WAVELET ANALYSIS OF BROADBAND SIGNALS TO EXTRACT AMPLITUDE AND FREQUENCY MODULATION: AN APPLICATION TO WALL TURBULENCE

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Abstract Large-scale structures in wall-bounded turbulent flows are known to exhibit a coupling with the small-scale energy in the flow. Besides a superposition of large-scale energy onto the near-wall dynamics, this coupling comprises an amplitude and frequency modulation of the small-scale fluctuations by the large-scale motions. In this work we use wavelet analysis to examine amplitude and frequency modulation. Albeit the wavelet-based approach for amplitude modulation condenses to analyses presented in earlier studies, the strength of the approach becomes evident from a convenient extension of the technique to extract local instantaneous frequency modulation. While discrete techniques were employed previously, an application of a continuous approach results in inherent advantages when phase lags between the large- and small-scale fluctuations in terms of amplitude and frequency modulation are investigated.

INTRODUCTION

Large-scale structures in wall bounded flows are known to modulate the amplitude of small-scale fluctuations. This amplitude modulation (AM) was evidenced initially by Brown & Thomas [4] and Bandyopadhyay & Hussain [1]. Simultaneously, Miksad *et al.* [14] investigated the role of amplitude and phase modulations in the context of laminar-to-turbulent transition. For wall-bounded flows, a considerable interest in AM has developed over the last decade [9, 13, 5, 2, 8]. In recent studies, the turbulent velocity signal in the boundary layer is decomposed into large- and small-scale components; the inner-normalized separation scale is typically taken as $\lambda_x^+ = 7000$ and the local mean velocity is used in determining the separation frequency [9, 13]. Subsequently, the impact of large-scales on the small-scale amplitude was explored using averages of the small-scale intensity conditioned on positive or negative fluctuations of large-scale events [9, 10]. Mathis *et al.* [13] introduced a robust scheme to quantify the degree of AM. The scheme relies on a correlation of the large-scale component with an envelope of the small-scale fluctuations, found through a Hilbert transform. Correlations across the boundary layer revealed how AM becomes stronger with increasing Reynolds number and how AM is present beyond the edge of the log-region [13, 2, 8] and for all three components of velocity [16]. Furthermore, large-scale superposition and modulation effects were found to be essential ingredients for a predictive model for wall-turbulence statistics [12].

In contrast to AM, a less robust scheme exists for quantifying frequency modulation (FM), which was addressed for wall turbulence by Ganapathisubramani *et al.* [8]. This work was based on a discrete technique where a count of the number of local extrema in the small-scale signal was used as a representation of the dominant small-scale frequency. Statistical views revealed that FM is absent for wall-normal locations above approximately 100 wall units ($Re_\tau = 14,150$). It was also shown that FM encompasses a lag –a phase modulation– between the large- and small-scale fluctuations, similar to the phase lag present in AM [1, 5]. Although a physical mechanism exists that includes the majority of the modulation characteristics [8], there continues to be a lack of understanding as to how these modulation lags are related, how they are arranged with respect to internal shear layers of strong vorticity, and how they scale with Reynolds number.

WAVELET ANALYSIS FOR STUDYING AMPLITUDE AND FREQUENCY MODULATION

Uniquely quantifying amplitude and frequency modulation embedded within a *broadband* signal remains a challenge. Here we extract the modulation via time-frequency analysis in the form of continuous wavelet transforms [7]. This approach is particularly suitable for investigating how a certain range of low frequencies modulate a range of higher frequencies. Modulations of this type are present in a variety of research areas, such as seismology [15] and wind turbine acoustics [11]. The ideas of an instantaneous frequency (IF) and time-preserving AM and FM have been around for decades. Many seminal contributions to the literature on this topic exist in the fields of signal processing and communications; see for example [3, 6]. Here we consider the technique in the context of wall-bounded turbulence and our approach is summarized as follows. At first, the signal's energy is decomposed in time-frequency space to preserve the temporal variations of the energy content in frequency space. The wavelet power spectrum (WPS) presents the signal's energy as a two-variable function of time and frequency and is computed using multiple wavelets; here we consider just the Morlet wavelet for brevity. Secondly, a frequency scale is chosen to demarcate the large- and small-scale components, as discussed before ($\lambda_x^+ = 7000$). For the range of small scales, the WPS can be represented by the total energy –or its square root– that is contained within the spectrum at each instance in time (Parseval's theorem) and an IF that is computed via the first spectral moment of the pre-multiplied spectrum. The constructed time traces of the small-scale energy and IF are then analysed through conventional correlation techniques. The advantage of this technique is the preservation of time and a physical representation of the instantaneous small-scale frequency, while previous techniques applied to wall

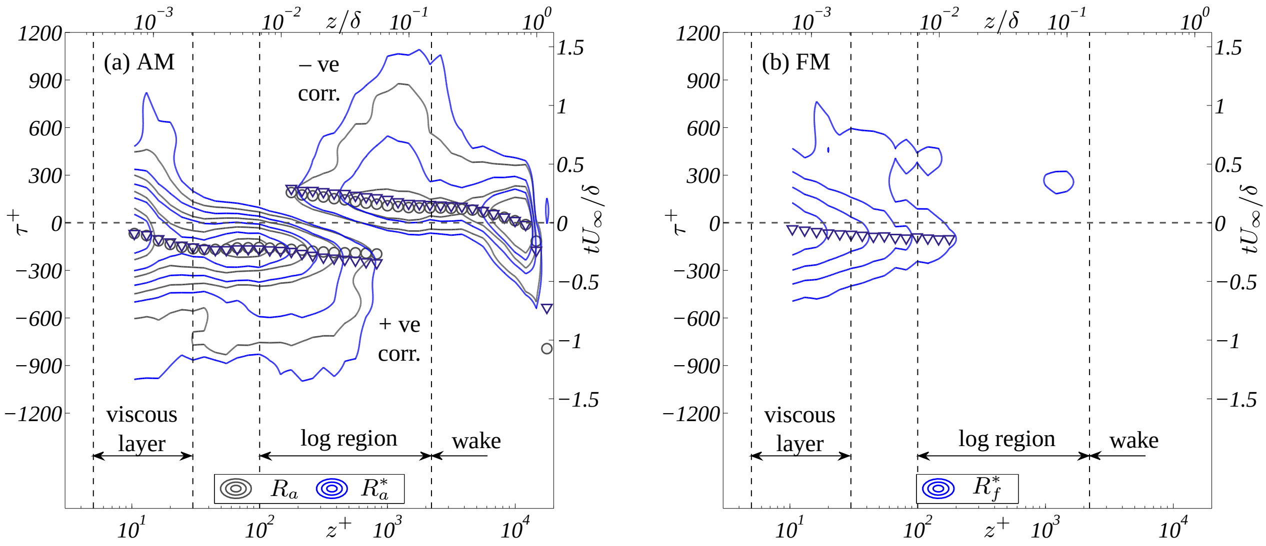

Figure 1. (a) Amplitude modulation coefficients as function of phase shift. Lowest level: -0.3, highest level: 0.5 (step 0.1; 0 contour omitted). (b) Frequency modulation coefficient as function of phase shift. Lowest level: 0.1, highest level: 0.4 (step 0.1). A negative phase shift ($\tau < 0$) represents a temporal lead of the small-scale signal with respect to the large-scale signal for a fixed observer.

turbulence can be susceptible to noise affiliated with experimental or numerical campaigns.

The technique is applied to velocity signals acquired in high-Reynolds-number turbulent boundary layers. Fig. 1a shows contours of the AM coefficient [13], which is the normalized correlation coefficient computed from the large-scale signal and small-scale amplitude time trace, across a boundary layer at $Re_\tau = 14,750$. The temporal phase shift between the large- and small-scale signals is denoted by τ. The trend is consistent with previous studies ([1, 9, 5] among others). The small-scale amplitude signal shows a lead ($\tau < 0$) of up to 200 inner-scaled time units and a phase reversal resides in the middle of the log-region. By employing the time trace of the IF in a similar manner, we obtain the FM coefficient (Fig. 1b). As identified before [8], FM is only significant near the wall ($z^+ < 200$). Moreover, a lead of the IF is observed, and equals approximately half the lag associated with AM. Thus, from a fixed-observer perspective, an increase of the small-scale amplitude is noticed at first, followed by an increase of the IF of these scales. Finally, the low-to-high speed inflection point of the large-scale signature passes by. The full scope of work will include an investigation of these phase lags for various Reynolds numbers, their association to the vorticity signature and their arrangement with respect to internal shear layers bounding uniform momentum zones. Henceforth, a physical model will be presented that builds upon established views [1, 12, 5, 8].

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