Technische Universiteit Delft
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De onzekerheidsanalyse in de positiebepaling van een boorkop met de covariance methode

(Engelse titel: Uncertainty analysis in determining the position of a drill bit with the covariance method)

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“De onzekerheidsanalyse in de positiebepaling van een boorkop met de covariance methode”

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Abstract

During oil drilling, the position of wells is indirectly determined by measurements near the drilling bit. However, the measurements are disturbed by noise causing errors in the measurements. It is important for large oil companies like Shell to determine the position of wells with a certain degree of uncertainty. Therefore, the main objective of this thesis is to assess the uncertainty in the final well position of MSA corrected survey data.

Two methods are investigated in this thesis to determine the positional uncertainty; Multi-Station Analysis in combination with the Minimum Curvature Method and Multi-Station Analysis in combination with the Covariance analysis method. In addition to Multi-Station Analysis, a second method has been investigated to determine sensor errors estimates and their uncertainty, namely the probabilistic estimation from Weighted Least Squares method.

The methods with their assumptions are discussed, whereby the Covariance analysis method is discussed in more detail.

Thereafter, the methods are tested with various test trajectories. The test results are compared to subsequently draw conclusions.
It is clear that Multi-Station Analysis is the most reliable method to estimate sensor errors with their uncertainty. However, it was striking when the azimuth angle is not fixed but varies, that certain errors are very poor.
In addition, it could be concluded that the choice to systematically or randomly consider positional uncertainty has a major influence on the positional uncertainty that arises.

The thesis is completed with recommendations for further research.

Preface

Before you lies the thesis "Uncertainty analysis in determining the position of a drill bit with the covariance method". It has been written to fulfill the graduation requirements of the Bachelor of Science in Applied Mathematics at Delft University of Technology (TU Delft). I was engaged in researching and writing this thesis from April to July 2019.

This research was carried out in a collaboration between Delft University of Technology and Shell Global Solutions International. I have always been interested in the possibilities of applying mathematics in practice in order to optimize processes. I was therefore very interested in researching the positional uncertainty that arises with oil drilling. It was a great challenge to do research in collaboration with a multinational like Shell Global Solutions International, where I got the opportunity to apply the knowledge I gained in my bachelor's degree and giving me the possibility to learn an incredible amount.

I would like to thank my supervisors prof.dr.ir K. Vuik from Delft University of Technology and ir. K.A Noy from Shell Global Solutions International for their time, valuable input and support during this process. I also wish to thank A. McGregor from HP Technologies for his help with the probabilistic estimation from Weighted Least Squares method. Without their cooperation, I would not have been able to conduct this analysis.

Contents

List of abbreviations

Abbreviation	Meaning	First mentioned
AHD	Along-Hole Depth	page 11
BHA	Bottom-Hole Assembly	page 9
MSA	Multi-Station Analysis	page 7
MCM	Minimum Curvature Method	page 7
MWD	Measurement While Drilling	page 9
TVD	True Vertical Distance	page 10
WLS	Weighted Least Squares	page 7

Table 1: List of frequent abbreviations.

List of symbols

Symbol	Quantity	Unit	Shorthand
A	azimuth angle	radians degrees	rad $\circ$
B	magnetic field strength	tesla microtesla nanotesla	T=Vs/m^2 μT=10^{-6}T nT=10^{-9}T
g	gravity force constant	m/s^2	g
I	inclination angle	radians degrees	rad $\circ$
ρ	correlation coefficient	-	-
T	toolface angle	radians degrees	rad $\circ$
θ	dip angle	radians degrees	rad $\circ$

Table 2: List of frequently used symbols.

Introduction

Shell Global Solutions is a division of Shell and one of their interests is the construction of oil and gas wells. The position of wells is indirectly determined through accelerometer and magnetometer sensor measurements near the drilling bit while constructing a well. These measurements are then converted into a survey of direction vectors by applying a series of coordinate transformations. With a method called Minimum Curvature Method (MCM), these outputs are then converted into the well position. As the sensor measurements contain errors, the calculated well position also contains an error.
Multi-Station Analysis (MSA) is a method to estimate and remove the sensor measurement errors and thus reducing the well position error. However, the MSA error estimate and removal is not exact, resulting an uncertainty in the final well position of MSA corrected survey data. The probabilistic estimation from weighted least squares (WLS) is another method to estimate the sensor measurement errors.

Shell uses MSA in combination with the Minimum Curvature Method to determine the final positional uncertainty of the drill bit. However, there is another method to determine the positional uncertainty known as the Covariance method. The following steps are undertaken:

- A short introduction to Shell's method and the Covariance method is given.
- A review is given of the underlying assumptions of Shell when MSA is applied in combination with the Minimum Curvature Method.
- A description is given of the probabilistic estimation from WLS.
- An analytical model has been delivered to estimate the positional uncertainty of a well survey that has been MSA corrected based on the method of covariances.
- A comparison has been made between the parameter uncertainty estimates of MSA, the method Shell uses, and the probabilistic estimation from WLS.
- A comparison has been made between the positional uncertainty when MSA is applied in combination with the Minimum Curvature Method, the method Shell uses, and when MSA is applied in combination with the analytical model based on the method of covariances.

Hence the objectives of this thesis are to assess the uncertainty in the final well position of MSA corrected survey data. See figure 1 for a overview of what will be covered in the thesis.

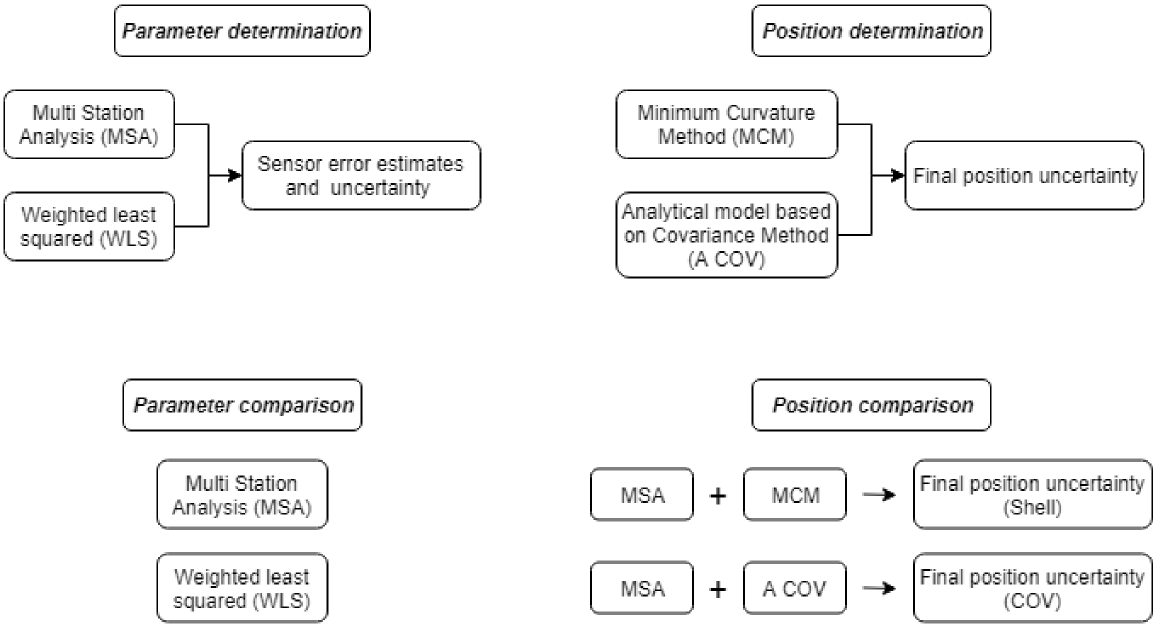

Figure 1: Overview of this thesis

1 Introduction to well-bore engineering

This chapter is a shortened introduction of bachelor thesis of H. V. Tan [1] to oil drilling.
Coordinates of wells need to be determined to ensure that wells are correctly positioned below the Earths surface. These coordinates are commonly determined with a specialized measurement tool, Measurement While Drilling (MWD) tool, near the drilling bit.
During drilling, the MWD sensors measures the drill bit acceleration and the Earths magnetic field in specified locations along the well, from which the well coordinates are derived. However, the sensor measurements contain systematic and random errors, which result in uncertainty in determining the well coordinates. This uncertainty can be expressed as an ellipsoid.
A mathematical technique called Multi-Station Analysis (MSA) is applied to reduce sensor measurement errors. The sensor measurement errors and its associated uncertainty can possibly be reduced when using MSA.

1.1 Components of a drilling rig

Oil wells are located far beneath the Earths surface. To extract the petroleum oil, a well-bore must be drilled through the surface. This is done by a drilling rig, placed in vicinity of the oil well. And important component of the rig is the Bottom-Hole Assembly (BHA). This is a part between the drill bit (the part that does the actual drilling) and drill pipe (steel piping) which contains drill collars as well as Measurement While Drilling tools:

- Non-Magnetic Drill Collar (NMDC) is protective casing which minimizes the effect of magnetic interference in the MWD tools.
- Measurement While Drilling (MWD) tools like accelerometers and magnetometers which are used to determine the direction of BHA.

The position of a drill bit is determined using indirect measurements collected from the MWD tools in the BHA. It is assumed that the position of BHA and drill bit coincide.

1.2 Position coordinates and orientation

The drill bit can be modelled as a cylinder. Its front face is called Tool Face. The turning angle around the middle axis with respect to the direction towards the Earths surface, High Side (HS), is called Tool Face angle, and the direction in which the drill bit is moving at any time is Downhole direction. High Side Right (HSR) is the remaining direction orthogonal to High Side lying across the Tool Face.
All measurements by the MWD tools are done with respect to the orientation of BHA, which is written in terms of x-axis (High Side + Tool Face angle), y-axis (High Side Right + Tool Face angle) and the z-axis (Downhole direction) coordinates, see figure 2.

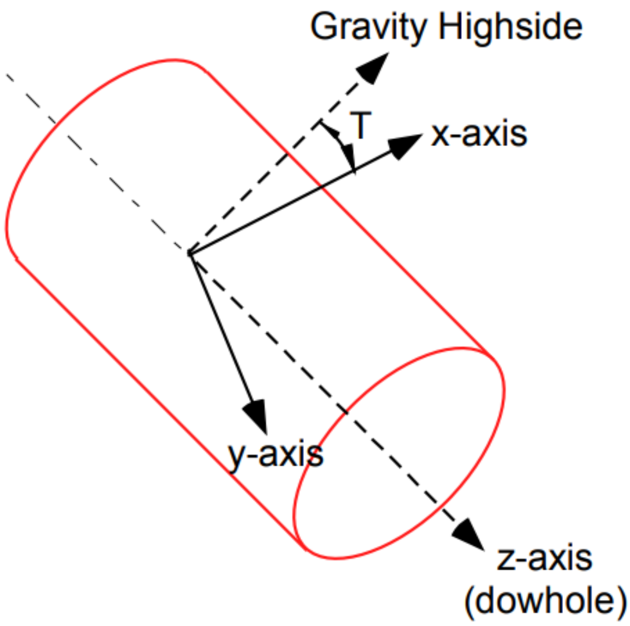

Figure 2: Orientation of BHA. Source: [2]

1.2.1 Direction vectors measured in term of angles

With coordinate transformations are measurement data converted into the direction vectors. The direction is measured in terms of two angles instead of the (x,y,z) plane:

- **Azimuth angle**: the clockwise angle in horizontal (x,y) plane with respect to Magnetic North.
- **Inclination angle**: the vertical clockwise angle (by right-hand rule) with respect to True Vertical Distance (TVD), the vertical component.

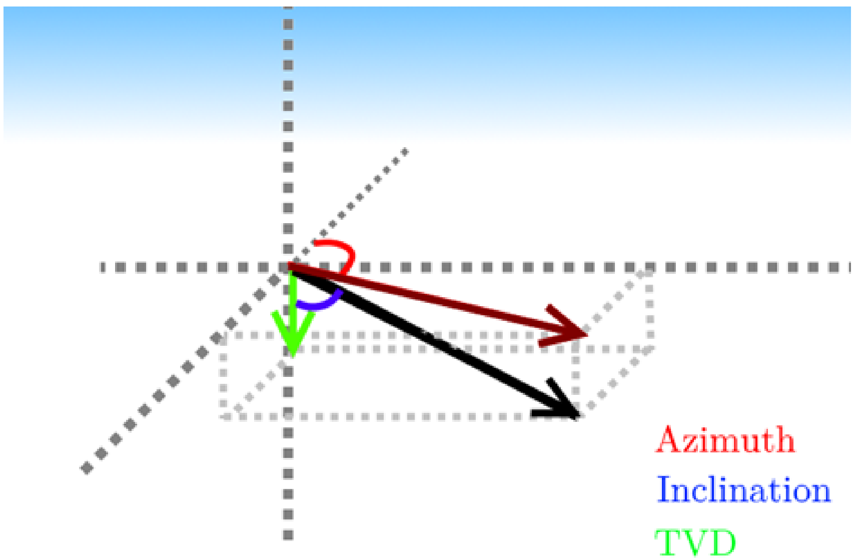

Figure 3: Direction of drill bit in Downhole direction. Source: [1]

1.2.2 Position expressed in North, East and Vertical coordinates

Once the size of TVD is known, then the vector representation of direction can be converted into its corresponding angular representation and vice versa. The drill bit position is expressed in North, East and Vertical coordinates. Additionally, any vector in general can also be represented with two vectors: a horizontal component along the Earths surface and a vertical component, which is Earths reference frame.

1.3 Step-by-step plan to determine position of BHA

There are 4 steps to determine the position of BHA:

1. A drilling trajectory is modelled in advance with a number of measurement stations and a starting and a final position. The actual travelled distance between starting position and position of drill bit is Along-Hole Measured Depth (AHD), and is always known in every station, see figure 4.

2. During drilling, measurements are made in each station using the MWD tools and it is assumed that the drill bit takes the modelled path. Three magnetometers measure the local Earths magnetic field strength which is used to calculate Azimuth. All magnetometers work independently from each other.
   Similarly, three accelerometers measure drill bit acceleration which is used to calculate both Azimuth and Inclination angle. Additionally, the TVD can be determined by this data.

3. MWD measurement data have to be converted into direction vectors using the coordinate transformations, which involve matrix rotation operations. These involve a Toolface matrix, an Inclination matrix and an Azimuth matrix. This model is fully described in the paper Boots & Coots International, Inc. 2010. The result is a survey containing the AHD, TVD, Inclination angle and Azimuth for each station.

4. The survey of direction data have to be converted into the overall (continuous) trajectory of the drill bit. The standard mathematical method to approximate the trajectory is the Minimum Curvature Method, which gives a close approximation of the overall trajectory.

The final position (at the last station) of the drill bit is the main interest for the well-bore industry.

Summary of this chapter
This chapter gave an introduction to well-bore engineering. The coordinates transformations and orientation of the drill bit are discussed. In addition is discussed how the position of the drill bit is determined.

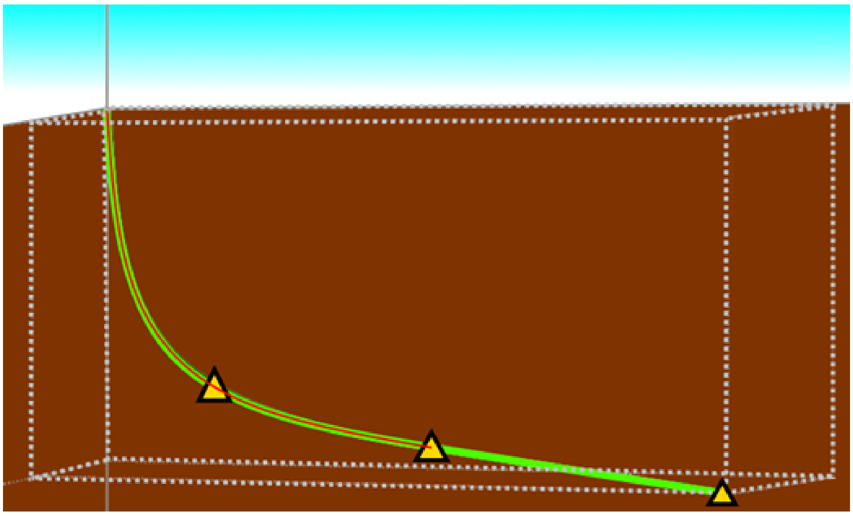

Figure 4: Modelled trajectory (green), including several stations (yellow). AHD between origin and second station is length of red curve. Source: [1]

2 Methods for uncertainty analysis of a drill bit

In this chapter, it will be briefly explained how the parameter uncertainties are determined by Multi-Station Analysis (MSA) and probabilistic estimation from Weighted Least Squares (WLS). In addition will also be explained how positional uncertainty is determined with the Minimum Curvature Method (MCM) and the Covariance analysis method. First a step-by-step procedure will be described to test the methods. After that, each method will be discussed briefly.

In reality there are error parameters, assumed systematic, and noise on the measurements during oil drilling, which means that positional errors and position of the drill bit cannot be determined perfectly. The error parameters and the noise in the measurements cause uncertainty that the well-bore industry wants to keep as small as possible.
Therefore, the well-bore industry has a great interest in quantifying typical well-bore positional errors and is especially interested in the uncertainty when estimating parameters and the position of the drill bit. There are various methods for predicting well-bore positional uncertainty caused by noise on the measurement data.
A commonly used method to determine the positional uncertainty uses Multi-Station Analysis. This method has remained the industry standard for a long time, but various shortcomings of the method have been identified. A number of factors have created the opportunity for the industry to develop an alternative method also referred to as the Covariance analysis method. The main motivation of this paper is to explain and compare the two different methods, whereby the Covariance analysis method will be explained in more detail.

To test the quality of the methods, modelled data is used to test and verify the models used in the methods. To simulate reality, noise is deliberately added to the modelled data. Note that exact noise is not known in reality. The methods will be explained in more detail in chapters 3, 4 and 5. Below is the procedure described to test the methods with test data.

Step 1: define model assumptions and survey traject
The first step is to define the geomagnetic reference model, in which the total magnetic field strength, B_{tot}, and the dip angle, θ, are defined. Then the model survey traject is defined, defining the Along Hole Depth, Inclination, Azimuth and Toolface. The drilling trajectory is modelled in advance with a number of measurement stations and a starting and final position. The Along Hole Depth, Toolface angle, the Inclination angle and the Azimuth are known in every measurement station.

Step 2: Generate survey data
The second step is to generate survey data Bx_c, By_c, Bz_c using transformation matrices on the raw or modelled data. The MWD measurement data have to be converted into direction vectors using the coordinate transformations, which involve matrix rotation operations. These involve a Toolface matrix, an Inclination matrix and an Azimuth matrix. This model is fully described in chapter two of Boots & Coots International [2].

According to the paper Boots & Coots International [2], using the Toolface, Inclination and Azimuth matrix, the magnetometer readings are:

$$B = [T] \cdot [I] \cdot [A] \cdot B' \tag{1}$$

$$
\begin{aligned}
\begin{bmatrix} Bx_c \\ By_c \\ Bz_c \end{bmatrix} &= \begin{bmatrix} cos(T) & sin(T) & 0 \\ -sin(T) & cos(T) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(I) & 0 & -sin(I) \\ 0 & 1 & 0 \\ sin(I) & 0 & cos(I) \end{bmatrix} \begin{bmatrix} cos(A) & sin(A) & 0 \\ -sin(A) & cos(A) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Bh \\ 0 \\ Bv \end{bmatrix} \\
&= \begin{bmatrix} B_h(cos(T)cos(I)cos(A) - sin(T)sin(A)) - B_v cos(T)sin(I) \\ B_h(-sin(T)cos(I)cos(A) - cos(T)sin(A)) + B_v sin(T)sin(I) \\ B_h sin(I)cos(A) + B_v cos(I) \end{bmatrix}
\end{aligned} \tag{2}
$$

Where:
$[T]$ is the Toolface matrix.
$[I]$ is the Inclination matrix.
$[A]$ is the Azimuth matrix.
T is the toolface angle.
I is the inclination angle.
A is the azimuth angle.

$$
\begin{aligned}
B_h &= B \cdot cos(\theta) \\
B_v &= B \cdot sin(\theta)
\end{aligned} \tag{3}
$$

Where:
B_h is the the horizontal component of the magnetometer data with B the magnetic field strength and θ the dip angle.
B_v is the the vertical component of the magnetometer data with B the magnetic field strength and θ the dip angle.

Therefore, synthetic survey data for the magnetic field strength, B, is generated by transforming the measured magnetic field strength, B', via the Toolface, Inclination and Azimuth matrix.

Step 3: applying bias and scale factor error to the survey data
Given synthetic survey data Bx_c, By_c, and Bz_c, the bias and scale factor errors can be applied to the synthetic data to generate measured data. This process is described with the following equations:

$$
\begin{aligned}
Bx_m &= Bx_c(1 + e_{xS}) + e_{xB} \\
By_m &= By_c(1 + e_{yS}) + e_{yB} \\
Bz_m &= Bz_c(1 + e_{zS}) + e_{zB}
\end{aligned} \tag{4}
$$

Where:
Bi_m is the measured survey data in direction i.
e_{iS} is the scale factor error in direction i.
e_{iB} is the bias error in direction i.

Step 4: applying noise and estimating parameters and uncertainty
The next step is to consider the noise on the measurement data. This can be done by applying random noise to the generated measured data or by considering a fixed uncertainty interval caused by noise. The final step is to estimate the parameters, which are in this case the sensor errors (three bias and three scale factors) and their uncertainty, also described as magnitudes,

which is caused by noise. The positional uncertainty of the drill bit can be determined with the estimates and uncertainty of the parameters with the Minimum Curvature Method or the Covariance analysis method. This fourth step is different between MSA and the probabilistic estimation from Weighted Least Squares.

2.1 Shell's method: MSA in combination with MCM

MWD measurement tools give values that are calculated from measurements of the local acceleration vector and the local magnetic vector. When the tool is stationary the acceleration vector is due to the Earths gravity. The direction and magnitude of this vector is well known. The local magnetic vector is due to the Earths magnetic field and is not well known because of magnetic interference of other objects or unbalanced electrical currents in the MWD instrument. This results in measurements that are not perfect.
The well-bore trajectory is calculated using the Minimum Curvature Method. This is only an approximation of the actual well path. A more detailed explanation of this method is given at the end of chapter 3.
One of the causes of well-bore positional uncertainty with MWD tools are sensor errors. There are two types of sensor errors namely bias errors and scale factor errors.
An advantage of Multi-Station Analysis is that it can reduce the effects of common sensor errors and magnetic interference. But MSA unfortunately also has limitations. MSA assumes that the measurement errors are systematic and not random. Moreover, there are limitations based on the orientation of the tool. For example: when drilling East or West and Horizontal the Bz magnetic interference (Drill String Interference) makes very little change in B_{tot} or dip angle, but it has maximum effect on the measured azimuth [3].

2.1.1 Parameter uncertainty with Multi-Station Analysis (MSA)

After performing step 3, where bias and scale factor errors are applied to generate measured data, random noise is added to the generated measured data. The noise has a value between 0 and 70 nanoTesla (nT). This process is described with the following equations:

$$\begin{aligned} Bx_{m^*} &= Bx_m + \text{random noise} \\ By_{m^*} &= By_m + \text{random noise} \\ Bz_{m^*} &= Bz_m + \text{random noise} \end{aligned} \tag{5}$$

In addition, random noise, as per ISCWSA standard, is added to the total magnetic field strength, B_{tot} and the dip angle θ:

$$\begin{aligned} B_{tot^*} &= B_{tot} + \text{random noise} \\ \theta^* &= \theta + \text{random noise} \end{aligned} \tag{6}$$

Then Multi-Station Analysis is performed to correct the magnetometer data and to estimate the sensor errors $e_{xB}, e_{yB}, e_{zB}, e_{xS}, e_{yS}$ and e_{zS} (the three bias and three scale factor errors). The magnetometer data, Bx_m, By_m, and Bz_m, is corrected every time the Multi-Station Analysis is performed. The horizontal and vertical components of the magnetometer data, b_h, b_v as mentioned in equation 3 are corrected likewise. More details how to determine the MSA solution is described in section 3.2.

The process of adding random noise and estimating the sensor errors is repeated multi times and is referred to as a Monte Carlo simulation. A Monte Carlo simulation is a simulation technique in which a physical process is simulated not once, but many times, each time with different starting conditions, in this case random noise. The result of this collection of simulations is a distribution function that displays the entire area of possible outcomes. The mean values are taken per error to estimate the sensor errors $e_{xB}, e_{yB}, e_{zB}, e_{xS}, e_{yS}$ and e_{zS}. The standard deviations are taken per error to give an uncertainty for every sensor error also referred to as the magnitudes.

2.1.2 Position uncertainty with the Minimum Curvature Method (MCM)

The corrected magnetometer data can be used to determine the corrected Toolface, Inclination and Azimuth using the following formulas [1]:

$$\begin{aligned} T &= tan^{-1}\left(\frac{Gy}{-Gx}\right) \\ I &= tan^{-1}\left(\frac{\sqrt{Gx^2+Gy^2}}{Gz}\right) \\ A &= tan^{-1}\left(\frac{-B_{hsr}}{B_{hs}cos(I)+Bz_m sin(I)}\right) \end{aligned} \tag{7}$$

With B_{hsr}, B_{hs} defined as:
$B_{hs} = Bx_m \cdot cos(T) - By_m \cdot sin(T)$.
$B_{hsr} = Bx_m \cdot sin(T) + By_m \cdot cos(T)$.
$G_x = -g \cdot cos(T) \cdot sin(I)$
$G_y = g \cdot sin(T) \cdot sin(I)$
$G_z = g \cdot cos(I)$
Where:
A is Azimuth angle.
I is Inclinaton angle.
T is Toolface angle.
g is the gravity constant.

The Minimum Curvature Method can then be used to determine the position of the drill bit in the Earth-referenced frame (North, East and Vertical). This process of correcting the magnetometer data and performing the Minimum Curvature Method to determine the position of the drill bit gives again a distribution of possible outcomes. The mean values of these distributions (for North, East and Vertical) are estimates for the position of the drill bit. The standard deviations of the distributions give the uncertainty for the position of the drill bit.

The Industry Steering Committee on well-bore Survey Accuracy, ISCWSA, has as main goal to produce and maintain standards for the industry relating to well-bore survey accuracy. They created a new method, the Covariance analysis method, that is mainly based on linear algebra.

2.2 Parameter uncertainty with probabilistic estimation from Weighted Least Squares (WLS)

Given the generated measured data (modelled data) as described in equation 4, the estimates for sensor errors can be calculated in the following way [4]:

$$\Delta\hat{x} = PH^T(HPH^T + R)^{-1}\Delta y \tag{8}$$

Where:
L is the length of the survey trajectory.
$\Delta\hat{x}$ is a vector of size 6 by 1 with the bias and scale error estimates $e_{xB}, e_{yB}, e_{zB}, e_{xS}, e_{yS}$, and e_{zS}.
P is the a priori covariance matrix of size 6 by 6 with the variances of the sensor error estimates on the diagonal.
H is the measurement matrix of size 3*L by 6 where the partial derivative of the generated measured data is taken with respect to each sensor error.
R is the measurement noise matrix of size 3*L by 3*L, with measurement noise on the diagonal.
Δy is a vector of size 3*L by 1 with the difference between the expected parameter estimates and the true parameter value.

The magnetometer readings Bx_c, By_c and Bz_c are depending on the magnetic field strength, B, the dip angle, Θ, and the Toolface, Inclination and Azimuth angle which differ in each measurement station. The measured data, Bx_m, By_m and Bz_m are constructed using the magnetometers readings. The values depend on the Toolface, Inclination and Azimuth angle which differ in each measurement station. This has as result that the values in matrix H are changing in every measurement station in the survey trajectory. Therefore, the matrix H has to be extended for every measurement station. The result of the matrix and vector multiplications is the vector $\Delta\hat{x}$ with the estimates for the bias and scale factor errors:

$$\Delta\hat{x} = \begin{bmatrix} e_{xB} & e_{yB} & e_{zB} & e_{xS} & e_{yS} & e_{zS} \end{bmatrix}^T \tag{9}$$

The matrix R can be generated with random noise between 0 and 70nT on the diagonal every time. This process of matrix and vector multiplication with every instance a different measurement noise matrix is repeated multi times. The mean values are taken per error to estimate the sensor errors $e_{xB}, e_{yB}, e_{zB}, e_{xS}, e_{yS}$ and e_{zS}.

The uncertainty for every sensor error is given by the following matrix [5]:

$$U = (H^T R^{-1} H + P^{-1})^{-1} \tag{10}$$

Where:
U is a matrix of size 6 by 6 that represents the error covariance bound.

Taking the square root of the diagonal elements of U gives the uncertainty for the sensor errors.

2.3 Position uncertainty with Covariance analysis method

The standard deviations (uncertainty) for each sensor error are necessary to determine the positional uncertainty. The positional uncertainty is described with the following matrix [6]:

$$COV = \begin{bmatrix} var(N,N) & cov(N,E) & cov(N,V) \\ cov(N,E) & var(E,E) & cov(E,V) \\ cov(N,V) & cov(E,V) & var(V,V) \end{bmatrix} \tag{11}$$

Where:
$var(N,N), var(E,E)$ and $var(V,V)$ are the variances of the North, East and Vertical positional uncertainties.
$cov(N,E), cov(E,V)$ and $cov(V,N)$ are the covariances between them.

The covariance matrix described in equation 11, that describes the positional uncertainty, can be constructed in the following way [7]:

$$[C_k] = \sum_{errors i} \sum_{k1 \leq k} \sum_{k2 \leq k} \rho(\epsilon_{i,l_1,k_1}, \epsilon_{i,l_2,k_2}) \mathbf{e}_{i,l_2,k_2} \cdot \mathbf{e}^T_{i,l_2,k_2} \tag{12}$$

where:
$\rho(\epsilon_{i,l_1,k_1}, \epsilon_{i,l_2,k_2})$ is the correlation coefficient between the value of the ith error source at the k_1th station (in l_1th leg) and the the k_2th station (in l_2th leg).
$\mathbf{e}_{i,l_2,k_2}$ is the vector error of the ith error source at the k_1th station (in l_1th leg) and the the k_2th station (in l_2th leg).

The survey leg is the distance between two survey stations. The error due to the presence of the ith error source at the kth survey station (in the lth survey leg) can be expressed as the sum of the effects on the preceding and following calculated displacements:

$$\mathbf{e_{i,l,k}} = \sigma_{i,l} \left(\frac{d\Delta\mathbf{r}_k}{d\mathbf{p}_k} + \frac{d\Delta\mathbf{r}_{k+1}}{d\mathbf{p}_k} \right) \frac{\partial \mathbf{p}}{\partial \epsilon_\mathbf{i}} \tag{13}$$

Where:
$\sigma_{i,l}$ is the magnitude (standard deviation) of the ith error source over the lth survey leg and p_k the instrument measurement vector at the kth survey station.
$\frac{d\Delta\mathbf{r}_k}{d\mathbf{p}_k} + \frac{d\Delta\mathbf{r}_{k+1}}{d\mathbf{p}_k}$ are the differentials of the displacement between survey station $k-1$ and k of the instrument measurement vector at the kth survey station.
$\frac{\partial \mathbf{p}}{\partial \epsilon_\mathbf{i}}$ are the weighting functions for an error source i.

Therefore, the covariance matrix can be computed given the necessities to calculate the error vector and the correlation coefficient of each error source at each measurement station.

Summary of this chapter
In this chapter is discussed that there are two methods to determine positional uncertainty of the drill bit. This are the Multi-Station Analysis in combination with the Minimum Curvature Method what has been the industry standard for a long time and the Covariance analysis method.
There is discussed how parameter (bias and scale factor errors) and positional uncertainty can be determined in 4 steps whereby the fourth step is discussed for the methods separately.

3 Multi-Station Analysis method (Shell method)

In this chapter, the method used by Shell to determine the positional uncertainty of the drill bit will be discussed. First their model assumptions will be discussed. Then, Multi-Station Analysis will be explained which Shell uses to determine the parameter uncertainty. Finally, the Minimum Curvature Method will be explained that gives Shell a positional uncertainty as a result.

3.1 Model assumptions of Shell's method

Multi-Station Analysis is used to estimate the sensor errors and the associated uncertainties of the estimated sensor error while drilling. To use this method, the following modelling assumptions are made which are also described in equation 4:

$$\begin{aligned} Bxk_m &= Bxk_c(1 + e_{xS}) + e_{xB} \\ Byk_m &= Byk_c(1 + e_{yS}) + e_{yB} \\ Bzk_m &= Bzk_c(1 + e_{zS}) + e_{zB} \end{aligned} \tag{14}$$

Where:
Bxk_c, Byk_c, Bzk_c are the x, y and z component of the magnetometer data in station k.
Bxk_m, Byk_m, Bzk_m are the uncorrected x, y and z component of the magnetometer data in station k.
e_{xS}, e_{yS}, e_{zS} are the scale factor errors in the x, y and z component.
e_{xB}, e_{yB}, e_{zB} are the bias errors in the x, y and z component.

MSA will give a perfect estimate for the bias and scale factor errors if there is no noise, but in reality, there is always noise on the measurement data. Therefore random noise is added to the magnetometer data in every station and also to the magnetic field strength B_{tot} and the dip angle Θ. This is the model assumption of Shell and it is important to understand that this assumption is different from the model assumption of the probabilistic estimation from Weighted Least Squares. Shell's model assumption is described by the following equations:

$$\begin{aligned} Bxk_{m^*} &= Bxk_c(1 + e_{xS}) + e_{xB} + \text{random noise} \\ Byk_{m^*} &= Byk_c(1 + e_{yS}) + e_{yB} + \text{random noise} \\ Bzk_{m^*} &= Bzk_c(1 + e_{zS}) + e_{zB} + \text{random noise} \\ B_{tot^*} &= B_{tot} + \text{random noise} \\ \theta^* &= \theta + \text{random noise} \end{aligned} \tag{15}$$

3.2 Multi-Station Analysis

MSA assumes the presence of systematic errors in the three magnetometers. These systematic errors are the MSA solution, the errors that MSA estimates, and are mathematically modelled by bias and scale factors. The magnetometer data is corrected in the following way [1]:

$$
\begin{aligned}
Bxk_m &= \frac{Bxk_{m^*} - \epsilon_{xB}}{\epsilon_{xS}} \\
Byk_m &= \frac{Byk_{m^*} - \epsilon_{yB}}{\epsilon_{yS}} \\
Bzk_m &= \frac{Bzk_{m^*} - \epsilon_{zB}}{\epsilon_{zS}}
\end{aligned} \tag{16}
$$

Where the bias and scale factor errors $e_{xB}, e_{yB}, e_{zB}, e_{xS}, e_{yS}$ and e_{zS} are estimated with MSA. But not only the magnetometer data has to be corrected but also the horizontal and vertical components of the magnetometer data B_h, B_v have to be corrected.

The MSA solution associated with erroneous magnetometer measurements is found by minimizing the following non-linear least squares error function, where corrected measurement data is compared against provided reference measurements [1]:

$$
L(\epsilon_{xB}, \epsilon_{yB}, \epsilon_{zB}\epsilon_{xS}, \epsilon_{yS}, \epsilon_{zS}) = \sum_{i}^{N}(B_{hk} - B_{vr})^2 + (B_{vk} - B_{vr})^2 \tag{17}
$$

Where:
B_{hr} and B_{vr} are the horizontal and vertical component of the erroneous magnetometer data in all stations.
B_{hk} and B_{vk} are the corrected horizontal and vertical component of the magnetometer data in station k.

Equation 17 is minimized by setting its partial derivatives to zero, yielding a system of 6 non-linear equations with 6 unknowns (the bias and scale errors), which is then solved with Newton-Raphson. In each iteration, Newton-Raphson involves the calculation of multiple real symmetric Jacobian matrices.

3.3 Uncertainty analysis of the parameters

The process whereby the sensor errors are estimated with Multi-Station Analysis is repeated multi times resulting in a distribution of all possible outcomes. The mean values are taken per error to estimate the sensor errors $e_{xB}, e_{yB}, e_{zB}, e_{xS}, e_{yS}$ and e_{zS}. The standard deviations are taken per error to give an uncertainty for every sensor error also referred to as the magnitudes.

3.4 Uncertainty analysis of the position of the drill bit

Every time Multi-Station Analysis is performed to estimate the sensor errors is the corrected magnetometer data also used as input in the Minimum Curvature Method to determine the position of the drill bit in the Earth-referenced frame.

3.4.1 Minimum Curvature Method

As mentioned earlier in section 2.1.2 is the Minimum Curvature Method used to approximate the actual well path. The idea behind the MCM is that it smooths two straight-line segments by using the Ratio Factor, denoted as RF, which is defined by the curvature of the well-bore. The Minimum Curvature method is the best method recommended for the calculating well-bore paths because it is applicable to any trajectory path, but differences are very small hence any method can be used for calculating the well trajectory.

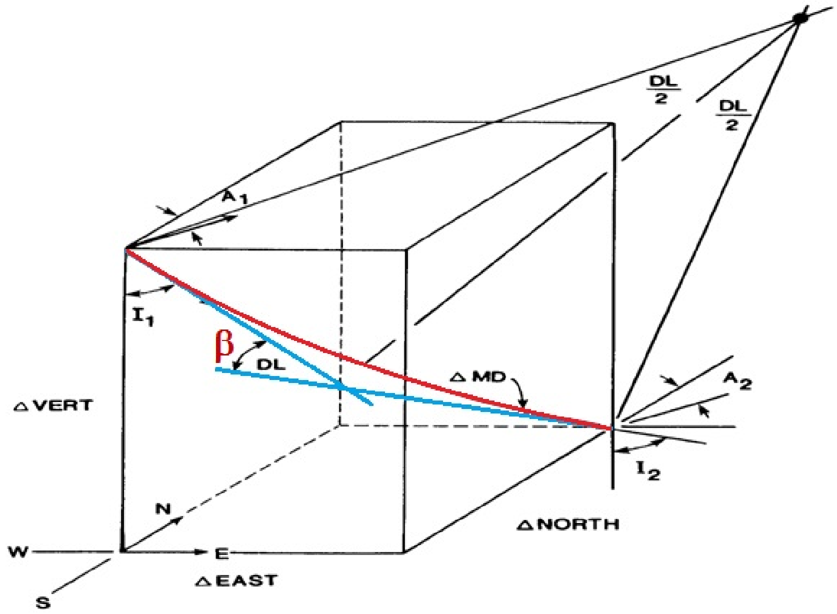

Figure 5: Schematic sketch of Minimum Curvature Method with two straight-line segments (blue) smoothed in the actual well path (red). Source: [8]

The Minimum Curvature Method has become the most common and accepted method for the industry. The formulas for the MCM are given as [9]:

$$\begin{aligned}
\Delta North &= \frac{\Delta MD}{2}\big[sin(I_1)cos(Az_1) + sin(I_2)cos(A_2)\big] * RF \\
\Delta East &= \frac{\Delta MD}{2}\big[sin(I_1)sin(Az_1) + sin(I_2)sin(A_2)\big] * RF \\
\Delta TVD &= \frac{\Delta MD}{2}\big[cos(I_1) + cos(I_2)\big] * RF \\
\beta &= cos^{-1}\big[cos(I_1 - I_2) - (sin(I_1)sin(I_2)(1 - cos(A_2 - A_1))\big] \\
RF &= \frac{2}{\beta}tan(\frac{\beta}{2})
\end{aligned} \tag{18}$$

Where:
ΔMD = Measured Depth between 2 station points in ft.
I_1 = Inclination (angle) of upper station point in degrees.
I_2 = Inclination (angle) of lower station point in degrees.
Az_1 = Azimuth direction of upper station point in degrees.
Az_2 = Azimuth direction of lower station point in degrees.
β = the dog leg angle, DL, in radians (dogleg severity), equivalent to the overall angle change of the drill pipe between any two stations.
RF = ratio factor.

The Minimum Curvature Method is essentially the Balanced Tangential Method, with each equation multiplied by a ratio factor. The Balanced Tangential Method is another method to

approximate the actual well path.
The Minimum Curvature Method gives likewise a distribution of possible position outcomes for the drill bit. The mean values of these distributions (for North, East and Vertical) are estimations for the position of the drill bit. The standard deviations of the distributions give the uncertainty for the position of the drill bit.
An example position calculation can be found in the appendix.

3.5 Conclusion

The bachelor thesis of H. V. Tan [1] states the following conclusion. Erroneous measurements in the magnetometer data results in an uncertainty of the final position of the drill bit. The uncertainty of the final position is bounded and can be represented as an ellipsoid centered around the actual final position. The Multi-Station Analysis process gives a smaller final position uncertainty ellipsoid.

Summary of this chapter
In this chapter, the Multi-Station Analysis method is discussed. The model assumptions are given followed by a detailed description how Multi-Station Analysis is used to correct erroneous magnetometer data, whereby the parameters (sensor errors) are estimated. After that is discussed how the corrected magnetometer data is used in the Minimum Curvature Method to determine the positional uncertainty of the drill bit, which can be described by an ellipsoid according to the bachelor thesis of H. V. Tan [1].

4 Probabilistic estimation from Weighted Least Squares (WLS)

In addition to MSA, there is another way to determine the parameters with their uncertainty. This method is the probabilistic estimation from Weighted Least Squares as already mentioned in chapter 2. In this chapter, the second method will be discussed with the model assumptions and how the parameters and their uncertainties are determined.

The paper of F. Hanak [4] provides a detailed description of the mathematical model, referred to as the consider covariance analysis technique, to determine the positional uncertainty. However, the parameters in that paper are split into two groups, the estimated and the considered parameters. A separation of the parameters would make it impossible to compare the Multi-Station Analysis method and the probabilistic estimation from Weighted Least Squares. Therefore, in this report, no separation of the parameters has been made with the probabilistic estimation from WLS. The paper also gives a validation for their approach but our main interest is to reproduce their work to compare this method with the Multi-Station Analysis method.

4.1 Model assumptions of the probabilistic estimation from Weighted Least Squares

As already mentioned in chapter 3 is the model assumption of Shell to add random noise to the magnetometer data and also to the magnetic field strength and the dip angle.
The paper of F. Hanak makes a different assumption. The paper assumes a standard uncertainty interval caused by noise on the parameter estimates giving the parameter uncertainty. The uncertainty interval with the standard deviations of the parameters are described in table 3. The paper takes a fixed uncertainty into account instead of a random uncertainty that Shell assumes.

Parameter description	Bias	Scale Factor Error
Standard deviation	70 nT	0.0016

Table 3: Magnetometer standard deviations from paper of F. Hanek.

4.2 Uncertainty analysis of parameters

Appendix A of the paper of F. Hanak has as goal to justify the use of probabilistic estimation from Weighted Least Squares to find a solution to the minimum variance estimation problem via orthogonal transformations. The equations and steps in this appendix are considered correct and will not be examined in detail. The only equation of interest is equation A-28 that describes the parameter estimates which is already mentioned in equation 8:

$$\Delta\hat{x} = PH^T(HPH^T + R)^{-1}\Delta y \tag{19}$$

Where:
$\Delta\hat{x}$ is a vector with the bias and scale error estimates $e_{xB}, e_{yB}, e_{zB}, e_{xS}, e_{yS}$ and e_{zS}.
P is the a priori covariance matrix with the variances of the sensor error estimates on the diagonal.
H is the measurement matrix where the partial derivative of the generated measured data is taken with respect to each sensor error.
R is the measurement noise matrix, with measurement noise on the diagonal.
Δy is the difference between the expected parameter estimates and the true parameter value.

The matrices and vectors can be described in the following way:

$$\begin{aligned}
\Delta\hat{x} &= \begin{bmatrix} e_{xB} & e_{yB} & e_{zB} & e_{xS} & e_{yS} & e_{zS} \end{bmatrix}^T \\
P &= \begin{bmatrix} \sigma^2_{e_{xB}} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma^2_{e_{yB}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma^2_{e_{zB}} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma^2_{e_{xS}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma^2_{e_{yS}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma^2_{e_{zS}} \end{bmatrix} \\
R &= \begin{bmatrix} \sigma^2_{Bx_m} & 0 & 0 \\ 0 & \sigma^2_{By_m} & 0 \\ 0 & 0 & \sigma^2_{Bz_m} \end{bmatrix} \\
H &= \begin{bmatrix} Bx_c(k) & 0 & 0 & 1 & 0 & 0 \\ 0 & By_c(k) & 0 & 0 & 1 & 0 \\ 0 & 0 & Bz_c(k) & 0 & 0 & 1 \end{bmatrix} \\
\Delta y &= \begin{bmatrix} Bx_m(k) - Bx_c(k) & Bx_m(k) - Bx_c(k) & Bx_m(k) - Bx_c(k) \end{bmatrix}^T
\end{aligned} \tag{20}$$

Where:
k is the station of the survey trajectory.

Matrices H and R and vector Δy can be extended for the whole survey trajectory by extending the old matrix with the new matrix and repeating this process for all stations.
The sensor error estimates can be determined by taking the noise in the matrix R random, giving slightly different estimates for the sensor errors each time. By repeating this matrix multiplication several times and taking the average of the estimates, the sensor error estimates are determined.
Although the interpretation of each vector and matrix has been described, it is still not very clear what the input should be for each matrix or vector. The results are therefore not reliable. The comparison between these results and the results with MSA will be discussed in more detail in chapter 6.

The uncertainty of the parameter estimates was already given by the equation 10:

$$U = (H^T R^{-1} H + P^{-1})^{-1} \tag{21}$$

Where:
U is a matrix that represents the error covariance bound.

Taking the square root of the diagonal elements of U gives the uncertainty for the sensor errors. The uncertainties are however much smaller than expected. This is probably due to the fact that some matrices for the parameter estimates are incorrectly interpreted having improper uncertainties as result. The results will be discussed in more detail in chapter 6.

Summary of this chapter

In this chapter, the probabilistic estimation from Weighted Least Squares is discussed with the focus on the parameter estimates and their uncertainty. The matrices used to estimate the parameters (sensor errors) and their interpretation are given. However, it appears that the interpretation of the matrices and vector input could be incorrect, resulting in unreliable results and prediction of the parameter uncertainties.

5 Covariance analysis method

In this chapter, the Covariance analysis method will be discussed to determine the positional uncertainty. Model assumptions will be described, followed by a detailed explanation how to construct the covariance matrix.

The Covariance analysis method describes the final position uncertainty of the drill bit with a covariance matrix with on the diagonal elements the uncertainty expressed as variances in the North, East and Vertical directions. The paper of H.S. Williamson [7] gives a detailed mathematical description how the covariance matrix should be constructed.

5.1 Model assumptions of Covariance analysis method

The mathematical model to determine the positional uncertainty of a drill bit presented in the paper of H.S. Williamson [7] has some assumptions that are implicit in the mathematics. These assumptions are as follows:

Assumptions for mathematical Covariance analysis method

- Errors in calculated well position are caused exclusively by the presence of measurement errors at the wellbore survey stations.
- Wellbore survey stations can be moddeled as three-element measurement vectors, the elements being Along-Hole Depth, Inclination and Azimuth.
- Errors from different error sources are statistically independent.
- There is a linear relationship between the size of each measurement error and the corresponding change in calculated well position.
- The combined effect on calculated well position of any number of measurement errors at any number of survey stations is equal to the vector sum of their individual effects.

5.2 How to construct the covariance matrix?

An error occurring at a survey station will result in an error, in the form of a vector, in the calculated well position which can be described with the following equation [7]:

$$\mathbf{e_i} = \sigma_i \frac{d\mathbf{r}}{d\mathbf{p}} \frac{\partial \mathbf{p}}{\partial \epsilon_\mathbf{i}} \tag{22}$$

Where:
$\mathbf{e_i}$ is a vector error of the ith error source.
σ_i is the magnitude of the ith error source.
$\frac{\partial \mathbf{p}}{\partial \epsilon_\mathbf{i}}$ is the weighting function of the ith error source.
$\frac{d\mathbf{r}}{d\mathbf{p}}$ describes how changes in the measurement vector affect the calculated well position.

For the purposes of computation, the error summation terminates at the survey station of interest and is given by [7]:

$$\mathbf{e}^*_{\mathbf{i},\mathbf{L},\mathbf{K}} = \sigma_{i,L} \frac{d\Delta\mathbf{r}_K}{d\mathbf{p}_K} \frac{\partial \mathbf{p_K}}{\partial \epsilon_\mathbf{i}} \tag{23}$$

$\Delta\mathbf{r}_k$ is the displacement between survey stations $k-1$ and k. The error due to the presence of the ith error source at the kth survey station (in the lth survey leg) can be expressed as the sum of the effects on the proceeding and following calculated displacements [7]:

$$\mathbf{e_{i,l,k}} = \sigma_{i,l}\left(\frac{d\Delta\mathbf{r}_k}{d\mathbf{p}_k} + \frac{d\Delta\mathbf{r}_{k+1}}{d\mathbf{p}_k}\right)\frac{\partial\mathbf{p}}{\partial\epsilon_\mathbf{i}} \tag{24}$$

Where:
$\sigma_{i,l}$ is the magnitude of the ith error source over the lth survey leg.
p_k is the instrument measurement vector at the kth survey station.

The total position error at a particular survey station k in survey leg l will be the sum of the vector errors $\mathbf{e_{i,l,k}}$ taken over all error sources i and all survey stations up to and including k. The uncertainty in this position error is expressed in the form of a covariance matrix [7]:

$$[C_k] = \sum_{errorsi}\sum_{k1\leq k}\sum_{k2\leq k}\rho(\epsilon_{i,l_1,k_1},\epsilon_{i,l_2,k_2})\mathbf{e}_{i,l_2,k_2}\cdot\mathbf{e}^T_{i,l_2,k_2} \tag{25}$$

Where:
$\rho(\epsilon_{i,l_1,k_1},\epsilon_{i,l_2,k_2})$ is the correlation coefficient between the value of the ith error source at the k_1th station (in l_1th leg) and the the k_2th station (in l_2th leg).

Substituting equation 24 in equation 25 gives:

$$[C_k] = \sum_{errorsi}\sum_{k1\leq k}\sum_{k2\leq k}\rho(\epsilon_{i,l_1,k_1},\epsilon_{i,l_2,k_2})\left(\sigma_{i,l}\left(\frac{d\Delta\mathbf{r}_k}{d\mathbf{p}_k}+\frac{d\Delta\mathbf{r}_{k+1}}{d\mathbf{p}_k}\right)\frac{\partial\mathbf{p}}{\partial\epsilon_\mathbf{i}}\right)\cdot\left(\sigma_{i,l}\left(\frac{d\Delta\mathbf{r}_k}{d\mathbf{p}_k}+\frac{d\Delta\mathbf{r}_{k+1}}{d\mathbf{p}_k}\right)\frac{\partial\mathbf{p}}{\partial\epsilon_\mathbf{i}}\right)^T \tag{26}$$

Therefore, the following components are required to determine the covariance matrix:

- The correlation coefficients, $\rho(\epsilon_{i,l_1,k_1},\epsilon_{i,l_2,k_2})$, between the value of the ith error source at the k_1th and k_2th station (in l_1th and l_2th leg).
- The vector error, $\mathbf{e_{i,l,k}}$, of the ith error source at the kth survey station over the lth survey leg determined by:
  - The magnitude, $\sigma_{i,l}$, of the ith error source over the lth survey leg.
  - The differentials of the displacement, $\frac{d\Delta\mathbf{r}_k}{d\mathbf{p}_k}+\frac{d\Delta\mathbf{r}_{k+1}}{d\mathbf{p}_k}$, between survey station $k-1$ and k to the instrument measurement vector at the kth survey station.
  - The weighting functions, $\frac{\partial\mathbf{p}}{\partial\epsilon_\mathbf{i}}$, for an error source i.

Multiplying vector e_i with e_i^T gives a matrix of size 3 by 3. Multiplying this matrix with the correlation coefficients gives a matrix of the following form [6]:

$$COV = \begin{bmatrix} var(N,N) & cov(N,E) & cov(N,V) \\ cov(N,E) & var(E,E) & cov(E,V) \\ cov(N,V) & cov(E,V) & var(V,V) \end{bmatrix} \tag{27}$$

Where:
$var(N,N), var(E,E)$ and $var(V,V)$ are the variances of the north, east and vertical position

uncertainties.
$cov(N, E), cov(E, V)$ and $cov(V, N)$ are the covariances between them and give the skew or rotation of the ellipse with respect to the principle axes.

$var(N, N)$ can be written as σ_N^2. The uncertainty in north axis (at 1-standard deviation) is $\pm\sqrt{\sigma_N^2}$. In the same way, the other terms on the diagonal are uncertainties along the other principle axes.

5.3 The correlation coefficients

The covariance matrix describes the position uncertainty in each axis on the main diagonal and the correlations between these values in the off-diagonal terms. The correlations could have in principle any value between -1 and 1, including zero for uncorrelated terms and also non-integer values. In practice however, the majority of the errors in borehole survey are either uncorrelated ($\rho = 0$) or fully correlated ($\rho = 1$) between different survey stations. This means there are two cases [10]:

1. **Correlated errors**: The errors between survey stations are said to be correlated if they are directly linked and would have the same underlying error value from station to station. The uncertainty contributions are added in the following way:

$$e_{total} = e_1 + e_2 \tag{28}$$

   An example is the z-axis accelerometer bias error. Since the same tool is used throughout a survey leg, this bias is expected to have the same value from survey station to survey station. Hence the effects of the error will build all the way down the well bore.

2. **Uncorrelated errors**: Errors are uncorrelated or statistically independent if the errors are not linked from station to station. Two independent error sources could both cause a positive inclination error and add together but it is also possible that one might create a positive inclination error and the other a negative error.
   In this case, a random value from survey station 1 and a random value from survey station 2 are taken and the error contributions must be root sum square (RSS) together:

$$e_{total} = \sqrt{e_1^2 + e_2^2} \tag{29}$$

   It is a basic assumption of the model framework that the statistics of the various error sources are independent so they will be first squared, then summed to then take the root from it together. The correlation $\rho(\epsilon_{i_1}, \epsilon_{i_2})$ is by assumption always 0 between different sources i_1 and i_2.

Although the different error sources are independent from each other, an individual error source can be statistically correlated from survey station to survey station along the well. The possible correlation between measurements depends as much on the tool configuration and measurement mode, as on the error source itself. For example, an error source may behave correlated between survey legs in the same well, but independent between survey legs in different wells. The lowest degree of correlation occurs when any two measurements are independent, in which case the error source is termed random.

Therefore the ISCWSA Error Model defines four propagation modes for the errors [10]:

Propagation mode	Identifier	ρ_1	ρ_2	ρ_3	
Random	R	0	0	0	always independent
Systematic	S	1	0	0	correlated from survey station to survey station
Well by Well	W	1	1	0	correlated from leg to leg
Global	G	1	1	1	correlated over all wells

Table 4: Correlation coefficients per error source.

Where the separate correlation coefficients ρ_1, ρ_2, ρ_3 are defined as:
ρ_1 is the correlation between survey stations within the same survey leg.
ρ_2 is the correlation between survey stations in different legs in the same well.
ρ_3 is the correlation between survey stations within different wells in the same field.

The propagation mode is a property of the error source and is defined in the tool model. In practice, most error sources are systematic within a leg or are random and only a limited number of well by well or global sources have been identified.

5.4 The derivation of the vector errors

As mentioned before, the vector error due to the presence of error source i at station k is the sum of the effect of the error on the preceding and following survey displacements:

$$\mathbf{e_{i,l,k}} = \sigma_{i,l}\left(\frac{d\Delta\mathbf{r}_k}{d\mathbf{p}_k} + \frac{d\Delta\mathbf{r}_{k+1}}{d\mathbf{p}_k}\right)\frac{\partial\mathbf{p}}{\partial\epsilon_\mathbf{i}} \tag{30}$$

The magnitudes, differentials of the displacement and weighting functions have to be determined first to be able to derive the vector errors.

5.4.1 The calculation of the magnitudes

The magnitudes, or in other words the uncertainty of the sensor errors, can be calculated via a Monte Carlo simulation. The probabilistic estimation from WLS to determine the sensor error uncertainties, described in chapter 4, unfortunately gave poor results. But the sensor error uncertainties can also be determined via the Multi-Station Analysis as described in section 2.1.1. Therefore MSA will be used to determine the magnitudes.

5.4.2 The determination of the differentials of the displacement

The two differentials can be expressed as [7]:

$$\frac{d\Delta\mathbf{r}_j}{d\mathbf{p}_k} = \left[\frac{d\Delta\mathbf{r}_j}{d\mathbf{D}_k}\,\frac{d\Delta\mathbf{r}_j}{d\mathbf{I}_k}\,\frac{d\Delta\mathbf{r}_j}{d\mathbf{A}_k}\right] \tag{31}$$

Therefore

$$\frac{d\Delta\mathbf{r}_k}{d\mathbf{p}_k} + \frac{d\Delta\mathbf{r}_{k+1}}{d\mathbf{p}_k} = \left[\frac{d\Delta\mathbf{r}_k + \mathbf{r}_{k+1}}{d\mathbf{D}_k}\,\frac{d\Delta\mathbf{r}_k + \mathbf{r}_{k+1}}{d\mathbf{I}_k}\,\frac{d\Delta\mathbf{r}_k + \mathbf{r}_{k+1}}{d\mathbf{A}_k}\right] \tag{32}$$

Both the Minimum Curvature Method as the Balanced Tangential Method can be used, but taking partial derivatives of the azimuth or inclination is very complicated with the Minimum Curvature Method. Therefore the Balanced Tangential Method is used, because there is no significant loss of accuracy in using the balanced tangential model [7]:

$$\Delta r_j = \frac{D_j - D_{j-1}}{2} \begin{bmatrix} sin(I_{j-1})cos(A_{j-1}) + sin(I_j)cos(A_j) \\ sin(I_{j-1})sin(A_{j-1}) + sin(I_j)sin(A_j) \\ cos(I_{j-1}) + cos(I_j) \end{bmatrix} \tag{33}$$

Then

$$\frac{d\Delta r_k}{dD_k} = \frac{1}{2} \begin{bmatrix} sin(I_{k-1})cos(A_{k-1}) + sin(I_k)cos(A_k) \\ sin(I_{k-1})sin(A_{k-1}) + sin(I_k)sin(A_k) \\ cos(I_{k-1}) + cos(I_k) \end{bmatrix} \tag{34}$$

$$\frac{d\Delta r_{k+1}}{dD_k} = \frac{1}{2} \begin{bmatrix} -sin(I_k)cos(A_k) - sin(I_{k+1})cos(A_{k+1}) \\ -sin(I_k)sin(A_k) - sin(I_{k+1})sin(A_{k+1}) \\ -cos(I_k) - cos(I_{k+1}) \end{bmatrix} \tag{35}$$

$$\frac{d\Delta r_j}{dI_k} = \frac{1}{2} \begin{bmatrix} (D_j - D_{j-1})cos(I_k)cos(A_k) \\ (D_j - D_{j-1})cos(I_k)sin(A_k) \\ -(D_j - D_{j-1})sin(I_k) \end{bmatrix} \tag{36}$$

$$\frac{d\Delta r_j}{dA_k} = \frac{1}{2} \begin{bmatrix} -(D_j - D_{j-1})sin(I_k)sin(A_k) \\ (D_j - D_{j-1})sin(I_k)cos(A_k) \\ 0 \end{bmatrix} \tag{37}$$

for $j = (k, k+1)$.
Substituting equations 34, 35, 36 and 37 in equation 32 gives a survey displacement matrix D_{3x3}:

$$\begin{aligned} D &= \frac{d\Delta \mathbf{r}_k}{d\mathbf{p}_k} + \frac{d\Delta \mathbf{r}_{k+1}}{d\mathbf{p}_k} \\ &= \frac{1}{2} \begin{bmatrix} sin(I_{k-1})cos(A_{k-1}) - sin(I_{k+1})cos(A_{k+1}) & Pcos(I_k)cos(A_k) & -Psin(I_k)sin(A_k) \\ sin(I_{k-1})sin(A_{k-1}) - sin(I_{k+1})sin(A_{k+1}) & Pcos(I_k)sin(A_k) & Psin(I_k)cos(A_k) \\ cos(I_{k-1}) - cos(I_{k+1}) & -Psin(I_k) & 0 \end{bmatrix} \end{aligned} \tag{38}$$

with $P = (D_{k+1} - D_{k-1})$

5.4.3 The derivation of the weighting functions

The weighting function for a particular error source is a 3 by 1 vector with one term for each measurement. These are the partial derivatives of the survey measurements with respect to that error source. The elements describe the effect of a unit error on the measured Along-Hole Depth, Inclination and Azimuth.
There are 12 basic sensor error sources (a bias and scale factor for each of the three accelerometers and three magnetometers) and requires its own weighting function. Therefore the weighting function can be defined as:

$$\frac{\partial p}{\partial \epsilon_i} = \begin{bmatrix} \frac{\partial AHD}{\partial \epsilon_i} \\ \frac{\partial I}{\partial \epsilon_i} \\ \frac{\partial A_m}{\partial \epsilon_i} \end{bmatrix} \tag{39}$$

Taking the partial derivatives of the survey measurements with respect to that error source gives the following six weighting functions for the bias (mbx, mby and mbz) and scale factor errors (msx, msy and msz) in the x, y and z direction:

$$
\begin{aligned}
mbx &= \frac{\partial p}{\partial \epsilon_{xB}} = \begin{bmatrix} 0 \\ 0 \\ \frac{-cos(I)sin(A)}{Bcos(\theta)} \end{bmatrix} \\
mby &= \frac{\partial p}{\partial \epsilon_{yB}} = \begin{bmatrix} 0 \\ 0 \\ \frac{cos(A)}{Bcos(\theta)} \end{bmatrix} \\
mbz &= \frac{\partial p}{\partial \epsilon_{zB}} = \begin{bmatrix} 0 \\ 0 \\ \frac{-sin(I)sin(A)}{Bcos(\theta)} \end{bmatrix} \\
msx &= \frac{\partial p}{\partial \epsilon_{xS}} = \begin{bmatrix} 0 \\ 0 \\ \frac{sin(I)sin(A)(tan(\theta)cos(I)+sin(I)cos(A))}{\sqrt{2}} \end{bmatrix} \\
msy &= \frac{\partial p}{\partial \epsilon_{yS}} = \begin{bmatrix} 0 \\ 0 \\ \frac{sin(A)(tan(\theta)sin(I)cos(I)-cos(I)^2cos(A)-cos(A))}{2} \end{bmatrix} \\
msz &= \frac{\partial p}{\partial \epsilon_{zS}} = \begin{bmatrix} 0 \\ 0 \\ \frac{(cos(I)cos(A)^2-cos(I)(sin(A)^2-tan(\theta)sin(I)cos(A))}{2} \end{bmatrix}
\end{aligned}
\tag{40}
$$

5.5 Total position covariance

There are three types of errors to consider: well by well and global errors, random errors and systematic errors with each their own covariance matrix. The definition and the contribution of these covarinace matrices is given in the paper of Williamson [7] including the following equations in the remainder of this chapter.

For **random errors**:

$$[C]_{i,l}^{rand} = \sum_{k=1}^{K1} (e_{i,l,k})(e_{i,l,k})^T \tag{41}$$

And the total contribution over all survey legs is:

$$[C]_{i,K}^{rand} = \sum_{l=1}^{L-1} [C]_{i,l}^{rand} + \sum_{k=1}^{K-1} (e_{i,l,k})(e_{i,l,k})^T + (e^*_{i,L,K})(e^*_{i,L,K})^T \tag{42}$$

For **systematic errors**:

$$[C]_{i,l}^{syst} = \left(\sum_{k=1}^{K1} e_{i,l,k} \right) \left(\sum_{k=1}^{K1} e_{i,l,k} \right)^T \tag{43}$$

And the total contribution over all survey legs is:

$$[C]^{syst}_{i,K} = \sum_{l=1}^{L-1}[C]^{syst}_{i,l} + \left(\sum_{k=1}^{K-1} e_{i,L,k}e^*_{i,L,K}\right)\left(\sum_{k=1}^{K-1} e_{i,L,k}e^*_{i,L,K}\right)^T \tag{44}$$

For **well by well and global errors**:

$$E_{i,K} = \sum_{l=1}^{L-1}\left(\sum_{k=1}^{K1} e_{i,l,k}\right) + \sum_{k=1}^{K-1} e_{i,L,k} + e^*_{i,L,K} \tag{45}$$

And the total contribution over all survey legs is:

$$[C]^{well}_{i,K} = E_{i,K}E^T_{i,K} \tag{46}$$

Vector errors are summed into the positional uncertainty matrix. The total position covariance at survey station K is the sum of the contributions from all the types of error source:

$$[C]^{survey}_K = \sum_{i\in R}[C]^{rand}_{i,K} + \sum_{i\in S}[C]^{syst}_{i,K} + \sum_{i\in W,G}[C]^{well}_{i,K} \tag{47}$$

5.6 Transformation into borehole reference frame

The results are in an earth-referenced frame (north, east and vertical) denoted as "nev". The transformation of the covariance matrices and bias vectors into the borehole referenced frame (highside, lateral, along hole) denoted as "hla" is:

$$[C]_{hla} = [T]^T[C]_{nev}[T] \tag{48}$$

Where

$$[T] = \begin{bmatrix} cos(I_K)cos(A_K) & -sin(A_k) & sin(I_k)cos(A_k) \\ cos(I_K)cos(A_K) & cos(A_k) & sin(I_k)sin(A_k) \\ -sin(A_k) & 0 & cos(I_k) \end{bmatrix} \tag{49}$$

is a rotation matrix.

Summary of this chapter

In this chapter, the model assumptions of the Covariance analysis method are discussed to determine the positional uncertainty.

Then it was discussed how the covariance matrix is constructed. Four components are required to construct the covariance matrix; the correlation coefficients, the magnitudes, the differentials of the displacement and the weighting functions. More details about the four components are given in the sections that followed.

Thereafter, the total position covariance matrix is discussed based on different types of error that can occur and how these errors contribute via summation to the covariance matrix.

Finally, it is discussed how the results can be transformed back to the borehole reference frame.

6 Results of the methods

Four test trajectories have been used to determine the parameter estimates and their uncertainty. These test trajectories consist of the Along-Hole Depth and the Inclination, Azimuth and Toolface angles per survey station. The test trajectories can be found in the appendix.

Information about the test trajectories

- The following geomagnetic reference data was used in the test trajectories:
  $B_{tot} = 51\ \mu$T
  $\theta = 72°$
- Test trajectories 1, 2 and 3 are used to test the methods and to show that the methods work. Test 4 involves real values where the bias and scale factor errors are unknown.
- Test trajectories 1 and 2 have been chosen to determine whether the direction of the survey trajectory influences the parameter estimate and the parameter estimate uncertainty. The expectation is that the more variance there is in toolface, inclination and azimuth, that the results will improve.
- In test trajectories 1, 2 and 3, the inclination angle and the toolface angle are increased whereby the toolface angle always changes between a few fixed values.
- In test trajectories 1 and 2 is the azimuth angle fixed, in comparison to test trajectory 3 wherein the azimuth angle varies.

The results are divided into two parts: results for parameter estimates and their uncertainty, and results for position estimates and their uncertainty. The results for parameter uncertainty of Multi-Station Analysis, the probabilistic estimation from Weighted Least Squares and the results of an external party that also uses probabilistic estimation from Weighted Least Squares are given on the following pages. These results are followed by the positional uncertainty results of MSA combined with the Minimum Curvature Method, MSA combined with Covariance analysis method and the results of an external party that also uses MSA combined with Covariance analysis method. First some comments will be given about the results followed by a comparison and conclusions.

6.1 Parameter estimates and uncertainty results

Some comments on the parameter estimates and uncertainty results.

- It is immediately clear from tables 5 and 6 that the standard deviation of the bias error in the z direction, ϵ_{xB}, is much greater than for the other five sensor errors in all tests. The reason for this is that the toolface has a spread of 360 degrees, making it possible to calculate ϵ_{xB} and ϵ_{yB} very precisely. For example $Bx_m = Bx_c(1 + \epsilon_{xS}) + \epsilon_{xB}$ + noise at a toolface x. Now take a toolface x + 180 degrees, then $Bx_m = -Bx_c(1 + \epsilon_{xS}) + \epsilon_{xB}$ + noise. If these two are added together and divided by two and the scale factor and noise are ignored, the result is a very good estimate for ϵ_{xB}. Because Bz_m is independent on the toolface becomes the estimate of ϵ_{zB} more unreliable, and that is reflected in the results. In table 7, it is also visible that the standard deviation of the bias error in the z direction, ϵ_{xB}, is much greater than for the other five sensor errors in all tests the but the difference is less extreme.

- The estimates for the scale factor errors in test 3 in tables 5 and 6 are very poor.
- The estimates for the bias and scale factor error in the z direction are always worse than the other sensor error estimates in table 7.
- The estimates for the bias and scale factor errors in test 4 in table 7 are much bigger than expected and also bigger than the estimates in tables 5 and 6.
- The parameter uncertainty in all tests with the Covariance analysis method in table 7 is much smaller than expected and also smaller than the uncertainties in tables 5 and 6, see figure 6.
- The confidence interval of MSA almost always covers the true input value, in contrast to the probabilistic estimation from Weighted Least Squares method because it has an extremely small confidence interval, see figure 6.

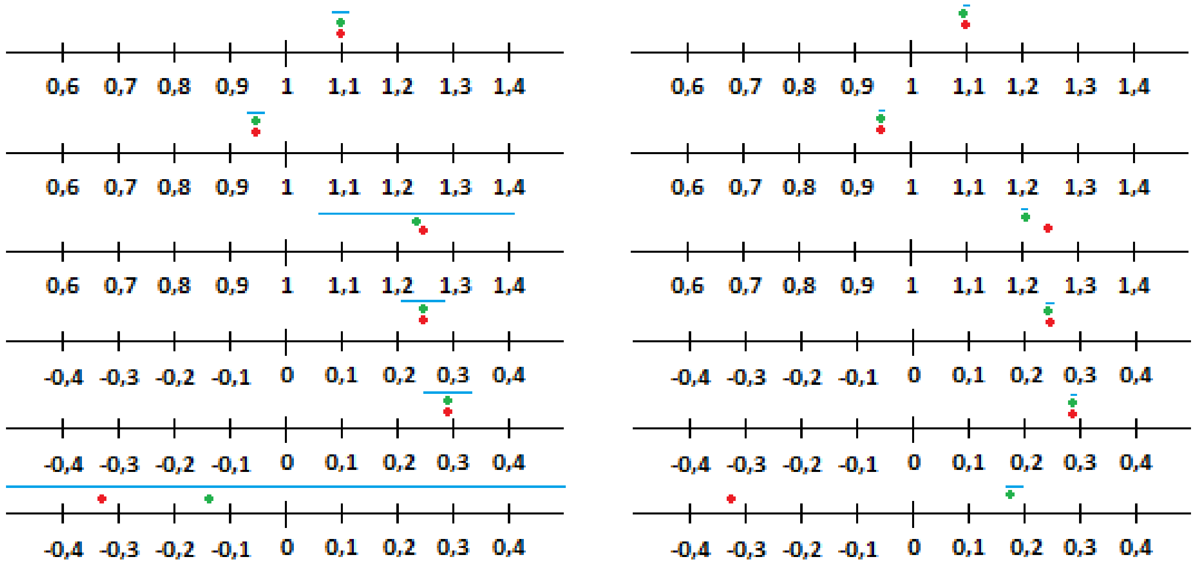

Figure 6: The parameter uncertainty of test 1 of MSA (left) and the probabilistic estimation from Weighted Least Squares (right) with the inputs (red), the estimates (green) and the confidence intervals (blue).

6.1.1 Multi-Station Analysis (Shell)

Test 1	Input	Estimate	3*Stdev	Confidence interval	Score
$1+\epsilon_{xS}$	1.100	1.100	0.013	(1.087; 1.113)	good
$1+\epsilon_{yS}$	0.950	0.950	0.011	(0.939; 0.961)	good
$1+\epsilon_{zS}$	1.250	1.236	0.175	(1.061; 1.411)	good
ϵ_{xB}	0.250	0.249	0.038	(0.211; 0.287)	good
ϵ_{yB}	0.295	0.296	0.041	(0.255; 0.336)	good
ϵ_{zB}	-0.335	-0.137	4.644	(-4.780; 4.507)	good
Test 2	Input	Estimate	3*Stdev	Confidence interval	Score
$1+\epsilon_{xS}$	1.008	1.008	0.033	(0.976; 1.041)	good
$1+\epsilon_{yS}$	0.992	0.992	0.032	(0.961; 1.024)	good
$1+\epsilon_{zS}$	1.008	1.010	0.089	(0.921; 1.098)	good
ϵ_{xB}	0.250	0.250	0.049	(0.201; 0.299)	good
ϵ_{yB}	-0.350	-0.349	0.048	(-0.397; -0.301)	good
ϵ_{zB}	0.850	0.773	4.374	(-3.601; 5.147)	good
Test 3	Input	Estimate	3*Stdev	Confidence interval	Score
$1+\epsilon_{xS}$	1.100	1.000	0.012	(0.989; 1.012)	bad
$1+\epsilon_{yS}$	0.950	1.000	0.012	(0.988; 1.012)	bad
$1+\epsilon_{zS}$	1.250	1.001	0.037	(0.963; 1.038)	bad
ϵ_{xB}	0.250	0.250	0.058	(0.192; 0.308)	good
ϵ_{yB}	0.295	0.295	0.077	(0.222; 0.369)	good
ϵ_{zB}	-0.335	-0.366	1.828	(-2.194; 1.465)	good
Test 4	Input	Estimate	3*Stdev	Confidence interval	Score
$1+\epsilon_{xS}$	N/A	1.000	0.018	(0.983; 1.018)	
$1+\epsilon_{yS}$	N/A	1.000	0.015	(0.984; 1.015)	
$1+\epsilon_{zS}$	N/A	1.018	0.038	(0.979; 1.056)	
ϵ_{xB}	N/A	-0.070	0.057	(-0.127; -0.013)	
ϵ_{yB}	N/A	-0.054	0.039	(-0.094; -0.015)	
ϵ_{zB}	N/A	-0.811	1.851	(-2.663; 1.040)	

Table 5: Test results for the parameter uncertainty with Multi-Station Analysis after MSA correction.

Where:
Estimate: is the mean values of all estimates.
Stdev: is the standard deviation of all estimates.
Conf. interval: is the 99,7% confidence interval which is the estimate plus and minus 3 standard deviations.

6.1.2 Probabilistic estimation from Weighted Least Squares

Test 1	Input	Estimate	3*Stdev	Confidence interval	Score
$1+\epsilon_{xS}$	1.100	1.091	0.010	(1.081; 1.101)	good
$1+\epsilon_{yS}$	0.950	0.947	0.012	(0.936; 0.959)	good
$1+\epsilon_{zS}$	1.250	1.200	0.085	(1.115; 1.285)	good
ϵ_{xB}	0.250	0.227	0.056	(0.172; 0.283)	good
ϵ_{yB}	0.295	0.311	0.057	(0.254; 0.367)	good
ϵ_{zB}	-0.335	-0.268	3.071	(-3.339; 2.803)	medium
Test 2	Input	Estimate	3*Stdev	Confidence interval	Score
$1+\epsilon_{xS}$	1.008	1.008	0.031	(0.977; 1.039)	good
$1+\epsilon_{yS}$	0.992	0.992	0.031	(0.960; 1.023)	good
$1+\epsilon_{zS}$	1.008	1.008	0.087	(0.921; 1.095)	good
ϵ_{xB}	0.250	0.248	0.076	(0.172; 0.325)	good
ϵ_{yB}	-0.350	-0.353	0.078	(-0.431; -0.274)	good
ϵ_{zB}	0.850	0.843	4.425	(-3.582; 5.268)	good
Test 3	Input	Estimate	3*Stdev	Confidence interval	Score
$1+\epsilon_{xS}$	1.100	1.000	0.005	(0.995; 1.005)	bad
$1+\epsilon_{yS}$	0.950	1.000	0.005	(0.995; 1.005)	bad
$1+\epsilon_{zS}$	1.250	1.000	0.014	(0.986; 1.014)	bad
ϵ_{xB}	0.250	0.250	0.093	(0.157; 0.343)	good
ϵ_{yB}	0.295	0.295	0.099	(0.196; 0.394)	good
ϵ_{zB}	-0.335	-0.335	1.973	(-2.308; 1.638)	good
Test 4	Input	Estimate	3*Stdev	Confidence interval	Score
$1+\epsilon_{xS}$	N/A	1.000	0.018	(0.982; 1.017)	
$1+\epsilon_{yS}$	N/A	0.999	0.016	(0.983; 1.015)	
$1+\epsilon_{zS}$	N/A	1.017	0.038	(0.979; 1.055)	
ϵ_{xB}	N/A	-0.055	0.085	(-0.139; 0.030)	
ϵ_{yB}	N/A	-0.066	0.063	(-0.129; -0.002)	
ϵ_{zB}	N/A	-0.773	1.829	(-2.602; 1.056)	

Table 6: Test results for the parameter uncertainty with probabilistic estimation from Weighted Least Squares of an external party.

Test 1	Input	Estimate	3*Stdev	Confidence interval	Score
$1+\epsilon_{xS}$	1.100	1.0998	0.0001	(1.0997; 1.0999)	good
$1+\epsilon_{yS}$	0.950	0.9501	0.0002	(0.9499; 0.9503)	good
$1+\epsilon_{zS}$	1.250	1.2066	0.0013	(1.2053; 1.2079)	bad
ϵ_{xB}	0.250	0.2489	0.0047	(0.2442; 0.2536)	good
ϵ_{yB}	0.295	0.2943	0.0019	(0.2924; 0.2962)	good
ϵ_{zB}	-0.335	0.1787	0.0185	(0.1602; 0.1972)	bad
Test 2	Input	Estimate	3*Stdev	Confidence interval	Score
$1+\epsilon_{xS}$	1.008	1.007	0.0017	(1.0053; 1.0087)	good
$1+\epsilon_{yS}$	0.992	0.9929	0.0016	(0.9913; 0.9945)	good
$1+\epsilon_{zS}$	1.008	1.0115	0.0029	(1.0086; 1.0144)	good
ϵ_{xB}	0.250	0.2428	0.0360	(0.2068; 0.2788)	good
ϵ_{yB}	-0.350	-0.3385	0.0361	(-0.3746; -0.3024)	good
ϵ_{zB}	0.850	0.6846	0.1207	(0.5639; 0.8053)	bad
Test 3	Input	Estimate	3*Stdev	Confidence interval	Score
$1+\epsilon_{xS}$	1.100	1.0735	0.0025	(1.0710; 1.0760)	good
$1+\epsilon_{yS}$	0.950	0.9619	0.0023	(0.9596; 0.9642)	good
$1+\epsilon_{zS}$	1.250	1.1457	0.0031	(1.1426; 1.1488)	bad
ϵ_{xB}	0.250	0.2212	0.0438	(0.1774; 0.2650)	good
ϵ_{yB}	0.295	0.276	0.0438	(0.2322; 0.3198)	good
ϵ_{zB}	-0.335	4.1879	0.1391	(4.0488; 4.3270)	bad
Test 4	Input	Estimate	3*Stdev	Confidence interval	Score
$1+\epsilon_{xS}$	N/A	1.7925	0.0017	(1.7908; 1.7942)	
$1+\epsilon_{yS}$	N/A	1.8188	0.0014	(1.8174; 1.8202)	
$1+\epsilon_{zS}$	N/A	1.1535	0.0030	(1.1505; 1.1565)	
ϵ_{xB}	N/A	15.5692	0.0290	(15.5402; 15.5982)	
ϵ_{yB}	N/A	-10.8773	0.0287	(-10.9060; -10.8486)	
ϵ_{zB}	N/A	-10.3187	0.1318	(-10.4505; -10.1869)	

Table 7: Test results for the parameter uncertainty with probabilistic estimation from Weighted Least Squares.

6.2 Position estimate and uncertainty results

Some comments on the position estimate and positional uncertainty results.

- The standard deviation in tables 8, 9 and 10 for test 1 is quite large compared to the other test 2, 3 and 4.
- The position estimates and position standard deviations have comparable results for all tests via the Multi-Station Analysis method and the Covariance analysis method of an external party in tables 8 and 9.
- The position standard deviations calculated via the Covariance analysis method, if the sensor errors are considered random, in table 10 are smaller than in the tables 8 and 9, see figure 7.
- The position standard deviations calculated via the Covariance analysis method, if the sensor errors are considered systematic, in table 10 are bigger than in the tables 8 and 9, see figure 7.
- The convariance analysis method to determine the position uncertainty described in this paper in chapter 5 gives no estimate for the position, only and position uncertainty.
- The are no results for the position in the z direction because there is no uncertainty in that direction, only in the x and y direction.

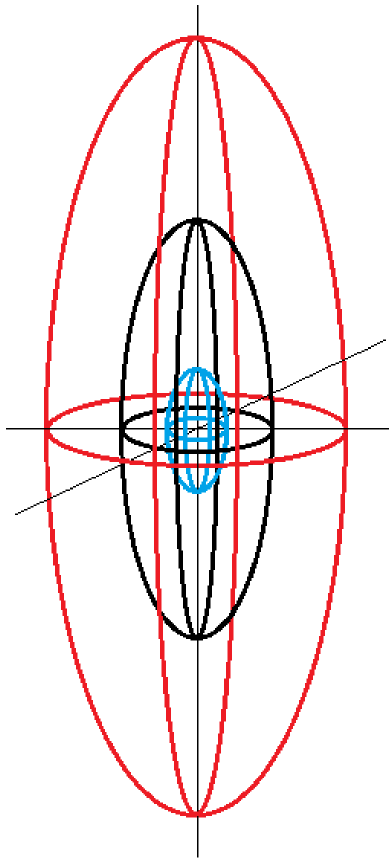

Figure 7: The position uncertainty ellipsoid of test 1 of MSA combined with the MCM (black), MSA combined with the Covariance method with sensor errors considered systematic (red) and MSA combined with the Covariance method with sensor errors considered random (blue).

6.2.1 MSA combined with Minimum Curvature Method (Shell)

Test 1	Estimate	3*Stdev	Confidence interval
final pos x	128.15	472	(-343.85; 600.15)
final pos y	-381.06	102	(-483.06; -279.06)
final pos z			
Test 2	Estimate	3*Stdev	Confidence interval
final pos x	3.45	65.43	(-61.98; 68.88)
final pos y	-29.80	5.81	(-35.61; -24.00)
final pos z			
Test 3	Estimate	3*Stdev	Confidence interval
final pos x	6.565	15.560	(-9.00; 22.13)
final pos y	-5.460	11.575	(-17.04; 6.11)
final pos z	-0.026	0.015	(-0.041; -0.011)
Test 4	Estimate	3*Stdev	Confidence interval
final pos x	0.300	10.142	(-9.842; 10.442)
final pos y	-0.620	19.330	(-19.950; 18.710)
final pos z	0.002	0.0045	(-0.003; 0.007)

Table 8: Test results for the positional uncertainty of MSA combined with the Minimum Curvature Method after MSA correction.

Where:
Estimate: is the mean values of all estimates in meters.
Stdev: is the standard deviation of all estimates in meters.
Conf. interval: is the 99,7% confidence interval which is the estimate plus and minus 3 standard deviations.

6.2.2 MSA combined with the Covariance analysis method

Test 1	Estimate	3*Stdev	Confidence interval
final pos x	121.9	367.9	(-246.07; 489.79)
final pos y	-348.2	98.5	(-446.68; -249.76)
final pos z			

Test 2	Estimate	3*Stdev	Confidence interval
final pos x	3.55	68.62	(-65.07; 72.17)
final pos y	-30.93	5.34	(-36.27; -25.59)
final pos z			

Test 3	Estimate	3*Stdev	Confidence interval
final pos x	7.42	24.37	(-16.95; 31.79)
final pos y	-5.95	22.15	(-28.10; 16.20)
final pos z	0.020		

Test 4	Estimate	3*Stdev	Confidence interval
final pos x	0.27		
final pos y	-0.55	16.94	(-17.49; 16.39)
final pos z		32.36	

Table 9: Test results for the positional uncertainty of MSA combined with the Covariance analysis method of an external party after MSA correction.

Test 1	Estimate	3*Stdev	Confidence interval
final pos x		164.8163	
final pos y		44.1624	
final pos z			

Test 2	Estimate	3*Stdev	Confidence interval
final pos x		38.6220	
final pos y		3.3790	
final pos z			

Test 3	Estimate	3*Stdev	Confidence interval
final pos x		17.4167	
final pos y		12.0197	
final pos z			

Test 4	Estimate	3*Stdev	Confidence interval
final pos x		6.1023	
final pos y		11.5852	
final pos z			

Table 10: Test results for the positional uncertainty of MSA combined with the Covariance analysis method if the sensor errors are considered random after MSA correction.

Test 1	Estimate	3*Stdev	Confidence interval
final pos x		913.8391	
final pos y		244.8624	
final pos z			
Test 2	Estimate	3*Stdev	Confidence interval
final pos x		165.6688	
final pos y		14.4941	
final pos z			
Test 3	Estimate	3*Stdev	Confidence interval
final pos x		54.1808	
final pos y		42.9227	
final pos z			
Test 4	Estimate	3*Stdev	Confidence interval
final pos x		39.0387	
final pos y		74.0197	
final pos z			

Table 11: Test results for the positional uncertainty of MSA combined with the Covariance analysis method if the sensor errors are considered systematic after MSA correction.

6.3 Comparison of results and methods

There a number of clear differences and similarities when the parameter and position results are compared. The following conclusions can be drawn about the parameter estimates and their uncertainty:

Conclusions about parameter estimates and uncertainty

- **Multi-Station Analysis:** The parameter estimates are good and as expected. Only the results for the scale factor errors in test trajectory 3 are poor.
- **Probabilistic estimation from Weighted Least Squares:** The parameter estimates are quite well. Only the estimates for the bias and scale factor errors in the z direction are bad in all tests. The standard deviation however is very small and smaller than expected. It can therefore be concluded that parameter uncertainty cannot be accurately determined and is unreliable with this method.
- **Probabilistic estimation from Weighted Least Squares (external):** An external party got good results and comparable with the results of Multi-Station Analysis. Those results are therefore included in the comparison.
- Overall are the parameter estimates fairly accurate with both the probabilistic estimation from Weighted Least Squares and Multi-Station Analysis. There are a few exceptions. The scale factor error estimates can be poor if the azimuth angle varies instead of being fixed. Also are the estimates in the z direction sometimes worse than in the x and y direction. Therefore it must be borne in mind that a fixed or variable azimuth angle gives different results for the scale factor errors and that the accuracy of the estimates can depend on the direction.
- Overall is the parameter uncertainty of Multi-Station Analysis good but from the probabilistic estimation from Weighted Least Squares too small and unreliable.

The following conclusion can be drawn about the position estimates and their uncertainty.

Conclusions about position estimates and uncertainty

- **MSA combined with the Minimum Curvature Method:** The position estimates and position standard deviations are good.
- **MSA combined with the Covariance analysis method:** The position standard deviations calculated via the Covariance analysis method are much smaller if the sensor errors are considered random and much bigger if the sensor errors are considered systematic compared with the position standard deviations in the tables 8 and 9. Considering the sensor errors random gives an underestimate (an too optimistic uncertainty) and considering the sensor errors systematic gives an overestimate (an a too pessimistic uncertainty).
- **MSA combined with the Covariance analysis method (external):** An external party got good results in line with the results of MSA combined with the Minimum Curvature Method.

- Overall is the positional uncertainty correct but it must be taken into consideration that the choice to consider the sensor errors random or systematic has a major influence on overestimating or underestimating the positional uncertainty.

- Secondly, it is clear that the uncertainty for the parameter estimate with the covariance method as described in this paper is considerably smaller than with the Multi-Station Analysis method and the covariance method of the external party.

- The position estimates and position standard deviations have comparable results for all tests via the Multi-Station Analysis method and the covariance method of an external party. The position standard deviations calculated via the Covariance analysis method described in this paper are smaller or bigger.

The methods are very similar in the first instance. The three basic steps as explained in chapter 2 are the same for both methods. The difference lies in step 4 where noise is added to the measurements and estimates are made for the parameters and the position and their uncertainty.

7 Conclusions

The main interest of this thesis was to assess the uncertainty in the final position of the drill bit of MSA corrected survey data. In addition, the aim was to reproduce two methods to determine parameter and positional uncertainty in order to compare results and to create more clarity and consensus on determining positional uncertainty in the well-bore industry.

Firstly, a model is delivered to estimate the positional uncertainty of a well survey that has been MSA corrected based on the method of covariances. This model corresponds to the results of commercial software from Shell, but a comment has to be made.
A choice must be made to which category the parameters belong. The choice to consider the positional uncertainty randomly or systematically gives a large underestimate or overestimate of the positional uncertainty and this must therefore be consciously considered.
In this thesis it is decided to systematically consider the parameter and position estimates and to randomly consider the parameter and position uncertainty.
The choice to systematically consider parameter and position estimates is because it is assumed that sensor error and position estimates between survey stations are directly linked, and therefore correlated, and would have the same underlying value from station to station.
On the other hand, it was decided to randomly consider parameter and position uncertainty, because there is no reason why it should be linked between survey stations and therefore are considered independent.

Secondly, the MSA estimates are taken into account. MSA gives perfect estimates, exactly the same as the input, for the six MWD magnetometer sensor error parameters $e_{xB}, e_{yB}, e_{zB}, e_{xS}, e_{yS}$ and e_{zS} when there is no noise on the measurement data. The estimates are still good if there is noise but not perfect. However, the following points should be taken in consideration:

a) The MSA estimates are perfect and are irrespective of the variation and direction in the survey toolface and inclination angle but it seems that a variation in the azimuth angle causes worse estimates for the scale factor errors, of which one must be aware.

b) The MSA estimates are perfect if there is no combined noise present in the sensors, the telemetry and surface data acquisition system.

c) The MSA estimates are perfect if the exact geomagnetic field parameters (magnetic field strength and dip angle) are known and used.

Thereafter, the MSA uncertainties are considered. It is important to remember that accelerometer errors are not included in this thesis. The MSA estimate for the six MWD magnetometer sensor error parameter uncertainties is dependent on some factors:

a) The MSA uncertainties are dependent of the variation and direction in the survey toolface, inclination and azimuth.

b) The MSA uncertainties are dependent on the combined noise in the sensors, the telemetry, the surface data acquisition system. The MSA uncertainties increase if there is more noise.

c) The MSA uncertainties are dependent on the value and the uncertainty of the geomagnetic field parameters (magnetic field strength and dip angle). The MSA uncertainties grow if this uncertainty is bigger.

d) The MSA uncertainties are dependent on the assumption that the accelerometers measurement are perfect. If the measurement is not perfect, in other words there is noise, the MSA uncertainties will increase.

Then, the two methods used to estimate the sensor errors with their uncertainty can be compared, MSA and probabilistic estimation from Weighted Least Squares.
Shell assumes that the MSA estimate for the six MWD magnetometer sensor error uncertainties is not dependent on the six MWD magnetometer sensor error values themselves. This is contrary to the method of probabilistic estimation from Weighted Least Squares, whereby the uncertainty in the sensor error estimates is not determined by noise in the sensors but by the calibration uncertainty in the sensors (70nT for bias and 0.0016 for scale factor errors).

As second to last comment, replication of the published ISCWSA error model to estimate the survey positional uncertainty through the method of covariances after survey data is MSA corrected learns the following:

a) The survey positional uncertainty is significantly overestimated when the positional uncertainty is considered systematic. The decision to consider the positional uncertainty systematic can be based on a certain dependence between different survey stations, which may not be the case, which, however, results in an overestimate of uncertainty.

b) The survey position uncertainty is significantly underestimated when the six MWD magnetometer sensor error parameter uncertainties are treated as random. If the position uncertainty is considered randomly, independence between survey stations is assumed, causing the uncertainty to rise slower and therefore gives a small uncertainty.

It seems that there is a certain balance between systematic and random consideration of positional uncertainties, but a good choice for the balance has not yet been found.

As final comment, the method of probabilistic estimation from Weighted Least Squares is able to estimate the six sensor error parameters, $e_{xB}, e_{yB}, e_{zB}, e_{xS}, e_{yS}$ and e_{zS}, quite well. However, the method generates at least 10 times or more smaller magnetometer sensor error uncertainties than the Monte Carlo method. Due to the extremely small uncertainties, do the uncertainty intervals of 99.7% not always overlap the actual value of the sensor error. This is unacceptable, which means that the method cannot be considered reliable.
Why the uncertainties are extremely small is not entirely clear. It is true that the parameter uncertainty only depend on the matrices H, P and R. Increasing the values in those matrices increases the parameter uncertainty, but this is often at the expense of the sensor error estimates that deteriorate more than the sensor error uncertainty improves in the sense that the uncertainty intervals are reasonable.

Everything considered, MSA is a reliable method to estimate parameters with their uncertainty, contrary to the probabilistic estimation from Weighted Least Squares method which gives unreliable uncertainties.
Considering the positional uncertainty, MSA in combination with the Minimum Curvature method and MSA in combination with the Covariance analysis method are both reliable methods. However, the position uncertainty from the Covariance analysis method significantly depends on the choice to consider positional uncertainty systematically or randomly.

8 Recommendations

There are a number of recommendations that can be made regarding follow-up research.

a) Mature the analytical ISCWSA model to estimate the survey positional uncertainty through the method of covariances, described in chapter 5, after survey data is MSA corrected.

b) Investigate a valid foundation to consider the positional uncertainty systematically or randomly or a balanced combination of both for the Covariance analysis method.

c) Investigate why the method of probabilistic estimation from Weighted Least Squares, described in chapter 4, generates significant smaller magnetometer sensor error uncertainties when compared with the Monte Carlo method using MSA.

Appendix

8.1 Example calculation of minimum curvature method

Take for example **station 1**:
Depth = 3500 ft
Inclination = 15 degrees (I_1)
Azimuth = 20 degrees (Az_1)

And **Station 2**:
Depth = 3600 ft
Inclination = 25 degree (I_2)
Azimuth = 45 degree (I_2)

Then MD = 3600 3500 = 100 ft.
Enter the data in formula 18 gives:
$\Delta North$ = 27.22 ft
$\Delta East$ = 19.45 ft
ΔTVD = 93.01 ft
β = 0.22605 radian (12.95 degrees)
RF = 1.00408

8.2 Test trajectories

Where:
AHD is the Along Hole Depth in meters.
Tlf is the Toolface angle in degrees.
Inc is the Inclination angle in degrees.
Azi is the Azimuth angle in degrees.

AHD	Tlf	Inc	Azi
0	25	75,5	75
20	115	76	75
40	205	76,5	75
100	295	77	75
200	25	77,5	75
300	115	78	75
400	205	78,5	75
500	295	79	75
600	25	79,5	75
700	25	77,5	75
800	115	78	75
900	25	77,5	75
1000	115	78	75
1100	205	78,5	75
1200	295	79	75
1300	25	79,5	75
1400	115	80	75
1500	205	80,5	75
1600	295	81	75
1700	25	81,5	75
1800	115	82	75
1900	205	82,5	75
2000	295	83	75
2100	25	83,5	75
2200	115	84	75
2300	205	84,5	75
2400	295	85	75
2500	25	85,5	75
2600	115	86	75
2700	205	86,5	75
2800	295	87	75
2900	25	87,5	75
3000	115	88	75

Table 12: Test trajectory 1.

AHD	Tlf	Inc	Azi
0	25	0	85
1000	25	0	85
1500	115	0	85
1550	205	2	85
1600	295	4	85
1650	25	6	85
1700	115	8	85
1750	205	10	85
1800	295	12	85
1850	25	14	85
1900	115	16	85
1950	205	18	85
2000	295	20	85
2050	25	22	85
2100	115	24	85
2150	205	26	85
2200	295	28	85
2250	25	30	85
2300	115	32	85
2350	205	34	85
2400	295	36	85
2450	25	38	85
2500	115	40	85
2550	205	42	85
2600	295	44	85
2650	25	46	85
2700	115	48	85
2750	205	50	85
2800	295	52	85
2850	25	54	85
2900	115	56	85
2950	205	58	85
3000	295	60	85

Table 13: Test trajectory 2.

AHD	Tlf	Inc	Azi
0	25	0	2
1000	115	0	5
1100	205	3	8
1200	295	6	11
1300	25	9	14
1400	115	12	17
1500	205	15	20
1600	295	18	23
1700	25	21	26
1800	115	24	29
1900	205	27	32
2000	295	30	35
2100	25	33	38
2200	115	36	41
2300	205	39	44
2400	295	42	47
2500	25	45	50
2600	115	48	53
2700	205	51	56
2800	295	54	59
2900	25	57	62
3000	115	60	65

Table 14: Test trajectory 3.

AHD	Tlf	Inc	Azi
1602,25	57,47	37,09	26,65
1641,19	157,34	38,52	26,89
1683,90	334,17	40,23	28,09
1725,06	278,49	42,49	28,32
1766,64	53,12	44,77	27,58
1807,38	87,39	47,28	27,40
1848,90	201,18	49,3	27,58
1891,08	289,05	51,52	28,13
1932,16	55,01	53,88	27,41
1972,99	288,32	55,3	27,56
2014,02	122,17	55,52	26,99
2055,03	242,83	55,47	27,10
2095,73	135,84	55,11	26,54
2137,24	84,44	55,38	26,26
2177,30	245,98	55,58	26,22
2219,06	141,63	55,55	24,97
2259,79	177,01	55,33	25,66
2301,06	67,73	55,37	26,31
2342,25	310,48	55,93	27,58
2383,66	58,27	56,1	27,10
2423,75	317,02	56,17	27,91
2468,07	268,62	55,87	28,34
2508,95	280,56	55,88	29,03
2550,01	187,08	56,02	29,82
2591,14	293,47	55,78	29,42
2631,93	196,99	55,22	30,14
2673,35	134,46	55,2	29,53
2714,18	31,71	55,17	27,12
2755,74	141,39	54,2	27,96
2797,20	83,97	52,87	27,74
2837,85	282,26	50,41	27,64
2879,04	209,04	48,11	28,17
2920,34	40,39	46,37	26,20
2961,35	154,3	43,94	27,35
3000,38	161,35	42,85	27,88
3033,51	87,36	40,5	28,51
3075,63	211,01	37,22	28,80
3113,23	175,54	34,82	29,16

AHD	Tlf	Inc	Azi
3156,61	133,09	32,44	29,49
3197,55	134,01	30,63	30,45
3237,99	21,85	28,85	31,20
3280,17	29,48	27,38	31,60
3321,07	200,05	25,97	28,84
3362,05	74,79	23,51	26,80
3403,11	243,22	20,75	28,92
3444,14	170,14	19,05	29,12
3485,36	259,13	17,4	27,04
3526,42	49,5	15,63	24,82
3567,26	317,62	13,87	22,57
3608,15	274,28	11,19	25,91
3648,77	274,96	9,97	24,67
3690,49	352,47	8,77	18,74
3703,58	15,08	7,72	17,65

Table 15: Test trajectory 4 with real survey data.

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