Archives
Steel Structures

Report 6-82-7

A design method for bolted beam to column connections with flush end plates and haunched beams

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CONTENTS Page

Page

List of symbols

Symbol	Definition
A_d	compressed cross sectional area of beam-web
A_f	cross sectional area of beam flange
A_s	tensile stress area of bolt
$\hat{B}_t$	design strength of a bolt
E	modulus of elasticity
$\hat{F}$	force in the design state
$\hat{F}_d$	design force of the compressed part of the column web
$\hat{F}_i$	design strength of the i^{th} boltrow (lower value of design strengths of end plate, column flange and bolts)
$\hat{F}_s$	design value for the shear strength of the column web
$\hat{F}_{1e}$	design strength of the end plate at the first boltrow
$\hat{F}_{1f}$	design strength of the column flange at the first boltrow
$\hat{F}_{1k}$	design strength of end plate or column flange when the design formula can be applied for both components of the connections)
$\hat{F}_{1t}$	lower value of the strengths of end plate, column flange and bolts added in the first boltrow
$\hat{F}_{1w}$	design strength of the column web at the first boltrow
$\hat{F}_{fl}$	design strength of the beam flange
$\hat{F}_{ws}$	design value for the shear strength of the beam web over the length of the haunch.
$\hat{F}_{1et}$	design strength of the end plate at the bolts added in the first boltrow
$\hat{F}_{1ft}$	design strength of the column flange at the bolts added in the first boltrow
$\hat{F}_{1kt}$	design strength of end plate or column flange when the design formula can be applied for both components
$\hat{F}_{s,d}$	design strength of the column web when shear or compression governs
$\hat{F}_{1sum}$	$\hat{F}_1 + \hat{F}_{1t}$
I	moment of inertia
I_v	moment of inertia of the connection

$\hat{M}_c$	design moment capacity in the beam section where the haunch flange is connected to the beam flange
$\hat{M}_e$	elastic moment of a cross section
$\hat{M}_p$	plastic moment of a cross section
$\hat{M}_v$	design moment capacity of the connection
T_f	tensile force in the beam flange
W_e	elastic section modulus
a	throat thickness of fillet weld
a_c	leg size of fillet weld at the end of the haunch flange
a_{wc}	throat thickness of fillet weld between haunch web and beam
a_d	throat thickness of weld between the end plate of a column and the column flange
a_s	throat thickness of the fillet weld between stiffener and column flange
a_w	throat thickness of the fillet weld between end plate and beam web
b	generally the width of column or end plate
b_c	width of the haunch flange
b_d	width of the end plate on top of the column
b_f	width of the beam flange
b_m	effective width or length
f_c	factor which depends on the required rotational capacity
f_i	factor used in the computation of the connection stiffness which depends on the number of boltrows.
f_{pr}	reduction factor for prying action
f_v	distribution factor
h_c	depth of the haunch
h_d	depth of the compressive zone of the haunch
h_i	lever arm of the i^{th} boltrow
i	index of the botlrows, numbered from the boltrow adjacent to tensile beam flange
k	flexibility factor or compliance of the connection ($\phi = k.M$)
l_c	haunch length
m_1	horizontal distance between the centre of the bolt and the root of the fillet

m_2	vertical distance between the centre of the bolt and the root of the fillet
m	$= m_1 + 0.2r_k$ or $m = m_1 + 0.2\sqrt{a_w}$
m_p	plastic moment per unit length of column flange or end plate
n'	distance between the centre of the bolt and the edge of column flange or end plate
n	distance between the centre of the bolt and the assumed point of application of the prying action
p	vertical pitch between the boltrows
t_c	haunch flange thickness
t_d	thickness of the end plate on top of the column
t_e	end plate thickness
t_f	flange thickness
t_w	web thickness
t_{wc}	web thickness of haunch
t_s	thickness of stiffener
w_1	horizontal distance between the bolts located in one half of the column flange
w_2	horizontal distance between the bolts located on both sides of the column web
y	centre distance between bolt and stiffener or beam flange
z	distance between two horizontal yield lines
ϕ	angle of rotation
σ_i	comparison stress
σ_e	design value of yield stress
σ_t	tension strength

The formulae of the design method have been tabulated in tables 1 to 9 inclusive to facilitate the application.
In each table it is referred to sections where explanations of the formulae and examples of the application are given.
Sometimes the results of reports 6-81-15 and 6-81-23 are also used as examples.
In the formulae the index k is used for the design strength when the formula is valid for the column flange as well as for the end plate. Otherwise either the index f is used when it is only valid for the column flange or the index e is used when it is only valid for the end plate.

The design method is based on the philosophy that semi-rigid connections may be applied in steel structures provided that the reduced stiffness of the connection is taken into account and the connection possesses sufficient rotational capacity (see Witteveen et al (1980) |1| and Bijlaard (1981) |2|).
The latter conditions are not necessary for structures with rigid connections because then the stiffness is assumed to be infinitely and the strength of the connection is such that a plastic hinge may be formed in the cross section of beam or column adjacent to the connection so that the required rotation is delivered by this plastic hinge. However, connections with flush end plates without a haunch generally do not possess sufficient strength and stiffness to be assumed rigid, so that special attention is given to the rotational capacity of this type of connection in the design method reported here.

The formulae with which the design strength of a particular component of the connection can be computed have been developed using theoretical models according to the theory of plasticity and the failure mechanisms observed in the tests in the ultimate limit state. Generally the tests were stopped because large plastic deformations occurred.
It has appeared that the limit state of large deformations reached due to bending of column flange or end plate is predicted sufficiently accurately with the yield line theory.

0.2. Features of the existing and proposed design methods

0.2.1. Existing design methods

In the Netherlands, two versions of the existing design method are valid. In the first version a completely rigid column flange and end plate are assumed.

0.2.1.1. First version

A location of the centre of rotation is assumed in line with the compressed flange of the beam. With these assumptions it is possible to compute the bolt forces because then the bolt elongations are proportional to the distances between the horizontal boltrows and the centre of rotation.
The connection reaches its ultimate limit state when the uppermost bolts (the first boltrow) reach this state.
These assumptions yield the design formula:

$$\hat{M}_v = 2\hat{B}_t \frac{\Sigma h_i^2}{h_1} \qquad (0.1)$$

Where:

$2\hat{B}_t$ is the design strength of two bolts in the uppermost boltrow

$\hat{M}_v$ is the design moment capacity of the connection

h_i is the distance between boltrow and the location of the compressive force.

A disadvantage of this design method is that the designer often has to apply stiffeners as shown in figure 2a because adequate design criteria are lacking with which may be proved that stiffeners may be avoided whereas completely rigid flanges are assumed.
The result of so many stiffeners may be fracture of the bolts before any deflection of column flange or end plate has taken place. In that case the connection may have insufficient rotational capacity. This may be disastrous when the adjacent material does not yield..
With flush end plates this is generally the case, because the strength of the connection is insufficient to cause a plastic hinge to form in the adjacent beam material.

0.2.1.2. Second version

In the Netherlands the research of bolted beam-to-column connections has always been aimed on the avoidance of stiffeners in order to decrease the welding costs of the structure and to obtain rotational capacity when the design strength of the connection is smaller than that of the adjacent beam. That is why the second version of the existing design method was introduced in which the design method of the connection may also be governed by failure of the column flange without stiffeners. The design formula for this version is (see fig. 2b):

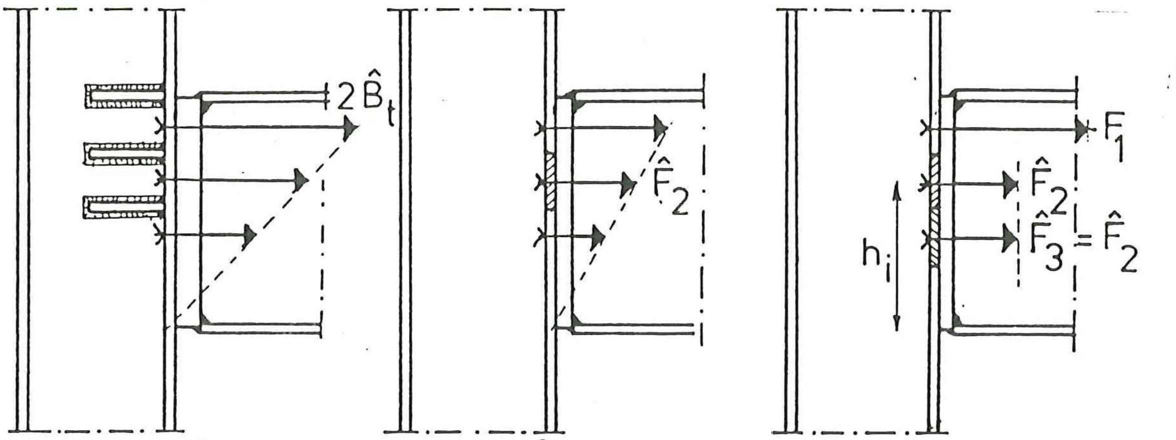
$$\hat{M}_v = \hat{F}_2 \frac{\Sigma h_i^2}{h_2} \qquad (0.2)$$

Where:

$\hat{F}_2$ is the design strength of the column flange without stiffeners at the second boltrow. The latter design strength is determined with the formulae as given in table 4 and developed in reference |4|.

The effective length of the column flange at the second boltrow is chosen equal to the vertical pitch.

The end plate thickness is adapted to the design load F_2, with the same formulae.



$\hat{M}_v = 2\,\hat{B}_t \cdot \frac{\Sigma h_i^2}{h_1}$ (0.1)
2a

$\hat{M}_v = \hat{F}_2 \cdot \frac{\Sigma h_i^2}{h_2}$ (0.2)
2b

$\hat{M}_v = \Sigma(\hat{F}_i \cdot h_i)$ (0.3)
2c

Fig. 2 : Formulae of the old and new design methods.

Despite that the column flange and end plate are not completely rigid a boltforce distribution is assumed which is also proportional to the distances between the horizontal boltrows and the centre of rotation. This approach is on the safe side as is shown by test results |9, 10|. This will be explained for different situations.

In the first place the uppermost (first) horizontal boltrow transmits more force than is assumed in the existing design method.
With a column flange stiffened at the first boltrow, the flange is far more rigid than at the second row; because the force is distributed to the stiffener as well as to the column web. The same situation exists on the beam-side of the connection where the end plate is supported by beam web and beam flange at the first boltrow.

With an unstiffened column flange it is possible that the forces transmitted by the first and second boltrow are equal if the flush end plate and bolts have sufficient strength to cause the yield line mechanism as observed in connections with extended end plates |4|.
In that case the effective length of the second boltrow is equal to that of the first row and larger than the pitch between the bolts. This implies that the forces transmitted by the first as well as the second boltrow are larger than assumed.
When at the first and second boltrows sufficient deformations occur, the other boltrows may also transmit more force than with the rectilinear proportional boltforce distribution is assumed.
That is why in the proposed method the latter assumption is abandoned.

0.2. Proposed or new design method

The design strength of each boltrow is computed with the formulae given in tables 2 to 4 inclusive.
Subsequently the design moment capacity is computed by adding up the products of design strength and lever arms.

Hence: $$\hat{M}_v = \Sigma(\hat{F}_i \cdot h_i) \qquad (0.3)$$

In figure 2 the assumed bolt force distributions and formulae of the various design methods are summarized.

0.3. The proposed design method contemplated as an extension of the existing design methods

The condition for the proposed design method is that at the first boltrow the column flange (or end plate) deform so much that an extension of the yield line mechanism caused by the other boltrows can be assumed.

This is only possible when the ultimate limit state of column flange (or end plate) is reached before the bolts fail. This statement will be discussed with the help of the graph on top of table 1 (see figure 3). In this graph the horizontal axis represents the ratio between the design strength of the column flange (or end plate) and bolts at the first boltrow; thus:

$$\frac{\hat{F}_{1k}}{2\hat{B}_t} = \frac{\text{design strength of column flange (or end plate) at the first boltrow}}{\text{design strength of two bolts in the first boltrow}}$$

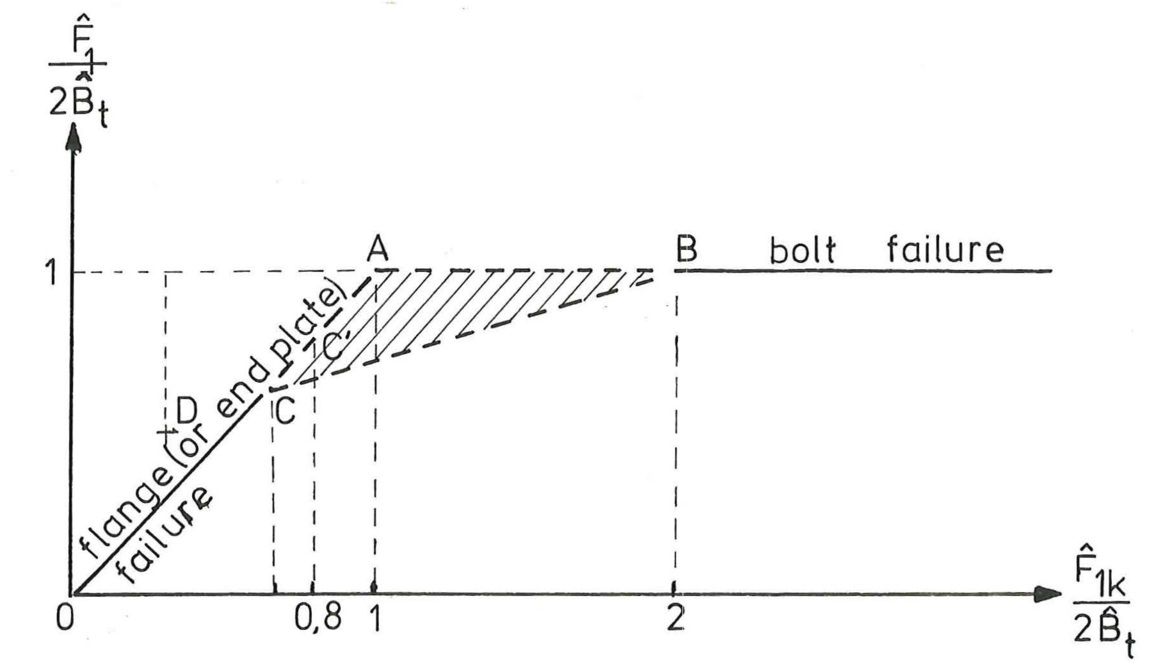


Fig. 3 : Relations between the design strengths of column flange, bolts and connection.

It is evident that this ratio increases proportionally with the quadrate of the thickness of column flange (or end plate) when the bolt strength is kept constant.

The vertical axis represents the design strength of the connection $\hat{F}_1$, also expressed as a ratio with respect to the design strength of the bolts.

The design strength of the connection, $\hat{F}_1$, is equal to that of the column flange (or end plate), $\hat{F}_{1k}$, as long as the design strength of the bolts is larger than that of column flange (or end plate).
Thus when:

$$\frac{\hat{F}_{1k}}{2\hat{B}_t} \leqslant 1$$

then the relation between the design strengths is represented by line OA in the graph. If the design capacities of column flange and end plate are larger than that of the bolts then the design strength of the connection is restricted by that of the bolts. Thus:

$$\frac{\hat{F}_1}{2\hat{B}_t} = 1 \quad \text{when} \quad \frac{\hat{F}_{1k}}{2\hat{B}_t} > 1$$

This latter relation is represented by the horizontal line through point A. In that area the first version of the existing method is valid. The validity of the proposed design method is limited to the situations represented by line OA. In those situations the limit state of large deformations of column flange (or end plate) is reached before the bolts fail, so that the deformation at the first boltrow is sufficient to cause the extension of the yield line mechanism at the other boltrows. However, a reservation should be made for a transitional area indicated by the hatched area in figure 3.

0.4. Influence of prying action

The deformation of column flange (and/or end plate) causes extra forces (prying action) as shown in figure 4.

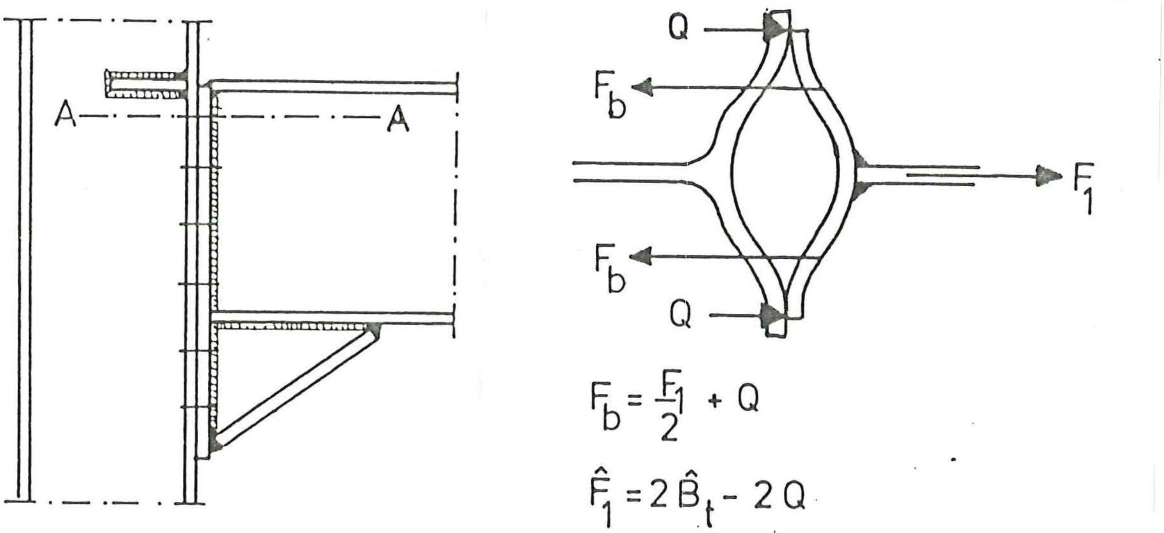


Fig. 4. : The deformation of column flange or end plate causes prying action (extra forces in the bolts).

These prying actions are incorporated in the design formulae developed for unstiffened column flanges and extended end plates |2, 4|
These prying actions are also measured in tests with stiffened column flanges and flush end plates, but it has not been possible to incorporate them in the design formulae.
The prying actions may cause premature failure of the bolts because they enlarge the bolt load. This phenomenon is taken into account by the hatched area in the graph on top of table 1.
The result is that the separation between the proposed design method and the existing elastic method becomes less clear because a transitional area CB arises in which at the left hand side much deformation of column flange (or end plate) occurs but premature failure may happen due to prying action.
This is taken into account by reduction of the design strength of the bolts with a factor f_{pr}.

Thus: $$\hat{F}_1 = \frac{2\hat{B}_t}{f_{pr}} \qquad (0.4)$$

In section 1.1.2. rough experimentally found values are given for the factor f_{pr}. At this moment the available testresults are mainly situated in the area on the left hand side of point C.
In this area prying forces have been measured which were equal to the applied load. In that case f_{pr} = 2 and the design strength would be reduced to 50% of that of the bolts e.g. indicated by point D in the graph. However, this is not important as long as this reduced design strength of the bolts is larger than the design strength of column flange (or end plate).

Based on the test results |5, 6| a factor f_{pr} = 1.5 is accepted in point C when the bolt is located near to the edge of column flange (or end plate), whereas f_{pr} = 1.25 is taken into account (point C') when the bolts are located near to the column web and stiffener.

Prying action does not occur when column flange and end plate are far stronger and stiffer than the bolts. For the time being it is assumed that this happens at a value:

$$\frac{\hat{F}_{1k}}{2\hat{B}_t} \geq 2 \text{ (point B in graph)}$$

It seems to make sense that future tests are executed of which the results are situated in area CB, to get more accurate values for f_{pr} and to investigate the transition from the existing to the proposed design method.
For the time being it is advised to avoid the transitional area in the design or to make a computation which is on the safe side e.g. by neglecting the cooperation of the second and other boltrows when it appears that the column flange (or end plate) yields at these boltrows according to the formulae valid for the unstiffened column flange (see the formulae in table 4 and example 8).
In test 2 of the reports 6-81-15 and 6-81-23: $0{,}8 < \frac{\hat{F}_{1f}}{2\hat{B}_t} < 1$.

According to the latter advise, the cooperation of the other boltrows should have been neglected because failure at these rows is governed by yielding of the flange.
However, the testresults show a cooperation of these boltrows; thus it is possible that the design method should be changed when more testresults with this favourable behaviour come available.

0.5. Restriction for the existing design method

Table 1 gives a summary of the properties of the existing and proposed design method. Herein it is stated that is is not allowed to apply the existing design method in statically undeterminated structures unless the design strength of the connection is larger than that of the connected beam. This is based on the following considerations.
From the foregoing it will be evident that the connection itself has practically no deformation capacity when the existing design method is used. Column flange and end plate are made infinitely stiff and bolts grade 8.8. fracture at a plastic elongation of about 2 mm whereas this elongation for bolts grade 10.9 is only about 1 mm.

The deformation of column flange and end plate is negligible. This may result in bolt fracture when a connection computed according to the existing design method is used in a statically undeterminated structure and unexpected deformations occur e.g. by settlement of the foundation. When the design strength of the connection is larger than that of the connected beam, the required rotation can be delivered by yielding of the beam flanges.
If this is not the case, the rotation should be delivered by the connection. This is only possible when the column flange or end plate yields before failure of the bolts occur, thus when the column flange or end plate is not infinitely stiff i.e. designed in accordance with the proposed design method.

0.6. Advantage of the proposed design method when the compressive or shear zone governs failure

In the foregoing only the tensile zone of the connection has been contemplated. However, the sum of the design forces of the boltrows may be restricted by the design capacity of the shear zone or compressive zone in the column web.
According to the existing design method, which is based on the theory of elasticity the design moment is then determined by the formula:

$$\hat{M}_v = \hat{F}_{s,c} \frac{\Sigma h_i^2}{\Sigma h_i} \tag{0.5}$$

where:

$\hat{F}_{s,c}$ is the design capacity of the shear or compressive zone and

$\frac{\Sigma h_i^2}{\Sigma h_i}$ is the distance between the centre of gravity of the bolt forces and the location of the compressive force.

The latter distance and thus the moment capacity M_v becomes smaller when more boltrows are applied. This seems odd, but it is in fully agreement with the conditions based on the theory of elasticity.
With the proposed design method, formula (0.3.) remains valid but another condition should be fulfilled too. This latter method is based on the philosophy that the first boltrow causes to form a yield line mechanism in the column flange (or end plate) and the other boltrows cause an extension of this mechanism when the load increases. This extension ceases when either the compressive zone or shear zone of the column web fails. That is why the condition should be fulfilled that:

$$\Sigma \hat{F}_i \leqslant \hat{F}_{s,c} \tag{0.6}$$

If the condition of formula (0.6) is not met, then a part of the design strengths of the boltrows should be neglected.
Based on the knowledge that the yield line mechanism extenses from the first bolt row, reduction of the sum of the design strength, $\Sigma \hat{F}_i$, may occur by neglecting the bolt rows with the smaller lever arms.

0.7 Comparison of design methods with test results

Table 10 gives the results of the computations carried out with the first and second version of the old design method and the computation with the new design method for the testspecimens of reports 6-81-15 |9|, and 6-81-23 |10|.
All values can be compared with the design strengths computed with the formulae of the new design method because these are adequate as being shown by comparison of these results with the moment rotation curves in figure 13 and figure 14 of report 6-81-23 |10|.

The design strength computed with the formulae of the first version of the old design method are generally too high because failure of the column flange and failure of the beam web have not been taken into account.
If failure of the column flange due to bending is taken into account according to the second version, then the design capacities are too low. They are also too low in the first version of the old design method when it appears from the proposed design method that the mechanism due to shear or compression of the column web governs.
In the proposed method, the failure of the beam web due to shear in the part over the haunch is specifically defined.
This does not mean that this type of failure would be neglected in the old design method but it could not occur due to the application of very thick end plates as followed from the design method.
Thus the new design method augments the design labour, but give the possibility to reduce the labour of manufacturing.
The new design method gives adequate design capacities for the tests of reports 6-81-15 and 6-81-23.

Test number	Existing or old design method												Proposed or new design method	
	1° version			2° version							Conclusion		Connection	Beam-web
	only boltfail. contemplated			flange failure			failure of column-web				1° version	2° version		
	Design strength 1° boltrow	$\frac{\Sigma h_i^2}{h_1}$	$\hat{M}_v$	Design strength 2° boltrow	$\frac{\Sigma h_i^2}{h_2}$	$\hat{M}_v$	Shear	Compr.	$\frac{\Sigma h_i^2}{h_2}$	$\hat{M}_v$	$\hat{M}_v$	$\hat{M}_v$	$\hat{M}_v$	$\hat{M}_v$
	kN	m	kNm	kN	m	kNm	kN	kN	m	kNm	kNm	kNm	kNm	
1-15	350	1.75	612	84	1.98	166		598	0.41	245	245 c	166 B	275	199
1-20	294	1.75	514	92	1.98	182		598	0.41	245	245 c	182 B	281	222
2-13	350	0.90	316	60	1.07	64		598	0.30	179	179 c	64 B	173	170
2-20	294	0.90	264	92	1.07	98		598	0.30	179	179 c	98 B	181	204
3-18	470	1.48	696	92	1.71	157		1020	0.41	418	418 c	157 B	344	366
3-21	456	1.48	674	92	1.71	157		1020	0.41	418	418 c	157 B	377	418
4-18	470	1.48	696	92	1.71	157		1020	0.41	418	418 c	157 B	265	206
4-21	456	1.48	674	92	1.71	157		1020	0.41	418	418 c	157 B	265	234
5-18	464	1.52	706	118	1.76	206		1020	0.42	428	428 c	206 B	378	325
5-21	464	1.52	706	138	1.76	242		1020	0.42	428	428 c	242 B	378	358
1	294	1.75	514	92	1.98	182	411	598	0.41	169	169 s	169 s	200 s	222
2	294	1.75	514	92	1.98	182	617	598	0.41	245	245 c	182 B	283 c	418
3	456	1.48	674	92	1.71	157	411	598	0.41	169	169 s	157 B	204 s	418
4	456	1.48	674	92	1.71	157	617	598	0.41	245	245 c	157 B	276 c	234
5	456	1.80	820	92	2.04	188	617	1020	0.47	290	290 s	188 B	324 s	417

Table 10: Design strengths computed with the old and new design methods for the testspecimens of reports 6-81-15 |9| and 6-81-23 |10|.

c = compression of the column-web
s = shear of the column web
B = bending of the column flange

} is the mechanism which determines the design capacity.

1. Design strength of the column flange and the flush end plate at the first boltrow

1.1. Two bolts in the boltrow (table 2)

Formula (4a) example 1

Doornbos (1979) |5| tested specimens as shown in figure 5. With the help of yield line theory he composed a chart with which the design strength of a plate can be computed.

The tests show good agreement between the limit states of large deformations and the design strengths computed with the chart.

The chart is given in table 2. In the chart the contemplated yield line mechanisms are indicated.

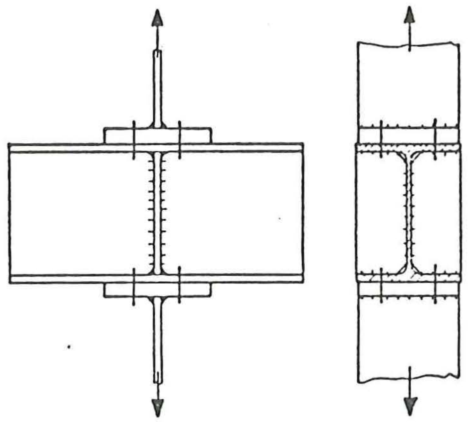


Fig. 5 : Testspecimen of a column section with stiffeners; the flanges are loaded transversely.

The chart gives the design strength of column flange (or end plate) in the following way. Determine the distances m_1 and m_2 between the boltcentre and the flange toes of the fillets (or fillet welds). Make the distances m_1 and m_2 dimensionless by dividing them by the width $m_1 + n'$; this gives the values λ_1 and λ_2.

Plot a point with the coordinates λ_1 and λ_2 and read the value α at

the end of the curve indicated with the plotted point.
The value α multiplied with the plastic moment per unit length gives the design strength of the column flange (or end plate). Formula (4a) gives the design strength of the column flange for a boltrow with two bolts (see e.g. the computation of the limit state loads at the first boltrow of test 1 to 5 inclusive for the column flange and the end plate in report 6-81-23).

1.1.2. Limitation due to prying action (table 1) example 2

It is already explained in the introduction that extra forces (prying forces) arise due to deformation of colomn flange (or end plate).
These prying forces may reduce the design strength of the connection.
In contradiction to the design formulae developed for the extended end plate |4| these prying forces have not been incorporated in the design formulae for the flush end plate.
However, the testresults show the presence of these prying actions.
Moreover, it appears that the increase of prying action becomes larger with respect to the increase of the applied load when the design strength of column flange (or end plate) is exceeded.

In figure 6 a representative example is given of the measured current of the boltforce (horizontal axis) with the increase of the applied bending moment (vertical axis) on the connection.
The dotted line gives the relation between the applied moment and the expected boltforce. Initially the boltforce does not increase due to the preload in the bolt. After exceeding of the preload, the difference between dotted and solid line gives the prying action which increases fast when the design strength of the connection is exceeded.

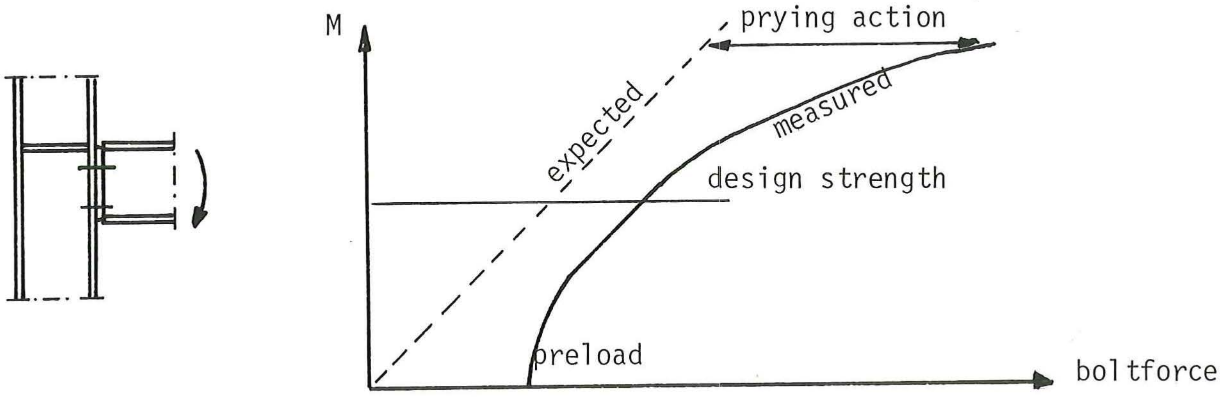


Fig. 6 : Relation between applied moment and measured boltforce.

In a structure the design strength may be exceeded by several causes.

Firstly when the structure has been designed in accordance with the theory of plasticity. In that case it is accepted that the design strength of a connection is exceeded under working load provided that the deformations of the whole structure remain within certain limits.
It is required that the connection in which the design strength is reached firstly, give so much deformation that the failure mechanism of the whole structure may be reached. This implies that the design strength is exceeded because the ratio between failure load and design strength is larger than unity. (For most beam to column connections which do not fail due to buckling of the compression side or fracture of the bolts, the ratio between failure load and design strength appears to be about 1.7.).

Sometimes it is stated that connections do not need rotational capacity if used in structures computed in accordance with the theory of elasticity. This is only true if all loading situations which occur during lifetime have been contemplated in the design . Generally this is impossible e.g. with unexpected settlements of foundations. Thus the design strength may also be exceeded in connections applied in statically indeterminated structures computed in accordance with the theory of elasticity.

That is why the magnitude of the prying action should be determined in the situation that the design strength is exceeded and large deformations occur(thus at the end of the test).

The influence of the prying action is only important with connections where bolt failure may occur. However, the design strength of the test specimens was mainly reached due to yielding of column flange (or end plate) before bolt failure could occur.

A clear view of the influence of prying action on the deformation capacity and the strength of the connections can only be developed when more test results are known in which bolt failure is the determining factor.
That is why the values stated here are on the safe side.

For the determination of the magnitude of prying action two areas of bolt location are distinguished. One area in which the bolt is located adjacent to the supports of column flange (or end plate). It is assumed that this area is limited by λ_1 and $\lambda_2 \leqslant 0.5$.

Outside this area, the second area is assumed . The distinguish is made because different mechanisms of prying action are observed in the different areas.

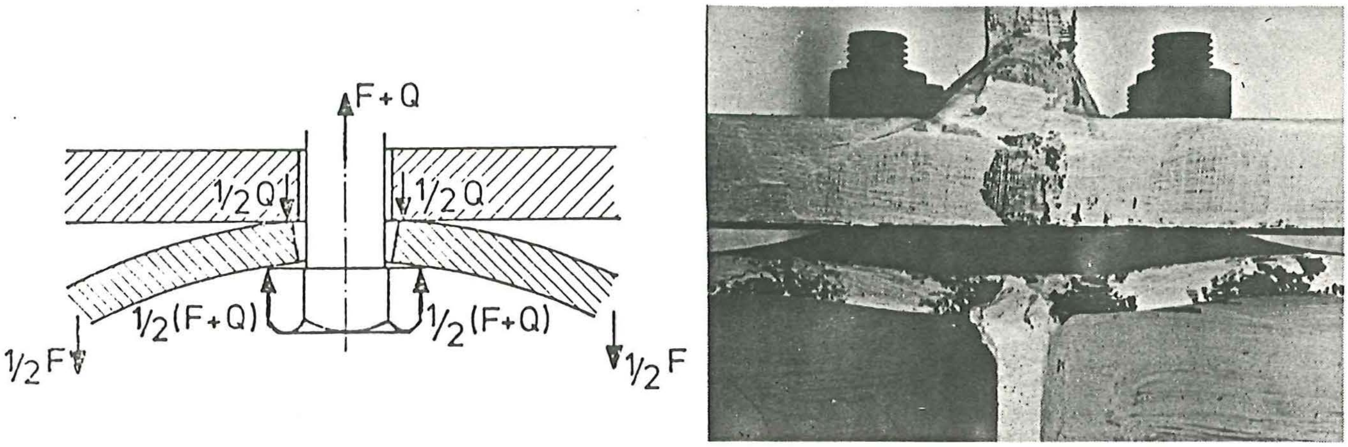


Figure 7 : Bending of the flange causes prying action.

Within the former area, the mechanism of figure 7 arises in which the plate forms a lever arm between the bolt head and end plate due to the deformations. For the time being, based on the testresults, it is stated that within the area with λ_1 and $\lambda_2 \leqslant 0.5$;

$$f_{pr} = \frac{6}{4 + \frac{\hat{F}_{1k}}{2\hat{B}_t}} \qquad (2a)$$

when: $0.8 \leqslant \frac{\hat{F}_{1k}}{2\hat{B}_t} \leqslant 2$

The magnitude of the prying force is made dependent on the ratio $\frac{\hat{F}_{1k}}{2\hat{B}_t}$, because the chance of occurrence of prying action decreases with the decrease of the deflection of column flange (or end plate) with respect to the plastic bolt elongation, thus with the increase of the ratio mentioned.

At values of: $\frac{\hat{F}_{1k}}{2\hat{B}_t} \leqslant 0.8$

the ultimate limit state of column flange (or end plate) governs, When: $\frac{\hat{F}_{1k}}{2\hat{B}_t} \geq 2$

it is assumed that prying action does not occur and bolt failure governs.

In the area where λ_1 and $\lambda_2 \geq 0.5$.

$$f_{pr} = \frac{4}{2 + \frac{\hat{F}_{1k}}{2\hat{B}_t}} \qquad (2b)$$

when: $0.67 \leq \frac{\hat{F}_{1k}}{2\hat{B}_t} \leq 2$

In this area prying action arises at the edge of column flange (or end plate) see figure 8.

Fig. 8 : Prying action arises at the edge of column flange (or end plate).

In example 2 some applications of formula (2a) are given. Example 3 gives an application of formula (2b).

In examples 4 and 8 attention is given to the influence of prying action on the deformation capacity. Example 8 is also used to show the neglect of the cooperation of the second horizontal boltrow when the first bolt row fails prematurely due to prying action.

It is already stated that this latter measure may be too conservative as is shown by the results and computation of test 2 of reports 6-81-15 and 6-81-23.

1.1.3. Flush end plate

Formula (4a) table 2, example 3 and 4

The ultimate limit state load of the first boltrow in the flush end plate will mostly be determined by that of the column flange, because the end plate thickness shall be adapted to the design strength of the latter mechanism.

That is why formula (4b) is given although it is identical to formula (4a) but only written in another form.

The value α will mostly be less than that of the column flange because the distance m_1 is larger due to the fact that the magnitude of the fillet weld is smaller than the fillet radius of the column section.

The application of formulae (1) to (3) inclusive, see table 1, is similar to that for the column flange.

See example 3 for the design of an end plate without deformation capacity and example 4 for the design of an end plate which has to give deformation capacity.

1.2. Design strength of the column flange and the flush end plate at the first boltrow with four bolts (table 3).

1.2.1. Formulae (5), (6), (7) and (10), example 5.

Adding more bolts in the first bolt row does only make sense when the bolts in the corner between web and stiffener, either:

- have sufficient strength to form a circular yield line mechanism, or
- fail before the column flange (or end plate) deforms.

Thus only when, either $\hat{F}_1 = 8\pi\ m_p$ or

or $\hat{F}_1 = 2\hat{B}_t$

In all other cases it is not certain which mechanism will develop and it is possible that the added bolts are located within the yield line mechanism already formed by the bolts in the corner and do not give an extra contribution to the transfer of forces.

When a circular mechanism is formed, then the effective width available for an added bolt: $b_m = \dfrac{b - w2 - 2m_{1,2}}{2}$

where:

$m_{1,2}$ is the smaller value of the distances m_1 and m_2.

Formulae (5), (6) and (7) are identical to the formulae developed for the unstiffened column flange and T-stub connection |4|. However, here the value of n is always taken: n = 1.25 m. In the case that yielding of column flange (or end plate) is avoided, the previous design method with the proposed effective width is conservative.

Formula (10) implies that the design strength of the first bolt row is equal to the sum of design strengths of corner and added bolts (see example 5 and the computation of tests 2-13 and 2-20 in appendix A4 of report 6-81-23).

1.2.2. Formulae (8), (9) and (10), example 6.

Formulae (8) and (9) are identical to formulae (5) and (6) with the difference that the plastic hinge may be formed in the end plate on top of the column when the column does not continue (or in the beam flange on the flush end plate side). (see example 6 and the computation of the end plate of tests 2-13 and 2-20 in appendix A4 of report 6-81-23).

2. Design strength of column flange and flush end plate at the second bolt row (table 4)

2.1. Formulae (11) and (12), example 7.

The second horizontal bolt row is located in a part of the column flange where no stiffeners are applied.
The formulae developed for the unstiffened column flange are valid.
The effective length of the column flange available for the second bolt row consists of two different parts. The part on the side of the first bolt row is limited by the vertical pitch and reduced by the length already used for the mechanism at the first bolt row.
The part on the other side of the bolt is equal to $2m + 0{,}625n'$ as shown in |4| , provided that the distance between bolt row and centre of rotation (compressive side) is sufficient in order that the yield line mechanism can develop. That is why the contribution of the second bolt row is neglected when $h_2 < 2(2m + 0{,}625n')$, unless an elastic bolt force distribution can be assumed. The latter assumption is possible when the force computed with formula (11) or (12) is larger than:

$$\frac{h_1 - p}{h_1} \; 2\hat{B}_t \quad \text{thus when } \hat{F}_{2k} \geq \frac{h_1 - p}{h_1} \; 2\hat{B}_t .$$

When it follows from the computation that a circular mechanism is formed at the first bolt row ($\alpha = 4\pi$), then the value of $m_{1,2}$ is equal to the lower value of m_1 and m_2, otherwise the larger value of m_1 and m_2 is chosen. (see example 7 and the computation of tests 2-13 and 2-20 in appendix A4 of report 6-81-23).

The previous explanation is also valid for the third and other bolt rows as far as the effective length is concerned. But when the third bolt row is located within the effective length of the second row, then this length is reduced to the overlap of both lengths that is equal to the vertical pitch, see e.g. the computation of the second bolt rows in appendices A1 and A4 of report 6-81-23.

2.2. Formula (13), example 8

Formulae (11) and (12) are based on the assumption that the flange deflects sufficiently at bolt row 1 in order that the mechanism of bolt row 2 can develop.
This is not possible when the bolts at the first bolt row fail prematurely, thus when $\hat{F}_1$ is determined by formula (2) or (3).
In that case a linear distribution of bolt forces occur. However, then the rotational capacity will be small.
In the tests of report 6-81-15 and 6-81-23 this situation did not occur because the deflection of the column was always sufficient even in the situation of test 3 and 5 of report 6-81-15 where stiffeners and backing plates were used respectively.

3. Design strength of column- and beam web (table 5).

3.0. General

The formulae are rather complicated, but the computation can be avoided if with a rough computation is shown that the sum of the design strengths of the bolt rows determined by yielding of column flange (or end plate) is smaller than the tensile strength capacity of the column web between the outermost boltrows, provided that the mechanism of figure 9 does not occur between the bolt rows.

Thus: $\Sigma \hat{F}_i \leqslant n.p.t_w.\sigma_e$ (3.1.)

where:

$\hat{F}_i$ is determined with the formulae of tables 2, 3 and 4

n is the number of bolt rows

p is the vertical pitch

t_w is the web thickness

If the requirement of formula (3.1.) is not fulfilled, then it may be shown with a more complicated computation in accordance with formulae (14) to (20) inclusive that the strength of the column web is sufficient. In that case the support of the stiffener at the first bolt row and a larger effective length at the lower bolt row is taken into account.

3.1. Formulae (14), (15) and (16), example 9.

With the computation of the chart of table 1 it is assumed that the supports of column flange (or end plate) do not deform. However, it is possible that the dimensions of the section is such that the mechanism of figure 9 occurs.
In this mechanism, the column web and flanges fail simultaneously..
The ultimate limit state load of this mechanism is reached theoretically when adjacent to the boltrow and at a distance:

$$z = t_f \sqrt{\frac{b}{t_w}}$$

yield lines are caused to form in the flanges.

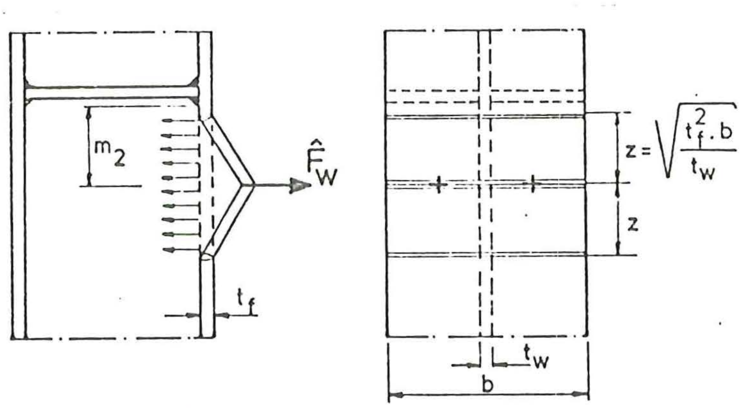


Fig. 9 : Failure mechanism in which the column web and flanges yield simultaneously.

The design strength of this mechanism is given by formula (15).
When the distance m_2 is smaller than z, then the web is supported by the stiffener and the design strength increases with a decreasing value of m_2 because bending of the flange becomes more difficult.
This behaviour is taken into account with formula (16). Some computations have shown that with European rolled section and a distance $m_2 < 1.5d_n$ (d_n = nominal bolt diameter), the value of z is always larger than m_2 so that formula (15) never governs in that case.
However, formula (15) is given for the sake of completeness (see example 9).
See example 13 for the application in the design of the flush end plate.

3.2. Formula (17), example 10.

The column flange on the unstiffened side of the bolt line shows the same behaviour as the unstiffened column flange of which is shown that the effective length is (2m + 0,625n'). Thus this effective length is also adapted for the column web.

3.3. Formulae (18) and (19), example 11.

When the column does not continue beyond the upper beam flange, a plastic hinge may be formed in the end plate of the column. In that case formulae (18) and (19) may give lower values than formulae (16) and (17), unless the end plate is made much thicker than the column flange.
The same reasoning is valid for the part of the connection formed by the

end plate and the flange of the beam.
See example 13 for the application in the design of the flush end plate.

3.4. Formula (20), examples 12 and 13.

When more bolt rows are applied, the situations of formulae (15) to (19) inclusive remain valid for the first bolt row. The other bolt rows cause only yielding of the part of the column web between the bolt rows. It is possible that the column web has some over capacity of strength at the first bolt row due to the influence of the stiffener. It is assumed that the bolt forces are distributed uniformly over the web length when yielding of the web occurs. Then it is sufficient when the sum of the design strengths of the bolts determined by yielding of the column flange (or end plate) is smaller than the total strength of the column web (see example 12 for the check of the column web, and example 13 for the check of the beam web).

3.5. Formula (21), example 14.

The design strength determined by shear of the column web ($\hat{F}_s$) or yielding and buckling of the column web on the compression side of the connection ($\hat{F}_d$) ought to be larger than the sum of the design strengths on the tension side, because otherwise the equilibrium requirement can not be fulfilled. The design formulae for $\hat{F}_s$ and $\hat{F}_d$ are given in |1| and |2|, but are repeated here for the sake of completeness.

$$\hat{F}_s = 0{,}58\ \sigma_y . t_{wk} . (h_k - 2t_{fk}) \qquad (3.5.1.)$$

$$\hat{F}_d = \sigma_y . t_{wk} \{ t_{fl} + 5(t_{fk} + r_k)\} \qquad (3.5.2.)$$

where:

t_{wk} = column-web thickness
t_{fk} = column-flange thickness
t_{fl} = beam-flange thickness
r_k = fillet radius of column
h_k = column depth

Reduction of the sum of the forces can be executed by neglecting the bolt rows with the smaller lever arms as explained in the introduction in section 0.6.

See e.g. the computation of test 1 and 3 in appendix A1 of 6-81-23 where the shear force capacity determines the design moment and test 2 where the design strength of the compression side determines the design moment.

4. Dimensions of stiffeners and welds (table 6)

4.1. Formulae (22) to (25) inclusive, examples 15 and 16.

From measurements in tests |5| it appeared that the force transferred by the corner bolt is distributed over web and stiffener reversed proportional to the distances m_1 and m_2.
That is why a distribution factor :

$$f_v = \frac{m_1}{m_1 + m_2}$$

is introduced.

Stress distributions as shown in figure 10 are also measured in tests .

It appears that the stresses are not uniformly distributed.
In three tests where $\lambda_1 > 0.5$, weld fracture occurred on the very edge of the stiffener (point A in figure 10).

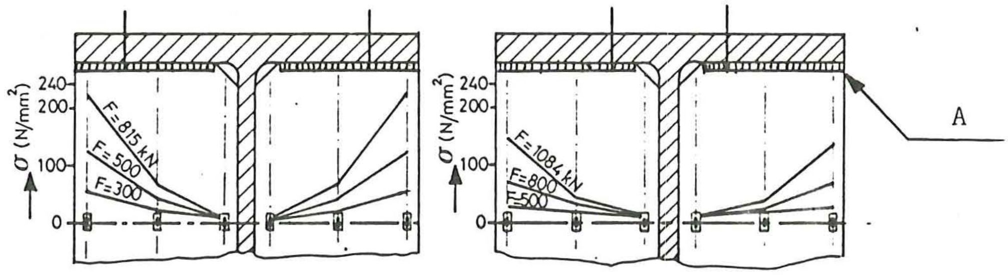


Fig. 10 : Measured stress distributions in stiffener and column-web.

At that instant, the stresses in the stiffener were still low. Probably failure was caused by bending of the weld |6| .

An excessive stiffener-thickness has small financial consequences.
That is why no research has been executed on this part of the connection and a rough method of computation is given.
Hence $f_v = 1$, when $\lambda_1 > 0.5$, moreover, it is assumed that the force is transferred by half the stiffener-thickness.

The effective width of the stiffener is taken $2m_2$ (formula (22)) provided that this width is available, thus if $m_2 > n'$. Otherwise the effective width is equal to $m_2 + n'$ (formula (23)).

An excess of the required weld size has large financial consequences especially when the throat thickness of the fillet becomes larger than 6 mm. In that case more welding runs are necessary.
That is why a good approach of the weld size is required. The proposed method is based on the test results (6-81-15 and | 6|) where weld fracture occurred.
In the situations that the rotational capacity should be delivered by bending of end plate or column-flange, bending occurs too in the fillet weld (see figure 11).
The fillet size should be sufficient in order that the weld can deliver the required deformations.

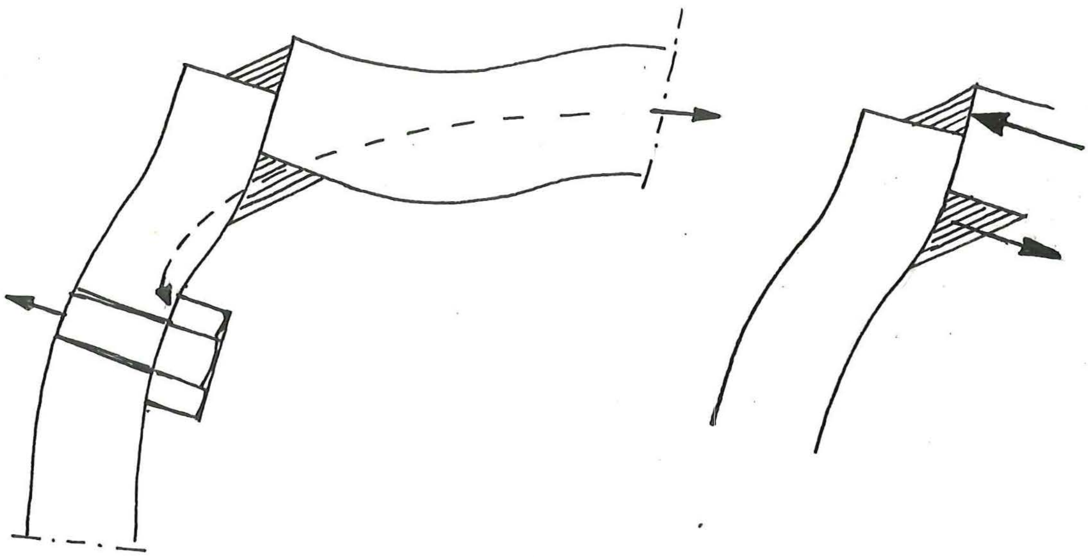
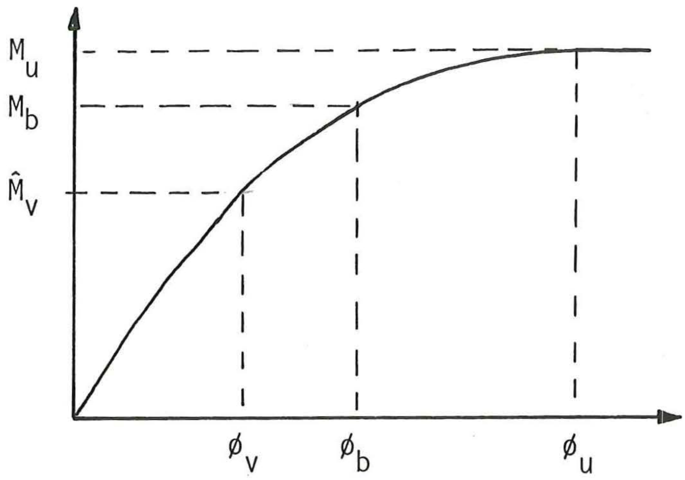


Fig. 11 : Bending of the end plate causes bending of the inner fillet weld.

The required deformation depends on the statical system of the structure. This is expressed with a magnification factor f_c, where:
$f_c = 1$ for statically determinated structures
$f_c = 1,4$ for braced frames
$f_c = 1,7$ for unbraced frames.

The values of f_c are based on moment rotation curves found in tests and they represent the ratio between the bending moment at the required rotation and the design bending moment capacity as indicated in figure 12.



ϕ_b = required rotation for braced frames →

$$\frac{M_b}{\hat{M}_v} = f_c = 1{,}4$$

ϕ_u = required rotation for unbraced frames →

$$\frac{M_u}{\hat{M}_v} = f_c = 1{,}7$$

Fig. 12 : Relationship between f_c and the statical system of the structure.

Formulae (24) and (25) are based on the assumption that the force will be transferred by the weld at one side of the stiffener.
However, a reduction factor of 0.7 is applied based on the following two considerations:

- The weld on the unloaded side of the stiffener will also contribute when yielding of the weld on the loaded side occurs.
- Loading of the weld until fracture is allowed in order to reach the required rotation.

See e.g. the computation of test 1 in appendix A1 of report 6-81-23 which shows that formula (25) is conservative.
When yielding of the web behind the first bolt row occurs as it is assumed in the mechanisms represented by formulae (16) to (19) inclusive, it is proved in report 6-81-15 on page 32 that yielding of the flange is reached when $a_d = 0.4t_d$ or $a_f = 0.4t_f$.
See e.g. the discussion of test 1 in report 6-81-15, pp 53-55.

5. The stiffness of a connection with flush end plate and a column with or without stiffeners (table 7)

5.1. Formula (26), example 17

Formula (26) represents the summation of all rotations caused by the deformations of various components of the connections on the levels of the first bolt row and the compression side.
The rotation caused by bolt elongation is e.g.:

$$\phi = \frac{F_1}{h_1} \frac{l_b}{2E.A_s} \qquad (5.1.1.)$$

where:

F_1 = the loading of the first bolt row with two bolts
l_b = the bolt length
h_1 = the distance between the first bolt row and the compression side
A_s = the stress-area of a bolt
E = modulus of elasticity

The rotation in accordance with formula (5.1.1.) is the elongation of the bolt according to Hooke's law divided by the distance h_1 between the first bolt row and the compression side. Because the bolt force:

$$F_1 = \frac{M}{h_1} , \quad \phi = \frac{M}{Eh_1^2} \frac{l_b}{2A_s} = kM \qquad k = \frac{\alpha_3}{Eh_1^2}$$

where:

$$\alpha_3 = \frac{l_b}{2A_s} \text{ when } F_1 = \hat{F}_1 \text{ (see table 7).}$$

When more bolt rows are present $F_1 = \frac{M}{f_i h_1}$

where:

f_i = factor which takes into account the contribution of the other bolt rows.

Here another approach of the stiffness is chosen than in |2|.
The changes are:

- the factor k instead of the moment of inertia I_v of the connection
- the factor k increases quadratically with the bending moment.

In |2| the moment of inertia is given for an arbitrarily chosen length equal to half the column depth.
This choice appeared necessary for computer applications. Now most of the computer programmes have the opportunity to compute with the moment rotation relations of the connections represented by the compliances k.
The designer who has not available such a program can still use the method with a substitute beam with length $l_v = \frac{1}{2} h_k$ and moment of inertia $I_v = \frac{\frac{1}{2} h_k}{k.E}$.

The stiffness formula given in |2| is mainly applicable for connections with extended end plates.
The mistake in the prediction of the stiffness behaviour of a structure depends on the difference between the computed and actual stiffness of the connections. This mistake increases with decreasing connection stiffness (see appendix A3 of report 6-81-23).

The semi-rigid connection with extended end plate has generally a considerable stiffness. Thus with these connections the mistake in the stiffness behaviour of the structure is generally small.
The stiffness of connections with flush end plates is less than that of connections with extended end plates. A mistake in the computed stiffness of a flush end plate connection has a larger effect on the determination of the moment distribution and deflections than the extended end plate. That is why is strived for a better approach.

The factor α_1 to α_6 inclusive are approached theoretically and if necessary adapted experimentally.
The quadratic term $(\frac{F1}{\hat{F}})^2$ in the factor α_1 to α_6 inclusive give a reasonable approach to the force deformation characteristic until yielding of the specific component of the connection.
Formula (26) gives the rotation at the design moment capacity.
The rotations at other bending moments are obtained by multiplying the result with the factor $(\frac{M_v}{\hat{M}_v})^2$. (see e.g. the computations in appendix A1 of report 6-81-23).

The factors α_1 to α_6 inclusive are determined experimentally with tests in which no special attention was given to the tightening (preload) of the bolts and the location of the contact force.

Research |12| shows that the rate of tightening as well as the location of the contact force is of great importance for the stiffness of the connection.

Another point is that various possible failure mechanisms are not incorporated in the stiffness formula. That is why formula (26) may only give an indication of the stiffness. With special measures the stiffness may be enhanced considerably.
Moreover, stiffness formula (26) mostly gives an underestimation of the stiffness as is shown for the tests of report 6-81-23 in appendix A3. In order to reach a more accurate prediction of the connection stiffness more research is necessary.

6. Connection with haunch and flush end plate

6.0. General

The haunch on the compression side of the connection according to figure 1 serves to improve the stiffness and enlarge the design strength of the connection.
In the case that the ultimate limit state moment of the connection is large enough to cause yielding of the beam flanges, no rotational capacity is required from the connection. In that situation the end plate thickness is not restricted and a simple formula based on the assumption that the tensile force in the beam-flange should be transferred by bending of the end plate gives the end plate thickness.

$$T_f * y = \tfrac{1}{4} b . t_e^2 \rightarrow t_e = 2\sqrt{\frac{T_f . y}{b\,\sigma_e}} \qquad (6.0.1.)$$

where:

T_f = tensile force in the beam flange
y = distance between centres of beam flange and first bolt row
b = width of the end plate
t_e = end plate thickness

However, the result of this formula is generally a large end plate thickness.
In that case the capacity of the bolts should be such that fracture does not occur before the plastic moment of the beam is reached and the moment capacity of the connection can be computed by assuming an elastic bolt force distribution.

When the strength of the connection is too small to cause yielding of the beam flanges, rotational capacity is required from the connection and the design method explained in the previous sections should be applied with addition of a check of the following components:

- shear of the beam web
- failure of the beam section at the end of the haunch
- failure of the haunch flange and welds or
- failure of the haunch web when a haunch without a flange is applied.

The formulae which enable this check are given in tables 8 and 9 and discussed hereafter.

6.1. Shear of the beam web

Formulae (27)

The force in the beam flange should be transferred to the bolts in the end plate. If the bending capacity of the end plate is not sufficient then the force will be transferred via the beam web (see figure 13). This may cause failure of the beam web due to shear stresses in the section just above the first bolt row.

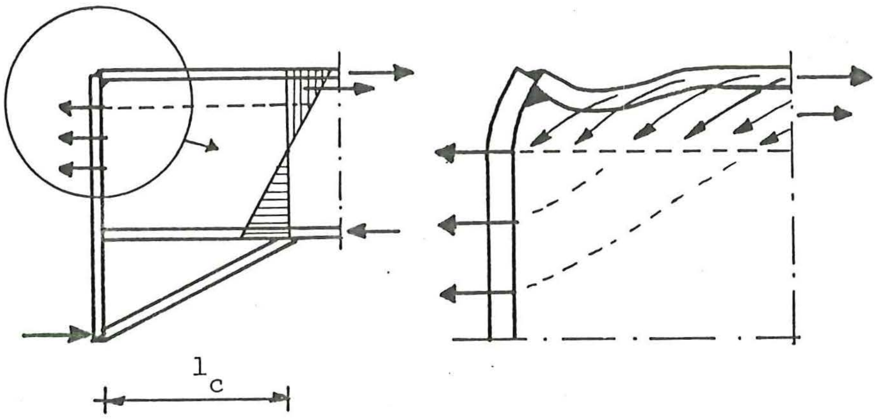


Fig. 13 : The force in the beam web is transferred by the beam web if the bending capacity of the end plate is insufficient.

The bending moment in the beam section at the end of the haunch ($\hat{M}_c$) is determined by:

- failure of the beam web due to shear over the length of the haunch in combination with
  - failure of the end plate due to bending and shear

The load transmitted by beam web and end plate in the ultimate limit state situation can be computed with:

$$\hat{F}_{ws} = \{ \frac{bt_e^2}{4y} + \frac{2t_w.t_e}{\sqrt{3}} + \frac{t_w.l_c}{\sqrt{3}} \} \sigma_y \qquad (6.1.1.)$$

where:

l_c = length of the haunch as indicated in figure 13.

and the other parameters as already mentioned,
The first term in formula 6.1.1. gives the bending moment capacity of the end plate, the second term is the load determined by shear of the end plate and the last term the shear force capacity of the beam web. Theoretically the second term is not possible because the complete section area of the end plate is already used for the bending capacity, but the resistance delivered by bending of the beam flange is neglected in this computation.
The results of tests 1, 4 and 5 of report 6-81-15 and test 4 of report 6-81-23 confirm the correctness of this approach.
The force $\hat{F}_{ws}$ should be in equilibrium with the resultant of the stresses in the beam flange and beam web above the first bolt row in the beam section at the end of the haunch (see figure 14).

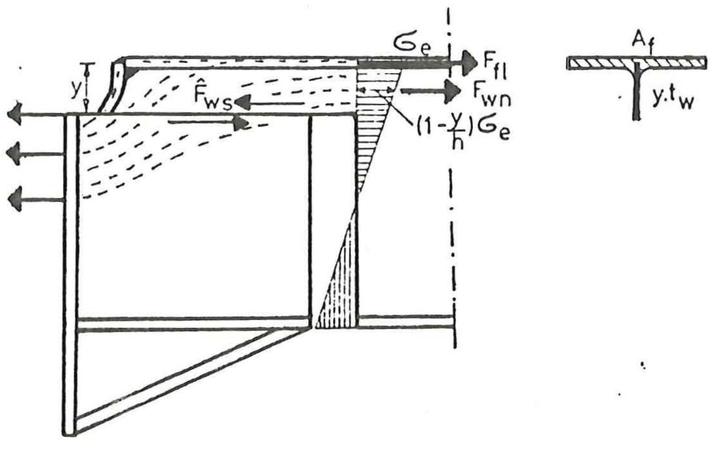


Fig. 14: In the beam section at the end of the haunch equilibrium should exist between the force transmitted by shear of the web and the resultant of stresses in the beam section.

Hence: $\hat{F}_{ws} = F_{fl} + F_{wn}$

If the situation is assumed, that the flange yields whereas the material below the flange behaves elasticly, then the forces $F_{fl} + F_{wn}$

can be approached with:

$$F_{fl} + F_{wn} = \{A_f + t_w.y\ (1 - \frac{y}{h})\}\ \sigma_y$$

where:

A_f = the section area of the beam flange

h = the beam depth

If : $\hat{M}_e = S.\sigma_y$

where: S = elastic section modulus of the beam, then the limit state bending moment $\hat{M}_c$ at the end of the haunch can be expressed as follows:

$$\hat{M}_c = \frac{0.58 t_w\ (l_c + 2t_e) + \frac{b t_e^2}{4y}}{A_f + t_w.y\ (1 - \frac{y}{h})}\ \hat{M}_e \tag{27}$$

This formula is rather complicated. That is why it is advised to check first whether:

$$\hat{M}_c \leqslant \frac{t_w . l_c}{2A_f}\ \hat{M}_e \quad \text{or} \tag{6.1.5.}$$

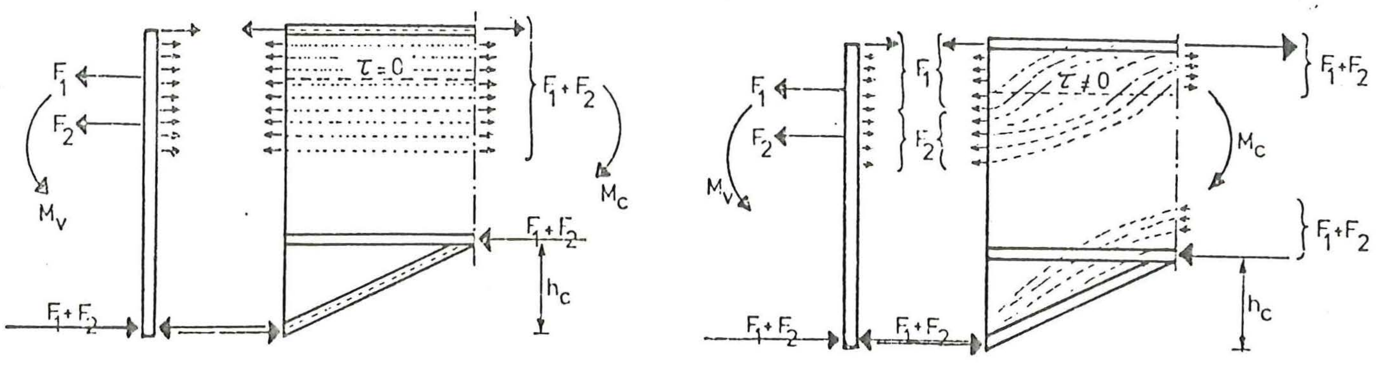
$$\hat{M}_c \leqslant \hat{M}_v - h_c . \Sigma\hat{F}_i \tag{6.1.6.}$$

If one of these conditions is fulfilled, then formula (27) does not govern.

Condition (6.1.5.) is developed from formula (27) by neglecting the cooperation of the end plate and assuming : $A_f + t_w.y = 1.16\ A_f$. The latter assumption can be made when normal dimensions and European rolled sections are applied.

Condition (6.1.6.) is explained with the help of figure 15. If at the end of the haunch the same force distribution exists as just behind the end plate, then shear forces do not exist in the cross section just above the first bolt row. With the assumed distribution of figure 14a:

$$\hat{M}_c = \hat{M}_v - (\hat{F}_1 + \hat{F}_2)\ h_c \tag{6.1.7.}$$



15a: Shear stress = 0

$$M_c = M_v - (F_1+F_2)\ h_c$$

15b: Shear stress ≠ 0

$$M_c > M_v - (F_1+F_2)\ h_c$$

Fig. 15 : Conditions for shear stresses in the beam web.

If: $M_c > \hat{M}_v - (\hat{F}_1+\hat{F}_2)\ h_c$ (6.1.8.)

then the level arm between tensile and compressive forces at the end of the haunch has to become larger than at the end plate. This is only possible when the bolt forces are transferred to the tensile flange via the beam web. Generally the situation of (6.1.8.) will apply because otherwise the function of the haunch is small.
With insufficient end plate thickness or haunch length the shear stresses will be too large.

If: $M_c \leqslant \hat{M}_v \cdot (\hat{F}_1+\hat{F}_2)\ h_c$ (6.1.9.)

then also shear stresses are present in the beam web, but these stresses are checked with the formula:

$$\tau = \frac{D}{t_w(h-2t_f)} \qquad (6.1.10)$$

where: D is the shear force in the beam
h is the beam depth
t_f is the flange thickness
t_w is the web thickness

According to the existing code of practice formula (6.1.10) will always be checked. That is why no special attention is given to the shear forces behind the flush end plate when a haunch is not applied.

Formula (27) indicates that the stress distribution in the cross section of the beam just at the end of haunch is also determined by the end plate thickness. This is illustrated by the result of test 4-18 and 4-21 of report 6-81-15.
The computation of these tests is given in appendix A4 of report 6-81-23. Figure 16 shows the result.

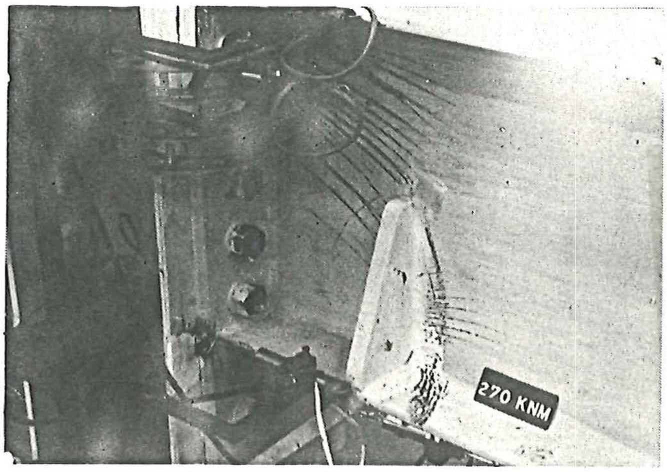


Fig. 16 : Despite the stiffening of the compressed flange, yielding occurred due to shear of the beam web.

Despite the stiffening of the compressed beam flange and beam web, failure of the beam flange occurred prematurely with respect to the computational result.
Actually the neutral axis shifted to the compressive side due to yielding of the beam web and end plate on the tensile side, which implied a larger loading of the compressed beam flange.

6.2. Haunch with flange

6.2.1. Joint of haunch- and beam flange

Formula (28)
The force transmitted by the haunch flange can be resolved in components as shown in figure 17.

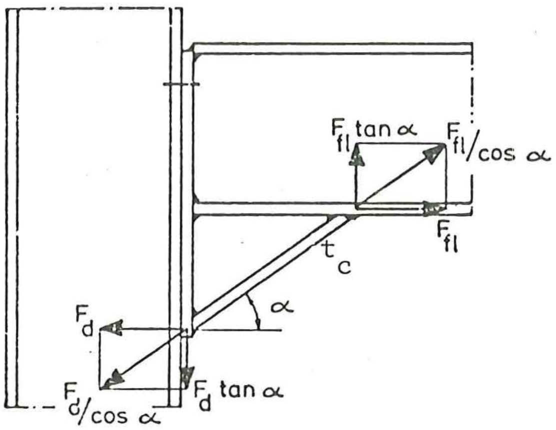


Fig. 17 : The force transmitted by the haunch flange is resolved in a web force and a flange force.

If all forces are expressed in the flange force, then the following relations exist:

flange force : F_{f1}
web force : F_{f1} tan α
haunch force : $F_{f1}/\cos$ α

The web force may cause yielding, buckling or crippling of the beam web when stiffeners are avoided. This can be checked with the same formula as used for the check of the column web |1|.
Hence:

$$F_{f1} \tan \alpha = \{5(t_f + r) + t_c\} \; t_w \sigma_e \qquad (6.2.1.)$$

where:
t_c = the thickness of the haunch flange

If in the longitudinal direction of the beam flange compressive stresses exist larger than half the yield strength, then the force $\hat{F}_{f1}$tan α should be reduced with a factor:

$$(1.25 - 0.5)\frac{|\sigma_{f1}|}{\sigma_y} \qquad (6.2.2.)$$

where: σ_{f1} = the maximum axial stress in the flange.

If $\{5(t_f + r) + t_c\}\ t_w = A_d$

and $b.t_f = A_f$

then formulae (6.2.1.) and (6.2.2.) can be combined to

$$\hat{F}_{f1} \tan \alpha = A_d\, \sigma_e \left(1.25 - 0.5\, \frac{|\hat{F}_{f1}|}{A_f \sigma_e}\right) \qquad (6.2.3.)$$

This may be rewritten as:

$$\hat{F}_{f1} = \frac{1.25 \cot \alpha}{\frac{A_f}{A_d} + 0.5 \cot \alpha}\ A_f . \sigma_e \qquad (6.2.4.)$$

When the left and right hand side of the latter equation is multiplied with the depth of the beam, then it follows that:

$$\hat{M}_c = \frac{1.25 \cot \alpha}{\frac{A_f}{A_d} + 0.5 \cot \alpha} \ . \ \hat{M}_e \qquad (28)$$

However, it should be checked whether:

$$\hat{M}_c \leqslant \frac{\cot \alpha}{\frac{A_f}{A_d}} \ . \ \hat{M}_e$$

to avoid an abusive application of the reduction factor.

6.2.2. Dimensions of haunch flange and fillet welds

The haunch flange should be dimensioned on a force $\hat{F}_{f1/\cos\alpha}$ (see figure 17).

If it is assumed that this force is uniformly distributed over the width of the haunch flange, then:

$$t_c \geqslant \frac{\hat{F}_{f1}}{b_c\, \sigma_y \cos\alpha} \qquad (6.2.5.)$$

Because:

$$\hat{F}_{f1} = \frac{\hat{M}_c}{\hat{M}_e}\ A_f\, \sigma_y$$

Formula (6.2.5.) can be rewritten as:

$$t_c \geqslant \frac{\hat{M}_c}{\hat{M}_e} \; \frac{A_f}{b_c \cos \alpha} \qquad (29)$$

If stiffeners are avoided, then the situation of figure 18 arises. The force is concentrated in the centre of the cross section because the flanges bend upwards.

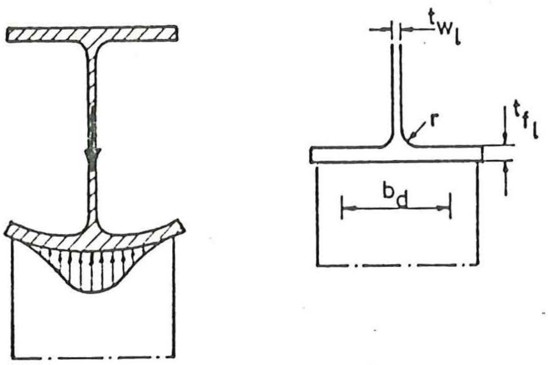


Fig. 18 : Without stiffeners, the force is concentrated in the centre of the haunch flange

The Dutch welding code of practice |13| gives a formula for the effective width of the haunch flange, viz.:

$$b_d = 10t_f + 2t_w$$

If this effective width is taken into account then formula (29) changes in:

$$t_c \geqslant \frac{\hat{M}_c}{\hat{M}_e} \; \frac{A_f}{(10t_f+2t_w)\cos \alpha} \qquad (6.2.7.)$$

However, when the haunch flange is compressed as shown in figure 18, then the haunch web is compressed too. That is why it may be assumed that a part of the force in the haunch flange is transmitted by the haunch web. (see figure 19)

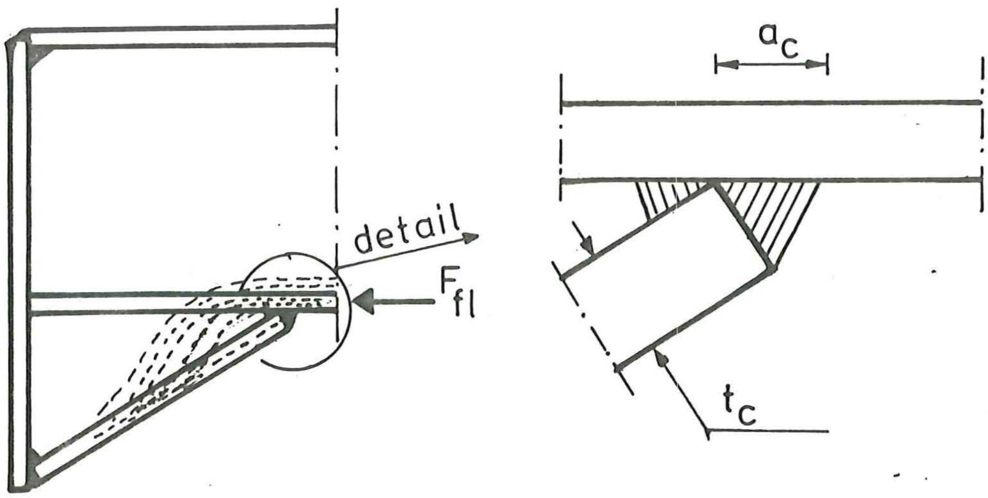


Fig. 19 The force in the beam web is partly transmitted by the haunch web into the haunch flange.

This is taken into account with a reduction factor 0.7,so that formula (6.2.7.) changes in:

$$t_c \geqslant \frac{\hat{M}_c}{\hat{M}_e} \quad \frac{0.7\ A_f}{(10t_f + 2t_w)\ \cos\alpha} \qquad (30)$$

To avoid buckling of the haunch flange it is required that:

$$t_c \geqslant \frac{b_c}{17} \qquad (31)$$

The welds between haunch- and beam flange should be dimensioned such that the parent material is not overloaded at the interface of weld and beam flange.

If the leg size is a_c (see figure 19) then the shear stress on the interface is :

$$\tau = \frac{\hat{M}_c}{\hat{M}_e} \quad \frac{0.7\ A_f\ \sigma_y}{(10t_f + 2t_w)a_c} \qquad (6.2.9)$$

The normal stress in the lateral direction to the flange is approached with:

$$\sigma = \frac{\hat{M}_c}{\hat{M}_e} \frac{0.7\ A_f\ \sigma_y\ \tan\alpha}{(10t_f+2t_w)a_c} \qquad (6.2.10)$$

Combination of the stresses according to the Huber-Hencky von Mises criterion gives:

$$a_c \geq \frac{\hat{M}_c}{\hat{M}_e} \frac{0.7\ A_f}{(10t_f+2t_w)} \sqrt{3+\tan^2\alpha} \qquad (32)$$

6.3. Haunch without a flange (table 9)

If the use of stiffeners is avoided, then the application of a haunch flange is strictly speaking irrational.
For, the distribution of the forces in beam and column web is perpendicular to that in the haunch flange.

In figure 20 a structure is shown, which is more in agreement with the current of the forces.
A thick web plate (25 mm) is welded between beam- and column web so that the transfer of forces can occur in the plane of the webs.

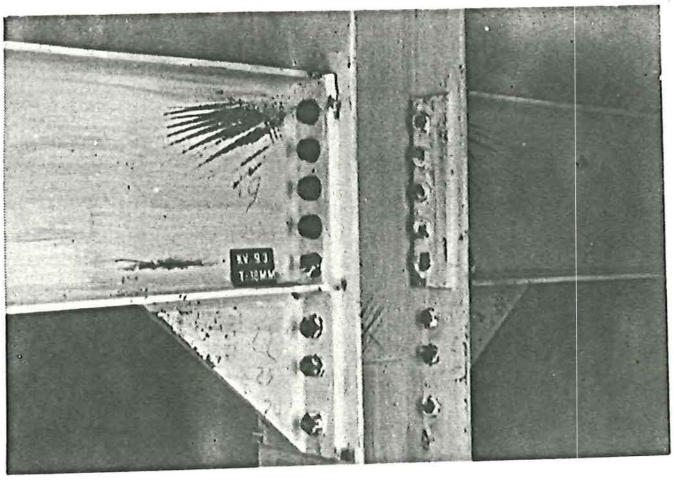
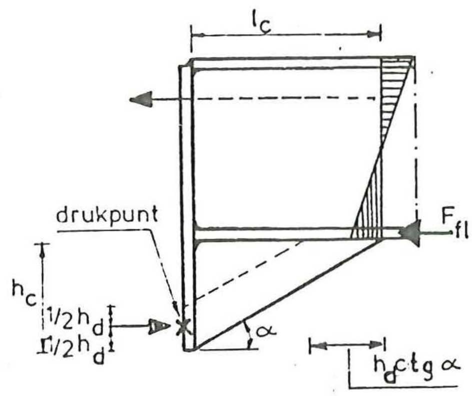


Fig. 20 : Connection with a haunch without a flange.

The computation of the web plate is based on the assumption of a depth $h_d \cos\alpha$ over which the force distributes in the haunch (see figure 21)



$h_d = 0{,}3$ à $0{,}5\ h_c$

Fig. 21 : Parameters used in the computation of the haunch without a flange.

More research is necessary to be able to give a solid advice for the magnitude of the depth h_d.
For the time being, the following assumptions are accepted for design purposes:

- $h_d = 0.3 - 0.5\ h_c$
- the resultant of the compressive force is located half-way the depth h_d.
- the transmission of the force between haunch and beam occurs over a length $h_d \cot\alpha$

Hence:

$$\hat{F}_{fl} = t_{wc} . h_d \cos^2\alpha\ \sigma_y \qquad (6.3.1.)$$

where: t_{wc} = the thickness of the haunch web.

Starting from the situation that the vertical component ($\hat{F}_{fl} \tan\alpha$) of the haunch force should be transmitted over the length $h_d \cot\alpha$ of the beam web it follows that:

$$t_{wc} \leqslant \frac{t_w}{\sin^2\alpha} \qquad (33)$$

It is allowed to apply a larger thickness than computed with formula (33), provided that the restriction of the thickness is taken into account when the design strength of the connection is computed, otherwise the yield strength of the beam web may be exceeded.

Formula (6.3.1.) can be rewritten as:

$$\frac{\hat{M}_c}{\hat{M}_e} = \frac{t_w . h_d \cos^2 \alpha}{A_f} \qquad (34)$$

If a specific bending moment should be reached then the thickness of the haunch web can be computed with formula (6.3.1.) rewritten as:

$$t_{wc} \geqslant \frac{\hat{M}_c}{\hat{M}_e} \quad \frac{A_f}{h_d \cos^2 \alpha} \qquad (35)$$

It is allowed to apply a larger thickness than with formula (33) is computed, in order to avoid buckling of the haunch web. The buckling mechanism can be avoided by choosing:

$$t_{wc} \geqslant \frac{h_d \cos \alpha}{8.5}$$

or by computing a web plate thickness with the buckling formula of a pin ended column with a width $h_d \cos \alpha$ and a length $l_c / \cos \alpha$.

The size of the fillet welds between haunch plate and beam determines mainly the costs and thus the attractiveness of this type of haunch.
If it is assumed that no gap exists between haunch plate and beam flange before the welding procedure is started, then contact stresses arise, due to shrinkage of the welds. In that case the welds are only loaded by shear forces.
If it is assumed that the shear force is distributed over the length l_c, then the weld size can be computed with:

$$a_{wc} \geqslant \frac{\hat{M}_c}{\hat{M}_e} \quad \frac{A_f}{1.65 l_c} \qquad (37)$$

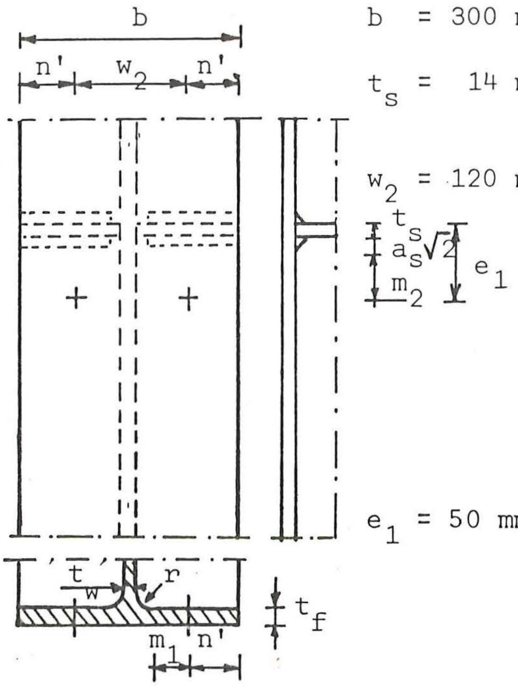
The results of tests 5 of reports 6-81-15 and 6-81-23 indicate that the approach of formula (37) may be followed, but more tests are necessary to show the adequacy.

7 EXAMPLES

Example 1

Formula (4a)

Column section HE 300-A



$b = 300$ mm, $t_w = 8{,}5$ mm, $t_f = 14$ mm, $r = 27$ mm

$t_s = 14$ mm, $a_s = 6$ mm

$$w_2 = 120 \text{ mm} \rightarrow m_1 = \frac{120 - 8.5 - 2 \times 27}{2} = 28.75 \text{ mm}$$

$$n' = \frac{300 - 120}{2} = 90 \text{ mm}$$

$$\lambda_1 = \frac{28{,}75}{28.75 + 90} = 0.24$$

$e_1 = 50$ mm $\quad t_s = 14 \text{ mm} \rightarrow m_2 = 50 - 14 - 6\sqrt{2} = 27.5$ mm

$$\lambda_2 = \frac{27.5}{28.75 + 90} = 0.23$$

From the chart in table 2 it follows that $\alpha = 4\pi$

$\hat{F}_{1f} = 2.4\pi.\frac{1}{4}.14^2.240 = 296$ kN

Example 2

Formulae (1),(2) and (3)

The situation is simular to that in example 1,so $\hat{F}_{1f} = 296$ kN with

bolts M24,grade 8.8

$2\hat{B}_t = 395$ kN $\rightarrow \dfrac{\hat{F}_{1f}}{2\hat{B}_t} = \dfrac{296}{395} = 0{,}74$

$\lambda_1 = 0{,}24$ en $\lambda_2 = 0{,}23$

$\Big\}\quad \hat{F}_1 = \hat{F}_{1f} = 296$ kN (1)

Bolts M20,grade 8.8 $\qquad 2\hat{B}_t = 275$ kN $\rightarrow \dfrac{\hat{F}_{1f}}{2\hat{B}_t} = \dfrac{296}{275} = 1{,}07$

$0{,}8 \leqslant \dfrac{\hat{F}_{1f}}{2\hat{B}_t} \leqslant 2, \longrightarrow \hat{F}_1 = \dfrac{2\hat{B}_t}{f_{pr}}$ (2) see the explanation in 0.4 and 1.1.2

$f_{pr} = \dfrac{6}{4 + 1{,}07} = 1{,}18$ (2a)

$\hat{F}_1 = \dfrac{2\hat{B}_t}{f_{pr}} = \dfrac{275}{1{,}18} = 232$ kN (the rotational capacity is not certain)

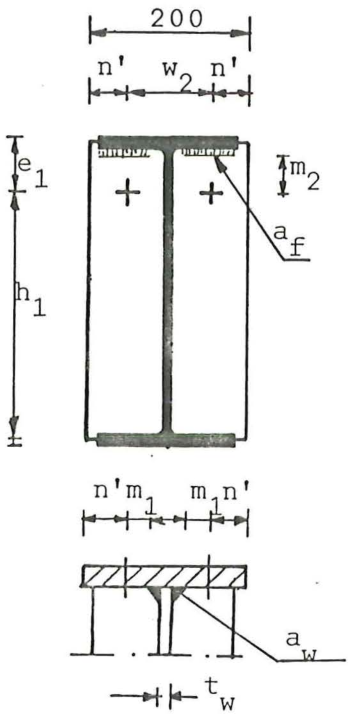
Bolts M16,grade 8.8 $\quad 2\hat{B}_t = 176 \text{ kN} \rightarrow \frac{\hat{F}_{1f}}{2\hat{B}_t} = \frac{296}{176} = 1.68$

$$0,8 \leq \frac{\hat{F}_{1f}}{2\hat{B}_t} \leq 2, \longrightarrow \hat{F}_1 = \frac{2\hat{B}_t}{f_{pr}} \quad (2) \qquad f_{pr} = \frac{6}{4 + 1.68} = 1.06 \quad (2a)$$

$\hat{F}_1 = \frac{176}{1.06} = 166 \text{ kN}$ (rotational capacity is small)

Example 3

Formulae (1) to (4) inclusive applied for the design of the flush end plate.The flush end plate is a component in the connection of a beam section IPE-400 and the column section HE-300A of example 1



$\hat{F}_1 = 296$ kN (see example 1) $\qquad e_1 = 50$ mm

$w = 120$ mm $\quad n' = \frac{200 - 120}{2} = 40$ mm $\quad a_f = 6$ mm

$a_w = 4$ mm $\quad t_w = 8.6$ mm $\quad t_f = 13.5$ mm

$$m_1 = \frac{120 - 8.6 - 8\sqrt{2}}{2} = 50 \text{ mm}$$

$$\lambda_1 = \frac{50}{50 + 40} = 0.56$$

$$m_2 = 50 - 13.5 - 6\sqrt{2} = 28 \text{ mm}$$

$$\lambda_2 = \frac{28}{50 + 40} = 0.31$$

From the chart in table 2 it follows that $\alpha = 10.5$

Now the end plate thickness should be designed such that the design strength $\hat{F}_1 = 296$ kN with bolts M24,grade 8.8,of which $2\hat{B}_t = 395$ kN

Hence: $\frac{\hat{F}_1}{2\hat{B}_t} = \frac{296}{393} = 0.75$

$$\left.\begin{array}{l} \lambda_2 > 0.5 \\ 0,67 \leq \frac{\hat{F}_{1f}}{2\hat{B}_t} \leq 2 \end{array}\right\} \quad f_{pr} = \frac{4}{2 + 0.75} = 1.45 \qquad (2b)$$

$$\hat{F}_1 = \frac{2\hat{B}_t}{f_{pr}} = \frac{396}{1.45} = 272 \text{ kN} \qquad (2)$$

This value is too low, thus the end plate thickness should be increased so that: $f_{pr} = \frac{395}{296} = 1.34$

Hence: $\frac{4}{2 + \frac{\hat{F}_{1k}}{2\hat{B}_t}} = 1,34 \quad (2b) \rightarrow \quad \frac{\hat{F}_{1k}}{2\hat{B}_t} = 1 \rightarrow \hat{F}_{1k} = 396\ \text{kN}$

Now the end plate thickness can be computed with formula (4b).

$$t_e = 2\sqrt{\frac{396000}{2 * 10.5 * 240}} = 17.72\ \text{mm} \longrightarrow t_e = 18\ \text{mm}$$

However, with this end plate thickness all rotational capacity should be delivered by the deformation of the column flange, because the bolt will fail before yielding of the end plate occurs.

Example 4

Application of formulae (1) to (4) inclusive for the design of an end plate which can give rotational capacity. In example 3, the end plate thickness was adapted to the force which could be transmitted by the column flange. There the rotational capacity of the end plate was not important because the bolts had sufficient strength to cause yielding of the column flanges.
In this example the end plate has to deliver the rotational capacity. The situation is simular to that of example 3. Thus the end plate is a component of a connection between a column section HE 300-A and a beam section IPE 400 , however now with bolts M20, grade 8.8, at a distance w_2=105 mm and e_1=45 mm.
The computation of the column flange is as follows:

$$\left.\begin{aligned} m_1 &= \frac{105 - 8.5 - 2 * 27}{2} = 21.25 \text{ mm} \\ n' &= \frac{300 - 105}{2} = 97.5 \text{ mm} \\ \lambda_1 &= \frac{21.25}{21.25+97.5} = 0.18 \\ m_2 &= 45 - 14 - 7\sqrt{2} = 21.1 \text{ mm} \\ \lambda_2 &= \frac{21,1}{21.25+97.5} = 0.18 \end{aligned}\right\} \quad \alpha = 4\pi$$

Formul (4a) $\hat{F}_{1f} = 8\pi * \frac{1}{4} * 14^2 * 240 = 296$ kN

$$\frac{\hat{F}_{1f}}{2\hat{B}_t} = \frac{296}{275} = 1.08 \rightarrow 0.8 \leqslant \frac{\hat{F}_{1f}}{2\hat{B}_t} \leqslant 2 \rightarrow f_{pr} = \frac{6}{4+1.08} = 1.18 \quad (2a)$$

$\hat{F}_1 = \frac{275}{1.18} = 233$ kN, but the magnitude of the rotational may be small

With one bolt row the design strength of connection becomes

$$\hat{M}_v = 233 * (0,4 - 0.045 - \frac{0.014}{2}) = 81 \text{ kNm}$$

The plastic moment of a section IPE 400 is M_p = 314 kNm.

Thus the connection cannot transmit the plastic moment of the connected beam, which implies that the connection should have sufficient rotational capacity, which in this case should be delivered by the end plate.

Computation of the end plate:

$$m_1 = \frac{105 - 8.6 - 8\sqrt{2}}{2} = 42.54 \text{ mm}$$

$$m_2 = 45 - 14 - 6\sqrt{2} = 22.5 \text{ mm}$$

Assume that a small width of end plate is chosen with n' = 1.5 times the boltdiameter. In that case n'= 30 mm. and

$$\lambda_1 = \frac{42.54}{42.54+30} = 0.59$$

$$\lambda_2 = \frac{22.5}{42.54+30} = 0.31$$

Because $\lambda_1 > 0.5$ the end plate thickness should be chosen such, that

$\frac{\hat{F}_{1k}}{2B_t} \leq 0.67$ in order to reach that the end plate gives sufficient rotational capacity. (see graph in table 1)

But in that case : $\hat{F}_1 \leq 0.67 * 275 = 184$ kN.

In order to reach a larger design strength of the connection, a larger width of the end plate is chosen.

$$n' = 50 \text{ mm} \rightarrow \left.\begin{array}{l} \lambda_1 = \frac{42.54}{42.54+50} = 0.46 \\ \lambda_2 = \frac{22,5}{42,5+50} = 0.24 \end{array}\right\} \quad \alpha = 4\pi$$

Now: $\hat{F}_{1e} \leq 0.8 * 275 = 220$ kN

Hence: $t_e \leq 2\sqrt{\frac{220000}{2*4*240}} = 12$ mm (4b)

In that case $t_e = 12$ mm

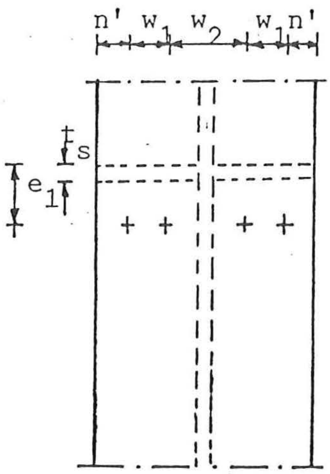
and $\hat{M}_v = 220 * (0.4-0.045- \frac{0.014}{2}) = 77$ kNm

Thus, in order to reach rotational capacity, the end plate thickness should be limited; with the concequence that the design strength is lower than the column flange capacity.

Example 5

Formulae (5) to (7) inclusive and applied for a connection with bolts added in the first bolt row. Column section HE 300A, bolts M20, grade 8.8.

b = 300 mm t_w = 8,5 mm t_f = 14 mm r = 27 mm t_s=14 mm a_s = 7 mm



$w2 = 105$ mm

$e_1 = 45$ mm

$$m_1 = \frac{105 - 8.5 - 2 * 27}{2} = 21.25 \text{ mm}$$

$$n' = \frac{300 - 105}{2} = 97{,}5 \text{ mm}$$

$$\lambda_1 = \frac{21.25}{21.25 + 97.5} = 0\ 18$$

$$m_2 = 45 - 14 - 7\sqrt{2} = 21.1 \text{ mm}$$

$$\lambda_2 = \frac{21.1}{21.25 + 97.5} = 0.18$$

$\alpha = 4\pi$

Formula (4a) $\hat{F}_{1f} = 8\pi * \frac{1}{4} * 14^2 * 240 = 296$ kN

Formula (2) $2\hat{B}_t = 275$ kN $\rightarrow \hat{F}_1 = 232$ kN (see example 2)

$$b_m = b - w2 - 2m_{1.2} = 300 - 105 - 2 * 21.2 = 152{,}6 \text{ mm}$$

$$\hat{F}_{1ft} = \frac{2b_m\, t_f^2 \sigma_e}{4m_2} \qquad (5)$$

$$\hat{F}_{1ft} = \frac{152.6 * 14^2 * 240}{4 * 21.1} = 170 \text{ kN}$$

$$\hat{F}_{1ft} = \frac{b_m\, t_f^2 \sigma_e + 10\hat{B}_t m_2}{9\ m_2} \qquad (6)$$

$$\hat{F}_{1ft} = \frac{152.6 * 14^2 * 240 + 10 * 137250 * 21.1}{9 * 21.1} = 190 \text{ kN}$$

$\hat{F}_{1t}$ = 170 kN

$$\hat{F}_{1\ som} = 232 + 170 = 402 \text{ kN} \qquad (10)$$

In example 4 it is concluded that bolts M20 can not cause yielding of the column flange in order to achieve sufficient rotational capacity. That was why a 12 mm thick end plate was applied. According to formula (5) the added bolts cause yielding of the column flange before bolt failure occurs. It is questionable whether the capacity of the added bolts is also sufficient to cause yielding of the column flange at the bolts in the corner. The test results known are insufficient to answer this question positively. That is why it is advised here to maintain an end plate thickness of 12 mm to accomplish sufficient deformation capacity from the end plate. In that case the computation of the end plate becomes (see example 4 for the data) as follows:

$$b_m = b - w2 - 2m_{1,2} = 300 - 105 - 2 * 22.5 = 150 \text{ mm}$$

$$\hat{F}_{1et} = \frac{2b_m t_e^2 \sigma_e}{4m_2} \qquad (5)$$

$$\hat{F}_{1et} = \frac{2.150.12^2.240}{4.22.5} = 115 \text{ kN}$$

$$\hat{F}_{1et} = \frac{b_m.t_e^2.\sigma_e + 10\hat{B}_t.m_2}{9.m_2} \qquad (6)$$

$$\hat{F}_{1et} = \frac{150.12^2.240+10.137250.22.5}{9.22.5} = 178 \text{ kN}$$

$$\hat{F}_{1et} = \frac{b_m(t_e^2 + t_{fl}^2)\,\sigma_e}{4y} \qquad (8)$$

$$\hat{F}_{1et} = \frac{150(12^2+13.5^2)\ 240}{4\ .\ 38} = 77 \text{ kN}$$

$$\hat{F}_{1et} = \frac{b_m.t_{fl}^2.\sigma_e + 10\hat{B}_t.y}{9y} \qquad (9)$$

$$\hat{F}_{1et} = \frac{150.13.5^2.240+10.137250.38}{9\ .\ 38} = 172 \text{ kN}$$

Thus, it appears that formula (8) governs with $\hat{F}1_t = 77$ kN, so that $\hat{F}1_{sum} = 220 + 77 = 297$ kN.

Example 6

Formulae (8), (9) and (10) (added bolts with an end plate on top of the column), see figure.
The situation is similar to that of example 5, with the exception that the column does not continue.

$e_1 = 45$ mm $\quad t_d = 14$ mm $\quad y = e_1 - \frac{1}{2}t_d$

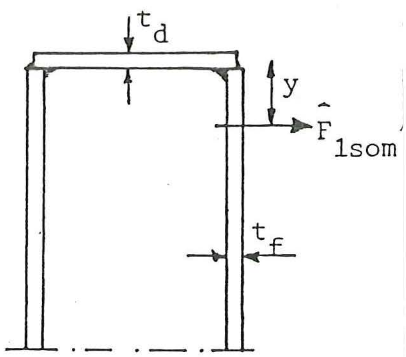
$y = 45 - 7 = 38$ mm

$$\hat{F}_{1ft} = \frac{(b - w2 - 2m_{1,2})(t_d^2 + t_f^2)\,\sigma_e}{4y} \qquad (8)$$

$$\hat{F}_{1ft} = \frac{(300 - 105 - 2 * 21.1)(14^2 + 14^2) * 240}{4 * 38} = 94 \text{ kN}$$

$$\hat{F}_{1sum} = 232 + 94 = 326 \text{ kN}$$

This latter value is considerably lower than the value computed in example 5 for the column flange. However, this is not serious because the end plate is the failing component.



Example 7

Formulae (11) and (12) (design forces of the second boltrow).
A reinforcement of the connection with more boltrows is only possible when either:

- the boltforce capacity is sufficient to cause yielding of the column flange (or end plate) at the first boltrow.

or:

- The bolts in the first boltrow fail before they can cause yielding of the column flange or end plate and the value computed with formula (12) is larger than that computed with formula (13) which is based on a linear proportional bolt force distribution.

But in the latter case the connection has not sufficient rotational capacity and either:

- the connection can only be applied in statically determinated structures or
- the design strength of the connection ought to be larger than the plastic moment of the connected beam.

In this example the boltforce capacity is sufficient to cause yielding of the column flange at the first boltrow.
In example 8 the other possibility is given. The situation for the first boltrow is similar to that of example 1.
The vertical pitch between the first and second boltrow is 70 mm.
Hence:

w_2 = 120 mm ; p = 70 mm ; m_1 = 28.75 mm ; m_2 = 27.5 mm ,

n' = 90 mm ; $\alpha = 4\pi$; $\hat{F}_1$ = 296 kN ; $m_{1,2}$ = 27.5 mm ,

m = 28.75 + 0.2 x 27 = 34.15 mm.

$$b_m = p - m_{1,2} + 2m + 0.625n' = 70 - 27.5 + 2 * 34.15 + 0.625 * 90 = 167.05$$

$$\hat{F}_{2f} = \frac{b_m \cdot t_f^2 \cdot \sigma_e}{m} \qquad (11)$$

$$\hat{F}_{2f} = \frac{167.05 * 14^2 * 240}{34.15} = 230 \text{ kN}$$

$$\hat{F}_{2f} = \frac{b_m t_f^2 \cdot \sigma_e + 4\hat{B}_t}{2(m+n)} \qquad (12)$$

$$\hat{F}_{2f} = \frac{167.05 * 14^2 * 240 + 791000 * 1.25 * 34.15}{2 * 2.25 * 34.15} = 271 \text{ kN}$$

Thus $\hat{F}_2$ = 230 kN

Condition is that: $2m+0.625n' < \frac{1}{2}h_2$ thus $h_2 > 2(2m + 0.625n') =$
$= 4 * 34.15 + 1.25 * 90 = 249$ mm

With a beam section IPE-400 and the end plate of example 3 the situation is as follows:

w2 = 120 mm p = 70 mm $m = 50 + 0.2 - 4\sqrt{2} = 51$ mm

n' = 40 mm n = 40 mm

$$b_m = p - m_{1,2} + 2m + 0.625n' = 70 - 50 + 2 \cdot 51 + 0.625 \cdot 40 = 147 \text{ mm}$$

$$\hat{F}_{2e} = \frac{b_m \cdot t_e^2 \cdot \sigma_e}{m} \qquad (11)$$

$$\hat{F}_{2e} = \frac{147 \ . \ 18^2 \ . \ 240}{51} = 224 \text{ kN}$$

$$\hat{F}_{2e} = \frac{b_m t_e^2 \sigma_e + 4\hat{B}_t \ . \ n}{2(m+n)} \qquad (12)$$

$$\hat{F}_{2e} = \frac{147 \ . \ 18^2 \ . \ 240 + 791000 * 1{,}40}{2(51+40)} = 236 \text{ kN}$$

Conditions are that: $h_2 > 4m + 1.25n' = 4 * 51 + 1.25 * 40 = 254$ mm and $\hat{F}_1 = \hat{F}_{1k}$.

The situation is that: $h_2 = 400 - 50 - 70 - 13.5 = 267$ mm and $\hat{F}_1 = \hat{F}_{1f} = 296$ kN. Thus the latter conditions are satisfied. If this would not be the case, then the next step would be the check whether:

$$\hat{F}_{2e} \geqslant \frac{h_1 - p}{h_1} \ 2\hat{B}_t \qquad (13)$$

If the latter condition would not be satisfied then the end plate thickness might be enlarged to accomplish the linear proportional bolt force distribution for the end plate side. Here, this is possible because the rotational capacity is delivered by yielding of the column flange. The latter measure would not be possible when the end plate had to deliver the rotational capacity. The following is valid for the end plate side:

$\hat{F}_1 = \hat{F}_{1f} = 296$ kN $\qquad h_1 = 337$ mm $\qquad$ and

$\hat{F}_2 = 230$ kN $\qquad h_1 = 267$ mm $> 4m + 1.25n' = 249$ mm

In this example the design strength is reached with yielding of the column flange at the first boltrow and yielding of the end plate at the second bolt row whereas the bolts behave elasticly.

$$\hat{M}_v = 296 * 0.34 + 224 * 0.27 = 161 \text{ kNm.}$$

Example 8

Application of formulae (11) and (12) whereas the end plate has to deliver the rotational capacity. Beam section IPE 400, column section HE 300A, with end plate of example 4, where $t_e = 12$ mm and $\hat{F}_1 = 220$ kN determined by yielding of the end plate.

Further the following is valid for the end plate.

$m_1 = 42.54$ mm

$m_2 = 22.5$ mm

$n = 50$ mm

$m = 42.54 + 0.2 * 4\sqrt{2} = 43.7$ mm

$b_m = p - m_{1,2} + 2m + 0.625n' = 60 - 22.5 + 2 * 43.7 + 0.625 * 50 = 156$ mm

$$\hat{F}_{2e} = \frac{b_m \cdot t_e^2 \cdot \sigma_e}{m} \qquad (11)$$

$$\hat{F}_{2e} = \frac{156 \cdot 12^2 \cdot 240}{43.7} = 123 \text{ kN}$$

$$\hat{F}_{2e} = \frac{b_m \cdot t_e^2 \cdot \sigma_e + 4\hat{B}_t \cdot n}{2(m+n)} \qquad (12)$$

$$\hat{F}_{2e} = \frac{156 \cdot 12^2 \cdot 240 + 550000 \cdot 50}{2(43.7+50)} = 176 \text{ kN}$$

$4m + 1.25n' = 4 * 43.7 + 1.25 * 50 = 237.3$ mm

$h_2 = 400 - 45 - \frac{14}{2} - 60 = 288$ mm

Thus $h_2 = 288$ mm $4m + 1.25n' = 237$ mm, so the yield line mechanism can develop in the end plate.

Conclusion: $\hat{F}_2 = 123$ kN as far as the end plate side is concerned.

For the column flange side the following is valid.

$$\left.\begin{array}{l} m_1 = 21.25 \text{ mm} \\ n' = 97.5 \text{ mm} \\ m_2 = 21.1 \text{ mm} \end{array}\right\} \quad \alpha = 4\pi$$

$m = m_1 + 0.2r = 21.25 + 0.2 * 27 = 26.7$ mm

$n = 1.25 * 26.7 = 33.4$ mm

$4m + 1.25n' = 4 * 26.7 + 1.25 * 97.5 = 228$ mm

$h_2 = 278$ mm $4m + 1.25\,n' = 228$ mm en $\hat{F}_1 = \hat{F}_{1e}$

Thus also in the column flange the yield line mechanism can develop because the distance h_2 is sufficient and the end plate yields at the first boltrow (see example 4).

$b_m = p - m_{1,2} + 2m + 0.625n' = 60 - 21.2 + 2 \ . \ 26.7 + 0.625 \ . \ 97.5 = 153$ mm

$$\hat{F}_{2f} = \frac{b_m \ . \ t_f^2 \ . \ \sigma_e}{m} \qquad (11)$$

$$\hat{F}_{2f} = \frac{153 \ . \ 14^2 \ . \ 240}{26.7} = 270 \text{ kN}$$

$$\hat{F}_{2f} = \frac{b_m \ . \ t_f^2 \ . \ \sigma_e + 4\hat{B}_t \ . \ n}{2(m+n)} \qquad (12)$$

$$\hat{F}_{2f} = \frac{152 \ . \ 14^2 \ . \ 240 + 550000 \ . \ 33.4}{2(26.7 + 33.4)} = 212 \text{ kN}$$

Conclusion: $\hat{F}_2 = 123$ kN determinated by the end plate side according to formula (11).

$\hat{M}_v = 220 * 0.35 + 123 * 0.29 = 113$ kNm.

If the end plate is made thicker than 12 mm, then the end plate does not yield at the first boltrow and $\hat{F}_1 = \hat{F}_{1f} = 232$ as computed with formula (2) in example 4.

In that case the second boltrow may also be taken into account because:

$$\hat{F}_{2f} = 212 \text{ kN} > \frac{h_2}{h_1} \ \hat{F}_1 = \frac{0.29}{0.35} * 232 = 192 \text{ kN}$$

and as far as the strength of the column flange is concerned a linear bolt-force distribution may be assumed.

However, then the end plate must be able to transmit a force of 192 kN at the second boltrow, thus:

$$t_e = \sqrt{\frac{\hat{F}_2 \ . \ m}{b_m \ . \ \sigma_e}} \qquad \text{(as follows from formula (11))}$$

$$t_e = \sqrt{\frac{192000 \ . \ 43.7}{156 \ . \ 240}} = 15 \text{ mm}$$

At this end plate thickness: $\hat{F}_{2e} = 192$ kN, according to formula (12).

In this case: $\hat{M}_v = 232 * 0.35 + 192 * 0.29 = 137$ kNm.

This design strength of the connection is smaller than the plastic moment of the connected beam and the connection has no rotational capacity. Hence, this connection with an end plate thickness of 15 mm and bolts M20, grade 8.8. can not be applied in a statically indeterminated structure.

Now an end plate thickness of 12 mm is assumed again. If a third bolt row is added, the following computation is valid for the end plate side:

$b_m = p = 60$ mm

$$\hat{F}_{3e} = \frac{b_m \, t_e^2 \, \sigma_e}{m} \qquad (11)$$

$$\hat{F}_{3e} = \frac{60 \cdot 12^2 \cdot 240}{43.7} = 48 \text{ kN}$$

$$\hat{F}_{3e} = \frac{b_m \cdot t_e^2 \, \sigma_e + 4\hat{B}_t \cdot n}{2(m+n)} \qquad (12)$$

$$\hat{F}_{3e} = \frac{60 \cdot 12^2 \cdot 240 + 550000 \cdot 50}{2(43.7 + 50)} = 158 \text{ kN}$$

provided that: $h_3 = 228$ m $> 1.25n' = 4 * 43.7 + 1.25 * 50 = 237.3$ mm

The latter is not the case thus: $\hat{F}_3 = 0$, unless:

$$\hat{F}_{3e} > \frac{h_1 - 2p}{h} \; 2\hat{B}_t = \frac{228}{348} \cdot 275 = 180 \text{ kN}$$

Again this is not true and the end plate can not be made thicker for the sake of rotational capacity, thus $\hat{F}_3 = 0$.

<u>Summary of the results of examples</u>

Column HE-300A, Beam IPE-400

Example	Bolts	w2 mm	Rot. capacity obtained from	t_e mm	$\hat{F}_1$ kN	$\hat{F}_2$ kN	$\hat{M}_v$ kNm
3	M24 8.8	120	column flange	18	296	-	99
4	M20 8.8	105	end plate	12	220	-	77
7	M24 8.8	120	column flange	18	296	224	161
8	M20 8.8	105	end plate	12	220	123	113

Example 9

Formulae (14), (15) and (16) (column web at first boltrow) with a column which continues beyond the connection. Further the situation is similar to that in example 1. Thus a column section HE-300A with bolts M24, grade grade 8.8.

$$z = t_f \sqrt{\frac{b}{t_w}} = 14 \sqrt{\frac{300}{8.5}} = 83 \text{ mm} \qquad (14)$$

m_2 = 27.5 mm < z = 83 mm, dus formule (15) overslaan.

$$\hat{F}_{1w} = \{(d_n + t_f \sqrt{\frac{b}{t_w}} + \frac{m_2}{2})\ t_w + \frac{bt_f^2}{2m_2}\}\ \sigma_e \qquad (16)$$

$$\hat{F}_{1w} = \{(24 + 14 \sqrt{\frac{300}{8.5}} + \frac{27.5}{2})\ 8.5 + \frac{300 * 14^2}{2 * 27.5}\}\ 240 = 503 \text{ kN}$$

But according to formula (1), $\hat{F}_1$ = 296 kN (see example 1).

Example 10

Formula (17) (column web with continuing column).
Situation similar to examples 1 and 9. Thus column section HE-300A with bolts M24, grade 8.8. and w2 = 120 mm.

m_1 = 28.75 mm   n' = 90 mm m_2 = 28.8 mm

$$\hat{F}_{1w} = \{(2m + 0.625\ n' + \frac{m_2}{2})\ t_w + \frac{bt_f^2}{2m_2}\}\ \sigma_e \qquad (17)$$

$$m = m_1 + 0.2 * r = 28.75 + 0.2 * 27 = 34.15 \text{ mm}$$

$$\hat{F}_{1w} = \{(2 * 34{,}15 + 0{,}625 * 90 + \frac{27.5}{2})\ 8.5 + \frac{300 * 14^2}{2 * 27.5}\}\ 240 = 539 \text{ kN}$$

It appears that formula (16) gives a lower value than formula (17) in this case. The difference would be larger when the value m_1 would be taken larger.

Example 11

Formulae (18) and (19) (column web of a column with an end plate).
The situation is similar to that in examples 1, 9 and 10. With the exception that the column does not continue. The end plate has a thickness $t_d = 14$ mm. Here, formula (18) is applied only, because in the continuing column formula (16) governed.

$$\hat{F}_{1w} = \left\{ (d_n + t_f \sqrt{\frac{b}{t_w}} + \frac{y}{2})\, t_w + \frac{b_f t_f^2 + b_d t_d^2}{4y} \right\} \sigma_e \qquad (18)$$

$$\hat{F}_{1w} = \left\{ (24 + 14 \sqrt{\frac{300}{8.5}} + \frac{50 - 7}{2})\, 8.5 + \frac{300 * 14^2 + 300 * 14^2}{4(50 - 7)} \right\} 240 = 427 \text{ kN}$$

The latter value is lar less than $\hat{F}_{1w} = 503$ kN computed with formula (16) as valid for a continuing column (see example 9).

Example 12

Formula (20) (column web when more boltrows are applied). The situation is similar to that in example 7 with a column section HE-300A which continues. Bolts M24, grade 8.8. Beam section IPE-400.

Formula (14) $z = 83.15$ mm $> m_2 = 27.5$

↓

Conclusion: neglect formula (15) — see example 9

Formula (16) → $\hat{F}_{1w} = 503$ kN — see example 9

Formula (17) → $\hat{F}_{1w} = 539$ kN — see example 10

$\hat{F}_{pw} = p \cdot t_w \sigma_e \rightarrow \hat{F}_{pw} = 70 * 8.5 * 240 = 142$ kN

Thus: $\hat{F}_{1w} + \hat{F}_{pw} = 503 + 142 = 645$ kN

Formulae (1) to (3) inclusive → $\hat{F}_1 = 296$ kN — see examples 1 and 2

Formulae (11) to (13) inclusive → $\hat{F}_2 = \underline{224}$ kN — see example 7

Hence: $\hat{F}_1 + \hat{F}_2 = 520$ kN, thus okay. (20)

Example 13

Check of the strength capacity of the beam web behind the flush end plate. This check is carried out for the end plates of examples 3 and 7 where t_e = 18 mm and the end plates of examples 4 and 8 where t_e = 12 mm because the latter end plates had to deliver the rotational capacity of the connection.

End plates of examples 3 and 7

t_e = 18 mm $\hat{F}_1$ = 297 kN (column flange fails)

$\hat{F}_2$ = 224 kN (end plate fails)

m_1 = 50 mm m_2 = 28 mm e_1 = 50 mm   m = 51 mm

t_f = 13.5 mm b_e = 200 mm   b = 180 mm   n' = 40 mm

t_w = 8.6 mm   p = 70 mm

} see examples 3 and 7.

$$z = t_e \sqrt{\frac{b_e}{t_w}} \quad (14)$$

$$z = 18 \sqrt{\frac{200}{8.6}} = 86.8 \text{ mm}$$

$$z = 86.8 > m_2 = 28 \text{ mm}$$

Formulae (16) and (17) combined with formula (20) give higher values than the combinations of formula (20) with formulae (18) and (19) because:

t_e = 18 mm > t_f = 13.5 mm en y = 50 - 7 = 43 mm > m_2 = 28 mm

That is why the latter combinations are checked only.

$$\hat{F}_{pw} + \hat{F}_{1w} = \{(p + d_n + t_e\sqrt{\frac{b_e}{t_w}} + \frac{y}{2})\, t_w + \frac{b_e t_e^2 + bt_f^2}{4y}\}\sigma_e \quad (20) + (18)$$

$$\hat{F}_{pw} + \hat{F}_{1w} = \{(70 + 24 + 18\sqrt{\frac{200}{8.6}} + \frac{43}{2})\, 8.6 + \frac{200.18^2 + 180.13.5^2}{4\ .\ 43}\}\ 240 = 554 \text{ kN}$$

$$\hat{F}_{pw} + \hat{F}_{1w} = \{(p + 2m + 0.625n' + \frac{y}{2})\, t_w + \frac{b_e t_e^2 + bt_f^2}{4y}\}\sigma_e \quad (20) + (19)$$

$$\hat{F}_{pw} + \hat{F}_{1w} = \{(70 + 2\ .\ 51 + 0,625\ .\ 40 + \frac{43}{2})\, 8,6 + \frac{200.18^2 + 180.13.5^2}{4\ .\ 43}\}240 =$$

$$= 587 \text{ kN}$$

The requirements of formulae (18) and (20) are fulfilled, because:

$\hat{F}_1 + \hat{F}_2 = 296 + 224 = 520 \text{ kN} < \hat{F}_{pw} + \hat{F}_{1w} = 554 \text{ kN}$

End plates of examples 4 and 8.

$t_e = 12$ mm $\quad \hat{F}_1 = 220$ kN (end plate fails)

$\hat{F}_2 = 123$ kN (end plate fails)

$m_1 = 42.5$ mm	$m_2 = 22.5$ mm	$e_1 = 45$ mm	$m = 43.7$ mm
$t_f = 13.5$ mm	$b_e = 205$ mm	$b = 180$ mm	$n' = 50$ mm
$t_w = 8.6$ mm	$p = 60$ mm	$y = 45-7= 38$ mm	

see examples 4 and 8.

$$z = t_e \sqrt{\frac{b_e}{t_w}} \qquad (14)$$

$$z = 12 \sqrt{\frac{205}{8.6}} = 59 \text{ mm}$$

$$z = 59 > m_2 = 22.5 \text{ mm}$$

Now it is not certain which formulae give the determinant value, so that all combinations of formula (20) with the other formulae will be checked.

$$\hat{F}_{pw} + \hat{F}_{1w} = \{(p + d_n + t_e\sqrt{\frac{b_e}{t_w}} + \frac{m_2}{2})\, t_w + \frac{b_e t_e^2}{2m_2}\}\sigma_e \qquad (20) + (16)$$

$$\hat{F}_{pw} + \hat{F}_{1w} = \{(60 + 20 + 12\sqrt{\frac{205}{8.6}} + \frac{22.5}{2})\; 8.6 + \frac{205.12^2}{2\,.\,22.5}\}\; 240 = 468 \text{ kN}$$

$$\hat{F}_{pw} + \hat{F}_{1w} = \{(p + 2m + 0.625n' + \frac{m_2}{2})\,\sigma_w + \frac{b_e t_e^2}{2m_2}\}\sigma_e \qquad (20) + (17)$$

$$\hat{F}_{pw} + \hat{F}_{1w} = \{(60 + 2.\ 43.7 + 0.625\ .\ 50 + \frac{22.5}{2})\; 8.6 + \frac{205.12^2}{2.\ 22.5}\}\; 240 =$$

$$= 549 \text{ kN}$$

The combination of formulae (20) and (18) gives:

$$\hat{F}_{pw} + \hat{F}_{1w} = \{(60 + 2\ .\ 43.7 + 0.625\ .\ 50 + \frac{43}{2})\; 8.6 + \frac{205.12^2+180.13.5^2}{4\ .\ 38}\}\; 220 =$$

$$= 512 \text{ kN}$$

The web of the beam is not the determinant component because:

$$\hat{F}_{pw} + \hat{F}_{1w} = 424 \text{ kN} > \hat{F}_1 + \hat{F}_2 = 220 + 123 = 343 \text{ kN}$$

Example 14

The sum of the design forces of the tension zone of the connection of example 7 is:

$$\hat{F}_1 + \hat{F}_2 = 296 + 224 = 520 \text{ kN}$$

According to formula (3.5.2.) a column section HE-300A has a design strength for the compression side of the connection $\hat{F}_d = 418$ kN.

Hence: $\hat{F}_2 = 418 - 296 = 122$ kN.

In this case: $\hat{M}_v = 296 * 0.34 + 122 * 0.27 = 134$ kNm.

The sum of the design forces of the tension zone of the connection of example 8 is:

$$\hat{F}_1 + \hat{F}_2 = 220 + 123 = 343 \text{ kN.}$$

In this case a reduction is not necessary to fulfil the requirement of equilibrium.

Example 15

Application of formulae (22) to (25) inclusive for the determination of the dimensions of stiffeners and welds if rotational capacity is not required.

For this example the situation of example 5 is chosen. Thus a column section HE-300A with 4 bolts in the first boltrow.

The bolts in the corner have a design strength $\hat{F}_1 = 232$ kN and the end plate has not been chosen such that this can give rotational capacity, thus: $f_c = 1$.

The added bolts caused yielding in the column flange at $\hat{F}_{1ft} = 170$ kN (see example 5).

$$\lambda_1 = 0{,}18 \text{ thus } t_s \geqslant \frac{m_1}{m_1 + m_2} \; \frac{\hat{F}_1}{2\, m_2\, \sigma_e} \qquad (22)$$

$$t_s \geqslant \frac{21.25}{21.25 + 21.1} \; \frac{220000}{2 * 21.1 * 240} = 10.8 \text{ mm}$$

The design force $\hat{F}_{1ft}$ = 170 kN should be transmitted by the rest of of the stiffener thus:

$$t_s \geqslant \frac{2\, \hat{F}_{1ft}}{(b - w2 - 2\, m_{1,2})\, \sigma_e}$$

Also here it is assumed that only one half of the stiffener thickness is active, so:

$$t_s \geqslant \frac{2 * 170000}{(300 - 105 - 2 \times 21.2) * 240} = 9.28 \text{ mm}$$

Thus t_s = 11 mm

The weld size is 0.7 times one half the stiffener thickness when f_c = 1, thus $a_s = 0.7 * \frac{1}{2} * 10.9 = 3.8$ mm round off a_s = 4 mm.

If this connection with 4 bolts M20 should be made suitable for application in a statically indeterminated structure, then the end plate should have to give the rotational capacity. This will be explained in example 16.

Example 16

Dimensions of stiffeners and welds when rotational capacity is required. Application of formulae (1) to (10) and (16) to (19) inclusive.

Assume that a beam section HE-300A is connected to a column section HE-300A then the computation of the end plate is as follows:

$$m_1 = 105 - 8.5 - 8\sqrt{2} = 42.6 \text{ mm}$$

$$n' = \frac{300 - 105}{2} = 92.5 \text{ mm}$$

$$\lambda_1 = \frac{42.6}{42.6 + 92{,}5} = 0{,}35$$

$$e_1 = 45 \text{ mm}$$

$$m_2 = 45 - 14 - 6\sqrt{2} = 22.5 \text{ mm}$$

$$\lambda_2 = \frac{22,5}{42.6 + 92.5} = 0.17$$

$$\alpha = 4\pi$$

To be sure of sufficient rotational capacity delivered by the end plate it is necessary that:

$$\frac{\hat{F}_{1e}}{2\hat{B}_t} < 0.8 \text{ because } \lambda_1 \text{ and } \lambda_2 < 0.5 \text{ (see table 1).}$$

$2\hat{B}_t = 275$ kN, thus $\hat{F}_{1e} < 0.8 * 275 = 220$ kN.

Formula (4b) applied, gives:

$$t_e \leqslant 2\sqrt{\frac{\hat{F}_1}{2\alpha\sigma_e}} = 2\sqrt{\frac{220000}{8\pi * 240}} = 12 \text{ mm} \qquad (4b)$$

Now the design strength of the added bolts is as follows:

$$\hat{F}_{1et} = \frac{2(b - w2 - 2m_{1,2})\, t_f^2 \sigma_e}{4m_2} \qquad (5)$$

$$\hat{F}_{1et} = \frac{2(300 - 105 - 2 * 22.5)\, 12^2 * 240}{4 * 22.5} = 115 \text{ kN}$$

$$\hat{F}_{1et} = \frac{(b - w2 - 2m_{1,2})\, t_f^2\, \sigma_e + 10\hat{B}_t * m_2}{9m_2} \qquad (6)$$

$$\hat{F}_{1et} = \frac{(300 - 105 - 2 * 22.5)12^2 * 240 + 10 * 137250 * 22.5}{9 * 22.5} = 178 \text{ kN}$$

$$\hat{F}_{1et} = \frac{(b - w2 - 2m_{1,2})\,(t_d^2 + t_f^2)\sigma_e}{4y} \qquad (8)$$

$$\hat{F}_{1et} = \frac{(300 - 105 - 2 * 22.5)(12^2 + 14^2)\sigma_e}{4 * (45-7)} = 80 \text{ kN}$$

$$\hat{F}_{1et} = \frac{(b - w2 - 2m_{1,2})t_d^2\sigma_e + 10\hat{B}_t \cdot y}{9y} \qquad (9)$$

$$\hat{F}_{1et} = \frac{(300 - 105 - 2 * 22.5)\ 14^2 * 240 + 10 * 137250 * 22.5}{9 * (45 - 7)} = 201 \text{ kN}$$

Here formula (8) governs with $\hat{F}_{1t} = 80$ kN

This implies that: $\hat{F}_{1sum} = 220 + 80 = 300$ kN.
The beam does not yield as appears from a computation with formulae (16) to (19) inclusive.

The size of the fillet weld between end plate and flange becomes with the application of formula (24) for statically loaded structures.

$$a_f \geqslant 0.7 * \frac{42.6}{42.6 + 22.5} * \frac{220000}{4 * 22.5 * 240} = 4.66 \text{ mm} \rightarrow a_f = 5 \text{ mm}$$

For a braced frame:

$$a_f \geqslant 1.4 * 4.66 = 6.53 \text{ mm} \qquad \rightarrow a_f = 7 \text{ mm}$$

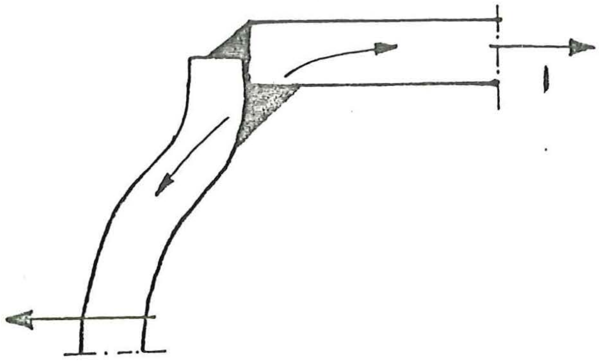
For an unbraced frame:

$$a_f \geqslant 1.7 * 4.66 = 7.92 \text{ mm} \qquad \rightarrow a_f = 8 \text{ mm}$$

It seems rather strange that in an unbraced frame the value a_f is larger than half the flange thickness ($t_f = 14$ mm). However, due to the required bending of the end plate the inner fillet weld is loaded by bending and tension (see the figure on page 66).

When one of the formulae (16) to (19) inclusive would be determinant and yielding of the beam web would occur then bending of the beam flange would occur too, so that : $a_f = 0.4\ t_f = 6$ mm.

Now $a_f = 8$ mm, when an unbraced frame is concerned.
The size of the outer fillet weld can be restricted to a single weld with a = 4 mm.



The inner fillet weld between end plate and beam flange is loaded by bending.

Now the welds between stiffeners and column flange should also be adapted, thus:

$a_s = 1.4 * 3.8 = 5.32$ mm $\rightarrow$ $a_s = 6$ mm for a braced frame

$a_s = 1.7 * 3.8 = 6.46$ mm $\rightarrow$ $a_s = 7$ mm for a side sway frame

Another aspect may not be neglected. As far as the column side is concerned, boltfailure is the determining factor, whereas the end plate fails due to yielding.
The magnification factor $f_c = 1.7$, applied for an unbraced or side sway frame is based on test-results where boltfracture did not occur with bolts.

In this example the problems arise because the bolts M20 do not have sufficient strength to cause yielding of the column flange which is necessary to get the rotational capacity.
In this case it would be better to apply bolts M24 where $\hat{F}_1 = 296$ kN.
Then the computation is as follows:

$m_1 = 28.75$ mm $\qquad m_2 = 27.5$ mm $\qquad \lambda_1 = 0.24$

$$t_s > \frac{m_1}{m_1 + m_2} \; \frac{\hat{F}_1}{2m_2\,\sigma_e} = \frac{28.75}{28.75 + 27.5} * \frac{296000}{2 * 27.5 * 240} = 11.46 \text{ mm}$$

Thus $t_s = 12$ mm

With a statically determinated structure $a_s = 0.7 * \frac{1}{2} * 11.46 = 4$ mm
In a braced frame $a_s = 1.4 * 4 = 5.6 \rightarrow 6$ mm
In an unbraced frame $a_s = 1.7 * 4 = 6.8 \rightarrow 7$ mm

Now the rotational capacity is obtained from yielding of the flanges. It is not necessary that the end plate give rotational capacity so that this may be made as thick as wanted to fulfil the strength and stiffness requirements.

Example 17

Formula (26) (stiffness formula)

The connection between the column section HE-300A and beam section IPE-400 as computed in examples and 14 is used to show the application of the stiffness formula. The rotation of the connection is computed at the design moment $\hat{M}_v$.

The latter rotation is multiplied with the factor $(\frac{M_v}{\hat{M}_v})^2$ when the rotation at an specific moment M_v is needed.

$\hat{F}_1 = 296$ kN $\quad \hat{F}_1 + \hat{F}_2 = 418$ kN (see example 14)

$\hat{F}_{1w} = 503$ kN (see example 9)

$\hat{F}_{1f} = 296$ kN (see example 1)

$\hat{F}_{1b} = 395$ kN (see example 2)

$\hat{F}_{1e} = 396$ kN (see example 3)

$\hat{F}_d = 418$ kN (see example 14)

$\hat{F}_{ws} = \infty$ symmetrically loaded connection, thus shear loading of the column web is not present.

Column web on the tension side

$$\alpha_1 = (\frac{296}{503})^2 \cdot \frac{1}{0.8 * 8.5} = 0.05093 \text{ mm}^{-1}$$

Column flange on the tension side

$$\alpha_2 = (\frac{296}{296}) *12 * 0.23 * \frac{28^2}{14^3} = 0.78857 \text{ mm}^{-1}$$

Bolts

$$\alpha_3 = (\frac{296}{395})^2 \frac{14 + 18 + 4 + 0,5 * 1.5 * 24}{2 * 353} = 0.04295 \text{ mm}^{-1}$$

End plate

$$\alpha_4 = (\frac{296}{396})^2 * 12 * 0.31 * \frac{50^2}{18^3} = 0.89096 \text{ mm}^{-1}$$

Column web compression side

$$\alpha_5 = \left(\frac{418}{418}\right)^2 * \frac{1}{0.8 * 8.5} \qquad = 0.14706 \text{ mm}^{-1}$$

$$1.92047 \text{ mm}^{-1}$$

$$f_i = \frac{296 * 0.34 + 122 * 0.27}{296 * 0.34} = 1.33 \qquad E = 210 \text{ kN/mm}^2$$

$$k = \frac{1,92047}{1.33 * 210 * 340^2} = 5.9481 * 10^{-8} \text{ rad/kNmm} = 5.948 * 10^{-5} \text{ rad/kNm}$$

At a moment: $\hat{M}_v = 134 \text{ kNm} \rightarrow \phi = 7.95 * 10^{-3}$ rad.

At a moment: $\frac{2}{3}\hat{M}_v = 89 \text{ kNm} \rightarrow \phi = 3.53 * 10^{-3}$ rad.

Example 18

Formula (27) (shear loading of the beam web)

The values $\frac{\hat{M}_c}{\hat{M}_e}$ are computed for various end plate thicknesses, haunch lengths and values of y.

The results are gathered in the following table

ℓ_c	$\frac{\hat{M}_c}{M^*_e}$			
	t_e = 10 mm		t_e = 25 mm	
	y = 50 mm	y = 100 mm	y = 50 mm	y = 100 mm
200 mm	0,42	0,35	0,63	0,46
400 mm	0,76	0,65	0,98	0,77

It follows from this table that the haunch length has a significant influence on the moment $\hat{M}_c$. The influence of the end plate thickness decreases with increasing haunch length. The influence of the distance y increases with increasing end plate thickness.

Example 19

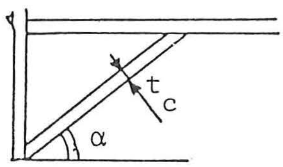
Formula (28) (compression side of the beam at the end of the haunch).
Beam section IPE 400 with a haunch of which the angle is varied and the

flange plate thickness $t_c = 20$ mm $t_f = 13.5$ mm $b = 180$ mm $A_f = 2430$ mm^2.

$r = 21$ mm $t_w = 8.6$ mm

$A_f/A_d = \frac{2430}{1665} = 1.47$ $A_d = \{20 + 5(13.5 + 21)\}\, 8.6 = 1655$ mm^2.

$$\frac{\hat{M}_c}{\hat{M}_e} = \frac{1.25 \cot\alpha}{1.47 + 0.5 \cot\alpha} \rightarrow \cot\alpha = \frac{1}{2} \rightarrow \frac{\hat{M}_c}{\hat{M}_e} = 0.36 \leqslant \frac{0.5}{1.47} = 0.34 \text{ thus } 0.34$$



$$\cot\alpha = 1 \rightarrow \frac{\hat{M}_c}{\hat{M}_e} = 0.64 \leqslant \frac{1}{1.47} = 0.68 \text{ thus } 0.64$$

$$\cot\alpha = 2 \rightarrow \frac{\hat{M}_c}{\hat{M}_e} = 1.01 \leqslant \frac{2}{1.47} = 1.36 \text{ thus } 1.0$$

Example 20

Formulae (29) to (32) inclusive (haunch flange and weld)
Beam with a haunch of which the angle α is varied (see example 19).

$\cot\alpha$	$\frac{\hat{M}_c}{\hat{M}_e}$	$\cos\alpha$
1	0.64	0.7071
2	1	0.8944

$t_f = 13.5$ mm $\quad t_w = 8.6$ mm

$A_f = 180 * 13.5 = 2430$ mm^2

<u>With cot α = 1</u>

$$t_c \geqslant 0.64 * \frac{2430}{180 * 0.7071} = 12.2 \text{ mm} \tag{29}$$

$$t_c \geqslant 0.64 * \frac{0.7 * 2430}{(10 * 13.5 + 2 * 8.6)0.7071} = 10.1 \text{ mm} \tag{30}$$

$$t_c \geqslant \frac{180}{7} = 10.5 \text{ mm} \tag{31}$$

Conclusion: $t_c > 12.2$ mm $\rightarrow t_c = 13$ mm

$$a_c > 0.64 * \frac{0.7 * 2430}{(10 * 13.5 + 2 * 8.6} \sqrt{3 + 1} = 14.3 \text{ mm} \tag{32}$$

If a complete connection is assumed, then: $a_c = 13 * \sqrt{2} = 18.3$ mm.
Thus with a computation based on formula 20 a reduction is possible.

<u>With cot α = 2</u>

$$t_c \geqslant 1 * \frac{2430}{180 * 0.8944} = 15 \text{ mm} \tag{29}$$

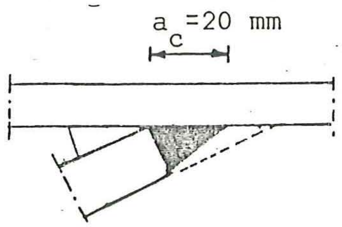
$$t_c \geqslant 1 * \frac{0.7 * 2430}{(10 * 13.5 + 2 * 8.6)0.8944} = 12.5 \text{ mm} \tag{30}$$

$$t_c \geqslant 10.5 \text{ mm} \tag{31}$$

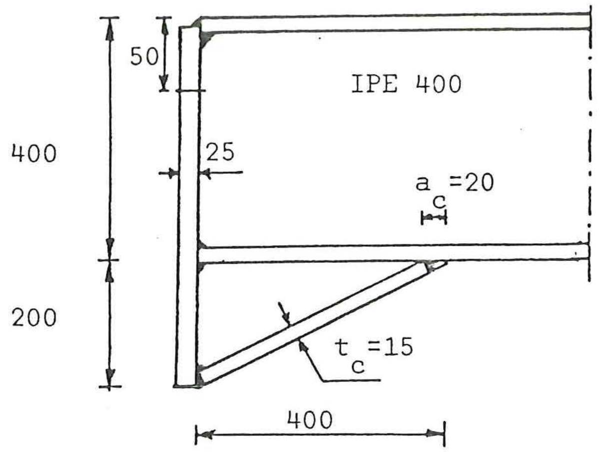
Conclusion: $t_c \geqslant 15$ mm $\rightarrow t_c = 15$ mm

$$a_c \geqslant 1 * \frac{0.7 * 2430}{(10 * 13.5 + 2 * 8.6)} \sqrt{3 + 0.25} = 20.15 \text{ mm} \tag{32}$$

If a complete connection is assumed then $a_c = 2 * 15 = 30$ mm, thus here a reduction is possible too.



If the results of examples 18, 19 and 20 are combined, then the haunch and end plate are dimensioned as shown in the figure in the case that 98% of the elastic bending moment of the connection may be transferred by the cross section of the beam at the end of the haunch, whereas no stiffeners are required and the rotational capacity is delivered by the connection.



Example 21

Formulae (33) to (37) inclusive (compression side haunch without a flange).

IPE-400 $\quad h_d = 100$ mm $\quad h_c = 200$ mm $\quad t_{wc} = 20$ mm

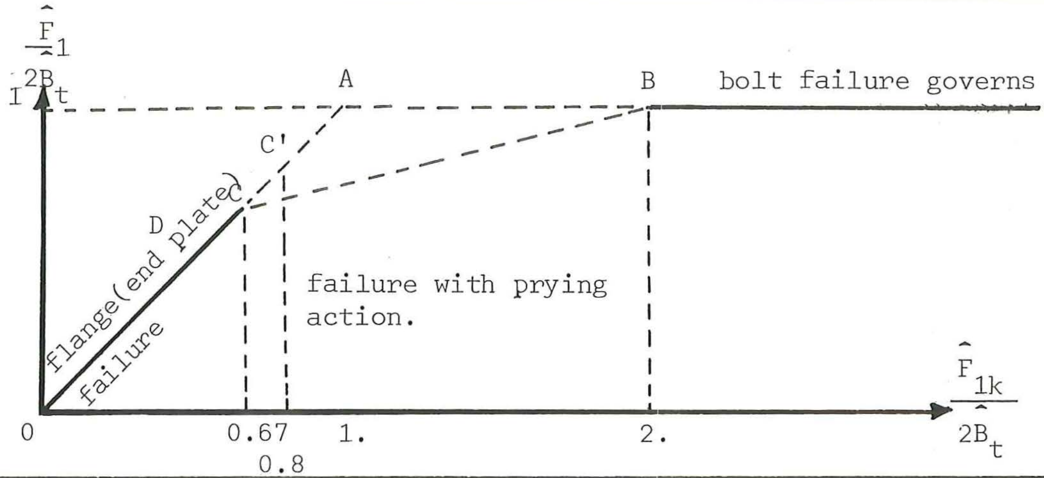
$\tan\alpha = 0.50 \quad t_w = 8.6$ mm $\quad t_f = 13.5$ mm $\quad b = 180$ mm

$\cos\alpha = 0.89 \quad \cos^2\alpha = 0.8$

$\sin^2\alpha = 0.20 \quad A_f = 13.5 * 180 = 2430\ mm^2 \quad l_c = 400$ mm

$$t_{wc} \leqslant \frac{8.6}{0.2} = 43 \text{ mm} \qquad (33)$$

$$\frac{\hat{M}_c}{\hat{M}_e} = \frac{20 * 100 * 0.8}{13.5 * 180} = 0.66 \qquad (34)$$



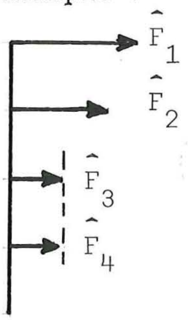
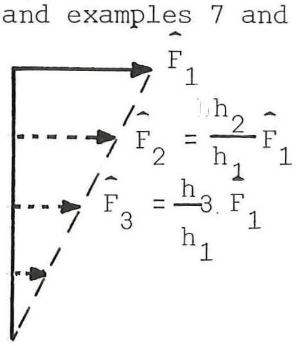
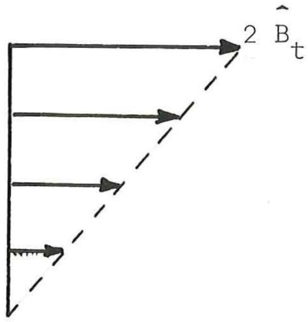
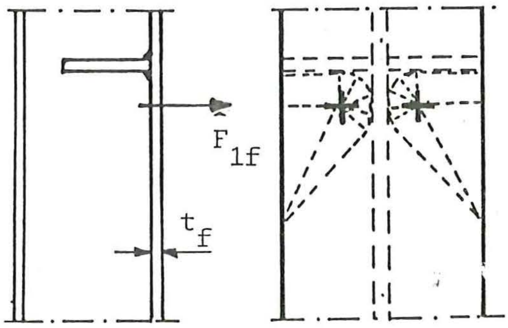
DESIGN LOAD AT THE FIRST BOLTROW, $\hat F_1$		
$\hat F_1 = \hat F_{1k}$ (1) for the value of $\hat F_{1k}$ see the chart in table 2 Explanation : 1.1.1 Example : 1.	$\hat F_1 = \frac{2\hat B_t}{f_{pr}}$ (2) Explanation: 0.5;1.1.2 Example : 2;3 and 4	$\hat F_1 = 2\hat B_t$ (3) Explanation: Introduction
DESIGN MOMENT OF THE CONNECTION, $\hat M_v$		
$\hat M_v = \Sigma(\hat F_i . h_i)$ as explained in section 0.3 and example 7  New method	$\hat M_v = \Sigma(\hat F_i . h_i)$ with the condition as explained in section 0.5 and examples 7 and 8 	$\hat M_v = 2\hat B_t . \frac{\Sigma h_i^2}{h_1}$  Old method
FAILURE CRITERIA		
Yielding of the column flange (or end plate) bolts elastic	Partly yielding of the column flange (or end plate) and failure of the bolts with prying action.	Failure of the bolts Column flange and end plate behave completely elastic.
CONNECTION NOT ALLOWED IN		
	More research required See the explanation in the introduction (0.5)	Statically indeterminated structures or structures with impact loads.
	Unless the design strength of the connection is larger than that of the connected member.	

Table 1 : Summary of the features of the design methods.

Formulae	Explanation	Example
Column flange at the first boltrow		
$\hat{F}_{1f} = 2\ \alpha\ m_p$ (4a) For α see the chart	1.1.1	1
End plate at the first boltrow		
$t_e = 2\sqrt{\dfrac{\hat{F}_1}{2\ \alpha\ \sigma_e}}$ (4b)	1.1.3	4

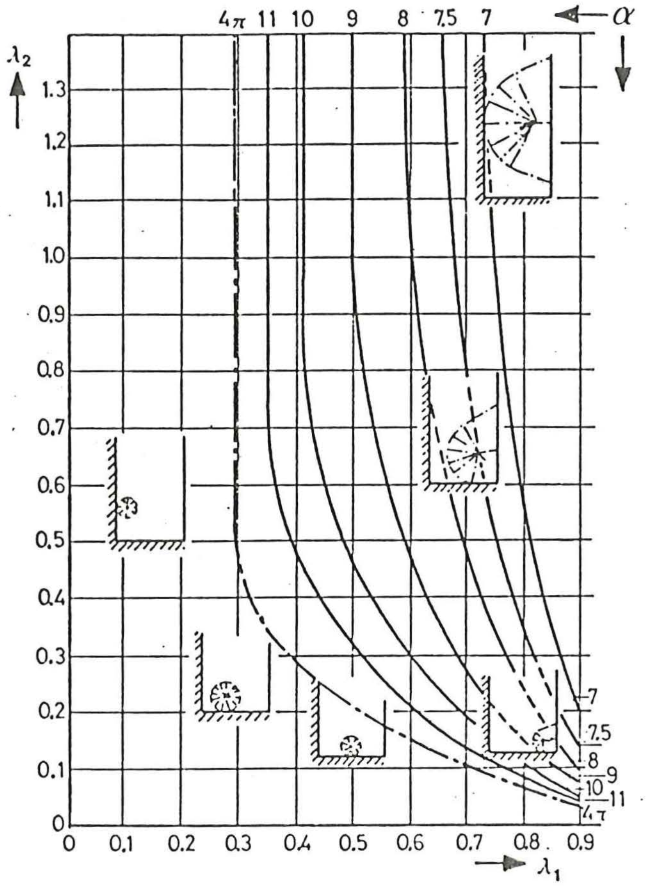
Column flange at the first boltrow

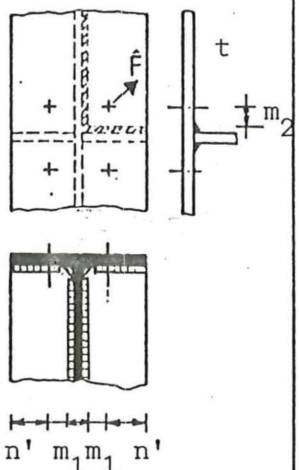


$\hat{F}_{1f} = 2\ \alpha\ m_p$ (4a)

For α see the chart

Chart for the determination of α





$\lambda_1 = \frac{m_1}{m_1 + n'}$

$\lambda_2 = \frac{m_2}{m_1 + n'}$

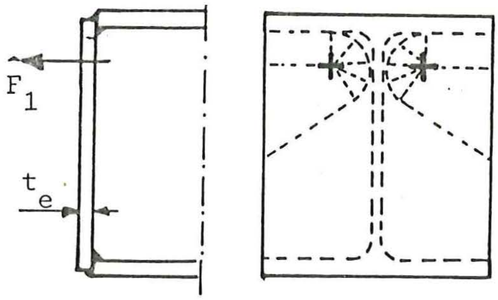
$m_p = \frac{1}{4}\, t^2 \sigma_e$

σ_y = yield strength

t = plate thickness

α = factor from chart

End plate at the first boltrow



$t_e = 2\sqrt{\dfrac{\hat{F}_1}{2\ \alpha\ \sigma_e}}$ (4b)

Table 2 : Formulae and chart for the computation of the design strengths of column flange and flush end plate at the first boltrow.

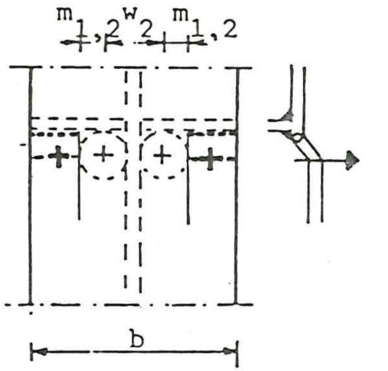
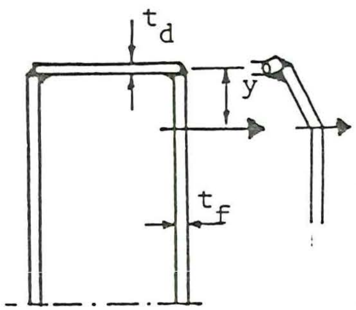
Formulae	Explanation	Example
Column flange (or end plate) at boltrow 1 with added bolts. Condition: $\hat{F}_1 = 8\pi . m_p$ or $\hat{F}_1 = 2\,\hat{B}_t$  $b_m = b - w_2 - 2\,m_{1,2}$ $m_{1,2} = m_1$ if $m_1 < m_2$ otherwise $m_{1,2} = m_2$		
$\hat{F}_{1kt} = \dfrac{2\,b_m . t_f^2 . \sigma_e}{4\,m_2}$ (5) $\hat{F}_{1kt} = \dfrac{b_m . t_f^2 . \sigma_e + 10\,\hat{B}_t . m_2}{9\,m_2}$ (6) $\hat{F}_{1kt} = 2\,\hat{B}_t$ (7)	1.2.1	5 en 16
 $\hat{F}_{1kt} = \dfrac{b_m . (t_d^2 + t_f^2) . \sigma_e}{4\,y}$ (8) $\hat{F}_{1kt} = \dfrac{b_m . t_d^2 . \sigma_e + 10\,\hat{B}_t . y}{9\,y}$ (9)	1.2.2	6 en 16
$\hat{F}_{1t}$ is the lower value of formulae (5) to (9) inclusive $\hat{F}_{1sum} = \hat{F}_1 + \hat{F}_{1t}$ (10)		

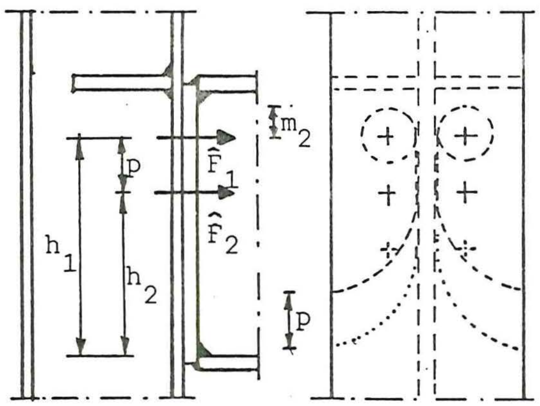
Table 3 : Formulae for the computation of the design strength of column flange (or end plate) if bolts are added in the first (uppermost) bolt row.

<table>
<tr><th>Formulae</th><th>Explanation</th><th>Example</th></tr>
<tr><td>

<u>Column flange(or end plate)</u>

$$\hat{F}_{2k} = \frac{b_m t_f^2 \sigma_e}{m} \qquad (11)$$

$$\hat{F}_{2k} = \frac{b_m t_f^2 \sigma_e + 4\hat{B}_t n}{2(m+n)} \qquad (12)$$



where : $b_m = p - m_{1,2} + 2m + 0,625n'$

$m = m_1 + 0,2r$

$n \leqslant 1,25\, m \leqslant n'$

if $\alpha = 4\pi$; $m_{1,2} = m_1$ if $m_1 < m_2$ otherwise $m_{1,2} = m_2$

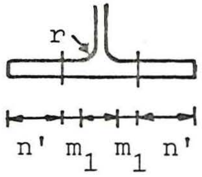
if $\alpha < 4\pi$; $m_{1,2} = m_1$ if $m_1 > m_2$ otherwise $m_{1,2} = m_2$

Condition for (11) and (12) is, that:

$h_2 \geqslant 4m + 1,25n'$ and $\hat{F}_1 = \hat{F}_{1k}$

If the latter condition is not fulfilled then $F_2 = 0$, unless:

$\hat{F}_{2k} \geqslant \frac{h_1 - p}{h_1} 2\hat{B}_t$, for then $\hat{F}_2 = \frac{h_1 - p}{h_1} 2\hat{B}_t$



</td><td>2.1.</td><td>7</td></tr>
<tr><td>

<u>Bolts in boltrow 2</u>

If $\hat{F}_1$ is determinated by formula (2) or (3) (see table 1), then:

$$\hat{F}_2 = \frac{h_1 - p}{h_1} 2\hat{B}_t \qquad (13)$$

</td><td>2.2.</td><td>8</td></tr>
<tr><td>

<u>Boltrow 3</u>

For this boltrow the same formula are valid as for the second boltrow, unless this boltrow is located within the effective length of the second row. In that case $b_m = p$

</td><td></td><td></td></tr>
</table>

Table 4 : Formulae for the computation of the design forces of column flange (or end plate) at the boltrows not located adjacent to stiffeners.

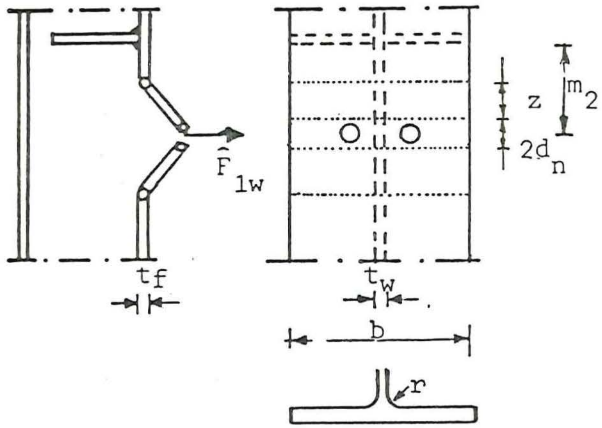
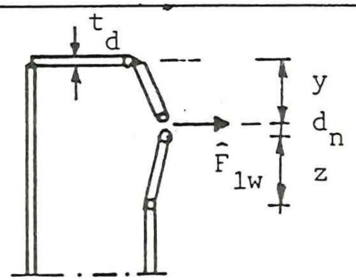
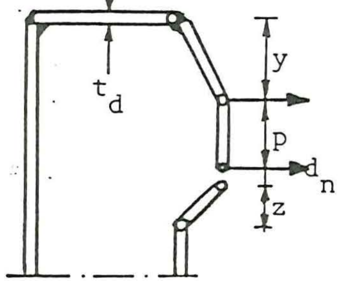
Formulae		Explanation	Example
Column web at boltrow 1 	Sometimes formulae (14) to (20) inclusive may be neglected $z = t_f \sqrt{\frac{b}{t_w}}$ (14)	3.0	
	If $z < m_2$ (generally not) $\hat{F}_{1w} = (2d_n + 2t_f\sqrt{\frac{b}{t_w}})\, t_w \sigma_e$ (15)	3.1	9
	If $z \geq m_2$ $\hat{F}_{1w} = \{(d_n + t_f\sqrt{\frac{b}{t_w}} + \frac{m_2}{2})\, t_w + \frac{bt_f^2}{2m_2}\}\sigma_e$ (16) $\hat{F}_{1w} = \{(2m + 0{,}625\, n' + \frac{m_2}{2})t_w + \frac{bt_f^2}{2m_2}\}\sigma_e$ (17)	3.2	10
	$\hat{F}_{1w} = \{(d_n + t_f\sqrt{\frac{b}{t_w}} + \frac{y}{2})t_w + \frac{b\, t_f^2 + b_d t_d^2}{4y}\}\sigma_e$ (18) $\hat{F}_{1w} = \{(2m + 0{,}625n' + \frac{y}{2})t_w + \frac{b\, t_f^2 + b_d t_d^2}{4y}\}\sigma_e$ (19) $m = m_1 + 0{,}2r$	3.3	11
If more boltrows are available 	$\Sigma \hat{F}_i \leq \hat{F}_{1w} + (n-1)\hat{F}_{pw}$; where $\hat{F}_{pw} = p.t_w.\sigma_e$ (20) n= number of boltrows $\hat{F}_{1w}$ is the lower value of formulae (16) to (19) incl.	3.4	12
	The same formulae are valid for the check of the beam web behind the flush end plate.		13
Compression or shear	$\Sigma \hat{F}_i \leq \hat{F}_d$ en $\Sigma \hat{F}_i \leq \hat{F}_s$ (21)	3.5	14

Table 5 : Formulae for the computation of the design strength of column and beam web (a rough estimate may be sufficient)

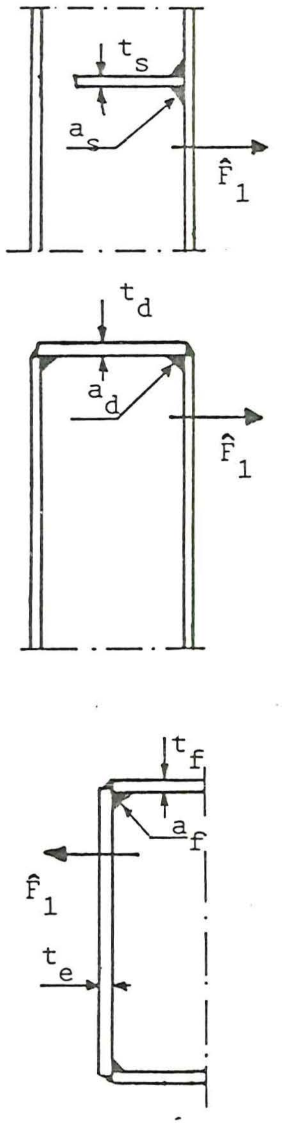
Formulae	Explanation	Example
 <u>Dimensions of stiffener</u> t_d of $t_s \geq f_v \frac{\hat{F}_1}{2m_2\sigma_e}$ (22) t_d of $t_s \geq f_v \frac{\hat{F}_1}{(m_2+n')\sigma_e}$ (23) $f_v = 1$ if $\lambda_1 > 0,5$ $f_v = \frac{m_1}{m_1+m_2}$ if $\lambda_1 \leq 0,5$ Remarks: Take into account formulae (16) to (19) inclusive. <u>Weld size</u> a_s either a_d or $a_f \geq 0,7\ f_c . f_v . \frac{\hat{F}_1}{4m_2\sigma_e}$ (24) a_s either a_d or $a_f \geq 0,7\ f_c . f_v . \frac{\hat{F}_1}{2(m_2+n')\sigma_e}$ (25)	4.1	15
where: $f_c = 1$ for statically determinated structures. $f_c = 1,4$ for braced frame (partial mechanism in beam) $f_c = 1,7$ for sway frame. but : a_s either a_d or $a_f \geq 4$ mm if the mechanism of formula (15) occurs. and : $a_d \geq 0,4\ t_d$, $a_f \geq 0,4\ t_f$ } if one of the mechanisms of formulae (16) to (19) inclusive occurs.		16

Table 6 : Formulae for the determination of the dimensions of stiffeners and fillet welds.

Formulae	Explanation	Example
Rotation $\phi = kM$ (26) Where: $k = \frac{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6}{f_i E h_1^2}$ $I_v = \frac{h_k}{2k E}$ Column web tension side $\alpha_1 = (\frac{\hat{F}_1}{\hat{F}_{1w}})^2 \frac{1}{0,8\, t_w}$ Column flange tension side $\alpha_2 = (\frac{\hat{F}_1}{\hat{F}_{1f}})^2 \frac{4m_k^2}{t_{fk}^3}$ for a column without stiffeners $\alpha_2 = (\frac{\hat{F}_1}{\hat{F}_{1f}})^2 \frac{12\, \lambda_2' m_{1k}^2}{t_{fk}^3}$ for a column with stiffeners Bolts: $\alpha_3 = (\frac{\hat{F}_1}{2\hat{B}_t})^2 \frac{l_b}{2A_s}$ End plate tension side $\alpha_4 = (\frac{\hat{F}_1}{\hat{F}_{1e}})^2 \frac{12\lambda_{2e} m_{1e}^2}{t_e^3}$ Column web compression side $\alpha_5 = (\frac{\Sigma\hat{F}_i}{\hat{F}_d})^2 \frac{1}{0,8\, t_{wk}}$ for a column without stiffeners Column web shear panel $\alpha_6 = (\frac{\Sigma\hat{F}_i}{\hat{F}_s})^2 \frac{1}{0,24\, t_{wk}}$ More boltrows $f_i = \frac{\Sigma(\hat{F}_i h_i)}{\hat{F}_1 h_1}$	5.1	17

Table 7 : Formulae for an approach of the flexibility of a beam to column connection with flush end plate and a column either with or without stiffeners.

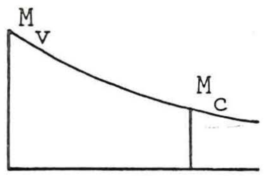
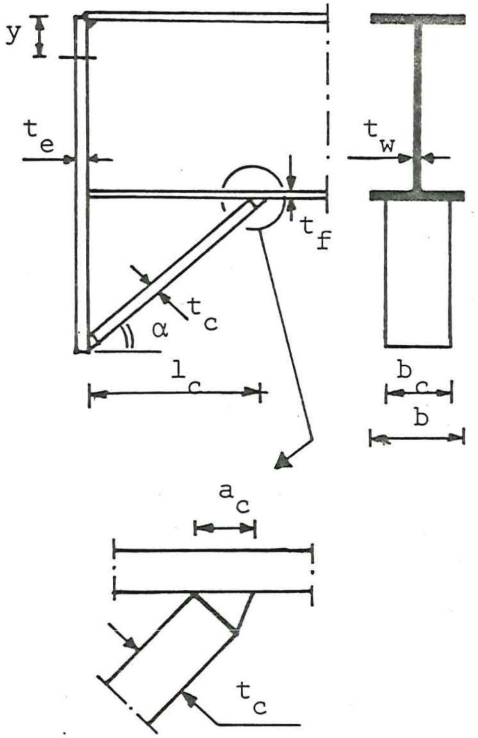
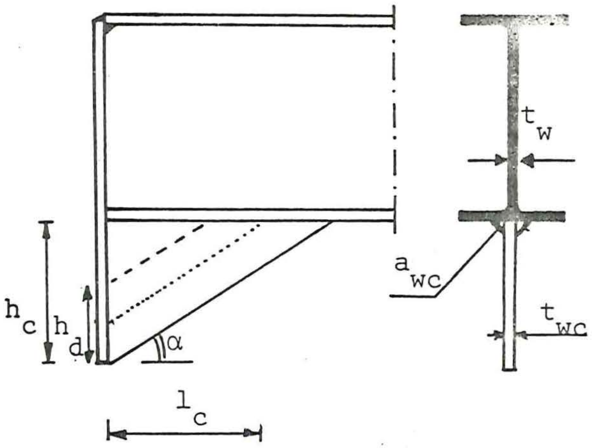
Formulae	Explanation	Example
The tension side does not become the determinating factor if $M_c \leqslant \hat{M}_v - h_c . \Sigma \hat{F}_i$ or $M_c \leqslant \frac{t_w . l_c}{2A_f} \hat{M}_e$		
 M-line $$\frac{\hat{M}_c}{\hat{M}_e} = \frac{0{,}58 t_w \; (l_c + 2t_e) + \frac{b t_e^2}{4y}}{A_f + t_w \cdot y} \qquad (27)$$ where: $\hat{M}_e = W_e . \sigma_e$ W_e = elastic modulus $A_f = b.t_f$	6.1.	18
 Compression side (haunch with flange) $$\frac{\hat{M}_c}{\hat{M}_e} = \frac{1{,}25 \cot \alpha}{\frac{A_f}{A_d} + 0{,}5 \cot \alpha} \leqslant \frac{\quad}{\frac{A_f}{A_d}} \qquad (28)$$ where: $A_d = \{t_c + 5(t_f + r)\} \; t_w$	6.2.1.	19
$$t_c \geqslant \frac{\hat{M}_c}{\hat{M}_e} \; \frac{A_f}{b_c \cos\alpha} \qquad (29)$$ $$t_c \geqslant \frac{\hat{M}_c}{\hat{M}_e} \; \frac{0{,}7 \; A_f}{(10 \; t_f + 2t_w)\cos\alpha} \qquad (30)$$ $$t_c \geqslant \frac{b_c}{17} \qquad (31)$$ $$a_c \geqslant \frac{\hat{M}_c}{\hat{M}_e} \; \frac{0{,}7 \; A_f}{(10t_f + 2t_w)} \sqrt{3 + \tan^2\alpha} \qquad (32)$$	6.2.2.	20

Table 8 : Formulae for the computation of the dimensions of the haunch with flange.



Formulae		Explanation	Example
h_d = assumption (0,3 à 0,5h_c)			
Compression side (haunch without flange)			
$t_{wc} \leqslant \dfrac{t_w}{\sin^2\alpha}$	(33)		
$\dfrac{\hat{M}_c}{\hat{M}_e} = \dfrac{t_{wc}\ h_d\ \cos^2\alpha}{A_f}$	(34)		
of			
$t_{wc} \geqslant \dfrac{\hat{M}_c}{\hat{M}_e} \dfrac{A_f}{h_d\ \cos^2\alpha}$	(35)	6.3.	21
$t_{wc} \geqslant \dfrac{h_c\ \cos\alpha}{8,5}$	(36)		
$a_{wc} \geqslant \dfrac{\hat{M}_c}{\hat{M}_e} \dfrac{A_f}{1,65\ l_c}$ provided that a gap between plate and beam is avoided.	(37)		

Table 9 : Formulae for the determination of the dimensions of a haunch without a flange.

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