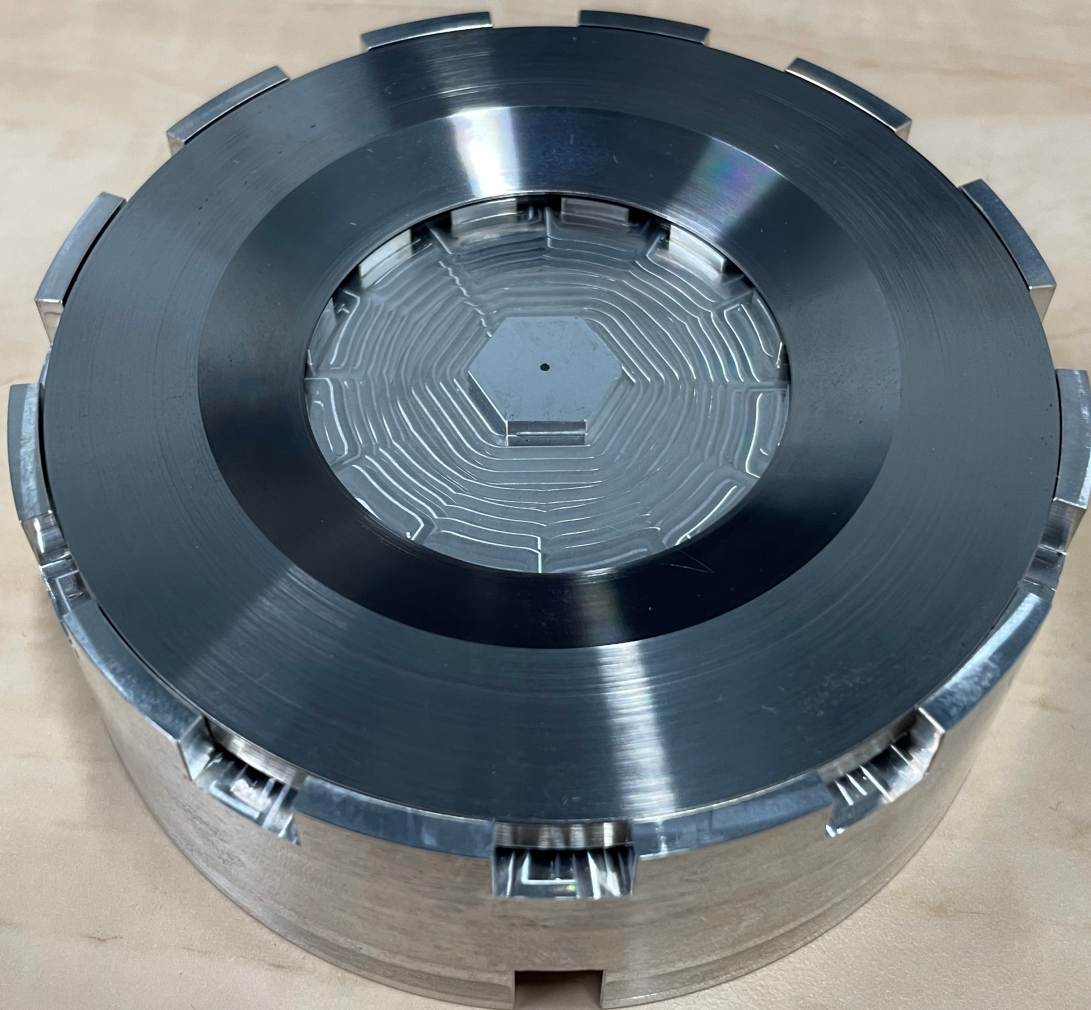


Department of Precision and Microsystems Engineering

Thermal distribution of wafers during contactless handling on active air bearings

Abel Versloot

Report no : 2025.013
Coach : Dr. Ir. R.A.J. van Ostayen & Dr. R. Delfos
Professor : Dr. Ir. R.A.J. van Ostayen & Dr. R. Delfos
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Thermal distribution of wafers during contactless handling on active air bearings

by

A.T. Versloot

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Student number:	4882911	
Thesis committee:	Dr. Ir. R.A.J. van Ostayen,	TU Delft, supervisor
	Dr. R. Delfos,	TU Delft, supervisor
	Dr. L. Botto,	TU Delft

This thesis is confidential and cannot be made public until April 29, 2026.

An electronic version of this thesis is available at <http://repository.tudelft.nl/>.

Abstract

Current wafer handling methods in the high-tech industry rely on mechanical contact to transport and position wafers. This leads to high breakage rates and contamination, ultimately reducing product yield and limiting further reductions in wafer thickness. To overcome these challenges, TU Delft has developed several air-based contactless handling systems. These systems operate by maintaining a thin air film between the air bearing surface and the substrate, generating a controllable viscous traction force. However, the strict thermal uniformity and stability requirements of modern lithography introduce new challenges, as even millikelvin-scale temperature variations can lead to overlay errors at the nanometer scale. Therefore, this research investigates the thermomechanical effects of active air bearings on wafers during contactless handling.

The surface of these air bearing systems consists of an array of unit cells, each fed with pressurized air at the inlet and connected to a vacuum at the outlet. This geometry drives airflow through the thin film, which can be tuned to achieve a net force on the substrate while also influencing heat transfer in the wafer. A schematic representation of the cross-sectional view of a typical unit cell is shown in Figure 1.

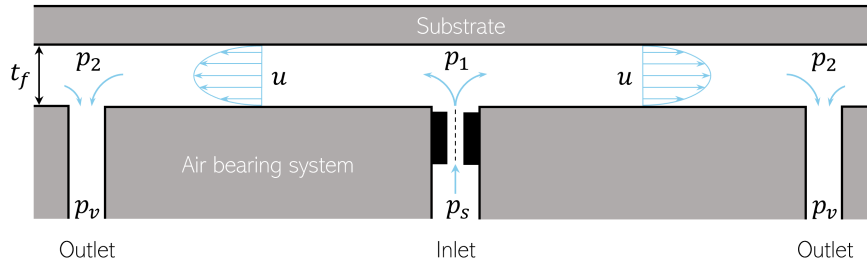


Figure 1: Schematic representation of the cross-sectional view of a typical radial flow unit cell.

Initially, a comprehensive theoretical analysis is performed on fluid flow and heat transfer phenomena in simplified geometries. Both rectilinear and axisymmetric models are developed to capture essential aspects of laminar thin film flows. Analytical expressions reveal the balance between expansion and viscous dissipation effects, particularly for rectilinear Poiseuille flow. Following the analytical work, the thesis further advances to complex numerical modelling using COMSOL Multiphysics, refining the geometry to better mirror actual operating conditions and incorporating the influence of mechanical wafer deformation.

In parallel, an experimental test setup is designed and implemented to replicate the operational environment of an air bearing system in a simplified form. This setup enables controlled measurement of temperature profiles and wafer deformation using complementary sensor techniques. One technique provides absolute temperature calibration, while another maps the relative temperature distribution across the wafer surface. Additionally, an optical method inspired by free-surface synthetic schlieren is used to quantify wafer deformation. A schematic representation of the experimental setup is provided in Figure 2. This experimental setup not only serves to validate the analytical models and numerical COMSOL simulations but also aims to explore measurement techniques applicable to operational air bearing systems.

Overall, this study demonstrates that air-based contactless handling systems have the potential to meet strict thermal requirements, provided that system parameters such as the inlet shape, thermal material properties, film height and pressure, and temperature conditions of the inlet and ambient air are carefully optimized. While the analytical and numerical models offered valuable insights into the thermomechanical behaviour of the system, the experimental test setup played a key role in validating predictions and highlighted points for improvements. These results provide a solid basis for further research and development of thermally stable contactless handling systems.

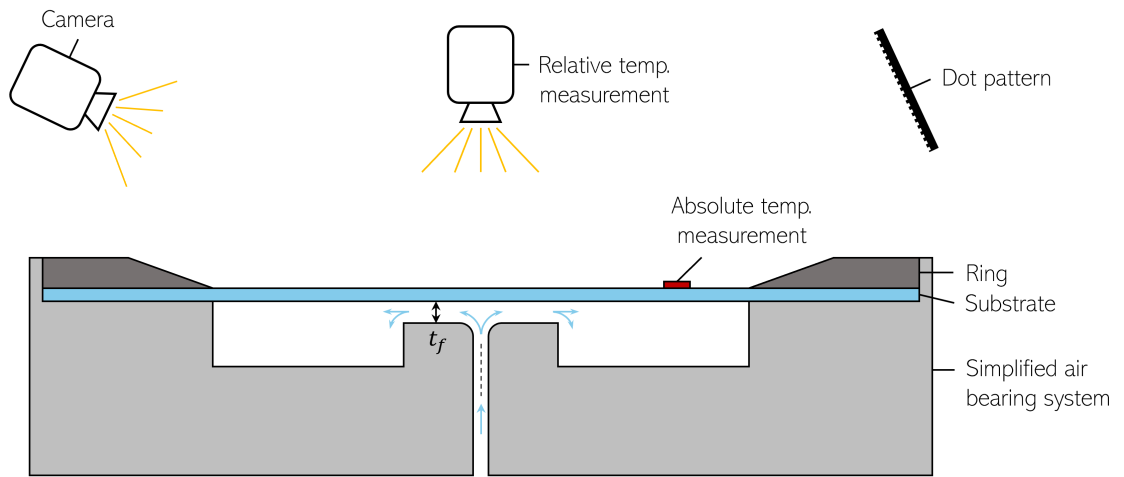


Figure 2: Schematic representation of the experimental test setup's cross-sectional view, including the temperature and deformation measurement techniques.

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Introduction

1.1 Background & Motivation

Within the high-tech industry, thin substrates serve as pivotal components in numerous production steps. The thin substrates undergo processes such as etching, deposition, and inspection in order to be applied in many high-tech products. Between and during these production processes, the thin substrates need to be transported and positioned with ever increasing speed and accuracy. On top of that, the ongoing need for miniaturization and cost reduction requires thinner substrates with a larger surface area and smaller feature sizes.

Currently, mechanical contact is the prevalent method for handling thin substrates like silicon wafers. However, this approach poses challenges due to the fragility of the wafers, resulting in high breakage rates, ultimately limiting further reductions in thickness. Figure 1.1 shows the evolution of the thickness and silicon usage of wafers used in the solar cell industry since 1990 [1]. As can be seen, the wafer thickness has not significantly decreased since 2008, which is primarily attributable to the high breakage rates due to the current wafer handling methods.

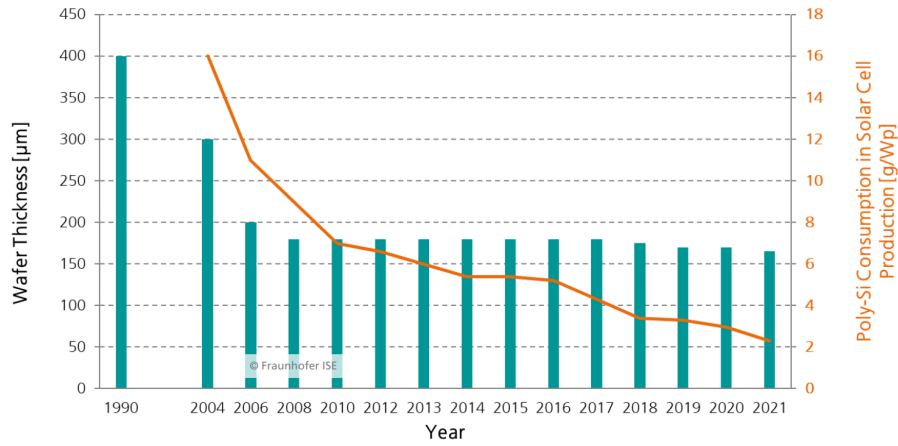


Figure 1.1: Wafer thickness and silicon usage in the solar cell industry [1].

Another consequence of using mechanical contact during wafer handling is contamination. At a particular production facility, an incurred yield loss of product of 0.5 - 1% was found to be caused by the "tripod" contamination pattern, illustrated in Figure 1.2, located on the backside of the wafer. Furthermore, Carey et al. [2] found that 30% of this contamination is transferred to the front side of the wafer in a thermal processing step, causing substrate degradation in specific cases. The metal-ion contamination was found to be caused by stainless-steel wafer pins used in wafer handling. In an attempt to circumvent the contamination, non-metallic pins were used instead of stainless-steel pins. However, the contamination effects were still observed, indicating the need to circumvent mechanical contact during wafer handling.

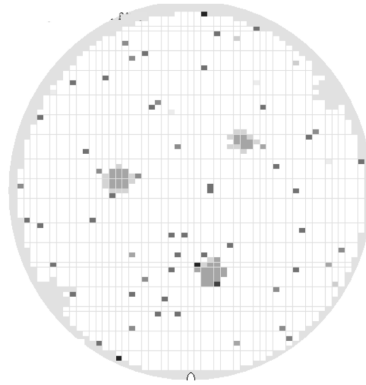


Figure 1.2: Wafer contamination pattern due to mechanical contact during handling [2].

The examples described above showcase the need for a contactless wafer handling solution. A wide variety of contactless handling technologies is available, based on physical principles such as electromagnetism, electrostatics, and acoustics [3]. At TU Delft, extensive research efforts have been dedicated to the development of air-based contactless handling systems, yielding several innovative concepts.

Air-Based Contactless Handling Systems

Each of the air-based contactless handling systems developed at TU Delft operates by maintaining a thin air film between the air bearing system and the substrate. The top surface of the air bearing is composed of an array of unit cells that are fed with pressurized air at the inlet and connected to a vacuum at the outlet. This pressure difference results in an airflow in the thin film, which in turn exerts a viscous traction force on the substrate. By controlling the airflow across the unit cells, actuation in multiple degrees of freedom (DOFs) can be achieved.

The way in which this airflow is generated and controlled varies for each generation. Figure 1.3 schematically depicts the typical geometry of an air bearing system using radial flow, where the substrate spans multiple unit cells to ensure even support and facilitate multi-DOF actuation. While the figure depicts unit cells with circular outlets, alternative shapes, such as hexagonal cells, have been implemented in earlier designs. Additionally, actuation can be achieved using rectilinear flow instead of radial flow, requiring different unit cell layouts.

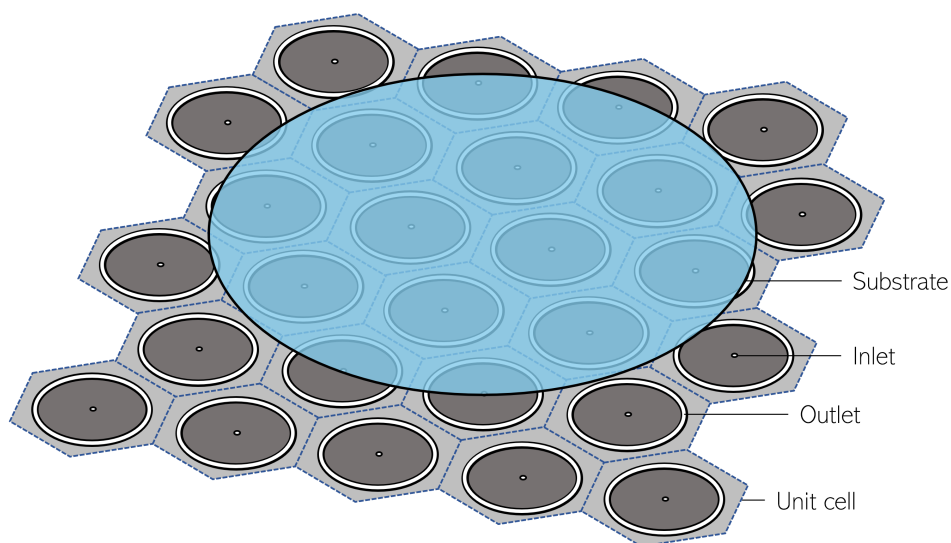


Figure 1.3: Schematic representation of the geometry of a typical air bearing system.

Figure 1.4 provides a cross-sectional view of a typical radial flow unit cell. Air is introduced at a pressure p_s and first passes through an inlet restriction designed to achieve a passively stable system with positive stiffness, as further discussed in Section 3.1.1. After passing through this restriction, the airflow enters the thin film at a reduced pressure p_1 and flows toward the outlet with a velocity profile denoted as u . Since the thin film height, t_f , is typically on the order of $10\ \mu\text{m}$, it imposes significant flow resistance, causing the pressure to drop further to p_2 . Depending on the outlet's flow restriction, the pressure eventually approaches the vacuum level p_v .

To actuate the substrate, a net force must be exerted on it, which can be achieved by creating an asymmetric flow profile around the unit cell inlet. The methods used to generate this asymmetric flow, along with the specifications and performance of each design generation, are detailed in Appendix A. Another application of the contactless handling system is also presented there, namely transportation.

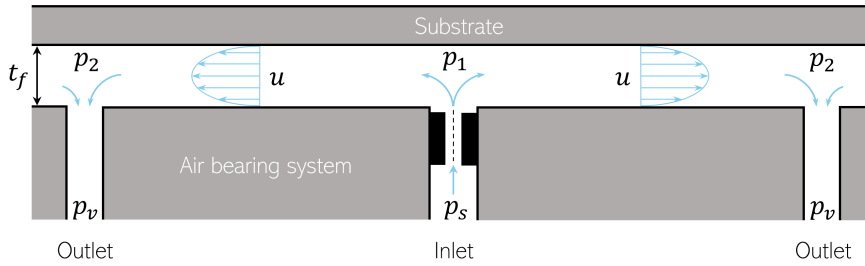


Figure 1.4: Schematic representation of the cross-sectional view of a typical radial flow unit cell.

Thermal Considerations

The demanding criteria for substrate positioning in the high-tech industry also translate to rather tight thermal tolerances for the temperature distribution of the wafers. Performing a simple calculation of the thermal expansion of a 300 mm silicon wafer, using a linear thermal expansion coefficient of $\alpha_L = 2.57 \times 10^{-6}\ \text{K}^{-1}$ at 293 K [4], shows that a uniform temperature increase of 10 mK results in a thermal expansion of 7.71 nm:

$$\Delta L = \alpha_L L \Delta T = 2.57 \times 10^{-6} \cdot 0.3 \cdot 10 \times 10^{-3} = 7.71\ \text{nm} \quad (1.1)$$

Even though this quantity seems negligible compared to the length scale of the wafer, it does play a significant role. In current ASML extreme ultraviolet (EUV) lithography machines, the minimum feature size has now shrunk to only a few nanometers. Furthermore, the total overlay error has reached a mere 1.1 nm [5], emphasizing that the thermal overlay error must now be within the sub-nanometer range.

To achieve such a tight tolerance, maintaining temperature uniformity across the wafer within a few millikelvin is essential. However, the air-based contactless handling technology, which generates airflow beneath the wafer, may influence its temperature distribution. Up until now, no research has been performed on the thermal aspects of the air-based contactless handling technology on wafers.

1.2 Research Objective

The stringent requirements on the temperature distribution of wafers in the high-tech industry pose a significant challenge for the implementation of an air-based contactless handling system. In addition to the thermal effects, the pressure distribution between the active air bearing and the wafer induces mechanical deformations. To assess the impact of this novel technology, both the thermal and mechanical effects need to be studied comprehensively, combining theoretical and experimental approaches. The primary goal of this research is therefore defined as follows:

Investigate the thermomechanical effects of active air bearings on wafers during contactless handling.

To achieve this goal, the following sub-objectives have been defined:

- Analytically model the thermomechanical phenomena that arise during contactless handling of wafers on active air bearings, using simplified geometries.
- Develop a numerical model that can be used to predict the thermomechanical effects of active air bearings on wafers.
- Explore measurement techniques to experimentally assess the thermomechanical effects of active air bearings on wafers.
- Develop an experimental test setup that incorporates several measurement techniques to assess the thermomechanical effects of a simplified air bearing system on wafers.
- Validate the theoretical models through physical experiments.

1.3 Thesis Outline

This thesis is structured as follows. First, [Chapter 2](#) presents a thermal analysis of the system using analytical, semi-analytical, and numerical methods to determine the flow and temperature characteristics of the thin film flow in a single unit cell. Both rectilinear and axisymmetric geometries are considered to model the flow between p_1 and p_2 shown in [Figure 1.4](#), with the axisymmetric case providing an accurate approximation of the radial thin film flow in unit cells at their equilibrium position.

Next, numerical modelling is conducted in COMSOL to handle more complex geometries, as outlined in [Chapter 3](#). The initial COMSOL geometry corresponds to the one used in [Chapter 2](#), for the purpose of model verification. The geometry is then gradually refined to achieve a more accurate representation of the system, incorporating both the wafer and air bearing system. This chapter also discusses several important factors influencing the thermomechanical response of the wafer, which are used for the parameter selection for the test setup. These parameters are selected to facilitate the use of relatively straightforward measurement techniques, for example, by amplifying the measurable temperature difference.

After that, [Chapter 4](#) describes the experimental test setup, including its design, manufacturing, and modelling. The test setup, schematically shown in [Figure 1.5](#), consists of a simplified air bearing system designed to obtain equivalent thermomechanical effects observed in operational air bearing systems. The test setup is simplified by excluding substrate levitation and instead using a fixed geometry, where the wafer is clamped at a predefined height above the air bearing surface. Furthermore, only a single unit cell with an atmospheric outlet pressure is used. To measure the wafer's temperature profile, two complementary measurement techniques are employed. One captures the relative temperature distribution across the wafer, while the other provides an accurate absolute temperature reading for calibration. Additionally, the deformation of the wafer is measured using an optical method inspired by free-surface synthetic schlieren, which relies on a camera and dot pattern. Further details on these measurement techniques are provided in [Chapter 4](#) and their corresponding appendices. The chapter concludes with the presentation and discussion of experimental results.

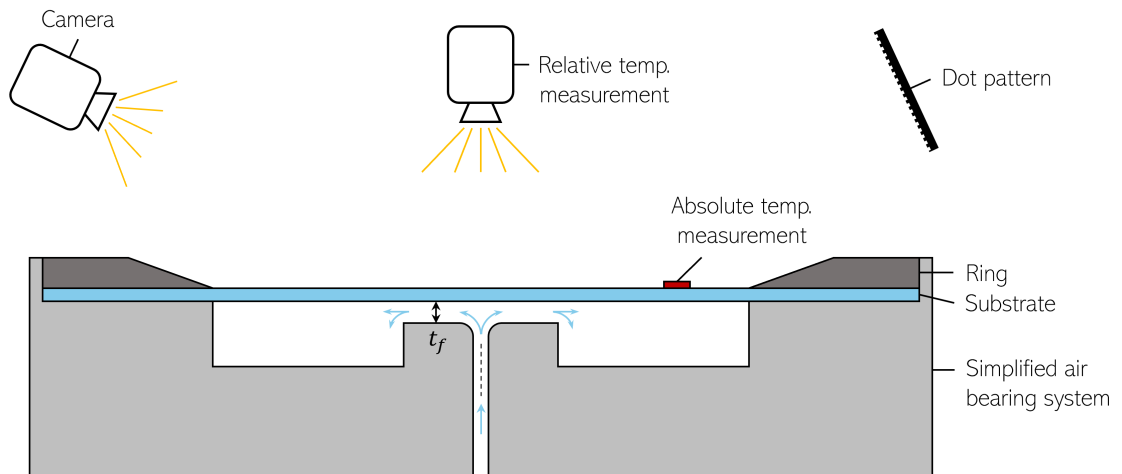


Figure 1.5: Schematic representation of the experimental test setup's cross-sectional view, including the temperature and deformation measurement techniques.

To conclude this thesis, [Chapter 5](#) summarizes the main findings obtained throughout the research and provides recommendations for future research on this topic.

Additionally, supplementary material is presented in the appendices at the end of the thesis. These appendices are referenced throughout the report whenever additional information might be required.

2

Thermal Analysis

This chapter provides a theoretical analysis of the key fluid mechanics and heat transfer phenomena involved in handling a wafer with an active air bearing system. The primary advantage of this analysis is that it offers valuable insights into the system's thermal behaviour, which can later be used to verify the numerical modelling results.

The general fluid mechanics and heat transfer theory used in this analysis is presented in [Appendix B.1](#). Furthermore, a simplified flow case of rectilinear flow between parallel plates is analyzed in detail in [Appendix B.2](#). This rectilinear flow analysis is first summarized below, serving as the foundation of the axisymmetric flow analysis discussed in [Section 2.2](#).

2.1 Rectilinear Flow

As a starting point for the thermal analysis of the thin film flow between the wafer and air bearing system, a simple 2D geometry consisting of two closely spaced parallel plates was used. This geometry aims to represent the region between the pressurized inlet (p_1) and the vacuum outlet (p_2) of a single unit cell. To further simplify the analysis, laminar incompressible flow was assumed with no-slip boundary conditions at both plates. Additional assumptions made during the analysis are presented in [Appendix B.2](#).

In this context, two distinct flow types can be identified: plane Poiseuille flow and plane Couette flow. Plane Poiseuille flow is driven by the pressure gradient between the inlet and outlet of the unit cell, producing a symmetrical, parabolic velocity profile $u(y)$ about the midplane ([Figure 2.1a](#)). Plane Couette flow, on the other hand, is induced by the relative motion of the top plate at velocity V with respect to the stationary bottom plate. This shear-driven flow results in a linear velocity profile ranging from zero at the stationary surface to V at the moving boundary ([Figure 2.1b](#)).

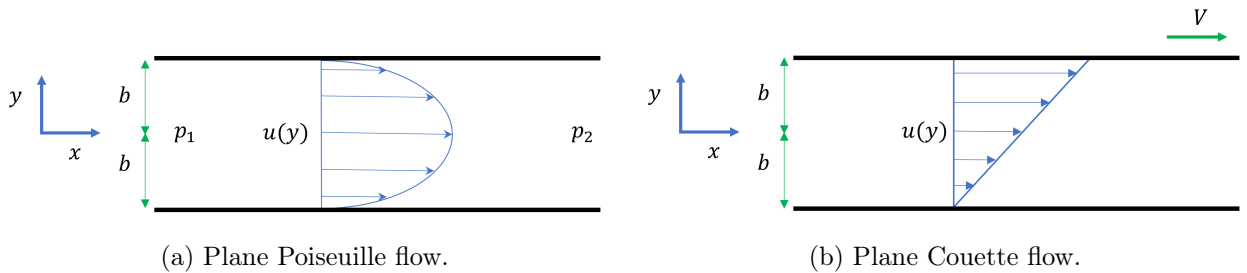


Figure 2.1: Schematic representation of two rectilinear flow types between parallel plates.

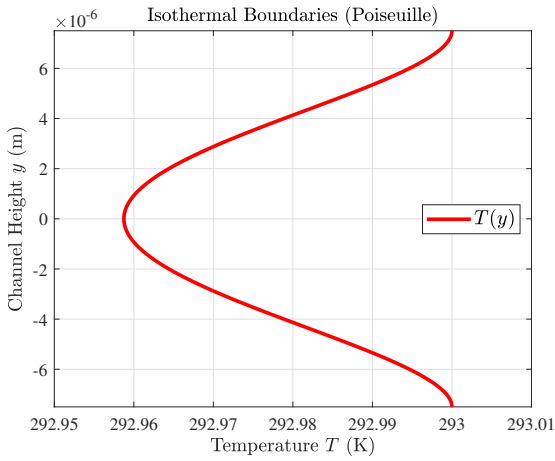
In typical air bearing systems, the velocity induced by plane Couette flow is negligible compared to that of plane Poiseuille flow, primarily due to the relatively low substrate speeds. As a result, the thermal effects associated with plane Couette flow are also significantly smaller than those caused by plane Poiseuille flow. This behaviour was further examined in [Appendix B.2](#), where the thermal energy equation ([Equation 2.1](#)) was solved for various thermal boundary conditions under both flow regimes.

$$\rho c_p u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial y^2} + u \frac{dp}{dx} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad (2.1)$$

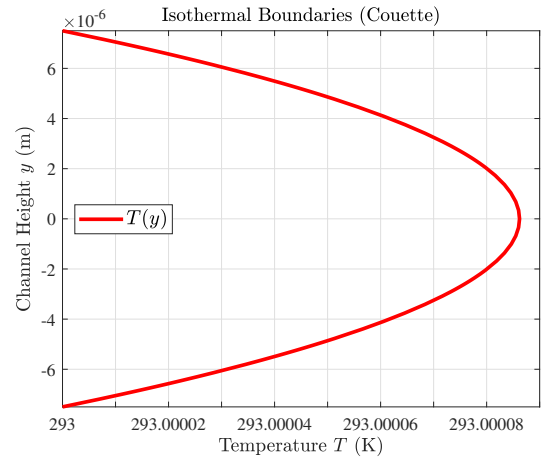
Figure 2.2 presents the computed temperature profiles across the thin film height for plane Poiseuille and plane Couette flow, assuming both boundaries are isothermal at 293 K. The results were generated using the MATLAB scripts provided in Appendix H.1 and Appendix H.2, with the parameters specified in Table B.2. Due to the assumption of fully developed steady-state flow, the convective term $\rho c_p u \frac{\partial T}{\partial x}$ was neglected.

In the case of Poiseuille flow, the air temperature decreases by approximately 40 mK at the channel center due to air expansion, represented by the $u \frac{dp}{dx}$ term. Near the channel boundaries, where the viscous dissipation term $\mu \left(\frac{\partial u}{\partial y} \right)^2$ dominates, the temperature gradient is perpendicular to the walls. The absence of a temperature gradient at the boundaries indicates no heat transfer between the walls and the airflow, making the boundaries both isothermal and adiabatic.

For Couette flow, no pressure gradient is present along the flow direction, leaving the viscous dissipation term and conduction term $k \frac{\partial^2 T}{\partial y^2}$. The resulting temperature profile reveals a slight temperature rise at the channel center due to viscous heating, along with a nonzero temperature gradient at the boundaries, indicating heat exchange with the walls.



(a) Plane Poiseuille flow.



(b) Plane Couette flow.

Figure 2.2: Temperature profiles between the parallel plates for two isothermal boundaries for the two rectilinear flow types.

The main takeaway from the thermal analysis of the rectilinear flow between parallel plates is that Poiseuille flow on its own does not cause any energy dissipation across the channel height, whereas Couette flow does. This can be explained by examining the graphs of the pressure derivative term and the viscous dissipation term over the channel height for plane Poiseuille flow, as shown in Figure 2.3. The pressure derivative term is most negative at the center of the channel, while the viscous dissipation term is dominant near the boundaries of the channel. When integrating the sum of both terms across the channel height, they perfectly balance out (Equation 2.2), indicating that no net energy is dissipated.

For the thin film temperature profile, this means that it is irrelevant whether there are two isothermal boundaries at the same temperature, two adiabatic boundaries, or one isothermal and one adiabatic

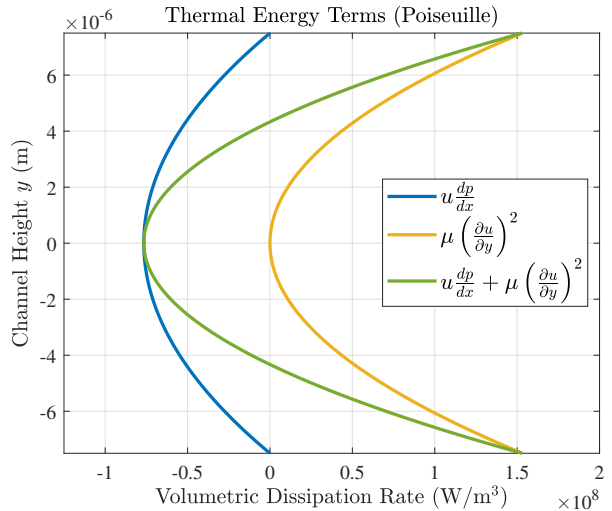


Figure 2.3: Relevant terms of the thermal energy equation for plane Poiseuille flow.

boundary. Ultimately, both boundaries will become isothermal and adiabatic.

$$\int_{-b}^b u \frac{dp}{dx} + \mu \left(\frac{\partial u}{\partial y} \right)^2 dy = 0 \text{ W/m}^2 \quad (2.2)$$

For Couette flow there is no pressure gradient, simplifying the integral to [Equation 2.3](#). The resulting value is a constant that is unequal to zero, meaning energy is dissipated for Couette flow.

$$\int_{-b}^b \mu \left(\frac{\partial u}{\partial y} \right)^2 dy = \frac{\mu V^2}{2b} \quad (2.3)$$

2.2 Axisymmetric Flow

In order to obtain a more accurate representation of radial flow air bearing systems, this section analyzes the velocity and temperature profiles for an axisymmetric geometry. Given that the thermal effects of Couette flow are significantly smaller in magnitude, only plane Poiseuille flow is considered here. The axisymmetric geometry consists of two parallel disks, with one disk centrally supplied with air at pressure p_1 through a circular inlet with radius r_1 . The air is assumed to flow purely radially outward toward the outer edges of the disks, which have an outer radius r_2 . The parallel disks are separated by a distance $2b$, representing the film height. The pressure at the outer edges of the disks is p_2 . [Figure 2.4](#) shows a schematic representation of the axisymmetric geometry.

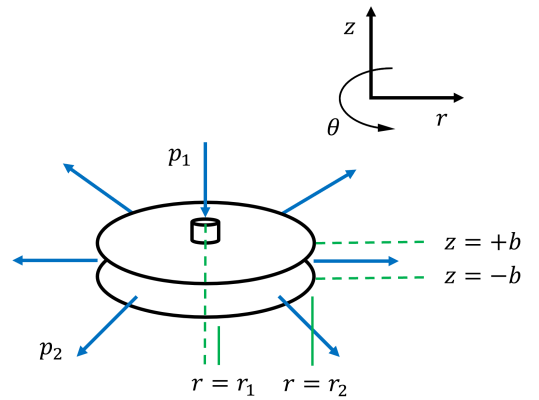


Figure 2.4: Schematic representation of the axisymmetric geometry.

For both velocity and thermal analyses, the flow is assumed to be fully developed in the region between r_1 and r_2 , and the influence of the vertical inlet region is neglected. Unlike the rectilinear flow analysis, compressibility effects are incorporated here by allowing the air density to vary with pressure. However, density variations due to temperature fluctuations are assumed negligible, and air density is treated as uniform across the film height. The MATLAB script used to perform the calculations in this section is included in [Appendix H.3](#).

2.2.1 Velocity Profiles

To account for the axisymmetric nature of the geometry, the continuity equation and Navier-Stokes equations are applied on a cylindrical coordinate system. The continuity equation in cylindrical coordinates is provided in [Equation 2.4](#) [6]. Assuming steady-state conditions and purely radial flow (i.e., $V_\theta = 0$ and $V_z = 0$), this equation simplifies to [Equation 2.5](#). In contrast to the rectilinear flow analysis, the air density can no longer be assumed constant due to compressibility effects and must therefore be retained in the continuity equation.

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(\rho r V_r)}{\partial r} + \frac{1}{r} \frac{\partial(\rho V_\theta)}{\partial \theta} + \frac{\partial(\rho V_z)}{\partial z} = 0 \quad (2.4)$$

$$\frac{1}{r} \frac{\partial(\rho r V_r)}{\partial r} = 0 \quad (2.5)$$

With:

$$\rho = \rho(r) \quad (2.6)$$

An initial assessment suggests that the radial velocity depends on both z and r . From the simplified continuity equation, it is evident that the radial velocity varies inversely with the radial coordinate r (Equation 2.7). However, the dependence of the radial velocity on the z -coordinate still requires further investigation.

$$V_r(r, z) = \frac{f(z)}{\rho(r)r} \quad (2.7)$$

The next step is to determine the functions $\rho(r)$ and $f(z)$ by applying the Navier-Stokes equations in cylindrical coordinates, as presented below [6]:

$$\begin{aligned} \rho \left(\frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_r}{\partial \theta} + V_z \frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r} \right) = -\frac{\partial p}{\partial r} + \\ \mu \left(\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r V_r) \right) + \frac{1}{r^2} \frac{\partial^2 V_r}{\partial \theta^2} + \frac{\partial^2 V_r}{\partial z^2} - \frac{2}{r^2} \frac{\partial V_\theta}{\partial \theta} \right) + \rho g_r \end{aligned} \quad (2.8a)$$

$$\begin{aligned} \rho \left(\frac{\partial V_\theta}{\partial t} + V_r \frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_\theta}{\partial \theta} + V_z \frac{\partial V_\theta}{\partial z} + \frac{V_r V_\theta}{r} \right) = -\frac{1}{r} \frac{\partial p}{\partial \theta} + \\ \mu \left(\frac{\partial}{\partial r} \left(\frac{1}{r} \frac{\partial}{\partial r} (r V_\theta) \right) + \frac{1}{r^2} \frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{\partial^2 V_\theta}{\partial z^2} + \frac{2}{r^2} \frac{\partial V_r}{\partial \theta} \right) + \rho g_\theta \end{aligned} \quad (2.8b)$$

$$\begin{aligned} \rho \left(\frac{\partial V_z}{\partial t} + V_r \frac{\partial V_z}{\partial r} + \frac{V_\theta}{r} \frac{\partial V_z}{\partial \theta} + V_z \frac{\partial V_z}{\partial z} \right) = -\frac{\partial p}{\partial z} + \\ \mu \left(\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial V_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 V_z}{\partial \theta^2} + \frac{\partial^2 V_z}{\partial z^2} \right) + \rho g_z \end{aligned} \quad (2.8c)$$

For thin films, the variations in the radial velocity profile across the film height are typically much greater than those across the radius. By additionally assuming steady-state, purely radial flow with negligible gravitational effects, the Navier-Stokes equations simplify to the boundary-layer equations for radial flow, as shown below [7]:

$$\rho V_r \frac{\partial V_r}{\partial r} = -\frac{\partial p}{\partial r} + \mu \frac{\partial^2 V_r}{\partial z^2} \quad (2.9a)$$

$$\frac{\partial p}{\partial \theta} = 0 \quad (2.9b)$$

$$\frac{\partial p}{\partial z} = 0 \quad (2.9c)$$

The last two equations indicate that the pressure profile remains constant across both the film height and the azimuthal coordinate. Thus, the pressure depends only on the radial coordinate:

$$p = p(r) \quad (2.10)$$

Consequently, the expected radial velocity profile (Equation 2.7) can be substituted into Equation 2.9a to derive the governing flow equation for radial flow between parallel disks as a function of $\rho(r)$ and $f(z)$:

$$\mu f''(z) \frac{1}{\rho(r)r} + f(z)^2 \frac{\rho(r) + \rho'(r)r}{\rho(r)^2 r^3} = \frac{\partial p}{\partial r} \quad (2.11)$$

The equation above is a coupled nonlinear differential equation, which cannot be solved exactly using analytical methods. Therefore, alternative approximation techniques are necessary to obtain a solution [8] [9].

Creeping Flow Assumption

For flows with a very small Reynolds number, $Re \ll 1$, the inertia terms in the Navier-Stokes equations can be neglected [10]. This assumption, known as creeping flow, is commonly applied in the field of tribology and forms the basis of both the viscous flow equations and the Reynolds equation [7]. Although the Reynolds number in air bearing systems, as described in [Appendix A](#), is typically larger than one, the creeping flow assumption is often used for analytical derivations. This approach simplifies the calculations significantly while still providing reasonable accuracy. By applying the creeping flow assumption, the term on the left-hand side of [Equation 2.9a](#) can be omitted, yielding the following equation:

$$\frac{\partial p}{\partial r} = \mu f''(z) \frac{1}{\rho(r)r} \quad (2.12)$$

Where the air pressure and density are related through the ideal gas law:

$$p(r) = RT\rho(r) \quad (2.13)$$

By substituting the ideal gas law into [Equation 2.12](#), the following separable differential equation is obtained:

$$RT\rho'(r)\rho(r)r = \mu f''(z) \quad (2.14)$$

$$RT \int_{\rho_1}^{\rho_2} \rho(r) d\rho = \mu f''(z) \int_{r_1}^{r_2} \frac{1}{r} dr \quad (2.15)$$

With no-slip boundary conditions:

$$f(-b) = 0 \quad f(b) = 0 \quad (2.16)$$

Solving the separable differential equation subject to the boundary conditions stated above yields the following solutions for $\rho(r)$ and $f(z)$:

$$\rho(r) = \sqrt{\frac{(\rho_2^2 - \rho_1^2) \ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)} + \rho_1^2} \quad (2.17)$$

$$f(z) = \frac{b^2 RT(\rho_1^2 - \rho_2^2)}{4\mu \ln\left(\frac{r_2}{r_1}\right)} \left(1 - \left(\frac{z}{b}\right)^2\right) \quad (2.18)$$

Next, the pressure profile $p(r)$ can be determined, and $f(z)$ can be expressed as a function of the inlet and outlet pressures:

$$p(r) = \sqrt{\frac{(p_2^2 - p_1^2) \ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)} + p_1^2} \quad (2.19)$$

$$f(z) = \frac{b^2(p_1^2 - p_2^2)}{4\mu RT \ln\left(\frac{r_2}{r_1}\right)} \left(1 - \left(\frac{z}{b}\right)^2\right) \quad (2.20)$$

Using the creeping flow assumption, the radial velocity profile is consequently described by [Equation 2.21](#).

$$V_r(r, z) = \frac{b^2(p_1^2 - p_2^2)}{4\mu r \ln\left(\frac{r_2}{r_1}\right) \sqrt{\frac{(p_2^2 - p_1^2) \ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)} + p_1^2}} \left(1 - \left(\frac{z}{b}\right)^2\right) \quad (2.21)$$

Perturbation Method

Another method for obtaining an approximate solution to the nonlinear differential equation shown in Equation 2.11 is the perturbation method [8]. To simplify the analysis, the density profile is assumed to follow the same form as the solution derived under the creeping flow assumption:

$$\rho_{pert}(r) = \rho_{creep}(r) \quad (2.22)$$

First, a perturbation parameter ϵ is introduced:

$$\mu f''(z) \frac{1}{\rho(r)r} + \epsilon f(z)^2 \frac{\rho(r) + \rho'(r)r}{\rho(r)^2 r^3} = \frac{\partial p}{\partial r} \quad (2.23)$$

Next, both sides of the separable differential equation can be integrated (see Equation 2.24), and the integrals can be evaluated in MATLAB using the known density function.

$$\mu f''(z) \int_{r_1}^{r_2} \frac{1}{\rho(r)r} dr + \epsilon f(z)^2 \int_{r_1}^{r_2} \frac{\rho(r) + \rho'(r)r}{\rho(r)^2 r^3} dr = \int_{p_1}^{p_2} dp \quad (2.24)$$

This simplification introduces a small mismatch between the pressure and density profiles. As a result, the pressure profile is no longer solely a function of r , but also exhibits slight variations along z to account for the assumed density profile.

For $\epsilon \neq 0$, the function $f(z)$ is assumed to be of the following form:

$$f(z) = f_0(\eta) + \epsilon f_1(\eta) + \epsilon^2 f_2(\eta) + \dots \quad (2.25)$$

Where η represents the normalized film height:

$$\eta = \frac{z}{b} \quad (2.26)$$

The second derivative of $f(z)$ can consequently be calculated:

$$f''(z) = \frac{1}{b^2} (f_0''(\eta) + \epsilon f_1''(\eta) + \epsilon^2 f_2''(\eta) + \dots) \quad (2.27)$$

For clarity, Equation 2.24 is rewritten as Equation 2.28 by introducing the parameters A and P :

$$f''(z) + \epsilon A f(z)^2 = -P \quad (2.28)$$

$$A = \frac{b^2 \int_{r_1}^{r_2} \frac{\rho(r) + \rho'(r)r}{\rho(r)^2 r^3} dr}{\mu \int_{r_1}^{r_2} \frac{1}{\rho(r)r} dr} \quad P = \frac{b^2(p_1 - p_2)}{\mu \int_{r_1}^{r_2} \frac{1}{\rho(r)r} dr} \quad (2.29)$$

After substituting Equation 2.25 and Equation 2.27 into Equation 2.28, the equation can be separated based on powers of ϵ , resulting in several differential equations. For this particular case, a second-order approximation was applied, leading to the following differential equations:

$$\epsilon^0 : \quad f_0''(\eta) + P = 0 \quad (2.30a)$$

$$\epsilon^1 : \quad f_1''(\eta) + A f_0(\eta)^2 = 0 \quad (2.30b)$$

$$\epsilon^2 : \quad f_2''(\eta) + 2A f_0(\eta) f_1(\eta) = 0 \quad (2.30c)$$

This system of equations was then solved using MATLAB to obtain the radial velocity profile, which will be presented later in this section.

Numerical Solution

Finally, numerical integration is applied to obtain a solution, specifically with the ode45 method. To apply ode45 in MATLAB, the second-order differential equation presented in Equation 2.24, excluding the perturbation parameter ϵ , must first be converted into two first-order equations, as outlined below:

$$y_1' = y_2 \quad (2.31a)$$

$$y_2' = \frac{(p_2 - p_1) - y_1^2 \int_{r_1}^{r_2} \frac{\rho(r) + \rho'(r)r}{\rho(r)^2 r^3} dr}{\mu \int_{r_1}^{r_2} \frac{1}{\rho(r)r} dr} \quad (2.31b)$$

Additionally, the ode45 function requires initial conditions for both y_1 and y_2 , meaning the derivative of the velocity profile at one of the boundaries must be known. Since this information is not available, a for loop was incorporated into the MATLAB script to iterate over an array of initial conditions until a symmetric velocity profile was achieved.

Comparison

To compare the three methods described in this section, the parameters shown in Table 2.1 were used. The resulting pressure profile along the radius of the unit cell, obtained with the creeping flow assumption, is depicted in Figure 2.5. As expected, the absolute pressure decreases monotonically from the inlet pressure p_1 to the outlet pressure p_2 . Since the flow is assumed to be isothermal up to now, the density follows the same trend, scaled by the inverse of the specific gas constant R and temperature T according to the ideal gas law (Equation B.7).

Table 2.1: MATLAB parameters used for the radial flow analysis.

Parameter	Value	Unit
r_1	0.5	mm
r_2	7	mm
p_1	1.5×10^5	Pa
p_2	0.8×10^5	Pa
b	7.5	μm
μ	1.8×10^{-5}	Pa s
R	287	J/kg K
T	293	K

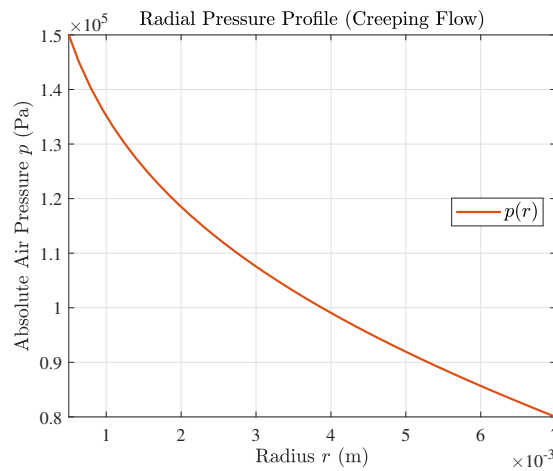


Figure 2.5: Radial pressure profile obtained with the creeping flow assumption.

The velocity profiles across the normalized channel height z/b at the inlet r_1 , obtained using the three approximation methods, are shown in Figure 2.6. All methods predict a similar general trend, with the maximum velocity occurring in the center of the channel. However, as highlighted in the zoomed-in view in Figure 2.6b, the creeping flow solution deviates notably from the results of the

numerical integration and perturbation method at full density ($\epsilon = 1$). Therefore, the creeping flow assumption was deemed to be insufficiently accurate for this case, and the nonlinear inertia term must be considered to obtain reliable results.

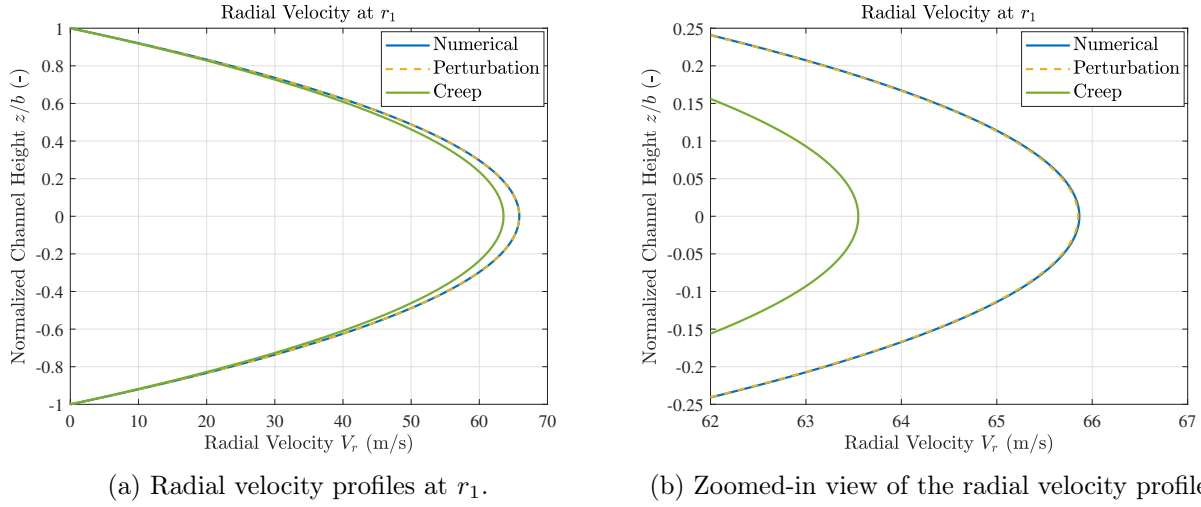


Figure 2.6: Radial velocity profiles obtained using the perturbation method, creeping flow assumption, and numerical integration.

Figure 2.7 shows the evolution of the velocity in the center of the channel over the radius of the axisymmetric domain. Due to the inverse proportionality of the radial velocity with respect to the radius, found using the continuity equation (Equation 2.7), the velocity decreases with increasing radius. However, since the velocity is also inversely proportional to the decreasing air density function, the velocity drop is reduced.

The centerline radial velocity is at its highest at $r = r_1$ and at its lowest at $r = r_2$. Using the perturbation solution and the numerical solution, the minimum and maximum velocities within the domain were evaluated for this particular set of parameters:

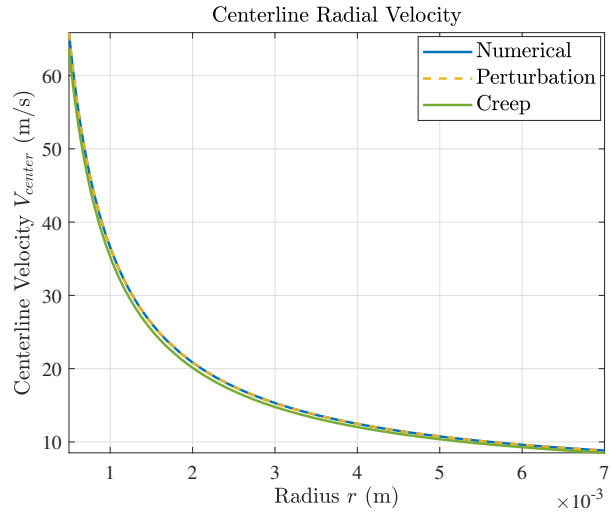


Figure 2.7: Radial centerline velocity over the radius of the axisymmetric domain.

$$V_r(r_1, 0) = 65.9 \text{ m/s} \quad V_r(r_2, 0) = 8.8 \text{ m/s} \quad (2.32)$$

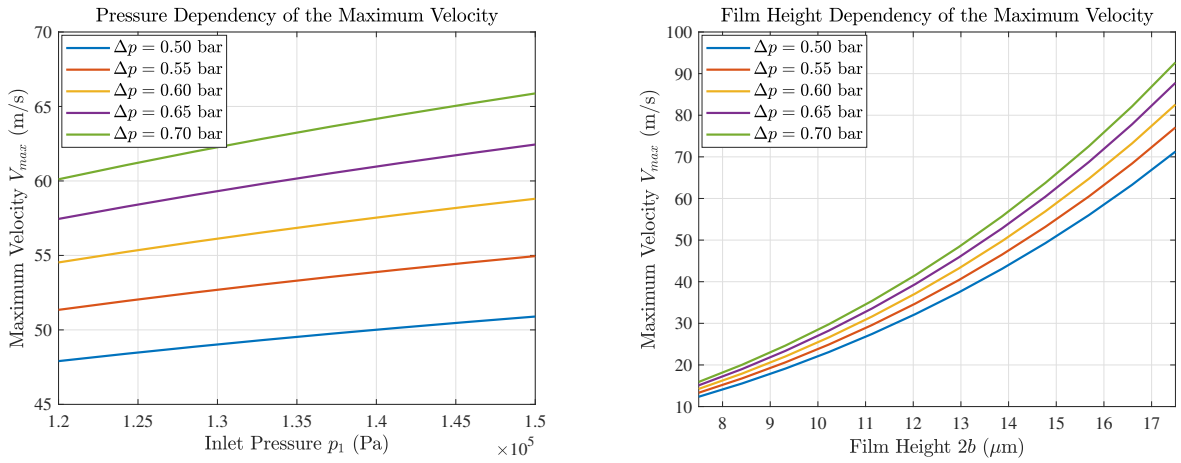
To gain further insight into the behaviour of the radial velocity profile, the maximum radial velocity, as determined by the perturbation method, was plotted for varying inlet pressures, pressure differences, and film heights in Figure 2.8.

Figure 2.8a illustrates that both the inlet pressure, p_1 , and the pressure difference, $\Delta p = p_1 - p_2$, significantly affect the maximum velocity. However, the impact of the film height, $2b$, is even more pronounced, as shown in Figure 2.8b. This can be explained by examining the definition of the maximum velocity for the creeping flow velocity profile:

$$V_{max,creep} = V_{r,creep}(r_1, 0) = \frac{b^2(p_1^2 - p_2^2)}{4\mu r_1 \ln\left(\frac{r_2}{r_1}\right)p_1} \quad (2.33)$$

Assuming fixed inlet and outlet radii, the equation above can be rewritten to derive the proportionality presented in Equation 2.34. While the inertia term is not included in this expression, it still predicts the behaviour of the more accurate perturbation solution reasonably well. As shown, the maximum velocity increases quadratically with film height, explaining the rapid rise in velocity with increasing b .

$$V_{max,creep} \propto b^2 \Delta p \left(2 - \frac{\Delta p}{p_1}\right) \quad (2.34)$$



(a) Pressure dependency of the maximum velocity for $b = 7.5 \mu\text{m}$. (b) Film height dependency of the maximum velocity for $p_1 = 1.5 \text{ bar}$.

Figure 2.8: Maximum radial velocity for varying system parameters using the perturbation solution.

2.2.2 Temperature Profiles

For the axisymmetric geometry consisting of two parallel disks with purely radial flow between them, the thermal energy equation presented in Equation B.18 simplifies to Equation 2.35. The inclusion of temperature derivatives with respect to both the radius and the channel height complicates the derivation of an analytical solution. Furthermore, simplifications applicable to the rectilinear flow analysis, such as $\frac{\partial T}{\partial x} = 0$ for isothermal boundaries and $T(x, y) = f(x) + g(y)$ for adiabatic boundaries, cannot be applied in the axisymmetric case. Consequently, a numerical approach was employed to obtain the desired temperature profiles.

$$\rho c_p V_r \frac{\partial T}{\partial r} = k \frac{\partial^2 T}{\partial z^2} + V_r \frac{dp}{dr} + \mu \left(\frac{\partial V_r}{\partial z} \right)^2 \quad (2.35)$$

The finite difference method was chosen as the most suitable numerical approach for this application. This method involves approximating first- and second-order derivatives with discrete equations, which are then applied to a mesh grid. To begin with, a mesh grid is defined to discretize the thin film domain between the parallel disks. A schematic of this mesh grid is shown in Figure 2.9. As illustrated, both the radius and the channel height are divided into small segments, Δr and Δz , respectively:

$$\Delta r = \frac{r_2 - r_1}{N_r - 1} \quad \Delta z = \frac{2b}{N_z - 1} \quad (2.36)$$

Here, N_r and N_z represent the number of grid points in the radial and channel height directions, respectively. Given that the channel height is much smaller than the radius, numerical instability may arise if the grid is not properly chosen. Therefore, it is crucial to select an appropriate number of grid points, which will be addressed later in this section. The thermal boundary conditions can be incorporated into the mesh grid by assigning the desired values to the boundary nodes. Additionally, the temperature profile of the inlet air can be specified by setting the initial values for the first column of the mesh, from $(1,1)$ to $(1,N_z)$.

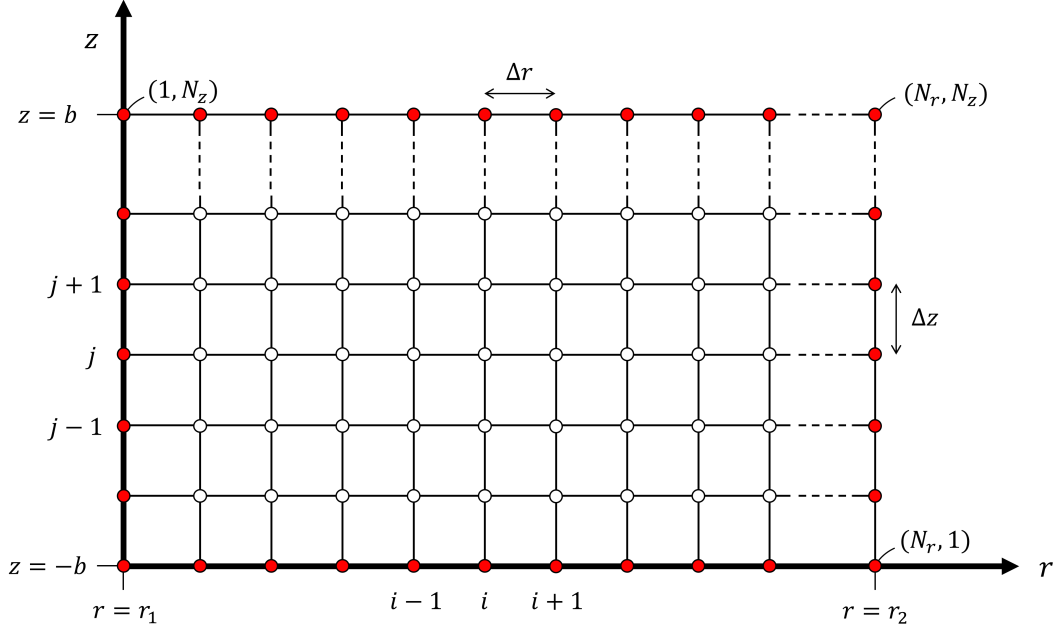


Figure 2.9: Schematic representation of the mesh grid used for the finite difference method.

Since the inlet air conditions are assumed to be known, the most logical approach is to apply an upwind finite difference scheme. This approach estimates derivatives using data points located upstream of the point of interest. The mesh grid is then filled column by column, following the flow direction.

For the first-order temperature derivative with respect to the radius, the forward finite difference method is used. Alternative approaches, such as the backward or central difference methods, are not as suitable in this context, as they require either information from downstream points that have yet to be computed or a predefined outlet temperature profile. For an arbitrary function $f(x)$, the forward difference equation is defined as [11]:

$$\left. \frac{\partial f}{\partial x} \right|_{x_i} = \frac{f_{i+1} - f_i}{\Delta x} \quad (2.37)$$

To approximate the second-order temperature derivative with respect to the channel height, the commonly used second-order central difference equation is applied. For an arbitrary function $f(x)$, the second-order central difference equation is given by [11]:

$$\left. \frac{\partial^2 f}{\partial x^2} \right|_{x_i} = \frac{f_{i+1} - 2f_i + f_{i-1}}{\Delta x^2} \quad (2.38)$$

The numerical errors introduced by this approach depend on the grid resolution Δr and Δz . The first-order forward difference scheme for the radial temperature derivative is associated with a truncation error of order $O(\Delta r)$, meaning the error decreases linearly as the grid resolution improves. On the other hand, the second-order central difference scheme for the temperature derivative with

respect to the channel height has a truncation error of order $O(\Delta z^2)$, which provides a more accurate approximation as the mesh is refined.

When applying the finite difference equations stated above to $T(r, z)$ in the thermal energy equation presented in Equation 2.35, the following equation is obtained:

$$\rho c_p V_r \Big|_{r_i, z_j} \frac{T_{i+1,j} - T_{i,j}}{\Delta r} = V_r \frac{dp}{dr} \Big|_{r_i, z_j} + k \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta z^2} + \mu \left(\frac{\partial V_r}{\partial z} \Big|_{r_i, z_j} \right)^2 \quad (2.39)$$

This equation can be rearranged to isolate the $T_{i+1,j}$ term:

$$T_{i+1,j} = T_{i,j} + \frac{\Delta r}{\rho c_p V_r \Big|_{r_i, z_j}} \left(V_r \frac{dp}{dr} \Big|_{r_i, z_j} + k \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta z^2} + \mu \left(\frac{\partial V_r}{\partial z} \Big|_{r_i, z_j} \right)^2 \right) \quad (2.40)$$

To optimize computation time, the equation above was vectorized, enabling the evaluation of an entire column at once instead of calculating each point individually. The implementation details of the finite difference method in MATLAB can be found in Appendix H.3. For the radial velocity profile, $V_r(r, z)$, the solution derived from the perturbation method was used. This solution not only provides sufficient accuracy but also benefits from being analytically derived, which means a differentiable formula is readily available for use. In contrast, the numerical solution for the velocity profile requires an array of values, making it necessary to apply more complex differentiation techniques, such as the finite difference method. The radial pressure derivative, $\frac{dp}{dr}$, was eliminated from the equation by substituting it with the simplified form of the Navier-Stokes equation as presented in Equation 2.9a.

The upwind finite difference scheme applied here is a non-iterative and steady-state method, meaning it does not introduce time-dependency or related stability concerns. Nonetheless, the mesh resolution influences both the stability and accuracy of the solution. As the finite difference approximation introduces local discretization errors at each grid point, these errors can accumulate progressively. If the mesh is too coarse, this accumulation may lead to numerical instability, potentially resulting in non-physical, diverging values.

Given that the film height is much smaller than the radial extent of the geometry, a significantly finer mesh is required in the radial direction. Additionally, since the accumulated error scales linearly with the radial direction but quadratically in the height direction, the radial grid spacing must be chosen finer to preserve accuracy. However, increasing grid resolution raises computational costs, resulting in a trade-off between accuracy and computational efficiency.

The parameters used for the thermal analysis in MATLAB are presented in Table 2.1 and Table 2.2. In this analysis, T_{s1} and T_{s2} represent the isothermal boundary temperatures. To account for both similar and dissimilar isothermal boundary conditions, two values were used for T_{s2} . Additionally, T_{air} denotes the air temperature at the inlet of the geometry. For simplicity, the air temperature was assumed to be uniform across the film height at the inlet.

Table 2.2: Thermal parameters used in MATLAB for the radial flow analysis.

Parameter	Value	Unit
c_p	1005	J/kg K
k	0.0261	W/m K
T_{s1}	293	K
T_{s2}	293/293.5	K
T_{air}	293	K

Isothermal Boundaries

For the case with two isothermal boundaries, the initial values of the bottom and top rows of the mesh grid were set to T_{s1} and T_{s2} , respectively. Additionally, the values in the first column of the

mesh grid, between the two outer rows, were set to T_{air} . This setup resulted in the initial mesh configuration shown in [Figure 2.10](#).

$$j \begin{pmatrix} T_{s2} & T_{s2} & \cdots & T_{s2} \\ T_{air} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ T_{air} & 0 & \cdots & 0 \\ T_{s1} & T_{s1} & \cdots & T_{s1} \end{pmatrix} \quad N_z \times N_r$$

Figure 2.10: Initial mesh used for the case of two isothermal boundaries.

Centerline Temperature

Two types of simulations were conducted using this mesh. The first simulation focused on evaluating the air temperature along the centerline of the channel across the entire radius of the parallel disks (r_1 - r_2) with two isothermal boundaries at the same temperature. Since only the centerline temperature is of interest, a smaller number of grid points in the z -direction was sufficient. However, the accuracy of the centerline temperature is slightly affected by the number of grid points used, meaning the grid cannot be arbitrarily set to the minimum of three points (including the boundaries). After experimenting with a smaller subdomain of the geometry, it was determined that using $N_z = 11$ grid points yielded reasonable accuracy without resulting in excessively long simulation times. First, the resolution in the z -direction can be calculated by applying [Equation 2.36](#):

$$\Delta z = \frac{2b}{N_z - 1} = \frac{2 \cdot 7.5 \times 10^{-6}}{11 - 1} = 1.50 \mu\text{m} \quad (2.41)$$

Through trial and error, an appropriate number of grid points in the radial direction was determined to be 30000. Using fewer grid points resulted in unrealistic temperature values at larger radii, while increasing the number beyond this did not yield significant improvements in accuracy. The corresponding resolution in the radial direction is:

$$\Delta r = \frac{r_2 - r_1}{N_r - 1} = \frac{7 \times 10^{-3} - 0.5 \times 10^{-3}}{30000 - 1} = 0.217 \mu\text{m} \quad (2.42)$$

For the resulting 11×30000 mesh grid, the total simulation time was approximately ten minutes. The majority of this time was spent on generating the matrices containing the densities, radial velocities, pressure gradients, and velocity gradients across the domain. To further reduce computation time, the symmetry of these matrices about the $z = 0$ axis was exploited. By computing only one half of each matrix and mirroring the results across $z = 0$, the number of required calculations was effectively halved.

The completed mesh was subsequently converted to Cartesian coordinates and visualized using a surface plot, shown in [Figure 2.11](#). The corresponding side and top views are provided in [Figure 2.12a](#) and [Figure 2.12b](#), respectively. At the center of the channel, the $V_r \frac{dp}{dr}$ term plays a dominant role, with its influence being most noticeable near the inlet of the geometry. Upon entering the channel, the air rapidly cools from 293 to approximately 292.3 K around $r = 0.66$ mm due to the high radial velocity. As the radius increases and the velocity diminishes, the centerline temperature gradually rises toward the isothermal wall temperature, T_s . When examining the surface plot from a side view, the inverse relationship between velocity and radius, as described in [Equation 2.7](#), becomes clearly apparent.

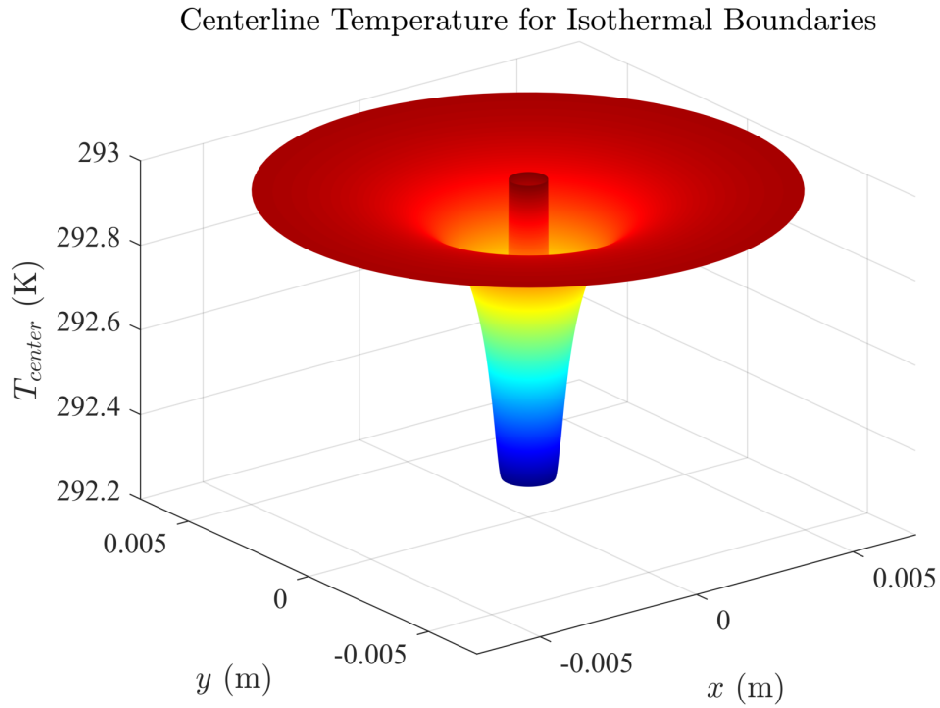


Figure 2.11: Surface plot of the centerline temperature for two isothermal boundaries at 293 K.

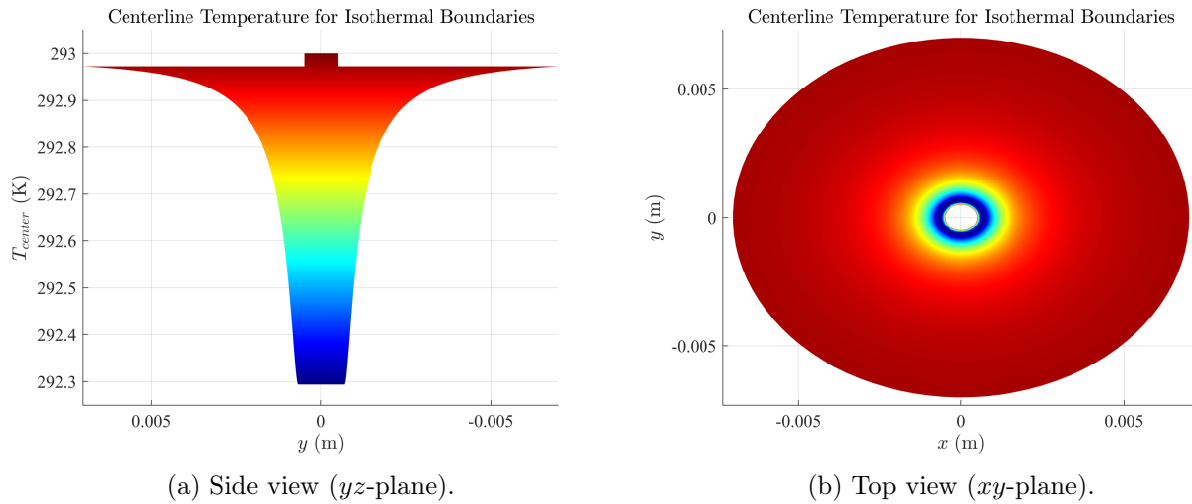


Figure 2.12: Side and top view of the surface plot for two isothermal boundaries at 293 K.

Film Height Temperature

The second simulation focused on determining the air temperature profile along the channel height. To obtain a well-resolved temperature distribution, a sufficiently fine grid was required in the z -direction, while keeping computational time within reasonable limits. After several trial-and-error iterations, an N_z of 41 was found to provide an appropriate balance between accuracy and computational efficiency, resulting in the following step size along the channel height:

$$\Delta z = \frac{2b}{N_z - 1} = \frac{2 \cdot 7.5 \times 10^{-6}}{41 - 1} = 0.375 \mu\text{m} \quad (2.43)$$

To reduce computation time, the simulation was performed on a smaller subdomain of the geometry. However, to ensure a fully developed temperature profile, the subdomain had to be sufficiently large

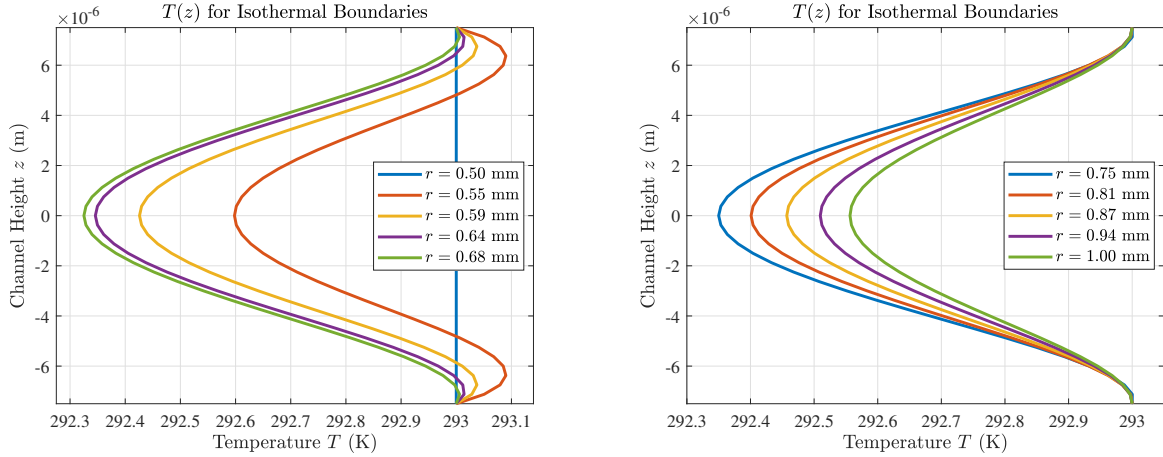
to minimize the influence of the uniform inlet condition. A domain size of $r_{2,sub} - r_1 = 0.5$ mm was found to be adequate.

A corresponding grid size of 20000 points in the radial direction yielded sufficiently accurate results, resulting in the following radial resolution:

$$\Delta r = \frac{r_{2,sub} - r_1}{N_r - 1} = \frac{1 \times 10^{-3} - 0.5 \times 10^{-3}}{20000 - 1} = 0.0250 \text{ } \mu\text{m} \quad (2.44)$$

The total simulation time for the 41×20000 mesh was approximately twenty-five minutes. The resulting mesh was used to generate temperature profiles at various radial positions within the domain, for the case of two isothermal boundaries set to the same temperature ($T_{s1} = T_{s2} = 293$ K). Figure 2.13a presents the temperature profiles in the region between the inlet and the location of the minimum centerline temperature, i.e., $0.5 \leq r \leq 0.68$ mm. As shown, the profiles are not fully developed in this region due to the influence of the uniform inlet temperature. In addition, the profiles are not perpendicular to the boundaries, indicating heat transfer from the airflow to the walls.

For the region beyond the minimum centerline temperature up to the outer radius of the subdomain ($0.68 < r \leq 1$ mm), the temperature profiles are shown in Figure 2.13b. Comparing these profiles to those obtained in the rectilinear flow analysis (Figure 2.2a) reveals a clear resemblance. Due to the high velocity in the center of the channel, the pressure derivative term $V_r \frac{dp}{dr}$ is dominant, causing a significant temperature drop. As the radius increases, the velocity decreases due to the expanding air film, leading to a subsequent rise in the centerline temperature. However, due to limitations in grid resolution imposed by computational constraints, the resolution of the temperature profiles is insufficient to confirm whether the temperature gradients become fully perpendicular to the boundaries. As a result, it cannot be definitively concluded whether the boundaries behave as both adiabatic and isothermal beyond the temperature minimum, leaving the presence of dissipation uncertain.



(a) Isothermal boundaries at $0.5 \leq r \leq 0.68$ mm.

(b) Isothermal boundaries at $0.68 < r \leq 1$ mm.

Figure 2.13: Temperature profiles over the film height at several radii for two isothermal boundaries at the same temperature.

Using the same mesh size, the simulation was conducted for two dissimilar isothermal boundary temperatures: $T_{s1} = 293$ K and $T_{s2} = 293.5$ K. The resulting temperature profiles at similar radial coordinates as before are shown in Figure 2.14. In this case, the evolution of the centerline temperature differs from the scenario with two identical isothermal boundaries, with the location of the lowest centerline temperature shifting to $r = 0.65$ mm. Prior to the temperature minimum (Figure 2.14a), the temperature gradients at both boundaries point in the same direction, indicating energy dissipation from the airflow to the boundaries. Beyond the temperature minimum (Figure 2.14b), the direction of the temperature gradient differs between the two boundaries, suggesting a possible reduction in dissipation, potentially reaching zero across the film height.

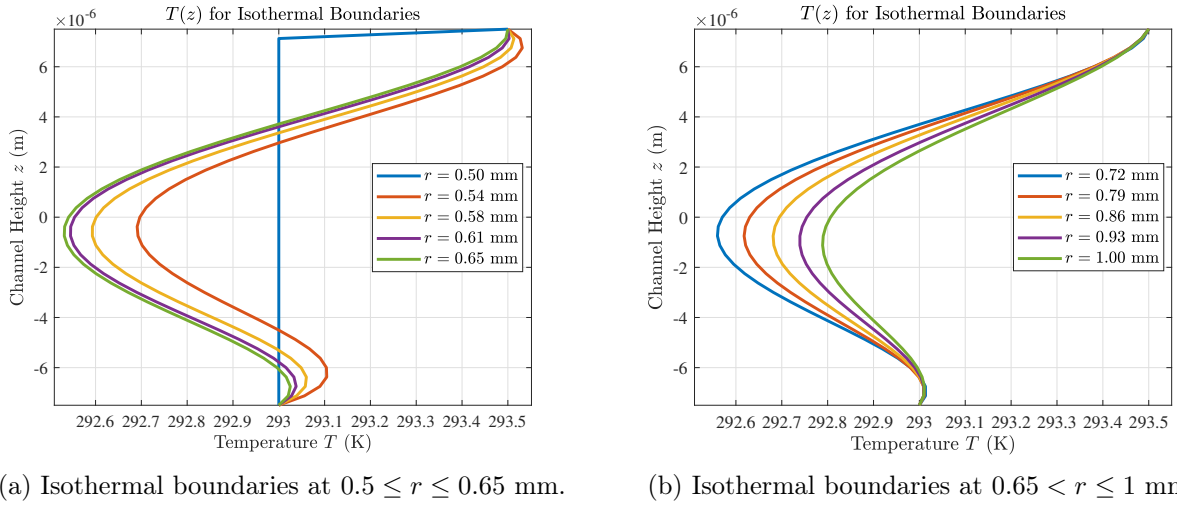


Figure 2.14: Temperature profiles over the film height at several radii for two isothermal boundaries at different temperatures.

Adiabatic Boundaries

For adiabatic boundaries, implementing the boundary conditions is more complex than for isothermal boundaries. Since the temperature at the boundaries is not fixed and varies across the radius, the initial mesh used for isothermal conditions is no longer adequate. Instead, the temperature derivative at both boundaries should be zero, meaning the temperature values in the top two and bottom two rows should match. However, in reality, this condition only holds for points infinitely close to the adiabatic boundaries. Consequently, to obtain accurate results using the finite difference method, a large number of grid points in the height direction is required. Since the number of grid points in the height direction should be significantly increased compared to the mesh used for the isothermal analysis, the computational effort quickly becomes immense.

Several attempts were made to achieve reliable results using different mesh configurations. It was found that the number of grid points in the height direction significantly influenced the simulation outcome: for a low number of grid points, the adiabatic wall temperature shows more radical fluctuations. This can be explained by the fact that the temperature variations are larger near the center of the channel. By dividing the channel height into fewer steps, the finite difference method takes grid points closer to the channel center into account for the calculation of the wall temperature, yielding larger temperature fluctuations. In order to obtain more realistic results, the amount of grid points would have to be increased even further, which is not feasible due to computational limitations. Additionally, further reducing the radial domain was not an option, as the flow was not yet thermally developed. As a result, the adiabatic boundary analysis was deferred to COMSOL, where a more refined approach could be implemented.

Conclusion

Finite Difference Method

Even though the resolution and accuracy of the finite difference method in this section are limited by the available computational power, it proved to be a very useful method for obtaining a general understanding of the evolution of the temperature profile over the radius of the axisymmetric geometry. Additionally, the method allowed for an analysis of the contribution of individual thermal energy equation terms, providing further insights into the flow dynamics.

To enhance the performance of the algorithm, mesh refinement near the boundaries of the domain could be implemented, while using a coarser mesh for the rest of the domain. This approach would significantly improve local resolution and overall accuracy, particularly for the adiabatic boundary

analysis. However, implementing such a procedure would add complexity, defeating the purpose of this analysis.

For simplicity, the airflow at the inlet was assumed to have a fully developed velocity profile with a uniform temperature. This led to a mismatch between the velocity and temperature profiles at the inlet. Given that the entrance length for laminar thin film flow is typically very short, the impact of this assumption is expected to be minimal. Ultimately, the COMSOL analysis presented in [Chapter 3](#) will provide more accurate results and facilitate the implementation of mesh refinements near the boundaries.

Energy Dissipation

The energy dissipation in the thin film can be determined by assessing the integral of the pressure derivative term and the viscous dissipation term over the parallel disks domain. For the rectilinear flow analysis, it was sufficient to simply take the integral over the channel height, since no flow variations were assumed along the length of the plates. However, in the axisymmetric analysis, the velocity profile in the thin film varies both along the channel height and the radius. Therefore, to assess the total dissipation in the domain, the following triple integral in cylindrical coordinates must be used:

$$\dot{Q}_{tot} = \int_0^{2\pi} \int_{r_1}^{r_2} \int_{-b}^b V_r \frac{dp}{dr} + \mu \left(\frac{\partial V_r}{\partial z} \right)^2 r dz dr d\theta \quad (2.45)$$

The pressure derivative term can be substituted with [Equation 2.9a](#) to obtain the dissipation integral stated below:

$$\dot{Q}_{tot} = \int_0^{2\pi} \int_{r_1}^{r_2} \int_{-b}^b \mu V_r \frac{\partial^2 V_r}{\partial z^2} - \rho V_r^2 \frac{\partial V_r}{\partial r} + \mu \left(\frac{\partial V_r}{\partial z} \right)^2 r dz dr d\theta \quad (2.46)$$

Next, [Equation 2.7](#) is used to eliminate V_r :

$$\dot{Q}_{tot} = \int_0^{2\pi} \int_{r_1}^{r_2} \int_{-b}^b \frac{\mu}{(\rho(r)r)^2} f(z) f''(z) + \frac{\rho'(r)r + \rho(r)}{\rho(r)^3 r^4} f(z)^3 + \frac{\mu}{(\rho(r)r)^2} f'(z)^2 r dz dr d\theta \quad (2.47)$$

I) Creeping Flow

Under the creeping flow assumption, the inertia terms in the Navier-Stokes equations can be neglected, leaving the following dissipation integral:

$$\dot{Q}_{tot,creep} = \int_0^{2\pi} \int_{r_1}^{r_2} \int_{-b}^b \frac{\mu}{(\rho(r)r)^2} f(z) f''(z) + \frac{\mu}{(\rho(r)r)^2} f'(z)^2 r dz dr d\theta \quad (2.48)$$

Using the previously determined creeping flow velocity profile, presented in [Equation 2.20](#), the integral can be evaluated:

$$\frac{\dot{Q}_{creep}}{A} = \frac{b^2(p_1^2 - p_2^2)^2}{8\mu r^2 p(r)^2 \ln\left(\frac{r_2}{r_1}\right)^2} \int_{-b}^b \left(\frac{3z^2}{b^2} - 1 \right) dz = 0 \quad (2.49)$$

Similar to the rectilinear flow analysis, no energy dissipation occurs in the thin film for the axisymmetric geometry when assuming creeping Poiseuille flow. However, especially near the inlet of the domain, where the velocity is highest, the creeping flow assumption lacks accuracy. Therefore, the velocity profile obtained through the perturbation method will be used next to provide a more realistic estimation of the energy dissipation.

II) Perturbation Method

Since the velocity profile equation obtained through the perturbation method is significantly more complex than the creeping flow solution, the energy dissipation was instead evaluated using MATLAB. The energy dissipation was found to be entirely attributed to the third-order inertia term in

the dissipation integral shown in Equation 2.47. The other two terms still cancel each other out over the film height. To gain further insight into the energy dissipation along radius of the disk, the third-order term was integrated over the film height:

$$\frac{\dot{Q}_{pert}}{A} = \frac{\rho'(r)r + \rho(r)}{\rho(r)^3 r^4} \int_{-b}^b f(z)^3 dz \quad (2.50)$$

The integral mentioned above was plotted across the axisymmetric domain to visualize the local energy dissipation per unit area (Figure 2.15). The corresponding side and top views of the surface plot are presented in Figure 2.16a and Figure 2.16b, respectively. Due to the radius raised to the fourth power in the denominator, the energy dissipation decreases rapidly with increasing radius. Furthermore, the magnitude of $f(z)$ determines by how much the dissipation graph is scaled. A larger $f(z)$ results in a higher local energy dissipation. At the inlet of the domain, the dissipation per unit area reaches its peak value of 6012 W/m². Just 1 mm further into the domain, at $r = 1.5$ mm, the dissipation per unit area has already dropped significantly to 98.7 W/m².

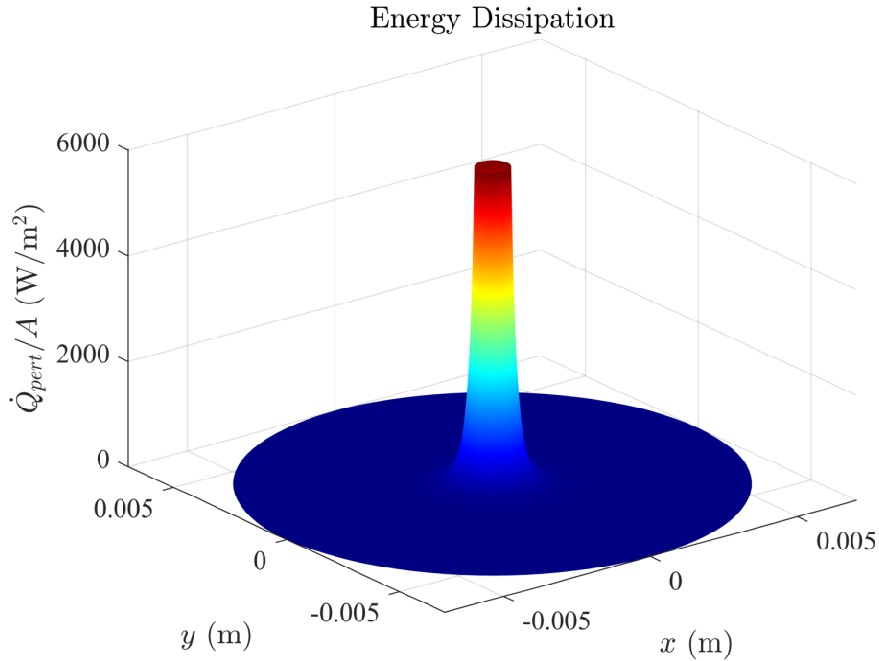


Figure 2.15: Surface plot of the local dissipation across the axisymmetric domain for the perturbation solution.

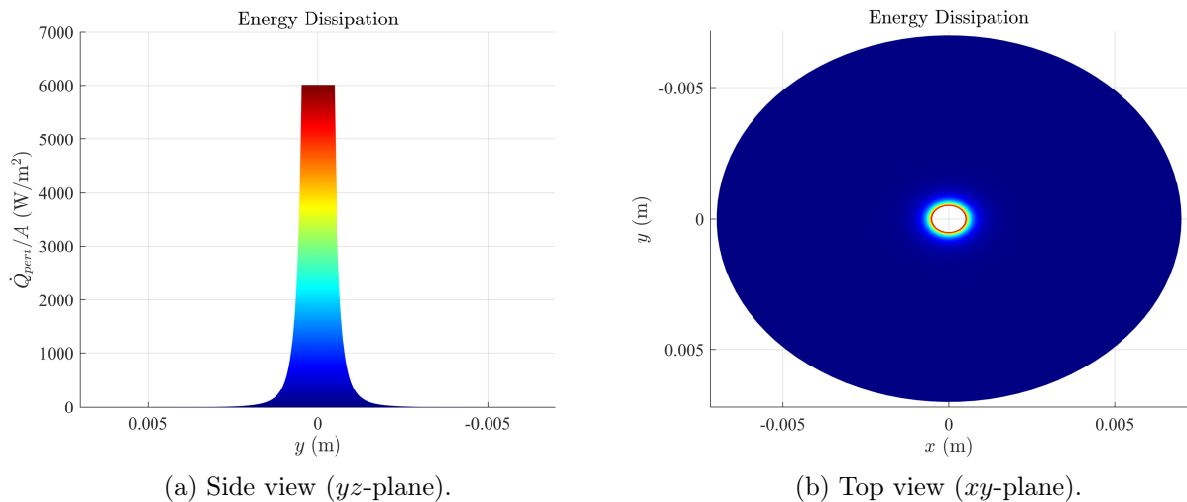


Figure 2.16: Side and top view of the local energy dissipation for the perturbation solution.

To determine the total dissipation in the axisymmetric domain, the surface plot should be integrated over the total area, as shown in Equation 2.47. Since only the nonlinear term contributes to the energy dissipation, the total dissipation integral simplifies to:

$$\dot{Q}_{tot,pert} = \int_0^{2\pi} \int_{r_1}^{r_2} \frac{\rho'(r)r + \rho(r)}{(\rho(r)r)^3} \int_{-b}^b f(z)^3 dz dr d\theta \quad (2.51)$$

This integral was evaluated in MATLAB, yielding a total dissipation of 5.36 mW. Notably, for this set of parameters, approximately 86% of the total dissipation originates from the first millimeter from the inlet, indicating significant local heating of the connected surfaces in that region. These results are consistent with the temperature profiles shown in Figure 2.13a, where a pronounced temperature gradient was observed at the flow boundaries near the inlet.

Similar to the maximum radial velocity, the total energy dissipation was plotted for varying inlet pressures, pressure differences, and film heights (Figure 2.17). The total dissipation is particularly sensitive to the film height, which can be attributed to the presence of the third-order $f(z)$ term in the total dissipation integral. Since the $f(z)$ term depends on the film height to at least the second order for the perturbation method, the total dissipation shows a dependence on the film height up to at least the seventh order after integration.

The pressure dependence of the total dissipation exhibits a less pronounced sensitivity. As the inlet pressure increases, the total dissipation rises approximately linearly. The magnitude of the pressure difference between the inlet and outlet determines the slope of this linear relationship. As the pressure difference increases, the total dissipation also increases.

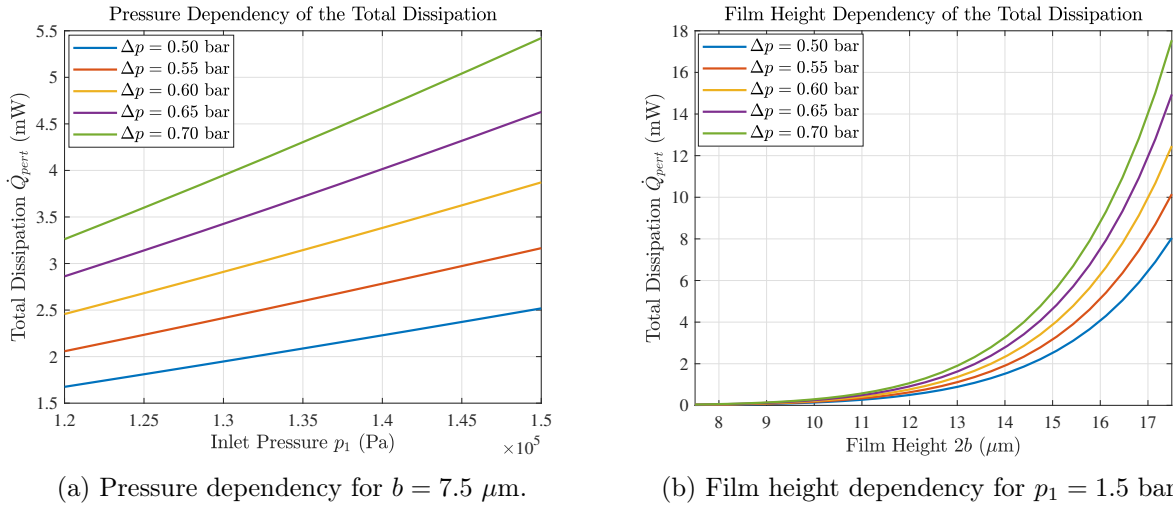


Figure 2.17: Total energy dissipation for varying system parameters for the perturbation solution.

3

COMSOL Modelling

In the previous chapter, a detailed theoretical analysis was conducted on the thin air film between the air bearing system and the wafer. Although the analytical, semi-analytical, and numerical solutions provided valuable insights into the thermal behaviour of the thin air film, extending these models to include the vertical inlet and outlet, as well as the complete air bearing system and wafer, proved too complex and computationally inefficient.

Therefore, the modelling was continued using COMSOL Multiphysics 6.2. The modelling decisions are first outlined in [Section 3.1](#). The model is then incrementally developed, starting with a simple axisymmetric air layer and progressing toward the complete system geometry. Based on these models, the key factors influencing the wafer's temperature distribution are further analyzed in [Appendix C](#). Finally, the conclusions of this analysis, along with the selected parameters for the experimental setup, are presented in [Section 3.4](#).

3.1 Modelling Decisions

Since each of the air-based contactless handling systems outlined in [Appendix A](#) operates with distinct principles and geometries, a key step in the modelling process is to define which geometric features are incorporated.

To develop a general solution applicable to centrally fed radial flow air bearing systems (such as Generations 2, 3, and 4), an axisymmetric simplification is employed throughout this chapter. The analysis is further limited to the equilibrium positions of Generations 2, 3, and 4, corresponding to a stationary wafer. Couette flow is thereby not considered. This is justified, as the thermal impact of Couette flow is minor compared to the influence of Poiseuille flow and does not contribute meaningfully to the understanding of the wafer's thermal behaviour. Moreover, since wafer processing occurs under stationary conditions, Couette flow lies outside the scope of this study.

3.1.1 Inlet Restriction & Recess

A necessary component of an air bearing system is the addition of an inlet restriction. Several types of restrictions are commonly used, including orifice, capillary, and porous restrictions. The primary function of these components is to create a pressure drop between the supply pressure and the pressure at the entrance of the thin air film. This pressure drop necessitates a higher supply pressure to achieve the same load-carrying capacity. However, in return, a passively stable system with a positive stiffness is obtained. A schematic illustration of the air bearing inlet, both with and without an inlet restriction, is shown in [Figure 3.1](#).

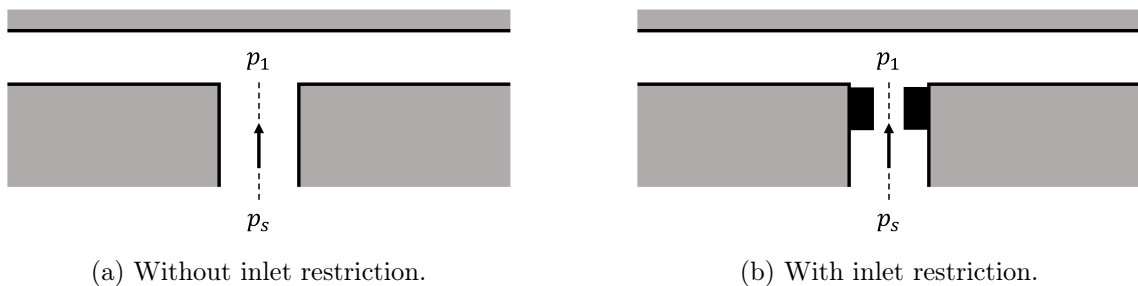


Figure 3.1: Schematic representation of the air bearing inlet with and without an inlet restriction.

In addition to an inlet restriction, the surface of an air bearing is often modified with a rectangular or conical recess, as illustrated in Figure 3.2. The primary advantage of these surface features is an increase in the static load-carrying capacity, allowing the bearing to support higher loads at the same supply pressure, p_s . Furthermore, the use of a conical recess has been shown to enhance the static stiffness of the bearing. However, recessed air bearing surfaces also present certain drawbacks. They require a higher mass flow, leading to increased operational costs, and increase the risk of pneumatic hammering, a phenomenon where pressure fluctuations induce vibrations on the substrate [3].

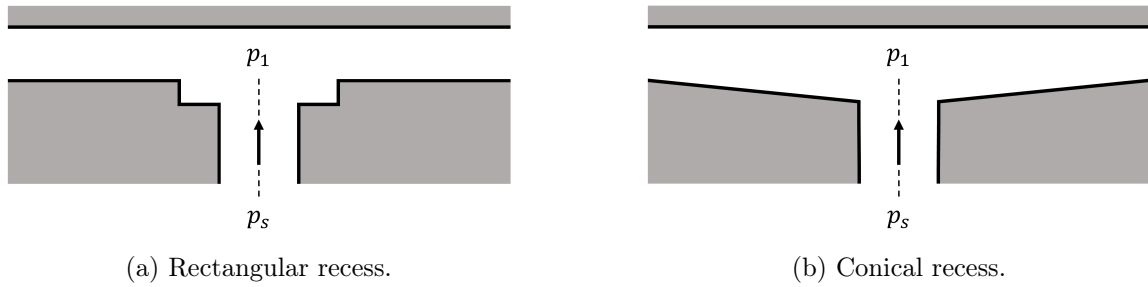


Figure 3.2: Schematic representation of the air bearing surface with two types of recesses.

As this research focuses specifically on the thermomechanical behaviour of air bearing systems, aspects such as bearing stiffness and load-carrying capacity fall outside the scope of the study and were omitted to reduce complexity. Consequently, this chapter investigates the basic inlet configuration shown in Figure 3.1a. However, the impact of design variations, such as inlet restrictions and surface recesses, on the system's thermomechanical response remains relevant for future research.

3.1.2 Time Dependency

All simulation results presented in this chapter were obtained using stationary simulations. While time-dependent simulations can sometimes reach a steady-state solution more efficiently, stationary simulations proved to be more effective for this study. Solving the stationary problem directly allowed for a quicker assessment of the system's equilibrium behaviour without the need to resolve transient dynamics.

However, the system's settling time is important for experimental validation. To address this, an indicative time-dependent simulation was performed to estimate the expected settling time, as discussed in Section 4.2.2.

3.1.3 Thermal Expansion

As noted in Section 1.1, thermal expansion of wafers is a major contributor to overlay errors in wafer processing. In this research, thermal expansion could potentially induce additional wafer deformation, affecting the thin film height. However, based on the theoretical analysis performed in the previous chapter, only sub-Kelvin temperature gradients are expected across small length scales. Combined with the low thermal expansion coefficient of the wafers used in this research, this results in thermal expansions of the wafer on the order of nanometers. This is negligible compared to the pressure-induced deformations, which were previously modelled to be on the order of microns [3]. Therefore, thermal expansion effects were not included in the COMSOL modelling.

3.2 Axisymmetric Air Layer Geometry

3.2.1 Horizontal Inlet

As a starting point for the COMSOL simulations, a simple axisymmetric air layer was investigated. Similar to the axisymmetric flow analysis presented in Section 2.2, the geometry consists of a thin air film with a thickness of $2b$ and a radius ranging from r_1 at the inlet to r_2 at the outlet. Pressurized air with pressure p_1 and uniform temperature T_{air} is supplied at the inlet, while a pressure of p_2 is maintained at the outlet. Finally, T_s represents the isothermal wall temperature.

A schematic representation of the axisymmetric air layer geometry is presented in Figure 3.3, with the corresponding COMSOL parameters listed in Table 3.1. The additional parameters r_3 , t_b , and t_w are relevant for subsequent simulations. The COMSOL parameters were chosen to match those used in the theoretical analysis, facilitating a direct comparison between the two approaches.

Table 3.1: COMSOL parameters used for the axisymmetric air layer simulations.

Parameter	Value	Unit
r_1	0.5	mm
r_2	7	mm
r_3	7.5	mm
p_1	1.5×10^5	Pa
p_2	0.8×10^5	Pa
b	7.5	μm
t_b	1	mm
t_w	525	μm
T_{air}	293	K
T_s	293	K



Figure 3.3: Schematic representation of the axisymmetric air layer geometry.

Using this geometry, a 2D axisymmetric analysis was conducted in COMSOL. For such a simulation, COMSOL requires a 2D geometry input, as described above. The governing flow and heat equations are solved as if the 2D geometry were rotated around the $r = 0$ axis to form a 3D geometry. This approach significantly reduces computational effort while providing results equivalent to a full 3D simulation of the circular domain. The airflow was assumed to be laminar and compressible ($M < 0.3$). The *Laminar Flow* and *Heat Transfer in Fluids* interfaces were coupled using the *Nonisothermal Flow* multiphysics to account for all relevant physics. For the mesh, a mapped mesh with boundary layer refinements was applied to ensure accuracy without excessively long simulation times.

Pressure & Velocity Profiles

The resulting density and pressure profiles are compared with those obtained previously using the creeping flow assumption in Figure 3.4. The COMSOL results align well with the MATLAB results, with the latter showing slightly lower density and pressure values across the domain. Additionally, for finer meshes, the COMSOL results remain consistent. The minor deviation between the COMSOL and MATLAB results can be attributed to the absence of the inertia term in the creeping flow solution. Equation 2.9a can be rewritten to isolate the pressure gradient term:

$$\frac{\partial p}{\partial r} = \mu \frac{\partial^2 V_r}{\partial z^2} - \rho V_r \frac{\partial V_r}{\partial r} \quad (3.1)$$

In this equation, the viscous term is negative, resulting in a negative pressure gradient. However, the inertia term, which is also negative due to the decreasing velocity over the radius, counteracts this effect. Since the COMSOL simulation accounts for inertia, the resulting density and pressure profiles exhibit a less steep gradient compared to the creeping flow solution.

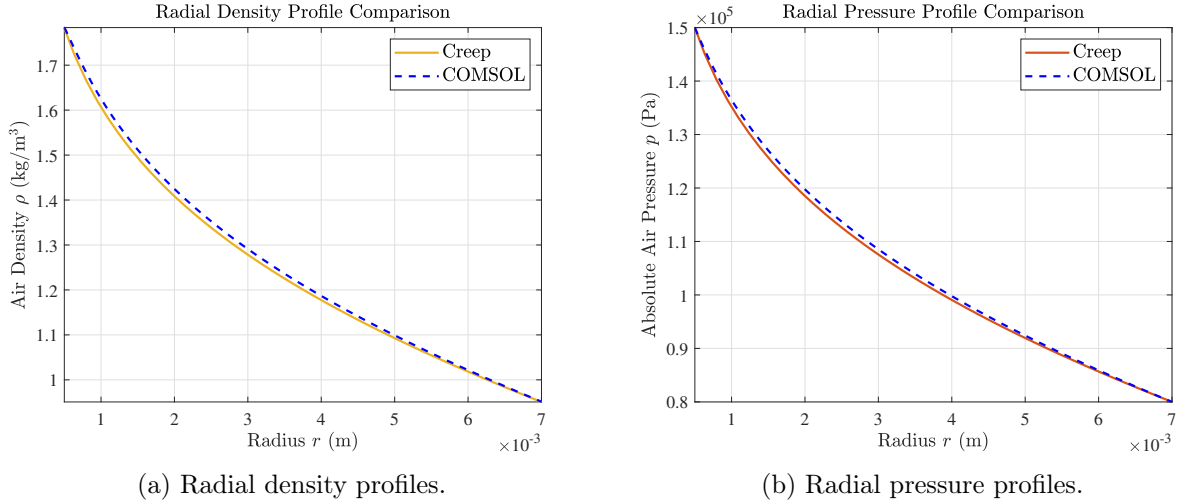


Figure 3.4: Comparison of the radial density and pressure profiles obtained using the creeping flow assumption in MATLAB with those from COMSOL.

The comparison of the radial velocity profiles at r_1 and the evolution of the centerline radial velocity are shown in Figure 3.5a and Figure 3.5b, respectively. Since the perturbation solution and the numerical MATLAB solution were found to be virtually identical, the numerical solution has been omitted from the plots for clarity. The COMSOL simulation produced a slightly higher maximum velocity and a somewhat steeper decline in the centerline velocity compared to the perturbation method. This discrepancy is attributed to differences in the density and pressure profiles. The maximum and minimum centerline radial velocities from COMSOL are as follows:

$$V_r(r_1, 0) = 66.9 \text{ m/s} \quad V_r(r_2, 0) = 8.8 \text{ m/s} \quad (3.2)$$

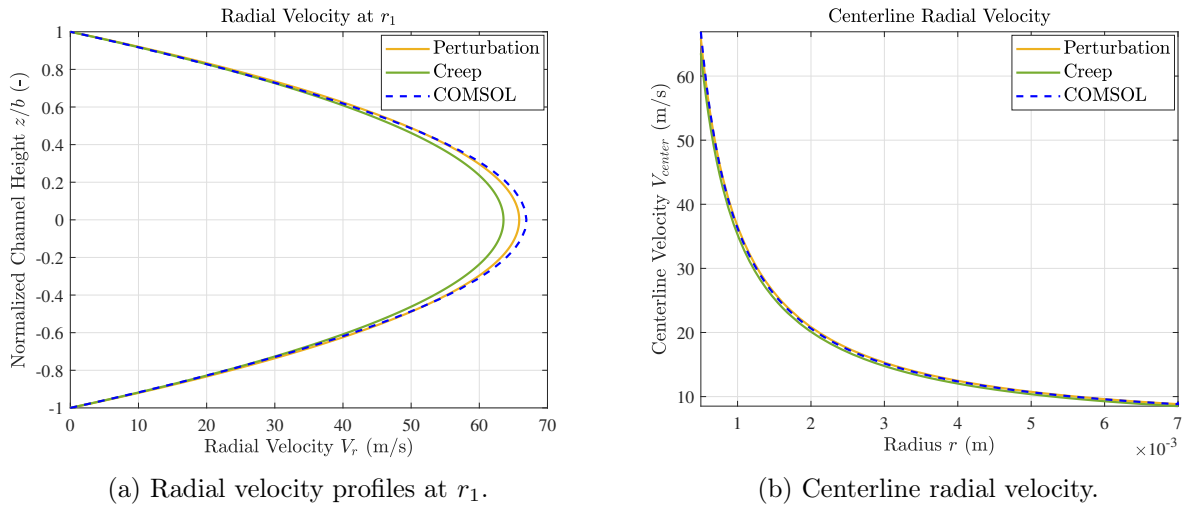


Figure 3.5: Comparison of the radial velocity profiles obtained using the creeping flow assumption and the perturbation method in MATLAB with those from COMSOL.

Temperature Profiles

Isothermal Boundaries

Figure 3.6 compares the temperature profiles obtained using the finite difference method in MATLAB with those from COMSOL, both with isothermal boundaries at 293 K. The centerline temperature over the entire domain is shown in Figure 3.6a, with both profiles exhibiting a minimum at $r = 0.68$ mm. The temperature profile along the film height at this radial coordinate is presented in Figure 3.6b. A discrepancy in the minimum centerline temperatures is observed between the two figures, resulting from the difference in mesh size used for the finite difference plots. Since the

centerline temperature was evaluated across the full domain, fewer grid points were used along the film height compared to the film height temperature profile, leading to lower accuracy for the centerline temperature. In Figure 3.6a, the minimum centerline temperature is 292.29 K using the finite difference method, compared to 292.32 K in the COMSOL result. However, in Figure 3.6b, both methods yield a minimum of 292.32 K, confirming that the finite difference method achieves higher accuracy when a finer mesh is applied.

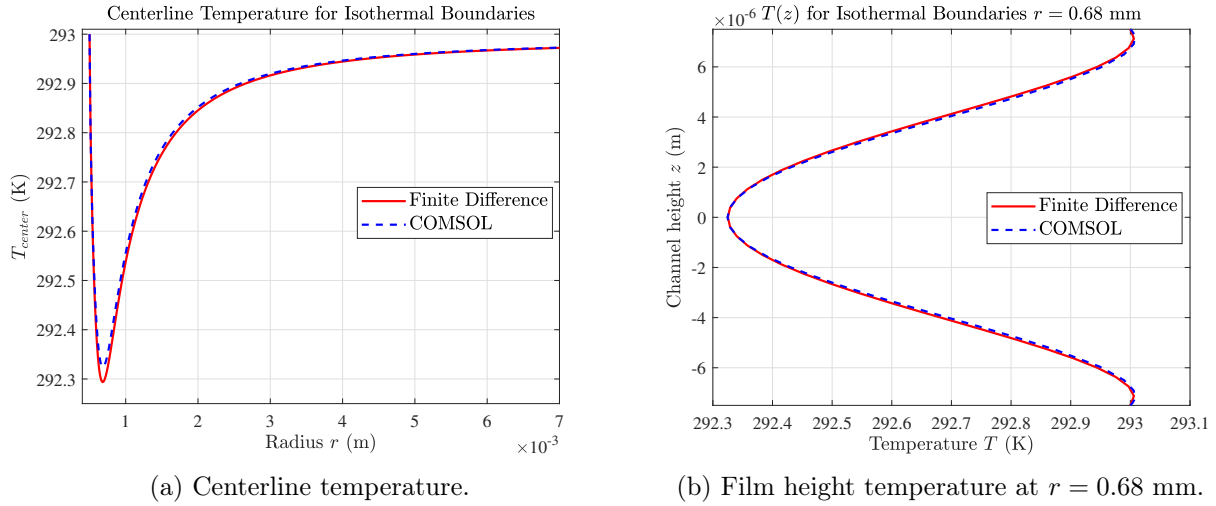


Figure 3.6: Comparison of the temperature profiles obtained using the finite difference method in MATLAB with those from COMSOL for two isothermal boundaries at 293 K.

Adiabatic Boundaries

The simulation was also run for a domain with two adiabatic boundaries, and the resulting temperature profiles are shown in Figure 3.7. Since the finite difference method was not used for this case, only the COMSOL results are presented. In Figure 3.7a, the temperature variations of the walls and the centerline along the radial coordinate are displayed. As expected, the wall temperature rises sharply near the inlet due to significant energy dissipation in the high-velocity region. After this initial increase, the wall temperature stabilizes, indicating minimal dissipation, which aligns with the energy dissipation analysis presented in Section 2.2.2. The centerline temperature initially decreases slightly before approaching the adiabatic wall temperature. The minimum centerline temperature occurs at $r = 0.525$ mm, with the corresponding temperature profile along the film height shown in Figure 3.7b. This profile is similar in shape to the isothermal film height temperature profile, differing mainly in its perpendicularity to the boundaries.

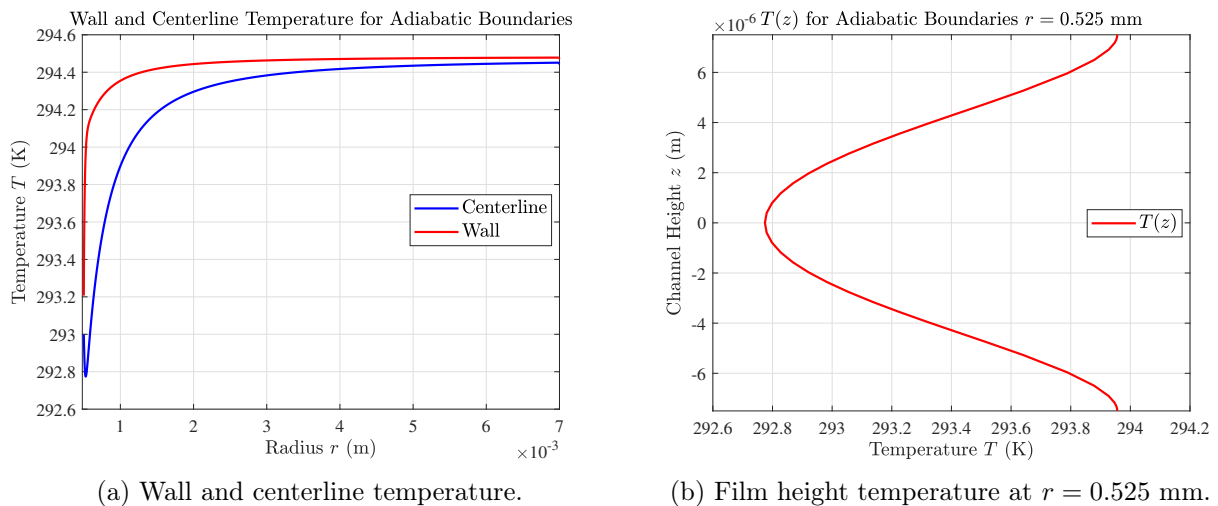


Figure 3.7: Temperature profiles obtained from COMSOL for two adiabatic boundaries.

Another interesting quantity is the difference between the wall temperature and the centerline temperature, $T_{wall} - T_{center}$. This difference is shown in Figure 3.8 for both adiabatic and isothermal boundaries. For adiabatic boundaries, the profile exhibits a sharper drop compared to the isothermal case. Since adiabatic boundaries prevent heat dissipation, all the energy remains within the flow. The dominant viscous effects near the boundaries cause significant heating in the high dissipation region. In the center of the channel, expansion effects prevail, leading to a larger temperature contrast compared to the heated walls.

Isothermal boundaries, in contrast, allow for continuous heat dissipation, maintaining a constant temperature. Despite this, viscous effects still cause a slight increase in temperature near the walls (see Figure 2.13a). However, since the heat generated by these effects is quickly conducted away, the temperature difference between the wall and the centerline remains relatively small. Ultimately, expansion and viscous effects are more localized for adiabatic boundaries, while these effects are more evenly distributed for isothermal boundaries.

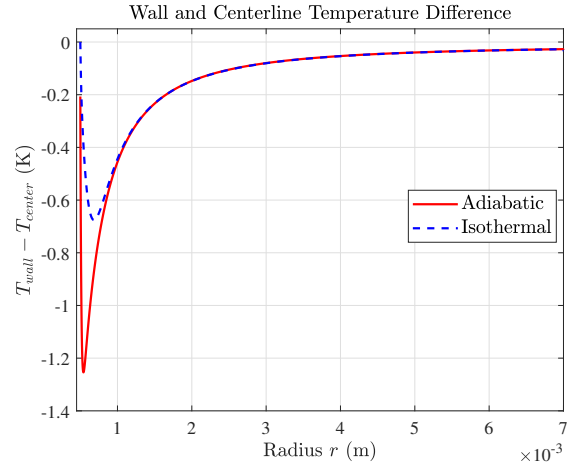


Figure 3.8: Difference between the wall and centerline temperature for adiabatic and isothermal boundaries.

3.2.2 Vertical Inlet

Next, the vertical inlet and outlet of the unit cell were incorporated into the COMSOL geometry. Figure 3.9 shows a schematic representation of the updated geometry, with the corresponding COMSOL parameters presented in Table 3.1. The height of the vertical inlet and outlet is represented by the thickness of the air bearing system t_b . The parameter r_3 denotes the center of the vertical outlet between two unit cells. Given the expected symmetry of the velocity profile around this axis, a *Symmetry* boundary condition was applied. For this geometry, a free triangular mesh was chosen over a mapped mesh to better accommodate corner refinements at the thin film entrance. Apart from the geometry and mesh changes, the rest of the simulation setup remained the same as in the axisymmetric horizontal air layer simulations.

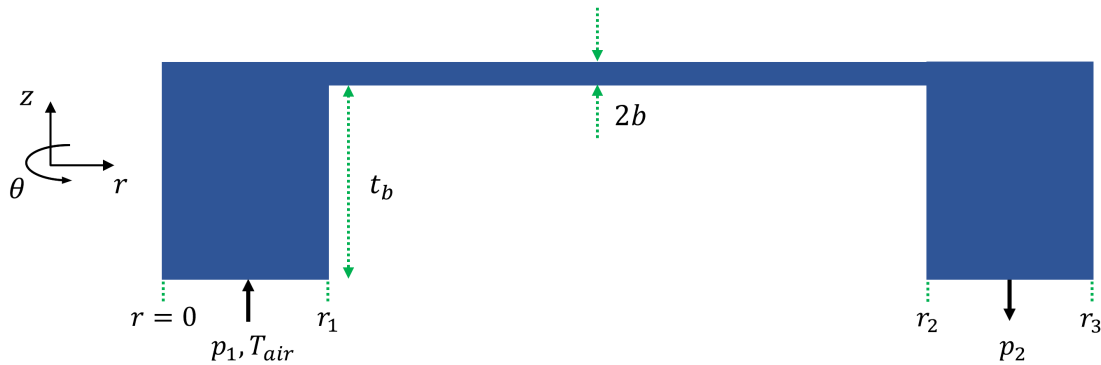


Figure 3.9: Schematic representation of the axisymmetric air layer geometry with a vertical air inlet and outlet.

Pressure & Velocity Profiles

For the air layer geometry with a vertical inlet and outlet, the pressure profile behaves differently from the horizontal inlet geometry. As shown in Figure 3.10, the pressure experiences a sharp drop

at the inlet of the thin film due to the rapid acceleration of the airflow. The isentropic flow relations presented in [Appendix B.1](#) provide an accurate approximation of this pressure drop and will be used in the calculations below.

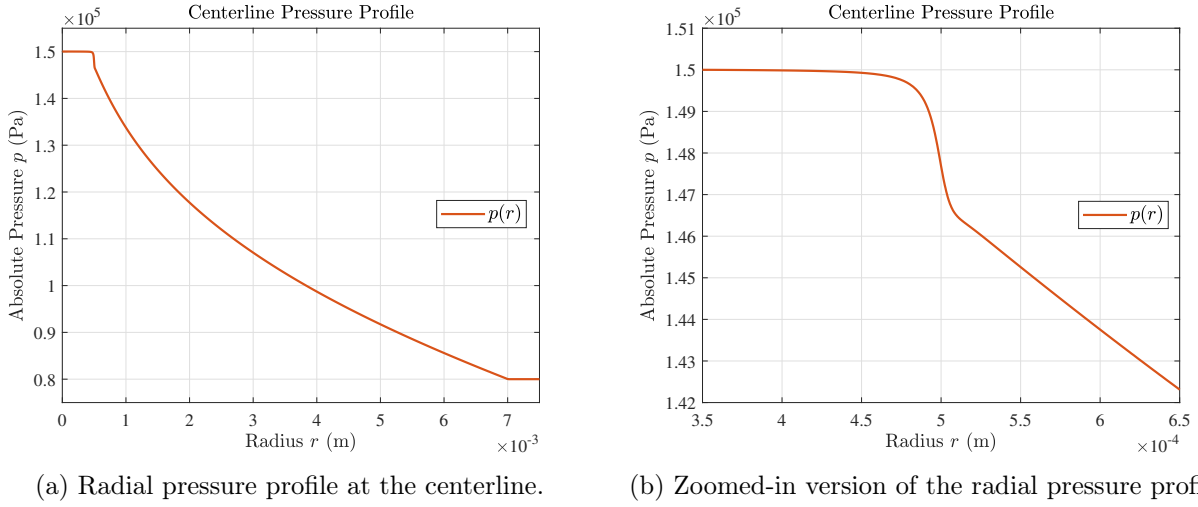


Figure 3.10: Radial pressure profile at the centerline for the air layer geometry with a vertical inlet and outlet.

The calculation of the pressure drop requires both the pressure at the vertical inlet and the centerline velocity at the inlet of the thin film. As shown in [Figure 3.11](#), the velocity profile exhibits the following: at $r = 0$, the velocity is negligible as the flow is almost entirely diverted to the sides. At $r = r_1$, the centerline velocity reaches its maximum of 61.1 m/s due to the sudden change in flow area. Between r_1 and r_2 , the velocity profile resembles that of the horizontal geometry ([Figure 3.5b](#)), with a new minimum of 8.2 m/s at the outer radius of the thin film. Beyond r_2 , the air expands into the large vertical outlet, causing the velocity to decrease further.

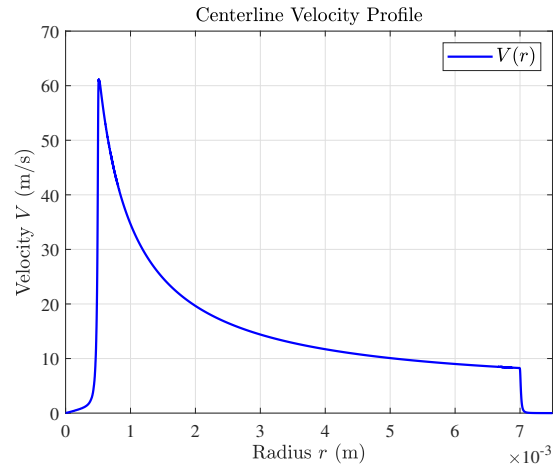


Figure 3.11: Air velocity over the axisymmetric domain at the height of the film centerline.

Using a speed of sound at 293 K of $a = 343$ m/s and a specific heat ratio of $\gamma = 1.4$, the observed pressure drop can be calculated [\[10\]](#):

$$M = \frac{v}{a} = \frac{61.1}{343} = 0.178 \quad (3.3)$$

$$P = P_i \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{-\frac{\gamma}{\gamma - 1}} = 1.5 \times 10^5 \cdot \left(1 + \frac{0.4}{2} \cdot 0.178^2 \right)^{-\frac{1.4}{0.4}} = 1.467 \times 10^5 \text{ Pa} \quad (3.4)$$

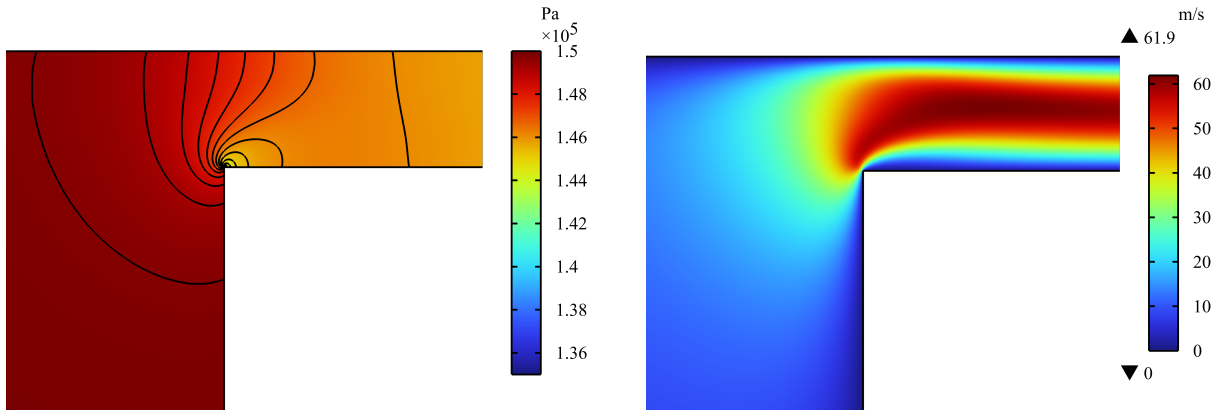
The calculated pressure drop closely matches the observed value in [Figure 3.10a](#), which is 1.465×10^5 Pa. Thus, the isentropic flow relations provide a reasonable approximation of the changes in thermodynamic properties of the air as it enters the thin film. It is important to note, however, that the maximum centerline velocity is also dependent on the pressure drop, indicating an interdependency

between the two properties. This relationship should be carefully considered when selecting parameters for the test setup.

At the entrance of the thin film, the airflow is diverted past the sharp corner of the unit cell. Due to the significant reduction in flow area, both the velocity and pressure distributions undergo local alterations. A colour map of the pressure distribution at the thin film inlet, including isobars, is shown in Figure 3.12a. The colour limits were adjusted to highlight the local pressure differences. Each isobar represents a pressure drop of 0.5 kPa. Just downstream of the sharp edge of the unit cell, the isobars are closely spaced at the bottom of the thin film, where a minimum absolute pressure of approximately 1.37 bar is observed, corresponding to a pressure drop of 0.13 bar compared to the inlet pressure. A little further downstream, an almost vertical isobar indicates the short entrance length of the thin film flow.

A colour map of the velocity magnitude is shown in Figure 3.12b, illustrating that as the airflow is squeezed past the sharp corner, it separates from the bottom of the channel. This point, where the flow area reaches its minimum, is known as the Vena Contracta [10]. Around the Vena Contracta, a low-pressure recirculation zone typically forms, which corresponds to the previously discussed isobars.

At the outlet of the thin film, where the flow area increases upon entering the vertical unit cell outlet, the local variations in the velocity and pressure profiles are negligible. The flow velocity and Mach number at the outlet of the thin film are much lower than at the inlet (see Figure 3.11), meaning the isentropic relations do not yield any significant deviations.



(a) Colour map of the absolute pressure with isobars at the thin film inlet. (b) Colour map of the flow velocity at the thin film inlet.

Figure 3.12: Pressure and velocity distribution at the thin film inlet for the axisymmetric geometry with vertical inlet and outlet.

Temperature Profiles

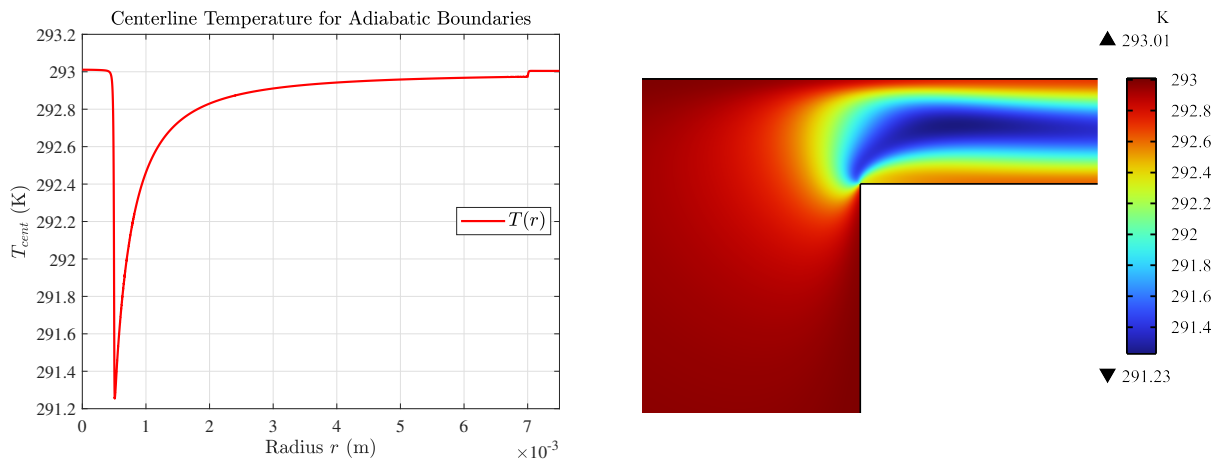
After thoroughly analyzing the effects of the vertical inlet and outlet on the pressure and velocity distribution, the next step is to investigate the thermal effects. Since the temperature of the wafer on top of the air bearing system is ultimately affected by the energy dissipation at the top boundary of the thin film, this will be the main point of interest in this section. Similar to the pressure drop calculation, the isentropic relations can first be used to estimate the centerline temperature at the entrance of the thin film. Using the same Mach number and specific heat ratio as before, the expected temperature drop is calculated as follows:

$$T = T_i \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{-1} = 293 \cdot \left(1 + \frac{0.4}{2} \cdot 0.178^2 \right)^{-1} = 291.2 \text{ K} \quad (3.5)$$

The calculated temperature drop corresponds well with the COMSOL results shown in Figure 3.13,

which yielded a minimum centerline temperature of 291.26 K. When comparing the centerline temperature for the geometry with a vertical inlet and outlet (Figure 3.13a) to the horizontal inlet geometry (Figure 3.7a), a difference in temperature drops of about 1.5 K can be observed at r_1 . This is due to the concept of static temperature explained through the isentropic flow relations. For the horizontal geometry, a uniform inlet temperature was assumed at the thin film inlet, where the flow velocity was already at its maximum. For the vertical inlet geometry, a uniform inlet temperature was assumed at the bottom of the vertical inlet, where the flow velocity is minimal. This results in an additional temperature drop due to the subsequent increase in flow velocity for the vertical inlet geometry.

The corresponding colour map of the temperature at the inlet of the thin film is depicted in Figure 3.13b. The similarities with the velocity colour map are evident, where the high-velocity regions show a more significant temperature drop than the low-velocity regions. Additionally, a temperature drop can be observed in the low-pressure zone below the Vena Contracta.



(a) Centerline temperature profile for the axisymmetric domain with vertical inlet and outlet.

(b) Colour map of the temperature at the thin film inlet.

Figure 3.13: Centerline temperature profile and temperature distribution at the thin film inlet for the axisymmetric domain with vertical inlet and outlet, assuming adiabatic boundaries.

To further investigate the local dissipation along the top boundary of the thin film, the temperature distribution of the top wall is plotted over the domain for an adiabatic boundary simulation. Figure 3.14 shows the relative temperature of the top wall of the axisymmetric domain compared to the inlet air temperature under these conditions. The plot reveals four distinct regions:

- I) *Vertical inlet/outlet region*: In this region, near the vertical inlet and outlet, the pressure remains nearly constant. Only the viscous dissipation term in the thermal energy equation contributes to heating, which is minimal due to the relatively low flow velocity in this area.
- II) *Cooling region*: At the inlet of the thin film, the pressure drops significantly, as described by the isentropic relations and the Vena Contracta effect. This causes a rapid drop in the wall temperature of over 300 mK.
- III) *Dissipation region*: In this section of the thin film, where flow velocities are high, energy dissipation becomes dominant. Particularly in the first 2 - 3 millimeters downstream from the inlet, the substantial energy dissipation causes a significant increase in the wall temperature.
- IV) *Negligible dissipation region*: After the first 2 - 3 millimeters from the thin film inlet, the flow velocity decreases to the point where energy dissipation becomes negligible. In this region, the wall temperature remains approximately constant.

A schematic representation of the temperature regions described above is presented in [Figure 3.15](#). It is important to note that this image is not to scale and does not depict the exact boundaries of the regions. The figure purely illustrates the approximate location and nature of each temperature region.

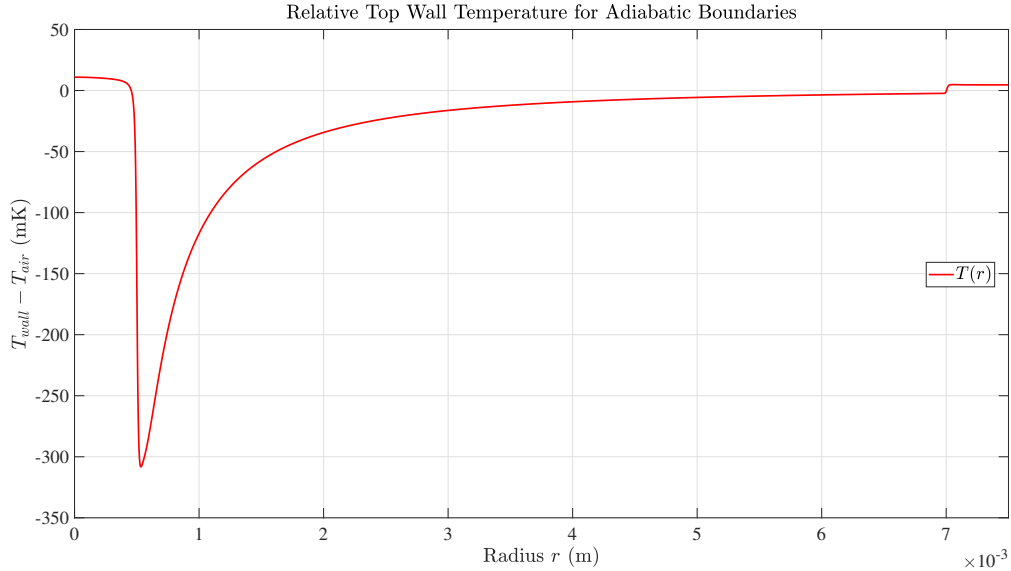


Figure 3.14: Relative temperature distribution along the top wall of the axisymmetric domain with vertical inlet and outlet, assuming adiabatic boundaries.

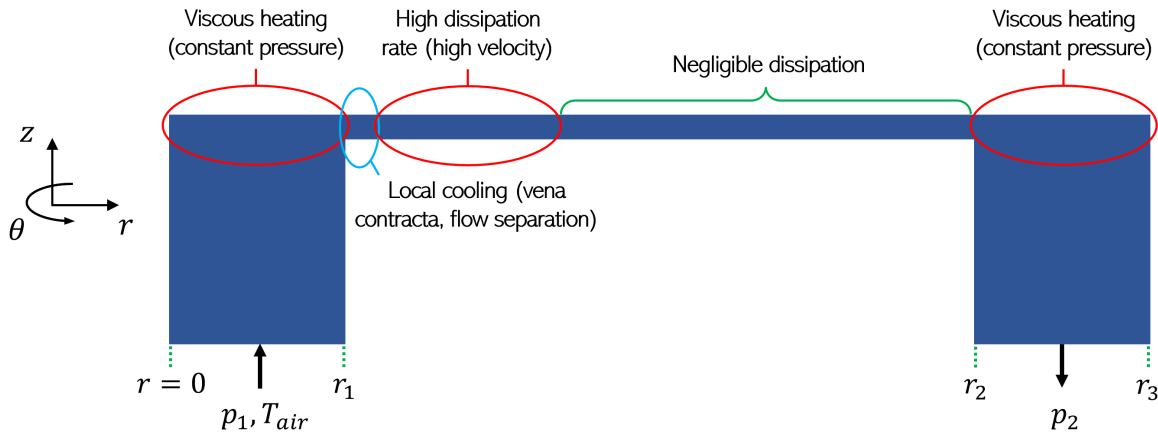


Figure 3.15: Schematic representation of the different temperature regions in the axisymmetric domain with vertical inlet and outlet.

3.3 Unit Cell Geometry

The final addition to the COMSOL geometry involves incorporating the air bearing system between the vertical inlet and outlet and the wafer positioned on top. A schematic representation of the unit cell geometry is shown in [Figure 3.16](#), with the corresponding COMSOL parameters listed in [Table 3.1](#). The thickness of the wafer, t_w , was set to $525 \mu\text{m}$, which is the standard thickness of a 100 mm wafer. Initially, the materials for the air bearing system and wafer were chosen as aluminium and silicon, respectively, with their thermal properties provided in [Table 3.2](#). The effects of these materials on the thermal distribution of the wafer are further discussed in [Appendix C.2](#).

Table 3.2: Thermal properties of aluminium and silicon used in COMSOL.

Material	Thermal conductivity k (W/m K)	Specific heat capacity c_p (J/kg K)	Density ρ (kg/m ³)
Aluminium	238	900	2700
Silicon	131	700	2329

To simulate the air above the wafer, an external natural convection heat flux boundary condition was applied. The surrounding air was assumed to be at atmospheric conditions, and the characteristic length (area/perimeter) was calculated as follows:

$$L = \frac{A}{P} = \frac{\pi r_3^2}{2\pi r_3} = \frac{r_3}{2} = 3.75 \text{ mm} \quad (3.6)$$

At the bottom of the air bearing system, both adiabatic and isothermal boundary conditions were applied to evaluate their impact on the wafer's thermal distribution. On the right side of the geometry, *Symmetry* boundary conditions were used to simulate the connection to an adjacent unit cell. For the full geometry mesh, a free triangular mesh was chosen. Similar to previous geometries, corner refinements and boundary layers were added to the air domain to enhance simulation performance. The solid components were meshed more coarsely, as heat conduction within these components is relatively simple compared to the more complex fluid mechanics in the airflow.

Finally, the *Laminar Flow* and *Heat Transfer in Solids and Fluids* interfaces were coupled through the *Nonisothermal Flow* multiphysics to model the temperature distribution of both the solid and fluid components.

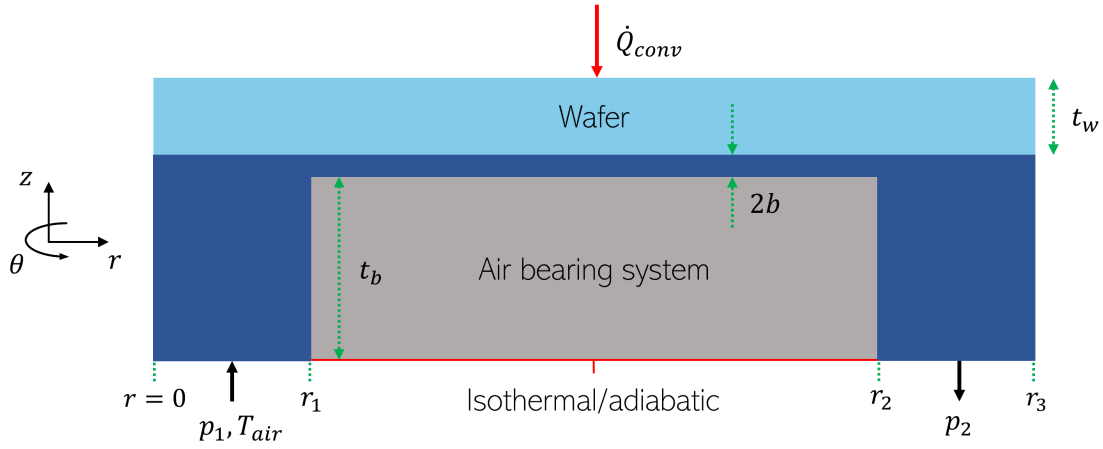


Figure 3.16: Schematic representation of the unit cell geometry with the air bearing system and wafer.

3.3.1 Temperature Profiles

Isothermal Air Bearing System

For an isothermal air bearing system, the colour map of the relative temperature distribution ($T - T_{air}$) is shown in Figure 3.17. Furthermore, a zoomed-in version of this colour map and the temperature profiles at the top and bottom of the wafer are presented in Figure 3.18a and Figure 3.18b, respectively. The lowest temperature in the domain is found at the sharp edge of the air bearing unit cell, where a temperature drop of 1.61 mK is observed relative to the inlet air temperature of 293 K. The rest of the air bearing system maintains a temperature of approximately 293 K due to the isothermal boundary condition.

At the bottom of the wafer, the minimum temperature is observed at the inlet of the thin film, with

a temperature drop of 0.868 mK. These local cooling effects correspond to the findings shown in Figure 3.15. However, it is also apparent that the relative temperature of the wafer drops below zero for $r < r_1$, a region previously found to be subject to viscous heating. This is attributed to heat conduction through the wafer, which causes the local temperature drop to propagate to surrounding areas. As a result, the minimum temperature at the top of the wafer occurs at $r = 0$, with a temperature drop of 0.167 mK.

After this local cooling region, the high dissipation rate in the thin film causes a maximum relative wafer temperature of 0.347 mK at the bottom and 0.341 mK at the top of the wafer. In the vertical outlet region, the wafer temperature increases slightly due to viscous dissipation, though the relative temperature remains lower than the maximum value near the thin film inlet.

Ultimately, a temperature difference of $\Delta T_{top} = 0.341 + 0.167 = 0.508$ mK is observed at the top side of the wafer, with a spacing of 2.2 mm between the two extremes. Due to the excellent thermal conductivity of the silicon wafer, the more significant temperature difference of $\Delta T_{bottom} = 0.347 + 0.868 = 1.22$ mK at the bottom of the wafer is smoothed out, which complicates the measurement procedure.

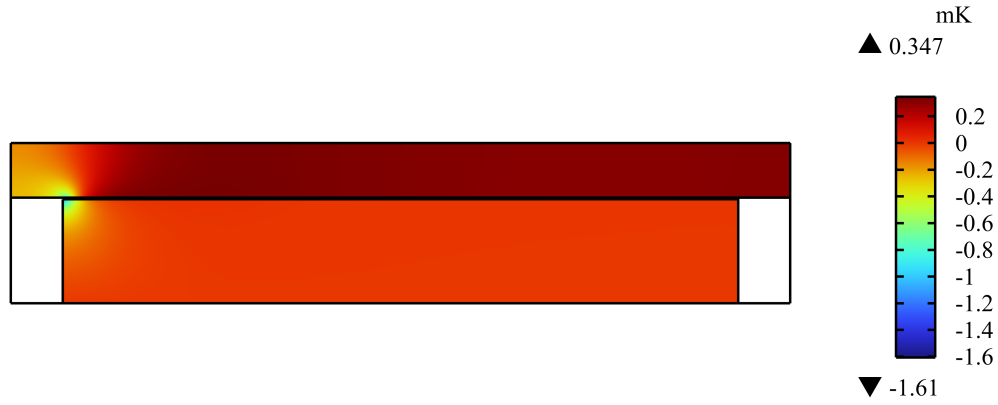
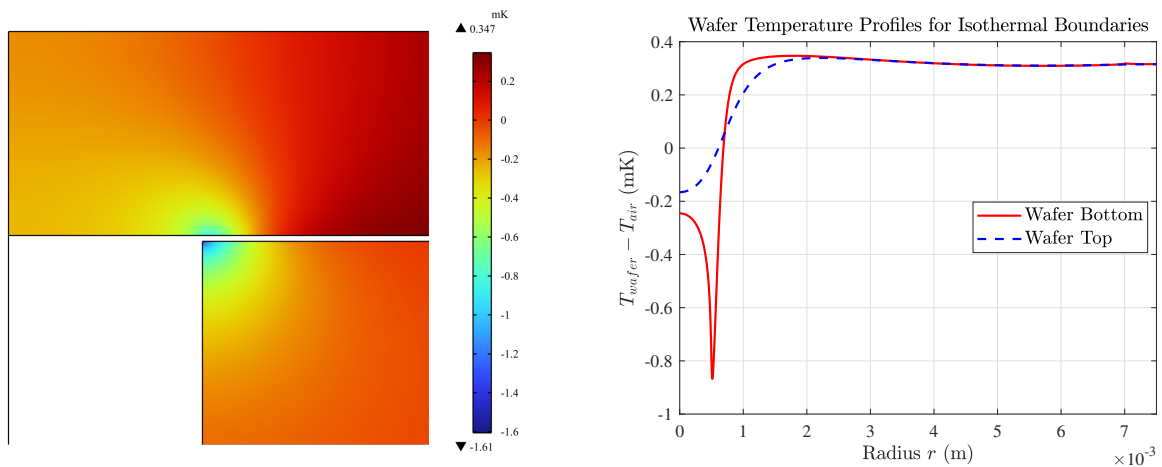


Figure 3.17: Colour map of the relative temperature distribution ($T - T_{air}$) of the air bearing system and wafer for an isothermal air bearing system.



(a) Zoomed-in view of the relative temperature distribution at the thin film inlet region.

(b) Radial temperature profiles at the top and bottom surfaces of the wafer.

Figure 3.18: Relative temperature distribution and wafer temperature profiles for the isothermal air bearing system configuration.

Adiabatic Air Bearing System

For the adiabatic air bearing system, the relative temperature distribution ($T - T_{air}$) is illustrated in Figure 3.19. A zoomed-in view of the thin film inlet region and the corresponding radial temperature profiles at the top and bottom surfaces of the wafer are shown in Figure 3.20a and Figure 3.20b, respectively. Compared to the isothermal case, the similarities between the colour maps and temperature profiles are evident. The general distribution of the colour map and the shape of the temperature profiles have remained the same, only differing in their extreme values.

The minimum temperature is again located at the sharp edge of the unit cell, measuring 2.38 mK above the inlet air temperature of 293 K. The remainder of the air bearing system maintains a relative temperature of approximately 4 mK, highlighting that heating effects outweigh cooling effects under these conditions. At the bottom of the wafer, the lowest relative temperature is found near the thin film inlet, reaching 3.15 mK. The top of the wafer still shows its minimum at $r = 0$, with a temperature of 3.86 mK above the inlet air temperature. The maximum relative temperatures at the top and bottom of the wafer are 4.38 mK and 4.39 mK, respectively. Similar to the isothermal boundary case, the temperature drops slightly in the negligible dissipation zone and increases marginally in the vertical outlet region.

In summary, a temperature difference of $\Delta T_{top} = 4.38 - 3.86 = 0.528$ mK is found at the top of the wafer, with a spacing of 2.1 mm between the two extremes. The bottom of the wafer shows a more significant temperature difference of $\Delta T_{bottom} = 4.39 - 3.15 = 1.24$ mK. Compared to the isothermal air bearing system, these temperature differences are marginally higher.

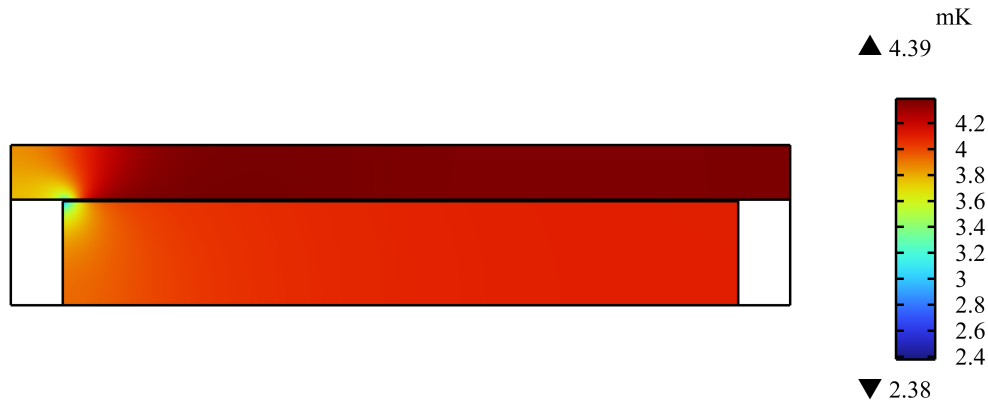
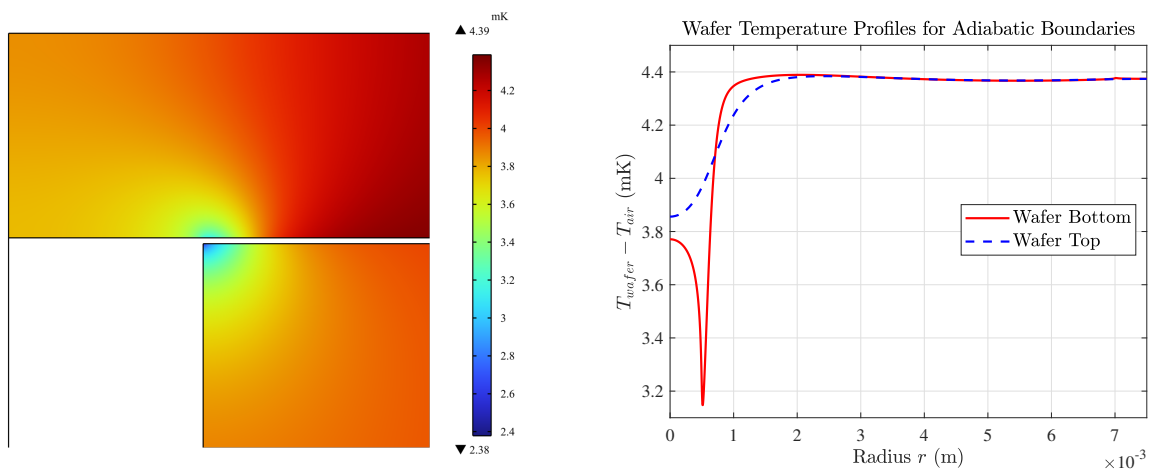


Figure 3.19: Colour map of the relative temperature distribution ($T - T_{air}$) of the air bearing system and wafer for an adiabatic air bearing system.



(a) Zoomed-in view of the relative temperature distribution at the thin film inlet region.

(b) Radial temperature profiles at the top and bottom surfaces of the wafer.

Figure 3.20: Relative temperature distribution and wafer temperature profiles for the adiabatic air bearing system configuration.

3.4 Test Setup

Up until now, the chosen parameters were selected to closely resemble the dimensions of an actual air bearing system, with the goal of gaining insight into the thermal behaviour of a wafer during operation. However, measuring the temperature distribution of a wafer during contactless handling on an active air bearing system does not necessarily require replicating a fully operational setup. Instead, the system can be greatly simplified to one that has a similar thermal impact on the wafer while facilitating a more straightforward temperature measurement.

[Appendix C](#) discusses several important factors that need to be taken into account for the parameter selection of the test setup. It was found that the inlet shape, material choice, the combination of inlet pressure and film height, and the thermal conditions of the inlet and ambient air have a significant influence on the wafer's temperature distribution. However, the measurable temperature difference cannot simply be maximized without regard for other constraints, such as limits on flow velocity and wafer deformation. Therefore, the parameters need to be selected in such a way that the measurable temperature difference is maximized while ensuring the flow velocity and deformation remain within acceptable limits. In theory, an optimization algorithm could be employed to identify the optimal parameter set. However, based on prior results, some parameter choices are evident, reducing the options to a handful of viable combinations. Below, the main findings of [Appendix C](#) are summarized:

- **Inlet Shape:** The sharp edge of the unit cell inlet locally disrupts the airflow, causing abrupt pressure and temperature drops. Using a chamfered or rounded inlet with a characteristic length on the order of the film height smooths these disturbances. Additionally, increasing this characteristic length reduces temperature gradients on the wafer.
- **Thermal Material Properties:** The wafer material strongly influences its temperature distribution. Materials with low thermal conductivity, such as quartz glass, substantially increase the measurable temperature difference. Although low thermal conductivity is typically associated with reduced thermal diffusivity and slower thermal response, the wafer's small thickness suggests that its thermal response time is unlikely to be substantially affected. The air bearing system material, on the other hand, has only a minor impact on the wafer's temperature.
- **Film Height & Pressure:**
 - **Velocity & Temperature:** Increasing either the film height or inlet pressure raises the flow velocity, which in turn increases the measurable temperature difference. A lower film height at the same flow velocity yields a slightly larger measurable temperature difference, but requires a significantly higher inlet pressure.
 - **Wafer Deformation:** The pressure field beneath the wafer causes it to deform upward. To limit deformation, both the applied load and clamping distance should be minimized. The load can be minimized by using only a single unit cell and reducing the inlet pressure. Additionally, wafers with lower flexural rigidity deform more. The deformation due to wafer sag (from its own weight) is negligible compared to the deformation induced by the pressure field.
- **Temperature Conditions:** The temperature difference between the inlet air and ambient air has a strong influence on the wafer's thermal profile. For the selected set of parameters, the smallest measurable temperature difference occurs when the inlet air is approximately 60 mK warmer than ambient. Beyond this threshold, the wafer's temperature difference increases approximately linearly with further deviations between the inlet and ambient air, both above and below the 60 mK difference.

Based on these findings, several parameters can already be determined. The test setup will consist of a single unit cell rather than multiple ones. This choice not only reduces the wafer's deformation

but also simplifies the test setup considerably. While using only one unit cell prevents the study of potential cross-effects between cells, the primary temperature effects occur near the unit cell inlet, and therefore, any cross-effects are expected to be minimal.

The air outlet of the test setup will be kept at atmospheric conditions rather than vacuum conditions. This eliminates the need for a vacuum pump, significantly simplifying the test setup. Additionally, since the thin film flow is driven by pressure differences, the overall thermal response of the system should remain comparable to that of an operational air bearing system. A 100 mm quartz glass wafer will be used instead of a silicon wafer to amplify the measurable temperature difference. This comes at the cost of a larger deformation of the wafer. This deformation will be counteracted by clamping the quartz glass wafer at a distance of 25 mm from its center. Theoretically, this distance could be reduced even further. However, this would leave insufficient space for invasive temperature sensors to be placed on the surface of the wafer. Furthermore, to further reduce the deformation of the wafer, a maximum inlet pressure of 1.2 bar will be applied. This pressure provides flexibility during testing, as it can be reduced if necessary while still maintaining wafer deformation within acceptable limits.

To determine an appropriate film height in combination with the inlet pressure, a COMSOL simulation that incorporates wafer deformation was performed. This simulation helps identify a film height that results in a flow velocity of approximately Mach 0.3 at an inlet pressure of 1.2 bar, thus maximizing the measurable temperature difference.

To accurately represent the expected wafer deformation pattern, the entire wafer up to the clamping radius was included in the geometry. A schematic of the updated geometry is shown in [Figure 3.21](#). The updated system parameters are listed in [Table 3.3](#), where r_4 denotes the clamping radius and r_f refers to the radius of the rounding at the thin film inlet. The remaining system parameters are unchanged from those in [Table 3.1](#). As noted earlier, the outlet pressure p_2 is set to atmospheric pressure for simplicity in the test setup.

Table 3.3: COMSOL parameters used for the unit cell geometry with wafer clamping.

Parameter	Value	Unit
r_4	25	mm
r_f	120	μm
p_1	1.2×10^5	Pa
p_2	1.0×10^5	Pa
b	15	μm

To determine the film height listed in the table, several simulations were performed to achieve a maximum flow velocity of approximately Mach 0.3. This condition was satisfied with an initial film height of 30 μm . The radius of the rounding was selected based on an analysis of the pressure distribution at the thin film inlet. A 120 μm radius was found to effectively smooth out pressure disruptions. Further increases in the rounding radius provided no significant improvement. While a rounding of one to two times the film height was sufficient for the configuration in [Appendix C.1](#), the higher velocity in the test setup configuration led to more pronounced pressure disruptions, necessitating a larger rounding to ensure a smooth transition.

The thermal boundary condition at the bottom of the air bearing system was set to isothermal, as the system is expected to act as a heat sink in the test setup. To simulate the wafer clamping, a fixed constraint was applied to the right edge of the wafer. Additionally, the vertical air boundary at r_3 was set to an outlet with atmospheric pressure to closely mirror the test setup. An external natural convection heat flux boundary condition was applied to the bottom of the wafer. While in reality the airflow will still have a nonzero velocity after leaving the thin film, such a boundary condition is considered suitable for this analysis. The thermal effects near the outlet are minimal compared to those in the thin film and do not significantly affect the temperature distribution in the measurement region. By using a heat flux boundary condition rather than extending the air domain over the entire length of the wafer, the computational effort is reduced.

$$t_{f,a} = t_{f,p} + w_{max} = 37.73 \mu\text{m} \quad (3.9)$$

Where $t_{f,p}$ is the passive film height, equal to $2b$. Ideally, the active film height would remain uniform across the entire unit cell. However, due to the wafer's deformation profile, the film height increases toward the center and decreases near the edges. Figure 3.23 illustrates this deformation relative to the air bearing system. At point A , located at the center of the wafer, the deformation reaches its maximum. In contrast, at point B , located at the unit cell's outer radius r_2 , the deformation is reduced to a fraction f of the maximum. For linear elastic materials, this fraction remains constant, regardless of the magnitude of the central load.

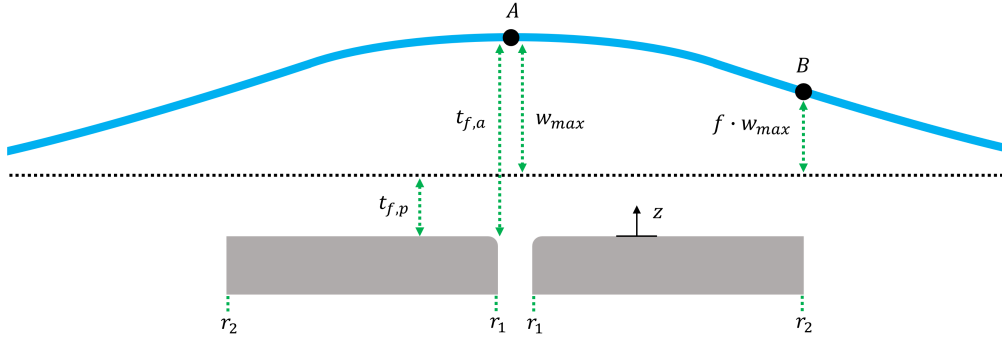


Figure 3.23: Schematic representation of the deformation of the wafer with respect to the unit cell.

The ratio of the active film heights at points B and A is given in Equation 3.10. Achieving a ratio close to unity requires either maximizing the passive film height or minimizing the maximum deformation. This consideration justifies the choice of a relatively large passive film height of $30 \mu\text{m}$ in combination with a low inlet pressure of 1.2 bar. Reducing or removing the passive film height would significantly decrease the uniformity of the active film height. Furthermore, a larger passive film height, in combination with a lower inlet pressure, helps to reduce the sensitivity of the system to manufacturing tolerances.

$$Z = \frac{z_B}{z_A} = \frac{t_{f,p} + f \cdot w_{max}}{t_{f,p} + w_{max}} \quad (3.10)$$

Figure 3.24 illustrates the distance between the bearing surface and the bottom of the wafer along the radial path from r_1 to r_2 , which represents the active film height. Local deviations in the film height caused by inlet rounding are not taken into account in this particular graph. At r_1 , the active film height measures $37.72 \mu\text{m}$. At r_2 , the active film height decreases slightly to $35.87 \mu\text{m}$, resulting in a total variation of just $1.85 \mu\text{m}$ across the surface of the unit cell. Given the small scale of this variation compared to the overall active film height, its influence is expected to be negligible.

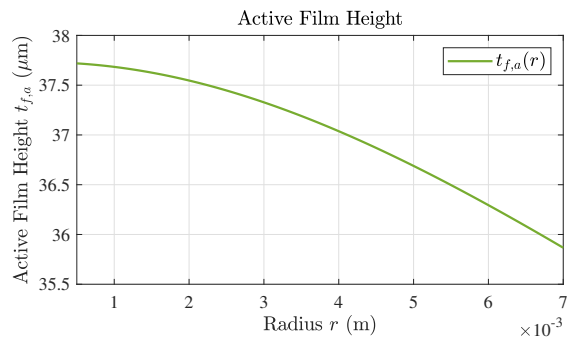


Figure 3.24: Active film height between the inner and outer radius of the unit cell.

3.4.2 Pressure & Velocity Profiles

Following the analysis of the wafer deformation pattern, the pressure and velocity profiles were examined. Figure 3.25 presents the absolute pressure distribution along the outer edges of the air

bearing system, comparing configurations with and without an inlet rounding of $120\ \mu\text{m}$. The x -axis represents the arc length instead of the radius. At $x = 1\ \text{mm}$, the simulated height of the air bearing system, t_b , is reached, corresponding to the location of the sharp edge of the unit cell. The influence of the inlet rounding is evident, as it smooths the sharp pressure dip found at the thin film inlet. Nevertheless, the pressure drop caused by the velocity increase at the inlet remains, explained through the isentropic flow relations. A minor disturbance is also visible at the sharp edge at the thin film outlet. While this could theoretically be mitigated by rounding the outlet, the disturbance is minor and does not affect the wafer's temperature distribution.

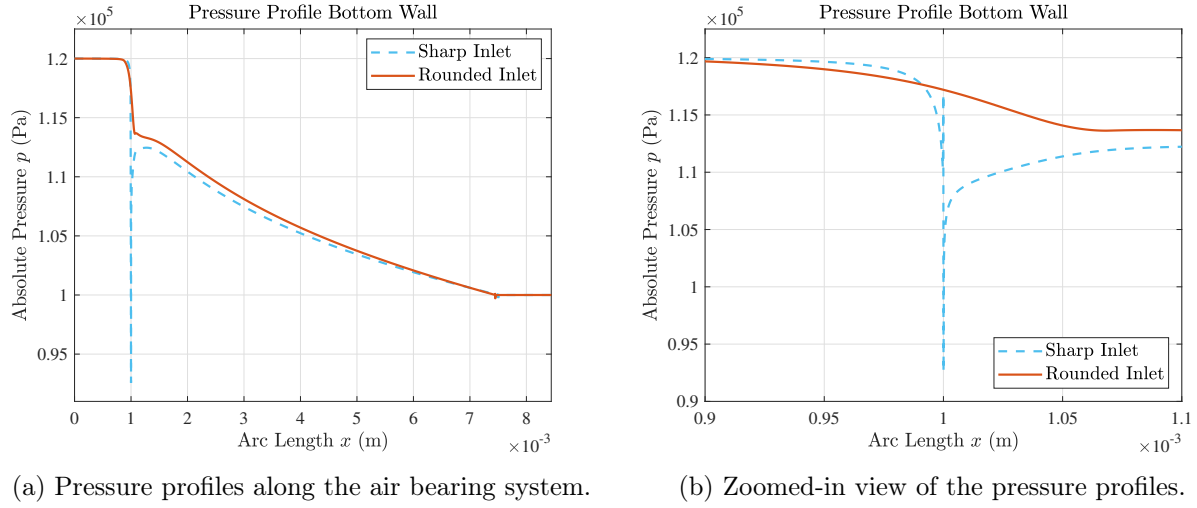


Figure 3.25: Pressure profile along the outer edges of the air bearing system for a sharp inlet and a rounded inlet.

Figure 3.26 illustrates the evolution of the centerline velocity across the entire radius of the geometry, comparing configurations with and without an inlet rounding of $120\ \mu\text{m}$. The rounded inlet results in a smoother velocity transition at the thin film entrance, reducing the peak velocity observed with the sharp inlet. This smoothing is attributed to the more gradual decrease in flow area introduced by the inlet rounding. However, the overall flow restriction in the thin film is slightly reduced due to the increased flow area at the inlet, leading to a higher flow velocity across the remaining thin film. With the rounded inlet, the maximum centerline velocity reaches $96.4\ \text{m/s}$, remaining well below a Mach number of 0.3, while the minimum centerline velocity in the thin film is $11.5\ \text{m/s}$.

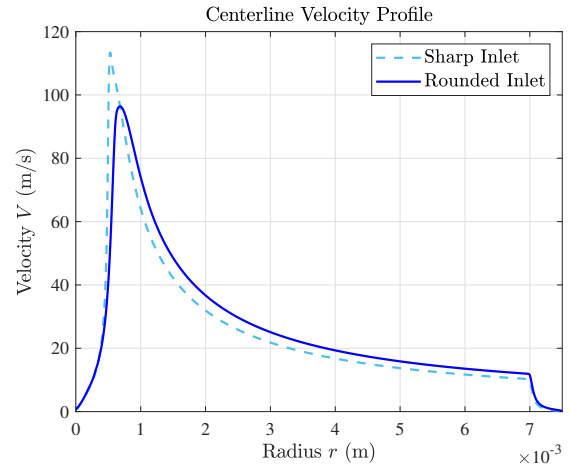


Figure 3.26: Radial flow velocity along the film centerline for both sharp and rounded inlets.

A colour map of the pressure distribution at the thin film inlet, including isobars is presented in Figure 3.27a. Each isobar represents a pressure drop of $0.5\ \text{kPa}$, and the colour limits were adjusted to clarify local pressure differences. Due to the rapid increase in flow velocity, the spacing between the isobars near the inlet rounding is relatively small. However, the isobars remain nearly perpendicular to the boundaries, indicating that the overall pressure field across the thin film inlet is minimally disturbed.

The corresponding colour map of the velocity distribution is shown in Figure 3.27b. Despite the significantly higher velocity compared to Figure 3.12, flow separation from the bottom boundary is

no longer observed due to the rounded inlet, effectively eliminating the low-pressure region below the Vena Contracta.

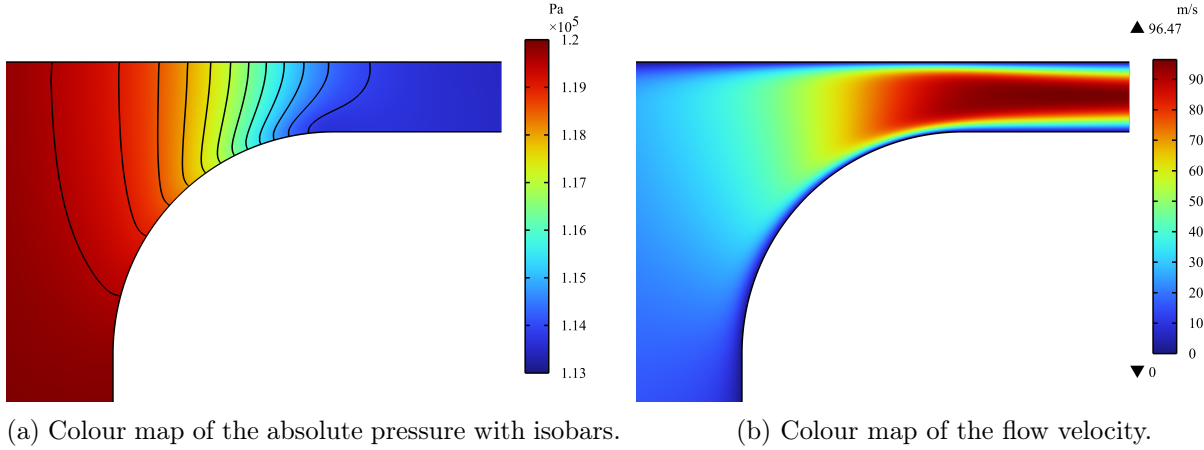


Figure 3.27: Pressure and velocity distributions at the thin film inlet from the fluid-structure interaction simulation.

3.4.3 Temperature Profiles

The relative temperature distribution ($T - T_{air}$) of the wafer and air bearing system is shown in Figure 3.28. A detailed view of the temperature distribution near the thin film inlet, as well as the temperature profiles at the top and bottom surfaces of the wafer, are presented in Figure 3.29a and Figure 3.29b, respectively. Due to the isothermal boundary condition at the bottom of the aluminium air bearing system, coupled with its excellent thermal conductivity, the system maintains a stable temperature of 293 K with negligible fluctuations.

The lowest temperature in the domain is observed at the bottom of the quartz glass wafer near the thin film inlet, where a temperature drop of 237 mK relative to the inlet air temperature of 293 K occurs. The maximum relative temperature of 60.88 mK also appears at the bottom of the wafer, slightly downstream, where heat is dissipated from the thin film flow. Consequently, the total temperature difference on the bottom surface of the wafer is $\Delta T_{bot} = 237.00 + 60.88 = 297.88$ mK. As expected, the temperature variation at the top of the wafer is less extreme, with a minimum relative temperature of -125.28 mK and a maximum of 46.95 mK relative to the inlet air temperature. Thus, the total measurable temperature difference on the top surface of the wafer is $\Delta T_{top} = 125.28 + 46.95 = 172.23$ mK, with a spacing of about 1.8 mm between the two extremes. Compared to the initial temperature difference of 0.508 mK on the top surface of the wafer, as shown in Figure 3.17 without any amplification measures, the temperature variation is now much more pronounced, simplifying the temperature measurement.

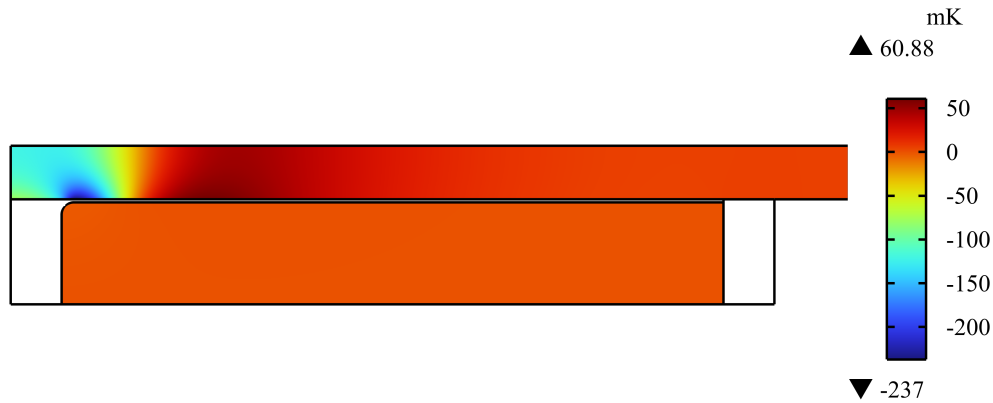
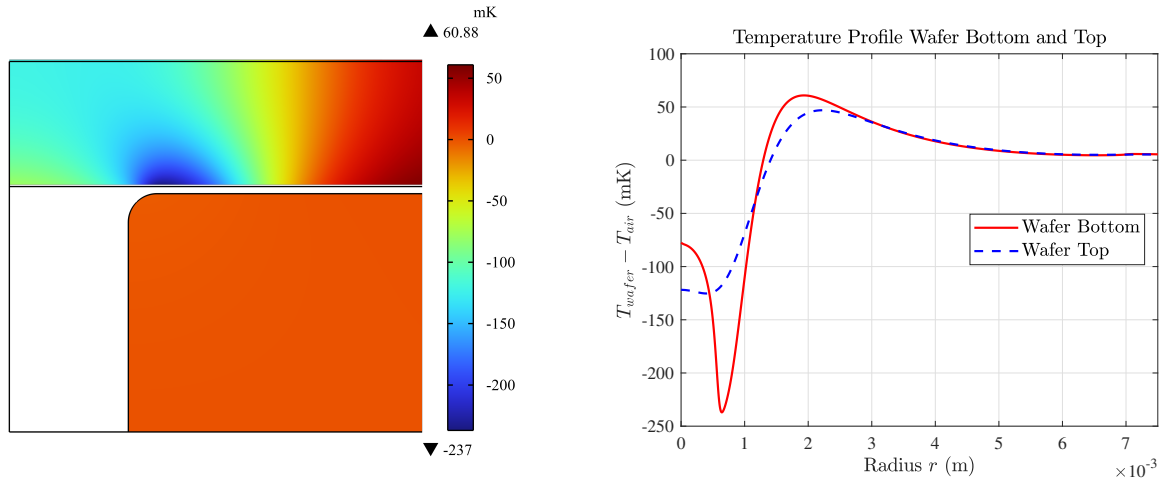


Figure 3.28: Colour map of the relative temperature distribution ($T - T_{air}$) of the air bearing system and wafer from the fluid-structure interaction simulation, assuming an isothermal air bearing system.



(a) Zoomed-in view of the relative temperature distribution at the thin film inlet region. (b) Radial temperature profiles at the top and bottom surfaces of the wafer.

Figure 3.29: Relative temperature distribution and wafer temperature profiles from the fluid-structure interaction simulation, assuming an isothermal air bearing system.

3.4.4 Discussion

The parameter selection for the test setup in this section is based on an idealized geometry with perfectly applied boundary conditions. However, due to manufacturing tolerances, environmental variations, and clamping imperfections, the actual test setup will differ from the designed geometry, affecting measurement accuracy. To validate the COMSOL models despite these imperfections, it is essential to conduct tests across a range of system parameters rather than relying on a single set. The results presented here should therefore be interpreted as indicative of the expected order of magnitude for key physical quantities, including temperature, pressure, deformation, and flow velocity.

Ideally, each of the four relevant factors affecting the thermomechanical behaviour of the wafer, namely inlet shape, thermal material properties, film height and pressure, and temperature conditions, should be varied separately during testing to assess their impact. However, such an extensive study was not feasible within the project's timeframe. Therefore, an air bearing system with fixed passive parameters, including film height, inlet shape, and clamping distance, was designed. Additionally, only quartz glass wafers were used. By varying the inlet air conditions, the wafer's thermomechanical response can be studied across various combinations of active film height, pressure drop, and temperature conditions.

The top view of the relative temperature distribution of the wafer up to its clamping radius is shown in Figure 3.30. As shown, the temperature gradients remain highly localized near the thin film inlet, positioned at the center of the wafer, while the remainder of the wafer surface closely follows the isothermal temperature of the air bearing system, maintained at 293 K.

This colour map represents the expected temperature profile in the experimental test setup when observing the wafer from above. Although the absolute temperature values will likely differ due to environmental variations and limited control over boundary conditions, the presence of a distinct, localized thermal gradient near the center and a largely uniform temperature elsewhere are key features that are expected to be reproduced in the experimental measurements.

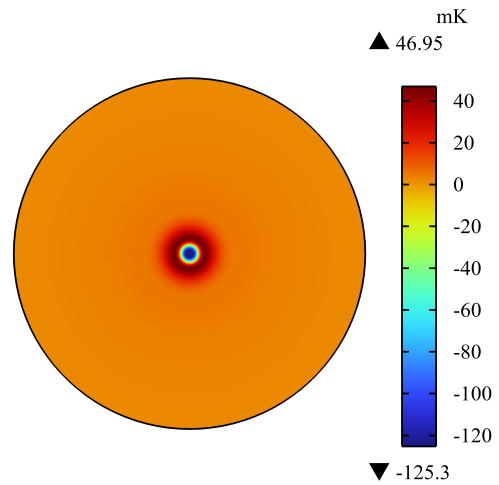


Figure 3.30: Top view of the relative temperature distribution of the wafer.

4

Test Setup

Based on the numerical COMSOL modelling discussed in the previous chapter, a thorough understanding was developed of the key factors influencing the thermomechanical response of the wafer. To validate these models, a set of parameters was selected for the experimental test setup. This chapter presents a detailed description of the design, implementation, and experimental results of this test setup.

First, [Section 4.1](#) outlines the design process, including the design goals, requirements, and final configuration. Next, [Section 4.2](#) presents the final modelling steps required before performing the experimental validation. Finally, the experimental results obtained with various measurement techniques are presented and discussed in [Section 4.3](#).

4.1 Design

4.1.1 Design Goal

An essential step in any design process is the formulation of a clear design goal. In this case, the primary objective is to validate the analytical, semi-analytical, and numerical models outlined in [Chapter 2](#) and [Chapter 3](#) for a simplified air bearing system. If the experimental results correspond to the theoretical findings, these models can be used to predict the thermal effects of an operational air bearing system on a wafer.

Beyond validation, this experimental study also aims to evaluate different measurement techniques and determine their suitability for more complex or fully operational air bearing systems.

In order to properly validate the models, the design of the physical system should closely match the geometry used in the models. Therefore, the geometry of the model described in [Section 3.4](#) will serve as the foundation for the design of the setup. Since each system parameter affects the wafer's temperature distribution, measuring the temperature at the wafer's top surface alone is insufficient. In addition to the temperature distribution, it is also necessary to measure the system's passive dimensions, the active film height, and the air supply conditions. Furthermore, practical manufacturing constraints must be considered during the setup design to avoid creating an infeasible design.

4.1.2 Design Requirements

The design requirements outline the essential functionalities and characteristics for the design of the test setup. Each subsection focuses on a specific subsystem within the test setup, including the air bearing system, air supply, substrate, air bearing system, substrate constraint, and the measurement techniques for temperature and film height.

Air Bearing System

The air bearing subsystem should consist of a single unit cell, with a centrally located circular, rounded inlet. To achieve a compact design, the substrate and/or substrate constraint must be directly integrated into the air bearing system, minimizing any potential misalignment between the substrate and the air bearing. The dimensions of the unit cell must align with the specifications

outlined in [Section 3.4](#), ensuring consistency across the simulation and testing environments. Additionally, the air bearing system should include an accessible interface for the air supply.

Air Supply

The air supply is responsible for providing a stable airflow to the air bearing system, of which the temperature, pressure, and flow rate must be monitored. This requires the integration of sensors and control components capable of measuring and regulating these properties. To minimize disturbances in the airflow, the interface between the air supply and the air bearing system should be designed to ensure a smooth transition, preventing local flow disruptions.

Substrate

To provide a good correspondence between the simulation and testing phases, the design requires a 100 mm wafer, with a thickness close to 525 μm . In addition, the thermomechanical material properties of the wafer should be similar to those of quartz glass. Furthermore, the wafer must accommodate both a temperature distribution measurement at its top surface and an active film height measurement. Multiple substrates with varying thicknesses or materials could be used to provide additional data points.

Substrate Constraint

The substrate constraint must maintain the substrate at a fixed height above the unit cell of the air bearing system, ensuring it stays securely in place. While it does not need to rigidly fix the substrate, the constraint should provide stable support that prevents movement without significantly affecting the wafer's deformation. Additionally, the constraint must offer uniform support along the circumference of the substrate to preserve the axisymmetric simplification. An adjustable film height could be considered as an additional feature, although this would add complexity to the design.

Temperature Measurements

To measure the temperature distribution on the top surface of the substrate, multiple measurements across the surface are required. Given that the theoretical temperature minimum and maximum are closely spaced (approximately 2 mm apart), the thermometry technique must provide a high spatial resolution. Additionally, the technique should be sensitive enough to detect small temperature differences on the order of 0.1 - 0.2 K at room temperature, requiring a sufficiently fine temperature resolution. While the primary focus is on capturing relative temperature variations across the surface, the absolute temperature values must also be determined with reasonable accuracy to validate the models. Furthermore, the temperature measurement technique must not interfere with the temperature distribution or the deformation pattern of the substrate.

For measuring the temperature of the inlet air, the key requirement is to determine the temperature difference relative to the ambient air, as shown in [Figure C.17](#). To ensure consistency in the measurements, it is preferable to apply the same thermometry technique for both inlet and ambient air temperature measurements.

Film Height Measurements

As previously discussed, the total film height consists of two components: the passive film height and the active film height. The passive film height refers to the distance between the surface of the unit cell and the bottom of the substrate when no air supply is present. The active film height represents this distance when the air supply is activated. Both of these components are required to properly validate the models.

Height distribution measurements should ideally be conducted for both the passive and active film heights, rather than relying on point measurements. Since these heights are on the order of tens of microns, the measurement technique must offer accuracy and resolution that are a fraction of this

scale. Furthermore, similar to the temperature measurements, the film height measurement must not interfere with the substrate's temperature distribution or deformation pattern.

4.1.3 Final Design

Based on the design requirements outlined in the previous section, the final design was iteratively developed, resulting in the complete test setup shown in [Figure 4.2](#). Additionally, a schematic representation of the cross-sectional view of the test setup is provided in [Figure 4.1](#). The setup was constructed on a Thorlabs breadboard, which facilitates secure mounting of components while providing vibration damping. Additionally, LINOS X 95 profiles, carriers, and mounting components were used to rigidly secure the subsystems to the breadboard, while still allowing for flexible and precise positional adjustments. Each subsystem in the test setup is labeled with a circled number, which will be discussed in detail below.

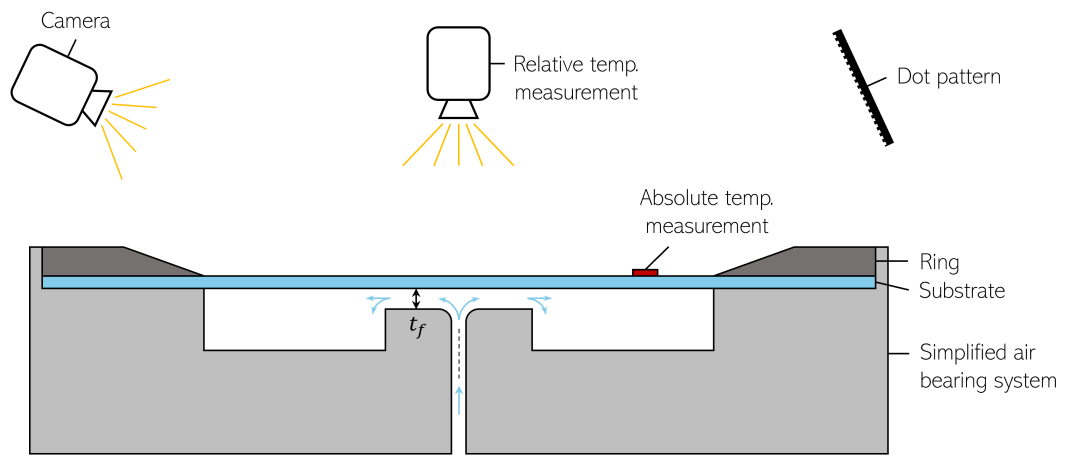


Figure 4.1: Schematic representation of the experimental test setup's cross-sectional view. *This image is repeated from [Figure 1.5](#) in [Section 1.3](#).*

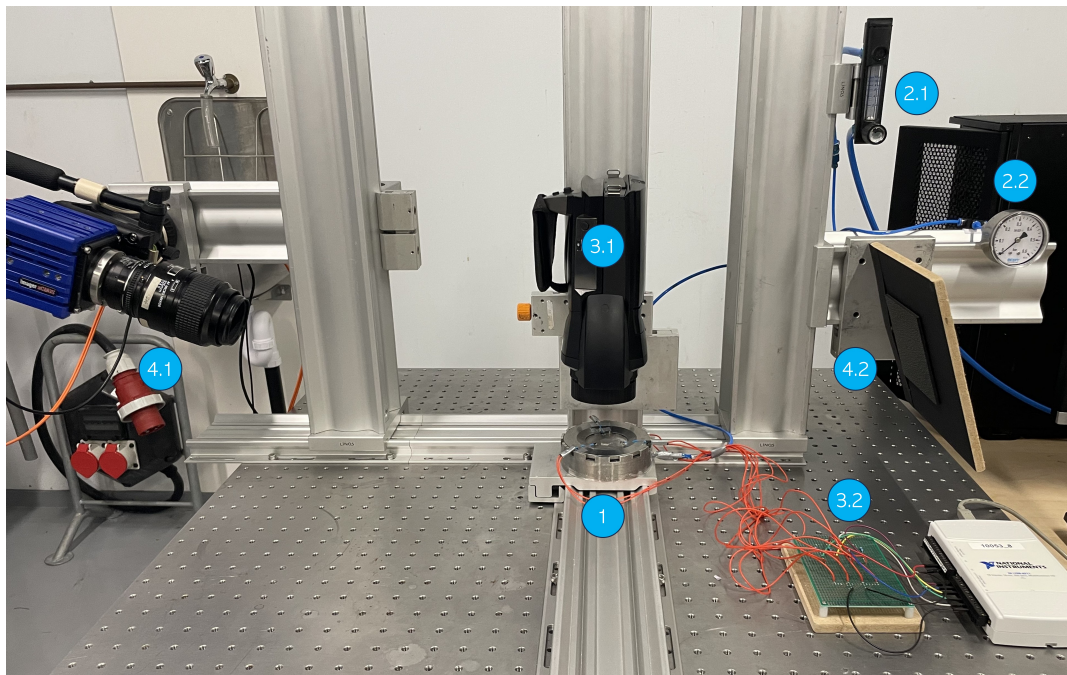


Figure 4.2: Complete test setup, where the circled numbers indicate the separate subsystems. (1) Air bearing system + substrate + substrate constraints, (2.1) Rotameter, (2.2) Pressure gauge, (3.1) Infrared camera, (3.2) NTC thermistors + DAQ device, (4.1) sCMOS camera, and (4.2) Dot pattern.

① Air Bearing System + Substrate + Substrate Constraint

For clarity, the air bearing system, substrate, and substrate constraint will be explained together. [Figure 4.3](#) illustrates the air bearing system with the (transparent) substrate and substrate constraint positioned on top. Additionally, the top and bottom view of the air bearing system are displayed in [Figure 4.4](#). The detailed dimensioning of the air bearing system and substrate constraint is discussed in [Appendix D](#).

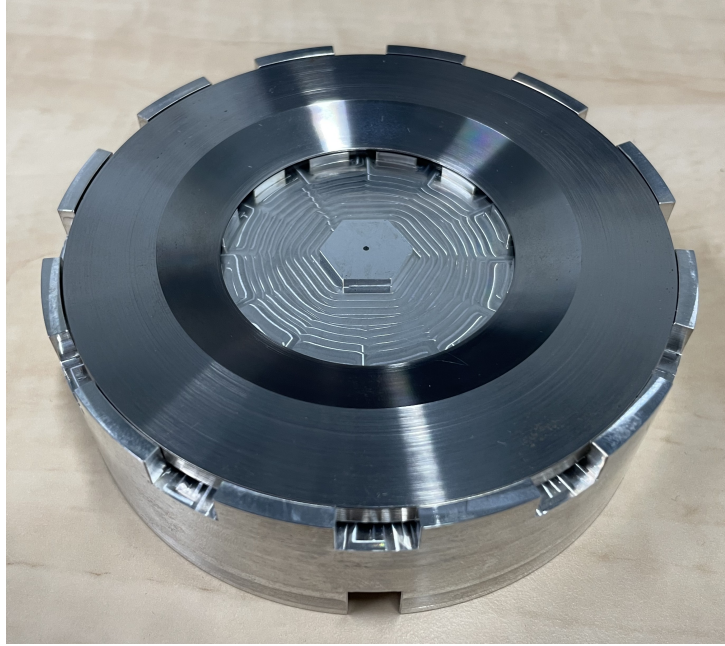
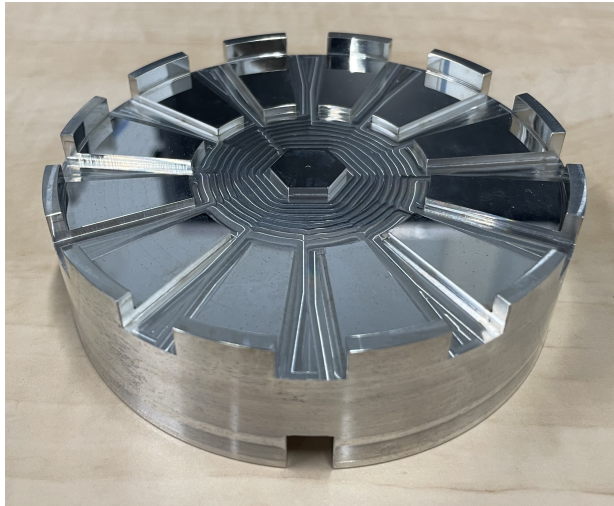
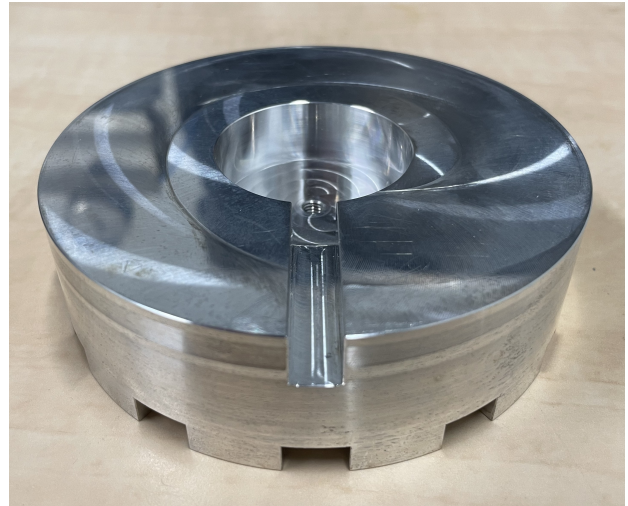


Figure 4.3: Air bearing system with the quartz glass wafer and constraining ring on top.



(a) Top view.



(b) Bottom view.

Figure 4.4: Top and bottom view of the manufactured air bearing system.

The top surface of the aluminium air bearing system features a central hexagonal unit cell, surrounded by 12 equally spaced segments. The hexagonal unit cell was chosen to resemble the variable pocket restriction concept presented in [Appendix A.2](#). While this design affects the system's axial symmetry, it is not expected to introduce significant deviations in the measurement results.

The top surface of the outer segments was designed to lie $30\text{ }\mu\text{m}$ above the top surface of the unit cell, representing the passive film height. Between these segments, the surface height is lowered by 2 mm, allowing air to exit the system with negligible flow velocity. The system includes 12 segments

to closely approximate an axisymmetric configuration, ensuring both uniform substrate support and consistent airflow. A lower segment count with the same total outlet area could lead to uneven substrate support and local flow disruptions, potentially affecting the thermomechanical response of the substrate. Additionally, each segment features a raised edge along the outer radius of the air bearing system, designed to keep the substrate concentrically positioned.

The air bearing system features a circular cutout at the bottom, with an M5 threaded hole positioned centrally to accommodate a pneumatic L-fitting. A slot extends from the cutout toward the outer radius to guide the pneumatic hose of the air supply.

For the substrate, a 100 mm quartz glass wafer with a thickness of $500 \pm 25 \mu\text{m}$ was chosen as the primary substrate. Additionally, a 100 mm quartz glass wafer with a thickness of $350 \pm 25 \mu\text{m}$ was selected for supplementary testing. To ensure proper model validation, the thicknesses of both wafers were measured at multiple points using a micrometer. The primary substrate's thickness was determined to be $518 \mu\text{m}$, while the secondary substrate measured $362 \mu\text{m}$.

To constrain the wafer, a thick 4 mm steel ring was placed on top of the wafer, which simply keeps the wafer in place by its weight. Along the inner edge of the ring, a large chamfer was added to improve the visibility of the wafer surface from more shallow angles.

Before placing the wafer and constraining ring onto the air bearing system, all contact surfaces were cleaned with isopropanol to prevent any contamination between them.

Material Selection

The specific materials used in the test setup and their relevant thermomechanical properties are summarized in Table 4.1. The air bearing system was constructed from aluminium 7075-T6, selected for its excellent machinability.

The constraining ring was made from 34CrNiMo6 steel, chosen primarily for its high density and stiffness, providing a robust constraint for the wafer.

The wafers are composed of fused silica JGS2, commonly known as quartz glass. This particular type of quartz glass provides excellent transmission in the UV and near-infrared range ($0.7 \mu\text{m}$ - $2.5 \mu\text{m}$), while being opaque to infrared radiation above approximately $4 \mu\text{m}$. Additionally, the thermomechanical properties of fused silica JGS2 align with those used in the modelling phase, ensuring consistency and simplifying model validation.

Table 4.1: Relevant thermomechanical properties of the materials used for the test setup [12].

Material	Thermal conductivity k (W/m K)	Specific heat capacity c_p (J/kg K)	Density ρ (kg/m ³)	Young's modulus E (GPa)	Poisson's ratio ν
Aluminium 7075-T6	130	960	2810	71.7	0.33
Steel 34CrNiMo6	40	460	7800	210	0.3
Fused silica JGS2	1.4	730	2210	72	0.17

Manufacturing

The air bearing system was manufactured at TU Delft's in-house workshop, DEMO, using a CNC milling machine. Since the thin film flow is rather sensitive to the flatness, parallelism, and surface roughness of the top surface of the air bearing system, strict manufacturing tolerances were set on both the top surface of the unit cell and the top surfaces of the outer segments.

The flatness of these surfaces and their relative parallelism were set to $1 \mu\text{m}$. Furthermore, the surface roughness was limited to $R_a 0.2 \mu\text{m}$, ensuring a maximum average deviation of the reference surface of $0.2 \mu\text{m}$. The remainder of the air bearing system was given a rougher surface finish, resulting in the visible grooves around the unit cell in Figure 4.3. Additionally, a tolerance of ± 5

μm was applied to the $30\ \mu\text{m}$ height difference between the unit cell and segment surfaces, while a tolerance of $\pm 10\ \mu\text{m}$ was used for the $120\ \mu\text{m}$ inlet rounding.

After manufacturing, the flatness, parallelism, and relative height difference of the unit cell and outer segments were measured using point measurements. Both the flatness and parallelism were verified to be within the specified tolerances. Furthermore, the relative height difference between the outer segments and the unit cell was measured to be $30\ \mu\text{m}$, also complying with the set tolerance. However, since this verification relies on point measurements, local surface deviations were possibly not captured. Therefore, no definitive comments can be made regarding the surface height profile of the air bearing system. The $30\ \mu\text{m}$ height difference is therefore regarded as the minimum passive film height. In practice, the film height is prone to local surface height deviations caused by the manufacturing process and contamination of either the air bearing surface or the wafer surface. Due to the rigidity of the wafer, it will lie on the three highest points of the surface of the outer segments, which are likely to be higher than the measured $30\ \mu\text{m}$.

The constraining ring was manufactured using a lathe due to its axisymmetric design. Unlike the air bearing system, the ring does not directly influence the thin film flow, so its manufacturing tolerances were less stringent. However, to ensure the wafer is evenly constrained, a flatness tolerance of $10\ \mu\text{m}$ was applied to the bottom surface of the ring.

② Air Supply

At the testing location, a 7-bar air supply is available. However, since the air bearing system is designed to operate at a supply pressure of approximately 1.2 bar, the air pressure is reduced in two stages. The first pressure drop, from 7 to 2 bar, is achieved using a pressure regulator located close to the 7-bar supply. This step serves two key purposes: managing pressure fluctuations and ensuring that the downstream regulator receives air at a manageable pressure level, which allows for more precise flow control. Additionally, the rapid pressure reduction at this stage causes the air to cool. To mitigate this, sufficient spacing is provided between the first and second pressure regulation steps, allowing the air to approach room temperature before reaching the system.

The second pressure drop is facilitated by a rotameter, also known as a variable area flow meter, with a range of 0.1 - 1.2 standard liters per minute (SLPM). The rotameter is positioned approximately 8 meters downstream of the first pressure regulator. This spacing ensures that the air has time to stabilize thermally and provides room for integrating an optional air-heating element.

Shortly after the rotameter, the hose diameter transitions from 8 to 4 mm, reducing the flow area while minimizing the pressure drop. The 4 mm hose section has an inner diameter of 2.6 mm. Within this section, a pressure gauge is installed to monitor the relative pressure, with a measurement range of 0 - 0.6 bar. A temperature sensor is placed on the outside of the supply hose, just before the air bearing system, with insulation foam wrapped around the hose to improve the accuracy of the air temperature readings. Finally, the air supply enters the air bearing system through an L-fitting.

A schematic representation of the air supply is shown in [Figure 4.5](#). It is important to note that the relative dimensions in the image are not to scale. The pressure regulator, rotameter, and pressure gauge are analog components, enabling manual adjustment and direct readings. Details regarding the temperature sensor will be discussed in the next section. Additional information regarding the pressure and temperature drops in the air supply is provided in [Appendix D.3](#).

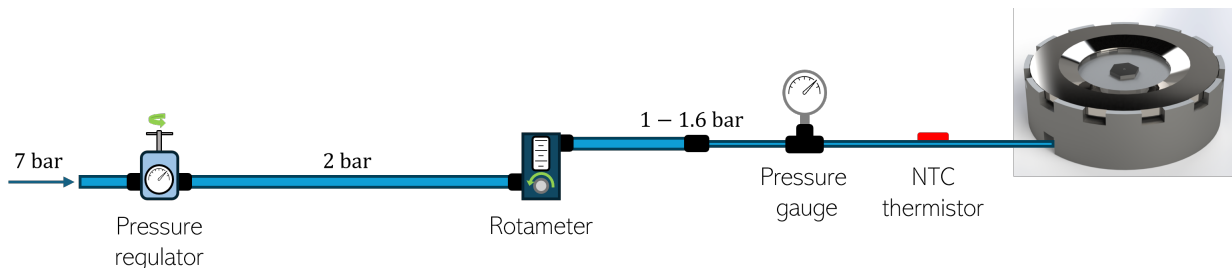


Figure 4.5: Schematic representation of the air supply with absolute pressure values.

③ Temperature Measurements

Based on the literature review of thermal sensing techniques presented in [Appendix E](#), coupled with the measurement requirements outlined in [Section 4.1.2](#), two techniques were selected: infrared thermography and NTC thermistors. Infrared thermography was employed to map the temperature distribution (relative temperatures) across the top surface of the wafer, while NTC thermistors were used to measure the absolute temperatures, enabling calibration of the infrared measurements.

Relative Temperatures

For measuring the temperature distribution of the wafer, it was quickly determined that only a non-invasive technique would be suitable, as both invasive and semi-invasive methods interfere with the temperature distribution to some extent. Additionally, invasive techniques often lack the spatial resolution required for this application. Among the available non-invasive methods, several offer adequate spatial and temperature resolution, but many require a scanning mechanism or a moving stage to obtain a 2D temperature measurement, adding complexity to the system. In contrast, an infrared camera can capture the entire top surface of the wafer in a single image with a compact setup. Additionally, a suitable infrared camera was readily available at the university, making it the most practical choice.

The infrared camera used for this research was the Testo 885, equipped with a detector size of 640×480 pixels. It has a thermal sensitivity of < 30 mK at 30°C , allowing the detection of minor temperature differences. Operating within a spectral range of $7.5 - 14\ \mu\text{m}$, the camera is suitable for measurements involving quartz glass wafers. As shown in [Figure 4.2](#), the infrared camera is positioned directly above the air bearing system at an adjustable height, enabling sharp imaging of the relevant area of the wafer. Although quartz glass wafers are opaque to infrared radiation within the relevant spectral range, they are not perfect black bodies. As a result, a portion of the incident infrared radiation is reflected off the surface, causing reflections of the infrared camera and other heat sources to appear in the captured images. This must be considered during the experimental validation.

Absolute Temperatures

A limitation of the infrared camera is that it measures only the infrared radiation emitted by an object, rather than its actual temperature. To determine the temperature, the surface emissivity must be known and correctly set on the device. Even with an exact emissivity value, the Testo 885 has an accuracy of only $\pm 2^\circ\text{C}$. To achieve a more accurate representation of the wafer's temperature distribution, three invasive temperature sensors were attached to its surface, as shown in [Figure 4.6](#). The advantages and disadvantages of RTDs, thermistors, and thermocouples are summarized in [Table 4.2](#). Theoretically, all three sensor types could be used for the intended purpose if properly wired and calibrated. However, the complexity of wiring, measurement, and calibration varies significantly between them.

Table 4.2: Advantages and disadvantages of three types of invasive temperature sensors.

Invasive sensor type	+	-
RTD	Accuracy Linearity Stability	Complex wiring Self-heating Low output
Thermistor	High output Sensitivity Simple wiring	Nonlinearity Self-heating Stability
Thermocouple	No driving current Temperature range Robustness	Nonlinearity Accuracy Low output

RTDs are known for their high accuracy, stability, and linearity. However, they require a complex four-wire configuration and have a relatively low output and sensitivity, making them less suitable in environments with measurement noise. Thermocouples, while robust and capable of operating in extreme conditions without a driving current, also suffer from low output and sensitivity.

Given the available measurement equipment and the uncontrolled measurement environment, high output and sensitivity are crucial to minimize the impact of measurement noise. Thermistors fulfill these requirements and are simpler to wire compared to RTDs. While thermistors require a driving current and are susceptible to self-heating, these effects can be mitigated through careful calibration and operation. As NTC thermistors generally provide higher sensitivity to temperature changes than PTC thermistors, they were selected as the most suitable sensor type for this application.

Detailed information on the operation, wiring, and calibration of the NTC thermistors is provided in [Appendix F](#). To ensure optimal thermal contact between the thermistors and the wafer surface, Arctic Silver thermal compound was used. Both the thermistors and the wafer surface were first cleaned with isopropanol, after which a small amount of Arctic Silver was applied between them. The thermistors were then secured to the constraining ring using tape to prevent disconnection.

A similar approach was used for the thermistor on the inlet air hose. After applying Arctic Silver, the thermistor was wrapped with insulation foam and secured with tape to minimize heat exchange with the environment. The inlet air thermistor is visible in the bottom left corner of [Figure 4.6](#).

Even after ensuring good thermal contact between the thermistors and wafer or hose, they remain influenced by the ambient temperature surrounding them. As a result, the thermistors will measure a temperature somewhere between the wafer or hose temperature and the ambient temperature.

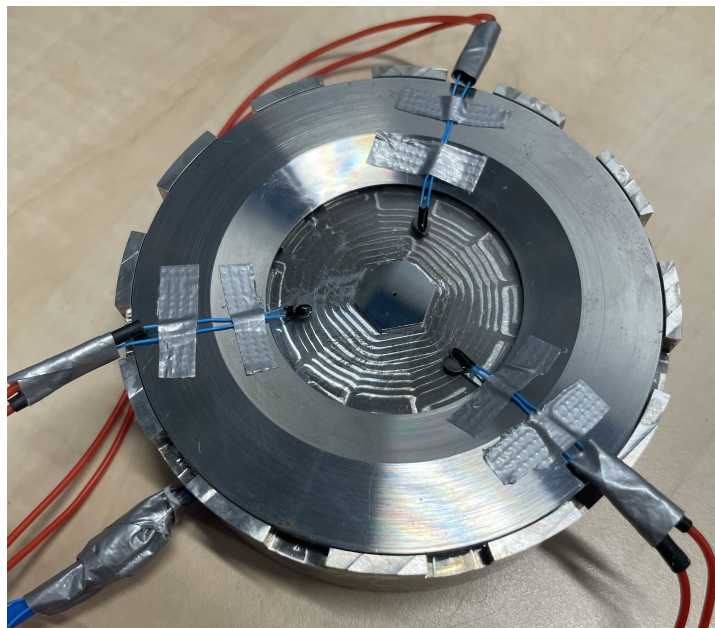


Figure 4.6: Air bearing system with the quartz glass wafer with thermistors attached and constraining ring on top.

④ Film Height Measurements

For the designed air bearing system, the passive film height is defined as the height difference between the surface of the unit cell and the surface of the outer segments, assuming the wafer is perfectly flat. However, in reality, wafers exhibit a total thickness variation (TTV) of less than 10 μm and an unspecified bow typically around 30 μm or less. The constraining ring is intended to mitigate the influence of bowing on the measurements. The effectiveness of this ring will be further evaluated in [Section 4.3.2](#). Moreover, no significant thickness variations were observed in micrometer measurements taken across the surface of the wafer. However, these variations might still exist and

could influence the distance between the bottom of the wafer and the surface of the unit cell.

In previous research conducted by He [13], as presented in [Appendix A.4](#), a confocal sensor was used to measure the film height. This sensor is capable of measuring both the local thickness of the wafer and the distance between the bottom of the wafer and the surface of the unit cell. While it requires the use of a transparent wafer, such as the one employed here, it is limited to point-by-point measurements. As a result, extensive measurements would be necessary to map the entire film height across the unit cell. However, at the time of performing this research, the confocal sensor was no longer available for use.

Instead, it was opted to use the surface height measurements provided by the air bearing system's manufacturer as the passive film height data. These measurements were combined with wafer deformation data, obtained using a technique inspired by free-surface synthetic schlieren, to determine the total film height. This approach was adopted based on the recommendation of a supervisor and follows the methodology outlined in [14].

In this setup, the wafer deformation measurement requires a wafer reflective in the visible light spectrum, with a random dot pattern positioned at a shallow angle on one side and a camera placed at the same angle on the other side. The camera captures the reflection of the dot pattern on the wafer's surface. When the wafer deforms, individual dots in the image exhibit translational shifts. By comparing a reference image with an image of the deformed state, a vector field can be generated. This vector field is then correlated to the local height gradients of the surface and integrated to obtain the wafer's height distribution. Unlike the confocal sensor, this technique allows for simultaneous mapping of the entire visible surface. Further details regarding the implementation and calibration of this technique are provided in [Appendix G](#).

4.2 Modelling

4.2.1 Wafer Deformation

Ring Constraint

In [Section 3.4](#), the wafer was assumed to be perfectly clamped at the clamping radius r_4 . However, by using the weight of a ring as the clamping method, the wafer is not fully constrained. Even if the weight of the ring exceeds the upward force exerted on the wafer by the pressure supply, the wafer is expected to deform more than with a perfect constraint. To account for this, the deformation of the wafer was analyzed using the *Solid Mechanics* physics interface in COMSOL, incorporating contact modelling. A schematic representation of the axisymmetric geometry used for the contact modelling is illustrated in [Figure 4.7](#), with the corresponding system parameters shown in [Table 4.3](#). For the thickness of the wafer, the measured value of $518 \mu\text{m}$ was used. The thickness of the ring at its inner radius r_4 is denoted by $t_{r,1}$, while r_5 represents the radius at which the ring reaches its full thickness $t_{r,2}$.

The outer radius of the quartz glass wafer is represented by r_6 . Although the air bearing system does not uniformly support the wafer around its entire circumference due to the wedge design, this simplification still provides valuable insight into the deviations caused by the imperfect constraint.

Table 4.3: COMSOL parameters used for the modelling of the designed test setup.

Parameter	Value	Unit
r_1	0.5	mm
r_2	7	mm
r_3	7.5	mm
r_4	25	mm
r_5	35	mm
r_6	50	mm
r_f	120	μm
p_1	1.2×10^5	Pa
p_2	1.0×10^5	Pa
b	15	μm
t_b	5	mm
$t_{r,1}$	0.5	mm
$t_{r,2}$	4	mm
t_w	518	μm
T_{air}	293	K
T_s	293	K

Since the wafer is not rigidly connected to the ring and air bearing system, an assembly was formed instead of a union, with *Contact Pairs* on both boundaries. These *Contact Pairs* allow the wafer to locally lose contact with the ring and air bearing system, using the *Augmented Lagrangian* method. A fixed constraint was added to the bottom of the air bearing system and a prescribed displacement of zero in the z -direction was set for the bottom right corner of the wafer in the 2D axisymmetric geometry. The pressure distribution, derived from the creeping flow assumption, was applied as a boundary load between the center of the wafer and r_2 . Gravity was also enabled to account for the weight of the ring.

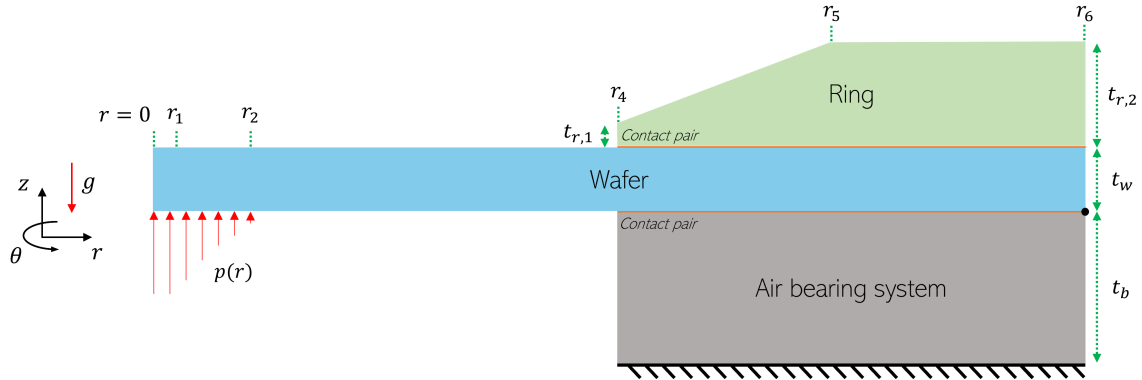


Figure 4.7: Schematic representation of the axisymmetric geometry including the outer wedges of the air bearing system and the ring used for the contact modelling simulation.

The resulting 3D deformation field of the axisymmetric geometry with the constraining ring is presented in Figure 4.9. At the center of the wafer, the maximum deformation reaches $10.2 \mu\text{m}$ for this specific set of parameters. Furthermore, a detailed 2D view of the deformation field near the clamping radius is depicted in Figure 4.8, showing localized detachment of the wafer from the air bearing system. On top of that, it can be seen that the ring only remains in contact with the wafer at its inner radius, resulting in a concentrated ring load on the wafer at the clamping radius. The remainder of the bottom surface of the ring remains fully detached from the top surface of the wafer due to the high stiffness of the ring.

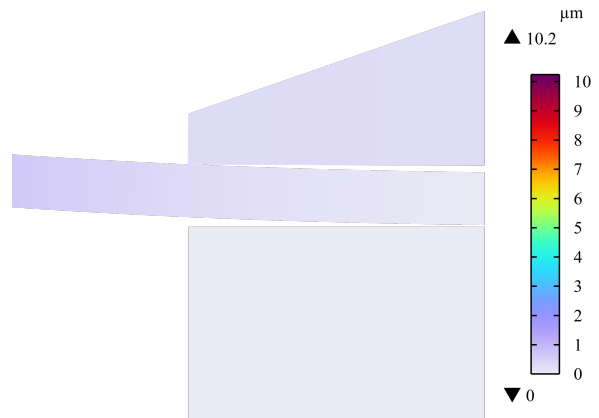


Figure 4.8: Detailed 2D view of the deformation field near the clamping radius.

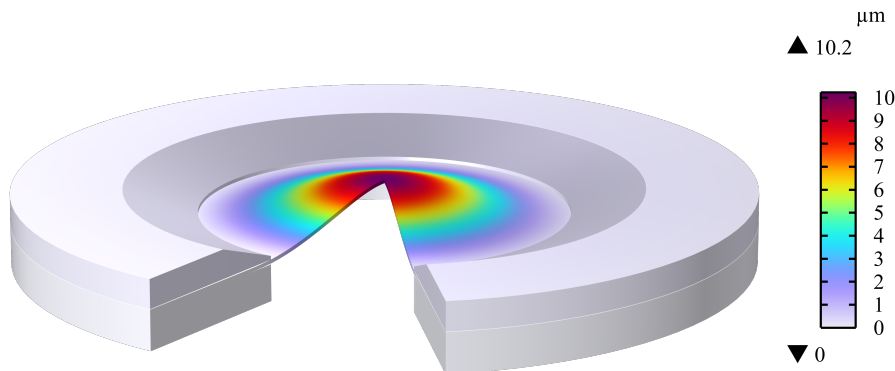


Figure 4.9: 3D deformation field resulting from the contact modelling simulation of the axisymmetric geometry.

Ring-Equivalent Constraints

Since the contact modelling of the full assembly substantially increases the computational effort, an equivalent set of boundary conditions was established, replicating the constraints imposed by the ring and air bearing system. At the inner radius of the ring, a *Ring Load* was applied with a magnitude equal to the weight of the ring, $F_{W,ring}$ (Equation D.2). To represent the air bearing system, a *Spring Foundation* boundary condition was applied at the bottom of the wafer between r_4 and r_6 , with its stiffness characteristic shown in Equation 4.1. Since the air bearing system only restricts downward deformations of the wafer in this region, the spring foundation was assigned a high stiffness for negative deformations ($w < 0$), while no stiffness was applied for upward deformations ($w \geq 0$). A stiffness of $k = 1 \times 10^{12}$ N/m was found to be suitable. As before, a prescribed displacement of zero in the z -direction was applied to the bottom edge of the wafer. A schematic representation of the geometry with the ring-equivalent constraints is depicted in Figure 4.10.

$$k_z(w) = \begin{cases} k & \text{if } w < 0 \\ 0 & \text{if } w \geq 0 \end{cases} \quad (4.1)$$

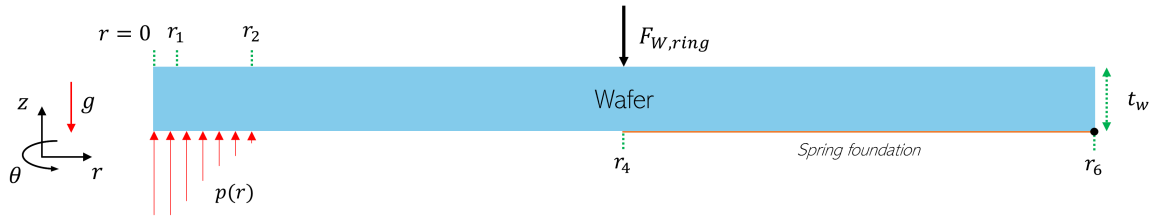


Figure 4.10: Schematic representation of the geometry with ring-equivalent constraints.

To compare the deformation of the wafer under three conditions: a fixed constraint at the clamping radius, a constraining ring on top of the wafer, and a ring-equivalent set of constraints, the maximum wafer deformation for each case was plotted against varying inlet pressures, as shown in Figure 4.11. The results reveal that the deformation under the ring and ring-equivalent constraints differ only slightly for inlet pressures below 1.175 bar. This is because, below this pressure threshold, the constraining ring has not fully detached from the top surface of the wafer, thereby deviating from the ideal ring load scenario. Above this threshold, the deformation for both ring constraints becomes virtually identical. The fixed constraint case, on the other hand, deviates notably from the other two, showcasing how the ring does not perfectly constrain the wafer at the clamping radius.

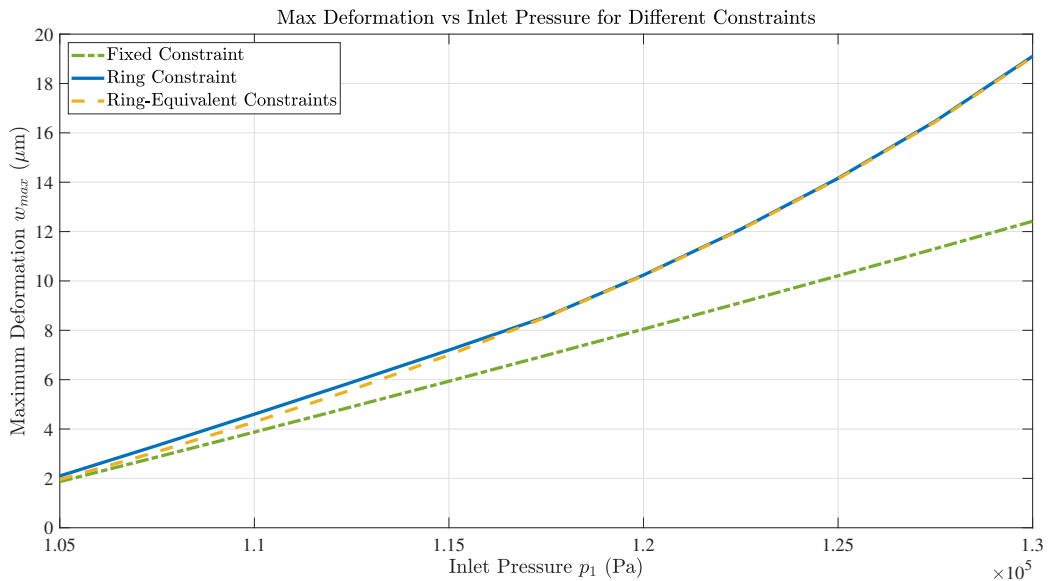


Figure 4.11: Maximum wafer deformation for a varying inlet pressure for three types of constraints.

Test Setup Model

Based on the wafer deformation results outlined above, the final geometry used to model the experimental test setup was constructed as shown in Figure 4.12. This geometry combines the fluid-structure interaction geometry shown in Figure 3.21 with the mechanical boundary conditions outlined in Figure 4.10.

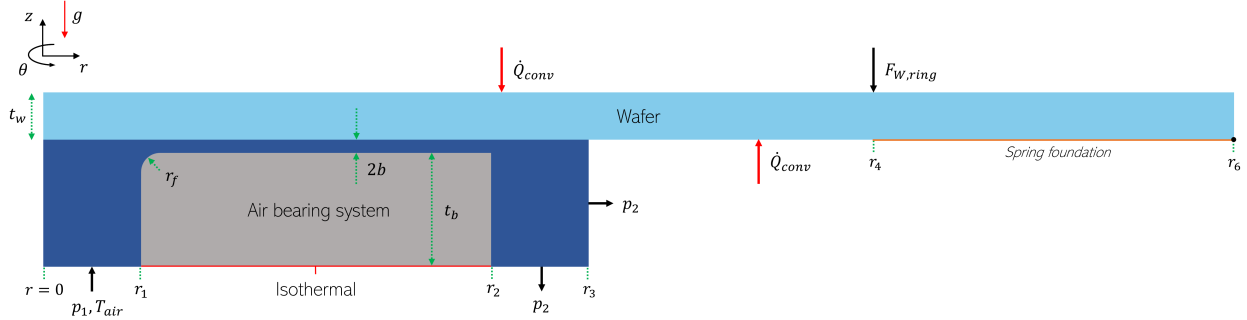
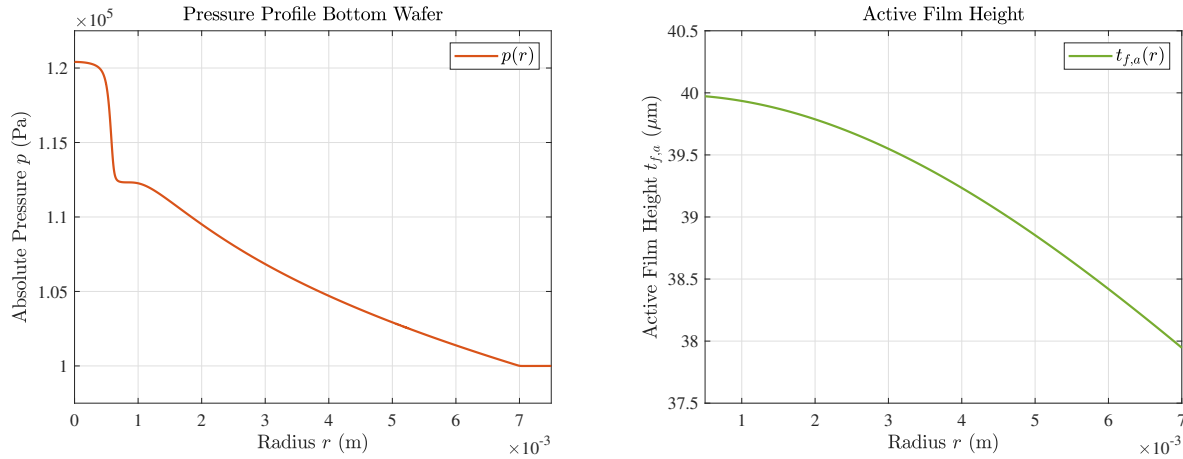


Figure 4.12: Schematic representation of the final geometry used to model the experimental test setup.

The pressure profile along the bottom of the wafer between $r = 0$ and r_3 , obtained using this geometry, is presented in Figure 4.13a. Additionally, the corresponding active film height between r_1 and r_2 is shown in Figure 4.13b. At the inner radius of the unit cell, the active film height is $40.0 \mu\text{m}$, corresponding to a wafer deformation of $10.0 \mu\text{m}$. This deformation is slightly lower than the value found for the ring-equivalent constraint in Figure 4.11, as the creeping flow pressure distribution used in that simulation does not account for the pressure drop when entering the thin film. At the outlet of the thin film, r_2 , the wafer deformation reaches $37.9 \mu\text{m}$. The total variation in film height is thus expected to be approximately $2.1 \mu\text{m}$ at an absolute inlet pressure of 1.2 bar.



(a) Pressure profile at the bottom of the wafer between $r = 0$ and r_3 .

(b) Active film height over the unit cell surface between r_1 and r_2 .

Figure 4.13: Pressure profile and active film height along the radius of the unit cell.

The remaining modelling results obtained using this geometry, including temperature profiles and flow rates, will be presented alongside the experimental results in Section 4.3.

4.2.2 Time-Dependent Study

Up to this point, only stationary simulations have been conducted in COMSOL to assess the system's steady-state behaviour. However, the transient response of the system also needs to be taken into account when performing physical tests. Although the primary focus of the measurements will

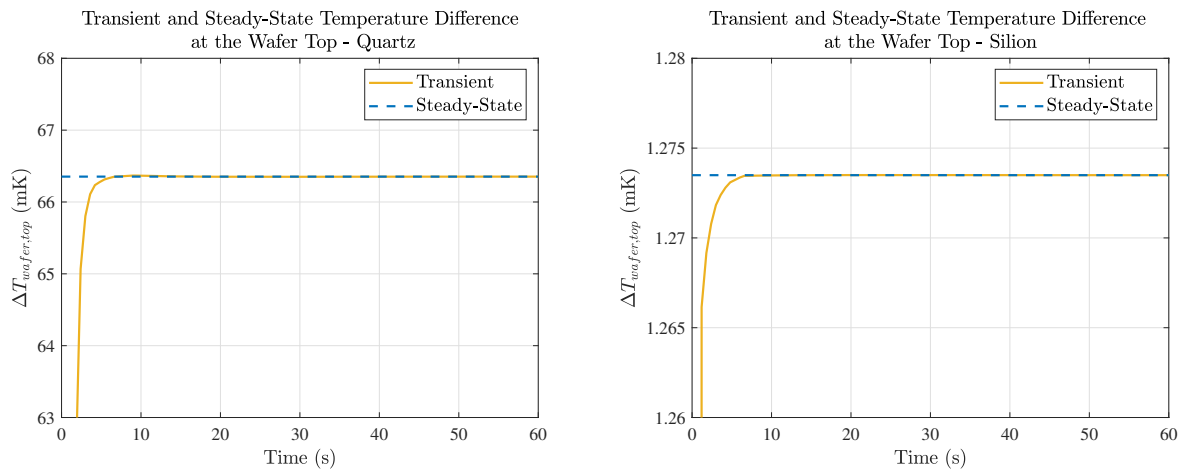
be on the steady-state performance, determining the system's settling time is crucial. To estimate this, a time-dependent study was conducted in COMSOL.

Given the computational demands of transient simulations, a simplified geometry was used, excluding the full size and deformation of the wafer. The geometry closely resembles that shown in Figure 3.16, but includes a rounded inlet with a radius of $120\ \mu\text{m}$ and the measured wafer thickness. Additionally, the air bearing system was assumed to be isothermal. Since wafer deformation was not considered, the active film height is smaller than in reality. However, this simulation is intended solely to estimate the settling time.

To facilitate faster convergence, the simulation was initialized using a pressure ramp. A linear ramp was used, increasing the pressure from 1 bar absolute at $t = 0\ \text{s}$ to 1.2 bar absolute at $t = 1\ \text{s}$. This ramped approach helps mitigate excessive pressure gradients within the domain, significantly reducing simulation times.

To better understand the system's transient behaviour, the temperature difference at the top surface of the wafer was plotted over time, as shown in Figure 4.14. Simulations were performed for quartz glass and silicon wafers, showing similar short settling times. The steady-state temperature distribution was reached within approximately 10 seconds for both wafer materials. This rapid stabilization is attributed to the wafer's low thickness and thermal mass, allowing it to quickly absorb or release energy to reach equilibrium.

Furthermore, the impact of the wafer material on the magnitude of the temperature difference is evident. Due to its lower thermal conductivity, the quartz glass wafer exhibited a temperature difference of approximately 66.4 mK, whereas the silicon wafer showed a much smaller difference of 1.27 mK.



(a) Transient temperature response of the quartz glass wafer. (b) Transient temperature response of the silicon wafer.

Figure 4.14: Transient and steady-state temperature difference at the top surface of the quartz glass and silicon wafers.

Since only stationary measurements will be performed, the short settling time is beneficial. Under ideal conditions, with a uniform and steady inlet air temperature and pressure, measurements could begin after approximately 10 seconds. In practice, however, additional components such as the hosing, thermistors, and air bearing system will also require time to stabilize after the pressure supply is activated, extending the overall time required to reach steady-state conditions.

4.3 Experimental Results

This section presents the experimental results obtained from the test setup. The primary goals are to validate the numerical models, gain insights into the thermomechanical behaviour of the wafer under different operating conditions, and explore the effectiveness of various measurement techniques. Multiple measurements were performed, focusing on four key quantities: pressure, flow rate, deformation, and temperature. These quantities were evaluated through three types of measurements: the relationship between pressure and flow rate, pressure and deformation, and pressure and temperature. Additionally, the in-situ self-heating of the thermistors was assessed. The results of these measurements are outlined in [Section 4.3.1](#), followed by a discussion in [Section 4.3.2](#), where the experimental findings are compared with theoretical predictions. This analysis provides a deeper understanding of the system's behaviour, evaluates the reliability of the measurement approaches, and highlights discrepancies between the modelled and experimental data.

4.3.1 Measurements

Pressure vs Flow Rate

The initial experiments with the completed test setup consisted of determining the relationship between the pressure drop across the unit cell and the air flow rate through the supply hose. Since this relationship primarily depends on the dimensions of the thin film, it serves as an indicator of the final film height in the test setup.

To measure the relationship between the pressure drop and the flow rate, the flow rate was varied from 0.2 to 1.2 L/min using the rotameter, using increments of 0.1 L/min. At each flow rate, the corresponding pressure drop was read from the pressure gauge. Both the rotameter and the pressure gauge are subject to measurement uncertainty, which arises from the device accuracy and the read-off error associated with the analog readings.

According to the rotameter manufacturer, the flow rate is measured with an accuracy of $\pm 4\%$ of its full scale. This introduces the following uncertainty in the flow rate measurement:

$$u_{a,Q} = 0.04 \cdot 1.2 \text{ L/min} = 48 \text{ mL/min} \quad (4.2)$$

The read-off error depends on the readability of the rotameter, which is influenced by the smallest flow rate division and the spacing between these divisions. For this rotameter, the smallest division is 0.05 L/min. Given the narrow spacing between divisions, half of the smallest division is used as an estimate for the read-off uncertainty:

$$u_{r,Q} = 25 \text{ mL/min} \quad (4.3)$$

The combined uncertainty on the flow rate measurement is calculated by adding the squares of the individual uncertainties and taking the square root of the sum [15]:

$$u_Q = \sqrt{u_{a,Q}^2 + u_{r,Q}^2} = 54 \text{ mL/min} \quad (4.4)$$

The uncertainty in the pressure measurement is calculated using a similar approach. Given the pressure gauge's accuracy of $\pm 1.6\%$, the associated uncertainty is:

$$u_{a,p} = 0.016 \cdot 0.6 \text{ bar} = 9.6 \text{ mbar} \quad (4.5)$$

The smallest division on the pressure gauge is 0.02 bar. Due to the relatively large spacing between divisions, pressure values can be interpolated more easily. As a result, a quarter of the smallest division is taken as the read-off uncertainty:

$$u_{r,p} = 5.0 \text{ mbar} \quad (4.6)$$

The total uncertainty in the pressure measurement is calculated as:

$$u_p = \sqrt{u_{a,p}^2 + u_{r,p}^2} = 11 \text{ mbar} \quad (4.7)$$

The measurement results for the 350 μm and 500 μm quartz glass wafers are shown in [Figure 4.15](#), including error bars accounting for measurement uncertainties. At low flow rates, the measured pressure drop is similar for both wafers, suggesting a comparable active film height. However, at higher flow rates, the 350 μm wafer exhibits a lower pressure drop than the 500 μm wafer. This suggests that the 350 μm wafer deforms more due to its lower flexural rigidity, increasing the active film height.

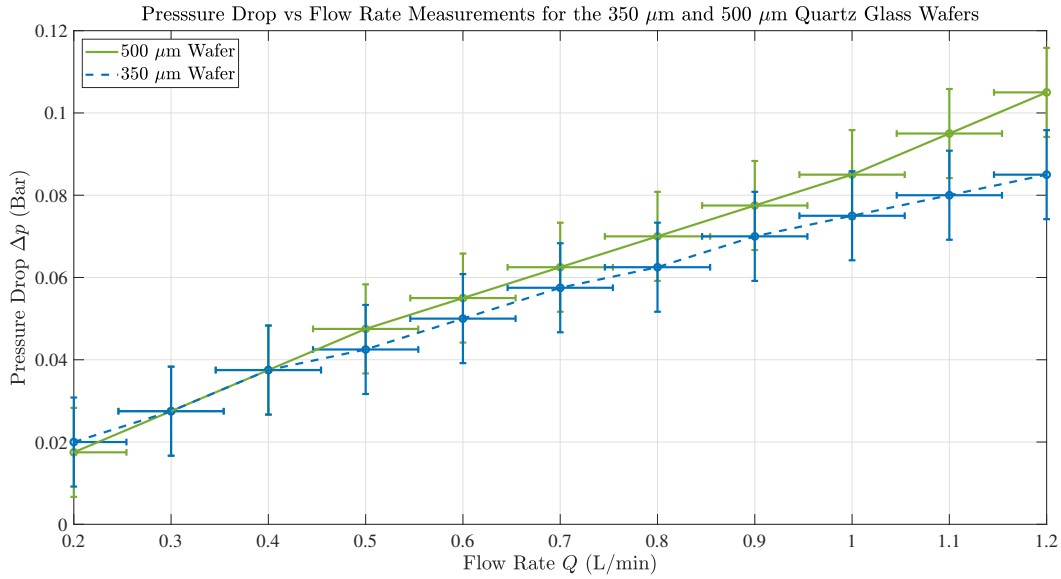


Figure 4.15: Pressure drop measurements for the 350 μm and 500 μm quartz glass wafers at varying flow rates.

Wafer Deformation vs Pressure

After measuring the relationship between the pressure drop across the unit cell and the flow rate through the supply hose, the wafer deformation was recorded at varying pressures and flow rates. By combining these measurements with the pressure drop–flow rate relationship, the active film height can theoretically be determined.

The wafer deformation was measured using the optical technique described in [Appendix G](#). Once the calibration procedure outlined there was completed, deformation tests were conducted for both the 350 μm and 500 μm quartz glass wafers. Since the relationship between inlet pressure and deformation is generally more straightforward than that between flow rate and deformation, measurements were taken at evenly spaced pressure levels. The pressure was varied from 0 to 0.2 bar in increments of 0.02 bar. However, since the flow rate was measured to be much higher than expected at each pressure level, the flow rate exceeded the maximum measurable value of the rotameter for some measurements.

For each wafer, a reference image was first captured with the air supply disabled. Subsequently, images were taken at each pressure level and compared to the reference image to construct the vector fields. The vector fields were then exported to MATLAB for further processing. A typical raw vector field obtained with DaVis is shown in [Figure 4.16a](#), with the image specifically depicting the 350 μm wafer at a pressure drop of $\Delta p = 0.1$ bar. To eliminate noise outside the visible wafer area, a mask was applied, indicated by the red ellipse. The resulting masked vector field, shown in [Figure 4.16b](#), reveals a maximum dot displacement of approximately 9 pixels.

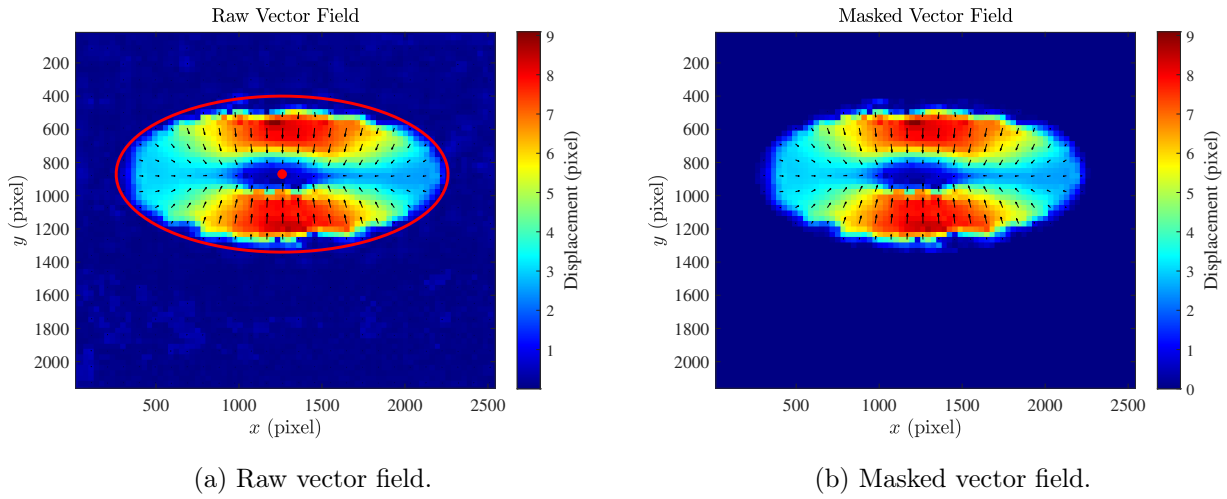


Figure 4.16: Vector field of the 350 μm wafer for $\Delta p = 0.1$ bar, before and after applying the elliptical mask.

Next, the surface height reconstruction was performed using the *PIVMat* toolbox to obtain the deformed surface height in pixels. Figure 4.17 presents the resulting surface height profile from both an isometric and a side view. The wafer was expected to exhibit a perfectly axisymmetric deformation profile, with maximum deformation at the center and approximately zero deformation at the clamping radius. While the overall deformation closely follows this expectation, slight variations are observed along the wafer's circumference, where a constant value was expected. This pattern is observed in all deformation measurements.

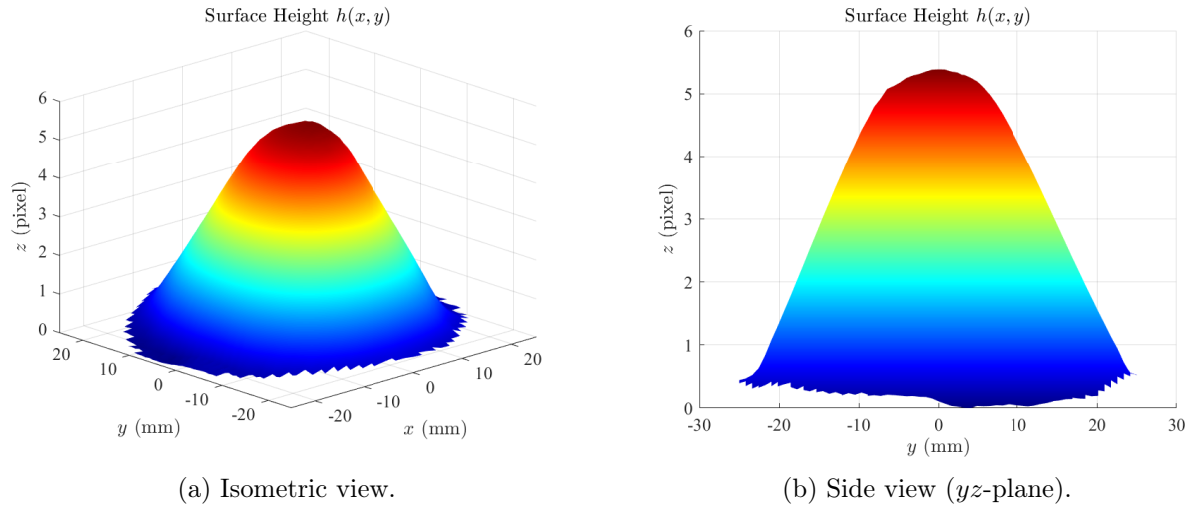


Figure 4.17: Reconstructed surface height of the 350 μm wafer for $\Delta p = 0.1$ bar in isometric and side views.

Then, the surface height profiles were converted to micrometers using the scaling factor obtained through the calibration procedure:

$$S = \frac{\Delta z_{\text{real}}}{\Delta z_{\text{pixels}}} \approx 2.3 \frac{\mu\text{m}}{\text{pixel}} \quad (4.8)$$

The maximum deformation was consequently determined by subtracting the minimum surface height from the maximum surface height. The resulting maximum wafer deformation was plotted for each pressure level, incorporating the pressure measurement uncertainty calculated in Equation 4.7.

Figure 4.18 presents the measured wafer deformation as a function of the pressure difference across the unit cell for the 350 μm and 500 μm quartz glass wafers. Both graphs show a similar trend, but the 350 μm wafer exhibits a significantly larger maximum deformation at each pressure level, as expected due to its lower flexural rigidity.

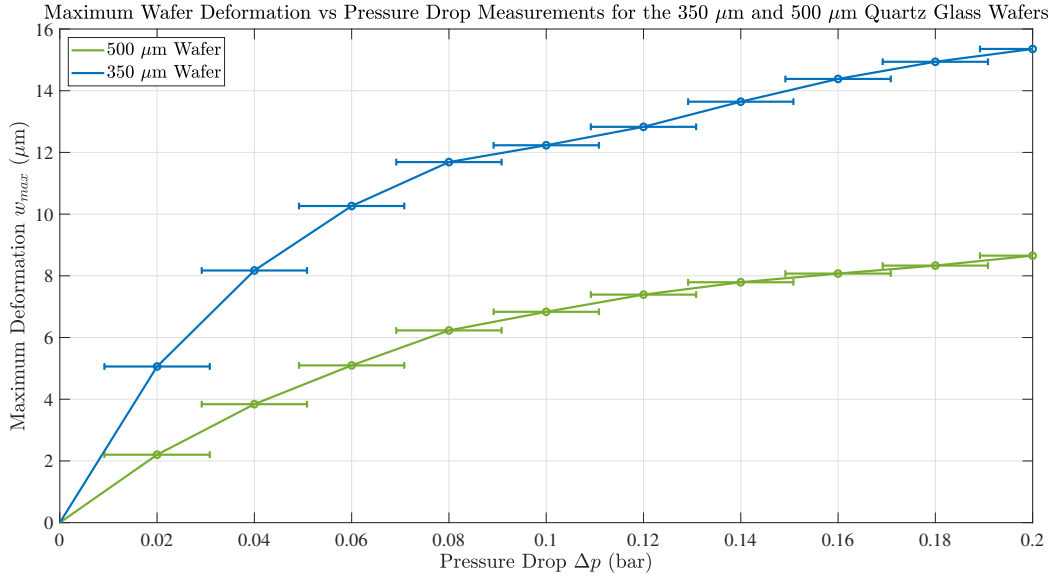


Figure 4.18: Maximum deformation measurement for the 350 μm and 500 μm quartz glass wafers at varying pressure differences across the unit cell.

Temperature

The final measured quantity was temperature. Before assessing the wafer temperature distribution, the in-situ self-heating of the thermistors was evaluated by combining the two thermometry techniques outlined in Section 4.1.3. Subsequently, the temperature distribution of both wafers was measured at varying pressures and temperature differences between the inlet and ambient air. In principle, these measurements, along with flow rate and deformation measurements, can be used to close the loop on the COMSOL model.

Self-Heating

As noted in Appendix F.5, self-heating of the thermistors in the test setup could significantly affect the accuracy of the temperature measurements. In water, self-heating was measured at 7.6 mK when the supply voltage was increased from 1 V to 2.5 V and 45.8 mK when further increased to 5 V. In air, self-heating was estimated to range between 0.60 K and 0.73 K at 2.5 V, depending on the operating temperature.

To assess self-heating in the test setup, temperature readings from the thermistors on the 500 μm quartz wafer and air supply hose were logged over a period of 22 minutes. A supply voltage of 1 V was applied for the first 5 minutes, then increased to 2.5 V for the next 5 minutes, and finally raised to 5 V for the remaining 12 minutes.

Figure 4.19 shows the thermistor measurement data, with black dotted vertical lines marking each voltage transition. As expected, the temperature increased with each voltage step, with a more pronounced rise from 2.5 V to 5 V. However, due to the lack of an accurate reference temperature, the exact self-heating effect at each voltage level could not be determined. Since all subsequent temperature measurements were performed at 2.5 V, a small temperature correction was applied: 50 mK was subtracted from all thermistor readings, as each thermistor showed at least a 50 mK increase when transitioning from 1 V to 2.5 V.

The thermistor on the inlet air hose showed more self-heating than those on the wafer surface, especially at 5 V, where its readings differed the most.

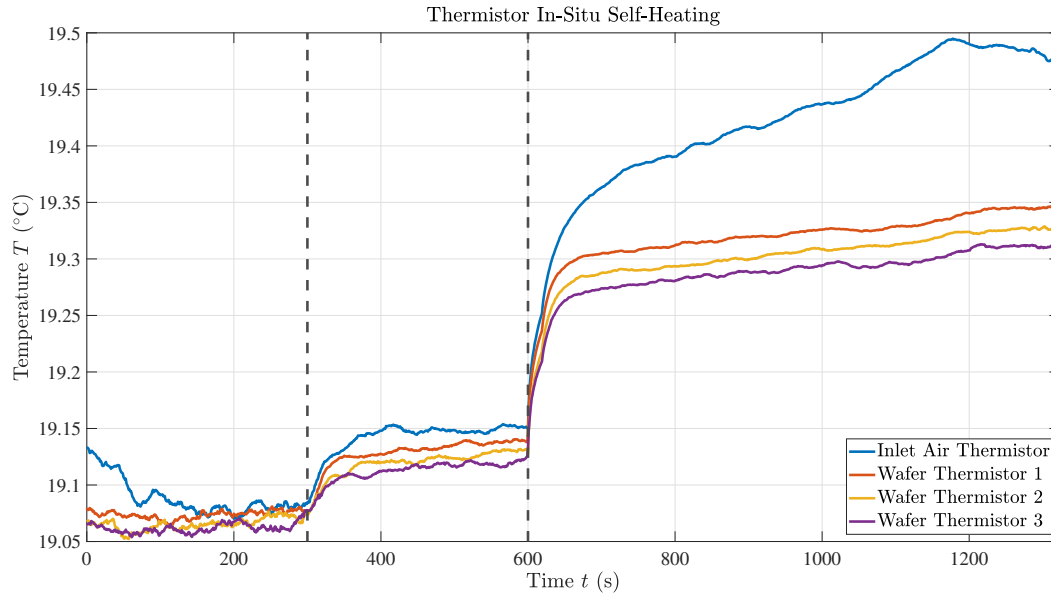


Figure 4.19: Thermistor in-situ self-heating measurements for the 500 μm quartz glass wafer.

To examine the impact of thermistor self-heating on the wafer's temperature distribution, infrared images were captured just before powering the thermistors and at each supply voltage level. Figure 4.20 presents the infrared images taken at supply voltages of 0 V and 5 V. The absolute temperatures in both images differ notably due to the limited accuracy of the infrared camera. Additionally, both images reveal a hotspot beneath the wafer's center, which is attributed to the reflection of the infrared camera on the wafer surface. Finally, the ring appears significantly hotter than the wafer, due to its higher reflectivity.

However, at the locations of the thermistors, the temperature distribution remains unchanged between the two images. Even after a detailed analysis of the corresponding pixel readings, no evidence of thermistor self-heating on the wafer surface was found. The same applies at lower supply voltages. This suggests that the self-heating effect is primarily internal.

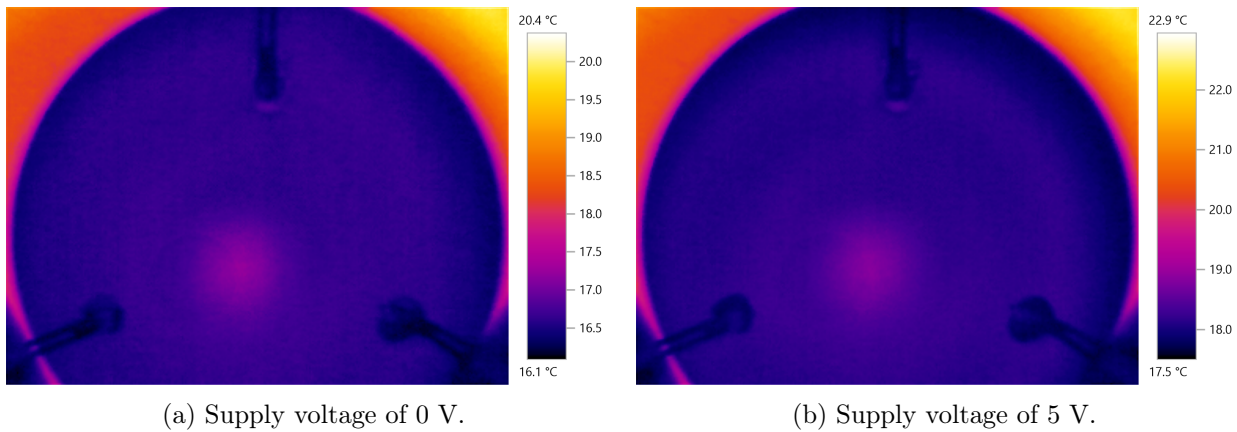


Figure 4.20: Infrared images captured of the 500 μm quartz glass wafer at two supply voltages.

Temperature vs Pressure Drop

To measure the temperature distribution of the wafer as a function of the pressure drop between the unit cell inlet and outlet, the supply pressure was varied between 0 bar and 0.2 bar, in increments of 0.02 bar. Since no measurable temperature difference was expected at 0.02 bar, this pressure level was omitted. For the first 15 minutes of the measurement, no air was supplied to the system, serving as a reference measurement. The pressure was then increased to 0.04 bar for the following ten minutes to allow the system to adjust to the inlet air temperature. After that, the pressure was further increased by 0.02 bar every 5 minutes.

The thermistor readings for both the 350 μm and 500 μm quartz glass wafers over the full 65-minute

measurement period are shown in Figure 4.21 and Figure 4.22, respectively. The vertical black dotted lines indicate each pressure increase, with the current pressure value marked at the top. For both wafers, the three wafer thermistors show close to identical temperature readings throughout the measurement period. Furthermore, the temperature readings show a gradual increase between the start and end of the measurement, due to the uncontrolled measurement environment. However, the inlet air thermistor shows a temperature decrease as the supply pressure increases, indicating that the inlet air is colder than the ambient air. To estimate this temperature difference, the average deviation between the three wafer thermistors and the inlet air thermistor was calculated for two conditions: when no air was supplied and when the supply pressure was between 0.1 and 0.2 bar. The difference between these two average deviations represents the temperature difference between the inlet and ambient air. This method yielded a temperature difference of $\Delta T = T_{air} - T_{amb} = -0.13^\circ\text{C}$ for the $350\ \mu\text{m}$ wafer and $\Delta T = -0.12^\circ\text{C}$ for the $500\ \mu\text{m}$ wafer.

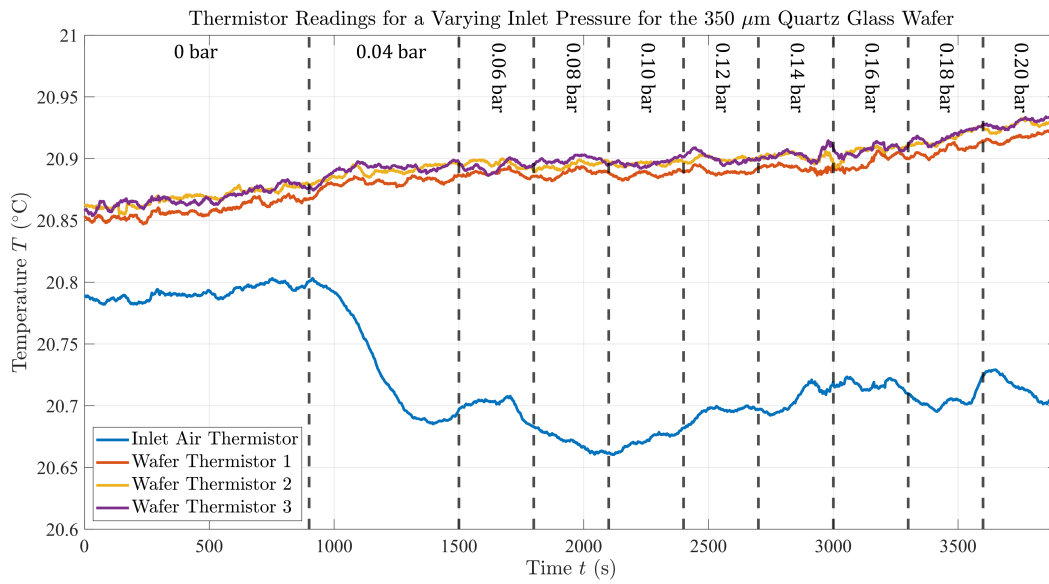


Figure 4.21: Thermistor readings of the wafer temperature distribution measurement at varying pressure drops for the $350\ \mu\text{m}$ quartz glass wafer.

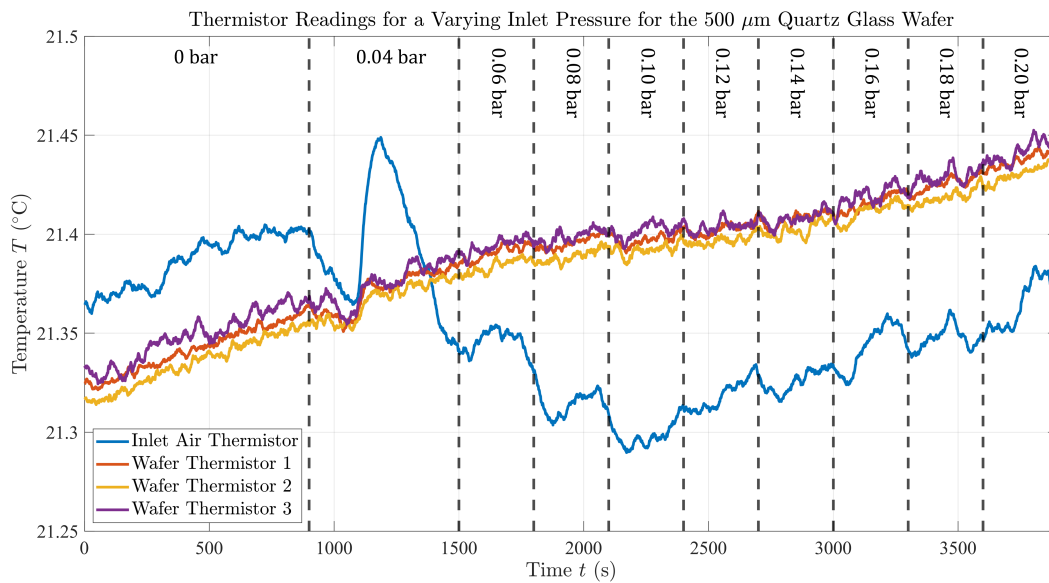


Figure 4.22: Thermistor readings of the wafer temperature distribution measurement at varying pressure drops for the $500\ \mu\text{m}$ quartz glass wafer.

At each pressure drop level, an infrared image was captured shortly before the next pressure increase. Figure 4.23 shows the resulting images at a pressure drop of 0 bar and 0.2 bar for the 500 μm wafer. At a pressure drop of 0 bar, no temperature fluctuations can be observed above the unit cell. However, at 0.2 bar, the unit cell inlet can clearly be recognized on the wafer surface by the circular cold spot in its center. Furthermore, just around this cold spot, a slightly hotter ring can be observed. Outside of the inlet area of the unit cell, the temperature distribution remains unchanged.

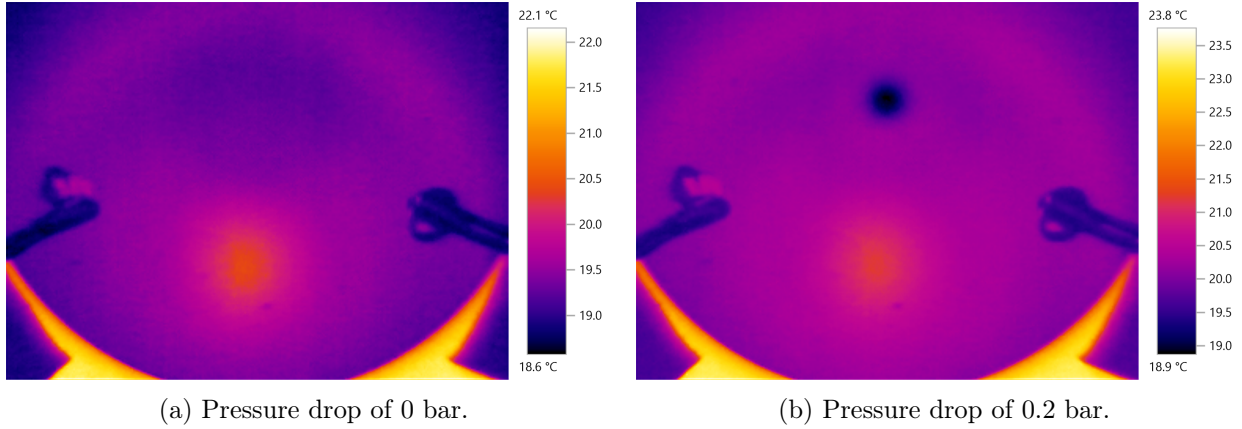


Figure 4.23: Infrared images captured of the 500 μm quartz glass wafer at two pressure drops.

To better quantify the temperature distribution of the wafer above the unit cell surface, pixel data from the infrared images was exported to MATLAB for post-processing. The pixel size was first converted to millimeters using the known distance between the unit cell inlet and the inner edge of the constraining ring, yielding a spatial temperature resolution of 66.7 μm . The temperature data was then plotted along a 14 mm horizontal line centered at the unit cell inlet to obtain the temperature profile across the entire unit cell diameter.

Since the absolute temperature values of the infrared images varied significantly, the profiles were calibrated using thermistor data. The average of the three wafer thermistors over the entire measurement period was used to align the infrared profiles at the outer two points, ensuring consistency between both methods. The calibrated profiles for the 350 μm and 500 μm wafers are shown in Figure 4.24 and Figure 4.25, using 0.04 bar pressure increments for clarity.

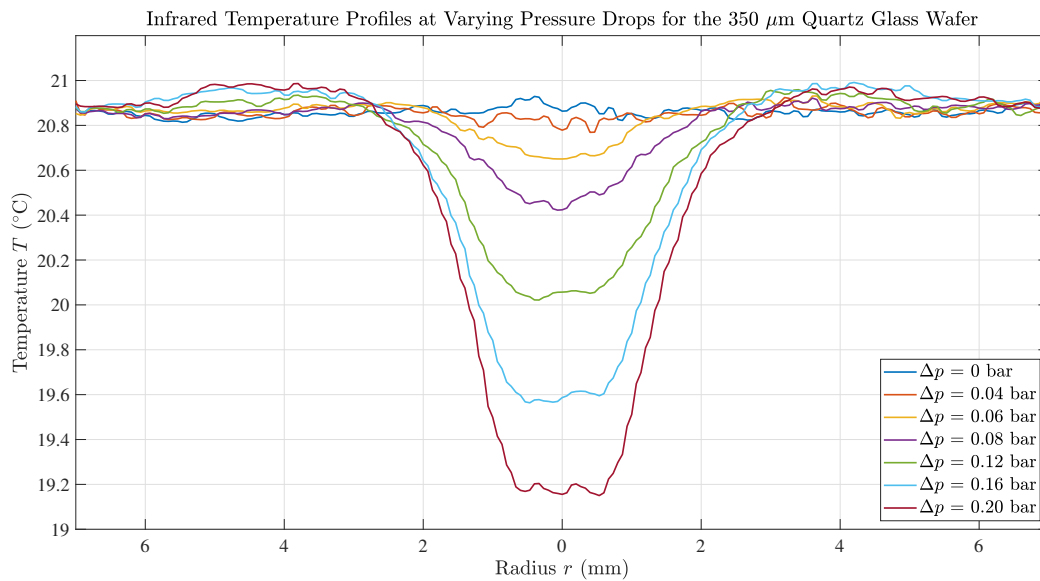


Figure 4.24: Infrared temperature profiles above the unit cell at varying pressure drops for the 350 μm quartz glass wafer.

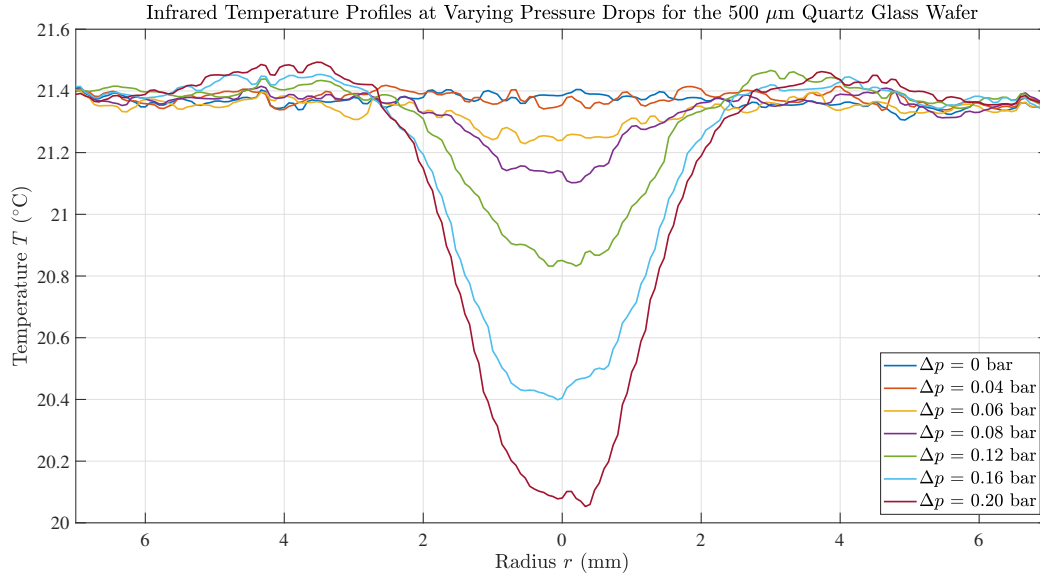


Figure 4.25: Infrared temperature profiles above the unit cell at varying pressure drops for the 500 μm quartz glass wafer.

At a pressure drop of 0 and 0.04 bar, no clear temperature fluctuations were observed among the inherent fluctuations of the infrared camera. However, for higher pressure drops, the temperature drop above the unit cell inlet can be seen to increase for an increasing pressure drop across the unit cell. Additionally, at higher pressure drops, the temperature can be seen to shortly increase above the baseline temperature outside of this cooling region, due to energy dissipation in the thin film. As expected, the temperature profiles are approximately axisymmetric around the unit cell inlet.

Comparing the infrared profiles of the two wafers, the temperature minima in the 350 μm wafer profiles appear broader and flatter than in the 500 μm wafer profiles. Additionally, the temperature drop above the unit cell is notably higher for the 350 μm wafer at each pressure value. To quantify this trend, the difference between the maximum and minimum temperature in each infrared profile was plotted against the corresponding pressure drop in Figure 4.26 for both wafers. Since no coherent temperature fluctuations were observed below a pressure drop of 0.06 bar, this data was excluded. As shown, the 350 μm wafer consistently exhibits a larger temperature difference compared to the 500 μm wafer. Moreover, both graphs reveal a steady increase in temperature difference with each pressure increment.

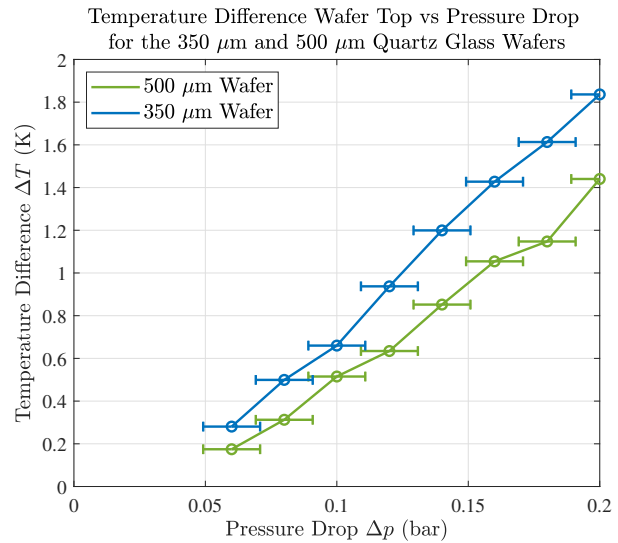


Figure 4.26: Temperature difference at the top surface of the 350 μm and 500 μm wafers for varying pressure drops.

Temperature vs Pressure Drop with Heated Inlet Air

To study the influence of the difference between the inlet air temperature and the ambient air temperature, an additional experiment was performed using heated inlet air. The inlet air was heated by extending the supply hose between the rotameter and pressure gauge by approximately 5 meters and coiling it inside a water-filled beaker. The beaker was placed on the heated magnetic stirrer, shown in Figure F.6, which was set to 220 $^{\circ}\text{C}$. Due to the poor heat transfer efficiency of the glass beaker, the water temperature was estimated to be around 80 $^{\circ}\text{C}$. The experiment

was conducted using the 500 μm wafer. Measurements were only conducted at a single pressure, as condensation formed inside the inlet hose, causing small water droplets on the wafer's bottom surface. When varying the inlet pressure, condensation tended to be released from the unit cell inlet, affecting the temperature measurements.

The measurement consisted of a 10-minute period with no air supply, followed by 30 minutes at a pressure drop of 0.10 bar across the unit cell. The corresponding thermistor readings are presented in Figure 4.27. A sharp increase in the inlet air thermistor temperature was observed when the air supply was activated, peaking at approximately 29 °C before gradually decreasing to a steady-state value of about 21 °C. The wafer thermistor readings remained stable throughout the measurement, suggesting that temperature variations were localized near the wafer's center. The temperature difference between the inlet air and ambient conditions was determined using the average deviation of the inlet air thermistor from the three wafer thermistors before air activation and after the 2000-second mark. The resulting temperature difference is given by $\Delta T = T_{air} - T_{amb} = 2.18$ °C.

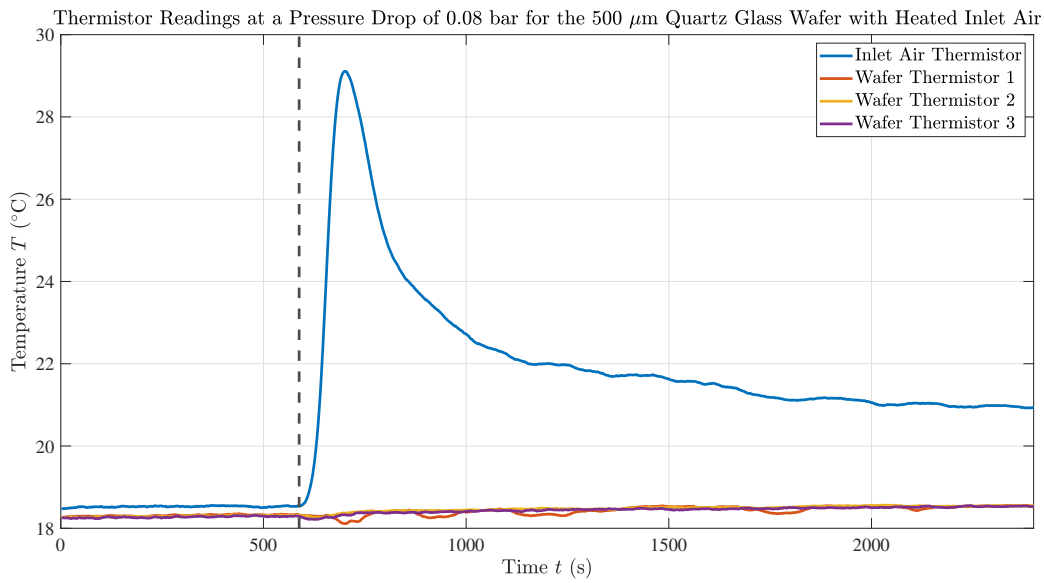


Figure 4.27: Thermistor readings of the wafer temperature distribution measurement at a pressure drop of 0 and 0.10 bar for the 500 μm quartz glass wafer with heated inlet air.

The corresponding infrared images at a pressure drop of 0 bar and 0.10 bar are shown in Figure 4.28. At a pressure drop 0.10 bar, the image shows a hotspot at the location of the unit cell inlet, instead of the previously observed coldspot. This indicates that the inlet air was indeed significantly warmer than the ambient air.

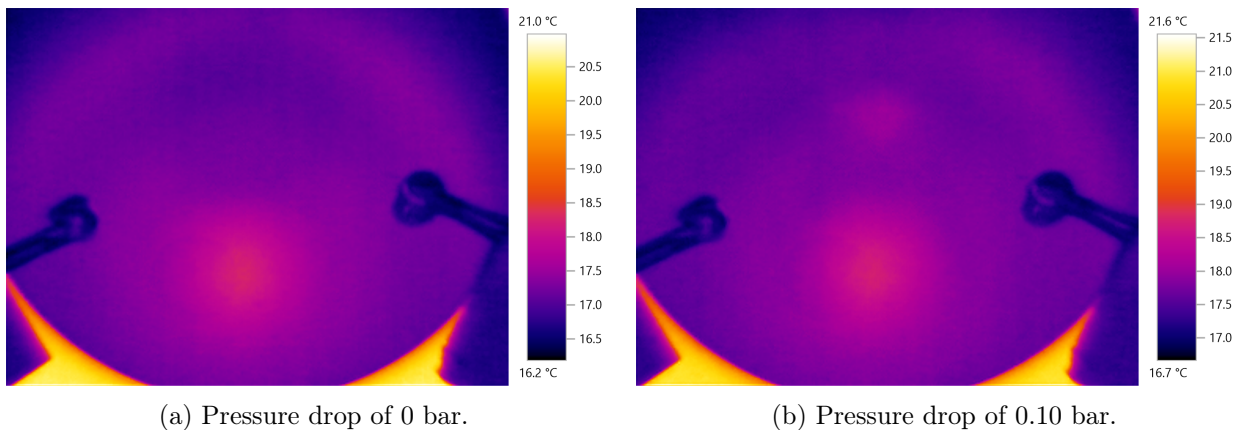


Figure 4.28: Infrared images captured of the 500 μm quartz glass wafer at two pressure drops with heated inlet air.

Using the same post-processing technique in MATLAB as described earlier, the temperature profiles shown in Figure 4.29 were obtained. Compared to the reference profile at a pressure drop of 0 bar, the 0.10 bar profile exhibits a notable temperature increase across the unit cell surface. The temperature difference between the minimum and maximum across the unit cell area was determined to be $\Delta T = 0.45$ °C.

Compared to the infrared profiles without heated inlet air, this profile exhibits a significantly broader temperature extreme, likely due to the larger temperature difference between the inlet air and ambient. This is further analyzed in the results discussion.

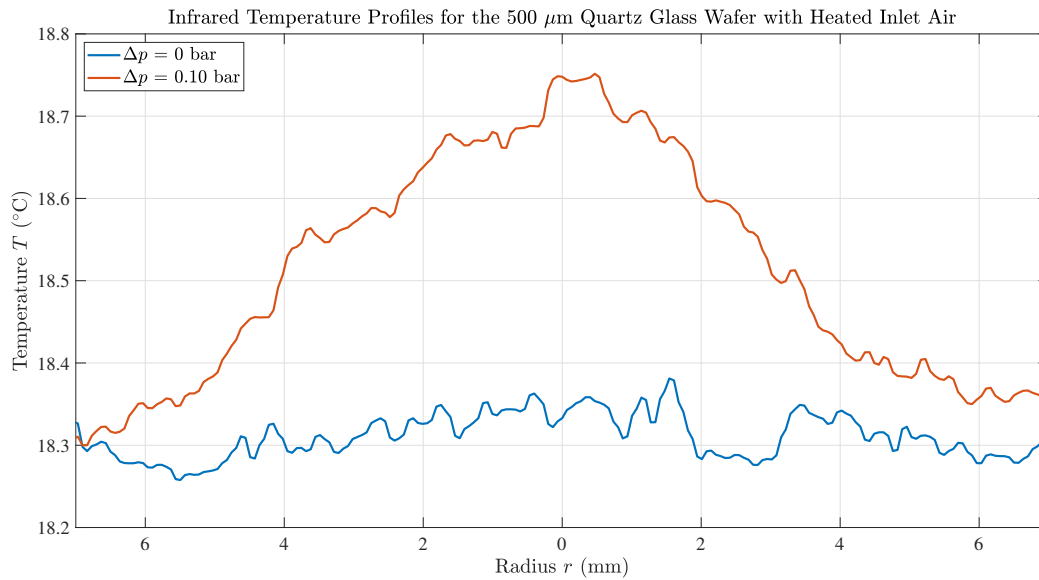
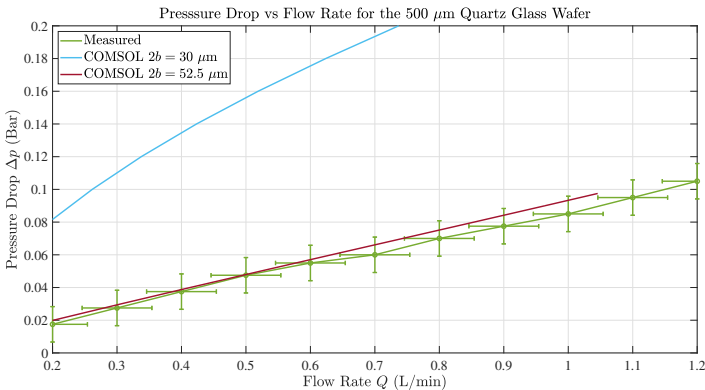
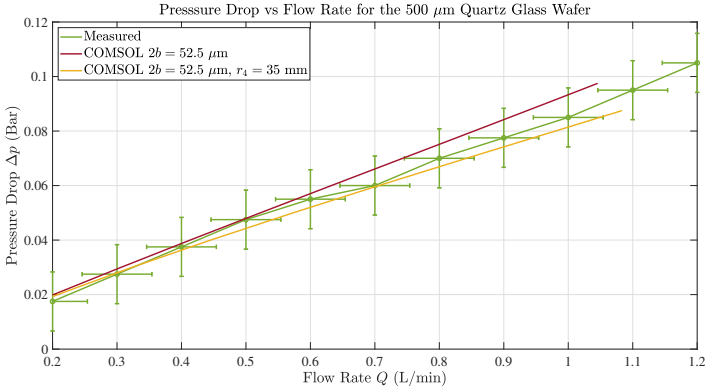
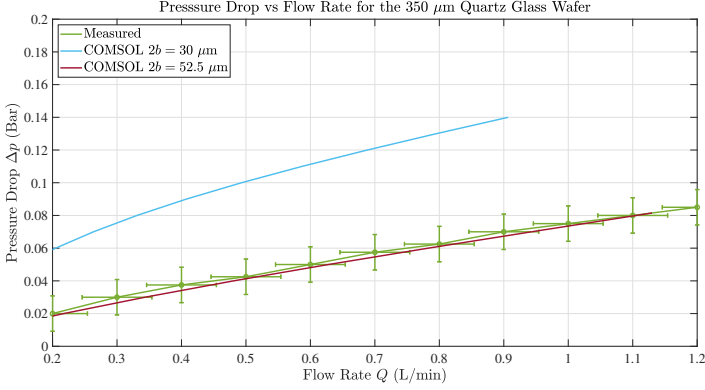
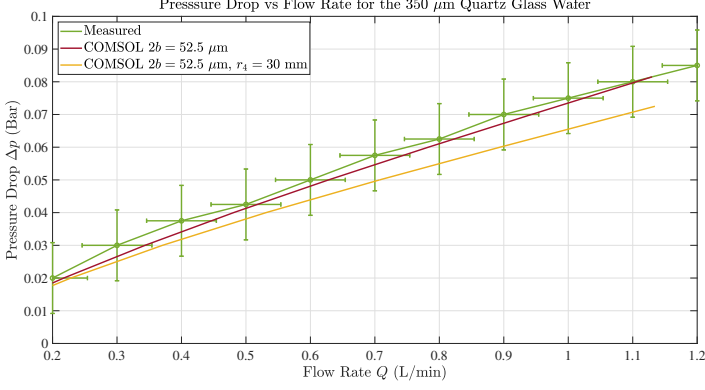


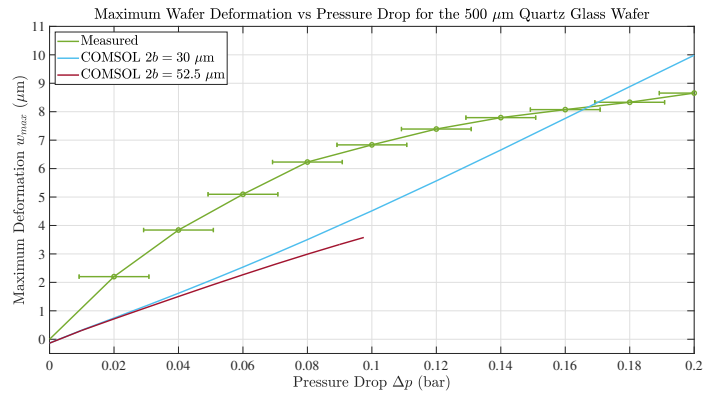
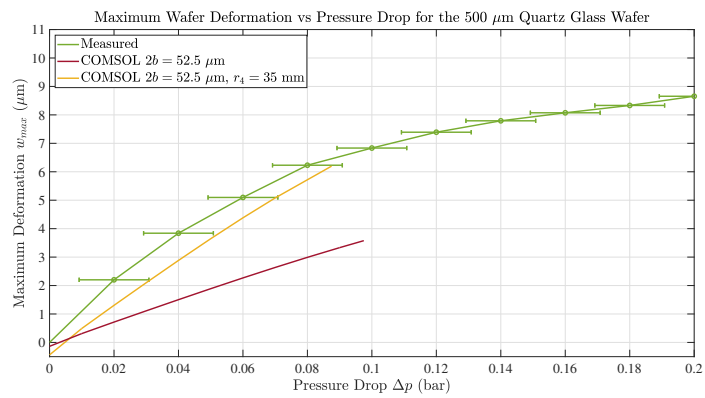
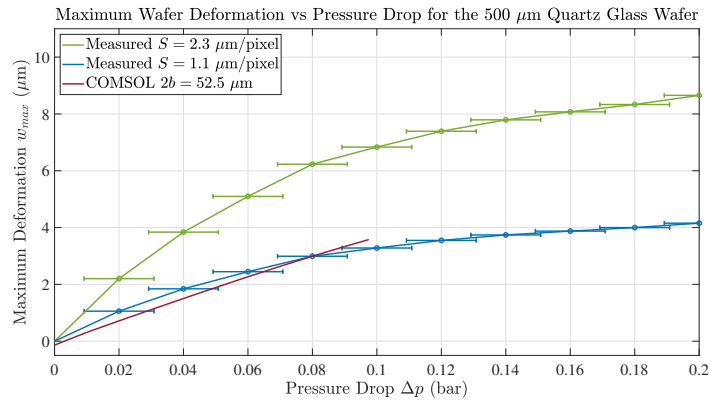
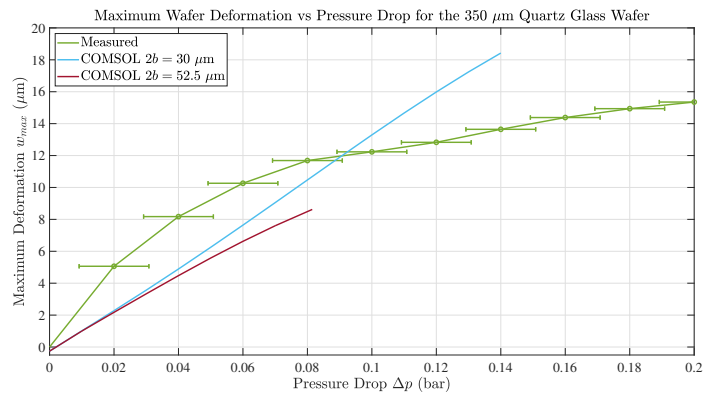
Figure 4.29: Infrared temperature profiles across the unit cell at a pressure drop of 0 and 0.10 bar for the 500 μm quartz glass wafer with heated inlet air.

4.3.2 Discussion

To evaluate the experimental data presented above, the discussion is structured as follows. First, an overview of all performed measurements, along with the COMSOL modelling results, is provided in Table 4.4. This overview is then followed by a detailed description of the possible sources of the discrepancies between the modelled and experimental data. Similar to the layout of the measurement results, each measurement is discussed separately. However, the cross-effects between the different measurements are also highlighted.

Table 4.4: Overview of all measurement and COMSOL simulations.

Measurements		COMSOL Simulations
Pressure vs Flow Rate	500 μm Wafer	 Figure 4.30: Varying film heights, 500 μm wafer.
		 Figure 4.31: Varying clamping radius, 500 μm wafer.
	350 μm Wafer	 Figure 4.32: Varying film heights, 350 μm wafer.
		 Figure 4.33: Varying clamping radius, 350 μm wafer.

500 μm
WaferFigure 4.34: Varying film heights, 500 μm wafer.Figure 4.35: Varying clamping radius, 500 μm wafer.Figure 4.36: Varying calibration scaling factors, 500 μm wafer.Figure 4.37: Varying film heights, 350 μm wafer.

350 μm
Wafer

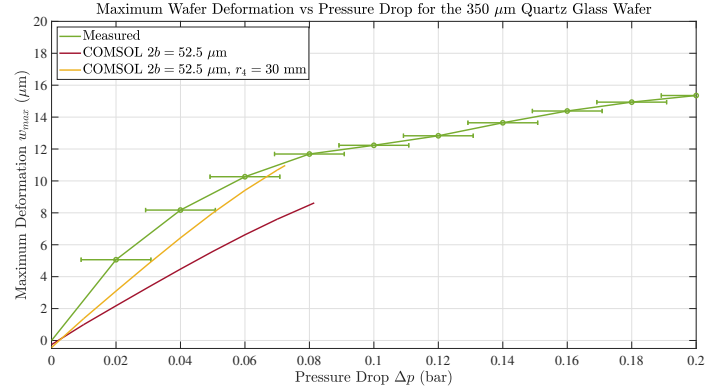


Figure 4.38: Varying clamping radius, 350 μm wafer.

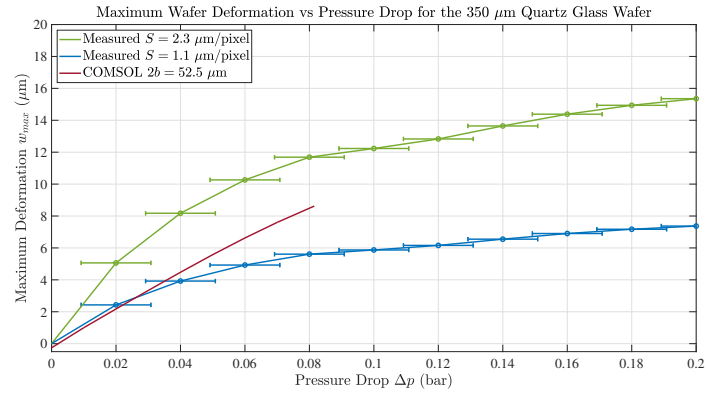


Figure 4.39: Varying calibration scaling factors, 350 μm wafer.

Temp.
vs Press.

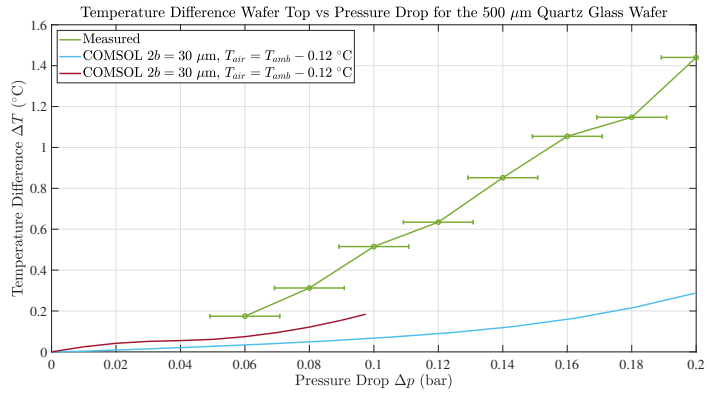


Figure 4.40: Varying film heights, 500 μm wafer.

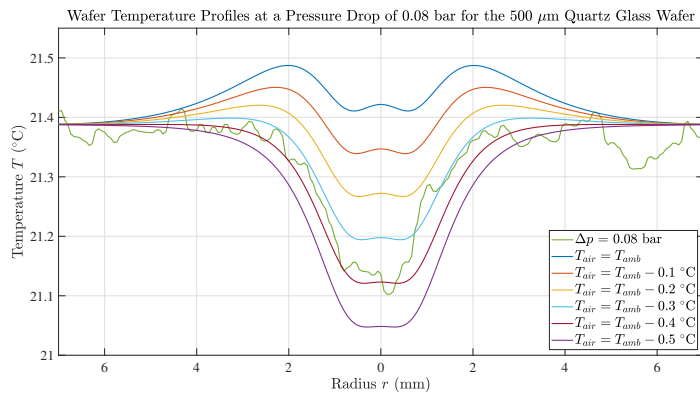


Figure 4.41: Varying inlet air temperatures (cold), 500 μm wafer.

Temperature vs Pressure	350 μm Wafer	Temp. vs Press. Heated Inlet Air	Figure 4.42: Varying inlet air temperatures (hot), 500 μm wafer.
		Temp. vs Press.	Figure 4.43: Varying film heights, 350 μm wafer.
			Figure 4.44: Varying inlet air temperatures (cold), 350 μm wafer.

Pressure vs Flow Rate

To compare the measured pressure-flow rate relationships with the modelled ones, COMSOL simulations were conducted for both wafers using the designed film height of 30 μm . The simulation results, along with the measurements data, are presented in Figure 4.30 and Figure 4.32 for the 500 μm and 350 μm wafers, respectively.

As shown, the simulations deviate significantly from the measurements when assuming a film height of 30 μm . This discrepancy in the pressure drop-flow rate relationship may arise from several factors, which are discussed below:

- *Dimensions:*

A likely explanation for the discrepancy between the modelled and measured results is a deviation in the system's actual dimensions from the design. To investigate this, all relevant

dimensions of the manufactured air bearing system were measured, except for the passive film height and inlet rounding, which could not be assessed with the available equipment. All measured dimensions were found to be within the specified tolerances.

To evaluate the potential influence of the unmeasured parameters, COMSOL was used to estimate the passive film height through trial and error. A passive film height of $52.5\ \mu\text{m}$ was found to provide the best fit with the measurement results for both wafers, as shown in Figure 4.30 and Figure 4.32. This theoretically derived passive film height will also be used for the discussion of the deformation and temperature measurement results later on. Since the discrepancy between the modelling and measurement results are rather large, the inlet rounding dimension is not expected to be the error source.

- *Modelling Assumptions:* The main assumption used throughout the modelling of the system is its axisymmetry. To simplify the geometry, a circular unit cell shape was used instead of the actual hexagonal shape. The smallest dimension of the hexagonal unit cell, 14 mm, was thereby applied along the entire circumference, as shown in Figure 4.45. However, for a hexagonal unit cell, the length of the thin film varies along its perimeter, with a maximum of 16.2 mm. This additional length results in a larger flow restriction, ultimately increasing the pressure drop at each flow rate. Since the measured pressure drop is actually lower than the modelled pressure drop, this simplification cannot be the cause of the measurement deviation.

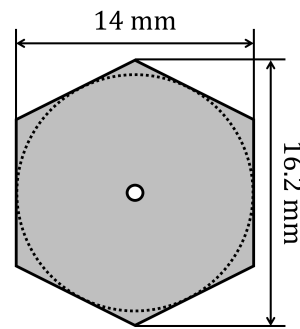


Figure 4.45: Dimensions of a circular and hexagonal unit cell.

Other modelling assumptions, such as the use of laminar compressible flow, are also unlikely to explain the discrepancy in the pressure drop-flow rate relationship. Notably, even at low flow rates, where the laminar flow assumption is most valid, the experimental pressure drop already deviates significantly from the modelled values.

- *Instrumentation:*
To determine the pressure drop-flow rate relationship, two analog instruments were used: a rotameter and a pressure gauge. Since a defect in either of these components directly affects the measurement outcome, the proper functioning of both components was tested. To verify the accuracy of the rotameter, a simple volumetric method was used. A cup filled with water was placed upside down in a large bowl of water, and the air supply hose was inserted into the cup. The flow rate was set on the rotameter, and the time required to displace the water with air was recorded to determine the actual flow rate. At several flow rates, the measured flow rate closely matched the value on the rotameter, confirming its accuracy. The pressure gauge was tested by comparing it with a second pressure gauge, connected in parallel. At multiple flow rates, the readings of both gauges were consistent, ruling out faulty instrumentation as a cause of the measurement deviation.
- *Air Supply Conditions:*
After the rotameter and pressure gauge, the supply air is directed through a 4 mm outer diameter hose, approximately 1-2 meters in length, to the air bearing system. Before reaching the wafer, air passes through several fittings, which could potentially lead to air leakage. Any

leak in the supply hose would reduce the supply pressure at a given flow rate, contributing to measurement deviations similar to those observed. To rule out this potential error source, the critical components were submerged to check for air bubbles, but no leaks were detected. Additionally, the supply hose could cause a pressure drop due to viscous friction between the air and the hose walls. However, the calculations presented in [Appendix D.3](#) suggest that the resulting pressure drop is negligible compared to the observed deviations. Furthermore, a pressure drop in the supply hose would cause a measured pressure drop higher than the actual one, whereas the observed discrepancies show the opposite trend.

- *Wafer Shape/Orientation:*

In addition to the dimensions of the air bearing system, the film height is influenced by the shape of the wafers. The 100 mm wafers used in testing typically exhibit a bow on the order of a few tens of microns. Given that the designed film height is only 30 μm , such a bow could significantly impact the actual film height. Additionally, local thickness variations may introduce small deviations in the film height. To assess this potential error source, both wafers were placed separately on top of the air bearing system in different orientations, and the corresponding pressure drop-flow rate relationships were measured. Only minor deviations were observed, likely due to instrument accuracy and reading uncertainties. This suggests that the wafers either have minimal bow and thickness variations or that the constraining ring effectively compensates for the wafer bow. In either case, shape and orientation of the wafers were ruled out as the cause of the measurement deviations.

In addition to shape and orientation, the deformation of the wafer also affects the active film height. If the wafer deforms more than expected, the active film height increases, leading to a decrease in the pressure drop at the same flow rate. This will be discussed in the next section.

Wafer Deformation vs Pressure

Similar to the pressure drop-flow rate measurements, the maximum deformation-pressure relationships were also compared to the simulation results using a film height of 30 μm , as shown in [Figure 4.34](#) and [Figure 4.37](#). As observed previously, the simulation results show a significant deviation from the measurements. Additionally, it is important to highlight that the simulated deformation is negative at a pressure difference of 0 bar, which is attributed to wafer sag. Since the wafer sag is minimal, it was not accounted for in the measurements.

As previously discussed, the wafer deformation is influenced by several factors: the pressure distribution between the unit cell and the wafer, the clamping method, and the wafer's material characteristics and geometry. Each of these factors will be discussed below, along with additional potential sources of error:

- *Dimensions:*

The pressure distribution between the unit cell and the wafer is highly dependent on the physical system dimensions, with the passive film height playing a particularly critical role. For a given pressure difference between the unit cell inlet and outlet, an increased film height leads to a higher flow velocity, which in turn increases the pressure drop at the inlet of the thin film. In other words, a greater film height at the same supply pressure results in reduced wafer deformation.

To investigate this effect, the maximum wafer deformations were also plotted for the theoretically derived film height of 52.5 μm , obtained from the flow rate measurements, for both wafers in [Figure 4.34](#) and [Figure 4.37](#). These simulations were conducted over a smaller pressure range to prevent the maximum flow velocity from exceeding a Mach number of 0.3 at higher pressures. While the modelled graphs still differ from the measurements, their overall trends align better. However, discrepancies remain in the trend and scaling between the graphs, which will be further addressed in the next bullet point.

Besides the physical dimensions of the air bearing system, variations in the dimensions of the wafers could also contribute to discrepancies. The thickness of the wafers was measured at multiple locations across their surfaces, and no significant deviations were observed. Additionally, based on the pressure drop–flow rate measurements, both the bow and total thickness variation (TTV) of the wafers were determined to be minimal in the setup. While wafer thickness appears in the flexural rigidity equation (Equation C.3) as a third-order term, these small variations are insufficient to explain the discrepancy between the modelled and measured wafer deformation.

Furthermore, the physical dimensions of the constraining ring were verified against its design specifications. The ring thickness was found to be slightly larger than intended, measuring 4.25 mm instead of 4 mm, which also resulted in a slightly higher weight. However, this deviation was accounted for in the COMSOL model and did not significantly affect the simulated results. One aspect that was not verified due to the lack of appropriate measurement equipment is the parallelism of the ring’s bottom surface relative to the wafer’s top surface. If the ring were slightly elevated at its inner radius, the wafer would be clamped at a radius further outward, increasing its maximum deformation.

This effect was analyzed by varying the clamping radius in the COMSOL model for both wafers. For the 500 μm wafer, a clamping radius of 35 mm provided the best fit, as shown in Figure 4.35. For the 350 μm wafer, a clamping radius of 30 mm gave the best match, as depicted in Figure 4.38. Apart from a zero-offset caused by wafer sag, the simulations with these adjusted clamping radii align well with the measurement results.

Additionally, the pressure drop–flow rate relationships obtained using the adjusted clamping radii are shown in Figure 4.31 and Figure 4.33 for the 500 μm and 350 μm wafers, respectively. These results remain within the measurement uncertainties. However, the need for different clamping radii in the simulations suggests that each wafer may exhibit unique clamping behaviour, possibly due to variations in their shape.

- *Calibration:*

The measurement technique used to determine wafer deformation was calibrated by applying a known displacement to a mirror and correlating this displacement to the measured pixel displacement. This calibration yielded a scaling factor of $S = 2.3 \mu\text{m}/\text{pixel}$. However, the discrepancy between the modelled and measured wafer deformation suggests that this conversion factor may be inaccurate.

To investigate this, the scaling factor was varied iteratively in MATLAB to assess its impact on the measurement results. When a scaling factor of $S = 1.1 \mu\text{m}/\text{pixel}$ was applied, the modelled wafer deformations fell within the measurement uncertainty for the 500 μm wafer, as shown in Figure 4.36. However, using the same scaling factor for the 350 μm wafer did not result in a satisfactory match, as shown in Figure 4.39. Unlike the clamping radius, the calibration scaling factor should be identical across both wafers, indicating that the calibration factor alone cannot fully explain the discrepancy.

Potential errors in the calibration factor could arise from an incorrectly assumed thickness of the metal sheet used in the calibration. The sheet thickness was measured at multiple locations using a micrometer and was verified to be approximately 50 μm . Additionally, if either the bottom surface of the mirror or the top surface of the plateau on which it was placed was not sufficiently flat, the actual displacement of the mirror could be smaller than the expected 50 μm .

To assess this, the mirror was positioned in different orientations and at various locations on the plateau. While slight variations in displacement were observed depending on the orientation and position, the scaling factor was determined using the maximum measured pixel displacement of 24 pixels. Since Figure 4.36 suggests that this pixel displacement should be more than twice as high for an applied displacement of 50 μm , surface flatness is unlikely to

be the primary cause of error.

However, this test did highlight limitations in the robustness of the calibration method, which should be addressed in future research.

- *Modelling Assumptions:*

The wafer deformation was modelled using both analytical and numerical approaches, as described earlier in this report. In the final geometry used to represent the test setup (Figure 4.12), a set of ring-equivalent constraints was applied. These constraints included a ring load at the top of the wafer at the clamping radius, a spring foundation beneath the wafer to approximate the outer segments of the air bearing system, and a prescribed vertical displacement of zero along the wafer's bottom circumference.

While this boundary condition setup produced wafer deformations nearly identical to those obtained from a full contact model of the constraining ring, the underlying assumptions have not been experimentally validated. In particular, simplifying the support beneath the wafer to a uniform foundation, rather than accounting for the actual outer segments with spacing between them, could potentially distort the relationship between the model and the measurements. To confirm that these boundary conditions do not contribute to errors in the wafer deformation measurements, a dedicated model validation experiment should be carried out in future research, as discussed further in Chapter 5.

- *Data Processing:*

After capturing images of both the reference and deformed states of the wafers, vector fields were constructed in DaVis. To validate the accuracy of the multigrid interrogation approach, the dot displacement was manually tracked for several dots across the visible wafer surfaces. In each case, the observed displacements aligned well with those in the generated vector field, confirming the proper functioning of the interrogation method.

During the post-processing of these vector fields in MATLAB, several assumptions were made to simplify the surface reconstruction process. For example, when calculating the surface gradients shown in Equation G.20 and Equation G.21, displacements caused by overall surface translations were excluded. However, since the same assumptions were applied during the calibration measurements, any resulting errors should already be accounted for in the scaling factor. Additionally, the minimum deformation at the clamping radius was set to zero. In reality, the wafers are slightly elevated at the inner radius of the ring due to imperfect clamping. However, this offset was modelled to be negligible compared to the maximum wafer deformation, as shown in Figure 4.9.

As previously mentioned, exploring an alternative calibration method in future research would be beneficial to better understand how physical wafer displacements are captured by the surface reconstruction algorithm.

- *Material Properties:*

The wafer deformation is directly affected by the following material properties: density, Young's modulus, and Poisson's ratio. The density was easily verified by weighing both wafers and calculating their corresponding volumes. However, verifying the Young's modulus and Poisson's ratio is more complex and requires specialized measurement techniques. Given that the wafers are made from high-purity fused silica (JGS2), the material properties are unlikely to deviate more than a few percent from their specified values, making this an unlikely error source.

Temperature

First, the temperature measurements were compared to the simulation results using a film height of $30\text{ }\mu\text{m}$ and the calculated temperature differences between the inlet and ambient air. The resulting maximum temperature differences at the top of the wafer for varying pressure drops are shown in

Figure 4.40 and Figure 4.43 for the 500 μm and 350 μm wafers, respectively.

It is evident that the measured temperature drops are consistently higher than the modelled ones, which could be attributed to the following factors:

- *Dimension:*

The temperature distribution of the wafers is influenced to a large extent by the dimensions of the thin film. A larger film height at the same pressure drop results in a higher flow velocity, which in turn amplifies the observed temperature gradients.

Similar to previous measurements, the temperature profiles were compared to simulation results obtained using the theoretically derived film height of 52.5 μm . The modelled results are plotted alongside the measured temperature profiles and the simulation results for a 30 μm film height in Figure 4.40 and Figure 4.43. While the modelled results for the increased film height align better with the measurements, notable deviations remain. Additionally, due to the limited temperature resolution of the infrared camera, temperature differences at pressure drops below 0.06 bar could not be determined, complicating the comparison. Nevertheless, these results indicate that the increased film height alone does not account for the observed discrepancies.

- *Calibration:*

The NTC thermistors were calibrated for an operating range between 18 °C and 22 °C, using a reference thermometer with an accuracy of 5 mK, as outlined in Appendix F.4. Subsequently, the infrared images were calibrated using the thermistors to obtain an accurate representation of the surface temperature of the wafers. This calibration is unlikely to have introduced discrepancies. Additionally, the absolute temperature level has minimal influence on the observed temperature gradients across the wafer, effectively eliminating it as a source of error.

However, the relative temperature measurement was not independently calibrated. The infrared camera is an off-the-shelf component, and its sensitivity was assumed to be accurate. If the camera incorrectly translates a radiation intensity offset into a temperature offset, the measured temperature gradients may deviate from the actual ones. This could result in either an overestimation or underestimation of the true temperature differences across the wafer surface and is a possible explanation for the observed discrepancies. A dedicated calibration of the infrared camera's sensitivity would be necessary to confirm its accuracy in capturing relative temperature variations.

- *Modelling Assumptions:*

In the COMSOL simulations, the air bearing system was assumed to be isothermal at 293 K. Natural convection was assumed on both the top and bottom surfaces outside the unit cell region, with an ambient temperature of 293 K. However, the validity of these thermal boundary conditions was not validated.

During testing, it was observed that the infrared camera heated up during extended operation, which also influenced the thermistor readings. This effect was uniform across the wafer surface, as all three thermistors showed a similar temperature increase over time. However, a uniform heat flux at the top surface of the wafer would not explain the observed increase in temperature gradients.

Further investigation into the validity of the thermal boundary conditions is recommended to better understand their impact on the system.

- *Air Supply Conditions:*

The possibility of an incorrectly assumed air pressure at the unit cell inlet was previously assessed in Section 4.3.2. However, the effect of the inlet air temperature has not yet been considered. As highlighted in Appendix C.4, the difference between the inlet and ambient air temperatures significantly influences the temperature distribution across the wafer, with larger temperature differences leading to more pronounced temperature gradients.

To assess the impact of the inlet air temperature on the measurement results, parametric sweeps were conducted for both wafers at a pressure drop of 0.08 bar, with varying inlet air temperatures. The modelled results are presented alongside the measured infrared profiles in [Figure 4.41](#) and [Figure 4.44](#) for the 500 μm and 350 μm wafers, respectively.

For the 500 μm wafer, the best agreement between the model and measurement is achieved with an inlet air temperature of 0.40 $^{\circ}\text{C}$ below ambient. For the 350 μm wafer, the closest match was observed between 0.45 $^{\circ}\text{C}$ and 0.60 $^{\circ}\text{C}$ below ambient, with an estimated optimum of 0.525 $^{\circ}\text{C}$.

A similar parametric sweep was conducted for the 500 μm wafer at a pressure drop of 0.10 bar using heated inlet air. As shown in [Figure 4.42](#), the measured profile most closely matched the simulation for an inlet air temperature 0.50 $^{\circ}\text{C}$ above ambient.

A summary of the measured ($\Delta T_{meas} = T_{air,meas} - T_{amb,meas}$) and simulated ($\Delta T_{sim} = T_{air,sim} - T_{amb,sim}$) temperature differences between the inlet and ambient air is presented in [Table 4.5](#). The measured temperature differences deviate significantly from the modelled values.

For the 0.08 bar tests, the inlet air thermistor records a temperature that falls between the measured ambient temperature and the simulated inlet air temperature. This suggests that the sensor does not fully capture the inlet air temperature, likely due to its mounting location on the exterior of the hose, where it is influenced by both the internal airflow and the surrounding environment. As a result, it appears to register a weighted average of the inlet and ambient air temperatures.

In contrast, during the 0.10 bar test with heated air, the measured temperature difference exceeds the simulated one. A plausible explanation is condensation forming at the thermistor's location inside the inlet hose, which could affect the reading. Given that the sensor was positioned at a low point in the hose relative to the unit cell, this interpretation is consistent with the observed data.

Table 4.5: Measured and simulated temperature differences between inlet and ambient air.

Measurement		ΔT_{meas} ($^{\circ}\text{C}$)	ΔT_{sim} ($^{\circ}\text{C}$)
500 μm Wafer	0.08 bar	-0.12	-0.40
	0.1 bar, heated inlet air	2.18	0.50
350 μm Wafer	0.08 bar	-0.13	-0.525

Overall, while the inlet air thermistor correctly detects whether the inlet air is hotter or colder than the ambient temperature, its accuracy is insufficient. For future research, an alternative method for measuring the inlet air temperature is recommended.

- *Material Properties:*

As discussed in [Appendix C.2](#), the material properties of the wafer, particularly its thermal conductivity, have a significant impact on its temperature distribution. If the thermal conductivity of the wafers were lower than specified, the observed temperature gradients would increase. However, as previously mentioned, the thermal conductivity of high-purity fused silica (JGS2) is unlikely to deviate more than a few percent from its specified value, making this an unlikely source of error.

Conclusions and Recommendations

This chapter summarizes the key findings derived from the theoretical and experimental investigations into the thermomechanical behaviour of wafers during contactless handling on active air bearings. Based on these findings, several recommendations are proposed to guide future research and development in this field.

5.1 Conclusions

5.1.1 General

- The combination of analytical, numerical, and experimental methods provided consistent and complementary insights into the thermomechanical effects of wafers on active air bearings. While the experimental setup was intentionally simplified to facilitate temperature measurement, the COMSOL models, when adapted to realistic operating parameters, indicate that thermal requirements could potentially be met under actual operating conditions. This integrated approach not only validates the methodology but also forms a strong foundation for further research and development of thermally stable contactless handling systems.

5.1.2 Regarding Modelling

Rectilinear Flow Analysis

- The rectilinear flow study demonstrated that, under plane Poiseuille flow conditions, the pressure-driven and viscous dissipation terms are exactly balanced over the thin film, resulting in zero net energy dissipation. For plane Couette, on the other hand, there is no pressure gradient along the flow direction, resulting in a minor temperature rise due to viscous heating.
- These simplified flow cases offered initial insight into the interplay between the various terms in the thermal energy equation within a confined thin air film, serving as a foundation for more advanced analyses.

Axisymmetric Flow Analysis

- Transitioning to an axisymmetric geometry enabled a more realistic representation of the radial flow in an air bearing system. The velocity distribution was derived through three methods: creeping flow assumption, perturbation method, and a numerical method. This analysis illustrated that while the creeping flow assumption (used for the commonly applied Reynolds equation) offers a reasonable approximation, it underestimates the velocity field when inertial effects become significant. To accurately predict the velocity profiles and associated thermal behaviour, the nonlinear inertia terms should be included through either the perturbation or numerical method.
- The implementation of a finite difference method in MATLAB provided valuable insights into the understanding of the evolution of the thin film temperature profile over the radius of the axisymmetric geometry.

- The performance of the finite difference method was limited by the available computational power. To improve the performance of the thermal results, mesh refinements are required, which also allows for an accurate adiabatic boundary analysis.
- The energy dissipation analysis showed that, for creeping Poiseuille flow, no energy is dissipated across the axisymmetric domain, similar to rectilinear Poiseuille flow. The inclusion of the nonlinear terms through the perturbation method did indicate very localized energy dissipation near the thin film inlet. For the selected set of parameters, a total dissipation of 5.36 mW was found, of which approximately 86% originates from the first millimeter from the inlet.

COMSOL Modelling

- The initial COMSOL model, based on the simplified axisymmetric geometry from the analytical work, successfully reproduced the key thermal and velocity trends identified in both the rectilinear and axisymmetric analyses.
- The inclusion of the vertical air inlet and outlet provided a more accurate representation of the actual system, also highlighting the significant cooling at the inlet of the thin film. Together with the high dissipation rate observed near the thin film inlet, these factors are the most critical for the temperature distribution of the wafer.
- Adding the air bearing system and wafer enabled a detailed analysis of the temperature distribution in the solid components. Under realistic operating conditions, the predicted maximum temperature difference across the wafer was 1.22 mK for an isothermal air bearing system and 1.24 mK for an adiabatic one.
- The temperature distribution of the wafer was found to be affected by four critical factors: inlet shape, thermal material properties, film height and pressure, and the temperature difference between the inlet and ambient air. By tuning these, the theoretical temperature difference on the top surface of the wafer was increased to 172.23 mK, while maintaining acceptable limits for flow velocity and wafer deformation.
- Overall, the COMSOL modelling proved to be a robust tool to investigate and predict the complex interplay between airflow, thermal gradients, and wafer deformation. Furthermore, it provides a solid foundation for future design optimizations and experimental validations.

5.1.3 Regarding Test Setup

- The test setup was designed as a simplified air bearing system with a clamped wafer, effectively replicating the key thermomechanical conditions predicted by the analytical and numerical models.
- A combination of measurement techniques was employed to characterize the wafer's thermo-mechanical behaviour. Surface deformation was measured using an optical method inspired by free-surface synthetic schlieren, while the temperature distribution was captured using an infrared camera calibrated with thermistors attached to the top of the wafer. This approach enabled both spatially resolved relative temperature measurements and accurate absolute temperature readings. While all techniques operated reliably and provided repeatable results, the temperature resolution and noise level of the infrared camera were not as good as expected, limiting its sensitivity to small temperature differences.
- A flow rate measurement was performed to characterize the overall air throughput in the system. The measured flow rate was higher than expected based on the nominal geometry and simulation inputs, suggesting that the actual film height may have been larger, likely due to

manufacturing imperfections. This deviation likely also contributed to the differences observed in both the temperature distribution and wafer deformation.

- Experimental results demonstrated that the wafer experiences minor, yet measurable, deformation under the imposed airflow conditions. However, the observed deformation patterns deviated notably from simulation predictions. These discrepancies are likely due to a combination of geometric uncertainties, limitations in the optical measurement setup, and unmodelled clamping effects.
- Temperature measurements on the top surface of the wafer revealed a clear spatial distribution, but also showed significant deviations from the predicted thermal profiles. These differences underscore the sensitivity of the system to actual thermal boundary conditions and the importance of an accurate measurement of the temperature difference between the inlet and ambient air.
- The discussion of the experimental data underscored the challenges in achieving ideal thermo-mechanical performance. Measurements of temperature, deformation, and flow rate all showed significant deviations from the simulations. This highlights the need for further refinement of the test setup to reduce uncertainties and improve measurement resolution. In particular, clamping the wafer instead of levitating it introduced uncertainties in the mechanical boundary conditions, complicating direct validation of the simulation results.
- Overall, the test setup not only validated the key predictions of the analytical and COMSOL models but also provided critical feedback for future design optimizations and experimental validations, setting a solid groundwork for subsequent studies on contactless wafer handling.

5.2 Recommendations

5.2.1 General

- It is recommended that a new generation of air-based contactless handling systems be developed, incorporating both the thermomechanical effects identified in this study and the actuation performance of the air bearing system. These aspects should be combined into a comprehensive optimization scheme to improve wafer temperature uniformity and mechanical stability, while maintaining or enhancing the system's actuation capabilities.

5.2.2 Regarding Modelling

- The inlet restriction and recess geometry, which are critical components for the performance of air bearing systems, should be included in future models to more closely resemble operational air bearing systems.
- The actuation performance of the air bearing system should be incorporated into the model. Coupling the thermomechanical behaviour with the actuation performance will allow for a more complete representation of the system.
- To capture potential cross-effects between adjacent unit cells, future models should eliminate the 2D axisymmetric simplification and incorporate multiple unit cells in a 3D geometry. This would also allow the exploration of different unit cell shapes, such as hexagonal configurations.
- In this study, system parameters were deliberately tuned to amplify the temperature gradients for easier measurement and validation. However, in an operational setting, minimizing these gradients is essential for thermal stability. Therefore, further modelling should focus on identifying how the four critical thermal factors can be tuned to minimize wafer temperature variation under real operating conditions.

- A more extensive sensitivity analysis is recommended to assess the impact of small perturbations in geometry and material properties. This should include the effect of potential manufacturing imperfections, as such deviations can influence the thermomechanical response and overall performance of the system.

5.2.3 Regarding Test Setup

- A more controlled measurement environment is recommended to reduce the influence of ambient temperature fluctuations, air currents, and light variation. This would improve the accuracy and repeatability of both thermal and optical measurements.
- The current method for measuring the inlet air temperature could be improved or replaced with a more direct approach, such as placing a temperature sensor inside the airflow, closer to the thin film inlet. This would reduce uncertainty in the thermal boundary conditions used for simulation validation.
- An alternative air heating method, such as a controlled inline heater, could offer more stable and precise control of the inlet air temperature, improving repeatability.
- The current wafer clamping method introduces uncertainty in the film height due to imperfect contact and unclear constraint conditions. To impose a more predictable and defined clamping load, a small downward-facing ridge could be added to the inner radius of the ring, thereby creating a ring load condition. Alternatively, a vacuum-based clamping method could be used to constrain the wafer more reliably.
- It is recommended to test wafers made from different materials to examine how varying thermal and mechanical properties affect the overall thermomechanical response. This would help in validating the model across a wider range of use cases.
- Future test setups should explore different unit cell configurations by varying the shape, size, and number of unit cells. Expanding beyond a single hexagonal unit cell will better represent operational air bearing systems and allow the study of cross-effects between adjacent cells. Additionally, variations in inlet geometry should be tested to validate their influence on the thermomechanical response of the wafer.
- Adding a vacuum supply at the unit cell outlet would more accurately replicate operational air bearing systems. The vacuum could also be used to constrain the wafer, potentially replacing the current clamping method.
- The current wafer deformation measurement technique would benefit from a more robust and in-situ calibration procedure. A recommended approach is to apply a known force or displacement at the center of the wafer and relate the resulting pixel displacement to this known input to determine the optical scaling factor, S . This calibration can also be used to validate the mechanical boundary conditions assumed in the COMSOL models.
- With appropriate modifications, the current optical deformation measurement technique could be extended to simultaneously capture both wafer deformation and air film height by leveraging the multiple dot reflections observed above the unit cell surface (see [Figure G.14](#)). This would require the use of a wafer that is transparent in the visible spectrum, as well as several enhancements to the optical setup:
 - Smaller dots to reduce overlap between multiple reflections and enable distinct identification of each reflection layer.
 - A smaller field of view (FOV) to increase spatial resolution and maintain sufficient pixel density per dot.

-
- A higher dot density to capture finer deformation and film height gradients above the unit cell.
 - The temperature resolution of the infrared camera was found to be insufficient to detect the small temperature gradients resulting from low pressure drops. Therefore, an alternative relative temperature measurement method is recommended. Ideally one capable of detecting temperature differences on the order of millikelvins, making it suitable for use in operational systems.
 - For the absolute temperature measurements, an alternative to NTC thermistors may be desirable, especially if a non-invasive method is available to avoid interference with the deformation measurements. If thermistors continue to be used, their performance can be improved by upgrading the data acquisition system and extending their calibration over a wider temperature range.
 - Ultimately, a future test setup should include a fully operational air bearing system with an actively levitated wafer. This would provide insight into thermomechanical interactions under realistic conditions and eliminate the uncertainties associated with the current clamping method.

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A

Air-Based Contactless Handling Systems

As mentioned in the introduction, multiple generations of air-based contactless handling systems have been developed at TU Delft. This appendix explores their evolution, providing a detailed description of their working principles, specifications, and performance.

A.1 Generation 1: Pressure Control

The first generation of the air-based contactless handling system was presented by Wesselingh in 2011 [16]. His concept consists of a static geometry in which the airflow can be altered by actively controlling the inlet and outlet pressures using valves. A schematic representation of the pressure-controlled handling system, including a substrate on top, is provided in Figure A.1. As can be seen, square unit cells are used, separated by so-called dams. In order to achieve multi-DOF actuation, four unit cells need to cooperate, as illustrated in Figure A.2a.

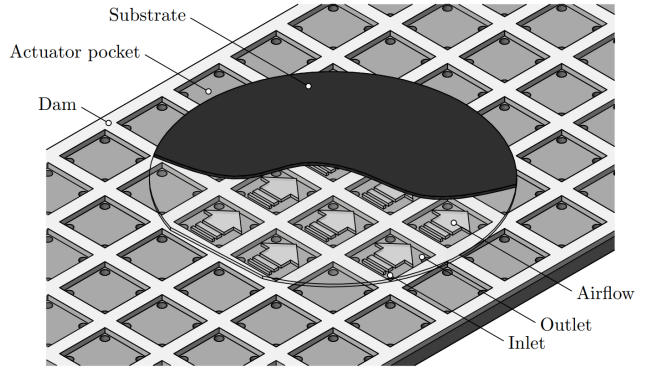
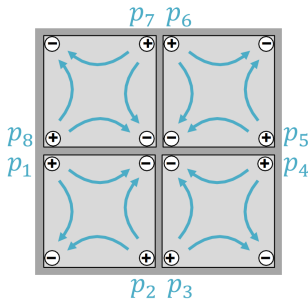
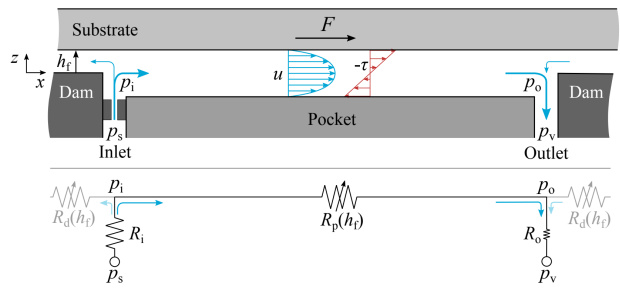


Figure A.1: Pressure-controlled contactless handling system [16].

The cross-sectional view of a unit cell is shown in Figure A.2b. Most of the airflow is directed over the pocket, generating the desired traction force. However, another parasitic flow is generated over the dams between the pockets, reducing the net force on the substrate.



(a) Top view of four unit cells [17].



(b) Cross-section of a unit cell [18].

Figure A.2: Schematic representations of the unit cells for the pressure-controlled concept.

Performance and Specifications

To evaluate the performance of his design, Wesselingh developed a demonstrator capable of actuating, sensing, and controlling the position of a 100 mm wafer. The demonstrator, depicted in Figure A.3, consists of 36 unit cells arranged in a square array, each with a dimension of $10 \times 10 \text{ mm}^2$. Using this setup, he achieved a positioning bandwidth of 50 Hz, with a servo error of 6 nm (1σ) for the planar DOFs. Furthermore, a maximum acceleration of 600 mm/s^2 and velocities of up to 30 mm/s were achieved.

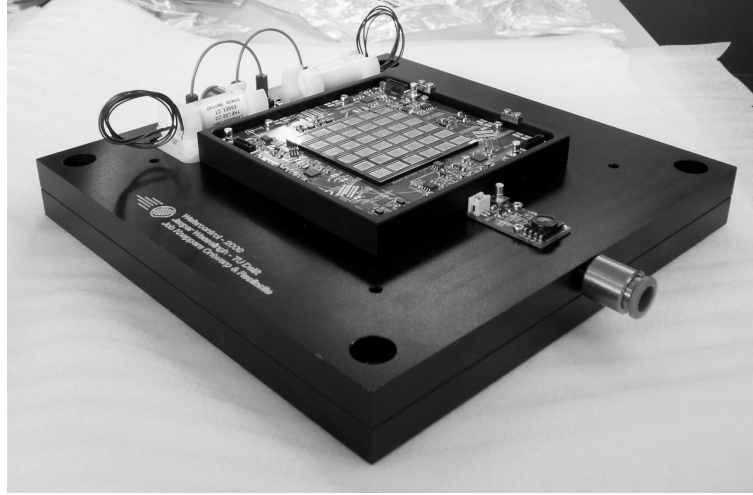


Figure A.3: Demonstrator developed by Wesselingh [16].

Due to manufacturing limitations, the achieved fly height was $15\ \mu\text{m}$ instead of the intended $10\ \mu\text{m}$, resulting in a reduced actuation force. Additionally, the system experienced delays due to the relatively large distance between the valves and the bearing surface.

A.2 Generation 2: Variable Pocket Restriction

To overcome the delay introduced by the valves in the first generation, Vuong developed a new concept featuring variable pocket restrictions, published in 2016 [3]. Instead of adjusting the in- and outlet pressure, this design utilizes a surface composed of deformable unit cells, commonly referred to as the flowerbed (Figure A.4). Each hexagonal unit cell is centrally fed with pressurized air and surrounded by a vacuum chamber. By tilting the unit cell, the airflow is redirected toward the side with the largest gap between the bearing surface and the substrate, thereby creating the desired pressure distribution.

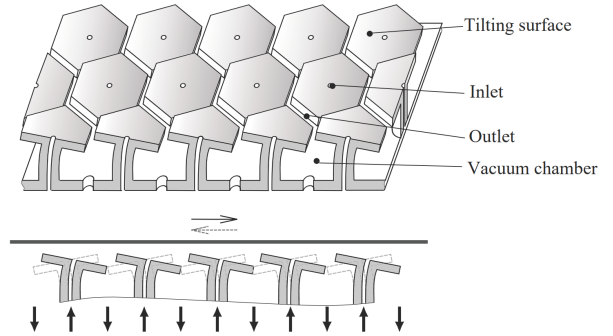


Figure A.4: Deformable surface of the variable pocket restriction concept [3].

Figure A.5 shows the cross-sectional view of a unit cell of the flowerbed and its corresponding electrical analogy. By adjusting the tilting angle α , the direction of the resultant viscous traction force on the substrate is adjusted accordingly, without introducing delay.

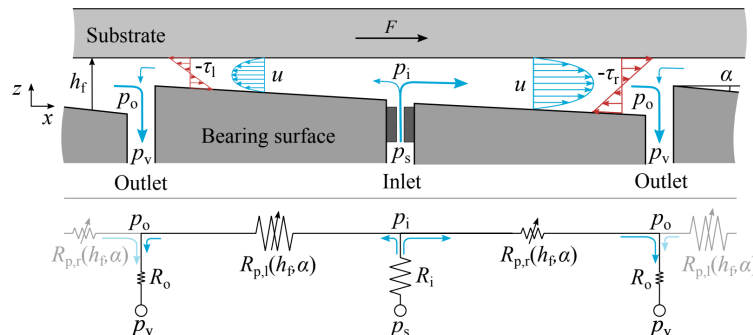


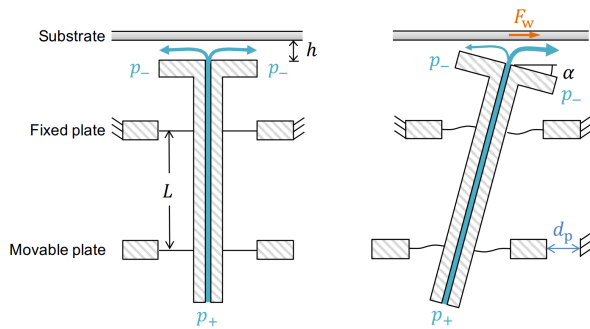
Figure A.5: Cross-sectional view of a unit cell of the flowerbed [18].

Performance and Specifications

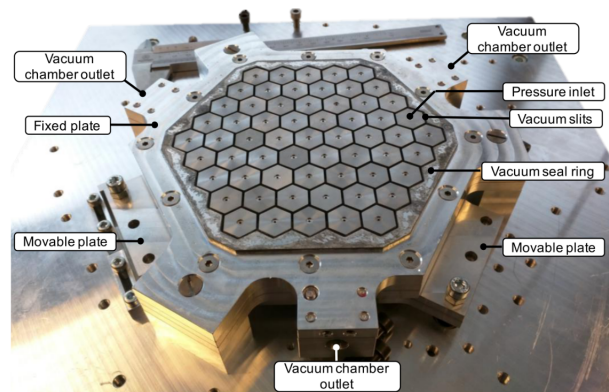
The demonstrator developed by Vuong is depicted in Figure A.6b. It consists of 61 individual flowers, each with an edge-to-edge dimension of 14 mm, a circular inlet of 1 mm in diameter, and a spacing of 1 mm between them. Furthermore, each flower is connected to a fixed plate at the top and a movable plate at the bottom. By displacing the bottom plate, three-DOF actuation of the substrate can be achieved. Additionally, the fixed and movable plates serve to seal the vacuum chamber and pressurize the inlet, respectively.

Vuong observed that the tilting angle of the flowers is proportional to the displacement of the movable plate up to 1000 Hz. Furthermore, he experimentally proved that the pneumatics can be modelled as a proportional gain up to 400 Hz. In continuation, Krijnen [19] and Jansen [17] further developed the control and actuation systems for the flowerbed, achieving a positioning error of 104 nm (2σ) and a maximum practically achievable acceleration of 1.17 m/s^2 . Jansen ultimately achieved a closed-loop positioning bandwidth of 233 Hz. More recently, Hooijschuur improved the positioning bandwidth to 300 Hz and implemented contactless substrate sensing using charge-coupled devices [20].

The fly height, defined as the minimum distance between the actuator and the substrate when the flowers are fully tilted, was assumed to be $10 \text{ }\mu\text{m}$ for this design. Consequently, the film thickness at rest is approximately $35 \text{ }\mu\text{m}$. However, due to the complexity of the flowerbed's design, manufacturability proved challenging. The bearing surface was not perfectly leveled, which affected both the fly height and overall system performance.



(a) Schematic representation of one flower at rest position and tilted [17].



(b) Demonstrator developed by Vuong [3].

Figure A.6: Demonstrator of the variable pocket restriction concept.

A.3 Generation 3: Variable Outlet Restriction

The third generation of the air-based contactless handling system was introduced by Verbruggen in 2017 [21]. He developed a concept in which the desired pressure distribution is achieved by varying the outlet restrictions of the unit cells. The design consists of circular pillars that are centrally supplied with pressurized air (Figure A.7). A top plate is installed around these pillars, forming dams between them. A small gap is maintained between the dams and the pillars, acting as the vacuum outlet of the system. By adjusting this gap, the outlet restriction can be controlled, enabling the generation of the intended traction force on the substrate.

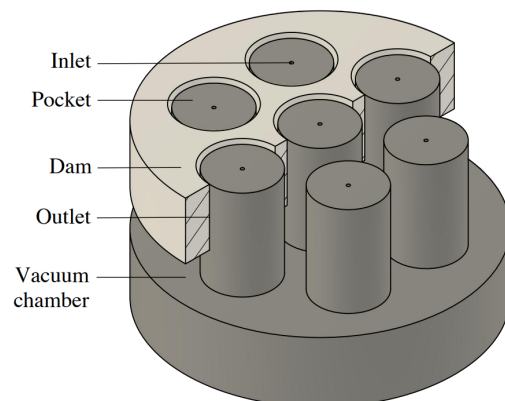


Figure A.7: Variable outlet restriction concept [18].

The cross-sectional view of a unit cell of the variable outlet restriction concept is shown in Figure A.8. As illustrated, changing the eccentricity of the pillars relative to the hole in the top plate creates an asymmetric outlet restriction, with a larger restriction on one side and a smaller one on the other. This asymmetry results in a net force on the substrate.

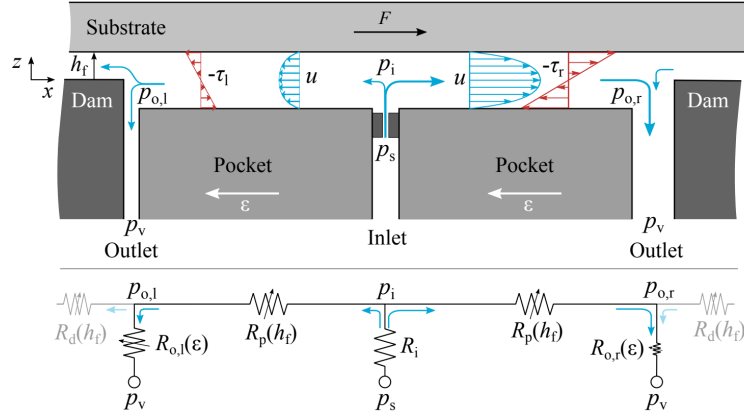
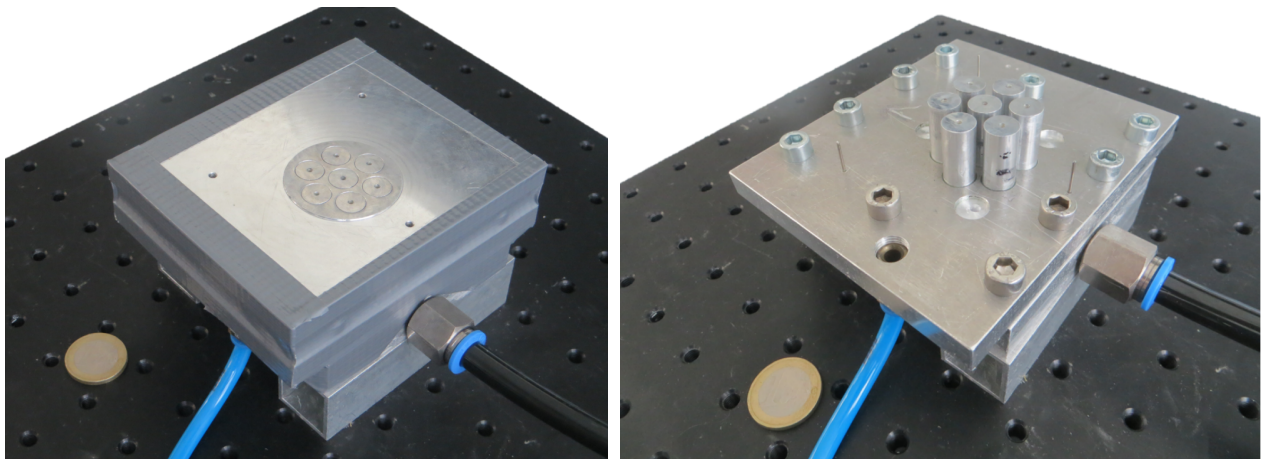


Figure A.8: Cross-sectional view of a unit cell of the variable outlet restriction concept [18].

Performance and Specifications

A demonstrator of the variable outlet restriction concept was developed by Verbruggen, as shown in Figure A.9. The system consists of seven pillars, each with a diameter of 10 mm. The holes in the top plate were designed with a diameter of 10.1 mm, creating a gap width of 50 μm at rest. The test setup did not include any control or actuation systems, instead relying on manual actuation for experimentation. Verbruggen reported an average angular acceleration of 11 rad/s^2 , which corresponds to a theoretical translational acceleration of 2.8 m/s^2 for a 200 mm wafer.

Initially, a fly height of 10 μm was intended for the system. However, stringent tolerance requirements posed challenges during manufacturing, resulting in a measured fly height of 19 μm . To address the manufacturability of this concept, Pijnenburg developed a novel manufacturing method, potentially decreasing the system's sensitivity to manufacturing errors [18]. However, this method also did not meet the tolerance requirements, meaning additional research remains needed.



(a) Fully assembled demonstrator.

(b) Demonstrator without its top plate.

Figure A.9: Demonstrator of the variable outlet restriction concept, developed by Verbruggen [21].

A.4 Generation 4: Continuous Deformable Surface

To address the manufacturability issues encountered with the variable pocket restriction concept discussed in [Appendix A.2](#), He developed the continuous deformable surface concept, published in 2024 [13]. Compared to the complex nature of Vuong’s flowerbed design, He’s concept consists of only two monolithic parts, ensuring ease of assembly. The top surface of the system, as shown in [Figure A.10](#), consists of compliant-based unit cells that are centrally fed with pressurized air. To generate the desired traction force on the substrate, the surface between the circular inlet and the vacuum outlet slits can be rotated in the out-of-plane direction by actuating the bottom plate in the xy -plane.

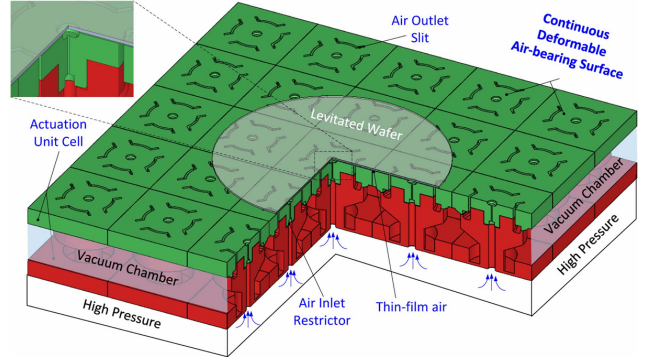


Figure A.10: Continuous deformable surface concept [13].

A schematic representation of a unit cell of the continuous deformable surface concept, including a cross-sectional view, is presented in [Figure A.11](#). As shown, the vacuum chamber is located between the top and bottom plates, while the high pressure chamber is located beneath the bottom plate. By actuating the bottom plate, the pocket restriction becomes variable across the unit cell surface, similar to the flowerbed concept ([Figure A.5](#)).

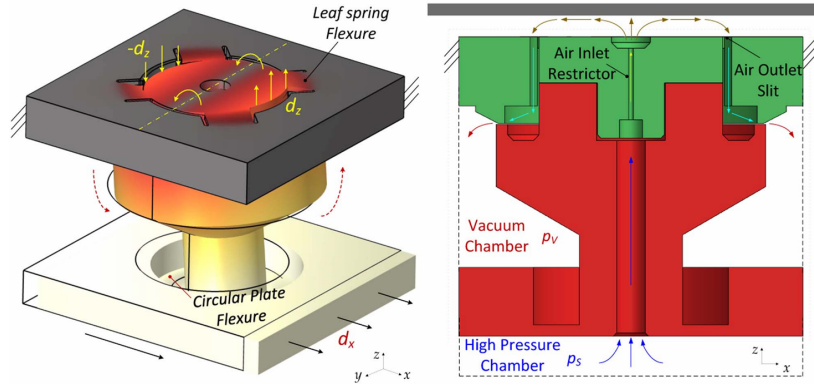
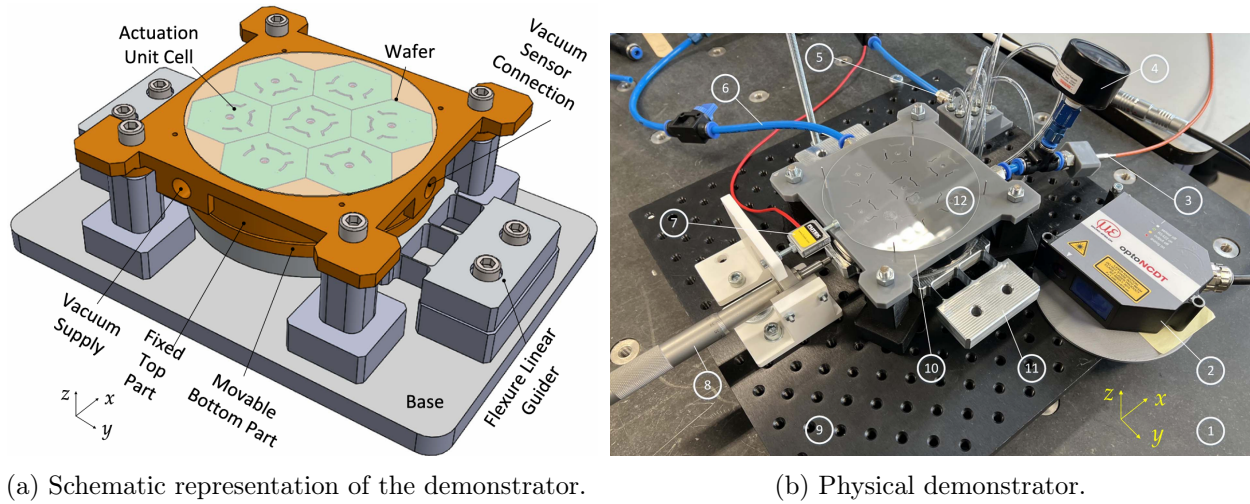


Figure A.11: Schematic representation of a unit cell for the continuous deformable surface concept, including a cross-sectional view [13].

Performance and Specifications

To validate his design, He developed a one-DOF modular demonstrator, depicted in [Figure A.12](#), using stereolithography (SLA) 3D printing. The demonstrator consists of seven hexagonal unit cells, each oriented differently to maintain a consistent ambient pressure boundary condition at the wafer’s edge. The demonstrator’s performance was assessed by levitating and actuating a 100 mm quartz wafer with a thickness of 0.5 mm. The system achieved a maximum traction force of 90 mN, corresponding to an acceleration exceeding 0.9 g for the wafer.

For the demonstrator, the film height was established to be $42.5 \mu\text{m}$ in the undeformed configuration, measured using a confocal sensor. However, due to the coupling effect between the air bearing pressure and the deformation/alignment of the wafer, the experimental results deviated from the numerical simulations, indicating the need for further design optimization.



(a) Schematic representation of the demonstrator.

(b) Physical demonstrator.

Figure A.12: Demonstrator of the continuous deformable surface concept, developed by He [13].

A.5 Contactless Handling Systems for Transportation

The previously discussed designs primarily focus on accurately positioning substrates using air bearing technology. However, research has also been conducted on the transportation capabilities of this promising technology. In 2016, Vagher first looked into the transportation aspect by developing a passive air conveyor for thin substrates, as well as a low-cost vision sensor for substrate detection [22]. The conveyor demonstrated successful transport of substrates of various materials and dimensions. The sensor achieved a resolution of $30\ \mu\text{m}$ at a frequency of 60 Hz.

In continuation, Snieder explored the possibility of combining an active bearing surface with a passive one to reduce the total required mass flow in 2017 [23]. The active bearing surface has both actuating and levitating capabilities, while the passive surface only levitates the substrate. He developed two demonstrators based on the design shown in Figure A.13. Experimental test demonstrated a smooth transition between the active and passive bearing surface, emphasizing the potential of this conveyor concept.

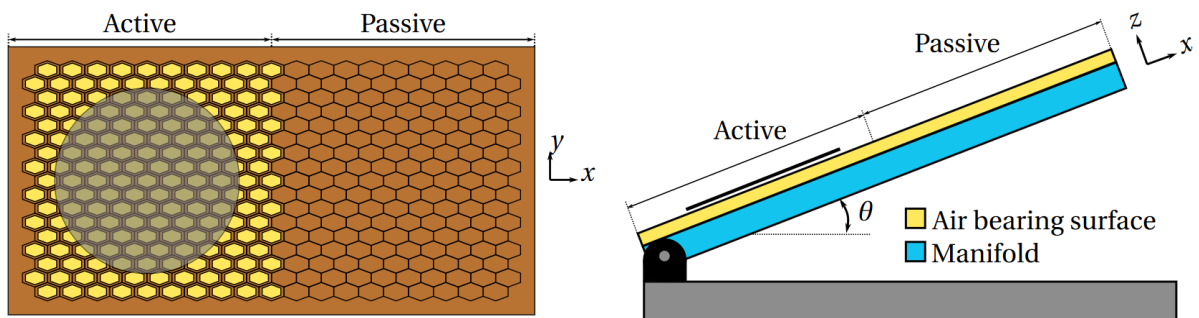


Figure A.13: Demonstrator developed by Snieder, shown from two different viewpoints [23].

B

Thermal Analysis: Background

B.1 General Theory

As outlined in [Appendix A](#), the air bearing systems consist of an array of unit cells that generate a viscous traction force on the substrate. [Figure B.1](#) presents a 2D parallel plate simplification of one such unit cell. In this model, the temperatures of the wafer and air bearing system are denoted by T_w and T_s , respectively. The velocity and temperature profiles of the thin film flow are represented by $u(y)$ and $T(y)$, while the wafer's velocity is denoted by V . The following heat transfer mechanisms are considered:

- Radiation between the wafer and surrounding surfaces.
- Convection at the top side of the wafer.
- Convection between the air bearing system and the wafer.
- Conduction within the wafer.

In reality, vertical inlets and outlets are positioned below p_1 and p_2 , respectively. These geometric features introduce more complex flow patterns, which are addressed in the numerical analysis presented in [Chapter 3](#).

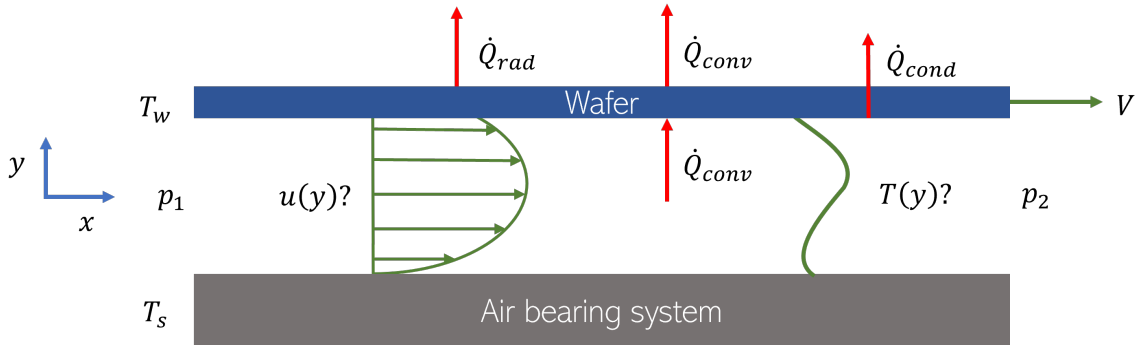


Figure B.1: 2D parallel plate simplification of a unit cell.

Radiation

Radiative heat transfer is driven by the temperature difference between the wafer and its surrounding surfaces. Within radiation, a distinction can be made between black surface radiation and gray surface radiation. For gray surface radiation, a portion of the irradiated energy is reflected depending on the emissivity of the surface, ε . In contrast, black surface radiation assumes that none of the irradiated energy is reflected, corresponding to an emissivity of one. The governing equation for radiative heat flux is given by [\[24\]](#):

$$q = \varepsilon \sigma (T_w^4 - T_s^4) \quad (\text{B.1})$$

Where σ denotes the Stefan Boltzmann constant ($\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$). The temperature appears to the fourth power in this equation, indicating that radiation becomes particularly dominant

at high temperatures and large temperature differences. Since all the surfaces surrounding the wafer will be at roughly the same low temperature as the wafer itself (room temperature), radiation will not play a significant role in the grand scheme of things.

Conduction

Conduction is a heat transfer mechanism stemming from temperature differences, i.e., temperature gradients within or between materials. When considering one dimensional steady-state conduction, Fourier's Law can be used to describe the heat flux [24]:

$$q_x = -k \frac{\partial T}{\partial x} \quad q_y = -k \frac{\partial T}{\partial y} \quad q_z = -k \frac{\partial T}{\partial z} \quad (\text{B.2})$$

Where k is the thermal conductivity of the material in which the heat is being conducted. Conduction is not only relevant for solid materials, such as silicon wafers, but also for gaseous mediums such as air. However, there is a significant difference in the rates of conductive heat transfer between the two. As shown in Table B.1, silicon has a much higher thermal conductivity, making it efficient at conducting heat and thereby evening out temperature differences within the material. In contrast, air is a much less efficient conductor, acting more as an insulator.

Table B.1: Thermal conductivity of air and silicon at 300 K [24].

Material	Thermal conductivity k (W/mK)
Air	0.0267
Silicon	148

A more comprehensive representation of multidimensional unsteady conduction in a material is given by Fourier's equation [24]:

$$\frac{\partial T}{\partial t} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad (\text{B.3})$$

With:

$$\alpha = \frac{k}{\rho c_p} \quad (\text{B.4})$$

Here α represents the thermal diffusivity, which depends on the thermal conductivity (k), specific heat capacity (c_p), and density (ρ). The implications of this parameter on the heat transfer within a material is further discussed in Appendix C.2.

Convection

Convection refers to the heat transfer from a surface to a moving fluid. Near the surface, heat is initially transferred to the fluid through conduction and radiation. Once this heat is absorbed by the fluid, it is carried away by the moving flow. Convection can be categorized into two types: natural convection and forced convection.

Natural convection occurs when fluid motion is driven by a temperature difference within the fluid. As the fluid heats up, its density decreases, causing it to rise. Cooler, denser fluid then moves in to replace the heated fluid. This continuous process of warm fluid rising and cooler fluid replacing it creates a circulation pattern.

In contrast, forced convection is driven by an external flow, such as pressure differences or mechanically induced flows. This type of convection occurs when a fluid is moved by an external force, such as a pump or fan, rather than by temperature-induced density variations.

The general formula for convective heat flux is given by [24]:

$$q = h\Delta T \quad (\text{B.5})$$

Here, ΔT represents the temperature difference between the surface and the fluid, while h denotes the heat transfer coefficient. Determining h is often complex, as it depends on various factors such as flow velocity, surface geometry and roughness, boundary conditions, and more. Due to this complexity, analytical solutions are typically limited to simplified cases and specific geometries. Instead, experimental data are often used to develop equations that estimate the Nusselt number for different system parameters. The Nusselt number can then be used to determine the heat transfer coefficient, as shown in Equation B.6 [24]. Additionally, many relations for heat transfer coefficients across a wide range of geometries and conditions are available in literature [25].

$$h = \frac{k\text{Nu}}{L} \quad (\text{B.6})$$

Where L is the characteristic length of the geometry. In the air bearing-wafer assembly, several convective heat transfer mechanisms are at play. At the top of the wafer, air flows across the surface due to the wafer's motion. When the wafer is at rest, only natural convection contributes to heat transfer. However, as the wafer starts moving, forced convection becomes more dominant and increases with velocity. Between the air bearing system and the wafer, forced convection, driven by the relatively high velocity of the thin air film, will be the dominant mechanism.

Overall, the heat transfer in the thin air film is of primary importance. Not only is the flow velocity in the air film much higher than at the top of the wafer, but the small film height also introduces some interesting effects regarding viscous dissipation and pressure gradients.

For any convection problem, it is best to start by examining the basic principles governing the flow. Since air is the medium, the ideal gas law can be applied with reasonable accuracy. Additionally, the isentropic flow relations can be used to estimate the compressible flow behaviour in the air bearing system, particularly for large velocity changes. Finally, three general conservation laws can be formulated: conservation of mass, conservation of momentum, and conservation of energy.

Ideal Gas Law

For gases under conditions of high temperature and low pressure, where the gas molecules are widely spread and their interactions are minimal, the ideal gas law described in Equation B.7 accurately describes the relation between pressure (p), density (ρ), and temperature (T). These quantities are related through the ideal gas constant (R). For air at atmospheric conditions, the ideal gas law still offers a reasonably accurate solution [10].

$$p = \rho RT \quad R = c_p - c_v = \text{gas constant} \quad (\text{B.7})$$

Where c_p and c_v denote the specific heat at constant pressure and volume, respectively. Even though the air density in the thin film varies significantly along the length of the unit cells due to the applied pressure difference, constant density will be assumed for further calculations presented in this appendix. The implications of this assumption will be further discussed in Appendix B.2. Additionally, compressibility effects are considered in the axisymmetric analysis (Section 2.2) and in the numerical models (Chapter 3).

Isentropic Flow Relations

The changes in thermodynamic properties for compressible gas flows under isentropic conditions can be accurately described by the isentropic flow relations. These relations are particularly useful for flows where heat transfer and viscous dissipation effects are negligible, and the flow can be considered

both adiabatic and reversible. For an ideal gas, the relationships between the static and stagnation (or total) properties are given by the following equations [6]:

$$\frac{T}{T_i} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-1} \quad (\text{B.8})$$

$$\frac{P}{P_i} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma}{\gamma-1}} \quad (\text{B.9})$$

$$\frac{\rho}{\rho_i} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{1}{\gamma-1}} \quad (\text{B.10})$$

Where M denotes the Mach number and γ represents the specific heat ratio:

$$M = \frac{v}{a} \quad a = \text{Speed of sound in air} \quad (\text{B.11})$$

$$\gamma = \frac{c_p}{c_v} \quad (\text{B.12})$$

These equations indicate that as the Mach number increases, there is a corresponding rise in stagnation temperature, pressure, and density relative to their static values. The stagnation properties (T_i , P_i , ρ_i) represent the thermodynamic state that the gas would attain if it were brought to rest isentropically. Although the flow is not assumed to be isentropic, these relationships provide valuable insights and are widely used in practical scenarios where deviations from isentropic conditions are minimal.

Conservation of Mass

Since mass cannot be created or destroyed, it should remain constant over time for an isolated system. By applying this principle to a three dimensional control volume, Equation B.13 can be derived, also known as the continuity equation. Here, $\mathbf{v}$ represents the velocity vector, containing the velocities in the x , y , and z direction (u , v , and w). In cases where density is assumed to be constant, the equation simplifies to Equation B.14 [24].

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (\text{B.13})$$

$$\nabla \cdot \mathbf{v} = 0 \quad (\text{B.14})$$

Conservation of Momentum

Momentum conservation is based on the principle that the time rate of change of momentum within a control volume is equal to the sum of the forces acting on that control volume. In the absence of external forces, this implies that momentum remains constant within the control volume. When assuming constant density and constant viscosity (μ), the Navier-Stokes equations can be derived [24]:

$$\rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) = -\frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \rho g_x \quad (\text{B.15a})$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} \right) = -\frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) + \rho g_y \quad (\text{B.15b})$$

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) = -\frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) + \rho g_z \quad (\text{B.15c})$$

Where g_x , g_y , and g_z denote the gravitational acceleration along their respective directions. The three components of the Navier-Stokes equations can also be rewritten into a single vector equation [24]:

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \mu \nabla^2 \mathbf{v} + \rho \mathbf{g} \quad (\text{B.16})$$

Including compressibility effects in the Navier-Stokes equations increases their complexity, as the continuity equation presented in Equation B.13 must also be integrated.

Conservation of Energy

Finally, the principle of conservation of energy can be used to formulate a governing equation that also includes the relevant thermal effects. Conservation of energy states that the total change in internal energy of a system should equal the difference between the heat added to the system and the work done by the system:

$$\Delta E = Q - W \quad (\text{B.17})$$

By incorporating convection, conduction, gravitational work, energy storage, and internal heat generation, the following thermal energy equation is obtained for an ideal gas [24]:

$$\rho c_p \frac{DT}{Dt} = \frac{Dp}{Dt} + \nabla \cdot k \nabla T + \mu \Phi + \dot{Q}_v''' \quad (\text{B.18})$$

Where $\dot{Q}_v'''$ is the volumetric heat source term, and Φ is the dissipation function, given by:

$$\Phi = 2 \left[\left(\frac{\partial u}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial y} \right)^2 + \left(\frac{\partial w}{\partial z} \right)^2 \right] + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)^2 + \left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right)^2 \quad (\text{B.19})$$

B.2 Rectilinear Flow

The theory presented in the previous section forms a complex set of equations that cannot be solved analytically for complex geometries and flow cases. Therefore, the air bearing-wafer assembly needs to be simplified and assumptions have to be made regarding the flow characteristics.

To gain an understanding of the thermal effects in the thin film flow between the air bearing system and the wafer, the geometry is simplified to a 2D parallel plate configuration, as shown in Figure B.1. In this simplified model, the left side of the parallel plates represents the air inlet, where pressurized air at p_1 is introduced. The right side of the plates maintains a lower pressure, p_2 , representing the air outlet. The origin of the coordinate system is centered between the parallel plates on the left side. In addition to simplifying the geometry, the following assumptions are made:

- Fully developed flow.
- Incompressible flow.
- Steady-state flow.
- Constant air properties.
- Laminar flow.
- No velocity component normal to the surfaces.
- Negligible gravitational effects.
- No-slip boundary conditions.

The assumption of incompressibility represents a significant simplification in this analysis. In reality, the density of air is related to its pressure via the ideal gas law (Equation B.7). Assuming isothermal flow, this implies that if the inlet pressure is, for example, three times higher than the outlet pressure, the density at the outlet would be three times lower. According to the conservation of mass (Equation B.13), the velocity must adjust accordingly, leading to an outlet velocity that is

three times higher than the inlet velocity.

By assuming incompressibility, a linear pressure gradient along the flow direction is obtained:

$$\frac{dp}{dx} = \frac{p_2 - p_1}{L} \quad (\text{B.20})$$

Where L represents the distance between the inlet and outlet of a unit cell. For compressible flow, a nonlinear pressure gradient arises, meaning the pressure gradient becomes a function of x rather than being a constant.

The thermal energy equation for ideal gases described in [Equation B.18](#) does take compressibility effects into account through the $\frac{Dp}{Dt}$ term, which will not be neglected in the calculation of the temperature profiles.

B.2.1 Velocity Profiles

The analysis begins by examining the rectilinear velocity profile between the parallel plates, where two distinct flow types can be identified: plane Poiseuille flow and Couette flow. Each of these flow types is analyzed separately in this section.

Plane Poiseuille Flow

Plane Poiseuille flow is induced by a pressure gradient along the length of the parallel plates. In this scenario, the inlet pressure exceeds the outlet pressure, creating a negative pressure gradient. This flow results in a symmetric velocity profile around the x -axis while satisfying the no-slip condition at both boundaries. Under these conditions, the Navier-Stokes equations simplify to:

$$\mu \frac{\partial^2 u}{\partial y^2} = \frac{dp}{dx} \quad (\text{B.21})$$

With:

$$u(b) = u(-b) = 0 \quad \frac{\partial u}{\partial y}(0) = 0 \quad (\text{B.22})$$

Solving the second-order differential equation with its corresponding boundary conditions results in the following velocity profile:

$$u(y) = \frac{b^2}{2\mu} \left(-\frac{dp}{dx} \right) \left(1 - \left(\frac{y}{b} \right)^2 \right) \quad (\text{B.23})$$

By using the definition of bulk velocity, this equation can be further simplified:

$$u_b = \frac{1}{2b} \int_{-b}^b u dy = \frac{b^2}{3\mu} \left(-\frac{dp}{dx} \right) \quad (\text{B.24})$$

$$u(y) = \frac{3}{2} u_b \left(1 - \left(\frac{y}{b} \right)^2 \right) \quad (\text{B.25})$$

The obtained velocity profile for plane Poiseuille flow is parabolic in y , as schematically shown in [Figure B.2](#).

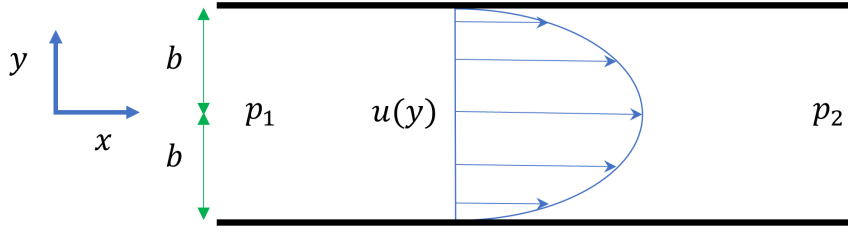


Figure B.2: Schematic representation of plane Poiseuille flow. *This image is repeated as Figure 2.1a in Section 2.1 of the main text.*

Couette Flow

Couette flow describes the motion of a fluid layer confined between two parallel plates, where the flow is induced solely by the relative motion of the top plate with respect to the stationary bottom plate. In this scenario, the movement of the top plate generates a shear-driven flow due to the viscosity of the fluid. Notably, there is no pressure gradient acting in the direction of the flow, making it a purely shear-driven phenomenon. By applying the Navier-Stokes equations to this specific case, the resulting governing flow equation can be derived:

$$\mu \frac{\partial^2 u}{\partial y^2} = 0 \quad (\text{B.26})$$

This second-order differential equation can be solved through two successive integrations, followed by the application of the appropriate boundary conditions:

$$u(-b) = 0 \quad u(b) = V \quad (\text{B.27})$$

$$u(y) = \frac{V}{2} \left(\frac{y}{b} + 1 \right) \quad (\text{B.28})$$

As can be seen, a simple linear velocity profile in y is obtained, as schematically depicted in Figure B.3.

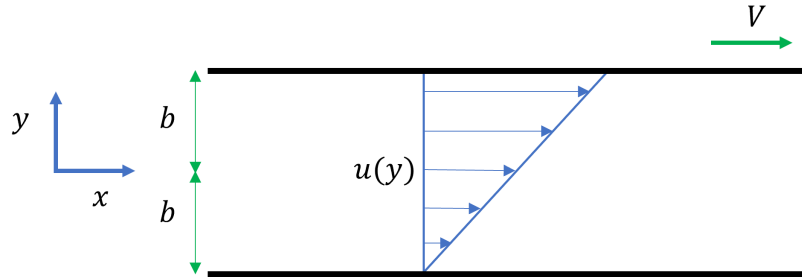


Figure B.3: Schematic representation of plane Couette flow. *This image is repeated as Figure 2.1b in Section 2.1 of the main text.*

B.2.2 Temperature Profiles

For the rectilinear flow analysis, the thermal energy equation presented in Equation B.18 simplifies to Equation B.29. Convection is accounted for by the term on the left-hand side of the equation. On the right-hand side, the first term signifies conduction, the second term denotes the material pressure derivative, and the third term represents viscous dissipation [7].

$$\rho c_p u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial y^2} + u \frac{dp}{dx} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad (\text{B.29})$$

Since this equation involves both first-order and second-order partial derivatives of temperature with respect to x and y , obtaining an analytical solution is only feasible for specific cases. By defining appropriate thermal boundary conditions for the confining plates, analytical solutions can be derived to provide deeper insights into the system's thermal behaviour.

In this section, two types of boundary conditions will be considered: isothermal and adiabatic boundaries. An isothermal boundary maintains a constant temperature, making it suitable for conductive bodies that can efficiently distribute heat, effectively acting as a heat sink. For the air bearing system, such a boundary condition appears to be well-suited. In contrast, an adiabatic boundary prevents heat exchange between the surface and its surroundings, implying that the temperature gradient at the boundary is zero. Adiabatic boundaries are commonly used for insulated systems or rapid processes where insufficient time is available for significant heat transfer to occur.

A MATLAB script was developed ([Appendix H](#)) to visualize temperature profiles under various boundary conditions and flow types. The system parameters used here are summarized in [Table B.2](#). These values do not directly correspond to those of the designs presented in [Appendix A](#), but rather serve to provide an initial understanding of the underlying problem. T_{s1} and T_{s2} represent the isothermal temperature of the bottom and top boundaries, respectively. Two different values for T_{s2} were selected to demonstrate the impact of having both similar and dissimilar boundary temperatures. For Couette flow a smaller delta was chosen, because of the low flow velocity. Additionally, T_{aw} denotes the adiabatic wall temperature at $x = 0$, while V represents the velocity of the top plate with respect to the bottom plate.

Table B.2: Parameters that were used in the MATLAB script.

Parameter	Value	Unit
b	7.5	μm
L	10	mm
p_1	1.5×10^5	Pa
p_2	0.8×10^5	Pa
ρ	1.22	kg/m^3
μ	1.8×10^{-5}	Pa s
c_p	1005	J/kg K
k	0.0261	W/m K
T_{s1}	293	K
$T_{s2,pois}$	293/293.2	K
$T_{s2,cou}$	293/293.0001	K
T_{aw}	293	K
V	1	m/s

Plane Poiseuille Flow

Using the parameters outlined above, the velocity profile for plane Poiseuille flow is shown in [Figure B.4](#). As observed, the maximum velocity in the channel reaches approximately 11 m/s.

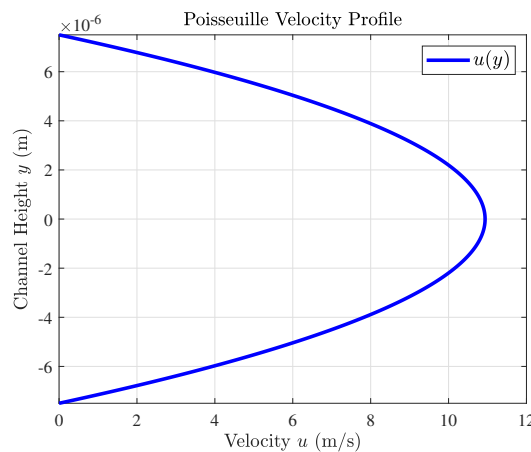


Figure B.4: Velocity profile for plane Poiseuille flow.

Isothermal Boundaries

First, the case of two isothermal boundaries is examined for plane Poiseuille flow. Since both boundaries maintain a constant temperature, the airflow will eventually reach thermal equilibrium, allowing the convection term in the thermal energy equation to be neglected. This simplification leads to the following governing equation:

$$0 = k \frac{\partial^2 T}{\partial y^2} + u \frac{dp}{dx} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad (\text{B.30})$$

By substituting the velocity profile (Equation B.25) and the pressure gradient (Equation B.21), the following second-order differential equation is derived:

$$\frac{\partial^2 T}{\partial y^2} = -\frac{9\mu u_b^2}{2b^2 k} \left(\frac{3y^2}{b^2} - 1 \right) \quad (\text{B.31})$$

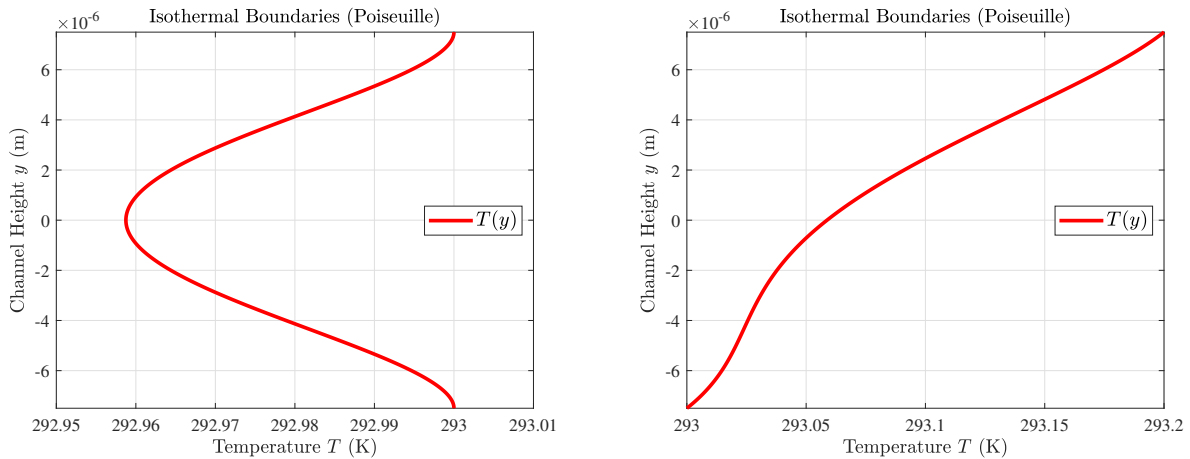
With boundary conditions:

$$T(-b) = T_{s1} \quad T(b) = T_{s2} \quad (\text{B.32})$$

Solving this differential equation yields:

$$T(y) = \frac{T_{s2} - T_{s1}}{2b} y + \frac{T_{s2} + T_{s1}}{2} - \frac{9\mu u_b^2}{4b^2 k} \left(\frac{y^4}{2b^2} - y^2 + \frac{b^2}{2} \right) \quad (\text{B.33})$$

Figure B.5 shows the resulting temperature profile for two isothermal boundaries. For the case of two similar isothermal boundaries (Figure B.5a), it is evident that the temperature profile is perpendicular to the boundaries, indicating a zero temperature gradient. This means that the boundaries are also adiabatic. Furthermore, at the center of the channel, a temperature drop of about 40 mK is observed, caused by the expansion of the air. On the other hand, for two dissimilar isothermal boundaries (Figure B.5b), the temperature profile becomes more linear, highlighting the effects of heat conduction. In this case, the boundaries are no longer adiabatic.



(a) Isothermal boundaries at the same temperature. (b) Isothermal boundaries at different temperatures.

Figure B.5: Temperature profile between the parallel plates for two isothermal boundaries for plane Poiseuille flow. Figure B.5a is repeated as Figure 2.2a in Section 2.1 of the main text.

Adiabatic Boundaries

When applying two adiabatic boundary conditions, a slightly different approach needs to be used. As suggested by Al-Bender [7], the temperature profile is assumed to be of the form:

$$T(x, y) = f(x) + g(y) \quad (\text{B.34})$$

Where $f(x)$ is the adiabatic wall temperature and $g(y)$ is the difference of the gas temperature with the wall temperature. Inserting this equation into the thermal energy equation (Equation B.29) yields:

$$\rho c_p u f'(x) = k g''(y) + u \frac{dp}{dx} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad (\text{B.35})$$

With boundary conditions:

$$g(b) = 0 \quad g'(b) = 0 \quad (\text{B.36})$$

First, Equation B.35 can be integrated twice with respect to y . By consequently applying the boundary conditions stated above, $g(y)$ can be written as a function of $f'(x)$:

$$g(y) = h(f'(x), y) \quad (\text{B.37})$$

With boundary conditions:

$$g'(-b) = 0 \quad f(0) = T_{aw} \quad (\text{B.38})$$

Using these additional boundary conditions, both $f(x)$ and $g(y)$ can be found. For this particular case $f(x)$ turns out to be a constant, meaning the adiabatic wall temperature does not change over x :

$$f(x) = T_{aw} \quad \rightarrow \quad f'(x) = 0 \quad (\text{B.39})$$

The resulting equation for the temperature profile is shown in Equation B.40. It is important to note that the terms following $\frac{\rho c_p}{6\mu u_b}$ are relatively small compared to the rest of the equation, allowing them to be neglected for simplicity. The simplified equation shows a clear correspondence with the temperature profile for two isothermal boundaries (Equation B.33).

$$T(x, y) = T_{aw} - \frac{9\mu u_b^2 (b-y)^2}{4b^4 k} \left(\left(\frac{y^2}{2} + by + \frac{b^2}{2} \right) + \frac{\rho c_p}{6\mu u_b} \left(\frac{b^2 y^2}{3} + \frac{2b^3 y}{3} - b^4 \right) \right) \quad (\text{B.40})$$

$$T(x, y) \approx T_{aw} - \frac{9\mu u_b^2}{4b^2 k} \left(\frac{y^4}{2b^2} - y^2 + \frac{b^2}{2} \right) \quad (\text{B.41})$$

As anticipated, the temperature profile between the parallel plates for two adiabatic boundaries (Figure B.6) equals that of the case with two isothermal boundaries (Figure B.5). Additionally, the temperature profile remains constant along the x -direction, indicating that no energy is dissipated in the flow.

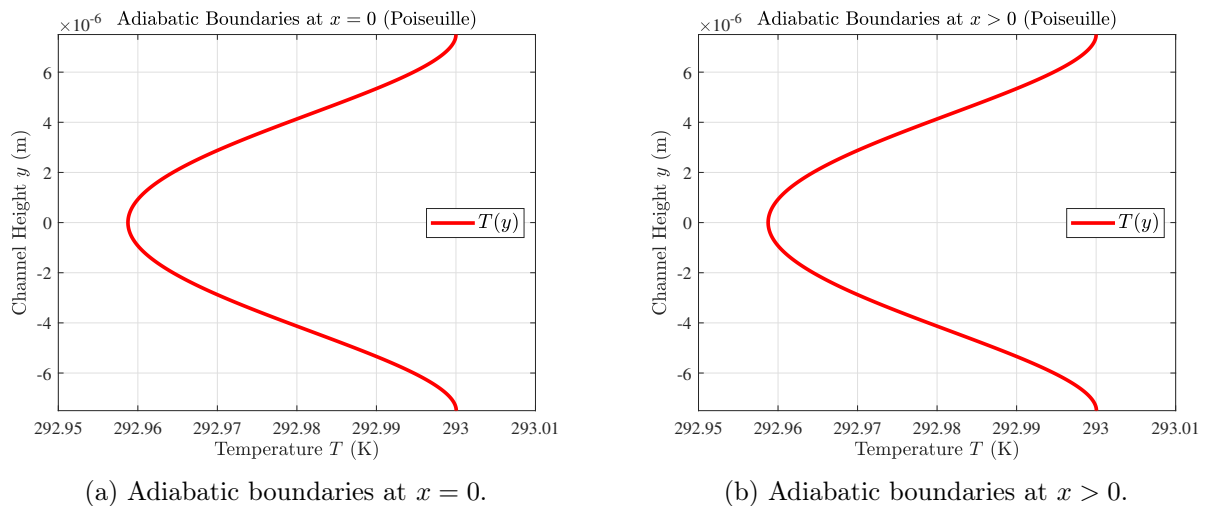


Figure B.6: Temperature profile between the parallel plates for two adiabatic boundaries for plane Poiseuille flow.

One Isothermal and One Adiabatic Boundary

In this case, the top plate is isothermal with a temperature of $T_{s2} = 293.2$ K, while the bottom plate is adiabatic with an initial temperature of $T_{aw} = 293$ K. The same approach as before can be applied, but with slightly adjusted boundary conditions. The thermal energy equation remains the same as in Equation B.35. Since the bottom plate is now adiabatic, the corresponding boundary conditions are as follows:

$$g(-b) = 0 \quad g'(-b) = 0 \quad (\text{B.42})$$

As before, $f(x)$ and $g(y)$ can be found by applying additional boundary conditions:

$$g(b) = T_{s2} - f(x) \quad f(0) = T_{aw} \quad (\text{B.43})$$

Solving the thermal energy equation subject to the boundary conditions stated above yields the adiabatic wall temperature function shown in Equation B.44. An exponential function is obtained that quickly reaches zero due to its large negative power. At $x = 0$, the adiabatic wall temperature corresponds to the initial condition T_{aw} . Slightly further downstream, the adiabatic wall temperature adjusts to the isothermal wall temperature. This transition is illustrated in Figure B.7. Initially, one boundary is isothermal and one is adiabatic, but once the flow is fully thermally developed, both boundaries are isothermal and adiabatic once again.

$$f(x) = T_{s2} + (T_{aw} - T_{s2}) e^{-\frac{kx}{2b^2 c_p \rho u_b}} \quad (\text{B.44})$$

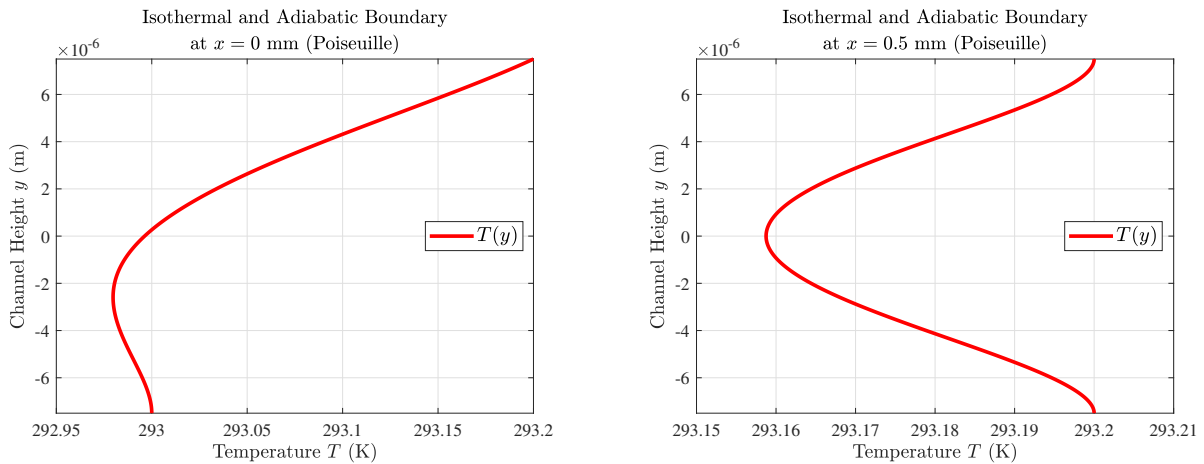
For small values of x , the temperature profile can be written as Equation B.45. As x increases, $h(x)$ (Equation B.46) quickly reaches zero, allowing the temperature profile to be approximated with Equation B.47.

$$T(x, y) = T_{s2} + h(x) - \frac{9\mu u_b^2 (b+y)^2}{4b^4 k} \left(\left(\frac{y^2}{2} - by + \frac{b^2}{2} \right) + \frac{kh(x)}{18\mu u_b^2} \left(-\frac{y^2}{2} + by + \frac{3b^2}{2} \right) \right) \quad (\text{B.45})$$

With:

$$h(x) = (T_{aw} - T_{s2}) e^{-\frac{kx}{2b^2 c_p \rho u_b}} \quad (\text{B.46})$$

$$T(x, y) \approx T_{s2} - \frac{9\mu u_b^2}{4b^2 k} \left(\frac{y^4}{2b^2} - y^2 + \frac{b^2}{2} \right) \quad (\text{B.47})$$



(a) One isothermal and one adiabatic boundary at $x = 0$ mm. (b) One isothermal and one adiabatic boundary at $x = 0.5$ mm.

Figure B.7: Temperature profile between the parallel plates for one isothermal (top) and one adiabatic boundary (bottom) for plane Poiseuille flow.

Couette Flow

The velocity profile for Couette flow was plotted in MATLAB using the parameters shown in [Table B.2](#). As expected, the velocity increases linearly from zero at the bottom boundary to V at the top boundary. The flow velocity in the thin air film is generally much lower for Couette flow compared to Poiseuille flow in this system, suggesting that the thermal effects in Couette flow are likely to be negligible. Nonetheless, examining the thermal behaviour of Couette flow remains important as it provides valuable insights into the thermal effects associated with a moving substrate.

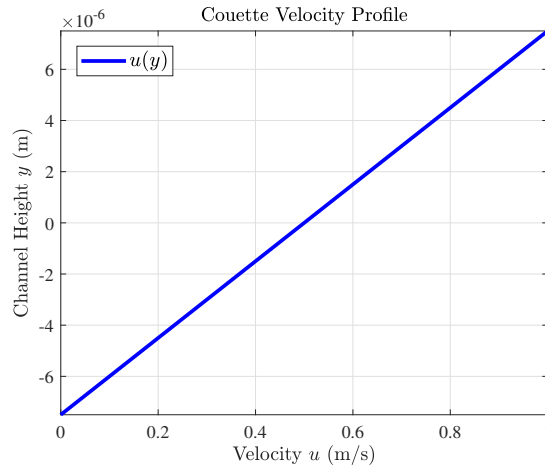


Figure B.8: Velocity profile for Couette flow.

Isothermal Boundaries

When analyzing Couette flow between two isothermal boundaries, the pressure gradient and convection terms can be omitted from the thermal energy equation, simplifying it to [Equation B.48](#):

$$0 = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad (\text{B.48})$$

Substituting the velocity profile for Couette flow ([Equation B.28](#)) into the equation, the following differential equation is obtained:

$$\frac{\partial^2 T}{\partial y^2} = -\frac{\mu V^2}{4b^2 k} \quad (\text{B.49})$$

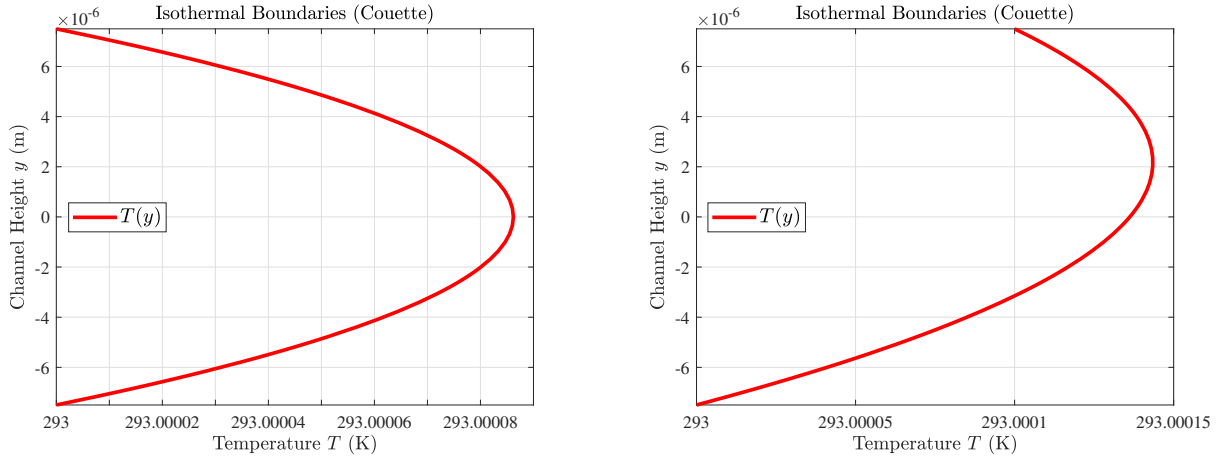
With boundary conditions:

$$T(-b) = T_{s1} \quad T(b) = T_{s2} \quad (\text{B.50})$$

Solving this differential equation yields the following temperature profile:

$$T(y) = \frac{T_{s2} - T_{s1}}{2b} y + \frac{T_{s2} + T_{s1}}{2} + \frac{\mu V^2}{8k} \left(1 - \left(\frac{y}{b} \right)^2 \right) \quad (\text{B.51})$$

The resulting quadratic temperature profile is plotted in [Figure B.9](#) for both similar and dissimilar isothermal wall temperatures. Compared to Poiseuille flow, the temperature difference at the center of the channel is significantly smaller. This is expected, as the flow velocity in Couette flow is only a fraction of that in Poiseuille flow. Moreover, a temperature increase is observed instead of a decrease. This can be explained by the absence of a pressure gradient, meaning viscous dissipation has a more pronounced effect in Couette flow. For the case of two dissimilar isothermal boundaries, the linear relation between the two temperatures is noticeable, with the parabolic profile superimposed.



(a) Isothermal boundaries at the same temperature. (b) Isothermal boundaries at different temperatures.

Figure B.9: Temperature profile between the parallel plates for two isothermal boundaries for Couette flow. *Figure B.9a is repeated as Figure 2.2b in Section 2.1 of the main text*

Adiabatic Boundaries

For the case of two adiabatic boundaries, only the pressure gradient term can be neglected, resulting in the thermal energy equation shown in Equation B.52. The solution approach is similar to the previous adiabatic boundary analysis, assuming a temperature profile of the form shown in Equation B.34.

$$\rho c_p u \frac{\partial T}{\partial x} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad (\text{B.52})$$

Substituting the assumed temperature profile into the simplified thermal energy equation results in the following governing equation:

$$\rho c_p u f'(x) = k g''(y) + \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad (\text{B.53})$$

With boundary conditions:

$$g(-b) = 0 \quad g'(-b) = 0 \quad (\text{B.54})$$

After solving for $g(y)$ as a function of $f'(x)$, the additional boundary conditions below can be applied to find the final solution.

$$g'(b) = 0 \quad f(0) = T_{aw} \quad (\text{B.55})$$

A linear function in x is obtained for the adiabatic wall temperature function:

$$f(x) = T_{aw} + \frac{\mu V x}{2 \rho c_p b^2} \quad (\text{B.56})$$

The resulting temperature profile equation is shown in Equation B.57, with the corresponding plots depicted in Figure B.10. As can be seen, the flow heats up rapidly. Even for the low flow velocity specified for Couette flow, the temperature increases at a rate of $dT/dx = 130.5$ K/m. This implies that the convection rate is considerably smaller than the dissipation rate, indicating the inefficiency of Couette flow in dissipating the heat generated by viscous dissipation. Additionally, the lower adiabatic boundary has a higher temperature than the top boundary. This can be explained by the lower flow velocity at the bottom of the channel, which reduces the convection rate.

$$T(x, y) = T_{aw} + \frac{\mu V x}{2\rho c_p b^2} - \frac{\mu V^2}{24b^2 k} \left(-\frac{y^3}{b} + 3by + 2b^2 \right) \quad (\text{B.57})$$

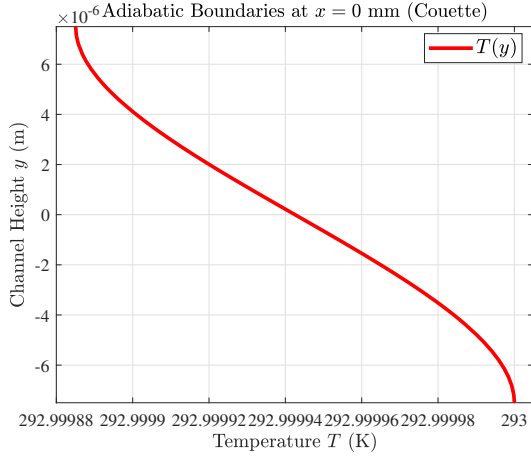
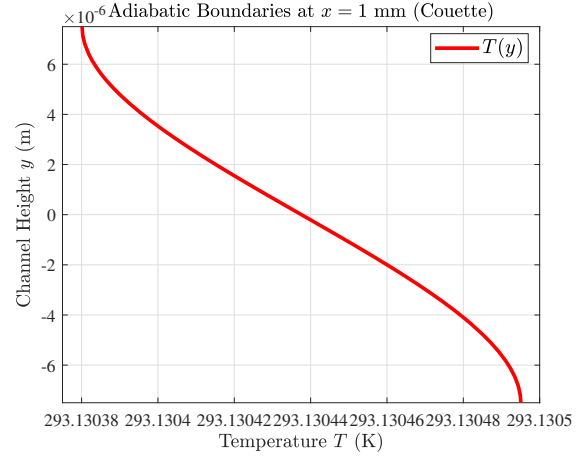
(a) Adiabatic boundaries at $x = 0$ mm.(b) Adiabatic boundaries at $x = 1$ mm.

Figure B.10: Temperature profile between the parallel plates for two adiabatic boundaries for Couette flow.

One Isothermal Boundary and One Adiabatic Boundary

Here, the lower boundary is assumed to be adiabatic with initial temperature $T_{aw} = 293$ K and the upper boundary is assumed to be isothermal with temperature $T_{s2} = 293$ K. The governing thermal energy equation is the same as for the case with two adiabatic boundaries (Equation B.53), with boundary conditions:

$$g(-b) = 0 \quad g'(-b) = 0 \quad (\text{B.58})$$

And additional boundary conditions:

$$g(b) = T_{s2} - f(x) \quad f(0) = T_{aw} \quad (\text{B.59})$$

The resulting adiabatic wall temperature function is shown in Equation B.60. An exponential function is obtained, where $f(x) = T_{aw}$ at $x = 0$ and $f(x) \approx T_s + \frac{\mu V^2}{2k}$ once the flow is fully thermally developed.

$$f(x) = T_s - \left(\frac{\mu V^2}{2k} + T_s - T_{aw} \right) e^{-\frac{3kx}{2\rho V c_p b^2}} + \frac{\mu V^2}{2k} \quad (\text{B.60})$$

For small values of x , the temperature profile can be described by Equation B.61. As x increases, $h(x)$ (Equation B.62) rapidly approaches zero, which allows the temperature profile to be approximated by Equation B.63.

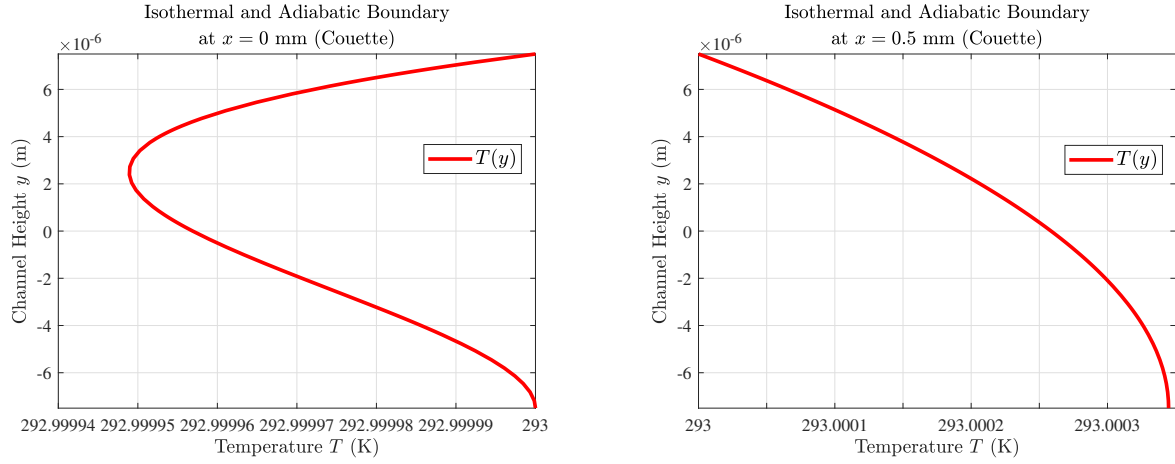
$$T(x, y) = T_{s2} - \frac{\mu V^2}{4k} \left(\frac{y^2}{2b^2} + \frac{y}{b} - \frac{3}{2} \right) + \frac{h(x)}{8} \left(\frac{y^3}{b^2} + \frac{3y^2}{b^2} + \frac{3y}{b} - 7 \right) \quad (\text{B.61})$$

With:

$$h(x) = \frac{e^{-\frac{3kx}{2\rho V c_p b^2}} (\mu V^2 - 2T_{aw}k + 2T_{s2}k)}{2k} \quad (\text{B.62})$$

$$T(x, y) \approx T_{s2} - \frac{\mu V^2}{4k} \left(\frac{y^2}{2b^2} + \frac{y}{b} - \frac{3}{2} \right) \quad (\text{B.63})$$

The resulting temperature profile has been plotted for two locations in the channel in Figure B.11. As observed, the entrance effects are minimal, and the adiabatic wall temperature quickly stabilizes at a value slightly higher than the isothermal wall temperature. If the isothermal and adiabatic boundaries were swapped, the adiabatic boundary would be cooler than the isothermal boundary, rather than warmer. This behaviour is attributed to the dependence of convection on the flow velocity, with the lower flow velocity near the bottom plate reducing the convective heat transfer.



(a) One isothermal and one adiabatic boundary at $x = 0$ mm. (b) One isothermal and one adiabatic boundary at $x = 0.5$ mm.

Figure B.11: Temperature profile between the parallel plates for one isothermal (top) and one adiabatic boundary (bottom) for Couette flow.

C

Supplementary COMSOL Modelling

Based on the COMSOL analysis of several geometries presented in [Chapter 3](#), a good understanding of the thermomechanical response of the wafer on top a radial flow air bearing system has been formed. This section discusses some essential factors affecting both the temperature distribution and mechanical deformation of the wafer. Based on these insights, the design parameters for the test setup are ultimately determined in [Section 3.4](#). All simulations discussed in this chapter are stationary and assume an isothermal air bearing system. Unless otherwise specified, the geometry and parameters presented in [Figure 3.16](#) and [Table 3.1](#) are used to model the system.

C.1 Inlet Shape

As shown in [Figure 3.12](#), the sharp edge of the unit cell locally disturbs the flow as it transitions from the vertical inlet into the thin film. To improve this transition, a rounding or chamfer can be introduced at the sharp edge of the unit cell. A rounding completely removes sharp corners from the geometry, while a chamfer still leaves two sharp edges, but reduces the angle between the intersecting surfaces. Chamfered edges are typically easier to manufacture. Given the short entrance length of the thin film, a rounding or chamfer with a characteristic dimension on the order of the film height may already be sufficient to achieve the desired flow behaviour.

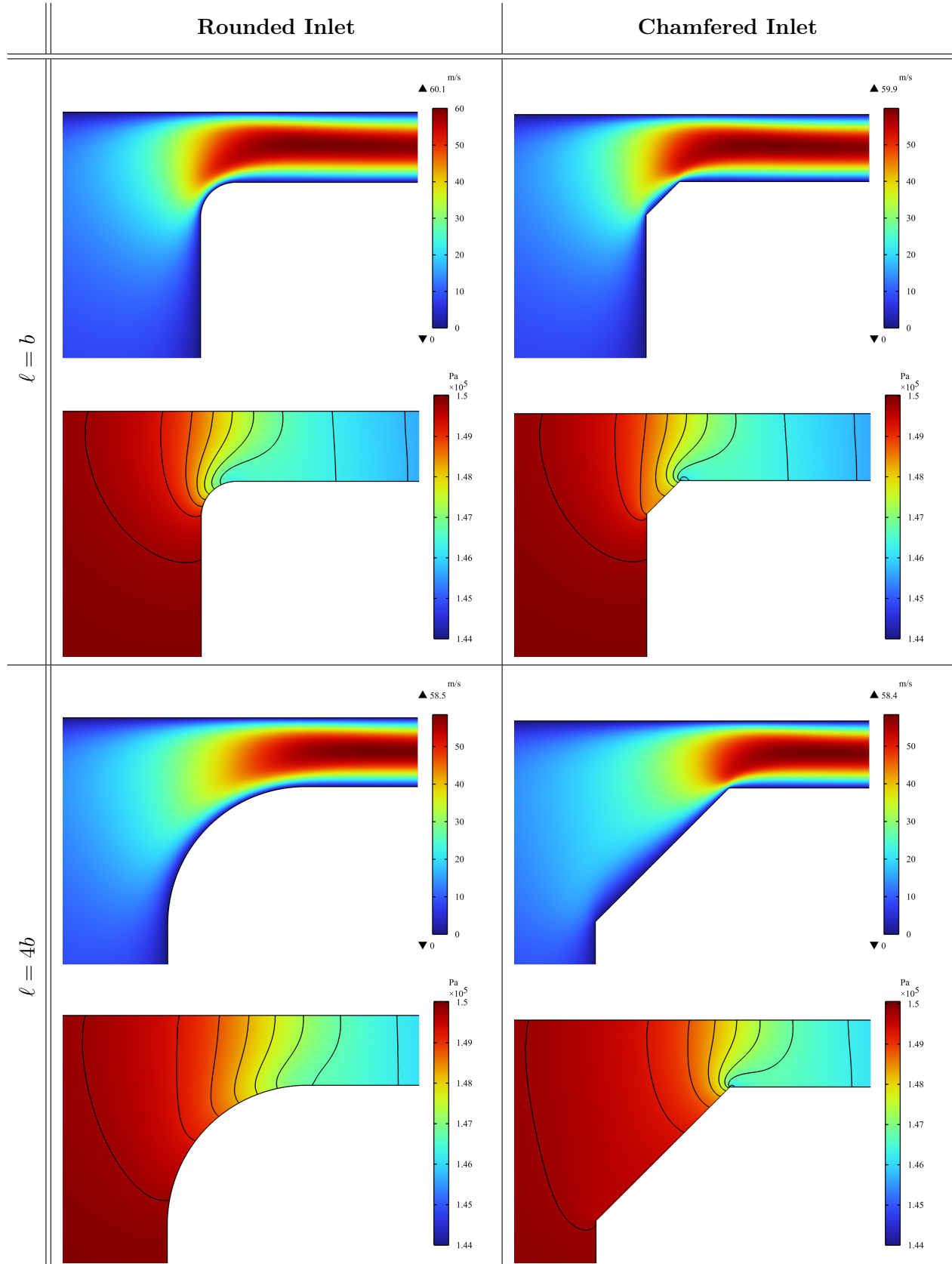
To study the impact of the inlet shape on the development of the flow, first the pressure and velocity profiles were compared for a sharp, chamfered, and rounded unit cell edge. Two characteristic lengths were considered: one equal to half the film height ($\ell = b$) and another equal to twice the film height ($\ell = 4b$). For a rounded edge, the characteristic length is the radius of the rounding. For a chamfered edge, the characteristic length is defined as the perpendicular distance from the original sharp edge to the new edge formed by the chamfer. Subsequently, the wafer's temperature distribution was analyzed for the same inlet shape variations. Finally, the influence of increasing the characteristic length on the air bearing system's performance is discussed.

C.1.1 Pressure & Velocity Profiles

[Table C.1](#) shows the colour maps of the pressure and velocity distribution at the inlet of the thin film for each inlet shape. The colour range of the pressure map was adjusted to clarify the pressure variations in the inlet region. Moreover, each isobar represents a pressure drop of 0.5 kPa. Compared to the sharp inlet geometry ([Figure 3.12](#)), the maximum velocity decreased for a modified inlet shape. This reduction is attributed to the increase in flow area at the entrance of the thin film. For inlet shapes with a larger characteristic length, the radial position where the thin film reaches its nominal height shifts further downstream, resulting in a lower maximum velocity near the inlet. However, by locally increasing the flow area at the thin film inlet, the overall flow restriction introduced by the thin film is slightly reduced. Therefore, the flow velocity after the inlet of the thin film will be higher for modified inlet shapes. This translates to a higher flow velocity at the outlet of the thin film. However, for a characteristic length on the order of the film height, this effect is barely noticeable. Regarding the pressure distribution, a smoother transition from the vertical inlet to the thin film is observed. This effect is especially prominent for rounded edges, where local pressure disturbances are minimized, and the isobars tend to align more perpendicularly to the boundaries. In contrast, the chamfered inlets still exhibit localized pressure disturbances at the sharp edges of the chamfer. However, dividing the original 90° edge into two 45° angles reduces these disruptions. Additionally, the

characteristic length of the inlet modifications shows a clear correlation with the flow development: larger characteristic lengths result in less significant local flow disruptions.

Table C.1: Colour maps of the pressure and velocity distribution at the thin film inlet for different inlet shapes and sizes.



To provide a more quantitative assessment of the pressure effects induced by the various inlet geometries, the pressure profiles at both the top and bottom surfaces of the thin film are presented in Figure C.1 and Figure C.2, respectively. At the top wall, the pressure gradients are largely similar across all cases, with only minor variations attributed to differences in local flow velocities. In Figure C.2, the absolute pressure along the entire circumference of the unit cell is plotted, where the x -axis represents the arc length rather than the radial coordinate. At $x = 1$ mm, the simulated height of the air bearing system, t_b , is reached, corresponding to the location of the sharp edge of the unit cell. Here, the sharp edge pressure profile shows an initial increase of approximately 0.01 bar, followed by a pronounced drop to a pressure of 1.37 bar. The pressure increase can be explained by the fact that the sudden change in flow direction and area causes the flow to locally decelerate. According to the isentropic flow relations or Bernoulli's law, a decrease in flow velocity results in an increase in pressure. The subsequent sharp pressure drop is primarily the result of flow separation at the Vena Contracta. For the chamfered inlets, the same effects can be observed as for the sharp inlet, but in twofold and with a smaller magnitude. While the magnitude of the second pressure drop is comparable between the two chamfered inlets, the first pressure drop varies noticeably. This variation stems from the differing velocities in the transition region between the vertical inlet and the thin film. A chamfer with a smaller characteristic length leads to a higher velocity in this zone, resulting in a more pronounced initial pressure drop compared to a chamfer with a larger characteristic length. The rounded inlets, on the other hand, create a much smoother transition from the vertical inlet to the thin film. The rounded inlet with a radius of half of the film height still results in a slight local pressure drop. However, the rounded inlet with a radius of twice the film height eliminates any local pressure disturbances, resulting in a more uniform flow and pressure distribution.

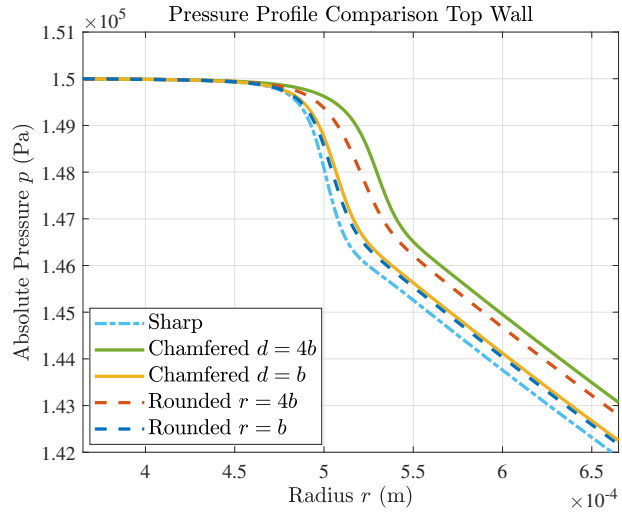


Figure C.1: Comparison of the pressure profiles at the top wall of the thin film for different inlet shapes.

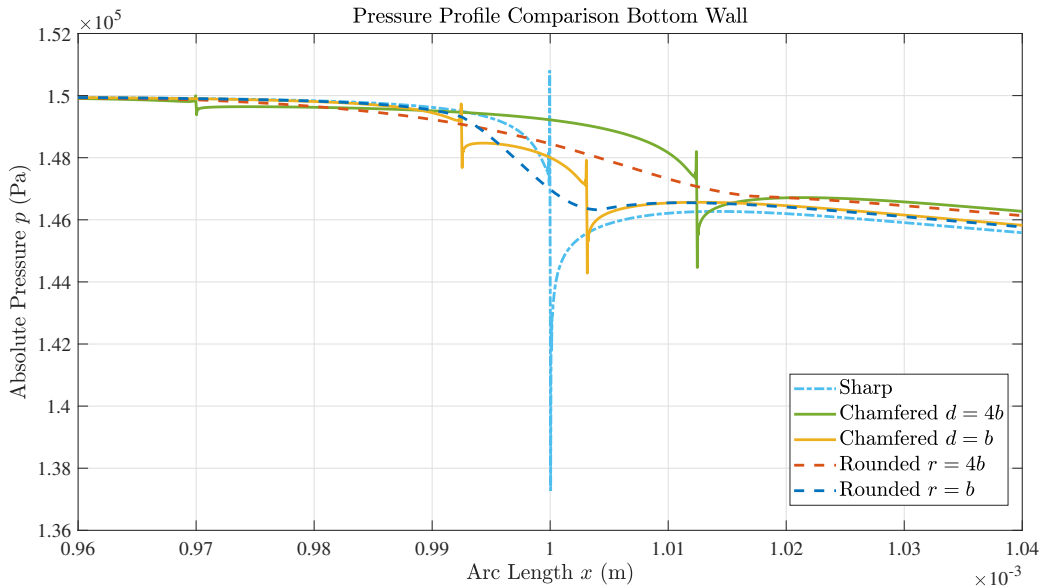


Figure C.2: Comparison of the pressure profiles at the bottom wall of the thin film for different inlet shapes.

C.1.2 Temperature Profiles

The corresponding temperature profiles at the bottom wall of the thin film and the top surface of the wafer are shown in Figure C.3 and Figure C.4, respectively. The temperature at the bottom of the thin film is plotted using the same x -axis data as the pressure profiles shown in Figure C.2, while the temperature profile at the top of the wafer is simply plotted over the entire radius of the geometry. It is evident that changing the shape of the inlet affects the temperature drop at the bottom of the thin film. For the chamfered inlets, two slight, but sharp temperature drops can be observed, instead of one more significant temperature drop for the sharp inlet. The rounded inlets result in a smoother transition from the vertical inlet to the thin film, without any abrupt temperature changes.

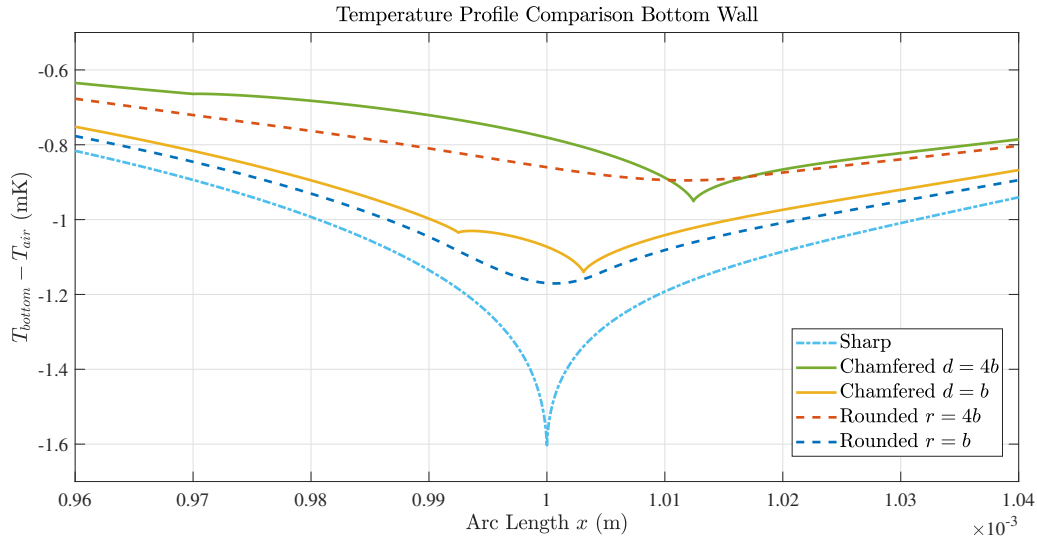


Figure C.3: Comparison of the temperature profiles at the bottom wall of the thin film for different inlet shapes.

Although the cooling effects at the thin film inlet were found to be less pronounced, Figure C.4 reveals that the absolute temperature at the top surface of the wafer decreases when a chamfer or rounding is applied to the inlet geometry. This reduction in temperature is attributed to the velocity-dependent nature of the dissipation rate, as previously discussed in Section 2.2.2. By introducing a chamfer or rounding, the flow velocity at the inlet of the thin film is reduced, leading to a significant decrease in heat dissipation. Consequently, it can be concluded that altering the inlet shape directly influences the balance between cooling and dissipation effects near the thin film inlet.

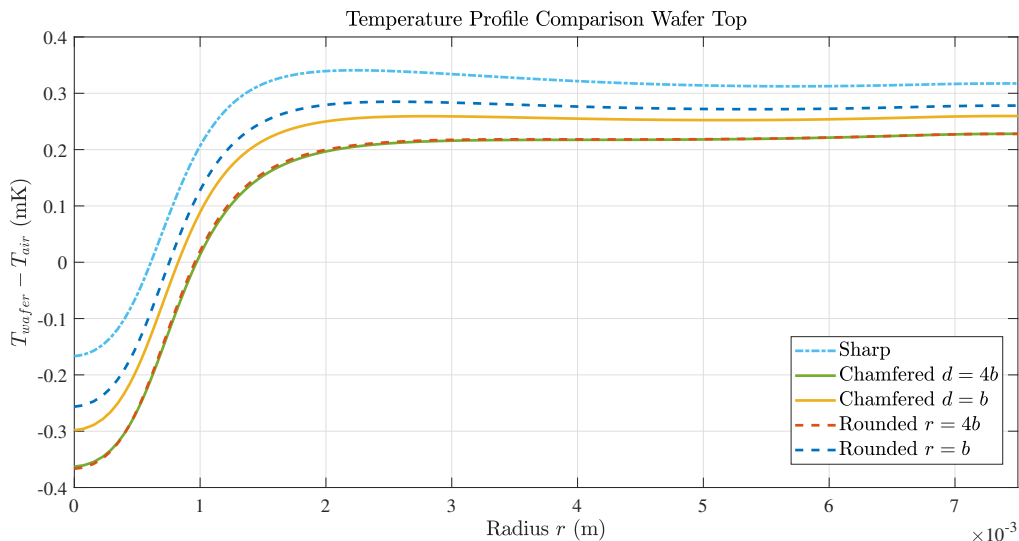


Figure C.4: Comparison of the temperature profiles at the top of the wafer for different inlet shapes.

The inlet shape impacts not only the absolute temperature of the wafer but also the relative temperature variations across its surfaces. Table C.2 presents the predicted temperature differences at both the top and bottom of the wafer for the various inlet configurations and characteristic lengths. It is evident that the temperature gradient becomes more pronounced when a chamfer or rounding is applied to the inlet, with larger characteristic lengths further amplifying this effect. At the bottom of the wafer, where the wafer shows its absolute maximum and minimum across its entire volume, the temperature differences increase slightly as well, although the effect is less significant than at the top surface.

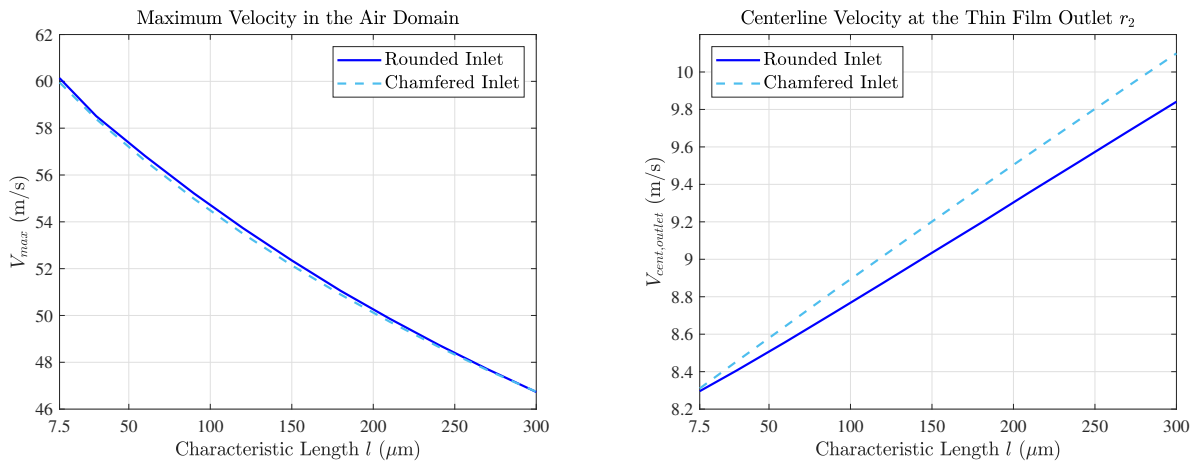
Table C.2: Relative temperature differences at the top and bottom of the wafer for various inlet shapes and dimensions.

Inlet Shape	$\Delta T_{wafer,top}$ (mK)	$\Delta T_{wafer,bottom}$ (mK)
<i>Sharp</i>	0.51	1.22
<i>Rounded $r = b$</i>	0.54	1.23
<i>Chamfered $d = b$</i>	0.56	1.25
<i>Rounded $r = 4b$</i>	0.59	1.26
<i>Chamfered $d = 4b$</i>	0.59	1.24

C.1.3 Performance

For the system parameters used in the simulations presented above, a characteristic length on the order of magnitude of the film height already showed sufficient smoothing at the inlet of the thin film. However, for a different set of parameters, resulting in a higher flow velocity, a larger characteristic length could be required to obtain similar effects. Therefore, a parametric sweep was performed, using a characteristic length varying from $l = b$ to $l = 40b$. Given the significant computational effort required for these simulations, the number of sweep steps was limited to eleven for both the rounded and chamfered inlet configurations.

The resulting velocity profiles for both inlet shapes are presented in Figure C.5. Here, V_{max} represents the maximum flow velocity in the air domain, and $V_{cent,outlet}$ denotes the flow velocity along the centerline at the outlet of the thin film. As expected, the maximum flow velocity reduces for an increasing characteristic length, due to the locally increased flow area. Furthermore, the centerline velocity at the thin film outlet rises for larger characteristic lengths, as the restrictive effect of the thin film diminishes with increasing inlet size.

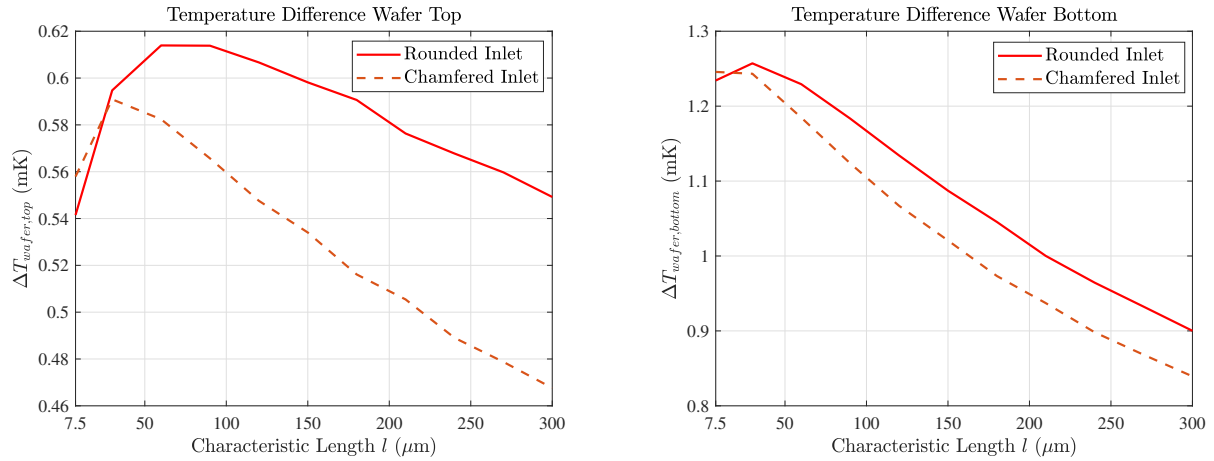


(a) Maximum flow velocity in the air domain.

(b) Centerline velocity at the outlet of the thin film.

Figure C.5: Velocity profiles for a varying characteristic length of the inlet rounding and chamfer.

The corresponding maximum temperature differences across the top and bottom surfaces of the wafer are presented in Figure C.6a and Figure C.6b, respectively. These results show that, following an initial increase in temperature differences for small characteristic lengths (consistent with the trends observed in Table C.2), the temperature gradients across the wafer progressively diminish as the characteristic length increases. This behaviour indicates that incorporating a chamfer or rounding with a larger characteristic length at the thin film inlet contributes to a more uniform thermal distribution, thereby reducing temperature gradients across the wafer.



(a) Maximum temperature difference on the top surface of the wafer.

(b) Maximum temperature difference on the bottom surface of the wafer.

Figure C.6: Maximum temperature difference on the wafer for a varying characteristic length of the inlet rounding and chamfer.

For the temperature measurements in this research, increasing the temperature differences across the wafer will facilitate the measurement process. However, for the practical implementation of the air bearing technology in a wafer handling system, minimizing temperature gradients across the wafer is crucial. Consequently, incorporating a relatively large chamfer or rounding at the thin film inlet could prove advantageous. Beside the beneficial effects of an alternative inlet shape on the pressure and temperature distribution, a large characteristic length is expected to have detrimental effects on both the mass flow density and the force density, which are two important performance metrics used in the previous research outlined in Appendix A. Therefore, future work should further investigate the influence of inlet shape on the actuation performance of the air bearing system. This could include exploring additional inlet geometries, such as elliptical profiles or chamfers with a shallower angle relative to the horizontal plane, which are anticipated to provide an even more gradual acceleration of the flow.

In conclusion, modifying the sharp edge of the unit cell at the thin film inlet has a pronounced impact on the flow behaviour. Introducing a chamfer or rounding with a characteristic length on the order of the film height creates a smoother transition from the vertical inlet to the thin film. This adjustment improves the uniformity of the pressure and velocity profiles and reduces temperature disturbances near the inlet. Additionally, using a larger chamfer or rounding further decreases temperature gradients across the wafer while preserving a similar pressure distribution within the thin film. However, as the effectiveness of these inlet modifications is influenced by other system parameters, their implementation in the test setup is addressed in more detail in Section 3.4.

C.2 Thermal Material Properties

Another key factor influencing the temperature distribution within the wafer and the air bearing system is the choice of materials. For unsteady heat conduction, three material properties are important: density (ρ), specific heat capacity (c_p), and thermal conductivity (k). As outlined in [Appendix B.1](#), these properties can be combined to determine the thermal diffusivity (α) of a material.

The thermal diffusivity is measured in m^2/s and indicates how quickly heat diffuses through a material. This parameter only plays a role in the transient behaviour of the system. Since all COMSOL simulations up until now were stationary, only the thermal conductivity k plays a role in the final temperature distribution. However, for the experimental phase, the thermal diffusivity remains a relevant parameter in determining the required settling time for each measurement.

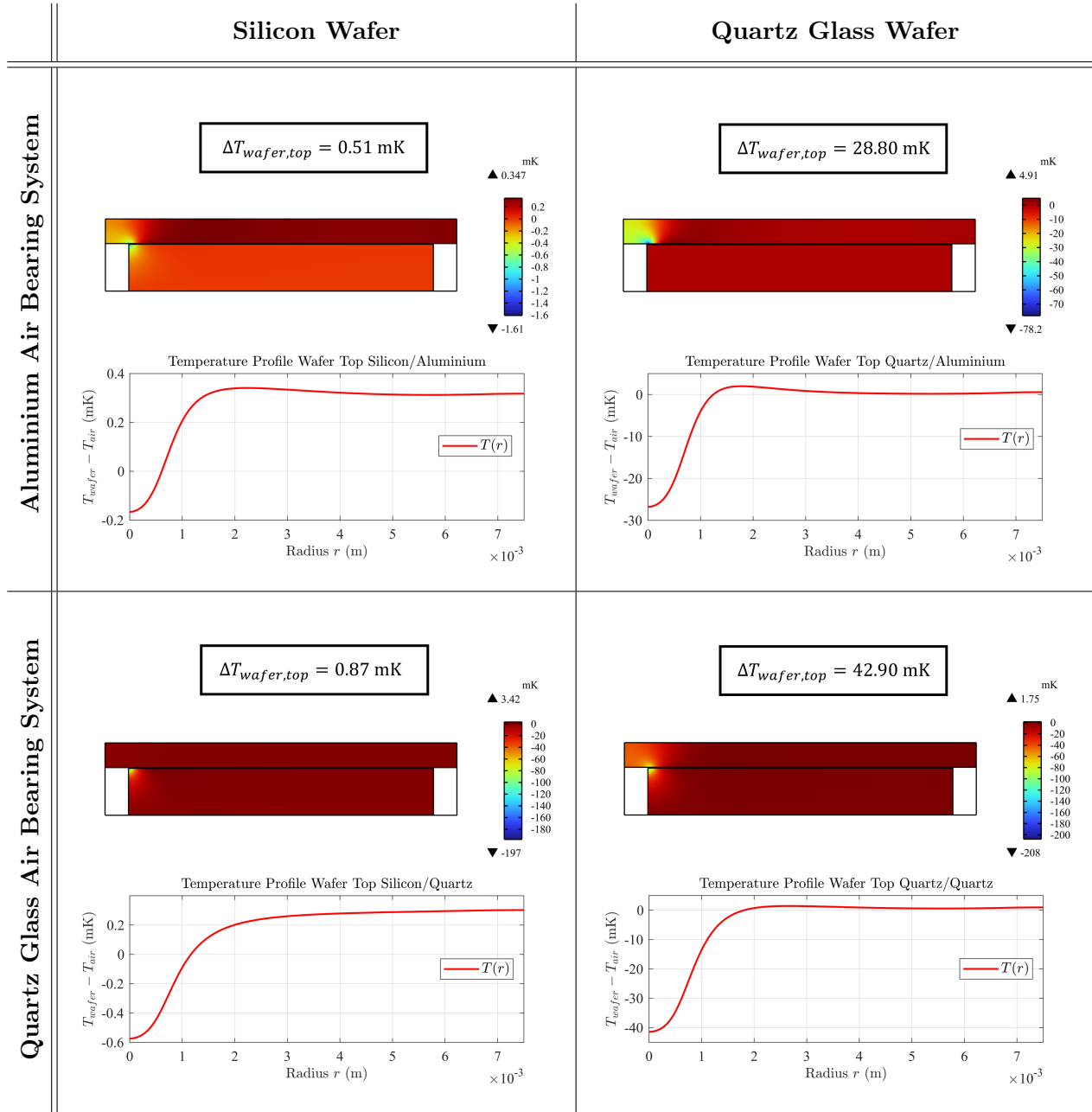
To assess the impact of material selection, several material combinations were evaluated for both the wafer and the air bearing system. The resulting solid temperature distributions and wafer surface temperature profiles are shown in [Table C.4](#). In addition to the commonly used aluminium (for the air bearing system) and silicon (for the wafer), quartz glass was also considered for both components. Quartz glass, a widely used alternative to silicon in the high-tech industry, exhibits significantly lower thermal conductivity, making it particularly relevant for this thermal analysis.

Although fabricating an air bearing system entirely from quartz glass is impractical due to manufacturing constraints, its inclusion provides valuable insights into how material properties affect the temperature distributions. A more feasible alternative with thermal conductivity similar to that of quartz glass could, for example, be a polymer such as PEEK. [Table C.3](#) gives an overview of the relevant thermal properties of aluminium, silicon, and quartz glass.

Table C.3: Thermal properties of aluminium, silicon, and quartz glass used in COMSOL [\[24\]](#).

Material	Thermal conductivity k (W/m K)	Specific heat capacity c_p (J/kg K)	Density ρ (kg/m ³)	Thermal diffusivity α (m ² /s)
Aluminium	238	900	2700	9.79×10^{-5}
Silicon	131	700	2329	8.04×10^{-5}
Quartz glass	1.40	730	2210	8.68×10^{-7}

Table C.4: Colour map and temperature profile of the relative temperature for several material combinations.



From the temperature distributions presented above, it is clear that the material of the wafer significantly influences its temperature distribution. This is particularly evident in the combination of the quartz glass wafer and aluminium air bearing system. In this case, the temperature profile at the top of the wafer shows a much lower minimum temperature of -26.82 mK, compared to the minimum of -0.17 mK for the original material combination. The maximum temperature also increases, though to a lesser extent, from 0.34 mK to 1.98 mK. The enhanced dominance of the cooling effect over the heating effect can be explained by the data in Figure 3.14. This graph illustrates that while the cooling effect at the thin film inlet is comparable in overall magnitude, it is much more localized than the heat dissipation effects further downstream. The low thermal conductivity of quartz glass prevents the heating and cooling effects from balancing each other, resulting in an increased measurable temperature difference of 28.80 mK for this material combination.

The material of the air bearing system has a less significant impact on the wafer's temperature

distribution. In the combination of a silicon wafer with a quartz glass air bearing system, the minimum temperature at the top of the wafer decreased to -0.57 mK, and the maximum temperature also decreased to 0.30 mK, resulting in a measurable temperature difference of 0.87 mK. A local temperature drop of -197 mK can be observed at the sharp edge of the unit cell. This contributes to a more notable temperature drop on the wafer as well, due to heat conduction through the thin air film. However, due to the low thermal conductivity of air ($k_{air} \approx 0.026$ W/m K), very little heat is transferred from the air bearing system to the wafer. Despite this, the silicon wafer's high thermal conductivity allows the increased cooling effects at the thin film inlet to more effectively counterbalance the dissipative heating effects, causing the maximum temperature to shift toward the outer radius of the geometry.

For the quartz glass wafer and quartz glass air bearing system combination, the previously described effects reinforce each other, leading to an even larger temperature difference. The minimum and maximum temperatures at the top of the wafer are -41.47 mK and 1.43 mK, respectively, resulting in a measurable temperature difference of 42.90 mK for this particular combination.

Overall, using a material with low thermal conductivity, such as quartz glass, offers significant advantages for the required temperature resolution of the temperature measurement. Notably, the material of the wafer was found to greatly increase the measurable temperature difference at the top of the wafer. However, as shown in Table C.4, quartz glass also has a much lower thermal diffusivity compared to aluminium and silicon, which slows the rate at which heat spreads through the material, resulting in a slower temperature response. Nevertheless, since the wafer is very thin, the thermal response is expected to remain largely unaffected. Furthermore, the test setup does not rely on rapid temperature changes, so the potentially slower thermal response of quartz glass is unlikely to influence the final results.

C.3 Film Height & Pressure

In Chapter 2, an analytical approximation of the maximum velocity and energy dissipation as a function of film height and pressure was presented. However, the impact of these parameters on the temperature distribution and mechanical deformation of the wafer was not discussed in detail. This section therefore explores the key considerations for selecting the film height and pressure for the test setup, using COMSOL modelling. Unlike in previous sections, the outlet pressure of the system is assumed to be atmospheric, in line with the actual test setup.

C.3.1 Velocity & Temperature Profiles

The maximum velocity in the axisymmetric domain was first determined for several combinations of inlet pressure and film height, as shown in Figure C.7. To obtain this figure, a *Parametric Sweep* was performed in COMSOL, using film heights ranging from 10 μm to 30 μm in 5 μm increments. Near a Mach number of 0.3, the compressibility effects of air change significantly, transitioning from incompressible flow to compressible flow [10]. Therefore, to avoid excessive complexity in the calculations and modelling assumptions, it is important to ensure that the maximum velocity stays below this threshold. For the parametric sweep, the inlet pressure range was selected such that the maximum velocity remained below this value for each film height. The inlet pressure range was set from 1.1 bar to 2.0 bar, with the maximum inlet pressure being reduced for film heights above 15 μm to ensure the maximum velocity remained within the desired limit. For each film height, ten pressure steps were included in the parametric sweep. As discussed in Section 2.2.1, increasing either parameter leads to a higher maximum velocity. The y -axis of the figure was capped at a maximum velocity of $V_{max} = 0.3 \cdot 343 = 103$ m/s to highlight only valid combinations.

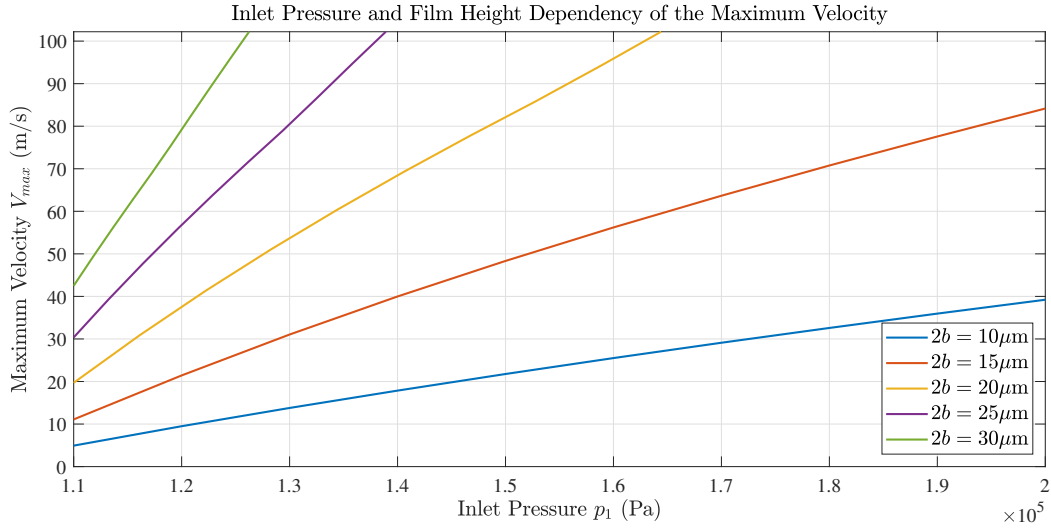


Figure C.7: Maximum velocity as a function of the inlet pressure and film height, using atmospheric outlet conditions.

The corresponding difference between the minimum and maximum temperature at the top of the wafer is shown in [Figure C.8](#) for the same combinations of parameters as above. Both an increased pressure difference and an increased film height lead to a higher measurable temperature difference. This aligns with the findings on energy dissipation in the thin film ([Section 2.2.2](#)) and the temperature drop at the thin film inlet ([Section 3.2.2](#)), both of which become more pronounced at higher flow velocities. However, the exact evolution of the separate effects for varying system parameters is unknown. Furthermore, it is unclear whether the inlet pressure and film height affect the temperature distribution separately or if only the combination (i.e., the velocity) is of importance. To gain further insight into these effects, the measurable temperature difference was plotted against the maximum velocity in the domain, as shown in [Figure C.9](#).

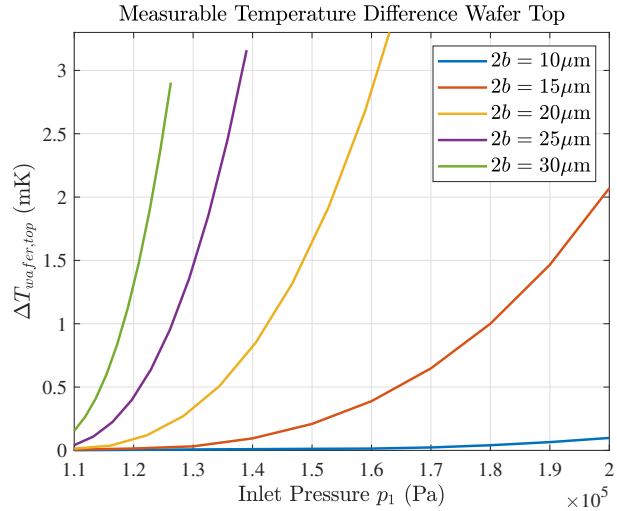


Figure C.8: Measurable temperature difference at the top surface of the wafer as a function of the inlet pressure and film height.

From this figure it becomes evident that at lower flow velocities, the measurable temperature difference is similar across all film heights. However, at higher velocities, the temperature difference increases for smaller film heights. This is likely due to the increased dominance of the cooling effect at the thin film inlet. For smaller film heights, the flow restriction is larger, resulting in more intense cooling effects at the Vena Contracta. Since heat dissipation depends on the flow velocity, its magnitude remains consistent for each film height at the same velocity. Therefore, the overall measurable temperature difference increases for smaller film heights with the same flow velocity.

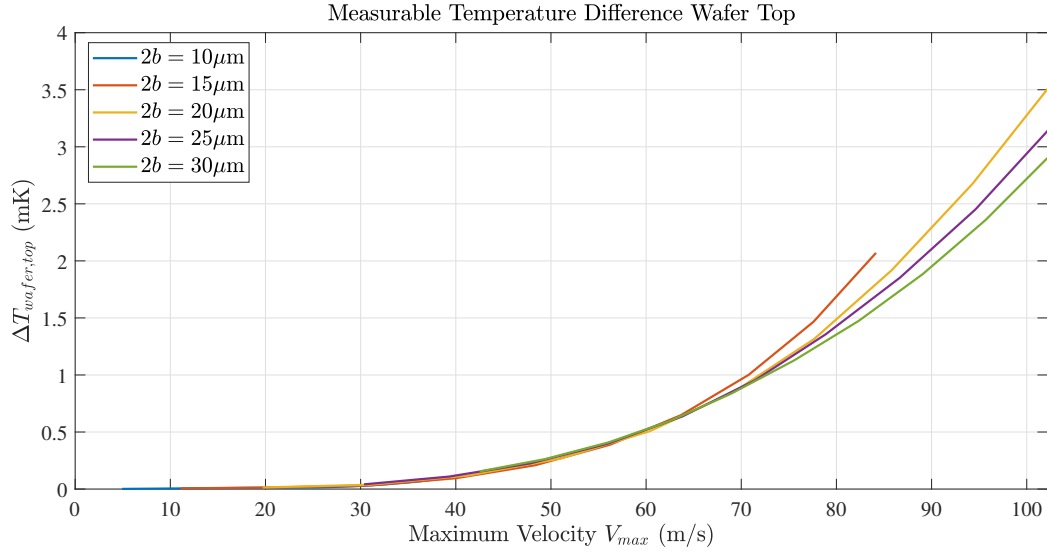


Figure C.9: Measurable temperature difference at the top surface of the wafer as a function of the maximum velocity in the domain.

Based on the results above, using a smaller film height could slightly amplify the measurable temperature difference. However, this would necessitate a higher inlet pressure, increasing the force exerted on the wafer. Additionally, when comparing the temperature effects of the film height/inlet pressure combination for a fixed velocity with the material selection discussed in the previous section, the influence of the former is much less significant. Therefore, the film height and inlet pressure will be chosen to achieve a specific flow velocity, without aiming to maximize the measurable temperature difference at this velocity.

C.3.2 Wafer Deformation

Besides the effects of the inlet pressure on the velocity and temperature distribution, it also affects the deformation of the wafer. The pressurized air below the wafer applies a distributed load to the surface, ultimately causing the wafer to bend upward. As mentioned previously, the wafer is clamped at a fixed distance from its center to simplify the test setup. Furthermore, the outlet pressure was chosen to be atmospheric. Compared to an operational air bearing system, this greatly simplifies the test setup. However, this also has detrimental effects on the wafer deformation, which will be further discussed in this section.

For a clamped circular plate with radius a subjected to a point load F at its center, the maximum deformation can be described using the following formula [26]:

$$w_{max} = \frac{Fa^2}{16\pi D} \quad (C.1)$$

For a uniformly distributed load q , the maximum deformation is calculated as:

$$w_{max} = \frac{qa^4}{64D} \quad (C.2)$$

Where D is the flexural rigidity:

$$D = \frac{Et_w^3}{12(1-\nu^2)} \quad (C.3)$$

Here, E represents Young's modulus and ν Poisson's ratio of the material. The relevant mechanical material properties used for the simulations presented in this section are shown in Table C.5. For silicon, the properties corresponding to the [1 0 0] crystal orientation were applied. Since both silicon

and quartz glass are brittle materials, their yield strength is not well-defined. As such, the tensile strength is considered the critical design limit instead.

Table C.5: Mechanical material properties of aluminium, silicon, and quartz glass used in COMSOL [12].

Material	Young's modulus E (GPa)	Poisson's ratio ν	Tensile strength (MPa)
Silicon	129.5	0.28	100
Quartz glass	72	0.17	48

Multiple Unit Cells

The initial conceptual design of the simplified air bearing system for the test setup consisted of seven hexagonal unit cells. By utilizing multiple unit cells, the design aimed to more closely replicate an actual air bearing system and study the interactions between the cells. For this system, the elastic deformation was calculated in COMSOL using the *Solid Mechanics* physics interface. A 100 mm silicon wafer with a thickness of 525 μm was clamped at its outer edge using a fixed constraint. The pressure distribution was simulated by applying the creeping flow pressure profile (Equation 2.19) as a boundary load to each unit cell. It is important to note that this pressure profile does not account for the pressure drop as the flow enters the thin film, so the simulated deformation may be slightly larger than in reality. For simplicity, circular unit cells were used in the simulation, and gravity was added to account for wafer sag.

Figure C.10 illustrates the boundary loads applied to the seven unit cells for an inlet pressure of $p_i = 1.5$ bar and atmospheric pressure $p_a = 1.0$ bar. The figure shows a bottom view of the wafer, with the load arrows pointing downward. As seen in the figure, the loads are maximal at the inlet of each unit cell, with a pressure difference of $\Delta p = p_i - p_a = 0.5 \times 10^5$ Pa. At the outlet of the unit cells, the distributed load is zero, as there is no pressure difference with the surrounding atmosphere.

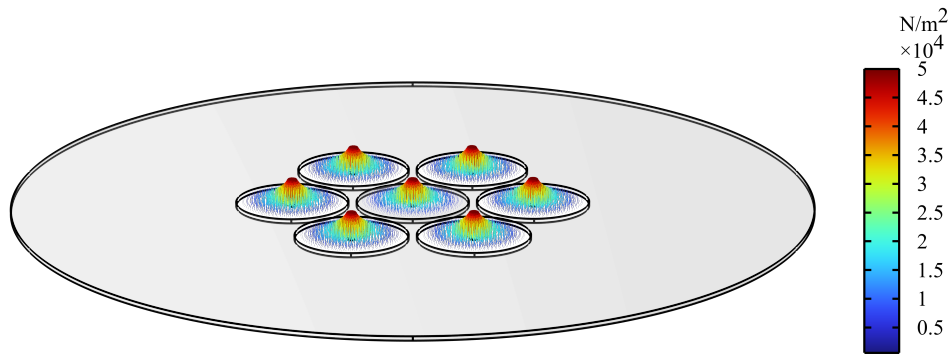


Figure C.10: Boundary load distribution applied by the pressure profile from seven unit cells, with arrows indicating load direction and magnitude (bottom view).

Figure C.11 shows a top view of the deformation field for the 100 mm silicon wafer. As anticipated, the maximum deformation occurs at the center of the wafer, with a value of 245.7 μm . Similarly, the maximum Von Mises stress is also concentrated at the center, reaching 25.3 MPa.

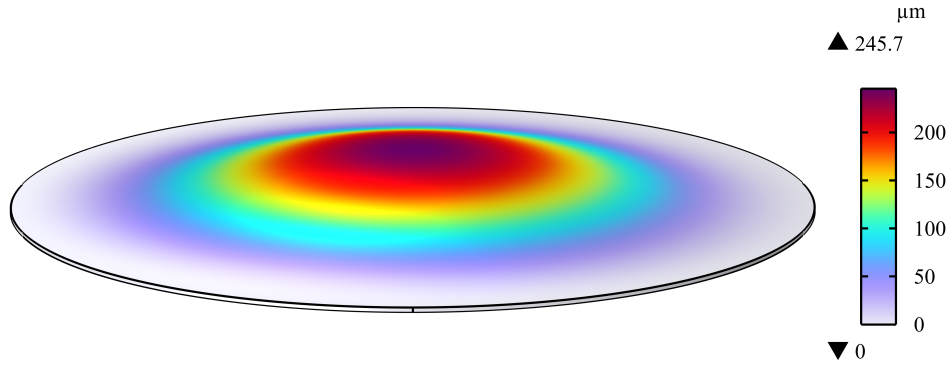


Figure C.11: Deformation field for a 100 mm silicon wafer clamped at its outer edge, subject to the pressure distribution from seven unit cells (top view).

The same simulation was also performed for a quartz glass wafer. As mentioned earlier, quartz glass offers more favorable thermal properties than silicon for temperature measurements. However, as shown in Table C.5, its mechanical properties lead to a significantly lower flexural rigidity, resulting in a higher maximum wafer deformation. Table C.6 summarizes the maximum deformation and Von Mises stress for both materials in the seven-unit-cell simulation. The quartz glass wafer exhibits a substantially greater maximum deformation of 466.1 μm , while the maximum Von Mises stress is slightly lower due to its reduced stiffness.

Table C.6: Maximum deformation and Von Mises stress for a clamped 100 mm silicon and quartz glass wafer subject to the pressure distribution of seven unit cells.

Material	Max. Deformation (μm)	Max. Von Mises stress (MPa)
Silicon	245.7	25.3
Quartz glass	466.1	23.2

For both wafer materials, the maximum Von Mises stress remains significantly below the tensile strength. However, given the desired film height, the maximum deformations are too high for the intended test setup. Consequently, several measures must be implemented to reduce the deformation to an acceptable level.

One Unit Cell

The first measure is reducing the number of unit cells from seven to one, which decreases the load on the wafer. For the single unit cell configuration, the point load assumption used in Equation C.1 should provide a reasonably accurate approximation. To verify this, the equivalent point load of the pressure distribution was calculated. The equivalent point load was calculated using the formula shown in Equation C.4. Here, the first integral represents the force exerted on the wafer at the vertical inlet, where the pressure is uniform. The second integral represents the force exerted on the wafer above the thin film, using the creeping flow pressure distribution.

$$F_{eq} = \int_0^{2\pi} \int_0^{r_1} (p_1 - p_a) r dr d\varphi + \int_0^{2\pi} \int_{r_1}^{r_2} \left(\sqrt{\frac{(p_2^2 - p_1^2) \ln\left(\frac{r}{r_1}\right)}{\ln\left(\frac{r_2}{r_1}\right)}} + p_1^2 - p_a \right) r dr d\varphi \quad (\text{C.4})$$

Here, p_a denotes the atmospheric pressure above the wafer, which was set equal to p_2 . This equation was evaluated in MATLAB, yielding an equivalent point load of 1.65 N. The maximum deformation

was then calculated analytically using [Equation C.1](#) and [Equation C.3](#) for both silicon and quartz glass wafers. Subsequently, COMSOL simulations were performed for both materials: one with the equivalent point load applied at the wafer center, and another using the distributed pressure load ([Figure C.12](#)). For both simulations, gravity was omitted. The results from all three approaches are summarized in [Table C.7](#) for both wafer materials.

As can be seen, the results of the analytical point load calculations and the COMSOL point load simulations correspond rather well. The analytical method can therefore be concluded to provide an accurate approximation of the deformation of a circular plate subject to a central point load in this particular deformation range. The COMSOL result of the distributed load simulations slightly differ from the point load simulations. Since the point load is more concentrated in the center of the wafer, the calculated deformations are slightly larger than for a distributed load. For finer mesh sizes and different input parameters these conclusions remain the same. Overall, [Equation C.1](#) serves as a useful tool for estimating the deformation of a wafer subjected to the distributed load of a single unit cell.

Table C.7: Maximum deformation of silicon and quartz glass wafers, calculated using three methods without gravity.

Material	Max. deformation (μm)		
	Analytical point load	COMSOL point load	COMSOL distributed load
Silicon	48.4	48.6	47.0
Quartz glass	91.8	92.2	89.1

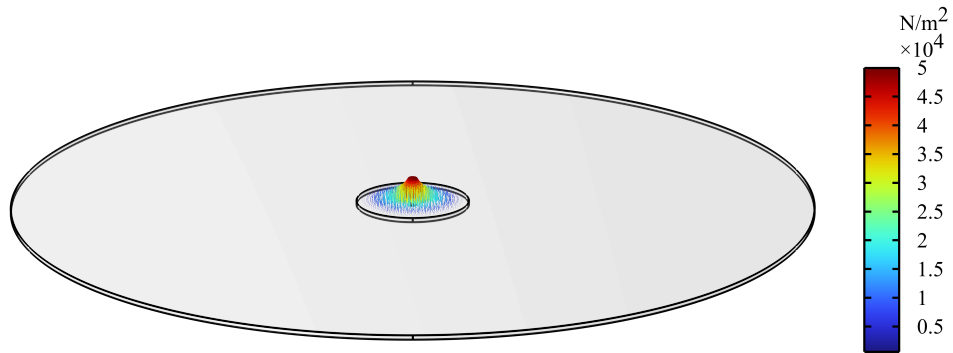


Figure C.12: Boundary load distribution applied by the pressure profile from one unit cell, with arrows indicating load direction and magnitude (bottom view).

Next, the same validation procedure was applied to assess wafer sag using [Equation C.2](#). In this case, the distributed load is due to gravity and can be calculated by dividing the wafer's weight by its surface area:

$$q = \frac{W}{A} = \frac{\rho g t_w \pi r^2}{\pi r^2} = \rho g t_w \quad (\text{C.5})$$

Using the densities of silicon and quartz glass shown in [Table C.3](#) and a gravitational acceleration constant of 9.81 m/s^2 , the distributed load was calculated for both wafer materials. The maximum deformation was then determined analytically using [Equation C.2](#) and numerically in COMSOL by activating the gravity load. The results are presented in [Table C.8](#).

As shown, the analytical and numerical results match exactly for both materials. Additionally, the wafer sag induced by gravity is significantly smaller than the deformation caused by the applied air

pressure. However, it is important to note that the wafer sag acts in the opposite direction to the pressure-induced deformation, resulting in a slight reduction of the overall deflection.

Table C.8: Distributed loads and maximum deformations of silicon and quartz glass wafers loaded with gravity.

Material	Distributed load q (N/m ²)	Max. deformation (μm)	
		Analytical	COMSOL
Silicon	12.0	0.69	0.69
Quartz glass	11.4	1.24	1.24

This effect was further confirmed in COMSOL by enabling gravity in the single unit cell simulation with the distributed pressure load. The resulting deformation field for the 100 mm silicon wafer is shown in Figure C.13. The maximum deformation is 46.3 μm , which is 0.7 μm lower than the corresponding simulation without gravity (Table C.7), due to the counteracting influence of wafer sag.

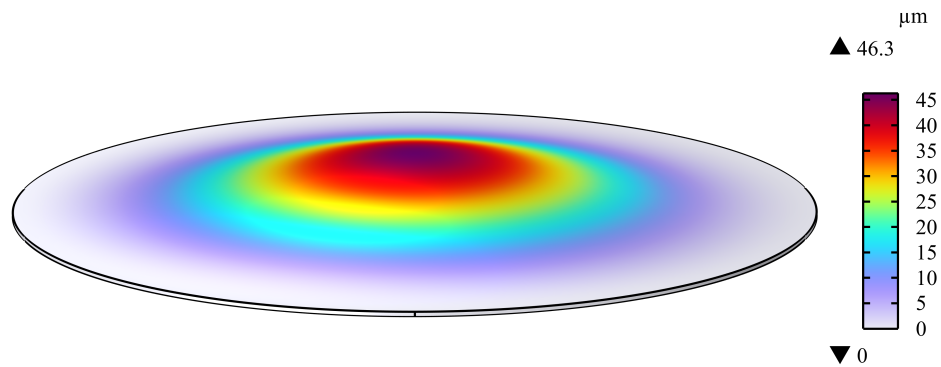


Figure C.13: Deformation field for a 100 mm silicon wafer clamped at its outer edge, subject to the pressure distribution from one unit cell and gravity (top view).

For the quartz glass wafer, the maximum deformation is 87.9 μm , consistent with the expected reduction from the inclusion of gravity. Overall, reducing the number of unit cells from seven to one led to more than a fivefold reduction in the maximum wafer deformation, as shown in Table C.9.

Table C.9: Comparison of the maximum deformation of silicon and quartz glass wafers for seven unit cells and one unit cell.

Material	Max. deformation (μm)	
	Seven unit cells	One unit cell
Silicon	245.7	46.3
Quartz glass	466.1	87.9

To further reduce the deformation, several parameters must be considered. Most notably, the wafer thickness appears to the third power in the flexural rigidity expression. Increasing the wafer thickness would therefore significantly reduce deformation. However, a thicker wafer would also diminish the transmission of the temperature profile from the bottom to the top surface, thereby reducing the measurable temperature difference. For this reason, the wafer thickness was left unchanged.

Another way to reduce the wafer deformation is by lowering the inlet pressure, thereby reducing the load exerted by the pressurized air. According to Equation C.1, the maximum deformation scales linearly with the applied force. However, the equivalent point load derived from the pressure distribution is not linearly related to the inlet pressure (Equation C.4). This relationship is shown in Figure C.14 for inlet pressures ranging from 1.1 to 2.0 bar, where a slight nonlinearity is observed.

The corresponding maximum deformations for the silicon and quartz glass wafers are presented in Figure C.15, using the three previously discussed methods. The analytical and numerical results based on the equivalent point load show good agreement across the full pressure range. However, at higher pressures, the equivalent point load results notably deviate from the distributed load results, with the latter providing a more accurate representation of wafer deformation in the experimental setup. Additionally, the quartz glass wafer exhibits a steeper deformation curve compared to silicon, owing to its lower flexural rigidity.

It is important to mention that these results do not yet account for the pressure drop as the air flows into the thin film. This pressure drop depends on the flow velocity, which is influenced by the film height. This aspect is further addressed in Section 4.3.1.

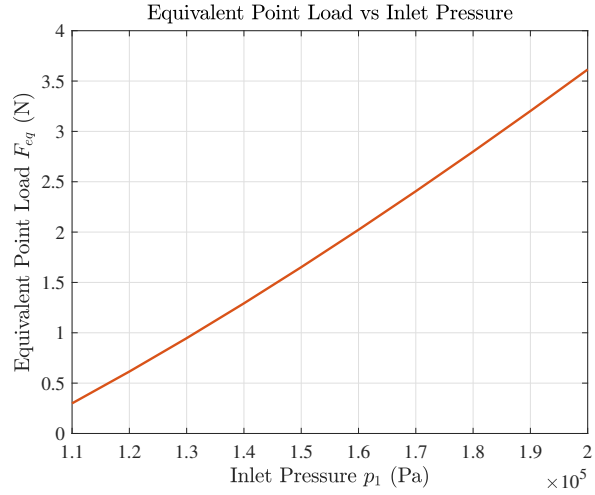


Figure C.14: Equivalent point load of the pressure distribution as a function of the inlet pressure.

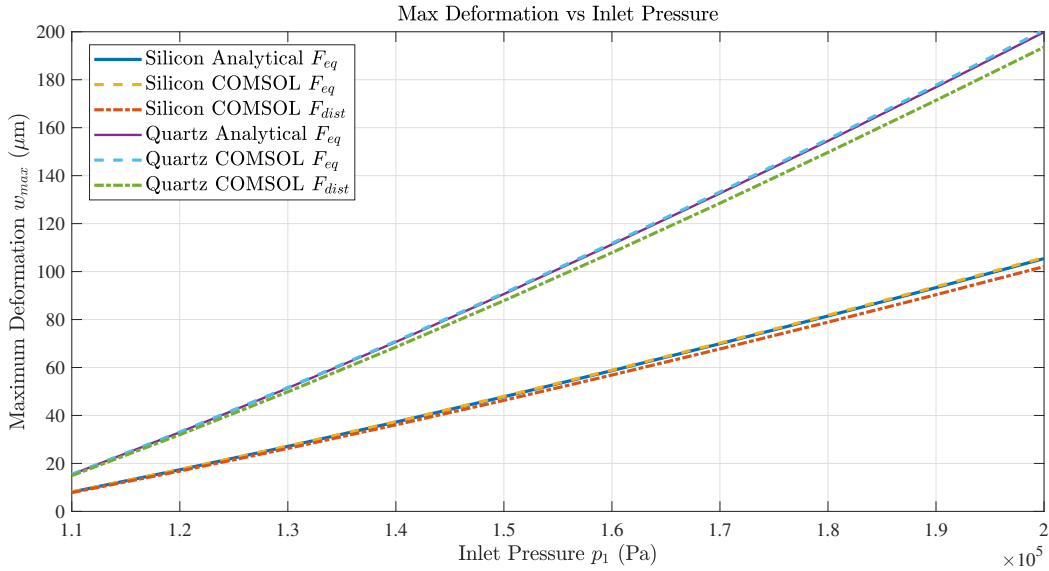


Figure C.15: Maximum wafer deformation as a function of the inlet pressure, calculated using three different methods for silicon and quartz glass wafers.

The final measure to reduce the maximum deformation of the wafer is to decrease the clamping distance. Until now, it was assumed that the 100 mm wafer was clamped at its outer edge, resulting in a clamping distance of 50 mm from the center. Since the clamping distance appears to the second order in Equation C.1, halving this distance would lead to a fourfold reduction in the maximum deformation. Using an inlet pressure of 1.5 bar, the maximum deformation was plotted as a function of the clamping distance in Figure C.16. As expected, the results for the distributed load slightly

deviate from those for the equivalent point load, but the overall quadratic trend in the distributed load graphs remains clear.

In this figure, the minimum clamping distance is set to 0.01 m, which is just 3 mm away from the outer edge of the unit cell. However, such a small distance could interfere with the wafer's temperature profile. Additionally, sufficient space is needed on the wafer surface to install invasive temperature sensors for calibration purposes. These factors are considered in the final parameter selection.

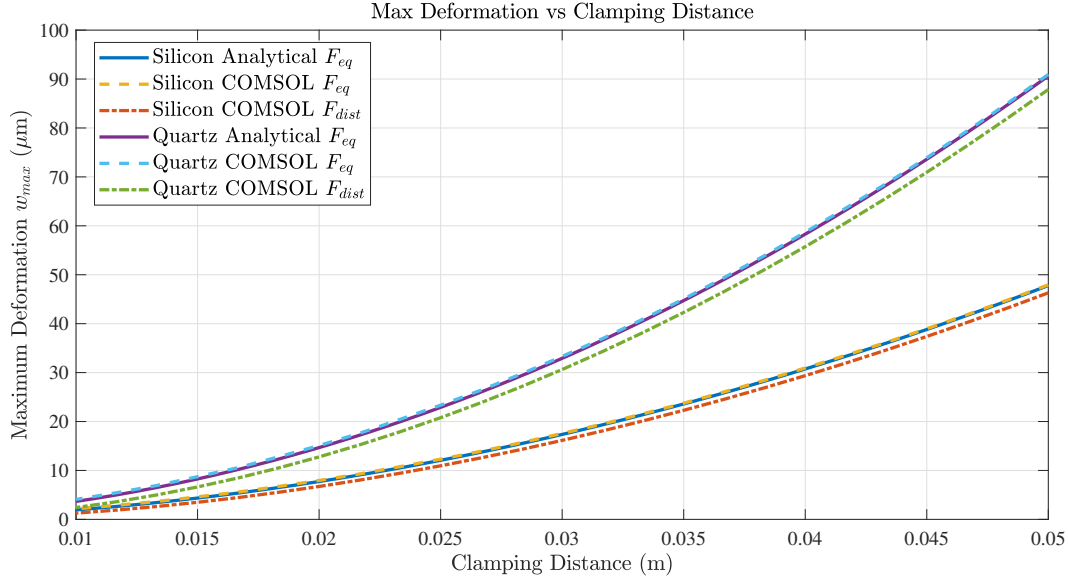


Figure C.16: Maximum wafer deformation as a function of the clamping distance, calculated using three different methods for silicon and quartz glass wafers.

To reduce the wafer deformation to an acceptable level, the three measures discussed in this section should be implemented together. Using one unit cell instead of seven already leads to a significant reduction in deformation. However, further reduction requires lowering both the inlet pressure and the clamping distance. Since the temperature profile is also highly sensitive to the inlet pressure and film height, these parameters cannot be set to their minimum values without consideration.

C.4 Temperature Conditions

Until now, the temperature of the inlet air, T_{air} , and the ambient temperature, T_s , were assumed to be equal at 293 K. However, variations in either of these parameters are likely to significantly impact the temperature distribution of the wafer. Understanding the sensitivity of the wafer's temperature distribution to changes in these conditions will therefore be crucial for validating the simulations experimentally. Additionally, varying either temperature during testing could provide valuable data points for further analysis.

To obtain a comprehensive overview of the effect of the temperature conditions on the temperature distribution of the wafer, an auxiliary parameter sweep was conducted. The ambient temperature, used for both the isothermal boundary at the bottom of the air bearing system and the heat flux boundary at the top of the wafer, was set to three values: 291 K, 293 K, and 295 K. For each of these ambient temperatures, the inlet air temperature was varied from 290 K to 296 K in increments of 0.2 K. After obtaining these initial results, the increments were locally refined in the critical temperature regions.

The maximum temperature difference at the top of the silicon wafer, $\Delta T_{wafer,top}$, is plotted in Figure C.17 for each combination of the inlet air and ambient temperature. Each graph shows its minimum temperature difference for an inlet temperature slightly above the ambient temperature.

Furthermore, the three graphs in the plot show a similar course, but are shifted over the x -axis toward these minima. This ultimately indicates that, for this particular temperature range, the temperature difference at the top surface of the wafer is purely driven by the difference in temperature between the inlet air and the ambient air, and not the absolute temperatures. The slope of the linear regions before and after the minima are similar, with a value of 7.7 mK/K.

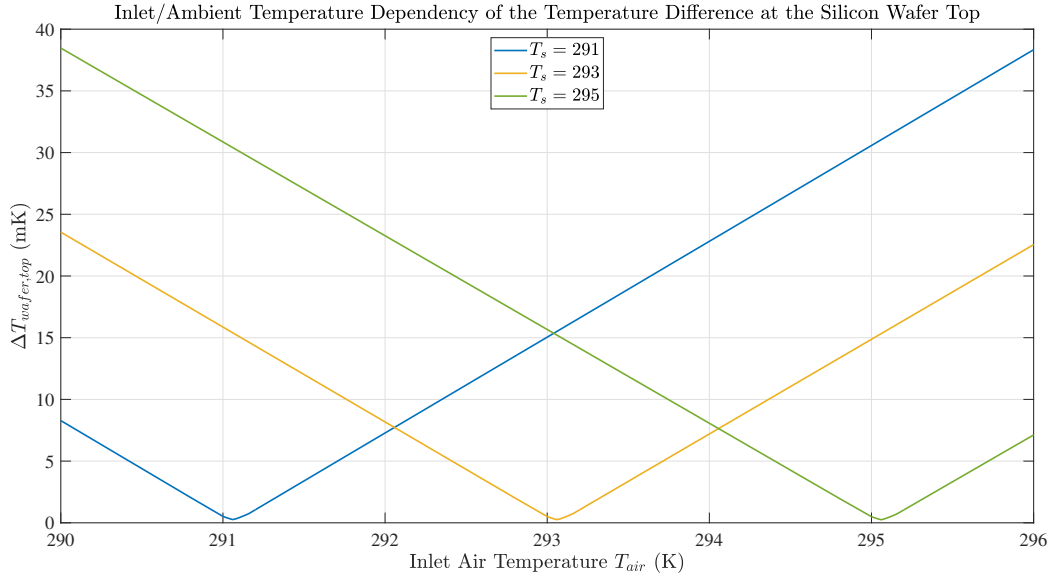


Figure C.17: Temperature difference at the top surface of the silicon wafer for varying ambient and inlet air temperatures.

As indicated, the temperature difference at the top surface of the silicon wafer reaches its minimum for an inlet air temperature slightly above the ambient temperature, around 60 mK higher. At this inlet temperature, the temperature difference at the top of the wafer is reduced to about 0.25 mK, compared to the initial 0.51 mK. This reduction is due to the dominance of the cooling effect at the thin film inlet. By using a slightly higher inlet air temperature, this cooling effects becomes more balanced with the energy dissipation further downstream. This is illustrated in Figure C.18, which demonstrates that for an inlet air temperature of $T_{air} = T_s + 0.06$ K, the temperature at the vertical inlet is closest to that at the vertical outlet. A substantially higher or lower inlet air temperature increases the dominance of the heating or cooling effects at the inlet, respectively.

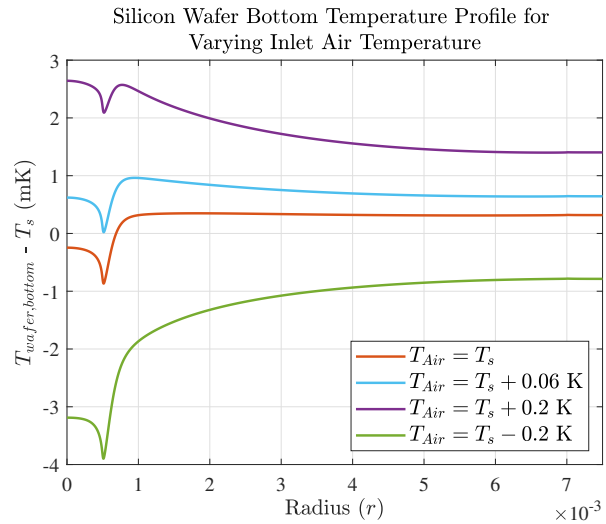


Figure C.18: Temperature profile at the bottom of the silicon wafer for a varying inlet air temperature.

A similar parametric sweep was performed for a quartz glass wafer, with the resulting temperature differences at the wafer's top surface shown in Figure C.19. As observed, the quartz glass wafer is significantly more sensitive to variations between the inlet air temperature and ambient air temperature, with a slope of 239 mK/K in the linear regions. Moreover, the temperature difference at the top surface is minimized when the inlet air temperature is 120 mK above the ambient temperature, where the difference is reduced to 8.47 mK compared to the initial 28.8 mK.

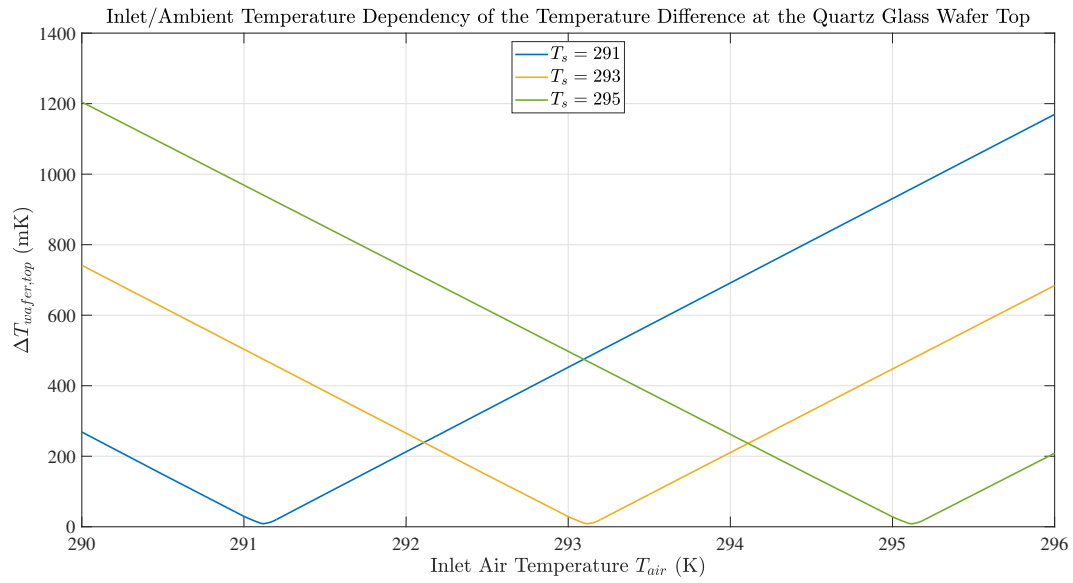


Figure C.19: Temperature difference at the top surface of the quartz glass wafer for varying ambient and inlet air temperatures.

To summarize, the wafer's temperature distribution is mainly influenced by the difference between the inlet air temperature and the ambient temperature, rather than their absolute values. A slightly higher inlet air temperature than the ambient temperature minimizes the temperature difference at the wafer's top surface, indicating a balance between cooling and heating effects near the inlet. These findings are important for both experimental validation and the implementation of active air bearing technology for wafer handling.

D

Detailed Design

D.1 Air Bearing System

Detailed drawings of the air bearing system's top and bottom views are shown in [Figure D.1](#) and [Figure D.2](#), respectively. The dimensions of the hexagonal unit cell were chosen to match those used in the modelling phase. The inner diameter of the outer segments was determined based on the clamping radius defined in [Section 3.4](#). To ensure the 100 mm wafer fits within the component, the outer diameter of the segments was set to 100.5 mm.

The dimensions of the cutout on the bottom of the air bearing system were designed to accommodate the pneumatic L-fitting and hose used for connecting the system to the air supply.

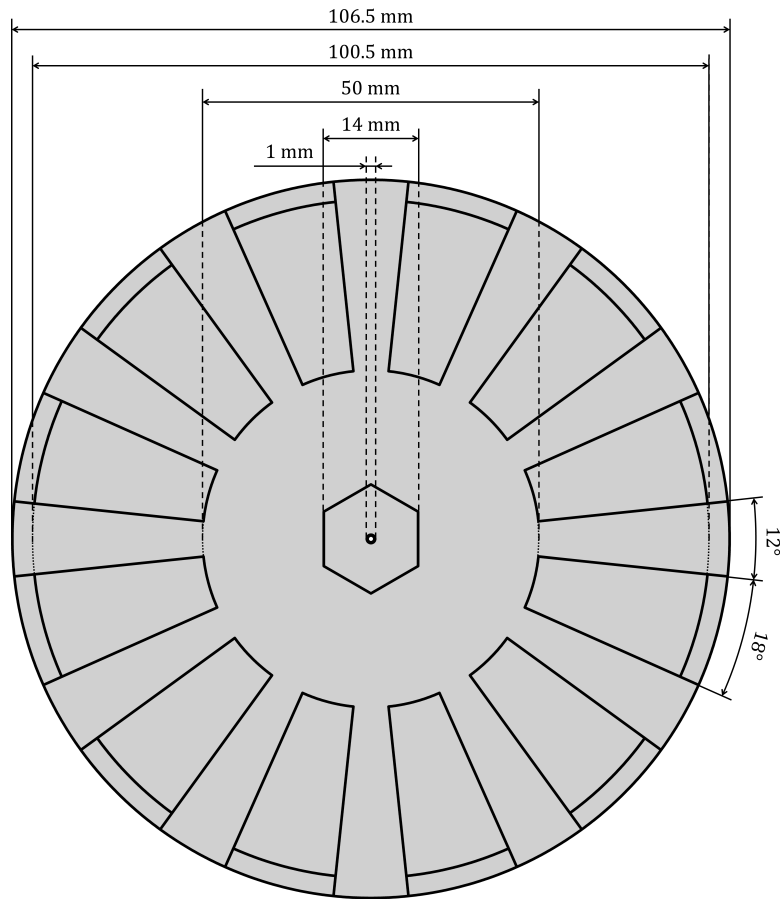


Figure D.1: Detailed drawing of the top view of the air bearing system.

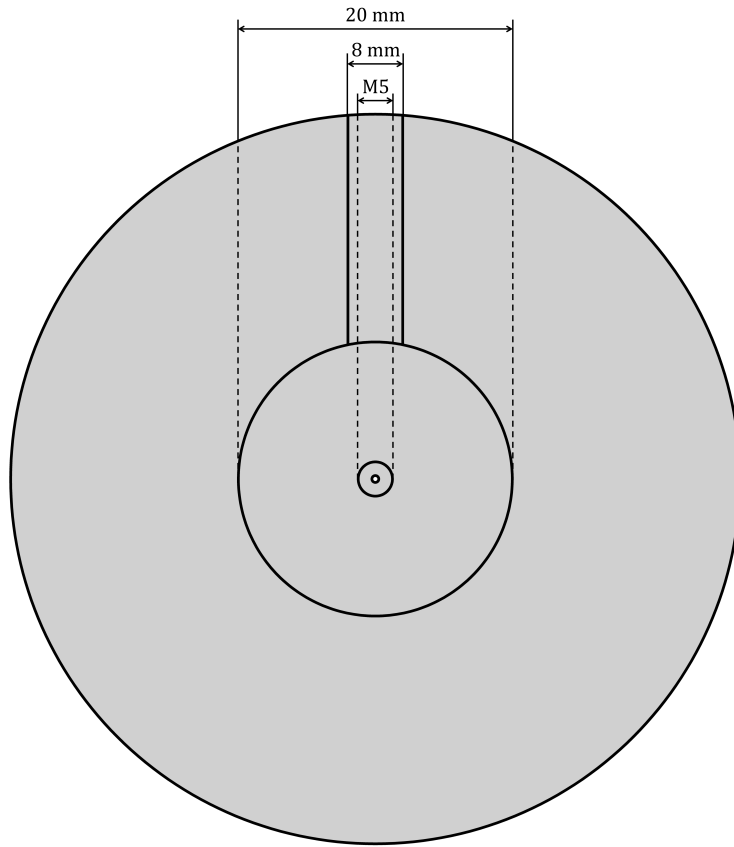


Figure D.2: Detailed drawing of the bottom view of the air bearing system.

A detailed drawing of the cross-sectional view of the air bearing system is illustrated in [Figure D.3](#), with a zoomed-in view of the air inlet shown in [Figure D.4](#). The base height of the air bearing system was selected to be 25 mm, leaving enough space for the pneumatic fitting to be inserted, while also providing a sufficiently long air inlet to obtain a well-developed flow. The top surface of the hexagonal unit cell is located 1.97 mm above this surface, while the outer segments are located 2 mm above the base surface. This leaves the desired passive film height of $30\text{ }\mu\text{m}$. The raised edges along the outer radius of the air bearing system are elevated 4.525 mm above the segments, representing the thickness of the wafer and constraining ring combined.

Furthermore, the zoomed-in cross-sectional view reveals a partially threaded M5 hole with a depth of 7 mm to accommodate the fitting. The hole is only partially threaded to enhance machinability. This leaves a length of 4.97 mm for the 1 mm diameter air inlet. Finally, a rounding of $120\text{ }\mu\text{m}$ was added to the sharp edge of the unit cell inlet, based on the modelling results.

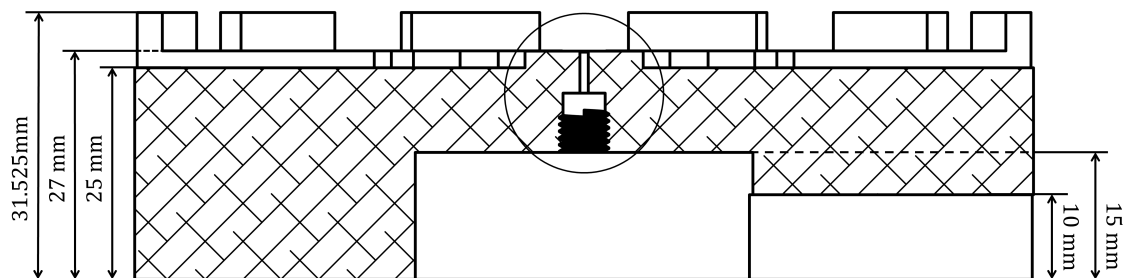


Figure D.3: Detailed drawing of the cross-sectional view of the air bearing system.

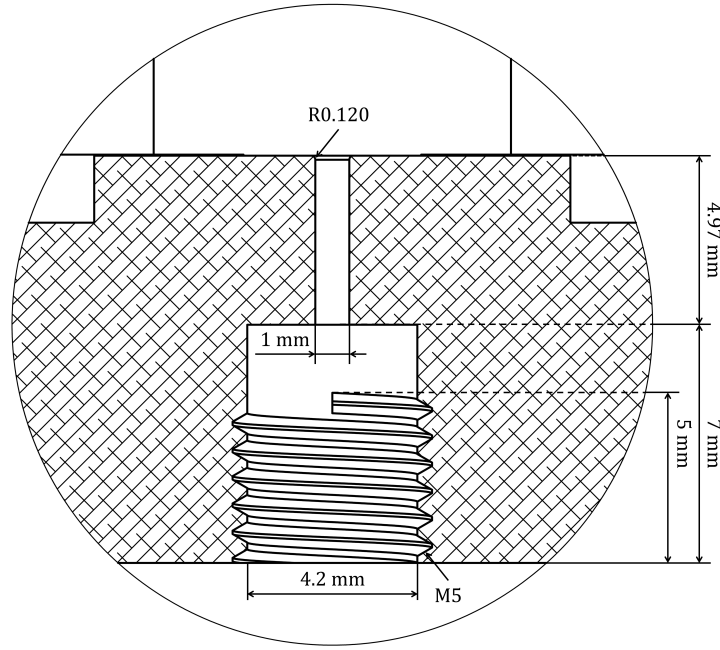


Figure D.4: Detailed drawing of the zoomed-in cross-sectional view of the air bearing system.

D.2 Substrate Constraint

The inner and outer radii of the ring used to constrain the wafer correspond to the previously defined clamping radius of 25 mm and the wafer's outer radius of 50 mm. To determine the minimum required ring thickness, a force equilibrium in the z -direction (normal to the wafer) was established:

$$+\uparrow \sum F_z = F_p - F_{W,wafer} - F_{W,ring} \quad (D.1)$$

Here, F_p represents the force exerted on the bottom of the wafer by the pressure distribution, as shown previously in Equation C.4. The weights of the wafer and the ring are denoted by $F_{W,wafer}$ and $F_{W,ring}$, respectively. The minimum required weight of the ring is determined by setting the sum of the forces to zero, ensuring force equilibrium. Using the material properties and parameters listed in Table 4.1 and Table 4.3, the minimum required weight of the ring was calculated to be 0.53 N.

Based on the steel material properties, the corresponding minimum thickness for a ring with a rectangular cross-section was calculated as 1.17 mm. However, at this thickness, the wafer is only marginally constrained, resulting in significantly higher deformation at its center. To address this, a thickness of 4 mm was used instead, effectively reducing the maximum deformation and enabling the use of inlet pressures higher than 1.2 bar during testing.

Additionally, a chamfer was introduced at the inner radius of the ring to improve visibility of the wafer surface at shallow angles. The ring's thickness varies from 0.5 mm at the inner radius to 4 mm at a point 10 mm outward, allowing a camera to be positioned at an angle of 19.3° relative to the wafer surface.

Using the specified dimensions, the total weight of the chamfered ring was calculated in Equation D.2 to be 1.56 N, which is nearly three times the minimum required weight.

$$F_{W,ring} = \pi \rho_{steel} g \left(t_{r2} (r_6^2 - r_4^2) - \frac{1}{3} (t_{r2} - t_{r1}) (r_5^2 + r_4 r_5 - 2r_4^2) \right) = 1.56 \text{ N} \quad (D.2)$$

A detailed drawing of the cross-sectional view of the constraining ring, including all relevant dimensions, is presented in Figure D.5.

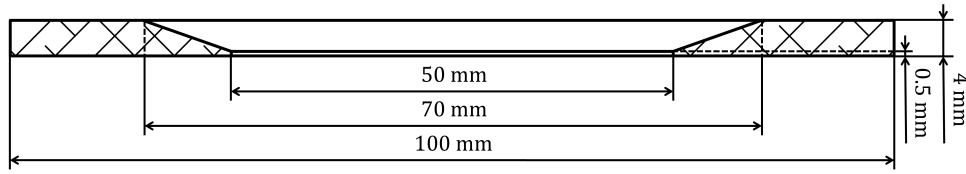


Figure D.5: Detailed drawing of the cross-sectional view of the ring used to constrain the wafer.

D.3 Air Supply

As previously shown in Figure 4.5, the pressure of the inlet air is reduced in two steps. First, the 7-bar supply air is regulated down to approximately 2 bar using a pressure regulator. Then, a rotameter further reduces the pressure to match the operating conditions of the air bearing system. Since the inlet air temperature significantly influences the wafer's temperature distribution, the temperature drop associated with each pressure reduction is estimated below. Additionally, the pressure drop in the supply hose after the pressure gauge is evaluated to determine the pressure of the air reaching the wafer.

D.3.1 Temperature

Both the pressure regulator and rotameter function as throttling device, meaning they reduce pressure without performing work. This type of expansion is typically considered an isenthalpic process, where the specific enthalpy remains constant [27]:

$$h_1 = h_2 \quad (\text{D.3})$$

Here, h represents the specific enthalpy, which is defined as the sum of the specific internal energy (u) and the product of pressure (p) and specific volume (v):

$$h = u + pv \quad (\text{D.4})$$

For an ideal gas, this equation simplifies to:

$$h_{ideal} = c_v T + RT = c_p T \quad (\text{D.5})$$

Since c_p is constant for an ideal gas, the air temperature also remains unchanged during an isenthalpic expansion.

However, for a real gas, the enthalpy is not just a function of temperature, introducing a temperature change upon expansion. The magnitude of this temperature change is described by the Joule-Thomson coefficient [27]:

$$\mu_J = \left(\frac{\partial T}{\partial p} \right)_h \quad (\text{D.6})$$

For air at 20 °C and a pressure of 1 bar, the Joule-Thomson coefficient is approximately 0.23 K/bar [28]. The temperature drop associated with the first pressure reduction from 7 to 2 bar and the second reduction from 2 bar to 1 - 1.6 bar can therefore be estimated as follows:

$$\Delta T_{7 \rightarrow 2 \text{ bar}} \approx \mu_J \Delta p = 0.23 \cdot 5 = 1.15 \text{ K} \quad (\text{D.7})$$

$$\Delta T_{2 \rightarrow 1 \text{ bar}} \approx \mu_J \Delta p = 0.23 \cdot 1 = 0.23 \text{ K} \quad (\text{D.8})$$

Both pressure reductions result in minor temperature drops. Additionally, due to the relatively long supply hoses, these temperature differences should even out before reaching the air bearing system.

Consequently, the temperature distribution of the wafer should not be affected by the pressure reductions.

However, the initial temperature of the supply air before reaching the first pressure regulator could have a significant effect on the temperature distribution. If the air is supplied at a temperature significantly different from ambient, it may still reach the air bearing system at a deviating temperature. Since the exact source of the supply air is unknown, no definitive conclusions can be drawn about its initial temperature.

D.3.2 Pressure

For laminar flow through a cylindrical hose, the pressure drop can be estimated using the Hagen-Poiseuille law, as shown in Equation D.9. This equation assumes incompressible flow, which offers a reasonable approximation at low flow velocities [10].

$$\Delta p = \frac{8\mu LQ}{\pi R^4} \quad (\text{D.9})$$

Here, L represents the length of the hose, R the inner radius of the hose, Q the volumetric flow rate through the hose, and μ the dynamic viscosity of air.

To confirm the validity of the laminar flow assumption, the Reynolds number is calculated. For an inner hose diameter D of 2.6 mm and a volumetric flow rate of 1.2 L/min, the flow velocity V and the Reynolds number Re are determined as follows:

$$V = \frac{Q}{A} = \frac{1.2 \cdot \frac{1}{60000}}{\pi \cdot (1.3 \times 10^{-3})^2} = 3.77 \text{ m/s} \quad (\text{D.10})$$

$$Re = \frac{\rho V D}{\mu} = \frac{1.2 \cdot 3.77 \cdot 2.6 \times 10^{-3}}{1.8 \times 10^{-5}} = 653 \quad (\text{D.11})$$

Since the transition from laminar to turbulent flow occurs at a Reynolds number of approximately 2300, the calculated value confirms that the flow remains well within the laminar regime.

Therefore, the pressure drop per meter of hose length can be calculated using the Hagen-Poiseuille equation:

$$\frac{\Delta p}{L} = \frac{8\mu Q}{\pi R^4} = \frac{8 \cdot 1.8 \times 10^{-5} \cdot \frac{1.2}{60000}}{\pi \cdot (1.3 \times 10^{-3})^4} = 321 \text{ Pa/m} \quad (\text{D.12})$$

For a hose length between 1 and 2 meters, the total pressure drop is estimated to be between 321 Pa and 642 Pa. Even at higher flow rates, the pressure drop across the hose is negligible compared to the pressure drop in the thin film.

E

Thermal Sensing Techniques

Measuring temperature accurately is a complex task that requires the use of temperature-dependent quantities. Due to the importance of temperature in many industries, such as the high-tech industry, many sensing techniques have been developed over the years. This appendix provides an overview of the most common thermal sensing methods, starting with key sensor characteristics and their relevance, followed by a discussion of invasive, semi-invasive, and non-invasive techniques.

E.1 Sensor Characteristics

For the experimental part of this research, the temperature distribution of a substrate on top of an air bearing system has to be measured. Additionally, the temperature difference between the inlet air and the ambient air must be determined to perform model validation.

First, it is important for these temperature measurements to be accurate and precise. Accuracy refers to how close the measurement is to the true value, while precision indicates the consistency of repeated measurements. Precision, often termed repeatability, is primarily influenced by instrumental noise and environmental factors. By maintaining a controlled environment, measurement repeatability can be enhanced. Accuracy, on the other hand, depends largely on the quality of the temperature sensor and the calibration of the measurement system. In the literature, typically only the accuracy of a thermometry technique is specified [15].

Besides accuracy and precision, temperature resolution and sensitivity are also crucial factors. Resolution refers to the smallest temperature increment that a sensor can detect. For 2D sensors, spatial resolution refers to the smallest distance between two temperature measurements, essentially the distance represented by a single pixel in a thermal image. Higher spatial resolution results in a more clearly defined temperature distribution. Sensitivity describes how much the sensor's output changes in response to temperature variations. A highly sensitive sensor produces a noticeable output change for even small temperature changes. However, a sensor with high sensitivity does not necessarily imply high resolution, and vice versa, a sensor with high resolution may not have a high sensitivity [15].

The response time of a temperature sensor refers to the time it takes for the sensor to register a significant change in temperature. For the current application, the sensor's response time is not critical, as only steady-state measurements will be conducted.

The temperature range of the sensor is less critical compared to other characteristics, as the measurements will be conducted at room temperature with only minor fluctuations expected. A sensor that operates reliably within this range will suffice.

The size and shape of the sensor, however, is rather relevant. Especially for invasive sensors, where the interference with the substrate should be minimized. Furthermore, in order to obtain a compact measurement setup, the footprint of the sensor needs to be kept to a minimum.

Finally, the cost of the temperature sensor can vary significantly between techniques, making it an important factor in sensor selection, especially with budget constraints.

While most relevant characteristics for the thermometry techniques discussed are provided, some data were unavailable for certain sensors, leading to occasional gaps in the information.

E.2 Invasive Sensors

Invasive temperature sensors are those that measure temperature through direct contact with the substrate. There are several types of invasive sensors, each based on different measuring principles. This section presents a selection of commonly used invasive temperature sensors, including the RTD, thermistor, and thermocouple. Other invasive sensors, such as those based on silicon or semiconductors [29], were found to be less suitable for the current application and are therefore not included.

The sensors discussed here measure temperature locally (0D), meaning that multiple measurements would be required at different locations to capture the temperature distribution across the substrate's surface. However, invasive sensors can also be combined with non-invasive sensors, for example, for calibration purposes.

E.2.1 Resistance Temperature Device (RTD)

RTDs measure temperature based on the temperature-dependent electrical resistance of materials. They typically consist of pure metals, known for their well-defined properties, which result in high accuracy and repeatability. The relationship between resistance and temperature varies depending on the material used. Materials like platinum exhibit a relatively linear relationship between resistance and temperature, while others may show less predictable behaviour. Due to their linearity, stability, high accuracy, and wide measurement range ($-200 - 850$ °C), platinum-based RTDs are often considered the best option [30].

The resistance-temperature relationship for platinum RTDs is typically described by the Callendar - Van Dusen equations [31]:

$$R(T) = R(0) (1 + AT + BT^2 + CT^3(T - 100)) \quad \text{for } T < 0 \quad (\text{E.1})$$

$$R(T) = R(0) (1 + AT + BT^2) \quad \text{for } T > 0 \quad (\text{E.2})$$

The most common type of RTD is the Pt100, a platinum-based sensor with a resistance of 100Ω at 0 °C. The corresponding coefficients for this specific type are defined by the IEC-60751 standard:

$$A = 3.9083 \times 10^{-3} \quad B = -5.775 \times 10^{-7} \quad C = -4.183 \times 10^{-12} \quad (\text{E.3})$$

Using these coefficients, the temperature can be determined by either solving Equation E.1 or Equation E.2, depending on the temperature [31]. The sensitivity of such an RTD is typically around $0.39 \Omega/\text{K}$ [32].

For the Pt100, tolerance classes have been centrally defined, specifying the minimum required accuracy. The most accurate class is the 1/10 DIN class, defined by a tolerance of $\pm(0.03^\circ\text{C} + 0.0005 \cdot |T|)$. Despite this tight tolerance, the accuracy of temperature measurements also depends on the RTD configuration used. Two-wire, three-wire, and four-wire configurations exist, as shown in Figure E.1. In the two-wire configuration, the resistance of the lead wires

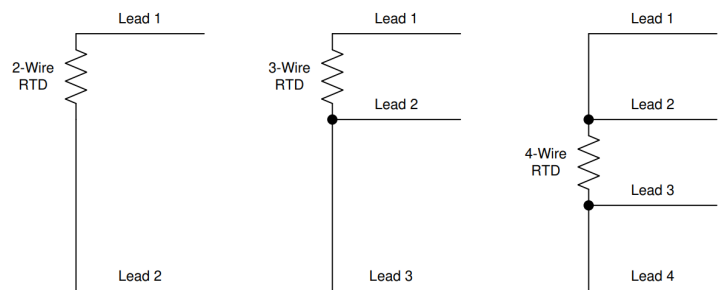


Figure E.1: Two-wire, three-wire, and four-wire RTD configurations [31].

introduces measurement errors. However, in the three- and four-wire configurations, the lead wire resistance can be compensated. The four-wire configuration provides the highest accuracy but is also the most expensive and complex. Another important factor for RTDs is self-heating. Depending

on the magnitude of the electrical current used for the temperature measurement, the resistance in the RTD element causes it to heat up. To minimize this effect, small currents on the order of one milliampère should be used [31].

RTDs are available in various forms, including probes, coils, and thin films. In this specific application, where space is limited, a thin film RTD is the most suitable option. While RTDs are often more expensive than other invasive temperature sensors due to their high precision, accuracy, and the materials used, their performance justifies the higher cost [33].

E.2.2 Thermistor

A thermistor is another type of electrical resistance sensor, distinct from RTDs in that it is made from mixtures of semiconducting materials rather than pure metals. Thermistors can be classified as having either a negative temperature coefficient (NTC) or a positive temperature coefficient (PTC). For NTC thermistors, resistance decreases with increasing temperature, while for PTC thermistors, resistance increases with temperature. The relationship between resistance and temperature is typically nonlinear, and thermistors have a more limited temperature range, typically from $-100\text{ }^{\circ}\text{C}$ to $350\text{ }^{\circ}\text{C}$. However, thermistors offer significantly higher temperature sensitivity than RTDs, with resistance changes of several percent per Kelvin, depending on the sensor type and temperature. The temperature dependence for NTC and PTC thermistors is approximated in Equation E.4 and Equation E.5, where B is a constant that varies with the type. The formula for PTCs is only valid within a limited temperature range ($T_1 < T < T_2$) [32].

$$R(T) = R(0)e^{B\left(\frac{1}{T} - \frac{1}{T_0}\right)} \quad (\text{E.4})$$

$$R(T) = R(0)e^{BT} \quad \text{for} \quad T_1 < T < T_2 \quad (\text{E.5})$$

Depending on the required accuracy of the measurement, higher order polynomials can also be used to approximate the temperature dependence [34].

Thermistors can achieve accuracies as high as $\pm 0.05\text{ }^{\circ}\text{C}$. However, commercially available thermistors typically have accuracies around $\pm 1\text{ }^{\circ}\text{C}$ [29]. Their resistance values are generally much higher than those of RTDs, making lead wire resistance negligible and eliminating the need for four-wire configurations [34]. However, thermistors are susceptible to self-heating and sensor drift, which can limit their accuracy [32].

Thermistors also come in various forms, such as probes, discs, or surface mounts. They are small in size and low in cost, and mostly applicable in cases where high accuracy is not critical [32].

E.2.3 Thermocouple

Thermocouples consist of two dissimilar electrical conductors joined at one end, with a voltage meter connected to their open ends. Figure E.2 shows a schematic of a typical thermocouple, where the left end represents the measurement (or hot) junction, and the right end represents the reference (or cold) junction. When there is a temperature difference between the hot and cold junctions, a voltage difference is generated across the two wires due to the Seebeck effect. The Seebeck effect describes the generation of an electromotive force (EMF) between two dissimilar conductors subjected to a temperature gradient.

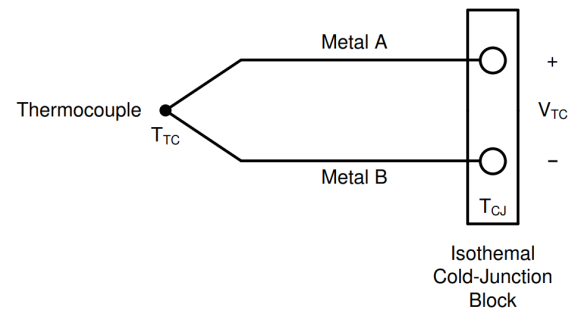


Figure E.2: Schematic representation of a thermocouple [35].

Each material has its own Seebeck coefficient, which determines the magnitude of the voltage induced by the temperature difference, i.e., its sensitivity. It is important to note that the Seebeck coefficient is temperature-dependent, leading to the inherent nonlinearity of thermocouples. The basic formula relating the generated EMF to the temperature difference is shown in Equation E.6, where S represents the Seebeck coefficient. However, in practice, higher-order polynomials are often used to describe this relationship to improve accuracy [29] [34] [35].

$$EMF = S\Delta T \quad (E.6)$$

Thermocouples are categorized into types based on the materials used for the two conductors. Table E.1 provides an overview of common types. As shown, most thermocouples have a wide temperature range, with some capable of measuring up to 3000 °C, covering a total range from −270 to 3000 °C [29]. Another key observation is the low voltage output and sensitivity, with typical EMF measurements in the millivolt range, requiring amplification and making them susceptible to electrical noise [34].

Table E.1: Common thermocouple types with their materials, temperature range, EMF, and Seebeck coefficient [35].

Type	Metal A	Metal B	Temperature Range (°C)	EMF over Temperature Range (mV)	Seebeck Coefficient ($\mu\text{V}/^\circ\text{C}$ at 0 °C)
J	Iron	Constantan	−210 to 1200	−8.095 to 69.553	50.37
K	Chromel	Alumel	−270 to 1370	−6.458 to 54.886	39.48
T	Copper	Constantan	−200 to 400	−6.258 to 20.872	38.74
E	Chromel	Constantan	−270 to 1000	−9.385 to 76.373	58.70
S	Platinum and 10% Rhodium	Platinum	−50 to 1768	−0.236 to 18.693	10.19

The accuracy of thermocouples depends largely on the type used, with Type T being the most accurate, achieving accuracies of up to ± 0.5 °C. Since each metal junction forms a separate thermocouple, it is crucial for all connections to the signal processing unit to be symmetric and ideally at the same temperature. Another key factor influencing performance is the cold junction, which must be kept at a known temperature. Cold-junction compensation is typically achieved using an additional sensor to measure the cold junction temperature [35].

Thermocouples are relatively low in cost, small in size, and reasonably stable. Furthermore, they are available in many shapes, including thin films, making them widely applicable [34].

E.3 Semi-Invasive Sensors

For semi-invasive sensors, the substrate or medium of interest undergoes treatment to enable remote sensing [29]. Based on preliminary research, the following techniques were selected for further exploration: thermochromic liquid crystals and thermographic phosphors, both of which are suitable for 2D temperature measurements. Other semi-invasive methods, such as temperature-sensitive paints (TSP) [36] and quantum dots [37], were not pursued due to their insufficient performance for the required application.

E.3.1 Thermochromic Liquid Crystals

Thermochromic liquid crystals change colour in response to temperature fluctuations. These organic materials have a molecular structure that is intermediate between a crystalline solid and an

isotropic liquid. The transition between these states occurs at specific temperatures, depending on the composition of the liquid crystals. When illuminated with white light, the liquid crystals reflect monochromatic light at a wavelength corresponding to the pitch of their helical molecular structure. As temperature increases, the wavelength of the reflected light decreases, causing a colour shift from red to blue across the visible spectrum. Key characteristics of thermochromic liquid crystals are as follows [34] [38]:

- Red-start temperature: the temperature at which the liquid crystal starts to reflect visible light.
- Colour play bandwidth: the temperature range over which the liquid crystal reflects visible light.
- Isochrome bandwidth: the temperature range over which the liquid crystal reflects a single colour.

By adjusting the composition of the liquid crystal, the properties can be tuned for each application. Typically, the colour play bandwidth is only a few degrees and the isochrome bandwidth can be as little as 0.1 °C. For narrow-band liquid crystals with a colour play bandwidth of 1 °C, a high accuracy of up to 0.1 °C can be achieved. Wide-band liquid crystals have a larger bandwidth of up to a few tens of degrees. However, a lower accuracy is obtained for wide-band liquid crystals [34].

Since thermochromic liquid crystals are made from organic materials, they are sensitive to UV light and chemical contamination. To address these concerns, they are often encapsulated in polymer spheres instead of being applied directly to a surface. This encapsulation allows the liquid crystals to be sprayed or painted onto the surface. Liquid crystals are also available in sheet form for easier application. To determine the temperature distribution on the treated surface, a light source and an imaging system are required. Typically, the imaging system consists of a CCD camera that captures an RGB image of the substrate. The captured image is then processed based on hue to determine the surface temperature at each pixel location [29].

A temperature resolution of at least 0.01 °C can be obtained over small temperature ranges, along with high spatial resolution [39]. However, it is important to note that accurate calibration is essential for reliable temperature measurements using thermochromic liquid crystals. While the liquid crystals themselves are relatively inexpensive, the costs associated with the imaging system are significantly higher [37] [38].

E.3.2 Thermographic Phosphors

Similar to thermochromic liquid crystals, thermographic phosphors enable temperature measurement through luminescence thermometry. These materials emit light when exposed to an external light source, typically ultraviolet (UV) light. Figure E.3 illustrates the underlying mechanism, where electrons transition from the ground state to an excited state upon absorption of the required energy. In this process, v and v' represent vibrational energy levels, and R denotes the distance between the atomic cores of the activator atom and the ligand for a single vibrational mode. After excitation, the electron relaxes to the lowest vibrational mode in the excited state before returning to the ground state by emitting a photon. This relaxation then shifts non-radiatively to the lowest vibrational mode of the ground state. Due to the

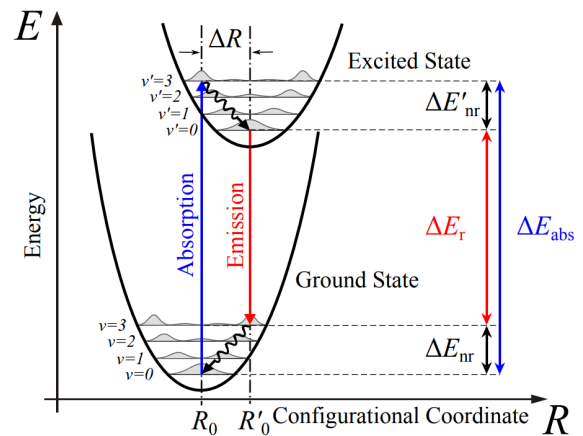


Figure E.3: Transition between the ground state and the excited state [40].

division between radiative and non-radiative relaxation, the energy emitted radiatively is lower than the absorbed energy, resulting in a red-shifted emission wavelength known as the Stokes shift. The larger the magnitude of ΔR , the greater the observed Stokes shift.

This process is highly temperature-dependent, making it useful for thermometry. The two most common thermometric methods using thermographic phosphors are the temporal and spectral methods, both of which can provide either 0D or 2D temperature measurements [40] [41].

Temporal Methods

Temporal methods rely on the decay times of phosphorescence. For pulsed excitation, the emitted intensity decays exponentially according to the following equation [41]:

$$I = I_0 e^{-t/\tau} \quad (\text{E.7})$$

where I_0 is the initial phosphorescence intensity, t is time, and τ is the decay time of the phosphor. The decay time decreases as the temperature increases and can be measured by observing the intensity decay over time. By calibrating these measurements with a theoretical model as shown in Equation E.7, the temperature can be determined. Alternatively, instead of using pulsed excitation, one can use a continuous, amplitude-modulated excitation signal with angular frequency Ω (Equation E.8). The emitted intensity signal (Equation E.9) will be phase-shifted relative to the excitation signal and exhibit a reduction in contrast (magnitude). By measuring either the phase shift or the loss of contrast, the temperature can be inferred [40] [41].

$$I_{ex} = I_0 (1 + a \sin \Omega t) \quad (\text{E.8})$$

$$I_{em} = bI_0 \left(1 + \frac{a}{\sqrt{1 + \Omega^2 \tau^2}} \sin(\Omega t + \phi) \right) \quad (\text{E.9})$$

2D temperature measurements can be obtained by capturing phosphorescence emissions at short time intervals using a framing camera equipped with multiple detectors [41].

Spectral Method

The spectral method determines temperature by calculating the intensity ratio between two spectral emission lines:

$$R = \zeta e^{-\frac{\Delta E}{kT}} \quad (\text{E.10})$$

where ΔE is the energy difference between the two emission lines, k is the Boltzmann constant, and ζ is an experimentally determined constant. The experimental setup for obtaining a 2D temperature measurement using the spectral method can be complex. It requires capturing two images simultaneously, each corresponding to different energy states. This typically involves either using two cameras with different filters or a single camera paired with an imaging stereoscope that enables two-colour operation [41].

Thermographic phosphors are typically composed of a ceramic carrier material doped with an activator. The phosphor can be applied to a substrate by mixing it with a liquid binder, which allows it to be painted or sprayed onto the surface [41]. These materials are suitable for temperature measurements across a wide range, from cryogenic temperatures to around 2000 °C, offering sensitivities of 0.05 °C and uncertainties ranging from 0.1% to 5% of the Celsius temperature reading [34]. However, the sensitivity and accuracy depend heavily on the specific thermographic phosphor used and the method of temperature determination. Additionally, measurement setups involving thermographic phosphors are often complex and expensive [29], making them a less attractive option for the application at hand.

E.4 Non-Invasive Sensors

Non-invasive temperature sensors measure temperature without making contact with the substrate. Unlike invasive sensors, they do not interfere with the measurement and allow for easier remote sensing. Non-invasive techniques are suitable for obtaining point measurements as well as temperature distributions by, e.g., scanning the surface, using optical components, or employing a moving stage. Typically the performance of non-invasive sensors is not as good as invasive sensors due to their sensitivity to environmental conditions and material properties. Furthermore, non-invasive temperature measurement setups are usually quite complex and costly, requiring components such as excitation lasers and optical elements.

This section will discuss several non-invasive thermometry techniques, including infrared thermography, Raman thermometry, refractive index methods/interferometry, fiber optic/lightpipe thermometry, acoustic thermometry, reflectance thermometry, and band edge thermometry.

E.4.1 Infrared Thermography

Infrared thermography is a thermometry technique that measures the thermal radiation emitted by an object. Any material with a temperature above absolute zero emits thermal radiation, which spans the electromagnetic spectrum from approximately 0.1 to 100 μm . However, most temperature measurements are performed within the 0.7 to 20 μm range due to limited device sensitivity outside of this specific spectrum. Infrared thermometers can be used over a wide temperature range, from 50 K to around 6000 K [29].

The spectral emissive power of a blackbody at wavelength λ and temperature T is given by Planck's law [42]:

$$E_{\lambda,b} = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)} \quad (\text{E.11})$$

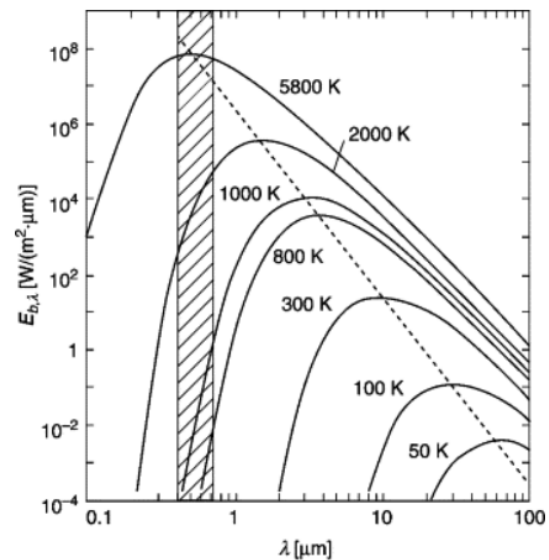


Figure E.4: Spectral emissive power of a blackbody plotted against the wavelength for different temperatures [42].

Where h is Planck's constant, k Boltzmann's constant, and c the speed of light in vacuum. A graphical representation of Planck's law has been depicted in Figure E.4 for several constant temperature lines. The shaded area represents the visible spectrum. In reality, objects do not match the blackbody simplification of Planck's law. As mentioned before in Appendix B.1, the emitted radiation of graybodies depends on the surface emissivity, ε . Therefore, accurate infrared thermography measurements require detailed knowledge of both the target and surrounding surface emissivities [34]. For materials such as silicon, the thermal optical properties are quite complex, making radiation thermometry challenging. Due to the bandgap energy structure of single-crystal silicon, its emissivity exhibits a strong temperature dependence at wavelengths longer than approximately 1 μm . Additionally, at ambient conditions, silicon's emissivity is relatively low, and it is semi-transparent to infrared radiation [43]. If infrared thermography were to be applied to the problem at hand, a different material would therefore need to be chosen to perform the measurements with.

Another critical factor influencing the performance of infrared thermography is the presence of background radiation. An infrared thermometer detects all radiation incident on its sensor, whether it is emitted, reflected, or transmitted. This becomes especially significant when the surroundings of the substrate are at high temperatures, as a considerable portion of the background thermal radiation may be reflected off or transmitted through the substrate's surface. This can lead to an artificially

elevated temperature reading for the substrate. To mitigate the effects of excessive background radiation, the substrate can be placed in a controlled environment free from hot objects, thus reducing unwanted interference.

A typical measurement setup for infrared thermography consists of the source, the propagation medium, and the detection system. The detection system usually contains optical elements, a detector, a processing circuit, and a display [34]. The most common type of detectors are thermal detectors, which convert the absorbed thermal radiation into heat. The temperature increase of the thermal detector is consequently determined using properties such as electrical resistance, thermoelectric EMF, and electrical polarization [29].

Infrared thermography is effective for both surface temperature distribution measurements and point temperature readings. For temperature distribution measurements, a sensitivity of 30 mK and an accuracy of 0.5% can be achieved. However, through careful calibration and accurate emissivity data, the performance can be further improved. However, high performance infrared thermography systems tend to be relatively costly [29].

E.4.2 Raman Thermometry

Raman spectroscopy is a versatile technique with a wide range of applications, including identifying chemical compounds, characterizing chemical bonding, and studying the solid-state structure of materials. A lesser-known application of Raman spectroscopy is temperature measurement [45] [46] [47]. In Raman spectroscopy, the substrate is exposed to monochromatic light, typically from a laser. When this incident light interacts with the molecules in the sample, most of it is elastically scattered (Rayleigh scattering), meaning it retains the same energy and frequency.

However, a small fraction of the light undergoes inelastic scattering, known as Raman scattering. In Raman scattering, the energy of the incident photons either increases (anti-Stokes scattering) or decreases (Stokes scattering), and the scattered light contains frequency components that differ from the incident light (Figure E.5). This energy exchange is linked to the vibrational and rotational modes of the molecules in the sample. The Raman spectrum is recorded by measuring the intensity of the scattered light as a function of its frequency, with peaks corresponding to different vibrational and rotational modes of the molecules in the sample [48].

There are two primary methods by which Raman spectroscopy can be used to determine the temperature of a substrate. The first method involves observing the shift in the peak position of a Raman band. As temperature increases, the bond length typically increases, which in turn decreases the energy of the vibrational mode. This increase in bond length results in a shift of the Raman peak position, which can be used to infer the temperature of the substrate. To use this method effectively, the Raman bands must be sufficiently narrow. The second method determines the temperature by calculating the intensity ratio between the anti-Stokes and Stokes signals, using either Equation E.12 (for energy-based detection) or Equation E.13 (for photon-based detection) [45].

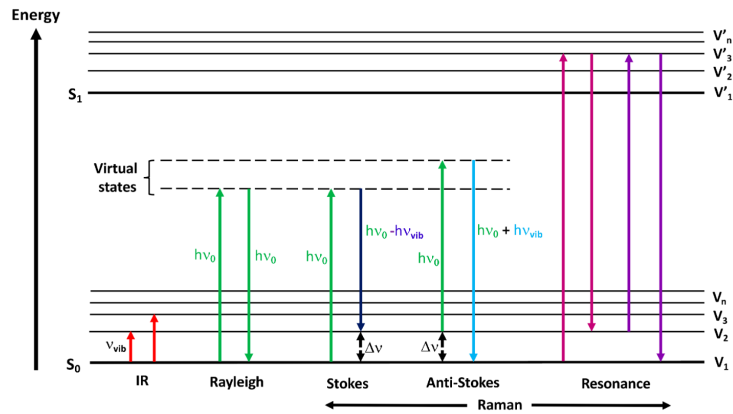


Figure E.5: Energy diagram showing the different scattering processes [44].

$$\frac{I_{AS}}{I_S} = \frac{(V_l + V_v)^4}{(V_l - V_v)^4} e^{-\frac{hV_v}{kT}} \quad (E.12)$$

$$\frac{I_{AS}}{I_S} = \frac{(V_l + V_v)^3}{(V_l - V_v)^3} e^{-\frac{hV_v}{kT}} \quad (\text{E.13})$$

Where V_l and V_v represent the frequencies of the laser and the vibrational mode, respectively, h is Planck's constant, k is Boltzmann's constant, and I_{AS} and I_S denote the anti-Stokes and Stokes scattering intensities, respectively.

A typical Raman spectroscopy measurement setup is relatively complex, comprising an excitation laser, focusing optics, collecting optics, a collector (such as a CCD), and a dispersion element. As a result, Raman spectroscopy tends to be an expensive measurement technique. Moreover, since Raman spectroscopy yields a point measurement, obtaining a 2D temperature distribution requires additional equipment, such as a scanning mechanism or a moving stage [47].

Raman thermometry can be applied across a broad temperature range, from cryogenic temperatures [47] to several thousand Kelvins [29], offering high spatial resolution [46]. An accuracy of 2 K has been reported for Raman thermometry [49].

E.4.3 Refractive Index Methods / Interferometry

The refractive index of a material is defined as the ratio of the speed of light in a vacuum to the speed of light in the material. It determines the degree to which light is bent when entering the material. The refractive index is influenced by both the temperature of the material and the wavelength of the incident light [50], making it a useful parameter for thermometry.

Refractive index thermometry techniques are commonly applied to gases, often using principles of interferometry. When a gas is illuminated with light, variations in its density and temperature cause corresponding changes in the refractive index, which in turn produces interference patterns in the outgoing light. By analyzing these interference patterns, the temperature of the gas can be inferred [29].

A similar approach can be applied to solid substrates, such as wafers. The changes in the optical path between the front and back surface of the substrate due to thermal expansion and refractive index changes can be observed based on the measured interference pattern. By consequently applying image processing techniques and fitting the observed interference pattern to a database, the temperature distribution of the substrate can be obtained with an accuracy of ± 1 K [51].

Refractive index methods can be applied over a broad temperature range [29], offering high spatial and temporal resolution [51]. However, these methods come with significant challenges, including the need for complex measurement setups and advanced image processing techniques. Additionally, precise calibration or fitting to a reference database is essential for achieving accurate results.

E.4.4 Fiber Optic / Lightpipe Thermometry

Fiber optic temperature sensors use light to convey temperature data employing a temperature-sensitive material as the sensor. As temperature fluctuates, the optical properties of the sensor, such as transmission, reflection, fluorescence, or absorption undergo changes. Subsequently, the generated optical signal is transported through optical fibers to a detection system, where it is converted into a temperature reading [52].

Optical fibers come in various types, including microstructured optical fibers (MOFs), conventional fibers, and grating-based fibers. Each type offers distinct advantages in terms of temperature range and sensitivity [53]. Fiber optic thermometry is typically implemented using a fiber optic probe (or lightpipe), which is placed in contact with or slightly above the surface of the object being measured.

The key benefits of fiber optic thermometry include its immunity to electromagnetic, vibrational, chemical, and thermal interferences [54]. Additionally, these sensors are relatively cost-effective and can be easily multiplexed, allowing for multiple measurements to be processed simultaneously. Since

the light signal is confined within the fiber through total internal reflection, fiber optic thermometry is particularly well-suited for long-distance applications [53]. However, the technique is generally limited to point measurements rather than temperature distributions. Furthermore, the accuracy of fiber optic thermometry is influenced by the optical properties of the substrate, necessitating careful calibration and a thorough understanding of the substrate's characteristics to achieve reliable results [54].

Fiber optic/lightpipe thermometry can be used over a broad temperature range, from approximately $-200\text{ }^{\circ}\text{C}$ to $2000\text{ }^{\circ}\text{C}$, with a typical accuracy of around $\pm 0.5\text{ K}$ [29].

E.4.5 Acoustic Thermometry

Another temperature-dependent quantity is the speed of sound, which can be utilized for temperature measurements in solids, liquids, and gases. For gases and liquids, the most straightforward method of acoustic thermometry involves measuring the time of flight between an acoustic transmitter and receiver placed at a fixed distance from each other. Alternatively, the phase shift of the acoustic wave can be analyzed to infer the temperature [55]. In contrast, for solids, more advanced techniques are generally employed, such as those involving surface acoustic waves (SAWs).

Surface acoustic waves (SAWs) are guided waves with wave vectors oriented parallel to the surface of a material. SAWs can be measured using either invasive or non-invasive techniques, depending on the application.

In invasive acoustic thermometry, SAW sensors are typically fabricated on a piezoelectric substrate. By leveraging the piezoelectric effect, an acoustic wave is generated by applying a voltage to the substrate. A second SAW sensor then detects the propagated wave, and the resulting frequency shift is used to infer the temperature of the material [56] [57].

A non-invasive approach to measuring SAWs involves techniques such as laser interferometry. In this method, ultrasonic SAWs are generated at two points on a surface using pulsed lasers. A laser interferometer then detects the ultrasonic echoes at another location, enabling the extraction of temperature data. This technique can be extended to determine the 1D temperature distribution of a surface [58]. However, obtaining a 2D temperature distribution with high spatial resolution is considerably more complex.

Acoustic thermometry is applicable across a wide temperature range, from $-269\text{ }^{\circ}\text{C}$ to $2000\text{ }^{\circ}\text{C}$. For solid materials, the accuracy of this technique is approximately $\pm 1\text{ K}$ [29].

E.4.6 Reflectance Thermometry

Reflectance refers to the portion of incident electromagnetic energy that is reflected by a surface. For normal incidence, the reflectance is determined by the refractive indices of the incoming medium (r_1) and the substrate (r_2) and can be expressed as [59]:

$$R = \left| \frac{r_1 - r_2}{r_1 + r_2} \right|^2 \quad (\text{E.14})$$

However, when a sample has an overlying dielectric layer with a thickness on the order of the incident light's wavelength, the reflectance relation becomes more complex. In such cases, the reflectance is influenced not only by the refractive indices but also by the thickness of the dielectric layer. As a

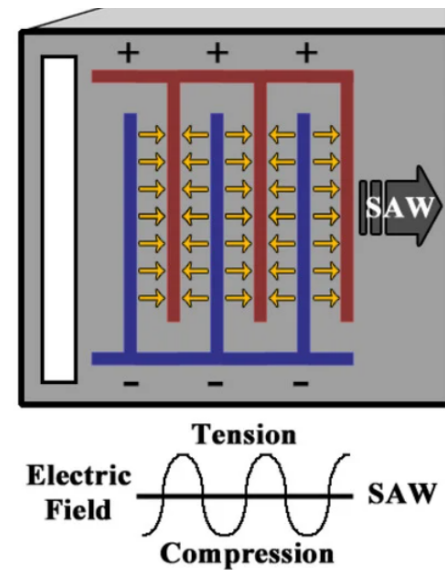


Figure E.6: Generation of a SAW wave [56].

result, both the reflectance and its temperature derivative exhibit periodic variations with respect to the layer thickness [60].

Like mentioned before, the refractive index of a material is temperature dependent. For a small temperature changes ΔT , the relationship between the change in temperature and reflectance is linear [59]:

$$\frac{\Delta R}{R} = \left(\frac{1}{R} \frac{\partial R}{\partial T} \right) \Delta T = C_{th} \Delta T \quad (\text{E.15})$$

In reflectance-based thermometry setups, temperature changes are typically measured by comparing the light intensity reflected from the surface of interest with a reference beam. This is achieved using a beam splitter and photodetector(s). A relatively simple reflectance thermometry setup is shown in Figure E.7. The system incorporates an optical microscope combined with a micropositioner to precisely focus the incident beam onto the substrate. To obtain a temperature distribution with micrometer-scale spatial resolution, a scanning mirror is employed. Additionally, a CCD camera is commonly used for accurate laser beam alignment on the desired area of the substrate [59] [60] [61].

Due to the numerous optical components involved, reflectance thermometry setups tend to be complex and expensive. However, this complexity is compensated by their exceptional performance in terms of spatial, temporal, and temperature resolution. As a result, reflectance thermometry is frequently used for temperature measurements in micro- and nanoscale materials [59].

This technique is applicable over a wide range of temperatures and wavelengths. However, to obtain accurate measurements, the optical properties of the substrate must be well characterized [60]. With careful calibration, reflectance thermometry can achieve an impressive accuracy of up to 10 mK [62].

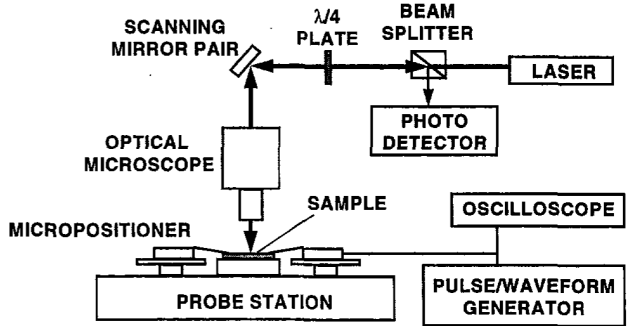


Figure E.7: Reflectance based thermometry setup [60].

E.4.7 Band Edge Thermometry (BET)

BET is a commonly used thermometry technique in the semiconductor industry for cases where IR thermometry is not applicable, namely at room temperature where Si and GaN wafers are semi-transparent to IR radiation [64]. This method relies on the temperature-dependent shift of the semiconductor bandgap, which is the energy difference between the valence band maximum and the conduction band minimum [65]. The temperature dependence of the bandgap is commonly described by the Varshni equation [66]:

$$E_g(T) = E_g(0) - \frac{\alpha T^2}{T + \beta} \quad (\text{E.16})$$

Here, $E_g(0)$ represents the bandgap energy at absolute zero, while α and β are material-dependent fitting parameters.

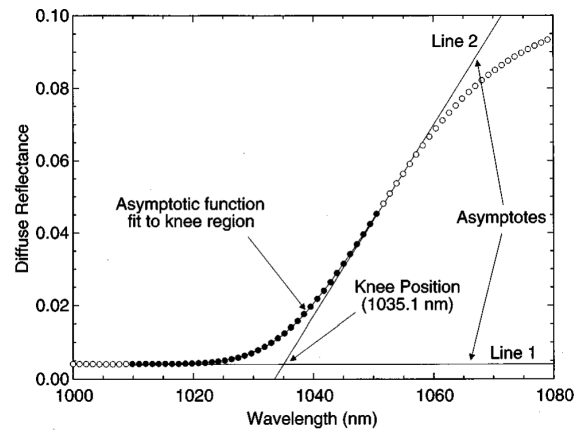


Figure E.8: Asymptotically fitted reflection spectrum of a semiconductor [63].

The bandgap can be measured by illuminating the semiconductor with a broadband light source, detecting the diffusely reflected light, and analyzing its spectrum. The reflected spectrum exhibits a distinct edge at the wavelength corresponding to the bandgap energy [67]. This bandgap wavelength can be approximated using either a linear or higher-order fit [68]. Figure E.8 presents an example of an asymptotically fitted reflection spectrum of a semiconductor. By referencing well-defined calibration curves, the substrate temperature can be determined.

Commercial band-edge thermometers, such as the kSA BandiT, offer a temperature resolution 0.1 °C and an accuracy of ± 2 °C for silicon wafers. Additionally, they can operate over a broad temperature range, from room temperature up to 1200 °C [68]. However, a key limitation of this technique is its reliance on precise knowledge of the substrate material's properties.

F

NTC Thermistors

F.1 Principles of Operation

Thermistor Characteristic

Measuring temperature using a thermistor involves determining its electrical resistance and applying it to the appropriate characteristic equation. As described in [Appendix E.2.2](#), a simple and effective method for calculating temperature is by using the Beta equation, shown in [Equation E.4](#). This equation requires only two parameters to determine the thermistor's resistance over its full range: the Beta constant (B or β) and the resistance at a reference temperature.

For the specific NTC thermistors utilized in this research, the reference resistance R_{25} at 25 °C and the Beta parameter $B_{25/100}$ are provided in [Table F.1](#). The subscript 25/100 indicates the two temperature points, 25 °C and 100 °C, used to calculate the Beta constant. The Beta equation provides a reasonable approximation of the thermistor characteristic for applications requiring moderate accuracy, particularly when operating within the reference temperature range used to derive the Beta parameter.

However, the Beta model is a simplified derivation of the more accurate Steinhart-Hart equation [69]:

$$\frac{1}{T} = A + B \ln R + C (\ln R)^3 \quad (\text{F.1})$$

The Steinhart-Hart equation is widely used for achieving a high-accuracy fit across temperature ranges spanning a few degrees to several hundred degrees Celsius. It requires resistance measurements at a minimum of three distinct temperatures to determine the constants A , B , and C . The calibration procedure used to determine these parameters over a suitable temperature range is described in [Appendix F.4](#).

Self-Heating

As mentioned before, thermistors are susceptible to self-heating, since they require an electrical current to perform temperature measurements. This increase in temperature (ΔT_{self}) is related to the power dissipation of the thermistor (P) and the dissipation factor in air (δ_{th}), as described by:

$$\Delta T_{self} = \frac{P}{\delta_{th}} \quad (\text{F.2})$$

The power dissipation is related to the current passing through the thermistor (I), the voltage across the thermistor (V), and its resistance (R) through the following expressions:

$$P = VI = I^2 R = \frac{V^2}{R} \quad (\text{F.3})$$

To ensure accurate temperature measurements, it is crucial to account for self-heating during both calibration and testing phases. Furthermore, to minimize the interference of the thermistors on the

Table F.1: Relevant thermistor parameters, obtained from the manufacturer's datasheet.

Parameter	Value	Unit
R_{25}	5	k Ω
$B_{25/100}$	3988	K
δ_{th}	1.5	mW/K

temperature field of the wafer, the self-heating should be confined within acceptable limits. This can be achieved by using an appropriate supply voltage. The self-heating effect and the selection of the supply voltage will be further discussed in [Appendix F.4](#).

F.2 Electrical Configuration

Several methods can be employed to determine the electrical resistance of a thermistor, one of which is using a multimeter. Multimeters measure resistance by passing a small, known current through the thermistor and measuring the resulting voltage drop. The electrical resistance can then be inferred using Ohm's law:

$$R = \frac{V}{I} \quad (\text{F.4})$$

While multimeters provide a simple and effective way to measure resistance, they are often limited by their inability to measure multiple thermistors simultaneously or perform data logging. To address these limitations, a 16-bit National Instruments USB-6211 data acquisition (DAQ) device was used, offering real-time control and data acquisition capabilities through the LabVIEW environment. The DAQ device was connected to a laptop via USB.

Instead of passing a small current, this device applies a known voltage over the thermistor. Since directly measuring the voltage drop over the thermistor would simply return this known voltage, the thermistor has to be placed in a circuit with reference resistor(s) to determine the thermistor's resistance. Two common circuit types are a voltage divider and a Wheatstone bridge.

For a voltage divider, the thermistor is placed in series with a known reference resistance, as shown in [Figure F.1a](#). The voltage drop across the thermistor can consequently be measured and converted to electrical resistance using the following formula:

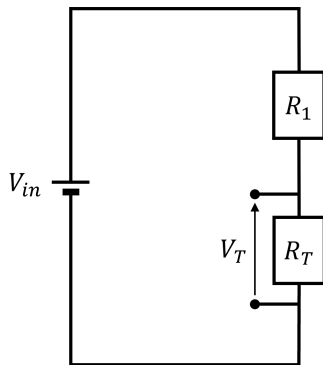
$$R_T = \frac{V_T}{V_{in} - V_T} \cdot R_1 \quad (\text{F.5})$$

A Wheatstone bridge, on the other hand, uses three known reference resistors, configured as in [Figure F.1b](#). The differential voltage can then be measured through the unbalance between the two legs of the Wheatstone bridge:

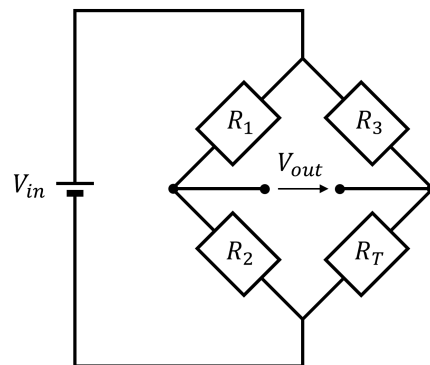
$$V_{out} = V_{in} \left(\frac{R_2}{R_1 + R_2} - \frac{R_T}{R_3 + R_T} \right) \quad (\text{F.6})$$

Assuming $R_1 = R_2 = R_3 = R$, the electrical resistance of the thermistor can be calculated as follows:

$$R_T = \frac{\frac{1}{2} - \frac{V_{out}}{V_{in}}}{\frac{1}{2} + \frac{V_{out}}{V_{in}}} \cdot R \quad (\text{F.7})$$



(a) Voltage divider configuration.



(b) Wheatstone bridge configuration.

Figure F.1: Two common types of electrical configurations to measure the resistance of a thermistor.

Through the use of a Wheatstone bridge, the common temperature drift and electrical noise observed in the two legs cancel out, theoretically leading to improved sensor accuracy. Additionally, the sensitivity of a Wheatstone bridge to changes in the electrical resistance of the thermistor is typically higher than that of a simple voltage divider. However, the Wheatstone bridge configuration is more complex than the voltage divider circuit [15].

Both configurations were tested, yielding similar results. Since the inherent sensitivity of thermistors is already sufficiently high, the Wheatstone bridge did not provide significant advantages in this regard. Furthermore, the electrical noise was not noticeably reduced when using the Wheatstone bridge, suggesting that the majority of the electrical noise originates from the voltage measurement process rather than from noise in the input voltage. As a result, the decision was made to use a voltage divider circuit, which offers greater simplicity without sacrificing performance.

The electrical configuration of the four thermistors is illustrated in Figure F.2. Each thermistor is part of a separate voltage divider, and the four voltage dividers are connected in parallel. This arrangement ensures that the resistance of one thermistor does not influence the voltage measurements of the others. The voltage drop across each thermistor is directly proportional to the ratio of the thermistor's resistance to the total resistance of its respective leg of the parallel circuit.

To select a suitable resistance value for the reference resistors, the sensitivity of the voltage measurement to changes in the thermistor resistance must first be analyzed. The sensitivity is given by:

$$\frac{dV_T}{dR_T} = \frac{R}{(R + R_T)^2} \cdot V_{in} \quad (\text{F.8})$$

The sensitivity is maximized by setting the derivative of the sensitivity with respect to the reference resistance to zero:

$$\frac{d}{dR} \left(\frac{dV_T}{dR_T} \right) = \frac{(R + R_T)^2 - 2R(R + R_T)}{(R + R_T)^4} = 0 \quad (\text{F.9})$$

Solving this equation yields a maximum sensitivity when $R = R_T$.

At a room temperature of 20 °C, where the thermistors are approximately expected to operate at, their resistance is approximately 6.25 kΩ. The closest available resistors to this value were 10 kΩ resistors, with a tolerance of 1 %. Each of the four reference resistors was first measured with a multimeter to obtain a more accurate resistance value. The resulting values can be found in Table F.2.

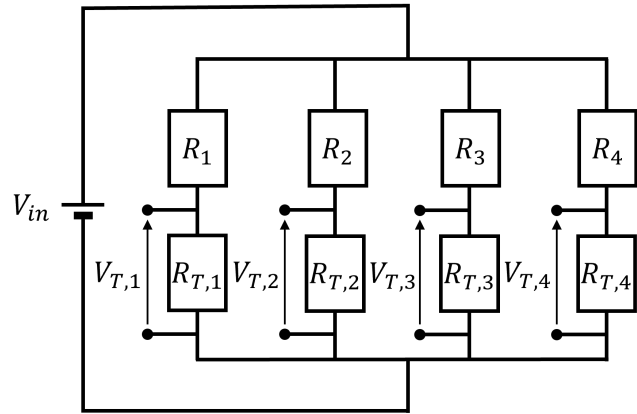


Figure F.2: Electrical configuration used to connect the four thermistors.

Table F.2: Electrical resistance of the reference resistors, measured using a multimeter.

Parameter	Value	Unit
R_1	10.04	kΩ
R_2	10.07	kΩ
R_3	10.09	kΩ
R_4	10.09	kΩ

To ensure proper connections between the electrical components, the circuit was soldered onto a prototyping board, as shown in [Figure F.3](#). The input voltage is provided through one of the analog output (AO) pins of the DAQ device, allowing for an adjustable voltage input. This input voltage is simultaneously measured using one of the analog input (AI) pins. Additionally, the voltage drop across each thermistor is monitored by connecting the output of each reference resistor to an AI pin. The thermistor outputs are connected to both the AI and AO grounds to ensure a common reference point.

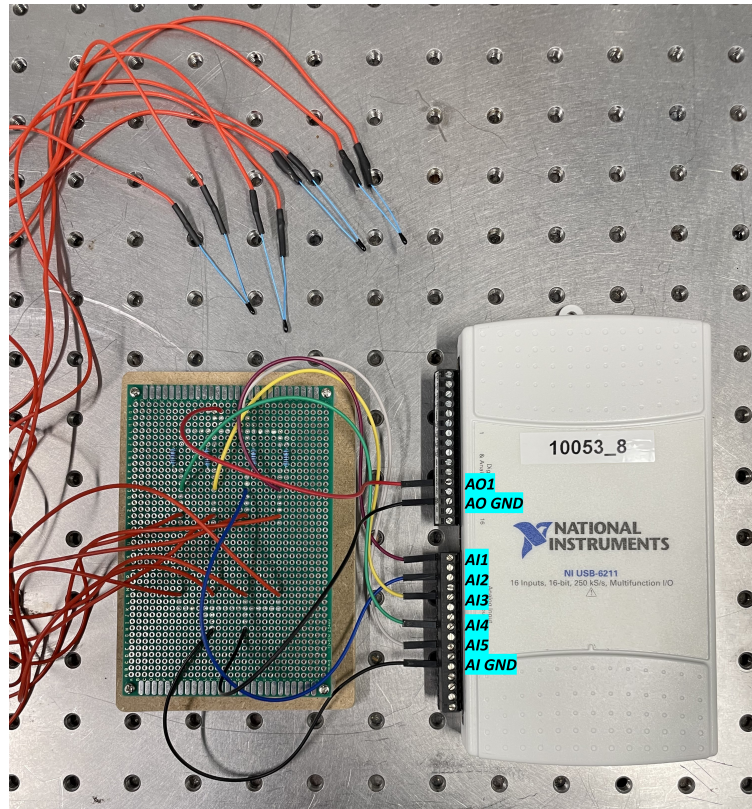


Figure F.3: Electrical circuit soldered onto a prototyping board and connected to the DAQ device, including port labels.

F.3 LabVIEW

To perform temperature measurements using the voltage divider circuit, the LabVIEW block diagram illustrated in [Figure F.4](#) was implemented, with its corresponding front panel shown in [Figure F.5](#). A numeric control block was used to regulate the voltage on the AO pin of the DAQ device, which powers the voltage divider circuit. The input data of the LabVIEW diagram consists of the 16-bit readings from the AI pins, obtained with the DAQ device and converted to a 2D array of scalars.

The raw voltage data is first converted to thermistor resistances using a formula node, based on the known reference resistor values. These resistances are then transformed into temperatures using the Steinhart-Hart equation. The coefficients for this conversion are discussed in [Appendix F.4](#).

To reduce measurement noise, a moving average was applied to the raw temperature data, averaging 20 consecutive measurements.

For visualization, a waveform chart plots the averaged temperature data from the four thermistors in real time. Both the raw voltage and raw resistance values, along with the averaged temperature data, are displayed using numeric indicators and logged to an Excel file for further processing.

An adjustable delay was included in the while loop to control the sampling rate.

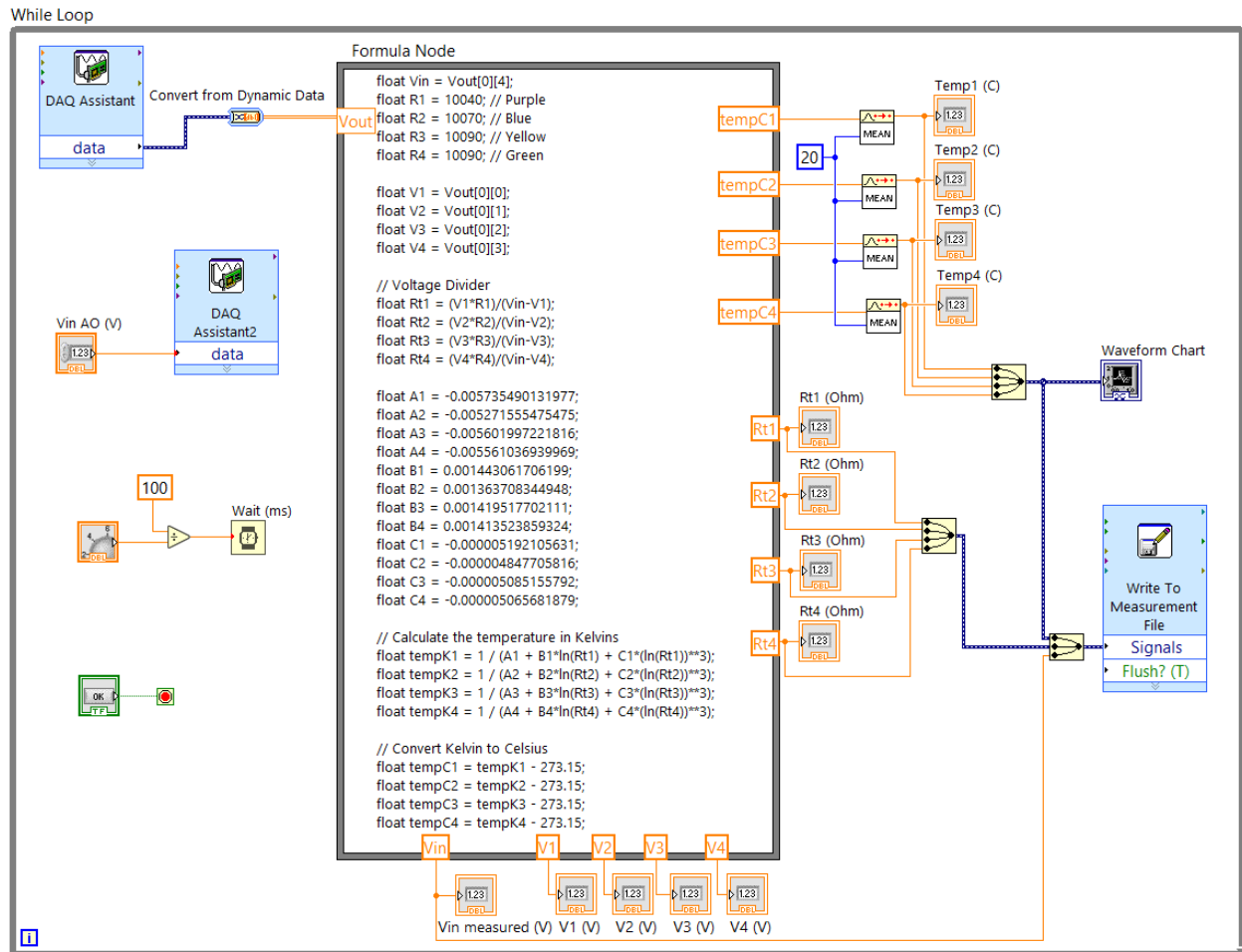


Figure F.4: LabVIEW block diagram used for the thermistor temperature measurements.

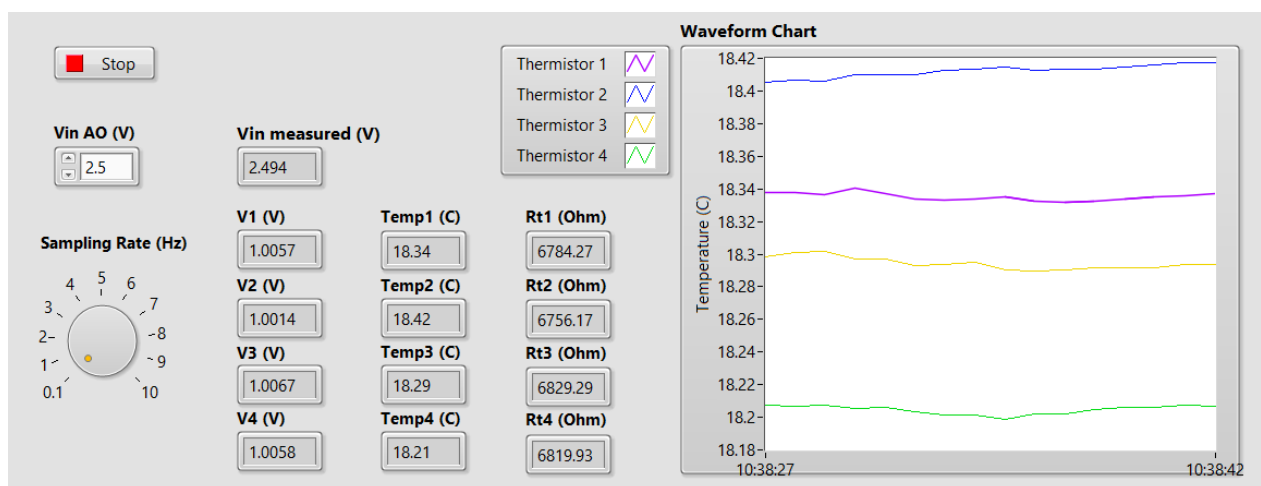


Figure F.5: LabVIEW front panel used for the thermistor temperature measurements.

F.4 Calibration

Process Description

As previously mentioned, the calibration of the thermistors requires resistance measurements at a minimum of three distinct temperatures to determine the Steinhart-Hart constants A , B , and C . These three temperatures should be selected based on the expected operating range. In this case,

the goal is to measure small temperature fluctuations around room temperature. Therefore, the calibration temperatures were selected as follows:

$$T_{ref,1} = 18\text{ }^{\circ}\text{C} \quad T_{ref,2} = 20\text{ }^{\circ}\text{C} \quad T_{ref,3} = 22\text{ }^{\circ}\text{C} \quad (\text{F.10})$$

The setup used to calibrate the four thermistors simultaneously is shown in Figure F.7. On the left side, the National Instruments DAQ device connected to the prototyping board is visible. At the center of the setup, a bowl of water is placed on top of a magnetic stirrer, which ensures the water temperature remains homogeneous. The stirrer has both heating and stirring functionalities, but only the stirring function was used during the thermistor calibration.

In the center of the bowl, an ASL F252 platinum resistance thermometer, with a verified accuracy of 5 mK, is submerged about two centimeters into the water. On either side of the reference thermometer, two thermistors are submerged to an equal depth into the water, as illustrated in Figure F.6. On the right side of the setup, the DAQ device of the reference thermometer is located, displaying the temperature readings with a resolution of 1 mK.

The water temperature was adjusted by adding either cold or hot water to the bowl and waiting until a steady and homogeneous temperature was achieved. This approach ensured consistent and accurate calibration conditions for all thermistors.

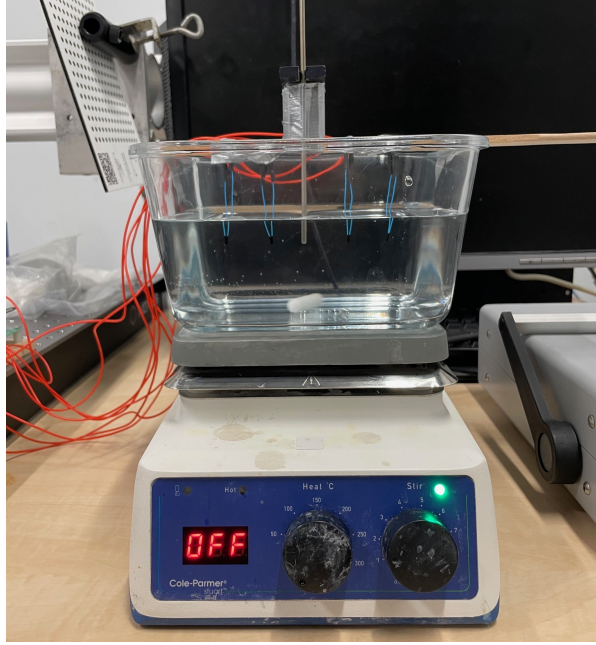


Figure F.6: Close-up view of the water bowl and magnetic stirrer used to calibrate the thermistors.



Figure F.7: Calibration setup used for the thermistors.

At each calibration temperature, measurements were conducted over a period of approximately 7.5 minutes. During the first 2.5 minutes, a supply voltage of 1 V was applied. This was increased to 2.5 V for the next 2.5 minutes, and finally, to 5 V for the remaining 2.5 minutes. This procedure allowed for an evaluation of both measurement noise and the self-heating effects of the thermistors, enabling the selection of an appropriate supply voltage for accurate and reliable measurements.

Pre-Calibration Results

The results of the calibration measurements at 18 °C, 20 °C, and 22 °C are presented in [Figure F.8](#), [Figure F.9](#), and [Figure F.10](#), respectively. In these graphs, T_{1-4} MA 20 represent the temperature readings of thermistors 1 to 4 with a moving average of 20 samples. The sampling rate for the thermistors was set to 1 Hz, with the first 10 seconds of data excluded to account for the moving average, resulting in a measurement duration of approximately 440 seconds for each calibration temperature. T_{ref} represents the more stable reference thermometer readings, for which a sampling rate of 0.2 Hz was used. The vertical black dotted lines indicate the timestamps at which the voltage was increased.

At 20 °C, the water temperature remained the most stable throughout the measurement, indicating the steady-state temperature of the water. At 18°C, the temperature gradually increased by approximately 60 mK over the measurement period, approaching the steady-state temperature. Conversely, at 22°C, the temperature decreased by around 60 mK, also converging toward the steady-state temperature.

Self-Heating

An additional observation is the temperature increase in the thermistors at points where the supply voltage was raised, attributed to self-heating. The effect is particularly pronounced after increasing the voltage from 2.5 V to 5 V. To quantify this phenomenon, the 1 V supply voltage was used as a reference where self-heating is assumed to be negligible. The average temperature deviation of each thermistor from the reference thermometer was calculated for each voltage and compared to the deviation at 1 V. To account for the transition period when switching from 2.5 V to 5 V, the first 20 measurements after increasing the voltage were excluded from the analysis.

Table F.3: Self-heating of each thermistor in water due to an increase in supply voltage.

Thermistor	$\Delta T_{1V \rightarrow 2.5V}$ (mK)	$\Delta T_{1V \rightarrow 5V}$ (mK)
1	4.1	45.3
2	11.8	48.4
3	8.0	43.7
4	6.6	45.7
Average	7.6	45.8

It is important to note that the magnitude of the self-heating depends on the thermistor's resistance, which varies with the calibration temperature. Nevertheless, for this analysis, the average deviation across all calibration temperatures was used to provide a general estimate of the self-heating magnitude in water.

The results are summarized in [Table F.3](#). Increasing the voltage from 1 V to 2.5 V caused an average temperature increase of 7.6 mK ($\Delta T_{1V \rightarrow 2.5V}$), while increasing it further to 5 V resulted in an average increase of 45.8 mK ($\Delta T_{1V \rightarrow 5V}$). Although doubling the voltage from 2.5 V to 5 V theoretically doubles the current and quadruples the power dissipation, the measured temperature increase shows a factor of $45.8/7.6 = 6.0$, likely due to measurement noise at lower voltages.

Finally, it is worth noting that the self-heating in water is significantly less pronounced than in air, primarily due to water's much higher specific heat capacity, thermal conductivity, and density. These properties enable water to absorb and dissipate heat more effectively, minimizing temperature rise of the thermistor.

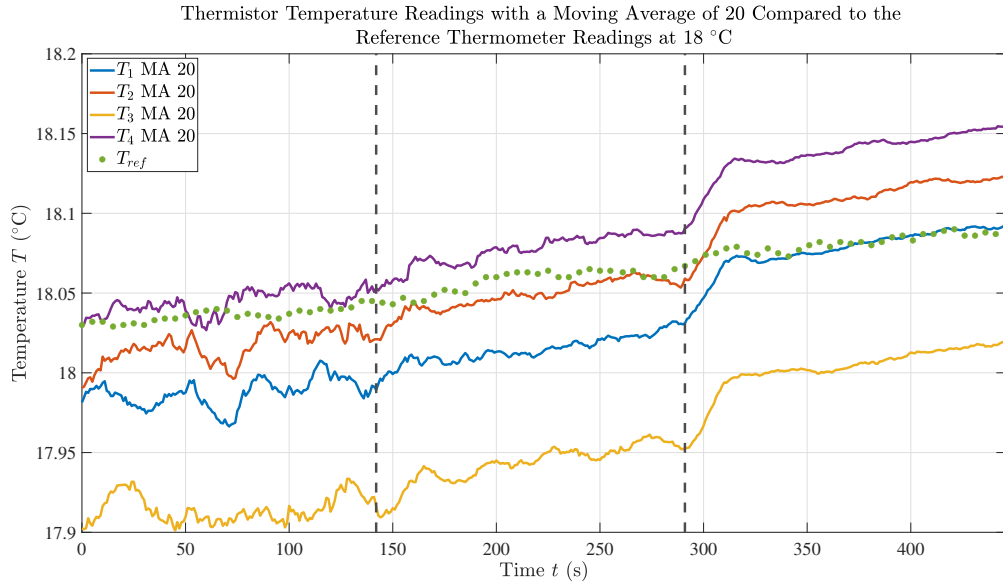


Figure F.8: Calibration measurements at 18 °C.

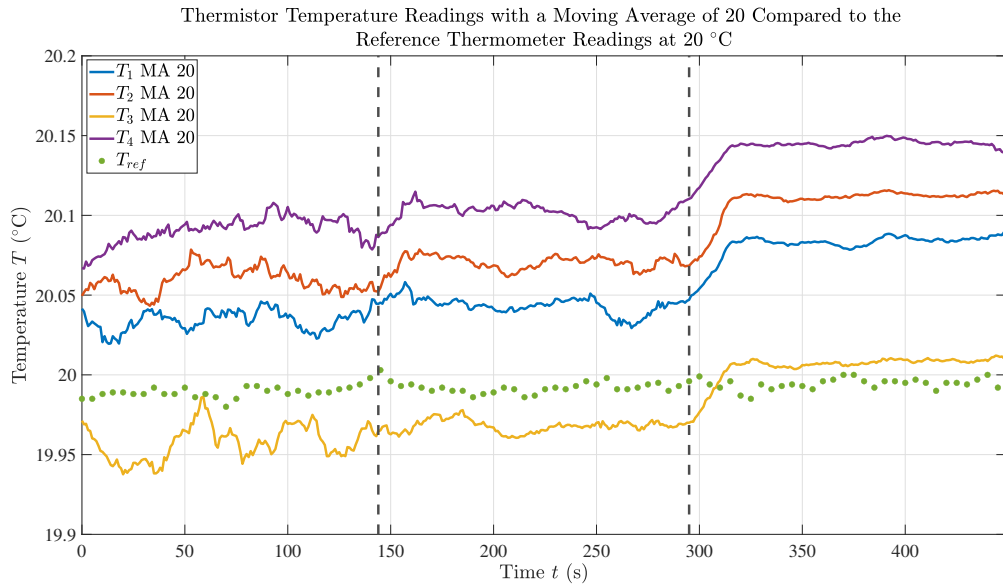


Figure F.9: Calibration measurements at 20 °C.

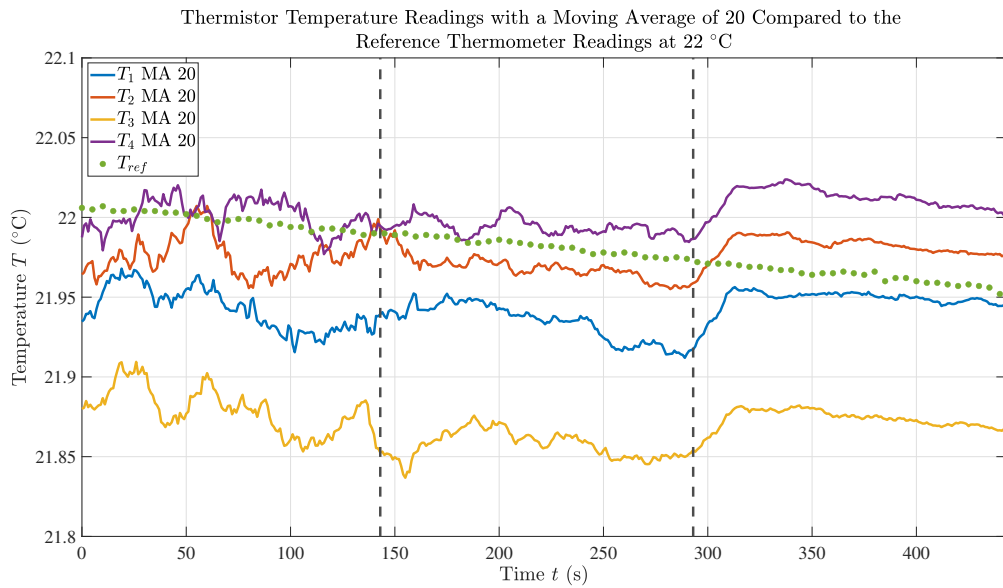


Figure F.10: Calibration measurements at 22 °C.

Measurement Noise

To evaluate the measurement noise in the calibration data, the raw temperature readings of each thermistor were plotted alongside the reference thermometer readings and the corresponding averaged thermistor readings. This resulted in a total of twelve plots, representing four thermistors at three calibration temperatures. As all these plots exhibit similar patterns, only one representative plot is shown (Figure F.11), specifically for thermistor 2 at 20 °C.

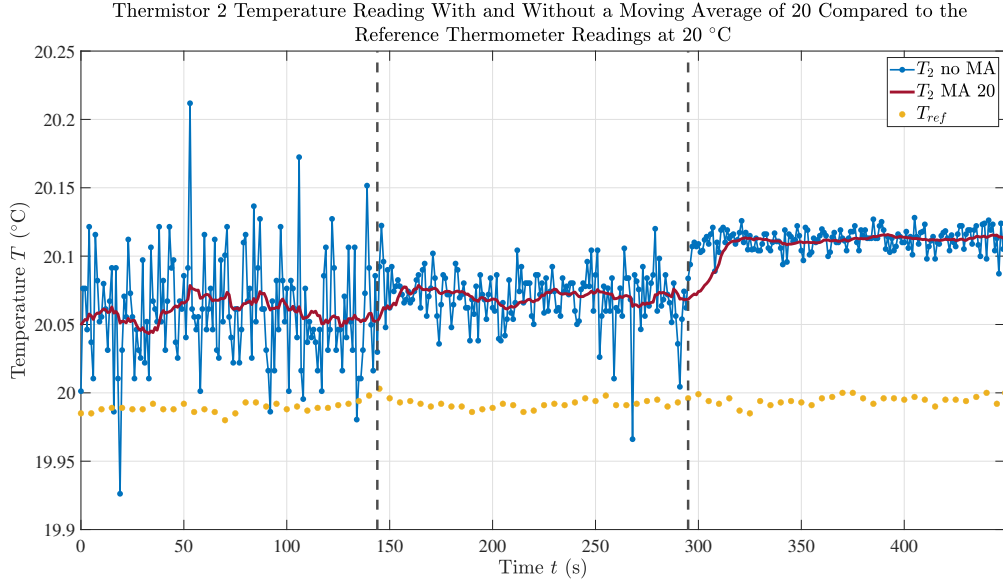


Figure F.11: Thermistor 2 readings with and without a moving average, compared to the reference thermometer readings at 20 °C.

At each voltage transition, a noticeable change in the magnitude of the measurement noise can be observed. To quantify this noise, the signal-to-noise ratio (SNR) was calculated. For approximately constant signals, such as the thermistor temperature readings, any deviation from the mean can be attributed to noise. The SNR is therefore defined as follows [70]:

$$\text{SNR} = \frac{\mu_s}{\sigma_n} \quad (\text{F.11})$$

Here, μ_s represents the mean value of the signal:

$$\mu_s = \frac{1}{N} \sum_{i=1}^N T_i \quad (\text{F.12})$$

And σ_n denotes the standard deviation of the noise:

$$\sigma_n = \sqrt{\frac{1}{N-1} \sum_{i=1}^N (T_i - \mu_s)^2} \quad (\text{F.13})$$

Where N is the total number of temperature measurements and T_i is the temperature reading at the i -th time step.

The SNR is typically expressed in terms of decibels:

$$\text{SNR}_{\text{dB}} = 10 \cdot \log_{10}(\text{SNR}) \quad (\text{F.14})$$

Since the temperature measurements at 20 °C yielded the most stable results, this data was used to compute the SNR for each voltage level across all thermistors. Both the raw temperature data

and the data smoothed with a moving average of 20 were analyzed, with the results presented in [Table F.4](#). Similar to the self-heating analysis, the first 20 measurements after increasing the supply voltage from 2.5 V to 5 V were excluded to avoid transient effects.

The results show that applying a moving average of 20 consistently improves the SNR across all thermistors and voltage levels, with an average increase of approximately 6.7 dB. Additionally, higher voltage levels yield a better SNR, regardless of whether a moving average is applied. The transition from 1 V to 2.5 V resulted in an average SNR improvement of 3.0 dB, while the increase from 2.5 V to 5 V led to a further improvement of 3.5 dB. All thermistors show similar improvements in SNR with increasing voltage and the use of a moving average, highlighting consistent performance across the dataset.

Table F.4: Signal-to-noise ratio expressed in decibels (SNR_{dB}) at each voltage with and without a moving average, at 20 °C.

Thermistor	SNR_{dB}					
	1 V		2.5 V		5 V	
	No MA	MA 20	No MA	MA 20	No MA	MA 20
1	27.2	35.0	29.6	36.3	33.2	39.4
2	27.1	34.1	30.0	37.2	34.2	40.8
3	26.7	32.7	30.3	37.3	33.5	40.4
4	26.6	33.8	30.4	36.2	33.8	40.0
Average	26.9	33.9	30.1	36.7	33.7	40.1

Calibration

For the calibration of the thermistors, an appropriate supply voltage was selected based on the self-heating and SNR analysis presented above. The analysis revealed a trade-off between self-heating and SNR: at lower supply voltages, self-heating is negligible, but measurement noise is more significant; at higher supply voltages, self-heating becomes more pronounced, though the SNR improves substantially. To balance these factors, a supply voltage of 2.5 V was selected. At this voltage, self-heating remains low enough not to interfere with the calibration, and the SNR is high enough that the noise is barely noticeable, ensuring reliable measurements without significant interference.

The resistance data for each thermistor was then fitted to the Steinhart-Hart equation using the matrix equation shown in [Equation F.15](#). In this equation, T_{1-3} represent the average reference thermometer readings at each calibration temperature at 2.5 V, while R_{1-3} denote the corresponding average thermistor resistances. The resulting Steinhart-Hart constants are presented in [Table F.5](#).

$$\begin{pmatrix} 1 & \ln R_1 & (\ln R_1)^3 \\ 1 & \ln R_2 & (\ln R_2)^3 \\ 1 & \ln R_3 & (\ln R_3)^3 \end{pmatrix} \begin{pmatrix} A \\ B \\ C \end{pmatrix} = \begin{pmatrix} \frac{1}{T_1} \\ \frac{1}{T_2} \\ \frac{1}{T_3} \end{pmatrix} \quad (\text{F.15})$$

Table F.5: Steinhart-Hart constants for each thermistors, determined through calibration.

Thermistor	A (1/K)	B (1/K)	C (1/K)
1	-5.74×10^{-3}	1.44×10^{-3}	-5.19×10^{-6}
2	-5.27×10^{-3}	1.36×10^{-3}	-4.85×10^{-6}
3	-5.60×10^{-3}	1.42×10^{-3}	-5.09×10^{-6}
4	-5.56×10^{-3}	1.41×10^{-3}	-5.07×10^{-6}

[Figure F.12](#) shows the comparison of the Beta model provided by the thermistor manufacturer and the calibrated Steinhart-Hart fits of each thermistor across the calibrated range. The Steinhart-Hart

better captures the nonlinear behaviour of the thermistors due to the introduction of the second- and third-order terms. Furthermore, the resistance response of each thermistor to a change in temperature is very similar, mainly showing a constant offset between each fit. This offset remains within the 1 % tolerance of the thermistor resistance specified by the manufacturer.

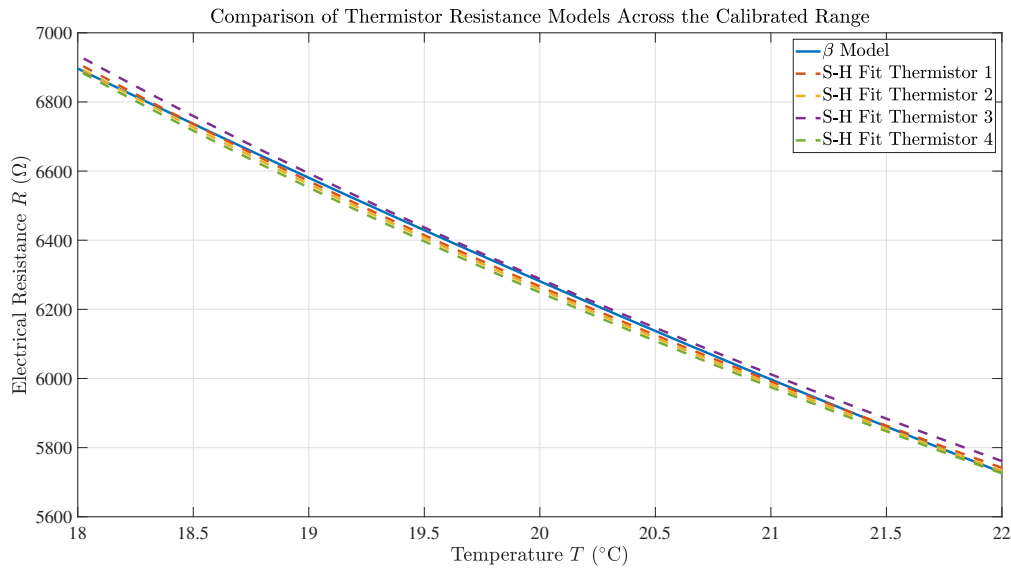


Figure F.12: Comparison of the Beta model and the Steinhart-Hart fits of each thermistor.

Post-Calibration Results

After determining the calibrated Steinhart-Hart constants, the same calibration measurement data was plotted using these constants, as shown in Figure F.13, Figure F.14, and Figure F.15. As expected, the thermistor temperature now aligns much more closely with the reference thermometer readings at each calibration temperature. This alignment is particularly accurate at a supply voltage of 2.5 V, the voltage used for the calibration of the thermistors.

The integration of the Steinhart-Hart fits into the LabVIEW diagram is illustrated in Figure F.4.

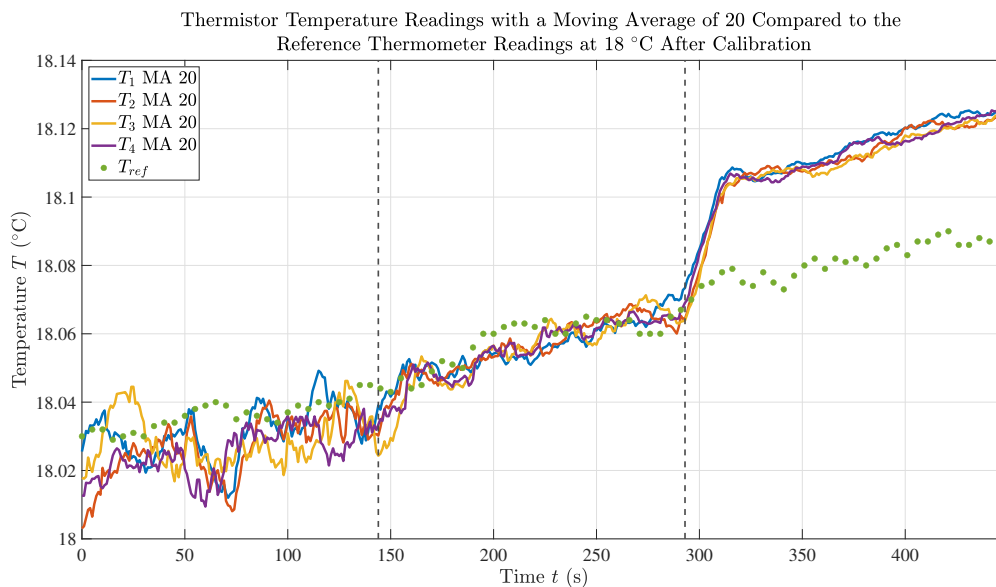


Figure F.13: Calibration measurements at 18 °C after calibration.

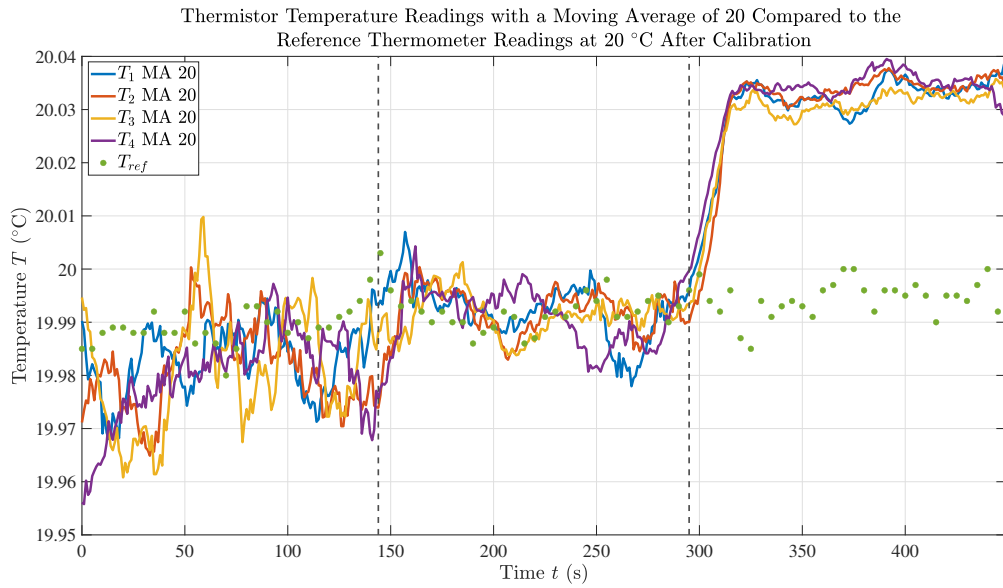


Figure F.14: Calibration measurements at 20 °C after calibration.

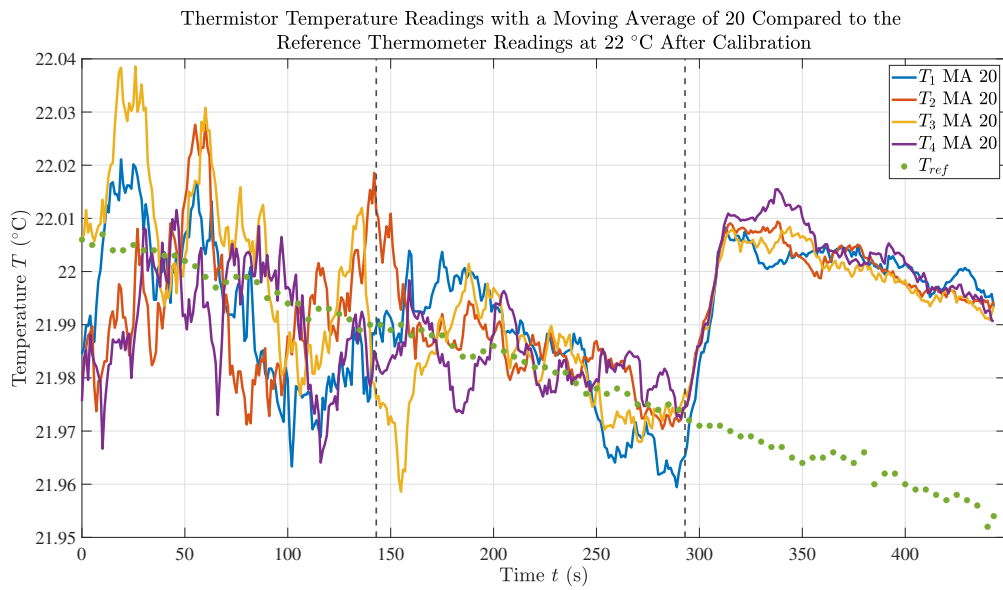


Figure F.15: Calibration measurements at 22 °C after calibration.

F.5 Performance

Theoretically, the precision and accuracy of the thermistors could be further assessed by performing measurements at several intermediate temperatures using the calibration setup. However, for the purposes of this research, this is not a primary concern. Since the calibration was conducted over a narrow temperature range, the thermistors' accuracy will be sufficiently high to provide reliable reference readings for infrared thermometry. Additionally, for measuring the ambient and inlet air temperature, only the temperature difference is crucial. By performing a reference measurement with no airflow and comparing it to a measurement with airflow, the temperature difference between the inlet and ambient air can be accurately determined, even if there is a slight initial offset between the sensors.

Self-Heating

Considering the impact of self-heating within the test setup is essential, as it can influence the accuracy of the temperature readings. The self-heating in air can be estimated by combining Equation F.2 and Equation F.3. Using the average resistance of the four thermistors at each calibration temperature, the self-heating in air was quantified in Table F.6. As expected, the predicted self-heating in air is significantly higher than that observed water. In the test setup, the magnitude of the self-heating is expected to fall between the cases of water and air, since both the wafer and the air hose have a high specific heat capacity, thermal conductivity, and density compared to air. However, the thermal contact between the thermistors and the test setup will also significantly affect the self-heating. Therefore, the self-heating should be assessed directly in the test setup for a more accurate analysis.

Table F.6: Average self-heating in air of the thermistors at a supply voltage of 2.5 V, at three temperatures.

T	$\Delta T_{self,air}$ (K)
18 °C	0.60
20 °C	0.67
22 °C	0.73

Sensitivity

The sensitivity of the voltage measurement over the thermistor to a change in temperature is expressed as follows:

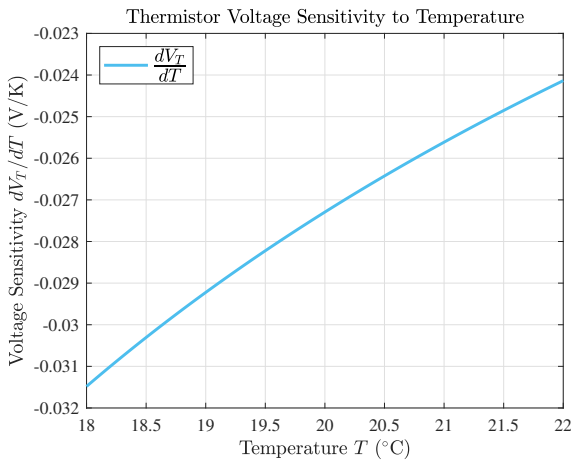
$$\frac{dV_T}{dT} = \frac{dV_T}{dR_T} \frac{dR_T}{dT} \quad (\text{F.16})$$

Where $\frac{dV_T}{dR_T}$ is given in Equation F.8, and $\frac{dR_T}{dT}$ is derived from the Steinhart-Hart equation.

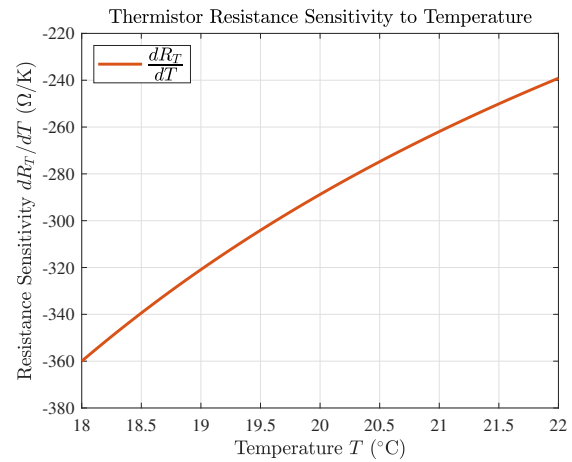
Figure F.16a and Figure F.16b illustrate the average voltage and resistance sensitivities to temperature variations for the four thermistors across the calibrated temperature range.

Both sensitivities are negative, reflecting the negative temperature coefficient (NTC) characteristic of the thermistors. This indicates that the resistance and the corresponding voltage drop over the thermistors decrease as the temperature increases. Additionally, the sensitivity magnitudes are higher at lower temperatures, consistent with the slope of the thermistor's resistance-temperature curve shown in Figure F.12.

The resistance sensitivity shows a substantial magnitude, averaging $-296 \Omega/\text{K}$, which highlights the pronounced change in resistance with temperature. This leads to a corresponding average voltage sensitivity of $-27.7 \text{ mV}/\text{K}$ at a supply voltage of 2.5 V.



(a) Voltage sensitivity.



(b) Resistance sensitivity.

Figure F.16: Thermistor voltage and resistance sensitivity to a change in temperature at a supply voltage of 2.5 V.

Resolution

The resolution of the DAQ device is determined by the number of bits, N , in its analog-to-digital converter (ADC). The National Instruments USB-6211 features a 16-bit ADC, which divides the specified voltage range into 2^{16} discrete steps. For a voltage range of 0 - 2.5 V, the voltage resolution can be calculated by:

$$V_{res} = \frac{V_{ref}}{2^N} = \frac{2.5}{2^{16}} = 38.1 \mu\text{V} \quad (\text{F.17})$$

Using this voltage resolution and the sensitivities discussed earlier, the temperature and resistance resolution of the system can be calculated. These resolutions represent the smallest measurable changes in temperature and resistance that can be detected by the DAQ device. The equations are as follows:

$$T_{res} = \frac{V_{res}}{\left| \frac{dV_T}{dT} \right|} \quad (\text{F.18})$$

$$R_{res} = \frac{V_{res}}{\left| \frac{dV_T}{dR_T} \right|} = \left| \frac{dR_T}{dT} \right| \cdot T_{res} \quad (\text{F.19})$$

Since the calculated resolutions represent magnitudes, the absolute values of the sensitivities are used to ensure the resolutions are expressed as positive quantities.

The variations in temperature and resistance resolutions of the thermistors with temperature are shown in Figure F.17a and Figure F.17b, respectively. Interestingly, while the resistance resolution improves (i.e. decreases) at higher temperatures, the temperature resolution worsens. This is because, at higher temperatures, the thermistor's resistance becomes less sensitive to changes in temperature ($\frac{dR_T}{dT}$), resulting in reduced temperature sensitivity despite improved resistance resolution.

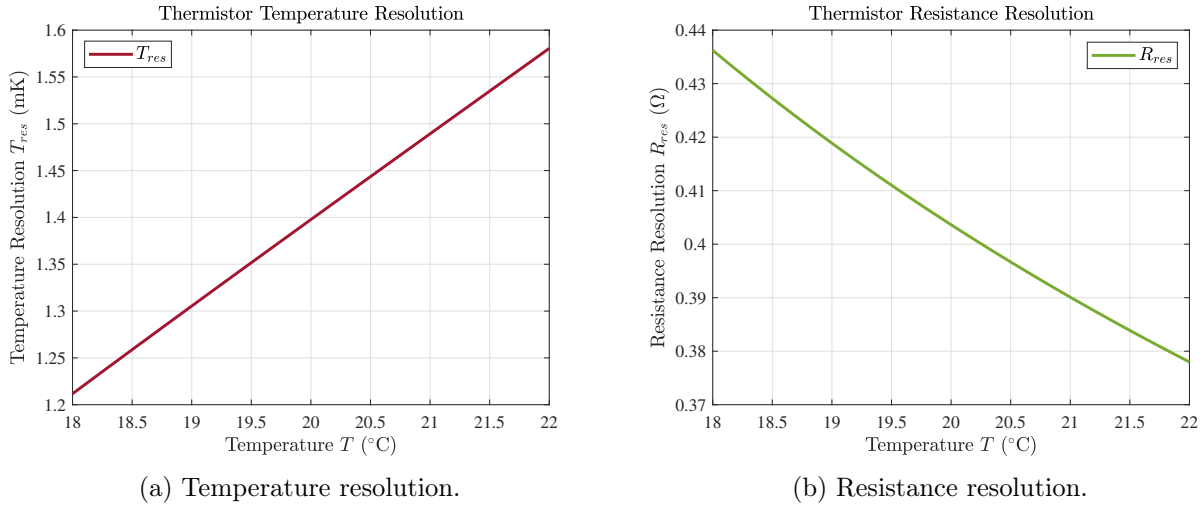


Figure F.17: Thermistor temperature and resistance resolution as a function of temperature.

G

Film Height Measurement

G.1 Principles of Operation

To determine the film height between the unit cell and the wafer, a deformation measurement technique based on an optical method called free-surface synthetic schlieren was employed. This method, previously applied in research by Greidanus et al. [14], studied the interaction between turbulent boundary layer flow and compliant surfaces using the refraction of light traveling through the boundary layer. In their setup, a random dot pattern was positioned beneath the flow and back-illuminated with an LED panel. The dot pattern was observed head-on from below using a camera. By capturing an image of the undeformed reference state and comparing it to the deformed state, the translation of individual dots could be detected. Using the translations of all the dots, a vector field was constructed through a multigrid interrogation approach. In their application, this vector field was directly proportional to the local gradients of the surface. By integrating these gradients over the deformed surface, the surface height could be reconstructed. The experimental setup used by Greidanus et al. is shown in Figure G.1.

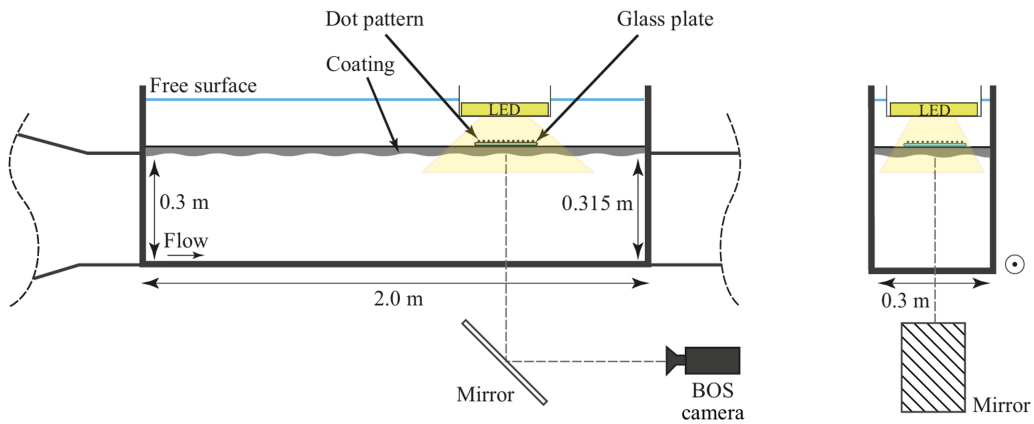


Figure G.1: Setup used to measure surface deformation of a compliant surface due to turbulent boundary flow, developed by Greidanus et al. [14].

In contrast to the application presented by Greidanus et al., the deformation of the wafer on the air bearing system is not caused by a water flow. Instead, it is induced by an air pressure distribution below the wafer's surface. Moreover, the setup does not allow for a dot pattern to be placed below the wafer without disrupting the thin film flow. To overcome this limitation, the reflective properties of the wafer were utilized instead of its refractive properties. By using the reflected light, it is possible to measure surface deformation without physically altering the wafer-air interaction.

A schematic of the setup used to measure the deformation of the wafer is shown in Figure G.2. The reflective wafer is positioned at the center of the setup. On the left side, a camera is placed at a shallow angle relative to the wafer surface to optimize light reflection. Opposite the camera, on the right side of the wafer, a random dot pattern is positioned at the same angle as the camera. This alignment ensures that the camera views the dot pattern head-on, simplifying the calibration process. The global coordinate system is defined with its origin at the center of the wafer: the z -axis points upward, the x -axis extends to the left (toward the camera), and the y -axis extends out

of the plane of the paper. Both the camera and the dot pattern have their own local coordinate systems. For these local systems, the y -axes point upward along their respective tilted planes, while the x -axes are directed into the plane of the paper.

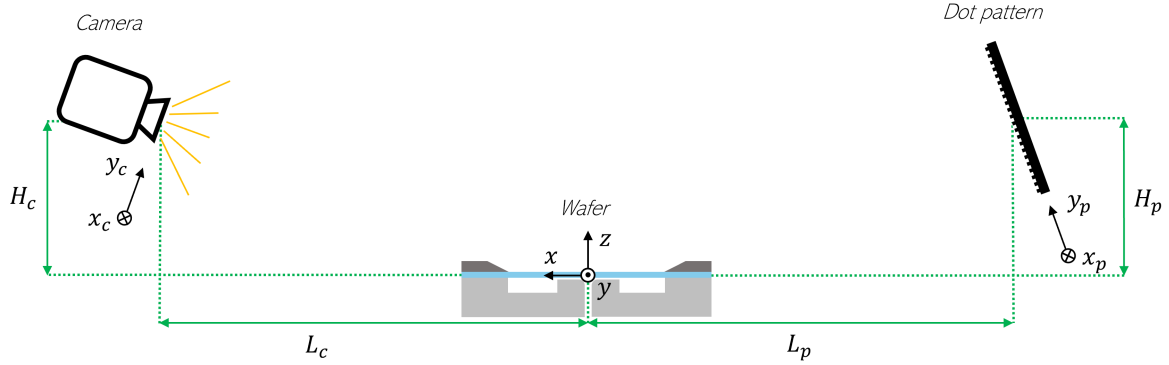


Figure G.2: Schematic representation of the setup used to measure the deformation of the wafer.

The camera and dot pattern are both positioned in the global xz -plane, with their respective distances summarized in Table G.1. The vertical distances from the wafer surface to the camera and dot pattern are denoted by H_c and H_p , respectively. Similarly, the horizontal distances along the x -axis from the center of the wafer to the camera and dot pattern are represented by L_c and L_p .

Table G.1: Distances of the camera and dot pattern with respect to the wafer.

Parameter	Value	Unit
H_c	200	mm
H_p	115	mm
L_c	550	mm
L_p	320	mm

To ensure proper alignment, both the camera and dot pattern were rotated approximately 20° toward the wafer's center. This configuration ensures that the center of the dot pattern points at the center of the wafer and aligns with the center of the camera's field of view.

G.1.1 Ray Tracing

To explain the principles of operation of this method, first, perfect reflection of all incoming light off the top surface of the wafer is assumed, as illustrated in Figure G.3. Based on the law of reflection, the angle of incidence of a light ray with respect to the surface normal is equal to the angle of reflection [71]:

$$\theta_i = \theta_r \quad (\text{G.1})$$

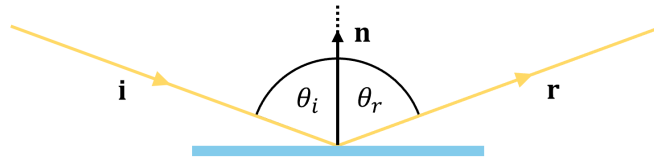


Figure G.3: Schematic representation of light reflecting off a surface.

The reflection of light rays can also be described in a more general form as a vector equation, shown in Equation G.2. The vector approach is especially useful for more complex reflection problems, where the trigonometry cannot easily be visualized.

$$\mathbf{r} = \mathbf{i} - 2(\mathbf{i} \cdot \mathbf{n})\mathbf{n} \quad (\text{G.2})$$

The deformation of the wafer can be described as a combination of three fundamental components: a pure translation in the z -direction, a pure rotation around the y -axis, and a pure rotation around

the x -axis. To determine how each of these components influences the displacement of the dots in the camera image, ray tracing was employed. In order to do so, a dot was simplified to a laser, of which the point of intersection with the camera plane was analyzed. Alternatively, the same result could be achieved by using a viewing ray approach, where the reflection is analyzed from the perspective of the camera.

For the z -translation and y -rotation, the problem can be reduced to two dimensions, allowing for a simplified geometric analysis through visual inspection. However, the x -rotation introduces a more complex reflection pattern, requiring the use of a vector-based approach to determine its effects.

z - translation

Figure G.4 illustrates the effect of a pure upward translation of the wafer by Δh on the path of a light ray. The angle between the incident ray and the xy -plane, denoted as φ , is approximately 20° for the camera and pattern distances described earlier. As shown in the figure, an upward translation of the wafer causes the incident ray to reflect at a point located left of the wafer's center. Since the wafer remains horizontal during this translation, the angle of the reflected ray relative to the xy -plane remains equal to the incident angle φ , ensuring that the reflected rays are parallel to one another.

This upward translation causes the reflected light ray to intersect the camera plane at a point located Δy higher in the camera's local coordinate frame. For a camera directly observing the dot pattern without the reflective wafer in between, this would manifest as an upward shift of the dot pattern in the camera image. However, due to the inverting properties of the reflective wafer, the observed shift appears in the opposite direction, resulting in a downward shift of the dots in the camera image. Conversely, a downward translation of the wafer would lead to an upward shift of the dots in the image. Assuming a constant incident ray angle φ across the surface of the wafer, a pure translation results in a uniform displacement of the dot pattern in the camera image.

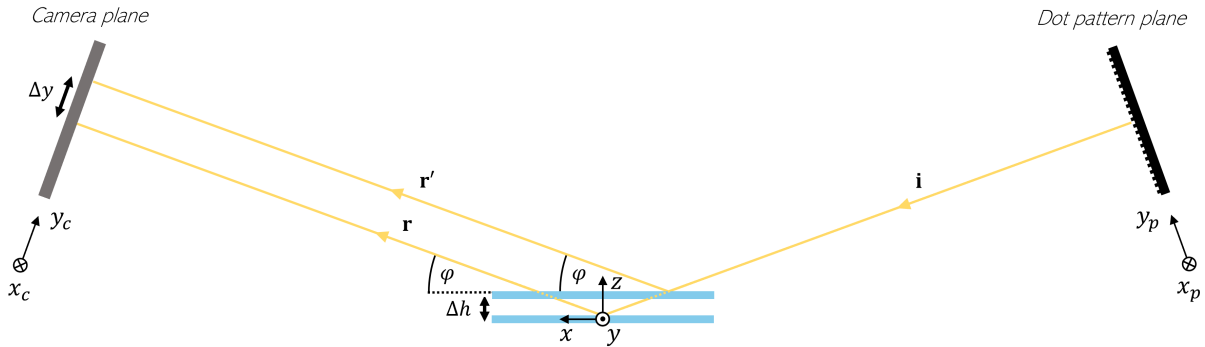


Figure G.4: Schematic representation of ray tracing for a pure upward translation in the z -direction.

The uniform shift in the dot pattern, resulting from a pure translation of the wafer, can be derived using the set of triangles illustrated in Figure G.5. Here, a represents the distance between the points of intersection of the incident ray with the wafer before and after the translation:

$$a = \frac{\Delta h}{\sin \varphi} = \frac{\Delta y}{\sin 2\varphi} \quad (\text{G.3})$$

From this, the dot shift Δy can be calculated as a function of the upward translation Δh and the incident ray angle φ :

$$\Delta y = 2\Delta h \cos \varphi \quad (\text{G.4})$$

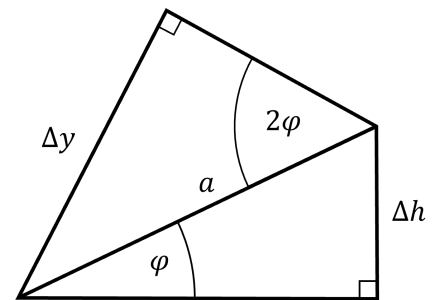


Figure G.5: Trigonometry used to solve the dot shift for a z -translation.

Notably, the dot shift Δy is independent of both the camera-wafer and pattern-wafer distances. This results from the parallelism of the reflected light rays.

y - rotation

For a counterclockwise (positive) rotation of the wafer around the global y -axis by an angle θ , the path of an incident light ray is reflected as shown in Figure G.7. The angle between the reflected rays is equal to 2θ , which can be derived using two methods.

The most straightforward approach employs basic trigonometry. For the horizontal wafer, the incident ray has an angle φ with the wafer surface, also resulting in an angle of φ for the reflected ray. When the wafer is tilted by θ , the incident ray has an angle of $\varphi - \theta$ with the tilted wafer, causing the reflected ray to also have an angle of $\varphi - \theta$ with the tilted wafer. Consequently, the reflected ray for the tilted wafer has an angle of $\varphi - 2\theta$ with the horizontal wafer. The angle between the two reflected rays is therefore $\varphi - (\varphi - 2\theta) = 2\theta$. The angles between the reflected rays and the wafer are visualized in Figure G.6.

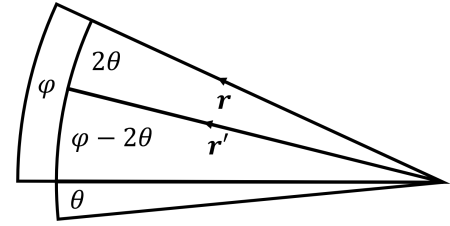


Figure G.6: Angles between the reflected rays and the wafer.

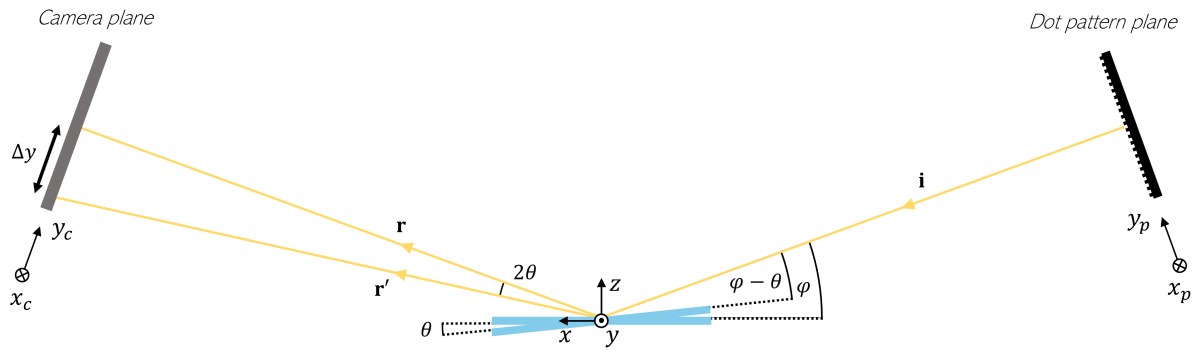


Figure G.7: Schematic representation of ray tracing for a counterclockwise rotation around the y -axis.

An alternative approach is the use of the vector equation presented in Equation G.2. A schematic representation of the light rays reflecting off the tilted and horizontal wafer, including their normal vectors, is shown in Figure G.8

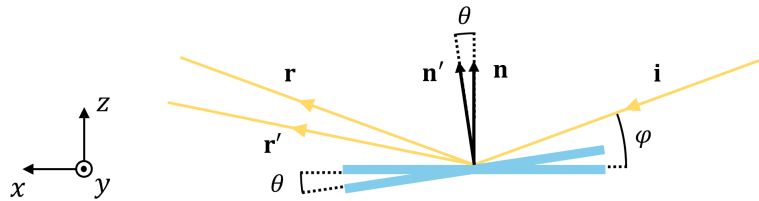


Figure G.8: Schematic representation of the ray reflection for a horizontal and tilted wafer, including their normal vectors.

For the horizontal wafer, the incident ray vector $\mathbf{i}$ and wafer normal vector $\mathbf{n}$ are:

$$\mathbf{i} = \begin{pmatrix} \cos \varphi \\ 0 \\ -\sin \varphi \end{pmatrix} \quad \mathbf{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (\text{G.5})$$

The reflected ray vector $\mathbf{r}$ is computed as:

$$\mathbf{r} = \mathbf{i} - 2(\mathbf{i} \cdot \mathbf{n}) \mathbf{n} = \begin{pmatrix} \cos \varphi \\ 0 \\ -\sin \varphi \end{pmatrix} - 2 \left(\begin{pmatrix} \cos \varphi \\ 0 \\ -\sin \varphi \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right) \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \cos \varphi \\ 0 \\ \sin \varphi \end{pmatrix} \quad (\text{G.6})$$

When the wafer is tilted by an angle θ , the normal vector becomes:

$$\mathbf{n}' = \begin{pmatrix} \sin \theta \\ 0 \\ \cos \theta \end{pmatrix} \quad (\text{G.7})$$

The reflected ray vector of the tilted surface is then calculated as:

$$\begin{aligned} \mathbf{r}' &= \mathbf{i} - 2(\mathbf{i} \cdot \mathbf{n}') \mathbf{n}' = \begin{pmatrix} \cos \varphi \\ 0 \\ -\sin \varphi \end{pmatrix} - 2 \left(\begin{pmatrix} \cos \varphi \\ 0 \\ -\sin \varphi \end{pmatrix} \cdot \begin{pmatrix} \sin \theta \\ 0 \\ \cos \theta \end{pmatrix} \right) \begin{pmatrix} \sin \theta \\ 0 \\ \cos \theta \end{pmatrix} \\ &= \begin{pmatrix} \cos \varphi \cos 2\theta + \sin \varphi \sin 2\theta \\ 0 \\ \sin \varphi \cos 2\theta - \cos \varphi \sin 2\theta \end{pmatrix} \end{aligned} \quad (\text{G.8})$$

The angle α between the two reflected rays is consequently given by:

$$\cos \alpha = \frac{\mathbf{r} \cdot \mathbf{r}'}{|\mathbf{r}| |\mathbf{r}'|} = \frac{\begin{pmatrix} \cos \varphi \\ 0 \\ \sin \varphi \end{pmatrix} \cdot \begin{pmatrix} \cos \varphi \cos 2\theta + \sin \varphi \sin 2\theta \\ 0 \\ \sin \varphi \cos 2\theta - \cos \varphi \sin 2\theta \end{pmatrix}}{\left| \begin{pmatrix} \cos \varphi \\ 0 \\ \sin \varphi \end{pmatrix} \right| \left| \begin{pmatrix} \cos \varphi \cos 2\theta + \sin \varphi \sin 2\theta \\ 0 \\ \sin \varphi \cos 2\theta - \cos \varphi \sin 2\theta \end{pmatrix} \right|} = \cos 2\theta \quad (\text{G.9})$$

Thus, the angle between the two rays is 2θ , confirming the visual trigonometric derivation. Since neither vector has a y -component, this angle is purely around the global y -axis. For this case, the vector calculation proved to be more complex than the visual approach. However, it was employed to evaluate the validity of the approach before applying it to the more complex x -rotation.

Since the two reflected rays are not parallel to each other for the tilted wafer case, this dot shift is not independent of the camera-wafer distance. [Figure G.7](#) reveals that the dot shift Δy in the local camera frame depends on the tilt angle and the camera-wafer distance in the following manner:

$$\Delta y = D_c \tan 2\theta \quad (\text{G.10})$$

Where the camera-wafer distance is defined as:

$$D_c = \sqrt{L_c^2 + H_c^2} \quad (\text{G.11})$$

For a counterclockwise tilt about the y -axis, the dots shift downward in the local camera frame, resulting in the camera capturing dots located higher on the pattern. Due to the inversion caused by the reflective wafer, the camera image shows an upward shift of the dots. Conversely, for a clockwise tilt, the dots appear shifted downward in the camera image.

x - rotation

A counterclockwise rotation of the wafer around the global x -axis by an angle γ causes a shift of the dots in both the x and y directions within the local camera frame. Due to the increased complexity of visual analysis for this rotation, only the vector-based approach was employed.

The incident ray vector, normal vector, and reflected ray vector for the horizontal wafer remain unchanged. However, when the wafer is tilted counterclockwise around the x -axis, the normal vector is modified as follows:

$$\mathbf{n}' = \begin{pmatrix} 0 \\ -\sin \gamma \\ \cos \gamma \end{pmatrix} \quad (\text{G.12})$$

The reflected ray of the tilted surface is calculated as:

$$\begin{aligned} \mathbf{r}' &= \mathbf{i} - 2(\mathbf{i} \cdot \mathbf{n}') \mathbf{n}' = \begin{pmatrix} \cos \varphi \\ 0 \\ -\sin \varphi \end{pmatrix} - 2 \left(\begin{pmatrix} \cos \varphi \\ 0 \\ -\sin \varphi \end{pmatrix} \cdot \begin{pmatrix} 0 \\ -\sin \gamma \\ \cos \gamma \end{pmatrix} \right) \begin{pmatrix} 0 \\ -\sin \gamma \\ \cos \gamma \end{pmatrix} \\ &= \begin{pmatrix} \cos \varphi \\ -\sin \varphi \sin 2\gamma \\ \sin \varphi \cos 2\gamma \end{pmatrix} \end{aligned} \quad (\text{G.13})$$

To determine the dot shifts in the x and y directions separately, the angle between the reflected rays, denoted as β , is decomposed into two components: β_z , representing the rotation around the global z -axis, and β_y , representing the rotation around the global y -axis. In order to do so, the projection of the reflected rays onto the xy - and xz -planes were first determined:

$$\text{proj}_{xy} \mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad \text{proj}_{xy} \mathbf{r}' = \begin{pmatrix} \cos \varphi \\ -\sin \varphi \sin 2\gamma \\ 0 \end{pmatrix} \quad (\text{G.14})$$

$$\text{proj}_{xz} \mathbf{r} = \begin{pmatrix} \cos \varphi \\ 0 \\ \sin \varphi \end{pmatrix} \quad \text{proj}_{xz} \mathbf{r}' = \begin{pmatrix} \cos \varphi \\ 0 \\ \sin \varphi \cos 2\gamma \end{pmatrix} \quad (\text{G.15})$$

The angles of rotation around the z -axis and y -axis, denoted as β_z and β_y , respectively, can then be calculated as follows:

$$\cos \beta_z = \frac{\text{proj}_{xy} \mathbf{r} \cdot \text{proj}_{xy} \mathbf{r}'}{|\text{proj}_{xy} \mathbf{r}| |\text{proj}_{xy} \mathbf{r}'|} = \frac{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \cdot \begin{pmatrix} \cos \varphi \\ -\sin \varphi \sin 2\gamma \\ 0 \end{pmatrix}}{\left| \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right| \left| \begin{pmatrix} \cos \varphi \\ -\sin \varphi \sin 2\gamma \\ 0 \end{pmatrix} \right|} = \frac{\cos \varphi}{\sqrt{\cos^2 \varphi + \sin^2 \varphi \sin^2 2\gamma}} \quad (\text{G.16})$$

$$\cos \beta_y = \frac{\text{proj}_{xz} \mathbf{r} \cdot \text{proj}_{xz} \mathbf{r}'}{|\text{proj}_{xz} \mathbf{r}| |\text{proj}_{xz} \mathbf{r}'|} = \frac{\begin{pmatrix} \cos \varphi \\ 0 \\ \sin \varphi \end{pmatrix} \cdot \begin{pmatrix} \cos \varphi \\ 0 \\ \sin \varphi \cos 2\gamma \end{pmatrix}}{\left| \begin{pmatrix} \cos \varphi \\ 0 \\ \sin \varphi \end{pmatrix} \right| \left| \begin{pmatrix} \cos \varphi \\ 0 \\ \sin \varphi \cos 2\gamma \end{pmatrix} \right|} = \frac{\cos^2 \varphi + \sin^2 \varphi \cos 2\gamma}{\sqrt{\cos^2 \varphi + \sin^2 \varphi \cos^2 2\gamma}} \quad (\text{G.17})$$

Consequently, the dot shifts Δx and Δy in the local camera frame can be determined using the camera-wafer distance:

$$\Delta x = D_c \tan \beta_z = D_c \tan \varphi \sin 2\gamma \quad (\text{G.18})$$

$$\Delta y = D_c \tan \beta_y = D_c \cdot \frac{\sin 2\varphi \sin^2 \gamma}{\cos^2 \varphi + \sin^2 \varphi \cos 2\gamma} \quad (\text{G.19})$$

Both a clockwise and a counterclockwise rotation of the wafer around the global x -axis result in a slight downward shift Δy of the dots in the local camera frame, leaving an upward shift in the camera image. A counterclockwise rotation of the wafer around the global x -axis results in a positive dot shift Δx in the local camera frame, leaving a negative dot shift in the camera image. In contrast, a clockwise rotation results in a positive dot shift in the camera image.

Evaluation

To evaluate the quantitative impact of each wafer motion, their magnitudes were plotted in Figure G.9. Each dot shift was analyzed for a height change ranging from 0 to 50 μm . For θ and γ , an angle of $\sin^{-1}(\Delta h/0.025)$ was used, representing a height change Δh over the 25 mm visible radius of the wafer.

The results indicate that the dot shift caused by pure upward translation ($\Delta y_{t,z}$) is negligible compared to shifts due to rotations about the y and x axes ($\Delta y_{r,y}$ and $\Delta x_{r,x}$, respectively). Additionally, the dot shift from the rotation around the x -axis is primarily in the x -direction of the camera plane, while the y -shift ($\Delta y_{r,x}$) is insignificant.

Based on these findings, the surface translations ($\Delta y_{t,z}$) and y -shifts from x -axis rotation ($\Delta y_{r,x}$) were excluded from the surface reconstruction. The local surface gradient ($\partial h/\partial x, \partial h/\partial y$) for a measured dot shift ($\Delta x, \Delta y$) is derived as follows:

$$\frac{\partial h}{\partial x} = \theta = \frac{1}{2} \tan^{-1} \left(\frac{\Delta y}{D_c} \right) \quad (\text{G.20})$$

$$\frac{\partial h}{\partial y} = \gamma = \frac{1}{2} \sin^{-1} \left(\frac{\Delta x}{D_c \tan \varphi} \right) \quad (\text{G.21})$$

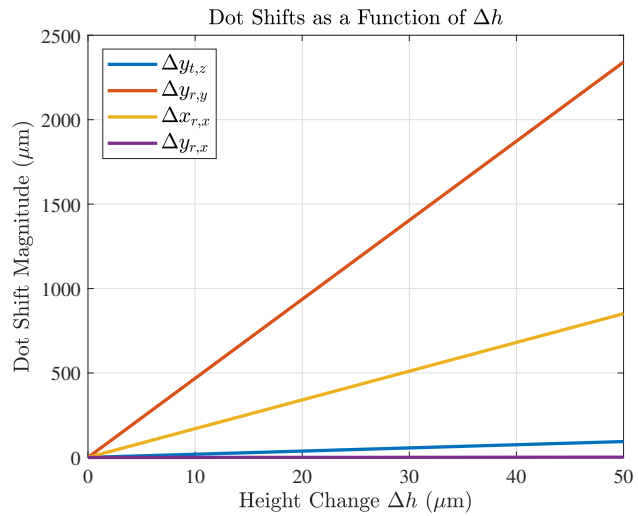


Figure G.9: Magnitude of the dot shifts for each wafer motion, as a function of the height change.

G.1.2 Surface Reconstruction

To find the local surface gradients across the whole surface of the wafer, a vector field is required, containing the local x and y displacements of the dots. To obtain this vector field, the Particle Image Velocimetry (PIV) tool in DaVis image acquisition and processing software was used. The software employs a multigrid interrogation approach, where cross-correlation is first performed on large interrogation windows to capture global displacement patterns. The window size is then gradually reduced over multiple passes to resolve finer displacement details. For this application, a final interrogation window of 64×64 pixels with an overlap of 50% was used, resulting in a displacement measurement every 32 pixels.

The dot pattern was captured using a LaVision Imager sCMOS CLHS camera, offering a resolution of 2560×2160 pixels with a pixel size of $6.5 \mu\text{m}$. Equipped with a 105 mm lens and positioned

according to the distances specified in [Table G.1](#), the camera provides a horizontal field of view (FOV) of approximately 65 mm. Due to the camera's angle, the vertical FOV is significantly larger. Within the 65 mm horizontal FOV, the 50 mm visible wafer region reflects an 85 mm section of the dot pattern.

The setup uses a 100 mm $\times$ 100 mm random dot pattern with non-overlapping dots. The dot diameter D_d was selected to be 0.4 mm, with a dot density of 15 %. Given the imaging conditions, each pixel corresponds to a physical distance of approximately 25.4 μ m on the wafer surface and 43.2 μ m on the dot pattern. Consequently, each dot is represented by approximately 9 pixels in the captured image. Within a 64 $\times$ 64 pixel interrogation window, this results in an estimated dot count of $N_I \approx 7$. Both a white plate with black dots and a black plate with white dots were tested, with the latter providing the highest contrast between the dots and the background.

Along the x -direction of the local camera frame, the 64 $\times$ 64 pixel interrogation window with 50% overlap yields a displacement measurement every 0.81 mm on the wafer surface. Due to the camera's perspective, the 50 mm visible wafer area appears as $50 \cdot \sin 20 = 17.1$ mm along the y -direction of the local camera frame. As a result, displacement measurements are spaced approximately 2.38 mm apart along this axis. This difference in measurement spacing indicates that the spatial resolution varies between the two camera axes.

To reconstruct the wafer's surface height, the vector field was exported to MATLAB for post-processing. First, an elliptical mask was applied to isolate the visible wafer region and eliminate noise outside this area. The masked vector field was then used as input for the *surfheight* function from the *PIVMat* toolbox [72], which was modified to incorporate the surface gradients defined in [Equation G.20](#) and [Equation G.21](#). The function utilizes the *intgrad2* algorithm [73] to reconstruct the surface height by integrating the 2D displacement field using finite difference methods. Additionally, to account for the camera perspective, the spacing between measurements along the y -axis of the local camera frame was scaled by a factor of $1/\sin 20$.

It is important to note that the surface reconstruction does not account for the perspective effect, where objects closer to the camera appear larger in the image than those farther away. This results in slight variations in measurement spacing along the local x -direction of the camera frame, depending on the position in the y -direction. However, these effects are expected to be minimal due to the relatively small diameter of the mirror. For future research, such distortions can be corrected by applying a perspective transformation to the obtained vector field.

G.2 Calibration

Since the output of the *surfheight* function is given in pixels, a conversion factor is required to express the surface height in micrometers. In theory, this conversion factor can be derived by ray tracing. However, this approach is quite complex, as it requires detailed knowledge of the internal optics of the camera. Instead, an experimental calibration procedure was performed by applying a known displacement to a reflective surface and determining the corresponding change in surface height in pixels.

The setup used for calibrating the deformation measurement technique is illustrated in [Figure G.10](#). A 1.2 cm thick mirror with a diameter of 50 mm was used, matching the visible area of the wafer. A flat steel sheet with a thickness of 50 μ m was placed under one side of the mirror, introducing a small tilt θ around the global y -axis.

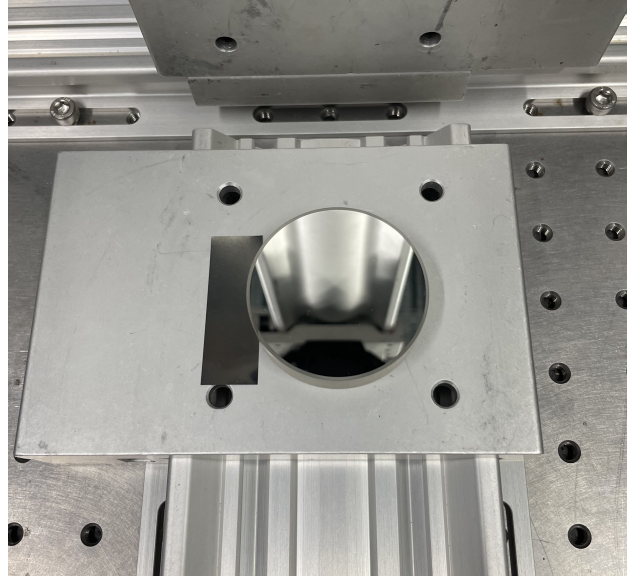


Figure G.10: Setup used to calibrate the deformation measurement technique.

To perform the calibration, a reference image was first captured with the steel sheet placed under the mirror, as shown in [Figure G.11a](#). The sheet was then removed, allowing the mirror to return to its original position, after which a second image was taken, shown in [Figure G.11b](#). The dot displacements between these two images provide the necessary data for determining the conversion factor. It is important to note that in the DaVis software, the y -axis in the camera image is oriented downward. This was properly accounted for during post-processing in MATLAB.

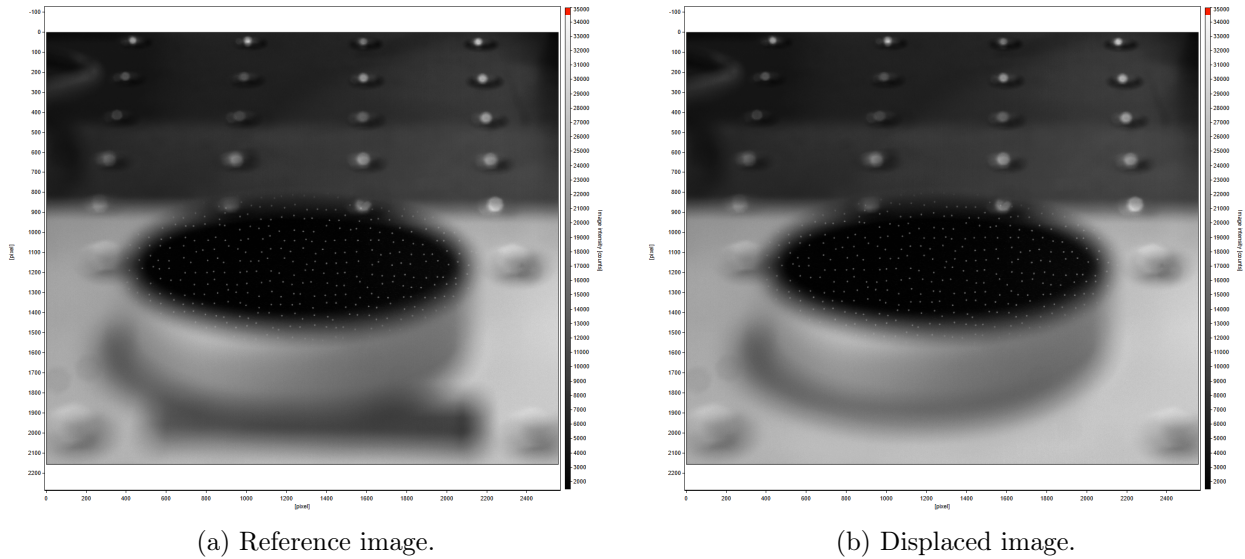


Figure G.11: Reference and displaced images used for calibration.

The two images were subsequently merged and processed in DaVis to generate the vector fields. In MATLAB, an elliptical mask was applied to the raw vector field (depicted by the red ellipse in [Figure G.12a](#)) to extract the relevant region, resulting in the masked vector field shown in [Figure G.12b](#). As can be seen, the vector field shows a uniform upward displacement of approximately 15 pixels, consistent with the expected result for a counterclockwise rotation about the global y -axis.

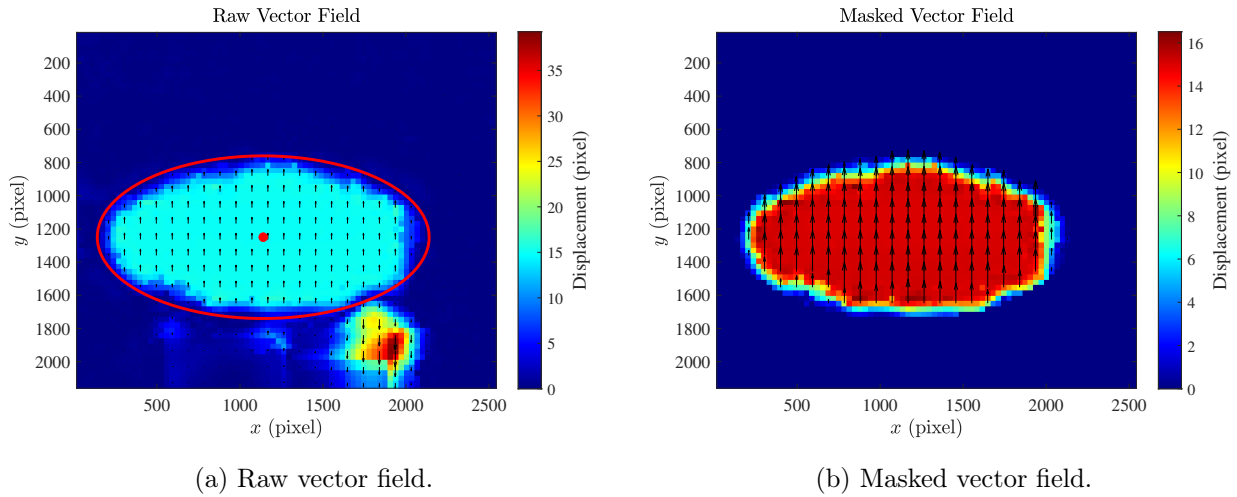


Figure G.12: Vector field of the calibration measurement before and after applying the elliptical mask.

Next, the surface reconstruction algorithm was applied to convert the masked vector field into a surface height map, shown in Figure G.13. The algorithm correctly interprets the upward dot displacement as a counterclockwise rotation around the global y -axis. Additionally, the side view reveals that the tilting effect diminishes near the outer edges of the mirror. This is attributed to image blurring at the mirror's circumference, as observed in Figure G.11, which reduces the local accuracy of the vector field.

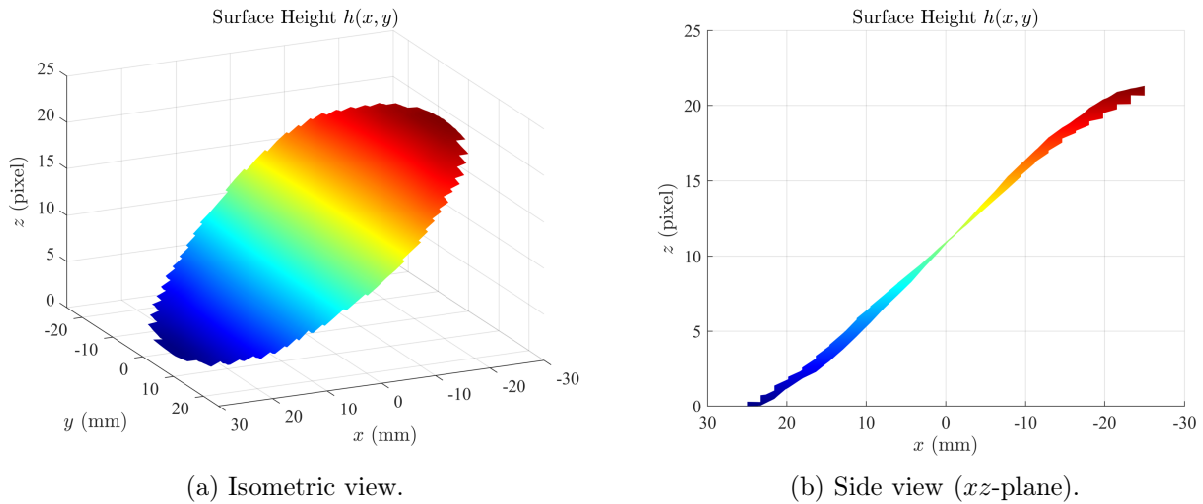


Figure G.13: Reconstructed surface height of the mirror in isometric and side views.

This calibration procedure was repeated three times, yielding an average z -displacement of approximately 22 pixels. This results in the following conversion factor:

$$S = \frac{\Delta z_{\text{real}}}{\Delta z_{\text{pixels}}} = \frac{50 \mu\text{m}}{22 \text{ pixels}} \approx 2.3 \frac{\mu\text{m}}{\text{pixel}} \quad (\text{G.22})$$

G.3 Wafer Tests

In contrast to the mirror used for calibration, the quartz glass wafer is partially reflective and transmits a portion of the incident light. This behaviour not only reduces the intensity of the reflected

light but also introduces additional reflections from the wafer's bottom surface and the top surface of the unit cell. Consequently, the camera image of the reflected dot pattern exhibits multiple reflections for each dot.

Figure G.14a displays the dot pattern reflection captured from the 500 μm wafer. This image was captured in a dark room with the dot pattern front-illuminated using a flashlight to enhance contrast. Compared to images taken in a brightly lit environment, this approach significantly reduces stray light, improving the vector field.

Around the visible area of the wafer a slight blur can be observed, also showing very weak dot reflections. These arise from reflections off the top surface of the constraining ring. However, these reflections do not affect the vector field derivation.

A zoomed-in view of the image is shown in Figure G.14b, highlighting the multiple dot reflections. In the center of the wafer, brighter dots can be observed compared to the outer regions. These correspond to reflections from the unit cell's top surface. Outside of the unit cell region, the air bearing system has a rougher surface finish, reducing its reflectivity and thereby eliminating these additional reflections. Nevertheless, across the entire visible surface of the wafer, two distinct reflections can be identified for each physical dot in the dot pattern.

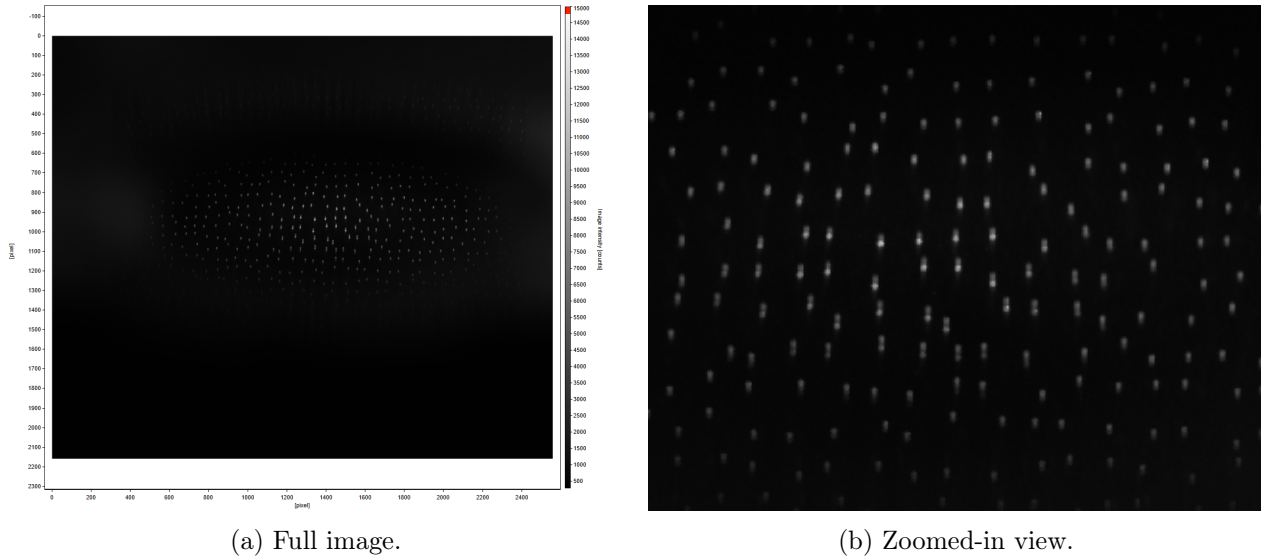


Figure G.14: Dot pattern reflection images for the 500 μm wafer, obtained in a dark room with a front-illuminated dot pattern.

To study the source and impact of each dot reflection, ray tracing was performed. Furthermore, the intensity of refracted and reflected light rays was calculated to determine which rays are captured by the camera. When light passes through the interface of two media with different refractive indices, it bends according to Snell's law [71]:

$$n_1 \sin \theta_i = n_2 \sin \theta_t \quad (\text{G.23})$$

Where n_1 and n_2 are the refractive indices of the respective media before and after refraction. A schematic representation of light refracting at the interface between two media is shown in Figure G.15. When $n_1 < n_2$, the transmitted ray $\mathbf{t}$ bends toward the normal, whereas for $n_1 > n_2$, it bends away from the normal. The refractive indices of air and quartz glass are taken as 1.00 and 1.46, respectively [12].

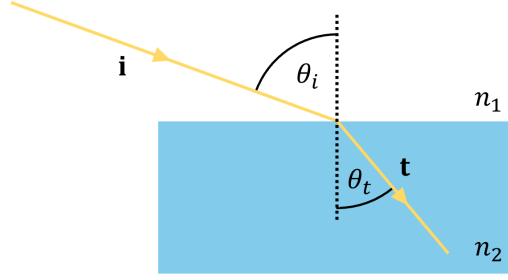


Figure G.15: Schematic representation of light refracting at the interface between two media.

Under the assumption of non-absorbing materials, the reflectance and transmittance satisfy the following relationship [71]:

$$T + R = 1 \quad (\text{G.24})$$

The reflectance of light at an interface depends on its polarization. R_s and R_p denote the reflectance for s -polarized and p -polarized light, respectively, and are given by the Fresnel equations:

$$R_s = \left| \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \right|^2 \quad (\text{G.25})$$

$$R_p = \left| \frac{n_2 \cos \theta_i - n_1 \cos \theta_t}{n_2 \cos \theta_i + n_1 \cos \theta_t} \right|^2 \quad (\text{G.26})$$

For unpolarized light, such as the ambient lighting at the testing location, the total reflectance is given by the average of the two polarization components:

$$R = \frac{R_s + R_p}{2} \quad (\text{G.27})$$

To estimate the intensity of light reflected from the top surface of the wafer, the reflectance components were plotted as a function of the angle of incidence in Figure G.16. As shown, the total reflectance R increases monotonically with the angle of incidence. Consequently, to achieve a high-contrast image of the reflected dot pattern, the camera should ideally be positioned at a shallow angle relative to the wafer surface. However, this comes at the cost of reduced spatial resolution in the local y -direction of the camera frame, as previously discussed.

For the camera-pattern distances listed in Table G.1, the reflectance is approximately 16.3%. The remaining 83.7% is transmitted through the top surface of the wafer.

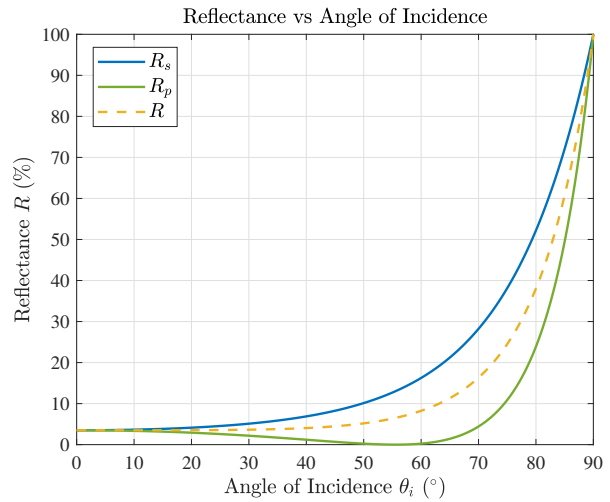


Figure G.16: Reflectance components at the top surface of the quartz glass wafer as a function of the angle of incidence.

A schematic representation of the relevant light reflections and refractions of the wafer and air bearing system is shown in Figure G.17. The intensity of each vector was calculated relative to the incident ray vector, using an angle of incidence $\theta_i = 70^\circ$. The corresponding angles of reflection and transmission are $\theta_r = 70^\circ$ and $\theta_t = 40^\circ$, respectively. The schematic image illustrates the scenario where the air bearing system is fully reflective, such as the top surface of the unit cell, and the wafer is in its undeformed state.

After the initial reflection of the top surface of the wafer, the transmitted portion of the light exhibits another reflection r_2 and transmission t_3 at the bottom surface of the wafer. The majority of the reflected portion is consequently transmitted through the top surface of the wafer again, after which 11.5% of the incident light intensity remains.

The transmitted portion first travels through the thin film, after which it's fully reflected of the top surface of the unit cell. The light ray then travels back through the wafer resulting in the transmitted ray t_5 with an intensity of 49% of the incident intensity. The remaining reflections and refractions do not have a sufficiently high intensity to be captured by the camera.

For an undeformed wafer, the three rays traveling toward the camera (r_1 , t_2 , and t_5) are parallel to each other, leaving a dot shift similar to that calculated for a pure translation of the wafer. However, due to the large thickness of the wafer compared to the film height, the spacing between the reflections of the top and bottom surfaces of the wafer is larger than the spacing between the reflections of the bottom surface of the wafer and the top surface of the unit cell. Theoretically, the spacing between the multiple dot reflections can be used to simultaneously determine the thickness of the wafer and the film height. However, this would require some alterations to the test setup, which are further discussed in [Section 5.2](#).

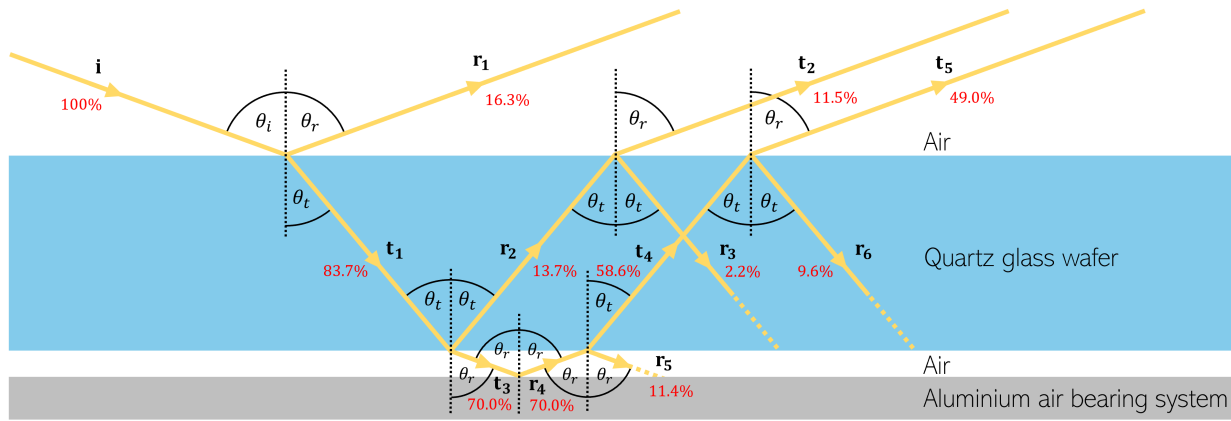


Figure G.17: Schematic representation of an incident light ray interacting with the wafer and air bearing system, including the relevant reflections and refractions.

When the wafer transitions to its deformed state, the light reflected from both the top and bottom surfaces of the wafer undergoes a rotation similar to that shown in [Figure G.7](#). This occurs because these reflections take place on surfaces that are tilted relative to the undeformed state. Due to the large distance between the camera and the dot pattern, this tilt results in a relatively significant dot shift in the camera image.

In contrast, for the portion of light transmitted through the wafer, the outgoing ray t_5 remains parallel to itself in both the deformed and undeformed states. This is because the light ultimately reflects off the horizontal air bearing system, which does not deform. However, it is important to note that the rays t_1 , r_2 , and t_4 inside the wafer are not parallel to themselves between the two states due to changes in the transmission angle. As a result, a dot shift occurs that is directly related to the wafer thickness. In the current test configuration, where the wafer thickness is negligible compared to the camera-pattern distance, this dot shift remains unnoticeable in the camera image.

All in all, only r_1 and t_2 result in a similar significant dot shift when the wafer deforms, while the dot reflections of the top surface of the unit cell (t_5) appear stationary between the two images. In DaVis, only the reflections from the top and bottom surface of the wafer were found to be used for the vector field composition. The resulting vector fields and reconstructed wafer surfaces are discussed in [Section 4.3.1](#). It is important to note that these wafer tests were performed before attaching the thermistors to the surface. After attaching the thermistors, slight blurring of the dots was observed in those regions, locally affecting the vector field. To mitigate this effect, one could employ multiple line integrals across the wafer surface instead of a surface integral.

H

MATLAB Code

H.1 Poiseuille Flow

```
1 clear all
2
3 %*****
4 %% Plane Poisseuille flow (Navier-Stokes)
5 %*****
6
7 syms dp_x mu b u_b y u(y)
8
9 % Specify the governing differential equation
10 Du = diff(u);
11 pde = diff(u,y,2) == dp_x/mu; % Navier-Stokes
12 cond1 = u(-b) == 0; % No-slip condition
13 cond2 = Du(0) == 0; % Symmetry
14
15 % Solve the velocity profile as a function of the pressure gradient
16 conds = [cond1 cond2];
17 uSol(y) = dsolve(pde,conds);
18 u(y) = simplify(uSol);
19
20 % Use the bulk velocity to rewrite the pressure gradient
21 u_b_eq = 1/(2*b) * int(@(y) u(y),-b,b);
22 dp_xeq = isolate(u_b == u_b_eq, dp_x);
23
24 % The velocity profile as a function of the bulk velocity
25 u(y) = rhs(subs(u(y), dp_x, dp_xeq));
26
27 b_n = 7.5e-6; % Half of the film height
28 mu_n = 1.8e-5; % Dynamic viscosity of air
29 dp_x_n = (0.8e5-1.5e5)/(10e-3); % The pressure gradient
30
31 % The velocity profile with numerical values
32 u_bulk = double(subs(u_b_eq, [b,mu,dp_x], [b_n,mu_n,dp_x_n]));
33 u_y = subs(u(y), [u_b, b], [u_bulk, b_n]);
34
35 % Plot the Poiseuille velocity profile over the channel height
36 figure;
37 fp = fplot(u_y, [-b_n b_n]);
38 plot(fp.YData,fp.XData, 'b', 'LineWidth', 2.5) % Swap the x and y axes
39 set(gca, 'FontSize', 12, 'FontName', 'Times');
40 set(gcf, 'Renderer', 'painters');
41 legend('$u(y)$', 'Interpreter', 'latex', 'FontSize', 14);
42 title('Poisseuille Velocity Profile', 'Interpreter', 'latex');
43 ylabel('Channel Height $y$ (m)', 'Interpreter', 'latex');
44 xlabel('Velocity $u$ (m/s)', 'Interpreter', 'latex');
45 ylim([-b_n b_n]);
46 grid on;
```

```

47
48 %*****
49 %% Isothermal Boundaries
50 %*****
51
52 syms T(y) rho cp k Ts1 Ts2 b mu u_b
53
54 % Fully developed flow with isothermal boundaries
55 DT_x = 0;
56
57 % Specify the governing differential equations with boundary conditions
58 pde = k * diff(T, y, 2) + mu * diff(u * diff(u, y), y) == DT_x * rho * cp
    * u;
59 cond1 = T(-b) == Ts1;           % Bottom isothermal boundary
60 cond2 = T(b) == Ts2;           % Top isothermal boundary
61
62 % Solve the differential equation
63 conds = [cond1 cond2];
64 Tsol(y) = dsolve(pde, conds);
65 T(y) = simplify(vpa(Tsol(y)));
66
67 Ts1_n = 293;                    % Bottom isothermal temperature
68 Ts2_n = 293.2;                  % Top isothermal temperature
69 rho_n = 1.22;                   % Density of air
70 cp_n = 1005;                   % Specific heat capacity of air
71 k_n = 26.1e-3;                  % Thermal conductivity of air
72
73 % The temperature profile with numerical values
74 T(y) = subs(T(y), [k, u_b, b, Ts1, Ts2, mu], [k_n, u_bulk, b_n, Ts1_n,
    Ts2_n, mu_n]);
75
76 % Plot the temperature profile
77 figure;
78 fp = fplot(T(y), [-b_n b_n]);
79 plot(fp.YData,fp.XData, 'r', 'Linewidth', 2.5);
80 set(gca, 'FontSize', 12, 'FontName', 'Times');
81 set(gcf, 'Renderer', 'painters');
82 legend('$T(y)$', 'Interpreter', 'latex', 'Location', 'best', 'FontSize',
    14);
83 title('Isothermal Boundaries (Poiseuille)', 'Interpreter', 'latex');
84 ylabel('Channel Height $y$ (m)', 'Interpreter', 'latex');
85 xlabel('Temperature $T$ (K)', 'Interpreter', 'latex');
86 ylim([-b_n b_n])
87 grid on;
88
89 %*****
90 %% Adiabatic Boundaries
91 %*****
92
93 syms df(x) f(x) g(y) rho cp k b x mu u_b Taw
94 % f(x): the adiabatic wall temperature
95 % g(y): the difference with the adiabatic wall temperature
96
97 dg = diff(g);
98 d2g = diff(dg, y);
99
100 % Specify the governing differential equation with a constant f'(x)
101 pde(y) = rho * cp * u * df(x) == k * d2g + mu * diff(u * diff(u, y), y);

```

```

102 cond1 = dg(b) == 0; % The top plate is adiabatic
103 cond2 = g(b) == 0; % Since the top plate is an adiabatic
    boundary, the difference with the adiabatic wall temperature (g(y)) is
    zero at this boundary
104
105 % Solve the differential equation to obtain g(y) as a function of f'(x)
106 conds = [cond1 cond2];
107 gsol(y) = dsolve(pde, conds);
108 g(y) = simplify(vpa(gsol(y)));
109 dg = diff(g);
110
111 % Specify the second differential equation used to solve f(x)
112 pde2(x) = subs(dg, [df(x), y], [diff(f(x)), -b]) == 0; % Zero
    temperature gradient at the bottom plate
113 cond3 = f(0) == Taw; % At x=0, the adiabatic wall temperature
    is equal to the starting condition Taw
114
115 % Solve to find f(x) and g(y)
116 fsol(x) = dsolve(pde2, cond3);
117 f(x) = simplify(vpa(fsol(x))); % f(x) turns out to be a constant->dT/dx=0
118 g(y) = subs(g(y), [df(x)], [diff(f(x))]);
119
120 % The temperature profile is the sum of the adiabatic wall temperature
    function plus the difference with the adiabatic wall temperature
    function
121 T(x,y) = f(x) + g(y);
122
123 Taw_n = 293; % Adiabatic wall temperature
124
125 % The temperature profile with numerical values
126 T(x,y) = subs(T(x,y), [cp, b, k, u_b, mu, rho, Taw], [cp_n, b_n, k_n,
    u_bulk, mu_n, rho_n, Taw_n]);
127
128 % Plot the temperature profile for x = 0
129 figure(1);
130 fp = fplot(T(0,y), [-b_n b_n]);
131 plot(fp.YData,fp.XData, 'r', 'Linewidth', 2.5)
132 set(gca, 'FontSize', 12, 'FontName', 'Times');
133 set(gcf, 'Renderer', 'painters');
134 legend('$T(y)$', 'Interpreter', 'latex', 'Location', 'best', 'FontSize',
    14);
135 title('Adiabatic Boundaries at $x = 0$ (Poiseuille)', 'Interpreter',
    'latex');
136 ylabel('Channel Height $y$ (m)', 'Interpreter', 'latex');
137 xlabel('Temperature $T$ (K)', 'Interpreter', 'latex');
138 ylim([-b_n b_n])
139 grid on;
140
141 % Plot the temperature profile for x > 0
142 figure(2);
143 fp = fplot(T(1,y), [-b_n b_n]);
144 plot(fp.YData,fp.XData, 'r', 'Linewidth', 2.5)
145 set(gca, 'FontSize', 12, 'FontName', 'Times');
146 set(gcf, 'Renderer', 'painters');
147 legend('$T(y)$', 'Interpreter', 'latex', 'Location', 'best', 'FontSize',
    14);
148 title('Adiabatic Boundaries at $x > 0$ (Poiseuille)', 'Interpreter',
    'latex');

```

```

149 ylabel('Channel Height $y$ (m)', 'Interpreter', 'latex');
150 xlabel('Temperature $T$ (K)', 'Interpreter', 'latex');
151 ylim([-b_n b_n])
152 grid on;
153
154 %*****
155 %% Adiabatic and Isothermal Boundary
156 %*****
157
158 syms df(x) f(x) g(y) rho cp k b x mu u_b Ts Taw
159
160 dg = diff(g);
161 d2g = diff(dg, y);
162
163 % Specify the governing differential equation
164 pde = rho * cp * u * df(x) == k * d2g + mu * diff(u * diff(u, y), y);
165 cond1 = dg(-b) == 0; % The lower plate is adiabatic
166 cond2 = g(-b) == 0; % No difference with the adiabatic wall
    temperature at the adiabatic wall
167
168 % Solve for g(y)
169 conds = [cond1 cond2];
170 gsol(y) = dsolve(pde, conds);
171 g(y) = simplify(vpa(gsol(y)));
172
173 % Specify the second differential equation used to solve f(x)
174 pde2 = subs(g(y), [df(x), y], [diff(f(x)), b]) == Ts - f(x); % At y=b
    the temperature difference with y=-b equals the minus of the
    isothermal wall temp and the adiabatic wall temp
175 cond3 = f(0) == Taw; % At x=0, the adiabatic wall temperature
    is equal to the initial condition
176
177 % Solve to find f(x) and g(y)
178 fsol(x) = dsolve(pde2, cond3);
179 f(x) = simplify(vpa(fsol(x))); % Exponential function quickly reaching 0
180 g(y) = simplify(vpa(subs(g(y), [df(x)], [diff(f(x))])));
181 f_x = subs(f(x), [cp, b, k, u_b, rho, Ts, Taw], [cp_n, b_n, k_n, u_bulk,
    rho_n, Ts2_n, Taw_n]);
182
183 % The complete temperature profile
184 T(x,y) = f(x) + g(y);
185
186 % The temperature profile with numerical values
187 T(x,y) = subs(T(x,y), [cp, b, k, u_b, mu, rho, Ts, Taw], [cp_n, b_n, k_n,
    u_bulk, mu_n, rho_n, Ts2_n, Taw_n]);
188
189 % Plot the temperature profile for x = 0
190 figure(1);
191 fp = fplot(T(0,y), [-b_n b_n]);
192 plot(fp.YData,fp.XData, 'r', 'Linewidth', 2.5), hold on;
193 set(gca, 'FontSize', 12, 'FontName', 'Times');
194 set(gcf, 'Renderer', 'painters');
195 legend('$T(y)$', 'Interpreter', 'latex', 'Location', 'best', 'FontSize',
    14);
196 title('Isothermal and Adiabatic Boundary', 'at $x=0$ mm (Poiseuille)',
    'Interpreter', 'latex');
197 ylabel('Channel Height $y$ (m)', 'Interpreter', 'latex');
198 xlabel('Temperature $T$ (K)', 'Interpreter', 'latex');

```

```

199 ylim([-b_n b_n])
200 grid on;
201
202 % Plot the temperature profile for x = 0.5 mm
203 figure(2);
204 fp = fplot(T(0.0005,y), [-b_n b_n]);
205 plot(fp.YData,fp.XData, 'r', 'LineWidth', 2.5), hold on;
206 set(gca, 'FontSize', 12, 'FontName', 'Times');
207 set(gcf, 'Renderer', 'painters');
208 legend('$T(y)$', 'Interpreter', 'latex', 'Location', 'best', 'FontSize',
14);
209 title('Isothermal and Adiabatic Boundary', 'at $x=0.5$ mm (Poiseuille)',
'Interpreter', 'latex');
210 ylabel('Channel Height $y$ (m)', 'Interpreter', 'latex');
211 xlabel('Temperature $T$ (K)', 'Interpreter', 'latex');
212 xlim([293.15 293.21])
213 ylim([-b_n b_n])
214 grid on;
215
216 %*****
217 %% Conclusion
218 %*****
219
220 % Specify the thermal energy equation terms
221 P = u_y * dp_x_n;
222 D = mu_n * diff(u_y, y)^2;
223
224 % Plot the separate and combined functions in one figure
225 figure(1);
226 fp = fplot(P, [-b_n b_n]);
227 plot(fp.YData,fp.XData, 'LineWidth', 2, 'DisplayName',
'$u\frac{dp}{dx}$'); hold on;
228 delete(fp);
229
230 fp = fplot(D, [-b_n b_n]);
231 plot(fp.YData,fp.XData, 'Color', [0.9290 0.6940 0.1250], 'LineWidth', 2,
'DisplayName', '$\mu\left(\frac{\partial u}{\partial y}\right)^2$');
232 delete(fp);
233
234 fp = fplot(P+D, [-b_n b_n]);
235 plot(fp.YData,fp.XData, 'Color', [0.4660 0.6740 0.1880], 'LineWidth', 2,
'DisplayName', '$u\frac{dp}{dx} + \mu\left(\frac{\partial u}{\partial y}\right)^2$');
236 delete(fp)
237
238 set(gca, 'FontSize', 12, 'FontName', 'Times');
239 set(gcf, 'Renderer', 'painters');
240 legend('Interpreter', 'latex', 'FontSize', 14, 'location', 'best');
241 title('Thermal Energy Terms (Poiseuille)', 'Interpreter', 'latex');
242 ylabel('Channel Height $y$ (m)', 'Interpreter', 'latex');
243 xlabel('Volumetric Dissipation Rate (W/m$^3$)', 'Interpreter', 'latex');
244 xlim([-1.25e8 2e8])
245 ylim([-b_n b_n]);
246 grid on;

```

H.2 Couette Flow

```

1 clear all
2
3 %*****
4 %% Couette flow (Navier-Stokes)
5 %*****
6
7 syms u(y) b v mu
8
9 % Specify the governing differential equation
10 pde = mu * diff(u, y, 2) == 0; % Navier-Stokes
11 cond1 = u(b) == v; % No-slip condition for a moving boundary
12 cond2 = u(-b) == 0; % No-slip condition
13
14 % Solve the symbolic velocity profile
15 conds = [cond1 cond2];
16 uSol(y) = dsolve(pde, conds);
17 u(y) = simplify(uSol(y));
18
19 b_n = 7.5e-6; % Half of the film height
20 v_n = 1; % Velocity of the top boundary
21
22 % The velocity profile with numerical values
23 u_y = subs(u(y), [v, b], [v_n, b_n]);
24
25 % Plot the Couette velocity profile over the channel height
26 figure;
27 fp = fplot(u_y, [-b_n b_n]);
28 plot(fp.YData,fp.XData, 'b', 'LineWidth', 2.5)
29 set(gca, 'FontSize', 12, 'FontName', 'Times');
30 set(gcf, 'Renderer', 'painters');
31 legend('$u(y)$', 'Interpreter', 'latex', 'Location', 'best', 'FontSize',
14);
32 title('Couette Velocity Profile', 'Interpreter', 'latex');
33 ylabel('Channel Height $y$ (m)', 'Interpreter', 'latex');
34 xlabel('Velocity $u$ (m/s)', 'Interpreter', 'latex');
35 ylim([-b_n b_n]);
36 grid on;
37
38 %*****
39 %% Isothermal Boundaries
40 %*****
41
42 syms T(y) rho cp k Ts1 Ts2 b mu
43
44 % Fully developed flow with isothermal boundaries
45 DT_x = 0;
46
47 % Specify the governing differential equation
48 pde = k * diff(T, y, 2) + mu * diff(u * diff(u, y), y) == DT_x * rho * cp
* u;
49 cond1 = T(-b) == Ts1; % Bottom isothermal boundary
50 cond2 = T(b) == Ts2; % Top isothermal boundary
51
52 % Solve the differential equation
53 conds = [cond1 cond2];
54 Tsol(y) = dsolve(pde, conds);

```



```

55 T(y) = simplify(vpa(Tsol(y)));
56
57 mu_n = 1.8e-5; % Dynamic viscosity of air
58 Ts1_n = 293; % Bottom isothermal temperature
59 Ts2_n = 293.0001; % Top isothermal temperature
60 rho_n = 1.22; % Density of air
61 cp_n = 1005; % Specific heat capacity of air
62 k_n = 26.1e-3; % Thermal conductivity of air
63
64 % The temperature profile with numerical values
65 T(y) = subs(T(y), [k, b, Ts1, Ts2, mu, v], [k_n, b_n, Ts1_n, Ts2_n, mu_n,
    v_n]);
66
67 % Plot the temperature profile
68 figure;
69 fp = fplot(T(y), [-b_n b_n]);
70 plot(fp.YData, fp.XData, 'r', 'LineWidth', 2.5)
71 set(gca, 'FontSize', 12, 'FontName', 'Times');
72 set(gcf, 'Renderer', 'painters');
73 legend('$T(y)$', 'Interpreter', 'latex', 'Location', 'best', 'FontSize',
    14);
74 title('Isothermal Boundaries (Couette)', 'Interpreter', 'latex');
75 ylabel('Channel Height $y$ (m)', 'Interpreter', 'latex');
76 xlabel('Temperature $T$ (K)', 'Interpreter', 'latex');
77 ylim([-b_n b_n])
78 grid on;
79
80 %*****
81 %% Adiabatic Boundaries
82 %*****
83
84 syms df(x) f(x) g(y) rho cp k b x mu Taw
85 % f(x): the adiabatic wall temperature
86 % g(y): the difference with the adiabatic wall temperature
87
88 dg = diff(g);
89 d2g = diff(dg, y);
90
91 % Specify the governing differential equation with a constant f'(x)
92 pde(y) = rho * cp * u * df(x) == k * d2g + mu * diff(u * diff(u, y), y);
93 cond1 = dg(-b) == 0; % The top plate is adiabatic
94 cond2 = g(-b) == 0; % Since the top plate is the adiabatic
    boundary, the difference with the adiabatic wall temperature (g(y)) is
    zero at this boundary
95
96 % Solve the differential equation to obtain g(y) as a function of f'(x)
97 conds = [cond1 cond2];
98 gsol(y) = dsolve(pde, conds);
99 g(y) = simplify(gsol(y));
100 dg = diff(g);
101
102 % Specify the second differential equation used to solve f(x)
103 pde2(x) = subs(dg, [df(x), y], [diff(f(x)), b]) == 0; % Zero
    temperature gradient at the bottom plate
104 cond3 = f(0) == Taw; % At x=0, the adiabatic wall temperature
    is equal to the initial condition Taw
105
106 % Solve to find f(x) and g(y)

```

```

107 fsol(x) = dsolve(pde2, cond3);
108 f(x) = simplify(vpa(fsol(x)));
109 g(y) = subs(g(y), [df(x)], [diff(f(x))]);
110
111 % The temperature profile is the sum of the adiabatic wall temperature
    function plus the difference with the adiabatic wall temperature
    function
112 T(x,y) = f(x) + g(y);
113
114 Taw_n = 293; % Adiabatic wall temperature
115
116 % The temperature profile with numerical values
117 T(x,y) = subs(T(x,y), [cp, b, k, mu, rho, Taw, v], [cp_n, b_n, k_n, mu_n,
    rho_n, Taw_n, v_n]);
118
119 % Plot the temperature profile for x = 0
120 figure(1);
121 fp = fplot(T(0,y), [-b_n b_n]);
122 plot(fp.YData,fp.XData, 'r', 'LineWidth', 2.5)
123 set(gca, 'FontSize', 12, 'FontName', 'Times');
124 set(gcf, 'Renderer', 'painters');
125 legend('$T(y)$', 'Interpreter', 'latex', 'Location', 'best', 'FontSize',
    14);
126 title('Adiabatic Boundaries at $x = 0$ mm (Couette)', 'Interpreter',
    'latex');
127 ylabel('Channel Height $y$ (m)', 'Interpreter', 'latex');
128 xlabel('Temperature $T$ (K)', 'Interpreter', 'latex');
129 ylim([-b_n b_n])
130 grid on; hold on;
131
132 % Plot the temperature profile for x = 1 mm
133 figure(2);
134 fp = fplot(T(0.001,y), [-b_n b_n]);
135 plot(fp.YData,fp.XData, 'r', 'LineWidth', 2.5)
136 set(gca, 'FontSize', 12, 'FontName', 'Times');
137 set(gcf, 'Renderer', 'painters');
138 legend('$T(y)$', 'Interpreter', 'latex', 'Location', 'best', 'FontSize',
    14);
139 title('Adiabatic Boundaries at $x = 1$ mm (Couette)', 'Interpreter',
    'latex');
140 ylabel('Channel Height $y$ (m)', 'Interpreter', 'latex');
141 xlabel('Temperature $T$ (K)', 'Interpreter', 'latex');
142 ylim([-b_n b_n])
143 grid on;
144
145 %*****
146 %% Adiabatic and Isothermal Boundary
147 %*****
148
149 syms df(x) f(x) g(y) rho cp k b x mu Ts Taw
150
151 dg = diff(g);
152 d2g = diff(dg, y);
153
154 % Specify the governing differential equation
155 pde = rho * cp * u * df(x) == k * d2g + mu * diff(u * diff(u, y), y);
156 cond1 = dg(-b) == 0; % The lower plate is adiabatic
157 cond2 = g(-b) == 0; % No difference with the adiabatic wall

```

```

    temperature at the adiabatic wall
158
159 % Solve for g(y)
160 conds = [cond1 cond2];
161 gsol(y) = dsolve(pde, conds);
162 g(y) = simplify(vpa(gsol(y)));
163
164 % Specify the second differential equation used to solve f(x)
165 pde2 = subs(g(y), [df(x), y], [diff(f(x)), b]) == Ts - f(x); % At y=b
    the temperature difference with y=-b equals the minus of the
    isothermal wall temp and the adiabatic wall temp
166 cond3 = f(0) == Taw; % At x=0, the adiabatic wall temperature
    is equal to the initial condition
167
168 Ts_n = 293.0001;
169
170 % Solve to find f(x) and g(y)
171 fsol(x) = dsolve(pde2, cond3);
172 f(x) = simplify(vpa(fsol(x)));
173 g(y) = subs(g(y), [df(x)], [diff(f(x))]);
174 f_x = subs(f(x), [cp, b, k, rho, Ts, Taw, v, mu], [cp_n, b_n, k_n, rho_n,
    Ts_n, Taw_n, v_n, mu_n]);
175
176 % The complete temperature profile
177 T(x,y) = f(x) + g(y);
178
179 % The temperature profile with numerical values
180 T(x,y) = simplify(subs(T(x,y), [cp, b, k, mu, rho, Ts, Taw, v], [cp_n,
    b_n, k_n, mu_n, rho_n, Ts_n, Taw_n, v_n]));
181
182 % Plot the temperature profile for x = 0
183 figure(1);
184 fp = fplot(T(0,y), [-b_n b_n]);
185 plot(fp.YData,fp.XData, 'r', 'LineWidth', 2.5)
186 set(gca, 'FontSize', 12, 'FontName', 'Times');
187 set(gcf, 'Renderer', 'painters');
188 legend('$T(y)$', 'Interpreter', 'latex', 'Location', 'best', 'FontSize',
    14);
189 title('Isothermal and Adiabatic Boundary', 'at $x=0$ mm (Couette)',
    'Interpreter', 'latex');
190 ylabel('Channel Height $y$ (m)', 'Interpreter', 'latex');
191 xlabel('Temperature $T$ (K)', 'Interpreter', 'latex');
192 ylim([-b_n b_n])
193 grid on;
194
195 % Plot the temperature profile for x = 0.5 mm
196 figure(2);
197 fp = fplot(T(0.0005,y), [-b_n b_n]);
198 plot(fp.YData,fp.XData, 'r', 'LineWidth', 2.5)
199 set(gca, 'FontSize', 12, 'FontName', 'Times');
200 legend('$T(y)$', 'Interpreter', 'latex', 'Location', 'best', 'FontSize',
    14);
201 title('Isothermal and Adiabatic Boundary', 'at $x=0.5$ mm (Couette)',
    'Interpreter', 'latex');
202 ylabel('Channel Height $y$ (m)', 'Interpreter', 'latex');
203 xlabel('Temperature $T$ (K)', 'Interpreter', 'latex');
204 ylim([-b_n b_n])
205 grid on;

```

H.3 Axisymmetric Analysis

```

1 clear all
2
3 %*****
4 %% Creeping Flow
5 %*****
6
7 syms n rho v_creep(r, n) r u(n) C
8
9 r1 = 0.0005;           % Inlet radius
10 r2 = 0.007;           % Outlet radius
11 p1 = 1.5e5;           % Inlet pressure
12 p2 = 0.8e5;           % Outlet pressure
13 mu = 1.8e-5;          % Dynamic viscosity of air
14 b = 7.5e-6;           % Half of the film height
15 R = 287;              % Ideal gas constant
16 T = 293;              % Reference temperature
17 rho1 = p1 / (R * T);  % Density of air at p1 and 293K
18 rho2 = p2 / (R * T);  % Density of air at p2 and 293K
19
20 % Specify and solve the ODE to find the normalized velocity profile
21 ode = diff(u,n,2) == b^2 * R * T * int(rho,rho,[rho1, rho2]) / (mu *
    int(1/r,r,[r1,r2]));
22 cond1 = u(1) == 0;
23 cond2 = u(-1) == 0;
24
25 conds = [cond1 cond2];
26 u(n) = simplify(dsolve(ode, conds));
27
28 % Specify and solve the ODE without boundaries to find the density
    function
29 eq = int(rho,rho) == mu * diff(u,n,2) * int(1/r,r) / (b^2 * R * T) + C;
30 sol = solve(eq,rho);
31 rho(r) = sol(2);           % Only the second solution is correct
32 C_n = solve(rho(r1) == rho1, C); % Find the integration constant
33
34 rho(r) = subs(sol(2), C, C_n); % Fill in the integration constant
35
36 % The velocity profile is inversely proportional to the radius and density
37 v_creep(r,n) = u(n) / (rho(r) * r);
38
39 % Plot the creep solution for the density profile
40 set(gcf,'Renderer','painters')
41 figure(1);
42 fplot(rho, [r1 r2], 'LineWidth', 1.5, 'Color', [0.9290 0.6940 0.1250],
    'DisplayName', 'Creep');
43 set(gca, 'FontSize', 12, 'FontName', 'Times');
44 title('Radial Density Profile Comparison', 'Interpreter', 'latex');
45 xlabel('Radius $r$ (m)', 'Interpreter', 'latex');
46 ylabel('Air Density $\rho$ (kg/m$^3$)', 'Interpreter', 'latex');
47 legend('Interpreter', 'latex', 'FontSize', 13)
48 grid on;
49
50 % Plot the creep solution for the pressure profile
51 figure(2);
52 fplot(rho*R*T, [r1 r2], 'LineWidth', 1.5, 'Color', [0.8500 0.3250
    0.0980], 'DisplayName', 'Creep');

```

```

53 set(gca, 'FontSize', 12, 'FontName', 'Times');
54 title('Radial Pressure Profile Comparison', 'Interpreter', 'latex');
55 xlabel('Radius $r$ (m)', 'Interpreter', 'latex');
56 ylabel('Absolute Air Pressure $p$ (Pa)', 'Interpreter', 'latex');
57 legend('Interpreter', 'latex', 'Fontsize', 13)
58 grid on;
59
60 % Plot the creep solution for the velocity profile at the inlet
61 figure(3);
62 creep = fplot(v_creep(r1, n), [-1 1]);
63 plot(creep.YData, creep.XData, 'LineWidth', 1.5)
64 set(gca, 'FontSize', 12, 'FontName', 'Times');
65 title('Radial Velocity Profile (Creeping Flow)', 'Interpreter', 'latex');
66 xlabel('Velocity $V_r$ (m/s)', 'Interpreter', 'latex');
67 ylabel('Normalized Channel Height $z/b$ (-)', 'Interpreter', 'latex');
68 legend('Creeping Flow $V_r(r_1, n)$', 'Interpreter', 'latex', 'Fontsize',
        13)
69 grid on;
70
71 %*****
72 %% Perturbation Method
73 %*****
74
75 syms A P n e f f0 f1 f2 ddf ddf0 ddf1 ddf2 u(n) v_pert(r, n)
76
77 % Perturbed ode
78 ode = ddf + e * f^2 * A + P;
79
80 % Fill in the second-order approximation and its second-order derivative
81 f_e = f0 + e * f1 + e^2 * f2;
82 ddf_sub = ddf0 + e * ddf1 + e^2 * ddf2;
83 sub_ode = simplify(subs(ode, [ddf, f], [ddf_sub, f_e]));
84
85 % Separate the equation base on like powers of e
86 eq = collect(sub_ode, e);
87 [coeffs_eq, terms_eq] = coeffs(eq, e);
88
89 % Isolate the terms with no e power and solve the ODE
90 e0 = subs(coeffs_eq(end) == 0, ddf0, diff(u,n,2));
91 cond1 = u(1) == 0;
92 cond2 = u(-1) == 0;
93 conds = [cond1 cond2];
94 sole0(n) = dsolve(e0, conds);
95
96 % Isolate the terms with the first e power and solve the ODE
97 e1 = subs(coeffs_eq(end-1) == 0, [ddf1 f0], [diff(u,n,2) sole0]);
98 cond1 = u(1) == 0;
99 cond2 = u(-1) == 0;
100 conds = [cond1 cond2];
101 sole1(n) = dsolve(e1, conds);
102
103 % Isolate the terms with the second e power and solve the ODE
104 e2 = subs(coeffs_eq(end-2) == 0, [ddf2 f0 f1], [diff(u,n,2) sole0 sole1]);
105 cond1 = u(1) == 0;
106 cond2 = u(-1) == 0;
107 conds = [cond1 cond2];
108 sole2(n) = dsolve(e2, conds);
109

```

```

110 % Add the three solution together to obtain the final symbolic solution
111 solu2(n) = sole0 + e * sole1 + e^2 * sole2;
112
113 r1 = 0.0005;
114 r2 = 0.007;
115 p1 = 1.5e5;
116 p2 = 0.8e5;
117 mu = 1.8e-5;
118 b = 7.5e-6;
119
120 % Perturbation parameter values
121 e_values = linspace(0, 1, 5);
122
123 % Estimate the required integrals of the creeping flow density function
124 C = 1/(rho*r);
125 int1 = int(C, r1, r2);
126 D = (rho+diff(rho,r)*r)/(rho^2*r^3);
127 int2 = int(D, r1, r2);
128
129 % Specify the constants A and P
130 A_n = b^2 * int2 / (mu * int1);
131 P_n = b^2 * (p1 - p2) / (mu * int1);
132
133 % Plot the radial velocity profile for several perturbation parameters
134 figure;
135 set(gca, 'FontSize', 12, 'FontName', 'Times');
136 hold on;
137 for e_n = e_values
138     u = vpa(subs(solu2, [A, P, e], [A_n, P_n, e_n]));
139
140     % The velocity profile is inversely proportional to the radius and
141     % density
142     v_pert(r, n) = u(n) / (rho(r) * r);
143
144     % Plot the velocity profile at the inlet
145     fp = fplot(v_pert(r1, n), [-1 1]);
146     plot(fp.YData, fp.XData, 'DisplayName', sprintf('$e = %.2f$', e_n));
147     delete(fp);
148 end
149 hold off;
150 title('Radial Velocity Profile (Perturbation)', 'Interpreter', 'latex');
151 xlabel('Velocity $V_r$ (m/s)', 'Interpreter', 'latex');
152 ylabel('Normalized Channel Height $z/b$ (-)', 'Interpreter', 'latex');
153 legend('Interpreter', 'latex', 'FontSize', 12)
154 grid on;
155
156 %*****
157 %% Numerical Solution
158 %*****
159
160 r1 = 0.0005;
161 r2 = 0.007;
162 p1 = 1.5e5;
163 p2 = 0.8e5;
164 mu = 1.8e-5;
165 b = 7.5e-6;
166

```

```

167 % Estimate the required integrals of the creeping flow density function
168 C = 1/(rho*r);
169 int1 = int(C, r1, r2);
170 D = (rho+diff(rho,r)*r)/(rho^2*r^3);
171 int2 = int(D, r1, r2);
172
173 % Specify the constants A and P
174 A_n = double(vpa(int2 / (mu * int1)));
175 P_n = double(vpa((p1 - p2) / (mu * int1)));
176
177 % Find the initial condition for which the numerical solution yields a
    symmetric velocity profile
178 y0_values = linspace(15450,15550,1000);
179
180 for y0 = y0_values
181     options = odeset('RelTol', 1e-6, 'AbsTol', 1e-6, 'MaxStep', 1e-7);
182     [z, y] = ode45(@(z, y) func(z, y, A_n, P_n), [-b b], [0 y0], options);
183     if y(end,1) >= -5e-7 && y(end,1) <= 5e-7
184         y(end,1)
185         y0
186         break
187     end
188 end
189
190 % The velocity profile is inversely proportional to the radius and density
191 v_num(r) = y(:,1) / (r * rho(r));
192
193 % The centerline velocity
194 v_num_center(r) = max(y(:,1)) / (r * rho(r));
195
196 % Plot the results of the three methods in this script in one plot at r1
197 figure(1);
198 plot(v_num(r1),z/b, 'DisplayName', 'Numerical', 'LineWidth', 1.5); hold
    on;
199
200 fp = fplot(v_pert(r1, n), [-1 1]); hold on;
201 plot(fp.YData, fp.XData, 'DisplayName', 'Perturbation', 'LineWidth', 1.5,
    'LineStyle', '--', 'Color', [0.9290 0.6940 0.1250]);
202 delete(fp);
203
204 creep = fplot(v_creep(r1, n), [-1 1]);
205 plot(creep.YData, creep.XData, 'DisplayName', 'Creep', 'LineWidth', 1.5,
    'Color', [0.4660 0.6740 0.1880]);
206 set(gca, 'FontSize', 12, 'FontName', 'Times');
207 set(gcf, 'Renderer', 'painters')
208 delete(creep);
209
210 title('Radial Velocity at $r_1$', 'Interpreter', 'latex');
211 xlabel('Radial Velocity $V_r$ (m/s)', 'Interpreter', 'latex');
212 ylabel('Normalized Channel Height $z/b$ (-)', 'Interpreter', 'latex');
213 legend('Interpreter', 'latex', 'FontSize', 13);
214 xlim([0 70]);
215 legend;
216 grid on;
217
218 % Plot centerline velocity over the whole domain for the three methods
219 figure(2);
220 fplot(v_num_center(r),[r1 r2], 'DisplayName', 'Numerical', 'LineWidth',

```



```

1.5); hold on;
221
222 fplot(v_pert(r, 0), [r1 r2], 'DisplayName', 'Perturbation', 'LineWidth',
    1.5, 'LineStyle', '--', 'Color', [0.9290 0.6940 0.1250]); hold on;
223
224 fplot(v_creep(r, 0), [r1 r2], 'DisplayName', 'Creep', 'LineWidth', 1.5,
    'Color', [0.4660 0.6740 0.1880]);
225
226 set(gca, 'FontSize', 12, 'FontName', 'Times');
227 set(gcf, 'Renderer', 'painters')
228 title('Centerline Radial Velocity', 'Interpreter', 'latex');
229 xlabel('Radius $r$ (m)', 'Interpreter', 'latex');
230 ylabel('Centerline Velocity $V_{center}$ (m/s)', 'Interpreter', 'latex');
231 legend('Interpreter', 'latex', 'FontSize', 13);
232 legend;
233 grid on;
234
235 %*****
236 %% Temperature Profiles - Isothermal Boundaries
237 %*****
238
239 syms z r
240
241 Ts1 = 293; % Bottom boundary isothermal temperature
242 Ts2 = 293; % Top boundary isothermal temperature
243 Tair = 293; % Inlet air temperature
244 r_sub = r2; % Outer radius of the simulated domain
245 k = 26.1e-3; % Thermal conductivity of air
246 cp = 1005; % Specific heat of air
247
248 % Use the perturbation solution to determine the temperature profiles
249 V_r(r,z) = subs(v_pert, n, z/b);
250
251 % Compute the required derivatives of the velocity profile
252 dV_rdz = diff(V_r, z);
253 d2V_rdz2 = diff(dV_rdz, z);
254 dV_rdr = diff(V_r, r);
255
256 % Determine the pressure derivative term
257 dp_dr(r,z) = mu * d2V_rdz2(r,z) - rho(r) * V_r(r,z) * dV_rdr(r,z);
258
259 % Define domain and mesh grid
260 Nz = 11; % Number of grid points in z
261 z = linspace(-b, b, Nz); % Discretized z coordinates
262 dz = z(2) - z(1); % Grid spacing in z
263
264 Nr = 30000 % Number of grid points in r
265 r = linspace(r1, r_sub, Nr); % Discretized r coordinates
266 dr = r(2) - r(1); % Grid spacing in r
267
268 % Track the computation time of the matrix formation
269 t_m0 = tic;
270
271 % Store the velocity, pressure derivatives, and velocity derivatives
    accros the domain in matrices by making use of their symmetry around z
    = 0
272 [R_half, Z_half] = meshgrid(r, z(z <= 0));
273 V_matrix_half = double(V_r(R_half, Z_half)).';

```

```

274 dp_matrix_half = double(dp_dr(R_half, Z_half)).';
275 dVdz_matrix_half = double(dV_rdz(R_half, Z_half)).';
276
277 % Reflect matrices to the other half of the domain to obtain the full
    matrices
278 V_matrix = [V_matrix_half, fliplr(V_matrix_half(:, 1:end-1))];
279 dp_matrix = [dp_matrix_half, fliplr(dp_matrix_half(:, 1:end-1))];
280 dVdz_matrix = [dVdz_matrix_half, -fliplr(dVdz_matrix_half(:, 1:end-1))];
281 rho_matrix = double(vpa(rho(r)));
282
283 % Display the computation time of the matrix formation
284 t_m = toc(t_m0);
285 disp(['Matrix Formation Time: ', num2str(t_m), ' s']);
286
287 % Initialize temperature matrices
288 T = zeros(Nr, Nz);
289 d2T_dz2 = zeros(Nr, Nz-2);
290
291 % Inlet condition
292 T(1, :) = Tair;
293
294 % Boundary conditions
295 T(:, end) = Ts2;
296 T(:, 1) = Ts1;
297
298 % Store the old temperature matrix to the check the boundary and inlet
    conditions
299 T_old = T;
300
301 % Track the computation time of the finite difference algorithm
302 t_fd0 = tic;
303
304 % Finite difference approximation for the thermal energy equation
305 for i = 1:Nr-1
306     % Compute the vectorized derivative approximation
307     d2T_dz2(i,:) = (T(i, 3:Nz) - 2 * T(i, 2:Nz-1) + T(i, 1:Nz-2)) / dz^2;
308
309     % Update temperature using the vectorized finite difference equation
310     T(i+1, 2:Nz-1) = T(i, 2:Nz-1) + dr .* (V_matrix(i, 2:Nz-1) .*
        dp_matrix(i, 2:Nz-1) + k .* d2T_dz2(i,:) + mu .*
        (dVdz_matrix(i, 2:Nz-1)).^2) ./ (rho_matrix(i) * cp .*
        V_matrix(i, 2:Nz-1));
311 end
312
313 % Display the computation time of the finite difference algorithm
314 t_fd = toc(t_fd0);
315 disp(['Finite Difference Computation Time: ', num2str(t_fd), ' s']);
316
317 % Convert the results to Cartesian coordinates for the surface plot
318 n_theta = 100;
319 theta = linspace(0, 2*pi, n_theta);
320 x_r = r .* cos(theta. ');
321 y_r = r .* sin(theta. ');
322 T0 = T(:, round((Nz+1)/2)); % Centerline temperature
323 T_r = repmat(T0.', [n_theta, 1]);
324
325 % 3D surface plot of the temperature in the center of the channel as a
    function of the radius

```

```

326 figure(1);
327 surf(x_r, y_r, T_r, 'EdgeColor', 'none');
328 colormap(jet(256));
329 set(gca, 'FontSize', 12, 'FontName', 'Times');
330 title('Centerline Temperature for Isothermal Boundaries', 'Interpreter',
      'latex');
331 xlabel('$x$ (m)', 'Interpreter', 'latex');
332 ylabel('$y$ (m)', 'Interpreter', 'latex');
333 zlabel('$T_{center}$ (K)', 'Interpreter', 'latex');
334 xlim([-r_sub r_sub]);
335 ylim([-r_sub r_sub]);
336 xticks([-0.005 0 0.005])
337 xticklabels({'-0.005', '0', '0.005'})
338 yticks([-0.005 0 0.005])
339 yticklabels({'-0.005', '0', '0.005'})
340
341 % T(z) for several radii in the domain before Tmin
342 figure(2);
343 set(gca, 'FontSize', 12, 'FontName', 'Times');
344 hold on;
345 [min_temperature, min_index] = min(T(:,(Nz+1)/2));
346 nrc = 5;
347 rc = linspace(1, round(min_index), nrc);
348 for i = 1:nrc
349     plot(T(round(rc(i)),:), z, 'Linewidth', 2, 'DisplayName', sprintf('$r$
      = %.2f$ mm', 1000*r(round(rc(i))))))
350 end
351 hold off;
352 title('$T(z)$ for Isothermal Boundaries', 'Interpreter', 'latex');
353 xlabel('Temperature $T$ (K)', 'Interpreter', 'latex');
354 ylabel('Channel Height $z$ (m)', 'Interpreter', 'latex');
355 xlim([min(T(round(rc(end)),:)) - 0.05, max(T(round(rc(2)),:)) + 0.05])
356 ylim([-b b]);
357 legend('Interpreter', 'latex', 'FontSize', 11.5, 'Location', 'best');
358 grid on;
359 box on;
360
361 % T(z) for several radii in the domain after Tmin
362 figure(3);
363 set(gca, 'FontSize', 12, 'FontName', 'Times');
364 hold on;
365 nrc = 6;
366 rc = linspace(min_index, Nr, nrc);
367 for i = 1 : nrc-1
368     plot(T(round(rc(i+1)),:), z, 'Linewidth', 2, 'DisplayName',
      sprintf('$r$ = %.2f$ mm', 1000*r(round(rc(i+1))))))
369 end
370 hold off;
371 title('$T(z)$ for Isothermal Boundaries', 'Interpreter', 'latex');
372 xlabel('Temperature $T$ (K)', 'Interpreter', 'latex');
373 ylabel('Channel Height $z$ (m)', 'Interpreter', 'latex');
374 xlim([min(T(round(rc(2)),:)) - 0.05, max(T(round(end),:)) + 0.05])
375 ylim([-b b]);
376 legend('Interpreter', 'latex', 'FontSize', 11.5, 'Location', 'best');
377 legend;
378 grid on;
379 box on;
380

```

```

381 %*****
382 %% Conclusion
383 %*****
384
385 syms z r
386
387 % Use the perturbation solution to estimate the energy dissipation
    'constant'
388 V_r(r,z) = subs(v_pert, n, z/b);
389 f(z) = r * rho(r) * V_r(r,z);
390
391 C = int(f^3, -b, b);
392
393 % The dissipation is inversely proportional to the fourth power of the
    radius
394 D(r) = C * (rho(r) + diff(rho(r),r) * r) / (rho(r)^3 * r^4);
395
396 % Determine and display the total dissipation in the axisymmetric domain
397 D_r = 2 * pi * int(D(r) * r, r1, r2);
398 D_mW = double(1000 * vpa(D_r));
399 disp(['Total Dissipation: ', num2str(D_mW), ' mW']);
400
401 % Convert the local dissipation function to Cartesian coordinates
402 n_r = 5000;
403 n_theta = 100;
404 r_n = linspace(r1, r2, n_r);
405 theta = linspace(0,2*pi, n_theta);
406 z0 = double(D(r_n));
407
408 x_r = r_n.*cos(theta. ');
409 y_r = r_n.*sin(theta. ');
410 z_r = repmat(z0,[n_theta, 1]);
411
412 % Calculate the percentage of the total dissipation in the first
    millimeter
413 perc = vpa(2 * pi * int(D(r) * r, r1, r1+0.001) / D_r)
414
415 % Plot the local dissipation over the axisymmetric domain
416 figure(1);
417 surf(x_r,y_r,z_r, 'EdgeColor', 'none');
418 colormap(jet(256));
419 set(gca, 'FontSize', 12, 'FontName', 'Times');
420 title('Energy Dissipation', 'Interpreter', 'latex');
421 xlabel('$x$ (m)', 'Interpreter', 'latex');
422 ylabel('$y$ (m)', 'Interpreter', 'latex');
423 zlabel('$\dot{Q}_{\text{pert}}/A$ (W/m$^2$)', 'Interpreter', 'latex');
424 xticks([-0.005 0 0.005])
425 xticklabels({'-0.005','0','0.005'})
426 yticks([-0.005 0 0.005])
427 yticklabels({'-0.005','0','0.005'})
428 xlim([-r2 r2]);
429 ylim([-r2 r2]);
430
431 %*****
432 %% Functions
433 %*****
434
435 % Define the second-order ODE as two first-order ODEs

```

```
436 function dydz = func(z, y, A_n, P_n)
437     dydz = [y(2); (-P_n - A_n * y(1)^2)];
438 end
```