waterloopkundig laboratorium
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on the magnitude of interfacial shear of subcritical stratified flows

G. Abraham
M. Karelse
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report on analysis of literature data

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ON THE MAGNITUDE OF INTERFACIAL SHEAR OF SUBCRITICAL STRATIFIED FLOWS

Summary

In this Report instability of internal waves is shown to explain why subcritical stratified flows, such as the arrested salt wedge, the temperature surface wedge and density induced return currents may be treated as two-layer flows without mixing. Further it is shown that in these flows interfacial shear does not depend on an internal Froude number, while it is influenced by turbulence, generated at the bottom.

1 Introduction

The following stratified flow phenomena may be characterized as subcritical steady stratified flows

- arrested salt wedge, (Fig. 1.a), (Schijf and Schönfeld, [1953], Harleman, [1961]),
- temperature surface wedge, (Fig. 1.b), (Bata, [1957],Polk et al, [1971]),
- density induced return currents, (Fig. 1.c), (Rigter, [1970], Holley and Waddell, [1976]).

These flows may be treated as two-layer flows without mixing at or through the interface.

In the above-mentioned subcritical stratified flows the conditions at the interface are, among other things, influenced by instability of internal waves. This instability is associated with mixing. The mixing is in itself a stabilizing factor as it causes a transition zone to be established at the interface. Once the transition zone has a given thickness it not only prevents further instability of internal waves, it also reduces the turbulent flux of a constituent through the interface. Hence the establishment of the transition zone is essentially the mechanism which allows the considered subcritical stratified flows to be treated as two-layer flows without mixing. This is elaborated upon in the first part of this Report.

In the preceeding paragraph distinction has been made between turbulence and instability of internal waves, the difference being that turbulence takes place at a scale, which is small in comparison with the scale of the internal waves. Further turbulence is a stochastic mechanism while internal waves form a deterministic mechanism. Both mechanisms may cause an exchange of momentum or a flux of a constituent through the interface. In formulae

$$\tau_{i,eff} = \tau_{i,inst} + \tau_{i,turb} \quad (1)$$

and

$$F_{i,eff} = F_{i,inst} + F_{i,turb} \quad (2)$$

where $\tau_{i,eff}$: effective interfacial shear stress, expressing the effect of exchange of momentum by instability of internal waves and turbulence per unit area of interface per unit time

$\tau_{i,inst}$: exchange of momentum through interface associated with instability of internal waves per unit area of interface per unit time

$\tau_{i,turb}$: exchange of momentum through interface associated with turbulence per unit area of interface per unit time

$F_{i,eff}$: flux of a constituent through interface both by instability of internal waves and turbulence per unit area of interface per unit time

$F_{i,inst}$: flux of a constituent through the interface because of instability of internal waves per unit area of interface per unit time

$F_{i,turb}$: flux of a constituent through interface because of turbulence per unit area of interface per unit time.

Once the above-mentioned transition zone prevents further mixing the conditions at the interface may be described by a more or less constant value of the internal Froude number, F'_Δ, defined as

$$F'_\Delta = \frac{(u_2-u_1)^2}{\frac{\Delta\rho}{\rho}\, g\, d} \qquad (3)$$

where u_1, u_2 : velocity of upper and lower layer

$\Delta\rho$: difference in density between upper and lower layer

ρ : density (either of upper layer or lower layer, the density of which is assumed to have approximately the same value)

d : thickness of transition zone

g : acceleration by gravity.

As explained below, this implies that the interfacial shear does not depend on the magnitude of an internal Froude number, defined otherwise.

When no further instability of internal waves occurs

$$\tau_{i,eff} = \tau_{i,turb} \qquad (4)$$

and

$$F_{i,eff} = F_{i,turb} \qquad (5)$$

Then it is reasonable to expect that the magnitude of the interfacial shear depends upon the level of turbulence as a consequence of which it may be influenced by turbulence generated at the bottom. This is illustrated by the experimental data presented in the second part of this Report.

The study described in this Report is performed by dr. G. Abraham, M. Karelse and A.G. van Os. It is incorporated in a basic research program T.O.W., project B, executed by Rijkswaterstaat (Public Works and Water Control Department), Delft Hydraulics Laboratory and other research institutes.

2 Characteristics of interface

2.1 Implication of a subcritical stratified flow

Stratified flows are solved as a two-layer system along the following procedure

(i) Deriving an expression for the slope of the interface as a function of
$\Delta\rho$: difference in density between the lower layer and the upper layer
h_1, u_1: thickness and velocity of upper layer
h_2, u_2: thickness and velocity of lower layer.

In formula

$$\frac{dh_1}{dx} \approx - \frac{dh_2}{dx} = f(h_1,h_2,u_1,u_2,\Delta\rho) \qquad (6)$$

where x : longitudinal co-ordinate.

(ii) Finding the magnitude of the thickness of the layers at one characteristic point from the fact that the considered stratified flow is critical at this point, which implies

$$\frac{u_1^2}{\frac{\Delta\rho}{\rho} g h_1} + \frac{u_2^2}{\frac{\Delta\rho}{\rho} g h_2} = 1 \qquad (7)$$

Figure 1 shows the sections where this condition is satisfied for an arrested salt wedge and for density induced return currents.

(iii) Solving h_2 as a function of x, using Equation 7 to determine the magnitude of the integration constant.

From the solutions obtained along this procedure it can be shown that in the considered stratified flows

$$\frac{u_1^2}{\frac{\Delta\rho}{\rho} g h_1} + \frac{u_2^2}{\frac{\Delta\rho}{\rho} g h_2} \leq 1 \qquad (8)$$

From Equation 8 it follows that

$$F_\Delta = \frac{(u_2-u_1)^2}{\frac{\Delta\rho}{\rho} g(h_1+h_2)} \leq 1 \qquad (9)$$

where F_Δ : densimetric Froude number, defined by Equation 9.

For the arrested salt wedge and for the temperature wedge Equation 9 follows directly from Equation 8 as either $u_2 = 0$ or $u_1 = 0$, while $(h_1+h_2) > h_1$ and $(h_1+h_2) > h_2$.
In problems of density induced return flows u_1 and u_2 have opposite signs. Hence some computational work is required to derive Equation 9 from the solution for this type of flows.

2.2 Findings of linear instability theory

Linear instability theory studies the behaviour of a periodic disturbance superimposed upon a background flow. It indicates under which circumstances the disturbance is stable and under which circumstances the disturbance is unstable. In the former case the amplitude of the disturbance decreases with increasing time, in the latter case the amplitude of the disturbance

increases with increasing time. The conditions of stability and instability are given as a function of the wave length of the disturbance and characteristics of the background flow by means of neutral stability curves corresponding to disturbances the amplitude of which remains constant with increasing time, separating the zones of stable and unstable solutions.
Several studies have been made on the stability of disturbances superimposed upon the background flow represented in Figure 2, (Yih, [1965], Thorpe, [1971], Dingemans, [1972], Hazel, [1972]). Results are shown in Figure 3. This figure shows neutral stability curves, the zone indicated by u being the zone of unstable conditions and the zone indicated by s being the one of stable conditions. In the figure the following dimensionless parameters occur

$\frac{1}{2\mu} \frac{\lambda}{(h_1+h_2)}$: measure for the ratio of the wave length, λ, of the disturbance and the total depth

$b = \frac{d}{h_1 + h_2}$: thickness of intermediate layer, d, over total depth

$F_\Delta = \frac{(u_2-u_1)^2}{\frac{\Delta\rho}{\rho} g(h_1+h_2)}$: densimetric Froude number based upon difference in velocity and density between upper and lower layer.

For a given value of F_Δ, satisfying Equation 9, there is a limiting value of $d/(h_1+h_2)$ above which disturbances of all wave lengths are stable. For values of $d/(h_1+h_2)$ below this limiting value there is a range of wave lengths leading to unstable disturbances. The limiting value of the thickness of the intermediate layer satisfies

$$\frac{d_L}{(h_1+h_2)} = \frac{1}{4} F_\Delta \qquad (10)$$

where d_L : smallest value of thickness of intermediate layer associated with disturbances of all wave length being stable.

The following reasoning may be based upon the above considerations.

Assume a stratified flow to be initially characterized by a given value of F_Δ and by a value of the thickness of the intermediate layer which is smaller than d_L. Then, the stratified flow being established smoothly, the unstable disturbances will generate mixing as a consequence of which d increases until its magnitude coincides with the limiting value d_L. Thus, the interface being too sharp (i.e. d being smaller than d_L) mixing occurs. This mixing in itself is a stabilizing factor, preventing further mixing because of instability of internal waves once the intermediate layer has the limiting thickness given by Equation 10.

For the stratified flows mentioned in the introduction Equations 9 and 10 imply

$$\frac{d_L}{(h_1+h_2)} \leq \frac{1}{4} \tag{11}$$

Equation 11 shows the thickness of the intermediate layer to be small in comparison with the total depth. This essentially explains why the stratified flows listed in the introduction may be treated as stratified flows without mixing.

2.3 Turbulence considerations

The conclusion given in Section 2.2 is compatible with turbulence considerations based upon the Richardson number, defined as

$$Ri = \frac{-g \frac{\partial \rho}{\partial z}}{\rho \left(\frac{\partial u}{\partial z}\right)^2} \tag{12}$$

where Ri : Richardson number

$\frac{\partial \rho}{\partial z}$: vertical density gradient

$\frac{\partial u}{\partial z}$: vertical gradient of horizontal velocity

z : vertical co-ordinate (measured positively upwards).

The Richardson number is a measure of the gain of potential energy associated with mixing leading to a turbulent flux of a constituent ($F_{i,turb}$), over the kinetic energy available to cause this type of mixing. This mixing is hampered when the Richardson number exceeds some not well defined critical limiting value (Monin, [1959], Long [1972]).

For the flows represented in Figure 1

$$\mathrm{Ri} = \frac{-g\,\frac{\partial\rho}{\partial z}}{\rho(\frac{\partial u}{\partial z})^2} :: \frac{g\,\frac{\Delta\rho}{d_\rho}}{\frac{(u_1-u_2)^2}{d_u^2}} :: \frac{\frac{\Delta\rho}{\rho}\,g\,\frac{d_u^2}{d_\rho}}{(u_1-u_2)^2} \qquad (13)$$

where d_ρ : thickness of intermediate layer from density profile
d_u : thickness of intermediate layer from velocity profile.

As a rule $d_u \geq d_\rho$. Hence for given values of $\Delta\rho$ and (u_1-u_2) Ri is equal to or larger than the value given by Equation 14

$$\mathrm{Ri} :: d_\rho \qquad (14)$$

indicating Ri to increase with increasing thickness of the intermediate layer. Thus the establishment of the intermediate layer by instability of internal waves is associated with an increasing value of the Richardson number. Hence the establishement of the intermediate layer is associated both with internal waves becoming stable and the turbulent flux of a constituent becoming small.

Plate, ([1971], pp. 70-76) distinguishes two critical values of the Richardson number:

$$\mathrm{Ri} \approx 1/4 \quad \text{and} \quad R \approx 1 \qquad (15)$$

The Richardson number of an initially quiescent fluid being below the value of 1/4, turbulence is likely to increase. The value 1 represents the value of the Richardson number above which turbulence is dampened. In the intermediate range turbulence can be maintained, provided that it has been generated in one way or another.

Deriving Equation 11 it was assumed that the considered stratified flows initially are characterized by a thickness of the intermediate layer which is smaller than d_L. The thickness of the intermediate layer was assumed to increase from $d = 0$ to $d = d_L$. This is associated with Ri increasing from Ri = 0, as can be seen from Equation 14. The final value of Ri is likely to be in the order of 1/4. There is no reason to expect an overshoot into a higher Richardson number range.

For the flow represented in Figure 2

$$Ri = \frac{\frac{\Delta\rho}{\rho} g \, d_L}{(u_1-u_2)^2} = \frac{d_L}{(h_1+h_2)} \frac{1}{F_\Delta} \qquad (16)$$

Substituting Equation 10, which is based on linear instability theory, into Equation 16 leads to Ri = 1/4. Thus also the linear instability theory leads to a value of Ri equal to 1/4. Therefore the above turbulence considerations are compatible with Equations 10 and 11. This observation seems to imply that in the considered subcritical flows turbulence only weakly affects the thickness of the transition zone.

The fact that in the above development the final value of Ri from turbulence considerations was taken equal to the value given by linear instability theory does not necessarily imply that the flow has been assumed to be laminar. This is due to the fact that nonlinear terms in the disturbance equations may lead to instability under conditions which from linear instability theory are found to be stable. [Joseph, 1976, Appendix E]. This could be the rationale behind the fact that Plate gives two critical values of the Richardson number in Equation 15.

2.4 Interfacial shear

When the transition zone prevents further mixing, the overall conditions at the interface can be described by the following parameters.

d : thickness of intermediate layer
(u_2-u_1) : difference in velocity between lower layer and upper layer
$\Delta\rho$: difference in density between lower layer and upper layer.

Hence for sufficiently large Reynolds numbers dimensional considerations and Equation 3 give

$$k_i = \frac{\tau_i}{\rho(u_2-u_1)^2} = f(F'_\Delta,\ L/L_L,\ \sigma) \qquad (17)$$

where τ_i : interfacial shear

k_i : interfacial shear coefficient, defined by first and second term of Equation 17

L : (length of) fetch between both layers

L_L : (length of) fetch required for the establishment of intermediate layer, having a thickness d_L given by Equation 10

σ : degree of turbulence.

For $L/L_L \geq 1$, k_i does no longer depend upon L/L_L. Then further from the above considerations $F'_\Delta \approx 4$, (the precise value depending on σ, which depends on the type of flow), as a consequence of which k_i does not depend upon F'_Δ either. Hence for $L/L_L \geq 1$ Equation 17 reduces to

$$k_i = f(\sigma) \qquad (18)$$

In the analysis leading to Equation 17 simplifying assumptions were made. For instance in Section 2.2 and 2.3 the thickness of the intermediate layer was assumed to have the same thickness in the velocity profile as in the density profile, an assumption which is not necessarily satisfied. Hence the result of the analysis must primarily be interpretated qualitatively.

In a previous analysis assuming entrainment to take place at the interface k_i was shown to depend upon a densimetric Froude number (Abraham and Eysink [1971]). This observation is incorrect for $L/L_L \geq 1$, as can be seen from Equation 18.

3 Experimental data on interfacial shear

The experimental data on the magnitude of the interfacial shear, given in this part of the Report, are primarily derived from a survey of literature (Delft Hydraulics Laboratory [1974]). Further experimental data supporting the basic assumption behind Equation 18 will be presented.

3.1 Experimental data on interfacial shear coefficient

3.1.1 General considerations

Turner [1973, Section 4.3] indicates that stratified flows may be influenced by turbulence and mixing generated internally, i.e. at the interface, and/or by turbulence and mixing generated externally, i.e. at the solid boundary such as the bottom. The significance of the former effect in comparison with the latter effect, and vice versa, varies with the type of stratified flow. Therefore experimental data on different types of stratified flows must be treated separately.

This Report gives data on the arrested salt wedge, the temparture surface wedge, density induced return currents and lock exchange flows. Details on how the experimental data were obtained are given in Table 1.

3.1.2 Arrested salt wedge

Figure 4 shows experimental results (k_i as a function of the Reynolds number for the upper layer, Re_1) derived from arrested salt wedge measurements both in the laboratory and in the field.

The few data available for large values of Re_1 suggest a value of k_i of about 4.10^{-4}.

3.1.3 Temperature surface wedge

Figure 5 shows experimental results (k_i as a function of the Reynolds number for the lower layer, Re_2) derived from temperature surface wedge measurements, performed both in the laboratory and in the field.

Again it seems that the value of k_i becomes constant for large values of Re_2, k_i being about $1,5.10^{-3}$.

3.1.4 Density induced return currents and lock exchange flows

Figure 6 shows experimental results (k_i as a function of the Reynolds number for the intermediate layer, Re_3) derived from measurements on density induced return currents and lock exchange flows, performed both in the laboratory and in the field. The available experimental data cover a substantial range of Reynolds numbers. Re_3 is defined in Figure 6.

For high values of the Reynolds number the average value of k_i was found to be about 7.10^{-4}.

3.1.5 Discussion

From the measurements it can be concluded that k_i decreases with increasing Reynolds number tending to a constant value for great values of Re, the constant value being

- arrested salt wedge $k_i \approx 4.10^{-4}$
- temperature surface wedge $k_i \approx 15.10^{-4}$ (19)
- density induced return currents, lock exchange flows $k_i \approx 7.10^{-4}$

The above values of k_i vary with the type of flow. As indicated in Section 1 this could be due to the effect of turbulence generated at the bottom on the degree of turbulence near the interface. This can be substantiated as follows:

The interfacial shear coefficient k_i is defined by Equation 17. Turbulence generated at the bottom playing a role, it is reasonable that for the types of stratified flows considered in this Report the interfacial shear satisfies an equation like

$$\tau_i = k_{i,i}\ \rho(u_2-u_1)^2 + k_{i,b}\ \rho\ u_2^2 \tag{20}$$

where $k_{i,i}$: interfacial shear coefficient, defined by Equation 20, expressing effect of turbulence generated at the interface

$k_{i,b}$: interfacial shear coefficient, defined by Equation 20, expressing effect of turbulence generated at the bottom.

Table 2 gives the relationship between $|u_2|$ and $|u_2-u_1|$ for the different subcritical stratified flows. Substituting these relationships into Equation 20, the above values of k_i can be shown to be compatible with

$$k_{i,i} \approx 4.10^{-4}$$
$$k_{i,b} \approx (11 \text{ to } 12).10^{-4} \qquad (21)$$

That the values of k_i for the different types of subcritical stratified flow can be reduced to one set of $k_{i,i}$ and $k_{i,b}$ values implies it to be a reasonable assumption that turbulence generated at the bottom influences the magnitude of interfacial shear.

The above observations imply that the degree of turbulence at the interface is an important parameter once the intermediate stabilizing layer has been established. Then nevertheless the turbulent flux through the interface ($F_{i,turb}$) is small, since in the critical Richardson number range the effect of density stratification upon the turbulent flux of mass is much greater than that upon the turbulent exchange of momentum (Delft Hydraulics Laboratory [1974]).

Since in the high Reynolds number range the bottom shear coefficient (k_b), defined as

$$\tau_b = k_b \; \rho \; u_2^2 \qquad (22)$$

is often about ten times larger than k_i, the ratio between the above $k_{i,i}$ and $k_{i,b}$ values is not an unreasonable one.

Equation 20 satisfies the condition that in the limiting case of $u_1 = u_2$ the interfacial shear-stress should not vanish. However, if in first approximation a linear shear-stress profile is assumed, in the limiting case one has

$$\tau_i = \frac{a_1}{a_1 + a_2} \tau_b \qquad (23)$$

This makes it a reasonable proposition to modify Equation 20 by writing it, as proposed by Vreugdenhil [1970], as

$$\tau_i = k_{i,i} \; \rho (u_2 - u_1)^2 + \frac{a_1}{(a_1 + a_2)} k'_{i,b} \; \rho \; u_2^2 \qquad (24)$$

where: $k'_{i,b}$: interfacial shear-stress coefficient, defined by Equation 24, expressing effect of turbulence generated at bottom.

Equation 24 shows an increasing influence of turbulence generated at the bottom with decreasing height of the lower layer. Qualitatively this has to be expected.
In the available experimental data the ratio of a_1 over (a_1+a_2) was not varied systematically. Because of this reason it was preferred to analyze the data on the basis of Equation 20 in stead of on the basis of Equation 24. This implies that further research is needed to describe the effect of turbulence generated at the bottom in greater detail.

The information presented on the experiments from which the values of k_i were derived does not allow to establish in which degree the values of k_i given are affected by inaccuracies in the measurements or otherwise.

It also must be observed that in the considered stratified flows F_Δ varies in the horizontal direction in a different manner for the different types of flow and in the different experiments. The variation of F_Δ in the horizontal direction may be associated with local deviations from Equation 10, and consequently may effect the magnitude of k_i in a different manner for the different flows.

Finally the length of an arrested salt wedge is inversely proportional to the magnitude of the interfacial shear-stress coefficient. This also holds for the temperature surface wedge. However, the length of a temperature surface wedge may also depend upon the loss of heat to the atmosphere, the length of the wedge being shorter with increasing magnitude of the loss of heat. Hence not recognizing the effect of the loss of heat to the atmosphere may lead to an overestimation of the magnitude of k_i.

3.2 Experimental data on thickness of intermediate layer

Several experimental studies designed to find the limiting value of the parameter Ri defined as

$$Ri = \frac{\frac{\Delta\rho}{\rho} g\, d}{(u_1-u_2)^2} \tag{29}$$

can be found in literature. In this context the limiting value is the value associated with no further mixing or breaking of internal waves. Experiments known from literature, give a limiting value of Ri in the order of 0.35 corresponding to a limiting value of F'_Δ in the order of 3 (Table 3).

The result of F'_Δ tending to a constant value provides a qualitative confirmation of the linear instability theory, and hence an indirect confirmation of the present analysis of interfacial shear based on this theory.

4 Conclusions

The following conclusions emerge for steady subcritical stratified flows

(i) In subcritical stratified flows the conditions at the interface are influenced by instability of internal waves which generates mixing through the interface. This mixing is a stabilizing factor leading to the establishment of a stabilizing intermediate layer. Once the establishment on this layer is completed internal waves are stable and turbulent mixing through the interface ($F_{i,turb}$) is small.

(ii) The establishment of the intermediate layer allows subcritical stratified flows to be treated as two layer flows without mixing.

(iii) After establishment of the stabilizing intermediate layer the conditions at the interface may be described by a more or less constant value of the internal Froude number, defined by Equation 3. This implies that the interfacial shear does not depend on the magnitude of an internal Froude number, defined otherwise.

(iv) After establishment of the stabilizing intermediate layer the magnitude of interfacial shear depends upon the level of turbulence, as a consequence of which it may be influenced by turbulence generated at the bottom.

(v) In accordance with conclusion (iv) it is reasonable to describe the interfacial shear by an equation of the type of Equation 20. The experiments summarized in this Report lead to the values of the coefficients contained in this equation which in the high Reynolds number range are given by Equation 21.

(vi) Equation 20 does not give a proper description of the limiting case of $u_1 = u_2$. Then an equation of the type of Equation 24 has to be preferred. This implies that further research is needed to describe the effect of turbulence generated at the bottom in greater detail.

(vii) Describing the interfacial shear by Equation 17, the experimental data lead to values of the coefficient k_i which in the high Reynolds number range are given by Equation 19. The effects of Reynolds number on k_i can be seen from Figures 4, 5 and 6.

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Table 1 Studies dealing with interfacial shear stress

Authors	Type of density current	Method for estimating k_i	h	U	Range of Re	Comments
Abraham/Eysink 1971	Lock exchange flow	Using linear shear stress profile	R_2	u_2	1.000-40.000	Laboratory
Barr 1967	Lock exchange flow	Analyzed by Abraham et al (1971)	R_2	u_2	115-14.000	Laboratory
Bata 1957	Arrested salt and thermal wedge, wedge with both layers flowing	Using linear shear stress profile			$\sim 10^4$	Laboratory
Boulot/Daubert 1969	Arrested salt wedge	From shape of interface	h_1	u_1	10^6	Field (Rhone)
Dick/Marsalek 1972, 1973	Thermal wedge with counter currents	Using linear shear stress profile	h_2	u_2	10^6	Field
Hendrikse 1965	Arrested salt wedge	From shape of interface	h_1	u_1	10^4	Laboratory
Holley/Waddell 1967	Exchange flow	From shape of interface	h^*	u^*		Field
Keulegan 1957	Lock exchange flow	Analyzed by Abraham et al (1971)	R_2	u_2		Laboratory
Keulegan 1966	Arrested salt wedge	From shape of interface	h_2	u_2	10^7	Field (Mississippi
Lanzoni 1959	Arrested salt wedge	From shape of interface	h_2	u_2	$\approx 5\ 10^3$	Laboratory
Maxwell/Holley 1957	Arrested thermal wedge	From shape of interface	h_2	u_2	500-50.000	Laboratory
Polk et al 1971	Arrested thermal wedge	Using linear shear stress profile	h_2	u_2	10^7	Field
Riddell 1970	Arrested salt wedge	From shape of interface (analyzed by Vreugdenhil 1971)	h_1	u_1	$< 10^3$	Laboratory
Rijkswaterstaat 1974	Exchange flow	Using linear shear stress profile	h^*	u^*	$\approx 10^6$	Field
Zanotti 1965	Arrested salt wedge	From shape of interface	h_1	u_1	10^5	Field

Note: R_2: hydraulic radius of underlayer

*: for definition see Figure 6

Re: Reynolds number defined as Uh/ν, U and h being taken from table

Table 2	Relationship between $|u_2-u_1|$ and $|u_2|$	
Arrested salt wedge	$u_2 \approx 0$	
Temperature surface wedge		$|u_2-u_1| = |u_2|$
Density induced return current, lock exchange flow	$|u_2| \approx |u_1|$	$|u_2-u_1| \approx 2|u_2|$

Table 3 Laboratory studies on limiting value of Ri (Eq. 29)

Authors	Limiting value of Ri	Type of flow	Comments
1 Dingemans (1972)	0.21 - 0.29	Lock exchange flow	6 experiments
2 Thorpe (1971)	0.32	Lock exchange flow	15 experiments
3 Koop (1976)	0.35	Forced density flow (see Fig. 7)	
4 Hopfinger (1972)	0.32	Forced density flow (see Fig. 7)	

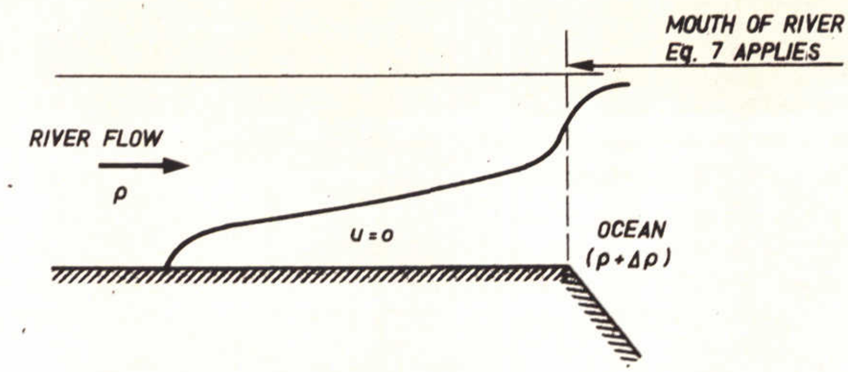


FIG. 1a ARRESTED SALT WEDGE

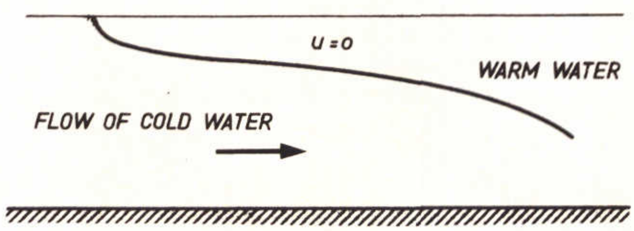


FIG. 1b TEMPERATURE SURFACE WEDGE

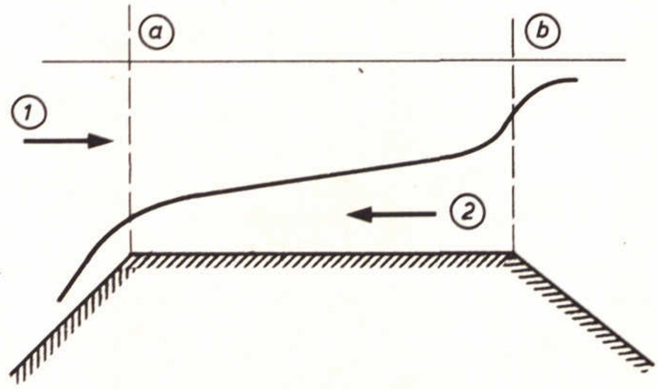
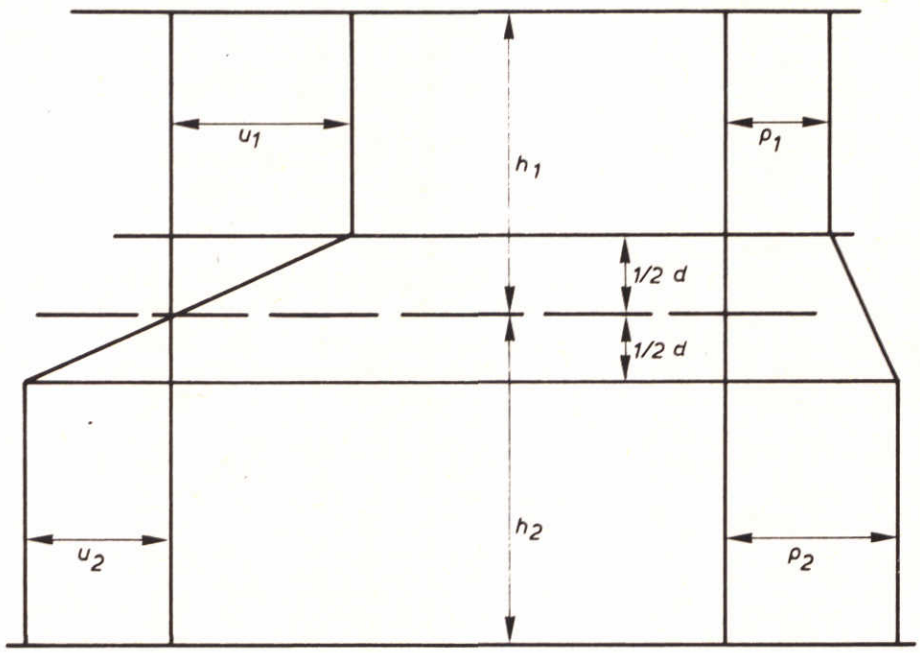


FIG. 1c DENSITY INDUCED RETURN CURRENT

(1) FLOW OF LIGHT FLUID
(2) DENSITY INDUCED RETURN CURRENTS OF HEAVY FLUID
(a) } Eq. 7 APPLIES AT THESE POINTS, WHEN RETURN
(b) } CURRENT HAS MAXIMUM VALUE

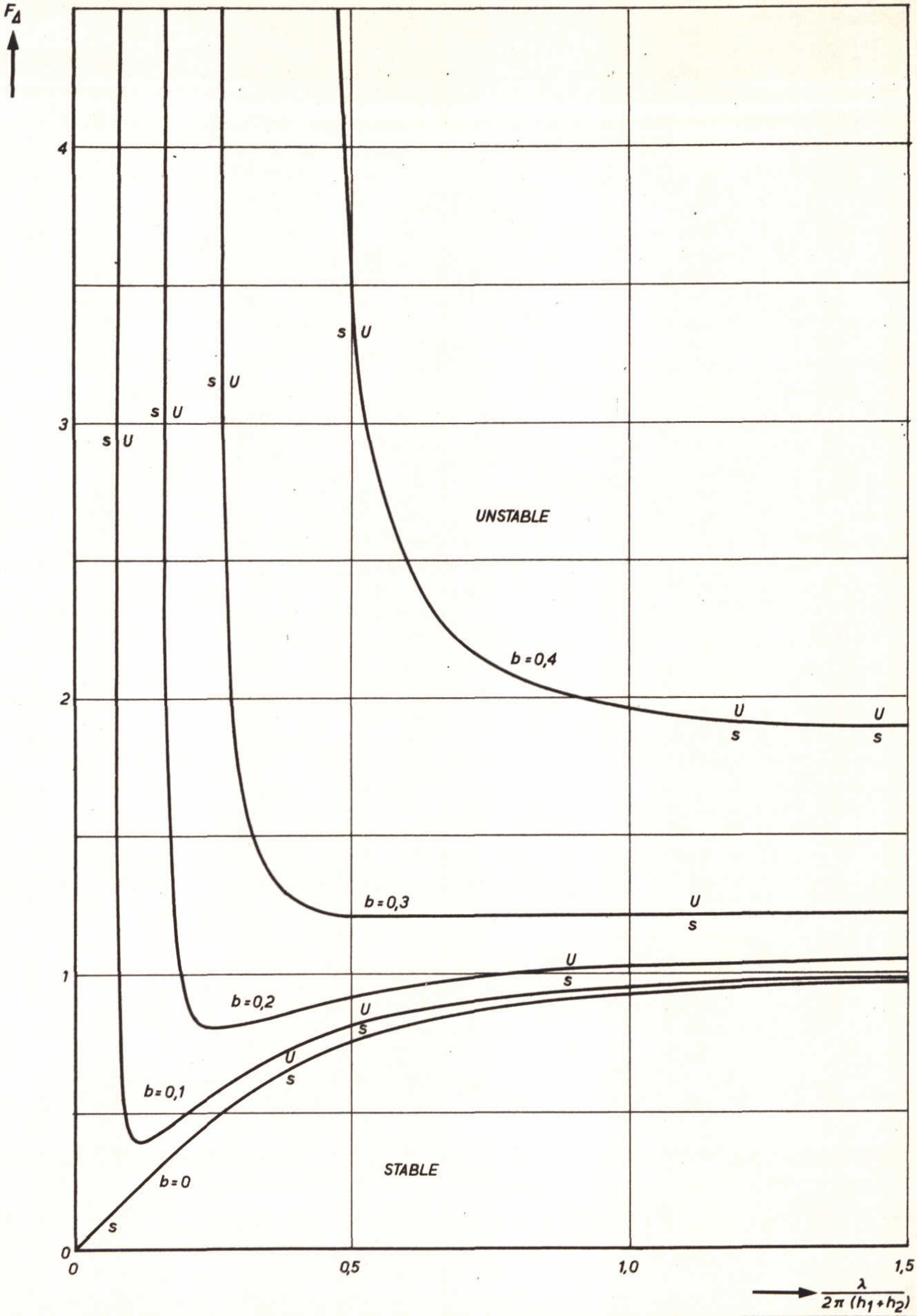
Subcritical stratified flows		R.R.
		A4
DELFT HYDRAULICS LABORATORY	R.880 - 1053	FIG. 1



Background flow for which the neutral stability curves represented in Figure 3 were derived ($h_1 = h_2$ and $|u_1| = |u_2|$)

R.R.	A4

DELFT HYDRAULICS LABORATORY | R. 880 - 1054 | FIG. 2



Neutral stability curves derived for the background flow given by Figure 2, the zone indicated by u being the zone of unstable conditions and the zone indicated by s being the one of stable conditions [Dingemans (1972)]

R.R.

A4

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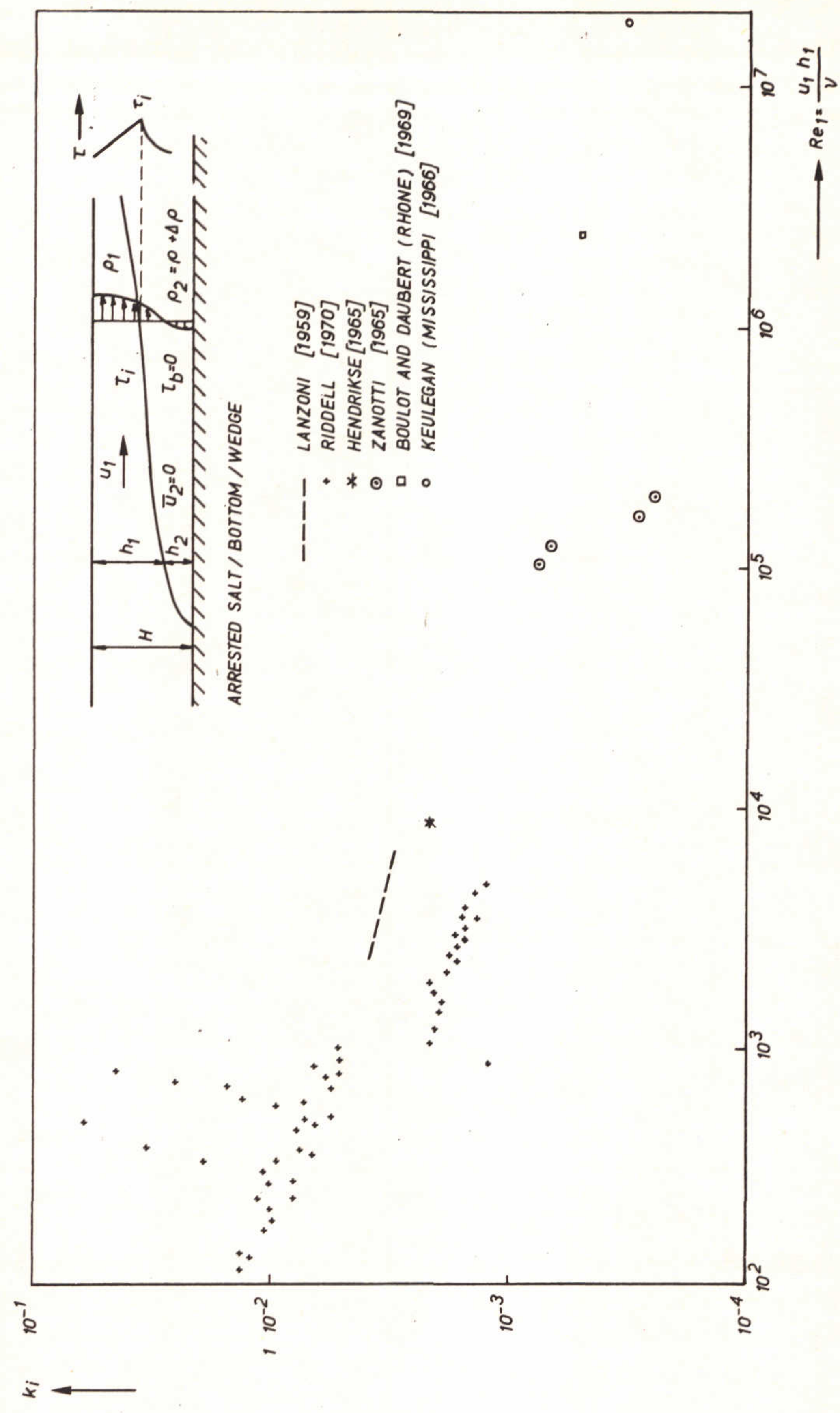
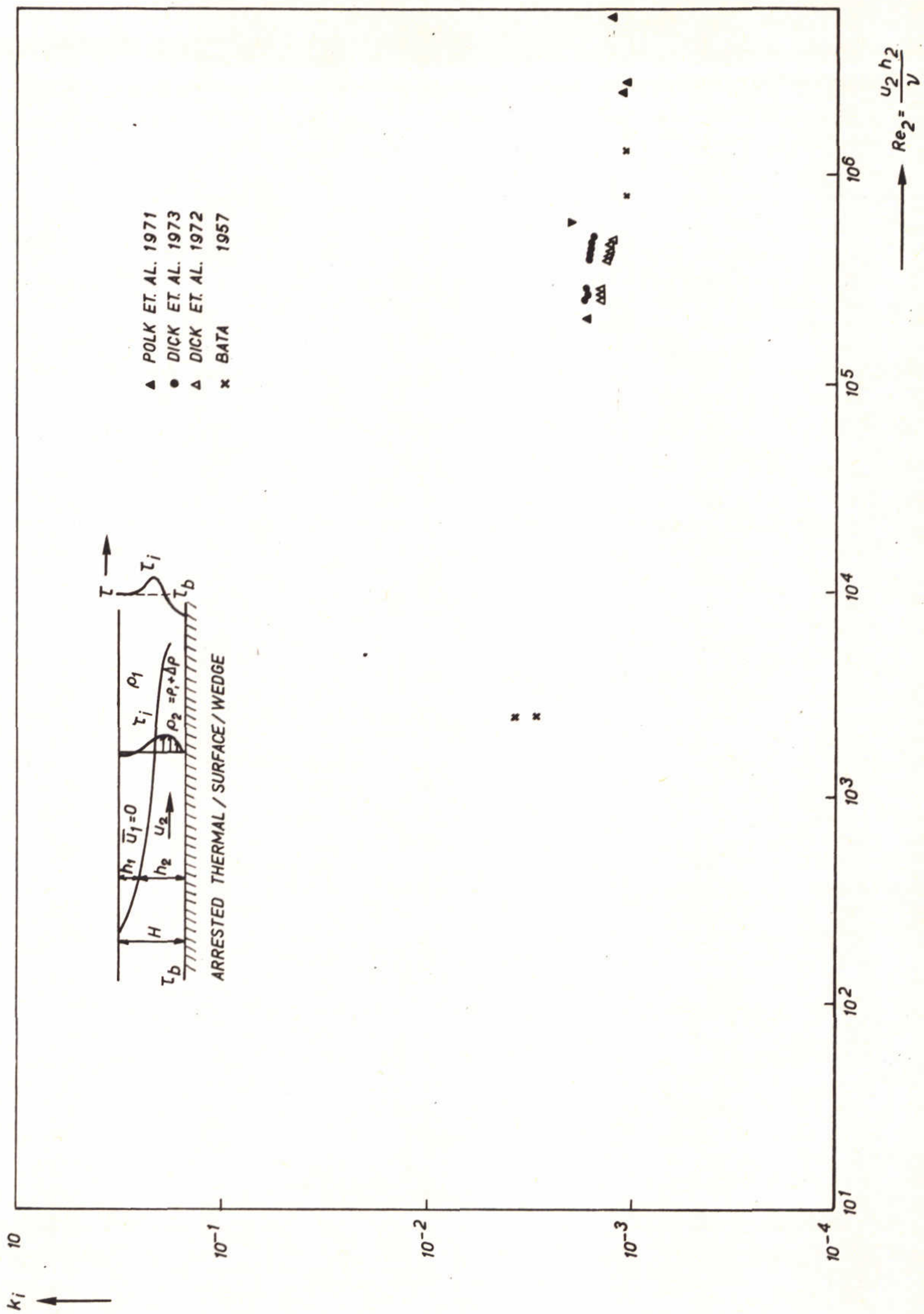
FIG. 3



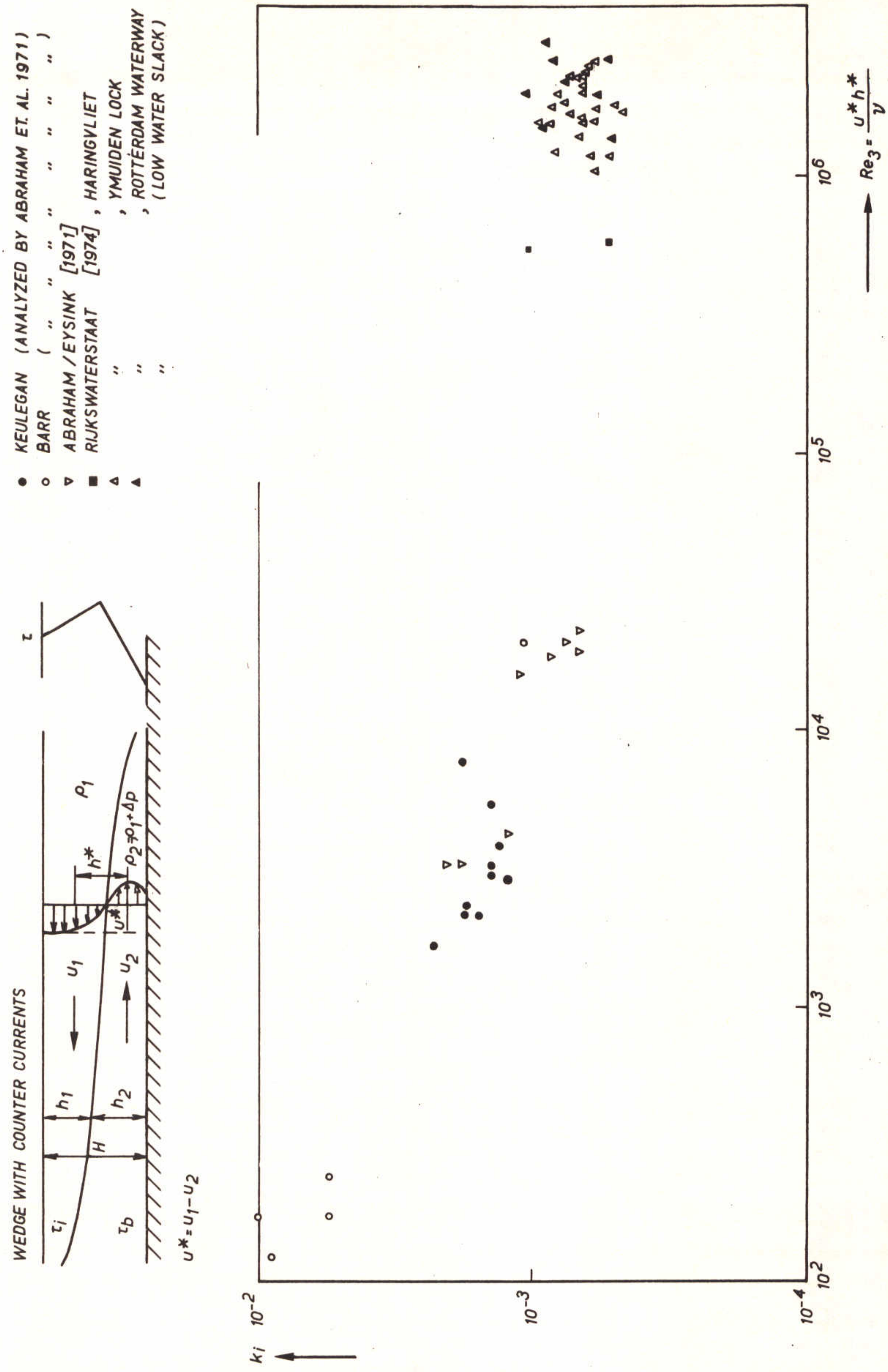
Arrested salt wedge: Variation of coefficient of interfacial shear with Reynolds number

A4

DELFT HYDRAULICS LABORATORY | R 880-1056 | FIG. 4



Temperature surface wedge: Variation of coefficient of interfacial shear with Reynolds number		A4
DELFT HYDRAULICS LABORATORY	R880 - 1057	FIG. 5



Lock exchange flows: Variation of coefficient of interfacial shear with Reynolds number

A4

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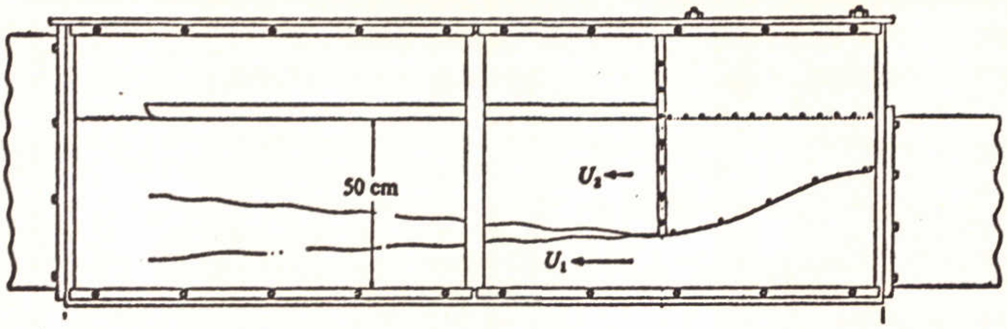
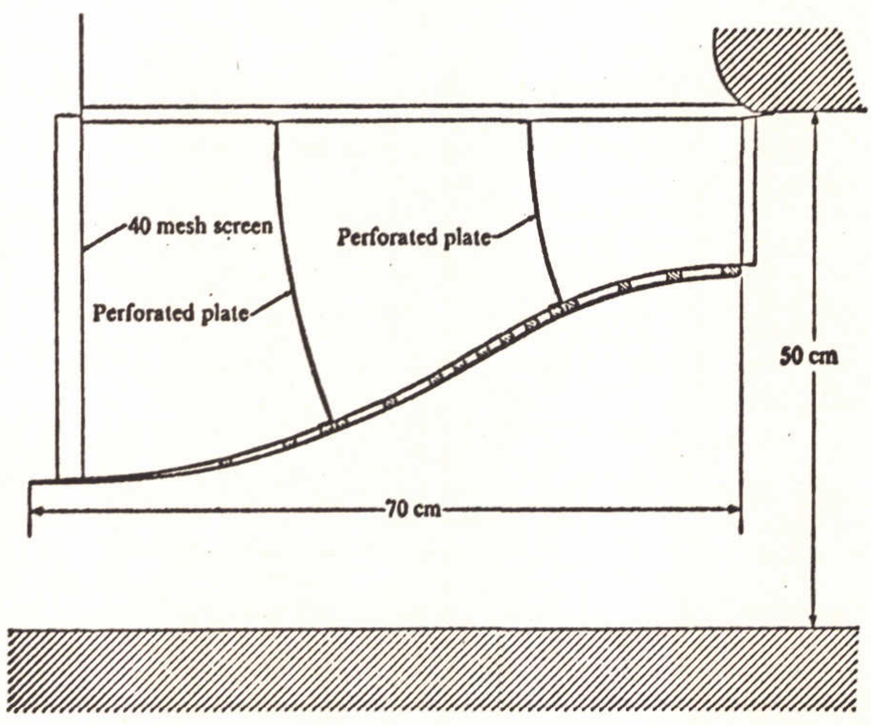
FIG. 6



a. Test section



b. Insert of the two layer system

Test section for forced density flow experiments
(Hopfinger [1972] and Koop [1976])

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FIG. 7

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