# A flow-based Integer programming approach to design an interurban shared automated vehicle system and assess its financial viability

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## Abstract

Shared automated vehicles (SAV) are expected to become part of on-demand transport systems in the future. In an interurban context, SAV systems may potentially contribute to significantly improve the availability and quality of public transport. In this paper, we propose a flow-based integer programming approach to design and assess the financial viability of a SAV interurban system. An integer programming model, based on profit maximization and allowing vehicle pooling, was developed to optimize the fleet size and vehicle movements of a typical working day. This model and its optimal results were the basis for assessing the financial viability of an interurban transport network in the central area of Portugal. Different levels of demand were tested and three different fleet compositions were considered: cars, minibuses, and a mixed fleet of both. The results showed that a mixed fleet of vehicles is the best option for profit maximization. The optimal share of each vehicle type, in this fleet composition, asymptotically converges to approximately 1/3 of cars and 2/3 of minibuses, as demand increases. The introduction of a SAV system in such a region for interurban trips can replace up to 18 private vehicles per SAV vehicle, and should be done with sequential expansions to guarantee a profit at the different stages. The break-even point, for the conditions of the case study, is between 1.5 and 2 passengers per vehicle, for a fleet of cars, and between 4 and 5 passengers per vehicle, for a fleet of minibuses.

**Keywords:** Shared automated vehicle, Fleet size, Optimization, Interurban transport, Linear programming

## 1. Introduction

Automated vehicles, vehicles with some level of automation to assist or replace human control in moving passengers and freight (Stocker and Shaheen, 2017), are still in development. Nowadays the more advanced vehicles available in the market are level 3 automation vehicles, considering the 5 levels of automation (Level 0 is conventional driving) defined by the Society of Automotive Engineers (SAE International, 2018). Level 5 vehicles, with full driving automation for all driving environments, also denominated as fully automated, autonomous, or driverless, due to the elimination of the need for a human driver, may become a reality in the next decades depending, among other things, on investment in R&D and people's interest in using shared mobility (Nieuwenhuijsen et al., 2018). This technology is expected to bring important benefits to users, namely increased in-vehicle productivity, mobility for non-drivers, and reduced transportation costs; as well as overall external benefits, such as improved safety, lowered congestion, lessened parking pressure, better fuel efficiency, and reduced emissions (Fagnant and Kockelman, 2015; Milakis et al., 2017; Litman, 2020). Shared fleet service providers are regarded to be the first adopters of driverless vehicles due to the high costs for the average consumer when vehicles enter the market (Wadud, 2017). In fact, carmakers are already making strategical adjustments, envisioning themselves as part of the future shared mobility players (Stocker and Shaheen, 2017).

The introduction of driverless vehicles into on-demand shared vehicle systems creates new degrees of freedom merging the current concepts by addressing their inherent challenges: solving rebalancing and the need for driver's license and insurance to use the system, in the case of carsharing services; and eliminating the need for a driver, in the case of ride-hailing or taxi services (Zhang et al., 2015). The resulting system, with different designations in the literature (e.g.: shared automated vehicle system, shared autonomous vehicle system, autonomous carsharing system, autonomous taxi system), is envisioned to have a central operator to optimally coordinate the movements of a fleet of shared driverless vehicles, serving personal mobility needs on request, individually or with ridesharing, as well as the rebalancing or charging movements of unassigned vehicles (Salazar et al., 2019; Narayanan et al., 2020). This system, hereinafter designated as shared automated vehicle (SAV) system, can be viewed as a mode of transport that could in the limit serve all ground transport demand in the future (Stocker and Shaheen, 2017). The vehicles' ability to move without a driver enhances the potential to change the way transportation is managed and perceived, enabling the emergence of more inclusive and efficient transport services (Childress et al., 2015). For instance, fully automated vehicles can be used by any person (even without a drivers' license) and the fact that they do not require staff to drive or relocate vehicles reduces system management costs (Santos and Correia, 2019), eventually enabling services to expand into lower demand areas. Vehicles with different capacities are being considered for on-demand SAV services. Besides the car, that has been widely studied (Narayanan et al., 2020), the minibus is viewed as another important option, since its seat capacity suits higher transport demand than the former, providing better mobility efficiency and flexibility when compared to the larger-sized traditional buses (Martinez and Viegas, 2017; Dai et al., 2020).

In an interurban context, SAV systems may potentially contribute to significantly improve the availability and quality of public transport, with the capability of breaking the car-public transport vicious cycle by which poor public transport accessibility leads to lower usage and an even lower quality system. Less dense (lower demand) areas would potentially be reconnected to the wider community, by replacing a poor public transport service that nowadays is mainly performed by buses that have to cover several Origin-Destination (OD) pairs in their low-frequency routes, with an on-demand service that uses smaller fully automated vehicles and faster routes. Such improvements in the level of interurban mobility would benefit local economies, by making rural areas and smaller cities, currently with low accessibility by other means than the private vehicle, more attractive to visitors and future residents (Bernhart et al., 2018). Despite the promising benefits, researchers have been focusing mostly on studying the potential of SAV systems inside urban areas (Soteropoulos et al., 2019), leaving a void in the literature concerning its implementation in a regional interurban context.

The design and analysis of SAV systems are associated with solving the problems of fleet size, vehicle-trip assignment, and rebalancing (Zhao and Malikopoulos, 2020). This is in line with the operator's goal of sizing their fleets to maximize profits while offering a proper level of service (Fagnant and Kockelman, 2018). Considering the latest publications, the two main approaches used to assess SAV systems are integer programming (IP) and agent-based modeling and simulation (ABMS).

The SAV system design and assessment through IP has been mainly associated with the vehicle routing problem (VRP) and its variants. The VRP approach optimizes vehicle movements by defining the route of each vehicle individually, though its NP-hardness makes it only viable for small networks without realistic size, even after recurring to heuristic methods (Lin et al., 2012; Macrina et al., 2019; Van Essen and Correia, 2019). Slicing the space or time dimensions of the optimization problem are strategies used to tackle the NP-hardness of VRP, allowing its application to larger size networks. An example of space slicing is the clustering process used by Masoud and Jayakrishnan (2017) to divide the spatial dimension of the VRP problem into smaller parts. The time-slicing is the most used by authors (Zhang et al., 2016; Ma et al., 2017; Alonso-mora et al., 2018; Bertsimas et al., 2019; Liang et al., 2020) and is performed recurring to model predictive control (MPC), also known as receding horizon control, or rolling horizon. The process is the following: an optimization problem is solved to determine a plan of action for a fixed time horizon (consisting of several time steps) by using the system status and the predicted demand; then, the first part of the plan, corresponding to the immediate actions, is implemented and the system status updated;

afterward, the time horizon is shifted one time step forward and the process is repeated (Mattingley et al., 2010). Instead of optimizing vehicle movements for the complete optimization period, the MPC approach allows decreasing the number of variables in each optimization process by looking into ahead finite horizons at a time.

A distinct IP approach to design a SAV system and assess its characteristics, not recurring to VRP or its variants, is the optimization of vehicle movements using flows. The flow-based optimization, previously applied to carsharing relocation operations (Jorge et al., 2014) and already used in the assessment of SAV systems (Liang et al., 2016; Tsao et al., 2018; Iglesias et al., 2018), aggregates vehicle movements into flows reducing the number of decision variables (when compared to VRP). Despite not having been demonstrated yet, this simplification has the potential to be applied to the assessment of large SAV systems, without the need for heuristic methods or MPC to find an optimal solution. Until now, flow-based optimization models were applied to small SAV networks (Liang et al., 2016) or to large SAV networks using MPC methods (Tsao et al., 2018; Iglesias et al., 2018).

ABMS uses the interaction between agents in a modeled environment to assess its effects as a whole (Abar et al., 2017). By using simple rules that regulate agents', a complex group behavior simulation is generated allowing the assessment of larger networks (when compared with VRP, and VRP using MPC) with details as close as possible to real-world context. The assessment of SAV systems is then made by exploring the relationship between input variables and model-simulated response through different scenarios (Fagnant and Kockelman, 2014, 2016; Martinez and Viegas, 2017; Wang et al., 2019). Algorithms, such as the golden section search can be used, in this process, to find near-optimal values (Fagnant and Kockelman, 2018). Indeed, the use of ABMS is capable to handle any type of network configuration and complexity, though a series of simulations are needed to retrieve near-optimal configurations.

In this work, we propose a flow-based integer programming approach to design and assess the financial viability of a SAV interurban system. An IP model, based on profit maximization and allowing vehicle pooling, is developed to optimize the fleet size and vehicle movements of a typical working day. This is done on a framework, possible due to the interurban scale, where vehicles are modeled through flow variables in a time-space network which avoids having to model each vehicle independently. The model is applied to an interurban transport network in the central area of Portugal, and the optimal solutions are used to produce a viability assessment. This assessment is particularly focused on: determining the profitable fleet configurations, identifying the optimal service coverage, performing a comparison with the current situation, finding the break-even point in terms of average vehicle occupation, and analyzing the potential of idle time to charge vehicle batteries. Experiments are set considering different demand levels for using SAV, and three different fleet compositions: cars, minibuses, and a mixed fleet of both. To the best of our knowledge, this is the first time that a flow-based optimization approach is applied to optimize the daily operations of a large scale SAV system without slicing time to reduce the original problem into smaller problems, and using a formulation that considers vehicle pooling. This is also the first paper where a financial viability assessment of a SAV system providing an interurban transport service is done.

The paper is organized as follows. Section 2 presents the developed mathematical model. Section 3 describes the case study of the Coimbra region (Portugal) and scenarios to which the model is applied. In section 4, the main results are presented. The article ends with the conclusions drawn from this work, in section 5.

## 2. Mathematical model

This section presents the formulation of the flow-based IP model to assess the viability of a SAV system in an interurban context. The model is based on profit maximization and optimizes the fleet size and the vehicle movements, with clients and relocations, of a typical working day.

## 2.1. Assumptions

We assume the existence of a SAV system that is controlled by an organization that supplies interurban trips in a certain region. The region is subdivided into zones (e.g.: municipalities), being each zone represented by a spatial node (e.g.: centroid). The traveling distance between nodes is large enough to consider that clients are willing to be detoured inside the same origin zone in order for the system to serve more demand per movement. The transported users inside each vehicle share the same origin and destination, meaning that the vehicles travel directly from one zone to the other without picking up another traveler on the way. This reduces the in-vehicle travel time and simplifies the way trips are modeled. Vehicles can be relocated empty if needed. It is considered that users always accept the vehicle provided to them (users' mode choice behavior is not modeled), and that the service price is constant. In future research, different service options (e.g.: with or without pooling, in a car or minibus) with different characteristics (e.g.: price, travel time), complemented with a user mode choice behavior, can be proposed to find an equilibrium between demand and supply. Bilevel programming formulations are one option to account for users' mode choice in optimization (Lu et al., 2021). Real-time information about the location of the vehicle and estimated arrival time, enables users to be at the specified pick-up point when the vehicle arrives. It is assumed that the vehicle idle time includes all refueling or recharging related operations (movements, waiting for charging/refueling), and that vehicles can always find a fuel or charging station close to their idle position (being the costs of driving specifically to refuel or recharge considered negligible). If electric vehicles are considered, it needs to be verified that battery capacity and charging speed can assure zero impact in the movement of clients and relocations, but this can only be achieved with a more detailed operational model (this work presents a simple and aggregated approach for the validation of this assumption in the case study).

The time taken to pick-up and deliver users inside the origin and destination zones is added to the vehicles' travel time but it is not modeled in the graph. The vehicles' travel time comprises the vehicle movement during the trip between the origin and destination zone, including the pick-up and drop-off of travelers. Two travel times are thus defined: $t_{ij}$ and $t_{pd_i}$. Being $t_{ij}$ the interzonal travel time between the representative nodes of zones $i$ and $j$, and $t_{pd_i}$ the average intrazonal travel time to pick-up or deliver the number of users that can be transported by the vehicle. For each zone, the pick-up time is considered equal to the delivery time.

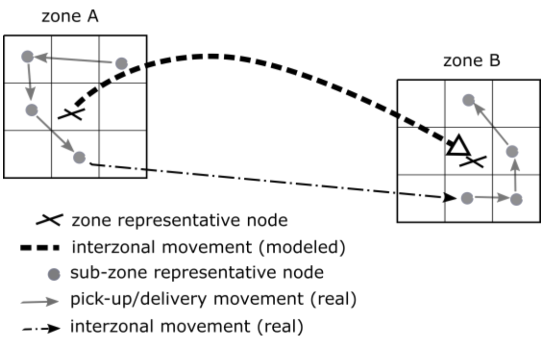



*Figure 1: difference between modeled and real interzonal movements*

In order to determine the intrazonal pick-up and delivery time ($t_{pd_i}$), requests are simulated inside each zone. For that purpose, each zone is subdivided into subzones (e.g.: small administrative units such as boroughs or parishes), being each subzone represented by its node. Requests are generated using the inverse transform sampling method applied to the population ratio of each subzone. The number of requests generated is equal to the vehicle capacity to assure that the pick-up or delivery times are always sufficient for any number of transported passengers. This is necessary because the optimization model does not track vehicles individually, and, therefore, the number of passengers inside each one. For instance, if 10 passengers are transported, in a certain movement, by a flow of 3 cars (considering that each car has a 4-seat capacity), the model, describing flows and not vehicles individually, does not know which cars transport less than 4 passengers (the possibilities are 4+4+2 and 4+3+3). To make sure that the modeled travel time, is always enough for all possibilities of passenger distribution per vehicle (not "controlled" by the model), we use the pick-up and delivery time values associated to filling up all the vehicles. The shortest distance between requests is determined using a simulated annealing algorithm (Eglese, 1990), assuming that requests at each subzone node are connected by straight lines (as the bird flies distances). The entire process is repeated $n$ times for each zone $i$, being the average value used as $t_{pd_i}$.

We assume that the relocation travel time, $t_{r_{ij}}$, is equal to $t_{ij}$, while the travel time of moving users in a vehicle between two zones, $t_{u_{ij}}$, is equal to $t_{pd_i} + t_{ij} + t_{pd_j}$. The latter contains a difference between the modeled movement and the real movement since it is considered that the distance traveled by the vehicle between the last pick-up point at the origin zone and the first drop-off point at the destination is equal to the distance between the nodes of the origin and destination zones (see Figure 1). We are assuming that this error is negligible for the distance ranges that will be modeled in the region, and eliminated by the discretization of time in time steps. This simplification allows to greatly reduce the needed travel time data, since it only requires to collect the travel time between zone centroids, and between sub-zones of the same zone (when compared to collecting the travel times between all the subzones).

In summary, the assumptions upon which the model is built are the following:

- A SAV System serves the interurban trips in a region composed of several cities;
- Vehicles travel directly from the zone of origin to the zone of destination, and ridesharing is allowed;
- Users' mode choice behavior is not modeled;
- Users and vehicles share real-time location information thus no waiting time is considered;
- Refueling and recharging are performed during the idle time of the vehicles;
- There is no shortage of fuel/charging stations nearby the vehicles' idle positions;
- The battery capacity and charging speed have no impact in the movement of the electric vehicles;
- The intrazonal travel time to pick-up and deliver passengers is included in the overall travel time;
- The intrazonal pick-up and delivery time is obtained by simulating as many requests as there is vehicle capacity;
- The error in travel time between modeled and real interzonal movements is assumed to be negligible;
- Vehicles are able to relocate without any occupant.

### 2.2. Mathematical formulation

Based on the assumptions defined above, we consider a two-dimensional time-space network (see Figure 2) and build an IP model that optimizes the SAV system movements by using flow variables instead of individual routing variables.

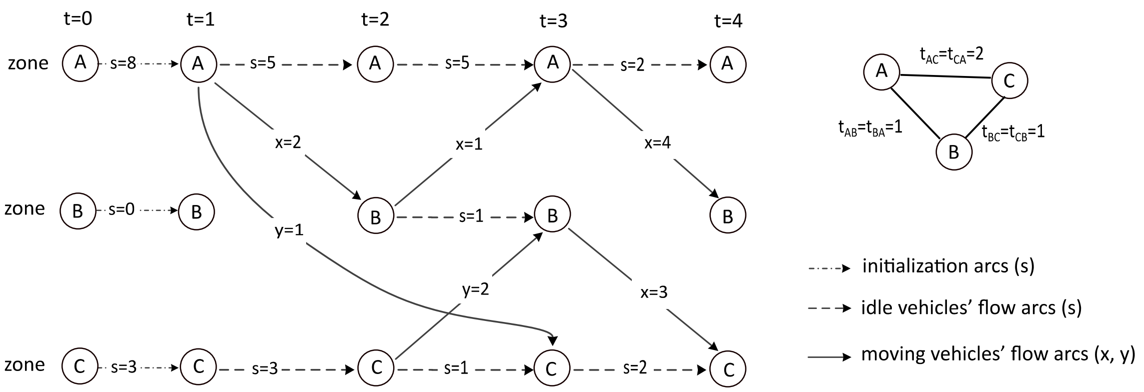



*Figure 2: Representation of vehicle flows in a two-dimensional time-space network for a single vehicle type*

A set of time instants $I = \{0, \dots, t, \dots, T\}$, discretizing time in steps, and a set of spatial nodes $N = \{1, \dots, i, \dots, S\}$, representing the considered zones, define the time-space network. The time step size represents the interval between system state updates. A set of vehicle types $Z = \{1, \dots, b, \dots, B\}$ allows to independently trace the flows of different categories of vehicles (e.g.: car and minibus). For this mathematical formulation, a vehicle type is a vehicle categorization considering the following characteristics: seat capacity, travel time, driving cost, and daily cost (the daily cost includes, for example, depreciation and maintenance costs).

Subsets of arcs are defined for each vehicle type $b \in Z$. The subset $A_1^b = \left\{\dots, \left(i_t, j_{t+t_{u_{ij}}^b}\right), \dots\right\}$ represents the arcs that connect zone $i$ at time instant $t$ and zone $j$ at time instant $t + t_{u_{ij}}^b$ with $t > 0$, being $t_{u_{ij}}^b$ the travel time moving users between zones $i$ and $j$ using a vehicle type $b$; subset $A_2^b = \left\{\dots, \left(i_t, j_{t+t_{r_{ij}}^b}\right), \dots\right\}$ represents the arcs that connect zone $i$ at time instant $t$ and zone $j$ at time instant $t + t_{r_{ij}}^b$ with $t$>0, being $t_{r_{ij}}^b$ the relocation time between zones $i$ and $j$ using a vehicle type $b$; and subset $A_3^b = \{\dots, (i_t, i_{t+1}), \dots\}$ represents the arcs that connect zone $i$ at time instant $t$ and zone $i$ at time instant $t$+1 for vehicle type $b$.

Decision variables:

$x_b\left(i_t, j_{t+t_{u_{ij}}^b}\right)$ number of vehicles of type $b$ moving with users from zone $i$ at time instant $t$ to zone $j$ at time instant $t$+$t_{u_{ij}}^b$;

$y_b\left(i_t, j_{t+t_{r_{ij}}^b}\right)$ number of vehicles of type $b$ relocating from zone $i$ at time instant $t$ to zone $j$ at time instant $t$+$t_{r_{ij}}^b$;

$s_b(i_t, i_{t+1})$ number of idle vehicles of type $b$ at zone $i$ from time instant $t$ to time instant $t$+1;

$v_b$ total number of vehicles of type $b$ that comprise the fleet.

Parameters:

$D(i_t, j)$ demand expressed in number of users at zone $i$ and time $t$ that want to go to $j$;

$t_{u_{ij}}^b$ travel time moving users by a vehicle of type $b$ between zones $i$ and $j$ including pick-up time and delivery time;

$t_{u_{ij}}^b = t_{pd_i}^b + t_{ij}^b + t_{pd_j}^b$, being $t_{ij}^b$ the interzonal travel time of vehicle of type $b$ between the representative nodes of zones $i$ and $j$, $t_{pd_i}^b$ the pick-up or delivery time (in this case pick-up) inside zone $i$ using a vehicle of type $b$, and $t_{pd_j}^b$ the pick-up or delivery time (in this case delivery) inside zone $j$ using a vehicle of type $b$, following the considerations of section 2.1.

$t_{r_{ij}}^b$ relocation time of vehicle of type $b$ between zones $i$ and $j$;

$t_{r_{ij}}^b = t_{ij}^b$ (as explained in section 2.1);
(different travel time matrices for $t_{u_{ij}}^b$ and $t_{r_{ij}}^b$ considering different periods of the day, can be used to contemplate traffic congestion effects);

$m_b$ seat capacity of a type $b$ vehicle;

$c_u^b(i, j)$ cost of a vehicle of type $b$ moving with users from zone $i$ to zone $j$, which includes the fuel/energy cost and infrastructure fees (e.g.: toll or congestion fees) for movements with users incorporating pick-up and drop-off;

$c_r^b(i, j)$ relocation cost from zone $i$ to zone $j$ for a vehicle of type $b$, which includes the fuel/energy cost and infrastructure fees (e.g.: toll or congestion fees) for relocation movements;

$c_v^b$ daily cost of a vehicle of type $b$, which includes license/registration fees, depreciation, and vehicle operating costs, such as: cleaning, and maintenance;

$p(i,j)$ price to transport one user from zone $i$ to zone $j$.

The model is formulated as follows:

$$\max(\Pi) = \sum_{(i_t,j)} p(i,j) \cdot D(i_t,j) - \sum_{b \in Z} \sum_{\left(i_t, j_{t+t_{u_{ij}}^b}\right) \in A_1^b} c_u^b(i,j) \cdot x_b\left(i_t, j_{t+t_{u_{ij}}^b}\right) - \sum_{b \in Z} \sum_{\left(i_t, j_{t+t_{r_{ij}}^b}\right) \in A_2^b} c_r^b(i,j) \cdot y_b\left(i_t, j_{t+t_{r_{ij}}^b}\right) - \sum_{b \in Z} c_v^b \cdot v_b \tag{1}$$

Subject to:

$$s_b(i_{t-1}, i_t)_{(i_{t-1}, i_t) \in A_3^b} + \sum_{\left(j_{t-t_{u_{ij}}^b}, i_t\right) \in A_1^b} x_b\left(j_{t-t_{u_{ij}}^b}, i_t\right) + \sum_{\left(j_{t-t_{r_{ij}}^b}, i_t\right) \in A_2^b} y_b\left(j_{t-t_{r_{ij}}^b}, i_t\right) - s_b(i_t, i_{t+1})_{(i_t, i_{t+1}) \in A_3^b} - \sum_{\left(i_t, j_{t+t_{u_{ij}}^b}\right) \in A_1^b} x_b\left(i_t, j_{t+t_{u_{ij}}^b}\right) - \sum_{\left(i_t, j_{t+t_{r_{ij}}^b}\right) \in A_2^b} y_b\left(i_t, j_{t+t_{r_{ij}}^b}\right) = 0, \forall i_t \in V | t > 0, \forall b \in Z \tag{2}$$

$$D(i_t, \mathrm{j}) \leq \sum_{b \in Z} m_b \cdot x_b\left(i_t, j_{t+t_{u_{ij}}^b}\right), \forall \left(i_t, j_{t+t_{u_{ij}}^b}\right) \in A_1^b \tag{3}$$

$$\sum_{i \in N} s_b(i_0, i_1) = v_b, \forall b \in Z \tag{4}$$

$$x_b\left(i_t, j_{t+t_{u_{ij}}^b}\right) \in \mathbb{N} \cup \{0\}, \forall \left(i_t, j_{t+t_{u_{ij}}^b}\right) \in A_1^b, \forall b \in Z \tag{5}$$

$$y_b\left(i_t, j_{t+t_{r_{ij}}^b}\right) \in \mathbb{N} \cup \{0\}, \forall \left(i_t, j_{t+t_{r_{ij}}^b}\right) \in A_2^b, \forall b \in Z \tag{6}$$

$$s_b(i_t, i_{t+1}) \in \mathbb{N} \cup \{0\}, \forall (i_t, i_{t+1}) \in A_3^b, \forall b \in Z \tag{7}$$

$$v_b \in \mathbb{N} \cup \{0\}, \forall b \in Z \tag{8}$$

The objective function (1) maximizes the total daily profit, a function of the revenues and costs. The revenues result from charging a price to users, while the costs come from vehicle movements and fleet fixed expenses. Constraint (2) ensures the conservation of vehicle flows in each time-space node for the different types of vehicles $b \in Z$. In other words, for all time-space nodes $(i,t)$, located after the initialization arcs $(t > 0)$, the vehicle flows entering a node need to be equal to the flows leaving the same node (see Figure 2). Constraint (3) associates requests with vehicle movements and guarantees that the number of persons transported by the vehicles does not surpass its capacity. This constraint simplifies the mathematical problem transforming demand into vehicle flows, therefore the model loses track of how many passengers travel inside each vehicle. Consequently, the assumption in 2.1 referring that "the intrazonal pick-up and delivery time is obtained by simulating as many requests as there is vehicle capacity" is used to guarantee that the travel time is always enough, on the conservative side, for any possible number of passengers. Constraint (4) sets the number and initial position of the vehicles for any type $b \in Z$, using the initialization arcs represented in Figure 2. Expressions (5) to (8) are the variables' domain constraints. Having the demand picked up at the time step of departure, as defined by constraint (3), does not mean that users are picked up at that exact instant. The number of vehicles represented by a certain flow will have, in reality, different departure times, anticipating or delaying their departure in relation to the time step considered. The modeled flow value starting at a certain time step is the representative flow of vehicles departing around the time that the system state is updated. Additionally, the size of the time steps is directly related to the number of requests to be served at each time instance, which is associated with the vehicle pooling efficacy in the model (a smaller time step size means a lower interval of time to accumulate demand requests to be served on the next time instant, and, therefore, lower chances of pooling).

Additional constraints, expressions (9) to (13), are included in some experiments, allowing the optimization model to decide which zones are worth exploring from a profit point of view. For that, the variable $D(i_t, j)$ should be replaced in the previous model by expression (9), where $a(i, j)$ is an auxiliary variable whose values are defined by expressions (10) to (12), being $M$ a big number and $r(i)$ a binary variable indicating if zone $i$ is included (value one) or not (value zero) in the system, as defined by expression (13). $D'(i_t, j)$ represents the demand data (constant) in number of users at zone $i$ and time $t$ that want to go to zone $j$, before applying the binary decision correspondent to the value of $a(i, j)$.

$$D(i_t, j) = D'(i_t, j) \times a(i, j) \tag{9}$$

$$a(i, j) \leq r(i), \forall i, j \in N \tag{10}$$

$$a(i, j) \leq r(j), \forall i, j \in N \tag{11}$$

$$a(i, j) \geq r(i) - M(1 - r(j)), \forall i, j \in N \tag{12}$$

$$r(i) \in \{0,1\} \tag{13}$$

### 2.3. Model size and computational speed

Model (1) - (8) when applied to only one type of vehicle (e.g.: $Z = \{\text{car}\}$) includes three sets of decision variables related to vehicle movements ($x_b$, $y_b$, and $s_b$), plus one decision variable related to fleet size ($v_b$). The number of decision variables related to sets $x_b$ and $y_b$ is equal to $2 \times (S^2 - S) \times (T^2 - \sum_{t=1}^{T} t)$. The number of variables is identical, justifying the multiplication by 2 to consider both sets. The subtraction of $S$ from $S^2$ is because we do not consider movements inside the same zone. And, the subtraction of $\sum_{t=1}^{T} t$ from $T^2$ is due to the fact that there are no trips occurring from time $t_2$ to $t_1$, being $t_2 > t_1$. The number of decision variables related to $s_b$ is $S \times (T - 1)$. The total number of decision variables is then equal to $2(S^2 - S)(T^2 - \sum_{t=1}^{T} t) + S(T - 1) + 1$. This number increases in proportion to the number of different types of vehicles adopted. For instance, considering two types of vehicles (e.g.: $Z = \{\text{car}, \text{minibus}\}$) doubles the number of decision variables. When the additional constraints to turn on and off zones are included, expressions (9) to (13), model (1) - (8) gains two new sets of variables: $r(i)$ and $a(i, j)$. The set of $r(i)$ has $S$ variables and the set of $a(i, j)$ has a size of $[S^2 - S]$, leading to a total increase in the number of variables of $S^2$. In this work, the zone constraints were just applied to the model with one vehicle type.

The solving speed was assessed by applying different demand levels to a network with 19 zones and 72 time steps (similar to the case study presented in this work). A laptop computer with an Intel Core i5 processor with 1.70GHz processing speed, a Solid-State Drive, and 10GB of RAM was used to run Fico Express solver. We set a maximum time limit of 10 thousand seconds to find an optimal solution. If an optimal solution has not been found within the time limit, then the current best feasible solution is accepted. Three variations of the model were tested: model (1) - (8) with only one vehicle type, model (1) - (13) with only one vehicle type, and model (1) - (8) with two vehicle types. Model (1) - (8) applied to only one type of vehicle reaches the optimal solution in an average of 11 seconds, considering all demand levels.

For model (1) - (13) applied to one vehicle type, the time to reach an optimal solution increases to an average of 45 seconds, considering all demand levels. This is a result of the referred increment of $S^2$ variables. Model (1) - (8) when applied to two vehicle types, doubling the size of the problem, is the most critical considering solving time. For this model variation, an optimal solution is not always achieved within the time limit (see Table 1). Having to decide the fleet sizes of two types of vehicles and corresponding movements expands the solution space, making it more difficult for the branch-and-cut algorithm used by Xpress to find the optimum, even though it rapidly converges to a gap under 1%. The referred difficulty increases with the number of trip requests.

*Table 1: optimal and feasible solutions' characteristics obtained for a fleet with two vehicle types*

| *Demand level (%)* | *Time to reach an optimal solution (s)* | | |
|---|---|---|---|
| *1* | *9* | | |
| *3* | 22 | | |
| *5* | *434* | | |
| | First solution with gap<1% | | Gap of accepted solution at 10,000s (%) |
| | *time (s)* | *gap (%)* | |
| *10* | 7634 | 0.44 | 0.41 |
| *15* | *4435* | *0.28* | *0.20* |
| *25* | 1750 | 0.79 | 0.79 |
| *50* | *117* | *0.87* | *0.74* |
| *75* | 126 | 0.61 | 0.42 |
| *100* | *76* | *0.55* | *0.33* |

## 3. Case study

The case study used in this paper for testing the flow-based approach is the Coimbra region, a level 3 NUTS[1] region of the Portuguese territory with 19 municipalities (see Figure 3 and Table 2 for a list of the municipalities and its characteristics). This region, despite having a main city which is Coimbra, with 143,396 inhabitants (INE, 2011), it has a polycentric type of occupation with significant distances between the different smaller cities. The system of cities does not have for most OD pairs enough demand to support building major infrastructure projects such as suburban rail or metro systems, making the inhabitants more dependent on their private cars, especially for interurban trips. The introduction of an interurban SAV system in such a region can potentially upgrade the transport system, improving quality and accessibility at a fair cost.

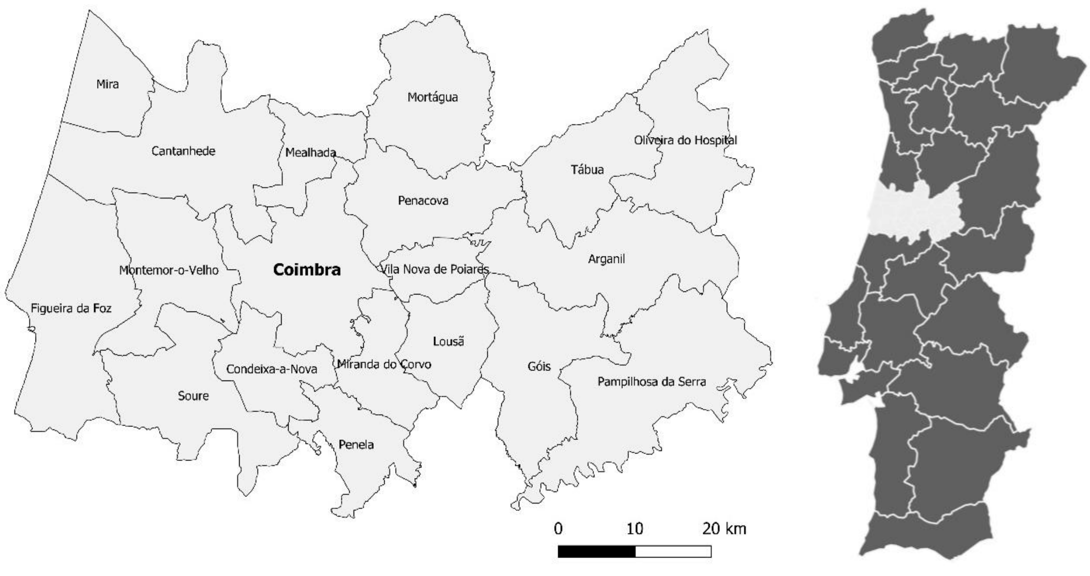



*Figure 3: Geographical representation of the Coimbra region and its municipalities*

[1] The NUTS classification is a hierarchical system for dividing the economic territory of the European Union.

In this study, we assess the implementation of a SAV service that transports passengers between municipalities. The intra-municipal trips are assumed to be well served by the local public transportation modes of each city and are not covered by the SAV service. The addition of intra-municipal trips would require a more complex approach since urban transportation modes and congestion effects would have to be modeled to account for the existence of competition with public transport in several municipalities of the region (e.g.: public transport, bicycle, walk) and the possible disruptions in traffic flow inside dense urban areas. To test the potential of establishing such a SAV intercity system in the Coimbra region, we use the intermunicipal trip demand data retrieved from the survey TIS.pt (2009) which is still the most complete one ever done in the region. The mobility patterns provided by this survey are designated as "current situation", and constitute a reference to the comparisons drawn in section 4.

*Table 2: statistical data: population, density and daily trips for Coimbra region* (INE, 2011), (TIS.pt, 2009)

| *Municipality* | *Population (inh)* | *Density (inh/km2)* | *# of parishes* | *Intermunicipal trips* using MT** as origin (#trips)* | *Intermunicipal trips* using MT** as destination (#trips)* |
|---|---|---|---|---|---|
| *Arganil* | 12145 | 36.5 | 14 | 2959 | 2916 |
| *Cantanhede* | 36595 | 93.6 | 14 | 9664 | 9727 |
| *Coimbra* | 143396 | 449.0 | 18 | 41530 | 41107 |
| *Condeixa-a-Nova* | 17078 | 123.2 | 7 | 6088 | 6096 |
| *Figueira da Foz* | 62125 | 163.9 | 14 | 8408 | 8389 |
| *Góis* | 4260 | 16.2 | 4 | 713 | 689 |
| *Lousã* | 17604 | 127.2 | 4 | 6095 | 6133 |
| *Mealhada* | 20388 | 184.6 | 6 | 7157 | 7307 |
| *Mira* | 12465 | 100.5 | 4 | 3185 | 3317 |
| *Miranda do Corvo* | 13098 | 103.6 | 4 | 5990 | 6049 |
| *Montemor-o-Velho* | 26171 | 114.3 | 11 | 5914 | 5875 |
| *Mortágua* | 9531 | 38.2 | 7 | 673 | 673 |
| *Oliveira do Hospital* | 20855 | 88.9 | 16 | 1843 | 1902 |
| *Pampilhosa da Serra* | 4481 | 11.3 | 8 | 535 | 535 |
| *Penacova* | 15251 | 70.4 | 8 | 5821 | 5843 |
| *Penela* | 5983 | 44.4 | 4 | 732 | 732 |
| *Soure* | 19245 | 72.6 | 10 | 4106 | 4240 |
| *Tábua* | 12071 | 60.4 | 11 | 1878 | 1878 |
| *Vila Nova de Poiares* | 7281 | 86.2 | 4 | 2888 | 2771 |
| *Total* | 460023 | -- | 168 | 116179 | 116179 |

**origin and destination inside the Coimbra region **MT - Motorized transport*

According to the survey, the Coimbra region has 116,179 intermunicipal daily trips (with origin and destination inside the region) performed by motorized transport. In terms of aggregated distance, this corresponds to 3.71 million kilometers. The hourly distribution of the demand is represented in Figure 4.

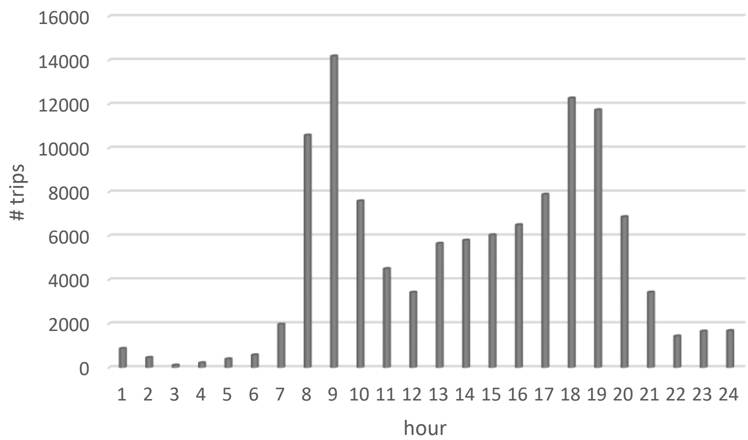
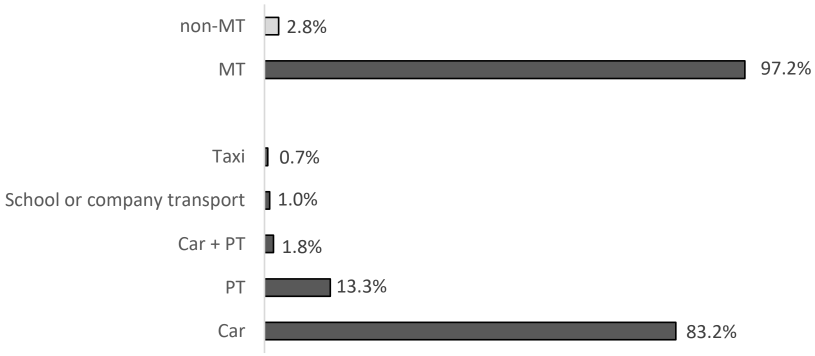



*Figure 4: hourly distribution of motorized transport trips inside the Coimbra region* (data from TIS.pt, 2009)

Regarding modal share, the car and public transport are used in 83.2% (96635 trips) and 13.3% (15462 trips) of the motorized transport trips, respectively. The remaining modes (school or company transport, taxi, and car plus public transport) represent only 3.5% of the motorized transport share (see Figure 5).




(MT - *Motorized transport*; PT - *Pubic transport*)

*Figure 5: Intermunicipal trips mode shares inside the Coimbra region* (data from TIS.pt, 2009)

If we consider that all the trips are commuting movements, we have around 48.3 thousand car commuting trips, and 7.7 thousand public transport commuting trips per day. Assuming a car occupancy rate of 1.5 passengers, we can estimate having more than 32 thousand private vehicles moving per day to satisfy the private vehicle demand share.

The Coimbra region was modeled using its administrative division (according to the statistical and surveyed data available), being the municipalities the modeled zones and parishes the subzones. Centroids were used as the representative nodes for both territorial entities. The distance between municipal centroids is in average 53km, being the minimum observed distance 12km and the maximum 137km (equivalent to crossing the region diagonally). The motorized trip data, retrieved from TIS.pt (2009) survey, was used to generate the demand between municipalities $D(i_t, j)$. It needs to be noted that the intention of using SAV in the future can be different than the current demand for motorized modes, though we use this demand as a basis to study the SAV application scenarios. Since we are not modeling the behavior of the travelers to assess the intention of using SAV, different levels of demand were generated to understand its influence on system design and financial impacts. For this purpose, a Poisson process was used with rates equal to 1, 3, 5, 10, 15, 25, 50, 75, and 100% of the hourly trip values observed in the survey (the process was applied to each hour of the 24-hour period for each origin and destination pair). The nominal value of the rate applied to the Poisson process designates the demand data scenario, for instance, the 25% demand level scenario resulted from applying a rate equal to 25%. It was considered that the introduction of driverless vehicles does not have an impact on travel speed values, remaining the same as today. The travel time between centroids, $t_{ij}$, was obtained selecting the shortest travel times between the municipality centers using Google Maps (Google, 2019). The travel time values are assumed to be constant throughout the day, since congestion is not significant in this region, namely for the considered routes. The pick-up and delivery times for each municipality $i$, $t_{pd_i}$, were calculated using the process described in 2.1 and the parishes' population values were retrieved from the bureau of statistics (INE, 2011). The total travel time values obtained using the simplification described in section 2.1 are on average 11% higher than the real ones, with a standard deviation of 7%. This difference is not important since it can be eliminated by the discretization of time in time steps.

In what respects to vehicles, we considered the use of electric technology, and two options were analyzed: car and minibus. Two models available in the market of known brands on each segment were chosen in order to get realistic values concerning price and performance (e.g.: vehicle price, seat capacity, battery capacity, engine consumption). The current model Renault Zoe with a 52kWh battery (Renault, 2019), a fully electric not driverless vehicle, was used as a reference to what could be the cost of a driverless car in the future. The capacity considered is 4 seats to comfortably transport passengers. This car model costs today about 24 thousand Euro (for companies, the price includes the battery), consumes 10-23 kWh/100km, and has a real autonomy of 300km. Considering that the vehicle in interurban trips consumes 20kWh to perform 100km, and that a unit of kWh costs 0.20 Euro (Observador, 2019), the energy cost is roughly 4€/100km, that is, 0.04€/km. In terms of depreciation costs, a new car loses 60% of its value in the first 3 years (TheAA.com, 2019). Considering a depreciation of 20% per year, we get a depreciation value per vehicle of 13€/day. Adding to this cost 7€/day for maintenance, yearly tax, insurance, and cleaning, the total daily cost of each car unit considered in this analysis is 20€.

For the minibus, we considered the Iveco Daily electric minibus with a 91kWh battery as a reference. This model has a capacity of 16 passengers, range of 250km, and costs today about 68 thousand Euro (Ansell, 2019). Considering the consumption of 36kWh per 100 km, the energy cost is approximately 7€/100km, that is, 0.07€/km (using the same kWh unit cost as for the car). The depreciation value is 37€/day, considering a 20% depreciation value per year for the first three years. Assuming a cost for maintenance, tax, insurance, and cleaning equal to 13€/day, the total cost per day for each minibus unit is 50€/day.

Since we are testing a system with electric vehicles, it is assumed that the adopted charging technology in the region is good enough to assure that vehicles can charge the energy needed to power their movements when idle and that there are no limitations of available charging sockets in each municipality. These conditions are necessary to assure the validity of the application of the optimization model (since there are no energy and battery-related constraints).

The costs of a vehicle moving with users and relocating are computed using the energy cost per kilometer and distances traveled (It is considered that no infrastructure usage fees are charged to the system provider). The distances driven with users include pick-up and delivery. The reference price charged per vehicle occupant is 0.10€/km, which is similar to the price charged by the bus company that nowadays serves the intermunicipal trips in the Coimbra region (Transdev, 2020). The price that was used is at the lower bound (normally associated with ridesharing) of the range of prices considered by other authors for SAV services, $0.11 (0.09€) to $1.03 (0.87€) per kilometer (Narayanan et al., 2020). To calculate the price for each origin and destination pair, we used the distance between the respective centroids.

For this study, we considered three different SAV systems:

1. A system with a fleet of cars;
2. A system with a fleet of minibuses;
3. A system with a mixed fleet of vehicles (cars and minibuses).

The optimization model was applied to different demand levels for each SAV system configuration and 20 minutes time steps (an acceptable value concerning the scope of this study). Additionally, we ran the model with the added constraints that allow the identification of which zones should be part of the system for the one-vehicle-type fleets (fleet of cars and fleet of minibuses).

## 4. Results

This section presents the analysis of the IP model results and the financial viability assessment for an interurban SAV system in the case study region.

### 4.1. Profitability of fleet configurations and optimal coverage

Analyzing the results of the three scenarios (only cars, only minibuses, and a mixed fleet of both) considering that all zones must be served, it can be seen that there are losses for low demand levels (see Table 3). This is due to the low and sprawled demand in the region, which affects the number of trips served per vehicle. The transition from losses to a profit happens for a demand level between 3% and 5% for both SAV car fleet and SAV mixed fleet (the referred transition can be observed in Table 3, though it is graphically unnoticeable in Figure 6), and for a demand level between 15% and 25% for a SAV minibus fleet (see Figure 6). In terms of profit, having a mixed fleet for the intermunicipal trips in the region of Coimbra is always better than a one-vehicle-type fleet. By analyzing the one-vehicle-type fleets, we verify that the car fleet leads to more profit for low demand levels, while the minibus leads to higher profit for higher levels of demand, being the turnover between 50% and 75% of demand.

*Table 3: Optimal results considering all municipalities covered by the SAV service*

| *Demand levels (%)* | *1* | *3* | *5* | *10* | *15* | *25* | *50* | *75* | *100* |
|---|---|---|---|---|---|---|---|---|---|
| | CAR FLEET | | | | | | | | |
| *Profit (€/day)* | -975 | -323 | 2144 | 10975 | 21218 | 41036 | 89921 | 138306 | 187214 |
| *fleet size* | 54 | 218 | 353 | 615 | 834 | 1303 | 2429 | 3595 | 4717 |
| *Revenues (€/day)* | 1051 | 8845 | 17190 | 36638 | 55804 | 93211 | 185446 | 278606 | 371466 |
| *Total costs (€/day)* | 2026 | 9168 | 15046 | 25662 | 34586 | 52174 | 95525 | 140300 | 184252 |
| *- costs moving users (€/day)* | 774 | 4475 | 7516 | 12704 | 16876 | 24516 | 43778 | 63637 | 83516 |
| *- relocation costs (€/day)* | 171 | 333 | 470 | 658 | 1030 | 1598 | 3166 | 4763 | 6396 |
| *- vehicle daily costs (€/day)* | 1080 | 4360 | 7060 | 12300 | 16680 | 26060 | 48580 | 71900 | 94340 |
| *Total trips served /day* | 313 | 2849 | 5454 | 11510 | 17486 | 29144 | 57926 | 87165 | 116050 |
| *Trips per vehicle /day* | 5.8 | 13.1 | 15.5 | 18.7 | 21.0 | 22.4 | 23.8 | 24.2 | 24.6 |
| *Average passengers /vehicle* | 1.2 | 1.8 | 2.0 | 2.5 | 2.9 | 3.3 | 3.6 | 3.8 | 3.8 |
| *Total relocations /day* | 99 | 242 | 325 | 485 | 736 | 1139 | 2216 | 3404 | 4516 |
| *Relocations per vehicle /day* | 1.8 | 1.1 | 0.9 | 0.8 | 0.9 | 0.9 | 0.9 | 0.9 | 1.0 |
| *Time moving users (%)* | 23.9 | 34.7 | 36.1 | 35.1 | 34.3 | 31.9 | 30.6 | 30.0 | 30.0 |
| *Relocation time (%)* | 9.0 | 4.6 | 3.9 | 3.3 | 3.8 | 3.7 | 3.9 | 4.0 | 4.1 |
| *Idle time (%)* | 67.0 | 60.7 | 60.0 | 61.6 | 61.9 | 64.3 | 65.5 | 66.0 | 65.9 |
| | MINIBUS FLEET | | | | | | | | |
| *Profit (€/day)* | -4945 | -15307 | -18040 | -12876 | -1791 | 24189 | 89426 | 151307 | 213627 |
| *fleet size* | 72 | 251 | 352 | 488 | 576 | 706 | 1022 | 1390 | 1740 |
| *Revenues (€/day)* | 1051 | 8845 | 17190 | 36638 | 55804 | 93211 | 185446 | 278606 | 371466 |
| *Total costs (€/day)* | 5995 | 24152 | 35230 | 49513 | 57595 | 69022 | 96020 | 127299 | 157839 |
| *- costs moving users (€/day)* | 2173 | 11256 | 17286 | 24716 | 28356 | 33067 | 43792 | 56263 | 68796 |
| *- relocation costs (€/day)* | 223 | 346 | 344 | 398 | 440 | 655 | 1128 | 1536 | 2043 |
| *- vehicle daily costs (€/day)* | 3600 | 12550 | 17600 | 24400 | 28800 | 35300 | 51100 | 69500 | 87000 |
| *Total trips served /day* | 313 | 2849 | 5454 | 11510 | 17486 | 29144 | 57926 | 87165 | 116050 |
| *Trips per vehicle /day* | 4.3 | 11.4 | 15.5 | 23.6 | 30.4 | 41.3 | 56.7 | 62.7 | 66.7 |
| *Average passengers /vehicle* | 1.2 | 1.9 | 2.4 | 3.4 | 4.6 | 6.5 | 9.8 | 11.6 | 12.6 |
| *Total relocations /day* | 70 | 145 | 152 | 174 | 196 | 280 | 467 | 657 | 834 |
| *Relocations per vehicle /day* | 1.0 | 0.6 | 0.4 | 0.4 | 0.3 | 0.4 | 0.5 | 0.5 | 0.5 |
| *Time moving users (%)* | 28.4 | 42.9 | 47.1 | 48.7 | 47.3 | 44.9 | 41.1 | 38.9 | 38.0 |
| *Relocation time (%)* | 7.7 | 3.7 | 2.6 | 2.2 | 2.1 | 2.5 | 3.0 | 3.0 | 3.2 |
| *Idle time (%)* | 63.9 | 53.4 | 50.3 | 49.2 | 50.6 | 52.5 | 55.9 | 58.1 | 58.9 |
| | MIXED FLEET | | | | | | | | |
| *Profit (€/day)* | -975 | -246 | 2801 | 13297 | 25166 | 48850 | 108910 | 169332 | 230203 |
| *cars fleet size* | 54 | 188 | 265 | 350 | 412 | 497 | 591 | 644 | 719 |
| *minibus fleet size* | 0 | 10 | 24 | 72 | 121 | 234 | 549 | 891 | 1212 |
| *Revenues (€/day)* | 1051 | 8845 | 17190 | 36638 | 55804 | 93211 | 185446 | 278606 | 371466 |
| *Total costs (€/day)* | 2026 | 9091 | 14389 | 23341 | 30638 | 44361 | 76536 | 109274 | 141263 |
| *- costs moving users (€/day)* | 774 | 4485 | 7416 | 12179 | 15676 | 21813 | 35850 | 49795 | 63633 |
| *- relocation costs (€/day)* | 171 | 346 | 473 | 562 | 672 | 908 | 1416 | 2049 | 2650 |
| *- vehicle daily costs (€/day)* | 1080 | 4260 | 6500 | 10600 | 14290 | 21640 | 39270 | 57430 | 74980 |
| *Total trips served /day* | 313 | 2849 | 5454 | 11510 | 17486 | 29144 | 57926 | 87165 | 116050 |
| *Trips per car/day* | 5.8 | 14.2 | 17.8 | 24.1 | 26.9 | 28.3 | 29.3 | 29.8 | 28.5 |
| *Trips per minibus /day* | 0 | 17.6 | 30.7 | 42.5 | 53.1 | 64.4 | 73.9 | 76.3 | 78.8 |
| *Average passengers /car* | 1.2 | 1.7 | 1.9 | 2.3 | 2.5 | 2.9 | 3.4 | 3.6 | 3.7 |
| *Average passengers /minibus* | 0 | 8.4 | 13.1 | 13.3 | 13.5 | 13.7 | 14.3 | 14.7 | 14.9 |
| *Total relocations /day* | 99 | 233 | 331 | 359 | 396 | 469 | 618 | 889 | 1119 |
| *Relocations per car /day* | 1.8 | 1.2 | 1.2 | 0.8 | 0.6 | 0.5 | 0.2 | 0.2 | 0.2 |
| *Relocations per minibus /day* | 0 | 0.1 | 0.5 | 0.9 | 1.2 | 1.0 | 0.9 | 0.8 | 0.8 |
| *Time moving users (%)* | 24.0 | 37.7 | 42.1 | 45.9 | 44.7 | 42.5 | 40.0 | 38.6 | 37.7 |
| *Relocation time (%)* | 9.0 | 5.2 | 4.8 | 3.9 | 3.6 | 3.4 | 3.3 | 3.6 | 3.6 |
| *Idle time (%)* | 67.1 | 57.2 | 53.0 | 50.2 | 51.7 | 54.0 | 56.7 | 57.8 | 58.6 |

For the demand level of 100%, the profit of having a mixed fleet is 23% higher than the one with the car fleet, and having a minibus fleet represents a 14% increase in profit when compared with the car fleet (see Table 3 and Figure 6). All in all, the estimated daily profits could reach 187 to 230 thousand Euro, serving the 116 thousand intermunicipal trips, being the most profitable solution the SAV mixed fleet followed by the minibus fleet and the car fleet, respectively. This represents an average profit of 1.61 to 1.98 Euros per passenger served. It is important to note that these profit values are a reference for design purposes and derive from assuming that all demand is served by the SAV system, which is, in this case, considered to be the only mode available for the intermunicipal trips. For more realistic values, all the modes should be considered and a mode choice model should be used. The mode choice model shall include the influence of traffic congestion on travel time, the service availability, and the different mode prices.

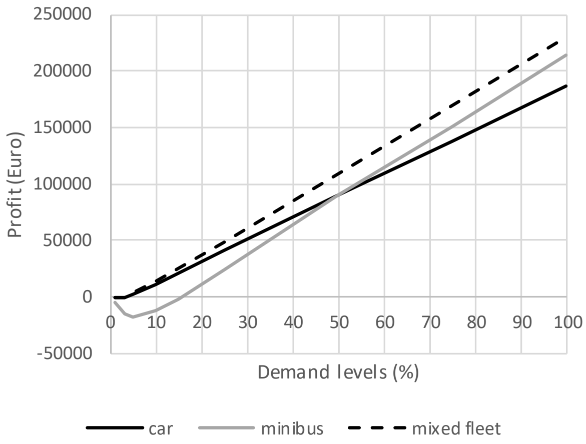



*Figure 6: Relation of daily profit with demand levels (serving all zones)*

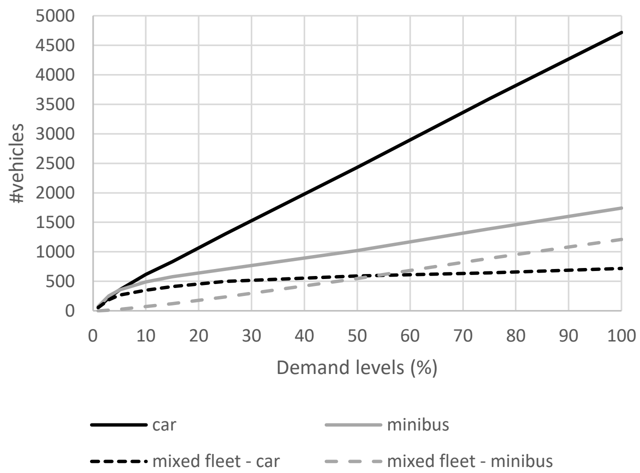



*Figure 7: Relation of number of vehicles with demand levels (serving all zones)*

Looking at the variation of the fleet size values for the different demand scenarios, it can be verified that the optimal fleet size increases at different rates for each fleet type (car, minibus, mixed), being the growth rates constant for demand levels above 10% (see Figure 7). At the referred demand level, the optimal solution comprises 615 vehicles for the car fleet and 488 vehicles for the minibus fleet. Along with the subsequent demand levels, the optimal fleet size increases at a linear rate of 45.7 for cars and 13.8 for minibuses per each percentual unit of demand added. The mixed fleet total number of vehicles (cars plus minibuses) has a constant growth rate of 16.6 vehicles per demand percentual unit growth, starting with 422 vehicles at 10% of demand. The share between cars and minibuses in the total number of vehicles follows an exponential trendline with a correlation coefficient of 0.99 (see Figure 8). As expected, the share of cars decreases as the demand level goes up, since higher capacity vehicles are more efficient. The cars start by representing 100% of the fleet (54 cars and no minibuses) for 1% demand level, and decrease to 37% (719 cars and 1212 minibuses) for the 100% demand level solution. The marginal share reductions of car over the minibus, in number of vehicles, per 1% demand increments are: 1.9% in the interval from 1% to 10% demand levels, 1.0% from 10% to 25%, 0.6% from 25% to 50%, 0.4% from 50% to 75%, and 0.2% from 75% to 100%. The car in the 100% demand level scenario is only used to satisfy the low demand movements. This flexibility of the mixed fleet leads to more trips being served per vehicle when compared to the car or bus fleets separately (see Table 3). As an example, for the 100% demand level scenario, the daily trips per car and per minibus for the mixed fleet are 28.5 and 78.8, respectively, while for the fleet of cars and fleet of minibuses separately, the trips per car are 24.6 and the trips per minibus are 66.7.

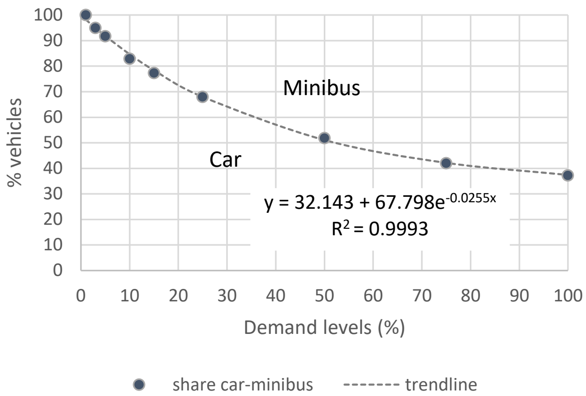



*Figure 8: optimal share between car and minibus for the mixed fleet*

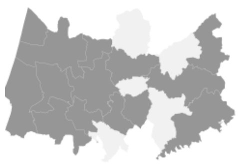
The optimal solutions for low demand levels lead to losses, as referred previously. This is because all demand requests are required to be fulfilled for the entire region. To investigate what would happen if we let the model choose the best combination of municipalities that leads to a more profitable service, we use the additional constraints presented in the above models (expressions 9 to 13). The additional constraints were only added to the one-vehicle-type fleets, and the main results are presented in Table 4 (the darker areas represent the municipalities with SAV service).

It can be observed that the model decides for no service in the Coimbra region for the 1% demand level in the case of a car fleet, and for the 1 and 3% demand levels in the case of a minibus fleet. For high demand levels, the model opts to serve all municipalities being the optimal solution equal to using the same model without the additional constraints (expressions 9 to 13).

The demand level scenarios between 3 and 15%, for the fleet of cars, and the ones between 5 and 50%, for the fleet of minibuses, have solutions with higher profit for configurations including just some of the municipalities, when compared to serving all municipalities. This can be related to the increase in the average number of passengers per vehicle resulting from the reduction of service coverage. For the car fleet, the difference in profit of having fewer municipalities covered by the SAV service is not that relevant, 1139 Euro per day for the 3% demand level scenario, and 58 Euro per day for the 15% demand level scenario (the presented values characterize the difference to serving all municipalities). The increase in vehicle costs related to the minibus fleet (when compared to the car fleet) makes the optimal solution of fewer covered municipalities more interesting for increasing profitability. This can be noted for the demand level scenarios of 5% and 10% where serving fewer municipalities leads to saving more than 18 and 15 thousand Euro per day, respectively, due mainly to the decrease in the fleet size and the increase in the number of average passengers per vehicle.

Note that to calculate the average number of passengers per vehicle for the mixed fleet (see Table 3), passengers were virtually distributed per vehicle type, being the smaller vehicles the first to be filled. The reason for this is that despite vehicle flows being expressed by vehicle type, the demand served is not, therefore this post-processing rule is necessary to determine the referred output result.

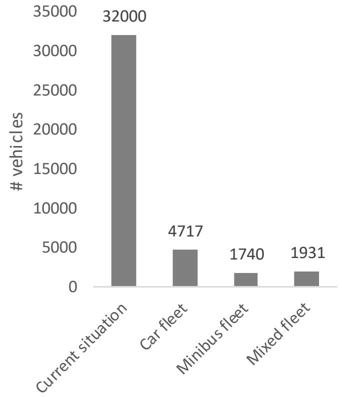
*Table 4: Optimal results using the additional constraints (expressions 9 to 13)*

| *Demand level (%)* | *Car fleet* | *Minibus fleet* |
|---|---|---|
| 1 | No service | No service |
| 3 |  Profit: 815€/day (+1139€) Vehicles: 88 Avp/v=1.9 Trips/v=15.8 | No service |
| 5 |  Profit: 3164€/day (+1021€) Vehicles: 239 Avp/v=2.2 Trips/v=18.4 |  Profit: 50€/day (+18091€) Vehicles: 14 Avp/v=3.2 Trips/v=21.1 |
| 10 |  Profit: 11231€/day (+256€) Vehicles: 570 Avp/v=2.5 Trips/v=19.5 |  Profit: 2594€/day (+15469€) Vehicles: 106 Avp/v=5.0 Trips/v=37.9 |
| 15 |  Profit: 21276€/day (+58€) Vehicles: 775 Avp/v=2.9 Trips/v=21.7 |  Profit: 7596€/day (+9387€) Vehicles: 191 Avp/v=5.8 Trips/v=42.1 |
| 25 | All municipalities<br>Profit: 41036€/day (+0€) Vehicles: 1303<br>Avp/v=3.3 Trips/v=22.4 |  Profit: 26056€/day (+1867€) Vehicles: 582 Avp/v=7.1 Trips/v=44.2 |
| 50 | All municipalities<br>Profit: 89921€/day (+0€) Vehicles: 2429<br>Avp/v=3.6 Trips/v=23.8 |  Profit: 89438€/day (+12€) Vehicles: 998 Avp/v=9.9 Trips/v=57.3 |
| 75 | All municipalities<br>Profit: 138306€/day (+0€) Vehicles: 3595<br>Avp/v=3.8 Trips/v=24.2 | All municipalities<br>Profit: 151307€/day (+0€) Vehicles: 1390<br>Avp/v=11.6 Trips/v=62.7 |
| 100 | All municipalities<br>Profit: 187214€/day (+0€) Vehicles: 4717<br>Avp/v=3.8 Trips/v=24.6 | All municipalities<br>Profit: 213627€/day (+0€) Vehicles: 1740<br>Avp/v=12.6 Trips/v=66.7 |

*(+ €) – difference relatively to the same scenario without using the additional constraints, expressions (9) to (13)*
Avp/v – *average passengers per vehicle*; Trips/v – *trips per vehicle /day*

## 4.2. Comparison with the current situation

The mobility patterns provided by the survey, hereby designated as “current situation", are the closest complete mobility data source available for the region and a reference for the following analysis. Comparing the 100% demand level scenario solutions with the current situation, it is estimated that each car in a SAV car fleet would be able to replace an average of 7 private cars, each minibus in a minibus SAV fleet would replace on average 18 private cars, and each vehicle in a SAV mixed fleet replaces on average 16 private cars (the number of cars in the current situation was estimated based on the assumption that all car intermunicipal trips in the survey dataset are commuter trips with an average vehicle occupancy rate of 1.5 passengers). This is a consequence of the ability of the shared system to improve vehicle usage, therefore decreasing vehicle idle time. For the current mobility situation, the average idle time is 93.6% (observed in the survey data), for the SAV car fleet the average value is 65.9%, for the SAV minibus fleet is 58.9%, and it is 58.6% for the SAV mixed fleet (see Figure 9). Important to note is that the increase in vehicle usage is fostered by relocations, which represent 3 to 4% of the vehicle time for the 100% demand level, and is limited by the demand peaks. Observing Figure 10, one can see that, even with the cushion effect made possible by relocations, more than half of the vehicles of the SAV car fleet are only used during the peaks. The values in this figure can differ for other scenarios with different system parameters, though the described behavior is recurrent and possibly observable in other networks since it is mainly related to the difference in demand between peak and off-peak (a result of having high levels of commuting trips) and the distance between the municipalities.



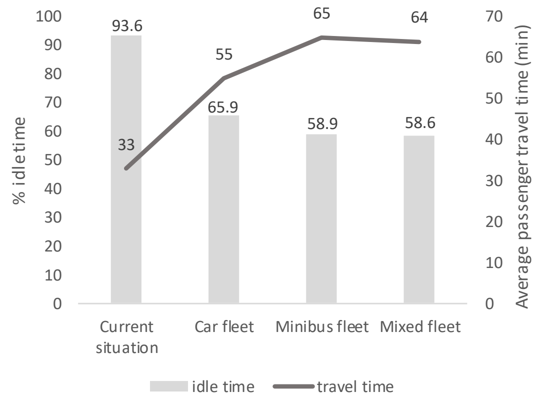




*Figure 9: Comparison with the current situation: number of vehicles, idle time and average passenger travel time*

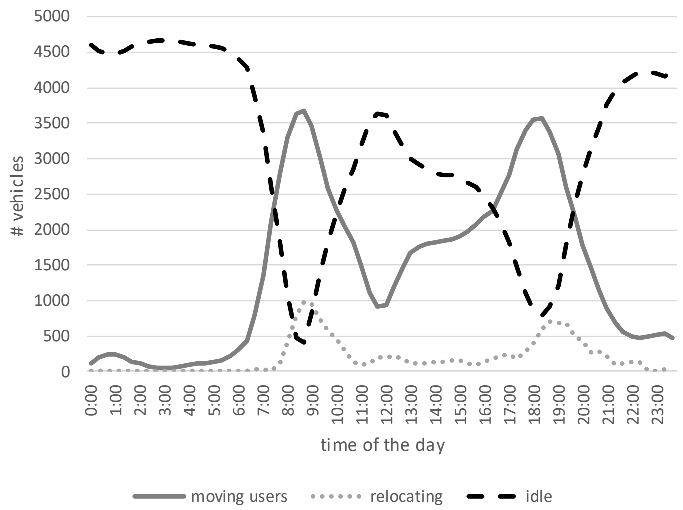



*Figure 10: Hourly distribution of the number of vehicles moving with clients, relocating and being idle of the SAV fleet of cars for the 100% demand level scenario.*

The described system improvements are sustained by a negative impact on passenger travel time values. The average trip duration experienced by passengers increases from 33 minutes for the current situation, to 55 minutes for the SAV car fleet, 65 minutes for the SAV minibus fleet, and 64 minutes for the SAV mixed fleet, representing an increment of 0.67, 0.97, and 0.94 times, respectively (see Figure 9). These increments are related to the sharing of vehicles, leading to an extension in trip duration due to picking-up operations at the zone of origin and the drop-off operations at the destination zone.

Note that the passenger average travel time was estimated by processing the optimal vehicle flows in order to differentiate the travel time experienced by each passenger. As a simplification, proportional weights were applied to pick-up and delivery travel time to emulate the time experienced by each passenger according to vehicle occupation in number of passengers ($k$) and vehicle capacity ($m$). For example, if a 4 seat vehicle is serving one request, the $t_{pd_i}$ is multiplied by a factor equal to zero on both origin and destination, since the vehicle goes directly from origin to destination without diverting its route; if the same vehicle is serving 2 requests, the vehicle needs to divert once at the origin and once at the destination, so one passenger can experience $0 \times t_{pd_i}$ and the other $\frac{1}{3} t_{pd_i}$ (since $t_{pd_i}$ was estimated for the number of passengers equal to capacity); if serving 3 requests, we have zero $t_{pd_i}$ for the first passenger, $\frac{1}{3} t_{pd_i}$ for the second passenger, and $\frac{2}{3} t_{pd_i}$ for the third passenger; for 4 requests, we have zero $t_{pd_i}$, $\frac{1}{3} t_{pd_i}$, $\frac{2}{3} t_{pd_i}$, and $\frac{3}{3} t_{pd_i}$. Expression (14) was defined, based on this arithmetic progression, to determine the accumulated passenger travel time, being $t_{ij}^{p}$ the travel time experienced by passenger $p$. The travel times $t_{ij}$, $t_{pd_i}$ and $t_{pd_j}$were defined in section 2.1.

$$\sum_{p=1}^{k} t_{ij}^{p} = k \times t_{ij} + \frac{k(k-1)}{2(m-1)}\left(t_{pd_i} + t_{pd_j}\right) \tag{14}$$

## 4.3. A fleet of cars carrying a maximum of one passenger

All the presented benefits are mostly due to sharing the vehicles seats, which is associated with the cost of increasing each passenger's travel time. We can argue that people are not willing to spend much more time in their commuter journeys in order to be able to leave their car at home. Moreover, there are no individual travel cost savings from carpooling with more or fewer people in the above scenarios. To understand people's sensitivity to these service parameters, one would need to perform a survey on the willingness to share and willingness-to-pay of the population of this intermunicipal community which is not the goal of this study.

*Table 5: Optimal results considering a fleet of cars carrying only one passenger*

| | ***All municipalities*** | ***The best profitable combination of municipalities*** |
|---|---|---|
| |  |  |
| *Profit (€/day)* | -134707 | 8341 |
| *Fleet size* | 13764 | 1357 |
| *Revenues (€/day)* | 371466 | 65096 |
| *Total costs (€/day)* | 506173 | 56755 |
| *- costs moving users (€/day)* | 197090 | 27330 |
| *- relocation costs (€/day)* | 33803 | 2286 |
| *- vehicle daily costs (€/day)* | 275280 | 27140 |
| *Total trips served /day* | 116050 | 16619 |
| *Trips per vehicle /day* | 8.4 | 12.2 |
| *Total relocations /day* | 22638 | 1950 |
| *Relocations per vehicle /day* | 1.6 | 1.4 |
| *Time moving users (%)* | 24.8 | 34.8 |
| *Relocation time (%)* | 4.3 | 2.9 |
| *Idle time (%)* | 70.9 | 62.2 |
| *Average passenger travel time (min)* | 32.8 | 33.2 |

To understand what would happen if vehicles were used by only one passenger at a time, meaning low willingness to share and spend time to pick-up other people, we ran the optimization model with a fleet of cars constraining their occupancy to only one passenger, using the same costs than those used for the car fleet and the 100% demand level scenario. In this case, there is no additional travel time related to pick-up and delivery of extra passengers, since there is only one passenger per trip. The optimal solution reveals losses of about 135 thousand Euro (see Table 5), which represents a loss of 1.2 Euro per passenger, for a fleet size of 13764 vehicles to support all requests. The profit difference is due to the increment in costs related to vehicle fixed daily costs (depreciation and maintenance) and vehicle usage (moving users and relocations). The number of vehicles is 2.9 times higher and the total costs from vehicle usage increased 2.6 times (see Table 3 and Table 5). Comparing this solution with the current situation for the Coimbra region, it can be seen that each SAV vehicle serving only one passenger at a time replaces 2 private cars for the intermunicipal trips, having a similar average passenger travel time of 33 minutes, though with a better idle time (70.9%).

To check if serving a reduced set of municipalities allow avoiding losses for a SAV car service carrying only one passenger at a time, we included the additional constraints (expressions 9 to 13) in the optimization model. The result was a profit for a combination of 10 municipalities, as can be seen in Table 5 (the darker areas represent the municipalities with SAV service). This combination of municipalities encompasses 14.3% of the intermunicipal trips and it does not include the municipality with the highest demand (Coimbra municipality). In order to sustain the high cost of having enough vehicles to transport each passenger at a time, the optimization model chooses a set of municipalities that cuts the demand peaks (see Figure 11). Smoothing the peaks leads to an increase in vehicle usage, resulting in a profit of around 8 thousand Euro. This shows that it is possible to serve the municipalities that have low demand, with a service level similar to private vehicles, maintaining a profit. Coincidently, these municipalities are those that have the poorest connections between each other which provides greater support to the implementation of such a system in the future.

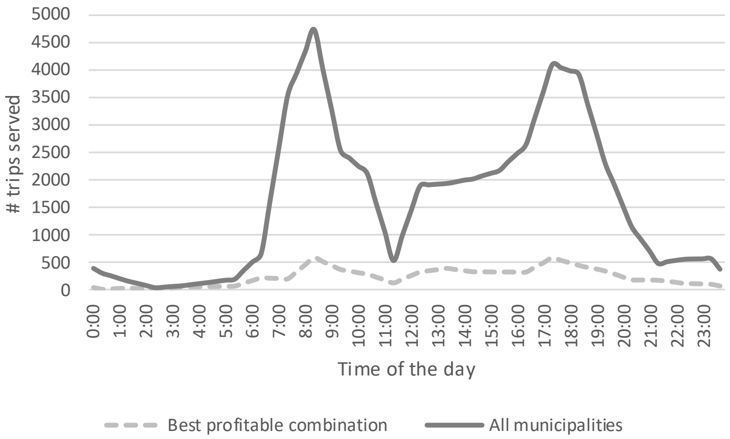



*Figure 11: Hourly distribution of trips served for the cases of best profitable combination and all municipalities*

### 4.4. Average passengers per vehicle and profit

It has been observed that the profit is directly proportional to the average number of passengers per vehicle, where some scenarios resulted in losses, while others in profit (see Table 3 and Table 4). The point of break-even can be analyzed using the previous outcomes. Looking at the variation of demand with full coverage (see Table 3), it can be seen that the fleet of cars break-even point is between 1.8 and 2.0 passengers per vehicle, and the minibus fleet has a break-even between 4.6 and 6.5 passengers per vehicle (approximately 4.7 using linear interpolation). The variation of demand with optimal coverage (see Table 4) leads to higher vehicle occupancy values than the full coverage for the same demand levels. The first results after the "no service" optimal decisions, which are the ones with profit, can reveal an upper limit for the break-even point. Hence, a possible upper limit for the car fleet is 1.9 and the minibuses fleet is 3.2 passengers per vehicle. The latter is not consistent with the results obtained for the variation of demand with full coverage. Though the results obtained for one passenger per vehicle were already pointing for the fact that choosing the right combination of municipalities can reduce the demand peaks, having a positive effect on the number of trips served per vehicle (total passengers served per vehicle in one day), which can lead to profit (see Table 5).

A different viewpoint on profit can be obtained with imposed variations in vehicle capacity for both vehicle types, forcing changes in the average vehicle occupation rate. This can be related to having different vehicle interiors that limit the maximum number of passengers (e.g.: decreasing seat capacity to increase comfort). New optimal results were added by applying the optimization model to the following scenarios with full regional coverage: for the car, it was considered capacities of 1, 2 and 3 seats and demand levels 25%, 50% and 100% (the scenarios with 4 seats were already considered for all demand levels, as well as the 1 seat for the 100% demand level); for the minibus, capacities varying from 3 to 15 seats, and the same demand levels used for the car (the scenarios with 16 seats were previously considered for all demand levels). The new intrazonal pick-up and delivery times for each zone $i$, $t_{pd_i}$, had to be calculated for the new capacities by using the process explained in section 2.1. The results are presented in Table 6 and Figure 12. In Figure 12, it can be seen that the break-even point assumes approximately the same position for the 25, 50, and 100% demand levels for both fleet types. The car fleet presents a break-even point approximately equal to 1.6 average passengers per vehicle (using linear interpolation), and the break-even for the minibus fleet is slightly above an average of 4 passengers per vehicle (approximately 4.1, using linear interpolation).

*Table 6: Optimal results considering unitary changes in capacity for car and minibus fleets*

| | 25% demand | | | 50% demand | | | 100% demand | | |
|---|---|---|---|---|---|---|---|---|---|
| Capacity | Avp/v | Trips/v | Profit | Avp/v | Trips/v | Profit | Avp/v | Trips/v | Profit |
| | CAR FLEET | | | | | | | | |
| 1 | 1 | 8.4 | -34371 | 1 | 8.4 | -67142 | 1 | 8.4 | -134707 |
| 2 | 1.9 | 15.2 | 20250 | 1.9 | 15.5 | 44333 | 2.0 | 15.7 | 91908 |
| 3 | 2.6 | 19.1 | 33273 | 2.8 | 20.0 | 72705 | 2.9 | 20.4 | 151324 |
| 4 | 3.3 | 22.4 | 41036 | 3.6 | 23.8 | 89921 | 3.8 | 24.6 | 187214 |
| | MINIBUS FLEET | | | | | | | | |
| 3 | 2.6 | 19.1 | -34525 | 2.8 | 20.0 | -55262 | 2.9 | 20.4 | -99043 |
| 4 | 3.3 | 22.4 | -17639 | 3.6 | 23.8 | -18157 | 3.8 | 24.6 | -21730 |
| 5 | 3.8 | 25.7 | -5269 | 4.4 | 28.0 | 9092 | 4.7 | 29.0 | 34407 |
| 6 | 4.3 | 28.5 | 2465 | 5.1 | 31.7 | 27599 | 5.5 | 33.4 | 74719 |
| 7 | 4.7 | 30.8 | 7968 | 5.7 | 34.9 | 40869 | 6.4 | 37.3 | 104379 |
| 8 | 5.0 | 31.4 | 8090 | 6.4 | 36.4 | 44733 | 7.2 | 39.8 | 116976 |
| 9 | 5.3 | 34.1 | 13681 | 6.9 | 40.5 | 57410 | 7.9 | 44.0 | 140297 |
| 10 | 5.6 | 35.6 | 16202 | 7.5 | 42.9 | 63818 | 8.7 | 47.0 | 154119 |
| 11 | 5.8 | 37.3 | 19606 | 7.9 | 45.5 | 71487 | 9.4 | 51.1 | 172507 |
| 12 | 6.0 | 38.2 | 20458 | 8.3 | 48.0 | 75603 | 10.1 | 54.4 | 182363 |
| 13 | 6.1 | 39.4 | 22230 | 8.8 | 50.5 | 80447 | 10.7 | 57.7 | 192411 |
| 14 | 6.3 | 40.9 | 24753 | 9.1 | 53.8 | 86493 | 11.4 | 62.0 | 204957 |
| 15 | 6.4 | 41.8 | 25351 | 9.5 | 56.2 | 89431 | 12.0 | 64.9 | 211028 |
| 16 | 6.5 | 41.3 | 24189 | 9.8 | 56.7 | 89426 | 12.6 | 66.7 | 213627 |

Avp/v – average passengers per vehicle; Trips/v – trips per vehicle /day

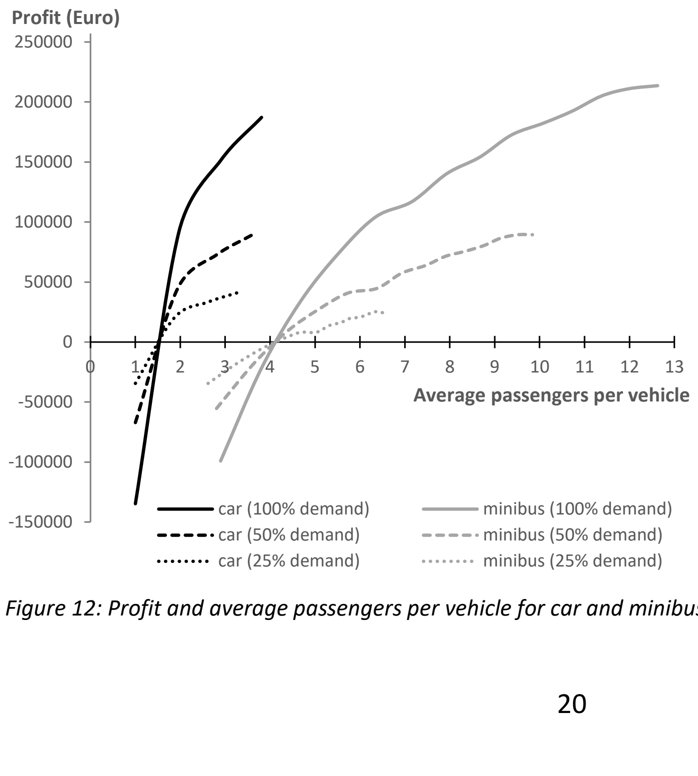



*Figure 12: Profit and average passengers per vehicle for car and minibus fleets with changes in capacity*

Plotting all the results (from Table 3, Table 4, Table 5, and Table 6), a linear relationship between the total trips per vehicle during one day and the average number of passengers per vehicle is verified (see Figure 13). The categorization of the graphical points in losses and profit for each fleet type allows an overview of the break-even points (see Figure 14) confirming the previous analysis. Overall, it can be stated that the car fleet break-even point is between 1.5 and 2.0 average passengers per vehicle, around 1.6 for high demand levels, and, for the minibus fleet, the break-even point is identified on an average vehicle occupancy between 4 and 5 passengers, at approximately 4.1 passengers per vehicle for high demand levels. The exceptions to this claim are the following profitable scenarios: the partial coverage set of municipalities for the fleet of cars carrying only one passenger considering 100% of demand (1 passenger per vehicle and 12.2 daily trips per vehicle), and the partial coverage set of municipalities for the minibus fleet considering 5% of demand (3.2 average passengers per vehicle and 21.1 daily trips per vehicle).

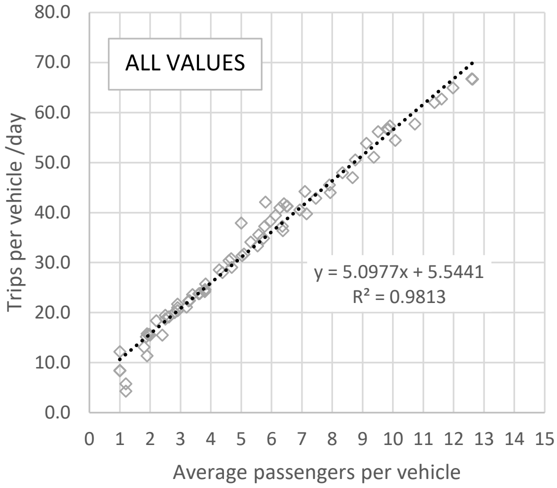
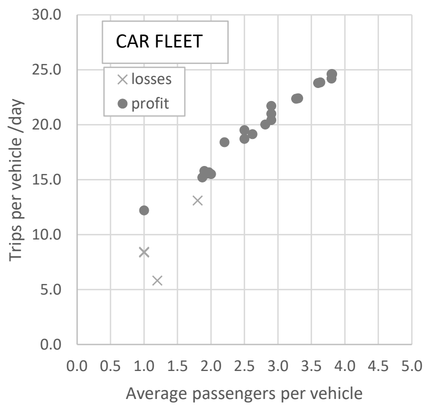



*Figure 13: Trendline for the relation of the daily trips per vehicle with the average passengers per vehicle considering all results for car and minibus fleets*



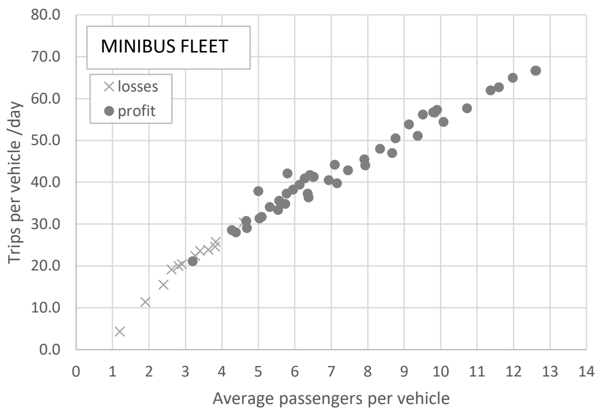




*Figure 14: Overview of the location of the break-even points using all the results obtained for the car and minibus fleets*

### 4.5. Validation of energy-related assumption

Since the SAV system under analysis in this study is comprised of electric vehicles, it is necessary to verify if the characteristics of the chosen vehicle (namely battery capacity and consumption rate) and charger type do not constrain the movement of vehicles, as it was assumed in 2.1. For the case study, we considered two types of vehicles with characteristics designed by the manufacturer: a car with 52kWh battery and a consumption rate of 20kWh per 100 km driven; and a minibus with 91kWh battery and a consumption rate of 36kWh per 100 km driven. Since the characteristics of the vehicles are already defined, we looked only at the charger type to validate the referred assumption. By choosing the charger type that keeps the vehicles with enough energy to move, we validate the energy-related assumption and, consequently, the optimal results. Three different types of chargers, which are currently in service in the study area (Chargemap, 2020), were included in the validation analysis: the three-phase AC current chargers with 11kw and 22kw power, and the rapid DC current charger with 50 kW power. The 100% demand level scenarios were the ones used since more energy is consumed to power the driving with less available idle time to charge the vehicles. For this analysis, we considered that there is no shortage of available power sockets (all vehicles can charge if required) and that vehicles start charging at the moment they become idle. This assumption allows showing the maximum potential of idle time in the charging process.

As a first approach we used the distance traveled and the vehicles' power consumption rates to determine the total energy spent in an aggregated way, and compared it with the charging potential calculated by multiplying the total idle time with the charger power. Looking at the charging potential and energy spent (see Table 7), we can anticipate problems for the fleet of minibuses and the mixed fleet by using the 11kW charger type, because, even if all idle time was used for charging, the accumulated energy would not be enough to satisfy the energy needed for the daily movements, and consequently, at the end of the day, the average battery level would be lower than the one verified 24 hours earlier. Though, not all charging potential flow (defined by idle time and charger power) can be stored due to the maximum limit of battery capacity available (vehicles cannot continue to charge after being fully charged) and probably some movements cannot be performed at some part of the day due to insufficient energy in the batteries. Therefore, the aggregated daily data on Table 7 is not a proper approach to validate the energy assumption. A better way to do so is to analyze the average battery level of vehicles throughout the day using the optimal results related to vehicle movements (see Figure 15).

*Table 7: Energy flows considering 11kW and 22 kW vehicle chargers for the 100% demand level scenario*

| | *Car fleet* | *Minibus fleet* | *Mixed fleet* | |
|---|---|---|---|---|
| | | | *cars* | *minibus* |
| # vehicles | 4717 | 1740 | 719 | 1212 |
| Total battery capacity (MWh) | 245 | 158 | 37 | 110 |
| distance traveled (km) | 2247800 | 1011980 | 410100 | 712560 |
| Energy spent (MWh) | 450 | 364 | 82 | 257 |
| Idle time (h) | 74584 | 24584 | 10433 | 16748 |
| **11 kW charger:** | | | | |
| - Charging potential (MWh) | 820 | 270 | 115 | 184 |
| - Charging potential after deducting energy spent (MWh) | 371 | -94 | 33 | -73 |
| **22 kW charger:** | | | | |
| - Charging potential (MWh) | 1641 | 541 | 230 | 368 |
| - Charging potential after deducting energy spent (MWh) | 1191 | 177 | 148 | 111 |
| **50 kW charger:** | | | | |
| - Charging potential (MWh) | 3729 | 1229 | 522 | 837 |
| - Charging potential after deducting energy spent (MWh) | 3280 | 865 | 440 | 580 |

For the fleet of cars, any of the charger types can be used without affecting vehicle operations. The idle time and number of idle vehicles are high enough when compared with vehicle consumption, keeping the average battery level always above 60%. For the fleet of minibuses, the 11kW charger type is not enough to keep the vehicles running (the total energy drops within the 24-hour period not recovering to the same level of the previous day, and, besides that, there is no energy in the batteries from 18h00 to 21h00), and the 22kW makes the average battery levels dropping below 20% (to a minimum of 16% at 20h20). A mixed solution with 22kW stations and some 50kW power stations is advisable. For the mixed fleet, having distinct decision variables for each vehicle type allows analyzing the set of cars and minibus separately. In terms of cars, any charger type can be used, for minibus vehicles, however, the provider needs to install 22kW chargers jointly with some 50kW charger stations to keep the vehicles moving. The use of 11kw chargers for the minibus vehicles of the mixed fleet does not deliver enough energy for the 24-hour period, resulting in zero energy in the batteries between 18h00 and 21h00. A solution using only 50kW chargers on the network, with enough sockets for idle vehicles to charge, supports the energy-related model assumption for all fleet types. The use of this charging power leaves enough extra idle time for waiting to charge if no power socket is available, and extra battery capacity to power daily movements if charging time is eventually reduced. It is important to notice that the charging station installation and maintenance costs can be attributed to the SAV company, costs which are not considered in the objective function of the models proposed in this work. Nevertheless, the importance of the energy-related assumption verification will probably be negligible at the time that the SAV system becomes a reality, due to the potential advances in vehicle and charging technology.

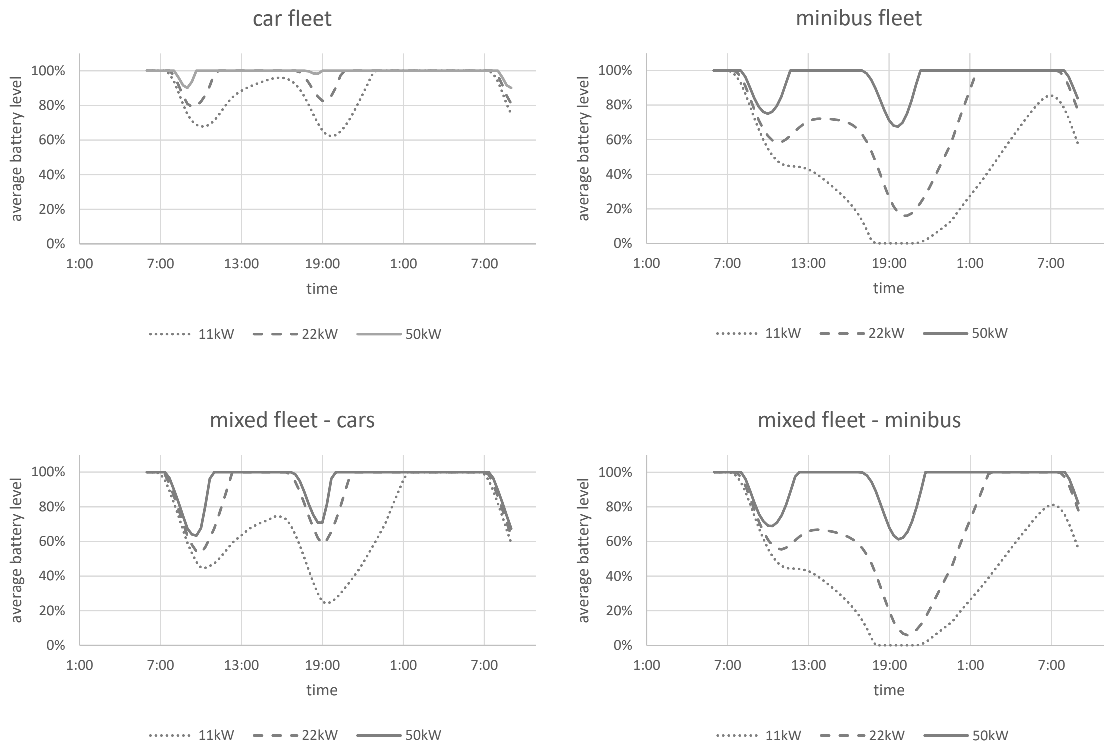



*Figure 15: Average battery level along the day for the different fleet types and the 100% demand level scenario*

## 5. Conclusions

In this study, we proposed an approach to design and assess the financial viability of a SAV system, allowing vehicle pooling in an interurban context. A flow-based Integer Programming (IP) model was developed, in which the objective function is set to maximize the profit by deciding on the vehicle fleet and the vehicle movements on a typical business day. Studying interurban movements reduces the needed detail, making it possible to use flows as decision variables instead of routing each vehicle individually. This simplification is important to solve large scale problems. The model is able to consider different types of vehicles (e.g.: car, minibus), and additional constraints are included to determine which zones should be part of the operational area. The use of electric vehicles in the model is subject to the validation of the assumption that battery capacity and charging speed have no impact in the movement of users and relocations. In terms of computational speed, it was verified that the model quickly converges to optimal solutions (with a gap under 1%) for a problem with 19 zones and 72 time steps (case study size).

A financial viability assessment of the SAV interurban system, using the optimal results from the IP model, was performed for the region of Coimbra, Portugal. Three different fleet compositions were tested: car fleet, minibus fleet, and a mixed fleet comprised of cars and minibuses. The results showed that the best fleet layout to serve all demand, covering the whole region, is the mixed fleet, which can return a profit of 2 Euro per passenger served. For this fleet composition, the variation of the optimal share in number of vehicles, between car and minibus, as demand increases, follows a non-linear trendline, in which the share of minibus vehicles is increasing. This share asymptotically converges to approximately 1/3 of cars and 2/3 of minibuses in the fleet, for the high demand level. Performing a set covering analysis, using the additional constraints to turn on and off zones, it was found that only selected subsets of municipalities lead to a profit for low demand levels. For high demand levels, the higher profit is returned when serving all municipalities of the region. Therefore, multistage planning where zones are added sequentially according to their demand is essential to support the profit of the company and guarantee a good level of service.

Comparing the optimal results, for the scenario where all demand is served, with the current situation, it was estimated that each vehicle in a SAV fleet can replace 7 to 18 private cars (depending on the fleet layout chosen). The improvements in sharing vehicle space are done at the cost of the passengers' travel time (which is not being considered in the objective function). Serving only one passenger per car leads to losses for covering the entire region. The reduction in profit, when compared with pooling people together, is due to the increment in the vehicles' daily costs and vehicles' usage costs. An assessment performed for the one-vehicle-type fleets has shown that the point of break-even occurs at an average number of passengers per vehicle between 1.5 and 2, for a fleet of cars, and between 4 and 5, for a fleet of minibuses. For the considered electric vehicle technology, a solution using only 50 kW chargers on the network, with enough sockets for idle vehicles to charge, supports the energy needs, without affecting the demand related or relocation movements.

All in all, this assessment approach is adequate and replicable to other regions where the introduction of SAVs can potentially enhance the availability of transportation alternatives for interurban travelling. Nevertheless, the presented results and conclusions should only be transferred to similar regions (in terms of distances, demand, and transport supply). Improvements can be implemented to overcome the limitations or to extend the contributions of this work by: accounting for traffic congestion impacts on travel times; considering a time window for user departures instead of a fixed instant; adding a users' mode choice behavior model to capture the equilibrium between supply and demand; using energy-related variables to allow the optimal design of charging operations and location of chargers; and, adding the demand for intra-zone (urban) trips in addition to the inter-zonal trips to enable an increase in vehicle usage during the day.

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