Surgery Scheduling

Dealing with uncertainty in surgery duration

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by

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Abstract

Scheduling surgeries in a hospital efficiently is a hard, but necessary task, because the Operating Rooms (ORs) contribute for about 40% of a hospitals total expenses. Therefore, we want to maximise the utilisation of the ORs. However, we want to prevent overtime, since there already is a lot of pressure on hospital employees. The overtime is not easily calculated, because of the stochastic nature of the surgery duration. In this thesis, we focus on maximising the utilisation of the ORs, without creating too much overtime. We model the scheduling of surgeries as integer linear programs (ILPs), which determine how many surgeries can be planned with the maximisation of the utilisation of the ORs as the objective. Different methods are used to include the overtime constraint and the resulting schedules are then compared and we take our conclusions.

In our research, we use data provided by an academic hospital in the Netherlands, which provides specialties, patient groups, a Master Surgery Schedule (MSS) and historical surgery durations. It also provides a minimum number of surgeries that have to be performed for each patient group.

Preferably, we want to avoid overtime altogether. This however is not possible, due to the stochastic nature of the surgery duration. Instead we formulate an overtime constraint, by setting a probability that a surgery has to end within the opening hours. We call this, the overtime constraint.

The premise of the first model is to create all possible combinations of surgeries which do not exceed the overtime constraint. In this model we create a variable which gives the distribution of the total surgery duration in a single OR on a single day. However, when adding multiple surgeries together we first need a distribution for the surgery duration. Unfortunately, we found that the surgery durations follow a log-normal distribution, which is supported by literature as well. The sum of log-normally distributed variables does not have a closed form, making it difficult to add the surgery durations together. To calculate the distribution of the total surgery duration, we use the Fenton-Wilkinson method.

The second model discretises the opening hours of each OR into time blocks. We then define the probability that a surgery ends within a certain number of time blocks to incorporate the surgery duration. This way we can define a Mixed Integer Linear Program (MILP) without relying on any specific distribution. At first, we use historical data to create the probability that a surgery ends within a certain number of time blocks. However, to ensure we compare the different models fairly, we use the same log-normal distributions as the previous model and discretises them.

We found that the Column Based Approach has a high utilisation and computes the columns and solves the ILP faster than the discrete model. Despite this, overtime can still occur with this approach. However, the number of times overtime occurs, remains well within acceptable limits. Additionally, the probability of no overtime directly impacts utilisation, as intuitively expected.

Lastly, due to time constraints, we did not find an optimal solution for the discrete models. Moreover, both suboptimal solutions seemed to have a lot more overtime. This needs us to believe, there is an error. This error could either be in our implementation of the model, or the model itself. Lastly, we found that the discrete solution using fitted data, generates a schedule which has less overtime and is faster to solve.

Finally, we could not conclude which method generates the best schedule according to their utilisation. However, the Column Based Approach is a lot faster and has less overtime compared to the discrete solution generated using fitted data.

Layman's Summary

In this thesis we compare different models for scheduling surgeries in a hospital, where we want to utilise the OR as much as possible. One of the constraints is that we want to restrict the overtime. However, the surgery duration is not set, because complications can happen. This resulted in finding different methods to deal with the variability of the surgery duration.
The first method creates all possible combinations for which the overtime constraint is met and use these combinations to create a schedule. The second method distributes the opening hours into time blocks and defines a probability that a surgery ends within a certain number of time blocks.
We found that creating all possible combinations results in a faster method, higher utilisation and less overtime.

Contents

1 Introduction

The operation room (OR) is one of the most expensive rooms in a hospital. The ORs contribute for about 40% of a hospitals total expenses and around 65% of all hospital admissions have a surgical component. This raises the need to maximise the utilisation of the ORs. This could improve both the patient flow, reduce waiting times for patients, prevent cancellations and even perform more surgeries or close an OR altogether, thus relieving some pressure for the medical staff.

Unfortunately complications can happen during surgery, which results into a variable surgery duration. Because of this variability it is hard to create a robust surgery schedule, since we want to prevent overtime as much as possible as well. Overtime occurs when a surgery is still in progress when the OR is supposed to be closed.

The problem is formulated as a Mixed Integer Linear Program (MILP), where all restrictions are linearly formed. The overtime constraint however is non-linear because the surgery duration is log-normal distributed. The objective of this thesis is to compare different methods that deal with this non-linear overtime constraint. To compare the different methods, we use the following three criteria:

1. Total utilisation of the OR
2. Total overtime created by the schedule
3. Total runtime of the program

The different methods compared are a column based approach and a discrete approach. In the column based approach, for each specialty we create all possible combinations of surgeries for which the overtime constraint is not exceeded. By solving the ILP using these columns we select the most optimal combination of these columns and create our schedule. To generate the columns, we first model the surgery duration with a log-normal distribution for each patient group, which is supported by literature. Using the Fenton-Wilkinson method, we can add the different distributions together and find the parameters for the distribution of the total surgery duration. This distribution is then used to check if the column do not exceed the overtime constraint.

In the discrete approach, we discretise the opening hours into time blocks, which leads to a discretised probability distribution for the surgery duration. Because the surgery duration is now discretised, we can reformulate the overtime constraint to be a linear constraint. This new reformulated ILP is solved for each different specialty, creating a schedule per specialty. The overall schedule is achieved by combining all these schedules. In the discrete approach we compare two different models for the surgery duration. We model it using historical data directly and we model it using fitted distributions from the historical data.

The structure of this thesis is as follows. First in Chapter 2 we give a literature review in which different methods to deal with the stochastic surgery duration are discussed. Next in Chapter 3, we create a general model in which we find a non-linear constraint, which is addressed in Chapter 4. There we discuss two methods to deal with the non-linear constraint. Here, we also give multiple options to model the surgery duration. In Chapter 5, we discuss the given data, explain how we use it and show the fitted data and how we use it as input for the models. Next in Chapter 6 we analyse the results of our methods. Finally, in Chapter 7 we draw our conclusions and give our recommendations for future research.

2 Literature

In the past few decades, there has been a lot of research focusing on maximising the utilisation of operating rooms (ORs). This literature study aims to further increase the knowledge needed to handle our research. For a more comprehensive understanding of the overall problem, we refer to Cardoen et al. [1], Guerriero and Guido [3], Zhu et al. [14] and Wang et al. [13].

2.1. Modelling Surgery Duration

Every operating room has certain opening hours for each day of the week. Surgeries can only be planned during these opening hours. Overtime occurs when a surgery is still being performed after the OR is supposed to be closed. This is caused by the stochastic nature of the surgery duration, which can be modelled in several different ways.

Most studies assume that the duration of surgeries are normally distributed [4, 5, 7, 11], even though it is widely known that the duration of most surgeries follow a log-normal distribution [8, 9]. The reasons why the log-normal distribution is not used vary, from a simpler implementation to not being the subject of their research.

One of the merits of a normal distribution is that the addition of two normal distributed variables is again normally distributed. However, with the log-normal distribution this is not so simple. When we add a large number of log-normal distributed variables together, the resulting variable is normal distributed. This however is not the case when adding just a small number of variables together. Fenton [2] assumes that the resulting variable is log-normal distributed and uses the Fenton-Wilkinson method to determine the parameters of this log-normal distribution.

2.2. Overtime Constraints

This thesis is not the first to compare different methods to deal with the overtime constraint. Three methods are discussed Van der Tuin [10].

The first method considered is the most simplified version of the constraint. Like Van Oostrum et al. [11], Hans et al. [4] and Lamiri et al. [5], this method considers the surgery duration to be deterministic and the expected duration is used. To prevent over-planning of the OR, the capacity of each OR is adjusted by an overtime factor. The biggest merit of this method is its simplicity. This model is simple, and therefore, easy to solve for considered instances. However, due to it being so simple, it is not realistic and does not model the reality accurately enough.

Secondly, [10] uses the same approach as Schneider et al. [7]. They first assume that the surgery duration has a normal distribution and they rewrite the overtime constraint using this distribution. In doing so, they found a square root function, which is approximated by fitting piecewise linear functions. Using these piecewise linear functions a closed form is achieved and used to solve the resulting ILP. This method models the reality a bit more accurately then the previous method and is still simple to compute. However as said before, it assumes that the surgery duration has a normal distribution, but by Wang et al. [13], we know that a log-normal distribution is more accurate.

Thirdly, Van der Tuin [10] uses a column based approach to create a new ILP with a different input parameter. The column based approach, proposed by Nguyen [6], generates for each specialty all possible combinations of surgeries for every OR with different capacities, which is then used as an input parameter for the new ILP. Each column has a total surgery duration, which is found by adding the surgery durations of the surgeries performed in the column. The MILP has to be rewritten to use these columns as an input, after which the resulting ILP is solved and a schedule is created using the chosen columns.

These columns can be created using both a normal distribution as well as a log-normal distribution for the surgery duration. When using a normal distribution, the total surgery duration also has a normal distribution and easy to calculate parameters. However, when using the log-normal distribution, these parameters are not so clear. Using the Fenton-Wilkinson method, these parameters can be approximated and columns can be generated.

The merit of using the column based approach is that it is versatile, it can be used with multiple different distributed variables, even distributions which are not mentioned in this thesis, without changing the overall model. However it might take a long time to generate these columns.

Finally, Vos [12] proposes a model which maximises the utilisation of the OR, whilst also maintaining a consistent bed occupancy level. First the OR-days are dissected into time blocks, considering post-surgery OR cleaning and specialty-specific requirements. In doing so the surgery duration also is discretised and as such the overtime constraint is also discretised. The surgery duration is calculated using the probability that a surgery ends within a certain number of time blocks.

The model created by [12] has a lot of variables and as such is solved using column generation. In column generation, the model is split into a Master Problem (MP) and pricing subproblems. In the MP a small subset of possible columns are considered and then solved. Using the pricing subproblems, new columns are generated to be considered in the MP. Vos [12] generates columns for each specialty, which allows most constraints to be considered in the pricing subproblems. The reason why column generation is used in this way, is because the Master Surgery Schedule used by Vos [12] allowed multiple specialties on the same OR-day. We rewrite the used model to more accurately compare column generation to the previous mentioned methods and as such we disregard the impact of a surgery on ward bed occupancy.

The main merit of using this approach is that it can be used with any probability distribution. The discretisation allows for an easier calculation of the probability that the surgeries finish in time. The drawbacks however, is that the number of variables and constraints is much larger, resulting in a longer computation time.

Van der Tuin [10] concludes that the column based approach using the log-normal distribution yields the best results. Thus, in this thesis, we compare this method to the discretised method proposed by [12].

3

Mathematical Model

The goal of this thesis is to create and use a model to schedule surgeries, such that the ORs are utilised as much as possible. This chapter starts with an introduction of the different sets. Next, the different constraints are explained and formulated. Finally, the objective function is formulated.

To create a more readable problem, we first determine and define the different sets, variables and parameters. First of all, every hospital has different specialties (S) which need OR time. Every hospital has a schedule which divides the ORs over the different specialties found in a hospital, which is found in the Master Surgery Schedule (MSS). This schedule shows which specialty has access to each OR on each day and repeats every few weeks. The other two sets are the different ORs (O) and the different days in one cycle of the MSS ($\mathscr{D}$). To speed up the model, we define the set $D \subset \mathscr{D}$ as all the weekdays in the MSS, since elective surgeries can only take place on weekdays. For an OR $o \in O$ and day $d \in D$, we call the pair (o, d) an OR-day.

Since there are a lot of different surgeries that can take place in a hospital, elective patients are clustered into patient groups based on the surgical procedure they need and the historical data about this procedure. We define G_s to be the set of these patient groups for specialty $s \in S$ and define $G = \bigcup_{s \in S} G_s$ to be the total set of patient groups.

To maximise the utilisation of the ORs, we want to formulate the problem into a Mixed Integer Linear Program (MILP). We start with the integer decision variable x_{odg} which indicates how many surgeries performed on patient group $g \in G$ are scheduled in OR $o \in O$ on day $d \in D$.

The first constraint is given by the MSS, by setting x_{odg} to zero if specialty $s \in S$ is not scheduled on OR-day (o, d), which is ensured in Constraints (3.1).

$$\sum_{g \in G_s} x_{odg} \le m_{ods} \cdot N_{ods}, \qquad \forall o \in O, \forall d \in D, \forall s \in S. \tag{3.1}$$

Here, m_{ods} is one if a surgery performed on a patient from patient group $g \in G_s$ from specialty $s \in S$ is allowed in OR $o \in O$ on day $d \in D$ and zero otherwise. N_{ods} is the maximum number of surgeries performed by specialty $s \in S$ on OR-day (o, d). By setting a maximum number of surgeries performed, we create an upper bound for the model, which results in a smaller feasible region. We determine N_{ods} by dividing the capacity of the OR-day and dividing this by the lowest average surgery duration of the patient group that is allowed to be scheduled in that OR-day.

Every OR-day (o, d) has different opening hours, which results in the capacity c_{od}. All surgeries should be performed within the opening hours of the ORs, however due to the variability of surgery durations this cannot be ensured. Thus, management has set a probability for which the surgeries should finish within the opening hours of OR $o \in O$ on day $d \in D$ and define this probability as α. We define the random variable Y_g as the surgery duration of patient group $g \in G$. Then, the total surgery duration of an OR-day is defined as $Z_{od} = \sum_{g \in G} x_{odg} \cdot Y_g$ for OR $o \in O$ on day $d \in D$. Constraints (3.2) ensure that the probability of overtime does not exceed $1 - \alpha$.

$$\mathbb{P}(Z_{od} \le c_{od}) \ge \alpha, \qquad \forall o \in O, d \in D. \tag{3.2}$$

As one can see, this constraint is not linear and thus not viable in a MILP, which is addressed in Chapter 4. Finally, to prevent some patient groups from being overlooked, management has set a minimum number of surgeries (β_g) which have to be performed during a cycle of the MSS, for each patient group $g \in G$. This is ensured by the following constraints:

$$\sum_{o \in O} \sum_{d \in D} x_{odg} \geq \beta_g, \qquad \forall g \in G. \tag{3.3}$$

Maximising the utilisation of the ORs is exactly the same as maximising the amount of time the ORs are being used. This is calculated by summing the duration of all performed surgeries during a single cycle through the MSS. The expected surgery duration is used in the objective function. This creates the linear objective function

$$\sum_{g \in G} \sum_{o \in O} \sum_{d \in D} x_{odg} \cdot \mathbb{E}[Y_g]. \tag{3.4}$$

4

Solution Methods

As mentioned before, the surgery duration is not deterministic, but stochastic, which leads to a non-linear constraint. In Section 4.1, we describe what the column based approach is, how this deals with the non-linearity and reformulate the mathematical model given in Chapter 3. Next in Section 4.2, we formulate the discretised model and finally, in Section 4.3, we discuss two different ways which use historical data to model the surgery duration.

4.1. Column Based Approach

In a Column Based approach, we generate per specialty all combinations of surgeries from the patient groups that can be scheduled on the OR-days. We define the set K_{od} as the set of all possible columns for one OR-day. The set K_{od} is inherently dependend on the specialty, because the MSS indicates which specialty can use OR $o \in O$ on day $d \in D$. Note that not all OR-days have the same capacity, thus in creating the columns, we have to take this into account. In example of a column is ENT: [1, 3, 0]. The column contains a single surgery performed on a patient from patient group ENT1, three surgeries performed on a patient from patient group ENT2 and no surgeries performed on a patient from patient group ENT3.

Using these columns, we can translate the original MILP. We need a different decision variable λ_k which is one if column $k \in K_{od}$ is used on OR-day (o, d) and zero otherwise.

There can at most be one of these columns used, leading to the constraints:

$$\sum_{k \in K_{od}} \lambda_k \leq 1 \qquad \forall o \in O, \forall d \in D. \tag{4.1}$$

To track the number of surgeries of group $g \in G$ for every column, we use input parameter x_g^k. Now that this parameter has been introduced, we can rewrite Constraints (3.3) to

$$\sum_{o \in O} \sum_{d \in D} \sum_{k \in K_{od}} \lambda_k \cdot x_g^k \geq \beta_g, \qquad \forall g \in G. \tag{4.2}$$

As for the objective function, the idea is the same. We calculate the total expected surgery duration, however, now we have a different decision variable. The idea is exactly the same as (3.4):

$$\sum_{g \in G} \sum_{o \in O} \sum_{d \in D} \sum_{k \in K_{od}} \lambda_k \cdot x_g^k \cdot \mathbb{E}[Y_g]. \tag{4.3}$$

4.2. Discretised Model

Instead of creating the possible combinations of surgeries beforehand, one can also use a different approach. Vos [12] proposes to discretise the OR-days into time blocks, which we call together the set T. This set of time blocks is defined as all the time blocks which are found on a single day.

Now lets define the other necessary sets. Let $T_{od} \subset$ be the set of all different time-blocks in which the OR $o \in O$ is open at day $d \in D$. Finally, let V_{od} be the possible positions of the surgery performed in OR $o \in O$ on day $d \in D$, i.e. the $v \in V_{od}$'th surgery on that OR-day. We define V_{od} to be the set $\{0, 1, \cdots, N_{ods}\}$.

Since the surgery duration is not deterministic, we define $p_{g\tau} \in [0, 1]$ to be the probability that a surgery performed on a patient from patient group $g \in G$ is finished within $\tau \in T$ time blocks.

4.2.1. Constraints

Due to this new approach, we need to define a new, but similar decision variable. Let f_{odgv} be the decision variable which is one if a patient from patient group $g \in G$ is scheduled to be the $v \in V$-th surgery in OR $o \in O$ on day $d \in D$ and zero otherwise. To ensure we do not add unnecessary decision variables, we only create the variable f_{odgv} if a surgery from patient group $g \in G_s$ from specialty $s \in S$ is scheduled in OR $o \in O$ on day $d \in D$.

We ensure the MSS by Constraints (4.4).

$$\sum_{v \in V} \sum_{g \in G_s} f_{odgv} \leq m_{ods} \cdot N_{ods}, \qquad \forall o \in O, \forall d \in D, \forall s \in S. \tag{4.4}$$

Naturally, there can only be a single surgery starting at each position $v \in V$, which is achieved using constraints (4.5)

$$\sum_{g \in G} f_{odgv} \leq 1, \qquad \forall o \in O, \forall d \in D, \forall v \in V. \tag{4.5}$$

To indicate when a surgery starts, we introduce the variable $b_{odgvt} \in [0,1]$, which gives the probability that the $v \in V$-th surgery performed on a patient from patient group $g \in G$ in OR $o \in O$ on day $d \in D$ starts at time $t \in T$. Obviously, the starting times of surgeries are dependent on the end time of their predecessor and as such we need to calculate the end time. To calculate the end time, the surgery duration is needed. This duration as stated before is stochastic and given by the parameters $p_{g\tau}$ for $\tau \in T$.

If f_{odgv} is zero, we need to ensure that b_{odgvt} is zero as well for all possible starting times. Similarly, the total probability that the start time exists should also be equal to one, when f_{odgv} is one. However, rounding errors can occur, thus we relax the constraints into:

$$\sum_{t \in T} b_{odgvt} \leq f_{odgv}, \qquad \forall o \in O, \forall d \in D, \forall g \in G, \forall v \in V, \tag{4.6}$$

$$\sum_{t \in T} b_{odgvt} \geq 0.9999 \cdot f_{odgv}, \qquad \forall o \in O, \forall d \in D, \forall g \in G, \forall v \in V. \tag{4.7}$$

To reduce the solution space further we formulate the MSS constraint for b_{odgvt} as well, which gives us the Constraints (4.8):

$$\sum_{g \in G_s} \sum_{v \in V} b_{odgvt} \leq m_{ods}, \qquad \forall o \in O, \forall d \in D, \forall s \in S. \tag{4.8}$$

To ensure that we start the first surgery of the day at the moment the OR opens, we introduce by Constraints (4.9).

$$b_{odg0t_{od}^{open}} = f_{odg0}, \qquad \forall o \in O, \forall d \in D, \forall g \in G. \tag{4.9}$$

The variable e_{odgvt} gives the probability that the $v \in V$-th surgery in OR $o \in O$ of day $d \in D$ performed on a patient from patient group $g \in G$ ends in time block $t \in T$. The end time of the $v \in V$-th surgery is dependent on both the starting time as well as the surgery duration. There are multiple ways where the v-th surgery ends in time block $t \in T$. One of which is if the starting time is t_0 and the duration of the surgery is $\tau = t - t_0$. However if the starting time is $t-1$ and the surgery takes just a single time block, we get the desired end time as well. Since everything in between also gives the desired end time, we combine these into one and get Constraints (4.10).

$$e_{odgvt} = \sum_{\tau=1}^{t-t_0} p_{g\tau} \cdot b_{odgv(t-\tau)}, \qquad \forall o \in O, \forall d \in D, \forall g \in G_s, \forall v \in V, \forall t \in T. \tag{4.10}$$

Note that t_0 is not necessarily the same as t_{od}^{open}.

We want to plan as much surgeries as possible on a single day, thus the next surgery is allowed to start directly after the previous one has ended. By setting the probability the next surgery starts to the probability that the previous surgery ended in the time block before. Constraints (4.11) and (4.12) ensure this is the case.

$$b_{odgvt} \leq \sum_{j \in G_s} e_{odj(v-1)(t-1)}, \qquad \forall o \in O, \forall d \in D, \forall g \in G_s, \forall v \in V \setminus \{0\}, \forall t \in T \setminus \{t_0\}, \tag{4.11}$$

$$b_{odgvt} \geq \sum_{j \in G_s} e_{odj(v-1)(t-1)} - (1 - f_{odgv}), \qquad \forall o \in O, \forall d \in D, \forall g \in G_s, \forall v \in V \setminus \{0\}, \forall t \in T \setminus \{t_0\} \tag{4.12}$$

To ensure that b_{odgvt} is set to $\sum_{j\in G_s} e_{odj(v-1)(t-1)}$, when $f_{odgv} = 1$ we need Constraints (4.12). Note that if $f_{odgv} = 0$, the right hand side will be negative, however Constraints (4.6) and Constraints (4.7) ensure that $b_{odgvt} = 0$. These constraints also ensure that there can only be a second surgery if there is a first, a third surgery if there is a second and so on. These constraints do not apply for $v = 0$ since there is no previous end time.

However to limit our search space, we also define this into Constraints (4.13)

$$F_{odgv} \leq \sum_{j\in G_s} F_{odj(v-1)}, \qquad \forall s \in S, \forall o \in O, \forall d \in D, \forall g \in G_s, \forall v \in V\setminus\{0\}. \tag{4.13}$$

There is a probability that a surgery is still being performed after opening hours of the OR, which is inevitable. Thus, management decides an $\alpha \in [0,1]$ such that the $v \in V$-th surgery performed finish within the opening hours of OR $o \in O$ on day $d \in D$ with a probability of at least α. Constraints (4.14) ensures this.

$$\sum_{t\in\{t_0,\cdots,t_{od}^{close}\}} e_{odgvt} \geq \alpha \cdot f_{odgv} \qquad \forall o \in O, \forall d \in D, \forall g \in G, \forall v \in V\setminus\{0\} \tag{4.14}$$

These constraints do not apply to the first surgery of the OR-day, because there could be patient groups for which only one surgery takes the maximum opening hours of that OR-day. By formulating the constraint in this way, these surgeries can still be planned, but only at the start of an OR-day.

To ensure that all surgeries are considered and planned, management has decided that a minimum number of surgeries are performed per surgery group $g \in G_s$, given by β_g. Constraints (4.15) ensure that these surgeries are performed.

$$\sum_{o\in O}\sum_{d\in D}\sum_{v\in V} f_{odgv} \geq \beta_g \qquad \forall g \in G \tag{4.15}$$

As for the objective function, this is in essence exactly the same as in (4.3). This time however we need to sum over all possible positions as well, resulting in the following objective.

$$\sum_{g\in G}\sum_{o\in O}\sum_{d\in D}\sum_{v\in V} f_{odgv} \cdot \mathbb{E}[Y_g] \tag{4.16}$$

4.2.2. Column Generation

In Column Generation the problem is split into a master problem and multiple pricing subproblems. In our case, we create different columns for each specialty. What this means, we use an initial schedule for each specialty, solve the master problem using these schedules and then find new possible schedules using the pricing subproblems. This is a great way to determine a schedule where multiple specialties share an OR on the same day. In our case study however, we do not have this. Thus, we get that the master problem is trivial, we simply solve the multiple pricing subproblems and add the resulting schedules to create a complete surgery schedule.

If the MSS would allow multiple specialties on the same OR-day, this would be a different case, then we would need the master problem to choose a linear combination of the given schedules.

4.3. Surgery Duration

To implement both the Column Based approach as well as the discretised model approach, we need to model the surgery duration with a distribution. There are two candidates we consider, a log-normal distribution and a discrete distribution where both are based on historical data.

Using the historical data, we can fit a log-normal distribution for every patient group. Since it is possible to plan multiple surgeries on a single day, the total surgery duration is a combination of more than one log-normal distributed variable. We assume that this new variable is also log-normally distributed and use the Fenton-Wilkinson method to calculate the parameters of this new distribution.

The Fenton-Wilkinson method is defined as follows: Let $X_0, X_1, \cdots, X_n$ be log-normally distributed variables with parameters $\mu_i, \sigma_i \forall i \in \{0, \cdots, n\}$, then we assume $X = \sum_{i=1}^{n} X_i$ is again log-normally distributed. We can calculate the expectation and the variance by:

$$\mathbb{E}[X_i] = e^{\mu_i + \frac{\sigma^2}{2}}, \tag{4.17}$$

$$\mathrm{Var}[X_i] = (e^{\sigma^2} - 1) \cdot \mathbb{E}[X_i]^2. \tag{4.18}$$

To calculate the parameters for X using the Fenton-Wilkinson method explained in Fenton [2] we need the expectation and variance of X first, which is simply the sum of the expectations and variances of X_i:

$$\mathbb{E}[X] = \sum_{i=0}^{n} \mathbb{E}[X_i], \tag{4.19}$$

$$\text{Var}[X] = \sum_{i=0}^{n} \text{Var}[X_i]. \tag{4.20}$$

Then the parameters μ_X and σ_X^2 are calculated by:

$$\sigma_X^2 = \log\left(\frac{Var[X]}{\mathbb{E}[X]^2} + 1\right), \tag{4.21}$$

$$\mu_X = \log(\mathbb{E}[X]) - \sigma_X^2. \tag{4.22}$$

Another method to model the surgery duration is to create a discrete distribution using the historical data. We look at how many surgeries ended within τ time blocks and divide this by the total number of surgeries.

$$p_{g\tau} = \frac{\text{\# Number of surgeries in } g \in G \text{ which took } \tau \text{ time blocks}}{\text{Total number of surgeries in } g \in G} \qquad \forall g \in G \tag{4.23}$$

There are merits for both approaches, the log-normal approach gives a smooth probability curve and allows surgery durations which might not have happened in the data. For instance, if in the historical data there were no surgeries that took one hour, but there were multiple surgeries that took longer. The log-normal approach would fit a probability the surgeries performed historically. Even though this approach gives a smooth curve, this might not be preferable. Some data in the tail might be flattened so much that it might not occur. Using the discrete distribution prevents this from happening and should be more historically accurate.

To give a better comparison between the discrete approach and the column based approach, we can fit the data to a log-normal distribution and then discretise it. In doing so, we lose some data in the tail, however, we get a smoother distribution for the surgery duration.

5
Data

In this chapter, we explain what data is used and how we used it. First, we describe the different specialties and their patient groups in Section 5.1 and in Section 5.2.

5.1. Specialties and Patient Groups

There are in total nine surgical specialties that we consider:

Table 5.1: Specialties

Abbreviation	Specialty
PLA	Plastic surgery
GYN	Gynaecology
URO	Urology
ENT	Ear, Nose and Throat
OB	Obstetrics
ORT	Orthopaedic surgery
OMS	Oral and maxillofacial surgery
EYE	Ophthalmology
GS	General surgery

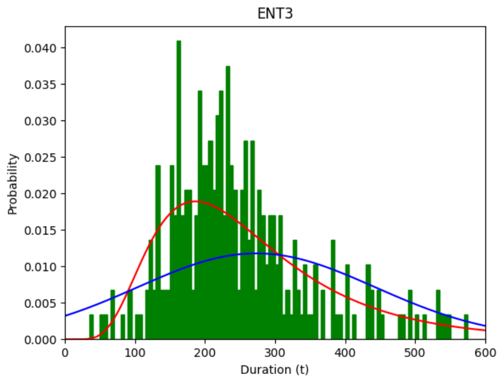
Instead of making a difference in every type of surgery, we are given patient groups which are grouped together. These groups have similar surgery durations and as such we can make a more robust surgery schedule. For each patient group, we fit the data to a log-normal distribution, which we then use as our input parameters in the solution methods. A list of these input parameters can be found in Appendix C. To prevent over-fitting, we use only 80% of the full data for each patient group, to fit the distribution and use the other 20% for the simulations.

In Figure 5.1, we can see the distribution of the data used for a couple of patient groups. As we can see, the log-normal distribution fits well for the patient groups ENT 3 (Figure 5.1a) and OMS 3 (Figure 5.1b). However, a normal distribution might fit better for the patient group URO 2 (Figure 5.1c) and we might not be able to say anything for the surgery duration of patient group URO 3 (Figure 5.1d).

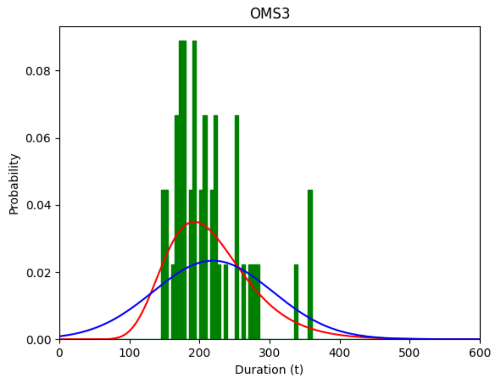
To determine which distribution fits best, we performed the Kolmogorov-Smirnov test. This test, when performed, tests the hypothesis that the given data came from a certain distribution and returns the p-value for this hypothesis. If the p-value is smaller than 0.05, the hypothesis is rejected. In Table 5.2, the results can be seen, and we see that most patient groups likely have a log-normal distribution. However, with some patient groups, we can only conclude that neither distribution fits well. Even though the log-normal distribution does not fit for some patient groups, we do use the fitted values for a log-normal distribution because the majority does follow a log-normal distribution. The higher p-value indicates that the given distribution is more likely.

5.2. Master Surgery Schedule

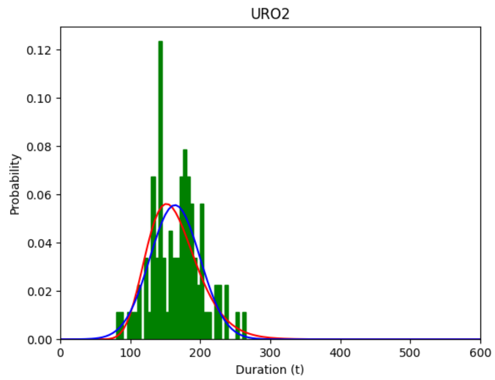
The MSS used in the LUMC is a bi-weekly schedule, meaning it repeats after two weeks. Both weeks do look quite alike, however there are two differences which are highlighted in blue. Notice that not all ORs are assigned a specialty for every day in the MSS, this is because emergency surgeries need to be able to be performed. Thus by leaving these ORs free, we have enough room for those to happen and thus we will not be assigning any surgeries to these OR-days. Table 5.3 shows the MSS we used and Table 5.4 shows the capacity for each OR-day.



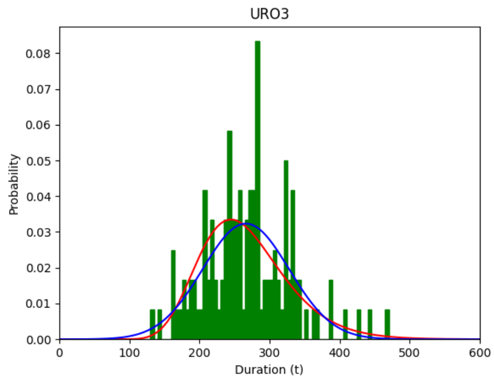
(a) Patient group ENT 3



(b) Patient group OMS 3



(c) Patient group URO 2



(d) Patient group URO 3

Figure 5.1: Distribution of patient groups. Red is the fitted log-normal distribution and blue is the fitted normal distribution

Table 5.2: p-values for normal and log-normal distributions. In **bold** is the highest p-value for the given patient group.

Patient group	p-value (normal distribution)	p-value (log-normal distribution)
GS1	0.00580558	**0.485827556**
GS2	0.000705125	**0.683982887**
GS3	0.000234102	**0.175788269**
GYN1	**0.319833922**	0.213761909
GYN2	0.001737893	**0.636357661**
GYN3	0.320501669	**0.856552221**
OMS1	0.450142484	**0.800702553**
OMS2	0.272111131	**0.858286222**
OMS3	0.025796049	**0.295157446**
ENT1	**0.020254328**	0.010086362
ENT2	6.58384E-08	**0.314740632**
ENT3	2.43301E-10	**0.033345333**
EYE1	0.001268573	**0.160428052**
EYE2	0.030482198	**0.046436399**
EYE3	3.57078E-06	**0.032209347**
ORT1	0.46933542	**0.514385706**
ORT2	0.035183111	**0.457013215**
ORT3	1.75005E-05	**0.045229139**
PLA1	0.79513287	**0.825140446**
PLA2	0.100493865	**0.679649333**
URO1	0.010856647	**0.599853495**
URO2	**0.628275498**	0.427141078
URO3	0.508468251	**0.537115095**
OB1	0.458736554	**0.835185142**
OB2	0.022518055	**0.370992922**

Table 5.3: Master Surgery Schedule for the first and second week.

	Monday	Tuesday	Wednesday	Thursday	Friday
OR1	ORT	ORT	ORT	ORT	
OR2	ORT	ORT / GS	ORT	ORT	ORT
OR3	URO	URO	URO	URO	URO
OR4			OB		
OR5	GS	GS		GS	GS
OR6	GS	GS	GS	GS	GS
OR7	GS	GYN	GS	GS	GS
OR8	GS	GS	GS	GS	GS
OR9	GYN	PLA / GYN	GYN	GYN	GYN
OR10	OMS	OMS	GS	OMS	
OR11	EYE	EYE	EYE	EYE	EYE
OR12					
OR13					
OR14					
OR15	ENT	ENT / -	ENT	ENT	ENT
OR16	ENT	ENT	ENT	ENT	ENT
OR10a	PLA		PLA		PLA

Table 5.4: Opening hours for the first and second week in minutes.

	Monday	Tuesday	Wednesday	Thursday	Friday
OR1	480	480	480	480	
OR2	480	480 / 480	480	480	480
OR3	480	480	480	480	480
OR4			480		
OR5	480	480		480	480
OR6	480	480	480	480	480
OR7	480	480	480	480	480
OR8	480	480	480	480	480
OR9	480	900 / 480	480	480	480
OR10	240	480	240	480	
OR11	480	480	480	480	480
OR12					
OR13					
OR14					
OR15	480	480 / 0	780	480	480
OR16	480	480	480	480	480
OR10a	240		240		480

6
Results

In this chapter, the results following the three different methods are presented and discussed. In Sections 6.1 and 6.2, we discuss our findings for the Columns Based Approach and the Discrete Model, respectively.

We use Python and Gurobi to implement and solve the different models. The resulting schedules are then simulated one thousand times using data from the validation sets.

6.1. Column Based

In the Column Based approach we first create all possible columns for the four different capacities: 240, 480, 780 and 900 minutes. This is done for robustness in the future if the MSS does change. The resulting columns can be found in Appendix B, note that they are displayed as rows. After these columns are made, the model described in Section 4.1 is solved and a schedule is formed. The resulting schedule can be found in Appendix D.

Now that we have a schedule in place, we can simulate it using historical data. For each scheduled surgery, we randomly select a surgery from the historical records corresponding to the same patient group and use its duration. This allows us to calculate the total utilisation and the total overtime for the schedule. The first schedule we simulated uses $\alpha = 0.95$ and the results are found in Table 6.1.

Regarding these results, we first note that the simulated average utilisation is similar to the expected utilisation, which is a 68% utilisation of the ORs. The expected utilisation is calculated by taking the average surgery duration of each planned surgery and add these expectations together. If using enough simulations, the average utilisation should look similar to the expected utilisation.

Next, we see that the number of cases with overtime is lower than the limit of 5%. This again is as we expected, the columns from which the schedule is build, have maximum probability of 5% that it creates overtime. When taking a closer look at the used columns in the schedule, we note that most columns have a probability of overtime of around 0.975%. This explains the number of OR-days which had overtime in the simulations.

Finally, the computing time of this model is fast, all possible columns are generated and the ILP is solved, within a second. This is faster than the results presented by Van der Tuin [10], because of a more efficient implementation of the models.

6.1.1. Varying α

To analyse what the impact of α is, we solve the column based model with different values of α and show the resulting expected utilisation, the simulated utilisation and the number of days which had overtime after simulations.

In Figure 6.1a, first, note that utilisation increases when we allow more overtime. Allowing more overtime enables more surgeries to be scheduled, thereby increasing the OR utilisation. Similarly, the number of days with overtime also increases. However, note that the number of days with overtime is still lower than the total number of days multiplied by α.

Table 6.1: Results using $\alpha = 0.95$ and time blocks of fifteen minutes, these results are the average of one thousand simulations

	Column Based	**Discreet (Historical)**	**Discreet (Fitted)**
Capacity (min)	56940	56940	56940
Expectation (min)	38878	36362	34717
Expectation of Utilisation percentage	68	64 %	61%
Utilisation (min)	38898	36448	34665
Utilisation percentage	68 %	64%	41%
OR-days with Overtime	3.454	8.64	5.28
OR-days with Overtime percentage	2.85 %	7.14&	4.36%
Total Overtime (min)	219	1219	843
Time Generating Columns and setting up ILP (s)	0.013	49	343
Time Solving ILP (s)	0.904	14405	2509

In Figure 6.1c, note that spread of the number of days with overtime increases, as α decreases. As when there are more surgeries scheduled on a single OR-day, the probability of complications increases as well. This directly results into an increased variability of the number of OR-days with overtime.

Note that as α decreases, the total overtime increases, which is exactly what is expected. However, the total overtime does not increase as much as the number of OR-days for which overtime occurred.

We found that almost every OR-day had an instance where overtime occurred, which is as expected. However, as we can see in Figures 6.2a and 6.2a, some OR-days have more overtime occurrences than others. To understand why, we take a look at the column corresponding to OR10 on the Thursday of the first week. This is the column, OMS: [4, 0, 0] and has a probability of 0.952 for which no overtime will occur. Given this probability we expect that only five percent would generate overtime. This is not the case however. It is possible that we cannot use the Fenton-Wilkinson method on random variables which are normally distributed. This is the case for patient group OMS1, as we can see in Table 5.2.

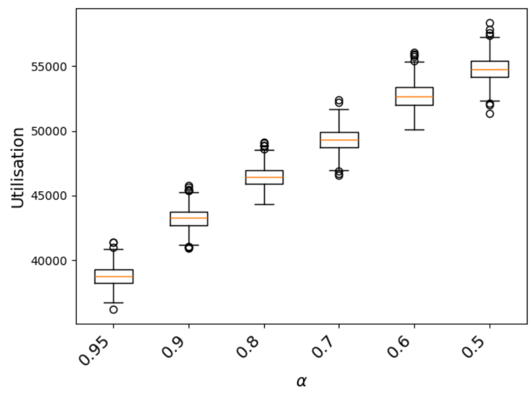
6.2. Discrete Model

We solve the ILP for every specialty $s \in S$ and then combine the generated columns to create a schedule. We discretise the opening hours in blocks of fifteen minutes and set the probability of no overtime to 95%. Note, due to time constraints, we had to cut off the ILP optimiser, which resulted in complete, but not optimal schedules.

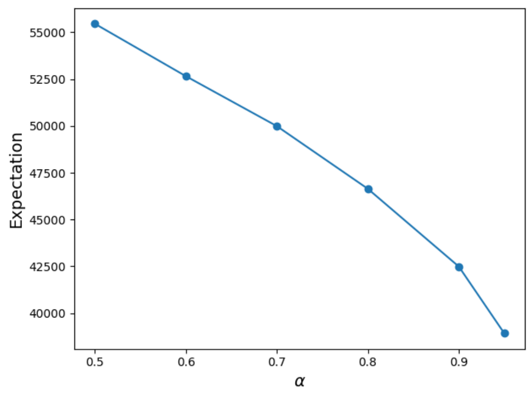
In Table 6.1, the number of days with overtime in the discrete model using historical data exceeded the set threshold of 5%. This is interesting, since we defined our model to not allow any overtime. This might be, because the probability that a surgery finishes, is over-fitted by the training set and there are some outliers in our validation set. These cases should also be in the fitted data however, thus the reason why the outliers exist, is not known.

In Figure 6.3b, we first note that the number of OR-days which had overtime using historical data is noticeably higher than the fitted data. This is as expected, since the fitted data does include surgery durations which did not occur in the historical data. As such, it could plan surgeries earlier than the historical model. Note that both models have a similar distribution in both their utilisation (Figure 6.3a) and number of OR-days for which overtime occurred (Figure 6.3b). Even though there are not a lot of days with overtime, the utilisation of both the historical and the fitted model can be incredibly high. Note that there are examples of weeks that have a utilisation of over one hundred and eighty thousand, while the total capacity is only fifty-seven thousand. These facts combined lead us to believe that the days where overtime happened, also gave a lot of overtime.

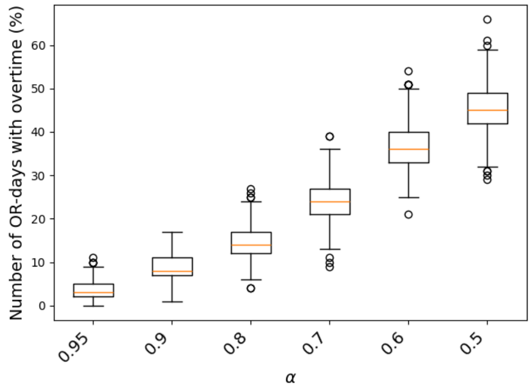
There are three remarks on these results. First, the schedules are not optimal, so utilisation could be higher with more computation time. Secondly, there are some errors regarding the overtime constraint. Although our model's definition seems sound, there might be an oversight. Lastly, the implementation of the model could be incorrect. Despite these potential errors, the computation time is much higher than the computation time of the Column Based model.



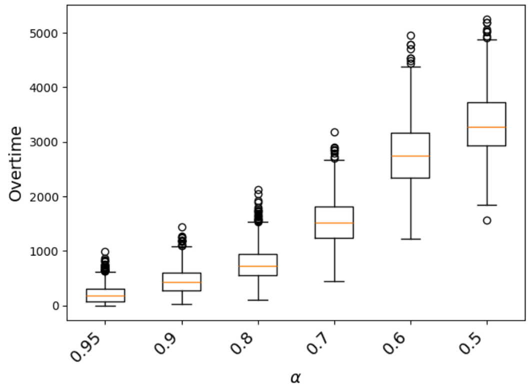
(a) Utilisation



(b) Expected utilisation

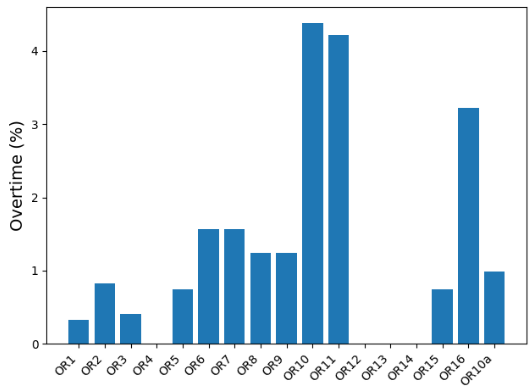


(c) Number of OR-days with overtime

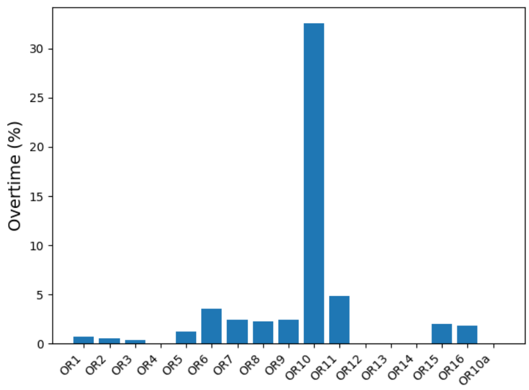


(d) Total overtime

Figure 6.1: Simulation results column based model

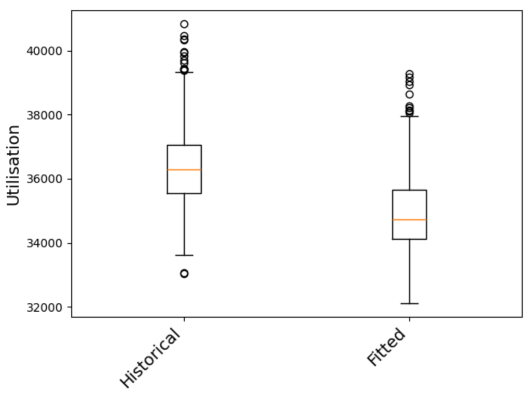


(a) Monday in the first week

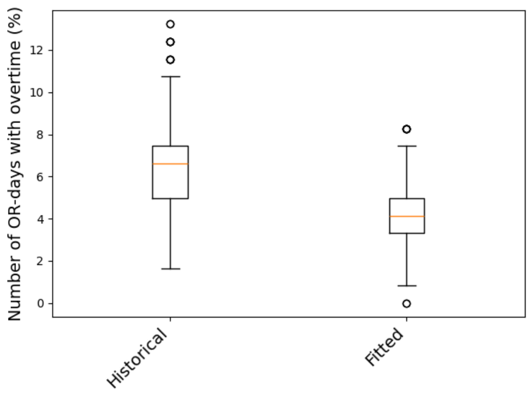


(b) Thursday in the second week

Figure 6.2: The number of instances with overtime. Note, there is a difference in the overtime axis.



(a) Utilisation for the discrete models



(b) Number of OR-days with overtime for the discrete models

Figure 6.3: Simulation results for the discrete models

7 Conclusions and Recommendations

The goal of this thesis is to compare different methods which maximises the utilisation of the OR, whilst dealing with a non-linear overtime constraint. Section 7.1 provides conclusions based on the results given in Chapter 6. In Section 7.2 we provide recommendations for future work.

7.1. Conclusions

As mentioned in Chapter 1 the methods are compared on:

1. Total utilisation of the ORs
2. Total overtime created by the schedule
3. Total runtime of the program

Because the discretised model was cut-off before an optimal model was found, we cannot take any conclusions on the total utilisation of the ORs. However, on the other two criteria, we can make some conclusions.

On both criteria, we found that the column based approach scored better then the discretised model. Even though we cut the discretised model off, it took much longer then the column based model. The total overtime created by discretised model using fitted data was close to the set boundary, however in the column based approach, it was much lower.

Unfortunately, without knowing the total utilisation of an optimal schedule created by the discretised model, we cannot say for certain that the column based approach yields a better result.

7.2. Recommendations

In this section, we first discuss what recommendations we have to adjust the research done in this thesis. After that, we state some recommendations for other research.

Firstly, the discretised model takes a lot longer to solve than the column based approach. This is because there are a lot more variables in the model to consider. We recommend to reduce the number of variables, for instance to remove the dependency on the starting position. In our implementation, we did not have the tools to solve the schedules per specialty at the same time, thus creating a parallel program would impact the solve time dramastically.

Secondly, we'd recommend to research the impact of adding more constraints, for instance the bed capacity to the given model and what the impact would be if these were added to both models. The discretised model would not change completely, but the impact it may have on the creation of all columns in the column based approach, might be significant.

Finally, the methods could be changed to incorporate the amount of overtime each day may have. When doing this, we might be able to fit even more surgeries into the schedules and reduce the overtime on days for which it is less ideal.

Regarding the data, at the moment we do not have any data on what the impact of the different surgeons or starting times have on the surgery duration. As one can imagine, more experienced surgeons could have a less distributed surgery duration then starting surgeons. In other words, experienced surgeons can perform routine surgeries in the same amount of time more often than starting surgeons. If this was taken into account when both fitting the data and creating the schedules, it could impact the model in a positive way. When the data is fitted with a higher accuracy, the probability of overtime would go down. There is a downside however, even with this extra element, we might not use the surgeons as an extra parameter, because the data is sparse. Another recommendation, use different data sets to compare the different models.

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Appendices

A
Models

In this appendix are the different models mentioned in this thesis.

A.1. General Model

$$
\begin{aligned}
\max \sum_{g\in G}\sum_{o\in O}\sum_{d\in D} x_{odg}\cdot \mathbb{E}[Y_g] & & & \\
\text{subject to} \sum_{g\in G_s} x_{odg} & \le m_{ods}\cdot N_{ods} & & \forall o\in O, \forall d\in D, \forall s\in S \\
\mathbb{P}(Z_{od}\le c_{od}) & \le \alpha & & \forall o\in O, d\in D \\
\sum_{o\in O}\sum_{d\in D} x_{odg} & \ge \beta_g & & \forall g\in G \\
x_{odg} & \in \mathbb{Z}^+ & & \forall o\in O, \forall d\in D, \forall g\in G
\end{aligned}
$$

A.2. Column Based Model

$$
\begin{aligned}
\max \sum_{g\in G}\sum_{o\in O}\sum_{d\in D}\sum_{k\in K_{od}} \lambda_k\cdot x_g^k\cdot \mathbb{E}[Y_g] & & & \\
\text{subject to} \sum_{k\in K_{od}} \lambda_k & \le 1 & & \forall o\in O, \forall d\in D \\
\sum_{o\in O}\sum_{d\in D}\sum_{k\in K_{od}} \lambda_k\cdot x_g^k & \ge \beta_g & & \forall g\in G \\
\lambda_k & \in \{0,1\} & & \forall o\in O, \forall d\in D, \forall k\in K_{od}
\end{aligned}
$$

A.3. Discrete Model

$$
\begin{aligned}
\max \ & \sum_{g\in G}\sum_{o\in O}\sum_{d\in D}\sum_{v\in V} f_{odgv}\cdot \mathbb{E}[Y_g] \\
\text{subject to } & \sum_{v\in V}\sum_{g\in G_s} f_{odgv} \le m_{ods}\cdot N_s, && \forall s\in S, \forall o\in O, \forall d\in D, \forall v\in V, \\
& \sum_{g\in G} f_{odgv} \le 1, && \forall o\in O, \forall d\in D, \forall v\in V \\
& \sum_{t\in T} b_{odgvt} \le f_{odgv}, && \forall o\in O, \forall d\in D, \forall g\in G, \forall v\in V, \\
& \sum_{t\in T} b_{odgvt} \ge 0.9999\cdot f_{odgv}, && \forall o\in O, \forall d\in D, \forall g\in G, \forall v\in V, \\
& \sum_{j in G_s}\sum_{v\in V} b_{odgv} \le m_{ods}, && \forall o\in O, \forall d\in D, \forall s\in S, \\
& b_{odg0t_{od}^{open}} = f_{odg0}, && \forall o\in O, \forall d\in D, \forall g\in G, \\
& e_{odgvt} = \sum_{\tau=1}^{t-t_0} p_{g\tau}\cdot b_{odgv(t-\tau)}, && \forall o\in O, \forall d\in D, \forall g\in G_s, \forall v\in V, \forall t\in T, \\
& b_{odgvt} \le \sum_{j\in G_s} e_{odj(v-1)(t-1)}, && \forall o\in O, \forall d\in D, \forall g\in G_s, \forall v\in V\setminus\{0\}, \forall t\in T\setminus\{t_0\}, \\
& \sum_{t\in\{t_0,\cdots,t_{od}^{close}\}} e_{odgvt} \ge (1-\alpha)\cdot f_{odgv}, && \forall o\in O, \forall d\in D, \forall g\in G, \forall v\in V\setminus\{0\}, \\
& \sum_{o\in O}\sum_{d\in D}\sum_{v\in V} f_{odgv} \ge \beta_g, && \forall g\in G, \\
& f_{odgv} \in \{0,1\} && \forall o\in O, \forall d\in D, \forall g\in G, \forall v\in V, \\
& b_{odgvt}, e_{odgvt} \in [0,1] && \forall o\in O, \forall d\in D, \forall g\in G, \forall v\in V, \forall t\in T.
\end{aligned}
$$

B
Columns

In this appendix are the different generated columns for $\alpha = 0.05$, i.e. the probability of overtime is 0.95. These are split into columns for each different capacity. The number in a column corresponds to the number of surgeries are planned for that given patient group. The probability that the given column gives overtime is also given. Note that every row is a single column.

B.1. Capacity of 240 minutes

Specialty	Group 1	Group 2	Group 3	Probability of no overtime
ORT	0	1	0	0,968569
ORT	1	0	0	0,999992
ORT	2	0	0	0,996263
OB	0	1	0	0,999979
OB	0	2	0	0,976267
OB	1	0	0	1
OB	1	1	0	0,998457
OB	2	0	0	0,999979
OB	3	0	0	0,996363
URO	0	1	0	0,963514
URO	1	0	0	0,99998
URO	2	0	0	0,994386
GS	1	0	0	0,999998
GS	2	0	0	0,997805
OMS	0	1	0	0,995973
OMS	1	0	0	0,996568
EYE	0	0	1	0,999466
EYE	0	1	0	0,999462
EYE	0	1	1	0,95877
EYE	0	2	0	0,984993
EYE	1	0	0	0,999994
EYE	1	0	1	0,992298
EYE	1	1	0	0,99718
EYE	2	0	0	0,99988
EYE	2	1	0	0,98037
EYE	3	0	0	0,997565
EYE	4	0	0	0,971754
PLA	0	1	0	0,999755
PLA	1	0	0	1
PLA	1	1	0	0,981561
PLA	2	0	0	0,999438

ENT	0	1	0	0,985359
ENT	1	0	0	0,999538
ENT	2	0	0	0,992406
GYN	0	1	0	0,961818
GYN	1	0	0	0,999975
GYN	2	0	0	0,995954

B.2. Capacity of 480 minutes

Specialty	Group 1	Group 2	Group 3	Probability of no overtime
ORT	0	1	0	0,999961
ORT	0	2	0	0,993477
ORT	1	0	0	1
ORT	1	1	0	0,999823
ORT	1	2	0	0,962535
ORT	2	0	0	1
ORT	2	1	0	0,998044
ORT	3	0	0	0,999999
ORT	3	1	0	0,978344
ORT	4	0	0	0,999887
ORT	5	0	0	0,993045
OB	0	1	0	1
OB	0	2	0	1
OB	0	3	0	0,999989
OB	0	4	0	0,996782
OB	1	0	0	1
OB	1	1	0	1
OB	1	2	0	1
OB	1	3	0	0,999654
OB	1	4	0	0,96558
OB	2	0	0	1
OB	2	1	0	1
OB	2	2	0	0,999979
OB	2	3	0	0,99355
OB	3	0	0	1
OB	3	1	0	0,999999
OB	3	2	0	0,999268
OB	4	0	0	1
OB	4	1	0	0,999955
OB	4	2	0	0,9871
OB	5	0	0	0,999999
OB	5	1	0	0,998406
OB	6	0	0	0,9999
OB	6	1	0	0,974606
OB	7	0	0	0,996474
OB	8	0	0	0,951443
URO	0	0	1	0,99367
URO	0	1	0	0,999999
URO	0	2	0	0,992654
URO	1	0	0	1
URO	1	0	1	0,964239
URO	1	1	0	0,999961
URO	2	0	0	1
URO	2	1	0	0,99827
URO	3	0	0	0,999998

URO	3	1	0	0,965694
URO	4	0	0	0,999755
URO	5	0	0	0,989543
GS	0	0	1	0,974729
GS	0	1	0	0,99978
GS	0	2	0	0,97657
GS	1	0	0	1
GS	1	1	0	0,99923
GS	2	0	0	1
GS	2	1	0	0,994289
GS	3	0	0	1
GS	3	1	0	0,956153
GS	4	0	0	0,99996
GS	5	0	0	0,995152
OMS	0	0	1	0,997753
OMS	0	1	0	1
OMS	0	1	1	0,951892
OMS	0	2	0	0,999864
OMS	1	0	0	0,999997
OMS	1	0	1	0,975232
OMS	1	1	0	0,999886
OMS	1	2	0	0,97236
OMS	2	0	0	0,99986
OMS	2	1	0	0,988486
OMS	3	0	0	0,994407
EYE	0	0	1	1
EYE	0	0	2	0,999996
EYE	0	0	3	0,998861
EYE	0	1	0	0,999999
EYE	0	1	1	0,999997
EYE	0	1	2	0,999588
EYE	0	1	3	0,975981
EYE	0	2	0	0,999994
EYE	0	2	1	0,999814
EYE	0	2	2	0,990616
EYE	0	3	0	0,999887
EYE	0	3	1	0,996179
EYE	0	4	0	0,998275
EYE	0	4	1	0,963777
EYE	0	5	0	0,983423
EYE	1	0	0	1
EYE	1	0	1	1
EYE	1	0	2	0,999955
EYE	1	0	3	0,992581
EYE	1	1	0	1
EYE	1	1	1	0,999977
EYE	1	1	2	0,99741
EYE	1	2	0	0,999982
EYE	1	2	1	0,998998
EYE	1	2	2	0,964
EYE	1	3	0	0,999538
EYE	1	3	1	0,984904
EYE	1	4	0	0,993512
EYE	1	5	0	0,95078
EYE	2	0	0	1

EYE	2	0	1	0,999999
EYE	2	0	2	0,999544
EYE	2	0	3	0,964841
EYE	2	1	0	0,999999
EYE	2	1	1	0,999828
EYE	2	1	2	0,986797
EYE	2	2	0	0,999914
EYE	2	2	1	0,99499
EYE	2	3	0	0,99795
EYE	2	4	0	0,977398
EYE	3	0	0	1
EYE	3	0	1	0,999986
EYE	3	0	2	0,996527
EYE	3	1	0	0,999991
EYE	3	1	1	0,998805
EYE	3	2	0	0,999523
EYE	3	2	1	0,978798
EYE	3	3	0	0,991367
EYE	4	0	0	1
EYE	4	0	1	0,999832
EYE	4	0	2	0,980804
EYE	4	1	0	0,999932
EYE	4	1	1	0,993145
EYE	4	2	0	0,997441
EYE	4	3	0	0,968448
EYE	5	0	0	0,999996
EYE	5	0	1	0,998495
EYE	5	1	0	0,99948
EYE	5	1	1	0,969448
EYE	5	2	0	0,988091
EYE	6	0	0	0,999944
EYE	6	0	1	0,99023
EYE	6	1	0	0,996646
EYE	6	2	0	0,955088
EYE	7	0	0	0,9994
EYE	7	0	1	0,955037
EYE	7	1	0	0,983013
EYE	8	0	0	0,995388
EYE	9	0	0	0,975038
PLA	0	1	0	1
PLA	0	2	0	0,999999
PLA	0	3	0	0,999374
PLA	1	0	0	1
PLA	1	1	0	1
PLA	1	2	0	0,999955
PLA	1	3	0	0,986055
PLA	2	0	0	1
PLA	2	1	0	0,999999
PLA	2	2	0	0,997941
PLA	3	0	0	1
PLA	3	1	0	0,999853
PLA	3	2	0	0,958383
PLA	4	0	0	0,999997
PLA	4	1	0	0,992585
PLA	5	0	0	0,999398

PLA	6	0	0	0,97326
ENT	0	1	0	0,999925
ENT	0	2	0	0,997959
ENT	0	3	0	0,967522
ENT	1	0	0	0,999999
ENT	1	1	0	0,999765
ENT	1	2	0	0,991928
ENT	2	0	0	0,999995
ENT	2	1	0	0,998822
ENT	2	2	0	0,969809
ENT	3	0	0	0,999945
ENT	3	1	0	0,993834
ENT	4	0	0	0,999448
ENT	4	1	0	0,972487
ENT	5	0	0	0,995668
ENT	6	0	0	0,975607
GYN	0	0	1	0,994483
GYN	0	1	0	0,999875
GYN	0	2	0	0,990408
GYN	1	0	0	1
GYN	1	0	1	0,976236
GYN	1	1	0	0,999591
GYN	1	2	0	0,959337
GYN	2	0	0	1
GYN	2	1	0	0,997189
GYN	3	0	0	0,999998
GYN	3	1	0	0,979245
GYN	4	0	0	0,999861
GYN	5	0	0	0,99504

B.3. Capacity for 720 minutes

Specialty	Group 1	Group 2	Group 3	Probability of no overtime
ORT	0	0	1	0,991637
ORT	0	1	0	1
ORT	0	1	1	0,9767
ORT	0	2	0	0,999997
ORT	0	3	0	0,999649
ORT	0	4	0	0,985327
ORT	1	0	0	1
ORT	1	0	1	0,987605
ORT	1	1	0	1
ORT	1	1	1	0,957724
ORT	1	2	0	0,999987
ORT	1	3	0	0,998122
ORT	2	0	0	1
ORT	2	0	1	0,978848
ORT	2	1	0	1
ORT	2	2	0	0,999908
ORT	2	3	0	0,990049
ORT	3	0	0	1
ORT	3	0	1	0,960559
ORT	3	1	0	0,999999
ORT	3	2	0	0,999242
ORT	3	3	0	0,955207

ORT	4	0	0	1
ORT	4	1	0	0,99999
ORT	4	2	0	0,994219
ORT	5	0	0	1
ORT	5	1	0	0,999816
ORT	5	2	0	0,965776
ORT	6	0	0	1
ORT	6	1	0	0,997445
ORT	7	0	0	0,999985
ORT	7	1	0	0,976869
ORT	8	0	0	0,999354
ORT	9	0	0	0,987632
OB	0	1	0	1
OB	0	2	0	1
OB	0	3	0	1
OB	0	4	0	1
OB	0	5	0	1
OB	0	6	0	0,999973
OB	0	7	0	0,997682
OB	1	0	0	1
OB	1	1	0	1
OB	1	2	0	1
OB	1	3	0	1
OB	1	4	0	1
OB	1	5	0	0,999997
OB	1	6	0	0,999588
OB	1	7	0	0,983021
OB	2	0	0	1
OB	2	1	0	1
OB	2	2	0	1
OB	2	3	0	1
OB	2	4	0	1
OB	2	5	0	0,999945
OB	2	6	0	0,995777
OB	3	0	0	1
OB	3	1	0	1
OB	3	2	0	1
OB	3	3	0	1
OB	3	4	0	0,999995
OB	3	5	0	0,999205
OB	3	6	0	0,971666
OB	4	0	0	1
OB	4	1	0	1
OB	4	2	0	1
OB	4	3	0	1
OB	4	4	0	0,99989
OB	4	5	0	0,992399
OB	5	0	0	1
OB	5	1	0	1
OB	5	2	0	1
OB	5	3	0	0,999989
OB	5	4	0	0,998469
OB	5	5	0	0,953848
OB	6	0	0	1
OB	6	1	0	1

OB	6	2	0	0,999999
OB	6	3	0	0,999776
OB	6	4	0	0,986546
OB	7	0	0	1
OB	7	1	0	1
OB	7	2	0	0,999977
OB	7	3	0	0,997074
OB	8	0	0	1
OB	8	1	0	0,999998
OB	8	2	0	0,999543
OB	8	3	0	0,976698
OB	9	0	0	1
OB	9	1	0	0,999951
OB	9	2	0	0,994479
OB	10	0	0	0,999997
OB	10	1	0	0,999068
OB	10	2	0	0,960681
OB	11	0	0	0,999894
OB	11	1	0	0,989772
OB	12	0	0	0,998114
OB	13	0	0	0,981488
URO	0	0	1	0,999996
URO	0	0	2	0,98701
URO	0	1	0	1
URO	0	1	1	0,999707
URO	0	2	0	1
URO	0	2	1	0,974982
URO	0	3	0	0,999807
URO	1	0	0	1
URO	1	0	1	0,999984
URO	1	0	2	0,951603
URO	1	1	0	1
URO	1	1	1	0,997912
URO	1	2	0	0,999999
URO	1	3	0	0,996989
URO	2	0	0	1
URO	2	0	1	0,99989
URO	2	1	0	1
URO	2	1	1	0,98694
URO	2	2	0	0,999959
URO	2	3	0	0,97207
URO	3	0	0	1
URO	3	0	1	0,999134
URO	3	1	0	1
URO	3	2	0	0,999071
URO	4	0	0	1
URO	4	0	1	0,993725
URO	4	1	0	0,999994
URO	4	2	0	0,987977
URO	5	0	0	1
URO	5	0	1	0,964548
URO	5	1	0	0,999773
URO	6	0	0	0,999999
URO	6	1	0	0,995648
URO	7	0	0	0,999959

URO	7	1	0	0,957642
URO	8	0	0	0,998735
URO	9	0	0	0,981648
GS	0	0	1	0,999896
GS	0	1	0	1
GS	0	1	1	0,996577
GS	0	2	0	0,99996
GS	0	3	0	0,997161
GS	1	0	0	1
GS	1	0	1	0,999734
GS	1	1	0	1
GS	1	1	1	0,988557
GS	1	2	0	0,999866
GS	1	3	0	0,989198
GS	2	0	0	1
GS	2	0	1	0,998992
GS	2	1	0	1
GS	2	1	1	0,96194
GS	2	2	0	0,999339
GS	2	3	0	0,960488
GS	3	0	0	1
GS	3	0	1	0,995477
GS	3	1	0	0,999996
GS	3	2	0	0,996287
GS	4	0	0	1
GS	4	0	1	0,979948
GS	4	1	0	0,999959
GS	4	2	0	0,980531
GS	5	0	0	1
GS	5	1	0	0,999467
GS	6	0	0	1
GS	6	1	0	0,994312
GS	7	0	0	0,999996
GS	7	1	0	0,958291
GS	8	0	0	0,999658
GS	9	0	0	0,98988
OMS	0	0	1	0,999996
OMS	0	0	2	0,99812
OMS	0	1	0	1
OMS	0	1	1	0,999969
OMS	0	1	2	0,975969
OMS	0	2	0	1
OMS	0	2	1	0,998825
OMS	0	3	0	1
OMS	0	3	1	0,965453
OMS	0	4	0	0,999559
OMS	1	0	0	1
OMS	1	0	1	0,999967
OMS	1	0	2	0,986328
OMS	1	1	0	1
OMS	1	1	1	0,999336
OMS	1	2	0	1
OMS	1	2	1	0,982646
OMS	1	3	0	0,999809
OMS	1	4	0	0,976917

OMS	2	0	0	1
OMS	2	0	1	0,999574
OMS	2	1	0	0,999999
OMS	2	1	1	0,991035
OMS	2	2	0	0,999902
OMS	2	3	0	0,990086
OMS	3	0	0	0,999999
OMS	3	0	1	0,995127
OMS	3	1	0	0,99994
OMS	3	2	0	0,995564
OMS	4	0	0	0,999955
OMS	4	0	1	0,962704
OMS	4	1	0	0,997872
OMS	5	0	0	0,998879
OMS	5	1	0	0,970672
OMS	6	0	0	0,985006
EYE	0	0	1	1
EYE	0	0	2	1
EYE	0	0	3	1
EYE	0	0	4	0,999999
EYE	0	0	5	0,999889
EYE	0	0	6	0,994259
EYE	0	1	0	1
EYE	0	1	1	1
EYE	0	1	2	1
EYE	0	1	3	1
EYE	0	1	4	0,999959
EYE	0	1	5	0,997747
EYE	0	1	6	0,955955
EYE	0	2	0	1
EYE	0	2	1	1
EYE	0	2	2	1
EYE	0	2	3	0,999983
EYE	0	2	4	0,999112
EYE	0	2	5	0,979411
EYE	0	3	0	1
EYE	0	3	1	1
EYE	0	3	2	0,999992
EYE	0	3	3	0,999639
EYE	0	3	4	0,990732
EYE	0	4	0	1
EYE	0	4	1	0,999996
EYE	0	4	2	0,999844
EYE	0	4	3	0,995897
EYE	0	5	0	0,999997
EYE	0	5	1	0,999927
EYE	0	5	2	0,998174
EYE	0	5	3	0,972992
EYE	0	6	0	0,999962
EYE	0	6	1	0,999165
EYE	0	6	2	0,986746
EYE	0	7	0	0,999599
EYE	0	7	1	0,993581
EYE	0	8	0	0,996874
EYE	0	8	1	0,966656

EYE	0	9	0	0,982502
EYE	1	0	0	1
EYE	1	0	1	1
EYE	1	0	2	1
EYE	1	0	3	1
EYE	1	0	4	0,999994
EYE	1	0	5	0,999348
EYE	1	0	6	0,979262
EYE	1	1	0	1
EYE	1	1	1	1
EYE	1	1	2	1
EYE	1	1	3	0,999997
EYE	1	1	4	0,999761
EYE	1	1	5	0,991178
EYE	1	2	0	1
EYE	1	2	1	1
EYE	1	2	2	0,999999
EYE	1	2	3	0,999907
EYE	1	2	4	0,996341
EYE	1	3	0	1
EYE	1	3	1	0,999999
EYE	1	3	2	0,999961
EYE	1	3	3	0,998485
EYE	1	3	4	0,972094
EYE	1	4	0	0,999999
EYE	1	4	1	0,999981
EYE	1	4	2	0,999358
EYE	1	4	3	0,98687
EYE	1	5	0	0,99999
EYE	1	5	1	0,999714
EYE	1	5	2	0,993946
EYE	1	6	0	0,999863
EYE	1	6	1	0,997208
EYE	1	6	2	0,965031
EYE	1	7	0	0,998685
EYE	1	7	1	0,982239
EYE	1	8	0	0,991126
EYE	1	9	0	0,958242
EYE	2	0	0	1
EYE	2	0	1	1
EYE	2	0	2	1
EYE	2	0	3	1
EYE	2	0	4	0,999952
EYE	2	0	5	0,996834
EYE	2	1	0	1
EYE	2	1	1	1
EYE	2	1	2	1
EYE	2	1	3	0,999982
EYE	2	1	4	0,998796
EYE	2	1	5	0,971236
EYE	2	2	0	1
EYE	2	2	1	1
EYE	2	2	2	0,999992
EYE	2	2	3	0,999533
EYE	2	2	4	0,987098

EYE	2	3	0	1
EYE	2	3	1	0,999996
EYE	2	3	2	0,999811
EYE	2	3	3	0,994376
EYE	2	4	0	0,999998
EYE	2	4	1	0,999917
EYE	2	4	2	0,997564
EYE	2	4	3	0,963314
EYE	2	5	0	0,99996
EYE	2	5	1	0,998928
EYE	2	5	2	0,982026
EYE	2	6	0	0,999509
EYE	2	6	1	0,991396
EYE	2	7	0	0,9959
EYE	2	7	1	0,955742
EYE	2	8	0	0,97679
EYE	3	0	0	1
EYE	3	0	1	1
EYE	3	0	2	1
EYE	3	0	3	0,999998
EYE	3	0	4	0,999687
EYE	3	0	5	0,987463
EYE	3	1	0	1
EYE	3	1	1	1
EYE	3	1	2	0,999999
EYE	3	1	3	0,999886
EYE	3	1	4	0,99488
EYE	3	2	0	1
EYE	3	2	1	0,999999
EYE	3	2	2	0,999956
EYE	3	2	3	0,997939
EYE	3	2	4	0,961517
EYE	3	3	0	1
EYE	3	3	1	0,999981
EYE	3	3	2	0,999161
EYE	3	3	3	0,98189
EYE	3	4	0	0,99999
EYE	3	4	1	0,999646
EYE	3	4	2	0,991741
EYE	3	5	0	0,999841
EYE	3	5	1	0,996274
EYE	3	5	2	0,953018
EYE	3	6	0	0,998302
EYE	3	6	1	0,976107
EYE	3	7	0	0,988161
EYE	4	0	0	1
EYE	4	0	1	1
EYE	4	0	2	1
EYE	4	0	3	0,99998
EYE	4	0	4	0,998327
EYE	4	0	5	0,959663
EYE	4	1	0	1
EYE	4	1	1	1
EYE	4	1	2	0,999992
EYE	4	1	3	0,99938

EYE	4	1	4	0,981867
EYE	4	2	0	1
EYE	4	2	1	0,999997
EYE	4	2	2	0,999763
EYE	4	2	3	0,992176
EYE	4	3	0	0,999998
EYE	4	3	1	0,999903
EYE	4	3	2	0,996687
EYE	4	3	3	0,950056
EYE	4	4	0	0,999957
EYE	4	4	1	0,99859
EYE	4	4	2	0,975448
EYE	4	5	0	0,999383
EYE	4	5	1	0,988331
EYE	4	6	0	0,994535
EYE	4	7	0	0,969036
EYE	5	0	0	1
EYE	5	0	1	1
EYE	5	0	2	0,999999
EYE	5	0	3	0,999856
EYE	5	0	4	0,992722
EYE	5	1	0	1
EYE	5	1	1	1
EYE	5	1	2	0,999948
EYE	5	1	3	0,997138
EYE	5	2	0	1
EYE	5	2	1	0,999979
EYE	5	2	2	0,998877
EYE	5	2	3	0,974841
EYE	5	3	0	0,999991
EYE	5	3	1	0,999549
EYE	5	3	2	0,988594
EYE	5	4	0	0,999809
EYE	5	4	1	0,994943
EYE	5	5	0	0,997759
EYE	5	5	1	0,967685
EYE	5	6	0	0,984048
EYE	6	0	0	1
EYE	6	0	1	1
EYE	6	0	2	0,999992
EYE	6	0	3	0,999155
EYE	6	0	4	0,974331
EYE	6	1	0	1
EYE	6	1	1	0,999997
EYE	6	1	2	0,999694
EYE	6	1	3	0,988976
EYE	6	2	0	0,999999
EYE	6	2	1	0,999883
EYE	6	2	2	0,995413
EYE	6	3	0	0,999952
EYE	6	3	1	0,998106
EYE	6	3	2	0,966291
EYE	6	4	0	0,999205
EYE	6	4	1	0,984014
EYE	6	5	0	0,99261

EYE	6	6	0	0,958537
EYE	7	0	0	1
EYE	7	0	1	1
EYE	7	0	2	0,999937
EYE	7	0	3	0,995949
EYE	7	1	0	1
EYE	7	1	1	0,999977
EYE	7	1	2	0,998461
EYE	7	1	3	0,964881
EYE	7	2	0	0,999991
EYE	7	2	1	0,999409
EYE	7	2	2	0,984085
EYE	7	3	0	0,999764
EYE	7	3	1	0,993031
EYE	7	4	0	0,996986
EYE	7	4	1	0,956151
EYE	7	5	0	0,978333
EYE	8	0	0	1
EYE	8	0	1	0,999997
EYE	8	0	2	0,999592
EYE	8	0	3	0,9843
EYE	8	1	0	0,999999
EYE	8	1	1	0,999854
EYE	8	1	2	0,993545
EYE	8	2	0	0,999945
EYE	8	2	1	0,997401
EYE	8	2	2	0,953592
EYE	8	3	0	0,998949
EYE	8	3	1	0,977924
EYE	8	4	0	0,989877
EYE	9	0	0	1
EYE	9	0	1	0,999974
EYE	9	0	2	0,997842
EYE	9	0	3	0,950867
EYE	9	1	0	0,99999
EYE	9	1	1	0,999205
EYE	9	1	2	0,977613
EYE	9	2	0	0,999699
EYE	9	2	1	0,990266
EYE	9	3	0	0,995871
EYE	9	4	0	0,970393
EYE	10	0	0	0,999999
EYE	10	0	1	0,999813
EYE	10	0	2	0,990786
EYE	10	1	0	0,999934
EYE	10	1	1	0,996363
EYE	10	2	0	0,998578
EYE	10	2	1	0,969336
EYE	10	3	0	0,985979
EYE	11	0	0	0,99999
EYE	11	0	1	0,998902
EYE	11	0	2	0,96833
EYE	11	1	0	0,999606
EYE	11	1	1	0,986248
EYE	11	2	0	0,994248

EYE	11	3	0	0,959391
EYE	12	0	0	0,999918
EYE	12	0	1	0,99482
EYE	12	1	0	0,998031
EYE	12	1	1	0,957265
EYE	12	2	0	0,980407
EYE	13	0	0	0,999467
EYE	13	0	1	0,98039
EYE	13	1	0	0,991869
EYE	14	0	0	0,997216
EYE	14	1	0	0,972438
EYE	15	0	0	0,988356
EYE	16	0	0	0,96107
PLA	0	1	0	1
PLA	0	2	0	1
PLA	0	3	0	1
PLA	0	4	0	1
PLA	0	5	0	0,999953
PLA	0	6	0	0,995677
PLA	1	0	0	1
PLA	1	1	0	1
PLA	1	2	0	1
PLA	1	3	0	1
PLA	1	4	0	0,999995
PLA	1	5	0	0,999083
PLA	1	6	0	0,963623
PLA	2	0	0	1
PLA	2	1	0	1
PLA	2	2	0	1
PLA	2	3	0	1
PLA	2	4	0	0,999859
PLA	2	5	0	0,988637
PLA	3	0	0	1
PLA	3	1	0	1
PLA	3	2	0	1
PLA	3	3	0	0,999985
PLA	3	4	0	0,997317
PLA	4	0	0	1
PLA	4	1	0	1
PLA	4	2	0	0,999999
PLA	4	3	0	0,999549
PLA	4	4	0	0,971358
PLA	5	0	0	1
PLA	5	1	0	1
PLA	5	2	0	0,99995
PLA	5	3	0	0,992253
PLA	6	0	0	1
PLA	6	1	0	0,999997
PLA	6	2	0	0,998521
PLA	7	0	0	1
PLA	7	1	0	0,999819
PLA	7	2	0	0,978598
PLA	8	0	0	0,999988
PLA	8	1	0	0,995201
PLA	9	0	0	0,999313

PLA	10	0	0	0,985133
ENT	0	0	1	0,994177
ENT	0	1	0	1
ENT	0	1	1	0,987106
ENT	0	2	0	0,999995
ENT	0	2	1	0,9649
ENT	0	3	0	0,999908
ENT	0	4	0	0,998534
ENT	0	5	0	0,985222
ENT	1	0	0	1
ENT	1	0	1	0,991897
ENT	1	1	0	1
ENT	1	1	1	0,979101
ENT	1	2	0	0,999987
ENT	1	3	0	0,999683
ENT	1	4	0	0,995337
ENT	1	5	0	0,961836
ENT	2	0	0	1
ENT	2	0	1	0,98751
ENT	2	1	0	0,999999
ENT	2	1	1	0,964766
ENT	2	2	0	0,999952
ENT	2	3	0	0,99885
ENT	2	4	0	0,986035
ENT	3	0	0	1
ENT	3	0	1	0,979368
ENT	3	1	0	0,999996
ENT	3	2	0	0,999796
ENT	3	3	0	0,995948
ENT	3	4	0	0,96218
ENT	4	0	0	1
ENT	4	0	1	0,964639
ENT	4	1	0	0,999978
ENT	4	2	0	0,999132
ENT	4	3	0	0,986909
ENT	5	0	0	0,999999
ENT	5	1	0	0,99988
ENT	5	2	0	0,996551
ENT	5	3	0	0,962625
ENT	6	0	0	0,999992
ENT	6	1	0	0,999376
ENT	6	2	0	0,987848
ENT	7	0	0	0,999936
ENT	7	1	0	0,997139
ENT	7	2	0	0,96319
ENT	8	0	0	0,999578
ENT	8	1	0	0,988853
ENT	9	0	0	0,997702
ENT	9	1	0	0,963893
ENT	10	0	0	0,989923
ENT	11	0	0	0,96476
GYN	0	0	1	0,999991
GYN	0	0	2	0,992332
GYN	0	1	0	1
GYN	0	1	1	0,999543

GYN	0	2	0	0,999986
GYN	0	2	1	0,988052
GYN	0	3	0	0,999236
GYN	0	4	0	0,981619
GYN	1	0	0	1
GYN	1	0	1	0,999976
GYN	1	0	2	0,975155
GYN	1	1	0	1
GYN	1	1	1	0,998272
GYN	1	2	0	0,999956
GYN	1	2	1	0,962591
GYN	1	3	0	0,99705
GYN	2	0	0	1
GYN	2	0	1	0,999895
GYN	2	1	0	1
GYN	2	1	1	0,993034
GYN	2	2	0	0,999793
GYN	2	3	0	0,988408
GYN	3	0	0	1
GYN	3	0	1	0,999428
GYN	3	1	0	0,999998
GYN	3	1	1	0,97366
GYN	3	2	0	0,998868
GYN	3	3	0	0,958679
GYN	4	0	0	1
GYN	4	0	1	0,99677
GYN	4	1	0	0,999976
GYN	4	2	0	0,993936
GYN	5	0	0	1
GYN	5	0	1	0,983806
GYN	5	1	0	0,999752
GYN	5	2	0	0,97215
GYN	6	0	0	1
GYN	6	1	0	0,99774
GYN	7	0	0	0,999985
GYN	7	1	0	0,984392
GYN	8	0	0	0,999591
GYN	9	0	0	0,993947
GYN	10	0	0	0,952016

B.4. Capacity of 900 minutes

Specialty	Group 1	Group 2	Group 3	Probability of no overtime
ORT	0	0	1	0,99644
ORT	0	1	0	1
ORT	0	1	1	0,991829
ORT	0	2	0	1
ORT	0	2	1	0,971629
ORT	0	3	0	0,999981
ORT	0	4	0	0,998749
ORT	0	5	0	0,970392
ORT	1	0	0	1
ORT	1	0	1	0,995428
ORT	1	1	0	1
ORT	1	1	1	0,985976

ORT	1	2	0	1
ORT	1	3	0	0,999902
ORT	1	4	0	0,994282
ORT	2	0	0	1
ORT	2	0	1	0,992964
ORT	2	1	0	1
ORT	2	1	1	0,97387
ORT	2	2	0	0,999997
ORT	2	3	0	0,999412
ORT	2	4	0	0,976303
ORT	3	0	0	1
ORT	3	0	1	0,987592
ORT	3	1	0	1
ORT	3	2	0	0,999978
ORT	3	3	0	0,996523
ORT	4	0	0	1
ORT	4	0	1	0,976166
ORT	4	1	0	1
ORT	4	2	0	0,999796
ORT	4	3	0	0,982236
ORT	5	0	0	1
ORT	5	0	1	0,952562
ORT	5	1	0	0,999998
ORT	5	2	0	0,998245
ORT	6	0	0	1
ORT	6	1	0	0,999959
ORT	6	2	0	0,98795
ORT	7	0	0	1
ORT	7	1	0	0,999354
ORT	8	0	0	0,999997
ORT	8	1	0	0,993065
ORT	9	0	0	0,999871
ORT	9	1	0	0,953121
ORT	10	0	0	0,997045
ORT	11	0	0	0,967454
OB	0	1	0	1
OB	0	2	0	1
OB	0	3	0	1
OB	0	4	0	1
OB	0	5	0	1
OB	0	6	0	1
OB	0	7	0	0,999989
OB	0	8	0	0,999095
OB	0	9	0	0,976736
OB	1	0	0	1
OB	1	1	0	1
OB	1	2	0	1
OB	1	3	0	1
OB	1	4	0	1
OB	1	5	0	1
OB	1	6	0	0,999999
OB	1	7	0	0,999839
OB	1	8	0	0,993066
OB	2	0	0	1
OB	2	1	0	1

OB	2	2	0	1
OB	2	3	0	1
OB	2	4	0	1
OB	2	5	0	1
OB	2	6	0	0,999978
OB	2	7	0	0,99835
OB	2	8	0	0,96316
OB	3	0	0	1
OB	3	1	0	1
OB	3	2	0	1
OB	3	3	0	1
OB	3	4	0	1
OB	3	5	0	0,999998
OB	3	6	0	0,999692
OB	3	7	0	0,988245
OB	4	0	0	1
OB	4	1	0	1
OB	4	2	0	1
OB	4	3	0	1
OB	4	4	0	1
OB	4	5	0	0,999956
OB	4	6	0	0,997019
OB	5	0	0	1
OB	5	1	0	1
OB	5	2	0	1
OB	5	3	0	1
OB	5	4	0	0,999995
OB	5	5	0	0,999412
OB	5	6	0	0,980442
OB	6	0	0	1
OB	6	1	0	1
OB	6	2	0	1
OB	6	3	0	1
OB	6	4	0	0,999912
OB	6	5	0	0,994683
OB	7	0	0	1
OB	7	1	0	1
OB	7	2	0	1
OB	7	3	0	0,99999
OB	7	4	0	0,998882
OB	7	5	0	0,968165
OB	8	0	0	1
OB	8	1	0	1
OB	8	2	0	0,999999
OB	8	3	0	0,999823
OB	8	4	0	0,990672
OB	9	0	0	1
OB	9	1	0	1
OB	9	2	0	0,99998
OB	9	3	0	0,997894
OB	10	0	0	1
OB	10	1	0	0,999998
OB	10	2	0	0,999644
OB	10	3	0	0,983959
OB	11	0	0	1

OB	11	1	0	0,999957
OB	11	2	0	0,996088
OB	12	0	0	0,999996
OB	12	1	0	0,99929
OB	12	2	0	0,973054
OB	13	0	0	0,999908
OB	13	1	0	0,992859
OB	14	0	0	0,998597
OB	14	1	0	0,955939
OB	15	0	0	0,987241
URO	0	0	1	1
URO	0	0	2	0,998839
URO	0	1	0	1
URO	0	1	1	0,999989
URO	0	1	2	0,964683
URO	0	2	0	1
URO	0	2	1	0,998457
URO	0	3	0	0,999998
URO	0	4	0	0,997627
URO	1	0	0	1
URO	1	0	1	0,999999
URO	1	0	2	0,994768
URO	1	1	0	1
URO	1	1	1	0,999917
URO	1	2	0	1
URO	1	2	1	0,990591
URO	1	3	0	0,999953
URO	1	4	0	0,980243
URO	2	0	0	1
URO	2	0	1	0,999996
URO	2	0	2	0,978467
URO	2	1	0	1
URO	2	1	1	0,999345
URO	2	2	0	1
URO	2	2	1	0,955459
URO	2	3	0	0,999192
URO	3	0	0	1
URO	3	0	1	0,999971
URO	3	1	0	1
URO	3	1	1	0,995415
URO	3	2	0	0,999991
URO	3	3	0	0,991177
URO	4	0	0	1
URO	4	0	1	0,999745
URO	4	1	0	1
URO	4	1	1	0,974765
URO	4	2	0	0,99977
URO	5	0	0	1
URO	5	0	1	0,997944
URO	5	1	0	0,999999
URO	5	2	0	0,996574
URO	6	0	0	1
URO	6	0	1	0,986736
URO	6	1	0	0,999947
URO	6	2	0	0,970511

URO	7	0	0	1
URO	7	1	0	0,998877
URO	8	0	0	0,999991
URO	8	1	0	0,986748
URO	9	0	0	0,999701
URO	10	0	0	0,994905
URO	11	0	0	0,955466
GS	0	0	1	0,999988
GS	0	0	2	0,98976
GS	0	1	0	1
GS	0	1	1	0,999605
GS	0	2	0	0,999997
GS	0	2	1	0,990015
GS	0	3	0	0,999738
GS	0	4	0	0,990323
GS	1	0	0	1
GS	1	0	1	0,999978
GS	1	0	2	0,97128
GS	1	1	0	1
GS	1	1	1	0,998714
GS	1	2	0	0,999993
GS	1	2	1	0,97019
GS	1	3	0	0,998999
GS	1	4	0	0,969045
GS	2	0	0	1
GS	2	0	1	0,99993
GS	2	1	0	1
GS	2	1	1	0,995305
GS	2	2	0	0,999968
GS	2	3	0	0,995803
GS	3	0	0	1
GS	3	0	1	0,999693
GS	3	1	0	1
GS	3	1	1	0,982903
GS	3	2	0	0,999817
GS	3	3	0	0,982985
GS	4	0	0	1
GS	4	0	1	0,998457
GS	4	1	0	0,999999
GS	4	2	0	0,998837
GS	5	0	0	1
GS	5	0	1	0,992348
GS	5	1	0	0,999992
GS	5	2	0	0,993066
GS	6	0	0	1
GS	6	0	1	0,966844
GS	6	1	0	0,99988
GS	6	2	0	0,965901
GS	7	0	0	1
GS	7	1	0	0,998499
GS	8	0	0	0,999999
GS	8	1	0	0,986589
GS	9	0	0	0,999945
GS	10	0	0	0,997961
GS	11	0	0	0,969183

OMS	0	0	1	1
OMS	0	0	2	0,999821
OMS	0	0	3	0,976717
OMS	0	1	0	1
OMS	0	1	1	0,999999
OMS	0	1	2	0,997483
OMS	0	2	0	1
OMS	0	2	1	0,999954
OMS	0	2	2	0,966362
OMS	0	3	0	1
OMS	0	3	1	0,997802
OMS	0	4	0	0,999997
OMS	0	5	0	0,998415
OMS	1	0	0	1
OMS	1	0	1	0,999998
OMS	1	0	2	0,998492
OMS	1	1	0	1
OMS	1	1	1	0,999967
OMS	1	1	2	0,981074
OMS	1	2	0	1
OMS	1	2	1	0,998883
OMS	1	3	0	0,999998
OMS	1	3	1	0,97295
OMS	1	4	0	0,999362
OMS	1	5	0	0,958363
OMS	2	0	0	1
OMS	2	0	1	0,999973
OMS	2	0	2	0,98917
OMS	2	1	0	1
OMS	2	1	1	0,999385
OMS	2	2	0	0,999999
OMS	2	2	1	0,986039
OMS	2	3	0	0,999718
OMS	2	4	0	0,98107
OMS	3	0	0	1
OMS	3	0	1	0,999627
OMS	3	1	0	0,999999
OMS	3	1	1	0,992652
OMS	3	2	0	0,99986
OMS	3	3	0	0,991381
OMS	4	0	0	0,999998
OMS	4	0	1	0,995986
OMS	4	1	0	0,999921
OMS	4	2	0	0,995969
OMS	5	0	0	0,999949
OMS	5	0	1	0,970871
OMS	5	1	0	0,998022
OMS	5	2	0	0,955604
OMS	6	0	0	0,998962
OMS	6	1	0	0,976901
OMS	7	0	0	0,987913
EYE	0	0	1	1
EYE	0	0	2	1
EYE	0	0	3	1
EYE	0	0	4	1

EYE	0	0	5	0,999999
EYE	0	0	6	0,999895
EYE	0	0	7	0,995618
EYE	0	1	0	1
EYE	0	1	1	1
EYE	0	1	2	1
EYE	0	1	3	1
EYE	0	1	4	1
EYE	0	1	5	0,999961
EYE	0	1	6	0,99823
EYE	0	1	7	0,968219
EYE	0	2	0	1
EYE	0	2	1	1
EYE	0	2	2	1
EYE	0	2	3	1
EYE	0	2	4	0,999985
EYE	0	2	5	0,999288
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GYN	5	1	0	0,999995
GYN	5	1	1	0,962994
GYN	5	2	0	0,998073
GYN	6	0	0	1
GYN	6	0	1	0,994529
GYN	6	1	0	0,999941
GYN	6	2	0	0,990081
GYN	7	0	0	1
GYN	7	0	1	0,97486
GYN	7	1	0	0,999407
GYN	7	2	0	0,958719
GYN	8	0	0	0,999997
GYN	8	1	0	0,995338
GYN	9	0	0	0,999916
GYN	9	1	0	0,973306
GYN	10	0	0	0,998589
GYN	11	0	0	0,986211

C
Group Data

Below are both the fitted data and the number of surgeries has to be performed on a patient from the corresponding patient group (β).

Table C.1: Group Data

Name	Mean	Std	Var	Shape log-normal	Loc log-normal	Scale log-normal	β
GS1	70,81875	19,79627	391,8921	0,27542	0	68,19924	14
GS2	151,3028	51,6008	2662,624	0,338764	0	142,9595	38
GS3	292,2068	85,79701	7361,127	0,270312	0	281,3119	15
GYN1	65,10674	20,55535	422,5223	0,329477	0	61,83744	7
GYN2	132,28	51,1609	2617,438	0,370667	0	123,3814	12
GYN3	248,1579	71,53724	5117,577	0,280722	0	238,5133	5
OMS1	95,62195	36,03238	1298,333	0,373327	0	89,33751	4
OMS2	131,9655	32,93164	1084,493	0,238664	0	128,1951	4
OMS3	219,2	85,29103	7274,56	0,284206	0	208,84	2
ENT1	54,17085	25,31532	640,8653	0,483262	0	48,71838	9
ENT2	104,4021	53,80984	2895,499	0,434195	0	94,41436	24
ENT3	273,3788	169,2877	28658,32	0,499745	0	239,0215	2
EYE1	39,76707	16,23355	263,5281	0,427917	0	36,5237	11
EYE2	63,53636	26,74694	715,3987	0,448038	0	57,95105	20
EYE3	97,08148	31,01757	962,0897	0,292311	0	92,85704	19
ORT1	69,24242	18,63246	347,1685	0,277642	0	66,72075	6
ORT2	136,1654	46,988	2207,869	0,335281	0	128,7906	19
ORT3	277,6383	160,1762	25656,4	0,465388	0	246,3412	0
PLA1	64,7619	16,10577	259,3957	0,256254	0	62,73098	4
PLA2	97,75342	29,50773	870,7063	0,295397	0	93,58149	7
URO1	70,23246	22,24807	494,9767	0,30743	0	66,99114	11
URO2	163,8876	35,83528	1284,167	0,228031	0	159,8311	4
URO3	266,175	61,65207	3800,978	0,23655	0	259,0044	6
OB1	49,64	16,15767	261,0704	0,31856	0	47,17075	2
OB2	84,07639	26,2266	687,8344	0,30603	0	80,37016	7

D
Schedules

This appendix contains the schedules, which resulted from the different models. The schedules contain the abbreviations of a specialty followed by a list of numbers. These numbers correspondent to the different patient groups corresponding to that specialty and how often a surgery is performed on that patient group. The ORs reserved for emergencies are left out.

D.1. Column Based

The following schedules correspond to the column based model using an α of 0.05.

Table D.1: First week of the schedule.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
OR1	ORT: [1, 2, 0]	ORT: [1, 2, 0]	ORT: [1, 2, 0]	ORT: [1, 2, 0]	
OR2	ORT: [1, 2, 0]	ORT: [1, 2, 0]	ORT: [1, 2, 0]	ORT: [1, 2, 0]	ORT: [1, 2, 0]
OR3	URO: [1, 0, 1]	URO: [1, 0, 1]	URO: [3, 1, 0]	URO: [1, 0, 1]	URO: [1, 0, 1]
OR4			OB: [1, 4]		
OR5	GS: [0, 2, 0]	GS: [0, 0, 1]		GS: [0, 2, 0]	GS: [3, 1, 0]
OR6	GS: [0, 2, 0]	GS: [0, 0, 1]	GS: [0, 0, 1]	GS: [3, 1, 0]	GS: [0, 0, 1]
OR7	GS: [0, 2, 0]	GYN: [1, 0, 1]	GS: [0, 0, 1]	GS: [0, 0, 1]	GS: [0, 0, 1]
OR8	GS: [0, 2, 0]	GS: [0, 0, 1]	GS: [0, 0, 1]	GS: [0, 0, 1]	GS: [0, 2, 0]
OR9	GYN: [1, 0, 1]	PLA: [9, 2]	GYN: [1, 0, 1]	GYN: [1, 0, 1]	GYN: [1, 0, 1]
OR10	OMS: [0, 1, 0]	OMS: [1, 2, 0]	GS: [2, 0, 0]	OMS: [4, 0, 0]	
OR11	EYE: [1, 2, 2]	EYE: [3, 2, 1]	EYE: [1, 2, 2]	EYE: [1, 2, 2]	EYE: [1, 2, 2]
OR15	ENT: [6, 0, 0]	ENT: [6, 0, 0]	ENT: [0, 2, 1]	ENT: [4, 1, 0]	ENT: [4, 1, 0]
OR16	ENT: [2, 2, 0]	ENT: [2, 2, 0]	ENT: [4, 1, 0]	ENT: [4, 1, 0]	ENT: [4, 1, 0]
OR10a	PLA: [1, 1]		PLA: [1, 1]		PLA: [6, 0]

Table D.2: Second week of the schedule.

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
OR1	ORT: [1, 2, 0]	ORT: [1, 2, 0]	ORT: [1, 2, 0]	ORT: [1, 2, 0]	
OR2	ORT: [1, 2, 0]	GS: [0, 0, 1]	ORT: [1, 2, 0]	ORT: [1, 2, 0]	ORT: [1, 2, 0]
OR3	URO: [1, 0, 1]	URO: [1, 0, 1]	URO: [3, 1, 0]	URO: [3, 1, 0]	URO: [3, 1, 0]
OR4			OB: [1, 4]		
OR5	GS: [0, 0, 1]	GS: [0, 2, 0]		GS: [0, 2, 0]	GS: [3, 1, 0]
OR6	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [3, 1, 0]	GS: [3, 1, 0]
OR7	GS: [0, 2, 0]	GYN: [1, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 0, 1]
OR8	GS: [0, 0, 1]	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [3, 1, 0]	GS: [0, 0, 1]
OR9	GYN: [1, 2, 0]	GYN: [1, 2, 0]	GYN: [1, 2, 0]	GYN: [1, 2, 0]	GYN: [1, 2, 0]
OR10	OMS: [0, 1, 0]	OMS: [1, 0, 1]	GS: [2, 0, 0]	OMS: [1, 0, 1]	
OR11	EYE: [1, 2, 2]	EYE: [1, 2, 2]	EYE: [1, 2, 2]	EYE: [1, 2, 2]	EYE: [1, 2, 2]
OR15	ENT: [4, 1, 0]		ENT: [0, 2, 1]	ENT: [2, 2, 0]	ENT: [2, 2, 0]
OR16	ENT: [4, 1, 0]	ENT: [6, 0, 0]	ENT: [2, 2, 0]	ENT: [2, 2, 0]	ENT: [4, 1, 0]
OR10a	PLA: [1, 1]		PLA: [1, 1]		PLA: [1, 1]

D.2. Discrete Model

The following schedules correspond to the discrete model using an α of 0.05.

Table D.3: First week of the schedule

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
OR1	ORT: [0, 0, 1]	ORT: [0, 0, 1]	ORT: [0, 0, 1]	ORT: [0, 0, 1]	
OR2	ORT: [0, 0, 1]	ORT: [0, 0, 1]	ORT: [0, 0, 1]	ORT: [0, 0, 1]	ORT: [0, 0, 1]
OR3	URO: [1, 0, 1]	URO: [0, 2, 0]	URO: [1, 0, 1]	URO: [1, 0, 1]	URO: [1, 0, 1]
OR4			OB: [6, 0]		
OR5	GS: [0, 2, 0]	GS: [0, 2, 0]		GS: [0, 2, 0]	GS: [0, 2, 0]
OR6	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]
OR7	GS: [0, 2, 0]	GYN: [1, 0, 1]	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]
OR8	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]
OR9	GYN: [1, 0, 1]	PLA: [7, 2]	GYN: [1, 0, 1]	GYN: [1, 0, 1]	GYN: [1, 0, 1]
OR10	OMS: [0, 0, 1]	OMS: [0, 2, 0]	GS: [0, 0, 1]	OMS: [0, 2, 0]	
OR11	EYE: [0, 3, 1]	EYE: [0, 3, 1]	EYE: [0, 3, 1]	EYE: [0, 3, 1]	EYE: [5, 0, 1]
OR12					
OR13					
OR14					
OR15	ENT: [0, 0, 1]	ENT: [0, 0, 1]	ENT: [3, 0, 1]	ENT: [0, 0, 1]	ENT: [0, 0, 1]
OR16	ENT: [0, 0, 1]	ENT: [0, 0, 1]	ENT: [0, 0, 1]	ENT: [0, 0, 1]	ENT: [0, 0, 1]
OR10a	PLA: [2, 0]		PLA: [2, 0]		PLA: [2, 2]

Table D.4: Second week of the schedule

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
OR1	ORT: [0, 0, 1]	ORT: [0, 0, 1]	ORT: [0, 0, 1]	ORT: [0, 0, 1]	ORT: [0, 0, 1]
OR2	ORT: [2, 1, 0]	GS: [0, 2, 0]	ORT: [0, 0, 1]	ORT: [0, 0, 1]	ORT: [0, 0, 1]
OR3	URO: [1, 0, 1]	URO: [1, 0, 1]	URO: [1, 0, 1]	URO: [1, 0, 1]	URO: [1, 0, 1]
OR4			OB: [4, 1]		
OR5	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]
OR6	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]
OR7	GS: [0, 2, 0]	GYN: [1, 0, 1]	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]
OR8	GS: [0, 2, 0]	GS: [0, 2, 0]	GS: [2, 1, 0]	GS: [0, 2, 0]	GS: [0, 2, 0]
OR9	GYN: [0, 2, 0]	GYN: [1, 0, 1]	GYN: [1, 0, 1]	GYN: [1, 0, 1]	GYN: [1, 0, 1]
OR10	OMS: [0, 0, 1]	OMS: [0, 2, 0]	GS: [0, 0, 1]	OMS: [1, 1, 0]	
OR11	EYE: [5, 0, 1]	EYE: [0, 3, 1]	EYE: [5, 0, 1]	EYE: [5, 0, 1]	EYE: [0, 3, 1]
OR12					
OR13					
OR14					
OR15	ENT: [0, 0, 1]		ENT: [1, 1, 1]	ENT: [0, 0, 1]	ENT: [0, 0, 1]
OR16	ENT: [0, 0, 1]	ENT: [0, 0, 1]	ENT: [0, 0, 1]	ENT: [0, 0, 1]	ENT: [0, 0, 1]
OR10a	PLA: [2, 0]		PLA: [2, 0]	PLA: [2, 0]	PLA: [2, 0]