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Uncertainty-based decision-making in fire safety: Analyzing the alternatives

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ABSTRACT

Large accidents throughout the 20th century marked the development of safety fields in engineering, devoted to better identify hazards, understand risks and properly manage them. As these fields evolved rather quickly and moved from a compliance to a risk-based approach, a significant delay in this transition was experienced in fire safety engineering (FSE). Devastating fires well into the 21st century and the restrictive nature of prescriptive codes signaled the need to transition towards a performance-based one. A performance-based approach provides flexibility and capitalizes on learning from accidental events and engineering disciplines such as process safety and FSE. This work provides an overview of the main alternatives to account for uncertainty in safety studies within the context of FSE, including traditional probabilistic analyses and emerging approaches such as strength of knowledge. A simple example is used to illustrate the impact of the uncertainty analysis on the results of a simple fire safety assessment. A structured evaluation is performed on each alternative to assess its ease of implementation and communication. The outcome is a compendium of advantages and disadvantages of the alternatives that constitute a toolbox for fire safety engineers to configure and use within their fire risk assessments. Process safety engineers are expected to gain an understanding of the similar and important challenges of FSE, being it directly relevant for process risk management and fire risk management in administrative buildings.

1. Introduction

1.1. Aim of this paper

Safety engineering covers a large range of specialized disciplines, including nuclear safety, chemical process safety and security, disaster and emergency management, reliability engineering and fire safety engineering (FSE). Cadena and Munoz (Jaime and Cadena, 2013) addressed the particular link between process safety and FSE, which despite addressing hardly intersecting systems, share common challenges and solutions. This paper presents chemical process safety practitioners with a view of the challenges of uncertainty accounting within FSE. As administrative and storage buildings constitute key elements of a chemical processing business, fire risks in these facilities must also be adequately managed to ensure process safety and business continuity.

PRAs are the typical way in which complex systems are assessed and their use in performance-based design in FSE is evident (Van Coile et al., 2017, 2018, 2019; Gernay et al., 2019). Gehandler (2017) highlights the issues with FSE's current theoretical framework, stating that the process to demonstrate an adequate safety level of a building is '*restricted by a linear design process where mainly quantitative data and methods matters*'. Following such a numerical and mechanical process might prevent safety engineers from solving the correct problems, e.g. demonstrating compliance rather than safety. Regardless of the approach to risk assessment, its outputs can yield potentially inaccurate results due to the uncertainties involved. Despite inaccurate, these results are not useless. The uncertainty involved in fire risk assessments cannot be a reason to abandon their use, on the contrary, it must be used as a tool to reinforce them and support better decision making. Managing uncertainty sources in risk assessments -particularly in PRAs- is a key challenge for FSE and to implement the recommendations identified by the mentioned enquiries.

This work aims at providing FSE practitioners with a picture of the alternatives available for analyzing and communicating uncertainty. Furthermore, the alternatives are evaluated to support their selection given a safety objective, available inputs and resource restrictions. The evaluation is done based on two dimensions: suitability and effectiveness. The results not only present FSE practitioners with a toolbox of

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alternatives, but also provide process safety engineers with reflections which are easily applicable to current challenges posed by uncertainty in quantitative risk assessments.

This introduction presents the mentioned challenges of accounting for uncertainty in fire risk assessments, starting with an analysis of those in process safety. Section 2 presents the main uncertainty analysis alternatives found in literature and describing them, section 3 implements them to a simple fire safety example. Section 4 evaluates the alternatives on a comparative basis and section 5 discusses the FSE challenges in light of the evaluation results, using input from practitioners and experts. Finally, section 6 presents the conclusions of this work and its role in addressing current FSE challenges.

1.2. Risk assessment supports decision making

Fire events in Table 1 show that complex systems can fail and lead to major loss for all involved stakeholders. Similar events have led to the development of safety engineering fields such as nuclear safety and chemical process safety. Safety engineering within its purposes to support decision-making and properly manage risk. The formal process to do so is a risk assessment, made of hazards (and scenarios) identification, risk analysis and risk evaluation. The input of this process is information about the system such as physical characteristics and hazardous elements, while the outputs depend on the definition of risk and the analysis method. In general, the outputs of a risk assessment is the prioritization of risks, identifying those which are not acceptable in relation to a pre-defined acceptance criterion. Such output depends on the risk definition choice. Diverse definitions are available (Aven and Renn, 2009; Analysis, 2015) and the one by ISO 31000 (Standardization, 2018) states that it is the effect of uncertainty on objectives. Aven et al. (2011) have stated that definitions of risk impact the way it is understood and evaluated. Detailed analyses of the ontological origin of the risk concept and its evolution throughout time are provided by Beck (Beck and Kewell, 2012) and Blokland and Reniers (2019). A popular definition for risk which enable many traditional risk assessment methodologies is that of consequences multiplied by likelihood, i.e. expected value.

Table 1
Examples of catastrophic events.

Event	Year	Location	Safety field	Human loss	Economic loss
Collapse of World Trade Centre (Usmani et al., 2003)	2001	New York, USA	Fire, security	2977 fatalities	US$8B (buildings value) (Amadeo, 2020)
Fire at Grenfell Tower (Torero, 2018)	2017	London, UK	Fire	72 fatalities	£50M (renovations and enquiry) (Booth, 2019)
New Zealand International Convention Centre fire (Johnston, 2019)	2019	Auckland, NZ	Fire	–	US$26M (liquidated damages over delays)
Campbell chemical warehouse fire	2019	Melbourne, Australia	Fire, process	–	Multi-million dollar clean up (Chris Vedelago, 2019)
Chemical warehouses fires in Dhaka	2019	Dhaka, Bangladesh	Fire	81 fatalities (India, 2019)	*Unknown*

1.3. Uncertainty in risk assessments

A recognized issue of risk assessment is the presence of uncertainty sources, which can not only be numerous but correlated in complex ways. Before the scientific foundations of risk assessment were structured around 1945–1980 (Dionne, 2013), many traditional engineering fields were already using its key features and managing the associated uncertainties. Structural engineering is an example relevant to FSE, where calculations needed to design a structure involve multiple complex variables and dependencies, which are modeled using well-known tools and parameters both of which involve considerable uncertainties, see Tallja et al. (Talja et al., 1997). In this particular example uncertainties are identified and managed through '*experience from practical analysis*', i.e. expert judgment. As complexity grows, new approaches to account for uncertainties, e.g. the comparison of the use of safety factors and reliability approach in structural design by Wang et al. (2019). Currently risk assessment is recognized as a formal process and the core of the risk management process as defined by ISO 31000 (Standardization, 2018) and Aven (2016a) presents a detailed analysis of its foundations, as well as of the way in which uncertainties are involved in the different steps. Using the example of the mature chemical process safety field, issues have been identified associated to the complacency of practitioners (Årstad and Aven, 2017), the limits of prediction in risk assessments (Goerlandt and Reniers, 2018), the limits of the typically used probabilistic approach (Aven, 2010) and communicating uncertainties in the studies to the stakeholders (Zeng and Zio, 2017). In particular, Pasman and Rogers (2018) conclude that risk analysts (engineers in charge of carrying out the risk assessment) are "*haunted*" by uncertainty while at the same time highlighting the vital role of risk assessments in supporting key decision making by stakeholders.

Safety science literature includes reviews on risk assessment methodologies, displaying approaches that adapt to different nature, complexity and magnitude of systems (Tixier et al., 2002; Marhavilas et al., 2011; Baecher, 2016). These reviews reflect not only the uniqueness and complexity of systems, but also the need to adapt the risk definition to better understand and manage uncertainty. Aven (2010) clarifies that the purpose of risk assessment is not obtaining a risk index, e.g. *Risk = Likelihood x Consequence*, but obtaining '*an objective description of unknown quantities*' or 'a scientific judgement about the unknown quantities' from the qualified safety engineers performing the risk assessment. A last and important consideration is that risk assessment foundations could lack coherence as they derive from reaction to catastrophic events and practical experience, rather than from a scientific approach (Yang et al., 2018). This also poses a challenge to benchmark results and to unify guidelines. As highlighted by Aven and Kristensen (2019) and the previously identified issues, it is a current priority of safety engineering fields to better understand uncertainty and its effect on risk assessments.

1.4. Risk assessment issues in fire safety

Most of the advancements of risk assessment have been undertaken in disciplines with a predominant performance-based approach, to which FSE has been transitioning since the appearance of the first performance-based construction codes, e.g. United States (Meacham, 1997). FSE has adapted risk assessment tools from other disciplines, however its framework does not provide the same basis for its structured and systematic implementation, as revealed by Hackitt (2018). An example is Approved Document A (Ministry of Housing, 2010), which requires an explicit and systematic risk assessment of buildings exceeding limits of area or number of stories. However, to implement such assessment there is limited guidance on the identification of fire scenarios for analyzing structural integrity (Bisby, 2019). The issues identified by Hackitt in the UK have also been identified in Australia (Peter Shergold, 2018) and could be extrapolated to other places where similar structures are in place. Identified issues include lack of shared risk assessment practices, poor data collection and sharing, inadequate documentation (Recommendations 5, 8, 12, 14 from Shergold and Weir (Peter Shergold, 2018)), availability, completeness and updating of fire risk assessments, lack a 'building safety manager', lack of a broad scope for risk assessments, management of changes and technical assumptions (Recommendations 3.2–3.4, 2.9, 9.3 from Hackitt (2018)). The identified issues highlight a vast range of challenges to be addressed –partly- by FSE and the way in which risk assessments are conducted, as well as their role in the decision-making process.

1.5. Uncertainty sources in risk assessment

The breakdown of uncertainty presented in Table 2 is based on the FSE context, but others exist and can be useful in the right context (Cooke and Bedford, 2001; Hayes, 2011; Walker et al., 2013). Such structure is product of the recompilation of the definitions provided by different sources and many of the works referenced in this study, mainly (Aven and Renn, 2009; Kaplan and Garrick, 1981; Notarianni and Parry, 2016). It can be argued that uncertainty is only divided into epistemic and aleatoric and that linguistic elements are included in the former (Zio and Pedroni, 2012); however, linguistic uncertainty as defined by Colyvan (2008) is particularly relevant in the context of safety engineering fields. The use in FSE of risk acceptability criteria such as ALARP (*as low as reasonably practicable*) (Van Coile et al., 2018) and of performance-based requirements (Meacham and. Van Straalen, 2018) which largely rely on linguistic elements, introduce this type of uncertainty which cannot be easily treated quantitatively. Johansen and Rausand (2015) capture linguistic uncertainty within a broader category of ambiguity and provide a detailed accounting of how to identify and treat it within the process of a risk assessment.

A generic risk assessment process is shown in Fig. 1, along with the typical uncertainty types fed to it. Propagating such uncertainties can be a procedural issue if all are deemed epistemic and expressed using classical or Bayesian probabilities; however, the process shows that different uncertainty types are mixed along the process, backing up Colyvan's (Colyvan, 2008) argument. Both Hackitt and Shergold-Weird enquiries (Hackitt, 2018; Peter Shergold, 2018) highlight the need of reducing vagueness and/or ambiguity from performance requirements used to evaluate a fire safety in a building. Such ambiguity and vagueness is not communicated and it is rather carried along the risk assessment process without explicit consideration. Notarianni (Notarianni and Parry, 2016) also notices this when discussing uncertainty sources in the FSE design process by stating that '*at present, performance criteria are not established or agreed on*'. The relevance of this type of uncertainty in FSE is not clear as reflected by the uncertainty definition of the Society of Fire Protection Engineering (SFPE): '*amount by which an observed or calculated value might differ from the true value*' (Hurley, 2012). This definition not only constitutes a narrow view of what risk and uncertainty are, but also greatly limits the capacity for practitioners to express uncertainty in studies where a probabilistic approach is not feasible or recommended.

Table 2
Uncertainty types in FSE.

Type of uncertainty	Epistemic	Aleatory	Ambiguity
Description	Lack of or incomplete knowledge	Natural variability	Vagueness, context dependency, linguistic, under-specificity, normative
Context	Complex systems and phenomena	Large populations	Performance-based regulation, criteria and guidelines
Reduction methods	New or improved theories based on experimental observation	Statistical studies to characterize probability distributions	Consensus and alternative interpretations
Residual uncertainty after reduction	Deep uncertainty associated to complex fire phenomena and interactions	Uncertainty associated to extreme variations	Misinterpretation

As presented in Table 2, epistemic uncertainty can be reduced through acquiring new information, i.e. evidence. With enough additional evidence, epistemic uncertainty can be reduced enough to make analysts comfortable with the outcomes of their risk assessments. Evidence can either update or improve prior knowledge on something already known, e.g. updating probabilities, or reveal new knowledge that did not exist, e.g. new failure modes. Hackitt's (Hackitt, 2018) enquiry was precisely motivated by this type of new findings, in which the facades used for energy consumption reduction contributed directly to the massive loss of the Grenfell Tower fire. Failure modes or faulty assumptions are typically identified when catastrophic events occur. Such uncertainty is referred to as 'deep uncertainty' or the highest level of uncertainty according to Walker et al. (2013) and cannot be treated in a probabilistic manner as Kaplan once proposed (Kaplan et al., 2001).

The variety of uncertainty sources and their type (including deep uncertainty) present a challenge for any risk assessment approach. Aven (2011a) provides a compilation of references in which risk assessments in general are deemed '*simplistic and unrealistic*' and even misleading when the background knowledge of the analyst is poor. This draws relevance to this work and the identification of the different uncertainty analysis alternatives available for FSE practitioners.

1.6. Quantitative risk assessment issues in FSE

The previous are particularly relevant for quantitative (probabilistic) risk assessments (known as QRA or PRA), in which an expected value approach is taken and the risk is quantified as a function of scenarios, their frequencies of occurrence and their consequences. PRAs have their foundation in the Rasmussen study (Commission, 1975) and the theoretical basis provided by Kaplan (Kaplan and Garrick, 1981). Several references provide technical details of PRAs (Benintendi, 2018; Benintendi; Ramachandran et al., 2011; Safetyo.C.f., 2000) and detailed examples include airports (Iervolino et al., 2019), liquid spill fires (Zhao et al., 2017), natural events triggering technological events (NaTech) (Antonioni et al., 2015; Cozzani et al., 2014), hydrogen refueling stations (Gye et al., 2019), land-use planning (Ltd, 2012), railway tunnels (Vanorio and Mera, 2012), urban road tunnels (Meng et al., 2011). In short, in PRAs risk is defined as the triplet $<s_i, p_i, x_i>$, where i refers to an integer number representing a given scenario (s), p refers to the probability of said scenario occurring and x to its consequences. Both Amundrud (Amundrud and Aven, 2015) and Zio (2018) highlight that PRAs are useful and their major advantage being scenario identification and risk sources comparison.

PRAs are applied in FSE through the BS 7974 (H247974:2019 and Appl, 2019) and rely on statistical data of fire ignition and probabilities of failure for safety equipment such as sprinklers or mechanical extraction. Similar approaches in process safety have already been analyzed after decades of implementation, highlighting the potential for large uncertainty margins which are seldom reported (Goerlandt et al., 2016; Rae and Alexander, 2012) and requiring new risk assessment perspectives (Aven and Kristensen, 2019; Goerlandt and Reniers, 2017; Bjørnsen et al., 2019; Flage and Aven, 2018; Khorsandi and Aven, 2017; Askeland et al., 2017; Bjerga et al., 2016; Aven, 2016b). Such uncertainties threat rendering risk assessments useless in supporting decision-making and the new perspectives call for a need to understand them not as a mechanistic process, but as a complex evidence gathering exercise than effectively supports decision-making. FSE is not strange to this potential problems, as a probabilistic nature to fire risk is found in early fire safety engineering literature (Castino, 1982).

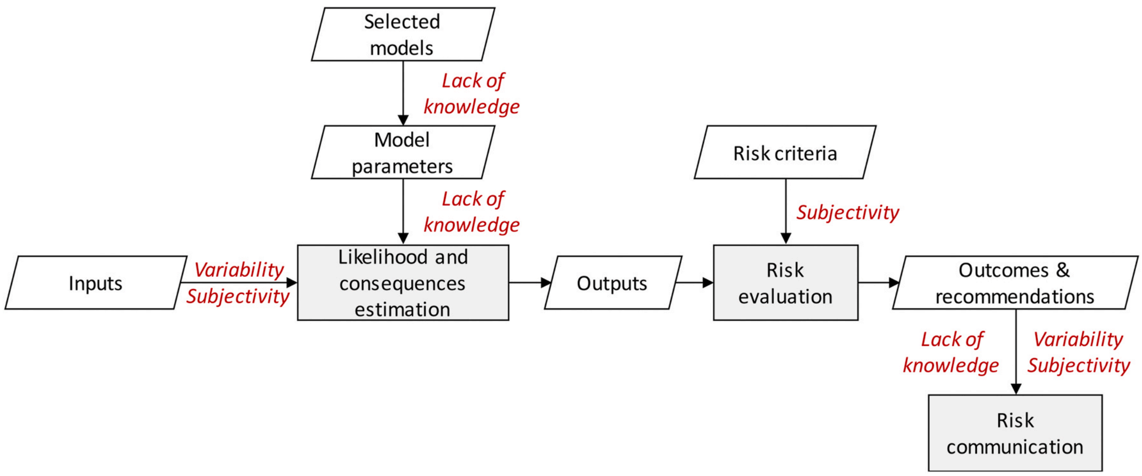

Fig. 1. Mapping of uncertainties along a generic risk assessment process.

2. Uncertainty analysis options

The approaches available for safety engineers to describe uncertainty go well beyond probability (Colyvan, 2008; Flage et al., 2014) and this section identifies the main approaches through a bibliographical search, using Dubois's work (Dubois, 2010) to guide the selection of search criteria (Table 3). The searches were conducted on Scopus® database which indexes key peer-reviewed literature much related to the mentioned safety engineering fields. The data was retrieved on the 13th of March of 2019 using a search timeframe between January 2000 and December 2018, limiting it only to journal articles. The table presents the search parameters, while Fig. 2 presents the predominance of the probabilistic approach in journal papers production and the growing contribution of the other main alternatives.

This initial set of approaches to uncertainty analysis is further expanded by exploring selected papers. One of these papers is the detailed and well-structured description of the Generalized Information Theory (GIT) presented by Klir (2004), and additional relevant work is also available (Hayes, 2011; Zio and Pedroni, 2012; Beer et al., 2013; Abdo et al., 2017; Aven, 2011b). Klir presents the approaches as tools to describe the degree of evidence that the true value of a variable of interest X is within a specified set, particularly the defined interval [0,1]. Klir calls these approaches uncertainty functions and include classical probability theory, possibility and evidence theory, and constitute the first approaches reviewed in this work, as presented in Table 4.

Uncertainty functions provide a numerical outcome of uncertainty contained in all the information available -or the lack of it- and this is not always possible to map onto the prescribed range. Klir recognizes alternative theories can fit within the GIT as long as they allow mapping uncertainty into the defined interval [0,1] and highlights that the "*choice of the uncertainty theory employed in dealing with each given problem should be determined by the nature of the problem itself*" (Klir, 2004).

Table 3
Searches and top results. .

Approach	Number of articles (2000–2018)	Top subject area	Top author (No. publications)
Probabilistic	749*	Engineering (28.4%)	Aven, T. (11)
Bayesian	316	Engineering (26.4%)	Khan, F. (6)
Fuzzy	306	Engineering (27.8%)	Huang, G.H. (23)
Possibility	150	Environmental (21.0%)	Zio, E. (5)
Belief	82	Engineering (11.6%)	Tesfamariam, S. (4)

*For the probabilistic, the results for 'non-probabilistic' were removed.

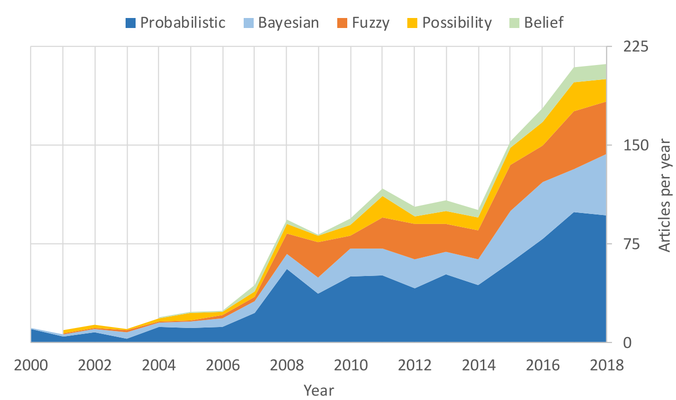

Fig. 2. Results and annual growth for the main approaches to uncertainty analysis.

Table 4
Uncertainty analysis alternatives within this work.

General approach	Specific approach	Date	Main feature
Uncertainty functions	Classical probability	1977	Additive, mapping onto [0,1]
	Possibility theory Evidence theory P-boxes	1978 1976 1987	Non-additive, mapping onto [0,1]
Robust Decision Making (Lempert Marchauet al., 2019)	Info-gap (Ben-HaimY.M., Nikolaides, 2005)	2005	Focus on robustness
	Strength of knowledge (Aven and Kristensen, 2019; Bjørnsen et al., 2019; Askeland et al., 2017; Aven, 2016b)	2017	Assumption testing
	Exploratory Model Analysis (Bankes, 1993)	2003	Scenario exploration

Beyond uncertainty functions, alternatives have arisen which focus on better supporting the decision-making despite the presence of considerable -unquantifiable- uncertainty both in the inputs used to define a scenario and in the assumptions used to model a system. Such alternatives are grouped under *Robust decision making* (Lempert Marchauet al., 2019), which is defined is a framework designed to support decision-making under deep uncertainty. An example of these alternatives is the info-gap approach (Ben-HaimY.M., Nikolaides, 2005) in which the robustness is defined as an uncertainty horizon, i.e. a measure of how much uncertainty the system can resist. Such an approach helps engineers define satisfying performance requirements and allowing for variability in expected loads to the system.

The info-gap approach applied to structural problems with uncertain load variables (Takewaki and Ben-Haim, 2005) is in line with the performance assessment done by Cadena et al. (2019) despite not specifying a robustness measure. In the latter, the assessment is accompanied by a judgment of its trustworthiness instead of a quantification of the robustness as in info-gap or of uncertainty as a probability. This trustworthiness judgment is done based on the approach formulated by Aven (Aven and Kristensen, 2019; Bjørnsen et al., 2019; Askeland et al., 2017; Aven, 2016b) in which all quantities and associated assumptions within the model used are judged according to their strength of knowledge and output sensitivity. Exploratory Modelling Analysis (EMA) is another alternative that has been formulated under the Robust Decision Making framework. EMA makes use of computational power to run a large set of potential realities (scenarios) and then provide the information as a whole for stakeholder to support their decision.

2.1. A note on inputs

The nature of the input is key to any risk assessment, and also to uncertainty analysis. The approaches considered in this work do not specify a particular type of input. In general, inputs can be classified as quantitative or qualitative. The former can be used in risk assessments if qualitative criteria are used to define the variables of risk, e.g. probability and likelihood. Qualitative inputs can also be transformed into the latter using fuzzy numbers, which are variables with a specific formalized language.

Fuzzy numbers can be used as inputs for most of the approaches presented in this work and have the added value of expressing the membership function of a quantity of interest within a specified range. This allows engineers to describe their knowledge about the quantity as a function of degree of belief, rather than as a crisp function. A review of the use of fuzzy numbers in safety engineering is presented by Kabir and Papadopolous (Kabir and Papadopoulos, 2018), while Shi (2009) presents a building fire risk analysis using fuzzy numbers. Fuzzy numbers also reflect the need for engineers to use additional information rather than just a crisp value.

Numeral Unit Spread Assessment Pedigree (NUSAP) (Funtowicz and Ravetz, 1990) constitute another type of input. NUSAP is a notational scheme that presents a numerical result along with information about the measurement and systematic error (Spread and Assessment, respectively) and the quality of what the number represents (Pedigree). NUSAP aims to provide decision makers with a more complete image of what a numerical result entails by taking into account its uncertainty as presented by Ellis et al. (2000). Given that pedigrees are subjectively defined categories, this constitutes an issue in propagating uncertainty and favor approaches such as fuzzy numbers or the recent alternative proposed by Zadeh (2011), Z-numbers.

In this work, none of these notations are prioritized, as we acknowledge that the type of number used depends on the knowledge and confidence of the analyst in defining a variable. For the rest of this work, typical quantitative inputs will be considered, but the previous referenced work present applications of the uncertainty analysis approaches using different inputs.

2.2. Probabilistic

The idea of having a number that expresses how likely an event is to happen is very powerful and is in line with the intuitive meaning of likelihood (Young, 2018). The probabilistic alternative has as main characteristic that the possible values of the variable of interest X are assumed independent and associated with random variation. By specifying the evidence theory framework, more precise estimates can be obtained, but less information is provided. In particular, the information associated to knowledge -and lack of it-is not explicitly presented in the estimates and a sense of completeness of knowledge is associated to the estimates; this is not always the case though. There are three distinct alternatives for a probability analysis: classical, frequentist and subjective. The former is analytical and evaluates the expected result against all possible results, e.g. rolling a dice, usually of little use in risk assessments. However, in fire risk assessments there are variables for which a uniform probability distribution is assumed, i.e. classical approach.

Frequentist alternative is suited to problems where a fully analytical approach is not possible, introducing random variables which can be probabilistically analyzed. This approach usually begins with recorded data that represents the behavior of the system to later analyze it and estimate future states of the system. This is done through logical constructions and random sampling. An example of the databases used in frequentist approaches is loss of containment databases such as (E and Offshore Hydrocarbon, 2001) in which oil & gas operators record and submit leaks and their main characteristics. Another relevant example of frequentist probabilistic analysis is the one carried out for the Cassini spaceship, where the aim was obtaining the probability of having a failure during the spacecraft's flight and then a reentry to Earth (Frank, 2000; Laboratory, 1997). This is a case of interest given that the hazard related to the reentry to Earth was not the impact (with most materials melting in reentry) but the nuclear fuel. The report that describes the calculation (Laboratory, 1997) presents a detailed accounting of the diverse possibilities explored for a reentry to earth, which could only happen under very specific conditions which were deemed "highly unlikely". In the Cassini case the assumptions are clear and explicit, being supported by the engineers to show they are reliable. Almost two decades ago Apostolakis (1991) presented the issue of focusing on obtaining estimates of the parameters that define a probability distribution which allows to gain insight to support a decision and claimed that such practice should be avoided. Not long ago Young (2018) advocated for the same argument when analyzing research results product of a large number of regressions. Apostolakis concluded that researchers should not only report the researchers' preferred statistics and regression, but the whole body of models that they used.

Subjective probabilities constitute a third approach, as mathematically structured by Savage (1972) and also known as the Bayesian approach. This expresses the degree of belief towards a particular event from a set of states of the system, which can be 'updated' as new observations are obtained. Cooke (Cooke and Bedford, 2001) describes that this intuitive approach allows for 'rational decision' and the incorporation new evidence, resulting in increased knowledge and improved probability estimations. In the context of fire risk assessments, the possibility of achieving such observations is not guaranteed and seldom the case.

Consider the design and construction of a mid-rise residential building which is intended for a life of no less than 40 years. Furthermore, consider that the architectural characteristics of this building do not match the boundaries of a prescriptive code and therefore a fire risk assessment must be done to explicitly demonstrate its safety level. Such assessment will depend on a large amount of variable from which information is limited and the best approach to account for them is the use of assumptions, as presented by Aven (Aven and Kristensen, 2019). The updating of risk assessments (and validation of their assumptions) through the life-cycle of a system is addressed by Hackitt (2018), who proposes a safety case approach for the approval process of high-risk buildings in the UK. In a chemical process plants requiring safety cases, dedicated stakeholders continuously works on the collection of new observations to optimize the operation and update the risk picture. This is not the case for most occupied buildings. The previous highlight that probabilistic –particularly Bayesian estimates-risk assessments in fire safety should not be the only option to express uncertainty.

Bayesian networks (BN) allow decomposing complex dependencies between multiple variables, representing them graphically through nodes and links. BN are based in Bayes theorem, using known probabilities of basic elements to estimate the conditional probabilities of dependent elements. This is a powerful use of known probabilities to model unknown probabilities in complex systems such as estimating the dynamic operational risk assessment in a chemical process setting (Barua et al., 2016) or estimating the risk of human fatality in building fires (Hanea and Ale, 2009). As this method is based on conditional probability theory, its weaknesses are similar. Probability distributions for the random variables in the basic nodes are required, which can introduce engineers ' biases or assumptions that do not correspond to the system's reality; their effect can only be accounted for if they are made explicit.

2.3. P-boxes

This approach is the result of using probability theory and interval analysis (Traub, 1967) to produce probability boxes (P-boxes) which represent a class of distributions, instead of a single one. The class is a representation of the associated epistemic uncertainty and the variability (aleatory uncertainty). In typical probabilistic analysis, a variable of interest can be represented by a random variable with a given probability distribution function and its related statistics, but if this function is unknown, the interval approach can be applied to its distribution. By doing so the variable is circumscribed within upper and lower boundaries. Each boundary is also a probability distribution function and these can have a distinct shape, e.g. Exponential, or not, i.e. non-parametric. Whether the shape is well defined or not is a function of the available information to the engineers. This information is also the key of the simple but effective P-box approach, as well defined bounds for the variables of interest must be known in order to analytically obtain the upper and lower boundaries.

For a variable of interest X with known minimum and maximum bounds, e.g. (Jaime and Cadena, 2013; Van Coile et al., 2019), the non-parametric p-box would be the one in Fig. 3a, while if information about X allows assuming it distributes normally with bound on its mean and standard deviation, e.g. $\overline{X} = [1, 1.75]$, $\sigma = [0.1, 0.3]$ the result is as in Fig. 3b. This evidently provides the probabilistic approach with a higher flexibility, particularly when there is no confidence in point value estimates and the engineers prefer to express uncertainty as an interval.

2.4. Evidence theory

This first approach, based on the Dempster-Shafer theory, allows using different sources of evidence to support an estimate of the degree of belief. In the context of risk assessment this degree of belief is associated to the true value of a quantity of interest (X). The uncertainty -or simply lack of knowledge-regarding the true value of this variable of interest is typically characterized in a probabilistic manner, as presented in section 2.2. In evidence theory the possible events -or values of the variable of interest- (x_1, x_2, …, x_i) are not constrained to single events, but to all possible subsets of events. Each of this possible events provide the analyst with a larger range of options to not only define a degree of belief of a single value, e.g. x_3, but to define the degree of uncertainty corresponding to combinations, e.g. (x_1, x_2).

The previous is a generalization of probability theory, where the latter have a probability of zero, i.e. $P(X = x_1, x_2) = 0$. Evidence theory provides a platform to gauge the strength of evidence by defining a basic probability assessment or masses for each subset, e.g. $m(x_1 \cup x_2) = 0.1$. These masses are either obtained through expert elicitation or using the available data, allowing to define the belief and plausibility measures (*Bel(X)*, *Pl(X)*). These measures can be interpreted as upper and lower boundaries of the degree of belief, which supports making a prediction between to possible values x_1, x_2 given that $Bel(x_1)$ is greater than the belief of all other possibilities.

Given the generalized form of this approach, it can be employed to process both probabilistic inputs and basic probability masses, as presented by Du (2006) for the structural analysis of a machine. This provides an additional layer of information denoting lack of certainty regarding singleton values, which can be mapped into the same space in which probabilities are represented.

2.5. Possibility theory

Possibility theory is closely related to fuzzy variables, as these allow translating qualitative expert criteria into numbers. As described by Dell'Orco (Dell'Orco and Kikuchi, 2004) and Darby (2004), possibility theory with its necessity and possibility measures constitute the application of Dempster-Shafer theory to a problem in which the possible sets of the variables of interest are nested, and therefore non-conflicting. This means that any possibility for the variable, includes or is included in another subset.

This alternative provides engineers with an additional tool to express uncertainty in a case in which no conflicting evidence exist, but not enough evidence is available to have completely exclusive and independent sets. The latter is the special case of probability. This has been found to be the case for a large set of studies in the field of process safety, in which possibility theory is used to compute the risk assessment of

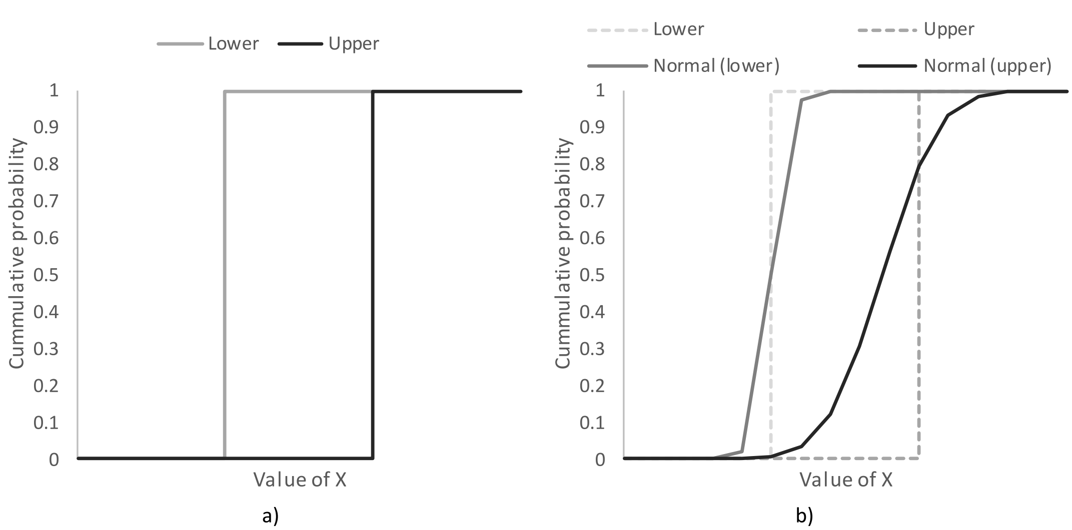

Fig. 3. a) P-box for known bounds of X, b) P-box assuming a normally distributed X with known bound on mean and standard deviation.

different experts for a set of scenarios.

Ouazari (Ouazraoui et al., 2013) presents the application to the Layers of Protection Analysis (LOPA), while Mandal (Mandal and Maiti, 2014) applies it to the Failure Modes and Effects Analysis (FMEA), both versatile and largely used process safety analyses. These examples show both the additional information conveyed by the results, in which risk is presented as a fuzzy number rather than a point-estimate (despite this can be is provided through different de-fuzzification methods). In these examples the outcomes reflect the possibility and necessity measures, analogous to the previously presented belief and plausibility of belief theory.

2.6. Info-gap

The approaches presented up to now rely on the concept of likelihood, degree of belief or directly to the probability. In these, the uncertainty of the analyses is provided within the output as in the case of evidence measures. Info-gap is an alternative approach which was first proposed by Ben-Haim (Ben-HaimY.M., Nikolaides, 2005). Info-gap's name points to the fact that there are information gaps within our models and therefore uncertainties that we need to manage, even in the extreme case in which the uncertainty level is so high that we cannot use one of the previously presented approaches.

Hayes (2011) describes info-gap as one of the main approaches to deal with uncertainty and highlights its non-probabilistic nature. This approach analyzes uncertainty from the perspective of how much uncertainty can the system handle, as a function of variations in the inputs and the parameters of the models used. This implies a considerable knowledge of the system and a model or set of models that allow obtaining an initial estimate, which in typical design approaches is used to obtain an optimized solution. Such solution is supported by a set of assumptions that in reality might not hold which if why info-gap provides a framework to test the resistance of the system to changes in them.

Ben-Haim (2005) exemplifies info-gap as a framework to establish a safety factor on the results of the system. This non-probabilistic approach -despite its requirement for considerable high levels of knowledge-provides a different approach to uncertainty. Such approach provides a solution that acknowledges the lack of knowledge and the impact this has on the system's response, instead of a frequency, probability or degree of belief.

2.7. Strength of knowledge

The idea of designing a system that involves uncertainty without addressing it as a probabilistic problem was presented with info-gap, but this is not the only approach. Strength of knowledge is another one of these approaches. It aims at identifying and managing assumptions and limitation used in an analysis and which could significantly affect its outcomes. Based on the studies published by its authors, Strength of Knowledge can be applied in different manners and it is flexible. In fact, it can be seen as a very general idea which is then tailored to suit the specific features of each problem, as presented in (Goerlandt and Reniers, 2017; Flage and Aven, 2018; Aven, 2011b, 2016b, 2017).

In a typical risk assessment -either in chemical process or fire safety-assumptions usually are found everywhere, beginning with the information defining the system, then the construction of the scenarios, the models to estimate risk indices and to evaluate them. The general idea of Strength of Knowledge is to identify key assumptions made within a risk assessment and then judge whether they need to be addressed before communicating the results to the stakeholders. To judge the strength of knowledge a set of ordinal categories of qualitative or semi-quantitative nature are defined (e.g. low/medium/high) with their corresponding criteria, which are then used to judge each assumption.

The supporting knowledge for an assumption is not the only aspect that can impact the outcomes, as the sensitivity of these to a variation in the inputs or in the assumption is also key. This aspect is also judged for each assumption using another set of ordinal categories, which can be defined with purely quantitative criteria based on sensitivity ranges on the outputs. Qualitative criteria can also be used for assumptions that cannot be quantitatively assessed, requiring a competent expert formulating them and overseeing the judgment. The results can help identify a narrow range of potentially problematic assumptions requiring treatment.

This treatment can be part of the risk assessment itself, such as further probabilistic analysis or the use of imprecise probabilities (e.g. evidence theory). If the resources are exhausted an at the end of the assessment there are still critical assumptions, these can be part of the decision making process and lead to establishing monitoring, control and mitigation measures that account for them not holding during the operation of the system.

2.8. Exploratory Model Analysis

Acknowledging the presence of deep uncertainties at different levels within and around a system, there is a need for uncertainty analysis alternatives that move further away from prediction and grow closer to the concept of robustness already introduced with the info-gap alternative. Exploratory Model Analysis presents a framework in which models are no longer used as the tool to find the exact answer or in the case of risk assessments, a failure prediction. Instead, models are recognized as the flawed constructions that they are and this is instead used to explore a wide range of plausible worlds, i.e. scenarios, and assess the response to particular decisions made by the stakeholders. In a risk assessment this means exploring how a large set of possible scenarios would react to different risk management strategies and decisions.

EMA can remove the mentioned burden on engineers and offer simpler approach to make the best out of the available models. Based on the work by Kwakkel (2017), EMA can be described as in Fig. 4 where the possible worlds are tested using different policies based on a model for the system. This constitutes a single computational experiment and each variation in the scenarios or the policy used generates different outcomes of interest. This differs from a typical engineering analysis in that all uncertainty sources are explicitly considered as part of the scenario, and allows analyzing the relation between these and the outcomes to identify those that produce higher sensitivity or breakpoints. Such process is the vulnerability analysis, which is complemented by the robustness evaluation of the outcomes that comes close to the robustness concept of the info-gap theory.

EMA can be interpreted as an intelligent exploration of unknown realities using imperfect models, from which a prediction is impossible. The value of this approach is that it allows providing the key stakeholders with a comprehensive set of results that identify a policy that results in successful outcomes based on a given set of conditions within the scenarios. This is clearly and brilliantly exemplified by the implementation of EMA for the policy selection of water management of a river in the United States (Groves et al., 2013). Given the simplicity of the compartment fire example EMA is not implemented as it would require increasing the complexity of the system to introduce the possibility of testing different risk management strategies.

3. Case study: compartment fire

To exemplify the differences between the presented alternatives a typical simple FSE example to illustrate them; both possibility theory and EMA are excluded. The former is excluded on the basis of its reliance on the additional fuzzy input from multiple practitioners, while the former is excluded as the simple nature of the example used does not support a wide range exploration as EMA intent.

The example used is the calculation of the descent of the smoke layer in a simple rectangular compartment of length 7 m, width 5 m and

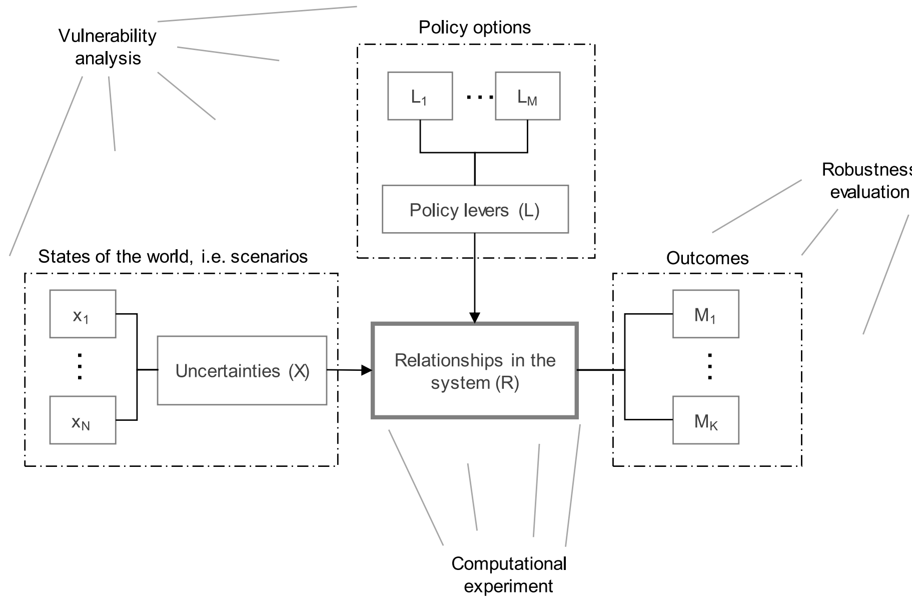

Fig. 4. EMA process and main components based on (Kwakkel, 2017).

height 3 m (total floor area 35 m^2) with a door of width 1.5 m and height 2 m. The compartment is of residential nature and expected to have a representative fuel load of polyurethane (PU) which is initially taken as 600 MJ/m^2 (Ocran, 2012) with a heat release rate of 400 kW/m^2 and a medium rate of fire growth ($\alpha = 0.0117$ kW/s^2). Using the set of equations of Annex 1 based on energy and mass conservation, as well as some key experimental correlations, a suitable model is constructed to perform the calculation. The model is applicable for pre-flashover conditions and limits the fire size to 1 MW, which is one of the assumptions also registered in Annex 1 and discussed through the next sections. With a critical height established at 2 m, the key output of the model is the time for the smoke layer to reach this level.

Assuming the previous input values without any uncertainty and applying them to the model, the result obtained is 58 s. Such an output is key for available vs required egress time calculations in performance-based analysis and it is discussed related to the outcome provided by each approach and its advantages and disadvantages.

3.1. *Probabilistic*

To illustrate a simple application of the probabilistic approach, we use the compartment fire example. Not enough information exists to define the exact value of the fire growth rate, which can range from a slow growing fire ($\alpha = 0.00293$ kW/s^2) to an ultra-fast one ($\alpha = 0.1874$ kW/s^2) (Bwalya et al., 2003). This leads to the engineers to formulate alpha as a continuous random variable, which according to Nilsson (Nilsson and Van Hees, 2014) distributes as a log-normal function with a mean of 0.01924 kW/s^2 and a specified 99.5th percentile of 0.219 kW/s^2, which are the result of analyzing 2965 fires. By using simple add-on to Microsoft Office Excel known as SIPmath (Sam and Savage, 2012), a Monte Carlo simulation is performed in order to sample the distribution an obtain ten thousand input fires, which are described by Fig. 5.

The obtained samples are the input for the compartment fire model in which the time for the smoke layer reaching 2 m can be calculated, yielding the results presented in Fig. 6. From this output it is possible to find obtain the time for untenable conditions and therefore the available time for egress in the compartment, which for a 50th percentile of 53 s and a 99th percentile close to 40 s. This indicates that there are fire scenarios in which extreme (but not impossible) scenarios can yield 18 s less than the previous output which did not account for uncertainty.

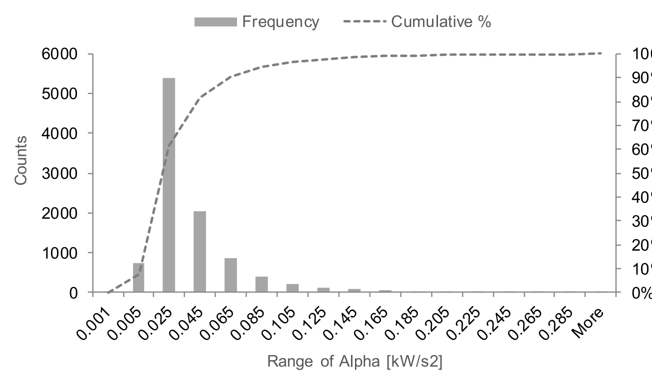

Fig. 5. Fire growth alphas sampled for input of the probabilistic analysis.

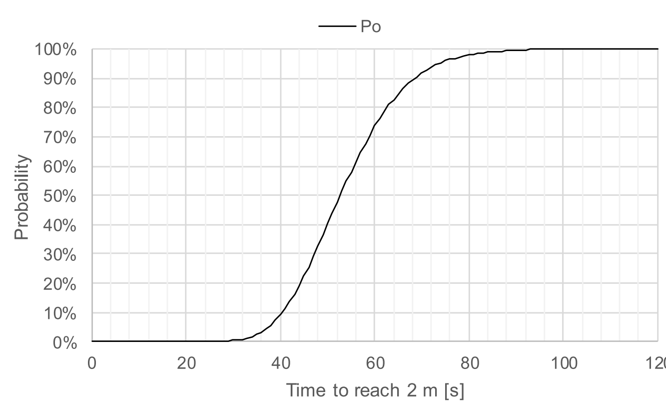

Fig. 6. Resulting distribution for descent time.

Likewise, there are scenarios in which the time is as large as 89 s, which if taken into account could lead to underestimate the risk.

3.2. *P-boxes*

To construct the P-boxes for the compartment fire example, a range of distributions replaces a single probability distribution. This allows incorporating uncertainty on the distribution parameters, which for the case of the compartment fire and the time for the smoke layer to descend to 2 m height are given by the 50th and 99.5th percentiles of the lognormal distribution that described the fire growth coefficient, alpha.

Fig. 7 presents the resulting P-box having P_o, the previous single distribution, P_1 a and P_2. The new distributions respond to the analyst considering different conditions than the ones initially introduced in the previous section. P_1 rules out possible arson in the compartment, which leads to a lognormal distribution with 50th and 99.5th percentiles of 0.011 kW/s^2 and 0.105 kW/s^2. P_2 is a pessimist view of the analyst, based on the fact that the data used for the previous distributions do not match exactly the nature of the compartment.

In order to present a more onerous scenario, the parameters of P_o are doubled, resulting in a lognormal distribution with 50th and 99.5th percentiles of 0.0385 kW/s^2 and 0.438 kW/s^2. Here, specifying a range becomes harder, as now there are three distributions for which the results could be reported. Using the results of Fig. 7, the original result of 58 s has a 45% probability in the pessimistic P_2 distribution, which could lead analyst to further hesitate in using such result. This of course translates into confidence boundaries and is an ideal tool to better understand the uncertainty of the calculation. With occupants' walking speed potentially under 1 m/s given they are sleeping or have a disability, the result of the P-box could help identify potential improvements to reduce egress path length.

The disadvantage for the use of such results is that their communication can mislead stakeholders in believing that the worst possible conditions are covered, which might not be the case depending on the engineers' trustworthiness on their assumptions such as the pessimistic parameters of P_2. As additional data becomes available such as that presented by Hopkin et al. (2019), the possibilities for incorporating different conditions further enable the use of P-boxes.

3.3. Evidence theory

Back to the compartment fire example, the evidence theory is applied to conflicting information regarding the fire growth rate. Two fire engineers are unsure of the correct rate to use in the calculation as the compartment's occupation might change through time. This provides a considerable large range of possible fire growth rates, leading to the use of probability masses for the slow (S), medium (m) and fast (F) rates. The frame of discernment is therefore $\theta = \{S, M, F\}$ and a power set: $2^\theta = \{\emptyset, S, M, F, (S, M), (S, F), (M, F), (S, M, F)\}$. For these eight focal elements, the two experts are asked to provide the basic probability assignments with $m(\emptyset) = 0$, which is presented in Fig. 8.

Disparity between the experts in their opinion is evident, which is common in the context of risk assessments (Yildiz et al., 2014) and could result in enhancement of the results (Bergmans et al., 2009). Particularly, discrepancies might be found between third party reviewers and the opinion of the fire services, which might want the consideration of more onerous fire scenarios. By applying Dempster's rule of

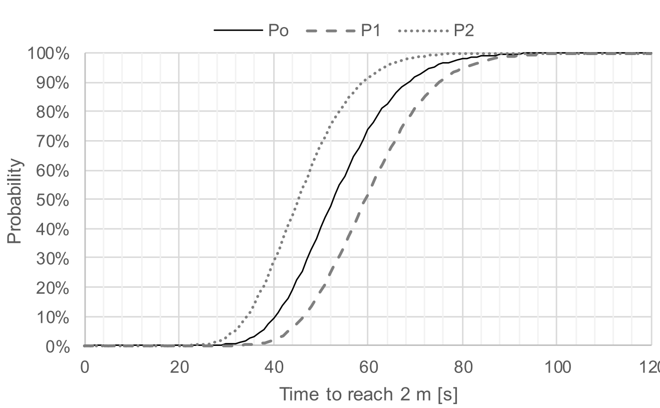

Fig. 7. P-box for the descend of the smoke layer height.

combination, these two independent structures can be combined and the resulting structure used to estimate the quantity of interest, i.e. time for the smoke layer to descend to 2 m height.

The results for the resulting structure for the quantity of interest are presented in Fig. 9. The results indicate that this time could be in the range between 41 and 17 s, with the former having a 40% likelihood and the range between 23 and 17 s one of 24%. Although this information -as with P-box analysis-does not provide a point-value answer, it enables combining evidence provided by multiple experts and obtaining a simple but clear result which could support the need to explore further design fires.

One possible application of the Dempster-Shafer theory is using structures such as those in Fig. 8 to construct probabilistic distributions and then assessing the belief and plausibility of a whole family of these. Such application can again be applied to our example, this time moving away from different engineers providing evidence, to a single one which provides a belief structure for the parameters of the lognormal probabilistic distribution considered in sections 2.2 and 2.3. The analyst defines the ranges for the parameters of the distribution as presented in Table 5, which are computed using the IP Toolbox add on for Matlab and yields the results of Fig. 10. This graph provides the result for the belief and plausibility measures for the evidence provided and it is compared to the possible P-box resulting from using the extreme values of the ranges of Table 5.

The comparison shows that indeed the evidence measurements provide additional insight to the possible ranges in which the 'real' distribution might be found, which are much narrower than those provided by the P-boxes. Using these results, the time for the smoke layer to reach 2 m height can be estimated for the 50th and 95th percentiles, yielding the ranges [25 s, 32 s] and [22 s, 25 s], respectively. It must be noted that in this example the conflicting information is minimum, with both experts not assigning a basic probability assignment to some subsets such as [S, F]. However, Dempster-Shafer theory does allow for the inclusion of this type of conflicting evidence, which could be useful for some fire safety studies where expert criteria largely differ (Dell'Orco and Kikuchi, 2004).

3.4. Info-gap

Ferson and Tucker (2008) provide an understanding on how to implement the info-gap approach using probability bounds, i.e. P-boxes. Info-gap is implemented following the safety factor concept. Specifically, four alternatives are presented and info-gap theory does not prescribe any particular one, as the analyst must decide which approach better fits each system. The approaches presented by Ferson and Tucker are practical and require a robustness measure (named alpha, for clarity referred here as α_R) which is applied for the inputs of the system and allow identifying the level at which the system's response is no longer acceptable. This measure is applied to confidence bounds, e.g. Kolmogorov-Smirnov for empirical distributions, proportional bounds, distribution shift and a validation metric.

The previous ideas are in line with that of Cadena (Cadena et al., 2019) when assessing fire risk through the concept of maximum allowable damage in a building for life safety, as an alternative to typical probabilistic approaches. These approaches are applied to the fire growth rate distribution presented in section 2.2 from the compartment fire example. First, an exponential function is defined which increases the P-boxes proportionally to the value of the fire growth rate (Fig. 11), followed by multiplying α_R and the critical value of the Kolmogorov-Smirnov test to the nominal distribution, both for a confidence of 1% and 20% (Fig. 12); the latter provides a wider P-box.

Applying the α_R to the nominal distribution original range and generates a simple but effective set of inputs, given that a fire growth step is established (a value of 0.01 kW/s^2 is selected). With this simple application of the robustness measure, the time for the smoke layer to reach 2 m can be tested (Fig. 13) until it reaches a critical value, defined

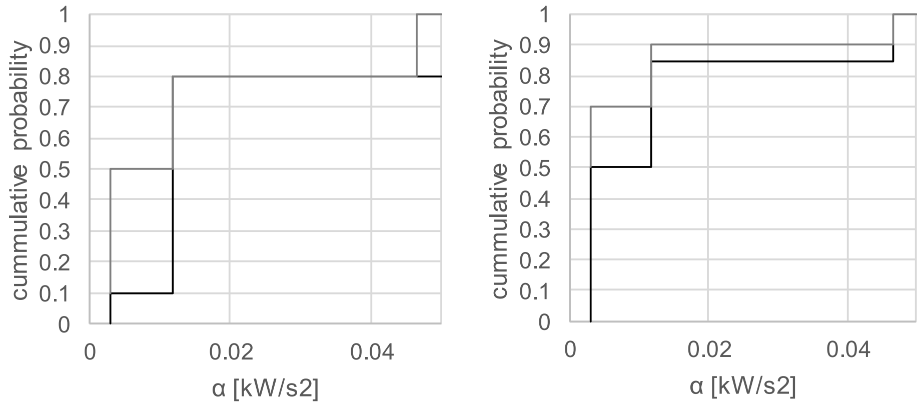

Fig. 8. Dempster-Shafer structures for fire growth rate based on two experts' opinion.

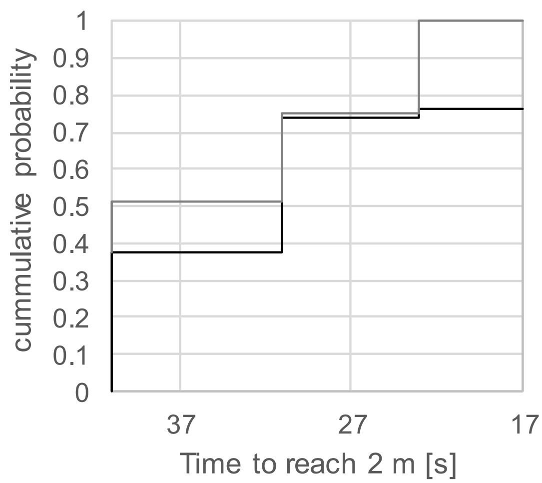

Fig. 9. Dempster-Shafer structure resulting for the time to reach 2 m based on two experts input.

Table 5

Evidence for constructing the distribution of the fire growth rate.

μ (kW/s^2)	σ (kW/s^2)	Basic probability assignment
[0.01, 0.03]	[0.03, 0.04]	0.9
[0.03, 0.05]	[0.04, 0.1]	0.1

as the time it takes for an occupant to egress from the room. Based on the maximum path distance of 12 m within the room and a walking speed of 0.7 m/s, this critical time is 16 s. A value of $\alpha_R > 17$ is required for the fire growth rate to reach ultra-fast (~0.19 kW/s^2) and yield a time of 16 s for the smoke layer to descend to 2 m height. As the value of α_R increases, so does the confidence on the result, which goes from 5% for $\alpha_R = 16.95$–90% for $\alpha_R = 17.17$ (Fig. 14).

This outcome would provide the designers of the building with an understanding of the robustness that can be expected from the compartment, which can then be extrapolated to the whole building. Although the result for robustness is accompanied by a notion of likelihood or chance, it is important to understand that info-gap analysis mainly focuses on the potential for the system to resist a load, rather than on the likelihood of said load actually occurring.

The application of info-gap theory and the robustness function in this example is simplified to provide the reader with a clear understanding of the potential for its use in fire safety. As complexity increases, info-gap robustness measurement allows evaluating two competing design alternatives which could imply significantly different trade-off in a fire safety strategy within a building. Such an evaluation is objective and is

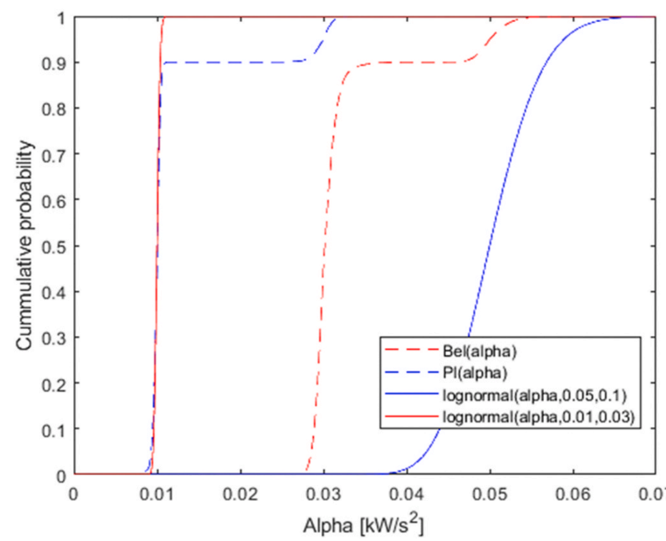

Fig. 10. Belief and plausibility measures for the lognormal distribution with uncertain parameters.

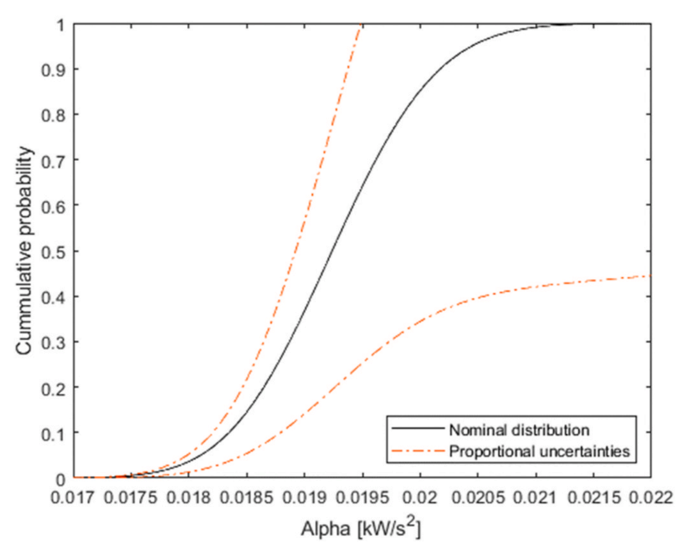

Fig. 11. (Left) Info-gap concept applied to P-boxes through proportional uncertainty.

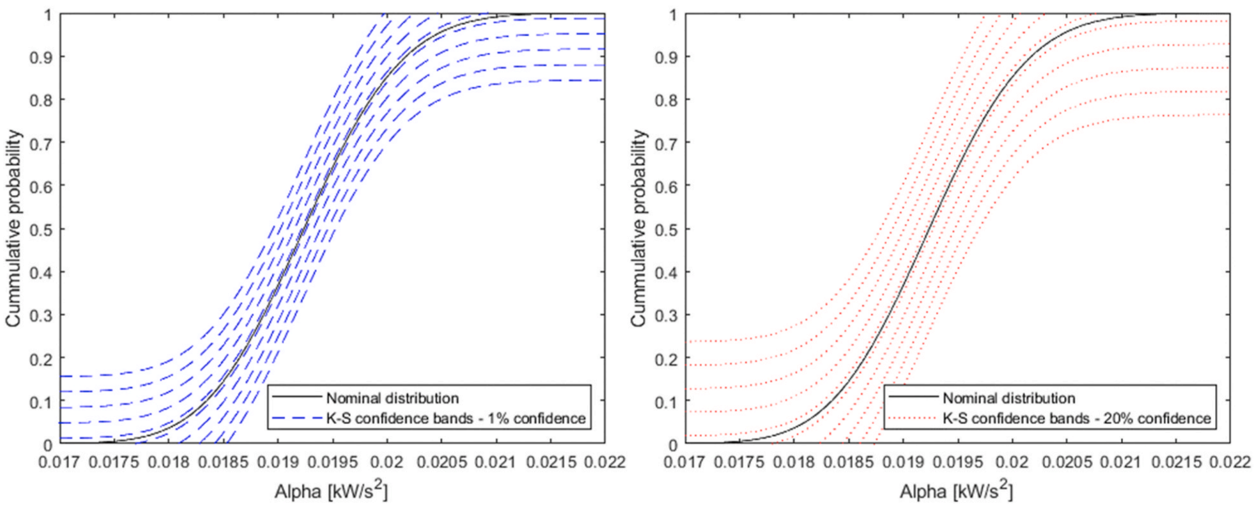

Fig. 12. Application of info-gap with Kolmogorov-Smirnov confidence bounds and α_R between 1 and 20.

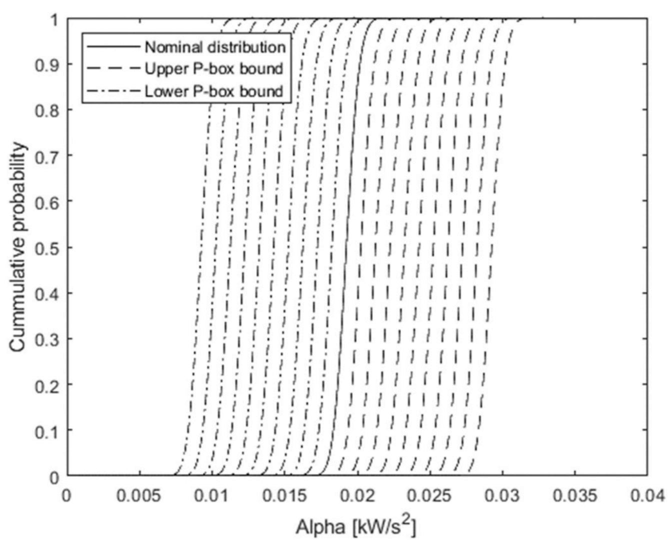

Fig. 13. Simple application of info-gap to generate fire growth rate P-boxes.

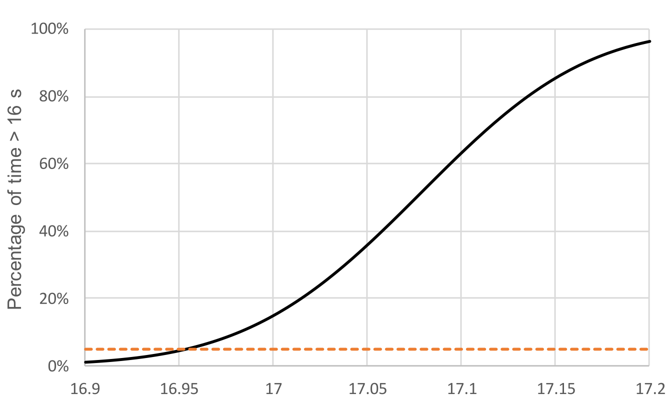

Fig. 14. Likelihood of time descending to 2 m being less than 16 s; 5% chance at a value of $\alpha_R = 16.95$.

not based on the likelihood of an unknown event or condition, but on the amount of uncertainty that the system can tolerate and still perform adequately.

3.5. *Strength of knowledge (SoK)*

This alternative is applied to the compartment fire example, which begins by defining the levels and criteria for the strength of knowledge and output sensitivity. The former is adapted from previous work, as strength of knowledge criteria is largely compatible with the features of this example. For output sensitivity, the criteria are defined based on the variable of interest, i.e. the time for the smoke layer to descend to 2 m height, compared to the time required for occupants to egress the compartment, which is 16 s based on the calculation of the previous section. The criteria for both aspects are presented in Table 6 and are applied to judge the list of assumptions within the analysis. The six assumptions and their SoK assessment are presented in Table 7, from which a wide range of potential issues are identified with the analysis.

From the list, all assumptions have the potential to influence the result, but those with lowest SoK and highest output sensitivity (OS) can be prioritized. This makes assumptions 3, 5 and 6 stand out and require a close analysis. Assumption 3 questions the validity of the chosen untenability criterion selected as a function of smoke height layer. This potential issue with the performance criterion itself is treated by analyzing the output of the compartment fire model used (Annex I) and comparing both criteria.

Table 6
Strength of knowledge and output sensitivity levels and criteria.

Aspect	Level	Criteria
Strength of knowledge	High	-Updated references back up the values or assumptions -Strong and relevant theoretical grounds -Subjective knowledge is backed up by theory or robust research
	Medium	Neither high nor low
	Low	-Poor theoretical grounds and references for the values and assumptions -Low consensus between personnel involved in the assessment -Knowledge sources are subjective and not validated
Output sensitivity	High	Variations within known ranges yield shorter available times than those required
	Medium	Neither high nor low
	Low	Large –unrealistic- variations required to yield shorter available times than those required

Table 7
List of assumption and SoK judgements.

Assumption	SoK	OS	Justification
1. Fuel is polyurethane foam	Medium	Medium	Representative of the fuel in a typical dwelling
2. Alpha t-squared fire growth is valid	High	Medium	Despite fire spread and growth having a complex behavior, in a dwelling this assumption is reasonable as long as different scenarios are analyzed
3. Smoke layer height criterion is valid	Low *(High)*	High *(Medium)*	Smoke layer temperature could reach a critical temperature before the smoke descends to 2 m height, thus yielding untenable conditions before it
4. No ventilation effect on smoke layer descent	High	High	Natural ventilation like doors and windows have a soffit below 2 m, therefore assuming it does not influence the smoke layer descent is onerous but valid for an initial analysis
5. Detection, notification and pre-movement times = 0 s	Medium	High	The occupants will react to the fire within the compartment and immediately evacuate
6. Walking speed = 0.75 m/s	Medium	High	Based on SFPE data (Gwynne et al., 2016) this is a slow speed, providing an onerous scenario for this simple analysis

The outcomes of the comparison are provided in Fig. 15, showing that smoke height layer is a more conservative criterion. Based on this result, the SoK and OS are assigned new values (High, Low) and is no longer a pressing concern for the output of the analysis. The remaining two assumptions, No. 5 and 6, point towards unreliable inputs that can be addressed in a simple and effective manner. Assumption 5 points out that no detection, notification and pre-movement times are considered for the value of 16 s representing the required safe egress time. A pre-movement time that cannot be exactly pre-defined. This means that both assumptions remain unchanged and constitute an important source of uncertainty which can increase the previously calculated 16 s, to times of 46 s or more (using a pre-movement time of 30 s based on the SFPE Handbook data (Gwynne et al., 2016)).

As pre-movement time increases due to lack of adequate means of notification or lack of understanding of the occupants due to low familiarity, the system reaches an unacceptable performance with lower fire growth rates. This evidently echoes the robustness approach of the info-gap theory, but also implies a much simpler course of action. Despite the lack of quantification, here it is easy to identify it is essential to ensure proper notification and a delivery of clear instructions to the occupants in order to maintain the system's performance.

4. Alternatives evaluation

The previous section presented a description of the main uncertainty analysis options, providing and objective and brief description. In this section an analysis of these alternatives is provided, which aims at helping risk analysis to select one of them according to their needs. To conduct the analysis, the Design Science Research Methodology (DSRM) (Peffers et al., 2007) is used with a modification.

The original DSRM includes six sequential steps in which a problem is identified, objectives for a solution are defined and an artifact is designed, demonstrated and evaluated. The last step is communicating the results of the process and the advantages of the artifact. In this work we don't produce an artifact, but evaluate existing artifacts that help describing uncertainty and therefore better understand risk. Based on the previous, the design and demonstration steps are omitted and Table 8 shows the summary of the modified DSRM.

Based on the proposed DSRM, this section addresses the evaluation of alternatives, for which a hierarchical structure is proposed following Prat's guidelines (Prat et al., 2014). The two dimensions selected for evaluation are suitability and effectiveness in the context of a fire risk assessment. These dimensions are broken down into evaluation criteria and sub criteria as shown in Fig. 16. The first dimension is suitability, which is assessed by evaluating whether (Fig. 16) the alternative provides significant advantages and insignificant limitations. The second dimension is effectiveness, evaluated based on the ease of implementation and of communication of each alternative. The evaluation criteria for both dimensions are defined in Table 9.

Implementing the DSRM to each alternative is done following the hierarchy structure and the evaluation criteria previously presented, with the results included in Annex II. The information consigned there not only presented the evaluation criteria and the level for each one, as well as the associated supporting comments, available software and

Table 8
DSRM steps applied to this work.

DSRM step	Application to this work
Identify the problem and motivate	Better risk understanding through the use of appropriate uncertainty analysis options (Section 1)
Objectives of a solution	Provide a clear picture of the uncertainty analysis alternative for fire safety engineers
Evaluation of alternatives	Use of hierarchy structure to perform the evaluation (Fig. 16)
Demonstration	Example of the compartment fire applied to the alternatives
Communication	Conclusions and opportunities based on the analysis are presented (section 0)

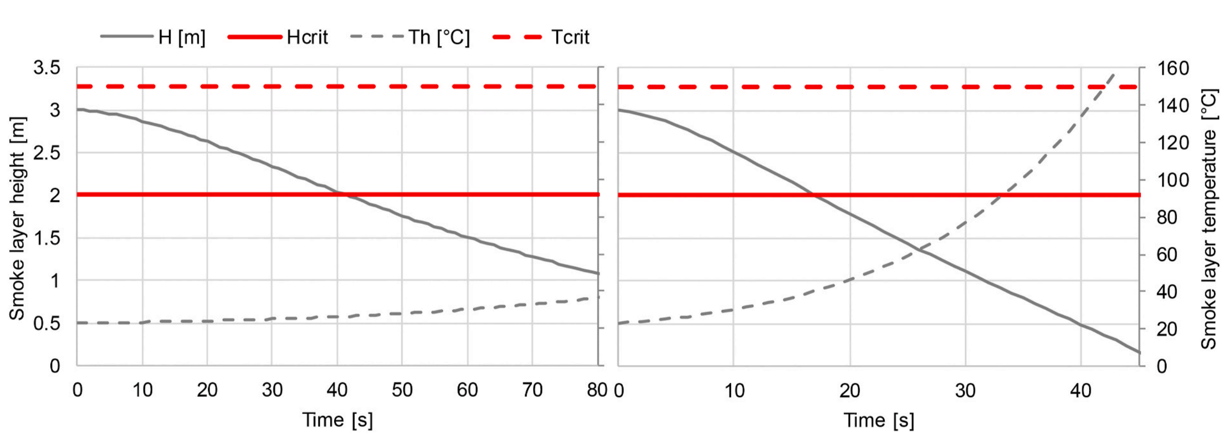

Fig. 15. Comparison of tenability criteria for a slow (left) and ultra-fast (right) fire growth.

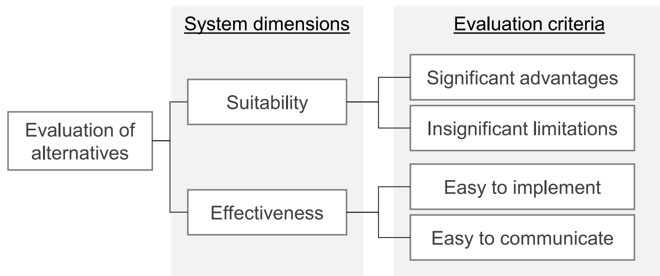

Fig. 16. Proposed hierarchy structure for the evaluation of alternatives; based on (Prat et al., 2014).

Table 9
Evaluation criteria -Suitability dimension.

Evaluation criteria	Level	Indicator of level attainment
Significant advantages	Fully achieved (FA)	The alternative is advantageous relative to the knowledge it provides and the potential to act upon it by relevant stakeholders.
	Partially achieved (PA)	Cannot be judged as FA nor NA
	Not achieved (NA)	Does not provide significant advantages or none can be associated to this incertitude space
Insignificant limitations (not related to implementation)	Fully achieved (FA)	Limitations exits but are clearly defined and minor changes in inputs or model parameters or assumptions do not make the approach inviable
	Partially achieved (PA)	Cannot be judged as FA nor NA
	Not achieved (NA)	It is possible to trespass the applicability limits with minor variations to inputs or changes to model parameters or assumptions
Easy to implement	Fully achieved (FA)	Clear sequence of steps in which data manipulation is clear and traceable; guidelines, algorithms and software are readily available
	Partially achieved (PA)	Cannot be judged as FA nor NA
	Not achieved (NA)	Lengthy and complex process with no clear guidelines for implementation. Software is not available or scarce.
Easy to communicate	Fully achieved (FA)	The output's uncertainty is easy to communicate and compare, being consistent with the quantity and detail level of the inputs.
	Partially achieved (PA)	Cannot be judged as FA nor NA
	Not achieved (NA)	The output does not clearly represent the uncertainty involved and can be inconsistent with the nature of the inputs.

implementation examples.

Once the evaluation is completed for all alternatives, the results can be synthesized as presented in Table 10. Notice that the objective of the evaluation is to show that none of the alternatives is perfect and all of them will imply important challenges for the practitioners implementing them either in FSE or other fields. It is also important to notice that the challenges that each alternative imply vary greatly in the dimension and criteria evaluated, which means that some of them will be better suited for some problems.

First, a general analysis of the results is done, beginning with those of the *suitability* dimension. The previous point of all approaches being useful is reflected by the results of the *significant advantages* criterion, where all alternatives partially or fully achieve it. It can be observed that a breakpoint exist in the *insignificant limitations* criterion, which can be decisive for practitioners exploring new options. The first four approaches score a *NA* for this criterion, as all of them are constructed on the basis of strong and often not justifiable assumptions such as following a particular shape of probability distribution function or on the subjective estimates of experts which can seldom be directly validated.

Although both info-gap and strength of knowledge present some important limitations, the basis of these approaches are physical phenomena and the recognition of the incomplete or imperfect knowledge, respectively. Such basis provides practitioners with imperfect but useful approaches in which uncertainty is explicitly formulated. In the effectiveness dimension, it can be observed that the more recent and alternative approaches present larger issues of implementation and communication. This can be a major obstacle for their implementation in the FSE context, as development of competences to correctly implement the alternatives might be needed. Communication is also an issue, as both the competences of the practitioners' play a crucial role, as well as the willingness of the AHJ and other stakeholders to analyze uncertainty from a non-probabilistic approach.

With the exception of software limitations that decrease the implementation ease for possibility theory and the opposite situation for probability, the first four approaches are considerably similar. Each one offers a unique way to understand the likelihood of an event occurring or of a condition existing, but approach the problem in a similar manner and therefore also offer similar challenges for practitioners. In particular, those different to probability have a heavy reliance on subjective estimates provided by more than one expert. This is reflected by the consistent NA level of achievement for the insignificant limitations criterion by all four approaches, which –again- does not render them useless, but impose significant challenges on the practitioners who wish to implement them. The mechanical nature of the calculations involved in these approaches can lead to believe that the results are trustworthy, but each individual problem should be treated on a case-by-case basis and the limitations explicitly stated. This treatment of the potential sources of uncertainty within the approach itself is seldom done, as the mechanical nature of the approaches usually leads practitioner to rely only on sensitivity analysis.

Info-gap and strength of knowledge approaches present two different ways to deal with uncertainty without necessarily using a probabilistic approach, however they are able to accommodate it. The results of the evaluation for the former indicate significant advantages for practitioners as it avoids dealing with uncertainty by focusing on measuring the robustness. In the case of a fire risk assessment this translates into understanding how much extra loads can a building take under fire condition before the objectives are compromised. However, there is not a standard way to do it and the available examples -although detailed and clear-need to be carefully extrapolated into each individual problem. Here lie the important limitations for this approach, as a deep analytical understanding of the problem is not always available in FSE; this triggers the use of methodologies as the one previously mentioned based on the concept of the maximum allowable damage (Cadena et al., 2019). This methodology acknowledges the presence of uncertainty in the risk assessment and triggers two simultaneous processes: assessing the system's performance as a function of the safety objectives and systematically record and judge the assumptions and limitations embedded in inputs, parameters and models employed.

The purpose of judging assumptions and limitations is not only to identify potential weaknesses of the assessment, but to identify actions that allow monitoring and controlling them during the life-cycle of the system. In the context of the ISO 31000 framework, this would be part of the risk treatment plan. The default tool to judge assumptions and limitations in the maximum allowable damage methodology is the strength of knowledge approach. The evaluation results for this

Table 10
Evaluation summary [].

Dimension	Criterion	Probability	P-boxes	Evidence	Possibility	Info-gap	SoK	EMA
Suitability	Significant advantages	FA	FA	PA	PA	FA	FA	FA
	Insignificant limitations	NA	NA	NA	NA	PA	PA	PA
Effectiveness	Easy to implement	FA	PA	PA	NA	NA	PA	PA
	Easy to communicate	PA	PA	PA	PA	PA	FA	NA

approach can be regarded as the best of all, as it fully achieves significant advantages and ease of communication. The bibliography available on this approach show how the simple idea of SoK can be applied in both a qualitative or quantitative format and effectively support decision making even in situations where knowledge is recognized to be incomplete or imperfect; this represents a significant tool for fire risk assessments, where much is known but even more assumed. The challenges of this approach lie on the need to properly setting it up in a case-by-case basis and with the use of categories that can evoke the same issues of constructing a risk analysis matrix (Duijm, 2015).

In the *suitability* dimension, only EMA fully achieves the *significant advantages* criteria given its explicit recognition of deep uncertainty and the treatment of a massive universe of possibilities in order to inform decision-makers. An approach like EMA and others belonging to Robust Decision Making (RDM) explore a wide range of possible futures based on a large scale exploration of variations of both conditions and solutions and both the theoretical construction of these futures and the complex relationships between variables involve represent a significant limitation. Even when implementation (rated *PA*) is supported by machine-learning or artificial intelligence, the complexity of these algorithms can be such to not allow traceability of the results or exploring the relationships between key variables. Furthermore, EMA does not fully achieve the criteria for effectiveness, as its implementation is still limited to complex problems which can be well characterized and populated with significant background information. Communicating EMA's outputs also poses a challenge, particularly for stakeholders used to a quantitative answer in the form of probabilities or frequencies. This approach can be considered at the extreme end of the possibilities to analyze uncertainty in fire safety risk assessments, and could be a powerful tool to assess the potential impact of fire safety policy changes such as proposed bans on specific materials.

5. Discussion on the practitioners' perspective

5.1. Discussion from the practitioners' perspective

As previously stated, the aim of the paper is to supply practitioners with a picture of the uncertainty analysis approaches and their evaluation based on ease of implementation/communication. In order to gauge and discuss the impact of the alternatives analysis an interview with two risk assessment experts were interviewed. The interviews do not intend to provide a statistical representation of the perception of all practitioners, but to incite the debate on key issues and needed changes in the role of risk assessments and their outputs in FSE. First the profile of the interviewees is presented, followed by the interview results and a brief discussion. Interviewee 1 (I1) has a doctorate in chemical engineering in the topic of systems theory applied to safety engineering, over 10 years of academic carrier in the process safety field and is currently a senior health and safety professional at a major oil & gas company. Interviewee 2 (I2) has a doctorate in civil engineering in the topic of structural fire, over 10 years of academic carrier in the FSE field and is currently a professor in a worldwide recognized university. Both interviewees were consulted due to the trajectories and recognition among peers of their contributions to their respective fields.

First, the interviewees were asked if they consider the probabilistic approach the best suited alternative to assess the performance of an engineering system, specifically the fire safety of an occupied building. I1 pointed out that identifying low probability scenarios is not always possible following a probabilistic approach, requiring complementary techniques. In particular, I1 drew attention to the outmost need of preliminary qualitative risk analysis that focus on identification of scenarios that then lead to refined quantitative analysis of prioritized scenarios. This is consistent with the approach presented by BS 7974:2019 (H247974:2019 and Appl, 2019), and differs from a more mechanistic approach as proposed by the verification methods (Johnson and Lobel, 2018). I1 states that following a probabilistic approach or an alternative is a decision to be made as a function of the analysis objectives and the available resources. He further suggest such a decision needs to be made by stakeholders at an early stage of the project and that in particular, the probabilistic approach lacks usefulness when the reliability changes in the system (also in its sub-systems and elements) are not managed through time. I2 answer is that the probabilistic approach is the best suited alternative for some systems, as it is the alternative that provides the highest resolution, i.e. most insight in the spectrum of possible performance of the system in case of fire. Contrasting this, I2 recognizes that some systems might only need a site visit from a competent professional to effectively account for uncertainties. A key point made by both I1 and I2 is that a risk assessment cannot be turned into the probabilistic analysis of all involved variables or uncertainties. I2 reinforces this mentioning the value of guidance and of prescriptive codes, which are applicable –and adequate-for a large range of projects. I2 states that codes and guidance -particularly in FSE-have the value of providing a common benchmark and alternatives to approach different problems. This range can be seen as to go from a purely guidance based (prescriptive) to depending fully on the competence of the practitioners (performance-based). This last point made by I2 poses an interesting issue, as both extremes of practice range in FSE (rule-based and performance based) have identified the danger posed by poorly managed uncertainties. In the context of chemical process safety, risk assessment guidelines point towards the probabilistic approach and the use of Paté-Cornell's (Paté-Cornell, 1996) uncertainty management levels to manage uncertainty in a quantitative manner. However, safety engineering presents a wide range of implicit and explicit manners to deal with uncertainty (Notarianni et al., 2016) including the way in which scenarios are identified, pre-defined rules for systems fitting a taxonomy (prescriptive approach), etc. The interviewees agreed on the need for a preceding qualitative assessment that guides the probabilistic one, and on the fact that using the former is no guarantee that uncertainty is adequately accounted for and managed.

The previous leads to the second question regarding the obstacles of implementing the alternatives in practice. I1 states that the alternatives constitute a 'toolbox', from which a selection must be made early in the assessment. This selection is based on three elements: 1) assessment objectives as a function of decision-making, 2) available information and 3) available resources. Given that these elements are clearly identified, I1 finds that any obstacle in selecting an alternative will be associated to the stakeholders with authorization (approval) power and the system users. This is coherent with the difficulties in implementation found in the evaluation, which are common to all alternatives and require informed stakeholders with the ability to interpret the outputs. Furthermore, I1 emphasizes the need to prioritize the uncertainty analysis alternatives for each assessment based on an initial qualitative analysis. To address the question, I2 states that in FSE the current preferred approach to risk assessments is implicit worst credible scenarios (deterministic), which is in fact backed up by the analysis of Johansson (Johansson et al., 2011) of Australian fire engineering reports and presented in a broader international perspective by Bjelland (Bjelland and Borg, 2013). Therefore, I2 highlights that practitioners should be more aware and willing to communicate about the uncertainties involved in deterministic assessments, as well as their limitations. However, I2 recognizes that the current construction focus on the end goal (i.e. getting a design approved and built) limits the possibility of communicating the uncertainties completely and effectively. This is consistent with the Shergold-Weir enquiry (Peter Shergold, 2018) in Australia and with the Hackitt (2018) enquiry in the UK. The former reflects it with several recommendations, including No. 9 and 10 associated to the lack of transparency and the need for a code of conduct for building surveyors. The latter referred to the issues associated to the construction regulatory system failure stating that "the primary motivation is to do things as quickly and cheaply as possible rather than to deliver quality homes which are safe for people to live in". I2 concludes that the main obstacle in FSE to implement the presented alternatives and better support decision-making is the lack of openness about the effects of uncertainties (within risk assessments) on objectives.

6. Conclusions

This work has described the need to better understand and employ available options for uncertainty analysis in the context of a fire risk assessment, starting from the premise that a single approach is not enough to tackle current challenges for FSE. Given the variability of uncertainty sources and their nature, practitioners cannot force all of them into a single analysis alternative, despite the lack of appropriate guidelines to do so. The lack of a clear perspective on the available alternatives has been addressed by this work, describing them and applying them to a simple compartment fire problem. Applying the considered alternatives to a simple fire safety problem the results vary significantly and imply supporting different decisions for the compartment. Despite its simplicity, the example illustrates the variability of the outputs based on the uncertainty analysis alternative. More importantly, no alternative provides the 'right' answer, while each one provides different pieces of valuable information.

The advantages and challenges of each alternative in relation to their implementation and the outputs they provide for the decision-making process were judged following the modified DSRM methodology. Using the suitability and effectiveness dimensions, each one of the eight alternatives were evaluated in the context of FSE. The results indicate –as the case study-that no alternative is the 'best' and each one must be considered carefully in the context of the assessment being carried out. One common challenge is the reliance on mathematical calculations that are increasingly complex, requiring competent users capable of handling them or a robust software package that aids the calculations. As with any tool used in engineering, the use of available software must be done within its applicability range and considering the assumptions that the tool itself introduces to the calculation. A versatile alternative is found in the strength of knowledge, however the calibration of its categories could also pose challenges for its implementation, i.e. how to define a 'high' strength of knowledge.

The work presented aims at widening the perspective of practitioners fire safety engineers involved in risk assessment, but capitalizes on the knowledge and experience of different disciplines facing the same challenges. Recognizing that very distinct disciplines share challenges and solutions, create a needed synergy that may build new, more trustworthy and effective, risk assessments. This work capitalizes on the work done by many others throughout decades and expects to provide the basis for future work, in which technically challenging alternatives such as EMA can be efficiently implemented to typical problems in chemical process and fire safety engineering.

Author statement

Jaime E. Cadena Gomez: Conceptualization, Formal analysis, Writing - Original Draft, Writing - Review & Editing, Project administration, Software. **Andres F. Osorio:** Conceptualization. **Jose L. Torero:** Conceptualization, Writing - Review & Editing. **Genserik Reniers:** Conceptualization, Supervision, Resources. **David Lange:** Supervision, Formal analysis, Writing - Review & Editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jlp.2020.104288.

Annex I. -Compartment fire model

The model for the compartment fire begins defining the heat release rate of the burning fuel, which is given by:

$$Heat\ \mathrm{Release}\ Rate = \dot{Q} = \ \Delta H_c \bullet \dot{m}_f$$

$\dot{Q}$ is the heat release rate of a material -and this is measured in Watts (W), ΔH_c is the ideal heat of combustion of the fuel measured in [J/kg] and $\dot{m}_f$ is the burning rate measured in [kg/s]. The latter can also be computed as a function of the area by defining is as:

$$\dot{m}_f = A \bullet \dot{m}_f''$$

where A is the area of the fire and $\dot{m}_f''$ is the burning rate per unit area. The fire growth can be extremely complex to model due to solid fuel combustion processes. A key -but onerous-simplification is that a fire can grow radially at an exponential rate. This means that the area of the fire can be described as a function of a radial fire spread (v_s):

$$A = \pi r^2 = \pi (v_s \bullet t)^2$$

where t is the time step. Integrating these three equations the heat release rate can be expressed as a function of a fire growth rate parameter known as alpha (α):

$$\dot{Q} = \left(\Delta H_c \bullet \pi \bullet {v_s}^2 \bullet \dot{m}_f'' \right) \bullet t^2 = \alpha t^2$$

Performing an energy balance based on the heat released by the fire and the heat transferred by radiation to the surroundings (taken as a 30% of the total released), the heat feedback into the fire and the heat transferred to the smoke, it can be found that the smoke temperature can be expressed as:

$$T_S = T_A + \frac{\dot{Q}_s}{\dot{m}_A C_p}$$

with $\dot{m}_A$the air entrainment is produced by the fire and the smoke plume, C_p is the specific heat capacity of the smoke, T_s is the temperature of the smoke, and T_A is the temperature of the ambient air. Based on experimental correlations, the air entrainment can be estimated as:

$$\dot{m}_A = E \left(\frac{g \rho_A^2}{C_p T_A} \right)^{1/3} \dot{Q}^{1/3} H^{5/3}$$

with g the gravity of the earth, E the entrainment constant (taken as 0.2), ρ_A the air's density and H the height at which the air entrainment is estimated. Assuming that the smoke produced is the same as air is entrained, it is possible to find the height of the smoke layer if its density is known (ρ_S). To estimate it, the ideal gas law is used:

$$\rho_S = \rho_A \frac{T_A}{T_S}$$

Using the compartment's ceiling area and the volume of the smoke layer, its height can be calculated:

$$H_S = \frac{V_S}{A_{Ceiling}} = \frac{\dot{m}_S / \rho_S}{A_{Ceiling}}$$

Annex II. -Detailed evaluation of each alternative

Probabilistic approach.

Table 11
Evaluation -Probabilistic approach

Dimension	Criterion	Evaluation	Supporting comments
Suitability	Significant advantages	FA	The main strength of this approach is that it considers a range of possible values for a quantity of interest and distributes the probability along it. This allows fitting a distribution that best fits the experimental data and obtaining its parameters, which allow for future estimations of the 'a posteriori' probability of failure. In practical terms, this approach allows experts selecting a range of values associated with different probability of occurrence instead of a single point value.
	Insignificant limitations	NA	This approach assumes that the probability of an event can be estimated as the result of infinite similar (equal) trials in which conditions remain the same, but this is hardly ever the case in real engineering systems. This largely depends on the availability (data set) to determine the distribution that fits the behavior of a quantity of interest. In practice, especially in complex systems, this information is seldom available and if so, it might need periodical updating. When this approach is used with subjective expert criteria, engineers might introduce uncertainty in the estimations by pre selecting a 'shape' of the probability distribution based on their judgment instead of using the best data available; this can also happen when no data is available and the experts choose a distribution in a subjective manner.
Effectiveness	Easy to implement	FA	Given required inputs and a clear workflow, the approach is easy to implement. However, this is dependent on having technical experts to structure the probabilistic assumptions, as well as competent practitioners to execute the analysis.
	Easy to communicate	PA	Outputs of numerical nature can be easily informed to stakeholders as in the case of individual risk indices, e.g. 1×10^{-6} fatalities/year; however such indices might not convey all the information associated to the source of probability distribution functions or to the assumptions behind their selection. Given that this alternative has been in practice for decades, this criterion is judged as partially achieved as it is not ensured that the uncertainty involved is explicitly accounted for and communicated.
Software available	MATLAB (https://www.mathworks.com/products/matlab.html) R (https://www.r-project.org/) Pelican (https://www.vosesoftware.com/products/pelican/) CristalBall (https://www.oracle.com/au/middleware/technologies/crystalball.html) SIPmath (https://www.probabilitymanagement.org/sipmath)		

P-boxes.

Table 12
Evaluation -P-box

Dimension	Criterion	Evaluation	Supporting comments
Suitability	Significant advantages	FA	Require fewer assumptions from the possible probability distribution functions that fit elicited data from experts, ranging from non-parametric, parametric to bounded parametric p-boxes. The less assumptions, the wider the bounds for the resulting p-box. This flexibility provides an alternative to Monte Carlo sampling methods, which require independency assumptions and therefore additional assumptions, which for complex systems might not hold.
	Insignificant limitations	NA	A p-box analysis requires information on the quartiles or key parameters of the distribution of the variables of interest and it does not solve the expert elicitation issues. Furthermore, explaining the difference between a parametric or non-parametric p-box and the technical details that define it might be a challenge when communicating it to decision-makers.
Effectiveness	Easy to implement	PA	Given required inputs and a clear workflow, the approach is easy to implement. However, this is dependent on having technical experts to structure the probabilistic assumptions, as well as competent practitioners to execute the analysis.
	Easy to communicate	PA	The result of P-boxes -typically a range, as in the example of the compartment fire- can contribute significantly to the communication of the knowledge limitations of the key inputs of the analysis, and therefore of the uncertainty involved. The margin between the P-box boundaries and the uncertainty it represents can be challenging to communicate to the stakeholders, as well as the implications of the upper bound on decision-making.
Software available	RAMAS Risk Calc 4.0 (http://www.ramas.com/riskcalc), MATLAB, SIPmath (https://www.probabilitymanagement.org/sipmath)		

Evidence theory.

Table 13
Evaluation -Evidence theory

Dimension	Criterion	Evaluation	Supporting comments
Suitability	Significant advantages	PA	Evidence theory does not try to describe uncertainty using a measurement, but it does provide a measure of the existing evidence that supports a particular subset of possibilities. A key advantage is allowing for non-mutually exclusive (e.g. overlapping subsets of a larger fundamental set) to be computed, which is not allowed by classical probability theory. Finally, the quantifiable gap between Belief and Plausibility is a measure of the uncertainty, which constitutes a valuable tool to evaluate different input sets for fire safety engineering such as the multiple -complex and unknown- design fire characteristics.
	Insignificant limitations	NA	This theory becomes less useful in cases where evidence is limited and where the assignment of basic probability assignments transform into a subjective exercise. As Denœux (Denœux, 2001) explains, "the complexity of aggregating pieces of evidence increases exponentially with the number of sources", which leads to restrictions on the size of the problem to be handled.
Effectiveness	Easy to implement	PA	There are important challenges to implement evidence theory, beginning with the -often-subjective- definition of the ranges and basic probability assignments used for the quantities of interest. The combination of multiple inputs and then the processing can be challenging if the quantities are considered dependent. This implementation requires a subject expert guiding the process and ensuring the desired outcome is obtained, although the availability of tools such as IP Toolbox increases the ease of use.
	Easy to communicate	PA	As an uncertainty function that maps into the [0, 1] range, the output is easy to communicate. However it presents similar challenges to the P-box's outputs, as it is not a point-value but bounded distributions. Furthermore, the use of the evidence measurements (Belief and Plausibility) increases the complexity of the information to be communicated.
Software available	MATLAB module DSI Toolbox (Auer, A Verified MATLAB Toolbox for the Dempster-Shafer Theory) IP Toolbox for MATLAB (Philipp Limbourg -https://www.mathworks.com/matlabcentral/fileexchange/9379-imprecise-probability-propagation-toolbox, https://www.uni-due.de/informationslogistik/ipptoolbox.php) R package 'EvCombR' (Alexander Karlsson, 2014) R packace 'evclust' or Evidential Clustering (Thierry Denoeux, https://cran.r-project.org/web/packages/evclust/index.html) IDRISI GIS Analysis in TerrSet (Clark University) Orfeo Toolbox, Fusion of Classifications application (https://www.orfeo-toolbox.org/CookBook/Applications/app_FusionOfClassifications.html) GRASS GIS program 'r.dst.combine' using Dempster's Rule of Combination (Benjamin Ducke, Gavin Powell, http://svn.osgeo.org/grass/grass-addons/grass6/dst/raster/r.dst.combine/description.html)		

Possibility theory.

Table 14
Evaluation -Possibility theory

Dimension	Criterion	Evaluation	Supporting comments
Suitability	Significant advantages	PA	Its output is a range of possible values for a variable of interest, regardless of their probability. This adds a layer of information to the risk picture (Guyonnet et al., 2003).
	Insignificant limitations	NA	There is the potential of obtaining over conservative solutions, as unlikely values can be found within the output. Furthermore, the subsets representing the values of the variable of interest should be nested; if this is not the case or cannot be ensured, evidence theory should be considered instead.
Effectiveness	Easy to implement	NA	There are important challenges to implement possibility theory, beginning with the -often-subjective- definition of the fuzzy sets used for the quantities of interest. The combination of multiple inputs and then the processing can be challenging depending on the sampling of the fuzzy inputs and also the defuzziphication technique selected. This implementation requires a subject expert guiding the process and ensuring the desired outcome is obtained.
	Easy to communicate	PA	As in evidence theory, the outputs of possibility theory might be unfamiliar to the stakeholders, specially the possibility and necessity measures.
Software available	PossibleRisk, LEDTools, IP Toolbox for MATLAB		

Info-gap.

Table 15
Evaluation -Info-gap

Dimension	Criterion	Evaluation	Supporting comments
Suitability	Significant advantages	FA	The outcomes of an info-gap analysis provides a quantification of uncertainties that cannot be assessed form a probabilistic approach and that are necessary to understand to support decision-making. In the presence of unstructured uncertainties such as in the case of fire safety concerns with new materials or assemblies, info-gap theory provides a functional alternative to support adequate decision-making.
	Insignificant limitations	PA	Most engineering analysis including those of fire safety engineering employ a large amount of models with irreducible -and unstructured- uncertainties, both key elements to apply the info-gap approach. Given the flexibility of this alternative, no significant limitations are identified except those related to its implementation.
Effectiveness	Easy to implement	NA	An info-gap analysis requires a detailed understanding of the system, the models involved and the inputs required, as well as an understanding of the uncertainty sources. Although these sources can be unstructured and unbounded, the info-gap is constructed based on those identified and an optimization scheme is used to estimate the robustness solution. This process can be mathematically demanding, requiring technical expertise for its implementation.
	Easy to communicate	PA	Despite it does not use probabilities, the results of the robustness functions should be fairly easy to communicate to stakeholder, in particular when two or more competing alternatives are compared. The significance of the robustness value can present significant challenges when taken out of a comparative analysis.

Strength of knowledge.

Table 16
Evaluation -Strength of Knowledge

Dimension	Criterion	Evaluation	Supporting comments
Suitability	Significant advantages	FA	Adding the additional layer of strength of knowledge into the risk assessment, two same subjective probabilities can be judged in a very different way and therefore lead to different decision making. This allows explicitly stating the uncertainty behind the assumptions and the expected outcomes in the assessment. A significant advantage of this approach is that it can be used at all levels of risk assessment (e.g. ongoing operations, design considerations) and compatible with tools such as QRA or Risk Matrices.
	Insignificant limitations	PA	Agreement is required to define the criteria for the strength of knowledge ordinal levels, which can be a complicated task when the assessment involves personnel with very different point of views or expertise. This constitutes the biggest challenge of the alternative, as the criteria can be largely subjective and can diverge in the presence of stakeholders with extremely different points of view.
Effectiveness	Easy to implement	PA	Initially, SoK approach could be considered the easiest approach to account for uncertainty as it relies on a flexible scheme of qualitative levels that measure the supporting evidence and knowledge of the engineers. However, its actual implementation -as in the adequate use of risk analysis matrices- must be tailored to each system and to the objectives of the analysis. This increases the potential for misusing the alternative or requiring considerable additional resources for properly applying it.
	Easy to communicate	FA	SoK yields a list of assumptions and limitations and the associated knowledge that supports them or questions them. This not only allows for ease of communication, but also constitutes an additional information layer which can help formulate risk management actions.

Exploratory Model Analysis.

Table 17
Evaluation -Exploratory Model Analysis

Dimension	Criterion	Evaluation	Supporting comments
Suitability	Significant advantages	FA	EMA is able to explore a vast set of possible scenarios including a large range of variables and variability in conditions. A statistical treatment of the resulting scenarios allow identifying key conditions that would make the decision alternatives inviable, hence finding robust strategies. Additionally, this methodology intakes new information, leading to adaptive strategies.
	Insignificant limitations	PA	Detailed knowledge of the system and the internal and external variables that might influence its performance is needed. Furthermore, advanced modelling tools and data base management is required in order to host the scenarios discovery and its statistical treatment, limiting the range of users for whom this methodology is viable. Not only specific competences in the field of computer experiments is required, including potential knowledge and skills of machine-learning and artificial intelligence, but a deep knowledge of the structure of the problem is required. The latter refers to the need for competent fire engineers to guide the construction process of the computational experiments. This in sum shows significant limitations to the methodology.
Effectiveness	Easy to implement	PA	Depending on the amount of conditions that require exploring and the technique chosen to generate the possible universes, the implementation will require more planning and technical resources such as computing power. Given the existence of a well characterized model, the EMA Workbench (Kwakkel, 2017) can be used to run it, given that a competent user leads the implementation. The latter constitutes the challenge for EMA implementation, as this technique is not typically used for fire risk assessments.
	Easy to communicate	NA	The ease of communication directly depends on the defined outcomes, as well as the complexity of the model. However, the communication is based on the evaluated policies and the scenario sets that yield either successful or unsuccessful results. For stakeholders used to probabilistic results, receiving such results could be challenging.
Software available	R, EMA workbench (TU Delft)		

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