Non-Linear Finite Element Analyses of Existing Reinforced Concrete Bridge Beams

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Summary

Three years ago, the Dutch Ministry of Infrastructure and the Environment initiated a project to re-evaluate the carrying capacity of existing bridges and viaducts (e.g. reinforced and pre-stressed concrete beams and slabs). Due to the increase of traffic and the reallocation of emergency lanes to traffic lanes, the safety verification of some concrete structures are not satisfied if the usual analytical procedures are followed. Nonlinear finite element analyses (NLFEA) are considered as one of the alternatives for the verification of the structural carrying capacity. Building codes hardly provide specific guidance on how NLFEA should be carried out and reported. Within the project guidelines for NLFEA have been developed in order to reduce model and user factors.
The aim of the project well fits with the philosophy of the new Model Code 2010 (MC2010) that proposes analytical and numerical calculation methods for the evaluation of the design resistance of reinforced concrete (RC) structures.
In the paper four reinforced concrete beams, characterized by different failure modes, have been analyzed through the analytical and numerical procedures proposed by the Model Code 2010 and following the Dutch guidelines. The results obtained have been compared with the experimental results available in literature.
Furthermore in order to focus on the main sensitive parameters that influence the results obtained from NLFEA and to obtain reliable and, at the same time, safe results, parametric studies have also been carried out on the beams. The main indications of the guidelines for reinforced concrete beams are presented in the paper.

Keywords: reinforced concrete beams, nonlinear finite element analyses, shear resistance evaluation, guidelines.

1. Introduction

The Dutch Ministry of Infrastructure and the Environment initiated a project to re-evaluate the carrying capacity of existing bridges and viaducts (e.g. reinforced and prestressed concrete beams and slabs) due to the increase of traffic and the reallocation of emergency lanes to traffic lanes in the last years. For some of the concrete structures, especially for shear critical structures, the safety verifications are not satisfied if the usual analytical procedures are followed. For this reason the Dutch Ministry of Infrastructure and the Environment proposed to develop guidelines, recently published in [1] for nonlinear finite element analyses (NLFEA) that users should follow in order to reduce model and users factors. Even now that NLFEA are more and more becoming a usual instrument in the current design process, building codes do not provide specific guidance on how to perform these analyses. For this reason the availability of guidelines on NLFEA becomes of interest for civil engineers.

The aim of the project well fits with the philosophy of the *fib* model code MC2010 [2] which proposes different calculation methods for the evaluation of the design shear resistance of RC structures. In particular for slender elements (e.g. reinforced and prestressed concrete beams and slabs) the design shear resistance can be evaluated through analytical and numerical procedures that belong to different levels of approximation: by increasing the level of approximation the complexity and the accuracy of the results obtained increases. Level of approximation I, II and III refer to analytical calculation methods (hand calculations) while the highest level of approximation, level IV, refers to numerical methods, performed with NLFEA. Within level IV the results obtained from NLFEA are properly reduced, in order to obtain the same safety level of analytical calculations, according to three alternative “safety format methods” denoted as Partial Factor method (PF), Global Resistance Factor method (GRF) and Estimation of Coefficient of Variation of resistance method (ECOV).

In the current paper four RC beams characterized by bending and shear failure have been analyzed with standard analytical procedures and with NLFEA, carried out with the software DIANA [3]. The results obtained have been compared with the experimental results available in literature.

The design resistance of the beams failing in shear has been evaluated analytically and numerically, covering the four levels of approximation of the MC2010 [2]. For beams failing in bending the design resistance has been evaluated analytically with standard sectional analyses and numerically applying the safety format methods (level IV) proposed by the MC2010.

This paper is part of an assessment of the applicability of NLFEA for structural verification of existing concrete bridges, following the safety formats of the MC2010 in combination with guidelines for the NLFEA. Aiming at reliable and safe results, various aspects of the cracking models used in NLFEA, including fixed versus rotating crack models, are investigated via a parametric study.

2. Design shear resistance following different levels of approximation

In section 2.1 and 2.2. the main instructions of the MC2010 for the evaluation of the design shear resistance with analytical and numerical procedures are presented. As mentioned in section 1, for beams failing in bending the design bending resistance has been calculated analytically with classical sectional analyses and numerically applying the prescriptions of the MC2010 described in section 2.2.

2.1 Analytical calculation methods (Level I, II, III)

As a general rule, level of approximation I may be used for the conception or the design of a new structure, level II is appropriate for the design of a new structure as well as for a general or brief assessment of an existing member and level III, or higher, may be used for the design of a member in a complex loading state or a more elaborate assessment of a structure.

In general the design shear resistance V_{Rd} is determined as the summation of concrete and steel contributions and it is limited to a maximum value $V_{Rd,max}$.

For beams without shear reinforcement the design shear resistance is calculated with level I and II while for beams with shear reinforcement the design shear resistance is calculated with level I, II and III. The detailed calculations of V_{Rd} are given in section 7.3 of MC2010.

2.2 Numerical calculation methods (Level IV)

For the application of the three safety format methods (GRF, PF, ECOV) different material properties are required as input data in NLFEA and the peak loads reached from the analyses are reduced in order to obtain the same safety level as analytical procedures.

In
Table *1* the mechanical properties required in the analyses and the calculation methods for the evaluation of the design shear capacity (P_d), starting from the peak load obtained in the analyses (P_u) are summarized. Further details are given in section 7.11 of MC2010.

Table 1: Safety format methods.

	Input mechanical properties	Design shear capacity
GRF	-mean	$P_d = \frac{P_{u,m}}{1.27}$
PF	-design	$P_d = P_{u,d}$
ECOV	-characteristic -mean	$P_d = \frac{P_{u,m}}{1.06 \cdot \gamma_R}$; $\gamma_R = \exp\left(0.8 \cdot 3.8 \cdot \left[\frac{1}{1.65} \ln\left(P_{u,m} / P_{u,c}\right)\right]\right)$

3. Case studies

In the paper four RC concrete beams have been analyzed as case study. The beams were tested in laboratory during different experimental programs and the experimental results are available in literature. In Fig. 1 an overview of the case studies is presented. The beams are denoted as RB1 [4], RB2 [5], RB3 [6] and RB3A [6].

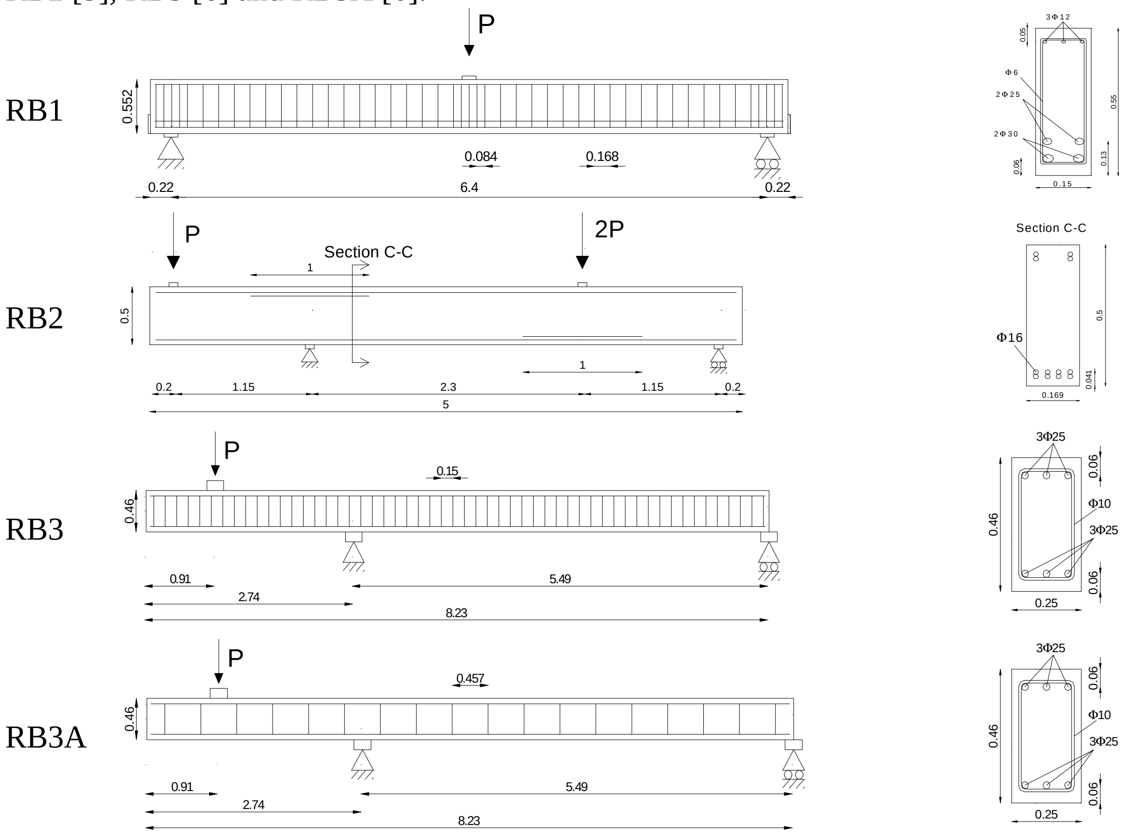

Fig. 1: Case studies: RB1, RB2, RB3, RB3A.

In Table 2 the main geometrical and mechanical features, the ultimate experimental load and the experimental failure mode detected are summarized.

Table 2: Geometrical and mechanical features of the cases study and experimental failure mode and ultimate load.

	L (m)	h (m)	A_c (m^2)	A_{sl} (m^2)	f_{cm} (MPa)	E_{cm} (MPa)	E_s (MPa)	Failure mode	$P_{u,exp}$ (KN)	
RB1	6.84	0.552	0.084	0.0024	43.5	34195	200000	B	265	
RB2	5.00	0.500	0.084	0.0008	53.0	36283	200000	DT	69	81
RB3	8.23	0.457	0.114	0.0015	31.2	30950	200000	B	142	
RB3A	8.23	0.457	0.114	0.0015	31.2	30950	200000	SC	156	

B=bending failure, DT=diagonal tension failure, SC=shear-compression failure

4. Finite element model

The beams have been analyzed with the finite element software DIANA [3] with a 2D modelling. The modelling of the beams follows the guidelines for NLFEA published in [1].
For concrete eight-noded quadrilateral isoparametric plane stress elements based on quadratic interpolation and (3x3) Gauss integration have been adopted. Embedded reinforcement elements, with the hypothesis of perfect bond, have been used to model reinforcement. Interface elements having no-tension behavior have been inserted between the beams and the loading/support plates.
The stress-strain relationship of the concrete uses an exponential softening law in tension, based on the definition of the fracture energy in tension, G_f, [2], [7] and a parabolic law in compression, based on the definition of the fracture energy in compression, determined as $G_c=250G_f$ according to [8]. For steel an elasto-plastic model with hardening law has been used.
The analyses have been carried out in load control using a regular Newton-Raphson scheme with a convergence criterion based on energy and force. Further modelling details of the beams can be found in [9], [10].

4.1 Crack model

The relations used in the crack model implemented in DIANA, like in most other commercial software, are for a wide range of use and therefore based on simplified modelling of the nonlinear behaviour of concrete and steel and concrete-to-concrete and concrete-to-reinforcement interface behaviour.
A total strain rotating crack model [11] and a total strain fixed crack model have been adopted in the analyses. Upon cracking the Poisson's coefficient linearly decreases from its initial value down to zero as the residual tensile stress becomes zero. Besides the Poisson's effect, also effects of biaxial stress states on the compressive strength of concrete are taken into account. The reduction of the compressive strength due to lateral cracking follows the Model B of Vecchio et al. [12]; only the compressive strength and not the peak strain is reduced, leading to a reduction of the Young's modulus already in the elastic phase.
Within the fixed crack model, the shear stiffness is gradually reduced after cracking through the shear retention factor. Both a constant or a variable shear retention factor, that decreases from 1 in the elastic phase, down to zero as the Young's modulus and Poisson's coefficient reduce, can be adopted.
A more refined structural assessment can in general be obtained if some important parameters of the crack model like aggregate interlock, tension stiffening, multi-axial stress-state etc. are taken into account in the material model [13], [14], [15], [16].

5. Results

A parametric study has been carried out on the beams by varying some aspects of the crack model in order to investigate their effects on the results of NLFEA. In Table 3 the variations of parameters and crack concepts are summarized.

Table 3: Parametric study.

RB1							
Analysis	Rotating crack model			Fixed crack model			
	ν	$f_{c,red}/f_c$	G_f	β	ν	$f_{c,red}/f_c$	G_f
Analysis 1	0.0	No limit*	MC1990	/	/	/	/
Analysis 2	Variable	No limit*	MC1990	/	/	/	/
Analysis 3	Variable	0.6#	MC1990	/	/	/	/
Analysis 4	Variable	0.6#	MC2010	/	/	/	/
Analysis 5	/	/	/	Variable	Variable	0.6#	MC1990
Analysis 6	/	/	/	Variable	Variable	0.6#	MC2010

RB2							
Analysis	Rotating crack model			Fixed crack model			
	ν	$f_{c,red}/f_c$	G_f	β	ν	$f_{c,red}/f_c$	G_f
Analysis 1	0.0	No limit*	MC1990	/	/	/	/
Analysis 2	Variable	No limit*	MC1990	/	/	/	/
Analysis 3	0.0	No limit*	MC2010	/	/	/	/
Analysis 4	Variable	No limit*	MC2010	/	/	/	/
Analysis 5	/	/	/	0.1	0.0	No limit*	MC1990
Analysis 6	/	/	/	Variable	0.0	No limit*	MC1990
Analysis 7	/	/	/	Variable	Variable	No limit*	MC1990
Analysis 8	/	/	/	Variable	Variable	No limit*	MC2010

RB3							
Analysis	Rotating crack model			Fixed crack model			
	ν	$f_{c,red}/f_c$	G_f	β	ν	$f_{c,red}/f_c$	G_f
Analysis 1	0.0	No limit*	MC1990	/	/	/	/
Analysis 2	0.0	0.6#	MC1990	/	/	/	/
Analysis 3	Variable	0.6#	MC1990	/	/	/	/
Analysis 4	Variable	0.6#	MC2010	/	/	/	/
Analysis 5	/	/	/	0.1	0.0	No limit*	MC1990
Analysis 6	/	/	/	Variable	0.0	No limit*	MC1990
Analysis 7	/	/	/	Variable	0.0	0.6#	MC1990
Analysis 8	/	/	/	Variable	Variable	0.6#	MC1990
Analysis 9	/	/	/	Variable	Variable	0.6#	MC2010

RB3A							
Analysis	Rotating crack model			Fixed crack model			
	ν	$f_{c,red}/f_c$	G_f	β	ν	$f_{c,red}/f_c$	G_f
Analysis 1	0.0	No limit*	MC1990	/	/	/	/
Analysis 2	Variable	No limit*	MC1990	/	/	/	/
Analysis 3	Variable	0.6#	MC1990	/	/	/	/
Analysis 4	Variable	0.6#	MC2010	/	/	/	/
Analysis 5	/	/	/	0.1	0.0	No limit*	MC1990
Analysis 6	/	/	/	Variable	0.0	No limit*	MC1990
Analysis 7	/	/	/	Variable	0.0	No limit*	MC1990
Analysis 8	/	/	/	Variable	Variable	0.6#	MC1990
Analysis 9	/	/	/	Variable	Variable	0.6#	MC2010

*If a lower limit to the reduction of the compressive due to lateral cracking is not set as input data the reduction can reach any value

the maximum reduction of the compressive strength due to lateral cracking allowed is set as 40% ($f_{c,red}/f_c$=0.6)

In Fig. 2 the load-deflection curves obtained from the parametric study are given. The failure mode of each beam has been well predicted both by NLFEA and by analytical calculations that gave consistent results. The RB1 beam failed in bending with yielding of bars after crushing of concrete, RB2 failed in diagonal-tension shear and RB3 failed in bending with crushing of concrete after yielding of bars. RB3A failed in shear with yielding of stirrups even if, by hand calculation it was detected that the load value corresponding to bending failure was almost equal to the load value corresponding to shear failure.

The analyses that best fit with the experimental results are Analysis 4, Analysis 3, Analysis 4 and Analysis 1, respectively for RB1, RB2, RB3 and RB3A. These analyses are highlighted in Table 3.

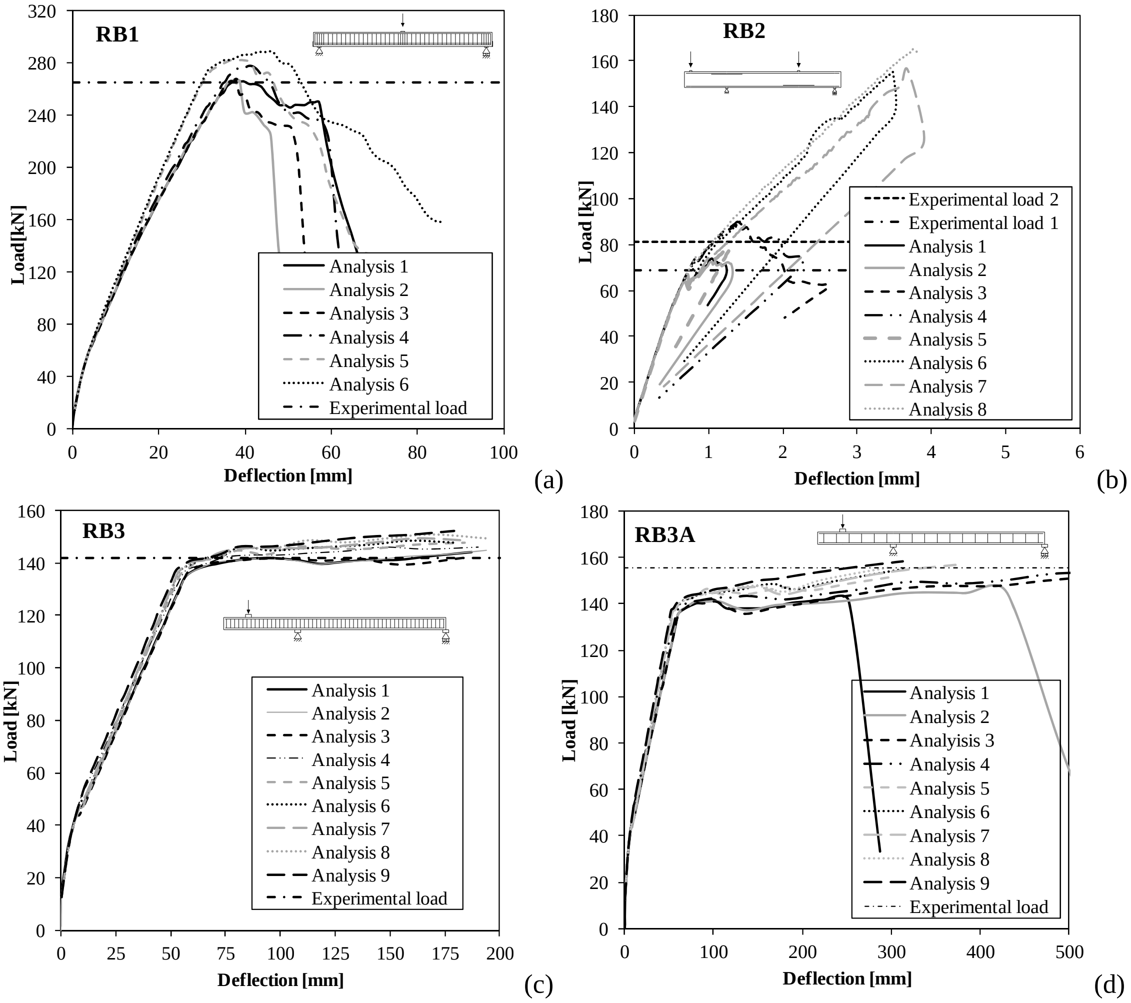

Fig. 2: Parametric study: load-deflection curves of (a)RB1, (b)RB2, (c)RB3, (d)RB3A beams.

From Fig. 2 it can be noted that, as expected, the sensitivity to the failure mode of the beams strongly depends on some aspects of the crack model used: the greatest effect of the modelling used is detected for shear critical beams.

For beams failing in bending (RB1, RB3) the peak load is well predicted for all analyses. For the RB1 beam, in which concrete crushed before reaching the peak, the greatest effect is given, as expected, by the fracture energy in compression in the post-peak behaviour (analysis 3-4 and 5-6).

For RB3A, that failed in shear but showed however a ductile behaviour, the sensitivity to the crack model is rather limited.

For the RB2 beam, which is without shear reinforcement and failed in diagonal-tension, the analyses carried out with the fixed crack model substantially overestimate the peak load. As a matter of fact, both the peak load and the stiffness in the cracked phase of shear critical structures without shear reinforcement are strongly influenced by the concrete-to-concrete interface behaviour,

especially by the aggregate interlock effect. For this reason, if the aggregate interlock effect is not properly modelled, the structural prediction can be wrongly interpreted. In particular the shear retention factor that models the aggregate interlock effect implemented in DIANA software is based on a simplified modelling (see section 4.1). More refined modelling of the aggregate interlock effect can be found for example in [14], [15], [16].
In Fig. 3 the design resistance of the beams, P_d, determined with analytical and numerical procedures proposed by the MC2010, are expressed as a percentage of the experimental load $P_{u,exp}$ (black histograms). The grey bars refer to the design load values P_d obtained by applying analytical and numerical procedures according to the MC 2010, while the white bars refer to the results obtained from NLFEA without applying any safety coefficients.

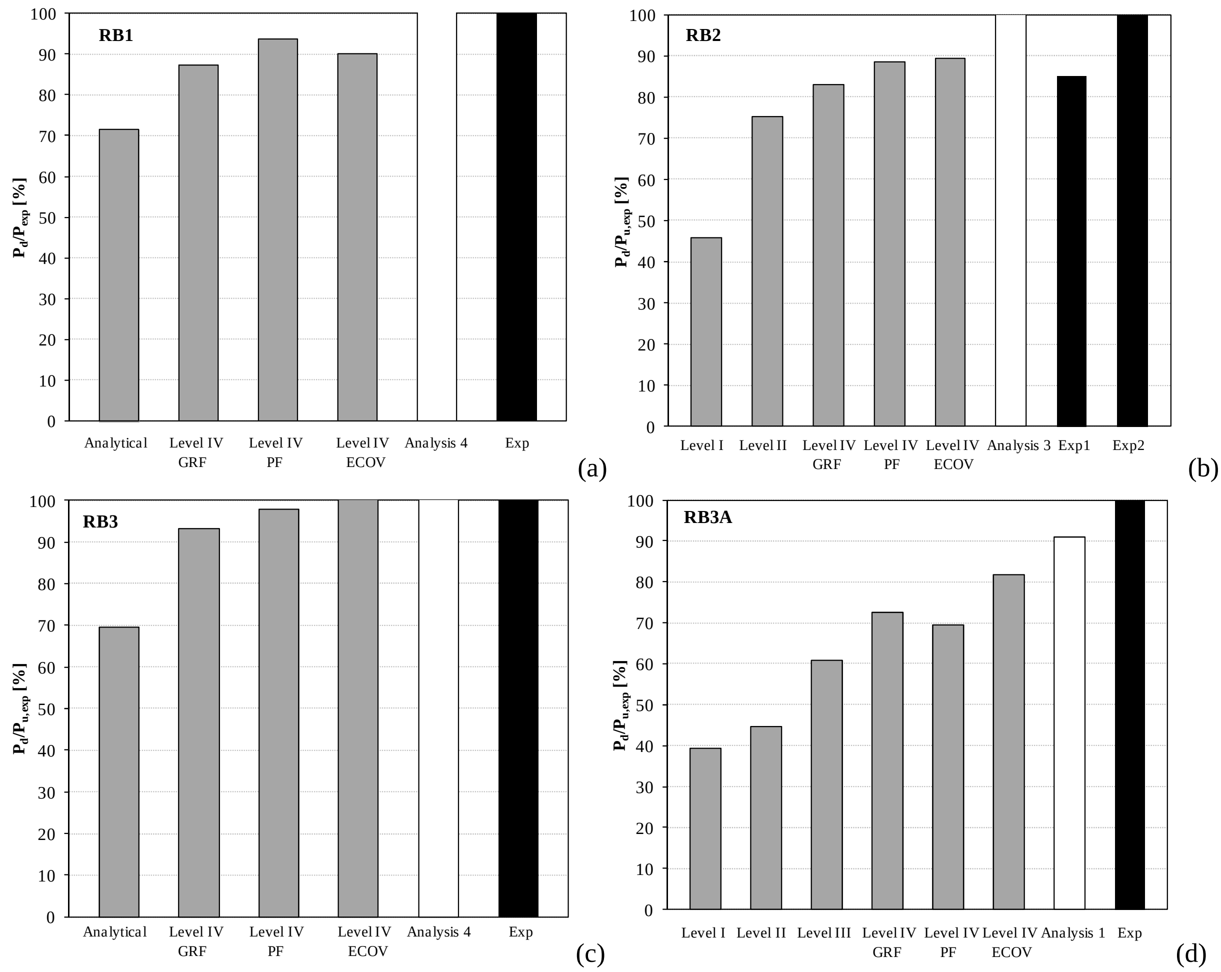

Fig. 3: Design resistances P_d obtained analytically and numerically expressed as a percentage of the experimental ultimate load $P_{u,exp}$ for (a) RB1, (b) RB2, (c) RB3, (d) RB3A.

Fig. 3 shows that for all beams the results well match with the philosophy of the MC2010: by increasing the level of Approximation the accuracy of the results and the design resistance increase. The design resistance obtained from NLFEA is higher than the design resistance obtained with analytical calculations (increases ranging from 17% to 33%). NLFEA reveal “hidden” capacities of the structure and give a refined structural assessment, assuming that the models have been verified adequately.

6. Final remarks

In the paper the nonlinear behaviour of four RC beams characterized by different failure modes has been investigated with NLFEA, following the Dutch guidelines for nonlinear finite element analyses recently published in [1]. The results obtained from NLFEA have been compared to the experimental results available in literature. The design resistance of the beams has been also evaluated with analytical and numerical procedures applying the calculation methods proposed by the MC2010.

The main conclusions of the research are summarized below.

- The MC2010 proposes analytical and numerical calculation methods for the evaluation of the design resistance of concrete structures. In the paper the MC2010 guidelines have been systematically applied to RC beams failing both in shear and in bending.
- The results obtained are in good agreement with the philosophy of the MC2010 for all type of failure modes. Level of approximation IV, determined from NLFEA and using safety formats, provides substantially higher design resistance values than the design resistance obtained from analytical calculations.
- NLFEA is a powerful calculation instrument for a refined structural assessment, able to take into account for real material properties and hidden capacities of the structure. Nevertheless the sensitivity of the results obtained from NLFEA depends on the modeling choices, like element type, crack model, convergence parameters, etc. In particular, some aspects of the crack model strongly influence the results obtained, especially for shear critical beams. If the concrete-to concrete and concrete-to-reinforcement interface behavior (e.g. aggregate interlock effect, tension stiffening effect, dowel action effect) are modeled with coarse and simplified constitutive relations, the structural prediction can be wrong.

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