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TRAIN TRAJECTORY OPTIMIZATION UNDER PARAMETRIC UNCERTAINTY AND ROBUST MAXIMUM PRINCIPLE ANALYSIS

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Abstract. In the paper the observed parameter variations of a single train into the trajectory optimization problem is incorporated. The computed trajectories should be energyefficient and robust with respect to the realized parameter configuration. A new approach for modelling the Train Trajectory Optimization problem under parametric uncertainty in terms of a minimax optimal control problem with a finite set of observed parameter configurations presented.

1. Introduction

Railways are among the most energy-efficient means of transport, and the research community is continuously aiming to further improve its reliability, punctuality and energy consumption. Particularly, train trajectory optimization is a well-known research topic to reduce the energy consumption in railway operation. This technique aims to calculate the speed of the train at each location of the track along its movement between two stations in the time prescribed by the timetable while minimizing the consumed energy. Several direct [6] and indirect [1] optimal control methods have been utilized to solve the train trajectory optimization problem. However, this topic has been mainly addressed deterministically, neglecting the stochastic nature of railway operation [5].

Train trajectory optimization considering model parametric uncertainty has been partially addressed by means of reinforcement learning [7, 9] and stochastic programming [8], but it has not been analyzed yet from a mathematical control theory perspective. Uncertainties in the parameters of the train motion model may impact railway operation. For instance, two units of the same kind of rolling stock may show differences in their dynamics [4]. Moreover, considering a particular rolling stock unit, different weather and engine wear conditions and fluctuations in the available catenary power may lead to non-negligible deviations in the dynamics over different trajectories in the same track. These different conditions may lower the reliability of the calculated optimal trajectories, causing delays and lowering the effectiveness and hindering the implementation of energy-efficient applications like Driver Advisory Systems and Automatic Train Operation.

Parameter estimation is therefore an essential way of gauging the impact of the different operating conditions in the train motion model that is included in the train trajectory optimization problem. To this end, historical operational data can be analyzed to calculate parameter variations [2].

In this research, we aim to incorporate the observed parameter variations of a single train into the trajectory optimization problem. The computed trajectories should be energy-efficient and robust with respect to the realized parameter configuration, therefore, their feasibility should be guaranteed independently of the realized parameter configuration, assuming that it takes its value among the set of observed parameter configurations. In this manuscript we show preliminary results of our research. We model the robust train trajectory optimization problem as a minimax optimal control problem that considers the set of observed parameter configurations. Then, we reformulate it as an optimal control problem and apply the Robust Maximum Principle [3] to study the properties of the robust-optimal control policies that solve the mentioned problem.

2. Train trajectory optimization under parametric uncertainty

Let $\mathcal{A} = \{\alpha_1, ..., \alpha_q\}$ be a finite set gathering the observed parameter configurations α. We consider a train running from s_o to s_e on a flat track. For any parameter configuration $\alpha \in \mathcal{A}$ let $v^\alpha(s)$ be the train speed, $t^\alpha(s)$ its running time and $E^\alpha(s)$ its energy consumption along the locations $s \in [s_o, s_e]$. Let $u_f(s)$ and $u_b(s)$ be the control variables that represent the applied tractive and brake effort, respectively. The units of all the variables defined in this document are SI units (s, m, kg). Moreover, we consider unity train mass. Therefore, the minimax train trajectory optimization problem is

$$\min_{u_f, u_b} J = \min_{u_f, u_b} \max_{\alpha \in \mathcal{A}} E^\alpha(s_e), \tag{1}$$

subject to

$$\frac{dv^\alpha}{ds} = f_v^\alpha(v^\alpha, t^\alpha, u_f, u_b, s) = \frac{u_f(s) - u_b(s) - r^\alpha(v^\alpha)}{v^\alpha(s)}, \ \alpha \in \mathcal{A}, \tag{2}$$

$$\frac{dt^\alpha}{ds} = f_t^\alpha(v^\alpha, t^\alpha, u_f, u_b, s) = \frac{1}{v^\alpha(s)}, \ \alpha \in \mathcal{A}, \tag{3}$$

$$\frac{dE^\alpha(s)}{ds} = u_f(s), \ \alpha \in \mathcal{A}, \tag{4}$$

$$u_f(s) v^\alpha(s) \leq p_{\max}^\alpha, \ \alpha \in \mathcal{A}, \tag{5}$$

$$0 \leq u_f(s) \leq u_{f,\max}^\alpha, \ \alpha \in \mathcal{A}, \iff 0 \leq u_f(s) \leq \bar{u}_{f,\max} \min_{\alpha \in \mathcal{A}} u_{f,\max}^\alpha, \tag{6}$$

$$0 \leq u_b(s) \leq u_{b,\max}^\alpha, \ \alpha \in \mathcal{A}, \iff 0 \leq u_b(s) \leq \bar{u}_{b,\max} \min_{\alpha \in \mathcal{A}} u_{b,\max}^\alpha, \tag{7}$$

$$v^\alpha(s_o) = v_o, \ v^\alpha(s_e) \leq v_e, \ t^\alpha(s_o) = 0, \ t^\alpha(s_e) \leq T, \ E^\alpha(s_o) = 0, \ \alpha \in \mathcal{A}, \tag{8}$$

where $r^\alpha(v^\alpha) = r_0^\alpha + r_1^\alpha v^\alpha + r_2^\alpha (v^\alpha)^2$ is the running resistance, and $\alpha_k = (r_0^{\alpha_k}, r_1^{\alpha_k}, r_2^{\alpha_k}, p_{\max}^{\alpha_k})^\top$, $k = 1, ..., q$, is the k-th parameter configuration, $\alpha_k \in \mathcal{A}$.

We assume $v^\alpha, \ v_o, \ v_e > 0, \ \forall \alpha \in \mathcal{A}$ in order to guarantee the Lipschitz condition of the dynamic equations (2), (3) with respect to v^α. Alternatively, the time could be used as the independent variable.

Moreover, from (1) and (4), we observe that $E^{\alpha_1}(s) = ... = E^{\alpha_q}(s), \ \forall s \in [s_o, s_e]$. Therefore, defining $E(s) E^{\alpha_1}(s) = ... = E^{\alpha_q}(s)$ allows us to eliminate (4) and to simplify the maximum term in the cost function.

$$\min_{u_f, u_b} J = \min_{u_f, u_b} \int_{s_o}^{s_e} u_f(s) ds, \tag{9}$$

Therefore, we may reformulate the optimal control problem by aggregating the state variables corresponding to all the parameter configurations $\alpha \in \mathcal{A}$ into the same state vector. Let $x = (v^{\alpha_1}, t^{\alpha_1}, ..., v^{\alpha_q}, t^{\alpha_q})^\top$ be the new state vector. Then, the robust train trajectory optimization problem with respect to the set of parameter configurations $\mathcal{A}$ is (9) subject to (5), (6), (7), (8) and

$$\frac{dx}{ds} = (f_v^{\alpha_1}, f_t^{\alpha_1}, ..., f_v^{\alpha_q}, f_t^{\alpha_q})^\top (x, u_f, u_b, s), \tag{10}$$

This turns into an optimal control problem that looks for the most energy-efficient trajectory that is feasible for all the considered parameter configurations $\alpha \in \mathcal{A}$ of the considered train.

3. Applying the robust maximum principle

Applying the Robust Maximum Principle [3] we obtain the following possible control phases:

- $\sum_{\alpha\in\mathcal{A}} \lambda_v^\alpha/v^\alpha > 1$. Then $u_f = \min\{\bar{u}_{f,\max}, p_{\max}^{\alpha_1}/v^{\alpha_1}, ..., p_{\max}^{\alpha_q}/v^{\alpha_q}\}$ and $u_b = 0$, which corresponds to a maximum traction phase.
- $0 < \sum_{\alpha\in\mathcal{A}} \lambda_v^\alpha/v^\alpha < 1$. Then $u_f = u_b = 0$, leading to a coasting phase.
- $\sum_{\alpha\in\mathcal{A}} \lambda_v^\alpha/v^\alpha < 0$. Then $u_f = 0$ and $u_b = \bar{u}_{b,\max}$, which is a maximum braking phase.

Furthermore, we find two singular control phases:

- $\sum_{\alpha\in\mathcal{A}} \lambda_v^\alpha/v^\alpha = 1$. Then, $0 < u_f < \min\{\bar{u}_{f,\max}, p_{\max}^{\alpha_1}/v^{\alpha_1}, ..., p_{\max}^{\alpha_q}/v^{\alpha_q}\}$ and $u_b = 0$, which is a cruising by applying partial traction phase.
- $\sum_{\alpha\in\mathcal{A}} \lambda_v^\alpha/v^\alpha = 0$. Then, $u_f = 0$ and $0 < u_b < \bar{u}_{b,\max}$, a cruising by applying partial brake phase.

4. Conclusion

We have presented a new approach for modelling the Train Trajectory Optimization problem under parametric uncertainty in terms of a minimax optimal control problem with a finite set of observed parameter configurations. The calculated trajectories should be robust under parametric uncertainty, provided that that the parameter configuration of the train will be drawn from the mentioned set of known parameter configurations. Furthermore, we have applied the Robust Maximum Principle [3] to analyze the potential solutions of the problem and showed that a robust-optimal control policy is a concatenation of maximum traction, coasting, maximum braking and maybe singular control phases.

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Keywords: Robust Train Trajectory Optimization, Train Trajectory Optimization, Minimax Optimal Control, Robust Maximum Principle, Parametric Uncertainty, Railways

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