Improved heat transfer modeling in a channel flow with rough surfaces using wall functions

Habiyaremye, V.; Mathur, A.; Roelofs, F.; Peeters, J. W.R.

Publication date
2024

Document Version
Final published version

Published in
Proceedings of Advances in Thermal Hydraulics, ATH 2024

Improved heat transfer modeling in a channel flow with rough surfaces using wall functions

V. Habiyaremye *, A. Mathur, and F. Roelofs
Nuclear Research and Consultancy Group (NRG)
Westerduinweg 3, 1755 LE, Petten, The Netherlands
habiyaremye@nrg.eu; mathur@nrg.eu; roelofs@nrg.eu

J.W.R. Peeters
Delft University of Technology
Mekelweg 5, 2628 CD, Delft, The Netherlands
J.W.R.Peeters@tudelft.nl

ABSTRACT

In flows over rough surfaces, the effect of roughness on momentum is different than that on heat transfer. Therefore, the standard Reynolds analogy, which is generally used for flows over smooth surfaces, is no longer valid. More specifically, the wall heat transfer to the fluid is overestimated when applying the Reynolds analogy in rough surface flows. In order to address this, several thermal correction models for rough surfaces have been proposed in literature. In this work, we investigate the applicability of these models to be used as wall functions in RANS simulations. For this, we use a channel flow geometry with rough surfaces and heat transfer at the walls, for which DNS data was produced at different Prandtl numbers. We show that the standard exponential damping function, which is used to constrain the thermal correction to the near-wall region, is not the best choice when using wall functions. Instead, we propose a new damping function which is better suited for the wall function approach. The newly proposed damping function also includes a dependency on the Prandtl number, which was found to make the thermal correction more accurate over a wider range of Prandtl numbers. The improvements are validated using the reference DNS results. The proposed damping function can allow for an easy adaptation of existing thermal correction models as wall functions for industrial scale RANS simulations.

KEYWORDS
RANS, surface roughness, wall functions, heat transfer

1. INTRODUCTION

In nuclear reactors, rust, as well as chemical and radiation-induced corrosion are known to affect surfaces of fuel rods, piping systems and other components [1]. These changes in surface topology can in turn have an effect on the flow of coolant and the heat transfer over these surfaces. It is therefore important to capture the effect of surface roughness in nuclear safety analyses [2]. In CFD modeling of nuclear reactors or reactor components, surfaces are generally modeled as perfectly smooth, which can lead to results that are not representative of real-life conditions. However, CFD models for surface roughness are becoming more and more readily available [3, 4]. The study of flow over rough surfaces is a well-established field of research with over a century of history. As a result, CFD models for the friction increase due to roughness have already been extensively validated. However, turbulent heat transfer over rough surfaces is a more

*corresponding author

complex and less mature topic of research in comparison [4, 5]. The Reynolds analogy, which states that there is a similarity between momentum and heat transfer in wall-bounded flows, is commonly used in smooth surface flows to compute heat transport. However, this analogy is known to no longer be valid when it comes to rough surfaces [6]. This can be explained by the fact that the presence of roughness elements increases the surface over which heat transfer can happen. Additionally, roughness can enhance flow mixing, which in turn enhances heat transfer. Secondary flows can lead to high local Nusselt numbers at points of impingement on roughness elements, but also low local Nusselt numbers inside recirculation bubbles [5], which could further decorrelate momentum and heat transport.

In order to overcome this issue, several correction models have been proposed in literature [7–10]. The approach used by these models is to correct the turbulent heat transport by modifying the turbulent Prandtl number in the near-wall region. However, such models are generally meant to be used with a wall-resolved mesh ($y^+ \leq 1$), leading to high computational costs and making them unsuitable to be used in nuclear applications. In this work, we investigate the applicability of these models to wall-modeled meshes ($y^+ \geq 30$) using wall functions. This is done by comparing RANS results to DNS of turbulent heat transfer in a periodic channel flow with rough walls. We show that the standard exponential damping function, which is commonly used to constrain the thermal correction to the near-wall region, is incorrect for wall-modeled meshes. Therefore, we propose a new damping function which ensures the correct behavior in the RANS simulations. This new damping function is shown to improve results compared to the exponential damping function in all of the channel flow cases for which DNS results were produced.

This paper is structured as follows. In section 2, the theory of turbulent flow over rough surfaces, including thermal correction models, is described. In section 3, the channel flow case setup is described in more detail, and the development of the new damping function is presented. Section 4 presents the results comparing the different RANS simulations to the reference DNS. Finally, in section 5, the conclusions of this work are presented, and an outlook is given for future work.

2. SURFACE ROUGHNESS THEORY

2.1. Flow over rough surfaces

Rough surfaces encountered in industrial applications are generally made of complex and heterogeneous roughness elements. This makes it difficult to implement a specific rough surface into a CFD model. One approach is to make a full three-dimensional scan of a real rough surface, and to use the measured topology as a flow geometry (see e.g., [11]). In this approach, the flow around each individual roughness element must be resolved using either DNS or LES, which can lead to an insurmountable computational burden for large geometries. Alternatively, the surface roughness can be parametrized, in order to model rather than resolve the flow around individual roughness elements. One approach is to use the roughness Probability Density Function (PDF) in order to describe the rough surface. The PDF represents the probability that a point on the rough surface has a certain height [3]. From the PDF, many statistical parameters can be extracted and used to parametrize the rough surface [12]. However, this approach is still too complex for real applications, where the exact surface topology may not be known. Using the *equivalent sand grain height*, we can instead characterize a rough surface using a single length scale. This length scale is based on the experimental measurements of Nikuradse [13], performed in a pipe whose internal surface was covered in sand grains to measure the effect of sand grain size on velocity profiles. The equivalent sand grain height of a given rough surface is the size of sand grains which, according to Nikuradse's experiments, produce a similar velocity profile to that of the rough surface [4].

Experimental and numerical observations have shown that the non-dimensional velocity profile undergoes a shift when surface roughness is introduced. In other words, the rough surface dimensionless velocity u_r^+

is given by $u_r^+ = u_s^+ + \Delta u^+$, with u_s^+, the smooth surface dimensionless velocity and Δu^+, the velocity shift caused by the surface roughness. Many models have been developed for Δu^+ as a function of the dimensionless equivalent sand grain height k_s^+. In this work, we use the model of Cebeci and Bradshaw [14]:

$$\Delta u^+ = \begin{cases} 0 & \text{if } k_s^+ < 2.25 \\ \frac{1}{\kappa}\log\left(\frac{k_s^+ - 2.25}{87.75} + C_s k_s^+\right)\sin(0.4258[\log(k_s^+) - 0.811]) & \text{if } 2.25 < k_s^+ < 90 \\ \frac{1}{\kappa}\log(1 + C_s k_s^+) & \text{if } k_s^+ > 90 \end{cases} \tag{1}$$

with $\kappa = 0.41$, the Von Karman constant, and C_s, the roughness constant indicating the uniformity of shape and spacing of roughness elements. We have found that $C_s = 0.253$ gives the most accurate velocity profile shift in the channel flow geometry.

2.2. Heat transfer over rough surfaces

As stated earlier, the Reynolds analogy is no longer valid in rough surface flows. In order to account for this, the turbulent Prandtl number, which is generally taken as a constant, needs an additional correction in the near-wall region. This corrected (or effective) turbulent Prandtl number is thus defined as:

$$Pr_{t,eff} = Pr_t + \Delta Pr_t. \tag{2}$$

Using this definition, modeling of heat transfer over a rough surface has been reduced to finding a suitable model for the thermal correction ΔPr_t as a function of the surface topology, and the distance from the wall. Most thermal correction models found in literature have the following general form:

$$\Delta Pr_t = F(k_s, ...) \cdot G(y, k_s) \tag{3}$$

with F, the correction function which computes the turbulent Prandtl number correction, and G, the damping function which limits the effect of the correction model to the near-wall region based on the wall distance y and the roughness equivalent sand grain height k_s.

Based on a literature study, we have selected two such models for further testing and validation. First, the model of Aupoix [8] has the following form:

$$F = A\Delta u^{+2} + B\Delta u^+ \tag{4}$$

$$A = (0.0155 - 0.0035 S_{corr})\left\{1 - \exp[-12(S_{corr} - 1)]\right\} \tag{5}$$

$$B = -0.08 + 0.25\exp[-10(S_{corr} - 1)] \tag{6}$$

$$G = \exp\left(-\frac{y}{k_s}\right) \tag{7}$$

where the corrected surface ratio S_{corr} represents the ratio of the rough surface area to the smooth surface area. The second selected model is that of Morency and Beaugendre [10], which takes the following form:

$$F = \frac{\kappa}{3.02}\frac{g(k_s^+)^\alpha \,\mathrm{Pr}^\beta}{C}, \tag{8}$$

where α, β and C are modifiable model constants, with default values $\alpha = 0.45$, $\beta = 0.8$ and $C = 1.92$, and where g is a function of the roughness size, defined in [10]. The damping function G can be set as Eq (7). Unlike the model of Aupoix, this model does not require any surface topology parameters other than the equivalent sand grain height. Additionally, it is a function of the molecular Prandtl number.
For both of the aforementioned models, the exponential damping function is equal to unity at the wall, and the full correction is applied, whereas the correction exponentially decreases as distance from the wall increases. However, when using a wall-modeled mesh, the correction is only applied in the first wall-adjacent cell, for which $y^+ \geq 30$. Not only does this mean that the thermal correction which is meant to be applied in the region between the first cell node and the wall is overlooked, but it also means that the region in which the thermal correction is applied becomes a function of the first cell size. Because of this, using the exponential damping function in a wall-modeled approach was found to underestimate the thermal correction term, and to suffer from strong mesh sensitivity. For more details on this, the reader is referred to [15].

3. ROUGH CHANNEL FLOW SETUP

In order to develop a damping function which is valid for the wall-modeled approach, the rough channel flow case was used. DNS results were produced at different Reynolds numbers, roughness sizes and Prandtl numbers in this geometry.

3.1. Direct Numerical Simulations

We use the rough channel flow setup as described in Peeters and Sandham [5]. It consists of a periodic channel flow with random roughness on the top and bottom walls. The walls are isothermal, while a uniform volumetric heat source drives the transport of heat. Three different rough surfaces were used in the simulations. The first surface is a filtered scan of a grit-blasted surface, while the second and third surfaces are 2×2 and 4×4 tiled versions of the first surface, used to obtain smaller roughness sizes.
In addition to the DNS results presented in [5], new DNS results obtained at higher Prandtl numbers (see [16]) were used in this work. A summary of the different cases is given in Table I. The results with $Re_\tau = 180$ presented in [5] were not used, since they could not be modeled in a RANS context using wall functions with sufficient resolution across the channel height.

Table I. DNS cases used in this work

Re_τ	k^+	Pr	Surface
360	0	[1, 2, 4, 6]	smooth
360	15	[1, 2, 4, 6]	4×4
360	30	[1, 2, 4, 6]	2×2
360	60	[1, 2, 4, 6]	1×1
540	0	1	smooth
540	90	1	1×1
720	0	1	smooth
720	120	1	1×1

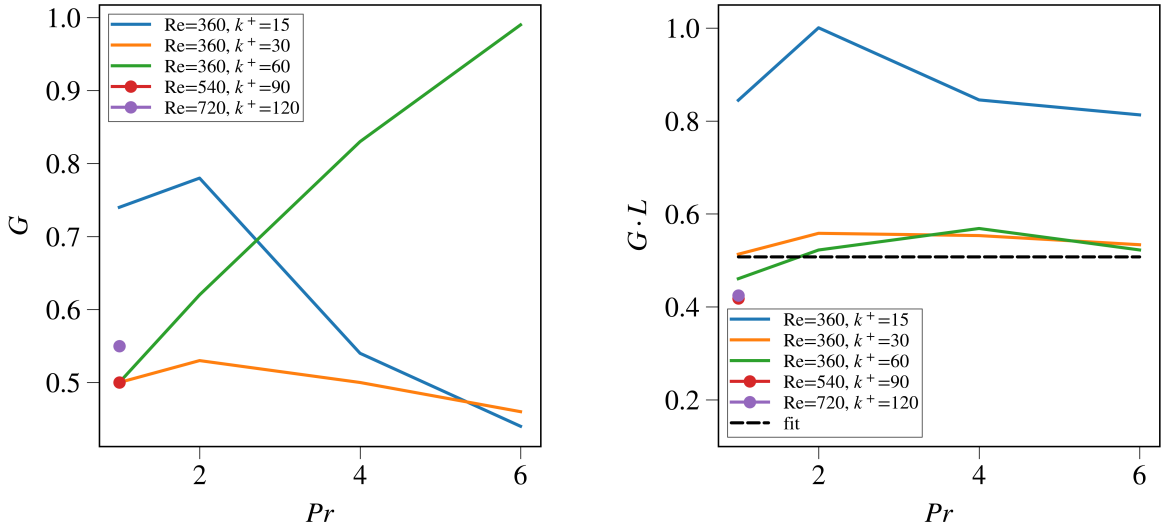

Figure 1. Optimal values of G for the different RANS simulations (left) and optimal values of G multiplied by Equation (9) (right)

3.2. RANS model

For the RANS simulations, a two-dimensional mesh was used with the same wall-normal and streamwise dimensions as the DNS. The RANS mesh is perfectly smooth at the walls, as the effect of surface roughness is entirely captured by the wall functions. For each case, the mesh resolution was chosen such that the first cell y^+ falls between 30 and 45. As a turbulence model, the standard $k-\varepsilon$ model was chosen. In the smooth wall simulations, the $k-\omega$ model gave slightly more accurate velocity and temperature profiles, but when wall roughness was added to the simulations, the $k-\omega$ model could no longer accurately reproduce the velocity profile shift, while the standard $k-\varepsilon$ model could. It was therefore chosen to proceed with the standard $k-\varepsilon$ model for this work. The simulations were carried out in OpenFOAM version 9, with the different thermal correction models implemented as wall function boundary conditions for the turbulent thermal diffusivity ('*alphat*' in OpenFOAM).

3.3. Wall function damping

The channel flow described above was used to develop a new damping function, which is meant to be used in combination with wall functions for the momentum and energy equations. The following approach was used. First, the optimal damping function value G was determined for each of the cases listed in Table I. The optimal damping function value is defined as the value G at which the error of the RANS temperature profile with respect to the DNS temperature profile is at its minimum. These optimal G values were found by running each case for a range of constant G values and finding the error minimum. When plotted directly, no clear pattern can be seen in the optimal G values. This is shown in Figure 1 (left). The next step was to find a function L such that multiplying the optimal G values by L would give a more discernable pattern in the results. In the process of finding such a function L, it was observed that the results at $k^+ = 15$ did not align with the other cases at higher roughness sizes. This may be explained by the fact that unlike the larger roughness sizes, the roughness elements with $k^+ = 15$ fall in the transition region between the viscous layer and the log-law region for both u^+ and T^+ [4, 5]. Therefore, the results at $k^+ = 15$ were omitted

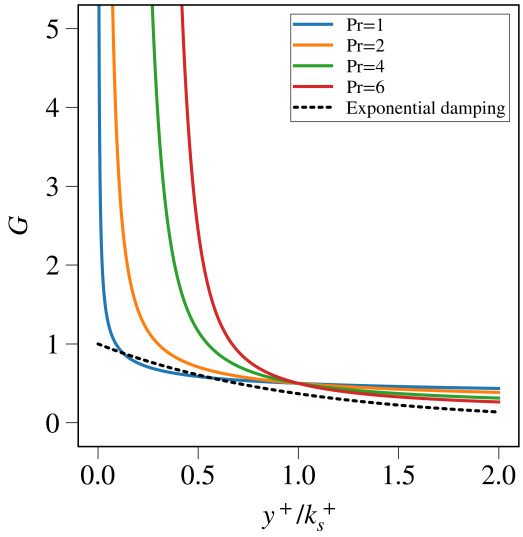

Figure 2. New damping function (10) **at different Prandtl numbers and exponential damping function** (7)**.**

in the development of the new damping function proposed in the present study. We propose the following expression for L:

$$L = 1 + \log_n \left[\left(\frac{y^+}{k_s^+} \right)^{Pr} + C \right] \tag{9}$$

where n and C are model constants given by $n = 100$ and $C = 1/n$. The optimal values of G multiplied by this function L are shown in Figure 1 (right). As was mentioned earlier, we observe that the results at $k^+ = 15$ follow a different trend from the other cases. Leaving out these results, we see that the different $G \cdot L$ values do not change much as a function of the Prandtl number, and can be reasonably approximated by a constant value $G \cdot L = 0.51$. Finally, we obtain the new damping function by dividing by L:

$$G = \frac{0.51}{L}. \tag{10}$$

A comparison between the new damping function given by Eq. (10) and the exponential damping function given by Eq. (7) is shown in Figure 2. Significant differences can be observed between the new damping function and the exponential one for $y^+/k_s^+ < 1$, especially when $Pr > 1$.

4. RESULTS

In this section, we show comparisons between the DNS temperature profiles and the different RANS simulation results. RANS simulations were run with the following thermal models: no roughness correction model, the Aupoix [8] and the Morency and Beaugendre [10] models with exponential damping function, and the Aupoix model with the new damping function proposed in this work (labeled 'WF damping' in the figures, for wall-function damping). The Aupoix, and Morency and Beaugendre models with exponential damping function were also applied as wall functions, despite the fact that they are initially designed for a wall-resolved approach. The temperature profiles are non-dimensionalized according to $T^+ = (T_w - T)/T_\tau$ with T_w, the wall temperature, T, the dimensioned temperature and T_τ, the friction temperature.

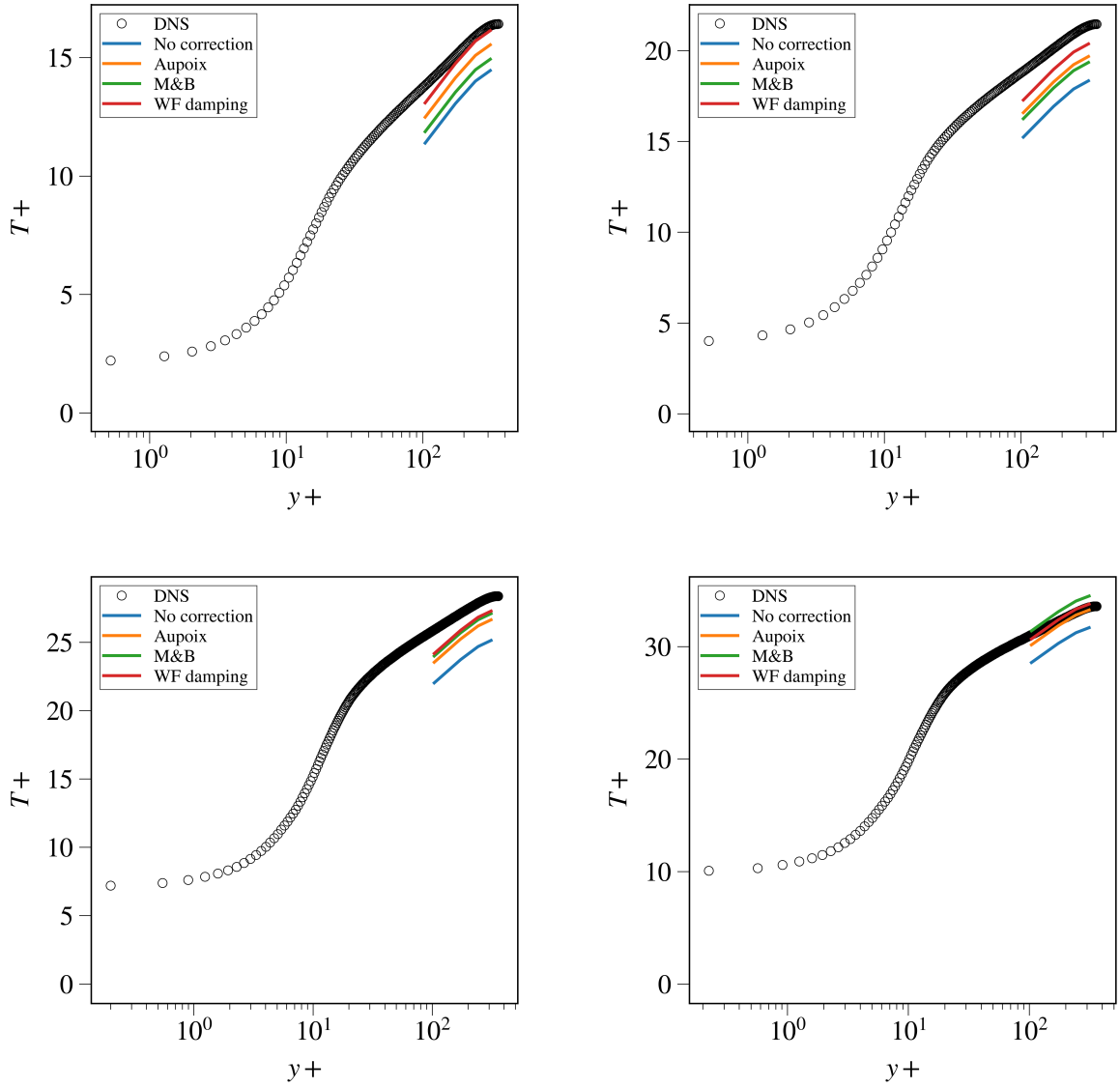

Figure 3. Temperature profiles over a rough surface with $k^+ = 30$ at $Re_\tau = 360$ and $Pr = 1$ (top-left), $Pr = 2$ (top-right), $Pr = 4$ (bottom-left) and $Pr = 6$ (bottom-right)

At $Re_\tau = 360$ and $k^+ = 15$ (results not included in this paper), the effect of roughness on heat transfer is still limited. The results with no thermal correction are very similar to those with the different correction models, and the error with respect to the reference DNS is still relatively small. Nonetheless, the results with the new damping function are closest to the reference DNS, despite the fact that these cases were not directly considered in the development of the new damping function. As the wall roughness increases, the influence of the thermal correction model becomes more clearly visible. This can be seen in Figures 3 and 4, showing the temperature profiles at $Re_\tau = 360$ and $k^+ = 30$ and 60, respectively. Here, the need for thermal correction models becomes more clear. When no thermal correction model is applied, the dimensionless temperature is significantly underestimated. The Aupoix model with the standard damping function improves on this but still underestimates T^+. The results with the new damping function show an improvement of results compared to the standard damping function across all cases. Finally, the results at higher

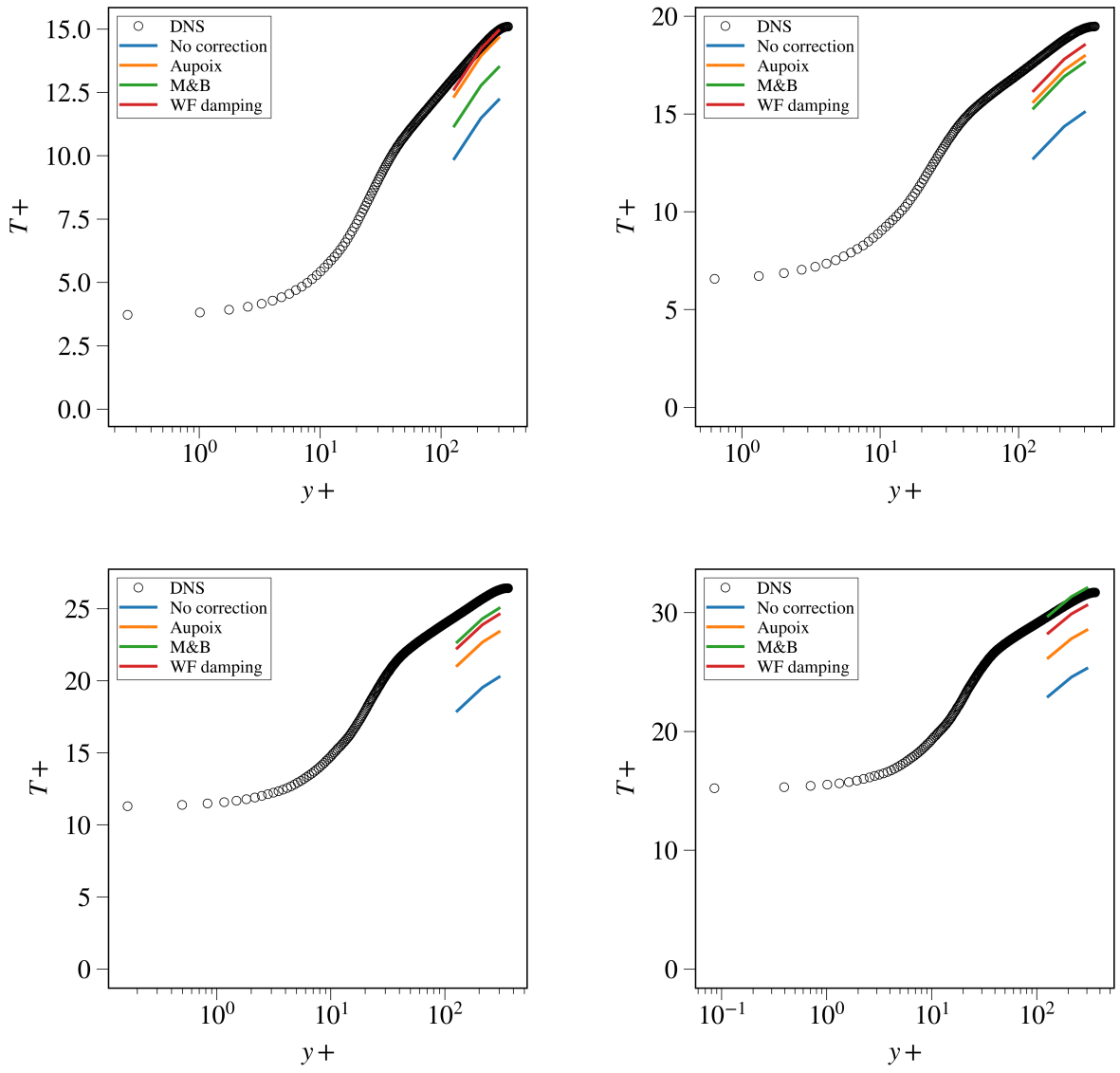

Figure 4. Temperature profiles over a rough surface with $k^+ = 60$ at $Re_\tau = 360$ and $Pr = 1$ (top-left), $Pr = 2$ (top-right), $Pr = 4$ (bottom-left) and $Pr = 6$ (bottom-right)

Reynolds numbers are shown in Figure 5. Here, the effect of roughness, and therefore also the error made with no thermal correction model is the largest. In these two cases, the results of the Aupoix model with exponenial damping are already in good agreement with the DNS. The results with the new damping function are therefore also very similar. This can also be deduced from Figure 2, as the new damping function is close to the exponential damping function at $Pr = 1$ and in the range of y^+/k_s^+ of these two cases.
The effect of the Prandtl number on the accuracy of the different models can also be observed with these results. In Figures 3 and 4, we can see that the accuracy of the Aupoix [8] model decreases as the Prandtl number increases. This model was mainly developed for aerospace applications ($Pr \approx 0.71$ for air), which could indicate that accuracy at higher Prandtl numbers may not have been a priority. On the other hand, the model of Morency and Beaugendre [10] has better accuracy at higher Prandtl numbers. The model's inclusion of a dependency on the Prandtl number is likely the reason for this. Similarly, the inclusion of a

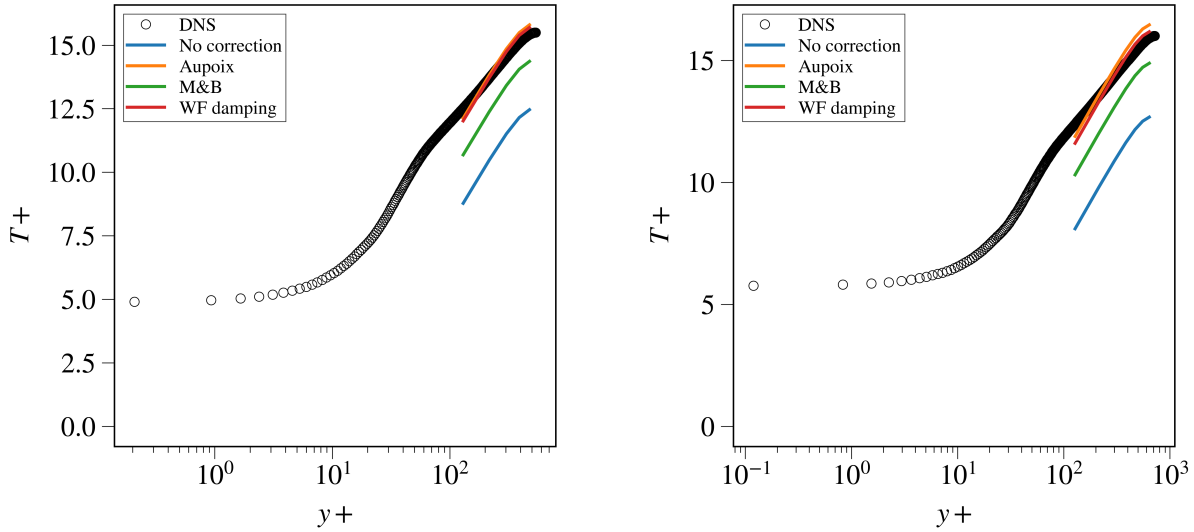

Figure 5. Temperature profiles over a rough surface with $k^+ = 90$, $Re_\tau = 540$ and $Pr = 1$ (left) and $k^+ = 120$, $Re_\tau = 720$ and $Pr = 1$ (right)

Prandtl number dependency in the new damping function is found to have a positive impact on the accuracy at higher Pr.

5. CONCLUSIONS

In this paper, we have investigated the applicability of different thermal correction models for turbulent heat transfer on rough walls as wall models for RANS simulations. In order to validate the models, a channel flow geometry with isothermal rough walls and a volumetric heat source was used, for which DNS results were made available at several Reynolds numbers, roughness heights and Prandtl numbers. We have shown that while improvements in accuracy can already be observed by implementing the selected models as wall functions directly, they still tend to underestimate the magnitude of the correction. Additionally, the exponential damping function which is often used in literature suffers from strong mesh sensitivity when used in wall-modeled simulations. This is due to the fact that the investigated models, and especially the exponential damping function, are designed for wall-resolved rather than wall-modeled simulations. In order to adress these shortcomings, a new damping function was proposed, which is specifically designed for the wall-modeled approach. This damping function and the thermal correction are only evaluated and applied in the first layer of wall-adjacent cells, which should have cell-centers in the log-layer ($y^+ > 30$). The damping function was obtained by finding the values at which the error in the temperature profile compared to the DNS was minimized for all of the available channel flow cases, and fitting a function through the obtained points. Compared to the exponential damping function, the newly proposed damping function gives a better agreement with the DNS in terms of the temperature profiles in the rough channel flow case.

Due to the limited amount of available DNS results for turbulent heat transfer on rough walls, the newly proposed damping function was only developed and validated in a limited number of cases. Additional DNS results at higher Reynolds numbers and roughness heights would allow us to gain more confidence in the data fitting, as well as to validate the new damping function more thoroughly. In order to better understand the difference in behavior of the simulations at $k^+ = 15$, more reference data is also needed at intermediate roughness sizes, i.e., $0 < k^+ < 15$ and $15 < k^+ < 30$.

REFERENCES

1. D. Féron, "Overview of nuclear materials and nuclear corrosion science and engineering," in *Nuclear Corrosion Science and Engineering*, pp. 31–56, Elsevier, (2012).
2. E. Laurien, "Implicit model equation for hydraulic resistance and heat transfer including wall roughness," *Journal of Nuclear Engineering and Radiation Science*, **2** (2), pp. 021016 (2016).
3. M. Kadivar, D. Tormey, and G. McGranaghan, "A review on turbulent flow over rough surfaces: fundamentals and theories," *International Journal of Thermofluids* (2021).
4. D. Chung, N. Hutchins, M. P. Schultz, and K. A. Flack, "Predicting the drag of rough surfaces," *Annual Review of Fluid Mechanics*, **53**, pp. 439–471 (2021).
5. J. Peeters and N. Sandham, "Turbulent heat transfer in channels with irregular roughness," *International journal of heat and mass transfer*, **138**, pp. 454–467 (2019).
6. S. Soleimani and S. Eckels, "A review of drag reduction and heat transfer enhancement by riblet surfaces in closed and open channel flow," *International Journal of Thermofluids*, **9**, pp. 100053 (2021).
7. J. Jiménez, "Turbulent flows over rough walls," *Annu. Rev. Fluid Mech.*, **36**, pp. 173–196 (2004).
8. B. Aupoix, "Improved heat transfer predictions on rough surfaces," *International Journal of Heat and Fluid Flow*, **56**, pp. 160–171 (2015).
9. F. Chedevergne, "Analytical wall function including roughness corrections," *International Journal of Heat and Fluid Flow*, **73**, pp. 258–269 (2018).
10. F. Morency and H. Beaugendre, "Comparison of turbulent Prandtl number correction models for the Stanton evaluation over rough surfaces," *International Journal of Computational Fluid Dynamics*, **34** (4), pp. 278–298 (2020).
11. P. Forooghi et al., "DNS of momentum and heat transfer over rough surfaces based on realistic combustion chamber deposit geometries," *International Journal of Heat and Fluid Flow*, **69**, pp. 83–94 (2018).
12. M. Thakkar, A. Busse, and N. Sandham, "Surface correlations of hydrodynamic drag for transitionally rough engineering surfaces," *Journal of Turbulence*, **18** (2), pp. 138–169 (2017).
13. J. Nikuradse, "Stromungsgesetze in rauhen Rohren," *VDI-Forschungsheft*, **361**, pp. 1 (1933).
14. T. Cebeci and P. Bradshaw, "Momentum transfer in boundary layers," *Washington* (1977).
15. J. Beitler, "Heat Transfer Correction Modelling for RANS Simulations on Rough Surfaces," *TU Delft* (2022).
16. J. W. Peeters, "Spectral Theory of Turbulent Heat Transfer in the Presence of a Rough Wall," *Physical review letters*, **131** (13), pp. 134001 (2023).