

DISCIPLINARY INTERACTION VARIABLE ELIMINATION (DIVE) APPROACH FOR MDO

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Abstract. *For multi-disciplinary optimization, we present a three level variables elimination approach involving, system level design variables Z , output variables Y_i of discipline i , for interaction with other disciplines and private variables X_i of discipline i . We first eliminate the private variables X_i by using meta models for each discipline. The output variables Y_i of discipline i are optimal with respect to its private variables X_i , for discipline specific objective function. Then an equilibrium system between interaction variables Y_i is solved. This system is viewed as state equations. This allows elimination of interaction variables Y_i using classical approaches like adjoint method. The main advantage of this approach is to accurately satisfy the state equation of the system. After elimination of interaction variables, a system level optimization problem restricted to variables Z , is solved. The reliability of each meta model is checked at optimal point. If the optimal point is on the boundary of the validity domain of particular meta model, then a new meta model for that particular discipline centered around the optimal point is created and the optimization cycle is reiterated with new meta model. We demonstrate our approach for two MDO test cases, problem of combustion of Propane and design of supersonic business jet.*

1 INTRODUCTION

The goal of Multi-Disciplinary Optimization (MDO) is to define a methodology for disciplines interaction with the aim of solving a global design problem. Optimization is only one part of the MDO process. The other inseparable part of MDO process is the framework within which different disciplines interact and share information. A good MDO framework must allow sufficient disciplinary autonomy and should align individual

disciplinary objectives to the global objectives. Different methodologies for interaction amongst disciplinary and interdisciplinary tasks gives rise to different MDO methods, for example, All At Once (AAO), Multi Disciplinary Feasible (MDF), Individual Discipline Feasible (IDF), Bi-Level Integrated System Synthesis (BLISS), Collaborative Optimization (CO), etc.. Number of survey articles provide details of different MDO methods and their comparison.^{1,2,3,4}

In this paper, we provide an alternative approach for MDO taking into account the reliability. It differs from existing MDO strategies by its methodology to handle different design parameters involved in the MDO process and autonomy provided to individual disciplines. Our approach for MDO is described in section 2. First the framework for disciplinary and interdisciplinary interactions is discussed and then the formulation for optimization problem is provided. Comparison between the new approach and existing methods is discussed in section 3. Finally, application of this new approach is presented for two MDO test cases, namely, the problem of combustion of Propane¹¹ and the design of super sonic business jet.¹²

2 NEW APPROACH FOR MDO

The common point between different publications is to consider two types of design parameters, public parameters (shared by disciplines) and private parameters (specific to the given discipline). Unlike most contributions, we consider only public parameters. Our framework assumes that for a given choice of public parameters, private parameters are fixed by each discipline at their optimal value. Private parameters are not visible outside the specific discipline. This allows complete autonomy to individual disciplines to handle its disciplinary variables. We denote private variables of i^{th} discipline by X_i . The public parameters are further divided into two types: interaction variables (or state variables as we shall see later), Y_i and system level variables, Z . Interaction variables Y_i are outputs of i^{th} discipline and are shared by other disciplines for disciplinary analysis and optimization. So in our approach design variables are divided into three groups. These groups are handled at two levels namely, the disciplinary or subsystem level and the system level.

2.1 The disciplinary level

The disciplinary level is responsible for the construction of meta models. Each discipline is supposed to construct a meta model that depends only on the public parameters. Also, for each set of public parameters $p_u = \{Z, Y_i\}$, the response of the meta-model is optimal with respect to it's private parameters X_i . The meta model can be constructed using techniques like neural network, kriging, reduced order models, etc.⁹ If required one can replace the meta model by the real model. The response also includes description about the reliability of the model. Each discipline provides some constraints on the public parameters to indicate the domain of validity of the meta model. To simplify

the presentation, we limit ourselves to two disciplines, however the generalization to n disciplines is straight forward.

Consider two disciplines represented by their meta models A_1 and A_2 .

$$a_1 = A_1(p_u, a_{21}) \quad (1)$$

$$a_2 = A_2(p_u, a_{12}). \quad (2)$$

The vector a_i represents the output of the i^{th} discipline. Vectors a_{12} and a_{21} represent the coupling between the two disciplines. It is convenient to split the output of the meta model A_1 into two parts, namely,

$$a_{11} = A_{11}(p_u, a_{21}) \quad (3)$$

$$a_{12} = A_{12}(p_u, a_{21}) \quad (4)$$

or in general, the output of the i^{th} discipline can be written as

$$a_i = (a_{ii}, a_{ii'}) = (a_i^{fc}, a_i^c, a_i^r, a_{ii'})$$

where,

- a_{ii} is an output of discipline i to the system level optimizer,
- $a_{ii'}$ is a vector of coupling variables between discipline i and i' ,
For the present case $i' = 3 - i$,
- a_i^{fc} is a scalar value of the disciplinary objective function,
- a_i^c is a vector of the disciplinary constraints of the type $(a_i^c \leq 0)$,
- a_i^r is a vector of constraints pertaining to the reliability of the meta model.

Each discipline transmits the following information to the system level:

- The meta model A_i in the form of a portable program.
- Constants b_i^l, b_i^s which indicate bounds on the system level design variables.

These bounds ensure the validity of meta models for given system level variables. Superscripts l and s indicate lower and upper bounds respectively. Bound constraints are added to the model reliability constraints a_i^r obtained from the output of the meta model and are of the form $a_i^r \leq 0$. Constraints $a_i^r \leq 0$ ensure that the optimal value of the objective function is within the reliability domain. It should be noted that this approach provides complete autonomy to individual disciplines. The interaction amongst subsystem and system level is oblivious to activities inside each disciplines.

2.2 System level

At system level we first eliminate interaction variables $Y_i = \{a_{ii'}\}$ and then solve optimization problem with respect to variables Z . The elimination approach for interaction variables depends on the type of coupling between different subsystems.

2.2.1 Strong coupling

The coupling implies a coupling between the disciplinary solvers. When coupling between disciplinary solvers is strong, it is more reasonable to re-group these two disciplines into one.

2.2.2 Weak coupling

Consider the terms in equations (1) and (2) representing coupling between the disciplines.

$$a_{12} = A_{12}(p_u, a_{21}) \quad (5)$$

$$a_{21} = A_{21}(p_u, a_{12}) \quad (6)$$

The system 5, 6 can constitute a triangular system and then is solved either by forward or backward substitution. If the system 5, 6 is not triangular, it is solved in an iterative manner. In principle, this set of equations admits a unique solution. The lack of unique solution indicates the failure of the meta model. For a given set of public parameters, one solves

$$\min_{a_{12}, a_{21}} ||a_{12} - A_{12}(p_u, a_{21})||^2 + ||a_{21} - A_{21}(p_u, a_{12})||^2$$

The crucial point, compared to classical methods is to give priority to the solution of the system described by 5, 6. Eventually, equations 5, 6 can be considered as state equations. This permits us to exploit classic tools such as adjoint methods available to optimal design. The use of adjoint method to solve state equation is more rewarding when number of interaction variables are large. These techniques then eliminate all the interaction variables Y_i . With the elimination of interaction variables, the system level optimization problem is then limited only to public variables Z . This makes the optimization problem small and notably global in nature. Accurate solution of state equations is a prime aspect of this new approach at which it differs from the existing methods.

2.2.3 System level optimization problem

At system level, following non-linear constrained optimization problem is solved.

$$\left\{ \begin{array}{l} \min_Z f(Z) = (\lambda_1 * a_1^{fc} + \lambda_2 * a_2^{fc}) \\ a_i^c \leq 0, i = 1, 2, \text{ disciplinary constraints} \\ a_i^r \leq 0, \text{ constraints for reliability} \\ b_i^i \leq Z \leq b_i^s, \text{ bounds on } Z \text{ for meta model validity} \\ + \text{ system level constraints} \end{array} \right. \quad (7)$$

where, λ_i are weights of the subsystem objective function.

When the optimal solution to above problem is obtained, the acceptability of the solution is checked. For given solution point, the location of the solution point with

respect to the boundary of the validity domain of each meta model is estimated. If the obtained solution is on the boundary of the validity domain of any of the meta models, an update in that meta model such that the new construction of a disciplinary meta model is centered around the solution point is demanded. With this updated meta-model it restarts the process of optimization. Meta model management framework is thus embedded in our approach. It permits the optimizer to move from one design space to another based on the reliability constraints. The new approach for MDO is schematically shown in Figure 1.

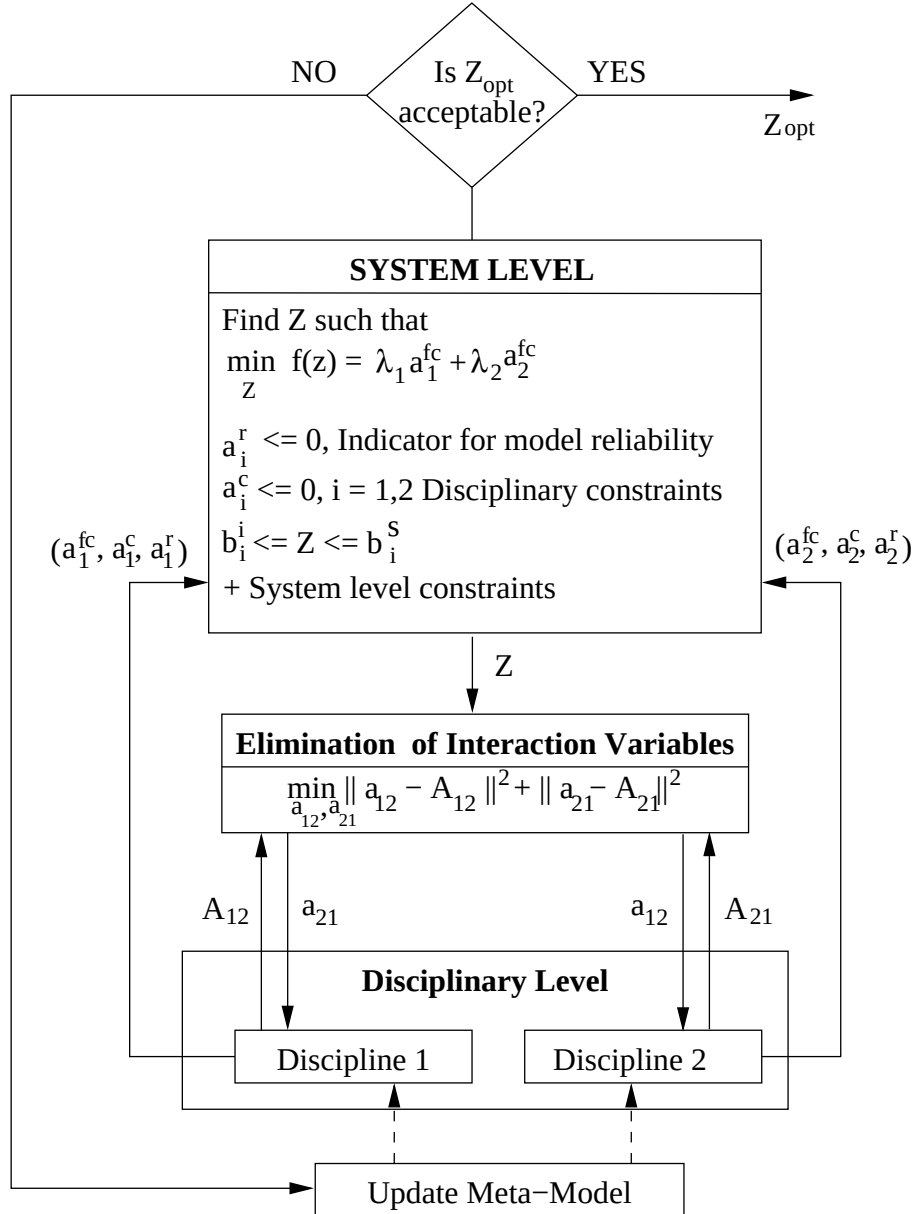


Figure 1: New approach for MDO

3 COMPARISON WITH OTHER METHODS

The approach presented in section 2 practically encompasses all known MDO methods:

- The proposed method is inspired extensively by the BLISS method (Bi - Level Integrated System Synthesis)¹². It differs from BLISS in the way it handles public and private variables. Our framework permits complete autonomy to individual disciplines to handle its private parameters and to decide individual disciplinary objectives. Unlike the original version of BLISS this method does not require evaluation of system sensitivity equations and thus the choice of sub-system objective function is left to individual disciplines.
- MDA (Multi - Disciplinary Analysis), the proposed method becomes the MDA method, when meta models are replaced by the real models.
- AAO (All At Once), the method proposed in this paper becomes the AAO method if one replaces meta models by the real models and that one solves equations (5), (6) and (7) simultaneously.
- CO (Collaborative Optimization), the CO method puts the equations (5) and (6) as a constraint on the system level optimization.

However, all these methods in a way rely on fixed point method to determine interaction variables and are not robust. Our approach to view system of equations governing interaction variables as state equations and to solve it using minimization technique is more robust and even efficient if number of interaction variables are large. Moreover, none of the above method propose any strategy for describing the quality of optimal solution. The meta model management framework we proposed permits to evaluate the acceptability criteria for optimal solution based on the reliability and model validity constraints.

4 APPLICATIONS

We illustrate our approach for MDO using two MDO test cases, namely, problem of combustion of Propane and conceptual design of super sonic business jet. These simulations were carried out using MATLAB optimization toolbox. MATLAB function *fmincon*, which is SQP based optimization solver, is used in these studies¹⁰. The primary aim of these studies is to show the accuracy with which the state equations of the system are satisfied. The meta models used in these studies does not permit to demonstrate strategy we proposed for meta model management. This aspect of our approach will be dealt elsewhere.

4.1 Problem of combustion of Propane

4.1.1 Description

This test case is obtained from NASA MDO test suite web site¹¹ and the problem description is reproduced here for completeness. This chemical equilibrium problem describes the combustion of Propane in air. Products of combustion are denoted by $x_i, i = (1 \text{ to } 10)$ and are unknowns to be determined. They represent the number of moles of each product formed for each mole of Propane burned. The 11th unknown x_{11} , used to simplify equations, is essentially the sum of the other 10 unknowns. There are 11 equations denoted by $f_i, i = (1 \text{ to } 11)$ relating products of combustion. The equation denoted by f_{11} is the requirement that the sum of the first 10 unknown (x) equals the 11th unknown x_{11} . Fixed parameters are P (pressure in atmospheres), R (the air to fuel ratio) and $K_i, (i = 5 \text{ to } 10)$ denote measured data. Ideally we want all the f_i 's ($i = 1 \text{ to } 11$) to be zero and all the x 's must be greater than zero. The set of equations governing the combustion process are given by equation 8

$$\begin{aligned}
f_1(x) &= x_1 + x_4 - 3 \\
f_2(x) &= 2x_1 + x_2 + x_4 + x_7 + x_8 + x_9 + 2x_{10} - R \\
f_3(x) &= 2x_2 + 2x_5 + x_6 + x_7 - 8 \\
f_4(x) &= 2x_3 + x_9 - 4R \\
f_5(x) &= K_5 x_2 x_4 - x_1 x_5 \\
f_6(x) &= K_6 x_2^{1/2} x_4^{1/2} - x_1^{1/2} x_6 \left(\frac{P}{x_{11}} \right)^{1/2} \\
f_7(x) &= K_7 x_1^{1/2} x_2^{1/2} - x_4^{1/2} x_7 \left(\frac{P}{x_{11}} \right)^{1/2} \\
f_8(x) &= K_8 x_1 - x_4 x_8 \left(\frac{P}{x_{11}} \right) \\
f_9(x) &= K_9 x_1 x_3^{1/2} - x_4 x_9 \left(\frac{P}{x_{11}} \right)^{1/2} \\
f_{10}(x) &= K_{10} x_1^2 - x_4^2 x_{10} \left(\frac{P}{x_{11}} \right) \\
f_{11}(x) &= x_{11} - \sum_{j=1}^{10} x_j
\end{aligned} \tag{8}$$

4.1.2 MDO formulation

The problem of combustion of Propane is formulated as MDO problem with system level optimization problem as defined by equation 9. Unknowns x_1, x_3, x_6, x_7 are treated as system level variables whereas $x_2, x_4, x_8, x_9, x_{10}$ are treated as interaction variables.

There are three subsystems and no private variables. Table 1 shows the subsystem level problem.

$$\begin{aligned} \min_Z f &= (f_2 + f_6 + f_7 + f_9) \\ \text{subjected to} \\ f_2, f_6, f_7, f_9 &\geq 0 \\ Z &\geq 0 \end{aligned} \tag{9}$$

Details	Input	Output	Constraints
Sub system 1	$Z = \{x_1, x_3, x_6, x_7\}$	$f_2, Y_1 = \{x_2, x_4\}$	$f_1 = 0, f_5 = 0$
Sub system 2	$Z = \{x_1, x_3, x_6, x_7\}$	$f_6, Y_2 = \{x_8, x_{10}\}$	$f_8 = 0, f_{10} = 0$
Sub system 3	$Z = \{x_1, x_3, x_6, x_7\}$	$(f_7 + f_9), Y_3 = \{x_5, x_9, x_{11}\}$	$f_3 = 0, f_4 = 0, f_{10} = 0$

Table 1: Details of subsystem input output relationship in the problem of combustion of Propane

The results obtained using our approach are shown in Table 2. Three different sets of initial values are used for system level variables. The case 2 represents starting point close to true solution where as case 3 is with initial value of system variables very far from true solution. Results from reference 11 for same set of initial values are reproduced in Table 3. Comparison of these two tables shows that state equations are satisfied more accurately in our approach. This also results in lesser number of iterations required at system level. Gradients for objective function and constraints were not explicitly provided but were computed by MATLAB *fmincon* routine. Number of subsystem function evaluations include those required for gradient computations. It is known that number of subsystem function evaluations can be brought down by explicitly providing gradients (even approximate gradients) to *fmincon* function but this approach is not explored here. Number of subsystem function evaluations were not available in reference 11.

4.2 Design of super sonic business jet (SSBJ)

This test case corresponds to the problem used by NASA to present the BLISS algorithm¹² and provides representative example of aircraft conceptual design. The global objective is to design supersonic business jet with maximum range. Subsystems include structure, aerodynamics, propulsion and range. The interaction amongst these disciplines is shown in Figure 2. Further details of this test case are referred to reference 12 and are not reproduced here.

4.3 MDO formulation

Table 4 shows list of public, interaction and private parameters as used in our approach. Meta models used in the original BLISS code were used without any modifications. As outlined in section 2, our method assumes that the choice of private parameters is an affair of individual disciplines. To facilitate the comparison with BLISS, private parameters of each discipline were set to the optimal values as obtained in BLISS. With this, the results obtained from our approach should match with those produced by BLISS which is indeed

Cases	case 1	case 2	case 3
Starting point Z	{2.0, 20.0, 0.0, 0.0}	{1.48620, 18.2890, 0.86022, 0.14533}	{2.0, 40.0, 5.0, 1.5}
Objective function value	3.99677×10^{-8}	3.49548×10^{-8}	4.06293×10^{-8}
No of iterations	5	3	7
No. of system objective function evaluations	30	20	42
No. of calls to sub-system meta-models	2880	1920	4032
Maximum residue	1.50263×10^{-8}	1.5112×10^{-8}	1.515507×10^{-8}
Value of constraint f_2	6.63687×10^{-9}	6.59901×10^{-9}	6.8746×10^{-9}
Value of constraint f_6	8.32817×10^{-9}	7.39945×10^{-9}	8.49799×10^{-9}
Value of constraint f_7	8.45813×10^{-9}	7.60243×10^{-9}	8.61474×10^{-9}
Value of constraint f_9	1.07169×10^{-8}	7.48939×10^{-9}	1.08254×10^{-8}

Table 2: Results obtained for the problem of combustion of Propane

Cases	case 1	case 2	case 3
Objective function value	5.544805×10^{-4}	8.7646×10^{-4}	1.6037×10^{-4}
No of iterations	50	50	60
No. of system objective function evaluations	551	522	666
Value of constraint f_2	1.2974×10^{-4}	3.5465×10^{-4}	5.3219×10^{-5}
Value of constraint f_6	6.9800×10^{-5}	7.0957×10^{-5}	7.7294×10^{-5}
Value of constraint f_7	3.1334×10^{-4}	2.6657×10^{-4}	9.7761×10^{-6}
Value of constraint f_9	4.1600×10^{-5}	1.8647×10^{-4}	2.0084×10^{-5}

Table 3: Values reported for the problem of combustion of Propane from reference 11

a case as seen from Table 5. The range of 3964 km is obtained in both the cases. The difference in approach is that, we have not carried out any system sensitivity analysis as in BLISS and have given complete autonomy to individual disciplines as to what should be the value of their private parameters.

5 CONCLUSIONS

In this paper, we have presented a new approach for multi disciplinary optimization considering the reliability aspect. The approach differs from existing MDO methods in number of ways.

1. By its treatment of different design variables involved in the MDO process as public and private parameters.
2. A three level variable elimination strategy to restrict the system level optimization problem to small size.

Parameter	Lower Bound	Upper bound	Initial	Optimal	BLISS Optimal
t/c	0.01	0.09	0.05	0.06	0.06
$h(km)$	30	60	45	60	60
M	1.40	1.80	1.60	1.40	1.4
AR	2.50	8.50	5.50	2.50	2.5
Λ	40.0	70.0	55.0	70.0	70.0
$S_{ref}(m^2)$	500	1500	1000	1500	1500
W_T	-	-	1	44748.5	44753.0234
W_f	-	-	1	19350.6	19350.6
Θ	-	-	1	1.02825	1.03
L	-	-	1	44748.5	44753.024
D	-	-	1	5552.84	5552.85
L/D	-	-	1	8.0587	8.0595
SFC	-	-	1	0.92394	0.92393
W_e	-	-	1	9435.9	9435.62
ESF	-	-	1	0.7327	0.7327
λ	0.10	0.40	0.40	0.40	0.4
x	0.75	1.25	0.75	0.75	0.75
C_f	0.75	1.25	0.75	0.75	0.75
T	0.10	1.00	0.16	0.16	0.16

Table 5: SSBJ test case results

We believe, these features makes this approach universal in nature and can be applied for any MDO process ranging from conceptual design to final stage design.

REFERENCES

- [1] Kodiyalam S., Evaluation of Methods for Multidisciplinary Design Optimization (MDO), Phase I, *NASA Contractor Report*, **NASA CR-1998-208716**, September 1998.
- [2] Kodiyalam S., and Yuan S., Evaluation of Methods for Multidisciplinary Design Optimization (MDO), Part II, *NASA Contractor Report* , **NASA CR-2000-210313**, November 2000.
- [3] Sobieszczanski-Sobieski, J. and Haftka, R. T. , Multidisciplinary Aerospace Design Optimization: Survey of Recent Developments, **AIAA Paper No. 96-0711**, (1996).
- [4] Masmoudi, M., Aurox, D. and Parte, Y., State of the Art in Collaborative Design, post SAROD-2005 workshop, Bangalore, India, 2005.

- [5] Padmanabhan, D. , Reliability Based Optimization for Multidisciplinary System Design, Ph.D Dissertation, Department of Aerospace and Mechanical Engineering, University of Notre Dame, Indiana, U.S.A., December 2004.
- [6] Du, X. and Chen, W., A Hierarchical Approach to Collaborative Multidisciplinary Robust Design, Proceedings of the 4th Congress of Structural and Multidisciplinary Optimization, Dalin, China, 2001.
- [7] Agrawal, H., Reliability Based Design Optimization: Formulations and Methodologies, Ph.D Dissertation, Department of Aerospace and Mechanical Engineering, University of Notre Dame, Indiana, U.S.A., December 2004.
- [8] Giunta, A.A., Eldred, M.S., Swiler, L.P., Trucano, T.G., and Wojtkiewicz, S.F., Jr., Perspectives on Optimization Under Uncertainty: Algorithms and Applications, AIAA-2004-4451, Proceedings of the 10th AIAA/ISSMO Multidisciplinary Analysis and Optimization Conference, Albany, NY, Aug. 30 - Sept. 1, 2004.
- [9] Queio, N. V., Haftka, R. T. et al, Surrogate-Based Analysis and Optimization, Progress in Aerospace Sciences, Vol. 41, pp. 1-28, 2005.
- [10] Anonymous, User guide for MATLAB optimization toolbox <http://www.mathworks.com/access/helpdesk/help/toolbox/optim/>, accessed May 2006.
- [11] Anonymous, NASA Multidisciplinary Design and Optimization branch <http://mdob.larc.nasa.gov/index.html>, accessed May 2006.
- [12] Sobieszczanski-Sobieski J., Agte J.S., and Sandusky R.R., Bi-Level Integrated System Synthesis (BLISS), *NASA Technical Manual*, **NASA TM-1998-208715**, 1998.