River regime based on sediment transport concepts

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Report No. IT 201
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ABSTRACT Rational regime relationships for the width, depth and slope of a river in equilibrium are developed using the Ackers and White sediment transport formula and the White, Paris and Bettess friction relationships, together with a principle of maximum sediment transporting capacity. This concept of maximising the sediment transporting capacity is shown to be equivalent to minimising the slope of the river. The relationships which are developed show good agreement with other empirically derived regime relationships and data from sand channels. Some comparisons are made with data from gravel rivers and the difficulties in applying regime concepts to these rivers are discussed.

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NOTATION

A	Value of F_{gr} at initial motion (Ackers and White)
B (m)	Width of channel
C	Parameter in Ackers and White sediment transport theory
D (m)	Sediment diameter
D_{50} (m)	Sediment diameter for which 50% of the sample is finer
d (m)	Depth of flow
D_{gr}	Dimensionless grain size (Ackers and White)
F_{gr}	Sediment mobility (Ackers and White)
F_{fg}	Sediment mobility, fine grains (Ackers and White)
G_{gr}	Dimensionless sediment transport (Ackers and White)
g (m/s^2)	Acceleration due to gravity
m	Parameter in Ackers and White sediment transport theory
n	Transition exponent (Ackers and White)
Q (m^3/s)	Discharge
R	Hydraulic radius
S	Channel slope
s	Specific gravity of sediment
V (m/s)	Velocity of flow
V_* (m/s)	Shear velocity
X	Sediment concentration
Z	Scale slope of trapezoidal channel (1 (vertical) to z (horizontal))
ν (m^2/s)	Kinematic viscosity

Subscripts

o	Observed value
c	calculated value

INTRODUCTION

The problem of determining a stable, cross-section geometry and slope of an alluvial channel has been the subject of considerable research over eighty years and continues to be of great practical interest (Ackers and Charlton, 1970; Charlton, et al, 1978; Ackers, 1980; Chang, 1980). An alluvial channel can adjust its width, depth and slope to achieve a stable condition in which it can transport a certain amount of water and sediment. It thus has three degrees of freedom and the problem is to establish relationships which determine these three quantities of width, depth and slope.

The various approaches to this problem fall into two broad categories: the regime and the physical methods. The regime method is an empirical method which relies on analysing available data and attempting to determine appropriate relationships from the data. An early attempt at this approach was made by Kennedy (1895) who collected data from stable canals and used this data to derive a relationship between the mean velocity and depth of flow. The usefulness of this method depends upon the quality of the data and the validity of the assumed form of the relationships. It has always been acknowledged that the various coefficients derived may not be truly constant but may vary slightly and that the equations should only be applied in situations similar to those for which the data was collected.

The physical method relies on specifying equations which describe the dominant individual processes such as sediment transport, flow resistance and bank stability. This approach can only be successful if the dominant processes are correctly identified and appropriate equations exist to describe them adequately. These approaches represent two extremes and obviously it is possible to combine aspects of both.

In this report we are concerned with the development of a physical approach. Two equations are readily available defining the sediment transport and the frictional characteristics but it is unclear what constitutes an appropriate third equation. Several proposals have been made for a suitable equation, some concerned with bank stability (Ackers, 1980), others based on some variational principle such as minimum stream power or minimum unit stream power (Chang, 1980; Yang, 1976). We advocate, in this report, a variational principle based on the assumption that an alluvial channel adjusts its geometric characteristics and gradient in such a way that the sediment transporting capacity is maximised. We can find no physical justification to support the principle of maximising the sediment transporting capacity but regard it as a useful hypothesis which, as will be shown, leads to acceptable predictions over a large range of flow conditions. Since formulating this hypothesis we have become aware of the work of Ramette (1980 a, b) who proposes a similar principle. Ramette, however, provides little justification for the approach and no comparisons with data. We also show that this principle is equivalent to the minimum stream power concept, (see Appendix). By using this variational principle together with the Ackers and White equations for sediment transport and the White, Paris and Bettess equations for flow resistance, the geometric and hydraulic parameters of a stable channel have been obtained for a wide range of practical applications. The results have been compared with available data and existing regime equations. This comparison confirms the validity of the presented method for a wide range of applications, extending the limited range of available regime equations.

FORMULATION OF THE METHOD

We consider six variables that describe the river system; the average velocity, V, average depth d, slope S, discharge Q, sediment concentration X and channel width B. Relating these variables we have an equation for the continuity of water flow, a sediment transport formula, a flow resistance formula and the condition that the sediment transport should be maximised, or equivalently, stream power should be minimised. In the work that follows we consider the discharge and slope to be imposed and determine the corresponding values of V, d, X and B.

Implicit in this work are the assumptions that the flow is steady and uniform and that the bed and bank material is non-cohesive.

Sediment transport

The Ackers and White equations (1973) have been used to calculate the sediment concentration. Ackers and White described the movement of sediment in terms of three dimensionless groups:

a) particle mobility, F_{gr}

b) sediment transport, G_{gr}

c) dimensionless particle size, D_{gr}

The particle mobility is the ratio of shear forces and immersed weight. For coarse sediments transport is considered to be a bed process and the particle mobility is expressed in terms of the net grain resistance. A fine sediment is considered to be transported in the main body of the flow where it is suspended by turbulence. As the intensity of the turbulence is dependent upon the total energy degradation, for fine sediments the particle mobility is expressed in terms of the total shear stress. The general definition of the particle mobility is

$$F_{gr} = \frac{V_*^n}{\sqrt{gD(s-1)}} \left(\frac{V}{\sqrt{32}\,\log_{10}(10d/D)}\right)^{1-n} . \qquad (1)$$

For coarse sediments n=0 while for fine sediments n=1. For intermediate, or transitional, sizes of sediment, n may take a value between 0 and 1 depending upon the value of D_{gr}.

Fine and coarse material is defined in terms of D_{gr}, where

$$D_{gr} = D \left(\frac{g\,(s-1)}{\nu^2}\right)^{1/3} . \qquad (2)$$

The right-hand term involves the cube root of the ratio of immersed weight to viscous forces. An extensive analysis of flume data led to the definition of $D_{gr} \geqslant 60$ for coarse sediments and $D_{gr} \leqslant 1$ for fine sediments. Sizes transitional between these two can exhibit both fine and coarse sediment behaviour.

The expression for sediment transport is based on the stream power concept. By combining the efficiency of transport with the mobility number a transport parameter is defined as

$$G_{gr} = \frac{Xd}{sD} \left(\frac{V_*}{V}\right)^n . \qquad (3)$$

A general transport equation is then obtained in the form

$$G_{gr} = C \left(\frac{F_{gr}}{A} - 1\right)^m , \qquad (4)$$

where C and m are coefficients depending on D_{gr}. A is the initial motion parameter, that is, the value of F_{gr} at the threshold of movement. The expressions for n, m C and A are given by:

for transitional sizes, $1 \leqslant D_{gr} \leqslant 60$

$$n = 1.0 - 0.56 \log_{10} D_{gr} \qquad (5)$$

$$m = \frac{9.66}{D_{gr}} + 1.34 \qquad (6)$$

$$\log C = 2.86 \log D_{gr} - \log_{10}{}^2 D_{gr} - 3.53 \qquad (7)$$

$$A = \frac{0.23}{\sqrt{D_{gr}}} + 0.14 \qquad (8)$$

for coarse sediments, $D_{gr} > 60$

$$n = 0.0 \qquad (9)$$

$$m = 1.50 \qquad (10)$$

$$C = 0.025 \qquad (11)$$

$$A = 0.17 \qquad (12)$$

Frictional characteristics

By utilising the same basic parameters as in the Ackers and White sediment transport theory, White, Paris and Bettess (1980) found that a linear relationship between mobilities related to the total shear stress F_{fg}, where

$$F_{fg} = \frac{V_*}{\sqrt{gD(s-1)}} \tag{13}$$

and the mobility related to the effective shear stress F_{gr} existed with coefficients depending upon D_{gr}. An extensive correlation exercise for a wide range of sediment sizes (0.04mm to 10mm) gave the equation

$$\frac{F_{gr} - A}{F_{fg} - A} = 1.0 - 0.76 \quad 1 - \frac{1}{\exp{(\log_{10} D_{gr})}^{1.7}} \quad . \tag{14}$$

This method has been favourably compared with the traditional methods of Einstein and Barbarossa, Engelund and Raudkivi and showed good agreement with data for sediment sizes in the range 0.04mm to 68mm (White, Paris and Bettess, 1980).

Variational principle

One extra equation was needed to solve the system. Various different approaches have been used to provide the necessary relationship, some of them relying on a type of variational argument in which the maximum or minimum of some quantity is sought. Previous experience led us to consider whether the system might maximise the sediment transporting capacity of the channel. More precisely the hypothesis is that, for a particular water discharge and slope, the width of the channel adjusts itself to maximise the sediment transport rate.

Alternative approaches have been proposed by Chang and Yang involving stream power.

Chang's hypothesis of minimum stream power is as follows:

> 'For an alluvial channel, the necessary and sufficient condition for equilibrium is when the stream power is a minimum subject to given constraints. Hence an alluvial channel with given water discharge and sediment inflow tends to establish its width, depth and slope such that the stream power or slope is a minimum,' (Chang 1980).

Yang's hypothesis is similar but his analysis is slightly different because he assumes that the cross-sectional area remains fixed during the minimisation (Yang 1978).

Numerical experiments indicated that maximising the transport rate and minimising the slope lead to the same results. Figure 2 shows both slope and sediment concentration as a function of width and shows that both extremes correspond to the same width, in this particular case, 43m. In the appendix we demonstrate analytically that the two principles are equivalent for a large range of sediment transport theories and friction equations. We show that where a maximum in the sediment concentration exists for a given discharge and slope, it corresponds to the minimum slope for the given discharge and the maximum sediment concentration previously calculated.

If one imposes values of discharge and slope but does not impose the condition of maximum sediment transport then there are a family of solutions each with different values of B, X, V and d, only one of which provides the maximum sediment rate.

All the remaining solutions have sediment transport rates less than the maximum and widths both less than and greater than that provided by the maximum transport rate. These all represent possible solutions of the system if it is constrained in some way, for example, by the relative erodibility of the bed and banks. Thus a channel with banks which are less erodible than the bed will have a width smaller than that corresponding to the maximum sediment transport case while a channel whose bed is more erodible than the banks will have a width correspondingly larger.

While the present study assumes that the flow is uniform and does not consider the plan geometry of the river it has been suggested that a principle of maximum sediment transport capacity is involved in determining the plan shape of a river. Orishi, Jain and Kennedy (1976) claim that 'a meandering channel can be more efficient than a straight one, in the sense that a given water discharge can transport a larger sediment load and, for some channel configurations and flow conditions, can require a smaller energy gradient'. Thus they postulate that the plan geometry of a river represents an attempt to maximise the transport rate. This should also be considered when studying the comparison of theory and observation for natural rivers presented later in the report. The effects of meandering may produce extremums different from those calculated on the basis of uniform flow.

COMPUTATIONAL PROCEDURE

A computer program was developed to determine the hydraulic and geometric characteristics of alluvial channels. For given values of water discharge, sediment concentration, bed-material size and water temperature it computes the width, depth, velocity and slope. A flow diagram showing the major steps of the computation is given in Figure 1. All the computations were performed to an accuracy of greater than 1%. A sample of the results obtained is shown in Table 1 for a particle diameter of 0.5mm.

The equations of Ackers and White and White, Paris and Bettess were based on flume experiments in which the channel shape was rectangular. For this work, however, we have assumed that the shape of the channel is approximately trapezoidal in cross-section with, in equation (1), the hydraulic radius R being used in place of the depth d and the shear velocity determined by the equation

$$V_* = \sqrt{gRS}. \tag{15}$$

The values of width and depth were then adjusted to give values corresponding to a trapezoidal section of the same cross-sectional area, where the side slope z (z horizontal to 1 vertical) of the trapezoid was given by Smith's (1974) empirically determined relationship:-

$$z = \begin{cases} 0.5 & \text{if } Q < 1\text{m}^3/\text{s} \quad (16) \\ 0.5\,Q^{1/4} & \text{if } Q \geqslant 1\text{m}^3/\text{s}. \quad (17) \end{cases}$$

Because the width to depth ratio is generally large, errors introduced by this simplification were not significant.

EVALUATION OF METHOD

The proposed method was compared with available data and with existing empirical regime relationships derived by fitting curves to data. The field data for sand channels came from the Punjab canals, CHOP (Canal and headworks observation programme) canals, UP (Uttar Pradesh) and Sind canals (ICID, 1966), Pakistan canals (ACOP) (Mahmood et al, 1979 a) and the Simons and Bender data from American canals (Simons, 1957). This provided a total of 213 observations. The selection of data was, in part, restricted by the requirement of having information on sediment size and concentration and hence some of the data traditionally used in regime analysis such as that used by Lacey, was not utilised here. The laboratory data was taken from the work of Ackers (1964), Ackers and Charlton (1970) and Ranga-Raju el at (1977). A summary of the canal data is given in Table 2.

For the comparison with observed data two different calculations were performed; in one the observed values of Q and S were taken and the width, depth and sediment concentration were calculated; in the other observed values of Q and X were taken and the width, depth and slope were determined.

Where the data included information on sediment grading curves the D_{35} size was chosen for computational purposes. This is in line with previous recommendations (Ackers and White, 1973 and White, Paris and Bettess, 1980).

The present method was also compared with data from gravel rivers in Alberta (Kellerhals et al, 1972). For rivers one has to select some discharge as the dominant or significant discharge. In the present work this was arbitrarily chosen to be the bankfull discharge. In considering the results it must be remembered that better agreement between prediction and observation might be obtained by a different choice of dominant discharge.

The regime equations considered herein have been divided into two groups, one group derived for sand channels and the other for gravels. The sand group includes Ackers' equations (Ackers, 1964) derived from small channel experiments, Lacey's equations (Lacey, 1930) and the ACOP equations (Mahmood et al, 1976b). These equations are summarised in Table 3 together with the range of data on which they were based. The gravel river equations are summarised in Table 4. The results of the comparison are described below, separately for sand and gravel channels.

Sand channels

General relationships

In Figures 3 and 4 existing regime relationships are compared with those produced by the present method. The ACOP relations have not been plotted in Figure 3 since they are substantially the same as the Lacey equations. There is a reasonable agreement with the existing empirically derived relationships. The agreement for depth and width is good for a wide range of discharges. The relationships for slope seem less satisfactory. It seems that the slope is strongly dependent upon the sediment transport rate, as has been observed by other investigators (Parker, 1979). The empirical relations of Lacey and ACOP do not take this into account and this could lead to errors as shown by Ranga-Raju (1977).

Comparison with data using the principle of minimum slope

Breadths, depths and slopes have been computed for sand channels using observed values of flows and sediment concentrations. Comparisons of observed and calculated data are shown in Figures 5 to 10.

Figures 5 and 6 relate to channel slope. Observed slopes are plotted against calculated slopes in Figure 5 and the slope discrepancy ratio (calculated value divided by observed value) is plotted against discharge in Figure 6. Calculated slopes are very sensitive to the observed sediment concentrations and much of the scatter in these plots may be attributed to errors in observation. Some data sets are better than others, the Punjab data being particularly consistent over a 1000 fold variation in discharge. The mean discrepancy ratio in Figure 6 is 2.201 with a standard deviation of 1.931.

Figures 7 and 8 relate to the depth of flow. The level of agreement is significantly improved in these plots, the only major exception being the laboratory measurements of meandering channels by Ackers and Charlton. The mean discrepancy ratio in Figure 8 is 1.031 with a Standard Deviation of 0.328.

Comparisons of channel widths are given in Figures 9 and 10. In the range $1 < B(m) < 20$ agreement between calculated and observed values is good. At greater breadths a tendency to underpredict becomes apparent. Flume data is once more anomalous. The mean discrepancy ratio in Figure 10 is 0.963 with a Standard Deviation of 0.328.

Comparison with data using the principle of maximum sediment transport

Sediment concentrations, depths and widths have been computed for sand channels using observed values of flows and slopes. These data are given in Figures 11, 12 and 13.

Figure 11 relates to sediment concentrations. The scatter in the data is similar to that recorded in Figures 5 and 6, and may, in part, result from errors in the observation of sediment concentrations.

Figures 12 and 13 relate to depths and widths. Comparisons of these figures with Figures 7 and 9 show almost identical patterns thus giving a practical demonstration of the equivalence of the two variational principles.

Gravel rivers

General relationships

The results from the present computational procedure are compared with the regime equations of Simons and Albertson (1957), Kellerhals (1972), Charlton et al (1978) and Bray (1980) which are summarised in Table 4. In Figures 14 to 17 these equations are plotted together with the curves which emerge from the present technique for a range of sediment concentrations from 10ppm to 50ppm and a range of sediment sizes from 20mm to 100mm.

The comparisons are not entirely satisfactory because the present method used the D_{35} size of the parent material (bulk sample in depth) as the representative size and the empirical relationships utilise, where appropriate, the D_{50}, D_{65} or D_{90}. Furthermore these latter sizes are usually based on surface sampling. In gravel rivers there is often a factor of 5 or more between the D_{35} size of the parent material and the D_{50} size of the surface material. This can be significant in terms of computed slopes, depths and, in particular, widths

Figures 14 and 15 relate to channel slope, the former being for a sediment size of 20mm and the latter being for 100mm material. The slope and position of the computed curves are in good agreement with the established empirical relationships. They are clearly sensitive to the assumed sediment concentrations and appear, on balance, to give better results on the assumption of low sediment concentrations.

Figure 16 shows the depth, discharge relationships for particle sizes of 20mm and 100mm. There is a systematic tendency to overestimate depths but the slope of the curves, ie the exponent of Q, is in line with the established empirical relationships.

Figure 17 relates to the width of channels. In an attempt to overcome the problem of representative sediment size mentioned above the D_{35} of the parent material was assumed to be in the range 5mm to 20mm. The tendency is for the new method to underpredict for low discharges. The discrepancy increases with increasing particle size but decreases with increasing discharge.

Minimisation of slope

In making a comparison with data using the principle of minimising slope it is necessary to utilise observed sediment concentrations. Unfortunately observations of sediment transport rates in gravel rivers at or near the dominant discharge are non-existant. Hence we have not been able to carry out any meaningful comparison along these lines.

Comparison with data using the principle of maximum sediment transport

As stated earlier we have taken the bankfull discharge to be the "dominant" discharge in gravel rivers. Additionally, the computational procedure utilises a D_{35} size for the parent material which has been estimated from quoted D_{50}, D_{65} and D_{90} values for surface samples. This procedure cannot be precise because, due to sediment sorting affecting the surface layer of the bed, there is no unique relationship between surface and parent gradings and, as a further complication, the sampling technique used for the Alberta data ignores materials less than 8mm in size. For each site the surface values of D_{50}, D_{65} and D_{90} were plotted and the appropriate surface value of D_{35}

for the parent (bulk sample in depth) material was obtained by dividing the surface value by two, a typical ratio for this type of river. The observed and estimated sizes are summarised in Table 5.

Figures 18 and 19 relate to the depth of flow. Agreement is good. The mean discrepancy ratio in Figure 19 is 0.97 with a Standard Deviation of 0.25.

Comparisons of width are given in Figures 20 and 21. The method appears to give reasonable results generally but there is more scatter in these plots than in those relating to depth. The mean discrepancy ratio in Figure 21 is 1.125 with a Standard Deviation of 0.54.

The results for calculated widths of gravel rivers are not as good as for sand bed rivers and this probably reflects the special features of gravel rivers which complicate the issue. These include:

(i) the widely graded sediment and the difficulty in specifying a significant particle size,

(ii) the shape of the sediment and the structural qualities of the bed,

(iii) the flash nature of flood events which make the choice of dominant discharge extremely difficult,

(iv) external constraints on the geometry of the channel including rock outcrops in the bed and banks.

CONCLUSIONS

1 A new method, based on physical principles, has been developed to predict the hydraulic and geometrical characteristics of alluvial channels. The method can be used without modification for both sand and gravel channels.

2 The method uses the sediment transport formula of Ackers and White (1973) and the frictional relationships of White, Paris and Bettess (1980). Additionally we use either the principle of maximum sediment transporting capacity or minimum channel slope (see appendix). These two variational principles are shown to be equivalent.

3 From comparisons with available data it has been shown that:-

(i) Predictions of slopes and/or sediment concentrations show scatter when compared with observations. This is not necessarily a deficiency in the method. There is a slight tendency to overestimate slopes and under-estimate sediment concentrations, see Figures 5, 6 and 11.

(ii) Good agreement is obtained for predicted depths with a slight tendency to overestimate. The results are consistent over a wide range of conditions, see Figures 7, 8, 12, 18 and 19.

(iii) Predictions of widths are excellent except for very large sand channels and for meandering laboratory channels where there is a tendency to underpredict, see Figures 9, 10, 13, 20 and 21.

4 Results from the new method can be presented in tabular form and Table 1 provides an example, (D = 0.5mm). It is hoped to publish a set of tables covering the range of particle sizes $0.06 < D(mm) < 100$ at a later date.

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APPENDIX

APPENDIX In this appendix we show that fixing the discharge and the slope and maximising the sediment transport rate is equivalent to fixing the discharge and the sediment transport rate and minimising the slope.

There are six variables that we must consider which are V, d, S, X, Q and B. We will denote the variables V, d and B by x_1, x_2 and x_3. There are three equations relating the six variables, a sediment transport equation, a resistance equation and a water continuity equation. The sediment transport equation and the resistance equation are assumed to be of the form

$$X = G(S, x_1, x_2, x_3) \tag{17}$$

and

$$S = F(x_1, x_2, x_3) \tag{18}$$

respectively, and we denote the continuity equation

$$Q = BVd \tag{19}$$

by

$$\phi(Q, x_1, x_2, x_3) = 0. \tag{20}$$

To give a geometric interpretation, imagine that equation (20) is used to eliminate one of the variables x_1, x_2, x_3, then X and S will depend upon just two independent variables x_1 and x_2. We can now draw contours of equal X and S as in Figure 22. For a particular value of X we require the values of x_1 and x_2 that provide the minimum value of S, that is, choosing a particular contour of X and moving along it to find the point with the minimum slope. It can be seen that if this value of S is fixed so that one moved along the contour of S the same values of x_1 and x_2 give the maximum value of X on that contour.

After this geometric demonstration we provide an analytic argument. We first consider the case where Q and X are fixed and S is minimised subject to satisfying equations (17) and (20). Using Lagrange multipliers this is equivalent to minimising the expression.

$$F(x_1, x_2, x_3) + \lambda\phi + \mu(-X + G(S, x_1, x_2, x_3). \tag{21}$$

The values of x_1, x_2, x_3, S, λ and μ which provide the extremum are solutions of the equations

$$\frac{\partial F}{\partial x_j} + \lambda \frac{\partial Q}{\partial x_j} + \mu \frac{\partial G}{\partial x_j} = 0, \; j = 1, 2, 3, \tag{22}$$

$$\phi = 0 \tag{23}$$

and

$$G(S, x_1, x_2, x_3) = X, \tag{24}$$

where

$$S = F(x_1, x_2, x_3). \tag{25}$$

Let us now consider the problem where Q and S are fixed and X is maximised subject to the constraints of equations (18) and (20). This is equivalent to maximising the expression.

$$G(S, x_1, x_2, x_3) + \lambda'\phi + \mu'(-S + F(x_1, x_2, x_3)). \tag{26}$$

The values of x_1, x_2, x_3, S, λ' and μ' which provide the extremum are solutions of the equations

$$\frac{\partial G}{\partial x_j} + \lambda' \frac{\partial Q}{\partial x_j} + \mu \frac{\partial F}{\partial x_j} = 0, \tag{27}$$

$$\phi = 0 \tag{28}$$

and

$$F(x_1, x_2, x_3) = S. \tag{29}$$

With the corresponding extremum value of X being given by

$$X = G(S, x_1, x_2, x_3). \qquad (30)$$

But equations (20) to (25) are identical to equations (27) to (30) provided

$\mu\mu' = 1$ and $\mu\lambda' = \lambda$

and provided that the value of X used in equations (20) to (25) is the same as that determined from equations (27) to (30).

Thus we have shown that if Q and X are given and the breadth, velocity and depth is calculated to give an extremum value of the slope then for that slope and the given Q the same values of breadth, velocity and depth give an extremum value of the sediment concentration. Similarly an extremum value of sediment concentration leads to an extremum value of the slope.

It should be noticed that we have not shown that an extremum value exists nor whether the extremums are maxima or minima. In the author's experience, using standard sediment transport and resistance equations, the only extremals are a maximum in the sediment concentration and a minimum in the slope, as the breadth changes.

TABLES

TABLE 1 CHANNEL CHARACTERISTICS AS A FUNCTION OF DISCHARGE AND SEDIMENT LOAD (D = 0.50mm)

SAND SIZE 0.50 MILLIMETRES

VELOCITY (METRES/SEC)
SLOPE *1000
DEPTH (METRES)
WIDTH (METRES)
FRICTION FACTOR *10

SEDIMENT CONCENTRATION (PPM)	DISCHARGE (CUMECS) 0.5	1.0	2.0	5.0	10.0	20.0	50.0	100.0	200.0	500.0	1000.0
10	**0.45** 0.24 0.46 2.4 0.323	**0.47** 0.19 0.62 3.4 0.321	**0.50** 0.16 0.81 5.0 0.322	**0.53** 0.12 1.15 8.2 0.320	**0.56** 0.10 1.51 11.8 0.324	**0.60** 0.09 1.96 17.1 0.329	**0.64** 0.07 2.76 28.2 0.338	**0.68** 0.06 3.57 41.0 0.346	**0.73** 0.05 4.59 59.8 0.354	**0.81** 0.04 6.60 93.8 0.370	**0.86** 0.04 8.24 141.9 0.376
20	**0.47** 0.31 0.42 2.5 0.372	**0.49** 0.26 0.56 3.6 0.372	**0.52** 0.21 0.73 5.3 0.375	**0.56** 0.17 1.05 8.5 0.381	**0.60** 0.15 1.36 12.4 0.387	**0.63** 0.13 1.76 17.9 0.395	**0.69** 0.11 2.47 29.3 0.405	**0.74** 0.09 3.20 42.2 0.415	**0.79** 0.08 4.12 61.1 0.424	**0.88** 0.07 5.74 99.2 0.436	**0.95** 0.07 7.35 143.3 0.445
40	**0.49** 0.43 0.38 2.7 0.441	**0.52** 0.36 0.51 3.8 0.445	**0.55** 0.31 0.66 5.5 0.449	**0.60** 0.25 0.93 9.0 0.457	**0.64** 0.22 1.21 13.0 0.465	**0.68** 0.20 1.57 18.6 0.473	**0.75** 0.17 2.20 30.2 0.485	**0.81** 0.15 2.84 43.3 0.494	**0.88** 0.14 3.64 62.3 0.502	**0.98** 0.12 5.06 100.4 0.513	**1.09** 0.11 6.67 137.7 0.522
60	**0.50** 0.52 0.36 2.8 0.490	**0.53** 0.45 0.47 4.0 0.494	**0.57** 0.39 0.61 5.7 0.500	**0.62** 0.33 0.87 9.2 0.509	**0.67** 0.29 1.13 13.2 0.517	**0.72** 0.26 1.46 19.1 0.526	**0.80** 0.22 2.03 30.8 0.536	**0.87** 0.20 2.63 43.8 0.545	**0.94** 0.19 3.38 62.6 0.553	**1.06** 0.17 4.70 100.3 0.561	**1.16** 0.16 6.02 143.0 0.567
80	**0.51** 0.61 0.34 2.9 0.529	**0.55** 0.53 0.45 4.0 0.534	**0.59** 0.46 0.59 5.8 0.540	**0.66** 0.40 0.87 8.8 0.554	**0.69** 0.35 1.07 13.5 0.557	**0.75** 0.32 1.38 19.3 0.565	**0.84** 0.28 1.93 31.0 0.576	**0.91** 0.25 2.48 44.3 0.583	**0.99** 0.23 3.20 62.8 0.589	**1.12** 0.21 4.46 99.9 0.597	**1.24** 0.20 5.71 141.8 0.600
100	**0.52** 0.70 0.33 2.9 0.562	**0.57** 0.61 0.45 3.9 0.571	**0.60** 0.54 0.56 5.9 0.573	**0.66** 0.46 0.79 9.5 0.582	**0.72** 0.41 1.02 13.8 0.589	**0.78** 0.37 1.32 19.5 0.597	**0.87** 0.33 1.85 31.1 0.607	**0.95** 0.30 2.38 44.3 0.613	**1.04** 0.28 3.05 63.1 0.618	**1.18** 0.25 4.26 99.7 0.624	**1.30** 0.24 5.47 140.9 0.627
200	**0.56** 1.08 0.29 3.0 0.677	**0.61** 0.96 0.38 4.3 0.682	**0.66** 0.86 0.49 6.2 0.687	**0.73** 0.75 0.69 9.9 0.694	**0.80** 0.68 0.89 14.0 0.700	**0.87** 0.63 1.15 19.9 0.704	**0.99** 0.57 1.61 31.6 0.709	**1.08** 0.53 2.06 44.8 0.711	**1.20** 0.49 2.67 62.5 0.712	**1.37** 0.45 3.72 98.1 0.711	**1.52** 0.43 4.76 137.9 0.709
400	**0.62** 1.73 0.26 3.2 0.813	**0.66** 1.57 0.32 4.8 0.829	**0.73** 1.43 0.43 6.3 0.817	**0.83** 1.27 0.61 10.0 0.820	**0.91** 1.18 0.78 14.2 0.820	**1.00** 1.09 1.00 20.0 0.820	**1.14** 1.00 1.39 31.6 0.817	**1.27** 0.94 1.81 43.4 0.813	**1.41** 0.88 2.32 61.5 0.808	**1.62** 0.82 3.23 95.3 0.798	**1.81** 0.78 4.14 133.2 0.789
600	**0.64** 2.34 0.23 3.4 0.919	**0.72** 2.13 0.31 4.5 0.902	**0.79** 1.95 0.40 6.4 0.900	**0.89** 1.76 0.56 10.1 0.897	**0.98** 1.63 0.71 14.2 0.894	**1.09** 1.53 0.93 19.7 0.889	**1.25** 1.40 1.29 31.1 0.881	**1.39** 1.32 1.66 43.3 0.873	**1.55** 1.25 2.13 60.4 0.863	**1.80** 1.17 2.95 94.1 0.848	**2.03** 1.11 3.82 129.1 0.836
800	**0.70** 2.90 0.23 3.1 0.970	**0.74** 2.66 0.29 4.7 0.986	**0.83** 2.45 0.37 6.5 0.970	**0.94** 2.22 0.52 10.2 0.954	**1.05** 2.07 0.68 14.0 0.947	**1.16** 1.94 0.87 19.8 0.939	**1.35** 1.80 1.23 30.0 0.926	**1.50** 1.70 1.57 42.6 0.915	**1.68** 1.61 2.01 59.2 0.902	**1.95** 1.51 2.81 91.2 0.883	**2.19** 1.44 3.61 126.4 0.867
1000	**0.71** 3.44 0.21 3.3 1.021	**0.79** 3.17 0.28 4.5 1.016	**0.85** 2.94 0.35 6.7 1.030	**1.00** 2.67 0.50 9.9 0.998	**1.10** 2.50 0.64 14.1 0.989	**1.20** 2.35 0.79 21.1 0.987	**1.42** 2.18 1.16 30.5 0.962	**1.58** 2.06 1.49 42.3 0.948	**1.79** 1.96 1.94 57.7 0.932	**2.08** 1.84 2.69 89.5 0.910	**2.34** 1.76 3.45 123.9 0.891
2000	**0.81** 5.97 0.18 3.4 1.196	**0.90** 5.56 0.24 4.7 1.181	**1.00** 5.20 0.31 6.5 1.164	**1.16** 4.79 0.43 10.1 1.140	**1.29** 4.52 0.56 13.9 1.129	**1.46** 4.28 0.72 19.1 1.099	**1.70** 3.99 1.01 29.3 1.070	**1.84** 3.81 1.22 44.4 1.061	**2.16** 3.63 1.67 55.4 1.022	**2.53** 3.43 2.33 84.6 0.989	**2.88** 3.29 3.03 114.4 0.962
4000	**0.94** 10.61 0.16 3.3 1.382	**1.05** 9.96 0.21 4.6 1.352	**1.18** 9.39 0.26 6.4 1.323	**1.37** 8.74 0.37 9.9 1.282	**1.55** 8.29 0.48 13.4 1.250	**1.74** 7.89 0.62 18.6 1.218	**2.06** 7.41 0.87 27.9 1.174	**2.32** 7.09 1.12 38.3 1.141	**2.71** 6.79 1.52 48.7 1.102	**3.13** 6.43 2.04 78.1 1.063	**3.54** 6.18 2.64 107.1 1.045

TABLE 2 SUMMARY OF FIELD AND LABORATORY DATA

Source Name	No. of observations	Sed. conc. X (ppm)	Discharge Q (m^3/s)	Slope x 1000	Water Surface width B (m)	Mean depth d (m)	Mean diam. D_{50} (mm)
Punjab	66	16 – 103	0.16 – 253	0.12 – 0.34	1.5 – 83	0.3 – 3.1	0.17 – 0.43
CHOP canals	12	190 – 4840	27.3 – 399	0.073 – 0.200	20.8 – 103	1.82 – 3.95	0.16 – 0.26
UP canals	91	19 – 1822	0.42 – 280	0.102 – 0.430	1.6 – 48.6	0.46 – 3.96	0.08 – 0.42
US canals (Simons)	13	44 – 447	1.2 – 29	0.058 – 0.330	3.2 – 22.1	0.8 – 2.6	0.096 – 7.0
Pakistan canals (ACOP)	17	95 – 4595	61.2 – 524	0.074 – 0.551	35.6 – 131.6	1.9 – 4.29	0.113 – 0.364
Sind canals	14	596 – 3508	1.3 – 248	0.057 – 0.165	3.8 – 77	0.79 – 3.58	0.033 – 0.079
Ackers, Charlton	11	52 – 612	0.0139 – 0.054	1.13 – 2.36	1.8 – 3.1	0.03 – 0.07	0.15
R Raju et al	6	150 – 242.5	0.0179 – 0.0197	0.73 – 2.83	1.4 – 1.6	0.06 – 0.09	0.27
Ackers	17	42 – 1288	0.0113 – 0.1507	0.67 – 2.23	0.8 – 2.9	0.06 – 0.18	0.15 – 0.18

TABLE 3 REGIME EQUATIONS FOR SAND CHANNELS

Investigator Source	Regime equations	Range of applicability
Ackers* (1964)	$B = 3.6\ Q^{0.42}$ $d = 0.28\ Q^{0.43}$ No slope/discharge equation	Flume experiments $0.16 < D_{50}$ (mm) < 0.34 $0.011 < Q$ (m^3/s) < 0.151 $52 < X$ (ppm) < 612
Lacey* (1930)	$B = 2.67\ Q^{0.5}$ $R = 0.473\ (Q/f)^{0.33}$ $S = (1/1828)\ f^{5/3}\ Q^{-1/6}$ $f = 1.76\ \sqrt{D_{50}}$, D_{50} in mm	Punjab canals $0.1 < D_{50}$ (mm) < 0.4 $2.8 < Q$ (m^3/s) < 280 $10 < X$ (ppm) < 50
ACOP* (1979)	$B = 2.63\ Q^{0.513}$ $d = 0.575\ Q^{0.311}$ $S = (1/3905)\ Q^{-0.087}$	Pakistan canals $0.09 < D_{50}$ (mm) < 0.37 $16 < Q$ (m^3/s) < 6688 $190 < X$ (ppm) < 3900

* Equations are in imperial units

TABLE 4 REGIME EQUATIONS FOR GRAVEL BED RIVERS

Investigator Source	Regime equations	Range of applicability
Simons Albertson (1957)	$B = 2.85\ Q^{0.5}$ $d = 0.31\ Q^{0.36}$ if $d < 2.1$m $d = 0.61 + 0.23\ Q^{0.36}$ if $d \geqslant 21.$m $S = 0.00617\ Q^{-0.24}$	Plane beds Low sediment transport rates Straight channels
Kellerhals* (1967)	$B = 1.80\ Q^{0.5}$ $d = 0.166\ Q^{0.4}\ D_{90}{}^{-0.12}$ $S = 0.120\ Q^{-0.4}\ D_{90}{}^{0.92}$	Plane beds $D_{50} \geqslant 6$mm $3 < d/D_{90} < 80$ Straight channels
Bray* (1980)	$B = 2.08\ Q^{0.528}\ D_{50}{}^{-0.07}$ $d = 0.256\ Q^{0.331}\ D_{50}{}^{-0.025}$ $S = 0.0965\ Q^{-0.334}\ D_{50}{}^{0.586}$	$5.5 < Q$(m^3/s) < 8920 $14.3 < B$ (m) < 566 $0.019 < D_{50}$ (m) < 0.145
Charlton, Brown and Benson (1978)	$B = 3.74\ K\ Q^{0.45}$, where $K = 1.3$ $d = 0.16\ Q^{0.42}\ D_{65}{}^{-0.38}\ D_{90}{}^{0.24}$ $S = 0.15\ Q^{-0.76}\ B^{0.76}\ D_{65}{}^{1.38}\ D_{90}{}^{-0.24}$	$2.7 < Q$ (m^3/s) < 550 $d/D_{90} > 3$ $d/D_{90} > 3$

* Equations are in imperial units

TABLE 5 ALBERTA GRAVEL RIVERS – SEDIMENT SIZE DATA

River	Location	Surface material*			Parent† material
		D_{90} (mm)	D_{65} (mm)	D_{50} (mm)	D_{35} (mm)
Peace	at Hudson Hope	81	53	46	19
	near Taylor	81	57	41	12
	At Dunvegan Bridge	127	70	53	18
Smoky	at Prudents Ranch	171	112	80	24
Little Smoky	near Guy	183	100	73	23
Athabasca	at Jasper	132	82	60	19
Wildhay	near Hinton	78	51	42	16
McLeod	above Embarras River	375	153	86	25
	near Wolf Creek	83	51	42	16
Wolf Creek	at Highway 16	95	62	51	20
Freeman	near Fort Assiniboine	138	83	70	28
Paddle	near Rochfort Bridge	76	54	46	19
North Saskatchewan	near Rocky Mountain House	127	79	63	23
	at Edmonton	70	40	31	11
Clearwater	above Limestone Creek	100	37	27	8
	near Rocky Mountain House	49	31	27	11
Prairie Creek	near Rocky Mountain House	66	51	43	17
Red Deer	at Drumheller	83	49	38	13
Little Red Deer	near Water Valley	127	77	63	24
	near the Mouth	105	58	47	18
Rosebud	at Redland	147	70	51	15
Bow	at Calgary	65	47	40	16
	below Carseland Dam	72	35	26	8
Pipestone	near Lake Louise	323	200	145	45
Sheep	at Black Diamond	105	59	43	13
Oldman	near Brocket	87	52	43	17
	near Fort MacLeod	101	61	49	18
	near Lethbridge	90	53	40	13
Crowsnest	at Frank	84	58	45	16
Crowsnest	near Lundbreck	177	118	96	37
Castle	near Cowley	202	113	78	22

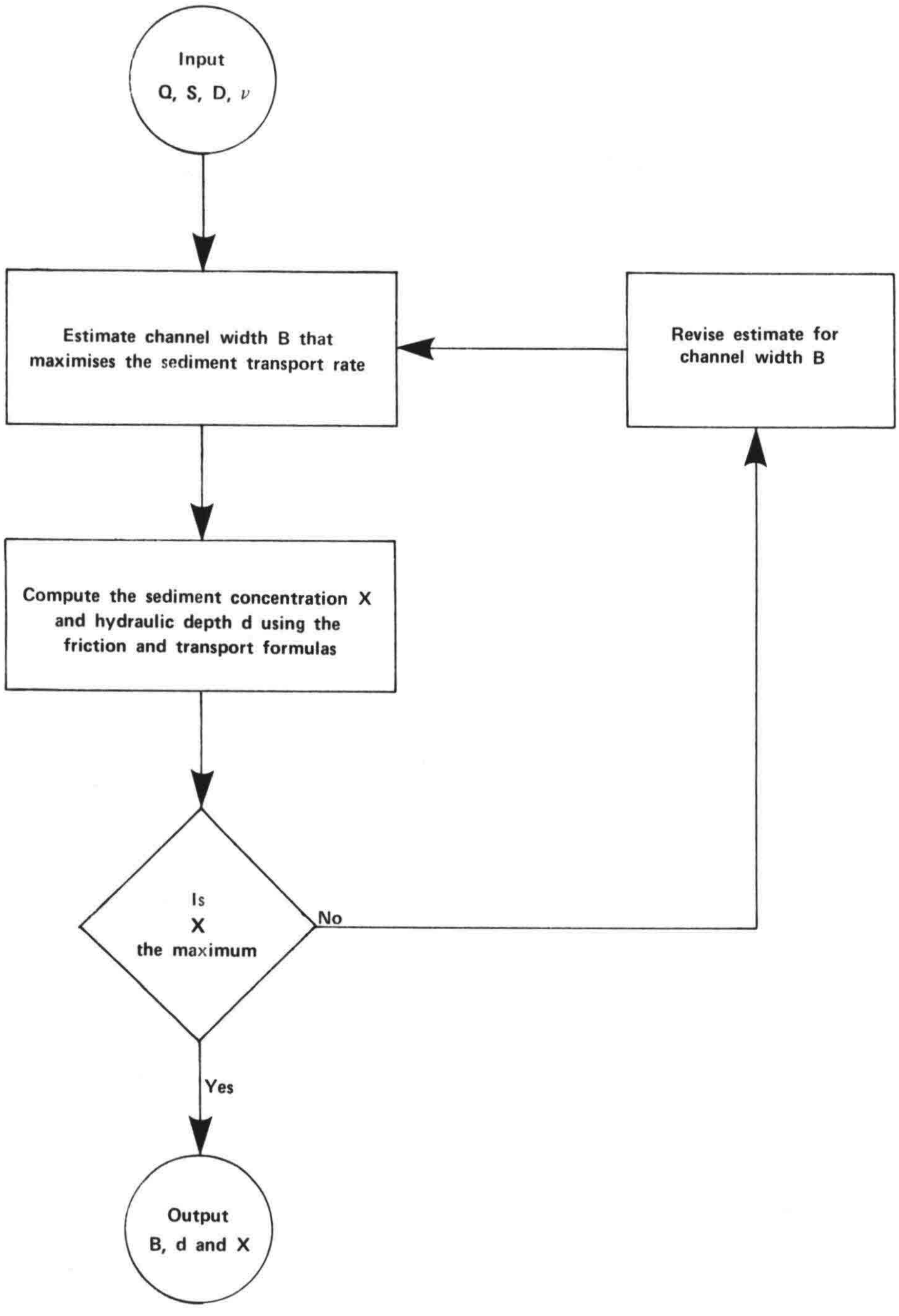
FIGURES

TABLE 5 (Cont'd)

River	Location	Surface material*			Parent† material
		D_{90} (mm)	D_{65} (mm)	D_{50} (mm)	D_{35} (mm)
Willow Creek	near Claresholm	45	29	23	8
Belly	near Stand Off	51	35	30	12
Drywood Creek	near Twin Butte	138	86	52	10
St Mary	near Lethbridge	171	98	63	16
Milk	at Milk River	36	25	19	6
Bow	at Lake Louise	168	115	89	31
Lobstick	near Entwistle	109	80	66	26

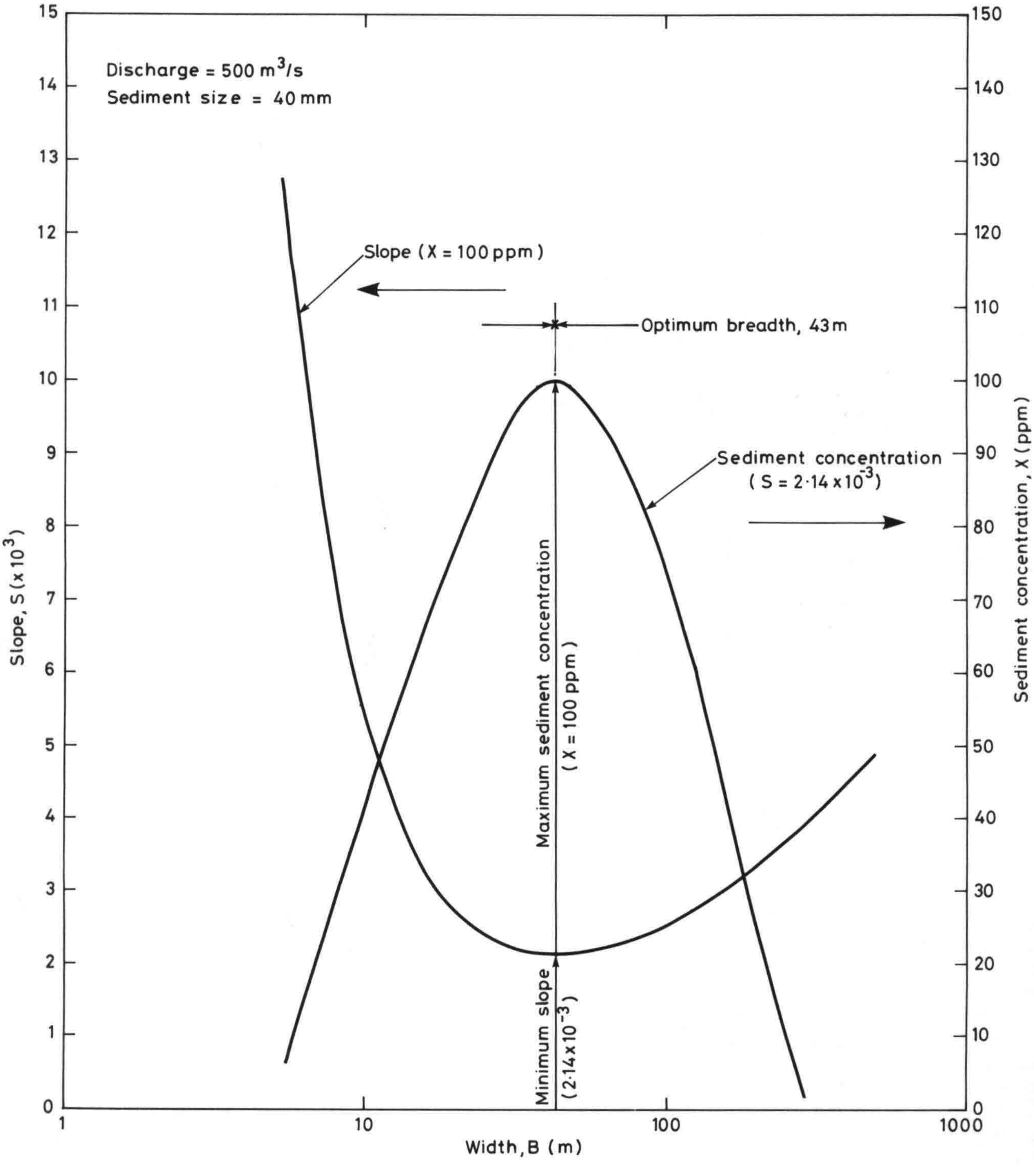
* observed values

† estimated value



Flow chart for calculating maximum sediment concentration

Fig 1



Slope and sediment concentration against width

Fig 2

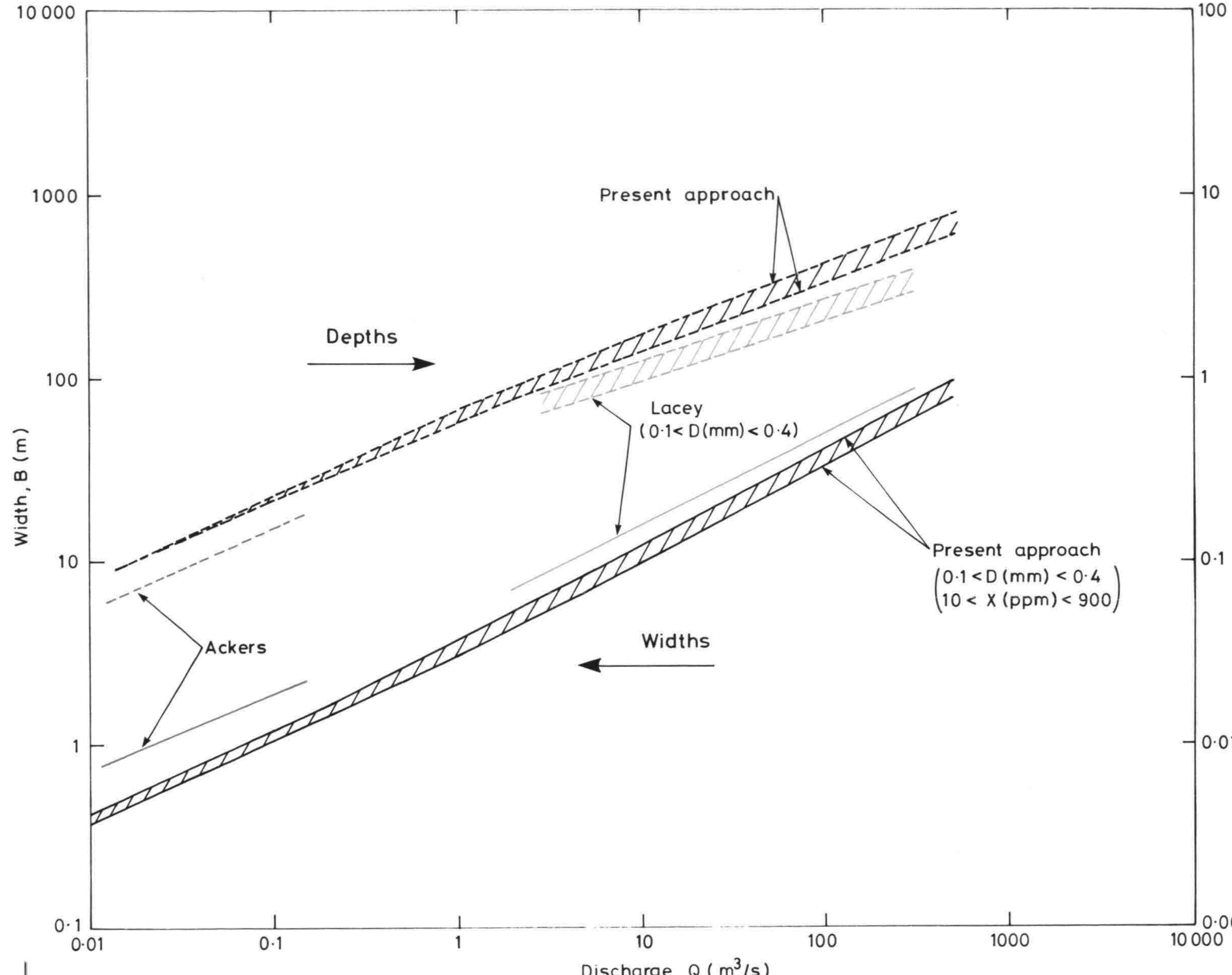


Fig 3

Regime depths and widths for sand channels

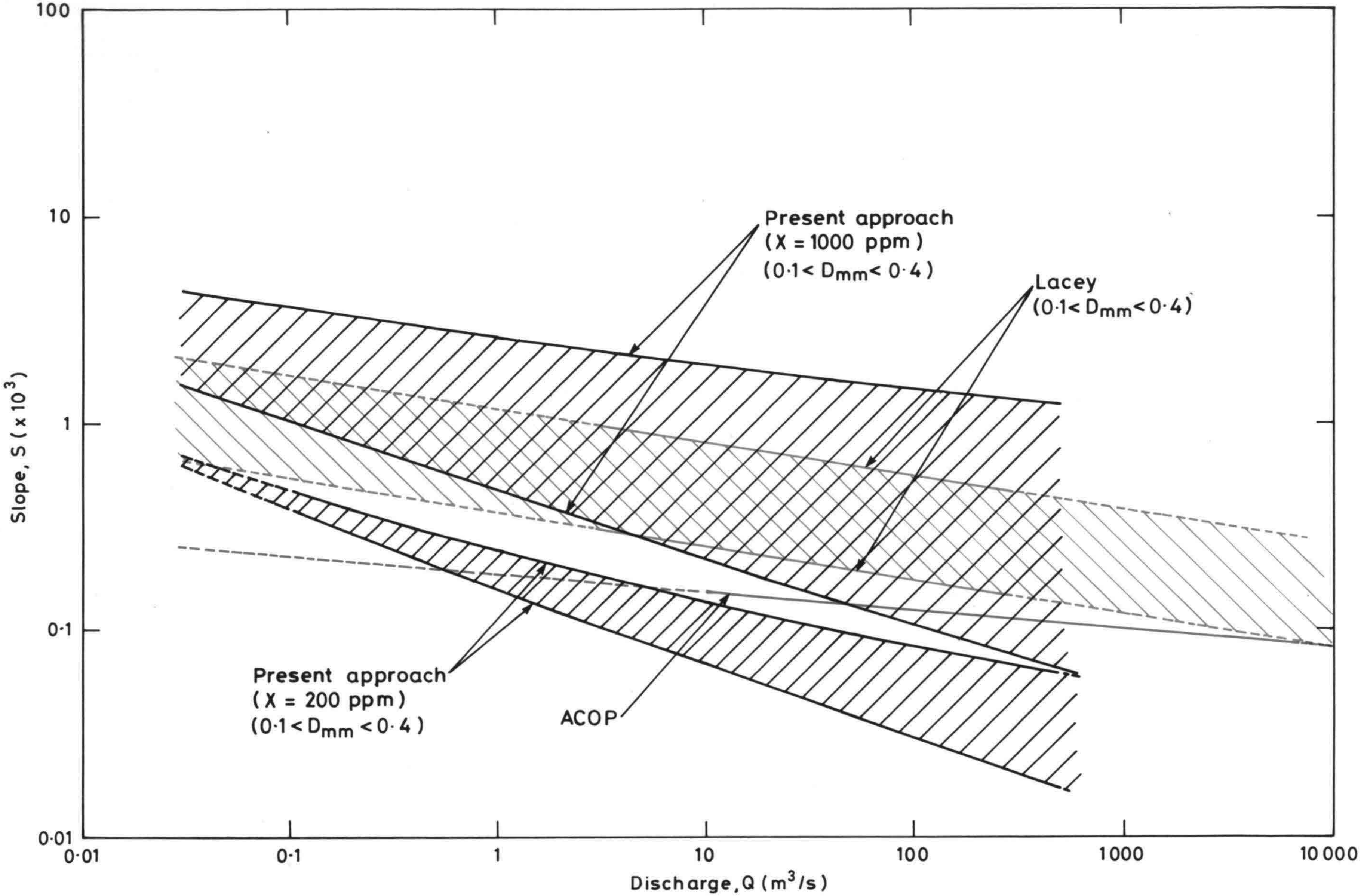
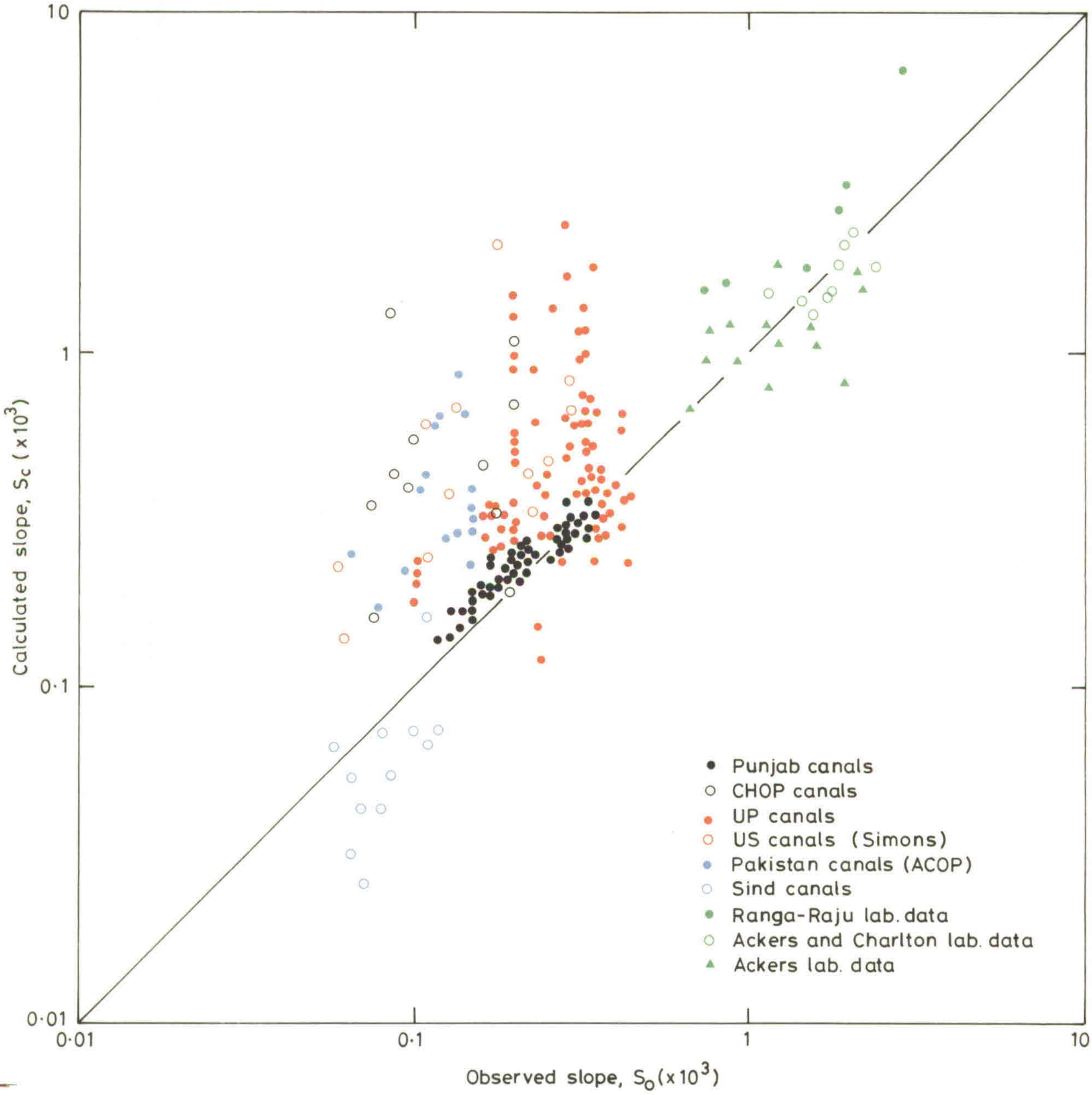


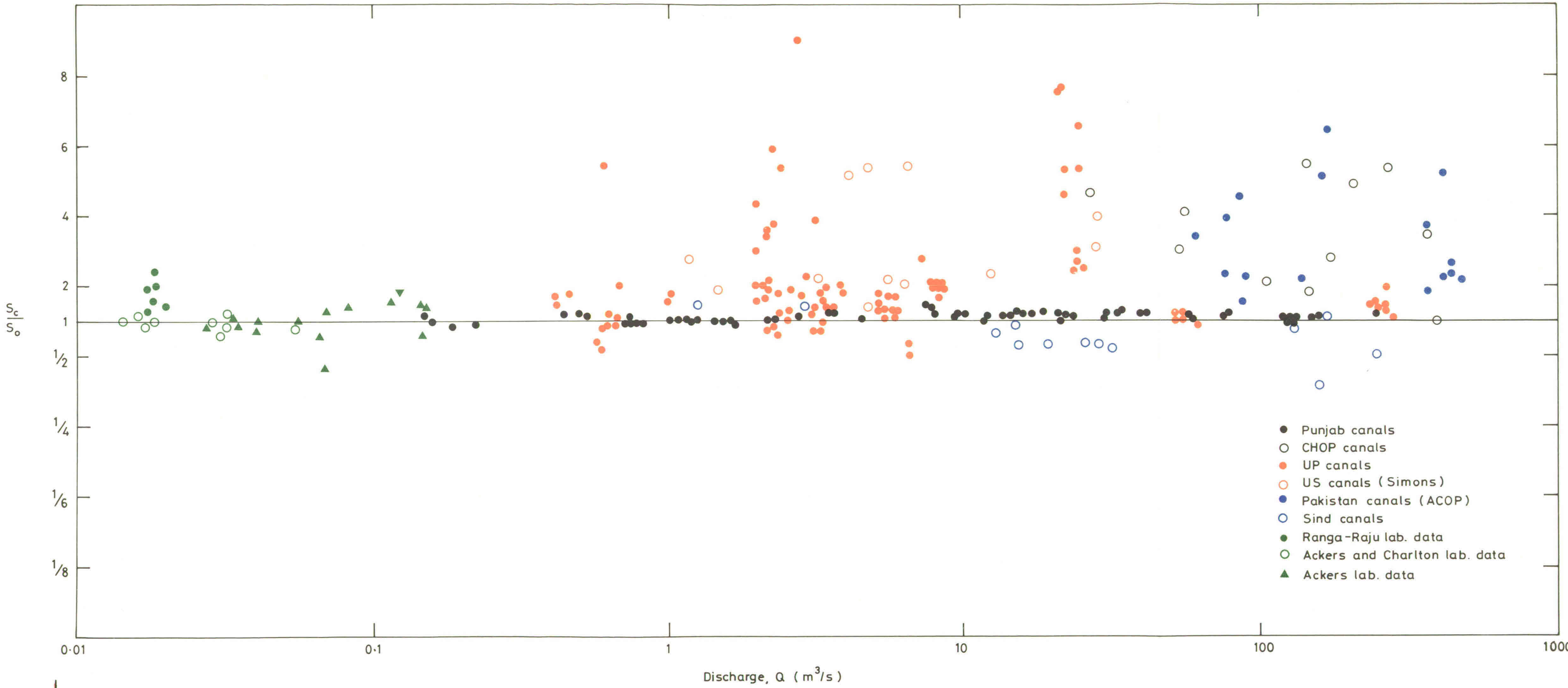
Fig 4

Regime slopes for sand channels



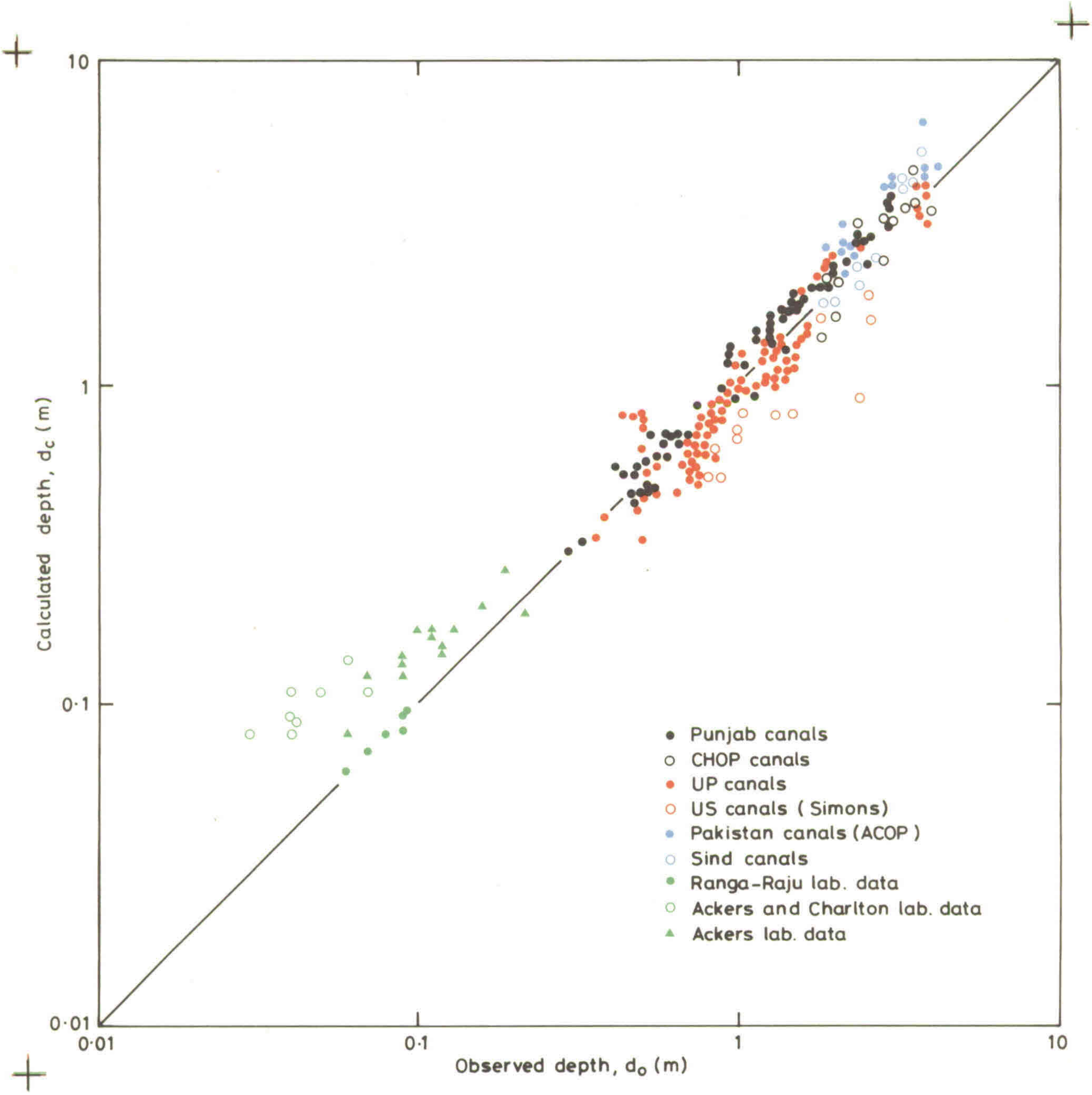
Calculated against observed slope using observed sediment concentrations, sand channels

Fig 5



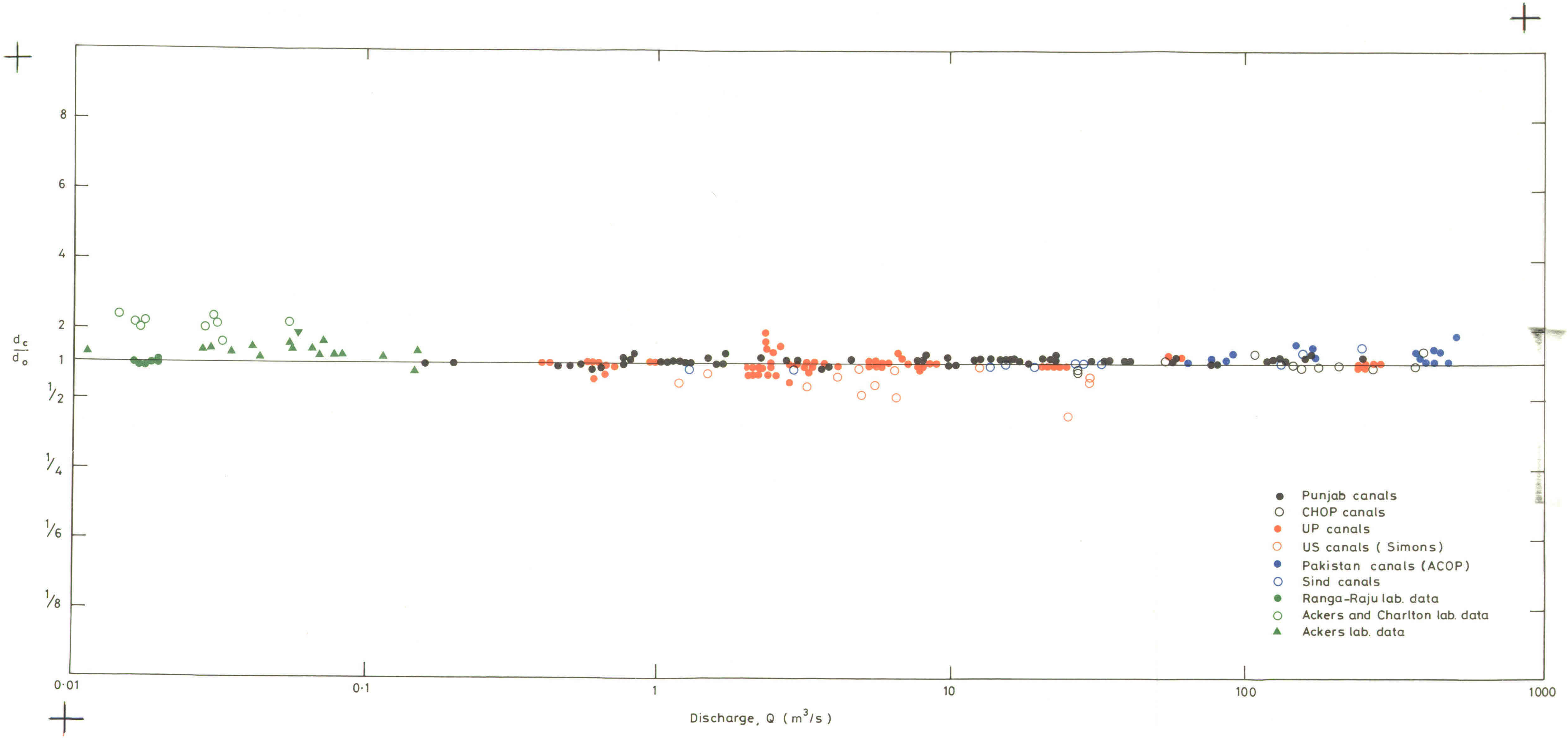
Discrepancy ratio for slope using observed sediment concentrations, sand channels

Fig 6



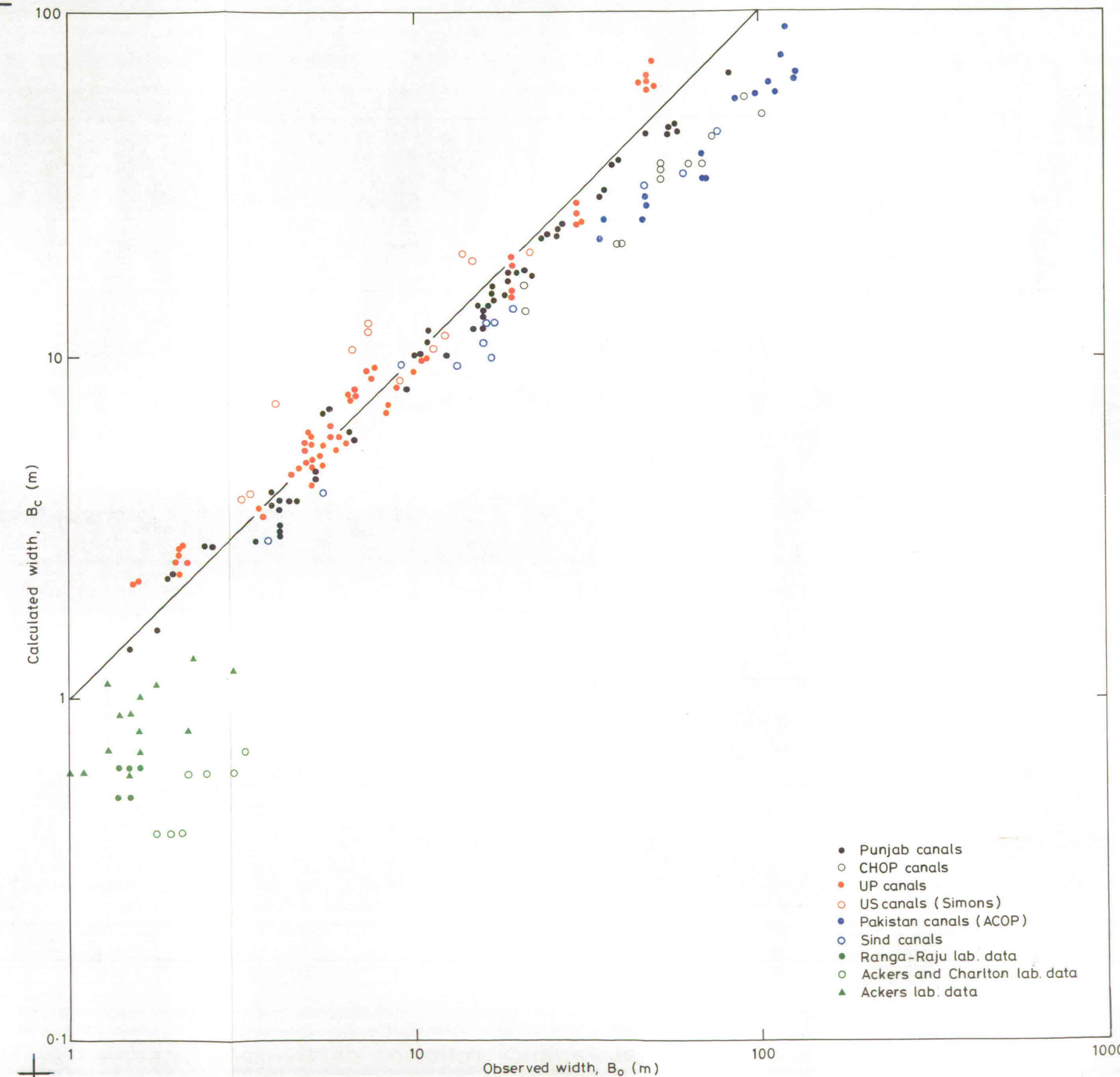
Calculated against observed depth using observed sediment concentrations, sand channels

Fig 7



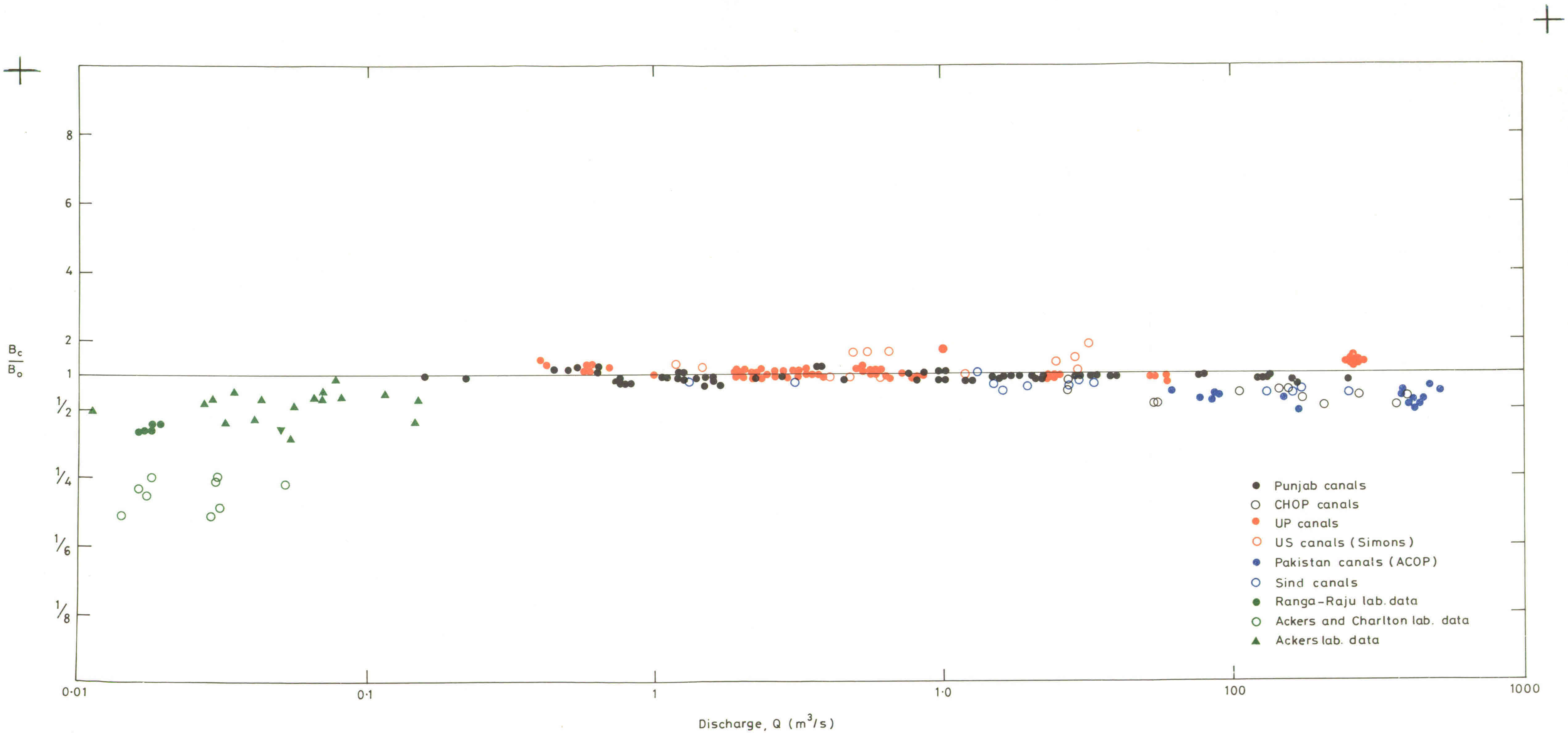
Discrepancy ratio for depth using observed sediment concentrations, sand channels

Fig 8



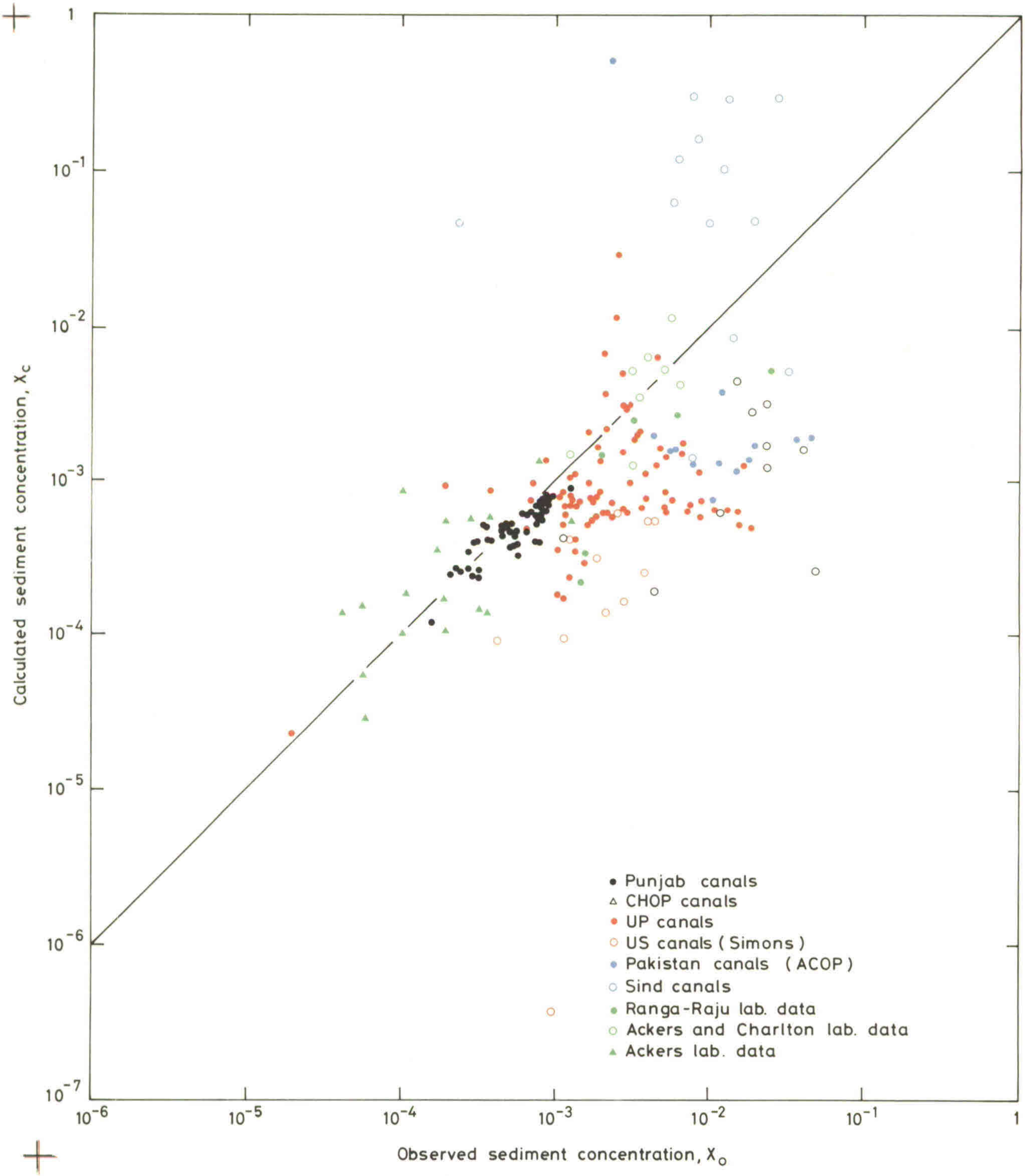
Calculated against observed width using observed sediment concentrations, sand channels

Fig 9



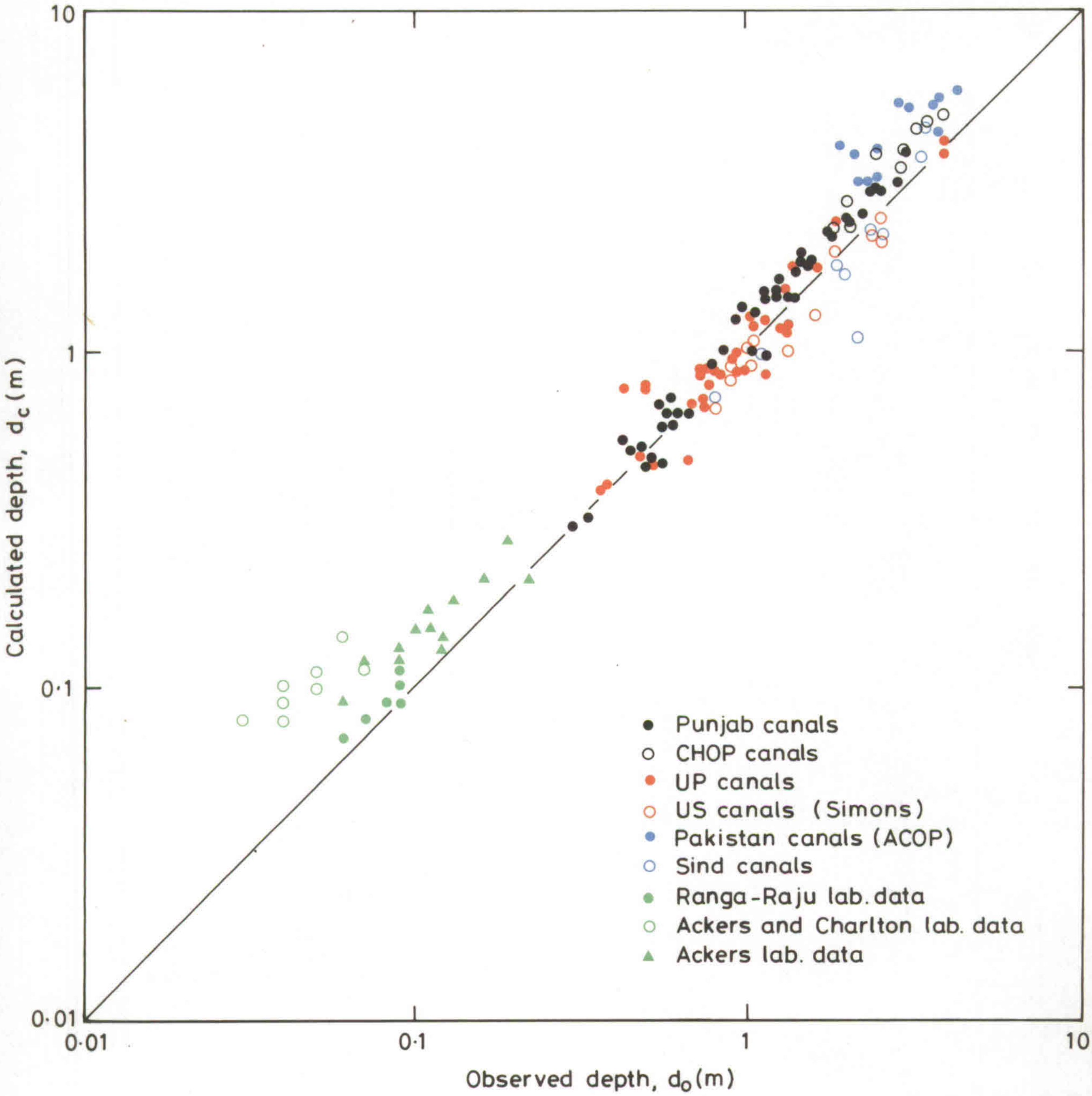
Discrepancy ratio for width using observed sediment concentrations, sand channels

Fig 10



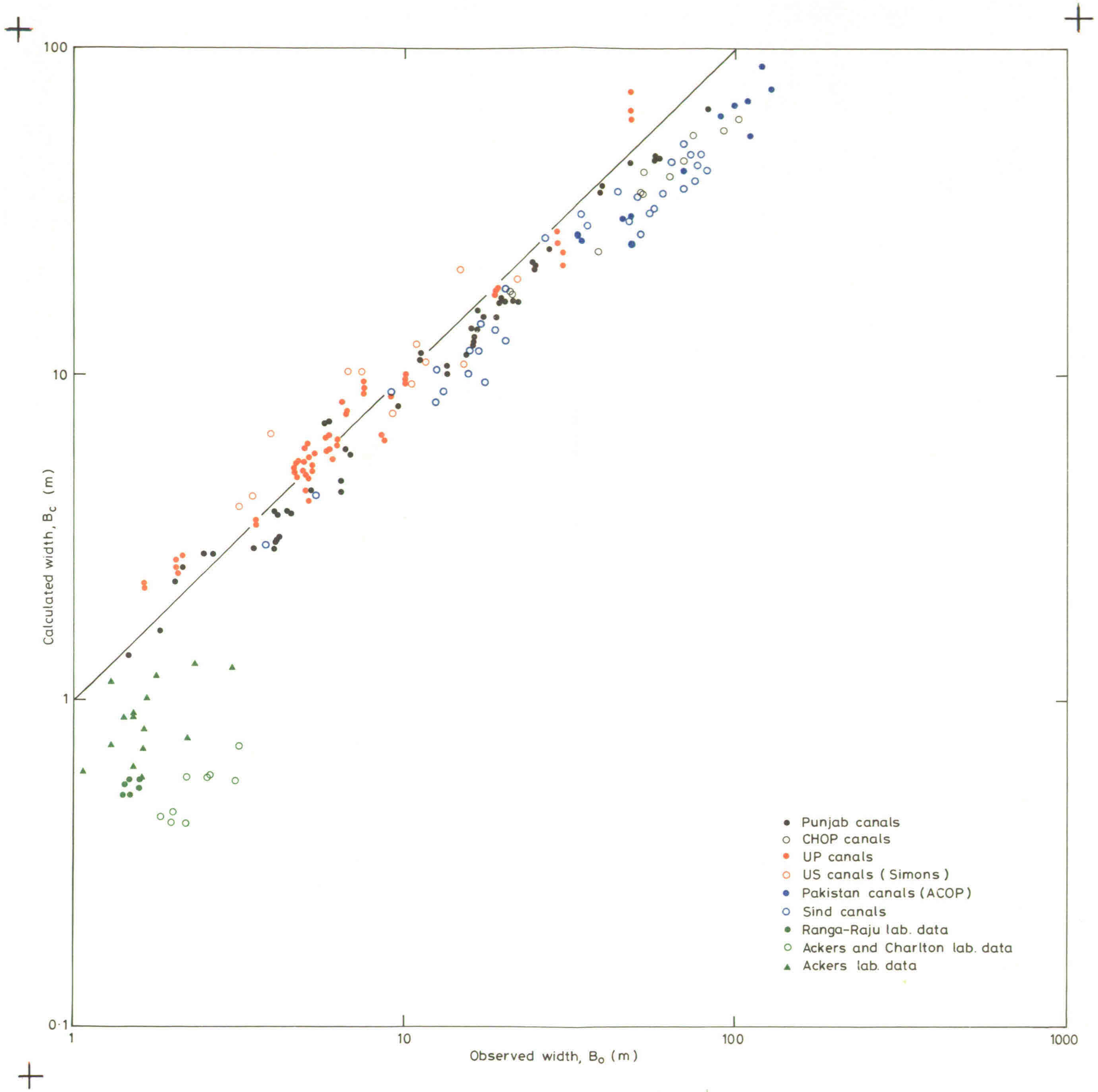
Calculated against observed sediment concentration using observed slopes, sand channels

Fig 11



Calculated against observed depth using observed slopes, sand channels

Fig 12



Calculated against observed width using observed slopes, sand channels

Fig 13

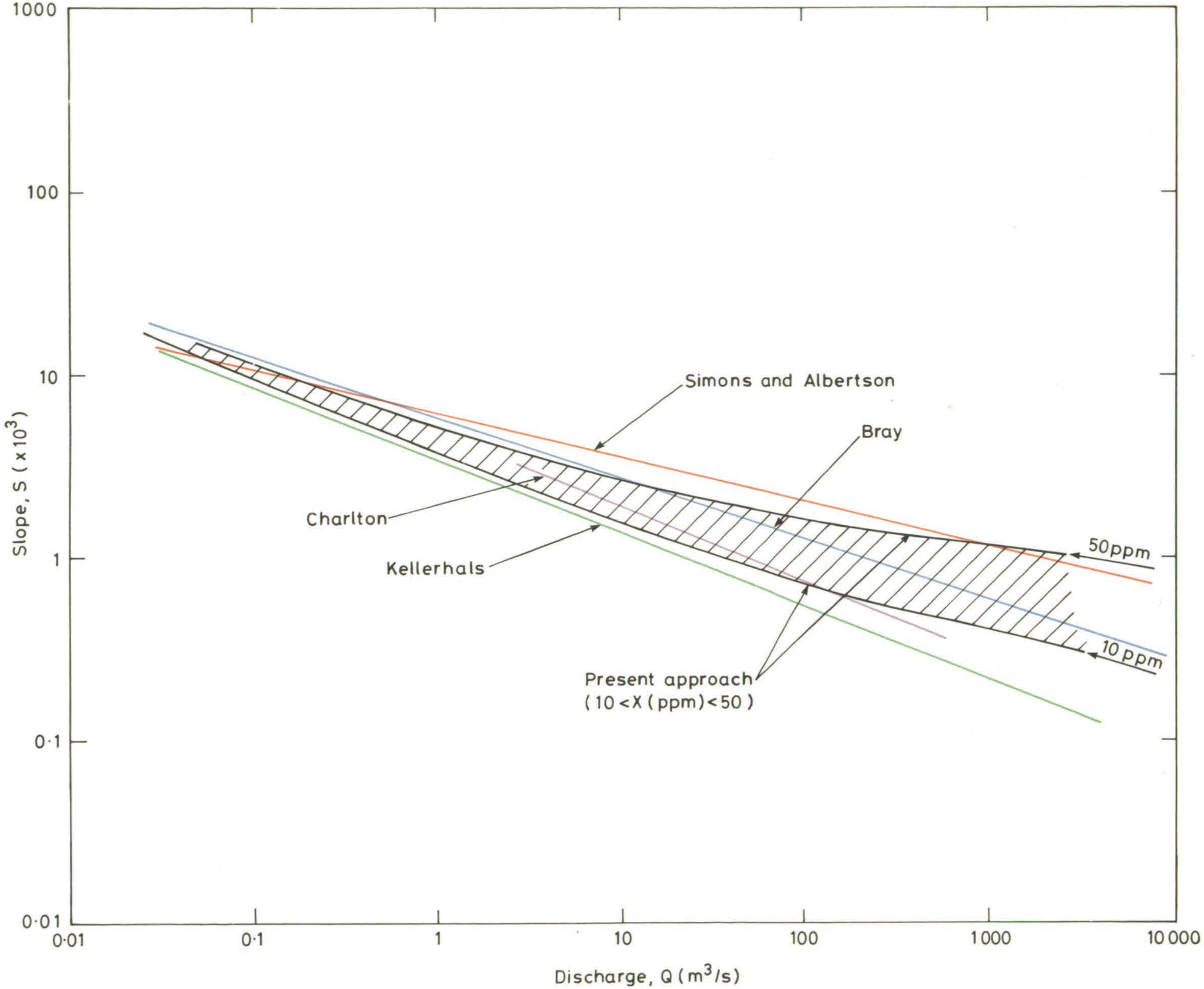
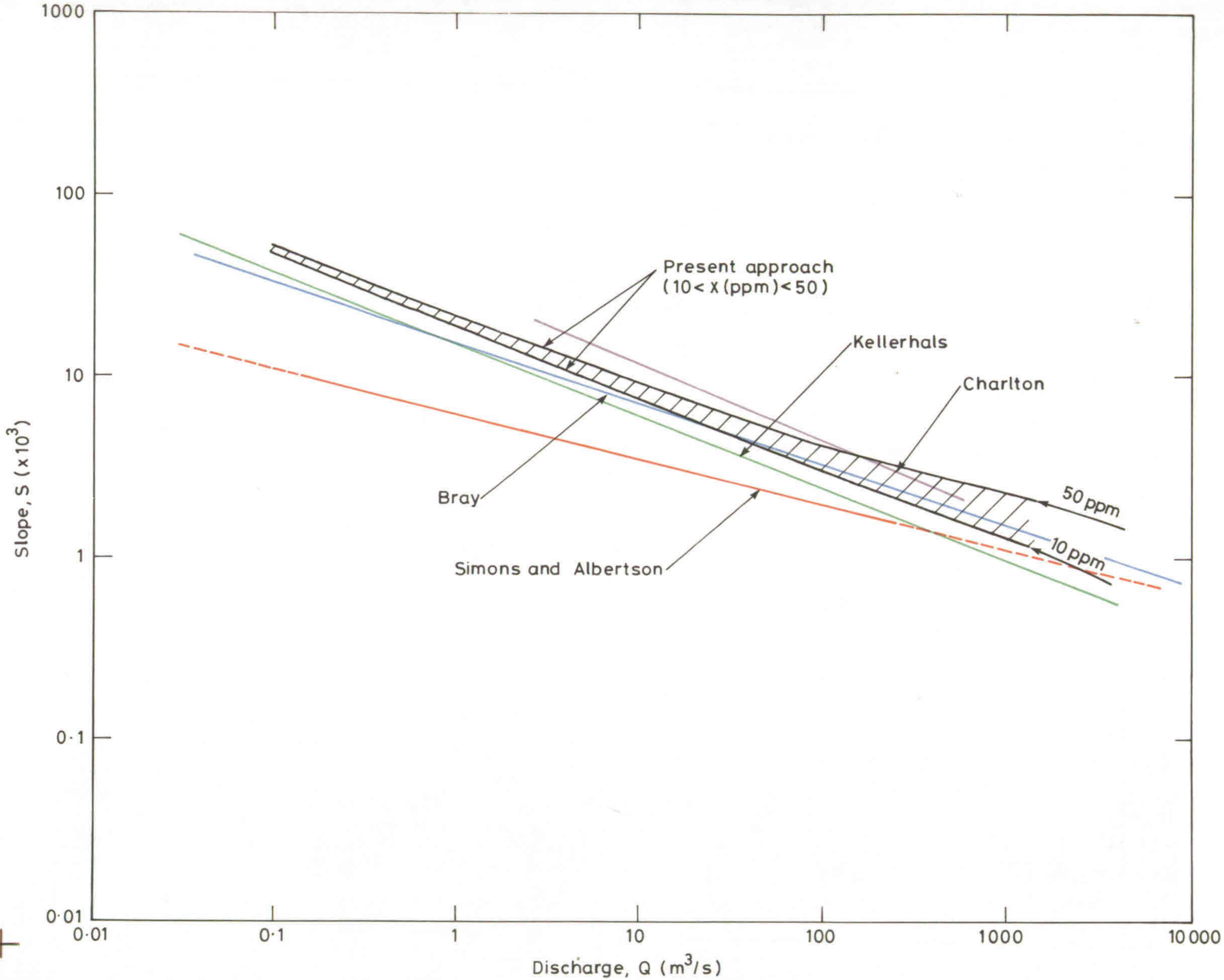


Fig 14

Regime slopes for gravel rivers, D = 20 mm



Regime slopes for gravel rivers, D = 100mm

Fig 15

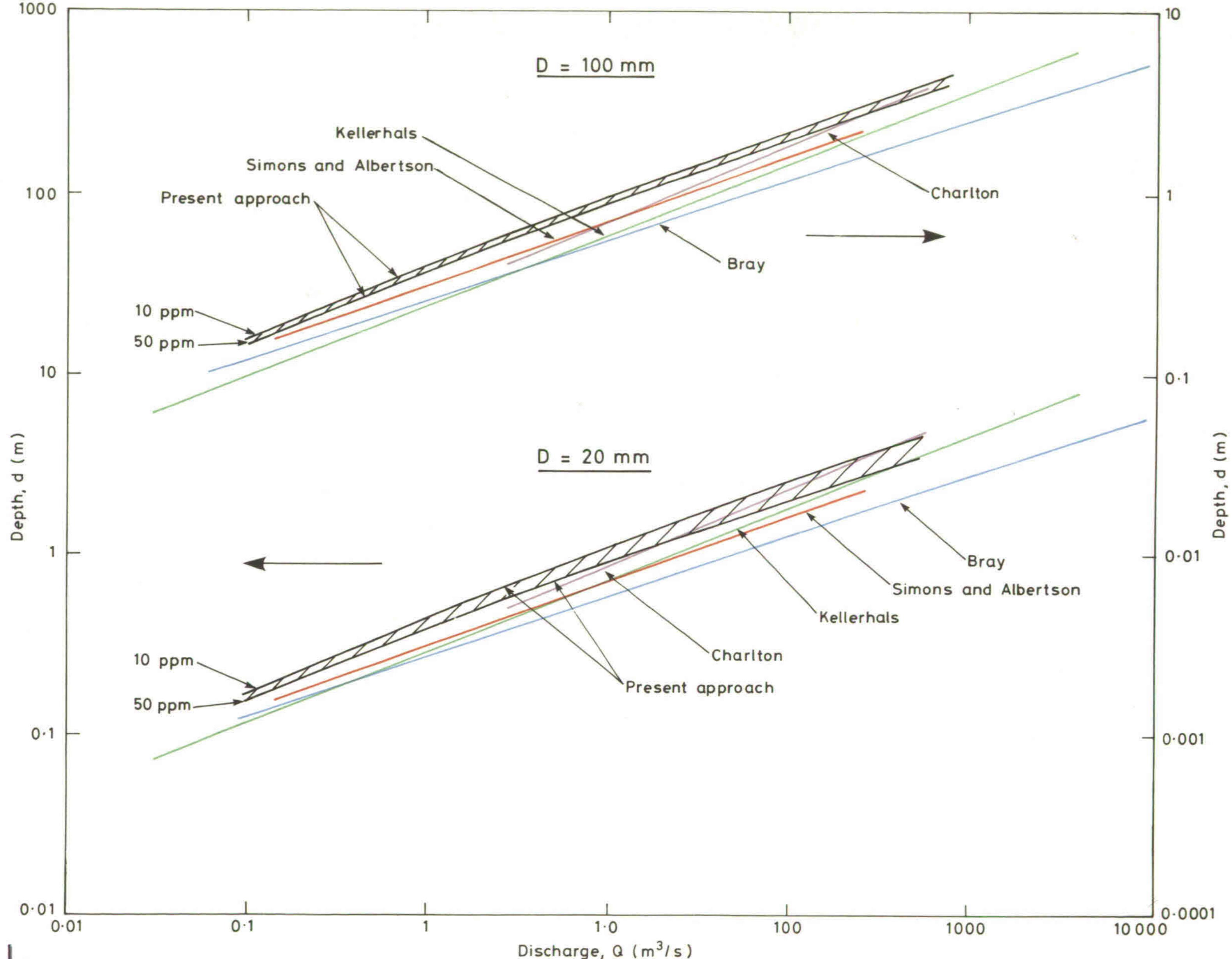


Fig 16

Regime depths for gravel rivers

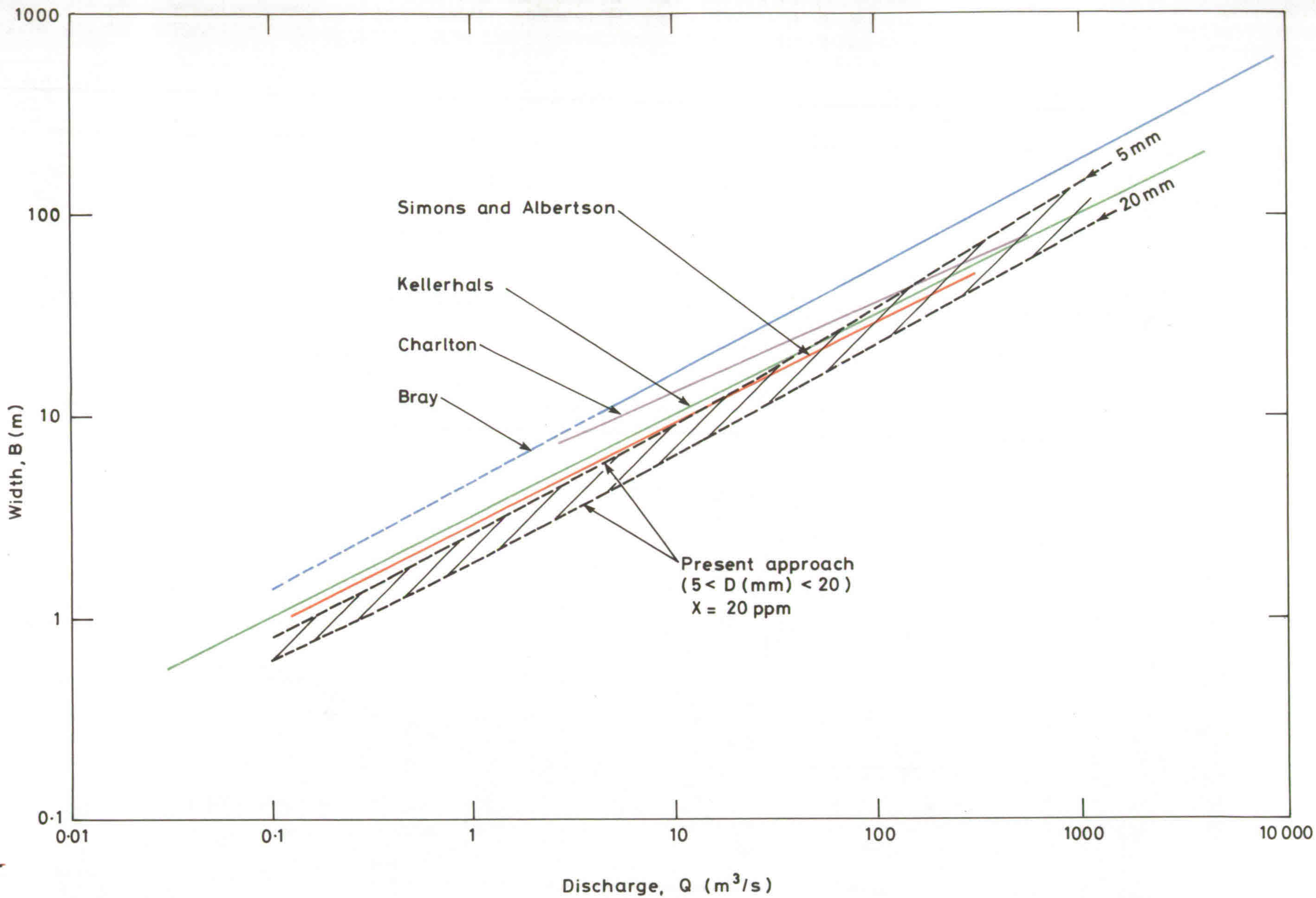
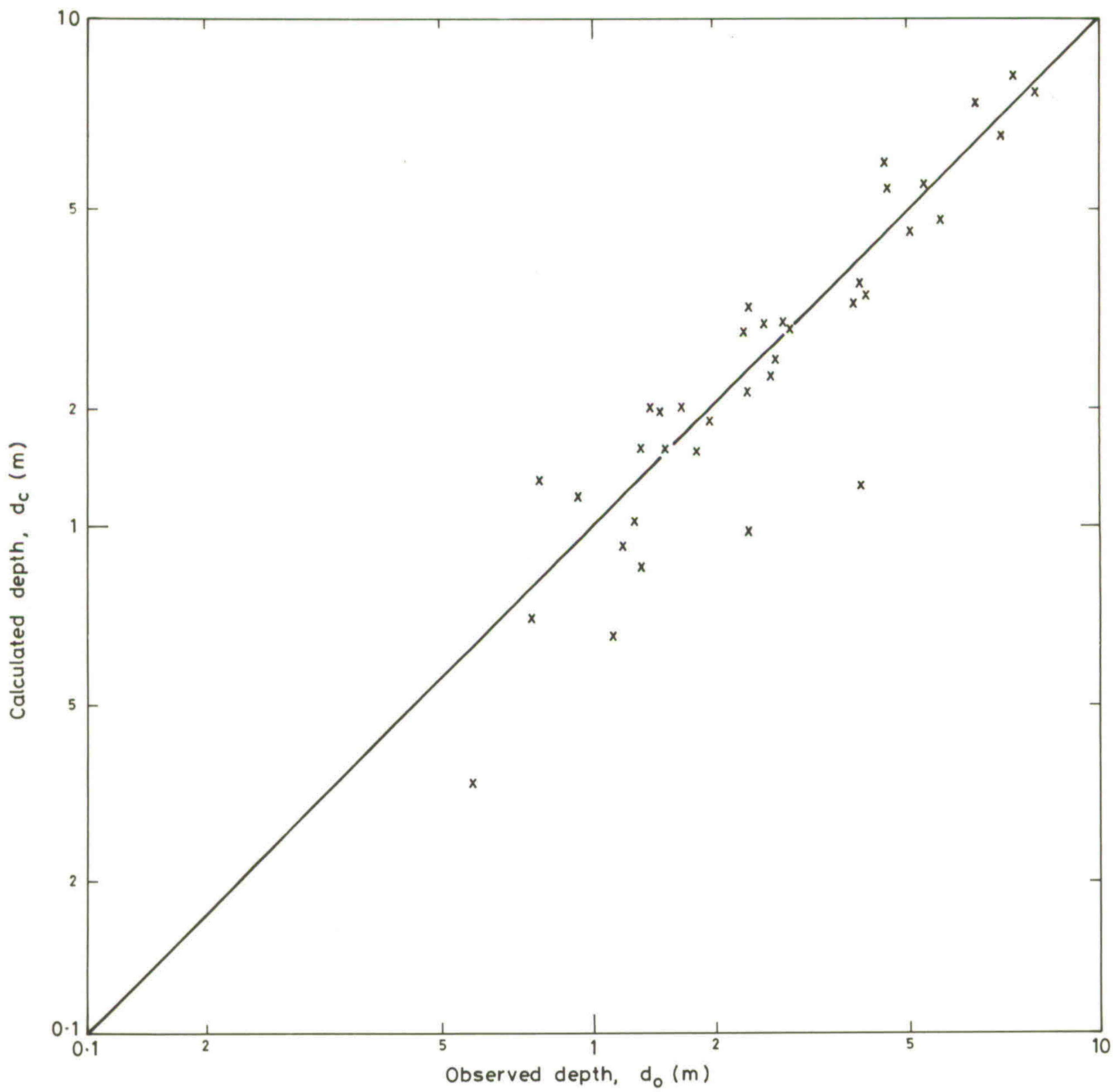


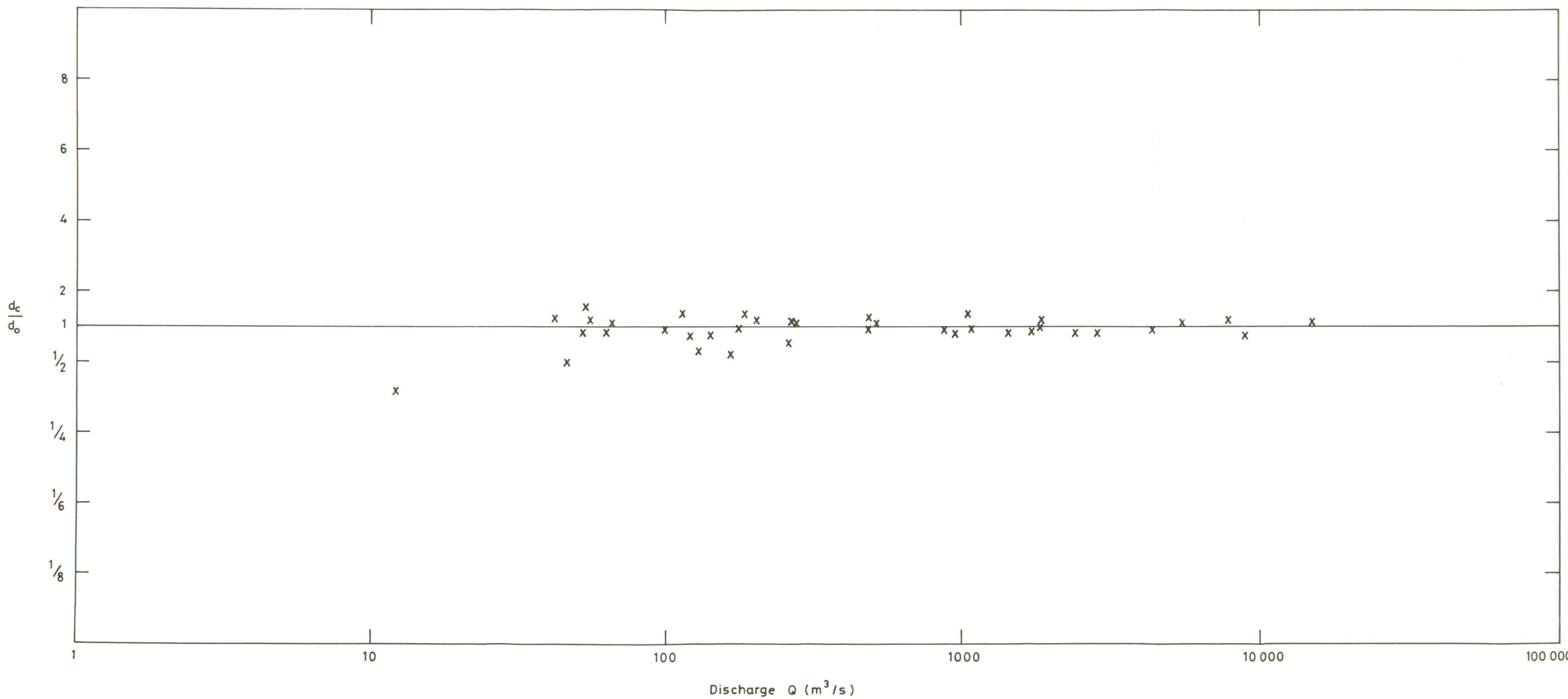
Fig 17

Regime widths for gravel rivers



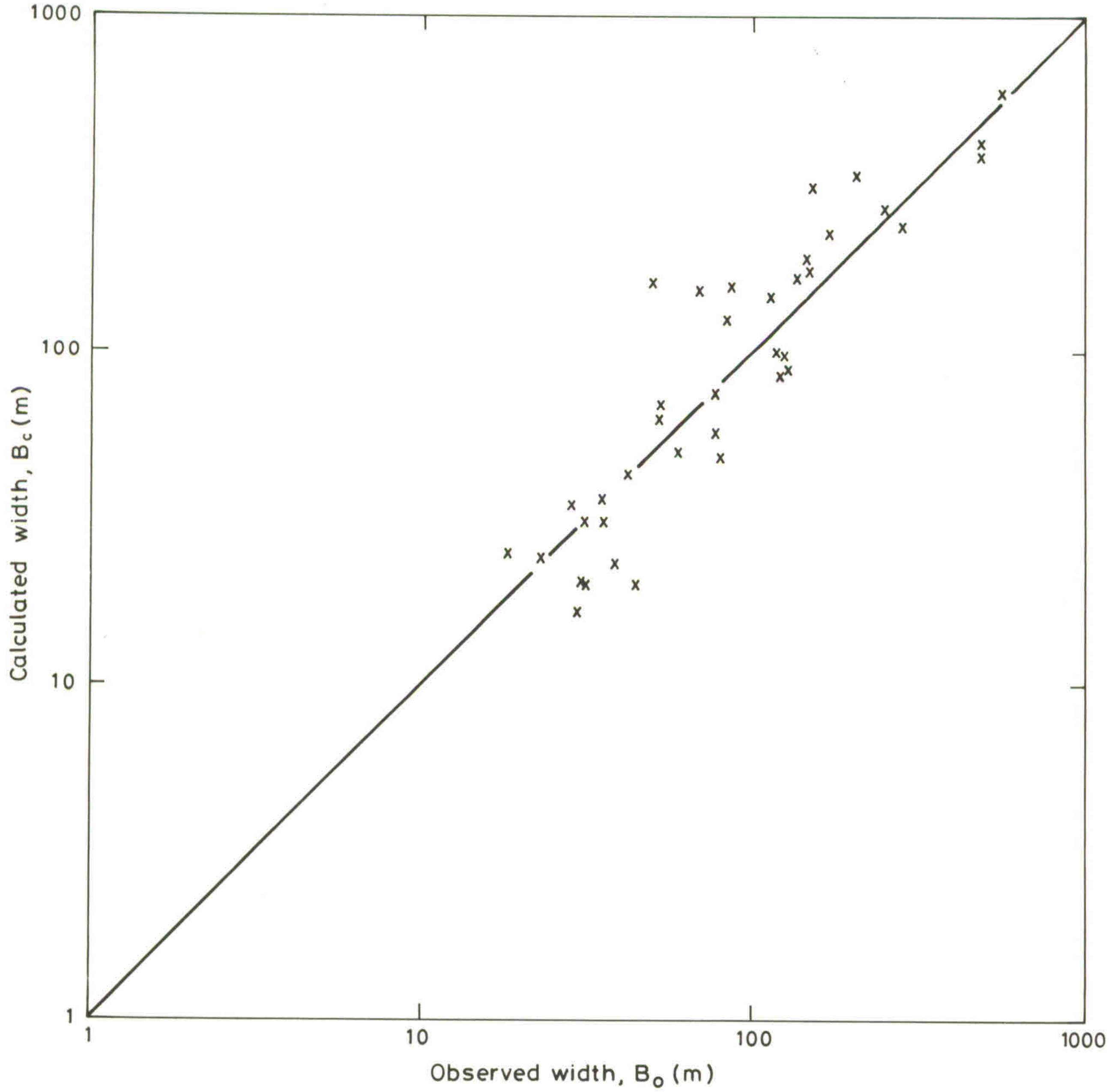
Calculated against observed depth using observed slopes, gravel rivers

Fig 18



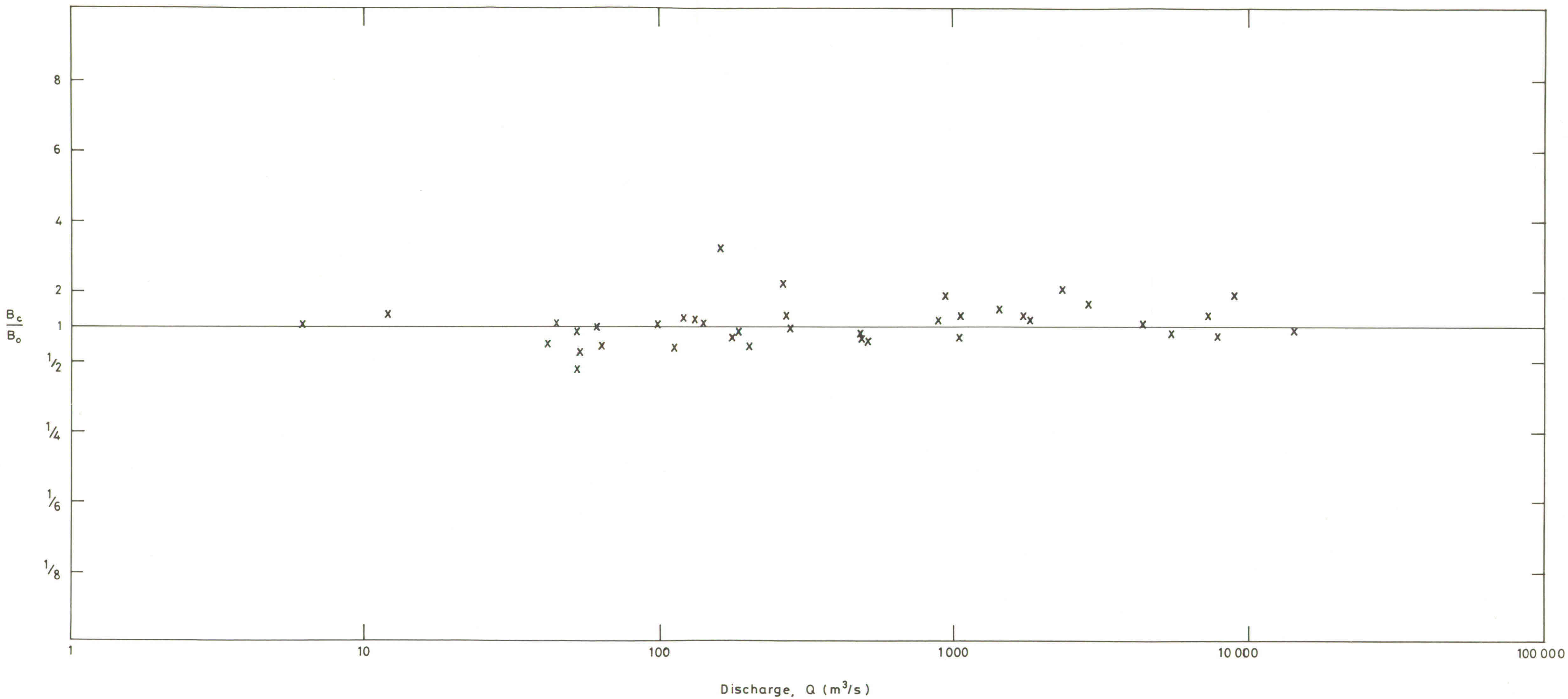
Discrepancy ratio for depth using observed slopes, gravel rivers

Fig 19



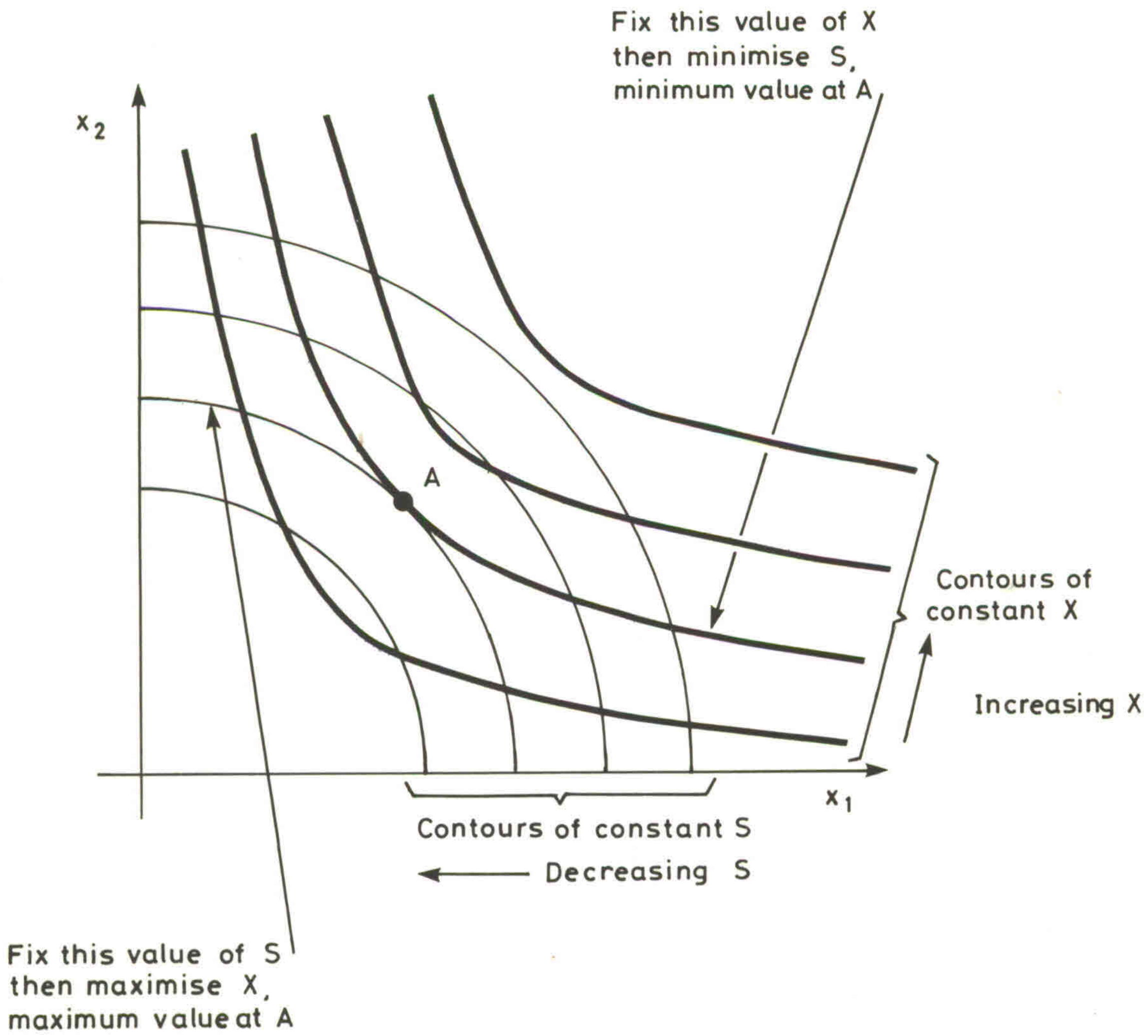
Calculated against observed width using observed slopes, gravel rivers

Fig 20



Discrepancy ratio for width using observed slopes, gravel rivers

Fig 21



Equivalence of two variational principles

Fig 22