presented to the customer in a transparent way, such as a price table. These attributes can be fixed costs that do not depend on the use of vehicles (e.g., registration fee or monthly membership fee) or variable costs that depend on the use of vehicles per trip. As for the variable costs per trip, users are often charged based on a combination of the rental time fee and travel distance fee. The rental time fee may differ during specific periods of day and week, e.g., some CSOs charge a lower rate at night to increase fleet utilization in low-demand periods. As for the travel distance fee, pricing plans often contain some free included distance per rental e.g., 10 kilometers (km), and, once the allowance of km is met, customers pay for the extra distance traveled.

Table 1 presents four pricing plans that Communauto carsharing offers to the customers in Ontario (Communauto, 2022). For instance, some customers may select the second pricing plan (i.e., Value plan) to join system, which requires them to pay \$5 per month as a fixed membership cost. Then, for each trip during weekdays, they should pay a combination of \$3.85 per hour and \$0.39 per km as the variable costs. Moreover, the rental time fee per trip during weekends is \$5.35 per hour, which is more expensive than weekdays since carsharing demand is higher during weekends in Ontario. Regarding the free included distance, for example, the first plan of Communauto carsharing (i.e., Open plan) involves 75 km free included distance for each trip, and customers who use this plan should pay \$0.17 for extra km traveled on that reservation. For details, we refer the reader to business review by Golalikhani et al. (2021b), where a comprehensive review and dataset of the business practices of CSOs, including the structure of their pricing plans, is presented.

Table 1
Pricing plans offer by Communauto carsharing in Ontario (Canada).

Name of plan	Fixed	Variable (per trip)			
	Monthly fee	Rental time fee (per hour)		Free included distance (km)	Travel distance fee (per km)
		Weekdays	Weekends		
OPEN	\$0	\$9	\$9	75	\$0.17 (first 75 km is free)
VALUE	\$5	\$3.85	\$5.35	0	\$0.39, after 50 km \$0.20
VALUE PLUS	\$12.5	\$3.45	\$4.95	0	\$0.35, after 50 km \$0.20
VALUE EXTRA	\$30	\$2.95	\$4.45	0	\$0.25, after 50 km \$0.20

3. Problem statement and mathematical model

This section defines the problem of designing proposed pricing plans in one-way carsharing systems.

3.1. Problem definition and assumptions

A CSO offers one-way services and faces the problem of deciding the values of pricing plans' attributes to maximize the system's profit. Our goal is to develop a model for optimizing a set of plans that compete to capture those segments of users that maximize the profit. The system's characteristics are clarified as follows:

Pricing plans: In line with business practices, plans should involve both fixed costs (i.e., a one-time registration fee) and variable costs. The variable costs are calculated based on rental time fee per minute and travel distance fee per km, taking into account the free included distance per trip. Note that it may also be possible to incorporate the free included distance into the travel distance fee to simplify the model. Nonetheless, considering the significance of the free included distance value in influencing customers' decisions when selecting from the offered plans, we distinguish these two plans' attributes to ensure that our model is fully aligned with market reality. The values of plans' attributes (i.e., registration fee, rental time fee, travel distance fee, and free included distance) must be selected from discrete sets that closely align with market values. In Section 5, these discrete sets represent the possible values that can be selected for plans' attributes as the decision variables of the problem. Using discrete variables in our model is motivated by several critical factors. Firstly, it aligns with real-world business practices where CSOs typically offer pricing plans in discrete increments due to some non-quantitative factors such as psychological and marketing considerations (Soppert et al., 2022; Eilertsen et al., 2024). This approach helps avoid overlapping values that are too similar, making the plans more comprehensible and practical for users. Secondly, this approach facilitates the linearization of the model, enabling us to also employ exact solution methods and thus ensuring computational efficiency. Additionally, discrete variables allow for effective approximation of various demand functions while maintaining a linear structure. Performing in-depth sensitivity analysis with optimal values further enhances the model's practical applicability, providing actionable managerial insights grounded in realistic, user-friendly pricing strategies

Business areas and time periods: We distinguish among different areas of the city and time periods according to their demand volume (i.e., high and low-demand areas/periods), following business practices, such as described in Communauto (2022). Each plan can have various rental time fees for different periods and areas based on the origin of the trips, as well as the starting time of rental. For instance, a plan may have a lower rental time fee for trips originating from a low-demand area during off-peak periods and a higher rate for trips starting from a high-demand area during peak periods.

Customers choice: In this study, every customer chooses between one of the offered plans (thus joining the carsharing system) or keeps his/her current choice (thus resorting to outside options such as a private car or public transportation). Each plan may attract different shares of the various segments of customers, and a discrete choice model estimates the choices of a given customer segment among different alternatives. Moreover, we make some assumptions regarding consumer choice behavior as follows:

A1- Customers demand segments: We identify customer segments based on the main characteristics of their trips, including their usage frequency, average travel distance per trip, and average travel time per trip, which are strongly related to plans' attributes (i.e., registration fee, rental time fee, travel distance fee, and free included distance). Thus, we do not directly consider the socio-demographic characteristics (we do not cluster users according to, e.g., income) since our focus is on the pricing model rather than on the data collection procedures for estimating those behavioral parameters for a concrete case study.

A2- Customers preferences: The users that are part of the same demand segment are considered to have a homogeneous travel behavior and, thus, the same utility function. The utility given by a user to a plan is the sum of a known component of the utility and an error term. The known component is a function of the plan's attributes, weighted by the importance of attributes for each customer segment, as well as an alternative-specific constant. The weights (i.e., coefficients) of attributes are instance-dependent parameters, which must be calculated based on the travel behavior of different segments. The utility of the status quo is set to 0.

Based on the system's characteristics and assumptions mentioned above, the problem can be briefly stated as follows. Given (a) different areas and time periods of a city according to their demand volume; (b) the demand data of each segment in different types of areas and periods; (c) maximum demand (in number of trips) that can be served in any area and period; (d) desired number of plans and levels for plans' attributes; (e) the importance (coefficient) of each attribute for each customer segments; and (f) the main costs of the system, the CSO is to decide the value of plans' attribute to maximize the system's profit.

3.2. Mathematical model and linear transformation

In the following, we first introduce the notations used in the paper and then present the mathematical model.

Indices and parameters

$p, p' \in \mathcal{P} = \{1, \dots, P\}$	Indices for the set of pricing plans $\mathcal{P}$, with P being the number of plans.
$g \in \mathcal{G} = \{1, \dots, G\}$	Index for the set of demand segments $\mathcal{G}$, with G being the number of segments.

$i \in \mathcal{A} = \{1, 2\}$	Index for the set of types of urban areas $\mathcal{A}$, based on their demand volume, with 1 being a high-demand urban area and 2 being a low-demand urban area.
$k \in \mathcal{K} = \{1, 2\}$	Index for the set of time periods $\mathcal{K}$, based on their demand volume, with 1 being a high-demand period and 2 being a low-demand period.
$v \in \mathcal{V} = \{1, \ldots, V\}$	Index for the set of possible registration fee levels, where V is the number of levels.
$s \in \mathcal{S} = \{1, \ldots, S\}$	Index for the set of possible free included distance amount levels per trip, where S is the number of levels.
$r \in \mathcal{R} = \{1, \ldots, R\}$	Index for the set of possible travel distance fee levels, where R is the number of levels.
$q \in \mathcal{Q} = \{1, \ldots, Q\}$	Index for the set of possible rental time fee levels, where Q is the number of levels.
Y_p^v	Registration fees at level v for pricing plan p.
L_p^r	Travel distance fee per km at level r for pricing plan p.
M_{pik}^q	Rental time fee per minute at level q for plan p for a trip starting from an area of type i in a period k.
F_p^s	Free included distance amount (km) per trip at level s for pricing plan p.
N_g	Number of customers in each segment g.
D_{gik}	Total demand (number of trips) for all customers of segment g starting from an area i in a period k.
O_{ik}	Maximum demand (number of trips) that can be served in an area of type i in a period k. As the periods and areas are considered as aggregate types (i.e., peak vs off-peak periods, high-demand vs low-demand areas), this upper bound is an approximation of the available capacity (related to fleet or other business constraints) to meet the demand from different segments. This allows distinguishing different types of periods and areas based on the expected overall capacity deployment.
AVT_{gik}	Average travel time (minutes) for each trip by customers of segment g starting from an area of type i in a period of type k. As customer segments aggregate similar travel behaviors, this parameter summarizes the average travel time of each segment for different types of trips.
AVD_{gik}	Average travel distance (km) for each trip by customers of segment g starting from an area of type i in a period of type k. This parameter summarizes the average of travel distance of each segment for different types of trips.
APD_{pgik}^s	Average payable distance (km) for each trip starting from an area of type i in a period k by customers of segment g who selected plan p when the level s of free included. $APD_{pgik}^s = \max\{AVD_{gik} - F_p^s, 0\}$.
VCR	Ratio between the number of vehicles and the number of customers in the carsharing system.
CO	Average monthly costs per shared car, including depreciation, maintenance, and parking costs.
CF	Fuel cost per km.
α	Coefficient of lexicographic ordering constraint to eliminate the symmetric solutions.
${\beta^0}_g$	Alternative-specific constant in the utility function of the demand segment g to measure the average unexplained utility of choosing a pricing plan compared to the current alternative.
${\beta^1}_g$	Marginal disutility of registration fee for segment g.
${\beta^2}_g$	Marginal utility of free included distance for segment g.
${\beta^3}_g$	Marginal disutility of travel distance fee for segment g.
${\beta^4}_{gik}$	Marginal disutility of rental time fee for segment g for a trip starting from an area i in a period k.

Decision variables

y_p^v	= 1 if registration fee at level v is chosen for pricing plan p, 0 otherwise.
f_p^s	= 1 if free included distance at level s is chosen for pricing plan p, 0 otherwise.
l_p^r	= 1 if travel distance fee at level r is chosen for pricing plan p, 0 otherwise.

m^q_{pik}	= 1 if rental time fee at level q is chosen for pricing plan p for a trip starting from an area of type i in a period k, 0 otherwise.

Auxiliary decision variables

u_{pg}	Utility obtained by customers of segment g when choosing pricing plan p
p_{pg}	Proportion of customers of segment g that choose pricing plan p.
n^v_{pg}	Number of customers in segment g that choose pricing plan p with a registration fee at level v.
d^q_{pgik}	Total satisfied demand (in number of trips) starting from an area of type i in a period of type k of customers segment g that choose pricing plan p with a rental time fee at level q.
z^{sr}_{pgik}	Total payable distance (km) for trips performed by the sum of customers of segment g that choose plan p with a free included distance amount at level s and a travel distance fee at level r, starting from an area of type i in a period k.

Problem formulation

The remainder of this section first presents the proposed mathematical model to maximize the profit of a one-way carsharing system through the design of optimal pricing plans. Developing this model required careful consideration of the problem's structure and constraints to incorporate relevant business practices and reflect the inherent challenges of one-way systems. Due to the nonlinearity of the original model, we then propose linear transformations to reformulate the original model as a linear problem. This linearization (reformulation of the MINLP as a MILP) involved the introduction of various new parameters to compute the utility of any possible plan as input parameters, ensuring a sufficiently strong linear formulation that allows the computation of optimal plans using standard MILP solvers.

$$Max(Z) = \sum_{p\in\mathcal{P},g\in\mathcal{G},i\in\mathcal{A},k\in\mathcal{K}} \left(\sum_{q\in\mathcal{Q}} d^q_{pgik} \times M^q_{pik} \times AVT_{gik} + \sum_{s\in\mathcal{S},r\in\mathcal{R}} z^{sr}_{pgik} \times L^r_p - \sum_{q\in\mathcal{Q}} d^q_{pgik} \times AVD_{gik} \times CF\right) + \sum_{p\in\mathcal{P},g\in\mathcal{G},v\in\mathcal{V}} n^v_{pg} \times (Y^v_p - VCR \times CO) \tag{1}$$

The objective function (1) maximizes the CSO's profit. It is calculated as the total revenues generated by rental time fee, travel distance fee, and registration fee paid by users, minus the total fuel costs of the satisfied trips and average monthly costs per shared car, determined by the ratio between vehicles and customers.

$$\sum_{v\in\mathcal{V}} y^v_p = 1 \qquad \forall p \tag{2}$$

$$\sum_{s\in\mathcal{S}} f^s_p = 1 \qquad \forall p \tag{3}$$

$$\sum_{r\in\mathcal{R}} l^r_p = 1 \qquad \forall p \tag{4}$$

$$\sum_{q\in\mathcal{Q}} m^q_{pik} = 1 \qquad \forall p,i,k \tag{5}$$

Constraints (2), (3), and (4) respectively ensure that for each pricing plan, only one registration fee, one free included distance, and one travel distance fee is selected. Constraint (5) assures that for each plan and each type of trip (i.e., trip starting from an area of type i in a period k), only one rental time fee is selected.

$$u_{pg} = \beta^0_g + \sum_{v\in\mathcal{V}} \beta^1_g(y^v_p \times Y^v_p) + \sum_{s\in\mathcal{S}} \beta^2_g(f^s_p \times F^s_p) + \sum_{r\in\mathcal{R}} (\beta^3_g \times L^r_p \times l^r_p) + \sum_{i\in\mathcal{A},k\in\mathcal{K},q\in\mathcal{Q}} (\beta^4_{gik} \times M^q_{pik} \times m^q_{pik}) \qquad \forall p,g \tag{6}$$

Constraint (6) calculates the utility obtained by customers in each segment for choosing a given pricing plan.

$$p_{pg} = \frac{\exp(u_{pg})}{\sum_{p'\in\mathcal{P}} \exp(u_{p'g}) + 1} \qquad \forall p,g \tag{7}$$

Constraint (7) calculates the proportion of customers of each segment that choose a plan. It is a multinomial logit (MNL) model that takes into account the utility obtained by each plan and the utilities obtained by other plans, as well as the utility of the status quo (set to 0), considering that the error term follows a Gumbel distribution. It should be noted that we might be overestimating the preferences for the carsharing plans versus the status quo due to the independence of irrelevant alternatives assumption of the MNL. We focus on plans' attributes as the most important factors of the proposed MNL model since the costs of using shared vehicles are the primary motivation to join the carsharing system (Rotaris, 2021; Caulfield and Kehoe, 2021). Hence, although we are aware of the error that might result from this simplification, we believe the MNL model brings suitable and realistic results since, in practice, the number of alternative pricing plans is limited, and the differences between them have significant relevance for consumer choice.

$$d^q_{pgik} \le p_{pg} \times D_{gik} \qquad \forall p,g,i,k,q \tag{8}$$

$$\sum_{p\in\mathcal{P},g\in\mathcal{G},q\in\mathcal{Q}} d^q_{pgik} \leq O_{ik} \qquad \forall i,k \tag{9}$$

$$\sum_{q\in\mathcal{Q}} d^q_{pg'ik} \leq 1.1\times(\frac{D_{g'ik}}{\sum_{g\in\mathcal{G}} D_{gik}})\times \sum_{g\in\mathcal{G},q\in\mathcal{Q}} d^q_{pgik} \qquad \forall p,g',i,k \tag{10}$$

$$d^q_{pgik} \leq m^q_{pik}\times O_{ik} \qquad \forall p,g,i,k,q \tag{11}$$

Constraints ((8)–(11)) are related to the satisfied demand. Constraint (8) ensures that the number of trips that start from an area of type i in a period k of customers of segment g that choose plan p with a rental time fee at level q is less than or equal to the proportion of customers of segment g, that choose plan p times the number of trips of segment g that start from an area of type i in a period k. Constraint (9) sets an upper bound on the maximum number of trips that can be served in an area of type i in a period k considering the available capacity of the system (related to fleet or other business constraints). In Section 5.2, this constraint is explained in detail, and an in-depth sensitivity analysis is undertaken to assess the influence of parameter O_{ik} on the model's performance. In this setting, the demand for a given trip type may include customers from different segments. Hence, considering the maximum number of trips that can be served, the model tends to assign the fleet capacity only to the most profitable segments. To avoid this, in line with business practices, we assume the CSO does not accept or reject trips based on the segment but allocates capacity as requests arrive. To this end, constraint (10) ensures that the satisfied demand of each segment is proportional to the weight of that segment in the potential demand. Constraint (11) ensures that d^q_{pgik} is greater than zero only when m^q_{pik} is set to 1 (i.e., when rental time fee at level q is chosen for plan p for a trip starting from an area of type i in a period k).

$$z^{sr}_{pgik} \leq f^s_p \times AVD_{gik}\times D_{gik} \qquad \forall p,g,i,k,s,r \tag{12}$$

$$z^{sr}_{pgik} \leq l^r_p \times AVD_{gik}\times D_{gik} \qquad \forall p,g,i,k,s,r \tag{13}$$

$$z^{sr}_{pgik} \leq \sum_{q\in\mathcal{Q}} d^q_{pgik}\times APD^s_{pgik} \qquad \forall p,g,i,k,s,r \tag{14}$$

Constraints (12)–(14) calculate the total payable distance of each segment. Constraint (12) ensures that z^{sr}_{pgik} can only take a value greater than zero when f^s_p is set to 1 (i.e., free included distance at level s is chosen for plan p). Constraint (13) assures that z^{sr}_{pgik} can only be greater than zero when l^r_p is set to 1 (i.e., travel distance fee at level r is chosen for plan p). Constraint (14) ensures that z^{sr}_{pgik} is less than or equal to the sum of total satisfied demands times APD^s_{pgik} (average payable distance).

$$\sum_{v\in\mathcal{V}} n^v_{pg} \leq \frac{\sum_{i\in\mathcal{A},k\in\mathcal{K},q\in\mathcal{Q}} d^q_{pgik}}{\sum_{i\in\mathcal{A},k\in\mathcal{K}} D_{gik}}\times N_g \qquad \forall p,g \tag{15}$$

$$\sum_{v\in\mathcal{V}} n^v_{pg} \geq \frac{\sum_{i\in\mathcal{A},k\in\mathcal{K},q\in\mathcal{Q}} d^q_{pgik}}{\sum_{i\in\mathcal{A},k\in\mathcal{K}} D_{gik}}\times N_g - 0.99 \qquad \forall p,g \tag{16}$$

$$n^v_{pg} \leq y^v_p \times N_g \qquad \forall p,g,v \tag{17}$$

Constraints (15)–(17) calculate the number of customers in each segment that chooses each pricing plan. Constraint (15) ensures that the number of customers of segment g that choose plan p is based on d_{pgik} and equal to the proportion of customers of segment g that choose plan p times the number of customers of segment g. Since the objective function (1) aims to maximize the profit, constraint (16) assures that the final value of n^v_{pg} is not affected by the objective function (i.e., is not set to zero) which means that if there is a demand this must be satisfied. Constraint (17) ensures that n^v_{pg} can only take a value greater than zero when the y^v_p is set to 1 (i.e., when registration fee at level v is chosen for pricing plan p).

$$\begin{aligned}\sum_{v\in\mathcal{V}} \alpha^3 Y^v_p y^v_p + \sum_{s\in\mathcal{S}} \alpha^2 F^s_p f^s_p + \sum_{r\in\mathcal{R}} \alpha^1 L^r_p l^r_p + \sum_{i\in\mathcal{A},k\in\mathcal{K},q\in\mathcal{Q}} \alpha^0 M^q_{pik} m^q_{pik} \\ - \sum_{v\in\mathcal{V}} \alpha^3 Y^v_{p+1} y^v_{p+1} - \sum_{s\in\mathcal{S}} \alpha^2 F^s_{p+1} f^s_{p+1} \\ - \sum_{r\in\mathcal{R}} \alpha^1 L^r_{p+1} l^r_{p+1} - \sum_{i\in\mathcal{A},k\in\mathcal{K},q\in\mathcal{Q}} \alpha^0 M^q_{p+1ik} m^q_{p+1ik} < -0.1 \quad \forall p = \{1,\dots,P-1\}\end{aligned} \tag{18}$$

Constraint (18) is a lexicographic ordering constraint to eliminate the symmetric solutions by ordering the pricing plans and also to develop unique plans. The symmetry arises because of the following: given a specific feasible solution, an alternative solution can be constructed by renumbering the plans, where they are the same and differ only by the numbering of the plans. Since the value of different attributes might be the same (e.g., registration fee and free included distance of two plans have the same value of 100), the lexicographic ordering constraint without coefficients (i.e., α^3, α^2, α^1 and α^0) does not eliminate two symmetry solutions when the sum values of two plans are the same. Thus, following the approach proposed by Jans (2009) we used coefficients to eliminate the symmetry solutions. It should be noted that Jans (2009) used the coefficients with values 4, 2, and 1 for the lot-sizing problem with binary variables, which are scaled in this study considering possible values of the plans' attributes to ensure the uniqueness of the solutions.

$$y^v_p \in \{0,1\} \qquad \forall p,v \tag{19}$$

$$f^s_p \in \{0,1\} \qquad \forall p,s \tag{20}$$

$$l_p^r \in \{0,1\} \qquad \forall p, r \tag{21}$$

$$m_{pik}^q \in \{0,1\} \qquad \forall p, i, k, q \tag{22}$$

$$d_{pgik}^q \in \mathcal{Z}_0^+ \qquad \forall p, g, i, k, q \tag{23}$$

$$n_{pg}^v \in \mathcal{Z}_0^+ \qquad \forall p, g, v \tag{24}$$

$$z_{pgik}^{sr} \in \mathcal{R}_0^+ \qquad \forall p, g, i, k, s, r \tag{25}$$

Constraints (19)–(25) set the domain for the decision variables.

Linearization of the multinomial logit model

The introduced model is nonlinear due to the nonlinearity of constraint (7). Hence, linear transformations are applied to reformulate the original model as a MILP problem. To this end, first, we introduce the following new parameters to calculate the utility of any possible plan as input parameters of the model:

$q_1, q_2, q_3, q_4 \in \mathcal{Q} = \{1, \dots, Q\}$	Index for the set of possible rental time fee levels for pricing plans, for different types of trips, where Q is the number of rental time fee levels.
$H_p^{q_1}$	Rental time fee per minute at level q_1 for plan p for a trip starting from a high-demand area (i.e., $i = 1$), in a high-demand period (i.e., $k = 1$).
$H_p^{q_2}$	Rental time fee per minute at level q_2 for plan p for a trip starting from a high-demand area (i.e., $i = 1$), in a low-demand period (i.e., $k = 2$).
$H_p^{q_3}$	Rental time fee per minute at level q_3 for plan p for a trip starting from a low-demand area (i.e., $i = 2$), in a high-demand period (i.e., $k = 1$).
$H_p^{q_4}$	Rental time fee per minute at level q_4 for plan p for a trip starting from a low-demand area (i.e., $i = 2$), in a low-demand period (i.e., $k = 2$).
$U_{pg}^{vsrq_1q_2q_3q_4}$	Utility given by a user of segment g, to plan p, when the registration fee, free included distance amount, and travel distance fee, are set at level v, s, r and rental time fee for trips from different types of areas and periods are set at level q_1, q_2, q_3, and q_4. $U_{pg}^{vsrq_1q_2q_3q_4} = \beta^0{}_g + (\beta^1{}_g \times Y_p^v) + (\beta^2{}_g \times F_p^s) + (\beta^3{}_g \times L_p^r) + (\beta^4{}_{g11} \times H_p^{q_1} + \beta^4{}_{g12} \times H_p^{q_2} + \beta^4{}_{g21} \times H_p^{q_3} + \beta^4{}_{g22} \times H_p^{q_4}) \quad \forall p, g, v, s, r, q_1, q_2, q_3, q_4$

Moreover, a new binary variable $w_{pg}^{vsrq_1q_2q_3q_4}$ is defined which takes the value 1 if registration fee at level v, free included distance at level s, travel distance fee at level r, and rental time fee at level q_1, q_2, q_3, and q_4 is chosen for different type of trips of pricing plan p, 0 otherwise. As a result, the auxiliary variable u_{pg} can be removed by replacing constraint (7), with constraint (26), and adding the new constraint (27).

$$p_{pg} = \frac{\sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_3,q_4\in\mathcal{Q}} \exp(U_{pg}^{vsrq_1q_2q_3q_4}) \times w_{pg}^{vsrq_1q_2q_3q_4}}{\sum_{p'\in\mathcal{P}}\left(\sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_3,q_4\in\mathcal{Q}} \exp(U_{p'g}^{vsrq_1q_2q_3q_4}) \times w_{p'g}^{vsrvsrq_1q_2q_3q_4}\right) + 1} \qquad \forall p, g \tag{26}$$

$$w_{pg}^{vsrq_1q_2q_3q_4} \in \{0,1\} \qquad \forall p, g, v, s, r, q_1, q_2, q_3, q_4 \tag{27}$$

We should also add constraints (28)–(34) to ensure that the selected levels of the registration fee, free included distance, travel distance fee, and rental time fees, of pricing plan p, which is offered to customers of segment g are equal to the levels that are selected in constraints (2)–(5).

$$\sum_{s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_3,q_4\in\mathcal{Q}} w_{pg}^{vsrq_1q_2q_3q_4} = y_p^v \qquad \forall p, g, v \tag{28}$$

$$\sum_{v\in\mathcal{V},r\in\mathcal{R},q_1,q_2,q_3,q_4\in\mathcal{Q}} w_{pg}^{vsrq_1q_2q_3q_4} = f_p^s \qquad \forall p, g, s \tag{29}$$

$$\sum_{v\in\mathcal{V},s\in\mathcal{S},q_1,q_2,q_3,q_4\in\mathcal{Q}} w_{pg}^{vsrq_1q_2q_3q_4} = l_p^r \qquad \forall p, g, r \tag{30}$$

$$\sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_2,q_3,q_4\in\mathcal{Q}} w_{pg}^{vsrq_1q_2q_3q_4} = m_{p11}^{q_1} \qquad \forall p, g, q_1 \tag{31}$$

$$\sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_3,q_4\in\mathcal{Q}} w_{pg}^{vsrq_1q_2q_3q_4} = m_{p12}^{q_2} \qquad \forall p, g, q_2 \tag{32}$$

$$\sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_4\in\mathcal{Q}} w_{pg}^{vsrq_1q_2q_3q_4} = m_{p21}^{q_3} \qquad \forall p, g, q_3 \tag{33}$$

$$\sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_3\in\mathcal{Q}} w_{pg}^{vsrq_1q_2q_3q_4} = m_{p22}^{q_4} \qquad \forall p, g, q_4 \tag{34}$$

Constraint (28) ensures that the level of registration fee of plan p which is offered to customers of segment g is equal to the selected level (i.e., v) of that plan for trips starting from an area of type i in a period k. Constraints (29) and (30) do the same for the level of free included distance and the level of travel distance fee, respectively. Constraint (31) assures that the level of rental time fee of plan p which is offered to customers of segment g for trips starting from a high-demand area in a high-demand period (i.e., q_1), is equal to the level (i.e., q) which is selected in constraint (5) for that type of trip (i.e., from a high-demand area in a high-demand period). Constraints (32)–(34) do the same for trips starting from a high-demand area in a low-demand period (i.e., q_2), from a low-demand area in a high-demand period (i.e., q_3), and from a low-demand area in a low-demand period (i.e., q_4), respectively. Accordingly, the nonlinear constraint (26) can be linearized based on the approach proposed by Sen et al. (2017) to tackle a constrained assortment optimization problem under the mixed MNL model. To this end, first, we define a new continuous variable h_g as follows:

$$h_g = \frac{1}{\sum_{p'\in\mathcal{P}}\left(\sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_3,q_4\in\mathcal{Q}}\exp(U_{p'g}^{vsrq_1q_2q_3q_4})\times w_{p'g}^{vsrvsrq_1q_2q_3q_4}\right)+1} \qquad \forall g \qquad (35)$$

$$h_g \in \mathcal{R}_0 \qquad \forall g \qquad (36)$$

Eq. (35) can be further rewritten as Eq. (37), and constraint (26) can be replaced by (38):

$$h_g + h_g \times \sum_{p'\in\mathcal{P}}\left(\sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_3,q_4\in\mathcal{Q}}\exp(U_{p'g}^{vsrq_1q_2q_3q_4})\times w_{p'g}^{vsrq_1q_2q_3q_4}\right) = 1 \qquad \forall g \qquad (37)$$

$$p_{pg} = h_g \times \sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_3,q_4\in\mathcal{Q}}\exp(U_{pg}^{vsrq_1q_2q_3q_4})\times w_{pg}^{vsrq_1q_2q_3q_4} \qquad \forall p,g \qquad (38)$$

Finally, the term $h_g \times w_{pg}^{vsrq_1q_2q_3q_4}$ in the Eqs. (37), and (38), can be linearized by adding the new continuous variable t_{pgik}^{vsrq}, and using the standard "big-M" approach: For any term yx, where y is continuous and non-negative and x is binary, we may define a new continuous variable $z = yx$ and add the following inequalities to the formulation: $y - z \le M(1-x)$, $\quad 0 \le z \le y$, $\quad$ *and* $\quad z \le Mx$, where M is a sufficiently large upper bound on y. Employing this technique, replacing, $h_g \times w_{p'g}^{vsrq_1q_2q_3q_4}$ by $t_{p'g}^{vsrq_1q_2q_3q_4}$ and selecting $\exp(0)$ for M, leads to the following equations:

$$h_g + \sum_{p'\in\mathcal{P},v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_3,q_4\in\mathcal{Q}}\exp(U_{p'g}^{vsrq_1q_2q_3q_4})\times t_{p'g}^{vsrq_1q_2q_3q_4} = 1 \qquad \forall g \qquad (39)$$

$$p_{pg} = \sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_3,q_4\in\mathcal{Q}}\exp(U_{pg}^{vsrq})\times t_{pg}^{vsrq_1q_2q_3q_4} \qquad \forall p,g \qquad (40)$$

$$t_{pg}^{vsrq_1q_2q_3q_4} \le h_g \qquad \forall p,g,v,s,r,q_1,q_2,q_3,q_4 \qquad (41)$$

$$t_{pg}^{vsrq_1q_2q_3q_4} \le w_{pg}^{vsrq_1q_2q_3q_4}\times\exp(0) \qquad \forall p,g,v,s,r,q_1,q_2,q_3,q_4 \qquad (42)$$

$$\exp(0)\times(h_g - t_{pg}^{vsrq}) \le 1 - w_{pg}^{vsrq_1q_2q_3q_4} \qquad \forall p,g,v,s,r,q_1,q_2,q_3,q_4 \qquad (43)$$

$$t_{pg}^{vsrq_1q_2q_3q_4} \in \mathcal{R}_0^+ \qquad \forall p,g,v,s,r,q_1,q_2,q_3,q_4 \qquad (44)$$

The complete linearized formulation of the model is presented in Appendix A.

4. Case study and data analyses

To validate the performance of the model, we conducted numerical experiments using taxi trip data in New York, USA. In the following, we first explain the key motivations for utilizing taxi trip data (Section 4.1). Next, we estimate the spatial and temporal demand parameters (Section 4.2) and identify parameters related to customer segments (Section 4.3). Finally, the estimation of other parameters is discussed in Section 4.4.

4.1. Taxi trip data

In a carsharing system, most trips are short to medium-term rentals, primarily for commuting, business, and recreational purposes (Schmöller et al., 2015; Habib et al., 2012). Hui et al. (2018) conducted a comparative analysis in China, finding that both taxi and carsharing trips share common features such as short duration of usage, and concluded that taxi service is the most similar mode to carsharing among different modes of transportation. Efthymiou and Antoniou (2016) observed similar trends in Athens, Greece, indicating that people who use taxis are more willing to join carsharing services. Survey-based studies by Lempert et al. (2019) and Yoon et al. (2017) also showed that taxi users are the most willing to shift to carsharing. Martı et al. (2022), Martí et al. (2021) analyzed the possibility of replacing taxis with carsharing fleets and concluded that users would be willing to use carsharing as it may suit them from an economic point of view. Building on the findings of these descriptive studies, several papers applied taxi trip data as the potential travel demand for carsharing to tackle various problems of CSOs at the strategic (Liu et al., 2018; Li et al., 2017), tactical (Brandstätter et al., 2017), and operational levels (Sayarshad and Chow, 2017; Wu et al., 2023).

In sum, more research has applied taxi data to tackle carsharing problems in the past few years due to the similarities between taxi and carsharing trips. Among them, many researchers have widely used the taxi trip data in New York as a representative sample of one-way carsharing trips (Huang et al., 2022b; Sayarshad and Chow, 2017; Çalık and Fortz, 2019). Hence, we utilized the trip dataset of the green taxis in Brooklyn, New York, related to the year 2019, which is made publicly available by the New York City Taxi and Limousine Commission (TLC, 2021). In this dataset, each row represents a taxi trip with attributes such as pick-up and drop-off date/time, pick-up and drop-off location, trip distance, and fare amount. However, we used a subset of these attributes, namely the pick-up and drop-off dates/times, pick-up and drop-off locations, trip distances, and trip duration. A data cleaning process was applied to eliminate trips with zero travel times or pick-up or drop-off locations in Brooklyn. As a result, 136,740 valid trips that can be assigned to several customers were identified to perform the required analyses.

4.2. Temporal and spatial analyses

As pointed out earlier, the pricing plans of the proposed model have rates that depend on the pick-up time period and area, which is in line with business practices. Thus, spatial and temporal analyses are performed to identify different periods/areas concerning their demand volume. To this aim, each day is divided into seven time intervals. The first is a 6-hour time interval from 1 a.m. to 7 a.m., and the remaining time intervals cover the rest of the day with periods of three hours from 7 a.m. to 1 a.m. (i.e., intervals 2 to 7). Then, the aggregate number of trips in each time interval during different weekdays is analyzed to identify temporal demand patterns.

Fig. 1 presents the temporal characteristics of taxi pick-ups in February 2019 by time intervals and days of the week. It can be seen that in the period 1:00–7:00, the number of trips on weekends is larger than that on weekdays, which shows that customers stay out late during weekends. However, as expected, the number of trips in the period 7:00–10:00 on weekdays is greater than that on weekends, mainly due to commuter and business trips. There are relatively similar demand patterns on weekdays and weekends during 10:00–16:00. Demand increases during 16:00–22:00 on almost all weekdays and Saturdays, and the high-demand periods become apparent. People usually return from work or use vehicles for recreational purposes during these periods. Finally, Fig. 1 depicts a reduction in the number of trips during the period 22:00–1:00 on both weekdays and weekends, which can be considered as the low-demand period. It should be noted that distinguishing among different time intervals across different days of the week in detail may lead to more precise plans. However, it may also lead to the design of complex plans that are inconvenient for customers. In this regard, we sought to identify different types of periods more clearly and conveniently to develop realistic plans that are often used in the market. As a result, the period 10:00–22:00 on both weekdays and weekends and 22:00–1:00 on Saturdays are considered the high-demand period. Then, periods 22:00–1:00 on Sundays and weekdays and 01:00–10:00 on both weekdays and weekends are considered the low-demand periods.

We also analyzed the spatial distribution of the trips within Brooklyn to distinguish areas according to their demand volume. In the dataset, the pick-up and drop-off locations of the trips fall into 61 unique IDs, representing taxi zones defined by TLC in Brooklyn. In line with several studies, we considered the aggregate number of trips started in each zone rather than density (i.e., $\frac{trips}{km^2}$) to identify various types of areas (Zhang et al., 2021; Lam and Liu, 2017). It should be noted that after identifying different types of areas (i.e., high and low-demand taxi zones), the zones' density can be considered by CSOs for assigning the number of required vehicles in the fleet deployment step. The natural breaks classification with ten classes was performed to break down the zones into separate classes by finding the maximum variance between individual classes and the least variance within each class (Liu et al., 2019; Brewer and Pickle, 2002). Fig. 2 shows the boundaries of Brooklyn taxi zones where the labels indicate the taxi zone IDs delimited by TLC, and the colors reflect the classification of zones obtained from natural breaks classification considering the frequency of trips. Investigating the trips' spatial distribution shows more or less a similar pattern across different periods of the week. In determining the boundary for high/low-demand areas, we considered the range of trip frequencies within each zone to ensure homogeneity across areas with distinct demand characteristics. As shown in Fig. 2, the range of trip frequencies for orange and red zones is approximately 500 trips, while it substantially increases (exceeding 1200) for other zones. Thus, to maintain homogeneity within low- and high-demand areas, zones with less than or equal to 2152 trips (i.e., orange and red zones) are considered to be low-demand areas, while those with more than 2152 are considered to be high-demand areas. Nevertheless, since comprehending the impact of this delineation on the optimal solutions and system profitability is beneficial for CSOs, we conducted a sensitivity

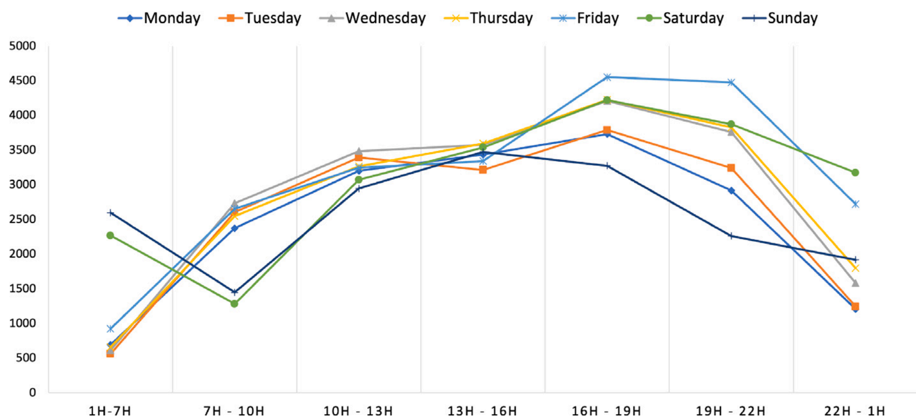

Fig. 1. Temporal rental patterns by time intervals and days of the weeks in Brooklyn.

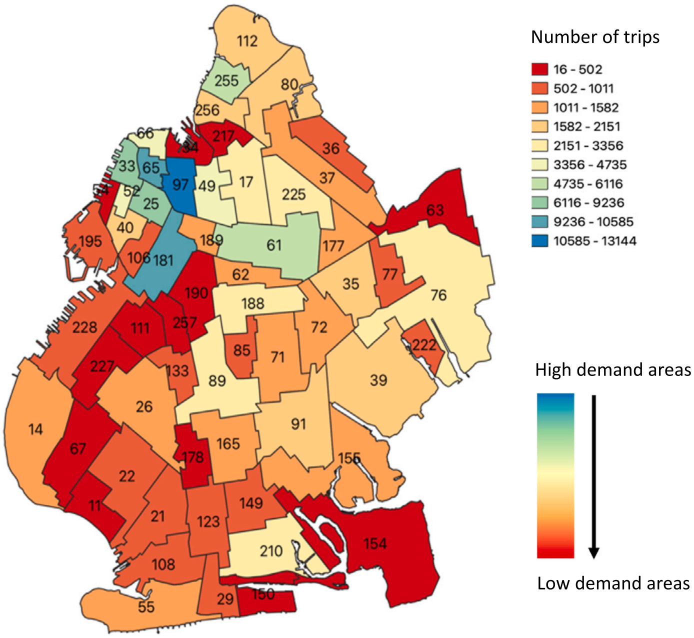

Fig. 2. Spatial rental patterns across different taxi zones in Brooklyn. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

analysis in Appendix F of the Supplementary Materials to analyze how pricing plans and CSO profits depend on different parameter settings.

The approaches we applied to identify high and low-demand areas and periods, such as considering the aggregate number of trips started in each zone rather than density and dividing each day into seven-time intervals, can also be used by different shared-mobility operators to provide user-friendly and easy-to-understand plans with varying rates for high and low-demand periods and areas.

4.3. Trip assignment and customer segmentation

We aim to consider customer travel heterogeneity to develop realistic pricing plans. However, the TLC taxi trip dataset does not have a customer ID field. Thus, we assigned the taxi trips to the potential customers based on the trip patterns of carsharing users obtained from the literature. In this regard, first, we considered three main types of carsharing users that are identified in the literature: commuters, professional users, and casual users (Perboli et al., 2018; Ferrero et al., 2015). Then, we followed a systematic approach that allowed us to assign all the 136,740 taxi trips available in the dataset to 19,643 potential carsharing users. The final dataset,[1] which now has customer IDs, can be applied for the intended analyses in this study and provide a sufficient and proper dataset for researchers in the field. The detailed procedure for assigning Brooklyn taxi trip data to a potential set of carsharing users is described in Supplementary Material in Appendix A. To categorize customers into demand segments based on the characteristics of their trips, we analyze three main characteristics of the trips: frequency of use (i.e., number of trips per month), average travel distance per trip, and average travel time per trip, which are significantly linked to the attributes of plans (i.e., registration fee, travel distance fee, and rental time fee). This segmentation is essential for identifying parameters required by the model (i.e., parameters related to customer segments) and developing pricing plans that compete to capture segments.

In this study, we applied partitioning clustering algorithms that are widely used in shared-mobility literature to identify different customer segments (Wu et al., 2020; Morency et al., 2007). In preliminary tests, different partitioning clustering methods were analyzed, and the k-means algorithm showed the best performance for splitting customers into K homogeneous clusters. An important step in k-means clustering is determining parameter k, which denotes the appropriate number of clusters. To determine parameter k, we examined various clusters ranging from 2 to 25 and evaluated their performance using the elbow method and Davies–Bouldin Index (DBI). The elbow and DBI index suggested 2 and 4 as the most appropriate cluster numbers for the given dataset, and 18 clusters achieved the third most suitable outcomes among the other alternatives. Hence, while the main focus is solving problems with 2 and 4 segments as real-size instances, we also tackle problems of up to 18 segments to investigate the potential impact of a more detailed segmentation on profit outcomes and to identify the main factors that affect the problem's complexity in finding optimal solutions. Appendix B in Supplementary Material presents the results of k-means algorithms with 2, 4, and 18 clusters.

[1] Available at: https://rb.gy/6piik

4.4. Parameter estimation

The values of input parameters are derived from different sources, such as Brooklyn taxi trip data, websites of CSOs, and relevant studies in the literature. The parameters related to types of urban areas and time periods are directly calculated through the temporal and spatial analyses presented in Section 4.2, whereas demand segments are derived from the customer segmentation of Brooklyn trip data in Section 4.3. Building on this, we measured the total demand (D_{gik}), the average travel time (AVT_{gik}), and the average travel distance (AVD_{gik}) of demand segments in different types of areas and periods (Appendix B in Supplementary Material). The values for the costs and other operational parameters are described in Appendix C in Supplementary Material, together with the corresponding sources. As mentioned, the attributes of plans must have values chosen from discrete sets (V, S, Q, and R). In Section 5.1, we solve instances using varying numbers of values within these discrete sets to analyze their impact on the computational time of the model and the system's profit. In terms of the number of pricing plans (P), we use three plans as the average number often used in the market. All numerical examples will be based on the same number of plans (i.e., three plans) since increasing/decreasing the number of plans will add/reduce the number of carsharing correlated alternatives, influencing the preferences for the carsharing plans versus the status quo.

The alternative-specific constants (i.e., ${\beta^0}_g$), and the coefficients of plans' attributes in the utility function (${\beta^1}_g$, ${\beta^2}_g$, ${\beta^3}_g$, ${\beta^4}_{gik}$) are instance-dependent parameters, which must be calculated based on the travel behavior of different segments. As for the alternative-specific constants (i.e., ${\beta^0}_g$), De Luca and Di Pace (2015) investigated the acceptability of a carsharing service and concluded that customers have a positive attitude towards the use of carsharing. However, the alternative specific coefficient is highly dependent on the existing modes that compete with carsharing and the willingness of the population to use other modes rather than their private cars. For experimentation purposes, we prefer to consider that there is a special preference for using a shared car, but this case can be easily changed in other applications of the model. Thus, in this study, we will use values of ${\beta^0}_g$ positive, scaling them for different segments as follows. In this regard, we first calculate the summation of average travel time and average travel distance for each segment. Then, the summation value is multiplied by the number of trips. Finally, the values are normalized between 0.01 and 1 to estimate carsharing's relative importance for each segment. As for the coefficients of plans' attributes (${\beta^1}_g$, ${\beta^2}_g$, ${\beta^3}_g$, ${\beta^4}_{gik}$), we calculate the importance of each attribute for different segments in three steps as follows.

1. First, according to the literature, we make three assumptions.
   (a) A user that makes a few trips per month is more price-sensitive to registration fees (i.e., fixed costs) compared to a user with more trips (Zheng et al., 2009; Martin et al., 2010; Zhou and Kockelman, 2011). In this regard, for instance, a survey of North American carsharing members conducted by Martin et al. (2010) showed that plans with no or low fixed cost are the most attractive for customers with few trips.
   (b) Users whose trips take a longer time are more price-sensitive to rental time fees compared to users with shorter rental times (Zhou and Kockelman, 2011).
   (c) Users who travel longer distances are more price-sensitive to travel distance fees and free included distance compared to users with shorter travel distances (Baumgarte et al., 2022; Zhou and Kockelman, 2011).
2. In the second step, we compared the average number of trips and average travel distance per trip between segments by normalizing them between −0.01 and −1 (i.e., negative impact on utility) to calculate the relative importance of the registration fee and the travel distance fee, respectively. Moreover, for all customer segments, the average travel distance per trip was normalized between 0.01 and 1 (i.e., positive impact on utility) to calculate the relative importance of free included distance for each segment. As pointed out earlier, each plan can have various rental time fees for different periods and areas. Hence, unlike the relative importance of other attributes that were calculated by comparing the corresponding values between segments, the importance of rental time fees is calculated by comparing the average travel times for different types of trips within each segment (i.e., AVT_{gik}). To this aim, the average travel time of different types of trips was normalized between −0.01 and −1 (i.e., negative impact on utility) within each segment to calculate the relative importance of the rental time fee of each type of trip. Thus, for each segment, the coefficients of the rental time fee for trips with the highest average travel time is set to −1, and for trips with the lowest average travel time is set to −0.01.
   Nevertheless, the utility obtained by customers from each plan is highly affected by the possible values of the plans' attributes (i.e., $Y_p^v, F_p^s, L_p^r, M_{pik}^q$). For instance, since the maximum value of the registration fee can be set to €30, and the maximum value of the travel distance fee can be set to €0.6, the utilities are highly dependent on the value of the registration fee compared to the distance fee.
3. Hence, in the third step, to scale the effect of different attributes, we initially calculated the average possible values of registration fees, free included km, travel distance fees, and rental time fees, respectively. Then, since the average value of possible rental time fees has the smallest value, the normalized values of ${\beta^1}_g$, ${\beta^2}_g$, and ${\beta^3}_g$ obtained from step 2, were normalized based on ${\beta^4}_g$ to scale the effect of different attributes in the utility function of plans. The final values of these parameters in different instances are listed in Appendix D in Supplementary Material.

The proposed approach not only enables the estimation of utilities based on the travel behavior of segments but also leads to the utilities that for some plans and segments are greater than zero and for some are less than zero (i.e., almost between −1 and 1), which make them comparable with the utility of status quo that is set to zero. In the next section, a numerical example is provided to illustrate the utility given by segments to optimal plans and the proportion of customers that choose each plan. Moreover, in Section 5.2, an in-depth sensitivity analysis is undertaken to assess the influence of key parameters on the model's performance.

Table 2
Main characteristics of the instances and the optimization results.

Instance	# Segments	Possible registration fees	Possible free included km	Possible travel distance fees	Possible rental time fees	Monthly profit	Running time (s)	MILP gap %
$I1$	2	$\{0, 30\}$	$\{0, 4\}$	$\{0.1, 0.6\}$	$\{0.1, 0.18\}$	672,217	24	0,00%
$I2$	4	$\{0, 30\}$	$\{0, 4\}$	$\{0.1, 0.6\}$	$\{0.1, 0.18\}$	680,105	65	0,00%
$I3$	10	$\{0, 30\}$	$\{0, 4\}$	$\{0.1, 0.6\}$	$\{0.1, 0.18\}$	731,241	961	0,00%
$I4$	18	$\{0, 30\}$	$\{0, 4\}$	$\{0.1, 0.6\}$	$\{0.1, 0.18\}$	730,051	314	0,00%
$I5$	2	$\{0, 15, 30\}$	$\{0, 2, 4\}$	$\{0.1, 0.3, 0.6\}$	$\{0.1, 0.15, 0.18\}$	674,697	36 000	52.83%
$I6$	4	$\{0, 15, 30\}$	$\{0, 2, 4\}$	$\{0.1, 0.3, 0.6\}$	$\{0.1, 0.15, 0.18\}$	688,685	36 000	59.56%
$I7$	10	$\{0, 15, 30\}$	$\{0, 2, 4\}$	$\{0.1, 0.3, 0.6\}$	$\{0.1, 0.15, 0.18\}$	696,073	36 000	90.26%
$I8$	18	$\{0, 15, 30\}$	$\{0, 2, 4\}$	$\{0.1, 0.3, 0.6\}$	$\{0.1, 0.15, 0.18\}$	682,408	36 000	105.60%

5. Experimental results and analysis

In the following, we initially present and discuss the computational results (Section 5.1), highlighting several valuable managerial insights derived from our findings. We then perform an in-depth sensitivity analysis to investigate how pricing plans and profit of CSOs interact with different parameter settings (Section 5.2), providing additional insights into the implications of these interactions for decision-makers. Finally, Section 5.3 presents a decomposition algorithm designed to yield high-quality results within a short time frame, particularly for large instances. All experiments are coded in Python, with a computation time limit of 10-hours, calling IBM ILOG CPLEX 22.1.0 on a Windows virtual machine with an Intel Xeon Gold 6148 CPU @ 2.40 GHz with 96 GB of installed RAM. This time limit was considered adequate (and can be even increased in practice) due to the strategic and long-term nature of the decisions tackled.

5.1. Computational results

This section aims to evaluate the performance of the proposed MILP model. First, while this study mainly focuses on solving problems with 2 and 4 segments as real-size instances, we will also tackle problems of up to 18 segments to understand the computational limitations of the model (Section 5.1.1). Then, Section 5.1.2 presents the results obtained for the base case to discuss the probabilities generated by the parameter estimation procedure addressed in Section 4.4. Finally, we discuss the potential for profit improvement when implementing time- and location-dependent rate plans compared to fixed-rate plans (Section 5.1.3).

5.1.1. Computational complexity

To evaluate the performance of the proposed model, we solved eight problem instances of different sizes. The goal is to determine the values of the plans' attributes (three plans) to maximize the system's profit. The instances are derived from Brooklyn taxi trip data and differ from each other in terms of (i) the number of customer segments and (ii) the number of possible values for the plans' attributes (i.e., size of sets V, S, Q, and R). The number of segments and possible values for the plans' attributes affect the size of the problem and its complexity. Thus, the main aim is to examine the model's performance in finding optimal solutions as the instance size increases and identify the main factors that affect the problem complexity. Table 2 presents the main characteristics of the instances and the optimization model results, including the best value of the objective function (i.e., profit), the running times, and the MILP gap values. All the other parameters of instances are set based on what has been discussed in Section 4.4 and presented in the Supplementary Material.

As shown in Table 2, instances $I1$-$I4$ only differ concerning the number of segments, ranging from 2 segments in instance $I1$ to 18 segments in $I4$. In the next four instances ($I5$-$I8$), we keep all the parameters similar to $I1$ to $I4$, respectively, and we only increase the number of potential values that can be used for the plans' attributes (i.e., three candidate values for each attribute).

Table 2 demonstrates that optimal solutions were obtained for instances $I1$ to $I4$. An important insight from the results is that classifying customers into more detailed segments does not significantly increase the computational time. This finding suggests that CSOs can use more detailed segmentation in their pricing strategies to capture customer heterogeneity better and design customized plans tailored to various target segments, potentially enhancing customers' satisfaction and increasing overall profitability.

Next, we analyze the results of the problems $I5$-$I8$ to illustrate the influence of the number of levels for plans' attributes on the model performance. From Table 2, it can be seen that none of these instances could obtain the optimal solution within the time limit. Comparing their running time and gap values with those obtained from instances $I1$-$I4$ shows that the number of levels for the plans' attributes has the most important effect on running time and computational complexity. As it can be seen, while the minimum and maximum attribute levels of plans in $I1$ and $I2$ remain consistent with those in instances $I5$ to $I6$, the introduction of additional potential values for the latter instances leads to designing more tailored plans for customer segments, thereby increases the system's profit. This increase is not apparent when comparing instance $I3$ and $I4$ with $I7$ and $I8$, mainly due to the significant MIP gap (i.e., 90.26% in $I7$ and 105.60% in $I8$) compared to other instances. It should be noted that since the upper and lower bounds are fixed, dividing the price domains into too many discrete points may not significantly affect customers' sensitivity. Hence, dividing the price domains into a significantly large number of possible values is not worthy for the objectives of this study.

In sum, the results of the proposed model for various sizes of problems revealed that the number of segments has a minor effect on the problem complexity compared with the number of levels for the plans' attributes. Meanwhile, the results also reflect that the greater number of potential values that can be used in the structure of plans can improve the system profit by designing well-tailored plans for target segments.

5.1.2. Choice for optimal pricing plans for the base case

In the remainder of this section, we use instance $I2$ with four segments as the base case since the Elbow method and DBI analysis suggested it as the most appropriate number of clusters for the given dataset. Table 3 summarizes the travel data related to all four segments in problem $I2$. In the following, detailed results for the base case are presented to better illustrate the procedure mentioned in Section 4.4 for estimating the coefficients of the utility function and to demonstrate their impact on the results.

Table 3
Travel data of customers in problem $I2$ for estimating the coefficients of the utility function.

Segment (g)	Avg. monthly trips per customer	Avg. travel distance (km) per trip	Avg. travel time per trip (minutes)			
			AVT_{g11}	AVT_{g12}	AVT_{g21}	AVT_{g22}
1	8	2.78	15.94	14.91	19.96	18.60
2	41	2.71	16.54	15.20	21.18	22.02
3	4	2.86	14.62	13.27	19.38	18.70
4	6	2.80	268.91	301.49	155.03	253.38

Applying the procedure presented in Section 4.4 to data presented in Table 3, we estimated the coefficients of the utility function, which are depicted in Fig. 3. As shown in Fig. 3, $\beta^0{}_g$, and $\beta^2{}_g$ take values between 0.01 and 1 with positive impacts on the utility, while $\beta^1{}_g$, $\beta^3{}_g$, $\beta^4{}_{g11}$, $\beta^4{}_{g12}$, $\beta^4{}_{g21}$, and $\beta^4{}_{g22}$ take the values between −0.01 and −1 with negative impacts on utility. Table 3 shows that segment three, with four trips, has the lowest average number of trips per customer, whereas segment two, with 41 trips, has the highest number. Hence, according to the procedure addressed in Section 4.4 and as depicted in Fig. 3, segment three is more price-sensitive to registration fees than segment two (i.e., −0.0096 versus −0.0001). Moreover, segments who travel longer distances are more price-sensitive to free included distance and travel distance fees than segments with shorter travel distances. Finally, the coefficients of rental time fees are calculated by comparing the average travel times for different types of trips within each segment. For instance, as shown in Table 3, customers of segment one have the highest average travel time for trips starting from low-demand areas in high-demand periods (i.e., AVT_{g21}) and the lowest for trips starting from high-demand areas in low-demand periods (i.e., AVT_{g12}). Thus, as depicted in Fig. 3, $\beta^4{}_{g21}$ takes the highest value, and $\beta^4{}_{g12}$ takes the lowest value.

Segment	Alternative-specific constant	Registration fee coefficient	Free included distance coefficient	Travel distance fee coefficient	Rental time fees in different areas/periods coefficients			
	β_g^0	β_g^1	β_g^2	β_g^3	β_{g11}^4	β_{g12}^4	β_{g21}^4	β_{g22}^4
1	0.0674	-0.0086	0.0369	-0.2582	-0.2124	-0.0100	-1.0000	-0.7333
2	0.5363	-0.0001	0.0007	-0.0050	-0.2047	-0.0100	-0.8782	-1.0000
3	0.0100	-0.0096	0.0714	-0.5000	-0.2284	-0.0100	-1.0000	-0.8902
4	1.0000	-0.0092	0.0457	-0.3201	-0.7798	-1.0000	-0.0100	-0.6748

Fig. 3. Coefficients of the plans' utility in problem $I2$, in which dark red colors reflect the coefficients with the most negative impact on utility and the dark green represents the most positive impact. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 4 presents the optimal value of plans' attributes (decision variables) obtained by solving instance $I2$, using the coefficients mentioned above. As can be seen, all three plans take the highest possible registration fee (i.e., €30). While P1 and P2 take the minimum free included distance of 0 km, P3, in contrast, offers the highest possible free included distance (i.e., 4 km). Moreover, P3 charges the lowest travel distance fee (i.e., €0.10) compared to other plans, and P1 offers lower rates in low-demand areas (i.e., $m^1_{121} = m^1_{122} =$ €0.10).

Table 4
The optimal values of the three plans' attributes by solving problem $I2$.

Plans	Registration fee	Free included distance	Travel distance fee	Rental time fees			
(P)	(y^v_p)	(f^s_p)	(l^r_p)	(m^q_{p11})	(m^q_{p12})	(m^q_{p21})	(m^q_{p22})
P1	30	0	0.60	0.18	0.18	0.10	0.10
P2	30	0	0.60	0.18	0.18	0.18	0.18
P3	30	4	0.10	0.18	0.18	0.18	0.18

Table 5 presents the utility values that lead to selecting these optimal values and the proportion of customers that consequently choose each alternative. As shown in Table 5, the small values of alternative-specific constants (i.e., $\beta^0{}_g$) for segments one and three lead to negative utilities for customers within these specific segments. The first plan (i.e., P1) is more tailored for customers of segment two since, unlike other segments, most customers (i.e., 28.90%) within this particular segment choose P1. In this regard, plan P1 has the highest possible travel distance fee (i.e., €0.6) and the lowest rental time fees for trips starting from the low-demand areas (i.e., $m^1_{121} = m^1_{122} =$ €0.10). As depicted in Fig. 3, among different segments, the customers within this particular segment are less price-sensitive to travel distance fees and are among the most price-sensitive segments to rental time fees for trips starting from the low-demand areas. From Table 4, it is clear that plan P2, as the most expensive one, is the less attractive plan for all segments,

Table 5
Utility given by users from a segment to an alternative and the probability of choosing each alternative (where "sq" denotes the status quo).

Segment	Utility P1	Utility P2	Utility P3	Utility sq	% customers choosing P1	% customers choosing P2	% customers choosing P3	% customers choosing sq
1	−0.55	−0.69	−0.42	0	21.00%	18.30%	24.10%	36.60%
2	0.30	0.15	0.15	0	28.90%	24.80%	25.00%	21.30%
3	−0.81	−0.96	−0.42	0	17.90%	15.40%	26.30%	40.40%
4	0.14	0.08	0.43	0	24.10%	22.80%	32.20%	20.90%

thereby attracting the fewest number of customers from each segment compared to other pricing plans as shown in Table 5. However, attracting fewer customers who pay higher fees is still profitable for the system. Next, we analyze plan P3, which is more tailored for customers of segments three and four. Illustrated in Fig. 3, it becomes evident that customers within these segments have the highest price sensitivity to travel distance fees and free included distance. As a result, most people in segments three and four (i.e., 26.30% and 32.20% respectively) have been attracted to P3 thanks to the most free included distance and the lowest travel distance fee.

In sum, analyzing the base case results shows that the proposed approach leads to the utilities that, for some plans and segments, are greater than zero and for some, are less than zero (i.e., between −1 and 1), making them comparable with the utility of status quo (i.e., 0).

5.1.3. Relevance of time- and location-dependent rates

As mentioned earlier, this is the first attempt to develop a combination of multi-attribute plans with time- and location-dependent rates for different segments of customers. Therefore, it is difficult to make a direct comparison with the pricing approaches addressed in the literature. Hence, we aim to investigate how incorporating time and location dependencies into rental time fees affects the model's performance. To generate plans where rates do not depend on location and time, we added new constraints to the MILP model, ensuring that each plan has only a unique rental time fee (Eqs. (45)–(46) in Appendix A). Table 6 shows the optimal plans obtained by solving $I2$ with new constraints, reaching the maximum profit of €662,866.

Table 6
The optimal values of the plans' attributes of problem $I2$ when each plan must take similar time fee.

Plans	Registration fee	Free included distance	Travel distance fee	Rental time fees			
(P)	(y_p^v)	(f_p^s)	(l_p^r)	(m_{p11}^q)	(m_{p12}^q)	(m_{p21}^q)	(m_{p22}^q)
P1	30	0	0.10	0.18	0.18	0.18	0.18
P2	30	0	0.60	0.18	0.18	0.18	0.18
P3	30	4	0.10	0.18	0.18	0.18	0.18

The results reveal that the total profit decreases from €680,105 in the base case to €662,866 (i.e., 3%) when using time- and location-independent plans. To provide a more comprehensive perspective, we also conducted a comparative analysis of results from instances $I1$, $I3$, and $I4$, wherein the model achieved optimal solutions. The results show that plans with time- and location-dependent rates can yield an additional 2%, 5%, and 6% in profits, respectively, compared with fixed-rate plans. It can be concluded that developing plans with rental time rates that depend on different areas and periods can increase the profit compared to plans with fixed-rates. This increase in profitability was expected since the model can adapt to the particular trip patterns of a city by adding its degrees of freedom. Nevertheless, this represents the trade-off for CSOs between the simplicity of fixed-rate plans and the potential for profit improvement through more complex pricing strategies.

5.2. Sensitivity analysis

The sensitivity analysis in this section assesses how customer choice and pricing plans interact with key parameters of the system and how this interaction affects the CSO's profit. To this aim, we focus on three groups of parameters that represent some key decisions and challenges in carsharing: the ratio between the number of vehicles and customers (VCR), the maximum demand that can be served in different areas and periods (O_{ik}), and the utility function parameters. The first two represent decisions related to fleet size and deployment (supply) and their ability to meet demand. The latter group represents customers' behavior, and we aim to analyze how the model reacts to different behaviors. We use instance $I2$ for the sensitivity analysis as a real-sized problem for which the model could find the optimal solution within the time limit.

5.2.1. Effects of the ratio between the number of vehicles and customers

In a carsharing system, different fleet sizes can be operated based on the tactical and operational decisions of CSOs (e.g., relocation strategy, target service level), which can increase or decrease the investment cost. Moreover, the number of active and inactive users may vary in different CSOs according to their registration and pricing policies, which affect vehicle utilization. For instance, a CSO can provide some free registration plans that do not require fixed fees, which may attract inactive users who may never use the vehicles. Thus, the VCR should be determined with caution. To investigate the influence of the VCR on the model and provide managerial insights for CSOs, we compared the results of $I2$ with four new cases where the VCR is modified to be twice and four times larger and smaller than its baseline (i.e., 0.02).

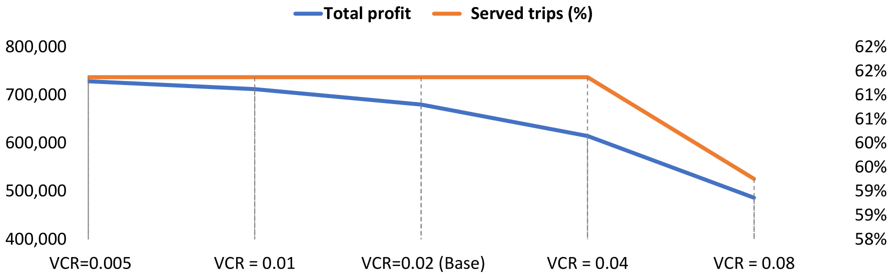

Fig. 4. Effects of VCR on the profit and percentage of served trips.

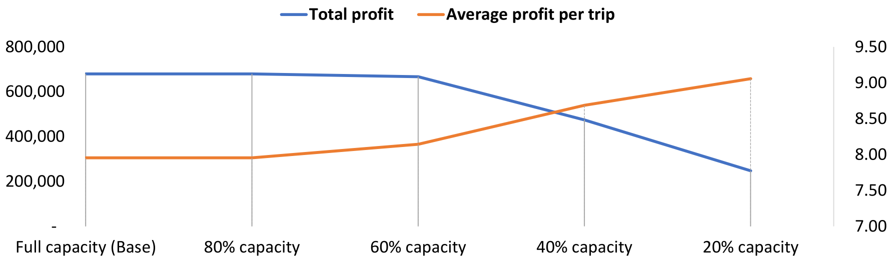

Fig. 5. Effects of maximum demand that can be served on the total profit and average profit per trip.

Fig. 4 shows how total profit and percentage of served trips vary with different VCR rates. It is clear that as VCR increases, the total profit consistently decreases due to higher costs incurred by CSOs in providing more vehicles. In terms of the percentage of served trips, when VCR increases from 0.005 to 0.04, the proportion of served trips remains constant. This range of VCR values (i.e., 0.005 to 0.04) leads to three distinct pricing plans that are consistent across all scenarios. This is because, within this range, the pricing plans have been optimized for maximum profitability, and the system does not tend to change plans to serve more demand. However, the situation changes when VCR is increased beyond 0.04, as depicted in Fig. 4. In this context, an increase in VCR from 0.04 to 0.08 imposes additional costs on the system (i.e., parameter CO) in this setting. Consequently, the system formulates more costly pricing plans tailored to satisfy the demand of the most profitable customer segments, leading to the loss of potential demand from unprofitable customers. Moreover, the values of the plans' attributes indicated that the optimal plans of all cases are very similar, and the pricing decision is robust to variations in this parameter. CSOs can leverage these insights to strategically adjust their fleet sizes and pricing plans to maximize profitability while efficiently serving customer demand.

5.2.2. Effects of maximum demand that can be served

As mentioned in Section 3.2, O_{ik} represents the maximum demand that can be served for trips originating from an area of type i during a period k. This upper bound approximates the available capacity (related to fleet or other operational constraints) to meet the aggregate number of trips in different types of areas and periods. For instance, this parameter can be set based on operators' relocation policies. Therefore, the deployment strategy will benefit from understanding the effect of this parameter on the system's profit. To this aim, we compare the result obtained by the base case with four alternative cases. In the base case, O_{ik} is equal to the maximum demand of the system, which means all potential demand can be served (i.e., full capacity). In contrast, in the four alternative cases, only 80%, 60%, 40%, and 20% of potential demand can be satisfied.

Fig. 5 shows that reducing Oik leads to a decrease in the total profit obtained by the CSO, from €680,105 in the base case to €247,718 in the 20% capacity case. However, when O_{ik} is decreasing, the average profit per trip increases from €7.95 in the base case to €9.06 in the 20% capacity case.

This suggests that under constrained capacity, the model prioritizes higher-profit trips, potentially optimizing revenue generation. The average profit per trip can be used as a useful metric for evaluating the profitability of serving additional trips under different Oik settings. For instance, with Oik set to 20% of potential demand, each additional trip yields €9.06 in profit. Companies can compare this profit with the costs associated with strategies like user-based relocation to assess the viability of meeting additional demands, ensuring efficient resource utilization, and maximizing profitability. Finally, the results showed that the values of the optimal plans in all cases are the same, which indicates that the model is robust to variations in this parameter.

5.2.3. Effects of the utility function parameters

In the remaining analyses, we assess the model's sensitivity to the parameters of the utility function. The main objective is to investigate the changes in the optimal solutions (i.e., value of the plans' attributes) resulting from changes in the coefficients of the utility function. For this purpose, the alternative cases are defined by reducing and increasing the weights of corresponding parameters that are used in the base case.

Table 7
Effects of $\beta^0{}_g$, $\beta^1{}_g$, $\beta^2{}_g$, $\beta^3{}_g$ and $\beta^4{}_{gik}$ on the pricing plans by comparing the results with base case.

Test	Registration fee	Free included distance	Travel distance fee	Rental time fees			
Parameter	(y^v_p)	(f^s_p)	(l^r_p)	(m^q_{p11})	(m^q_{p12})	(m^q_{p21})	(m^q_{p22})
$\beta^0{}_g \times -1$	0%	0%	0%	0%	0%	0%	0%
$\beta^0{}_g \times -0.5$	0%	0%	0%	0%	0%	0%	0%
$\beta^0{}_g \times 0.5$	0%	0%	0%	0%	−15%	17%	0%
$\beta^0{}_g \times 2$	0%	0%	0%	0%	0%	0%	0%
$\beta^1{}_g \times 0.1$	**0%**	0%	0%	0%	0%	0%	0%
$\beta^1{}_g \times 2$	**0%**	0%	0%	0%	0%	0%	0%
$\beta^1{}_g \times 5$	**−33%**	100%	−38%	0%	0%	0%	0%
$\beta^1{}_g \times 10$	**−100%**	−100%	0%	0%	−15%	17%	0%
$\beta^2{}_g \times 0.1$	0%	**−100%**	**0%**	0%	0%	0%	0%
$\beta^2{}_g \times 2$	0%	**0%**	**0%**	0%	0%	0%	20%
$\beta^2{}_g \times 5$	0%	**200%**	**−38%**	0%	−15%	17%	0%
$\beta^2{}_g \times 10$	0%	**200%**	**−38%**	0%	−15%	17%	0%
$\beta^3{}_g \times 0.1$	0%	**0%**	**0%**	0%	0%	0%	0%
$\beta^3{}_g \times 2$	0%	**100%**	**−77%**	0%	0%	0%	0%
$\beta^3{}_g \times 5$	0%	**100%**	**−77%**	0%	0%	0%	0%
$\beta^3{}_g \times 10$	0%	**100%**	**−77%**	0%	0%	0%	−17%
$\beta^4{}_{gijk} \times 0.1$	0%	0%	0%	**0%**	**−15%**	**17%**	**0%**
$\beta^4{}_{gijk} \times 2$	0%	0%	0%	**−15%**	**0%**	**0%**	**−35%**
$\beta^4{}_{gijk} \times 5$	0%	0%	−38%	**0%**	**0%**	**−35%**	**−35%**
$\beta^4{}_{gijk} \times 10$	0%	100%	−38%	**−30%**	**−30%**	**−35%**	**−35%**

To understand the effect of the alternative-specific constant (i.e., $\beta^0{}_g$) and the coefficients of the plans' attributes (i.e., $\beta^1{}_g$, $\beta^2{}_g$, $\beta^3{}_g$ and $\beta^4{}_{gik}$) four test problems are defined for each coefficient. Table 7 shows the results of this analysis, in which the values of y^v_p, f^s_p, l^r_p, m^q_{p11}, m^q_{p12}, m^q_{p21}, and m^q_{p22} represent the difference (in percentage) between the average levels of each attribute in three optimal plans using new parameters and the average levels of each attribute in three optimal plans in the base case.

Regarding the impact of $\beta^0 g$, as expected, this parameter mainly affects the total utility obtained from pricing plans. Consequently, as can be seen in Table 7, the average values of optimal plans' attributes are very close to the former plans and only slightly differ regarding the rental time fee in alternative case $\beta^0{}_g \times 0.5$.

In terms of the coefficients of the plans' attributes, as depicted in Table 7, alterations in each coefficient primarily influenced their corresponding attributes. For instance, across all experiments, modifications in the registration fee values (y^v_p) occurred exclusively when the corresponding coefficient ($\beta^1{}_g$) was changed. Similarly, the most noticeable shifts in values for free included distance and travel distance fee (i.e., f^s_p and l^r_p) were observed when changing the values of $\beta^2{}_g$ and $\beta^3 g$. A similar observation can be made for rental time fee values (m^q_{pik}), which mostly changed in response to variations in their corresponding coefficients (i.e., $\beta^4{}_{gik}$).

To better assess the model's sensitivity to the utility function parameters, we expanded the sensitivity analysis to encompass two additional problem instances, $I1$ and $I3$, where the model achieved optimal results. The computational results demonstrated that the impact of key parameters on the model's performance closely resembles that of our base case ($I2$). The detailed results of problem instances $I1$ and $I3$, along with the corresponding figures and tables, are presented in Appendix E in Supplementary Material.

In sum, this analysis indicates that the accurate estimation of coefficients of the plans' attributes is critical for achieving optimal plans since they have a significant effect on the value of the plans' attributes.

5.3. Decomposition algorithm

The results in Section 5.1.1 showed the method's ability to handle real-sized problems (i.e., $I1$ and $I2$) by successfully obtaining the optimal solutions. However, to further investigate the factors influencing problem complexity, we extended our analysis to encompass instances with up to 18 clusters and more levels for plans' attributes. Despite its success with real-sized instances, the exact method struggled to solve very large instances (i.e., $I5$-$I8$) within a 10-hours time limit. Hence, in this section, we propose a relax-and-fix-based (R&F) algorithm, which efficiently produces high-quality results within a shorter timeframe.

R&F is an effective approach to dealing with large-scale instances by decomposing the original MILP model into several less complex subproblems that can be solved sequentially, leading to solutions to the original problem (Pochet and Wolsey, 2006). The subproblems are easier to solve since the method relaxes a set of integer and binary variables to solve each subproblem, decreasing its complexity. We propose to decompose the original model into subproblems according to the pricing plan (creating, in this case, three subproblems or iterations). In the first iteration, only the binary variables of the first plan (P1) are kept as integers, while the binary variables of the other plans are relaxed as continuous variables. The solver then solves the model, obtaining a partially feasible solution to the original problem. In the second iteration, all the integer variables of P1 are fixed to the values optimized in the previous iteration, and the variables of plan P2 are kept as integers. Finally, in the last iteration, all the binary variables of P1 and P2 are fixed to values optimized in the corresponding subproblem, and the variables of P3 are integers. For these tests, we set a maximum time limit of 5000 s per iteration (whereas the exact method had a total of 36000 s).

Table 8
Comparison of the objective function (OF) values (i.e., monthly profit) and runtimes of the exact method and R&F. Gaps (%) are the difference between the value of the R&F algorithm and Exact method.

Instance	OF Exact method	OF R&F	OF gap between R&F and exact	Running time (s) Exact method	Running time (s) R&F	Time gap (s) between R&F and exact
$I1$	672,217	661,087	−1.66%	24	2	−93.38%
$I2$	680,105	670,750	−1.38%	65	6	−90.47%
$I3$	731,241	727,727	−0.48%	961	28	−97.09%
$I4$	730,051	726,589	−0.47%	314	109	−65.21%
$I5$	674,697	665,867	−1.31%	36 000	271	−99.25%
$I6$	688,685	677,797	−1.58%	36 000	5742	−84.05%
$I7$	696,073	732,113	4.49%	36 000	10 198	−71.67%
$I8$	682,408	665,569	−2.47%	36 000	10 493	−70.85%

Table 8 compares the results achieved by the exact method and the R&F algorithm in terms of the profit and running time. Notably, the R&F algorithm demonstrated the ability to achieve results very similar to the exact method in terms of monthly profit and even outperformed it in the case of the large instance $I7$. Additionally, when considering the running time, it was evident that the R&F algorithm significantly reduced the time needed to solve problems compared to the exact method in all instances. This suggests the R&F algorithm could be particularly valuable in larger instances where many segments and price levels need to be considered.

6. Conclusions and managerial insights

One-way CSOs often face significant challenges in planning the system to make the business more efficient and profitable. To tackle this, an optimal combination of customized pricing plans for the various segments of customers can adjust the interactions between demand and supply in a long-term perspective and yield higher profits for CSOs. To our knowledge, this is the first attempt to develop a set of optimal plans with realistic attributes for heterogeneous groups of customers. In addition, we incorporated several practical aspects into the model, such as varying rates for different areas and periods and a discrete choice model that estimates the customer response to the value of the different attributes in the plans. Due to the model's nonlinearity, standard solvers could not easily solve the proposed model to optimality. Hence, linear transformations were proposed to reformulate the original model as a MILP problem.

Extensive computational experiments with different sizes on the trip dataset of Brooklyn taxis were conducted to assess the model's performance. To apply the intended analyses, we followed a systematic approach to assign 136,740 taxi trip data to 19,643 potential carsharing users, which also provided a sufficient and proper dataset for researchers in the field. The numerical experiments demonstrated the ability of the proposed approaches to estimate the utility of multi-attribute pricing plans that can be applied in future studies. The computational results also showed the profit improvement of the proposed time- and location-dependent model compared to the profit obtained from time- and location-independent plans. Finally, a sensitivity analysis investigated the impact of key parameters on pricing plans and system profit to draw insights into the planning and operations of the carsharing system. From the study, the following main key insights can be taken:

- Using customer travel behavior as a source for building customer segments (instead of or building on socio-demographic information) can lead to relevant choice models that allow developing well-tailored plans and thus improving the system's profit.
- Considering the location and time dependency in developing pricing plans has a clear impact on complexity but it also has a significant impact on the profit of CSOs. Therefore, it is worth to pursue research on solution methods that allow tackling this complexity.
- The proposed model is shown to be robust to variations in the value of marginal disutility of registration fee. Nevertheless, changing the value of other parameters of the utility function results in significant differences in the optimal plans, which means that these parameters should be estimated with great accuracy.

The development of pricing plans for carsharing systems still has scope for improvement, and further research can be undertaken in several aspects. First, for simplicity, we applied a multinomial logit model to measure the proportion of customers of each segment that choose a pricing plan, while other models can be used to better capture the interaction between different alternatives. Second, in this study, we used the main characteristics of customers' travel behavior which are highly related to the attributes of pricing plans, to identify customer segments. Although this approach can calculate customer response to the value of the plans' attributes, in future studies, it would be interesting to also include socio-demographic characteristics (e.g., economic status) of customers on the customer segmentation procedures and compare the results with those found in this study. Third, it could be interesting to analyze the impact of choice complexity on customers' choice and acceptance. This research can be extended to develop different numbers of plans with different levels of complexity that could be assessed regarding user acceptance in an empirical study. Finally, while our study aimed to balance and optimize carsharing pricing plans across different periods at a strategic level, we acknowledge that considering the interactions between these periods could be a valuable area for further exploration. In this context, an interesting future research direction is to extend this model by integrating operational aspects, such as interactions between different periods, into consideration that provide feedback for strategic decisions. Due to the increased complexity, this integration would necessitate advanced solution methods, but it could ultimately support more tailored and effective decisions.

CRediT authorship contribution statement

Masoud Golalikhani: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Formal analysis, Data curation, Conceptualization. **Beatriz Brito Oliveira:** Writing – review & editing, Writing – original draft, Validation, Supervision, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Gonçalo Homem de Almeida Correia:** Writing – review & editing, Writing – original draft, Validation, Methodology, Investigation, Formal analysis, Conceptualization. **José Fernando Oliveira:** Writing – review & editing, Validation, Supervision, Resources, Project administration, Methodology, Conceptualization. **Maria Antónia Carravilla:** Writing – review & editing, Validation, Supervision, Project administration, Methodology, Funding acquisition, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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All authors provided critical feedback and helped shape the research and approved the submitted version.

Appendix A. Linear formulation of the pricing model

$$Max(Z) = \sum_{p\in\mathcal{P},g\in\mathcal{G},i\in\mathcal{A},k\in\mathcal{K}} \Big(\sum_{q\in\mathcal{Q}} d^q_{pgik} \times M^q_{pik} \times AVT_{gik} + \sum_{s\in\mathcal{S},r\in\mathcal{R}} z^{sr}_{pgik} \times L^r_p - \sum_{q\in\mathcal{Q}} d^q_{pgik} \times AVD_{gik} \times CF \Big) + \sum_{p\in\mathcal{P},g\in\mathcal{G},v\in\mathcal{V}} n^v_{pg} \times (Y^v_p - VCR \times CO) \quad (1)$$

Subject to:

$$\sum_{v\in\mathcal{V}} y^v_p = 1 \qquad \forall p \quad (2)$$

$$\sum_{s\in\mathcal{S}} f^s_p = 1 \qquad \forall p \quad (3)$$

$$\sum_{r\in\mathcal{R}} l^r_p = 1 \qquad \forall p \quad (4)$$

$$\sum_{q\in\mathcal{Q}} m^q_{pik} = 1 \qquad \forall p, i, k \quad (5)$$

$$\sum_{s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_3,q_4\in\mathcal{Q}} w^{vsrq_1q_2q_3q_4}_{pg} = y^v_p \qquad \forall p, g, v \quad (28)$$

$$\sum_{v\in\mathcal{V},r\in\mathcal{R},q_1,q_2,q_3,q_4\in\mathcal{Q}} w^{vsrq_1q_2q_3q_4}_{pg} = f^s_p \qquad \forall p, g, s \quad (29)$$

$$\sum_{v\in\mathcal{V},s\in\mathcal{S},q_1,q_2,q_3,q_4\in\mathcal{Q}} w^{vsrq_1q_2q_3q_4}_{pg} = l^r_p \qquad \forall p, g, r \quad (30)$$

$$\sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_2,q_3,q_4\in\mathcal{Q}} w^{vsrq_1q_2q_3q_4}_{pg} = m^q_{p11} \qquad \forall p, g, q = q_1 \quad (31)$$

$$\sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_3,q_4\in\mathcal{Q}} w^{vsrq_1q_2q_3q_4}_{pg} = m^q_{p12} \qquad \forall p, g, q = q_2 \quad (32)$$

$$\sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_4\in\mathcal{Q}} w^{vsrq_1q_2q_3q_4}_{pg} = m^q_{p21} \qquad \forall p, g, q = q_3 \quad (33)$$

$$\sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_3\in\mathcal{Q}} w^{vsrq_1q_2q_3q_4}_{pg} = m^q_{p22} \qquad \forall p, g, q = q_4 \quad (34)$$

$$d^q_{pgik} \le p_{pg} \times D_{gik} \qquad \forall p, g, i, k, q \quad (8)$$

$$\sum_{p\in\mathcal{P},g\in\mathcal{G},q\in\mathcal{Q}} d^q_{pgik} \le O_{ik} \qquad \forall i, k \quad (9)$$

$$\sum_{q\in\mathcal{Q}} d^q_{pg'ik} \leq 1.1\times(\frac{D_{g'ik}}{\sum_{g\in\mathcal{G}} D_{gik}})\times \sum_{q\in\mathcal{Q},g\in\mathcal{G}} d^q_{pgik} \qquad \forall p, g', i, k \tag{10}$$

$$d^q_{pgik} \leq m^q_{pik}\times O_{ik} \qquad \forall p,g,i,k,q \tag{11}$$

$$z^{sr}_{pgik} \leq f^s_p \times AVD_{gik}\times D_{gik} \qquad \forall p,g,i,k,s,r \tag{12}$$

$$z^{sr}_{pgik} \leq l^r_p \times AVD_{gik}\times D_{gik} \qquad \forall p,g,i,k,s,r \tag{13}$$

$$z^{sr}_{pgik} \leq \sum_{q\in\mathcal{Q}} d^q_{pgik}\times APD^s_{pgik} \qquad \forall p,g,i,k,s,r \tag{14}$$

$$\sum_{v\in\mathcal{V}} n^v_{pg} \leq \frac{\sum_{i\in\mathcal{A},k\in\mathcal{K},q\in\mathcal{Q}} d^q_{pgik}}{\sum_{i\in\mathcal{A},k\in\mathcal{K}} D_{gik}}\times N_g \qquad \forall p,g \tag{15}$$

$$\sum_{v\in\mathcal{V}} n^v_{pg} \geq \frac{\sum_{i\in\mathcal{A},k\in\mathcal{K},q\in\mathcal{Q}} d^q_{pgik}}{\sum_{i\in\mathcal{A},k\in\mathcal{K}} D_{gik}}\times N_g - 0.99 \qquad \forall p,g \tag{16}$$

$$n^v_{pg} \leq y^v_p \times N_g \qquad \forall p,g,v \tag{17}$$

$$h_g + \sum_{p'\in\mathcal{P},v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_3,q_4\in\mathcal{Q}} \exp(U^{vsrq_1q_2q_3q_4}_{p'g})\times t^{vsrq_1q_2q_3q_4}_{p'g} = 1 \qquad \forall g \tag{39}$$

$$p_{pg} = \sum_{v\in\mathcal{V},s\in\mathcal{S},r\in\mathcal{R},q_1,q_2,q_3,q_4\in\mathcal{Q}} \exp(U^{vsrq}_{pg})\times t^{vsrq_1q_2q_3q_4}_{pg} \qquad \forall p,g \tag{40}$$

$$t^{vsrq_1q_2q_3q_4}_{pg} \leq h_g \qquad \forall p,g,v,s,r,q_1,q_2,q_3,q_4 \tag{41}$$

$$t^{vsrq_1q_2q_3q_4}_{pg} \leq w^{vsrq_1q_2q_3q_4}_{pg}\times \exp(0) \qquad \forall p,g,v,s,r,q_1,q_2,q_3,q_4 \tag{42}$$

$$\exp(0)\times(h_g - t^{vsrq}_{pg}) \leq 1 - w^{vsrq_1q_2q_3q_4}_{pg} \qquad \forall p,g,v,s,r,q_1,q_2,q_3,q_4 \tag{43}$$

$$\begin{aligned}&\sum_{v\in\mathcal{V}}\alpha^3 Y^v_p y^v_p + \sum_{s\in\mathcal{S}}\alpha^2 F^s_p f^s_p + \sum_{r\in\mathcal{R}}\alpha^1 L^r_p l^r_p + \sum_{i\in\mathcal{A},k\in\mathcal{K},q\in\mathcal{Q}}\alpha^0 M^q_{pik} m^q_{pik} - \sum_{v\in\mathcal{V}}\alpha^3 Y^v_{p+1} y^v_{p+1}\\ &- \sum_{s\in\mathcal{S}}\alpha^2 F^s_{p+1} f^s_{p+1} - \sum_{r\in\mathcal{R}}\alpha^1 L^r_{p+1} l^r_{p+1} - \sum_{i\in\mathcal{A},k\in\mathcal{K},q\in\mathcal{Q}}\alpha^0 M^q_{p+1ik} m^q_{p+1ik} \leq -0.1 \qquad \forall p=\{1,\dots,P-1\}\end{aligned} \tag{18}$$

$$y^v_p \in \{0,1\} \qquad \forall p,v \tag{19}$$

$$f^s_p \in \{0,1\} \qquad \forall p,s \tag{20}$$

$$l^r_p \in \{0,1\} \qquad \forall p,r \tag{21}$$

$$m^q_{pik} \in \{0,1\} \qquad \forall p,i,k,q \tag{22}$$

$$d^q_{pgik} \in \mathcal{Z}^+_0 \qquad \forall p,g,i,k,q \tag{23}$$

$$n^v_{pg} \in \mathcal{Z}^+_0 \qquad \forall p,g,v \tag{24}$$

$$z^{sr}_{pgik} \in \mathcal{R}^+_0 \qquad \forall p,g,i,k,s \tag{25}$$

$$w^{vsrq_1q_2q_3q_4}_{pg} \in \{0,1\} \qquad \forall p,g,v,s,r,q_1,q_2,q_3,q_4 \tag{27}$$

$$h_g \in \mathcal{R}_0 \qquad \forall g \tag{36}$$

$$t^{vsrq_1q_2q_3q_4}_{pg} \in \mathcal{R}^+_0 \qquad \forall p,g,v,s,r,q_1,q_2,q_3,q_4 \tag{44}$$

The following constraints are not a part of the final model and are only added to the model in Section 5.1.3 to generate pricing plans that do not depend on location and time (time-and location-independent plans).

$$m^q_{pik} = m^q_{p(i+1)k} \qquad \forall p,k,q, \textit{and } i=1 \tag{45}$$

$$m^q_{pik} = m^q_{pi(k+1)} \qquad \forall p,i,q, \textit{and } k=1 \tag{46}$$

Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.tre.2024.103760.

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