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TECHNISCHE HOGESCHOOL DELFT

COMPARISON OF CALCULATED AND MEASURED HEAVING AND PITCHING MOTIONS OF A SERIES 60, C_B=.70 SHIP MODEL IN REGULAR LONGITUDINAL WAVES.

By Prof.ir J. Gerritsma and W. Beukelman.

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1. Introduction.

In view of recommendation 5.5. of the Seakeeping Committee of the I.T.T.C. 1963, the results of a motion test in regular longitudinal waves are compared with corresponding calculated motions.

In this report the frequency characteristics of the heaving and pitching motion, as determined by the experiment and the calculation are given in tabular and graphical form to facilitate the comparison .

2. The shipform.

Experiment and calculation were carried out for the C_B = .70 parent form of the Series 60, which is described in detail in [1].

The main particulars of the ship model, corresponding to this shipform, which was used for the experiments are given in Table 1. (See page 6).

3. Model tests.

3.1. Experimental procedure.

The experiments in waves were carried out with a towed model, which was free to pitch, heave and surge but restrained for roll, sway and yaw. A gravity type dynamometer with a 1 : 5 ratio between towing force and towing weight provided an almost constant towing force, acting through the centre of gravity of the model.

Pitch and heave were measured by micro-friction potentiometers, where a vertically sliding rod, guided in a light sub-carriage was used for reference. The wave height was measured about four meters in front of the model with a resistance type wave height meter.

To determine the phase angles of the motions with respect to the wave, the mean longitudinal position of the model for each run was read by eye from a linear scale attached to the towing carriage. Due to the small surging motion in head waves this method is sufficiently accurate.

Heave, pitch and wave were recorded on an UV recorder as a function of time and the records were analysed for motion amplitudes and phase angles.

The mean forward speed of the model was obtained by measuring the speed of the towing carriage and by adding the relative speed of the sub carriage to which the model is connected via the vertical sliding rod.

3.2. Test conditions.

The motion tests were carried out for two wave heights and a wide range of forward speeds and wave lengths. A summary of the test conditions is given in Table 2. (See page 6).

3.3. Experimental results.

The wave is defined by the vertical displacement of the water surface at the midship section, assuming that the surging motion is zero. The wave and the motions may then be described by:

$$\xi = \zeta_a \cos(\omega_e t + kx) \quad \text{- wave}$$

$$z = z_a \cos(\omega_e t + \varepsilon_{z\xi}) \quad \text{- heave}$$

$$\theta = \theta_a \cos(\omega_e t + \varepsilon_{\theta\xi}) \quad \text{- pitch}$$

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where the circular frequency of encounter follows from:

$$\omega_e = \omega + \frac{\omega^2}{g} V$$

The definition of wave and motions is illustrated in Figure 1.
The test results are given as dimensionless amplitude characteristics (z_a/ξ_a for heave, $\theta_a/k\xi_a$ for pitch) and phase angles ($\varepsilon_{z\xi}$, $\varepsilon_{\theta\xi}$, $\varepsilon_{\theta z}$).

The experimental results are summarized in Table 3. (See page 7).

4. Calculation of the heaving and pitching motions.

The equations of motion are:

$$\left.\begin{aligned} \rho g \nabla \ddot{z} &= F \text{ (heave)} \\ I_{yy}\ddot{\theta} &= M \text{ (pitch)} \end{aligned}\right\} \qquad (2)$$

where:

$$F = \int_L (F_1' + F_2' + F_3')\, dx_b$$

and:

$$M = -\int_L (F_1' + F_2' + F_3') x_b\, dx_b \qquad (3)$$

By using the strip theory and taking into account the effect of forward speed [3, 4, 5, 6] the following expressions for the sectional values F' can be found.

$$F_1' = -2\rho g y_w (z - x_b \theta - \xi^*)$$

$$F_2' = -N'(\dot{z} - x_b\dot{\theta} + V\theta - \dot{\xi}^*)$$

$$F_3' = -\frac{d}{dt}\left\{m'(\dot{z} - x_b\dot{\theta} + V\theta - \dot{\xi}^*)\right\} \qquad (4)$$

or:

$$F_3' = -m'(\ddot{z} - x_b\ddot{\theta} + 2V\dot{\theta} - \ddot{\xi}^*) + V\frac{dm'}{dx_b}(\dot{z} - x_b\dot{\theta} + V\theta - \dot{\xi}^*)$$

where:

y_w - the half width of the waterline
m' - the sectional added mass
N' - the sectional damping coefficient,

and:

$$\xi^* = \xi\left(1 - \frac{k}{y_w}\int_{-T}^{0} y_b\, e^{kz_b}\, dz_b\right)$$

This may be written as:

$$\zeta^* = \zeta e^{-kT^*}$$

where T^* is determined from:

$$T^* = -\frac{1}{k} \lg\left(1 - \frac{k}{y_w} \int_{-T}^{0} y_b \, e^{kz_b} \, dz_b\right)$$

A numerical analysis of F_1 shows that T^* is approximately equal to the mean draft of a section: $\bar{T} = \frac{A_x}{2y_w}$, at least for the shipform under consideration.

The equations of motion (2) are usually written in the form of two coupled second order differential equations with frequency dependent coefficients, namely:

$$\left.\begin{aligned} &\text{heave} \quad (a+\rho\nabla)\ddot{z} + b\dot{z} + cz - d\ddot{\theta} - e\dot{\theta} - g\theta = F_a \cos(\omega_e t + \varepsilon_{F\zeta}) \\ &\text{pitch} \quad (A+I_{yy})\ddot{\theta} + B\dot{\theta} + C\theta - D\ddot{z} - E\dot{z} - Gz = M_a \cos(\omega_e t + \varepsilon_{M\zeta}) \end{aligned}\right\} \quad (5)$$

From (1), (2) and (3) it follows that:

$$\left.\begin{aligned} a &= \int_L m' \, dx_b & d &= \int_L m' \, x_b \, dx_b \\ b &= \int_L N' \, dx_b & e &= \int_L N' \, x_b \, dx_b - Vm \\ c &= \rho g A_w & g &= \rho g S_w - Vb \\ A &= \int_L m' \, x_b^2 \, dx_b & D &= \int_L m' \, x_b \, dx_b \\ B &= \int_L N' \, x_b^2 \, dx_b & E &= \int_L N' \, x_b \, dx_b + Vm \\ C &= \rho g I_w - VE & G &= \rho g S_w \end{aligned}\right\} \quad (6)$$

and:

$$\frac{F_a}{\zeta_a} \begin{matrix} \cos\varepsilon_{F\zeta} \\ \sin\varepsilon_{F\zeta} \end{matrix} = +2\rho g \int_L y_w \, e^{-kT^*} \begin{matrix} \cos kx_b \\ \sin kx_b \end{matrix} \, dx_b +$$

$$-\omega(\omega + kV) \int_L m' \, e^{-kT^*} \begin{matrix} \cos kx_b \\ \sin kx_b \end{matrix} \, dx_b + \quad (7)$$

$$\mp \omega \int_L N' \, e^{-kT^*} \begin{matrix} \sin kx_b \\ \cos kx_b \end{matrix} \, dx_b .$$

$$\frac{M_a}{\zeta_a}\begin{matrix}\cos\varepsilon_{M\zeta}\\ \sin\varepsilon_{M\zeta}\end{matrix} = -2\rho g\int_L y_w x_b\, e^{-kT^*}\begin{matrix}\cos kx_b\\ \sin kx_b\end{matrix}\, dx_b +$$

$$+\omega(\omega + kV)\int_L m' x_b\, e^{-kT^*}\begin{matrix}\cos kx_b\\ \sin kx_b\end{matrix}\, dx_b +$$

$$\pm\omega\int_L N' x_b\, e^{-kT^*}\begin{matrix}\sin kx_b\\ \cos kx_b\end{matrix}\, dx_b + \qquad (7)$$

$$\pm\omega V\int_L m'\, e^{-kT^*}\begin{matrix}\sin kx_b\\ \cos kx_b\end{matrix}\, dx_b .$$

In the expressions (6) and (7) the sectional added masses m' and the sectional damping coefficients N' according to Tasai's method were used [7]. In this method the cross-sections of the ship are approximated only by a two coefficient transformation of the unit circle.

The results of the calculation are given in Table 4. (See pages 8, 9, 10).

5. Conclusions.

For comparison purposes the experimental and calculated frequency characteristics are plotted in Figure 2. Good agreement is shown for the motion amplitudes and the phases of the motions, especially for the larger wave-length/ship-length ratio's.

The experiment shows a satisfactory linearity of the motion in the considered wave height range.

The regions where water is shipped on deck are indicated in Figure 2 for the $\frac{L_{pp}}{40}$ wave height. Also in these regions the linearity as well as the calculation is satisfactory.

The results of earlier tests with a $L_{pp} = 2.438$ m model, as given in [2] agree very well with the present experiment in which a greater range of wave lengths was considered. The new heave values are slightly higher for wave lengths larger than 1.2 L.

Table 1. Main particulars of the ship model.

Length between perpendiculars	2.258	m
Length on the waterline	2.296	m
Breadth	.322	m
Draught	.129	m
Volume of displacement	.0657	m^3
Blockcoefficient	.700	
Coefficient of mid length section	.986	
Prismatic coefficient	.710	
Waterplane area	.572	m^2
Waterplane coefficient	.785	
Longitudinal moment of inertia of waterplane area	.1685	m^3
L.C.B. forward of $L_{pp}/2$	.011	m
Centre of effort of waterplane area aft $L_{pp}/2$	.038	m
Radius of gyration	.25	L_{pp}.

Table 2. Test conditions.

Speed	F_n = .15, .20, .25, .30.
Wave length ratio	λ/L_{pp} = .6, .8, 1.0, 1.2, 1.4, 1.6, 1.8.
Wave height	$2\,\zeta a/L_{pp}$ = 1/50, 1/40.

Table 3.

Measured heave and pitch frequency characteristics. Series .60, C_B=.70.

	λ/L_{pp}	$\omega_e\sqrt{\frac{L}{g}}$	$2\zeta_a/L_{pp}$ = 1/40					$2\zeta_a/L_{pp}$ = 1/50				
			z_a/ζ_a	$\theta_a/k\zeta_a$	$\varepsilon_{z\zeta}$	$\varepsilon_{\theta\zeta}$	$\varepsilon_{\theta z}$	z_a/ζ_a	$\theta_a/k\zeta_a$	$\varepsilon_{z\zeta}$	$\varepsilon_{\theta\zeta}$	$\varepsilon_{\theta z}$
F_n = .15	0.6	4.93	0.122	0.034	-	-	-	-	-	-	-	-
	0.8	3.98	0.181	0,231	- 37*	+173*	-150*	0.135	0.211	- 50*	+155*	-155*
	1.0	3.45	0.818	0.712	- 38	-158	-120	0.861	0.738	- 34	-158	-124
	1.2	3.07	0.945	1.063	+ 4	-127	-131	1.030	1.117	+ 6	-131	-137
	1.4	2.79	0.880	1.121	- 4	-112	-108	0.878	1.161	+ 10	-116	-126
	1.6	2.57	0.802	1.132	+ 5	-105	-110	0.905	1.198	+ 6	-112	-118
	1.8	2.39	0.875	1.013	+ 16	-100	-116	0.900	1.157	0	-120	-120
F_n = .20	0.6	5.33	0.081	0.031	-	-	-	-	-	-	-	-
	0.8	4.38	0.110	0.175	- 56*	+154*	-150*	0.098	0.167	- 51*	+142*	-167*
	1.0	3.77	0.626	0.602	- 66	-178	-112	0.741	0.638	- 64	-180	-116
	1.2	3.34	1.271	1.031	- 20	-144	-124	1.377	1.040	- 22	-149	-127
	1.4	3.02	1.228	1.337	- 10	-135	-125	1.238	1.319	+ 6	-128	-134
	1.6	2.77	1.019	1.220	+ 2	-116	-118	1.089	1.344	+ 6	-119	-125
	1.8	2.57	1.002	1.138	+ 14	-105	-119	1.023	1.253	+ 3	-120	-123
F_n = .25	0.6	5.86	0.062	0.021	-	-	-	-	-	-	-	-
	0.8	4.77	0.081	0,138	- 71*	+137*	-152*	0.073	0.123	- 48*	+130	+178*
	1.0	4.08	0.473	0.500	- 97	+162*	-101	0.542	0.526	- 86	+166	-108
	1.2	3.60	1.312	0.979	- 42	-155	-113	1.334	0.953	- 54	-166	-112
	1.4	3.24	1.578	1.220	- 23	-144	-121	1.689	1.283	- 21	-148	-127
	1.6	2.97	1.304	1.295	- 3	-128	-125	1.417	1.424	+ 1	-128	-129
	1.8	2.74	1.165	1.275	+ 9	-112	-121	1.200	1.378	+ 4	-121	-125
F_n = .30	0.6	6.38	0.062	0.012	-	-	-	-	-	-	-	-
	0.8	5.16	0.071	0.105	- 70*	+128*	-162*	0.057	0.095	- 66*	+118*	-176*
	1.0	4.39	0.330	0.392	-114	+140*	-106	0.370	0.408	- 98	+145*	-117
	1.2	3.86	1.135	0.938	- 81	-179	- 98	1.143	0.887	- 84	+174*	-102
	1.4	3.47	1.711	1.194	- 47	-152	-105	1.906	1.166	- 45	-160	-115
	1.6	3.16	1.602	1.311	- 20	-141	-121	1.852	1.348	- 15	-142	-127
	1.8	2.92	1.378	1.398	+ 2	-121	-123	1.419	1.437	+ 1	-126	-127

Phase angles are given in degrees. An asteric indicates that in Figure 2 the plotted phase angles were varied ± 360 degrees for ease of plotting.

Table 4.

Calculated heave and pitch frequency characteristics Series 60; C_B=.70.

	λ/L_{pp}	$\omega_e\sqrt{\frac{L_{pp}}{g}}$	z_a/ζ_a	$\theta_a/k\zeta_a$	$\varepsilon_{z\zeta}$	$\varepsilon_{\theta\zeta}$	$\varepsilon_{\theta z}$
F_n = 0	.60	3.24	.396	.017	+110	- 12	-122
	.65	3.11	.346	.071	+117	-130	+113
	.70	3.00	.278	.154	+118	-128	+114
	.75	2.89	.209	.233	+113	-124	+123
	.80	2.80	.152	.305	+ 98	-120	+142
	.85	2.72	.124	.372	+ 70	-117	+173
	.90	2.64	.138	.430	+ 40	-114	-154
	.95	2.57	.179	.482	+ 22	-112	-134
	1.00	2.51	.227	.528	+ 13	-110	-123
	1.20	2.29	.412	.668	+ 1	-105	-106
	1.40	2.12	.547	.758	- 1	-102	-101
	1.60	1.98	.644	.819	- 2	-100	- 98
	1.80	1.87	.715	.862	- 2	- 98	- 96
F_n = .15	.60	4.93	.101	.039	+ 13*	- 46*	- 59
	.65	4.56	.127	.030	+ 23*	0 *	- 23
	.70	4.34	.127	.054	+ 36*	+ 75*	+ 39
	.75	4.15	.088	.130	+ 59*	+108*	+ 49
	.80	3.98	.048	.243	+166*	+127*	- 39
	.85	3.83	.210	.389	-128	+145*	- 87
	.90	3.69	.446	.535	-102	+161*	- 97
	.95	3.56	.693	.658	- 81	+176	-103
	1.00	3.45	.894	.753	- 62	-173	-111
	1.20	3.07	1.051	1.004	- 14	-142	-128
	1.40	2.79	.926	1.056	- 3	-124	-121
	1.60	2.57	.895	1.060	- 2	-115	-113
	1.80	2.39	.901	1.057	- 2	-110	-108

Phase angles are given in degrees.

Table 4 continued.

Calculated heave and pitch frequency characteristics Series 60; C_B=.70.

	λ/L_{pp}	$\omega_e\sqrt{\frac{L_{pp}}{g}}$	z_a/ζ_a	$\theta_a/k\zeta_a$	$\varepsilon_{z\zeta}$	$\varepsilon_{\theta\zeta}$	$\varepsilon_{\theta z}$
F_n = .20	.60	5.33	.069	.032	+ 6*	- 58*	- 64
	.65	5.04	.090	.026	+ 15*	- 29*	- 44
	.70	4.79	.097	.029	+ 25*	+ 41*	+ 16
	.75	4.57	.086	.069	+ 40*	+ 84*	+ 44
	.80	4.38	.036	.137	+ 47*	+103*	+ 56
	.85	4.20	.086	.246	+172*	+118*	- 54
	.90	4.04	.230	.383	-154	+131*	- 75
	.95	3.89	.447	.543	-132	+145*	- 83
	1.00	3.77	.709	.697	-112	+160*	- 88
	1.20	3.34	1.412	.988	- 46	-159	-113
	1.40	3.02	1.289	1.129	- 14	-138	-124
	1.60	2.77	1.108	1.151	- 5	-124	-119
	1.80	2.57	1.132	1.136	- 3	-116	-113
F_n = .25	.60	5.86	.049	.027	+ 2*	- 66*	- 68
	.65	5,53	.067	.024	+ 10*	- 46*	- 56
	.70	5.24	.077	.019	+ 18*	+ 7*	- 11
	.75	4.99	.074	.039	+ 29*	+ 66*	+ 37
	.80	4.77	.058	.083	+ 51*	+ 88*	+ 37
	.85	4.57	.048	.151	+114*	+101*	- 13
	.90	4.39	.108	.245	+171*	+111*	- 60
	.95	4.23	.234	.369	-166	+121*	- 73
	1.00	4.08	.417	.521	-150	+132*	- 78
	1.20	3.60	1.445	1.054	- 83	-174	- 91
	1.40	3.24	1.662	1.096	- 36	-149	-113
	1.60	2.97	1.432	1.183	- 14	-135	-121
	1.80	2.74	1.236	1.196	- 6	-124	-118

Table 4 continued.

Calculated heave and pitch frequency characteristics Series 60; C_B=.70.

	λ/L_{pp}	$\omega_e\sqrt{\frac{L_{pp}}{g}}$	z_a/ζ_a	$\theta_a/k\zeta_a$	$\varepsilon_{z\zeta}$	$\varepsilon_{\theta\zeta}$	$\varepsilon_{\theta z}$
F_n = .30	.60	6.38	.037	.024	- 1*	- 72*	- 71
	.65	6.01	.053	.022	+ 6*	- 57*	- 63
	.70	5.69	.062	.017	+ 13*	- 21*	- 34
	.75	5.41	.064	.023	+ 22*	+ 45*	+ 23
	.80	5.16	.055	.051	+ 37*	+ 76*	+ 39
	.85	4.94	.040	.097	+ 74*	+ 90*	+ 16
	.90	4.74	.056	.161	+139*	+100*	- 39
	.95	4.56	.124	.246	+172*	+107*	- 65
	1.00	4.39	.232	.355	-172	+115*	- 73
	1.20	3.86	1.188	1.043	-118	+160*	- 82
	1.40	3.47	1.846	1.191	- 61	-155	- 94
	1.60	3.16	1.754	1.130	- 29	-142	-111
	1.80	2.92	1.510	1.190	- 13	-132	-119

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7. Nomenclature.

a b c d e g, A B C D E G	- Coefficients of the equations of motion for heave and pitch.
A_w	- Area of waterplane.
A_x	- Area of cross-section.
C_B	- Blockcoefficient.
F	- Total force on ship
F_1', F_2', F_3'	- Sectional hydromechanical forces.
F_a	- Wave force amplitude on restrained ship.
$F_n = \frac{V}{\sqrt{gL_{pp}}}$	- Froude number.
g	- Acceleration due to gravity.
I_w	- Longitudinal moment of inertia of waterplane area with with respect to the y_b axis.
I_{yy}	- Real moment of inertia of ship.
$k = \frac{2\pi}{\lambda}$	- Wave number.
L_{pp}	- Length between perpendiculars.
M	- Total moment on ship.
M_a	- Wave moment amplitude on restrained ship.
m'	- Sectional added mass.
N'	- Sectional damping coefficient.
S_w	- Statical moment of waterplane area.
t	- Time.
T	- Draught of ship.
V	- Speed of ship
x_b, y_b, z_b	- Right-handed body axis system.
$y_w(x)$	- Half width of designed waterline.
z	- Heave displacement.
z_a	- Heave amplitude.
ε	- Phase angle between the motions (forces, moments) and the waves.

ζ - Instantaneous wave elevation.

ζ_a - Wave amplitude.

θ - Pitch angle.

θ_a - Pitch amplitude.

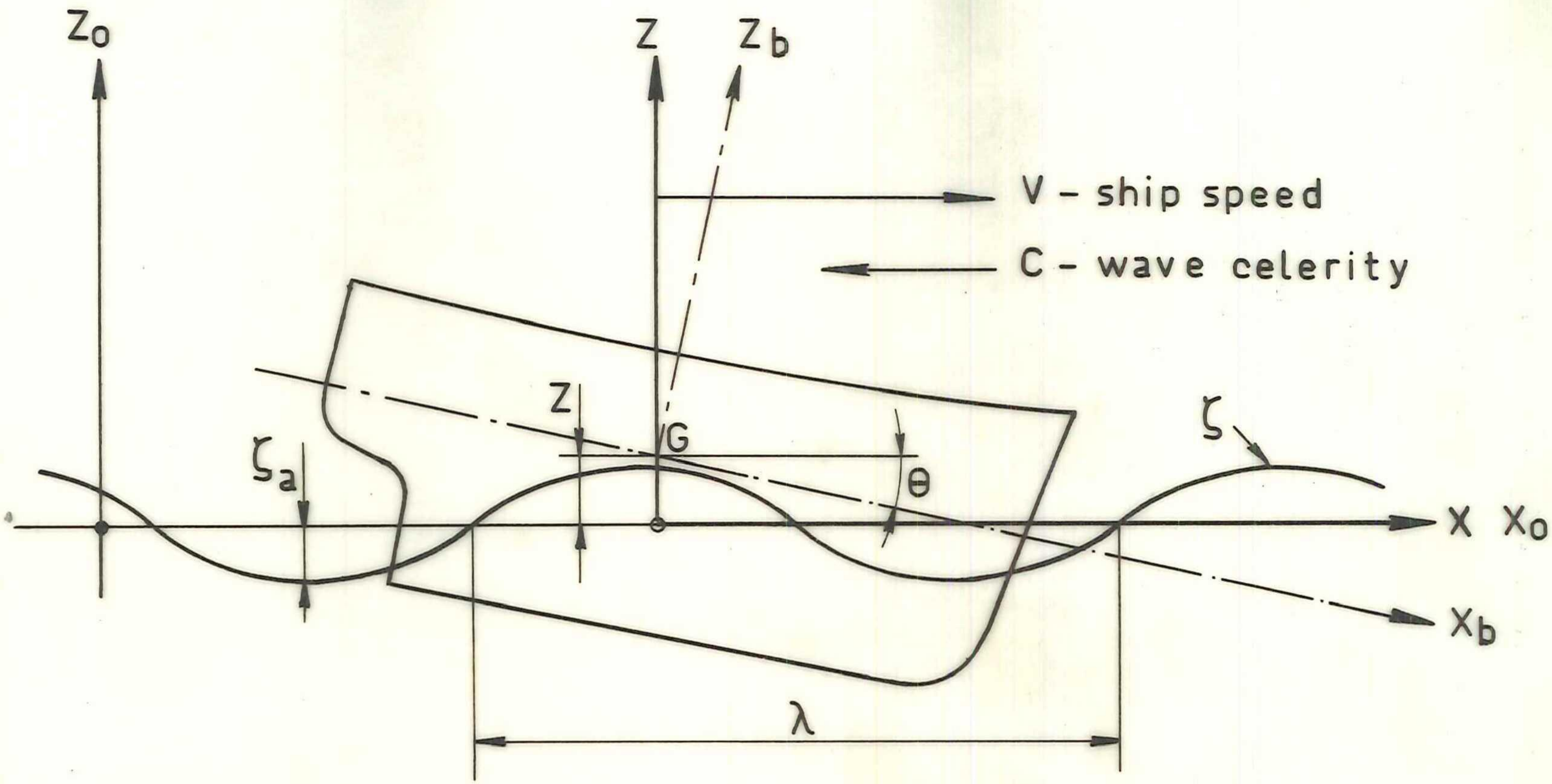
λ - Wave length.

ρ - Density of water.

∇ - Displacement volume.

ω - Circular frequency.

ω_e - Circular frequency of encounter.



wave – $\zeta = \zeta_a \cos(kx_0 + \omega t)$ t.o.v. $x_0\ y_0\ z_0$

$\zeta = \zeta_a \cos(\omega_e t)$ t.o.v. $x\ y\ z$, $x = o$

heave – $z = z_a \cos(\omega_e t + \varepsilon_{z\zeta})$

pitch – $\theta = \theta_a \cos(\omega_e t + \varepsilon_{\theta\zeta})$

$$\omega_e = \omega + \frac{\omega^2}{g} V$$

Figure 1 Definition of wave and motions

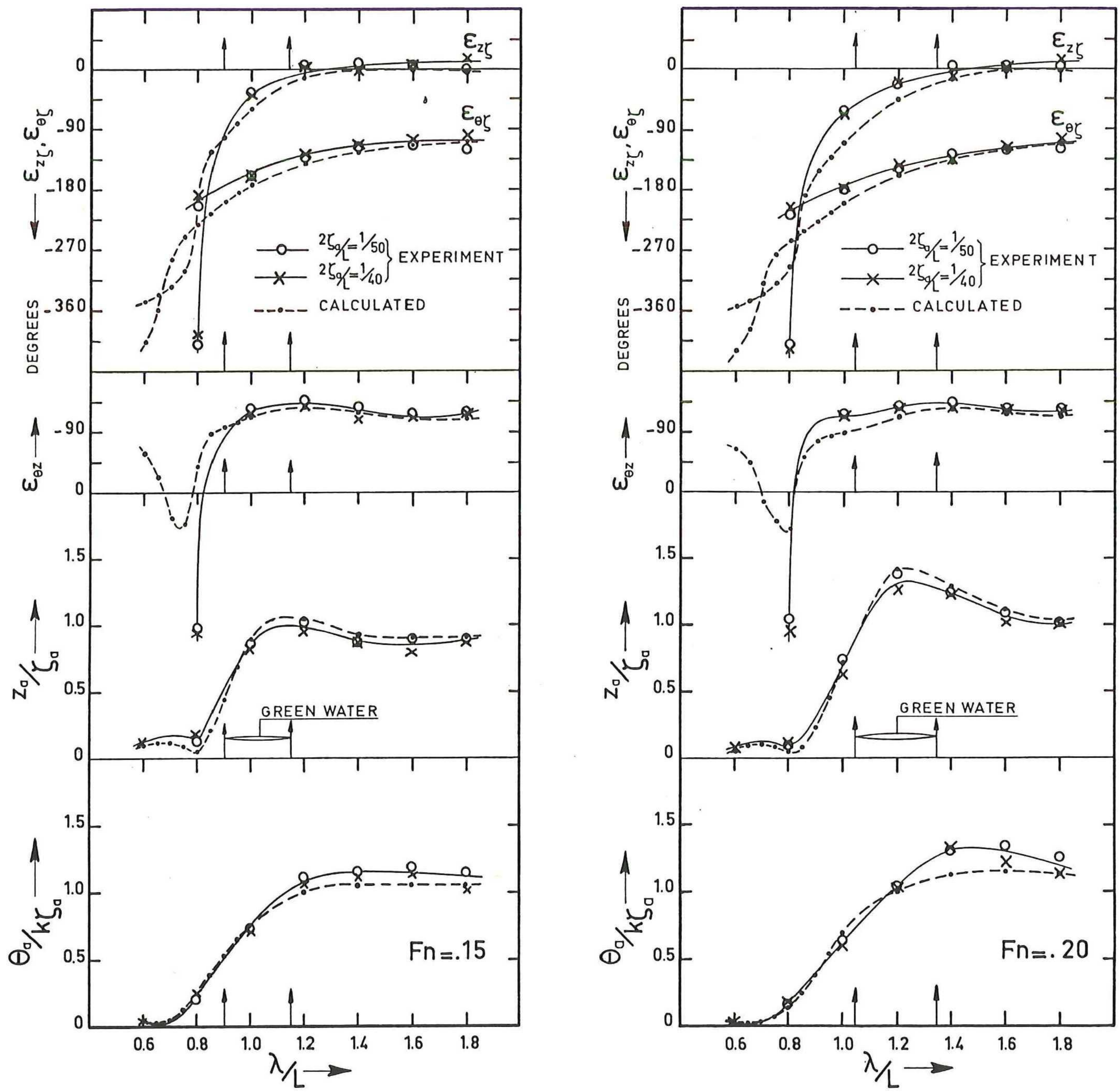


FIGURE 2a CALCULATED AND MEASURED AMPLITUDE – AND PHASE CHARACTERISTICS FOR HEAVE AND PITCH. SIXTY SERIES C_B=.70, Fn= . 15 AND Fn=.20

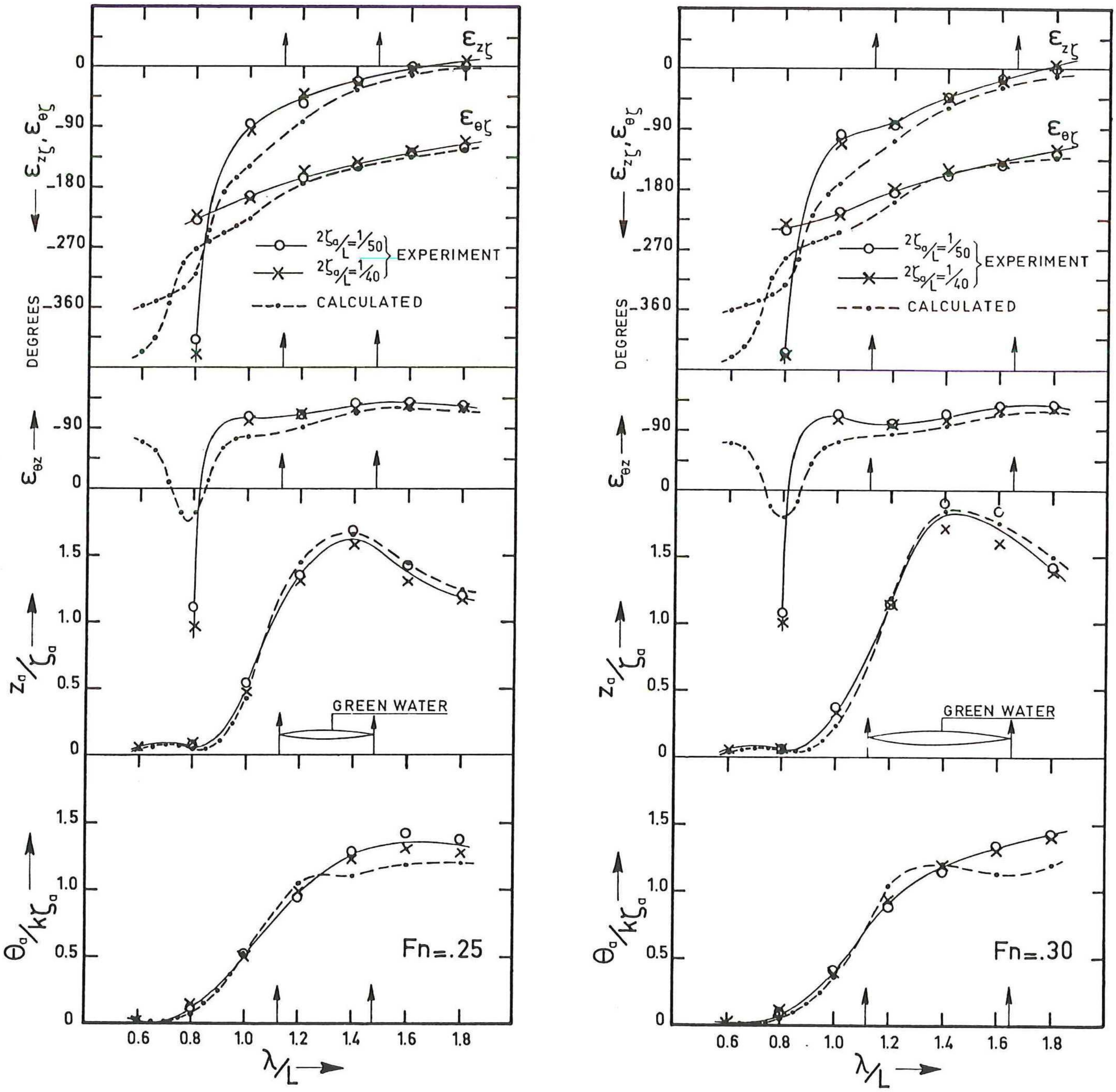


FIGURE 2b CALCULATED AND MEASURED AMPLITUDE– AND PHASE CHARACTERISTICS FOR HEAVE AND PITCH. SIXTY SERIES C_B=.70, Fn=.25 AND Fn=.30