



ELECTRIC VERTICAL TAKE-OFF AND LANDING VEHICLE COMMUNITY NOISE PREDICTION: FROM FLOW SIMULATION TO FLIGHT MISSION ANALYSIS

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ABSTRACT

This paper presents an overview of noise prediction capabilities available at Dassault Systèmes and Delft University of Technology in the field of electric vertical take-off and landing vehicle aeroacoustics. Three main aspects are covered: (i) noise source calculation via scale-resolving high-fidelity Lattice-Boltzmann flow simulations, (ii) noise propagation calculations in urban environments via Gaussian-beam tracing techniques, (iii) and flight mission analysis via a multi-fidelity model-based system engineering framework. Key features of the different numerical simulations techniques are discussed in more detail. Finally, a vision of a combined experimental/digital eVTOL noise certification process is outlined.

Keywords: *eVTOL noise, UAM noise, MBSE, PowerFLOW, OptydB*

1. OVERVIEW

In the last five years, at Dassault Systèmes (3DS) and Delft University of Technology (TU-Delft), we performed research in three areas related to the prediction

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of electric Vertical and Take-Off Landing (eVTOL) vehicle community noise. The first area is related to the prediction of the aerodynamic noise generated by the propulsive units and by their mutual interaction by using the Lattice-Boltzmann/Very Large Eddy Simulation (LB/VLES) solver SIMULIA PowerFLOW by 3DS. The second area is the prediction of sound propagation from the vehicle to the ground by means of the Gaussian Beam Tracing (GBT) solver UYGUR [1] by TU-Delft, which takes into account reflection on complex terrains, and refraction due to wind velocity and air temperature gradients. The third research area is related to the usage of a Model Based System Engineering (MBSE) framework on the 3DS 3DEXPERIENCE (3DX) platform to predict the noise footprint of an eVTOL undertaking a given mission, by combining low- and high-fidelity aerodynamic and aeroacoustic prediction techniques.

Concerning the prediction of aerodynamic noise generation from rotors, it should be argued that PowerFLOW solver has been extensively validated by simulating the Source Diagnostic Test (SDT) fan configuration at subsonic and supersonic tip conditions [2–5], the S-76 full-scale helicopter rotor aerodynamics [6], and the HART-II helicopter aerodynamics and aeroacoustics in Blade Vortex Interaction (BVI) conditions [7]. To address the specific challenge of rotor aeroacoustics in transitional boundary layer conditions, which is a key aspect for drone rotor aeroacoustics, an extensive benchmark activity was conducted by TU-Delft and 3DS [8–11]. In particular, we focused on the mechanisms of broadband noise generated



by a laminar separation bubble [12, 13], and on the effects related to flow recirculation in the test chamber [14].

Concerning the sound propagation in urban areas, we performed proof-of-concept studies using LBM [15] for a vehicle hovering in a urban area, and by using point-to-point curved-ray tracing [16] and GBT [1, 17] coupled with a Noise Hemisphere Database (NHD) approach [18] for a vehicle flying over a urban area.

Finally, the NHD approach was also used to compute the noise generated by an eVTOL flying different trajectories [18], some of them affected by Blade-Wake Interaction (BWI) noise due to wakes from front rotors interacting with rear rotors. The MBSE flight mission analysis framework presented hereafter makes use of the same approach to compute the noise on ground, with the only difference that the on-ground noise footprint is computed during the accomplishment of the virtual flight.

For the sake of an overview, only key features of these methodologies are discussed in the following three sections. Finally, the concept of a hybrid physical/digital eVTOL noise certification process is introduced in section 5 as a natural consequence of the presented state-of-the-art eVTOL noise simulation capabilities.

2. LB/VLES HIGH-FIDELITY NOISE SOURCE PREDICTION

This section summarizes the predictive capabilities of the high-fidelity LB/VLES solver PowerFLOW concerning two different rotor noise benchmarks, spanning from low to high Reynolds numbers and tip speeds, which are relevant in the context of eVTOL noise source prediction.

The first benchmark is about rotorcraft BVI noise, which is a phenomenon that occurs when a rotor blade interacts very closely with the tip vortices released by the other blades, the blade itself and/or by other rotors in multi-copter configurations. The noise resulting from the BVI-induced blade surface pressure fluctuation is a relevant source of annoyance for the community, and sets limits to the rotorcraft/eVTOL operations in urban areas. The considered benchmark is a scaled model of a BO-105 4-bladed main rotor tested in the framework of the HART-II project [19]. Hybrid high-subsonic/transonic PowerFLOW simulations are performed by considering a rigid blade motion, but a computational approach to account for the steady and unsteady aerodynamic effects associated to the blade elastic deformations is employed [7].

Figure 1 shows an instantaneous snapshot of the blade tip vortex system extracted according to the λ_2 criterion.

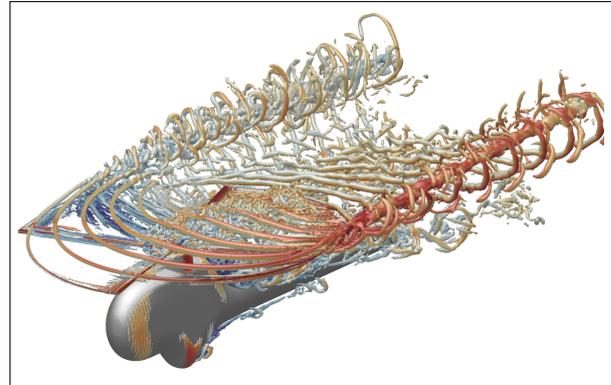


Figure 1. Instantaneous λ_2 iso-surfaces colored by the flow velocity magnitude.

This images qualitatively illustrate as the low dissipative properties of the LB allows to preserve the coherent character of the tip-vortex over a larger number of wake spirals, which represents a crucial aspect for the accurate prediction of the BVI noise, along with elastic deformation prediction capabilities.

Figure 2 shows the comparison between the experimental and numerical noise footprints evaluated on a carpet of microphones located 2.2 m below the rotor hub.

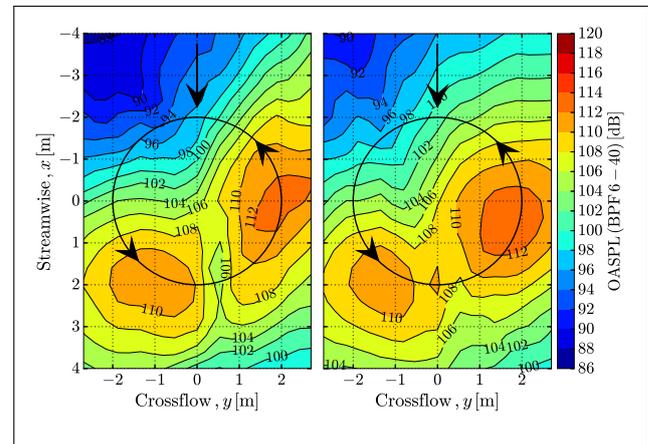


Figure 2. Comparison between experimental (left) and PowerFLOW (right) OASPL below the rotor.

To highlight the BVI noise contribution, the Overall Sound Pressure Level (OASPL) in the frequency range between the 6th and the 40th Blade Passing Frequency (BPF) is depicted. The PowerFLOW noise maps are com-

puted by integration of the Ffowcs-Williams & Hawkins (FWH) equation on a porous surface encompassing the whole helicopter geometry. The straight arrow indicates the direction of the free-stream velocity, while the circular one the direction of the rotor rotation. It can be noticed that, with the inclusion of the aerodynamic effects associated to the elastic deformation, PowerFLOW is able to accurately capture the overall noise directivity, as well as the high-noise lobes related to BVI occurrence in the advancing and retreating sides, in a quite satisfactory way.

The second benchmark reported in this overview is about a two-bladed drone propeller, designed and tested at TU-Delft, which is operated at a low Reynolds number (between $7 \cdot 10^4$ and $9 \cdot 10^4$) and characterized by the occurrence of a laminar separation bubble and a corresponding high-frequency broadband noise hump at high-advance ratio in the far-field noise spectra [12, 13]. The accurate prediction of low-Reynolds number propeller aeroacoustics can be a quite challenging problem to be addressed with high-fidelity CFD methods, especially hybrid ones, due their intrinsic difficulties to deal with shallow boundary-layer separation and re-attachment. A variant of PowerFLOW LB/VLES solver of PowerFLOW, with enhanced modeled-to-resolved turbulence transition capabilities, has been recently developed at 3DS to address this class of flow phenomena, without the usage of any physical or numerical triggering system to activate the scale-resolving turbulence mode.

Finally, Fig. 3 shows the comparison between the experimental and numerical far-field noise radiated by the propeller. Beside some experimental artifacts that are not reproduced in the ideal context of a numerical simulation (i.e. wind tunnel background noise, electric motor noise and tonal noise at multiples of the shaft frequency due to the non-perfect balance of the blades loading), PowerFLOW results show a fairly good agreement with the experimental data. In particular, the tone at the BPF-1, as well as the high frequency broadband hump due to the presence of the LSB in close proximity of the blade trailing-edge, is predicted in an excellent way. 3

3. GBT NOISE PROPAGATION, MULTIPLE REFLECTIONS AND REFRACTION IN URBAN AREAS

The UYGUR software developed at TU-Delft imports computed noise spectra around a source, terrain geometry and 3D wind flow profiles to calculate the noise footprint, as schematically illustrated in Fig. 4. The noise prediction

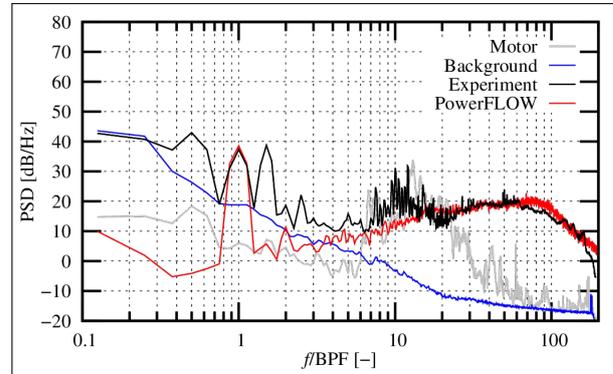


Figure 3. Comparison between experimental and numerical far-field noise radiate by the propeller.

consists in three steps. Firstly, ray path tracing (RPT) is performed to obtain ray trajectories. Secondly, dynamic ray tracing (DRT) is conducted to calculate the paraxial ray field in the vicinity of each ray trajectory. Finally, Gaussian beams are constructed based on the solution of the DRT procedure in the vicinity of each ray path.

UYGUR extends the conventional GBT approach to account for complex source directivity by determining ray-sphere intersection points within the RPT procedure. Noise data on the sphere are computed using high-fidelity approaches based on PowerFLOW, as discussed in section 2, or by means of low-fidelity methods [8, 20]. Then, each Gaussian beam initial phase and magnitude are updated with the ones stored on the ray-sphere intersection point and propagated towards the terrain. Compared to existing GBT-based propagation tools, UYGUR has two main advantages. Firstly, it does not require modifying the beam summation equation, which removes extra computational time. Secondly, it can accurately propagate the noise signals even in the presence of strong refraction due to horizontal and vertical variations in air temperature and wind velocity gradients.

Fig. 5 compare UYGUR noise predictions with results obtained using a Finite Element Method (FEM) solver *OptydB-GFD* [21–23] for a three-building environment with wind velocity of 5 m/s and wind blowing along the positive X-axis direction [1]. The results show that UYGUR is capable of capturing the general trend of the FEM results, which includes multiple reflections between building surfaces and the ground, and refraction above building rooftops.

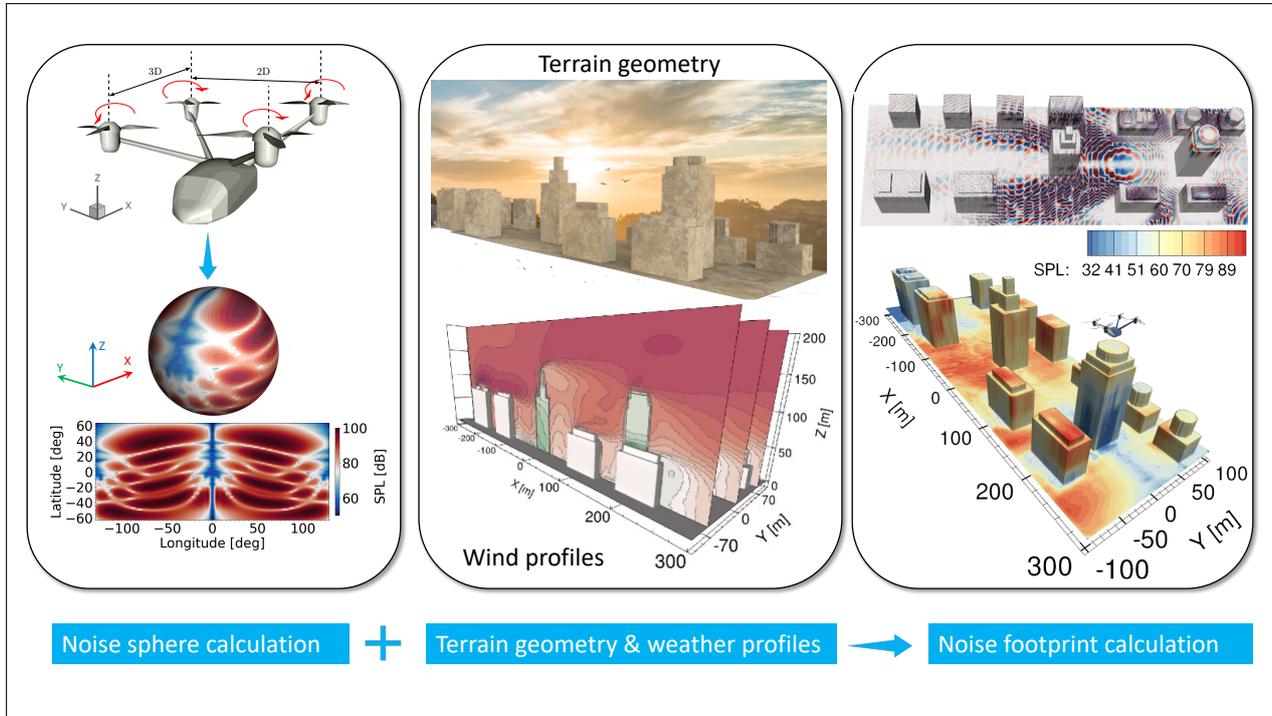


Figure 4. Schematic illustration of the noise footprint computational procedure, adapted from [17].

4. MBSE FLIGHT MISSION ANALYSIS AND NHD-BASED NOISE PREDICTION

The optimization of take-off and landing procedures of an eVTOL vehicle is a difficult and interesting task due to the higher number of flight and operational control parameters compared to helicopters. A target mission profile under consideration by several stakeholders is a 60 miles air-shuttle between an airport and the business city center. Considerations on the battery capacity reveal that lifting surfaces are required to restrict the electrically powered lift generation during take-off and landing, and thus cover the target cruise range. Therefore, the most viable eVTOL concepts are vehicles lifted by rotors that convert from a vertical to a horizontal flight thanks to tilting rotors and/or tilting lifting surfaces. Fig. 6 shows the eVTOL concept used by 3DS to showcase various simulation capabilities offered to the market. This vehicle presents several characteristics that can be found in different architectures recently presented to the public, such as counter-rotating rotors on the front, which are used only in vertical flight conditions, and shrouded counter-rotating tilting rotors on the rear, which supply lift in vertical flight, and thrust in

horizontal flight. The capability to tilt rotors and parts of the wing constitute an additional degree of freedom to be considered during the calculation of the aeromechanical trim conditions and during the design of low-noise take-off and landing flight procedures.

In multi-rotor configurations, BWI phenomena can take place at certain flight conditions when the wake from one rotor is ingested by another rotor. For this kind of architectures, therefore, a low-noise procedure is one for which the occurrence of BWI conditions is minimized.

Electrical motors allow to vary the rotor RPM, thus adapting the rotational speed to the different flight conditions to supply the required lift and thrust, while operating close to the optimal advance ratio. Compared to the control of the collective angle of a helicopter rotor blade, the variation of the rotational speed can be used to better distribute the lift between the wings and the rotors during a conversion stage.

Based on the above arguments, it can be argued that an eVTOL can fly from one point to another one following different trajectories, and some of them are noisier than others. Therefore, the main goal of using a MBSE flight mission analysis framework is to evaluate the on-

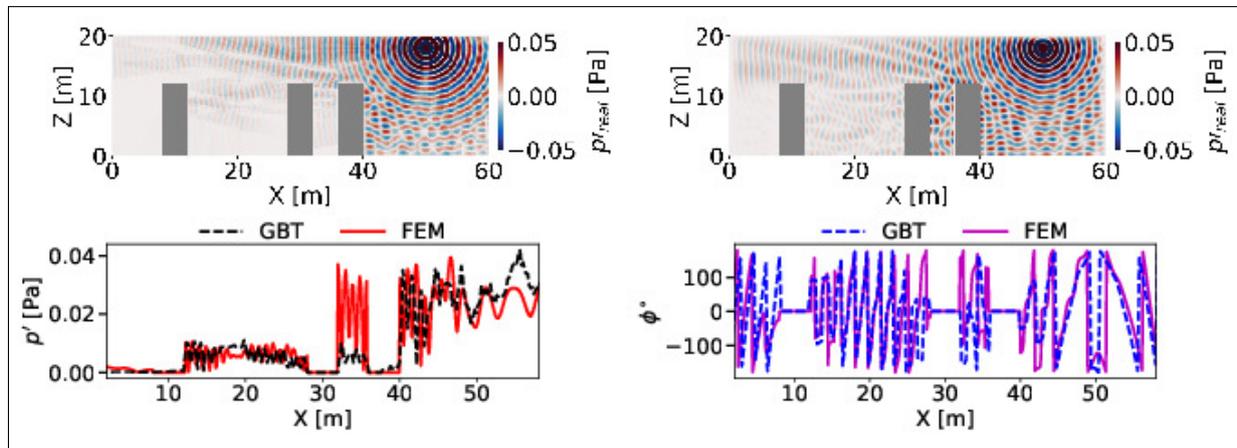


Figure 5. Comparison between GBT and FEM results. Pressure field calculated with the UYGUR (top-left) and *Opty∂B*-GFD (top-right) solvers. Pressure magnitude (bottom-left) and the phase (bottom-right) along a line at $Z = 10$ m (adapted from [1]).

ground noise footprint of an eVTOL during its flight, by taking into account both aeromechanical and aerodynamic effects.

The entire workflow for flight mission analysis and community noise assessment developed by 3DS is illustrated in Fig. 6, whose main phases have been numbered from 1 to 8. All data managed by the workflow are stored in a dedicate area of the 3DX project, thus guaranteeing digital continuity.

The first phase of the workflow, **Phase #1**, consists in designing the geometry of the main vehicle components using the 3DX geometry modeling capabilities, and calculating the mass distribution and the moments of inertia based on the specified material properties. In **Phase #2**, the rotor geometry is used by the low-fidelity *Opty∂B*-BEMT tool [8] to compute look-up tables of rotor thrust and torque coefficients for different values of RPM and rotor advance ratio. In **Phase #3**, the aerodynamic force and moments coefficients of the airframe are computed by using the 3DS Multi-Copter Aerodynamics and Aeroacoustics Simulation (MAAS) PowerFLOW automatic workflow to generate a computational setup for the full vehicle, by removing the propulsive components, and by setting the global resolution to a pre-defined coarse value. Simulations covering a broad range of flow conditions in terms of angle of attack, angle of sideslip and flight Mach number are then performed on a High Performance Computing (HPC) cloud system and force/moments coefficients are stored in look-up tables. In **Phase #4**, an aeromechan-

ical model is built in the 3DX Dymola Behavior Modeling (DBM) App, which solves the flight dynamics equations to determine the non-inertial equilibrium of the vehicle on every point of the trajectory. Look-up tables of the rotor and airframe aerodynamic coefficients and the inertial parameters stored in the project are used to solve the dynamic trim problem at every time step, and to compute the new vehicle position. Dymola has the capability to integrate the Modelica flight dynamics and rotor performance models with other Modelica models describing the operational scenario (World, Atmosphere, Terrain, and Vertiport), and to connect all these models with a third-party autopilot. The usage of the autopilot allows to track a target trajectory by acting on the control parameters (control surfaces and rotor RPMs) and receiving flight status from the DBM model in a closed-loop Software In the Loop (SIL) process. If the autopilot is able to interact with a virtual Ground Control Station (GCS) or a virtual graphic cockpit model, these graphical components can be connected with the DBM and the 3DX scenario visualization tool, the Creative Experience App. All these assembled components constitute a flight-mission simulator operated on the 3DX platform, as illustrated on the top-right corner of Fig. 6. Once a virtual flight mission has been accomplished, in **Phase #5**, the flight status recording, say the values of Mach number, angle of attack, angle of sideslip, rotor RPMs, rotor tilt angles, control surface angles, etc., sampled every 0.05 s, is down-sampled using a procedure consisting of three main steps. The

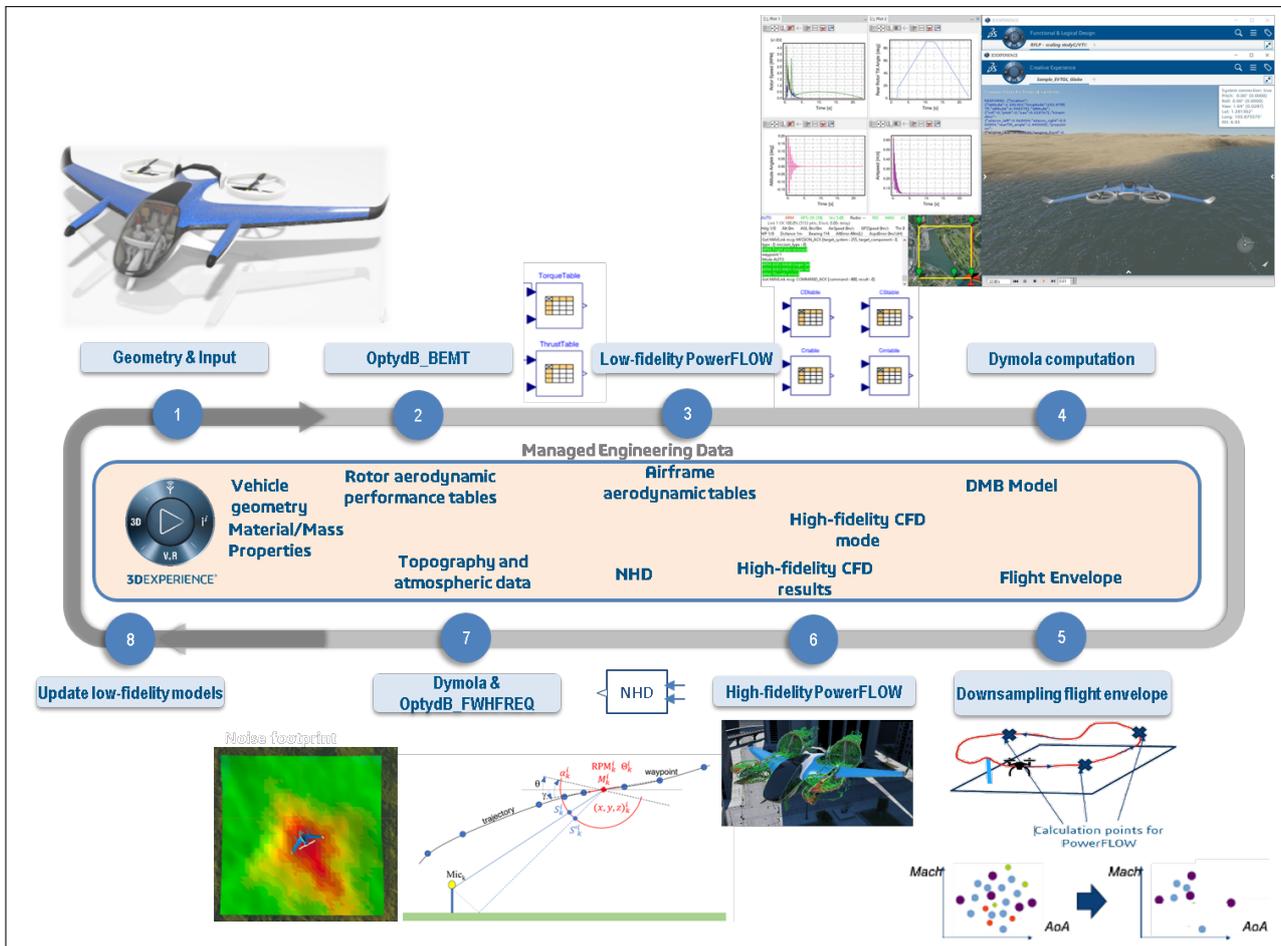


Figure 6. 3DS eVTOL flight mission analysis MBSE workflow.

first one is the filtering and averaging of the raw data; the second step consists in the identification of the most probable flight conditions and “corner” events based on vehicle type and prior knowledge; finally, the third step consists in a graphical check of the selected flight status in the whole flight envelope¹. In **Phase #6**, high-fidelity aerodynamics and aeroacoustic calculations are carried out by using PowerFLOW (flow simulation) and *OptydB-FWHFREQ* (integral noise calculations) for every point of the down-sampled flight envelope. Narrow-band

¹ The flight envelop down-sampling algorithm, as well as other aspects of the eVTOL flight mission analysis workflow are described in a patent request entitled “Assessing Vehicle Noise” submitted to the US Patent and Trademark Office (application number 18/167,956).

noise spectra at microphones distributed on a hemisphere around the vehicle are stored in the NHD and used after for on-the-fly noise footprint calculations. The key feature of *OptydB-FWHFREQ* is its capability to manage a large number of microphones (order of 1000), and to remove the spurious effects of the vehicle wake crossing the integration fluid surface, thanks to the employment of a specific Ffowcs-Williams & Hawkins (FWH) formulation including quadrupole noise corrections in the frequency domain. In **Phase #7**, a new virtual flight is simulated as in Phase # 3, but this time, every N number of time steps, the instantaneous flight status is used by the tool *OptydB-FOOTPRINT* to calculate the noise on a prescribed portion of the ground. The tool initially interpolates the current noise spectra on the hemisphere from the stored NHD

spectra, and then it extrapolates the on-ground noise levels using a straight-ray procedure, by taking into account ground reflection and absorption, atmospheric absorption, and Doppler effects. The noise calculation can be performed off-line, by importing the flight trajectory once the mission has been accomplished, or, in a real-time/on-the-fly modality, by using the last updated vehicle position. The rate N depends on the available computational power. A key element of this process is the capability to extract the on-ground points at the beginning of the calculation, for a short event flight, like for instance a vehicle landing, or on-the-fly, by considering the current vehicle position. The ground points are extracted from the Shuttle Radar Topography Mission (SRTM) earth topography database by means of a 3DS tool that takes as input the coordinates of the vehicle, in which the noise carpet is centered, and the dimensions of the carpet. When on-the-fly noise calculations are performed, the carpet follows the vehicle and the noise levels can be visualized on the 3DX platform. In the last and optional phase of the workflow, **Phase #8**, the high-fidelity CFD results of Phase #6 can be used to define a correction strategy to be applied to all low-fidelity aerodynamic coefficients stored in the rotor and airframe look-up tables used by the DBM calculation.

5. TOWARDS A HYBRID PHYSICAL/DIGITAL EVTOL NOISE CERTIFICATION PROCESS

Considering the state of the art in eVTOL noise prediction in real operational conditions, we can speculate on the usage of numerical simulations to support the noise certification process of eVTOL vehicles.

Our vision of a hybrid physical/digital noise certification process is based on the idea that, due to the high variety of eVTOL architectures and the variety of flyable procedures for every architecture, it is not judicious to leave to OEMs the decision about which trajectory to fly during certification, in the same way it is not possible to prescribe the same trajectory for all vehicles. A process aimed at guaranteeing that the community noise does not exceed a certain threshold should consider the vehicle in real operational scenarios. In other words, although the requisite of uniformity, which is applied to fixed-wing aircraft and partially to helicopters, cannot be applied to eVTOL vehicles, the ultimate goal of a certification should be guaranteed. Our idea would be, therefore, to apply a paradigm shift, from the concept of uniform test conditions (e.g. the same trajectory) to the concept of uniform process to define the test conditions. A numerical simula-

tion process similar to the one presented in this paper can be created by combining a reliable aeromechanical model of the vehicle, supplied by the OEM, with accurate prediction of community noise in different flight conditions. Such a process could be adopted by the certification agencies to define a set of flyable and potentially noisy procedures that, in agreement with the OEM, can be tested to obtain an average noise certification level in a more representative scenario.

Another potential usage of numerical simulations in the framework of a certification process should focus on the condition of hovering in proximity of a vertiport. Since it is not possible to define all the possible operational scenarios, CFD simulations can be used to predict the wind gust in the wake of a canonical vertiport and the noise generated by its interaction with the vehicle. Indeed, inflow turbulence can have a dramatic effect on noise, thus invalidating the meaningfulness of a certified noise level. On one side, a rotor operating in non constant flow conditions is much noisier than one operated in unperturbed ideal conditions; on the other side, the flight control system will constantly vary the RPM of the rotors to keep the hovering point, thus resulting in additional annoyance due to the low-frequency amplitude modulation of the radiated noise.

6. CONCLUSIVE REMARKS

A multi-fidelity flight simulation workflow for eVTOL community noise assessment was presented. The key software components of this workflow were discussed, followed by a vision of how numerical simulations can support eVTOL noise certification. Moreover, the capability to predict the noise propagation in urban areas by means of advanced GBT algorithms was discussed and placed in the context of a reliable community noise prediction. Future research will point in the direction of integrating GBT capabilities in a virtual flight framework for community noise assessment and long-range noise propagation.

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