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1pBB8. Modeling nonlinear acoustic waves in media with inhomogeneities in the coefficient of nonlinearity

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The refraction and scattering of nonlinear acoustic waves play an important role in the realistic application of medical ultrasound. One cause of these effects is the tissue dependence of the nonlinear medium behavior. A method that is able to model those effects is essential for the design of transducers for novel ultrasound modalities. Starting from the Westervelt equation, nonlinear pressure wave fields can be modeled via a contrast source formulation, as has been done with the INCS method. An extension of this method will be presented that can handle inhomogeneities in the coefficient of nonlinearity. The contrast source formulation results in an integral equation, which is solved iteratively using a Neumann scheme. The convergence of this scheme has been investigatedfor relevant media (e.g., blood, brain, and liver). Further, as an example, the method has been applied to compute the 1D nonlinear acoustic wave field in an inhomogeneous medium insonified by a 1 MHz Gaussian pulse propagatingup to 100 mm. The results show that the method is able to predict the propagation and the scattering effects of nonlinear acoustic waves in media with inhomogeneities in the coefficient of nonlinearity. This motivates a similar extension ofthe 3D INCS method.

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INTRODUCTION

Refraction and scattering of nonlinear pressure wave fields occur if inhomogeneities in the acoustic medium parameters are present. As these phenomena play an important role in medical diagnostic and therapeutical applications [1, 2, 3, 4], the design of new transducers and the development of novel applications requires the ability to model these effects accurately.
The Iterative Nonlinear Contrast Source (INCS) method is an excellent method to model nonlinear acoustic wave fields [5, 6, 7, 8]. Up to date this method has only been applied to homogeneous media. In this paper an extension of the INCS method, such that it can handle inhomogeneities in the coefficient of nonlinearity, is proposed.
After formulating the theory and introducing the contrast source formulation for the Westervelt equation in section II, the iterative Neumann solution method is presented in section III. In section IV the convergence of the iterative method is investigated with respect to relevant media (i.e. fat, liver, brain, water and blood). Further, as an example, the method has been applied to compute the one-dimensional nonlinear acoustic wave field in a medium with inhomogeneities in the coefficient of nonlinearity. Results are shown and discussed in section IV, followed by the conclusion in section V.

THEORY

Let the vector $\boldsymbol{x}$ denote a position in a three-dimensional spatial domain and let the scalar x denote a position in a one-dimensional domain. Let the scalar t denote the time coordinate. Let ∂_t denote the time derivative, ∇ the Nabla operator and ∇^2 the Laplacian operator. The acoustic properties of a specific medium are defined via the ambient speed of sound c_0, the ambient volume density of mass ρ_0 and the coefficient of nonlinearity β. Values are resumed in Table 1 [9] for fat, liver, brain water and blood.
The nonlinear propagation of the pressure wave field $p(\boldsymbol{x},t)$ is described by the Westervelt equation [10]. In order to take inhomogeneities in the nonlinear behavior into account, a space dependent coefficient of nonlinearity $\beta(\boldsymbol{x})$ is included. The resulting generalized form of the Westervelt equation reads

$$\nabla^2 p(\boldsymbol{x},t) - \frac{1}{c_0^2}\partial_t^2 p(\boldsymbol{x},t) = -S_{\mathrm{pr}}(\boldsymbol{x},t) - S_{\mathrm{nl}}\left[p(\boldsymbol{x},t)\right], \tag{1}$$

in which the primary source term $S_{\mathrm{pr}}(\boldsymbol{x},t)$ is defined via the sources which generate the acoustic field; the volume density of injection rate source q and the volume density of volume force source $\boldsymbol{f}$ [11], viz.

$$S_{\mathrm{pr}}(\boldsymbol{x},t) = \rho_0 \partial_t q(\boldsymbol{x},t) - \nabla \cdot \boldsymbol{f}(\boldsymbol{x},t). \tag{2}$$

The nonlinear source term $S_{\mathrm{nl}}\left[p(\boldsymbol{x},t)\right]$ describes the nonlinear behavior of the medium and reads

$$S_{\mathrm{nl}}\left[p(\boldsymbol{x},t)\right] = \frac{\beta(\boldsymbol{x})}{\rho_0 c_0^4}\partial_t^2 p^2(\boldsymbol{x},t). \tag{3}$$

For the forward problem, equation (1) represents a contrast source problem with known primary sources and medium parameters and unknown pressure wave field.

SOLUTION METHOD

The solution of the forward problem, represented by equation (1) is obtained by solving the integral equation [5, 6, 7, 8] which reads

$$p(\boldsymbol{x},t) = G(\boldsymbol{x},t) *_{\boldsymbol{x},t} \left\{S_{\mathrm{pr}}(\boldsymbol{x},t) + S_{\mathrm{nl}}\left[p(\boldsymbol{x},t)\right]\right\}, \tag{4}$$

with $*_{\boldsymbol{x},t}$ the convolution operator over space and time and G the Green's function. The Green's function describes the field generated by a delta source $\delta(\boldsymbol{x})\delta(t)$ in a homogeneous lossless linear background medium. For the three-dimensional case G equals

$$G(\boldsymbol{x},t) = \frac{\delta\left(t - \frac{||\boldsymbol{x}||}{c_0}\right)}{4\pi\,||\boldsymbol{x}||}, \tag{5}$$

TABLE 1. Acoustic medium parameters for water and several human tissues [9].

Medium	β	c_0 [ms^{-1}]	ρ_0 [Kgm^{-3}]
fat	6.150	1430.0	928
liver	4.375	1578.0	1050
brain	4.275	1562.0	1035
water	3.480	1482.3	1000
blood	4.000	1584.0	1060

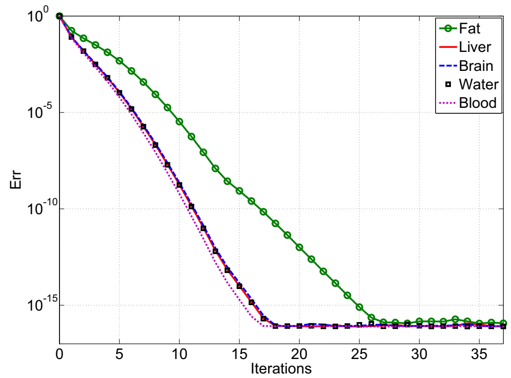

FIGURE 1. Normalized error functional Err for water and relevant human tissues in configuration A.

with $\delta(t)$ the Dirac delta function and $||\cdot||$ the length of a vector. For the one-dimensional case G equals

$$G(x,t) = \frac{c_0}{2} H\left(t - \frac{|x|}{c_0}\right), \tag{6}$$

with $H(t)$ the Heaviside step function.
After discretization with respect to space and time, the Neumann iterative solution scheme [11] used to solve the presented integral equation (4) reads

$$\mathsf{p}^{(n)} = 0 \quad \text{for } n < 0, \tag{7}$$

$$\mathsf{p}^{(n)} = \mathsf{G}\left[\mathsf{S}_{\text{tot}}\left[\mathsf{p}^{(n-1)}\right]\right] \quad \text{for } n \geq 0, \tag{8}$$

$$\mathsf{S}_{\text{tot}}\left[\mathsf{p}^{(n)}\right] = \mathsf{S}_{\text{pr}} + \mathsf{S}_{\text{nl}}\left[\mathsf{p}^{(n)}\right], \tag{9}$$

in which the vector $\mathsf{p}^{(n)}$ contains the n-th order approximation of the pressure field at discrete grid points. Further, G is the operator that convolves the discrete Green's function with the discrete total source. The latter is obtained by the source operator $\mathsf{S}_{\text{tot}}\left[\mathsf{p}^{(n)}\right]$.

Convergence of the iterative solution

To investigate the convergence of the presented method, a normalized error functional is introduced. This error functional is defined as

$$\text{Err}(n+1) = \frac{\left\|\mathsf{p}^{(n)} - \mathsf{G}\left[\mathsf{S}\left[\mathsf{p}^{(n)}\right]\right]\right\|}{\left\|\mathsf{p}^{(0)}\right\|}, \tag{10}$$

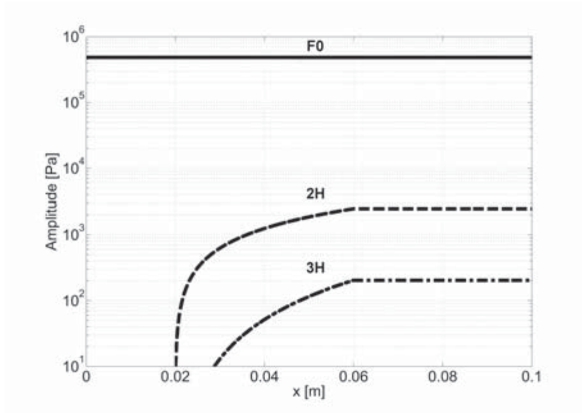

FIGURE 2. Amplitude profile of the fundamental (F0), second harmonic (2H) and third harmonic (3H) component in configuration B.

and indicates if the obtained solution converges toward a stable solution.

IN SILICO EXPERIMENTS

In this section the one-dimensional results obtained with the presented method are shown and discussed. First, the convergence of the presented contrast source method is investigated for homogeneous fat, liver, brain, water and blood. Second, in order to show the capability of the method to model scattering effects caused by inhomogeneities in the coefficient of nonlinearity, results obtained for a pressure wave field propagating through an inhomogeneous nonlinear medium are presented.

Configurations

Two configurations are used to demonstrate the presented method; configuration A, which is based on nonlinear homogeneous media and configuration B, which uses liver as a linear homogeneous background medium and which contains inhomogeneities in the coefficient of nonlinearity β. The coefficient equals

$$\beta(x) = \begin{cases} 0, & x < 20 \text{ mm or } x > 60 \text{ mm}, \\ 4.375, & 20 \text{ mm} \leq x \leq 60 \text{ mm}. \end{cases} \tag{11}$$

In both configurations the one-dimensional pressure wave field has been generated by a Gaussian pulse, resulting in a pressure jump at the location of the source which reads

$$\Delta P(t) = 2P_0 e^{-(2t/t_w)^2} \sin(2\pi f_0 t), \tag{12}$$

with center frequency f_0 = 1 MHz, time width $t_w = 3/f_0$ and pressure peak P_0 = 0.5 MPa.

Results

In Fig. 1 the normalized error functional Err, obtained for configuration A, is represented for fat, liver, brain, water and blood. In each case the normalized error functional flattens after it reaches the value $\mathrm{Err(n)} \approx 10^{-16}$. This value is related to the numerical precision of the computer. As expected the stronger the nonlinearity, and hence the contrast, the more iterations are required to reach convergence toward a stable solution.

In Fig. 2 the amplitude profiles of the fundamental (F0), second harmonic (2H) and third harmonic (3H) component, generated in configuration B, are shown with respect to depth x. The results clearly show that the harmonics start to

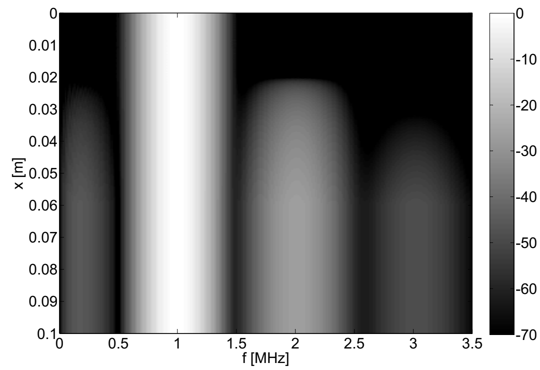

FIGURE 3. Normalized space-frequency profile of the nonlinear pressure wave field in configuration B. Amplitude values in dB relative to the maximum value of the fundamental.

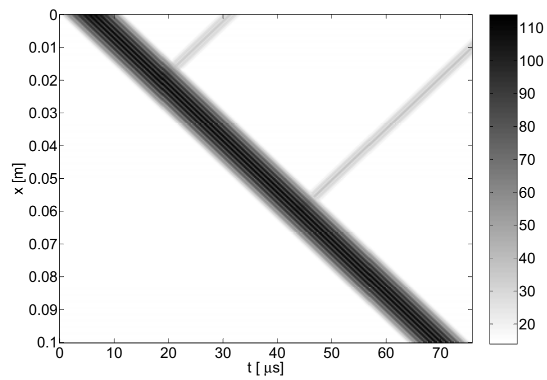

FIGURE 4. Space-time profile of the nonlinear pressure wave field in configuration B. Amplitude values in dB re 1 Pa.

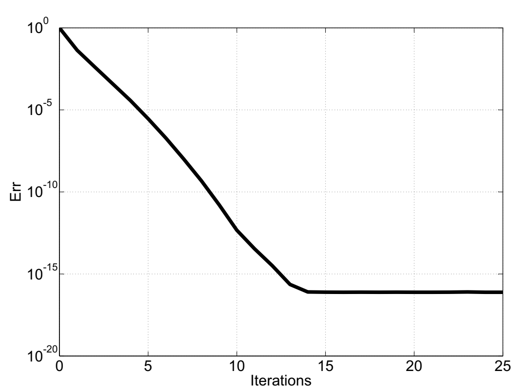

FIGURE 5. Normalized error functional Err for configuration B.

grow at 20 mm depth and remain constant after 60 mm. This is in agreement with the expectations as the coefficient of nonlinearity is only non zero in the region in between this depths. The same analysis is valid for Fig. 3 where the normalized amplitude profile of the pressure wave field generated in configuration B is plotted with respect to depth x and frequency f. The amplitudes are normalized with respect to the maximum of the absolute value of the fundamental component of the wave field in the frequency domain.
In Fig. 4 the amplitude profile of the nonlinear pressure wave field generated in configuration B is represented with respect to depth x and time t. As the coefficient of nonlinearity β changes from 0 to 4.375 and from 4.375 to 0, at respectively $x = 20$ mm and $x = 60$ mm, part of the pressure wave field is reflected at these points and propagates back to the source.
Figure 5 shows the normalized error function for configuration B. In this case the scheme needs lees iterations to converge with respect to the homogeneous nonlinear case as the part of the domain that contains the contrast is smaller.

CONCLUSION

In this paper, a method to compute the propagation of nonlinear ultrasound pressure wave fields through media with inhomogeneities in the coefficient of nonlinearity β has been presented. The method is based on a generalized form of the Westervelt equation in which a space dependent coefficient of nonlinearity $\beta(\boldsymbol{x})$ has been included. The resulting integral equation is solved using a Neumann iterative scheme.
The presented contrast source method has been developed from the INCS method which is known to produce accurate results for nonlinear acoustic wave fields in the presence of weak to moderate nonlinearity.
The presented method has been tested for the one-dimensional case. In silico experiments show that effects related to nonlinear propagation and scattering caused by inhomogeneities in the coefficient of nonlinearity are modeled correctly.
This motivates a similar extension of the method to the three-dimensional case.

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