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### Energy structure of hybrid Semiconductor-superconductor nanowire Based devices

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### ENERGY STRUCTURE OF HYBRID SEMICONDUCTOR-SUPERCONDUCTOR NANOWIRE BASED DEVICES

### ENERGY STRUCTURE OF HYBRID SEMICONDUCTOR-SUPERCONDUCTOR NANOWIRE BASED DEVICES

### Proefschrift

ter verkrijging van de graad van doctor aan de Technische Universiteit Delft, op gezag van de Rector Magnificus prof. dr. ir. T.H.J.J. van der Hagen, voorzitter van het College voor Promoties, in het openbaar te verdedigen op donderdag 28 maart 2019 om 10:00 uur

door

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Keywords:	Superconductor, Semiconductor, Josephson junctions, Radiation,
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"Scars are not injuries. A scar is a healing. After injury, a scar is what makes you whole."

> Tanner Sack, The Scar By China Miéville

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### **SUMMARY**

Materials possessing superconducting properties are highly sought after due to their potential technological applications. Much of the work has focused on utilising a thin insulating barrier separating a pair of superconducting electrodes, a Josephson junction, as a workhorse in the field of superconducting based quantum computation. Yet such structures are highly sensitive to their surrounding environment. Furthermore the associated energy scales, Josephson coupling and charging energy, are set by the junction geometry. Replacing the insulating barrier with a semiconducting material has the significant advantage of offering tunable energy scales with the aid of an applied electric field. Due to recent advancement in material development, hybrid combinations of superconductors and semiconductors have made it possible to devise various architectures. The present thesis focuses on investigating Josephson junctions and Cooper-pair transistors formed from semiconducting nanowires covered by a superconducting layer.

The first experimental work focuses on studying the high frequency radiation generated by an indium antimonide (InSb) niobium titanium nitride (NbTiN) based nanowire Josephson junction. The radiation is detected with the aid of an on-chip circuit consisting of the source capacitively coupled to a superconducting quantum interference device (SQUID), acting as a detector. The backaction and frequency range of detection is adjusted through the application of a flux through the SQUID. To limit radiation leakage out of the circuit resistive lines are implemented. The radiation incident upon the SQUID is detected via photon-assisted tunnelling (PAT) of quasiparticles visible in the current-voltage characteristics of the SQUID, with the onset of PAT directly related to the frequency of the radiation. The measured spectra revealed two contributions to the radiation; a component due to Josephson radiation and another due to white noise. The white noise is attributed to finite density of sub-gap quasiparticles present in the nanowire.

The following experimental work utilises a similar circuitry, where the detector is replaced by a single tunnel Josephson junction and an indium arsenide (InAs), aluminium (Al) based nanowire junction was embedded in a SQUID with another tunnel junction. The single tunnel junction now employs the role of a spectrometer which induces and subsequently detects transitions within the surrounding electromagnetic environment. The SQUID plays the role of environment. By ensuring a high asymmetry between the Josephson couplings of the two junctions in the SQUID, transitions between the energy levels in the nanowire junctions are resolved.

The second half of the thesis focuses on the investigation of an InAs-Al Cooper-pair transistor (CPT). By embedding the CPT in a similar on-chip spectroscopy circuit the underlying energy levels are revealed. Studies into quasiparticle dynamics within the spectroscopy circuit and through the measurements of switching currents of the CPT motivate future experiments in this direction.

The two systems under study in the present thesis have potential applications in the

field of quantum computation, either as Andreev based qubits or as building blocks in topological based quantum circuits. Of paramount importance is the underlying interplay of studied energy scales. Furthermore, quasiparticle dynamics reveal potential limitations of both systems.

## SAMENVATTING

Supergeleidende materialen zijn zeer gewild vanwege hun potentiële technologische toepassingen. Voor het ontwikkelen van een supergeleidende kwantumcomputer spelen Josephsonjuncties een centrale rol. Deze juncties bestaan uit twee supergeleidende electroden gescheiden door een dunne isolerende barrière, en zijn zeer gevoelig voor hun omgeving. De relevante energieschalen voor deze juncties, de Josephson- en oplaad-energie, worden bepaald door hun geometrie. Wanneer er een halfgeleidende in plaats van een isolerende barrière wordt gebruikt, heeft dit als voordeel dat de energieschalen afgestemd worden door een elektrisch veld aan te leggen. Recente ontwikkelingen in de materiaalkunde hebben het mogelijk gemaakt hybride halfgeleider supergeleider systemen op verscheidene manieren in elektronische circuits toe te passen. De focus van het onderzoek in dit proefschrift ligt op Jospehsonjuncties en Cooperpaartransistoren (CPT) gemaakt van halfgeleider nanodraden bedekt met een supergeleidende laag.

Het eerste experiment richt zich op de hoogfrequente straling uitgezonden door Josephsonjuncties gemaakt van indiumantionide (InSb) nanodraden en niobiumtitaniumnitride (NbTiN). Deze straling wordt gedetecteerd met een circuit op dezelfde chip waarbij een supergeleidend kwantuminterferentiedevice (SQUID) capacitief gekoppeld is aan de junctie. De terugslag en het frequentiebereik van de detector worden gecontroleerd door een flux aan te brengen door de SQUID. Resistieve connecties beperken het verlies van straling uit het detectiecircuit. De straling wordt in de detector omgezet in stroom door fotongeassisteerde tunneling van quasideeltjes. De frequentie van de straling bepaalt vanaf welk voltage er een stroom gaat lopen. De gemeten spectra laten twee soorten straling zien; de Josephsonstraling en straling vanwege witte ruis. Deze ruis wordt veroorzaakt door de eindige hoeveelheid quasideeltjes in de supergeleidende energiekloof van de nanodraad.

In het volgende experiment worden Josephsonjuncties van indiumarsenide (InAs) aluminium (Al) nanodraden, die samen met een reguliere junctie zijn ingebed in een SQUID, bestudeerd. Voor dit experiment is een enkele Josephsonjunctie in plaats van een SQUID gebruikt als detector. Deze detectorjunctie wordt ingezet als spectrometer voor het induceren en vervolgens detecteren van transities in de elektromagnetische omgeving, die bestaat uit de SQUID. Door een sterke asymmetrie in de Josephsonenergie van de juncties in de SQUID kunnen de transities in de nanodraadjunctie worden vastgesteld.

De tweede helft van dit proefschrift richt zich op InAs-Al CPT. De energieniveaus van de CPT zijn gemeten door deze in een vergelijkbaar spectroscopiecircuit op te nemen. Verder is de dynamiek van quasideeltjes in zowel dit circuit als door transitiestroommetingen waargenomen. Deze metingen bieden mogelijkheden voor toekomstige experimenten.

De twee in dit proefschrift bestudeerde systemen hebben potentiële toepassingen in kwantumcomputers, bijvoorbeeld als Andreevkwantumbits of als bouwsteen voor topo-

logische kwantumcircuits. De onderliggende energieschalen zijn hiervoor van uiterst belang. De dynamiek van quasideeltjes onthult potentiële limiterende factoren voor beide toepassingen.

# 

**INTRODUCTION** 

### **1.1.** QUANTUM MECHANICS

The dawn of the 20<sup>th</sup> saw the introduction of the Quantum theory, pioneered by minds such as Niels Bohr and Edwin Schrödinger. Since its inception, the predictions of quantum theory have been relentlessly applied in experimental facilities all over the world and verified across the entire spectrum of physics; from astro to particle physics.

Essentially quantum theory can be broken down into two main constituents; the unitary evolution of a quantum state as dictated by the Schrödinger equation and the probabilistic outcome of quantum measurements on a state as governed by Born's probability rule [1].

Despite its undoubted success and tremendous predictive power, its initial introduction was met with plenty of scepticism [2–4]. Although mathematically sound, the abrupt transitions between deterministic and probabilistic outcomes led to counter intuitive implications upon the nature of both the micro and macroscopic world. The notions of superposition, entanglement and non-locality steered further away from our classical viewpoint of the world [5, 6].

It then comes as no surprise that the nature of quantum theory is still heavily debated to this day. Many alternative interpretations haven been put forward claiming to offer solutions to existing inconsistencies but in doing so create their own [7-11].

Despite the ongoing debate, the delicate nature of the interpretation of quantum world has little significance on the predictive power of quantum theory. Indeed the theory itself is being harnessed for technological applications such as computation and communication with the hope that these technologies themselves, amongst other tasks, will shed light upon the nature of quantum theory.

### **1.2.** QUANTUM COMPUTING

Analogous to its classical counterpart, the building block of a quantum computer is the quantum bit, qubit. Similar to classical bits, qubits act as information carriers whose state can be manipulated in a controlled manner in order to perform logical operations. A qubit consists of a quantum two level system whose global state is put into a superposition of its constituents. The state of the qubit can then be adequately represented as a pointer on the Bloch sphere where the direction of the pointer will be determined by the relative phase between the states of the qubit. In order to ensure consistency, as per Born's probability rule, each state will have a probability weight associated with it which when squared and summed with rest of the weights will add up to unity. As such each qubit will have two basis functions associated with it [12, 13]. A multi-qubit system consisting of *N* qubits will in turn have  $2^N$  basis functions, highlighting its increased storage capability over the classical counterpart.

Undoubtedly the most popular platform investigated for the realisation of a quantum computer is based on superconducting materials [14]. At the heart of superconducting qubits is the Josephson tunnel junction, a nanoscale object consisting of two superconducting materials separated by a thin insulating barrier [15, 16]. Through the advancement of fabrication techniques, elaborate circuits based on such Josephson elements have been realised which are analogous to artificial atoms residing in optical cavities [17]. Due to their restricted dimensionality, superconducting qubits offer a signifi-

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cant advantage in terms of scaling up to larger systems. However, due to superconducting properties such qubits are highly sensitive to local perturbations in the environment. The interaction leads to a dephasing of the qubit state in a process commonly referred to as decoherence, yet the manipulation of a qubit can only be achieved through the interaction with the environment requiring the development of fault-tolerant schemes.

Alternatively one can look to replace the insulating barrier with a semiconducting material thus enabling the development of more exotic types of qubits. The advantages stem from the *in-situ* tunability of the semiconductor where the formation of the current carrying states, Andreev bound states, leads to the formation of an adjustable Andreev two level system [18, 19]. Furthermore under the necessary external combinations of electric and magnetic fields, the formation of Majorana bound states (MBS) is predicted to occur. These zero energy excitations come in pairs and are typically found at the interface between the semiconductor and superconductor. Such topological qubits rely on the non-Abelian statistics associated with MBS where the quantum two level system is formed from the degenerate ground state and is insensitive to local perturbations in the environment [20–22].

### **1.3.** Spectroscopy

As there are various material combinations one can imagine for the realisation of qubits based on semiconductor-superconductor heterostructures, a set of diagnostics is necessary to discern which are more favourable. Spectroscopy techniques look to reveal the internal energy structure of a given system. Arguably the most widely implemented, the d.c. spectroscopy reveals a highly exotic spectrum which can be cumbersome to interpret. On the opposite side, r.f. spectroscopy looks to induce well defined transitions between certain energy levels, however an elaborate experimental set-up is necessary. This thesis, in large part, is dedicated to developing an on-chip circuit that fuses the advantages of both techniques enabling broadband studies. The concept relies on implementing the Josephson tunnel junction as a d.c. to a.c. power converter which studies its surrounding electromagnetic environment. By carefully designing the environment and embedding the structure of interest within it, the energy structure of the said system can be addressed.

### **1.4.** OUTLINE OF THE THESIS

The focus of this thesis is to study the energy structure of two architectures; nanowire based Josephson junctions and Cooper-pair transistors (CPT). We begin by outlining the necessary tools in order to understand the following experimental chapters. In **chapter 2** we provide a theoretical overview beginning with the BCS theory of superconductivity. Through the aid of Andreev reflection we look at the formation of Andreev bound states in the weak link of a Josephson junction and subsequently discuss how such junctions can be used as detectors or sources of high frequency radiation.

In the following **chapter 3**, we outline the fabrication and characterisation of each individual component and finally the full circuit used to perform the on-chip microwave spectroscopy.

In chapter 4 we report upon the detection of radiation due to a nanowire based

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Josephson junction. The nanowire junction is capacitively coupled to a superconducting quantum interference device (SQUID) that acts as a radiation sensor.

In **chapter 5** we reveal the presence of Andreev bound states in a nanowire based Josephson junction. The nanowire junction is embedded in a SQUID with another tunnel junction where the phase dispersion of the ABS is monitored vie an applied flux through the SQUID. A similarly capacitively coupled tunnel junction is used to resolve the ABS. In addition we study the response of the ABS to an in-plane magnetic field.

We move away from the single nanowire junction in **chapter 6** and study the switching current response of the nanowire CPT. The evolution of the switching current as a function of temperature and an in-plane magnetic field is studied.

In the final experiment, **chapter 7**, we embed a similar CPT in an on-chip microwave spectroscopy circuit studying its energy dispersion as a function of phase and induced gate charge. Similar conclusions are reached from the temperature studies.

Finally we summarize the results in **chapter 8** and provide an outlook for further experimentation.

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# 2

# THEORY

The purpose of the present chapter is to outline the necessary tools in order to understand the following experimental work. We begin with a brief discussion of superconductivity, followed by the introduction of Andreev reflection. We expand upon these concepts and discuss the formulation and utilisation of Josephson junctions. We then discuss the basic principle of charging physics. Finally we combine all of these concepts to study the interaction of a Josephson junction with a well defined electromagnetic environment.

### 2.1. SUPERCONDUCTIVITY

### 2.1.1. INTRODUCTION TO SUPERCONDUCTIVITY

The emergent phenomenon of superconductivity has been observed in a variety of materials upon undergoing a phase transition associated with a temperature known as the critical temperature,  $T_C$ . After this transition the superconductor can carry a dissipationless current as was originally discovered by Kamerlingh Onnes in 1911 [1] and expel any external magnetic field up to a particular critical field,  $B_C$ , as was originally discovered by Meissner and Ochsenfeld in 1933 [2]. Associated with the emergence of superconductivity is the formation of a coherent, macroscopic state which can be described by a global wavefunction. The macroscopic nature of superconductivity was originally captured by the phenomenological theory of Ginzburg-Landau [3] introduced in 1950. However the microscopic formalism had to wait until 1957 when it was introduced by Bardeen, Cooper and Schrieffer (BCS) [4]. In the following we will focus on this BCS formalism.

### 2.1.2. BCS SUPERCONDUCTIVITY

The foundation of BCS formalism relies on the formation of electron-electron bound states upon the transition into the superconducting phase. Below  $T_C$  a weak attractive force between electrons dominates over the Coulomb repulsion. The value of  $T_C$  is material specific and has been found to be related to the Isotope Effect [5, 6], where  $T_C$  is directly related to the mass of the nuclei. Although in the original formulation the origin of attractive interaction was not specified it was late observed, due to the Isotope Effect, to be mediated by phonons. These phonons are generated when a negatively charged electron locally disturbs the positively charged ion lattice which in turn attracts another electron with an opposite spin and momentum. The original interaction between the first electron and the lattice causes the electron to be scattered from a state k to k', similarly the partnering electron is scattered from state -k to -k', as is illustrated in Fig. 2.1(b). Due to the necessity of available states for the electron to be scattered into, not all electrons can participate in the formation of electron-electron states, only those above the Fermi sea (Fig. 2.1(a)). Furthermore the electron-electron states are strongly overlapping leading to strong correlations between them giving rise to a strong rigidity of the superconducting phase. In the following we will follow [7–9] in the derivation of superconductivity in the bulk.

### **BOGOLIUBOV-DE GENNES EQUATION**

The Hamiltonian used to describe the afro-mentioned interaction is the so-called *pair-ing* Hamiltonian, which in momentum space is given by the following equation:

$$H_{pairing} = \sum_{k\sigma} \epsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + \sum_{kl} V_{kl} c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger} c_{-l\downarrow} c_{l\uparrow}$$
(2.1)

Where the first term accounts for the non interacting electrons with  $\epsilon_k = (\hbar k)^2/2m^* - E_F$ , with  $E_F$  the Fermi energy. The second term accounts for the attractive interaction between electrons of opposite spin and momentum, characterised by a potential strength  $V_{kl}$ . The creation (annihilation),  $c_k^{\dagger}(c_k)$ , operators are used to denote the addition (removal) of electrons.



Figure 2.1: (a) Cartoon representation of the scattering of electron states just outside the Fermi sea. (b) The scattering of an incident electron from state k to k' via the interaction with the lattice generates a virtual phonon causing an electron with an opposite -k to scatter to a new state -k'.

In the mean-field approximation,  $H_{pairing}$  is expressed as the *model* Hamiltonian as given by the following equation:

$$H_{k} = \sum_{k\sigma} \epsilon_{k} c_{k\sigma}^{\dagger} c_{k\sigma} + \sum_{kl} \left( \Delta_{k} c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger} + \Delta_{k}^{*} c_{-k\uparrow} c_{k\downarrow} \right)$$
(2.2)

Where  $\Delta_k$  is a complex vector potential responsible for the addition of electron-electron states, known as Cooper-pairs [10], commonly referred to as the superconducting pairing potential. It is important to note that although the first part of the  $H_k$  does conserve particle number, due to the quadratic nature of the  $c_k^{\dagger}c_{-k}^{\dagger}$ , the entire  $H_k$  does not. Nevertheless the spin and parity (even or odd number of particles) of the system are conserved.

For the remainder of this thesis we will assume an s - wave pairing potential expressed as  $\Delta_k = \Delta e^{i\phi}$  which is momentum independent with a magnitude  $\Delta$ , the superconducting energy gap, and characterised by a superconducting phase,  $\phi$ .

Before we can look for solutions we need to turn the Hamiltonian into a form that can be diagonalised, which we do through the introduction of a spinor field  $\Psi_k = \begin{pmatrix} c_{k\uparrow} \\ c_{-k\downarrow}^{\dagger} \end{pmatrix}$ . From which we utilise  $H_k = \sum \Psi_k^{\dagger} \mathscr{H} \Psi_k$  where we have introduced:

$$\mathcal{H} = \begin{pmatrix} \epsilon_k & \Delta_k \\ \Delta_k^* & -\epsilon_k \end{pmatrix}$$
(2.3)

The next step in looking for a solution to this Hamiltonian is to solve the one-particle Schrödinger equation with a plane wave solution of the form  $\begin{pmatrix} u_k \\ v_k \end{pmatrix} e^{ikr}$  where  $u_k$  and  $v_k$  are coherence factors associated with electrons and holes respectively. In doing so we arrive at the Bogoliubov-de Gennes (BdG) equation:

$$\mathscr{H}\begin{pmatrix} u_k\\ v_k \end{pmatrix} = E_k \begin{pmatrix} u_k\\ v_k \end{pmatrix}$$
(2.4)

2

Given the form of the Hamiltonian we can expect two solutions with eigenvalues of the form  $\pm E_k = \pm \sqrt{\epsilon_k^2 - \Delta^2}$  for two plane waves of the form  $|k_+\rangle = \begin{pmatrix} u_k \\ v_k \end{pmatrix} e^{ikr}$  and  $|k_-\rangle = \begin{pmatrix} -v_k^* \\ u_k^* \end{pmatrix} e^{ikr}$ . Given the normalisation condition of  $|u_k|^2 + |v_k|^2 = 1$  we find that the coherence factors expressed as follows:

$$u_{k} = e^{-i\phi/2} \sqrt{\frac{1}{2} \left( 1 + \frac{\epsilon_{k}}{\sqrt{\epsilon_{k}^{2} + \Delta^{2}}} \right)}$$

$$v_{k} = e^{i\phi/2} \sqrt{\frac{1}{2} \left( 1 - \frac{\epsilon_{k}}{\sqrt{\epsilon_{k}^{2} + \Delta^{2}}} \right)}$$
(2.5)

We transform this into a one-particle problem, where we look at excitations upon a given ground state, by introducing the following transformation:

$$\gamma^{\dagger}_{k_{+}} = u_{k}c_{k\uparrow} + v_{k}c^{\dagger}_{-k\downarrow}$$

$$\gamma^{\dagger}_{k_{-}} = -v^{*}_{k}c_{k\uparrow} + u^{*}_{k}c^{\dagger}_{-k\downarrow}$$
(2.6)

Here we have introduced a new set of fermionic creation (annihilation) operators,  $\gamma_k^{\dagger}(\gamma_k)$  describing elementary quasiparticle excitations<sup>1</sup> which are a linear superposition of electrons and holes. Therefore we can utilise these excitations to represent the Hamiltonian in the following way:

$$H = H_V + \sum_k E_k \gamma_k^{\dagger} \gamma_k \tag{2.7}$$

Here  $H_V$  describes the *vacuum* state,  $|V\rangle$ , i.e. a state in the absence of any excitations. Thus we can construct the ground state,  $|GS\rangle$ , of the superconducting state by looking at how the  $\gamma_k^{\dagger}(\gamma_k)$  operators act upon  $|V\rangle$ :

$$|GS\rangle = \prod_{k} \gamma_{k_{-}}^{\dagger} |V\rangle$$
(2.8)

$$= \prod_{k} \left( u_{k} + v_{k} c_{k\uparrow}^{\dagger} c_{-k\downarrow}^{\dagger} \right) |V\rangle$$
(2.9)

From this expression of  $|GS\rangle$  we see that it is made up of Cooper-pairs. We can further construct the excited states of a given system by acting  $\gamma_{k_+}^{\dagger}$  on  $|GS\rangle$ . From this point of view we see that each excitation leads to the addition of a single-particle with a particular energy and as a result is referred to as the single particle picture.

<sup>&</sup>lt;sup>1</sup>Commonly referred to as Bogolioubons [7].

### DENSITY OF STATES

From the construction of the  $|GS\rangle$  and the excited states we can construct a density of states (DOS), depicted in Fig. 2.2, [7] as:

$$\frac{N_s(E)}{N(0)} = \frac{d\epsilon}{dE} = \begin{cases} \frac{E}{\sqrt{E^2 + \Delta^2}} & \text{for} E > \Delta\\ 0 & \text{for} E < \Delta \end{cases}$$
(2.10)

Where  $N_s(E)$  is the superconducting quasiparticle DOS and N(0) is taken to be a constant normal DOS. From this expression and Fig. 2.2 we indeed see that  $\Delta$  plays the role of a superconducting energy gap in the DOS acting to separate the Cooper-pair condensate from single quasiparticle excitations.



Figure 2.2: (a) A cartoon representation of the DOS in a superconductor in the ground state. (a) Excited state consisting of a single excited quasiparticle.

### 2.1.3. ANDREEV REFLECTION

In the previous section we found that in ideal superconductors single quasiparticle excitations are excluded provided that the energy of these quasiparticles is  $E < \Delta$ . Therefore when a superconductor is brought into contact with a normal metal, the transport of charges from the normal metal to the superconductor is strongly modified, as shown in Fig. 2.3.

If we first consider an electron with an energy  $E > \Delta$  in a normal metal incident upon the superconductor, it is permitted to enter as a quasiparticle. However if the energy of the incident electron is  $E < \Delta$  then it will have a finite probability to be retro-reflected as a hole with an opposite momentum and energy, -E, back into the normal metal. This process, known as Andreev Reflection (AR) [11], results in the injection of a Cooper-pair into the superconductor. However, due to the possible presence of impurities at the normal metal-superconductor (NS) interface, the electron has a finite probability to undergo specular reflection back into the normal metal, thus resulting in no net transfer of charges between the two systems.

The process of AR was formally studied by Blonder, Tinkham and Klapwijk (BTK) [12], where the presence of impurities was accounted for by introducing a barrier of variable



Figure 2.3: (a) Cartoon representation of the reflection and transmission of an electron state in the normal metal incident upon the NS. The electron can enter the superconductor either as a quasiparticle (QP) or undergo Andreev Reflection and enter the superconductor as a Cooper-pair. The finite specular reflection is not represented. (b) The same picture but now in momentum space.

strength, *Z*. Here we make use of the plane wave solutions introduced in the previous section,  $|k_0\rangle = {\binom{v_0}{u_0}}e^{ikr}$ , as such we can write the plane wave propagating within the normal metal as:

$$\Psi_{N}^{\prime}(r) = \begin{pmatrix} 1\\0 \end{pmatrix} e^{ik_{n}^{+}r} + A \begin{pmatrix} 0\\1 \end{pmatrix} e^{ik_{n}^{-}r} + B \begin{pmatrix} 1\\0 \end{pmatrix} e^{-ik_{n}^{+}r}$$
(2.11)

where  $k_n^{\pm}$  are the wave-vectors, of opposing momentum, associated with electrons and holes and are given by:

$$k_n^{\pm} = \sqrt{k_F^2 \pm 2m^* E/\hbar^2}$$
(2.12)

with  $k_F$  being the Fermi wave-vector.

Here pre-factors *A* and *B* are the probability amplitudes associated with the retroreflection of a hole and the specular reflection of an electron respectively.

Similarly we can look for a plane wave propagating within the superconductor as:

$$\Psi_{S}^{\vec{}}(r) = C \begin{pmatrix} u_{0} \\ v_{0} \end{pmatrix} e^{ik_{s}^{+}r} + D \begin{pmatrix} u_{0} \\ v_{0} \end{pmatrix} e^{-ik_{s}^{-}r}$$
(2.13)

Similar to the normal case,  $k_s^{\pm}$  are the superconducting wave-vectors deifend as  $k_s^{\pm} = \sqrt{k_F^2 \pm 2m^* \sqrt{E^2 - \Delta^2}/\hbar^2}$  and *C* and *D* are the transmission probabilities associated with electrons and holes respectively. Following BTK, by viewing the interface as a discontinuity and matching the solutions of the plane-waves at the discontinuity, the general expression for the pre-factors is calculated to be:

$$|A| = \begin{cases} \frac{\Delta^2}{E^2 + (\Delta^2 - E^2)(1 + 2Z^2)^2} & \text{for} E < \Delta \\ \frac{u_0^2 v_0^2}{\gamma^2} & \text{for} E > \Delta \end{cases}, |B| = \begin{cases} 1 - |A|^2 & \text{for} E < \Delta \\ \frac{(u_0^2 - v_0^2)^2 Z^2(1 + Z^2)}{\gamma^2} & \text{for} E > \Delta \end{cases}$$
(2.14)

Utilising these definitions we can look for the current, carried by the charges involved, across the system as:

$$I_{NS} = \frac{2e}{h} \int dE[f(E - eV)][1 + A(E) - B(E)]$$
(2.15)

Where f(E) and f(E - eV) are Fermi-Dirac distribution functions, V is a bias Voltage applied across the interface.

We can consider two limiting cases when looking at the *V*, first we consider the scenario where the *V* is large in comparison to  $\Delta$ ,  $eV > \Delta$ . The current is then given by  $I_{NS} = TG_0V$  where *T* is the transmission associated with the barrier *Z* and  $G_0$  is the conductance quantum. From this we can define the normal state conductance of the system as  $G_N = TG_0$ , which is related to the transport of quasiparticles with energies above  $\Delta$  and is only related to strength of the transmission through the barrier. For smaller values of *V*, such that  $eV < \Delta$ , the conductance in the superconducting domain was found [13] to be given by:

$$G_S = G_0 \frac{2T}{(2-T)^2}$$
(2.16)

From this expression we see that for a perfectly transmitting NS interface the conductance associated with the superconducting state, referred to in literature as the *sub*-*gap* conductance, is twice that of the normal state,  $G_S = 2G_N$ . This then reaffirms that the charge transport in the superconducting state is maintained by Cooper-pairs.

However in the limit of a weakly transmitting interface, such that  $T \ll 1$ , sub-gap conductance is suppressed. In such a scenario  $G_N \gg G_S$  as  $G_N \propto T$  and  $G_S \propto T^2$  thus under a given bias the conductance across the NS interface would give the underlying DOS. This technique, commonly referred to as tunnelling spectroscopy, has been widely utilised to study the superconducting properties of hybrid superconducting systems [14, 15].

### 2.1.4. MULTIPLE ANDREEV REFLECTION

The phenonmenon of Multiple Anreev Reflection (MAR) [16, 17] arises when the NS interface discussed in the previous section is mirrored such that the normal conductor is now encapsulated between two superconducting electrodes. Then under a given voltage bias a quasiparticle from the lower band of one superconductor can effectively 'bounce' between the two NS interfaces until it reaches the upper band of one of the superconductors, as shown in Fig. 2.4.

Under a given voltage bias,  $V_{\text{bias}}$ , quasiparticles in the lower conduction band of a superconductor acquire a certain amount of kinetic energy. These quasiparticles are then permitted to enter the normal conductor either as an electron or a hole with a given energy *E* at the first NS interface. These electrons (holes) are then incident upon the second NS interface and will undergo AR, whereupon a Cooper-pair will be transferred into the second superconductor. The retro-reflected hole is then permitted to undergo further AR until it reaches the upper conduction of either of the superconductors, whereupon each successive AR a Cooper-pair is transferred from one superconductor to the other. This process, known as MAR of the order n + 1 where n is the number of AR, results in a finite current flowing through the system.



Figure 2.4: (a) A representation of the MAR process of order 2 in the semiconductor picture. Effective voltage bias across the device is represented in the form of a kinetic energy of a quasiparticle in the lower band of the left superconductor that enters the normal conductor as an electron, undergoes an AR at the first NS interface and the resulting hole recombines into a quasiparticle upon reaching the upper band of the original superconductor. Upon AR a Cooper-pair is transferred into the left superconductor. (b) MAR process of order 3.

Assuming the scenarios as presented in Fig. 2.4, the threshold voltage for MAR process of the lowest order will be given by:

$$V_{\rm th} = \frac{2\Delta}{en} \tag{2.17}$$

Where  $n(V_{\text{bias}}) = 2\Delta/eV_{\text{bias}}$ . Each AR process is sensitive to the transmission probability,  $\tau$ , at a given interface. For  $\tau < 1$  each MAR process will be sensitive to  $\tau^n$ , thus the presence of MAR phenomena for channels with  $\tau \ll 1$  is negligible.

### **2.2.** JOSEPHSON JUNCTIONS

In the following section we build up on the concept of AR by extending the NS interface with the addition of another superconductor on the opposite side of the normal metal thus forming an SNS system. By restricting the dimensions of the normal metal such that it acts as a *weak link* between the two superconductors we study the formation of localised bound states known as Andreev Bound States (ABS), Fig. 2.5. These states are then responsible for the charge transport across the SNS system and we briefly discuss the current associated with each ABS. Such systems are commonly referred to as Josephson junctions and we look at two adaptations where the weak link is replaced with either a piece of insulator or a semiconducting material. Furthermore we study how these Josephson junctions react to their surrounding environment and, finally, how their properties can be utilised to study physical effects.

### **2.2.1.** The Andreev Bound State

Previously we saw that when an electron plane wave in the normal metal is incident upon the NS interface, there is a finite probability that it will be Andreev reflected as a hole of opposite spin and momentum into the normal metal. By encapsulating the normal metal between two superconductors, the Andreev reflected hole incident now upon the opposing NS interface can also undergo AR. In such a scenario the incident hole will be Andreev reflected as the original electron back into the normal metal. From the perspective of the two superconductors this sequence of events is equivalent to the removal of one Cooper-pair from the second superconductor and the same Cooper-pair being injected into the first superconductor [11, 18].



Figure 2.5: Sketch of an Andreev bound state. An electron in the N part incident upon the right NS interface can undergo AR being retro-reflected as a hole that itself can undergo AR at the left NS interface, resulting in the formation of a bound state, ABS, in the weak link whose dimension,  $d < \xi_C$ .

Provided that the phase coherence between the two forward and backward travelling waves is preserved, their interference will lead to the formation of a pair of localised bound states, ABS, with a pair of energies  $\pm E_A$ . Since the two superconductors have a particular phase associated with them,  $\phi_1$  and  $\phi_2$ , the ABS that link the two superconductors will depend on the phase difference,  $\varphi = \phi_1 - \phi_2$ .

By restricting ourselves to a 1-dimensional case and requiring that the length of the normal metal is smaller than the superconducting coherence length,  $d < \xi_C$ , the normal metal takes the role of a perfect conductor, which in the Landauer formalism [19] can be completely characterised by a set of transmissions,  $\tau$ , for each conduction channel. As AR does not mix different conduction channels we deduce that the ABS are purely characterised by  $\varphi$  and  $\tau$ .

In order to evaluate the dispersion of ABS, the Scattering matrix approach was first utilised by Landauer and Büttiker [20] where the role of the 1-dimensional conductor was reduced to that of scattering impurity. We refer the reader to [21, 22] for the full derivation and only state the result here. For a short junction,  $\xi_C \gg d$ , with a single conduction channel the energy dispersion of a pair ABS is given by:

$$\pm E_A = \pm |\Delta| \sqrt{1 - \tau \sin^2\left(\varphi/2\right)} \tag{2.18}$$

From this relation we observe that the pair of ABS are  $2\pi$  periodic in  $\varphi$  and that for perfectly transmitting channel the energy of the ABS goes to zero at  $\varphi = \pi$ . However for  $\tau < 1$ , a finite gap opens at the crossing with the magnitude given by  $2\Delta\sqrt{1-\tau}$  as shown in Fig. 2.7(a).

By making use of the one-particle picture and general formalism that was used to construct  $|GS\rangle$  of the superconductor, we define the Hamiltonian associated with the SNS system as being composed of two parts, one is responsible for the ABS and the other for the set of continuum states in the superconductor:

$$H_{SNS} = H_A + H_{BdG} \tag{2.19}$$

$$= E_A \left( \gamma_{A_+}^{\dagger} \gamma_{A_+} - \gamma_{A_-}^{\dagger} \gamma_{A_-} \right) + \sum_{|E| > |\Delta|} E \gamma_E^{\dagger} \gamma_E$$
(2.20)

Where we have introduced a set of creation (annihilation) operators  $\gamma_{A_{\pm}}^{\dagger}(\gamma_{A_{\pm}})$  that are similar in nature to quasiparticles and are referred to as Andreevons. They are responsible fo the creation of a pair of ABS with energies  $\pm E_A$ . The second term is responsible for the continuum of states found in the superconductor. With the aid of this Hamiltonian we can construct the ground state by again populating the vacuum state:

$$|GS\rangle = \left(\prod_{E_A} \prod_{E_i} \gamma_{E_i}^{\dagger}\right) \gamma_{A_-}^{\dagger} |V\rangle$$
(2.21)

From which we obtain the ground state energy of the SNS system to be:

$$E_{GS} = -E_A + \sum_{E < -\Delta} E \tag{2.22}$$

Hence we see that in the one-particle picture the ground state configuration consists of a continuum of states filled up to an energy  $-\Delta$  and a pair of localised states of energies  $\pm E_A$  found within  $\Delta$ .

### THE ANDREEV TWO LEVEL SYSTEM

In order to determine the nature of ABS and the configurations within which they can exist we restrict ourselves to the case where  $\varphi = \pi$ . Here ABS live deep within the superconducting region,  $E_A \ll \Delta$ , resulting in an effective two-level system characterised purely the Andreev Hamiltonian:

$$H_A = E_A \left( \gamma_{A_+}^{\dagger} \gamma_{A_+} - \gamma_{A_-}^{\dagger} \gamma_{A_-} \right)$$
(2.23)

We already saw that the  $|GS\rangle$  of the ABS two level system can be constructed by operating  $\gamma^{\dagger}_{A_{-}}$  on the  $|V\rangle$ . Thus we can construct the four possible states of the ABS system from:

$$|GS\rangle$$
 (2.24)

$$\gamma_{A_{-}}|GS\rangle$$
 (2.25)

$$\gamma^{\dagger}_{A_{+}} |GS\rangle \tag{2.26}$$

$$\gamma_{A_{+}}^{\dagger}\gamma_{A_{-}}|GS\rangle \tag{2.27}$$

The four possible configuration are represented in Fig. 2.6(a) within the one-particle picture. Due to the fermionic nature of Andreevons, each excitation will have an associated spin, allowing us to express the excitations as  $\gamma_{A_-} = \gamma^{\dagger}_{A_{\downarrow}}, \gamma^{\dagger}_{A_+} = \gamma^{\dagger}_{A_{\uparrow}}$ . Therefore it is convenient to introduce the excitation [7] picture to account for the spin texture as shown in Fig. 2.6(b) and given by the following relations:

$$|GS\rangle = |-\rangle \tag{2.28}$$

$$\gamma_{A\downarrow}^{\mathsf{T}} |GS\rangle = |\downarrow\rangle \tag{2.29}$$

$$\gamma_{A\uparrow}^{\dagger} |GS\rangle = |\uparrow\rangle \tag{2.30}$$

$$\gamma^{\dagger}_{A\uparrow}\gamma^{\dagger}_{A\downarrow}|GS\rangle = |+\rangle \tag{2.31}$$

Here  $|\pm\rangle$  states correspond to a Cooper-pair either in a ground or an excited state of the ABS two level system with energies  $E_{GS}$  and  $E_{GS} + 2E_A$  respectively. Both of these states have an even parity and have no net spin associated with them and hence are the only states that can coherently transfer Cooper-pairs across the weak link. The two other states  $|\uparrow,\downarrow\rangle$  correspond to the so-called poisoned state of ABS, where a single quasiparticle excitation causes an odd population of the ABS two level system and gives it a finite spin [23–25].



Figure 2.6: (a) The one-particle picture representation of the 4 possible state configurations of an Andreev twolevel system. The first and last configuration refer to a Cooper-pair in the ground or excited state. The second and third configuration refer to the case of a poisoned ABS. (b) The equivalent excitation picture.

For the purpose of chapter 5, assuming that the ABS two level system can only take on an even configuration we can rewrite the Andreev two level Hamiltonian [26–28] as:

$$H_A = -E_A(\varphi)\sigma_z \tag{2.32}$$

Where we have taken into account the  $\varphi$  dependence of ABS and  $\sigma_z$  is the Pauli spin matrix.

### THE ANDREEV CURRENT



Figure 2.7: (a) Dispersion of ABS ground (red) and excited (blue) states as a function of  $\varphi$  for  $\tau = 0.9$ , in the case of a perfect transmission the two states cross at  $\varphi = \pi$  as shown by the dashed line. (b) The resulting supercurrent as carried by the ABS.

The manifestation of a supercurrent (Fig. 2.7(b)), a dissipationless current mediated by Cooper-pairs, is directly related to the dispersion of ABS by  $^2$  [29]:

$$I_A(\varphi,\tau) = -\frac{2e}{\hbar} \frac{\partial E_A}{\partial \varphi} = \frac{e\Delta}{2\hbar} \frac{\tau \sin(\varphi)}{\sqrt{1 - \tau \sin^2(\varphi/2)}}$$
(2.33)

Here each conductance channel carries its own supercurrent. For a pair of ABS we see that the two states  $|\pm\rangle$  carry a supercurrent of the same magnitude but in opposing directions. Since the conduction channels do not mix, for a junction of several channels and in the low temperature limit,  $E_A, \Delta \gg k_B T$ , the supercurrent of the entire system is then simply the sum of each channels contribution,  $I(\varphi, \tau) = \sum_i I_A(\varphi_i, \tau_i)$ . From this we can define the Current-Phase relation (CPR) [30] of a given junction as:

$$I(\varphi) = \frac{e\Delta}{2\hbar} \sum_{i} \frac{\tau \sin(\varphi)}{\sqrt{1 - \tau \sin^2(\varphi/2)}}$$
(2.34)

The nature of the weak link is not limited to any particular type of material. For the purpose of this thesis we will focus on only two types of weak links: a semiconducting nanowire (NW) and a piece of insulator (I). Although the underlying principles of ABS apply in both scenarios the junction specific properties,  $\tau$  and  $\varphi$ , can vary drastically. From this point forth we will adopt the following nomenclature: when we refer to a nanowire based Josephson junction, we will explicitly state S-NW-S junction. When talking about the SIS Josephson junction, we will refer to it as the widely adopted tunnel junction. When we refer to principles that can apply to both junctions we will simply talk in terms of junctions.

<sup>&</sup>lt;sup>2</sup>The present case is only valid in the case of a short, ballistic junction.

### **2.2.2.** The Tunnel Junction

In the tunnel junction limit the supercurrent is carried by a large number of weakly transmitting channels,  $\tau_i \ll 1$ , that share a global  $\varphi$ . As such the CPR in the tunnel junction limit reduces to:

$$I(\varphi) = I_C \sin(\varphi) \tag{2.35}$$

Where we have introduced a new parameter,  $I_C$ , known as the critical current of the junction. This is the maximum supercurrent that a tunnel junction can maintain. In the low temperature limit Ambegaokor and Baratoff [31] showed that the critical current is directly related to the normal state conductance,  $I_C = \pi \Delta G_N / 2e^3$ .

The tunnel junction Hamiltonian then becomes:

$$H_J = -E_J \cos\left(\varphi\right) \tag{2.36}$$

Where  $E_I$  is the Josephson coupling,  $E_I = \hbar I_C / 2e$ .

As will be discussed later, in certain configurations, the tunnel junction can be used as a non-linear inductor, with the Josephson inductance given by:

$$L_J = V \left(\frac{dI}{dt}\right)^{-1} = \frac{\hbar}{2eI_C \cos\left(\varphi\right)}$$
(2.37)

THE A.C. AND D.C. JOSEPHSON EFFECTS



Figure 2.8: (a) The flow of a quasiparticle current when the Josephson junction is subject to a bias,  $|eV| > 2\Delta$ . (b) At zero bias, Cooper-pairs tunnel coherently through the weak link with the direction of the flow given by  $\varphi$ , the d.c. Josephson effect. (c) At a small finite bias,  $|eV| < 2\Delta$ , Cooper-pairs tunnel across the weak link through emission and absorption of virtual photons, the a.c. Josephson effect.

In Section 2.1.3 we showed that there are two contributions to the current carried across the NS interface, one due to the quasiparticles and one due to Cooper-pairs, depending on the bias *V* that the junction is subject to. We extend upon this to account for the transport across a Josephson junction. As is shown in Fig. 2.8(a), if the junction is

 $<sup>{}^{3}</sup>G_{N}$  is the total conductance related to  $\Sigma_{i}\tau_{i}$ .

subject bias voltage such that  $|eV| > 2\Delta$  then a flow of quasiparticles is permitted across the weak link.

In the absence of any voltage the Cooper-pair condensates of two superconductors are aligned and Cooper-pairs carry a supercurrent across the junction as per the CPR. The direction of the supercurrent will be given by  $\varphi$  in what is known as the d.c. Josephson effect which is shown in Fig. 2.8(b) [32, 33].

When the junction is subject to a bias such that  $|eV| < 2\Delta$ , the phase difference  $\varphi$  starts to wind as per the a.c. Josephson effect:

$$V = \frac{\hbar}{2e} \frac{d\varphi}{dt}$$
(2.38)

Which causes  $\varphi$  to oscillate at the Josephson frequency,  $f_J = 2eV/h$ . This in turn causes Cooper-pairs to tunnel back and forth across the junction via the emission and absorption of virtual photons, as shown in Fig. 2.8(c). Where the energy of the photons is given by  $hf_J$ . This gives rise to an AC current at a frequency  $f_J$ . Although a virtual process, in the presence of an environmental mode that can admit radiation equivalent to the tunnelling of Cooper-pairs, a finite Cooper-pair current will flow for  $|eV| < 2\Delta$ . This current is commonly referred to as the inelastic Cooper-pair tunnelling current (ICPT).



Figure 2.9: Sketch of the I(V) characteristics of Josephson junction. A clear onset of the quasiparticle current is visible for  $|eV| > 2\Delta$  with a supercurrent present at eV = 0.

A schematic of the bias applied across the Josephson junction is shown in Fig. 2.9. The scenarios due to schematics of Figs. 2.8(a) and (b) are clearly visible and we, for now, omit the presence of environmental modes. The Josephson junction has been implemented as both a source and detector of radiation [34–36]. Before we discuss how this is done we will first discuss a common model used to understand the dynamics of a junction with realistic dimensions, the RCSJ model.

### 2.2.3. THE RCSJ MODEL

Following [7, 37] we model a realistic junction as an ideal junction that is shunted by a resistor and a capacitor in parallel, as is shown in Fig. 2.10. The capacitive shunt comes from the geometric structure and the resistive shunt comes from the finite dissipation in



Figure 2.10: A realistic Josephson junction subject to a current bias is represented as an ideal Josephson element in parallel with a resistive and capacitive shunt.

a finite voltage regime<sup>4</sup>. By applying a current bias between the two poles of the circuit, the total current is determined through Kirchoff's laws as:

$$I = I_C \sin\left(\varphi\right) + \frac{V}{R} + C\frac{dV}{dt}$$
(2.39)

Where the first term is responsible for the ideal Josephson element in the circuit, the second term is the parallel component of the current through the resistive shunt and the final term is the current contribution due to the capacitive shunt.

Using the a.c. and d.c. Josephson effects we can rewrite the equation for the current as follows:

$$0 = \left(\frac{\varphi_0}{2\pi}\right)^2 C \frac{\partial^2 \varphi}{\partial t^2} + \left(\frac{\varphi_0}{2\pi}\right) \frac{1}{R} \frac{\partial \varphi}{\partial t} + \frac{\partial}{\partial \varphi} U(\varphi)$$
(2.40)

with

$$U(\varphi) = -\frac{\varphi_0}{2\pi} \left( I_C \cos(\varphi) + I\varphi \right) \tag{2.41}$$

Where  $\varphi_0$  is the flux quantum.

This equation is remarkably similar to that of motion of an object on a spring subject to some damping. By equating the comparable terms in both equations we can give some intuitive meaning to the above equation. The capacitive term takes on the role of a mass term whilst the resistive term is associated with damping of a fictitious phase particle  $\varphi$  moving under a potential  $U(\varphi)$ .

In Fig. 2.11 (a) we show the general form of the potential in the absence of any current bias, *I*, and further define two properties. The phase particle remains in a metastable state residing at the bottom of one of the minima of  $U(\varphi)$  oscillating at what is known as the plasma frequency,  $\omega_P = \frac{1}{\sqrt{L_L C}}$ .

Under a finite *I* the potential  $U(\varphi)$  will experience an effective tilt, with the magnitude of the tilt directly given by  $i = I/I_C$ , as is shown in Fig. 2.11(b). From the shape of the potential and the relative tilt, one would anticipate that the phase-particle will remain in a given minima until *I* has reached  $I_C$  beyond which point the phase particle

<sup>&</sup>lt;sup>4</sup>For small bias,  $eV_{\text{bias}} < 2\Delta$  the shunt resistance is dominated by what is referred to as the quasiparticle resistance,  $R_{QP}$ , which arises due to finite sub-gap DOS. For large bias  $eV_{\text{bias}} > 2\Delta$  the shunt resistance is dominated by normal state resistance,  $R_N$ .


Figure 2.11: (a) Sketch of the phase particle oscillating at the bottom of one minima of  $U(\varphi)$  at a frequency of  $\omega_P$  in the absence of a current bias, *I*. (b) When the junction is subjected to a *I*,  $U(\varphi)$  experiences an effective tilt where the magnitude of the tilt is directly related to the ratio of  $I/I_C$ .

can traverse down the slope of the potential thus entering what is known as the running state. The phase-particle will then acquire a particular velocity, depending on I, which will result in a finite voltage V flowing across the junction as per the DC Josephson effect. However this is only true in a ideal experimental set-up where the current source is a perfect source. In realistic experimental set-ups however, the applied current will have some thermal fluctuations associated with it [38] hence the current bias I will fluctuate around the ideal value with  $I = \overline{I} + \delta I$ . As such the phase-particle can experience a kick where it can traverse down the potential for  $I < I_C$ . Upon reaching the next minimum the phase-particle can either be trapped or remain in the running state. Whether or not the phase particle gets trapped will depend on the depth of the potential wells and the effective friction felt by the phase-particle. As this is related to the effective mass of the phase-particle and damping it experiences, we define a quality factor associated with the Josephson junction;  $Q = \omega_P RC$ . Through the capacitive term we can define two different regimes: the underdamped and overdamped regime. In the underdamped regime the C is large enough that the Q > 1, the phase-particle will enter the running state when escaping the first minimum as there is not enough force to stop it at the next minimum. In the opposite case, the overdamped regime,  $Q \ll 1$ , and the phase particle can momentarily enter the running state but will have a finite probability to be retrapped in a given minimum after its escape. The phase-particle will traverse down an incremental

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set of wells leading to a small voltage flowing across the junction. Thus when studying the junction dynamics one can discern between the two regimes by looking for the presence/absence of voltage before the junction has entered into the running state  $^5$ .

This premature escape out of the potential gives rise to the concept of a switching current,  $I_{SW}$ , a property that is directly attainable in a measurement. From  $I_{SW}$  it is possible to retrace the original  $I_C$  by obtaining a distribution of switching events where the mean of  $I_{SW}$  will be given by:

$$\langle I_{SW} \rangle = I_C \left\{ 1 - \left[ \frac{k_B T}{2E_J} \ln \frac{\omega_P \delta t}{2\pi} \right]^{2/3} \right\}$$
(2.42)

Here *T* is the temperature associated with the surrounding environment and  $\delta t$  is the time associated with the sweeps necessary to attain the distributions. Due to the thermal dependence of  $I_C$ ,  $\omega_P$  and  $E_J$  it is difficult to extract the true  $I_C$  from a single distribution of  $I_{SW}$  for a given temperature setting. Thus one is required to perform the same experiment over a wide *T* range. With the acquired distributions a set of escape rates can be experimentally calculated which upon comparison with the theoretical variations would give a more accurate estimate for  $I_C$ .

#### **2.2.4.** JOSEPHSON RADIATION

We now turn our attention to application of the Josephson junction as a source of monochromatic radiation. Here we only outline the brief principle and refer the reader to [34] for a detailed treatment. Previously we introduced the concept ICPT where for given finite voltage bias,  $V_{\text{bias}}$ , Cooper-pairs oscillate across the junction at the Josephson frequency,  $f_J$ , via the emission and absorption of virtual photons. In the presence of an electromagnetic environment that can admit some radiation, the tunnelling of Cooper-pairs results in a generation of photons whose energy is directly related to applied bias via:  $2eV_{\text{bias}} = \hbar \omega_{ph}$ . Here the factor of 2 accounts for the charge of the Cooper-pairs and the process is shown in Fig. 2.12.

As was demonstrated [38] a current source will have some fluctuations associated with it. The presence of these fluctuations will allow us to model the junction as a source of frequency dependent current *noise* which is typically characterised by a current noise spectral density in frequency space as [38]:

$$S_{I}(\omega) = \int_{-\infty}^{\infty} e^{i\omega\tau} C(\tau) d\tau \qquad (2.43)$$

Where  $C_{\tau}$  is the current-current correlation function. In general  $S_I(\omega)$  will be composed of two components. One for positive frequencies,  $S_I(\omega)$ , corresponding to the *absorption* part of the spectrum and  $S_I(-\omega)$  corresponding to the *emission* part of the spectrum. In the low temperature limit,  $k_B T < eV_{\text{bias}}$ , there will be an asymmetry between the two,  $S_I(\omega) \neq S_I(-\omega)$ , where the absorption will be stronger than emission.

<sup>&</sup>lt;sup>5</sup>We note that experimentally one can generally distinguish between the two scenarios via the presence (underdamped) or absence (overdamped) of a hysteresis between the switching and retrapping process. A more careful treatment is necessary in the presence of more exotic R and C terms [39] and the self-heating of the junction.



Figure 2.12: Sketch of how a Josephson junction can be used as a source of HF radiation. Cooper-pairs can only flow provided that the environment can accept the associated photon radiation.

Later we will see how the same Josephson junction can be utilised to study the high frequency (HF) radiation incident upon it.

Depending on the bias the junction is subjected to, it can act as a noise source of two different origins. When  $eV_{\text{bias}} < 2\Delta$  the current across the junction is carried by Cooper-pairs via interaction with the environment. The junction then acts as a source of Josephson radiation characterised by  $S_I(\omega) = \frac{I_C^2}{4}\delta\left(f - \frac{2eV_{bias}}{h}\right)$ . On the other hand when the junction is biased at  $eV_{\text{bias}} \gg 2\Delta$  it can be approximated as a linear conductor. The shot noise emitted is frequency independent and is characterised by  $S_I(\omega) = eIF$  where F is the Fano factor associated with a conductor.

#### **2.2.5.** RADIATION DETECTION

As well as being utilised as a source of monochromatic radiation, the Josephson junction can also be used as a detector of high-frequency (HF) radiation. The detection mechanism is based on the process of Photon-assisted tunnelling (PAT) and is depicted in Fig. 2.13.



Figure 2.13: Sketch of how a Josephson junction can be used as a detector of HF radiation. The incoming HF can induce a quasiparticle current in the junction provided that  $eV_{bias} + \hbar \omega_{ph} \ge 2\Delta$ . The coupling environment is depicted by a set of capacitors.

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When the junction is subject to  $eV_{\text{bias}} < 2\Delta$  the transport of quasiparticles is forbidden. However if the junction is irradiated such that  $eV_{\text{bias}} + \hbar\omega_{ph} \ge 2\Delta$ , where  $\omega_{ph}$  is the frequency of the incoming radiation, a measurable quasiparticle current is detected. The effect of this incoming radiation demonstrates itself as a step-like onset, as shown in Fig. 2.14(b), where the location of the onset is directly set by  $\omega_{ph}$ .

In chapter 4 we outline how a tunnel junction is implemented to measure the Josephson radiation emitted from a S-NW-S junction. This method relies on the coupling between the source and detector mediated by an environment,  $Z(\omega)$ , allowing us to express the coupling as follows:

$$S_V(\omega) = |Z(\omega)|^2 S_I(\omega)$$
(2.44)

Here  $S_V(\omega)$  is the noise spectral density associated with the voltage oscillations developed across the detector due to the incoming radiation. From the form of  $Z(\omega)$  we obtain  $S_I(\omega)$ , the noise spectral density associated with the emitted current noise. For the purpose of the present thesis we focus on only the two afro-mentioned types of radiation.

In the presence of radiation the extra addition to the current is expressed as [34–36]:

$$I_{PAT}(V_{bias}) = I_{QP}(V_{bias}) + I_{QP,0}(V_{bias})$$
(2.45)

Where  $I_{QP}(V_{bias})$  is the contribution due to the pure Josephson radiation and  $I_{QP,0}(V_{bias})$  is the presence any finite current in the absence of any radiation, as shown in Fig. 2.14(b).

We make the assumption that absorption spectrum of the detector is a single frequency spectrum. Here the voltage fluctuations can be described by:

$$\delta V_{bias}(t) \propto V_{bias,0} \cos(\omega_0 t) \tag{2.46}$$

following we can express  $I_{QP}(V_{bias})$  as:

$$I_{QP}(V_{bias}) = \int_{-\infty}^{\infty} \left(\frac{e}{\hbar\omega}\right)^2 S_V(\omega) \, d\omega$$
 (2.47)

$$= \frac{1}{8\pi} \left( \frac{eV_{bias,0}}{\hbar\omega_0} \right)^2 \tag{2.48}$$

#### **2.2.6.** GENERAL $Z(\omega)$ ENVIRONMENTS

In general the true from of the electromagnetic environment surrounding the Josephson junction can be quite complicated. The external biasing circuit gives rise to stray capacitances that strongly modify the current-voltage (I(V)) characteristics. Therefore, we extend upon the RCSJ model and generalise the environment by a complex impedance  $Z_{env}(\omega)$  which takes on the role of the resistive shunt. From the junction perspective the impedance of the global system can be expressed as:

$$Z(\omega) = \frac{1}{i\omega C + Z_{env}^{-1}(\omega)}$$
(2.49)

Following Ingold and Nazarov [40] we model the energy modes in the environment as a set of harmonic oscillators with an associated frequency given by,  $\omega_n = 1/\sqrt{L_n C_n}$  where *n* denotes the index of the mode. The environment can then be viewed as linear combination of each independent harmonic oscillator<sup>6</sup>.

<sup>&</sup>lt;sup>6</sup>We note that  $Z_{env}^{-1}(\omega) = Y_{env}(\omega) = \sum_{n} Y_{env,n}(\omega, n)$ .

Each mode in the environment can then be excited by accepting an energy  $\hbar \omega_n$  where the transitions between the ground and excited states of the harmonic oscillators is calculated using Fermi's Golden rule.

The interaction between the environment and the junction is then accounted for by the energy exchange between the two systems. This interaction is best described by the P(E) theory which relates the interaction to the probability, P(E), of the environment to admit radiation of energy, E. In the case of the junction this radiation comes from the tunnelling of Cooper-pairs under a given  $V_{\text{bias}}$ , as such  $E = 2eV_{\text{bias}}$ . The forward tunnelling rate of the Cooper-pairs across the junction is then given by:

$$\vec{\Gamma} (V_{bias}) = \frac{\pi}{2\hbar} E_J^2 P (2eV_{bias})$$
(2.50)

The direction of the tunnelling events is given by sign of  $\varphi$  and the reverse tunnelling rate is  $\Gamma(V_{bias}) \propto P(-2eV_{bias})$ . From the expressions for  $\Gamma(V_{bias})$  and  $\Gamma(V_{bias})$  we can express a net current flowing through the junction as:

$$I(V_{bias}) = 2e\left(\overrightarrow{\Gamma}(V_{bias}) - \overleftarrow{\Gamma}(V_{bias})\right)$$
(2.51)

$$= \frac{\pi}{2\hbar} E_J^2 \left( P \left( 2eV_{bias} \right) - P \left( -2eV_{bias} \right) \right)$$
(2.52)

From this expression we gather that at  $V_{\text{bias}} < \Delta/2e$ , the tunnelling of Cooper-pairs transfers an energy of  $2eV_{\text{bias}}$  to the environment (as there are no other available states in the junction that can admit this energy). The current,  $I_{ICPT}$ , can then be used to directly study the surrounding environment for  $\omega$  ranging between 0 and  $\Delta/\hbar$ . Furthermore the junction can be seen as a source of d.c. power,  $P = I_{ICPT}V_{bias}$  which converts to an a.c. power felt by the environment, this implies that  $I_{ICPT}$  can be effectively used to study high-frequency environments. This conversion is more commonly expressed as<sup>7</sup>:

$$I_{ICPT}(V_{bias}) = \frac{I_C^2 Re[Z(\omega)]}{2V_{bias}}$$
(2.53)

In Fig. 2.14(a) we show the emergence of  $I_{ICPT}$  and its dependence on different environmental modes.

#### 2.2.7. SQUIDS

When a junction is embedded in a closed superconducting loop or placed in parallel with another junction it forms either an r.f. or d.c. superconducting quantum interference device (SQUID) [7]. For the purpose of the present thesis we briefly outline the working principle of a d.c. SQUID with a schematic representation shown in Fig. 2.15(a).

The total supercurrent through the SQUID is then the combination of the supercurrents flowing through the two junctions characterise by  $\varphi_{I1}$  and  $\varphi_{I2}$  respectively.

$$I_{SQUID} = I_{C,J1}CPR(\varphi_{J1}) + I_{C,J2}CPR(\varphi_{J2})$$

$$(2.54)$$

<sup>&</sup>lt;sup>7</sup>We note that we have limited our discussion to scenarios involving only single photon transitions where the above equations hold provided that  $Z(\omega) \ll R_Q$  where  $R_Q$  is the superconducting resistance quantum.



Figure 2.14: (a) The resulting  $I_{ICPT}$  in the presence of an environment,  $Z(\omega)$ , characterised by  $1/\sqrt{LC}$  where C is 20 fF. (b) I(V) characteristics of detection of HF radiation from Josephson (step-like) or quasiparticle (smooth onset) noise.



Figure 2.15: (a) Sketch of a d.c. SQUID consisting of two Josephson junctions connected in parallel via a superconducting loop. (b) The resulting modulation of  $I_{C,SQUID}$  with respect to  $I_{C,J1/J2}$  as a function of the SQUID flux  $\gamma$ . For a perfectly symmetric SQUID the  $I_{C,SQUID}$  goes to zero for  $\gamma$  at odd multiples of  $\pi$ .

For simplicity we assume sinusoidal CPR of both junctions which allows us to express  $I_{SOUID}$  as:

$$I_{SQUID} = I_{C,J1} \sin(\varphi_{J1}) + I_{C,J2} \sin(\varphi_{J2})$$
(2.55)

When the SQUID is subject to an externally applied magnetic field such that it is threading the SQUID, the two superconducting phases  $\varphi_{J1}$  and  $\varphi_{J2}$  are linked via the threading flux<sup>8</sup>:

$$\gamma = \varphi_{J1} - \varphi_{J2} = 2\pi \frac{\Phi}{\Phi_0} \mod |2\pi|$$
 (2.56)

Where the link between the externally applied flux,  $\Phi$ , and the SQUID flux,  $\gamma$ , is accounted for by the vector potential *A* due to the externally applied field. Utilising this relation we can express the supercurrent flowing through one junction with respect to the phase of the other.

<sup>&</sup>lt;sup>8</sup>This is valid in the limit of negligible kinetic and geometric loop inductance.

In the case of a symmetric SQUID,  $I_{C,J1} = I_{C,J2} = I_{C,J}$ , the critical supercurrent of the SQUID,  $I_{C,SOUID}$  is modulated with respect to the flux,  $\gamma$ :

$$I_{C,SQUID} = 2I_{C,J} |\cos(\gamma/2)| \tag{2.57}$$

From the above expression we see that the SQUID can be regarded as a Josephson junction with a flux tunable  $E_I$ .

In most realistic realisations of a d.c. SQUID there is some asymmetry present between the two junctions, which is parametrised as:

$$\alpha = \frac{I_{C,J1} - I_{C,J2}}{I_{C,J1} + I_{C,J2}} \tag{2.58}$$

With this consideration the expression for  $I_{C,SQUID}$  is modified to:

$$I_{C,SQUID} = (I_{C,J1} + I_{C,J2})\sqrt{(1 - \alpha^2)\cos^2(\gamma/2) + \alpha^2}$$
(2.59)

The effect of this asymmetry is captured in Fig. 2.15(b).

#### **2.3.** The Cooper-pair Transistor

Instead of placing two junctions in a parallel configuration, we now turn to the case where they are in series, with an isolated superconducting island separating them. This configuration is commonly referred to as the Cooper-pair transistor (CPT) and is of particular interest as it gives rise to an interesting interplay between three energy scales associated with it: the Charging energy,  $E_C$ , the Josephson energy,  $E_J$ , and the superconducting gap,  $\Delta$ . Here we give a brief summary of the CPT and refer the reader to [41, 42] for more details.

#### **2.3.1.** The Cooper-pair Transistor Hamiltonian

The CPT is schematically shown in Fig. 2.16(a) and it consists of a superconducting island that is tunnel coupled to the nearby source and drain contacts via two Josephson junctions. Furthermore there is a nearby electrostatic gate which is capacitively coupled to the island. We begin our discussion by assuming that the island is perfectly isolated.

In typical experiments the nanoscale dimensionality of the island and the cryogenic temperatures at which the CPT is investigated leads to the fact that the energy it costs to add or remove a charge to or from the island,  $E_C \gg k_B T$ . In the present section we adopt  $E_C = e^2/2C_{\Sigma}^{9}$  where  $C_{\Sigma} = C_{J1} + C_{J2} + C_g$  is the total capacitance of the system,  $C_g$  is the capacitance of the gate and  $C_{J1/2}$  are the capacitances of each junction. Although the charge on the island is quantized, it can be manipulated in a continuous manner by applying an electric field via a gate voltage,  $V_g$ . As such we can describe the superconducting island with the following Hamiltonian:

$$H_I = E_C \left( n - n_g \right)^2$$
 (2.60)

<sup>&</sup>lt;sup>9</sup>Note that we have adopted the expression for  $E_C$  when dealing single-*e* charges. For the case of Cooper-pairs  $E_{C,S} = 4E_C$ .



Figure 2.16: (a) Sketch of a CPT where two junctions of  $E_{J1/2}$  isolate a superconducting island whose charge occupation is manipulated via  $V_g$ . (b) The charge dispersion of the island.

Where  $n_g = C_g V_g / e$  is the gate-induced charge and *n* is the effective number of charges on the island. The ground state energy dispersion as a function of  $n_g$  is shown in Fig. 2.16(b), which is 2e-periodic in  $n_g$ .

We now turn on the coupling to the surrounding environment by introducing the junctions characterised by their mutual  $H_{J1/J2}$ . For the time being we assume symmetric junctions and introduce a global  $E_J = 2E_{J1/J2}$  and  $\varphi = \varphi_1 + \varphi_2$ . From this we arrive at the tunnel coupling Hamiltonian is given by:

$$H_I = -E_I \cos\left(\varphi\right) \tag{2.61}$$

Thus the total Hamiltonian of the system becomes:

$$H_{CPT} = H_I + H_J \tag{2.62}$$

$$= E_C \left(n - n_g\right)^2 - E_J \cos\left(\varphi\right) \tag{2.63}$$

At a first glance we observe that this Hamiltonian has two degrees of freedom, the charge on the island, *n*, and the phase,  $\varphi$ . In the quantum mechanical language these two variables take on the role of operators with the associated commutation relations,  $[n, \varphi] = i$ , which states that at any given configuration the knowledge of one variable leads to an ill-defined value for the other.

We now look for solutions to  $H_{CPT}$  via the eigenvalue problem and we transform the Hamiltonian into a charge basis which leads to:

$$E|n\rangle = E_C \left(n - n_g\right)^2 |n\rangle - \frac{E_J}{2} \left(|n - 2\rangle + |n + 2\rangle\right)$$
(2.64)

From this expression we observe that on top of the existing charge dispersion of  $H_I$  the the  $E_J$  term in the  $H_{CPT}$  mixes adjacent charge states, creating an avoided crossing at the intersection between two charge states.<sup>10</sup>

<sup>&</sup>lt;sup>10</sup>The avoided crossing leads to the re-normalisation of the charnging energy where the renormalized value is  $E_C^* < E_C$  [43].

Due to the commutation relation of *n* and  $\varphi$  we can look at two different limiting cases. First, when  $E_C \gg E_J$ , *n* is a well defined variable with energy dispersion as a function of  $n_g$ . Here the  $E_J$  term acts as a perturbation. In the opposing limit,  $E_C \ll E_J$ , the  $E_J$  term is dominant and the charge on the island is ill-defined. This is the so-called Transmon [44] limit where the CPT is insensitive to any charge noise. The two cases are represented in Fig. 2.17.



Figure 2.17: **a** Charge dispersion of the CPT in the  $E_J \gg E_C$ . **b** Charge dispersion in the opposing limit,  $E_C \gg E_J$ .

#### **2.3.2.** QUASIPARTICLES IN A COOPER-PAIR TRANSISTOR

So far we have limited our discussion to the case where charges present in the system are Cooper-pairs. However the island could host quasiparticle excitations provided that the energy associated with brining them onto the island is  $\epsilon > \Delta$ ,  $E_C$ . We account for the presence of a finite number of quasiparticles by introducing a new term to  $H_{CPT}$ :

$$H_{QP} = \sum_{k} \epsilon_k a_k^{\dagger} a_k \tag{2.65}$$

Where the creation(annihilation) operators  $a_k^{\dagger}(a_k)$  are responsible for quasiparticles with energies  $\epsilon_k$ . Although quasiparticles can come from a variety of sources they are generally grouped into two categories: equilibrium and non-equilibrium [45]. We will limit our discussion to the presence of equilibrium quasiparticles that can come from the finite temperature of the surrounding environment. In the present scenario the 2e-periodic charge dispersion is modified to take into account the presence of single charges on the island as shown in Fig. 2.18. This results in an effective shift of the two parabolas with respect to one another by 1e in  $n_g$ 

The energy difference between even and odd occupations of the island is known as the Free-energy difference,  $F_{o-e} = F_o - F_e$ , where  $F_{e/o} = -kT \ln (Z_{e/o})$  and  $Z_{e/o}$  is the partition function of the even/odd state. The presence of  $F_{o-e}$  has been studied in detail in [42] and we state the result for small temperatures,  $k_B T \ll \Delta$ , as:

$$F_{o-e} = -k_B T \ln \left[ \tanh \left( N_{\rm eff} e^{-\Delta/k_B T} \right) \right]$$
(2.66)



Figure 2.18: Charge dispersion of the CPT in the  $E_C > E_I$  limit. The solid balck lines depict the ground and first excited states of the CPT in the even-parity charge configuration. The dotted dark red line depicts  $\Delta \gg E_C$ . The dashed orange lines depict the odd-parity charge states. (a) In the low temperature limit  $T \ll T^*$  the first available state for quasiparticle occupation of the CPT is located at  $\Delta$  (b) For  $T < T^*$ ,  $F_{o-e}$  starts to decrease below  $\Delta$ . (c) For  $T > T^*$  The CPT becomes fully 1e-periodic.

Where  $N_{\text{eff}}$  is the effective number of quasiparticles in the system and depends on the specifics of the material as well as the volume of the island. From this expression we see that at zero temperature quasiparticles with  $\epsilon < \Delta$  are absent from the CPT and the ground state charge dispersion is perfectly 2e-periodic. As the temperature starts to increase, the charge dispersion associated with odd states in Fig. 2.18 begins to reduce in energy leading to an even-odd pattern. The CPT finally transitions into a 1e-periodic ground state at the crossover temperature  $T^* = \Delta/k_B \ln (N_{\text{eff}})$  [46–48]. Note that in most experimental systems  $T^* < T_C$  thus the loss of 2e-periodicity in the ground state is not symbolic of the loss of superconductivity.



Figure 2.19: (a) The different charge configurations the CPT can be in. The first two correspond to the case where although there is a quasiparticle present in the leads (middle), the island remains in an even-parity state. Finally a quasiparticle can reside on the island. (b) The different tunnelling mechanism between the lead and the island where the island acts as a barrier (first) or as a trap (second). Figure adapted from [46].

So far we have concerned ourselves with the presence of a quasiparticle on the island, however there are two possible configurations in which the CPT could be:

i A quasiparticle on the island with the associated energy  $E_i = E_0(n_g) + \Delta_i$  where

 $\Delta_i$  is the superconducting energy gap of the island and  $E_0(n_g)$  is the ground state energy of the island. Here the parity of the island as well as the CPT is odd.

ii A quasiparticle in the leads with the energy  $E_l = E_0(n_g) + \Delta_l$  where  $\Delta_l$  is the superconducting energy gap of the leads. Although the parity of the CPT is odd, the island is in an even configuration.

From these two relation we can infer two possible energy configurations that the CPT can find itself in. Either  $\Delta_l < \Delta_i$  or  $\Delta_l > \Delta_i$ , these two scenarios are shown in Fig. 2.19. In the case of  $\Delta_l < \Delta_i$  it is energetically favourable for the quasiparticle to reside in the leads and the island then takes on the role of the barrier. The island prefers to remain in an even-charge configuration and any excited quasiparticles are quickly evacuated. In the reverse case,  $\Delta_l > \Delta_i$ , the island acts as a quasiparticle trap and the quasiparticle prefers to remain on the island, leading to an odd-charge configuration. In the presence of finite thermal fluctuations we can associate a set of rates associated with hopping on/off the island as shown in Fig. 2.19. With the aid of these rates we can associate two scales with the addition or removal of a quasiparticle to and from the island; poisoning,  $\tau_o$ , and un-poisoning,  $\tau_e$ . For  $\Delta_l < \Delta_i$  we see that  $\tau_e > \tau_o$ , thus even if the island of the CPT is poisoned it will quickly *unpoison* itself. Thus by carefully engineering  $\Delta_l$  and  $\Delta_i$  it is possible to maintain the even parity of the island for a longer time-scale.

The dynamics of quasiparticles and the subsequent effect of  $\tau_o$  and  $\tau_e$  on the parity of the island are of crucial importance for potential CPT applications. Although a variety of experimental algorithms exist for their study, one of the more trusted methods is based on measuring the  $I_s(n_g)$  of the CPT. This technique is utilised in chapter 6 and we briefly outline  $I_s(n_g)$  here with its adaptation covered in chapter 3.

#### **2.3.3.** Supercurrent of the Cooper-pair Transistor

Analogous to the d.c. SQUID, the CPT can be viewed as a field-effect transistor where the supercurrent,  $I_s$ , is modulated by  $n_g$  as per the following expression:

$$I_s(n_g) = \frac{1}{\varphi_0} \frac{\partial E}{\partial \varphi}$$
(2.67)

Thus we see that  $I_s(n_g)$  will directly reproduce the oscillatory pattern of the ground state charge dispersion. Furthermore by utilising  $\tau_o$  and  $\tau_e$  we can anticipate that for any given  $n_g$  configuration the island can be found in an odd or even parity state, leading to a bimodal distribution of  $I_s(n_g)$ . This bimodal distribution can be used to study the parity lifetime,  $\tau_P$ , of the island (how long it remains in a given parity configuration).

#### **2.4.** ON-CHIP MICROWAVE SPECTROSCOPY

Due to the rising relevance and adaptations of semiconductor-superconductor hybrid structures the energetic structure of S-NW-S junctions and nanowire CPTs (NW-CPTs) and their subsequent manipulation has become desirable. Here we briefly outline a technique based on  $I_{ICPT}$  that induces transitions in the environment through microwave spectroscopy [21, 25, 49–52], revealing ABS excitations in S-NW-S junctions and excitations in the energy dispersion of the NW-CPT. This process of on-chip microwave spec-



Figure 2.20: (a) Schematic representation of the circuit used to perform on-chip microwave spectroscopy, with the components contained within the dashed line located on-chip. The two sides of the circuit are capacitively coupled through  $C_C$  and the radiation generated by the spectrometer is incident upon the h-SQUID acting as the environment. The h-SQUID consists of either an S-NW-S junction (b) or NW-CPT (c) in parallel with a reference tunnel junction such that  $E_{J,S-NW-S/NW-CPT} < E_{J,TJ}$ . The entire circuit is decoupled from the rest of the set-up through  $R_{SOUID/spec}$ .

troscopy builds upon most of the concepts already covered. We leave the specifics for chapters 5 and 7, giving a brief outline here.

In Fig. 2.20 we present the schematics of the circuit, it consists of two main components; a hybrid SQUID (h-SQUID) and a tunnel junction acting as a spectrometer, capacitively coupled together. The spectrometer gets its name through the interaction with the surrounding environment. The h-SQUID plays the role of the environment and is detected by the spectrometer through the  $I_{ICPT}$  of  $I_{spec}$  ( $V_{spec}$ ). We investigate two variations of h-SQUID, one containing an S-NW-S junction (Fig. 2.20(b)) and one with a NW-CPT (Fig. 2.20(c)). In both cases the other arm contains a tunnel junction that acts as a reference junction through  $E_{J,S-NW-S/NW-CPT} < E_{J,TJ}$ . As such upon the application of a flux through h-SQUID, most of the phase,  $\varphi$ , drops across S-NW-S/NW-CPT.

The two sides of the circuit are coupled via a large capacitance  $C_C$  and further decoupled through  $R_{SQUID}$  and  $R_{spec}$ , this limits the presence of any extra modes in the circuit. As such the energy exchange between the spectrometer is limited to h-SQUID only.

We are interested in studying the transitions as shown in Fig. 2.21. For S-NW-S junctions we are interested in the transitions between the ground and excited states of ABS. 2



Figure 2.21: (a) An incoming radiation of  $\hbar\omega$  induces excitations between the ground,  $-E_A$ , and excited,  $E_A$ , states of the ABS corresponding to  $\hbar\omega = 2E_A$  as a function of  $\varphi$ . (b) Energy excitations of the NW-CPT as a function  $n_g$  due to the same incoming radiation. (c) Energy excitations of NW-CPT as a function of  $\varphi$ .

The incoming radiation is due to photons and as such cannot alter the parity of the system thus the transitions that the spectrometer can detect will be  $\hbar \omega = 2E_A(\varphi)$  (similar considerations are true for the NW-CPT). However the reference junction of h-SQUID can also admit some radiation leading to the resulting spectrum as seen by the spectrometer consisting of modes of two origins. To understand the shape of the spectrum better we model h-SQUID with two components:

$$H_{env} = H_{TJ} + H_{ABS} \left( H_{CPT} \right) \tag{2.68}$$

The first term accounts for the reference junction and the second for S-NW-S (NW-CPT) junction. The true form  $H_{env}$  is rather complicated and is not analytically solvable, however under certain approximations it can be accounted for by the spin-boson model [53, 54]. The first approximation we make is that the phase drop over the reference junction is  $\delta \ll 1$  (this is valid in the case of an asymmetric h-SQUID as we have employed here). The tunnel junction can then expressed as harmonic oscillator [55]:

$$H_{TJ} = \hbar \omega_p \left( a^{\dagger} a + \frac{1}{2} \right) \tag{2.69}$$

Here the plasma mode  $\omega_P$  is set by junction capacitance,  $C_J$ , and inductance,  $L_J$ . The eigen-energies of the associated plasmon states<sup>11</sup> are then set by  $(n + \frac{1}{2})\hbar\omega_P$ . It is important to note that under this approximation we have reduced  $H_{TJ}$  to a ladder of constant energy levels where the magnitude of the inter-state excitations is set by  $\hbar\omega_P \approx \sqrt{2E_{J,TJ}E_{C,TJ}}$ .

We now turn to the other side of h-SQUID where we look for excitations in  $H_{ABS}$  or  $H_{CPT}$ .<sup>12</sup> Due to the asymmetry in both cases the induced excitations will depend heavily on  $\varphi$  or  $n_g$ , thus by varying either of these two parameters the two modes of h-SQUID can be discerned.

However, we are not solely limited to excitations in S-NW-S (NW-CPT) *or* the reference junction, we can also anticipate higher order transitions consisting of both modes. In practice these transitions are less likely to occur as the energy required to induce such them is at least  $\hbar \omega_P + 2E_A (\Delta E_{CPT})$ .

<sup>&</sup>lt;sup>11</sup>The plasmon states are bosonic.

<sup>&</sup>lt;sup>12</sup>In both cases the energy levels of S-NW-S and NW-CPT are fermionic.



Figure 2.22: The energy spectrum as a function of  $n_g$  for h-SQUID containing a NW-CPT and a reference junction. On the left the two excitations are manually superimposed on top of one another. On the right the finite coupling is taken into account. The excitations are induced from the ground state (black line). The flat region of the first excitation (blue) is attributed to that of the plasma mode whereas the region that varies with  $n_g$  is attributed to the NW-CPT. Higher excitations of the h-SQUID are shown in red, green and orange. Note that the dispersion of plasma modes varies weakly with  $n_g$  whereas the opposite is true for the NW-CPT. The shaded regions depict the interaction region caused by the hybridisation of the two modes.

The discussion thus far is only true in the limit when the two excitations are sufficiently far from one another (i.e.  $\hbar \omega_P \ll 2E_A(\Delta E_{CPT})$ ) and are thus decoupled. However for odd-values of  $n_g$  and  $\varphi = (n+1)\pi$ ,  $\hbar \omega_P \approx 2E_A(\Delta E_{CPT})$  leading to a strong coupling of the two modes. This coupling leads to the hybridisation between the two modes and is generally accounted for by introducing a coupling term to the Hamiltonian,  $H_g$ . The exact form of  $H_g$  will depend on the h-SQUID in question but can generally be expressed as,  $H_g \propto g(\varphi, n_g)(a^{\dagger} + a)$ . In experiment this coupling will represent itself as an anticrossing between the two modes [56], where the magnitude of the resulting splitting will be directly related to  $g(\varphi, n_g)$ .

From the excitation spectrum we can construct the energy spectrum of given h-SQUID and although we leave the details for chapters 5 and 7, we briefly show how a typical representation looks like in Fig. 2.22. Here the h-SQUID consists on a NW-CPT placed in a regime where  $E_C > E_J$ . On the left of Fig. 2.22 we see the usual CPT oscillations as a function of  $n_g$  and superimposed plasma mode excitations. For certain values of  $n_g$  it is visible that the two modes cross. By turning on the coupling between the two we obtain a spectrum shown on the right of Fig. 2.22. Here the hybridisation of the two modes has resulted in anti-crossings with the interaction region depicted by the shaded regions.

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# 3

### **METHODS**

The purpose of the present chapter is to outline the experimentally necessary pre-requisites to obtain the results discussed in the following experimental chapters. Most of the experimental results were obtained by using the on-chip radiation or microwave spectroscopy circuit and we will outline the fabrication and characterisation of each component. The devices were fabricated in the cleanroom facilities of the Kavli Institute of Nanoscience located at the TU Delft campus. Subsequently the devices were measured in a dilution refrigerator with a base temperature of 20 mK.

#### **3.1.** INTRODUCTION

In the present chapter we discuss the general development of the devices used to perform the experiments covered in this thesis. We in particular focus on the development of the on-chip circuit used to conduct both the microwave spectroscopy and radiation detection experiments. All of the fabrication was performed within the Kavli Institute of Nanoscience cleanroom facility.

We, first, give a brief introduction into the principles of a dilution refrigerator and the electronics used to conduct the experiment and refer the reader to [1, 2] for a detailed discussion. Finally, we briefly discuss the basic characterisation measurements performed to determine the suitability of a particular device.

#### **3.2.** Measurement Set-up

The measurements, as outlined in the following chapters, were performed in dry dilution refrigerators (DR) with base temperatures ranging between 12 and 30 mK. In order to place the samples into the cryogenic environment, in all cases, a probe was loaded into the top of DR. The probe itself mimicked the stages of DR and the sample was located at the bottom of the probe, at the cold finger. Here the sample was glued, using silver paint, into a sample holder that was in turn connected to the cold finger. In order to reduce the influence of any stray radiation, a 'box in a box' approach for the sample shielding was implemented. Here the sample holder was covered with a cap coated with Aeroglaze on the inside. Furthermore a Copper can with another layer of Aeroglaze was used to enclose the cold finger.

The electrical connection to the sample was achieved via 48 DC electric lines thermally anchored to each stage of the probe and accessible via a break-out box located on top of the probe. In order to suppress any incoming noise, a combination of RC-filters (active between 50kHz and 100MHz),  $\pi$ -filters (active between 100MHz and 1GHz) and copper powder filters (active above 1GHz) were implemented. The measurements themselves were carried out using the low-noise IVVI rack built within the TU Delft facilities.

For more information we refer the reader to [1, 2].

#### **3.3.** Semiconducting Nanowires

Central to the work presented is the semiconducting nanowire (NW) with two types of NWs implemented; indium antimonide (InSb) and indium arsenide (InAs). The InSb NWs were grown using the metal organic vapor phase epitaxy (MOVPE) technique on an InP (111) substrate with the aid of a gold catalyst. For a detailed explanation of the growth procedure we refer the reader to [3]. The InAs NWs were grown using molecular beam epitaxy (MBE) with a the thin layer of aluminium (Al), typically between 3 and 7 nm used in the present work, grown in - situ. We refer the reader to [4] for a detailed discussion.

The NWs in both cases were transferred from the growth chip to the sample with the aid of a nanowire manipulator [5]. Typically the sample contained a predefined layer of electrostatic gates covered by a 30 nm layer of  $SiN_x$  dielectric. To not disturb the dielectric a tungsten needle with an indium tip was used for the transfer.

#### **3.4.** FABRICATION

#### 3.4.1. JOSEPHSON JUNCTIONS

The Josephson junction as discussed in section 2.2 plays a central role in the present thesis. We focus our discussion on two types of Josephson junctions, the superconductorinsulator-superconductor (SIS) tunnel and the superconductor-NW-superconductor (S-NW-S) junction.

#### SIS JUNCTION

Figure 3.1(a) depicts a cartoon of the SIS junction, although a variety of superconducting materials can be implemented, we focus on  $Al/ALO_x/Al$  heterostructures. Here a thin layer of  $AlO_x$  acts as a weak link separating two Al leads. We adopt the widely used shadow evaporation technique to fabricate our SIS junctions [6], in a dedicated electronbeam evaporation tool. The sample and the source of Al were located in their own respective chambers (upper and lower respectively) with a mechanical valve used to isolate the two chambers when necessary. The first layer of Al is deposited at an angle of  $11^{\circ}$  with respect to the source of Al within a lower chamber of a base pressure of  $2 \times 10^{-8}$ mBarr. The  $AlO_x$  layer is then created through a controlled input of  $O_2$  into the upper chamber, raising the pressure to ~ 1.3 mBarr for a period of 4 minutes. The final layer of Al is then deposited at an angle of  $-11^{\circ}$  within similar conditions as the first layer. Figure 3.1(b) shows a scanning electron micrograph (SEM) of a typical device from a top-view perspective with the junction enclosed in the white rectangle.



Figure 3.1: (a) Cartoon representation of the SIS tunnel junction, the two Al layers are deposited sequentially with an oxidation step in between giving rise to the  $AlO_2$  layer. (b) An SEM image of a typical device with the SIS junction enclosed in the white rectangle.

The target Josephson energy,  $E_J$ , of a given SIS junction is achieved through either modifying its area or the thickness of the AlO<sub>x</sub> layer. The latter is achieved through the variation of the oxidation time. A detailed study on these considerations can be found in [7].

Due to the low critical field,  $B_C$ , of bulk Al, its superconducting properties degrade quickly in the presence of a magnetic field. However in order to study the effects discussed in the experimental results, a finite in and out of plane field is necessary. To combat the field induced degradation the geometry of the junction was investigated. Initially a combination of 30/50 nm thick (first/second) layers was employed, as was used in chapter 4, where the onset field for a finite sub-gap quasiparticle current (and the noticeable decrease in  $E_J$ ) was several 10s mT. By adjusting the geometry to a combination of 9/11 nm layers with long (1 $\mu$ m) arms, as was used in chapters 5 and 7, this onset field was increased to  $\sim$  300 mT. The purpose of the long arms, which eventually fan out into large Al pads, was to navigate any field induced vortices away from the junction.

#### S-NW-S JUNCTION





Figure 3.2: (a) An SEM image of S-NW-S junction made from NbTiN/InSb/NbTiN located on top of a set of electrostatic gates used to tune the junction Josephson coupling. (b) An SEM image of a typical S-NW-S junction made from InAs nanowire with an epitaxial Al layer. The Al (light blue in the inset) is then removed to form the junction. The local electrostatic gates on top of which the junction is located are used to tune its Josephson coupling. Image adapted from [8].

Figure 3.2 displays two types of S-NW-S junctions investigated in the present thesis. Figure 3.2(a) shows a junction where an InSb NW takes the role of the weak link with NbTiN the superconductor of choice for the leads. The junction itself is located on top of a 30 nm thick layer of  $SiN_x$  separating it from a layer of pre-defined Ti/Au electrostatic gates. The middle gate (directly below the weak link) is used to tune the conductance through the uncovered part of the NW and hence the transparency of the junction. The superconducting leads are made via a sputtering process from a NbTi target in a N atmosphere. In order to ensure a good contact is achieved between the NW and NbTiN, a 2 minute 30 second Ar dry etch is performed beforehand. Although this process is known to corrode the NW, the best induced supercurrent is achieved with this recipe [9].

Figure 3.2(b) shows an equivalent junction made from Al and InAs again located on top of a similar pre-defined gate configuration. Aside from the materials implemented, the main distinction here is that the Al layer is deposited in - situ during the growth process of the InAs NW [4]. Such a combination is known to give rise to favourable superconducting properties, such as a hard induced superconducting gap [10], and this is the basis of most of the experimental work covered in this thesis. The weak link in the junction is made by performing a Transene D wet etch to selectively remove the Al layer from the desired region [11, 12]. The transparency of the junction is similarly tuned via a nearby electrostatic gate.

#### SQUID

In the present thesis we implement two types of SQUIDs: a d.c. SQUID containing two SIS junctions of similar  $E_J$  (Fig. 3.3(b)) and an asymmetric hybrid SQUID consisting of



Figure 3.3: (a) An SEM image of a SQUID formed from an S-NW-S and an SIS junction. The two junctions are connected by two pieces of NbTiN. (b) An SEM image of a SQUID formed from two SIS junctions.

one SIS junction and an S-NW-S junction (Fig. 3.3(a)). The necessity of the asymmetry is to ensure that upon performing the experiments discussed in chapter 5 and 7, most of the phase drop upon the application of an external magnetic flux occurs across the S-NW-S junction. In chapter 7 the S-NW-S junction is replaced by a nanowire Cooper-pair transistor (NW-CPT). The d.c. SQUID is created in a single shadow evaporation process. The hybrid SQUID, however, is made in three steps. First the SIS junction is created, then the NW is deterministically deposited on top of the already existing gate pattern [5] (if necessary a Transene D etch is performed). Finally the connection between the two sides is established by sputtering a layer of NbTiN (with a 2 minute Ar dry etch beforehand).

#### **3.4.2.** NANOWIRE COOPER-PAIR TRANSISTOR

The NW-CPT as implemented in chapters 6 and 7 is fabricated in much the same way as the Al-InAs S-NW-S junctions. The NWs are locates on top of a predefined gate pattern and Al is removed from two sections of the NW using Transene D. Gates located directly below the regions where the Al was removed are used to tune the transparency of each junction independently. A larger gate located underneath the superconducting island created between the two junctions is used to tune the charge occupation on the island.

#### 3.4.3. ON-CHIP MICROWAVE SPECTROSCOPY

The circuit used to perform the radiation detection experiment discussed in chapter 4 differs from the circuit used for the on-chip microwave spectroscopy experiments (chapters 5 and 7) only in the finer details. We focus here on the fabrication of the on-chip microwave spectroscopy circuit as a result.

Figure 3.4 shows a typical device under consideration. The entire structure is fabricated on top of an intrinsic Si/SiO<sub>2</sub> (with a resistivity of  $2k\Omega$ -cm) wafer where the SiO<sub>2</sub> layer is 285 nm deep. This choice of substrate is motivated by the necessity to limit any potential stray capacitances within the circuit. The circuit itself begins with a 5/10 nm layer of Ti/Au for the electrostatic gates and the lower plates of the on-chip capacitors. This is followed by a 30 nm layer of sputtered SiN dielectric which covers both of the components. All of the connections made to the bond pads consist of 5/30 nm of Cr/Pt resistive lines. These are designed to be 80 nm wide and ~ 120  $\mu$ m long leading to a line



Figure 3.4: (a) An optical image of a typical device used in the on-chip microwave spectroscopy experiments. The outer NbTiN bond pads are used to establish a connection between the device and the electronics. The bond pads are connected to the resistive lines via NbTiN links. (b) An SEM image of the device is shown in the purple box where the two sides of the circuit are visible. On the right, located within the green box is the spectrometer SIS junction. On the left side the device under study is located within the orange box. The device is either a NW-CPT (top) or an S-NW-S junction (bottom) is embedded in a SQUID with an SIS junction (contained within the white box). A zoom in of the NW-CPT is shown in the red box, the light blue regions depict the presence of an Al shell. The two sides of the circuit are capacitively coupled via two capacitors as shown in the light blue box. The connection between the circuit and the resistive lines is shown in the dark blue box.

resistance of  $12k\Omega$ . This is followed by two SIS junctions, one acts as the spectrometer and the other acts as a reference junction in the hybrid SQUID. The spectrometer, as

shown in the green box in Fig. 3.4(b), has an area of approximately  $100 \times 100$  nm<sup>2</sup> leading to typical  $E_I \approx 60 \ \mu \text{eV}$ . Whilst the reference junction is approximately  $100 \times 750 \text{ nm}^2$  leading to typical  $E_I \approx 260 \,\mu\text{eV}$ . The contact to the spectrometer is achieved through a layer of 10/80 nm of Ti/Au with an Ar dry etch performed before to remove the native oxide from the junction. We motivate this material combination due to its ability to act as quasiparticle traps [13]. In the same step the upper plates of the capacitors are deposited as well as the connections to the resistive lines, as shown in the dark blue box in Fig. 3.4(b). An extra layer of Ti/Au is further placed on top of the biasing lines of the spectrometer thus reducing those line resistances from 12 to 2 k $\Omega$ . The area of the capacitors, as shown in the light blue box in Fig. 3.4(b), are designed such that the coupling capacitance,  $C_C$ , is approximately 400 fF. This ensures that the two sides of the circuit are sufficiently coupled in the high-frequency domain but that the circuit itself does not lead to unwanted resonances in the frequency range of interest. The Al-InAs NW is then placed on top of the gate pattern and the Al is removed from the regions where the junction(s) is to be formed as shown in the red box in Fig. 3.4(b). Finally the SQUID is formed by sputtering a layer of 120 nm of NbTiN, with another Ar dry etch beforehand. The exact line-shape of the SQUID connection is designed such that the enclosed area is approximately  $6 \times 3$  $\mu$ m<sup>2</sup> leading to a flux periodicity of the SQUID around 200  $\mu$ T. Simultaneously the bond pads are also created from NbTiN.

#### **3.5.** CHARACTERISATION

#### **3.5.1.** JOSEPHSON JUNCTIONS

Once the fabrication of the junction is finished, the normal state resistance of the junctions at room temperature,  $R_N$ , is checked. For the SIS junction its  $I_C$  ( $E_J$ ) can be estimated form  $R_N$  suing the Ambegaokar-Baratoff relation [14]:

$$\pi\Delta = 2eI_C R_N \tag{3.1}$$

Provided that the superconducting gap,  $\Delta$ , is known. The SIS junctions uitlised as spectrometers yielded typical  $R_N$  values in the range of 13 to 19 k $\Omega$  whilst the reference junctions in the hybrid SQUID have  $R_N \sim 2-3 \text{ k}\Omega$ . Although this relation does not hold for the S-NW-S junctions, the measure of  $R_N$  is still a good diagnostics tool to detect whether the junction will perform at cryogenic temperatures. The  $R_N$  values for S-NW-S junctions vary significantly however typically fall within 10-100 k $\Omega$ .

Once at 4K, the linear I(V) response of the SIS junctions is measured to see that its resistance has not deviated form  $R_N$  significantly. For the S-NW-S junction the gate response of the conductance is verified [15]. From these two characterisation measurements a decision is made on whether or not to proceed with the experiment.

#### **3.5.2.** NANOWIRE COOPER-PAIR TRANSISTOR

Similar room temperature and 4K tests are initially carried out for the NW-CPT. Once the device is cooled to the base temperature of the dilution refrigerator further checks are necessary. Figure 3.5 shows two exemplary scans performed on a NW-CPT with 600  $\mu$ m long island. Here the NW-CPT is placed in a strongly Coulomb-blockaded regime such that its Charging energy,  $E_{C,CPT}$ , is significantly larger than  $E_{J,CPT}$ . A magnetic field of



Figure 3.5: (a) Charge stability diagram of a NW-CPT in the Coulomb blockaded regime with  $E_{C,CPT} = 1.4$  meV and a  $n_g$  periodicity of 2 meV. Obtained at an a magnetic field of 2T as such the NW-CPT is in the normal state. (b) The same charge stability diagram obtained at zero magnetic field where a superconducting energy gap,  $\Delta = 190 \ \mu$ eV is observed.

2T is then applied in-plane of the device thus removing any superconductivity, as shown in Fig. 3.5(a). The NW-CPT is then biased at  $V_{CPT}$  and the charge state of the island is modulated by  $V_{pg}$ . The purpose of this is to extract the bare  $E_{C,CPT}$  and the length of the 1e-periodicity (and hence the 2e-periodicity) of  $V_{pg}$ . By removing the magnetic field the presence of the superconducting energy gap,  $\Delta$ , is observed.

We note that for the experiment discussed in chapter 7, due to the shunting of the NW-CPT by the reference junction in the hybrid SQUID, the only check that is available is the 1e-periodicity response at finite bias across the SQUID,  $|eV_{CPT}| > 2\Delta$ .



Figure 3.6: (a) The modulation of  $I_{sw}$  as a function of  $n_g$  in the weakly Coulomb blockaded regime. The figure is taken from chapter 6. (b) A schematic representation of the how the  $I_{sw}$  values are recorded. The red curve depicts a typical  $V_{CPT}(I_{CPT})$  response of the NW-CPT with the applied ramp shown in blue. The  $I_{sw}$  of the CPT is recorded by monitoring the voltage drop across the CPT and by recording the point when the voltage drop develops to the value of  $V_{th}$ .

In order to perform the measurements as outlined in chapter 6,  $E_{J,CPT}$  is gradually increased until the NW-CPT is in a weakly Coulomb blockaded regime,  $E_{C,CPT} \approx E_{J,CPT}$ , where a finite supercurrent  $I_s$ , is observed. Following section 2.2.3, we observe that the

switching current,  $I_{sw}$ , is periodic with the induced gate charge,  $n_g$ , as shown in Fig. 3.6(a). In order to experimentally study this modulation a few hundred of  $I_{sw}$  values are recorded for each value of  $n_g$ , from which a histogram of switching events is obtained. The  $I_{sw}$  events are recorded upon applying a  $I_{ramp}$  across the NW-CPT whilst monitoring the voltage drop that develops. A sharp transition occurs when the CPT switches from a superconducting to a dissipative state, as indicated by the  $V_{th}$  point in Fig. 3.6(b).

#### **3.5.3.** ON-CHIP MICROWAVE SPECTROSCOPY



Figure 3.7: (a) A schematic representation of the circuit used to carry out the on-chip microwave spectroscopy experiments. The two sides of the circuit are capacitively coupled via  $C_C$ . Radiation emitted by the spectrometer, shown in green, is then guided to the SQUID which plays the role of the environment. The circuit is further decoupled from room temperature electronics via  $R_{spec}$  and  $R_{SQUID}$ . The two sides are addressed by applying  $V_{SQUID}$  and  $V_{spec}$  independently. (b) A schematic representation of the SQUID used to conduct the microwave spectroscopy measurements of the ABS in an S-NW-S junction. The reference SIS junction is taken to have a finite, but small,  $E_{C,TJ}$ . (c) A schematic representation of the SQUID used to conduct the microwave spectroscopy measurements of the charge and phase dispersion of the NW-CPT.

Figure 3.7 shows a schematic representation of the on-chip microwave spectroscopy circuit with the two hybrid SQUIDs corresponding to the S-NW-S and NW-CPT experiments shown in Figs. 3.7 (b) and (c) respectively. At the base temperature of the dilution fridge, characterisation of each part of the circuit is done independently. First a voltage bias,  $V_{spec}$ , is applied across the spectrometer junction with the current,  $I_{spec}$ , measured simultaneously. The other side of the circuit is kept at zero bias and the NW is kept at zero conductance, resulting in an I(V) characteristic of the spectrometer as is shown in Fig.

3.8(b). We note that  $I_{spec}(V_{spec})$  displays a clear supercurrent branch with an  $I_{sw} \approx 5$  nA, smaller than the anticipated  $I_C \approx 20$  nA due to the measured  $\Delta = 250 \ \mu\text{eV}$ . The presence of backbending at  $|eV_{spec}| = 2\Delta$  is attributed to the finite self-heating of the junction. We now focus on the extra contribution to the current flowing through the junction observed just outside the retrapping current (a zoom in is shown in the inset of Fig. 3.8(b)). We attribute the extra current to the  $I_{ICPT}$  of the junction in an electromagnetic environment. Although the presence of  $I_{ICPT}$  should be visible on both sides of the supercurrent branch, due to the finite asymmetry in the switching,  $I_{sw}$ , and retrapping,  $I_r$ , current the presence of  $I_{ICPT}$  is masked by  $I_{sw}$  on the + $V_{spec}$  side. From this we conclude that the junction does indeed act as a spectrometer and has detected the presence of an environmental mode. The nature of the mode can be understood within an RLC network of circuit elements. We predict that this mode is due to the reference tunnel junction in the hybrid SQUID. To confirm this we fit the shape of  $I_{ICPT}$  by assuming C to be due to the geometric capacitance of the junction<sup>1</sup>, R to be due to the shunt resistance felt by the junction and L to be  $L_I$  which is calculated from  $I_C$ . We outline the details of the fit in chapters 5 and 7. Next the  $I_{SOUID}(V_{SOUID})$  response of the SQUID is recorded (again in the absence of the NW), as can be seen from Fig. 3.8(a), a clear supercurrent due to the hybrid SOUID tunnel junction is observed.



Figure 3.8: (a) An  $I_{SQUID}(V_{SQUID})$  response of the SQUID when the spectrometer is kept at zero bias,  $V_{spec}=0$ , and the NW is kept in the zero conductance state. A clear supercurrent due to the SIS junction of the SQUID is observed. (b) An  $I_{spec}(V_{spec})$  response of the spectrometer when the SQUID is kept at zero bias,  $V_{SQUID}=0$ , and the NW is kept in the zero conductance state. Aside from the supercurrent of the spectrometer, an extra enhancement in  $I_{spec}$  is observed, attributed to the presence of an electromagnetic environment.

The next step is to introduce the NW into the experiment via the local gates. By applying a more positive voltage to the gate located directly beneath the junction an enhancement in the SQUID supercurrent, as well extra current within the superconducting gap,  $\Delta$ , as shown in Fig. 3.9(a), is recorded. At a sufficiently large enough supercurrent through the S-NW-S junction the response of the SQUID to an external flux is monitored as shown in Fig. 3.9(b). At this point all of the characterisation measurements are complete and the experiments can be performed. In chapter 5 we discuss the observation

<sup>&</sup>lt;sup>1</sup>We note that this capacitance is made up from stray capacitances in the circuit as well as the geometric capacitance of the junction, which can be extracted through the aid of methods employed in [16].

of ABS in an S-NW-S junction and how they behave in the presence of a magnetic field applied along the junction. In chapter 7 we discuss the charge and phase dispersion of the NW-CPT.



Figure 3.9: (a) An  $I_{SQUID}(V_{SQUID})$  response of the SQUID as the NW conductance is increased. At  $V_{G,NW} = 1$  mV the supercurrent flowing through the SQUID has increased and a sizeable sub-gap current is present. (b) The flux periodicity of the SQUID. A clear modulation of the supercurrent is observed with a period of roughly 100  $\mu$ T.

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# 4

### JOSEPHSON RADIATION AND SHOT NOISE OF A SEMICONDUCTING NANOWIRE JUNCTION

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We measured the Josephson radiation emitted by an InSb semiconductor nanowire junction utilizing photon assisted quasiparticle tunnelling in an AC-coupled superconducting tunnel junction. We quantify the action of the local microwave environment by evaluating the frequency dependence of the inelastic Cooper-pair tunnelling of the nanowire junction and find the zero frequency impedance  $Z(0) = 492\Omega$  with a cut-off frequency of  $f_0 = 33.1$  GHz. We extract a circuit coupling efficiency of  $\eta \approx 0.1$  and a detector quantum efficiency approaching unity in the high frequency limit. In addition to the Josephson radiation, we identify a shot-noise contribution with a Fano factor  $F \approx 1$ , consistent with the presence of single electron states in the nanowire channel.

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#### 4.1. INTRODUCTION

The tunnelling of Cooper pairs through a junction between two superconducting condensates gives rise to a dissipationless current [1] with a maximum amplitude of the critical current,  $I_c$  [2]. Upon applying a finite voltage bias V, the junction becomes an oscillating current source;

$$I_s(t) = I_c \sin(2\pi f t), \tag{4.1}$$

with a frequency set by hf = 2eV where *h* is the Planck constant and *e* is the electron charge.

The Josephson radiation, defined by equation (4.1) has mostly been investigated for superconducting tunnel junctions [3–5], metallic Cooper-pair transistors [6] and in circuit QED geometries [7, 8]. Recently, it has also been proposed as a probe for topological superconductivity [9–11], which requires gateable semiconductor Josephson junctions [12].

In contrast to superconductor-insulator-superconductor (SIS) junctions, Josephson junctions with a semiconductor channel feature conductive modes of finite transmission probabilities [13, 14], leading to deviations from a sinusoidal current-phase relationship [15] and the universal ratio of the critical current and the normal-state conductance [2]. Furthermore, soft-gap effects [16] have been shown to result in excess quasiparticle current for subgap bias voltages, limiting prospective applications such as topological circuits [17] and gate-controlled transmon qubits [18].

Here we investigate the high-frequency radiation signatures of a voltage-biased semiconductor Josephson junction [12] by directly measuring the frequency-resolved spectral density. As a frequency-sensitive detector, we utilize a SIS junction, where the photonassisted tunneling current [5] is determined by the spectral density of the coupled microwave radiation [19]. In addition to the detection of the monochromatic Josephson radiation, we demonstrate the presence of a broadband contribution, attributed to the shot noise of the nanowire junction [20], similarly to earlier experiments on carbon nanotube quantum dots [21, 22].

#### **4.2.** DEVICE FABRICATION AND LAYOUT

Our setup follows the geometry of earlier experiments utilizing SIS junctions [5]. In contrast, our microwave radiation source is an InSb nanowire (NW) [23] Josephson junction (Fig. 4.1(d)) with a channel length of 100 nm. The junction leads (in brown in Fig. 4.1(d)) are created by removing the surface oxides by Ar ion milling and then *in-situ* sputtering of NbTiN superconducting alloy. Owing to the highly transparent contacts, this procedure enables induced superconductivity in the semiconductor channel [17, 18]. A predefined gate structure (purple regions in Fig. 4.1(d)) provides electrostatic control of the semiconductor channel and is covered by sputtering a 20 nm thick SiN<sub>x</sub> dielectric layer.

The I(V) characteristics of the two junctions are measured in a standard four point probe geometry via highly resistive Pt feedlines effectively decoupling the on-chip elements (Fig. 4.1) thermally anchored at 20 mK from the measurement setup. In order to gain access to a wider  $V_{\text{NW}}$  range, we use  $R_1 = 1 \,\text{k}\Omega$  in the nanowire biasing lines and  $R_2 = 6 \,\text{k}\Omega$  in the voltage measurement leads (see Fig. 4.1(b)).

The detector SIS split junction is shown in Fig. 4.1(f) and is fabricated using standard



Figure 4.1: (a) Photon emission due to the inelastic Cooper-pair tunnelling between condensate levels shifted by the bias voltage,  $V_{\text{NW}}$ . (b) The microwave equivalent circuit of the measurement setup, where R and C in the blue dashed box represent the microwave losses and stray capacitance, yielding a  $2\pi f_0 = (RC)^{-1}$  upper cutoff frequency. The  $C_c \gg C$  coupling capacitors have a negligible effect above a frequency of  $2\pi f_c = (RC_c)^{-1}$ with  $f_c \ll f_0$ , but allow for the application of independent DC bias voltages  $V_{\text{NW}}$  and  $V_{\text{det}}$ . The  $I_{\text{NW}}(V_{\text{NW}})$  and  $I_{\text{det}}(V_{\text{det}})$  characteristics are measured through the Pt feedline resistors, depicted by  $R_1$  and  $R_2$ , respectively. (c) Photon-assisted quasiparticle tunnelling for a detector voltage bias  $V_{\text{det}}$  and an incoming photon energy of hf. (d) False coloured scanning electron micrograph of the nanowire Josephson junction contacted with NbTiN after being placed on three electrostatic gates. (e) Bright field optical image of the coupling circuitry before the NbTiN deposition step with the nanowire junction (green box) and the detector junction (red box). (f) False coloured micrograph of the detector split junction with an applied magnetic flux  $\Phi$ . The scale bars depict  $1 \,\mu$ m (d),  $20 \,\mu$ m (e) and  $0.5 \,\mu$ m (f), respectively.

shadow evaporation techniques [24]. The typical normal state resistance was measured to be  $20 \text{ k}\Omega$  for a nominal junction area of  $100 \times 100 \text{ nm}^2$ . The bottom and top Al layer thicknesses are 9 and 11 nm, respectively. The split junction geometry enables the flux control of the total Josephson coupling of the detector. To measure the quasiparticle tunnelling response, we set  $\Phi = \Phi_0/2$ , with  $\Phi_0 = h/2e$  the flux quantum, to minimize the Josephson coupling. We note that the minimal detector critical current is negligible compared to that of the nanowire junction. Finally, we utilize two parallel plate capacitors of  $C_c \approx 400 \text{ fF}$  with sputtered SiN<sub>x</sub> dielectric which couple the nanowire junction to the detector in the frequencies of interest (Fig. 4.1(e)), yet enable independent voltage biasing and current measurements in the DC domain.

#### **4.3.** THEORY

The mesoscopic noise source under consideration is characterized by its current noise density,  $S_I(f)$  [20], which results in the voltage noise density  $S_V(f) = S_I(f)|Z(f)|^2$ , where Z(f) is the complex frequency-dependent impedance of the coupling circuit. In Fig. 4.1(b), we depict a parallel *RC* network resulting in  $Z(f) = R(1 - jf/f_0)/(1 + f^2/f_0^2)$  with  $2\pi f_0 = (RC)^{-1}$  in the limit of negligible detector admittance,  $r_{det}^{-1} = dI_{det}/dV_{det} \ll R^{-1}$ .

We deduce the voltage noise density  $S_V(f)$  starting from the equation for the photonassisted current in the SIS detector [5, 25]:



Figure 4.2: (a) Measured photon-assisted quasiparticle current  $I_{PAT}$  as a function of the detector bias voltage  $V_{det}$  and nanowire bias voltage  $V_{NW}$ . The orange dots denote the extracted frequency on the upper axis for a given  $V_{NW}$ . The solid black line is the best linear fit with  $f/V_{NW} = 475 \text{ MHz}/\mu V$ . (b) Horizontal line traces at different  $V_{NW}$  values. The inset shows the full  $I_{det,0}(V_{det})$  characteristics of the detector when the Josephson radiation is absent. Note the difference in the current scale. The applied flux  $\Phi = \Phi_0/2$  through the split junction results in a suppressed detector supercurrent branch which minimizes its Josephson radiation. The arrow depicts  $2\Delta/e = 480 \,\mu$ V, the onset of the quasiparticle current.

$$I_{\text{PAT}}(V_{\text{det}}) = \int_0^\infty S_V(f) \left(\frac{e}{hf}\right)^2 I_{QP,0}\left(V_{\text{det}} + \frac{hf}{e}\right) \mathrm{d}f,\tag{4.2}$$

which describes the DC current contribution at an applied voltage  $V_{det} < 2\Delta$ . Crucially, this equation holds if the quasiparticle current in the absence of radiation has a well-defined onset,  $I_{QP,0}(V_{det} < 2\Delta) = 0$  [5] and in the limit of weak coupling, where multiphoton processes do not contribute to the quasiparticle current [19]. In addition, a detector with a sharp quasiparticle current onset can reach the quantum limit [25] where each absorbed photon results in the tunnelling of one quasiparticle.

In the presence of a monochromatic radiation, where  $S_V(f) \sim \delta(f - \mathscr{F})$ , equation (4.2) describes the shift of the initial  $I_{QP,0}(V_{det})$  quasiparticle current by  $\delta V_{det} = h\mathscr{F}/e$ . This is the case of the Josephson radiation [5] with  $S_I(f) = \frac{I_c^2}{4}\delta(f - \mathscr{F})$ , where  $h\mathscr{F} = 2eV_{NW}$  with  $V_{NW}$  the applied voltage bias on the emitter junction with a critical current  $I_c$ . On the other hand, the *nonsymmetrized* quasiparticle shot noise is characterized by  $S_I = eIF$  in the zero frequency and zero temperature limit with I being the applied current. The Fano factor, F is characteristic to the mesoscopic details of the junction [20].

Note that equation (4.2) can be handled as a convolution of  $S_V(f)/(hf)^2$  and  $I_{QP,0}(V_{det})$ . However, the inverse problem leading to  $S_V(f)$  is unstable due to the noise in the experimental data. To this end, we use Tikhonov regularization [26] to extract the noise density measured by the detector. It is to be noted that the measured  $I_{det,0}$  (see inset of Fig. 4.2(b)) exhibits backbending due to the self-heating effects in the leads of the superconducting tunnel junction, therefore we used a monotonous  $I_{QP,0}(V_{det})$  centered around the same quasiparticle onset. However, the uncertainty of  $I_{QP,0}(V_{det})$  prevents the determination of the exact lineshape of  $S_V(f)$  which could indicate the linewidth of the Josephson radiation [27].

#### 4.4. DISCUSSION



Figure 4.3: (a) The measured  $\delta V(f) = I_c |Z(f)|$  voltage fluctuation on the detector junction. The solid line depicts the fitted cutoff with  $f_0 = (2\pi RC)^{-1} = 33.1 \text{ GHz}$ . Right vertical axis shows the impedance |Z(f)|, see text. (b) Experimental  $I_{\text{ICPT}}(V_{\text{NW}})$  trace of the nanowire junction exhibiting a current peak due to the supercurrent branch. The linear contribution with a resistance  $R_{\text{NW}} = 14.03 \,\text{k}\Omega$  (green solid line, see inset for raw  $I_{\text{NW}}(V_{\text{NW}})$  trace) is subtracted. The blue solid line depicts the fitted curve with  $I_c = 9.38$  nA critical current and a noise temperature T = 132 mK. (c) Variation of the nanowire junction current  $\Delta I_{\text{ICPT}}$  as a function of the detector voltage  $V_{\text{det}}$ . The extracted circuit efficiency  $\eta$  (d) and the detector quantum efficiency Q (e) as a function of  $V_{\text{NW}}$ , see text.

We demonstrate the detection of the Josephson radiation in Fig. 4.2. In panel (a), we plot the PAT current contribution as a function of the DC bias voltages  $V_{det}$  and  $V_{NW}$ . In Fig. 4.2(b), we show line traces  $I_{PAT}(V_{det})$  exhibiting well-defined onset values corresponding to a monochromatic Josephson radiation tuned by  $V_{NW}$ . Thus, we can extract the radiation frequency based on equation (4.2) (orange dots in Fig. 4.2(a)). By evaluating the relation between  $V_{NW}$  and the radiation frequency (black line in Fig. 4.2(a)), we find a ratio of  $475 \pm 4.2 \frac{\text{MHz}}{\mu \text{V}}$  which is in reasonable agreement with  $\frac{2e}{h} \sim 484 \frac{\text{MHz}}{\mu \text{V}}$  expected for the case of Cooper-pair tunnelling [28]. The intersect for f = 0 is set by the quasiparticle current onset to be  $2\Delta/e = 480 \,\mu \text{V}$  (see inset of Fig. 4.2(b)).

The impedance Z(f) of the environment results in a finite power dissipation  $I_c^2 \operatorname{Re}(Z(f))/2$ which gives rise to a DC current due to inelastic Cooper-pair tunnelling (ICPT) processes in the NW Josephson junction (see Fig. 4.1(a)) [4]. This effect has been first addressed to calculate the shape of the supercurrent branch in overdamped SIS junctions and purely resistive environments [29]. Later, the theory was adapted for high channel transmissions [30]. It has also been shown that for an arbitrary  $Z(f) \ll h/4e^2 \approx 6.5 \operatorname{k}\Omega$ , the ICPT



Figure 4.4: (a) Measured detector  $I_{PAT}(V_{det})$  line traces at  $V_{NW} = 65, 95$  and  $125 \,\mu$ V bias voltage from the bottom to top, respectively. (b) The measured  $dI_{PAT}/dV_{NW}$  (light gray line) and the fitted curves at the top (F = 1.3, red line) and the bottom (F = 0.8, blue line) of the confidence interval, respectively.

contribution can be evaluated as [4]:

$$I_{\rm ICPT} = \frac{I_c^2 {\rm Re}[Z(f)]}{2V_{\rm NW}},\tag{4.3}$$

with a critical current  $I_c$  and an applied voltage  $V_{\text{NW}}$ . Here, the junction effectively probes the real component of the impedance Z(f) at a frequency  $f = 2eV_{\text{NW}}/h$ .

In the following, we use a circuit model where the two independently measured current values  $I_{\text{PAT}}(V_{\text{det}})$  and  $I_{\text{ICPT}}(V_{\text{NW}})$  depend on the same microwave environment, characterized by Z(f). This model applies provided that the linear resistance of the nanowire and the impedance of the detector,  $r_{\text{det}}$ , are much higher than the effective shunt resistance of the circuit, depicted by R in Fig. 4.1(b). In addition, the lumped element description of Fig. 4.1(b) is valid if the circuit is much smaller than the characteristic wavelength  $c/f \sim 1 \text{ mm}$ . Our structure,  $50 \,\mu\text{m}$  in size (see Fig. 4.1(e)), fulfills this condition. Note that this is in contrast to a prior work [8] where the sample and detector were embedded in a transmission line resonator and thus the effective impedance values were measured to be different.

It is important to notice that the PAT current decreases with increasing frequency (Fig. 4.2(b)). By correcting for the ~  $f^{-2}$  dependence in equation (4.2), we find that the fluctuation amplitude  $\delta V = I_c |Z(f)| \sim \sqrt{S_V}$  exhibits a characteristic cutoff frequency (Fig. 4.3(a)), even though the current oscillation amplitude of the Josephson junction is constant, see equation (4.1). Thus, we can attribute this cutoff to the coupling circuit impedance, Z(f). We find a good agreement between the experimental data and the impedance of a single-pole *RC* network (solid blue line in Fig. 4.3(a)) yielding to a cutoff frequency  $f_0 = (2\pi R C)^{-1} = 33.1 \text{ GHz}.$ 

Next, we turn to the measured I(V) trace of the nanowire Josephson junction. The inset of Fig. 4.3(b) shows the raw curve, which exhibits a supercurrent peak around zero  $V_{\text{NW}}$  and a linear branch. The latter fits to a linear slope of  $R_{\text{NW}} = 14.03 \,\text{k}\Omega$  (solid green line). We then extract the  $I_{\text{ICPT}}(V_{\text{NW}})$  component by subtracting this slope from the raw

measured data (black dots in Fig. 4.3(b)), which is an additive component to the supercurrent peak unless the device has channels of transmission very close to unity [30]. In order to find the critical current and the noise temperature of the junction, we use the finite temperature solution of Ivanchenko and Zil'bermann [29] with substituting |Z(f)|as the impedance of the environment [31]. With this addition, we find an excellent agreement with the experimental data (blue solid line in Fig. 4.3(b)), with  $I_c = 9.38$  nA critical current. Notably, with the now determined value of  $I_c$ , we can extract  $R = 492 \Omega$ and C = 9.8 fF fully characterizing the microwave environment of the junctions. In addition, we find  $I_c R_{NW} = 132 \,\mu$ V, which indicates the induced superconducting gap in the nanowire channel. This value is close to the induced gap values measured earlier in similar devices [17, 32]. We also extract an effective noise temperature T = 132 mK, which is higher than the substrate temperature of 20 mK, similarly to earlier experiments [30].

Thus far, we evaluated  $I_{ICPT}(V_{NW})$  at  $V_{det} \approx 50 \,\mu V \ll 2\Delta/e = 480 \,\mu V$ , where  $I_{PAT} \approx 0$ , thus the detector load is negligible. However, depending on  $V_{NW}$ , we find a negative  $\Delta I_{ICPT}(V_{det})$ , i.e. a reduction of the emitter current, when the detector threshold is on resonance with the emitted frequency (Fig. 4.3(c)). We can understand this effect by the reduction of Z(f) in equation (4.3) in the presence of a finite  $r_{det}$  in parallel with R. In first order, we find  $\Delta I_{ICPT}/I_{ICPT} = -\text{Re}(Z(f))/r_{det} \approx -R/r_{det}$ . By using the measured DC current values, we evaluate the efficiency of the coupling circuit to be the ratio of the absorbed and emitted power  $\eta = P_{det}/P_{emi} = 2I_{PAT}/I_{ICPT}$  (Fig. 4.3(d)). We find typical values spanning 0.1 - 0.2, an order of magnitude improvement over earlier reported values [5, 33], however  $\eta < 1$  owing to the resistive losses of the device. Furthermore, the decrease of  $\eta$  with increasing f is consistent with the low-pass nature of the coupling circuit. We also calculate the detector quantum efficiency  $Q = P_{det}/\Delta P_{emi} = 2I_{PAT}/\Delta I_{ICPT}$  (Fig. 4.3(e)) and find values scattering around unity. This value directly measures the ratio of electron and photon rate passing the detector junction, thus confirming that it is in the quantum limit [25].

Finally, we note that the measured reduction  $\Delta I_{\text{ICPT}}/I_{\text{ICPT}} \ll 1$  directly confirms our initial assumption of negligible detector load on the circuit. This proves that the analysis based on a circuit model with the same Z(f) for the nanowire junction and the SIS detector is consistent.

We now turn to the shot-noise contribution to  $I_{PAT}$ . We observe a monotonous increase in  $I_{PAT}$  with increasing  $V_{NW}$  at any  $V_{det}$  consistently with the broadband  $S_I$  (Fig. 4.4(a)). Note that, in contrast with the data shown in Fig. 4.2(b), here the contribution of the Josephson radiation is negligible. To quantify the shot-noise contribution, we consider the derivative of the nonsymmetrized expression with respect to  $V_{NW}$  [34]:

$$\frac{dS_I(f)}{dV_{\rm NW}} = \frac{F}{R_{\rm qp}} \frac{d}{dV_{\rm NW}} \left( \frac{hf + eV_{\rm NW}}{1 - e^{-\beta(hf + eV_{\rm NW})}} + \frac{hf - eV_{\rm NW}}{1 - e^{-\beta(hf - eV_{\rm NW})}} \right)$$
(4.4)

where  $\beta = 1/k_{\rm B}T$  is the inverse temperature <sup>1</sup>. We can then calculate  $dI_{\rm PAT}/dV_{\rm NW}$  by substituting  $dS_I(f)/dV_{\rm NW}$  in place of  $S_I(f)$  in equation (4.2). Using the effective temperature T = 132 mK extracted earlier we find a confidence interval of F = 0.8...1.3 (Fig. 4.2(b)). Considering that the channel length of 100 nm is similar to the mean free path found earlier in the same nanowires [35], this result is consistent with ballistic transport

<sup>&</sup>lt;sup>1</sup>Note that we omitted the voltage-independent terms in [34].
which is dominated by single electron channels of low transmission where F = 1 [20, 36]. In contrast, F = 1/3 characteristic of diffusive normal transport [37] does not fit our data.

Furthermore, the measured  $I_{\text{NW}}(V_{\text{NW}})$  and  $I_{\text{PAT}}(V_{\text{NW}})$  do not agree with a transport dominated by multiple Andreev reflections, where a subgap structure is anticipated both in the current [38] and in the shot noise [39] depending on the channel transmissions. Our experiment thus provides insight into the nature of the charge transport at finite voltage bias in the nanowire Josephson junction and concludes that the finite subgap current can be attributed to single electron states inside the induced superconducting gap.

#### 4.5. CONCLUSION

In conclusion, we built and characterized an on-chip microwave coupling circuit to measure the microwave radiation spectrum of an InSb nanowire junction with NbTiN bulk superconducting leads. Our results clearly demonstrate the possibility of measuring the frequency of the Josephson radiation in a wide frequency range, opening new avenues in investigating the  $4\pi$ -periodic Josephson effect [40] in the context of topological superconductivity [41]. Based on the Fano factor, the shot-noise contribution to the measured signal demonstrates the presence of subgap quasiparticle states and excludes multiple Andreev reflection as the source of subgap current of the nanowire Josephson junction.

#### **CONTRIBUTIONS**

D.J.v.W., A.P. and T.K. fabricated the devices, performed the experiments. R.v.G. assisted with the data analysis. D.C., S.R.P. and E.P.A.M.B. contributed to the nanowire growth. L.P.K. and A.G. designed and supervised the experiments and analysed the data. The manuscript has been prepared with contributions from all authors.

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# 5

## MICROWAVE SPECTROSCOPY OF Spinful Andreev bound states in Ballistic Semiconductor Josephson junctions

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Through the use of the on-chip microwave spectroscopy circuit we have successfully studied the dispersion of Andreev bound states present in junctions made from Al-InAs nanowires. The circuit employs two tunnel junctions made out of Al/AlO<sub>x</sub>/Al as a reference junction in a hybrid SQUID and as a spectrometer. The hybrid SQUID consists of the reference junction in parallel with the nanowire junction, with the large asymmetry responsible for most of the phase drop to occur across the nanowire junction. We study the dispersion of Andreev bound states as a function of phase and gate voltage and reveal their spinful nature through the application of an in-plane magnetic field.

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#### 5.1. INTRODUCTION

The superconducting proximity effect in semiconductor nanowires has recently enabled the study of new superconducting architectures, such as gate-tunable superconducting qubits and multiterminal Josephson junctions. As opposed to their metallic counterparts, the electron density in semiconductor nanosystems is tunable by external electrostatic gates providing a highly scalable and *in-situ* variation of the device properties. In addition, semiconductors with large *g*-factor and spin-orbit coupling have been shown to give rise to exotic phenomena in superconductivity, such as  $\varphi_0$  Josephson junctions and the emergence of Majorana bound states. Here, we report microwave spectroscopy measurements that directly reveal the presence of Andreev bound states (ABS) in ballistic semiconductor channels. We show that the measured ABS spectra are the result of transport channels with gate-tunable, high transmission probabilities up to 0.9, which is required for gate-tunable Andreev qubits and beneficial for braiding schemes of Majorana states. For the first time, we detect excitations of a spin-split pair of ABS and observe symmetry-broken ABS, a direct consequence of the spin-orbit coupling in the semiconductor.

The linear conductance  $G = \frac{2e^2}{h} \sum T_i$  of a nanostructure between two bulk leads [1] depends on the individual channel transmission probabilities,  $T_i$ . Embedding the same structure between two superconducting banks with a superconducting gap of  $\Delta$  gives rise to Andreev bound states (ABS) [2]. If the junction length is much smaller than the superconducting coherence length,  $\xi$ , i.e. in the short junction limit, then the ABS levels depend on the phase difference  $\phi$  between the leads according to [3]:

$$E_{\text{ABS},i}(\phi) = \pm \Delta \sqrt{1 - T_i \sin^2 \frac{\phi}{2}}.$$
(5.1)

These subgap states with  $|E_{ABS}| \le \Delta$  are localized in the vicinity of the nanostructure and extend into the banks over a length scale determined by  $\xi$ . Note that equation (5.1) is only valid in the absence of magnetic field, when each energy level is doubly degenerate.

Direct microwave spectroscopy has recently demonstrated the occupation of the ABS by exciting a Cooper pair in atomic junctions [4]. Unlike quasiparticle tunnelling spectroscopy, which has also been used to detect ABS [5, 6], resonant excitation by microwaves is a charge parity-conserving process [7]. This property enables coherent control of ABS which is required for novel qubit architectures [8] and makes microwave spectroscopy a promising tool to detect Majorana bound states [9] in proximitized semiconductor systems [10–12].

#### **5.2.** DEVICE SET-UP

We investigate ABS excitations in Josephson junctions that consist of indium arsenide (InAs) nanowires covered by epitaxial aluminium (Al) shells [13]. The junction, where the superconducting shell is removed, is 100 nm (device 1, see the red box in Fig. 5.1(a)) and 40 nm long (device 2), respectively. The nanowire is then embedded in a hybrid superconducting quantum interference device (SQUID) whose second arm is a conventional Al/AlO<sub>x</sub>/Al tunnel junction (in yellow box), enabling the control of the phase drop  $\phi$  by means of the applied magnetic flux  $\Phi$  through the SQUID loop. In the limit of a negligible loop inductance and an asymmetric SQUID, where the Josephson coupling of the



Figure 5.1: **Device schematics and working principle.** (a) Equivalent circuit diagram: Bright field optical image of the hybrid SQUID with one InAs semiconductor nanowire weak link (scanning electron micrograph, in the red box) and an Al/AlO<sub>x</sub>/Al tunnel junction (enclosed by the yellow box). The SQUID is capacitively coupled to the spectrometer Al/AlO<sub>x</sub>/Al Josephson junction (scanning electron micrograph, in the green box) via  $C_c$ . The transmission of the semiconductor channel is tuned by the gate voltage,  $V_g$ . Additional gates near the electrodes are kept at a constant voltage  $V_{s1,2}$ . Circuit elements within the dashed box are located on-chip, thermally anchored to 12 mK. (b) and (c) excitations of the hybrid SQUID: the Andreev bound state at  $\hbar \omega = 2E_{\text{ABS}}$  (b) and the plasma oscillations at  $\hbar \omega = \hbar \omega_p$  (c) are excited by a photon energy  $\hbar \omega = 2eV_{\text{spec}}$  set by the DC voltage bias of the spectrometer (d) with a superconducting gap  $\Delta_{\text{spec}}$ . (e) Schematic circuit diagram of the hybrid SQUID. The total phase  $\varphi = \phi + \delta$  is determined by the applied flux  $\Phi$ . (f) The measured I(V) trace of the spectrometer junction with the nanowire in full depletion, i.e. in the absence of ABS excitations. The red solid line shows the fit to the circuit model of a single resonance centered at  $\hbar \omega_p$ , see text. Images and data were all taken on device 1.

nanowire is much smaller than that of the tunnel junction, the applied phase  $\varphi$  mostly drops over the nanowire link:  $\phi \approx \varphi = 2\pi\Phi/\Phi_0$ , where  $\Phi_0 = h/2e$  is the superconducting flux quantum. We measure the microwave response [4, 7] of the nanowire junction utilizing the circuit depicted in Fig. 5.1(a), where a second Al/AlO<sub>x</sub>/Al tunnel junction (in green box) is capacitively coupled to the hybrid SQUID and acts as a spectrometer. Further details on the fabrication process are given in section 5.9.1.

#### **5.3.** CIRCUIT CHARACTERISATION

In this circuit, inelastic Cooper-pair tunnelling (ICPT, Fig. 5.1(d)) of the spectrometer junction is enabled by the dissipative environment and results in a DC current,  $I_{spec}$  [14]:

$$I_{\rm spec} = \frac{I_{\rm c,spec}^2 \operatorname{Re}[Z(\omega)]}{2V_{\rm spec}}.$$
(5.2)

Here  $I_{c,spec}$  is the critical current of the spectrometer junction,  $V_{spec}$  is the applied voltage bias, and  $Z(\omega)$  is the circuit impedance at a frequency  $\omega = 2eV_{spec}/\hbar$ . Since  $Z(\omega)$  peaks at the resonant frequencies of the hybrid SQUID [4, 14], so does the DC current  $I_{spec}$ , allowing us to measure the ABS excitation energies of the nanowire junction (Fig. 5.1(b)), as well as the plasma frequency of the SQUID (Fig. 5.1(c)).

First we characterize the contribution of the plasma mode with the nanowire junction gated to full depletion, i.e. G = 0. We show the I(V) curve of the spectrometer junction of device 1 in Fig. 5.1(f), where we find a single peak centered at  $\hbar \omega_p / 2 = eV_{\text{spec}} = 46 \,\mu\text{eV}$  and a quality factor  $Q \approx 1$ . In the limit of  $E_C \ll E_J$ ,  $\hbar \omega_p = \sqrt{2E_C E_J}$ , where  $E_C$  is the charging energy of the circuit and  $E_J$  is the Josephson coupling of the tunnel junction (Fig. 5.1(e)). Estimating  $E_J = 165 \,\mu\text{eV}$  from the normal state resistance [15], this measurement allows us to determine  $E_C = 25.4 \,\mu\text{eV}$  (see section 5.9.4.). The choice of a low quality factor in combination with a characteristic impedance  $Z_0 = 551 \,\Omega \ll R_q = h/4e^2$  ensures the suppression of higher order transitions and parasitic resonances.

#### **5.4.** GATE VOLTAGE DEPENDENCE

Next, we investigate the spectrometer response as a function of the gate voltage  $V_g$  applied to the nanowire. Note that the spectrometer response to the ABS transitions is superimposed on the plasma resonance peak. In order to achieve a better visibility of the ABS lines, we display  $-d^2 I_{\text{spec}}/dV_{\text{spec}}^2$  ( $V_{\text{spec}}$ ) rather than  $I_{\text{spec}}(V_{\text{spec}})$  (see Fig. 5.10 for comparison). In the presence of ABS, the spectrum exhibits peaks at frequencies where  $\hbar\omega = 2E_{ABS,i}$  [7]. In Fig. 5.2(a), we monitor the appearance of these peaks for an applied phase  $\varphi = \pi$ , where the ABS energy of equation (5.1) is  $E_{ABS,i}(\pi) = \Delta \sqrt{1 - T_i}$ . Notably, for  $V_g$  values close to full depletion (see red bar in Fig. 5.2(a)), we see a gradual decrease of  $E_{ABS}(\pi)$  with increasing  $V_g$  (black circles in Fig. 5.2(e)). In this regime, we find a good correspondence with equation (5.1), assuming single channel transport,  $G = \frac{2e^2}{h}T$  (red solid line in Fig. 5.2(e), see section 5.9.5. on the details of the measurement of G). However, the observed  $\Delta = 122 \,\mu\text{eV}$  is smaller than the  $\Delta_{\text{Al}} \approx 200 \,\mu\text{eV}$  of the thin film Al contacts, in agreement with the presence of induced superconductivity in the nanowire [16]. Increasing  $V_{g}$  further, we observe a sequential appearance of peaks, which we attribute to the opening of multiple transport channels in the weak link and the consequent formation of multiple ABS [3] as the Fermi level,  $E_F$ , increases. We also find a strong variation of  $E_{ABS}$  with  $V_g$  similarly to earlier experiments [17–19]. We attribute this observation to mesoscopic fluctuations in the presence of weak disorder [3], such that the mean free path of the charge carriers is comparable to the channel length.



Figure 5.2: **Gate dependence of Andreev bound states.** (a)  $-d^2 I/dV^2$  of the spectrometer junction as a function of  $V_g$  at  $\varphi = \pi$ , where  $E_{ABS,i} = \Delta \sqrt{1 - T_i}$  in the short junction limit. Panels (b) and (c):  $-d^2 I/dV^2$  of the spectrometer junction as a function of  $\varphi = 2\pi\Phi/\Phi_0$  for one channel (b) and several channels (c). The qualitative agreement of the line shapes with equation (7.1) confirms the short junction behaviour. Arrows in panel (a) indicate  $V_g$  for these measurements. Weakly visible vertically shifted replicas of the ABS lines indicate higher order transitions, see text. (d) Strong hybridization between the ABS excitation and the plasma mode with a level repulsion of  $\varepsilon = 22 \,\mu$ eV at the yellow dashed line. (e)  $E_{ABS}(\varphi = \pi)$  as a function of the DC linear conductance *G* of the nanowire weak link in the gate span denoted by the red bar in panel (a). The error bars correspond to the linewidth of the measured signal. The solid red line shows the prediction of the single channel model with  $\Delta = 122 \,\mu$ eV $\pm 3 \,\mu$ eV, see text. All data was taken on device 1. Grey regions denote lack of data due to bias instability of the circuit.

#### **5.5.** FLUX DEPENDENCE

Now we turn to the flux dependence of the observed spectrum, shown in Fig. 5.2(b) and (c) for two distinct gate configurations. We find a qualitative agreement with equa-

tion (5.1) with one transport channel in Fig. 5.2(b) and several channels in Fig. 5.2(c), confirming that our device is in the short junction limit. In addition, we observe the plasma mode at  $eV_{\text{spec}} < 50 \,\mu\text{eV}$ . We also find that the plasma mode  $\hbar \omega_p$  oscillates with  $\varphi$  when the nanowire is gated to host open transport channels. This is expected due to the Josephson coupling of the nanowire becoming comparable to  $E_J$ , which also causes a finite phase drop,  $\delta$ , over the tunnel junction. We also note the presence of additional, weakly visible lines in the spectrum which could be attributed to higher order processes [4]. However, we did not identify the nature of these excitations, and we focus on the main transitions throughout the current work.

In addition, we observe the occurrence of avoided crossings between the Andreev and plasma modes, as shown in Fig. 5.2(d) at  $\varphi = \pi$ . These avoided crossings require  $\hbar \omega_p \approx 2\Delta \sqrt{1-T}$ , which translates to a high transmission probability  $T \approx 0.8 - 0.9$ , and demonstrates the hybridization between the ABS excitation and the plasma mode. The coupling between these two degrees of freedom has previously been derived [7, 20], leading to a perturbative estimate for the energy splitting  $\varepsilon \approx \Delta T \left( E_C/2E_J \right)^{1/4} \approx 40 - 70 \,\mu\text{eV}$ , similar to the observed value of 22  $\mu$ eV. The discrepancy is fully resolved in the numerical analysis of the circuit model developed below.

#### **5.6.** MODEL

We provide a unified description of the energy spectrum of the circuit as a whole, and consider the following Hamiltonian for the hybrid SQUID (Fig. 5.1(e)) [20]:

$$\hat{H} = E_C \hat{N}^2 + E_I (1 - \cos\hat{\delta}) + \hat{H}_{ABS}(\varphi - \hat{\delta}).$$
(5.3)

Here  $\hat{\delta}$  is the operator of the phase difference across the tunnel junction, conjugate to the charge operator  $\hat{N}$ ,  $[\hat{\delta}, \hat{N}] = i$ . The first two terms in equation (5.3) represent the charging energy of the circuit and the Josephson energy of the tunnel junction (Fig. 5.1(e)). The last term describes the quantum dynamics of a single-channel short weak link [21, 22], which depends on  $\Delta$  and T. For the analytic form of  $\hat{H}_{ABS}$ , see section 5.10. To fully account for the coupling between the ABS excitation and the quantum dynamics of the phase across the SQUID, we numerically solve the eigenvalue problem  $\hat{H}\Psi = E\Psi$  and determine the transition frequencies  $\hbar\omega = E - E_{GS}$  with  $E_{GS}$  being the ground state energy.

This procedure allows us to fit the experimental data, and we find a good quantitative agreement as shown in Fig. 5.3(a) for a dataset taken at  $V_g = -1410 \text{ mV}$  with the fit parameters  $\Delta = 122 \,\mu\text{eV}$  and T = 0.57. The previously identified circuit parameters  $E_I$ and  $E_C$  are kept fixed during the fit. We note that the observed ABS transition (orange solid line) only slightly deviates from equation (5.1) (black dashed line). The modulation of the plasma frequency (green solid line) is then defined by the model Hamiltonian with no additional fit parameters. We further confirm the nature of the plasma and ABS excitations by evaluating the probability density  $|\Psi(\delta, \sigma)|^2$  of the eigenfunctions of equation (5.3) at  $\varphi = \pi$  (Fig. 5.3(b)). In the ground state of  $\hat{H}$  (GS) and in the state corresponding to the plasma excitation (green line in Fig. 5.3(a)), the probability density is much higher in the ground state of the weak link ( $\sigma = g$ , blue line) than in the excited state ( $\sigma = e$ , red line). In contrast, the next observed transition (orange line in Fig. 5.3(a)) gives rise to a



Figure 5.3: **Theoretical description of the transitions.** (a) Solid lines denote the transitions identified by the model described in the text, with  $\Delta$  and *T* being free parameters. The experimental dataset is the same as the one shown in Fig. 5.2(b). The dashed line shows equation (5.1) for the fitted  $\Delta = 122 \,\mu\text{eV}$  and T = 0.57. (b) The probability density  $|\Psi(\delta,\sigma)|^2$  in the ground state of the hybrid SQUID (GS), and in the two excited states depicted in panel (a), respectively. The weight in the ABS ground state ( $\sigma = g$ ) and in the ABS excited state ( $\sigma = e$ ) distinguishes between the plasma mode and the ABS. (c) The measured relative intensity of the ABS transition (black circles) compared to the theoretical expectation based on equation (5.3) (orange solid line) and from [7] (black dashed line) with no additional fitting parameters.

higher contribution from  $\sigma = e$  confirming our interpretation of the experimental data in terms of ABS excitations. Furthermore, the model can also describe measurement data with *T* close to 1, where it accurately accounts for the avoided crossings between the ABS and plasma spectral lines (see section 5.9.6. for a dataset with *T* = 0.9).

In Fig. 5.3(c) we show the visibility of the ABS transition as a function of the applied phase  $\varphi$ , which is proportional to the absorption rate of the weak link, predicted to be  $\propto T^2(1-T)\sin^4(\varphi/2) \times \Delta^2/E_{ABS}^2(\varphi)$  [7]. We note that in the experimental data the maximum of the intensity is slightly shifted from its expected position at  $\varphi = \pi$ . This minor deviation may stem from the uncertainty of the flux calibration. Nevertheless, using T = 0.57, obtained from the fit in Fig. 5.3(a), we find a good agreement with no adjustable parameters (black dashed line). A similarly good correspondence is also found with the full numerical model (orange line) based on equation (5.3).

#### **5.7.** IN-PLANE MAGNETIC FIELD DEPENDENCE

We now discuss the evolution of the ABS as a function of an in-plane magnetic field *B* aligned parallel to the nanowire axis, which is perpendicular to the internal Rashba spin-orbit field (see the inset in Fig. 5.4(b) for measurement geometry). The applied field lifts the Kramers degeneracy of the energy spectrum, splitting each Andreev doublet into a pair  $E_{ABS}^{\pm}(\phi)$ . For small *B*, the splitting  $E_{ABS}^{+}(\phi) - E_{ABS}^{-}(\phi)$  is linear in *B*, due to the Zeeman effect. However, the spin-split single particle levels are not accessible by microwave spectroscopy, which can only induce transitions to a final state with two excited quasiparticles. Thus we can only measure  $E_{tot}(\phi) = E_{ABS}^{+}(\phi) + E_{ABS}^{-}(\phi)$  and expect no split of the measured spectral lines. The experimental data (Fig. 5.4(a)) shows that  $E_{tot}$  decreases with *B*, while the lineshape remains qualitatively intact.

In order to explain the field dependence of  $E_{tot}$ , we study the behaviour of ABS in a simple model consisting of a short Josephson junction in a one-dimensional quantum wire with proximity-induced superconductivity, Rashba spin-orbit and an applied Zeeman field parallel to the wire [10, 11, 23]. Within this model, we are able to find  $E_{ABS}^+$  and  $E_{ABS}^-$ , and reproduce the observed quadratic decrease of the measured  $E_{tot}(\pi)$  (black circles in Fig. 5.4(b)). Initially, as *B* is increased, the proximity-induced gap  $\Delta(B)$  is suppressed (black solid line), while the energy  $E_{ABS}^+(\pi)$  (blue solid line) increases due to the Zeeman split of the ABS. However, a crossing of the discrete ABS level with the continuum is avoided due to the presence of spin-orbit coupling, which prevents level crossings in the energy spectrum by breaking spin-rotation symmetry. The repulsion between the ABS level and the continuum causes a downward bending of  $E_{ABS}^+(\pi)$ , in turn causing a decrease in  $E_{tot}(\pi)$  (black dashed line).

We perform the calculations in the limit where the Fermi level  $E_F$  in the wire is well above the Zeeman energy  $E_Z = \frac{1}{2}g\mu_B B$  and the spin-orbit energy  $E_{SO} = m\alpha^2/2\hbar^2$  with *m* the effective mass and  $\alpha$  the Rashba spin-orbit coupling constant. In this case and in the short junction limit, the ratio  $E_{tot}(\pi)/\Delta$  is a function of just two dimensionless parameters:  $E_Z/\Delta$  and  $\sqrt{E_{SO}E_F}/\Delta$ . First we extract  $\Delta = 152 \,\mu$ eV and T = 0.56 at B = 0(leftmost panel in Fig. 5.4(a)). Then we perform a global fit on  $E_{tot}(\phi)$  at all *B* values and obtain a quantitative agreement with the theory for  $g = 14.7 \pm 0.6$ , which is in line with expected *g*-factor values in InAs nanowires [24–26] and  $\sqrt{E_{SO}E_F}/\Delta = 0.32 \pm 0.02$ . This model is consistent assuming  $E_F > E_Z \approx 100 \,\mu$ eV at 300mT. Thus we attain an upper



Figure 5.4: **Spectroscopy of spin-split Andreev bound states in a Rashba nanowire.** Panel (a) shows the flux dependence of the Andreev bound states at B = 0, 100 and 300mT, respectively, applied parallel to the nanowire. The zero-field fit yields to T = 0.56 and  $\Delta = 152 \,\mu\text{eV}$ . Dash lines depict the fit of  $E_{\text{tot}}(\phi) = E_{\text{ABS}}^+(\phi) + E_{\text{ABS}}^-(\phi) + E_{\text{ABS}}^-(\phi)$  to the model described in the text. (b) Black circles show the measured  $E_{\text{tot}}(\pi)$  as a function of *B*. The error bars correspond to the linewidth of the measured signal. The dashed line depicts the fit to the theory with  $g = 14.7 \pm 0.6$  and  $\sqrt{E_{\text{SO}}E_F}/\Delta = 0.32 \pm 0.02$ , see text. The Zeeman-split ABS levels  $E_{\text{ABS}}^{\pm}(\pi)$  and the proximity-induced gap  $\Delta(B)$  obtained from the model are shown as visual guides. The dotted line depicts the expected behavior of  $E_{\text{tot}}(B)$  in the presence of a strong orbital magnetic field with  $B_* = 400$  mT and weak spin-orbit coupling, see text. (c)  $E_{\text{ABS}}^{\pm}(\phi)$  computed at B = 100 mT are shown as blue and red solid lines, together with the calculated transition energy  $E_{\text{tot}}(\phi)$  (black dashed line). The experimental data was taken on device 2 at  $V_g = 140$  mV. Grey regions denote lack of data due to bias instability of the circuit.

bound  $E_{\rm SO} \lesssim 24 \,\mu$ eV, equivalent to a Rashba parameter  $\alpha \lesssim 0.12 \,\text{eV}$ Å in correspondence with earlier measurements on the same nanowires [26]. However, assuming the opposite limit,  $E_F \approx 0$ , the theory is not in agreement with the experimental data (see Fig. 5.16 for comparison).

The theoretical energy spectrum shown in Fig. 5.4(b) predicts a ground state fermionparity switch of the junction at a field  $B_{sw} \approx 400 \text{ mT}$ , at which the lowest ABS level  $E_{tot}^-(\pi) =$ 0 (red line in Fig. 5.4(b)). This parity switch inhibts the resonant excitation of the Zeemansplit ABS levels [27] thus preventing microwave spectroscopy measurements for  $B > B_{sw}$ . This prediction is in agreement with the vanishing visibility of the ABS line at  $B \approx B_{sw}$  in the experiment.

In addition to the interplay of spin-orbit and Zeeman couplings, the orbital effect of the magnetic field [28] is a second possible cause for the decrease of the ABS transition energy. Orbital depairing influences the proximity-induced pairing and results in a quadratic decrease of the induced superconducting gap:  $\Delta(B) = \Delta(1 - B^2/B_*^2)$ , where  $B_* \sim \Phi_0/A$  and A is the cross-section of the nanowire. A simple model which includes both orbital and Zeeman effect, but no spin-orbit coupling, yields  $B_* \approx 400 \,\text{mT}$  when fitted to the experimental data (see Fig. 5.16(c) for details). In this case, the fit is insensitive to the value of the *g*-factor. However, the model also predicts the occurrence, at  $\varphi = \pi$ , of a fermion-parity switch at a field  $B_{sw} < B_*$  whose value depends on the *g*-factor. Because agreement with the experimental data imposes the condition that  $B_{sw} > 300 \,\text{mT}$ , in section 5.10.4. we show that this scenario requires  $g \lesssim 5$ , which is lower than *g*-factor values measured earlier in InAs nanowire channels [24–26].

Furthermore, we can consider the qualitative effect of the inclusion of a weak spinorbit coupling  $(E_{SO} \ll \Delta)$  in this model containing only the orbital and Zeeman effects. We note that, without spin-orbit coupling, the upper Andreev level  $E_{ABS}^+(B)$  crosses a continuum of states  $\Delta(B)$  with opposite spin upon increasing the magnetic field (see Fig. 5.16(c)). The crossing happens at a field of  $B_{cross}$  whose value depends on the gfactor: using the upper bound for g derived in the last paragraph,  $g \approx 5$ , we can estimate  $B_{\rm cross} \approx 150 \, {\rm mT}$ . At this magnetic field, a weak spin-orbit coupling results in an avoided crossing between the Andreev level  $E_{ABS}^+(B)$  and the continuum. As a consequence, when  $B > B_{cross}$ , the energy  $E_{ABS}^+(B)$  is bounded by the edge of the continuum and it is markedly lower than its value in the absence of spin-orbit coupling. In turn, this results in a decrease of the transition energy  $E_{tot}(B)$  at  $B > B_{cross}$ , to the extent that such a model containing the joint effect of orbital depairing and weak spin-orbit coupling would depart from the experimental data in the range 150 mT < B < 300 mT (see dotted line in Fig. 5.4(b)). Thus, although based on the geometry of the experiment we cannot rule out the presence of an orbital effect of the magnetic field, these considerations imply that it does not play a dominant role in the quadratic suppression of the transition energy in the present measurements.

We finally note that in all cases we neglect the effect of *B* on the Al thin film, justified by its in-plane critical magnetic field exceeding 2T [29].

We present the ABS spectrum in the presence of several transport channels in Fig. 5.5. While at zero magnetic field (left panel) the data is symmetric around  $\varphi = \pi$ , in a finite magnetic field (right panel) the data exhibits an asymmetric flux dependence (see the yellow dashed line as a guide to the eye). This should be contrasted with Fig.



Figure 5.5: **Time-reversal symmetry-broken ABS in magnetic field.** The symmetry axis at  $\varphi = \pi$  at zero magnetic field is denoted by yellow dashed line. Note that at B = 40 mT the observed spectrum does not obey the mirror symmetry with respect to the same line. The data was taken on device 1 at  $V_g = -20 \text{ mV}$ . Grey regions denote lack of data due to bias instability of the circuit.

5.4(a) where the data for a single-channel wire are presented at different values of the magnetic field: each of the traces is symmetric around  $\varphi = \pi$ . This behaviour agrees with theoretical calculations in the short-junction limit, which show that this asymmetry can arise in a Josephson junction with broken time-reversal and spin-rotation symmetries as well as more than one transport channel [30]. While the data is asymmetric with respect to  $\varphi = \pi$ , there is no visible shift of the local energy minima away from this point. This observation is consistent with the absence of an anomalous Josephson current [31–33] for our specific field configuration (magnetic field parallel to the wire), in agreement with theoretical expectations [34–36].

#### 5.8. CONCLUSION

In conclusion, we have presented microwave spectroscopy of Andreev bound states in semiconductor channels where the conductive modes are tuned by electrostatic gates and we have demonstrated the effect of Zeeman splitting and spin-orbit coupling. The microwave spectroscopy measurements shown here could provide a new tool for quantitative studies of Majorana bound states, complementing quasiparticle tunnelling experiments [12, 24]. Furthermore, we have provided direct evidence for the time-reversal symmetry breaking of the Andreev bound state spectrum in a multichannel ballistic system. This result paves the way to novel Josephson circuits, where the critical current depends on the current direction, leading to supercurrent rectification effects [37, 38] tuned by electrostatic gates.

#### CONTRIBUTIONS

D.J.v.W., A.P. and D.B. performed the experiments. B.v.H., J.I.V. and L.I.G. developed the theory to analyse the data. P.K. and J.N. contributed to the nanowire growth. D.J.v.W., A.P. and D.B. fabricated the samples. L.P.K. and A.G. designed and supervised the experiments. D.J.v.W., B.v.H., L.P.K. and A.G. analysed the data. The manuscript has been prepared with contributions from all the authors.

#### 5.9. SUPPLEMENTARY INFORMATION

#### **5.9.1.** DEVICE FABRICATION

The devices are fabricated on commercially available undoped Si wafers with a 285 nm thick thermally grown SiO<sub>x</sub> layer using positive tone electron beam lithography. First, the electrostatic gates and the lower plane of the coupling capacitors are defined and Ti/Au (5 nm/15 nm) is deposited in a high-vacuum electron-beam evaporation chamber. Next, the decoupling resistors are created using Cr/Pt (5 nm/25 nm) with a track width of 100 nm, resulting in a characteristic resistance of  $100 \Omega/\mu m$ . Then, a 30 nm thick SiN<sub>x</sub> layer is sputtered and patterned to form the insulation for the coupling capacitors and the gates. We infer  $C_c = 400$  fF based on the surface area of  $6.5 \times 30 \mu m^2$  and a typical dielectric constant  $\varepsilon_r = 7$ .

In the following step, the tunnel junctions are created using the Dolan bridge technique by depositing 9 and 11 nm thick layers of Al with an intermediate oxidization step *in-situ* at 1.4 mbar for 8 minutes. Then, the top plane of the coupling capacitors is defined and evaporated (Ti/Au, 20nm/100nm) after an *in-situ* Ar milling step to enable metallic contact to the Al layers. Next, the InAs nanowire is deterministically deposited with a micro-manipulator on the gate pattern [39].

The channel of device 1 is defined by wet chemical etch of the aluminium shell using Transene D at 54 °C for 12 seconds. The channel of device 2 is determined by *in-situ* patterning, where an adjacent nanowire casted a shadow during the epitaxial deposition of aluminium [40]. The superconducting layer thickness was approximately 10 nm for both devices deposited on two facets.

Finally, the nanowire is contacted to the rest of the circuit by performing Ar plasma milling and subsequent NbTiN sputter deposition to form the loop of the hybrid SQUID. We show the design parameters of the devices in Table 5.1.

	Device 1	Device 2
Channel length (nm)	100	40
Tunnel junction area (nm <sup>2</sup> )	$400 \times 120$	$200 \times 120$
Flux periodicity ( $\mu$ T)	38	120
Spectrometer junction area (nm <sup>2</sup> )	$120 \times 120$	$120 \times 120$

Table 5.1: Geometry of the devices featured in the current study.

#### 5.9.2. MEASUREMENT SET-UP

The measurements were performed in a Leiden Cryogenics CF-1200 dry dilution refrigerator with a base temperature of 12 mK equipped with Cu/Ni shielded twisted pair



Figure 5.6: **Detailed schematics of the measurement setup.** The inset of panel (a) shows a bright field optical image of device 1. The solid black box denotes the radiation shielded environment thermally anchored to 12 mK. (b) On-chip lumped circuit elements attached to the hybrid SQUID (on the left) and the spectrometer Josephson junction (on the right).

cables thermally anchored at all stages of the refrigerator to facilitate thermalization. Noise filtering is performed by a set of  $\pi$ -LC filters (~ 100 MHz) at room temperature and copper-powder filters (~ 1 GHz) in combination with two-pole RC filters (~ 100 kHz) at base temperature for each measurement line. The schematics of the setup is shown in Fig. 5.6.

#### 5.9.3. DEVICE CIRCUIT PARAMETERS

We characterise the circuit based on the plasma resonance observed with the semiconductor nanowire gated to zero conductance, i.e. full depletion. In this regime, we infer the environmental impedance  $\text{Re}[Z(\omega)]$  based on equation (5.2) and assume the following form, which is valid for a parallel LCR circuit:

$$\operatorname{Re}[Z(x)] = \frac{Z_0 Q}{1 + \frac{Q^2}{r^2} (1 - x^2)^2},$$
(5.4)

with  $x = \omega/\omega_0$  the dimensionless frequency. The resonance of the circuit is centered at  $\omega_0 = (LC)^{-1/2}$  with a quality factor of  $Q = R\sqrt{\frac{C}{L}}$  and a characteristic impedance of  $Z_0 = \sqrt{L/C}$ . Consistently with this single mode circuit, we find one peak in the I(V) trace of the spectrometer that we fit to equation (5.4) (Fig. 5.7). We find a good quantitative agreement near the resonance peak, however the theoretical curve consistently deviates at higher voltages, i.e. higher frequencies. We attribute this discrepancy to additional losses or other resonant modes of the circuit not accounted for by equation (5.4).

In addition, we use the superconducting gap and the linear resistance of the junctions to determine the Josephson energy  $E_J$  and the Josephson inductance  $L_J$ . With these, we infer the circuit parameters listed in Table 5.2.



Figure 5.7: **Plasma resonance of the circuit**. The measured (black dots) and fitted (solid red line) I(V) trace of the spectrometer junction for device 1 (a) and for device 2 (b) respectively, with the nanowire in full depletion. The fits are based on equation (5.4), see text. Note that we omitted the supercurrent branch for clarity. In panel (b), the inset shows the spectrometer response to an in-plane magnetic field of 300 mT.

#### 5.9.4. Spectrum Analysis

Peaks in the I(V) trace of the spectrometer correspond to peaks in Re[ $Z(\omega)$ ], i.e. allowed transitions of the environment coupled to the spectrometer. In order to remove the smooth background of the plasma mode (see Fig. 5.7), we evaluate  $-d^2I/dV^2(V)$ , the second derivative of the I(V) to find peaks in Re[ $Z(\omega)$ ] after applying a Gaussian low pass filter with standard deviation of  $1.5 \mu$ V. We benchmark this method in Fig. 5.9, and find that the peaks where  $-d^2I/dV^2(V) > 0$  correspond to the peaks in I(V) and hence



Figure 5.8: **Large scale I(V) trace of the spectrometer junction.** The subgap features are shown in detail in Fig. 5.7(a). The back-bending near  $eV_{\text{spec}} \approx 2\Delta_{\text{spec}} = 482 \,\mu\text{eV}$  is attributed to a local overheating of the junction due to a large quasiparticle current density above the gap edge. The data was taken on device 1.

	Device 1	Device 2
Tunnel junction resistance $R_J$ (k $\Omega$ )	4.80	10.7
Tunnel junction gap $\Delta_J$ (µeV)	245	250
Tunnel junction critical current $I_{c,J} = \frac{\pi \Delta_J}{2eR_J}$ (nA)	80.2	36.7
$E_J = \frac{\hbar I_{\rm c,J}}{2e} \; (\mu \rm eV)$	165	75.5
Tunnel junction inductance $L_J = \frac{\Phi_0}{2\pi I_{c,JJ}}$ (nH)	4.10	8.94
Spectrometer resistance $R_{\rm spec}$ (k $\Omega$ )	17.1	18.4
Spectrometer gap $\Delta_{\text{spec}}$ ( $\mu$ eV)	241	249
Spectrometer critical current $I_{c,spec} = \frac{\pi \Delta_{spec}}{2eR_{spec}}$ (nA)	22.2	21.3
Shunt resistance $R(\Omega)$	634	743
Shunt capacitance C (fF)	12.6	11.1
Charging energy $E_c = \frac{2e^2}{C}$ (µeV)	25.44	29.1
Plasma frequency $f_p = \frac{1}{2\pi\sqrt{L_JC}}$ (GHz)	22.9	16.0
Characteristic impedance $Z_0 = \sqrt{\frac{L_f}{C}} (\Omega)$	551	897
Quality factor $Q = R \sqrt{\frac{C}{L_J}}$	1.15	0.83

Table 5.2: Circuit parameters of the devices featured in the current study.

 $-d^2I/dV^2(V)$  is a good measure of the transitions detected by the spectrometer junction.

Alternatively, the background can be removed by linewise subtracting the detector response at  $\varphi = 0$  [4], where the ABS does not contribute to the spectrometer response



Figure 5.9: **Spectrum analysis by second derivative.** The I(V) (red line, left axis) and the corresponding  $-d^2I/dV^2(V)$  trace (black line, right axis) of the spectrometer showing the same peaks denoted by dashed lines. Note that only peaks above  $-d^2I/dV^2(V) = 0$  (grey horizontal line) correspond to actual transitions. This dataset was taken on device 1, at  $V_g = -1410$  mV, phase biased to  $\varphi = \pi$ .

[7]. We show the result of this analysis in Fig. 5.10. Notably, the phase dependence of the plasma mode gives rise to additional features near  $\varphi = \pi$ . Furthermore, datasets exhibiting hybridization between the ABS and plasma mode cannot be evaluated by this method. However, the line subtraction and the second derivative are in agreement if there is sufficient spacing between the plasma mode and the ABS line (see Fig. 5.2(b) and Fig. 5.10 for comparison).



Figure 5.10: **Spectrum analysis by background subtraction.** (a)  $I_{sub}(\varphi) = I_{spec}(\varphi) - I_{spec}(\varphi = 0)$  spectrometer current after subtracting the line trace at  $\phi = 0$ . (b) Single linetrace of the raw data  $I_{spec}(\varphi = \pi)$  (red line, left axis) and  $I_{sub}(\varphi = \pi)$  (black line, right axis). This dataset was taken on device 1, at  $V_g = -1410$  mV.

#### 5.9.5. I(V) TRACE OF THE HYBRID SQUID

We measure the I(V) trace of the hybrid SQUID as a function of the gate voltage  $V_g$  at  $V_{\text{spec}} = 0$  (Fig. 5.11) and find that the subgap conductance increases with increasing gate voltage, in qualitative agreement with the contribution of multiple Andreev reflection (MAR). The zero voltage data corresponds to the supercurrent branch and the dashed lines denote the bias range where there is no data due to the bias instability of the driving circuit. In addition, we find a back-bending at the gap edge  $eV_{\text{SQUID}} = 2\Delta_J$ , attributed to self-heating effects in the tunnel junction.

We evaluate *G* in Fig. 5.2(e) in the bias voltage range  $-V_{SQUID} = 350...430 \,\mu V > 2\Delta$ . We note that due to the soft superconducting gap in the nanowire junction, we did not identify MAR features after subtracting the current background of the tunnel junction.



Figure 5.11: **The** I(V) **trace of the hybrid SQUID.** At  $V_g = -1.75$  V, the nanowire is in full depletion, thus the corresponding I(V) trace represents the Al/AlO<sub>x</sub>/Al tunnel junction in the hybrid SQUID. The bias voltage  $V_{\text{SQUID}}$  was swept from the left to the right. The data was taken on device 1.



#### 5.9.6. FIT OF ABS WITH HIGH TRANSMISSION

Figure 5.12: **Experimental data and fit to the theory for ABS with high transmission.** In this figure we show the numerical fit to the model of equation (5.3), similarly to Fig. 5.3(a), but for a different dataset taken at  $V_g = -1.525$  V on device 1. The figure shows that the model of equation (5.3) can accurately predict the avoided crossing originating in the coupling between the ABS and the plasma mode. Best-fit parameters are  $\Delta = 97.5 \pm 1.7 \,\mu$ eV and  $T = 0.90 \pm 0.01$ . Dashed line denotes the undressed Andreev level defined by equation (5.1). We note that the extracted value for  $\Delta$  is lower than in Fig. 5.3(a). This may stem from the fit underestimating the gap, since most of the datapoints are around  $\varphi = \pi$ , or due to a genuine dependence of  $\Delta$  on  $V_g$  because of the change in the wavefunction overlap as a result of the electrostatic gating [41]. In panel (b), we show the probability density for the ground state (GS) and the two observed excited states denoted by the green and orange lines, respectively in panel (a) at  $\varphi = \pi$ .

#### 5.9.7. TIME-REVERSAL SYMMETRY-BROKEN ABS IN BIPOLAR MAGNETIC FIELD



Figure 5.13: **Symmetry-broken ABS in a bipolar magnetic field.** The full spectrum is symmetric around  $\varphi = \pi$  at zero magnetic field (center panel) with the mirror axis denoted by the yellow dashed line. Note the asymmetry of the two lowermost ABS transitions at  $B = \pm 40$  mT. The antisymmetric contribution is most visible at  $V_{\text{spec}} \approx 100 \,\mu$ V, which develops an opposite shift for positive and negative magnetic fields, respectively. The data was taken on device 1 at  $V_g = -770$  mV. Grey regions denote lack of data due to bias instability of the circuit.

#### **5.10.** THEORY

**5.10.1.** ESTIMATE OF THE ABS-PLASMA RESONANCE AVOIDED CROSSING Before describing the quantum model of the circuit in detail, we discuss the estimate for the energy splitting at the avoided crossing between the ABS transition and the plasma frequency shown in Fig. 5.2(d).

For simplicity, we model the plasma oscillations as a bosonic mode with a flux-independent frequency given by  $\hbar \omega_p = \sqrt{2E_J E_C}$ , and the weak link as a two-level system, with energies  $\pm E_{ABS}(\varphi)$  defined by equation (5.1). This system with the two independent degrees of freedom is described by the Hamiltonian  $\hat{H}_0 = \hbar \omega_p (\hat{a}^{\dagger} \hat{a} + \frac{1}{2}) + E_{ABS} \hat{\sigma}_3$ . Next, we add the coupling term corresponding to the excitation of the weak link due to the voltage oscillations induced by the junction in the form

$$H_g(\varphi) = g(\varphi)\sqrt{z}\left(\hat{a}^{\dagger} + \hat{a}\right)\hat{\sigma}_1.$$
(5.5)

where  $z = \sqrt{E_C/2E_J}$ . This term describes a linear coupling between the two-level system and the phase difference across the junction.  $g(\varphi)$  is then given by the current matrix element between the ground and excited states of the weak link, which was derived in Ref. [7]:

$$g(\varphi) = \Delta T \sqrt{1 - T} \sin^2(\varphi/2) \frac{\Delta}{E_{\text{ABS}}(\varphi)}.$$
(5.6)

The square of this current matrix element gives the microwave absorption rate of the weak link, plotted in Fig. 5.3(c) (black dashed line). From the coupling Hamiltonian, we immediately obtain that at  $\varphi = \pi$ , the splitting is

$$\varepsilon = \Delta T \sqrt{z} \tag{5.7}$$

We note that equation (5.7) is the lowest-order estimate of the avoided crossing in the small parameter  $\sqrt{z}$ . The relatively high value  $\sqrt{z} \approx 0.52$  of device 1 may explain the discrepancy between this simple estimate and the observed value, which is captured by the full model, see Fig. 5.12. Finally, we note that the expression (5.6) was also derived in Ref. [20] starting from the full model (see next section). In particular, the quantity  $\Omega_x(\varphi)$  in Ref. [20] is equal to  $\sqrt{z} g(\varphi)$ .

#### **5.10.2.** HAMILTONIAN DESCRIPTION OF THE HYBRID SQUID

We now describe the theoretical model of the hybrid SQUID that was used to fit the experimental data. Our model is based on Refs. [21] and [22]. The Hamiltonian of the model is equation (5.3) repeated here for convenience:

$$\hat{H} = E_C \hat{N}^2 + E_J (1 - \cos\hat{\delta}) + \hat{H}_{ABS}(\varphi - \hat{\delta}), \qquad (5.8)$$

with  $[\hat{\delta}, \hat{N}] = i$ . The Hamiltonian of the weak link is [21]

$$\hat{H}_{\text{ABS}}(\phi) = \Delta \hat{U}(\phi) \left[ \cos(\phi/2) \,\hat{\sigma}_3 + \sqrt{1 - T} \sin(\phi/2) \,\hat{\sigma}_2 \right] \hat{U}^{\dagger}(\phi) \,, \tag{5.9}$$

with  $\hat{U}(\phi) = \exp(-i\sqrt{1-T}\hat{\sigma}_1\phi/4)$ . Here  $\hat{\sigma}_2$  and  $\hat{\sigma}_3$  are two Pauli matrices which act on a space formed by the ground state of the weak link and an excited state with a pair of

quasiparticles in the weak link. By expanding the product above, the Hamiltonian can be put in the form  $\hat{H}_{ABS}(\phi) = V_2(\phi) \hat{\sigma}_2 + V_3(\phi) \hat{\sigma}_3$ . The two functions  $V_2$  and  $V_3$  are:

$$V_2(\phi) = \Delta \sqrt{1 - T} \sin\left(\phi/2\right) \cos\left(\sqrt{1 - T}\phi/2\right) - \Delta \cos\left(\phi/2\right) \sin\left(\sqrt{1 - T}\phi/2\right), \quad (5.10)$$

$$V_3(\phi) = \Delta \sqrt{1 - T} \sin\left(\frac{\phi}{2}\right) \sin\left(\sqrt{1 - T} \frac{\phi}{2}\right) + \Delta \cos\left(\frac{\phi}{2}\right) \cos\left(\sqrt{1 - T} \frac{\phi}{2}\right), \quad (5.11)$$

We introduce the ground  $(|g\rangle)$  and excited states  $(|e\rangle)$  of the weak link in the presence of an equilibrium phase difference,

$$\hat{H}_{ABS}(\phi) |g\rangle = -E_{ABS}(\phi) |g\rangle,$$
 (5.12a)

$$\hat{H}_{ABS}(\phi) |e\rangle = +E_{ABS}(\phi) |e\rangle$$
, (5.12b)

where  $E_{ABS}(\phi)$  is given in equation (5.1). In the basis  $|\pm\rangle$  of eigenstates of  $\hat{\sigma}_3$ ,  $\hat{\sigma}_3 |\pm\rangle = \pm |\pm\rangle$ , they are given by

$$|g\rangle = c_{g+}(\phi) |+\rangle + c_{g-}(\phi) |-\rangle, \qquad (5.13a)$$

$$|e\rangle = c_{e+}(\phi) |+\rangle + c_{e-}(\phi) |-\rangle , \qquad (5.13b)$$

with the coefficients

$$c_{g+}(\phi) = i \frac{E_A(\phi) - V_3(\phi)}{\sqrt{2E_A(\phi)[E_A(\phi) - V_3(\phi)]}}, \qquad c_{g-}(\phi) = \frac{V_2(\phi)}{\sqrt{2E_A(\phi)[E_A(\phi) - V_3(\phi)]}}, \quad (5.14a)$$

$$c_{e+}(\phi) = -i \frac{E_A(\phi) + V_3(\phi)}{\sqrt{2E_A(\phi)[E_A(\phi) - V_3(\phi)]}}, \quad c_{e-}(\phi) = \frac{V_2(\phi)}{\sqrt{2E_A(\phi)[E_A(\phi) + V_3(\phi)]}}.$$
 (5.14b)

The coefficients are normalized:

$$|c_{g+}(\phi)|^{2} + |c_{g-}(\phi)|^{2} = |c_{e+}(\phi)|^{2} + |c_{e-}(\phi)|^{2} = 1.$$
(5.15)

To find the resonant frequencies of the hybrid SQUID, we solve the eigenvalue problem  $\hat{H}|\Psi\rangle = E|\Psi\rangle$  numerically. We adopt the basis  $|\delta, \pm\rangle \equiv |\delta\rangle \otimes |\pm\rangle$  for the joint eigenstates of the  $\hat{\delta}$  and  $\hat{\sigma}_3$  operators:  $\hat{\delta}\hat{\sigma}_3 |\delta, \pm\rangle = (\hat{\delta} |\delta\rangle) \otimes (\hat{\sigma}_3 |\pm\rangle) = \pm \delta |\delta, \pm\rangle$ . For the numerical solution, we use a truncated Hilbert space where the phase interval  $[-\pi, \pi)$  is restricted to *M* discrete points, with lattice spacing  $2\pi/M$ . A complete basis of the truncated Hilbert space is given by the 2M vectors  $|\delta_k\rangle \otimes |\pm\rangle$  with  $\delta_k = 2\pi k/M$  ( $k = 0, \pm 1, \pm 2, ..., \pm (M-$ 1)/2), and  $|\pm\rangle$  the eigenvector of  $\hat{\sigma}_3$ . The Hamiltonian is thus represented as a  $2M \times 2M$ matrix in this basis and diagonalized numerically. We choose the parameter *M* large enough to guarantee convergence of the eigenvalues.

Once the spectrum is known, we use the transition frequencies from the ground state,  $\omega_n = E_n - E_{GS}$ , to do a least-square fit to the experimental data. The details of the numerical procedure are listed in the Jupyter notebooks available at [42].

Once an eigenstate  $|\Psi\rangle$  is determined numerically, we represent its two-component wavefunction in the basis of the weak link eigenstates  $\{|g\rangle, |e\rangle\}$  from equation (5.13), evaluated at  $\phi = \varphi$ :

$$|\Psi\rangle = \sum_{\delta} \sum_{\sigma=g,e} \Psi(\delta,\sigma) |\delta,\sigma\rangle , \ \Psi(\delta,\sigma) = \langle \delta,\sigma \rangle \Psi,$$
 (5.16)



Figure 5.14: **Equilibrium phase drop**  $\delta$  **across the tunnel junction.** The black line is given by equation (5.20), the red line by the numerical solution of equation (5.19). In both cases, we use the same circuit parameters as in Fig. 5.3(a):  $\Delta = 122 \,\mu$ eV, T = 0.57,  $E_I = 165 \,\mu$ eV.

where

$$|\delta,\sigma\rangle = |\delta\rangle \otimes (c_{\sigma+}(\varphi)|+\rangle + c_{\sigma-}(\varphi)|-\rangle).$$
(5.17)

The probability densities  $|\Psi(\delta, \sigma)|^2$  plotted in Fig. 5.3(b) and Fig. 5.12(b) allow us to evaluate at a glance whether the eigenstate  $|\Psi\rangle$  has a large overlap with the excited state  $\sigma = |e\rangle$  of the (decoupled) weak link.

Finally, in Fig. 5.3(c) we show the numerical prediction for the visibility of the ABS transition as a function of the phase bias,  $\phi$ . The visibility is determined by the absolute square of current operator matrix element  $\langle GS | \hat{J}(\varphi) | \Psi \rangle$  between the ground state  $|GS\rangle$  and the excited state  $|\Psi\rangle$  of  $\hat{H}$  corresponding to the ABS transition. The current operator is [22]

$$\hat{J}(\varphi) = E_J \sin(\hat{\delta}) + \frac{\partial H_{\text{ABS}}(\varphi - \hat{\delta})}{\partial \hat{\delta}}.$$
(5.18)

#### **5.10.3.** EQUILIBRIUM PHASE DROP

We have often assumed that the equilibrium phase drop across the weak link,  $\phi$ , is close to the total applied phase,  $\phi \approx \phi$ . Here, we verify this assumption by calculating the equilibrium phase drop of the hybrid SQUID model we presented in the previous section.

Since  $\phi = \varphi - \delta$ , (see equation (5.8)), it is sufficient to show that the equilibrium phase drop  $\delta \equiv \langle GS | \hat{\delta} | GS \rangle$  across the tunnel junction is small.  $\delta$  is given by the position where the ground state Josephson energy of equation (5.8) is minimal for  $E_C = 0$ . From this condition, after taking a derivative of the Josephson energy, we obtain the following tran-

scendental equation for  $\delta$ :

$$E_J \sin(\delta) + \frac{\Delta T}{4} \frac{\sin(\delta - \varphi)}{\sqrt{1 - T \sin^2[(\varphi - \delta)/2]}} = 0.$$
(5.19)

We note that the above expression defines a zero net current through the hybrid SQUID with the two arms hosting the same supercurrent. For  $E_J \gg \Delta T/4$ , a good approximate solution is given by

$$\delta \approx \frac{\Delta T}{4E_J} \frac{\sin(\varphi)}{\sqrt{1 - T\sin^2(\varphi/2)}}.$$
(5.20)

up to quadratic corrections in  $(\Delta T/E_J)$ . In Fig. 5.14 we show that for the parameters used in Fig. 5.3(a), this approximate solution is very close to the exact, numerical one. Both exhibit a sinusoidal behavior with a maximum  $\delta \approx 0.12$  at  $\varphi \approx \pi/2$ . This confirms that the phase drop across the weak link,  $\phi = \varphi - \delta$ , remains very close to the applied phase  $\varphi$  everywhere. In particular,  $\phi$  is exactly equal to  $\varphi$  at  $\varphi = n\pi$ , where *n* is integer.

### **5.10.4.** ANDREEV BOUND STATES IN A PROXIMITIZED RASHBA NANOWIRE IN A PARALLEL MAGNETIC FIELD

In this Section, we introduce the model used to describe the behaviour of ABS as a function of the magnetic field *B*. We start from the standard Bogoliubov-de Gennes (BdG) Hamiltonian of a Rashba quantum wire with proximitized *s*-wave superconductivity and an external Zeeman field [10, 11]:

$$H_{\rm BdG} = -\left(\frac{\partial_x^2}{2m} - E_F\right)\tau_z - i\,\alpha\,\partial_x\,s_z\,\tau_z + E_Z\,s_x + \Delta\,e^{i\phi\theta(x)\,\tau_z}\,\tau_x + V\,\delta(x)\,\tau_z\,. \tag{5.21}$$

Here, the two sets of Pauli matrices  $\tau_{x,y,z}$  and  $s_{x,y,z}$  act in the Nambu and spin spaces, respectively;  $m = 0.023 m_e$  is the effective mass in InAs [43],  $\alpha$  is the Rashba spin-orbit coupling strength which defines  $E_{SO} = m\alpha^2/2$ .  $E_Z = \frac{1}{2}g\mu_B B$  is the Zeeman energy,  $\Delta$  is the proximity induced gap and  $\theta$  is the Heaviside step function. The Fermi level  $E_F$  is measured from the middle of the Zeeman gap in the normal state band dispersion, see Fig. 5.16. Note that starting with equation (5.21) we set  $\hbar = 1$ . The superconducting phase difference between the left lead (x < 0) and the right lead (x > 0) is denoted by  $\phi$ . The last term of equation (5.21) models a short-range scatterer at x = 0, accounting for the finite channel transmission.

We seek bound state solutions of the the BdG equations,

$$H_{\rm BdG}\Psi(x) = E\Psi(x), \qquad (5.22)$$

at energies  $|E| < \Delta$ . We will consider in particular two opposite regimes: (a)  $E_F \gg E_{SO}, E_Z, \Delta$  and (b)  $E_F = 0$ , see the two insets in the corresponding panels of Fig. 5.16. In order to find bound state solutions we proceed as follows:

1. We linearize the BdG equations for the homogeneous system ( $V = 0, \phi = 0$ ) around  $E = E_F$ . In this way, we obtain two effective low-energy Hamiltonians,  $H_{\text{eff}}^{(a)}$  and

 $H_{\rm eff}^{(b)}$ , which are linear in the spatial derivative. They can be written as:

$$H_{\rm eff}^{(a)} = -i v \partial_x \tau_z \sigma_z - v q_0 \tau_z \rho_z + \frac{\Delta \alpha k_F}{v q_0} \tau_x \sigma_z + \frac{\Delta E_Z}{v q_0} \tau_y \rho_y, \qquad (5.23a)$$

$$H_{\text{eff}}^{(b)} = -i\alpha \partial_x \tau_z \sigma_z + \Delta \tau_x + \frac{1}{2} E_Z \sigma_z (1 - \rho_z).$$
(5.23b)

We now have three sets of Pauli matrices:  $\tau_{x,y,z}$  (Nambu space),  $\rho_{x,y,z}$  [distinguishing the inner/outer propagating modes, and replacing the spin matrices  $s_{x,y,z}$  of equation (5.21)], and  $\sigma_{x,y,z}$  (distinguishing left- and right-moving modes, and not to be confused with the  $\sigma$  matrices used in the previous Section). For regime (a), we have also introduced the Fermi momentum  $k_F = \sqrt{2mE_F}$ , the Fermi velocity  $v = k_F/m$  and the energy difference  $vq_0 = \sqrt{\alpha^2 k_F^2 + E_Z^2}$  between the two helical bands at the Fermi momentum. Note that, in the regime (b) where  $E_F = 0$ , the linearization requires  $E_{SO} \gg \Delta$ ,  $E_Z$ , so it corresponds to the limit of strong spin-orbit coupling.

- 2. Using equation (5.21), we compute the transfer matrix  $\mathcal{T}$  of the junction in the normal state ( $\Delta = 0$ ), at energy  $E = E_F$ . The transfer matrix gives a linear relation between the plane-wave coefficients of the general solution on the left and right hand sides of the weak link. In computing  $\mathcal{T}$ , we neglect all terms  $\propto E_F^{-1}$  in regime (a). In regime (b), the transfer matrix is computed for  $E_Z = 0$ , since the effect of magnetic field on scattering can be neglected to due the small dwell time in the short junction. At  $E_Z = 0$ , the transfer matrix depends on the single real parameter T, the transmission probability of the junction. The latter is given by  $T = 4k_F^2/(4k_F^2 + V^2)$  in regime (a), and  $T = 1/(1 + V^2/\alpha^2)$  in regime (b).
- 3. Using the transfer matrix  $\mathcal{T}$  as the boundary condition at x = 0 for the linearized BdG equations, we obtain the following bound state equation for *E*:

$$\det\left[1-G(E)\,\tau_z\,\sigma_z\left(e^{-i\phi\,\tau_z/2}\,\mathcal{F}-1\right)\right]=0\,,\tag{5.24}$$

where G(E) is the integrated Green's function,

$$G(E) = \nu \int \frac{\mathrm{d}q}{2\pi i} \, e^{-iq \cdot 0} \left[ H_{\mathrm{eff}}(q) - E \right]^{-1} \,, \tag{5.25}$$

and  $H_{\text{eff}}(q)$  is the Fourier transform of either of the linearized Hamiltonians of equation (5.23). [In regime (b), v must be replaced by  $\alpha$  in the expression for G(E)]. In deriving the bound state equation, we have neglected the energy dependence of the transfer matrix, which is appropriate in the short junction limit. In regime (b), this also requires that the length of the junction is shorter than  $\alpha/E_Z$ , so that we can neglect resonant effects associated with normal-state quasi-bound states in the Zeeman gap, which would lead to a strong energy dependence of the transfer matrix of the ABS derived in Ref. [3], except that it is formulated in terms of the transfer matrix of the weak link, rather than its scattering matrix. Unlike its counterpart, equation (5.24) incorporates the effect of the magnetic field in the superconducting leads. It is

thus appropriate to study the effect of a magnetic field on the ABS in the limit of uniform penetration of the field in the superconductor.

4. After performing the integral for G(E), the roots of equation (5.24) can be determined numerically. For the two regimes, this leads to the typical behavior of the ABS shown in Fig. 5.16 against the experimental data. We find a better agreement with the experimental data for regime (a).

From G(E), we can also compute the proximity-induced gap of the continuous spectrum  $\Delta(B)$ :  $\Delta(B)$  is the minimum value of *E* such that the poles of G(E) touch the real axis in the complex plane [of course,  $\Delta(B)$  can also be found by minimizing the dispersion relation obtained by diagonalizing equation (5.23) in momentum space]. In regime (a), the relevant spectral gap is always at the finite momentum, so the behaviour of  $\Delta(B)$  depends on the strength of the spin-orbit coupling, as shown in Fig. 5.15. Two features are evident from the figure.

First, with increasing spin-orbit coupling, the linear behaviour  $\Delta(0) - \Delta(B) \propto B$  changes to to a quadratic suppression  $\Delta(0) - \Delta(B) \propto B^2$  for small *B*. This is due to the vanishing first-order matrix elements of the Zeeman interaction, due to the removal of the spin degeneracy of finite-momentum states by the spin-orbit interaction. Secondly, the proximity-induced gap  $\Delta(B)$  never closes – as long as the superconductivity in the aluminium shell is present – because spin-orbit interaction competes with the Zeeman effect and prevents the complete spin polarization of the electrons. These two facts explain the behaviour of  $\Delta(B)$  shown in Fig. 5.4(b). In regime (b) with  $E_F = 0$ , which is extensively discussed in the literature of Majorana bound states,  $\Delta(0) - \Delta(B) \propto B$  due to the Zeeman-induced suppression of the gap for states at zero momentum (where spin-orbit is not effective).

An in-depth theoretical study of equation (5.24), including a detailed analysis of its roots at finite magnetic fields and the code used in the numerical solution, is in preparation. It will also be interesting to extend the current model beyond the linearization to allow the calculation of the spectrum at arbitrary values of  $E_F$ .



Figure 5.15: The effect of the spin-orbit interaction and Zeeman field on the induced superconducting gap. The lack of spin-orbit interaction leads to a linear decrease of  $\Delta(B)$  (black line), which becomes parabolic in the limit of  $\sqrt{E_{\text{SO}}E_F} \gg E_Z = \frac{1}{2}g\mu_B B$  (blue and green lines). The green line corresponds to the best fit to the experimental data shown.

#### ORBITAL FIELD

Because a quadratic suppression of  $\Delta(B)$  and the ABS energies may also be due to the orbital effect of the magnetic field, without invoking spin-orbit interaction, it is important to compare the data with this scenario. In a simple model which includes orbital and Zeeman effect, the field-dependence of the Andreev bound states may be written down as follows:

$$E_{\text{ABS},\pm}^{(\text{orb})}(\phi,B) = \Delta(1-B^2/B_*^2)\sqrt{1-T\sin^2(\phi/2)} \pm (1/2)g\mu_B B.$$
(5.26)

Here,  $B_* \sim \Phi_0 / A$  is the magnetic field scale which governs the suppression of the proximityinduced gap due to the orbital field, A is the cross-section of the nanowire and  $\Phi_0 = h/2e$ . In writing equation (5.26), we have neglected the effect of the orbital field on the scattering at the junction. This should be a good approximation as long as the junction is modeled by a  $\delta(x)$  potential with no dependence on the radial coordinate of the nanowire. Thus, essentially, the phase dependent part of the Andreev bound state energies can be obtained by replacing  $\Delta$  with  $\Delta(1 - B^2/B_*^2)$  in equation (5.1). In the absence of spin-orbit coupling, the Zeeman term enters additively in equation (5.26).

Using equation (5.26), we can perform a fit to the experimental data to determine the optimal value  $B_* = 400 \pm 2$  mT. Note that the fit is insensitive to the value of *g*, since *g* drops out from the sum  $E_{ABS,+}^{(orb)} + E_{ABS,-}^{(orb)}$ . However, equation (5.26) predicts the occurrence of a fermion parity-switch at a field  $B_{sw} < B_*$  given by the condition  $E_{ABS,-}^{(orb)}(\phi, B_{sw}) = 0$ . From this condition, and assuming the knowledge of both  $B_{sw}$  and  $B_*$ , the *g*-factor can then be deduced by inverting equation (5.26) at  $\phi = \pi$ ,

$$g = \frac{\Delta\sqrt{1-T}}{\mu_B B_{\rm sw}} \left(1 - B_{\rm sw}^2 / B_*^2\right)$$
(5.27)

The occurrence of this fermion-parity switch must be accompanied by a drastic disappearance of the ABS transition [27]. In the experiment, such disappearance can be excluded up to at least 300 mT. Therefore, by requiring that  $B_{sw} > 300$  mT and using the values quoted for all other parameters, we obtain an upper bound of *g*,

$$|g| < 5.08$$
 (5.28)

In Fig. 5.16(c) we plot the energy spectrum resulting from equation (5.26), which includes only the orbital and Zeeman effects. The black line in Fig. 5.16(c) represents the edge of the continuous spectrum for states with spin down,  $\Delta(B) = \Delta(1-B^2/B_*^2) - \frac{1}{2}g\mu_B B$ . In Fig. 5.16(c), we choose g = 5, close to the upper bound of equation (5.28). The inclusion of a weak spin-orbit coupling in the model would not affect the curvature of  $\Delta(B)$  and  $E_{ABS}^{(orb),\pm}(B)$  at small fields  $g\mu_B B \ll \Delta$  (see the blue curve in Fig. 5.15): the curvature would still be entirely dictated by the orbital effect. As mentioned, the Andreev level and the continuum cross at a value of the field  $B_{cross}$  such that  $E_{ABS}^{(orb),+}(B_{cross}) = \Delta(B_{cross})$ . For  $B_* = 400 \text{ mT}$  and g = 5, the crossing happens at  $B_{cross} \approx 150 \text{ mT}$ , see Fig. 5.16(c). However, the inclusion of a weak spin-orbit coupling prevents the level crossing, causing the Andreev level to bend below the edge of the continuum. As a consequence, the transition energy  $E_{tot}(B)$  decreases sharply at  $B > B_{cross}$ , in contrast with its behavior in the absence of spin-orbit coupling (compare the dashed and dotted lines in Fig. 5.6(c)). The

behavior of  $E_{tot}(B)$  in the presence of weak spin-orbit coupling clearly disagrees with the experimental data in the field range 150 mT < B < 300 mT.

The considerations above motivate the approximation where we attribute the quadratic suppression of  $E_{tot}(B)$  to the joint effect of spin-orbit and Zeeman couplings; the orbital effect does not play a dominant role in the observed dispersion.

#### FITS TO THE DATA

We have presented three different scenarios that can be used to interpret the magnetic field dependence of the ABS transition energies. We have fitted all three models to the entire data set available, consisting of a flux bias sweep of the ABS spectra at six different magnetic fields (B = 50, 75, 100, 150, 200 and 300 mT). For each flux bias at which it was visible, we have extracted the position of the ABS transition. For each value of B we attributed to all the data points an error bar corresponding to the half-width at half-maximum of the ABS peak at  $\varphi = \pi$ , neglecting for simplicity the flux variation of the width. The total dataset consisted of more than 300 datapoints. We then performed a least-square fit to the ABS transition energies predicted by the three different models. The results are illustrated in Fig. 5.16.



Figure 5.16: **The magnetic field dependence of ABS in high and low Fermi level regimes and for orbital magnetic field.** The top row shows the evolution of the spin-split Andreev levels  $E_{ABS}^{\pm}(B)$  (blue and red lines), the transition energy  $E_{tot}(B) = E_{ABS}^{+}(B) + E_{ABS}^{-}(B)$  (dashed line) at  $\varphi = \pi$  and the proxitimity-induced gap  $\Delta(B)$  versus the magnetic field *B*. The theoretical curves for  $E_{tot}$  are compared against the experimental data (black dots). The three panels correspond to the three different theoretical models described in the text: high Fermi level (a), low Fermi level (b), and a model without spin-orbit coupling but only orbital and Zeeman effects of the field (c). For the latter, the dotted line in panel (c) depicts the qualitative behavior of  $E_{tot}$  if a weak-spin orbit coupling is included in the model. The middle and the bottom row show the resulting dispersion of the Andreev levels as a function of  $\varphi$  for the three different theoretical models, displayed on top of the measured spectrum at B = 100 mT and B = 300 mT, respectively. In each row, all three columns feature the same experimental dataset. The global fit parameters for the left column are  $g = 14.7 \pm 0.6$  and  $\sqrt{E_{SO}E_F}/\Delta = 0.32 \pm 0.02$ . The middle column is evaluated with a single fit parameter  $g = 11.2 \pm 0.1$ . Note the lack of dispersion in panel (h), due to the merging of the Andreev bound states with the continuum, which causes all the lines to fall on top of each other. In the right column we use the best-fit value  $B_* = 400 \pm 2 \text{ mT}$  and g = 5, the latter imposed by the lower bound on the parity switching field  $B_{SW} > 300 \text{ mT}$ , where  $E_{ABS}^{-}(\pi) = 0$ .

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## 6

### MAGNETIC FIELD DEPENDENT QUASIPARTICLE DYNAMICS OF NANOWIRE SINGLE-COOPER-PAIR TRANSISTORS

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Parity control of superconducting islands hosting Majorana zero modes (MZMs) is required to operate topological qubits made from proximitized semiconductor nanowires. We test this control by studying parity effects in hybrid InAs-Al single-Cooper-pair transistors (SCPTs) to evaluate the feasibility of this material system. In particular, we investigate the gate-charge modulation of the supercurrent and observe a consistent 2*e*periodic pattern indicating a general lack of low-energy subgap states in these nanowires at zero magnetic field. In a parallel magnetic field, an even-odd pattern develops with a gate-charge spacing that oscillates as a function of field demonstrating that the modulation pattern is sensitive to the presence of a single bound state. In addition, we find that the parity lifetime of the SCPT decreases exponentially with magnetic field as the bound state approaches zero energy. Our work shows that aluminium is the preferred superconductor for future topological qubit experiments and highlights the important role that quasiparticle traps and superconducting gap engineering would play in these qubits. Moreover, we demonstrate a new means by which bound states can be detected in devices with superconducting leads.

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# 6.1. INTRODUCTION

The interplay of charging energy  $E_C$  and the superconducting gap  $\Delta$  leads to the surprising result that the electrical transport in a mesoscopic superconducting island containing a macroscopic number of electrons is sensitive to the addition or removal of a single electron [1-4]. This parity effect has been extensively studied in Al-AlO<sub>x</sub> SCPTs by measurements of the 2*e*-periodic gate-charge modulation of the Coulomb peak spacing, the ground state charge, and the switching current [5–13]. In recent experiments, the presence of MZMs in hybrid semiconductor-superconductor nanowires was inferred from the field-induced 1e Coulomb blockade periodicity, illustrating the utility of this periodicity in understanding the low-energy spectrum of mesoscopic superconducting islands [14-21]. In contrast with these previous studies which utilized devices with normal metal leads, we investigate parity effects in gate-tuneable nanowire SCPTs which have superconducting leads by studying the junction gate, temperature, and parallel magnetic field dependence of the switching current modulation. These experiments not only give new insights into quasiparticle dynamics but also represent a first step towards implementing recent Majorana-based qubit proposals which require Josephson coupling to the leads to enable parity-to-charge conversion for MZM manipulation and readout [22-26].

The Hamiltonian of a SCPT consists of three terms:  $H = H_C + H_I + H_{BCS}$ . The Coulomb term,  $H_C = E_C (n - n_g)^2$ , stabilizes the excess charge n on the island which can be changed by varying the gate-charge  $n_g$ . The effective charging energy  $E_C = e^2/2C$  is given in terms of the electron charge *e* and a generalized capacitance *C* that takes into account the geometric capacitance and possible renormalization effects due to tunneling of quasiparticles [27–31]. The Josephson term for symmetric junctions  $H_I = -E_I \cos{(\phi/2)} \sum_n |n\rangle \langle n +$ 2| + h.c., with  $E_I$  the Josephson energy and  $\phi$  the superconducting phase difference across the island, couples adjacent, equal-parity states and results in energy level anticrossings when states with the same parity are degenerate. The third term describes the spectrum of the gapped BCS quasiparticles resulting in an energy offset  $\Delta$  for the odd ground state due to an unpaired electron in the superconductor. Figure 6.1(a) shows the resulting band structure of a SCPT. The corresponding gate-charge modulation of the critical current is shown in Fig. 6.1(b). We denote the amplitude of the (even) ground state charge dispersion  $E_{gs}(n_g)$  with  $\delta E_{eo} = E_{gs}(n_g = 1) - E_{gs}(n_g = 0)$ . When  $\Delta > \delta E_{eo}$ the ground state is always even. Consequently, the switching current modulation will be 2e-periodic at T = 0 in this simple model.

Quasiparticle poisoning, however, affects this 2*e*-periodic modulation. Previous studies have illustrated three important timescales, namely the poisoning rate  $\Gamma_{in}$  at which quasiparticles in the lead tunnel to the island, the non-equilibrium unpoisoning rate  $\Gamma_{out}^{neq}$  at which non-equilibrium quasiparticles on the island tunnel out to the leads, and the relaxation rate  $1/\tau$  at which non-equilibrium quasiparticles on the island relax to the gap edge or subgap states [32–35]. While the relaxation is important for the quasiparticle dynamics, the thermodynamics of the system can be described by equilibrium poisoning rates  $\Gamma_{in}$  and  $\Gamma_{out}$  alone; therefore, we leave the implications of relaxation in our devices to the discussion section below. The ratio  $\Gamma_{in}/\Gamma_{out}$  gives the relative occupation between the even and odd parity states in equilibrium  $p_{odd}/p_{even}$ . If  $\Gamma_{in}/\Gamma_{out} \approx 1$  as is expected to occur at high temperature, the switching current modulation deviates from 2*e* periodicity and exhibits a 1*e* periodicity instead.



Figure 6.1: **Theoretical background and device layout.** (a) The band structure of a SCPT as a function of gate-charge  $n_g$  for  $\phi = 0$ ,  $\Delta/E_C = 1.5$ , and  $E_J/E_C = 0.25$ . The charge dispersion of the odd parity branch (in red) is displaced from the even parity branch (in blue) by the superconducting gap  $\Delta$  and  $\delta n_g = 1$ . The amplitude of the ground state charge dispersion is denoted by  $\delta E_{eo}$  (b) The corresponding critical current modulation as a function of  $n_g$ . (c) A false-coloured scanning electron micrograph of a nanowire SCPT. The etched regions in the Al shell define the junctions and the island. By applying voltages to the electrostatic gates, we can tune the chemical potential (with  $V_{BG}$ ), the junction transparency (with  $V_{JG1,2}$ ) and the charge occupation of the island (with  $V_{PG}$ ). (d) 3-dimensional device schematic. The nanowire is deterministically placed on top of a SiO<sub>2</sub>/Si++ substrate. It is then contacted by a stack of NbTiN and Ti/Au 1  $\mu$ m away from the etched regions. Finally, the local gates are deposited. The arrow indicates the direction of the magnetic field for the data presented in Fig. 6.4.

Figure 6.1(c) presents a scanning electron micrograph (SEM) of one of our SCPTs, and a 3-dimensional schematic of the device is shown in Fig. 6.1(d). The SCPTs are fabricated from InAs nanowires covered with a thin aluminium shell on two of their facets. It has been shown that this material combination results in a hard, induced superconducting gap in the nanowire [36, 37]. The aluminium shell is etched in two regions along the nanowire in order to define the island together with the two Al-InAs-Al Josephson junctions. The wire is contacted 1  $\mu$ m away from each junction by NbTiN/Ti/Au contacts which are expected to act as quasiparticle traps due to the presence of normal metal and the large subgap density of states in NbTiN [38]. Previous studies have shown the effectiveness of such traps to reduce the quasiparticle density [7, 35, 39]. Voltages  $V_{JG1}$  and  $V_{JG2}$  applied to the side gates tune the transparency of the weak links while the plunger gate voltage  $V_{PG}$  tunes the chemical potential of the island, and the global backgate voltage  $V_{BG}$  tunes the chemical potential of the whole system. The SCPTs are mounted to the cold finger of a dilution refrigerator with a base temperature of 27 mK. Presently we report on the data obtained on a 500 nm long island device, additional data and discus-

sion can be found in section 6.5. Unless otherwise indicated, the presented data were obtained at 27 mK and at zero magnetic field.



Figure 6.2: **Gate dependence of the** 2*e*-**periodic switching current modulation. (a)** Charge stability diagram measured in the strongly Coulomb-blockaded regime with  $\Delta = 180 \ \mu\text{eV}$  and  $E_C^0 = 1.5 \ \text{meV}$ . **(b)** Histogram of the 2*e*-periodic switching current  $I_{sw}$  in the weakly Coulomb-blockaded regime, indicating that  $\Delta > \delta E_{eo}$ . At this gate setting,  $V_{JG1} = -4.1 \ \text{V}$  and  $V_{JG2} = -5.7 \ \text{V}$ ,  $R_N = 14.8 \ \text{k}\Omega$ . **(c)** Schematic representation of the current ramp (in red) used to obtain the  $I_{sw}$  histograms and the resulting voltage across the SCPT (in blue). The switching current  $I_{sw}$  is recorded when the voltage drop on the SCPT reaches a threshold value  $V_{th}$ . **(d)** Switching current histograms for varying normal state resistance. The normal state resistance is calculated as the average over the  $n_g$  range at high bias. Note the change of vertical scale for the two topmost panels. The peak height asymmetry seen for  $R_N = 8.9 \ k\Omega$  and  $R_N = 19.6 \ k\Omega$  is due to cross coupling between the junctions and  $V_{PG}$ .

# 6.2. RESULTS

**6.2.1.** COULOMB BLOCKADE AND SWITCHING CURRENT HISTOGRAMS We first tune the device into Coulomb blockade by increasing the heights of the barriers separating the island from the leads. The clear, regular Coulomb diamonds shown in Fig. 6.2(a) demonstrate the creation of a single, well-defined island. Moreover, a 1*e*-periodic conductance modulation appears when  $e|V_b| > 4\Delta$  and transport through the island is dominated by quasiparticles which enables us to identify the gate voltage periodicity corresponding to 1*e* [40]. The current at lower bias voltages is too small to resolve in the Coulomb blockade regime since it involves Cooper pair transport and is therefore higher order in the tunneling. Finally, we extract the superconducting gap  $\Delta = 180 \ \mu\text{eV}$  and the geometric charging energy  $E_C^0 = 1.5 \text{ meV}$  from the observed diamonds.

In order to generate a measurable supercurrent, we lower the barriers in order to increase  $E_I$  which simultaneously suppresses  $\delta E_{eo}$ . The switching current is recorded by triggering on the voltage step in the I - V curve as illustrated in Fig. 6.2(c); this is repeated N times for each  $n_g$  to gather statistics, typically N = 100 to 500. Figure 6.2(b) shows the resulting switching current histogram which is 2*e*-periodic, indicating that in this regime the charge dispersion has decreased at least an order of magnitude to the point that  $\delta E_{eo} < \Delta$ , consistent with the observed charging energy renormalization in a nanowire island with normal leads [16].

To establish that our observed 2*e* periodicity is robust, we investigate the gate-charge modulation for a wide range of gate settings, as is shown in Fig. 6.2(d). We characterize each gate setting by the normal state resistance of the device. Figure 6.2d shows that the modulation is observed for  $R_N$  ranging from 5.8 to 19.6 k $\Omega$ . At  $R_N = 5.8$  k $\Omega$ , the switching current was only modulated by 5%, indicating that the device is in the Josephson dominated regime where  $E_I > \delta E_{eo}$ .

The other devices behave similarly as can be seen in Fig. 6.5. Five out of the six measured SCPTs show a 2*e*-period modulation robust over different gate settings. The remaining SCPT (device 5) exhibits an even-odd pattern, indicating that  $\delta E_{eo} > \Delta$ . Nevertheless, the robustness of the 2*e*-signal across gate settings and devices suggests a general lack of low-energy subgap states inside the islands at zero field, consistent with the hard gap observed in bias spectroscopy experiments which locally probe the density of states [37, 42].

### **6.2.2.** TEMPERATURE DEPENDENCE AND MODELLING

To gain insight into the relevant poisoning mechanisms of the SCPT, we measure the temperature dependence of the 2*e*-periodic switching current modulation at  $R_N = 14.8$  $k\Omega$ . As can be seen in Fig. 6.3(a), we observe that the 2*e* periodicity persists up to  $T \approx 189$ mK at which point the oscillations develop local maxima at even  $n_g$  values and finally become fully 1*e*-periodic for  $T^* \approx 300$  mK. This is consistent with an expected level spacing of the Al shell  $\delta$  of a few mK when using the estimate for vanishing charge dispersion  $k_B T^* = \Delta / \ln(\Delta/\delta)$  [43]. For comparison to the histograms, Fig. 6.3(b) shows dV/dI data taken over the same temperature range, linecuts of individual I - V traces are shown in Fig. 6.6. At elevated temperatures the dV/dI characteristics show a similar behaviour as the histograms including the onset of local maxima at even  $n_g$ . This can be explained by a self-averaging that takes place in the overdamped regime due to a succession of multiple switching and retrapping events. Indeed, we note that for T > 189 mK, the dV/dItraces show negligible hysteresis, indicating that the SCPT is in the overdamped regime. At low temperatures the junction enters the underdamped regime where a single phase slip can drive the junction normal, which leads to increased fluctuations in the dV/dIdata at base temperature.

Our modelling of the dV/dI data, outlined in Section 6.5.2, focuses on the overdamped regime. We identify two limiting cases, depending on the ratio of the parity switching times controlled by  $1/\Gamma_{in}$ ,  $1/\Gamma_{out}$  and the response time of the SCPT given by



Figure 6.3: **Temperature dependence.** (a) Switching current modulation as a function of temperature. The experimental histograms shown in grayscale are overlaid by the theoretical fit to the average switching current  $\langle I_{sw} \rangle$  (red curves). Individual fits are for different values of  $\Delta$ ,  $E_J$ , and  $E_C$ . The average values for resulting the parameters are  $\Delta \approx 220 \ \mu eV$ ,  $E_J \approx 43 \ \mu eV$ , and  $E_C \approx 160 \ \mu eV$ . (b) dV/dI data for the same temperature range as in (a) obtained from numerical derivation of the I-V curves. The current bias is swept from negative to positive values, hence, the switching (retrapping) current at positive (negative) bias. At elevated temperatures the overdamped dV/dI data shows a similar behaviour as the histograms in (a) with local maxima appearing at even  $n_g$  at  $T \approx 189$  mK and a fully 1*e* periodic modulation at  $T^* \approx 300$  mK. At low temperatures the junction is in the underdamped regime as indicated by the asymmetric dV/dI and the increased fluctuations due the absence of self-averaging.

the Josephson time constant  $\tau_J = \hbar/2eI_c R_J$  [44], with  $R_J$  the effective shunt resistance of the device and  $I_c$  the critical current. For slow parity switches one expects a double peak structure in the dV/dI. In contrast we observe a parity-averaged single peak in the dV/dI which shows that at high temperatures the SCPT is in the fast parity switching regime  $\Gamma_{\rm in}$ ,  $\Gamma_{\rm out} \gg 1/\tau_J$ . At  $T \approx 189$  mK where the SCPT transitions into the overdamped regime,  $R_J \approx 180 \Omega$  and  $I_c \approx 3$  nA leading to  $\tau_J \approx 1$  ns in our experiment.

Given the fast (un)poisoning at high temperature, we model the observed switching currents as the weighted sum of the switching current of the even and the odd parity states, with the relative probabilities governed by the free energy difference of the two states. Our model includes the charging energy of the island, Josephson coupling of the island to the leads, and the entropic factor associated with bringing a quasiparticle into the island; see Section 6.6 for a more detailed discussion. We note that though the fast (un)poisoning is a necessary assumption to fit the data at high temperature T > 189 mK, at low temperatures the probability to find the system in the odd state becomes negligible, i.e.  $p_{\text{odd}}/p_{\text{even}} \propto \exp(-(\Delta - \delta E_{\text{eo}})/k_B T) \rightarrow 0$  for  $\Delta > \delta E_{\text{eo}}$ . Thus, for low temperatures the system is essentially only in the even state which yields the 2*e*-periodic histograms of Fig. 6.3(a(v)).

The fitting gives approximate values of the  $\Delta$ ,  $E_J$ , and  $E_C$ . These values have error bars of the order of half of their values due to the weak parameter dependence of the fitting function. The fitted value of the superconducting gap  $\Delta \approx 220 \ \mu$ m is, within its error bar, consistent with the value obtained from the Coulomb diamonds in Fig. 6.2(a). Similarly, the fitted  $E_J \approx 43 \ \mu$ m is consistent with the observed switching current. The fitted effective  $E_C \approx 160 \ \mu$ m, however, is smaller than  $E_C^0$  extracted from the Coulomb diamond data in Fig. 6.2(a). This indicates that, in the regime of open barriers,  $E_C$  is significantly renormalized by virtual quasiparticle tunneling processes relative to the geometric charging energy [27–31]. The set of consistent fit parameters, together with an excellent fit of the model to the observed switching current dependence on  $n_g$ , supports the validity of the model and the assumption of fast (un)poisoning at high temperatures. Similar fitting results for device 2 strengthen this conclusion, see Fig. 6.9.

### **6.2.3.** PARALLEL MAGNETIC FIELD DEPENDENCE

Next, we study the effect of a parallel magnetic field on the switching current modulation. In particular, we tune the gates such that  $R_N = 12.9 \text{ k}\Omega$  and  $I_{sw}$  shows a 2*e*-periodic modulation at zero field, as is shown in Fig. 6.4(b). The 2*e* periodicity implies that  $\Delta > \delta E_{eo}$  and thus that the ground state is always even. As a magnetic field is applied along the nanowire axis, the spinful, odd-charge states are split by the Zeeman energy, thereby reducing the minimal single-particle excitation energy  $E_0$  of the island. Here, we consider a bound state with energy  $E_0$ . This state is residing in the island since its energy is modulated by  $n_g$  [14]. The parity-dependence of the bound state energy suggests that its origin is superconductivity-related. Moreover, the effective g-factor of a bound state residing partially in the InAs nanowire may be larger than that of the states in the Al shell [45, 46]. This is why in Fig. 6.4(a) the bound state energy is detached from the quasiparticle continuum for finite magnetic fields. Interestingly, when the applied field is large enough so that  $E_0 < \delta E_{eo}$ , the parity of the ground state around  $n_g = \pm 1$  changes to odd. During the retrapping process of the switching current measurement the system tends to be reset to the ground state, indicated by the general lack of bimodal switching current distributions in our data. Hence, the corresponding parity-flip shows up as a dip in the switching current modulation around odd  $n_g$ , causing an even-odd pattern. Figures 6.4(c) and (d) show examples of this even-odd structure in the switching current modulation measured at 250 mT and 300 mT, respectively.

We investigate the field dependence of this even-odd pattern in more detail by defining the length in gate-charge over which the even (odd) state is stable as  $S_{\text{even}}$  ( $S_{\text{odd}}$ ). In Fig. 6.4(e) these spacings are tracked as a function of the magnetic field using both switching current histograms and I - V measurements, see Figs. 6.10 and 6.11 of Section 6.6. for the representative data. The even (odd) data points are obtained by averaging



Figure 6.4: **Parallel magnetic field dependence.** (a) The many body energy levels of the SCPT at finite magnetic field. The blue lines indicate the ground and first excited state of the even parity branch. The odd parity ground state is shown in red with the shaded red region emphasizing the quasiparticle continuum on the island. The green curves indicate the presence of a bound state on the island with energy  $E_0(B)$  leading to an even-odd structure of the ground state when  $E_0 < \delta E_{eo}$  (b-d) Switching current histograms at 0 mT, 250 mT, and 300 mT showing the field evolution of the even-odd structure. (e) The even and odd spacings as a function of the parallel magnetic field obtained from both histograms and I - V traces. The observed crossing at 420 mT and subsequent oscillation is attributed to the presence of an bound state that oscillates about zero energy as a function of magnetic field while the superconducting bulk on the island remains gapped. For the most part switching current histograms and I - V characteristics give the same spacings. At low fields the slow I - V measurements pick up rare poisoning events and thus do not recover full 2*e* periodicity (f) The even parity lifetime at  $n_g = 1$  as a function of the magnetic field. The solid line is a guide to the eye indicating an exponential dependence. The lower inset presents a typical dataset used for the extraction of  $\tau_{\text{even}}$ . The upper inset shows a schematic representation of the energy needed to add a single quasiparticle to different parts of the device.

over 2 (3) successive spacings. Earlier studies performed in metallic superconducting islands found a monotonous drop in  $S_{\text{even}}$  [7–9, 47]. In contrast, we find an oscillating behavior in the even and odd spacings with the first crossing at 420 mT. After the first crossing, the spacings oscillate around 1*e* with increasing oscillation amplitude. The crossings indicate a closing and reopening of the energy gap for single-particle excitations in the island. Therefore, we conclude that the oscillating pattern is caused by the field-induced zero energy crossings of a single bound state that is detached from the continuum as is illustrated in Fig. 6.4(a).

Similar to Fig. 6.3, the histograms and I - V characteristics mostly coincide. For small fields below 200 mT, however, the histograms indicate an even ground state, while the slower I - V traces display an even-odd pattern, see Fig. 6.4(e). This discrepancy

occurs because the slower I - V measurements are sensitive to rare trapping events of quasiparticles in the island [33]. The latter occur since even in the absence of subgap states the island acts as a metastable trap with energy  $\delta E_{eo}$  below the gap of the super-conducting lead around odd  $n_g$ . In rare cases the metastable state becomes occupied long enough by quasiparticles to cause switching to the resistive state.

In addition, we measure the parity lifetime of the SCPT in a parallel field by performing slow histogram measurements while fixing the gate-charge at  $n_g = 1$  so that the extracted lifetime corresponds to poisoning of the even state [11, 38]. For representative histograms see the lower inset of Fig. 6.4(f) and Fig. 6.12. At  $n_g = 1$ , we expect the worst-case scenario for poisoning since the energy difference between the even and odd state is maximal (i.e., favoring the odd state). We observe that this lifetime decreases exponentially with field between 225 and 300 mT, see Fig. 6.4(f). We are limited to this intermediate field range because the lifetime is too large to obtain useful statistics at lower fields and too small to be captured by the bandwidth of the measurement electronics at larger fields. Still, by extrapolating the lifetime to 415 mT where  $S_{\text{even}} = S_{\text{odd}} = 1e$ , one can estimate the parity lifetime when the bound state is at zero energy to be  $\approx 1$  ns.

# **6.3.** DISCUSSION

We begin by noting that the growth of the even-odd spacing oscillation as a function of field seen in Fig. 6.4(e) is reminiscent of one of the proposed signatures of overlapping Majorana zero modes [48]. However, this increasing oscillation amplitude was only observed in a narrow gate range in our device, as is illustrated in Fig. 6.13. This makes it difficult to map the amplitude of the first oscillation to a Majorana overlap, as was done in Ref. [15]. From our results we can only conclude that if this oscillation is indeed due to the presence of overlapping MZMs, the topological portion of the device parameter space is rather small. Nevertheless, mapping the even-odd peak spacing in this manner could be used in future experiments to signal the transition to the topological regime in devices with superconducting leads such as the ones proposed in Refs. [24–26]. This could be an attractive alternative to gap-edge spectroscopy [49–52] as a signature of the topological regime in these all-superconducting systems.

We also note that the splitting of the 2*e*-signal into an (oscillating) even-odd signal is not always observed. Measurements performed on device 4, which has a 3  $\mu$ m-long island, show a sharp transition of the 2*e*-signal to the 1*e*-signal at a parallel field of 100 mT, similar to the behaviour observed while increasing the temperature in device 1. This field evolution of the  $I_{sw}$  modulation indicates that the SCPT is in the fast (un)poisoning limit with  $\Gamma_{in}/\Gamma_{out} \approx 1$ , possibly caused by a field-induced softening of the superconducting gap in the island and/or leads.

To understand the exponential decrease of the even state lifetime with field seen in Fig 6.4(f), we model the system as an island connected to a gapped superconducting lead in contact with a normal metal quasiparticle trap as is shown in the upper inset of Fig. 6.4(f). In the field range where we measure the lifetime, the observed even-odd pattern indicates that the energy difference between the odd and even state at  $n_g = 1$  is always negative, as also depicted in Fig. 6.4(a) and the inset of Fig. 6.4(f). Therefore, at  $n_g = 1$  poisoning is only prevented by the quasiparticle filtering effect of the superconducting gap in the leads. Quasiparticles can cross this gap in two ways: by thermal excitation

to the gap edge or by tunneling through the gap. Both processes are exponentially suppressed by a factor that scales with the size of the gap in the leads  $\Delta_{\text{lead}}$ . However, quantitative estimates of the relative strength of the tunneling and thermal activation contributions require a microscopic knowledge of the device. Still, both processes lead to an exponential dependence of the lifetime with field since  $\Delta_{\text{lead}}(B) = \Delta_{\text{lead}}(0) - \frac{1}{2}g\mu_B B$ . In either case, the filtering effect should be enhanced by increasing the length of the superconducting leads as well as by increasing  $\Delta_{\text{lead}}$ . Since recent studies indicate that the size of the proximitized gap in semiconducting nanowires is gate-tunable [45, 46], we suggest enhancing this filtering effect by locally gating the leads of the SCPT. Additionally, the length of the leads could be varied in order to investigate the proximity effect of the traps on the gap in the leads [53, 54].

Next, for a Majorana-based qubit one is primarily concerned with poisoning events which change the state of the qubit - namely, poisoning of the MZMs [55]. If direct tunneling from the quasiparticle trap is the dominant poisoning mechanism, the subgap state is expected to be directly poisoned since it is the lowest energy state on the island. In this case, the measured  $\tau_{\text{even}}$  in Fig. 6.4(f) directly gives the bound state lifetime since the quasiparticle residence time in the subgap state is likely to be longer than the relevant switching timescale of the junction -  $\tau_I$  in the case of an overdamped junction and  $2\pi/\omega_p$  where  $\omega_p$  is the plasma frequency in the case of an underdamped junction. In the opposite case of thermally activated poisoning, quasiparticles are first transfered elastically from the superconducting lead to the continuum in the island before relaxing to the subgap state within a time  $\tau$  [33]. In this case, quasiparticles can escape from the island before they are detected if  $\Gamma_{out}^{neq}$  is faster than the SCPT response time. Note, however, that as long as quasiparticles can be detected faster than  $\tau$ , most of the poisoning events of the subgap states will be detected. The time  $\tau_{even}$  therefore again represents the parity lifetime of the subgap states while the overall parity of island might fluctuate faster. Our previous estimate of  $\tau_I \approx 1$  ns sets a lower bound on our poisoning detection bandwidth since the junction would switch even faster to the resistive state in the underdamped case which we observe at low temperature. Given that typical resonators in time-domain RF measurements have bandwidths of no more than a few 10's of MHz [12, 34, 35], switching current measurements are a promising alternative before Majorana poisoning times can be measured more directly via the coherence of MZM-based qubits.

Finally, with the design of future MZM-based qubits in mind it is worth comparing our results with those obtained with NbTiN islands [38]. Our observed gate-charge modulation of the switching current shows a robust 2e-periodic signal for a wide range of gate settings which indicates that there are no low-energy subgap states inside the SCPTs at zero magnetic field. This is in stark contrast to the case of NbTiN islands, where subgap states result in a 1*e*-periodic, bimodal switching current distribution. In that case, despite the large superconducting gap, the island parity is effectively randomized after each measurement when the island retraps after being flooded with quasiparticles.

# 6.4. CONCLUSION

We have investigated quasiparticle poisoning in hybrid InAs-Al SCPTs by measuring the gate charge modulation of the switching current as a function of temperature and mag-

netic field. In contrast to previous studies of NbTiN SCPTs, we observe a consistent 2eperiodic supercurrent at zero field despite having a similar gap in the island and leads. This highlights the fact that at zero field there are no subgap states in the island and places Al as the superconductor of choice for MZM qubit experiments despite its smaller gap and critical field relative to NbTiN. In addition, we have observed, for the first time, an oscillating pattern in the gate periodicity of the supercurrent due to the field-induced zero energy crossing of a bound state. This opens the door to using the switching current to identify MZMs in qubit devices with superconducting leads. This is a crucial proofof-principle demonstration as the superconducting leads are not compatible with the zero-bias peak measurements typically taken as evidence of MZMs. We have performed lifetime measurements on this subgap state and observed an exponential decay of the lifetime in magnetic field due to a collapsing filtering effect of the leads. This exponential decay highlights the importance of proper engineering of the superconducting gap via local gating and intentional quasiparticle traps to minimize the presence of quasiparticles in the leads in future topological qubits.

### CONTRIBUTIONS

J.v.V., A.P., A.G., and J.D.W. designed the experiment. J.N. and P.K. grew the nanowires. J.v.V., A.P., and J.D.W. fabricated the devices and performed the measurements. T.K., D.P., R.L., and J.v.V. performed the theoretical modelling. All authors contributed to analysing the data and writing the paper.

# **6.5.** Supplementary Information

## **6.5.1.** EXPERIMENTAL METHODS

### NANOWIRE GROWTH AND DEVICE FABRICATION

InAs nanowires are grown by molecular beam epitaxy via gold-catalyzed vapor-liquidsolid growth. A thin aluminium shell is then evaporated on two facets of the nanowire before breaking vacuum in order to form a pristine semiconductor-aluminium interface. The InAs-Al nanowires are deposited deterministically on a Si++ substrate covered with 285 nm thick thermal SiO<sub>2</sub> using a micro-manipulator. The Josephson junctions together with the island are created by etching the Al shell from the nanowire in two 70 nm wide windows using a 12 s Transene D dip at 48°C. The wire is contacted 1  $\mu$ m away from the junctions using an argon plasma etch at 100 W for 2 min and 45 s to remove the native oxide, followed by the *in situ* deposition of NbTi/NbTiN (5 nm/70 nm) and the *ex situ* deposition of Ti/Au (5 nm/35 nm). Finally, Ti/Au (10 nm/120 nm) side gates are deposited using evaporation. For all lithography steps electron beam lithography was used to pattern the resist (PMMA 950k A4 spin coated at 4000 rpm). For device 6, Ti/Au (5 nm/10 nm) local back gates and a 30 nm thick SiN<sub>x</sub> dielectric layer were deposited prior to the nanowire deposition.

### SWITCHING CURRENT HISTOGRAMS

The switching current histograms were measured using a Rigol DG4062 arbitrary waveform generator to supply the waveform of the ramp to an optically isolated current source which results in a time-dependent current bias of the device characterized by a constant current ramp rate dI/dt. The voltage across the SCPT is measured in a fourterminal configuration using a voltage amplifier that is optically isolated from the commercial electronics. A typical current bias waveform together with the resulting voltage are schematically depicted in Fig. 6.2(c). When the measured voltage crosses the preset voltage threshold  $V_{th}$ , the corresponding bias current is recorded using a custom sample-and-hold circuit and a Keithley 2000 digital multimeter. This reference voltage is tuned inside the voltage step that separates the supercurrent branch from the quasiparticle current branch so that the recorded current measures the switching current. This measurement is repeated *N* times to acquire the switching current histogram. The readout lines consist of Cu/Ni twisted pair cables, and are filtered using three stages of filtering: a  $\pi$  filter with a cutoff frequency of 100 MHz at room temperature, high frequency copper-powder filters at base temperature, and two-pole *RC* filters with a cutoff frequency of 50 kHz also at base temperature.

Device	L (µm)	Backgate layout	$t_{Al}$ (nm)	$E_C^0$ (meV)	$\Delta$ ( $\mu eV$ )	Cooldown
1	0.5	Global	5	1.5	0.18	2
2	1	Global	5	0.85	0.16	2
3	2	Global	5	0.44	0.22	2
4	3	Global	5	0.35	0.18	2
5	3	Global	5	No data	No data	2
6	1.2	Local	8	0.5	0.16	1

Table 6.1: Device overview. Parameters characterizing the devices are the length of the island L, the thickness of the aluminium shell  $t_{Al}$ , and the backgate layout. Moreover, the geometric charging energy  $E_C^0$  and superconducting gap  $\Delta$  are listed. These parameters are extracted from charge stability diagrams in the strongly Coulomb blockaded regime (data shown in Fig. 6.5). In addition, the cooldown at which the device was measured is specified.

### **6.5.2.** OVERDAMPED JUNCTION LIMIT

In this section, we discuss the I-V characteristics and switching dynamics of overdamped junctions in the presence of quasiparticles. For larger temperatures where the I-V characteristics depart from 2*e* periodicity the junction is typically in the overdamped regime which allows the following theory to capture the 2*e* to 1*e* crossover (see Fig. 6.3).

### OVERDAMPED RCSJ MODEL

We start with the standard RCSJ model of a junction with dissipative resistance  $R_J$ , capacitance C and Josphenson energy  $E_J = I_c \hbar/2e$ . The overdamped regime is reached once the damping rate  $\tau_R^{-1} = (R_J C)^{-1}$  exceeds the plasma frequency  $\omega_p = \sqrt{2eI_c/\hbar C}$ . The equation of motion then takes the form of the Langevin equation

$$\dot{\phi} + \sin\phi = \mathscr{I} + \sqrt{2\Gamma_T}\eta(t), \qquad (6.1)$$

where  $\phi$  is the phase differences across the junction,  $\mathscr{I} = I_{\text{bias}}/I_c$  is the current bias relative to the critical current, and time is measured relative to  $\tau_J = (2eI_cR_J/\hbar)^{-1} = \tau_R^{-1}\omega_p^{-2}$ . Fluctuations due to thermal noise  $\Gamma_T = k_B T/E_J$  are assumed to be short-time correlated  $\langle \eta(0)\eta(t) \rangle = \delta(t)$ . Note that in these units the renormalized voltage  $v = V/I_cR_J$  is given



Figure 6.5: Scanning electron micrographs, Coulomb blockade diamonds, and gate dependence of the 2*e*-periodic signal for device 2-6 (**a** – **e**). The  $E_C^0$  and  $\Delta$  extracted from the Coulomb blockade data are summarized in Table 6.1. Note that apart from device 5 which shows an even-odd pattern in the switching current modulation, all devices show a 2*e* modulation, illustrating that the 2*e* signal does not correspond to a fine-tuned gate setting. The gate settings corresponding to the measurements are specified in each subfigure. For device 2, 5, and 6 the backgate is fixed at  $V_{BC2} = -10$  V,  $V_{BC5} = -8.5$  V, and  $V_{BG6} = 0$ , respectively.

by  $v = \dot{\phi}$ . The Langevin form can be mapped to a Fokker-Planck equation

$$\partial_t p = \partial_\phi \left( [\partial_\phi u] p \right) + \Gamma_T \partial_\phi^2 p \tag{6.2}$$

in terms of the probability distribution  $p = p(\phi, t)$  and the potential  $u(\phi, \mathscr{I}) = -\cos\phi - \mathscr{I}\phi$ . Equation (6.2) is in the form of a continuity equation  $\partial_t p + \partial_\phi j_p = 0$  which defines the probability current

$$j_p = -[\partial_{\phi} u] p - \Gamma_T \partial_{\phi} p. \tag{6.3}$$



Figure 6.6: Linecuts of the I - V characteristics in the underdamped at 27 mK (a,b) and in the overdamped regime at 300 mK (c,d). The arrows in (b,d) indicate the sweep direction of the current bias. This data corresponds with the data shown in Fig. 6.3.

The I-V characteristics can be obtained from considering the stationary case  $\partial_t p = 0$  of constant current  $\partial_{\phi} j_p = 0$ . When the probability distribution is normalized with respect to the interval  $[0, 2\pi]$  using periodic boundary conditions in  $\phi$ , the current  $j_p$  is a measure for the rate at which the phase particle traverses the interval and is therefore related to the voltage  $v = 2\pi j_p$ . Solving equation (6.3) yields the I-V characteristics [56–58]:

$$\nu = 2\pi\Gamma_T \left( e^{\frac{2\pi\mathscr{I}}{\Gamma_T}} - 1 \right) \left\{ \int_0^{2\pi} d\phi \left[ \int_0^{\phi} d\phi' + e^{\frac{2\pi\mathscr{I}}{\Gamma_T}} \int_{\phi}^{2\pi} d\phi' \right] e^{\frac{u(\phi',\mathscr{I}) - u(\phi,\mathscr{I})}{\Gamma_T}} \right\}^{-1}.$$
 (6.4)

### QUASIPARTICLE DYNAMICS IN THE OVERDAMPED RCSJ MODEL

The overdamped RCSJ model can be readily extended to include quasiparticle dynamics by keeping track of the even  $\alpha = 0$  and odd  $\alpha = 1$  state of the island,

$$\partial_t \begin{pmatrix} p_0 \\ p_1 \end{pmatrix} + \partial_\phi \begin{pmatrix} j_0 \\ j_1 \end{pmatrix} = \begin{pmatrix} -\gamma_{\rm in} p_0 + \gamma_{\rm out} p_1 \\ +\gamma_{\rm in} p_0 - \gamma_{\rm out} p_1 \end{pmatrix}, \tag{6.5}$$

where  $j_{\alpha} = -[\partial_{\phi} u_{\alpha}]p - \Gamma_T \partial_{\phi} p_{\alpha}$  in terms of the parity dependent potential  $u_{\alpha}(\phi, \mathscr{I}) = -a_{\alpha} \cos \phi - \mathscr{I} \phi$  with  $a_0 = 1$  and  $a_1 = E_{J1}/E_{J0}$ . Here  $\gamma_{in/out} = \Gamma_{in/out} \tau_J$  are dimensionless rates of switching the system from even to odd or vice versa. To derive the I-V characteristics we again look at the steady state  $\partial_t p_0 = \partial_t p_1 = 0$ . We then observe that  $\partial_{\phi}(j_0 + j_1) = 0$  and, similar to the single state case, the constant is fixed by the voltage  $j_0 + j_1 = \nu/2\pi$ . The normalization condition of the probabilities is now in terms of  $\int d\phi(p_0 + p_1) = \bar{p}_0 + \bar{p}_1 = 1$ . We now consider the two limiting cases of slow and fast poisoning.

### SLOW POISONING LIMIT

In the slow poisoning limit corresponding to  $\gamma_{in}$ ,  $\gamma_{out} \ll 1$  the right hand side of equation (6.5) can be neglected which, in the steady state, sets both  $j_0$  and  $j_1$  independently to a constant and therefore recovers the standard RCSJ model. The result from section (6.5.2) can essentially be copied with voltages  $v_{\alpha}$  defined by equation (6.4) using the potential  $u_{\alpha}$  of the corresponding parity state. To calculate the total voltage drop one needs to take into account that the normalization of the probabilities  $\int_0^{2\pi} d\phi p_{0/1} = \bar{p}_{0/1} = \gamma_{out/in}/(\gamma_{out}+\gamma_{in})$  enters equation (6.4) so that the voltages  $v_{\alpha}$  are correctly weighted. This yields:

$$v = v_0 \bar{p}_0 + v_1 \bar{p}_1. \tag{6.6}$$

We therefore conclude that in the slow poisoning case the I-V characteristic is the weighted sum of the I-V characteristics of each parity where the weight is given by the average occupation. For significantly different even and odd switching currents we would therefore expect a double-kink in the I-V characteristics which is most clearly visible in the derivative dV/dI which subsequently turns into a double-peak (see Fig. 6.7).



Figure 6.7: Analytic solution of the slow and fast poisoning limit. Voltage drop *V* in terms of the bias current  $I_{\text{bias}}$  (left) and  $dV/dI_{\text{bias}}$  (right). The slow poisoning case (blue) is showing a double-kink behaviour while fast poisoning (yellow) leads to a single-kink at the averaged switching current. Parameters are  $I_c^{(0)} = 2.5$ nA,  $I_c^{(1)} = 7.5$ nA, and  $\Gamma_T = 0.05$ .

### FAST POISONING LIMIT

In the fast poisoning limit corresponding to  $\gamma_{\text{in}}, \gamma_{\text{out}} \gg 1$  the probabilities  $p_{\alpha}(\phi)$  have to cancel the leading order of the right hand side of equation (6.5) [up to terms  $\mathcal{O}(1)$ ]. This allows to separate the fast quasiparticle dynamics, that lock the ratio of  $p_0(\phi)/p_1(\phi)$  to  $\bar{p}_0/\bar{p}_1$  for each  $\phi$ , from the  $\phi$ -dependence of the probability distribution, i.e. use the ansatz  $p_{\alpha}(\phi) = \bar{p}_{\alpha}p_+(\phi)$  with  $p_+(\phi) = p_0(\phi) + p_1(\phi)$ . We can then solve for  $p_+(\phi)$  by looking at the sum of both components of equation (6.5)

$$\frac{\nu}{2\pi} = j_1 + j_2 = -[\partial_{\phi} u_0] p_0 - [\partial_{\phi} u_1] p_1 - \Gamma_T \partial_{\phi} (p_0 + p_1)$$
$$= -[\partial_{\phi} (\bar{p}_0 u_0 + \bar{p}_1 u_1)] p_+ - \Gamma_T \partial_{\phi} p_+,$$

which yields the same type of equation to solve as for the standard RCSJ model with an effective potential  $u_{\text{eff}} = \bar{p}_0 u_0 + \bar{p}_1 u_1$ . The solution corresponds to a single kink that signals the onset of the resistive state that lies in between the double step kink solution of the slow poisoning case, see Fig. 6.7.

### GENERAL CASE

In the general case the differential equation (6.5) can be solved numerically. Figure 6.8 shows a grey scale plot of the dV/dI-I characteristics for fast/intermediate/slow poisoning with different in/out rate ratios. Comparing the different poisoning scenarios with the experimental data of Fig. 6.3 clearly shows that at high temperatures, where the  $\Gamma_{\rm in}/\Gamma_{\rm out}$  ratio is sizable, slow and intermediate poisoning is incompatible with the experimental data while fast poisoning yields an excellent agreement. Note that at the level of equation (6.5) the critical currents  $I_{\rm c}^{(\alpha)}(n_g)$  are input parameters. For simplicity, we used a parabolic dependence with a minimum and maximum of 2.5nA and 7.5nA to resemble the behaviour of Fig. 6.2(b).



Figure 6.8: Numerical solution of the dV/dI characteristics in different parameter regimes. The columns from left to right correspond to fast, intermediate and slow poisoning, respectively. The rows correspond to different ratios of  $\Gamma_{in}/\Gamma_{out}$  and are therefore a measure of the temperature. For concreteness the plots assume a fixed ratio corresponding to a weak charge dispersion.

# **6.6.** TEMPERATURE DEPENDENCE

In this section, we discuss the model used to fit the switching current histograms in Fig. 6.3 for varying temperature. In our model, we assume that the quasiparticle poisoning and unpoisoning rate is fast compared to the characteristic switching time of the junction  $\Gamma_{\rm in}$ ,  $\Gamma_{\rm out} \gg 1/\tau_J$ . Under this assumption, which is justified above in Section I, the critical current of the SCPT is given by the weighted average of the critical currents in the even and odd states with the weighting coefficients given by the free energy difference between the parity states of the SCPT. In the opposite regime when the switching rate of the junction is much larger than the poisoning or unpoisoning rate  $\Gamma_{\rm in} \ll 1/\tau_J$  or  $\Gamma_{\rm out} \ll 1/\tau_J$ , we expect a bimodal switching current distribution. This is not observed in the temperature dependence data. It is important to note that the fast poisoning-unpoisoning assumption may be broken at small temperatures, where the probability of odd state occupation is very small. Even though we do not see bimodal distribution of the switching current at low temperatures, this may be due to weak spectral density of the peak corresponding to the odd occupations.

Thus, we need to compute the critical currents in the two parity states as functions of the gate charge  $n_g$ , as well as the free energy difference between the two states. We start with the Hamiltonian of the system which consists of three terms:  $H_J + H_C + H_{BCS}$ . The Josephson term has the form

$$H_J = -E_{J1}\cos(\hat{\varphi}_i - \phi/2) - E_{J2}\cos(\hat{\varphi}_i + \phi/2), \tag{6.7}$$

where  $E_{J1,2}$  are the Josephson energies of the two junctions,  $\hat{\varphi}_i$  is the superconducting phase of the island, and  $\phi$  is the phase difference between the superconducting leads. This expression can be rewritten as

$$H_J = -(E_{J1} + E_{J2}) \sqrt{1 - \frac{4E_{J1}E_{J2}}{(E_{J1} + E_{J2})^2} \sin^2 \phi / 2\cos(\hat{\varphi}_i - \varphi_0)},$$
(6.8)

where  $\varphi_0$  is a  $E_{J1,2}$ -dependent constant. We notice that the operator  $e^{i\hat{\varphi}_i}$  changes the number of particles on the island by two, thus the Josephson Hamiltonian in the charge basis  $|n\rangle$  can be written as

$$H_{J} = -(E_{J1} + E_{J2})\sqrt{1 - \frac{4E_{J1}E_{J2}}{(E_{J1} + E_{J2})^{2}}\sin^{2}\phi/2}\sum_{n}|n\rangle\langle n+2| + \text{h. c.}$$
(6.9)

For the fitting procedure we fix moderately asymmetric junction with  $E_{J1}/E_{J2} = 2$ . Moreover, the Josephson energy is defined as  $H_J(\phi = 0) - H_J(\phi = \pi)$ .

Next, the Coulomb term of the Hamiltonian is given by

$$H_C = \sum_n E_C (n - n_g)^2 |n\rangle \langle n|, \qquad (6.10)$$

where  $E_C = e^2/2C$  with *e* the elementary electron charge and *C* the effective capacitance of the island taking into account the geometric capacitance and possible renormalization effects due to (virtual) tunnelling of quasiparticles. Finally, we approximate the energy to add an unpaired quasiparticle to the superconducting island as

$$H_{\rm BCS} \approx \sum_{n} \Delta \frac{1 - (-1)^n}{2} |n\rangle \langle n|, \qquad (6.11)$$

where  $\Delta$  is the superconducting gap on the island, and  $\frac{1-(-1)^n}{2}$  is the parity of the SCPT. The approximation corresponds to the limit of small typical excitation energies of quasiparticles relative to the gap edge  $\Delta$ . Note that while this approach captures the dominant contribution to the energetics, it is important to include the full quasiparticle dispersion for the entropic contribution discussed below.

Using the total Hamiltonian  $H = H_J + H_C + H_{BCS}$  in the even parity sector, we can find the even ground state energy of the SCPT as a function of the phase difference between the superconducting leads  $\phi$  and gate charge  $n_g$ ,  $E^{\text{even}}(\phi, n_g)$ . The odd ground state energy is found by shifting  $E^{\text{even}}(\phi, n_g)$  by  $n_g = 1$  and adding the superconducting gap;  $E^{\text{odd}}(\phi, n_g) = E^{\text{even}}(\phi, n_g + 1) + \Delta$ . The zero-temperature supercurrent is given by the derivative of the ground state energy as a function of phase difference

$$I_c^{\text{even}}(\phi, n_g) = \frac{e}{\hbar} \frac{\partial}{\partial \phi} E^{\text{even}}(\phi, n_g).$$
(6.12)

The finite-temperature supercurrent is given by the weighted sum of the even and odd ones with the assumption of the fast (un)poisoning, where the weighting factors  $p_{odd}$  and  $p_{even} = 1 - p_{even}$  are calculated as follows

$$p_{\rm odd}(T,\phi,n_g) = \frac{Z_{\rm odd}}{Z_{\rm even} + Z_{\rm odd}} = \frac{1}{1 + \exp(\Delta F(T,\phi,n_g)/k_B T)},$$
(6.13)

where  $Z_{\text{odd,even}}$  are the partition functions in the state with and without a quasiparticle on the middle island, respectively,  $\Delta F$  is the free energy difference between the parity states,  $k_B$  is the Boltzmann constant, and T is the temperature.

The free energy difference between the two parity states can be computed as follows

$$\Delta F(T,\phi,n_g) = \delta E(\phi,n_g) - T \ln \tanh z_i(T,\delta_i), \qquad (6.14)$$

$$z_i(T,\delta_i) = \frac{2\pi\sqrt{k_B T \Delta}}{\delta_i} \exp(-\Delta/k_B T).$$
(6.15)

Here  $\delta E(\phi, n_g) = E^{\text{even}}(\phi, n_g + 1) - E^{\text{even}}(\phi, n_g)$  is the difference in ground state energy between the odd and even parity sectors (not including  $\Delta$ ),

 $z_i$  is the partition function difference between the quasiparticle being inside the island at energy  $\Delta$  and in the lead at zero energy, and  $\delta_i$  is the level spacing inside the island. In this expression we assume that the quasiparticle is tunnelling from a gapless, large quasiparticle trap. This means the lead has negligible level spacing and negligible change of entropy due to the removal of one electron from the trap. We assume  $\delta_i = 5$  mK. This gives the following expression which we use to fit the data

$$I_{c}(T, n_{g}) = \max_{\phi} \left[ I_{c}^{\text{even}}(\phi, n_{g})(1 - p_{\text{odd}}(T, \phi, n_{g})) + I_{c}^{\text{even}}(\phi, n_{g} + 1) p_{\text{odd}}(T, \phi, n_{g}) \right].$$
(6.16)

Equation (6.16) fits the data nicely, as shown in Fig. 6.3(a). It is important to mention that the same fit assuming non-equilibrium quasiparticles in the lead instead of equilibrium quasiparticles in the trap does not fit the data, since even at lowest temperatures it would produce even-odd or purely 1e periodicity. We thus conclude that the dominating poisoning effect is via direct tunnelling of the quasiparticles from the normal lead.

6



Figure 6.9: Switching current modulation as a function of temperature for device 2 at  $R_N = 10.5 \text{ k}\Omega$ . The experimental histograms shown in grayscale are overlaid by the theoretical fit to the average switching current  $\langle I_{\text{SW}} \rangle$  (red curves). Individual fits are for different values of  $\Delta$ ,  $E_J$ , and  $E_C$ . The resulting values for the parameters are  $\Delta \approx 245 \ \mu\text{eV}$ ,  $E_C \approx 192 \ \mu\text{eV}$ , and  $E_J \approx 111 \ \mu\text{eV}$ .



Figure 6.10: Representative switching current histograms as a function of parallel magnetic field for device 1 at  $V_{BG} = -7.55$  V,  $V_{JG1} = V_{JG2} = -1.5$  V. The S<sub>even</sub> and S<sub>odd</sub> spacings reported in Fig. 6.4(e) are extracted from the average S<sub>even</sub> and S<sub>odd</sub> of these histograms. Note that the histogram at  $B_{\parallel} = 400$  mT is distorted around  $n_g = 1.5$  and  $n_g = 3.5$  due to false triggers.



Figure 6.11: Representative I-V characteristics as a function of parallel magnetic field for device 1 at  $V_{BG} = -7.55$  V,  $V_{JG1} = V_{JG2} = -1.5$  V. The S<sub>even</sub> and S<sub>odd</sub> spacings reported in Fig. 6.4(e) are extracted from the average S<sub>even</sub> and S<sub>odd</sub> of these I-V characteristics.



Figure 6.12: Slow switching current histograms at  $n_g = 1$  at representative values of the magnetic field taken on device 1 at  $V_{BG} = -7.55$  V,  $V_{JG1} = V_{JG2} = -1.5$ . The even state lifetime  $\tau_{\text{even}}$ , presented in Figure 4f of the main text, was obtained from the exponential tail of the histograms, using the following model:  $N = N_0 \exp\left(-\frac{I-I_0}{dI/dt \tau_{\text{even}}}\right)$ , where  $N_0$  is the number of counts at  $I_0$  and dI/dt is the current ramp rate. The extracted lifetimes are tabulated in Table 6.2.

$B_{\parallel}$ (mT)	dI/dt (nA/s)	$\tau_{\rm even}$ (ms)	Ν
225	2.5	105	3500
	10	80	5000
237	2.5	50	5000
	10	30	10000
250	10	4.8	10000
	50	5.6	10000
262	50	1.6	10000
	100	2.4	25000
275	100	0.78	10000
	500	0.87	10000
	1000	0.39	10000
287	500	0.1	10000
	1000	0.26	50000
	2000	0.22	50000
300	250	0.11	10000
	500	0.073	10000
	1000	0.05	50000

Table 6.2: Overview of the even state lifetimes at  $n_g = 1$  as a function of the parallel magnetic field  $B_{\parallel}$  taken on device 1, this data is presented in Fig. 6.4(f). In addition, the current ramp rate dI/dt and the sample size N used to construct the switching current histogram are presented



Figure 6.13: Parallel magnetic field dependence of the even and odd spacing in device 1 at  $V_{BG} = -7.65$  V,  $V_{JG1} = V_{JG2} = -1.5$  V. Although it is very close to the used gate setting for the data presented in Fig. 6.4, we believe this corresponds to a different charge configuration in the SCPT because the of hysteresis in the gate response. (a) I-V characteristic used for the construction of (b) for representative values of the parallel magnetic field. (b) Seven and S<sub>odd</sub> as a function of the parallel magnetic field. Seven (S<sub>odd</sub>) is obtained by averaging over 2 (3) successive spacings respectively. At this gate setting, the spacings also cross confirming the data presented in Fig. 6.4(e). However, the shape of the oscillation pattern is different from Fig. 6.4(e).



Figure 6.14: Parallel magnetic field dependence of the I-V characteristics of device 4. Instead of an even-odd pattern that develops as a function of field as was observed for device 1, the I-V characteristics develops a peak in the switching current at odd gate charge similar to the behavior that was observed as a function of temperature. This indicates that the SCPT is in the fast unpoisoning limit possibly caused by a field-induced softening of the gap.

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# 7

# BROADBAND MICROWAVE SPECTROSCOPY OF SEMICONDUCTOR NANOWIRE-BASED COOPER-PAIR TRANSISTORS

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The Cooper-pair transistor (CPT), a small superconducting island enclosed between two Josephson weak links, is the atomic building block of various superconducting quantum circuits. Utilizing gate-tunable semiconductor channels as weak links, the energy scale associated with the Josephson tunnelling can be changed with respect to the charging energy of the island, tuning the extent of its charge fluctuations. Here we directly demonstrate this control by mapping the energy level structure of a CPT made of an indium arsenide (InAs) nanowire with a superconducting aluminium (Al) shell. We extract the device parameters based on the exhaustive modelling of the quantum dynamics of the phase-biased nanowire CPT and directly measure the even-odd parity occupation ratio as a function of the device temperature, relevant for superconducting and prospective topological qubits.

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# 7.1. INTRODUCTION

The energy landscape of a Cooper-pair transistor (CPT), a mesoscopic superconducting island coupled to superconducting leads via two Josephson junctions, is determined by the interplay of the electrostatic addition energy of a single Cooper pair,  $E_C = (2e)^2/2C$  [1, 2] and the coherent tunnelling of Cooper pairs, characterized by the Josephson energy,  $E_J$  [3, 4].

The electronic transport through CPTs have mostly been studied for metallic superconducting islands enclosed between tunnel junctions by voltage bias spectroscopy [5–7], switching current measurements [8–11], microwave reflectometry [12, 13], and broadband microwave spectroscopy [14]. Recent material developments [15, 16] made it possible to investigate superconducting transport in semiconductor nanowire weak links, which lead to Andreev level quantum circuits [17–19] and gate-tunable superconducting quantum devices [20–23]. In addition, semiconductor nanowire (NW) CPTs have been fabricated and measured [24–26], which are the atomic building blocks of proposed topological quantum bits based on Majorana zero-energy modes [27–30].

These applications require the control of the Josephson coupling via the semiconductor weak link [31]. In addition, the charging energy of a NW CPT can deviate from the predictions of the ortodox theory [1, 2] due to renormalization effects arising because of finite channel transmissions [32]. Therefore, understanding the quantum dynamics of CPTs with semiconductor weak links is crucial for these hybrid device architectures.

# 7.2. EXPERIMENTAL SET-UP

Here we directly measure the transitions between the energy levels of a NW CPT. The CPT embedded in the circuit is shown in Fig. 7.1(a). The superconducting island is created from an indium arsenide (InAs) nanowire with an epitaxial layer of aluminium (Al) [15] between two Josephson junctions, formed by removing two sections of the Al shell with a wet chemical etch. We investigated two devices, both with 100 nm long junctions and island lengths of 800 nm and 1.75  $\mu$ m for device 1 and device 2 (enclosed in the red box in Fig. 7.1(a)), respectively. The junctions were tuned via their respective local electrostatic gates,  $V_{tg1}$  and  $V_{tg2}$ . The gate charge,  $n_g$ , was set by the gate voltage  $V_g$  (see right panel in Fig. 7.1(a)). The nanowire CPT is embedded in a superconducting quantum interference device (SQUID) with an Al/AlO<sub>x</sub>/Al tunnel junction (in the yellow box in Fig. 7.1(a)) which exhibits a much higher Josephson energy than the CPT. This asymmetry ensures that the applied phase  $\varphi = 2\pi \Phi/\Phi_0$  drops mostly over the CPT. Here,  $\Phi$  is the applied flux and  $\Phi_0 = h/2e$  is the superconducting flux quantum. Full details of the fabrication process are given in Section 3.4.3.

# 7.3. CIRCUIT CHARACTERISATION

We utilized a capacitively coupled  $Al/AlO_x/Al$  superconducting tunnel junction as a broadband on-chip microwave spectrometer (green box in Fig. 7.1(b)) [14, 17, 33], where inelastic Cooper-pair tunnelling gives rise to a DC current contribution in a dissipative environment [34]:



Figure 7.1: **Device schematics and working principle. (a)** Left: Scanning electron micrograph of the nanowire CPT (in red box) and an Al/AlO<sub>x</sub>/Al tunnel junction (yellow box) forming the hybrid SQUID loop. Middle: false coloured micrograph of the nanowire CPT (device 2). Right: three dimensional sketch of the CPT on the three electrostatic gates. **(b)** Equivalent circuit schematics with the hybrid SQUID on the left and a single Al/AlO<sub>x</sub>/Al tunnel junction used as a spectrometer (green box) on the right side. The circuit elements within the black dashed box are on-chip and cooled to  $T \sim 18 \text{ mK}$ . **(c)** I(V) trace of the spectrometer with the CPT arm in full depletion (device 1). The red solid line shows the fit to the circuit model of a single resonance centred at  $\hbar \omega_p = 148 \,\mu\text{eV}$  driven by the photons with an energy of  $\hbar \omega = 2eV_{\text{spec}}$  emitted by the spectrometer junction of gate charge,  $n_g$  and total phase bias,  $\varphi$ .

$$I_{\text{spec}} = \frac{I_{\text{c,spec}}^2 \operatorname{Re}[Z(\omega)]}{2V_{\text{spec}}}.$$
(7.1)

Here,  $I_{c,spec}$  is the critical current of the spectrometer tunnel junction and  $Z(\omega)$  is the impedance of the environment at the frequency  $\omega = 2eV_{spec}/\hbar$ , determined by the spectrometer DC voltage bias,  $V_{spec}$  (Fig. 7.1(d)). This DC to microwave conversion allowed us to directly measure the excitation energies of the hybrid SQUID, where Re[ $Z(\omega)$ ] exhibits a local maximum [35]. To reduce microwave leakage, we applied the bias voltages to the hybrid SQUID and to the spectrometer junctions via on-chip resistors, yielding  $R_{SQUID} = 12 k\Omega$  and  $R_{spec} = 2.8 k\Omega$ . The chip (in black dashed box in Fig. 7.1(b)) was thermally anchored to the mixing chamber of the dilution refrigerator with a base temperature of  $\approx 18 \text{ mK}$ .

We begin by analysing the circuit while keeping both nanowire junctions in full depletion by applying large negative gate voltages  $V_{tg1}$  and  $V_{tg2}$ . The I(V) curve of the spectrometer of device 1 is shown in Fig. 7.1(c). A clear peak is observed with an amplitude of 3nA centred at  $\approx 75 \,\mu$ eV. We attribute this peak to the plasma resonance of the tunnel junction in the SQUID at  $\hbar \omega_p = \sqrt{2E_{JL}E_{CL}}$ . Here  $E_{JL} = \Delta_J h/(8e^2R_J) = 249 \,\mu$ eV is the Josephson energy [4], with  $\Delta_J = 245 \,\mu$ eV being the superconducting gap and  $R_J = 2.96 \,\text{k}\Omega$  the normal state resistance of the junction. This value yields  $E_{CL} = 2e^2/C_L = 44 \,\mu$ eV and a shunt capacitance  $C_L = 7.28 \,\text{fF}$ . Fitting the resonant peak using equation (7.1), we find a quality factor  $Q \approx 1$  and a characteristic impedance  $Z_0 = 610\Omega \ll R_q = h/4e^2$ , which together ensure the validity of equation (7.1) describing a direct correspondence between the measured  $I_{\text{spec}}$  and  $\text{Re}[Z(\omega)]$ . We note that we found very similar values for device 2 as well. For a detailed analysis and a list of all parameters see section 7.7.3.

# 7.4. NANOWIRE COOPER-PAIR TRANSISTOR SPECTROSCOPY

Next, we investigate the spectrometer response to the applied gate voltage  $V_g$  and phase  $\varphi$  (Figs. 7.2(b) and (c) and 7.3(a) and (b)) when the Josephson junctions are opened by setting positive gate voltages  $V_{tg1}$  and  $V_{tg2}$ . The excitations of the CPT are superimposed on that of the plasma resonance, so we display  $|d\text{Re}(Z)/dV_{\text{spec}}|$  to reach a better visibility of the transitions (see section 7.7.2 for comparison). Note that we show the excitation energy  $\hbar \omega = 2eV_{\text{spec}}$  on the vertical axis for all spectra. This measurement yields clear oscillations as a function of both  $n_g$  and  $\varphi$ , consistent with the expected periodic behaviour of the CPT energy levels [8]. We note that the finite load resistance of the spectrometer  $R_{\text{spec}}$  prevented us from measuring the transitions below  $2eV_{\text{spec}} = 103 \,\mu\text{eV}$ .

### 7.4.1. HYBRID SQUID MODEL

We model our device with the schematics depicted in Fig. 7.2(a) and build the Hamiltonian of the circuit based on conventional quantization procedures [36, 37]. We use the conjugate charge and phase operators which pairwise obey  $[\hat{\varphi}_{1,2}, \hat{N}_{1,2}] = i$  and note that



Figure 7.2: **Theoretical description of measured spectra.** (a) Equivalent schematics of the hybrid SQUID used to build the circuit Hamiltonian. Observed transitions for device 1 as a function of the gate charge,  $n_g$  (b) and applied phase bias,  $\varphi = 2\pi\Phi/\Phi_0$  (c). The transitions are identified at the local minima of  $|d\text{Re}(Z)/dV_{\text{spec}}|$  (yellow dots). The best fit is shown as solid lines, yielding  $E_{C1} = 168 \,\mu\text{eV}$ ,  $E_{C2} = 260 \,\mu\text{eV}$ ,  $E_{Cc} = 188 \,\mu\text{eV}$ ,  $E_{I1} = 132 \,\mu\text{eV}$  and  $E_{I2} = 16 \,\mu\text{eV}$ , see text. (d) The corresponding energy bands of the device as a function of  $n_g$  at  $\varphi = \pi$ . The two-component probability distributions of the ground state (e), first excited state (f) and second excited state (g) at  $n_g = 0$  and  $\varphi = \pi$ , denoted by circles of the corresponding colour in panel (d), see text. See Fig. 7.4 upper row for gate voltage values.

$$\hat{\delta} = \varphi - \hat{\varphi}_1 - \hat{\varphi}_2;$$

$$\hat{H} = \frac{1}{2} E_{C1} (\hat{N}_1 - n_g)^2 + \frac{1}{2} E_{C2} (\hat{N}_2 + n_g)^2 - \frac{1}{2} E_{Cc} (\hat{N}_1 - n_g) (\hat{N}_2 + n_g) - E_{J1} \cos(\hat{\varphi}_1) - E_{J2} \cos(\hat{\varphi}_2) - E_{JL} \cos(\varphi - \hat{\varphi}_1 - \hat{\varphi}_2). \quad (7.2)$$

Here the charging of the circuit is described by the effective parameters  $E_{C1}$ ,  $E_{C2}$  and  $E_{Cc}$  set by the capacitance values  $C_1$ ,  $C_2$ ,  $C_L$ ,  $C_{ig}$  and  $C_G$  with a functional form provided in section 7.7.3. The Cooper-pair tunnelling is characterized by the Josephson energies



Figure 7.3: **Theoretical description of measured spectra.** The observed transitions for device 1 as a function of the gate charge,  $n_g$  (a) and applied phase bias  $\varphi = 2\pi\Phi/\Phi_0$  (b). The transitions are identified at the local minima of  $|d\text{Re}(Z)/dV_{\text{spec}}|$  (yellow dots). The best fit is shown with solid lines, yielding  $E_{C1} = 93 \,\mu\text{eV}$ ,  $E_{C2} = 184 \,\mu\text{eV}$ ,  $E_{Cc} = 104 \,\mu\text{eV}$ ,  $E_{J1} = 148 \,\mu\text{eV}$ ,  $E_{J2} = 46 \,\mu\text{eV}$ . (c) The corresponding energy bands of the device as a function of  $n_g$  at  $\varphi = \pi$ . The two component probability distributions of the ground state (d), first excited state (e) and second excited state (f) at  $n_g = 0$  and  $\varphi = \pi$ , denoted by circles of the corresponding colour in panel (c). Here,  $V_{tg1} = 0.4V$ ,  $V_{tg2} = 1.92V$ , and  $V_g = 713.1...719.7 \,\text{mV}$ 

of the three junctions,  $E_{J1}$ ,  $E_{J2}$  and  $E_{JL}$ , respectively. We note that we set  $E_{JL} = 249 \,\mu\text{eV}$  for the analysis below.

To calculate the excitation spectrum, we solve the eigenvalue problem to find  $E_i(n_g, \varphi)$ , where  $\hat{H}\Psi_i = E_i\Psi_i$ , and compute the transition energies  $\hbar\omega_i = E_i - E_0$ , with  $E_0$  being the ground state energy of the system. This model allows us to fit the excitation spectra simultaneously as a function of  $n_g$  and  $\varphi$  based on the first two transitions (red and purple solid lines for  $\hbar\omega_1$  and  $\hbar\omega_2$ , respectively) against the measured data (yellow circles in Figs. 7.2 and 7.3). For illustration, we also display  $\hbar\omega_3$  (orange line) in Figs. 7.2(b) and 7.3(a) using the same fit parameters, however, this transition was not observed in the experiment.

To understand the nature of the excited levels, we calculate the energy bands of the hybrid SQUID using the fitted parameters (Figs. 7.2(d) and 7.3(c)) and evaluate the probability distribution  $p_i(N_1, N_2) = |\Psi_i(N_1, N_2)|^2$ , where  $N_1$  and  $N_2$  form the charge computational basis. However, it is more instructive to use the charge numbers  $N_{CPT} = N_1 - N_2$  and  $N_{Loop} = N_1 + N_2$ . Intuitively,  $N_{CPT}$  and  $N_{Loop}$  represent the excess number of Cooper pairs on the island and in the loop, respectively. Indeed, the ground state wavefunction is centered around  $N_{CPT} = N_{Loop} = 0$  (Figs. 7.2(e) and 7.3(d)). Conversely, the probability distribution of the first excited state (Fig. 7.2(f) and second excited state for Fig. 7.3(f))

exhibits a bimodal distribution in  $N_{\text{Loop}}$ , consistently with the first plasma mode excitation but no excess charge on the CPT (purple circle in Fig. 7.2(d) and red circle in Fig. 7.3(c)). This is in contrast with the wavefunction of the next energy level (Figs. 7.2(g) and 7.3(e) and red circle in Fig. 7.2(d) and purple circle in Fig. 7.3(c)), which is centered around  $N_{\text{CPT}} = \pm 1$ . This analysis demonstrates the coupling between the plasma and localized charge degrees of freedom [38].



### 7.4.2. EXCITATION SPECTRUM OF THE HYBRID SQUID

Figure 7.4: Charge and phase dispersion of the measured spectra. Upper row: The measured excitations spectrum of device 1 as a function of  $n_g$  and  $\varphi$  with  $V_{tg1} = 0.55$  V,  $V_{tg2} = 0.85$  V and  $V_g = 250...264.5$  mV, same as in Fig. 7.2. Panel (b) shows the full map of the second excitation whereas linecut data is shown at the positions denoted by the orange and red lines respectively in (a) and (c). Bottom row: measured data on the same device with  $V_{tg1} = 1.5$  V,  $V_{tg2} = 1.545$  V and  $V_g = 916.9...931.4$  mV. Note the weak dependence on  $n_g$  due to the more open semiconductor channels. The best fits of the first two excitation energies are overlain in panels (a), (c) and (d), (e). All data was taken on device 1, the parameters of the best fit are listed on the right for each setting.

Next, we investigate the impact of  $V_{tg1}$  and  $V_{tg2}$  on the CPT spectrum. In Fig. 7.4, we show the measured spectra for two distinct gate settings. Remarkably, almost a full suppression of the charge dispersion is achieved by an  $\approx 1$  V increase in  $V_{tg1}$  and  $V_{tg2}$ , showcasing the feasibility of topological quantum bit designs relying on the modulation of the charge dispersion in superconductor-semiconductor hybrid devices [28]. Furthermore, we observe a strong renormalization of the characteristic charging energies in the open regime [32, 39], which does not exist for the case of fully metallic CPTs with tunnel junctions, where the charging energy is fully determined by the device geometry. In addition, we find an increase in the Josephson energies  $E_{I1,2}$ , further contributing to the

suppression of the charge dispersion of the CPT in the limit of  $E_I \gg E_C$  [40].

# **7.5.** TEMPERATURE DEPENDENCE



Figure 7.5: **Temperature dependence of charge excitations** The measured excitation spectra of device 2 at  $V_{tg1} = 1.4$  V,  $V_{tg2} = 1.5025$  V as a function of  $n_g$  corresponding to  $V_g = -533... - 527.8$  mV at  $\varphi = \pi$  and at a temperature of 18mK (**a**) and 350 mK (**b**). (**c**) The extracted even charge parity state occupation  $p_{even}$  as a function of the temperature. The inset shows the modulation of the spectrometer current at  $2eV_{spec} = 180 \,\mu\text{eV}$  at these two temperatures, which defines  $\delta I_{odd}$  and  $\delta I_{even}$ , see text. The fit lines in (**c**) are based on equation (7.3), without (blue dashed line) and including overheating (solid red line).

Thus far, we only considered the even charge occupation of the island, where all electrons are part of the Cooper-pair condensate, and a single quasiparticle occupation is exponentially suppressed in  $\Delta/k_B T$ , where  $\Delta$  is the superconducting gap [41]. However, a residual odd population is typically observed in the experiments, attributed to a non-thermal quasiparticle population in the superconducting circuit. In our experiment, we also find an additional spectral line, shifted by  $\delta n_g = 0.5$  (see Figs. 7.2(b) and 7.4(a)), substantiating a finite odd number population of the island. We investigate this effect as a function of the temperature, and find that above a typical temperature of  $T^* \approx 300 \, \text{mK}$ ,

the measured signal is fully 1*e* periodic (Fig. 7.5(b)), in contrast to the 2*e* periodic data taken at 18 mK (Fig. 7.5(a)).

To quantify the probability of the even and odd occupations, we extract the gatecharge dependent component of the measured spectra  $\delta I_{\text{spec}}(n_g)$  to evaluate  $\delta I_{\text{odd}} = \delta I_{\text{spec}}(n_g = 0.5)$  and  $\delta I_{\text{even}} = \delta I_{\text{spec}}(n_g = 0)$ , see the inset in Fig. 7.5(c). We now make the assumption that the microwave photon frequency is much higher than the parity switching rate of the CPT. We evaluate the current response at  $hf = 2eV_{\text{spec}} = 180 \,\mu\text{eV}$ (see Figs. 7.5(a) and (b)) corresponding to  $f = 43.5 \,\text{GHz}$ , well exceeding parity switching rates measured earlier on similar devices [25, 42]. In this limit, the time-averaged spectrometer response is the linear combination of the signals corresponding to the two parity states and  $\delta I_{\text{even},\text{odd}} \sim p_{\text{even},\text{odd}}$ , respectively. From this linear proportionality,  $p_{\text{even}} = (1 + \delta I_{\text{odd}} / \delta I_{\text{even}})^{-1}$  follows.

We plot the extracted  $p_{\text{even}}$  in Fig. 7.5(c). We find that above a crossover temperature  $T^* \approx 300 \,\text{mK}$ ,  $p_{\text{even}}$  approaches 1/2, in agreement with the commonly observed breakdown of the parity effect at  $T^* < \Delta$  as a result of the vanishing even-odd free energy difference [11, 43, 44]:

$$\Delta F = -k_B T \ln \tanh\left(N_{\rm eff} e^{-\Delta/k_B T}\right). \tag{7.3}$$

Here,  $N_{\text{eff}} = \rho V \sqrt{2\pi k_B T \Delta}$  at a temperature of *T* with the island volume being *V*. We use the density of states at the Fermi level in the normal state  $\rho = 1.45 \times 10^{47} \text{ J}^{-1} \text{m}^{-3}$  for aluminium [12]. Then the even charge parity occupation is given by  $p_{\text{even}} = 1 - 1/(1 + e^{\Delta F/k_B T})$ .

While this analysis describes the breakdown of the even-odd effect (see blue dashed line as the best fit in Fig. 7.5(c)), it fails to account for the observed saturation  $p_{\text{even}} \sim 0.8 < 1$  in the low temperature limit, at T < 150 mK. This saturation can be be phenomenologically understood based on a spurious overheating of the island. We assume that the electron temperature  $T_e = (T_0^5 + T^5)^{1/5}$ , where the chip (phonon) temperature is T, and the electron saturation temperature is  $T_0$  due to overheating and weak electron-phonon coupling at low temperatures [45].

The resulting best fit is shown as a solid red line in Fig. 7.5(c). We find a metallic volume of  $V = 4.66 \times 10^{-23} \text{ m}^3$ , consistent with the micrograph shown in Fig. 7.1(a). The fit yields a superconducting gap  $\Delta = 140 \pm 3 \mu \text{eV}$ , slightly lower than the that of bulk aluminum, which is expected due to the presence of induced superconductivity in the semiconductor. The fitted saturation temperature  $T_0 = 244 \pm 4 \text{ mK}$  and limiting  $p_{\text{odd}} = 1 - p_{\text{even}} \approx 0.17$  demonstrates the abundance of non-equilibrium quasiparticles, in agreement with recent experimental findings [46, 47] on metallic devices. Our results substantiate the importance of controlling the quasiparticle population for hybrid semiconductor-superconductor CPTs in prospective topological quantum bits to decrease their rate of decoherence [48].

# 7.6. CONCLUSION

In conclusion, we performed broadband microwave spectroscopy on the gate charge and phase-dependent energy dispersion of InAs/Al hybrid CPTs, utilizing an on-chip nanofabricated circuit with a superconducting tunnel junction as a frequency-tunable microwave source. We understand the observed spectra based on the Hamiltonian of the circuit and find the characteristic charging and Josephson tunneling energy scales, both exhibiting strong modulation with the electrostatic gates coupled to the semiconductor channels. This broad tunability demonstrates the feasibility of prospective topological qubits relying on a controlled suppression of the charge modulation. Finally, we directly measure the time-averaged even and odd charge parity occupation of the CPT island, yielding a residual 0.17 odd parity occupation probability, which can be a limiting factor for topological quantum bit architectures that rely on charge parity manipulation and readout.

### CONTRIBUTIONS

A.P. and D.L. fabricated the devices, performed the experiments and analysed the data. B.v.H. assisted with the fabrication and measurements. P.G. and J.N. contributed to the nanowire growth. L.P.K. and A.G. designed and supervised the experiments and analysed the data. The manuscript has been prepared with contributions from all authors.

# 7.7. SUPPLEMENTARY INFORMATION

## 7.7.1. CIRCUIT PARAMETERS

For both devices, the circuit was analysed through the observation of the plasma peak in the I(V) response of the spectrometer with both junctions of the nanowire in full depletion. We insert the impedance  $\text{Re}[Z(\omega)]$  into equation (7.1) as:

$$\operatorname{Re}[Z(x)] = \frac{Z_0 Q}{1 + \frac{Q^2}{v^2} \left(1 - x^2\right)^2}$$
(7.4)

Here  $Q = R\sqrt{C/L}$  is the quality factor and  $Z_0 = \sqrt{L/C}$  is the characteristic impedance of the circuit. We introduced a dimensionless frequency  $x = \omega/\omega_0$  defined with  $\omega_0 = 1/\sqrt{LC}$ . We display the obtained fits of equation (7.4) in Fig. 7.6. We attribute the deviation between the fits and data at higher frequencies to additional losses or modes not accounted for by equations (7.1) and (7.4).



Figure 7.6: Fit to the plasma resonance of the circuit. The measured data is represented by black dots with the fit based on equation (7.4) shown as a red line for device 1 (a) and device 2 (b), respectively.

	Device 1	Device 2
Tunnel junction resistance $R_J$ (k $\Omega$ )	3.17	2.96
Tunnel junction gap $\Delta_J$ (µeV)	245	245
Tunnel junction critical current $I_{c,J} = \frac{\pi \Delta_J}{2eR_J}$ (nA)	121.4	130.3
$E_{JL} = \frac{\hbar I_{\rm c,J}}{2e} \; (\mu \rm eV)$	249	267.3
Tunnel junction inductance $L_J = \frac{\Phi_0}{2\pi I_{c,JJ}}$ (nH)	2.71	2.53
Spectrometer resistance $R_{\rm spec}$ (k $\Omega$ )	14.44	12.7
Spectrometer gap $\Delta_{\text{spec}}$ ( $\mu$ eV)	238	241
Spectrometer critical current $I_{c,spec} = \frac{\pi \Delta_{spec}}{2eR_{spec}}$ (nA)	25.9	30.3
Shunt resistance $R(\Omega)$	$598.1 \pm 0.3$	$493.3\pm0.87$
Shunt capacitance $C_L$ (fF)	$7.28\pm0.01$	$8.04\pm0.03$
Charging energy $E_{CL} = \frac{2e^2}{C_L}$ (µeV)	$43.96\pm0.07$	$39.8\pm0.15$
Plasma frequency $f_p = \frac{1}{2\pi \sqrt{L_J C_L}}$ (GHz)	$35.83 \pm 0.02$	$35.29 \pm 0.07$
Characteristic impedance $Z_0 = \sqrt{\frac{L_I}{C_L}} (\Omega)$	$610.1\pm0.42$	$561.0 \pm 1.05$
Quality factor $Q = R \sqrt{\frac{C_L}{L_J}}$	$0.98\pm0.001$	$0.88 \pm 0.0025$

Table 7.1: Circuit parameters of the devices featured in the current study.

## 7.7.2. Spectrum Analysis

The excitation energies of the circuit reveal themselves as peaks in the measured I(V) traces of the spectrometer. We are interested in discerning the modes that arise due to the nanowire CPT which manifest themselves as additional peaks observed on top of the plasma mode. In order to improve their visibility we evaluate  $|d\text{Re}[Z(\omega)]/dV_{\text{spec}}|$  after applying a Gaussian low pass filter (Fig. 7.7). Transitions superimposed on the plasma mode peak appear as minima in  $|d\text{Re}[Z(\omega)]/dV_{\text{spec}}|$ .

# **7.7.3.** THEORY

We establish the hybrid SQUID Hamiltonian based on the circuit shown in Fig. 7.2(a). We associate a voltage with each node and write the charging energy of the system as

$$T = \frac{1}{2}C_L(V_1 - V_2)^2 + \frac{1}{2}C_1(V_1 - \nu)^2 + \frac{1}{2}C_2(\nu - V_2)^2 + \frac{1}{2}C_G(V_1^2 + V_2^2) + \frac{1}{2}C_{ig}(\nu - V_g)^2.$$
(7.5)

Similarly, the total Josephson energy is as as follows:

$$U = -E_{J1}\cos(\varphi_1) - E_{J2}\cos(\varphi_2) - E_{JL}\cos(\varphi - \varphi_1 - \varphi_2).$$
(7.6)

Now we obtain the Lagrangian of the system as:

$$\mathscr{L} = T - U, \tag{7.7}$$


Figure 7.7: Analysis of the measured  $I_{\text{spec}}(V_{\text{spec}})$ . The blue and red lines represent  $\text{Re}(Z)(V_{\text{spec}})$  for device 1 obtained at two  $V_g$  values. The light blue and orange lines display  $|d\text{Re}[Z(\omega)]/dV_{\text{spec}}|$  with the dashed line indicating the positions of the resonances arising due to the nanowire CPT and the hybrid SQUID.

and use the phase  $\varphi_n$  and charge  $q_n$  as the canonical conjugate variables:

$$q_n = \frac{\partial \mathscr{L}}{\partial \dot{\phi}_n} \text{ and } \phi_n = \frac{\partial \mathscr{L}}{\partial \dot{q}_n}.$$
 (7.8)

Note that each voltage difference in equation (7.5) can be expressed with the phase  $V = \varphi_0 \dot{\varphi}$ , where  $\varphi_0 = \Phi_0/2\pi$  is the reduced flux quantum. Next, we obtain the Hamiltonian of the circuit:

$$H = \Sigma_i \dot{\varphi_i} \frac{\partial \mathscr{L}}{\partial \dot{\varphi_i}} - \mathscr{L}, \tag{7.9}$$

which we can express in the following form:

$$H = \frac{1}{2} \frac{\left(q_1 - q_g\right)^2}{C_{Det}^2 / C_{i2}} + \frac{1}{2} \frac{\left(q_2 + q_g\right)^2}{C_{Det}^2 / C_{i1}} - \frac{\left(q_1 - q_g\right)\left(q_2 + q_g\right)}{C_{det}^2 / C_{ic}} + U,$$
(7.10)

where  $q_g = C_{ig}V_g/(2 + C_{ig}/C_G)$  and

$$C_{i1} = C_L + C_1 + \frac{(1 + \frac{C_{ig}}{C_G})^2 C_G + C_G + C_{ig}}{(2 + \frac{C_{ig}}{C_{C_G}})^2}$$
(7.11)

$$C_{i2} = C_L + C_2 + \frac{(1 + \frac{C_{ig}}{C_G})^2 C_G + C_G + C_{ig}}{(2 + \frac{C_{ig}}{C_{C_G}})^2}$$
(7.12)

$$C_{ic} = C_L + \frac{C_G}{2 + \frac{C_{ig}}{C_C}}$$
(7.13)

$$C_{Det}^2 = C_{i1}C_{i2} - C_{ic}^2. ag{7.14}$$

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Finally, we write the Hamiltonian operator with the conjugate number and phase operators, which pairwise obey  $[\hat{\varphi}_{1,2}, \hat{N}_{1,2}] = i$ :

$$\hat{H} = \frac{1}{2}E_{C1}(\hat{N}_1 - n_g)^2 + \frac{1}{2}E_{C2}(\hat{N}_2 + n_g)^2 - \frac{1}{2}E_{Cc}(\hat{N}_1 - n_g)(\hat{N}_2 + n_g) - E_{J1}\cos(\hat{\varphi}_1) - E_{J2}\cos(\hat{\varphi}_2) - E_{JL}\cos(\varphi - \hat{\varphi}_1 - \hat{\varphi}_2)$$
(7.15)

With a set of effective charging energies defined as:

$$E_{C1} = (2e)^2 C_{i2} / C_{Det}^2 \tag{7.16}$$

$$E_{C2} = (2e)^2 C_{i1} / C_{Det}^2$$
(7.17)

$$E_{Cc} = (2e)^2 C_{ic} / C_{Det}^2. ag{7.18}$$

We diagonalize  $\hat{H}$  in the charge basis, span by  $N_1$  and  $N_2$ , where the Josephson terms act as stepping operators,  $e^{\pm i\hat{\varphi}_i} |N_i\rangle = |N_i \pm 1\rangle$ .

#### 7.7.4. MODEL SIZE

The eigenvectors and eigenvalues for the system at every gate charge value between -1 and 1 are numerically computed with a maximal value of  $N_i = \pm 4$ . This value has to be sufficiently large to ensure that the wavefunctions have negligible weight near the extremum values of  $N_i$ . As shown if Fig. 7.8, the extracted eigenenergies of the system for typical energy scales do not change if the system size is further increased.



Figure 7.8: The calculated energy levels as a function of  $n_g$  at  $\varphi = \pi$  of the hybrid SQUID using N = 4 and N = 9 for  $E_{C1} = 168 \,\mu\text{eV}$ ,  $E_{C2} = 260 \,\mu\text{eV}$ ,  $E_{Cc} = 188 \,\mu\text{eV}$ ,  $E_{J1} = 132 \,\mu\text{eV}$ ,  $E_{J2} = 16 \,\mu\text{eV}$  and  $E_{JL} = 249 \,\mu\text{eV}$ .

#### 7.7.5. PEAK EXTRACTION

First, all minimum values in  $|d\text{Re}(Z)/dV_{\text{spec}}|$  are collected, using both  $V_g$  and  $V_{\text{spec}}$  linecuts. Then, using  $E_{JL}$  extracted from the bare SQUID IV trace and initial guesses for  $E_{C1}, E_{C2}, E_{Cc}, E_{J1}$  and  $E_{J2}$ , a first iteration of the dispersion relation of the CPT is calculated from the model described in the previous section. It is important that this initial guess yields a good visual agreement between the calculated dispersion relation and the experimentally observed features in the measured data. Then, at each experimental gate charge and for each energy mode, the closest minima to this first estimate is used as the experimental dispersion relation.

This method was used to extract the experimental dispersion relation of all data sets in this article except for the data of Fig. 7.4(e). In this case, due to the flat dispersion relation of both the plasma and the CPT modes in gate space, minima in  $|d^2 \operatorname{Re}(Z)/dV_{spec}^2|$ are used to identify features in  $\operatorname{Re}[Z(\omega)]$ . To ensure that the right features were tracked in this manner, the expected position of the features is inferred from a separate data set (Fig. 7.4(f)) showing the phase dependence of  $\operatorname{Re}[Z(\omega)]$ .

#### 7.7.6. FITTING PROCEDURE

To determine the charging and Josephson energies of the hybrid system, a least-square minimization was performed on the difference between experimental data points and the calculated excitation spectra. This minimization was performed on the two lowest excitation energies for both a gate charge scan (eg. Fig. 7.3(a)) and a flux scan (eg. Fig. 7.3(b)) simultaneously. The least-square minimization was performed on 2 periods in gate space (range 4*e*) and 1 period in flux space (range 2 $\pi$ ). The minimization procedure was carried out iteratively by varying the set  $E_{C1}$ ,  $E_{C2}$ ,  $E_{Cc}$ ,  $E_{J1}$ ,  $E_{J2}$  through ~ 60000 combinations while keeping  $E_{JL}$  fixed at the value determined from the plasma mode fit (see Table. 7.1). The first iteration started from an initial guess and typically spanned over a range of  $\pm 40 \,\mu\text{eV}$  for  $E_{C1}$  and  $E_{C2}$  and  $\pm 30 \,\mu\text{eV}$  for  $E_{J1}$ .  $E_{Cc}$  was varied as a fraction of the geometrical mean of  $E_{C1}$  and  $E_{C2}$ , typically 0.87  $\pm$  0.09, and  $E_{J2}$  was varied as a fraction of  $E_{J1}$ , typically 0.25  $\pm$  0.1. This process iterated around the previous optimal value with progressively smaller parameter range until the optimal energies were changing by less than  $4\mu\text{eV}$ . The procedure was performed over a few different initial guesses, and it was verified that the solutions converged to the same set of parameters.

#### 7.7.7. EVEN-ODD OCCUPATION

After interpolation and applying a Gaussian filter to the measured spectra, a linecut, corresponding to a gate voltage value halfway between the odd and the even peaks, was subtracted. To account for the background of our data, another linecut, corresponding to a large  $V_{\rm spec}$  away from resonances, was subtracted. Following this, the spectrometer voltage value  $V_{\rm spec}^{max}$  for the maximum  $\delta I_{\rm spec}$  current oscillations was selected, and we find  $2eV_{\rm spec}^{max} \approx 180 \,\mu {\rm eV}$  for Fig. 7.5. The large and small current peaks are attributed to the even and odd occupation, respectively. Finally, the current values  $\delta I_{\rm even}$  and  $\delta I_{\rm odd}$  were averaged over 3 data points in  $V_{\rm spec}$  (centred about  $V_{\rm spec}^{max}$ ) and over 3 data points in gate voltage (centred about the peaks) for each peak. Assuming that the current response is proportional to the initial occupation probability, we can write  $p_{\rm even} = \delta I_{\rm even} / (\delta I_{\rm even} + \delta I_{\rm odd}) = (1 + \delta I_{\rm odd} / \delta I_{\rm even})^{-1}$ .

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## **CONCLUSIONS AND OUTLOOK**

#### 8.1. CONCLUSIONS

The focus of experimental investigation covered in this thesis is on the energetic structure of hybrid semiconductor-superconductor nanowire based devices. This is achieved primarily through the development and improvement of pre-existing techniques and we summarize the resulting conclusions reached in each experiment.

- In **chapter 4** we show that the radiation emitted from a nanowire, indium antimonide (InSb), based Josephson junction consists of two components; shot and Josephson noise. This is achieved through the utilisation of an on-chip circuit where the source of radiation is capacitively coupled to a superconducting quantum interference device (SQUID) acting as a high-frequency detector. The Josephson noise is characterised by the Josephson energy of the nanowire junction. The shot noise contribution to the measured signal is attributed to the presence of a finite number of subgap quasiparticle states.
- In **chapter 5** we reveal the presence of Andreev bound states (ABS) in an indium arsenide nanowire (InAs) with an epitaxial layer of aluminium (Al), where a nanowire Josephson junction is defined through selectively removing a region of Al. The nanowire junction is embedded in an asymmetric hybrid SQUID with a tunnel Josephson junction of a larger Josephson energy. The SQUID is in turn capacitively coupled to another tunnel Josephson junction, with a smaller Josephson energy as compared to the reference junction of the SQUID, acting as a spectrometer. The gate induced ABS of the nanowire junction are revealed along with their phase dependence achieved through flux biasing of the SQUID. Upon application of an inplane magnetic field the Zeeman effect and spin-orbit coupling are inferred from the observed evolution of ABS.
- In chapter 6 we investigate the effect of quasiparticle poisoning on an InAs-Al based Cooper-pair transistors (CPTs) through the measurement of a periodic modulation of the CPT switching current ( $I_{sw}$ ). The observation of a 2e-periodic supercurrent at zero magnetic field and at a base temperature of  $\approx$  30 mK reveals the absence of deep subgap states on the island of the CPT. Temperature studies, however, highlight the occurrence of fast poisoning and unpoisoning processes. Upon the application of a magnetic field in plane of the CPT a zero energy crossing is observed and attributed to an induced subgap state. The results motivate further studies into system quasiparticle dynamics.
- In **chapter 7** we adapt the circuitry used in chapter 5 to study the energy landscape of an InAs-Al CPT as a function of gate induced charge and the phase over the island of the CPT. Due to the gate tunability of Josephson junctions either side of the island, the competing nature of charging and Josephson energies is observed. Studies of temperature dependence reveal not only the quasiparticle dynamics of the island but also the role of microwave radiation on overheating the electronic system of the island.

#### **8.2.** OUTLOOK

The results presented in this thesis demonstrate the potential of two hybrid structures, the nanowire based Josephson junction and Cooper-pair transistor (CPT), for applications in the field of quantum computation [1]. The ability to vary the number of Andreev bound states (ABS) present in a given nanowire junction via a local electrostatic gating highlight the potential of such systems for the realisation of Andreev based qubits. Here the ability to tune the ABS such that their energies are small in comparison with the superconducting gap, leads to the formation of an Andreev two level system [2, 3]. Furthermore, both hybrid architectures are central to the development of topological qubits [4–6]. We reveal the ability to tune a nanowire CPT from a charging to a Josephson energy dominated regime, in line with the necessities for manipulation of topological qubits [4]. With the aid of switching current measurements and studies of temperature dependence we reveal the presence of quasiparticles (and their dynamics), which significantly impacts the performance of each hybrid as either qubit platform [7].

Here we build upon the results by outlining a set of modifications to the experimental circuits. We focus on the on-chip coupling circuit as introduced in chapters 4,5 and 7, which in turn would allow for the inclusion of switching current measurements. This allows us to propose a set of experiments to either further validate the observed results and build upon them.

#### **8.2.1.** CIRCUIT MODIFICATIONS

The on-chip coupling circuit had already been subjected to extensive investigation throughout the course of the experimental work and had subsequently been modified. However further improvement is necessary as evidenced by back-bending observed in almost all current-voltage characteristics of tunnel junctions at the transition from superconducting to normal state. The back-bending originates from self-heating, caused by a finite quasiparticle density, of the tunnel junction and significantly degrades its performance as a detector of radiation. The reduction of this self-heating can then be achieved through trapping of quasiparticles at artificially created vortex sites in the vicinity of the tunnel junction. Such trapping cites can come in a variety of flavours such as local defects, however a more realistic approach would be to etch pinning cites as is done in conventional circuit-QED architectures [8, 9]. A more elegant approach would be to implement on-chip coolers, such as SINIS structures. Here the tunnel junction is interupted by a small metallic island, usually made of copper (Cu), with the cooling dominated by the weak electron-phonon coupling between the island and the substrate [10, 11].

Thus far the nanowire based Josephson junctions and CPTs have been created through deterministically placing the nanowire on a pre-defined gate pattern covered by a dielectric. However the profile of the dielectric layer is rather rough, meaning when the nanowire sits on top there are pockets of air created between the two layers. Coupled with defects in the dielectric, the gate-induced performance of the nanowire can become hysteretic and susceptible to local charge fluctuations. A suggested improvement would be to implement *wrap-around* gates. Here the nanowire is first placed upon the substrate with the dielectric and the metallic gate deposited around the junction (or the island) [12].

The most important modification to the circuit comes, however, from its decoupling

from the electromagnetic environment. In all of the discussed experiments a combination of thin and long Chromium/Platinum (Cr/Pt) was used for resistive decoupling. This ensured that the high-frequency radiation would not leak out of the circuit as well as keeping the quality factor, Q, of the circuit  $\approx 1$ . However the presence of several k $\Omega$  in the biasing lines of the spectrometer in chapter 5 and 7, significantly limited the range of frequency resolution to  $\approx 10$  GHz. Furthermore the low Q suppressed the higher order excitations within the circuit and hence of the device of interest. By slightly raising Q to  $\approx 10$ , most of the radiation would still be expected to reach the device of interest however the signal would lead sharper linewidths in the observed excitations [13, 14]. Such a Q could be readily achieved reducing the resistive shunt to  $\approx 200 \Omega$  and implementing high kinetic inductance materials, such as NbTiN, with an inductance of a few nH [15].

#### 8.2.2. JOSEPHSON RADIATION

Upon being subjected to a voltage bias,  $V_{\text{bias}}$ , the radiation emitted from a Josephson junction gives direct access to its underlying dynamics [16, 17]. The characteristic feature of a conventional Josephson tunnel junction is that the current-phase relation (CPR) is  $2\pi$  periodic and this is revealed by the radiation through its relation to the Josephson frequency,  $f_J = 2eV_{\text{bias}}/h$ , here h is Planck's constant [18]. Strikingly, by taking a nanowire based Josephson junction and driving it into a topological state, identified in this case by the formation of two Majorana bound states (MBS) at either side of the weak link in the junction, the emitted radiation is modified [19]. Here the CPR deviates to become  $4\pi$  periodic with a characteristic emission frequency,  $f_M = eV_{\text{bias}}/h$ . Detecting such a radiation can then be a useful tool in distinguishing between different types of junctions. It should be noted, however, that non-topological Josephson junctions may also produce radiation equivalent to a  $4\pi$  periodic Josephson effect with the aid a finite probability of Landau-Zener tunnelling [20] as well as a variable electromagnetic environment [21].

In [22] a hybrid circuit of chapters 4 and 7, was used to study the radiation emitted from an InAs-Al based nanowire Josephson junction and its dependence on an in-plane magnetic field. The observed  $4\pi$  periodic Josephson effect was attributed to the transition into the topological phase corroborated with strenuous modelling. To further validate the claims, the nanowire junction was replaced by a conventional tunnel junction. Under the application of an in-plane magnetic field no  $4\pi$  periodic Josephson effect was observed. Although the conclusions reached appear to be consistent with a transition into a topological phase, further experimentation is necessary.

Taking into account the general suggested improvements upon the circuit itself we look into more specific requirements. The main criticism placed upon the experimental results is the broad linewidth of the detected signal, especially towards the higher magnetic fields. Undoubtedly the presence of a finite density of quasiparticles in the nanowire Josephson junctions leads to finite heating effects. This, in large, is governed by the interface between the superconductor and semiconductor. Thus trying different combinations of Al coverage of InAs or implementation of InSb instead could lead to a reduction in the necessary magnetic fields.

However, more conclusive evidence would be to perform complementary experiments. First by placing the nanowire junction in a highly asymmetric SQUID with a tunnel junction, the  $4\pi$  Josephson effect can be inferred from the switching current statistics

as a function of the applied flux [23]. Such a technique has further advantages as it is not parity conserving and can thus shed light upon the quasiparticle poisoning dynamics. The main limitation would be the frequency of applied current pulses, especially if the poisoning process are in the order of  $100\mu$ s to 10 ms, requiring MHz pulses. This can be achieved through modifying the filtering of fridge lines as discussed in chapter 3. This could in turn reduce the quality of detected signals, thus cryogenic amplification at 4K may be required.

Alternatively one can look to implement a room temperature high-frequency detection set-up as was done in [24] to measure the radiation emitted from a nanowire junction. However the surrounding electronics can further mimic the  $4\pi$  radiation and hence care must be taken when analysing the detected spectra.

#### 8.2.3. MICROWAVE SPECTROSCOPY

The Andreev bound state is the fundamental part of a Josephson junction, responsible for carrying supercurrent. The dispersion of ABS as a function of the phase difference across the weak link depends upon the nature of the weak link itself. On top of governing the junction dynamics, ABS act as a two level system, the Andreev two level system, which in turn has potential applications in the field of quantum computation as an Andreev qubit [2, 3].

Josephson junctions based on InAs-Al hybrid nanowire architectures are prime candidates for Andreev qubits due to their tunability [25, 26]. For a given junction, we have shown in chapter 5 that the number of ABS present can be accordingly adjusted from one to many with varying transmission probabilities. Further studies into the timedependent dynamics on the same junctions have revealed poisoning dynamics on  $\approx 200$  $\mu$ s, comparable to other state of the art systems [27, 28]. The work covered in chapter 5 shows the promise of this platform for applications in the field of topological quantum computation, due to the presence of spin-orbit interaction and Zeeman energy splitting in magnetic field. Further evidence of spin-orbit interaction was revealed in a traditional circuit-QED set-up, performed on junctions of a similar composition [29]. Motivated by this we propose further experiments along similar lines.

Most of the experimental results reported on Al-InAs junctions focus on the short to intermediate junction limit, where the length of the weak link, *d*, is smaller or comparable ot the superconducting coherence length,  $\xi_0$ . Indeed for junctions with  $d \approx 100-400$  nm, the induced superconducting energy gap,  $\Delta$ , is of the order 180-110  $\mu$ eV resulting in  $\xi_0 = \hbar v_F / \Delta \approx 200-800$  nm, here  $v_F$  is the Fermi velocity in the weak link. Here the ABS dispersion is parabolic as discussed in chapter 3. Increasing *d* to  $\approx 800$  nm to 1  $\mu$ m (and preferably longer) would lead to the long junction limit. The ABS dispersion changes to a chequerboard pattern. Furthermore junctions in the long limit are expected to reveal evidence consistent with the presence of MBS [31].

Building upon the suggested circuit modifications and implementing a switching current technique, supercurrent spectroscopy of ABS can be performed [14]. Combined with a higher *Q* and a narrower linewidth of the measured spectra would reveal the spin-split nature of ABS in magnetic field. Again, direct switching current measurements of the hybrid SQUID would reveal the underlying poisoning dynamics, corroborating existing results.



Figure 8.1: **a** Andreev bound state dispersion in a left Al-InAs nanowire Josephson junction as a function of the phase difference across,  $\varphi_L$ . **b** Similar diseprsion in the right junction. **c** Schematic representation of the two Andreev molecules, green and light blue, composed in a single nanowire. **d** For the separation of two molecules,  $d \gg \xi_0$ , there is no hybridization between the ABS. **d** By bringing the two molecules closer together, the resulting hybridization of ABS leads to a non-local supercurrent due to the modified dispersion. Figure adapted from [30].



Figure 8.2: A combination of inductors and resistors decouples the on-chip circuit from the surrounding environment. A spectrometer tunnel junction is capacitively coupled to two SQUIDs, one a hybrid SQUID of a nanowire and a tunnel junction and the other an rf-SQUID of one nanowire junction. The areas of the two SQUIDs differ by at least a factor of two in order to ensure that only the phase across the hybrid SQUID varies upon an application of external flux.

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So far we have limited our discussion to the behaviour of ABS constituted in a single junction, it is however instructive to think what happens when two junctions are brought in close proximity of one another. The hybridization between two sets of ABS can lead to exotic behaviour in the individual junction as well as modifying the transport through the resulting islands formed [30]. Figures 8.1(a) and (b), shows the dispersion of two junctions with a single ABS in each with a transmission probability  $\tau \approx 0.94$ , forming two Andreev molecules. The two junctions are separated by a superconducting island of length *d* (Fig. 8.1(c)). For situations where  $d \gg \xi_0$  the two Andreev molecules are fully independent as evidenced by Fig. 8.1(d) where only one set of ABS disperses with the phase across the right junction. When brought closer together, Fig. 8.1(e), the hybridisation between the two sets of ABS modifies the resulting dispersion in the right junction as a function of the phase across the right junction depends upon the phase difference across the left junction.

In Fig. 8.2 we propose a circuit in which such an experiment is possible. Most of the components remain unaltered however in addition to the hybrid SQUID we find an rf-SQUID. The hybrid SQUID consists of the right nanowire and a tunnel junction whilst the rf-SQUID consists of the left nanowire junction in Fig. 8.1 (c). The two SQUIDs are designed to have varying areas in order to ensure that upon application of a magnetic flux, a full flux period may pass through the hybrid SQUID whilst maintaining the rf-SQUID almost unaltered<sup>1</sup>. By modifying *d* the degree of hybridisation can be studied.

<sup>&</sup>lt;sup>1</sup>Alternatively flux biasing lines made of NbTiN directed towards each SQUID may be implemented.

#### **8.2.4.** COOPER-PAIR TRANSISTOR

In line with the circuit modifications, similar considerations already addressed for single nanowire Josephson junction apply to the nanowire Cooper-pair transistor (CPT). The added advantage of sharper linewidth and a reduced shunting of the spectrometer would reveal the presence of any subgap states on the island of the CPT in a finite magnetic field, thus confirming the observations presented in chapter 6. For future perspectives of the switching current measurements of CPTs we propose further investigation into the length of leads either side of the island. In chapter 6 we discuss the role of the filtering effect on fast poisoning and unpoisoning of the island, this effect is enhanced for larger superconducting leads. Further improvements can be achieved through local electrostatic gating of the leads, as this leads to an enhanced superconducting gap [32].

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## LIST OF PUBLICATIONS

- D. J. van Woerkom, A. Proutski, B. van Heck, D. Bouman, J. I. Väyrynen, L.I. Glazman, P. Krogstrup, J. Nygård, L.P. Kouwenhoven, A. Geresdi *Microwave Spectroscopy of Spinful Andreev bound states in Ballistic Semiconductor Josephson junctions*, published in Nature Physics 13, 876-881 (2017).
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- J. van Veen, A. Proutski, T. Karzig, D. I. Pikulin, R. M. Lutchyn, J. Nygård, P. Krogstrup, A. Geresdi, L.P. Kouwenhoven, J. D. Watson, *Magnetic field dependent quasiparticle dynamics of nanowire single-Cooper-pair transistors*, published in Phys. Rev. B 98, 174502.
- A. Proutski, D. Laroche, B. van 't Hooft, P. Krogstrup, J. Nygård, L.P. Kouwenhoven, A. Geresdi, Broadband microwave spectroscopy of semiconductor nanowire-based Cooper-pair transistors, published in arXiv:1901.10992.
- 1. D. Laroche, D. Bouman, D.J. van Woerkom, **A. Proutski**, C. Murphy, D.I. Pikulin, C. Nayak, R.J.J. van Gulik, J. Nygård, P. Krogstrup L.P. Kouwenhoven, A. Geresdi, *Observation of the*  $4\pi$ -periodic Josephson effect in indium arsenide nanowires, published in Nature Communications **10**, 245 (2019).