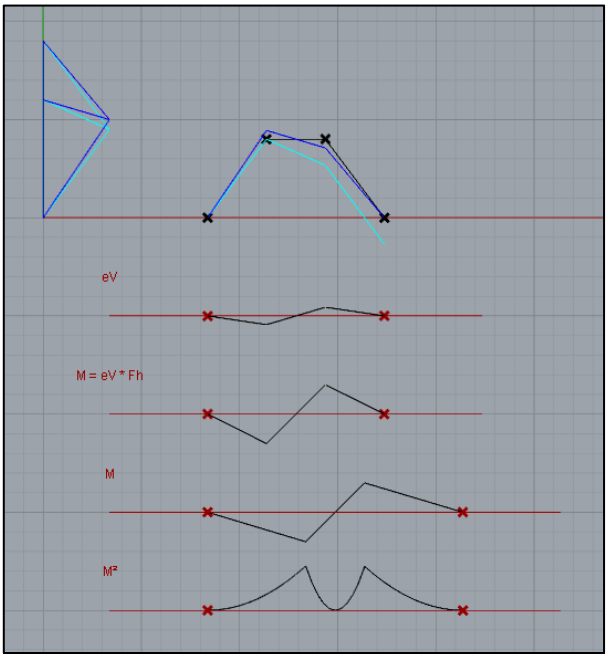
Graphically calculating arcs and shells by using the lowest complementary energy

P3 Report



Jeroen Ros

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Delft University of Technology

Faculty of Architecture

Department Building Technology

Main mentor - Andrew Borgart

Second mentor - Michela Turrin

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Introduction

The aim of this report is to present and explain the progress that has been made since the P2 Report (Ros, 2016). This paper will discuss how the model and calculations for finding the support reactions of an arc have been changed and whether these adjustments have been an improvement compared to the method presented earlier.

The current state of research on three dimensional shapes will be discussed as well, along with a first set of results to see whether the research is heading in a good direction. It will also be discussed whether the method used for achieving these results is sufficient enough to continue upon towards the P4 report.

P2 report

The P2 report (Ros, 2016) started out with a piece of history, as the concept of formfinding started as early as the Roman era. Domes were constructed as being half of a sphere. Many centuries later came the first 'real understanding' of stresses and force trajectories. From this point on, more complex shapes could be designed and with computational software the optimal thickness could be retrieved.

However, the main issue with these calculating methods (most notably the Finite Elements Method) is the amount of time it takes to get accurate results. Each small adjustment in the design needs a new three dimensional model for which the loads, supports, boundary conditions etc. need to be added every single time.

With this 'problem' in mind the numerical theory as described in a former TU Delft student's paper (van Dijk & Borgart, 2014) was made more graphical. A tool to aid future architects and/or designers should give them more insight in how a curved element is coping with stresses caused by loads.

Calculating an arc

The definition discussed in the P2 report (Ros, 2016) has been written in Grasshopper, a plugin for the three dimensional modelling software Rhinoceros 5.0.

The method started out with the forces acting on the arc. These forces are drawn as lines stacked on top of each other to create the start of the force polygon. By taking a random point next to these stacked forces the force polygon can be closed. The slopes of these closing lines are the same as the lines for the line of thrust. The horizontal distance between this point and the forces is the same as the horizontal support reactions of the arc (as the vertical support reactions are determined by the vertical distance between the random point and the top/bottom of the stacked forces):

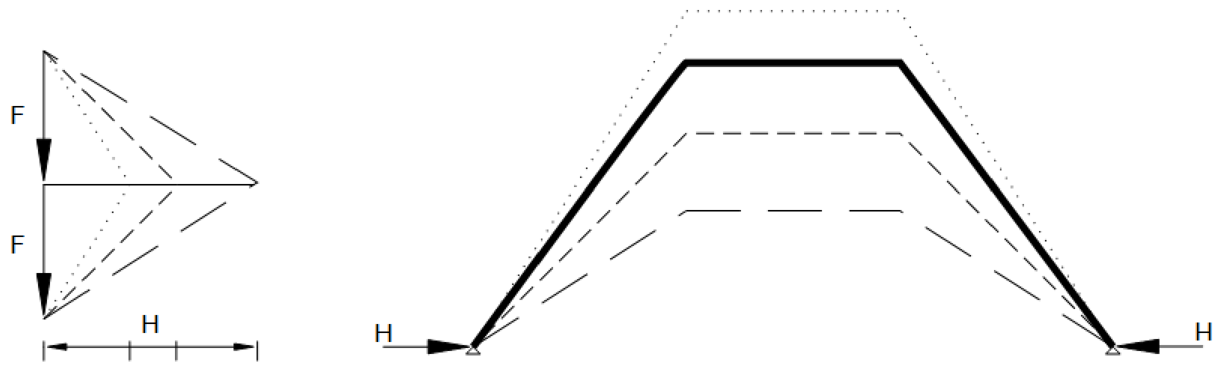


Figure 1. Infinite force polygons create an infinite amount of thrust lines (own work).

With the method of the lowest complementary energy it was possible to calculate the optimal line of thrust, and therefore retrieving the correct (horizontal) support reactions caused by the given loads on the designed arc. However, things were not as simple anymore when the forces were unequal:

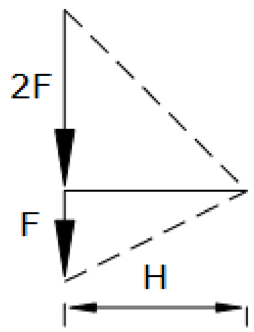


Figure 2. Force polygon with unequal forces (own work).

When drawn as a line of thrust the problem becomes clear:

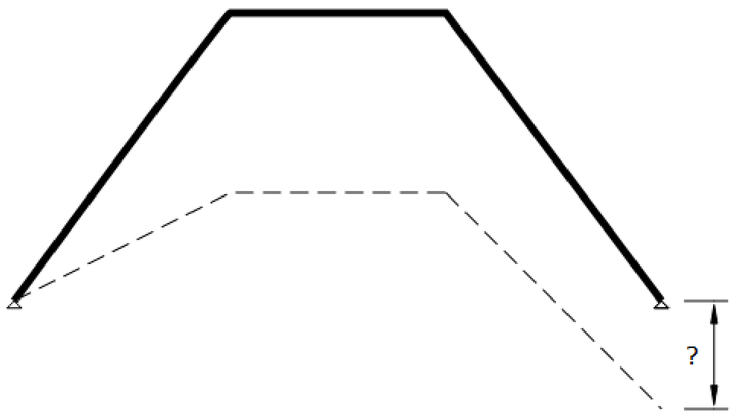


Figure 3. The line of thrust does not connect to the designed arc (own work).

However, this issue was very easily (and graphically) fixed by connecting the ends of the line of thrust and taking this slope to find a new height for the randomly chosen point:

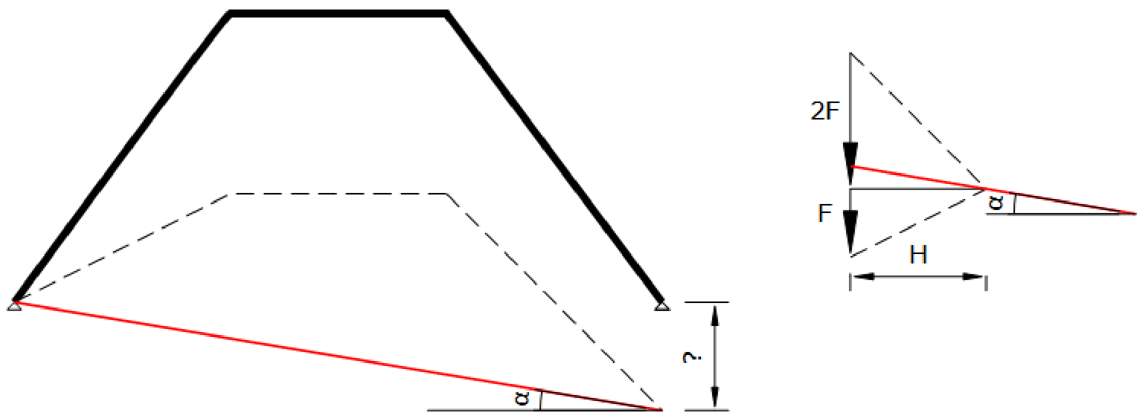


Figure 4. Finding the correct height for the randomly chosen point (own work)..

With the correct height for the randomly chosen point (and therefore for having the correct vertical support reactions) the force polygon can be constructed again, giving an infinite amount of 'correct solutions':

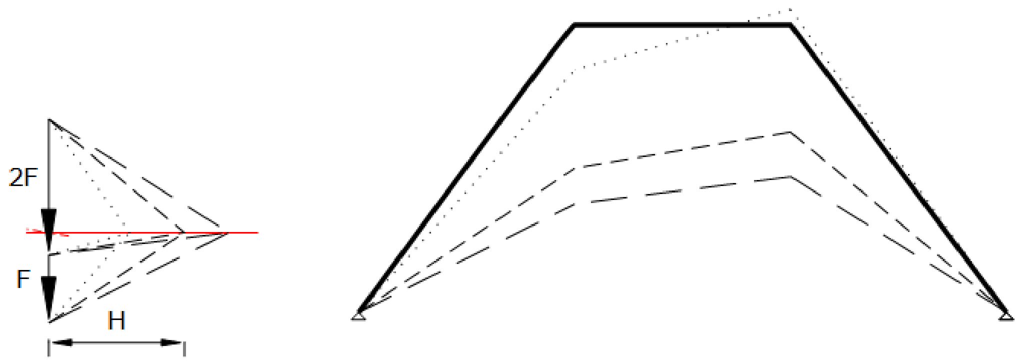


Figure 5. Corrected force polygons give correct lines of thrust (own work).

The finished definition gave some pretty accurate results (Ros, 2016) compared to the Finite Element Method:

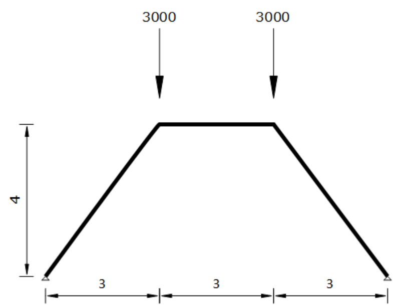


Figure 6. Example 1 - equal loads, thickness = 0,10m (Ros, 2016).

Grasshopper definition	2250 N
Oasys GSA Suite	2250 N

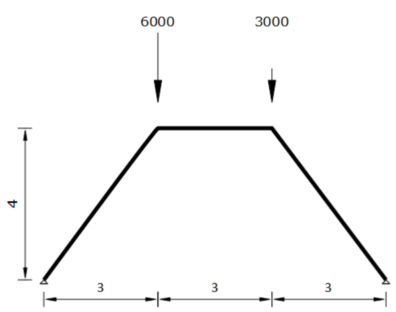


Figure 7. Example 2 - unequal loads, thickness = 0,10m (Ros, 2016).

Grasshopper definition	3374 N
Oasys GSA Suite	3375 N

Room for improvements

The definition however quickly ran into some minor trouble. To start with the definition was rather large (physically and because of needed computing power) because of its implemented feature to have a flexible amount of segments of the arc. This caused the definition to compute the maximum amount of segments, ignoring the results that were not requested. This caused the definition to be rather slow and crashing quite often on a less powerful computer.

What was not discussed in the P2 report (Ros, 2016) but was also one of the major things that the model was missing, was a function to cope with uneven supports. The written definition could only cope with an arc that was on a perfectly flat surface, as otherwise the line of thrust would not connect to the supports of the designed arc.

Both these complaints have been thought through and have been implemented in an updated version of the definition for calculating the support reactions of an arc. The possibility for changing the number of segments has been removed, as the lack of speed was more annoying than the extra possibilities it gave. Therefore this definition can only handle one different situation; a new model is needed for an additional case.

What also was quite important (especially for understanding the three dimensional shapes later on) was making the model capable of coping with uneven supports. This was one of the main focus points when rewriting the definition.

Calculating an arc – revisited

Finding the line of thrust for an arc is a statically indeterminate situation, as there are two variables that can be used for finding the line of thrust:

The horizontal support reaction

> This force is the main variable that changes the shape of the force polygon. When this polygon changes shape, the slopes of the connecting lines change as well. As these slopes change, the slopes of the line of thrust change as well, giving it a different shape.

The height of the line of thrust

> As the height of the line of thrust changes, the slopes change along with it. As these slopes change shape, the force polygon changes along with it. As the slopes of the force polygon change, the randomly chosen point changes it position, giving a different result for the horizontal support reaction.

The horizontal support reaction and the height of the line of thrust are very dependant on each other, but as they are both unknown in the beginning of the calculation, it is hard to find the optimal solution without having some other demand it needs to satisfy.

This is where the lowest complementary energy comes in. The most optimal value (the lowest value) needs to be found to see which value for the height of the line of thrust or the horizontal support reaction is best (Ros, 2016). The horizontal support reaction was used as the main variable in the previously built definition.

Renewed method

In the method presented in the P2 report (Ros, 2016) the line of thrust was created from the polar figure, and later corrected when the supports were not matching. In this renewed method it is not necessary anymore to correct the line of thrust, as the correct height for the closing point of the force polygon can also be calculated using the equation of equilibrium:

$$\sum F = 0$$

In this case:

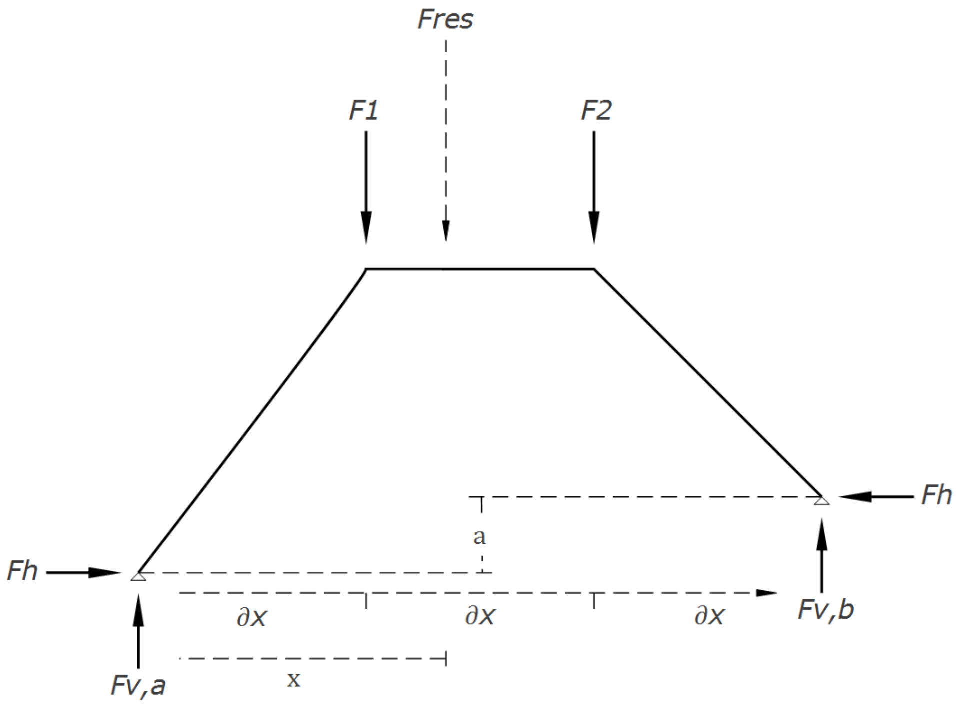


Figure 8. Example for uneven supports (own work).

In which:

$$F_{res} = \sum_{i=1}^{n} F_i$$

And

$$M_a = \sum_{i=1}^{n} F_i * (i * \partial_x)$$

Which leads to

$$x = \frac{M_a}{F_{res}}$$

When using the equation of equilibrium the difference in height must be calculated as a function for the horizontal support reaction:

$$F_{correction} = \frac{a}{n * \partial_x} * F_h$$

Following these formulas it is possible to calculate the vertical support reactions:

$$F_{v,a} = \left(\frac{(n * \partial_x) - x}{(n * \partial_x)} * F_{res}\right) + F_{corection}$$

And

$$F_{v,b} = \left(\frac{x}{(n * \partial_x)} * F_{res}\right) - F_{corection}$$

With these forces known it is possible to narrow down the amount of possibilities of the force polygon, as the height is now known.

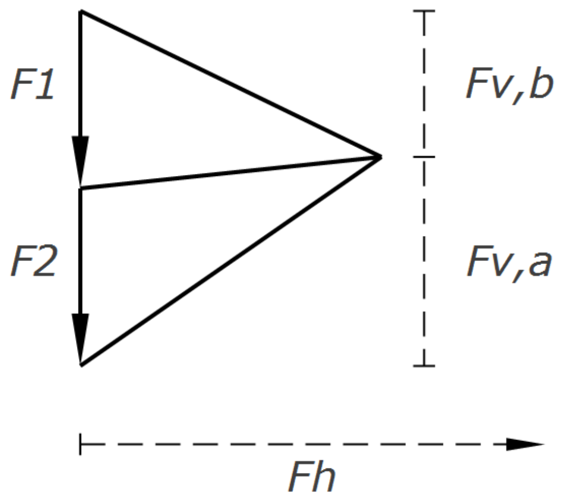


Figure 9. Possibilities for the force polygon (own work).

From this point on the same method as presented in the P2 report (Ros, 2016) can be used.

Normal forces

For each value for 'Fh', a different line of thrust appears. Therefore, the normal forces also change for each value of the horizontal support reaction.

The normal force can be found by comparing the angle of the line of thrust to the line of the designed arc. The angle needed (γ) is the angle between the designed arc (α) and the line of thrust (β), which is dependant of the horizontal support reaction. Angle γ is calculable by using simple geometric equations like the SOH-CAH-TOA-triangle.

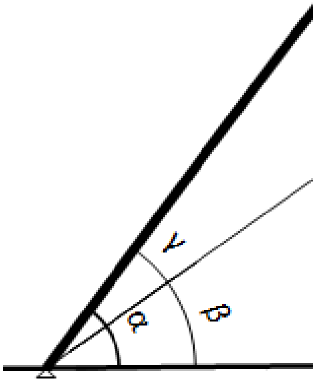


Figure 10. Finding the angle for calculating the normal forces (Ros, 2016).

The force polygon is resembling the forces acting on the designed arc, with each length of the segment of the polygon resembling the size of the force. When a valid line of thrust is drawn from the force polygon, it connects to both supports on the designed arc. The force flowing through the designed arc therefore is found by multiplying the length of the force polygon segments by calculated angle γ:

$$N_n = F_n * \cos(\gamma_n)$$

As the line of thrust is a visual way of showing what is going on, it is something imaginary and therefore cannot transfer actual forces. The normal forces have to be related to the length of the corresponding segments of the designed arc:

$$E_{c,N} = N^2 * l$$

In which

N = Normal force in the segment of the designed arc, adjusted by angle γ
l = Length of the segment of the designed arc

This gives a total complementary energy caused by the normal forces of:

$$\sum_{i=1}^{n} E_{c,N} = N_i^2 * l_i$$

In which

n = Number of segments

This is simple to calculate for a computerized script, as a computer can return a list of values for the calculated $E_{c,N}$ for a list of possible solutions for the variable value of 'Fh'.

Bending moments

When looking for the complementary energy caused by bending moments, the first thing that is of importance is the distance between the line of thrust and the designed arc, called the eccentricity:

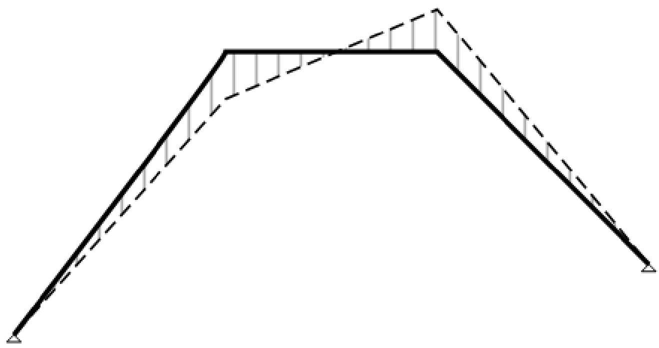


Figure 11. Finding the distance between the designed arc and the line of thrust (own work).

This eccentricity can be plotted:

Figure 12. Plot of the eccentricity (own work).

Which gives the bending moments when multiplied with the variable horizontal support reaction:

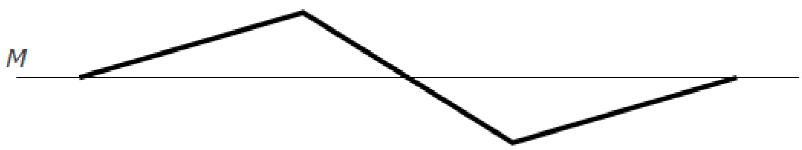


Figure 13. Plot of the bending moment, caused by the eccentricity.

However, as a bending moment is related to the structure it is caused upon, the previous plot needs to be adjusted so that the length of the designed arc matches the bending moments:

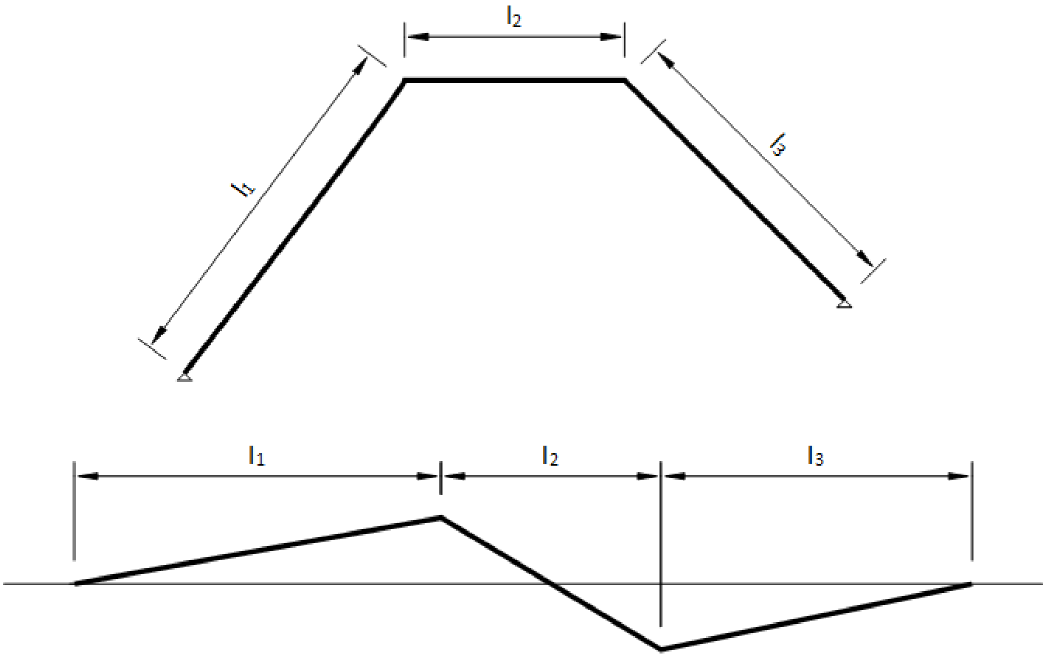


Figure 14. Bending moment related to the designed arc (own work).

This plot needs to be squared:

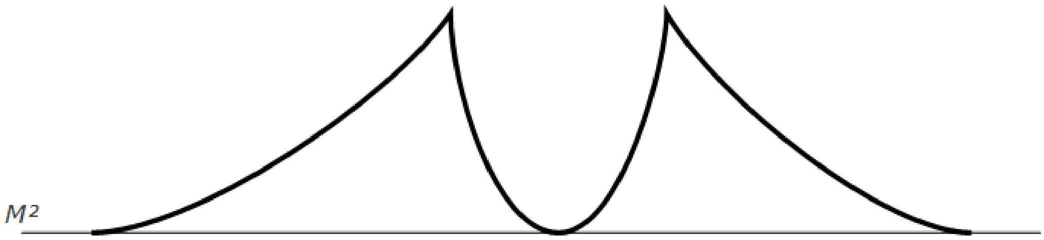


Figure 15. The bending moments squared (own work).

When the formula described in the P2 report (Ros, 2016) is looked upon

$$\sum E_c = E'_{c,N} + E'_{c,M}$$

It can be found that the complementary energy caused by the bending moment is a differential equation, so to bring it to the same level as the sum it has to be integrated. In the graphical way this means that the area underneath the previously found plot has to be used:

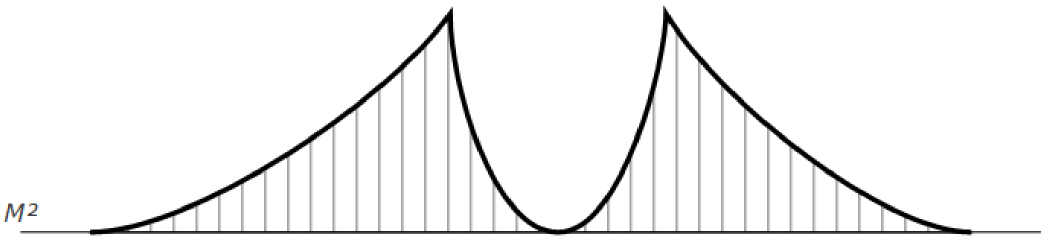


Figure 16. Area underneath the plot (own work).

When examining the formula for the total complementary energy (van Dijk & Borgart, 2014)

$$\sum E_c = \mathrm{l}\left(\overline{\mathrm{N}^2} + \frac{12 * \overline{\mathrm{M}^2}}{t^2}\right)$$

All that is needed for retrieving the total complementary energy for a certain horizontal support reaction is the thickness of the arc.

Validating the new method

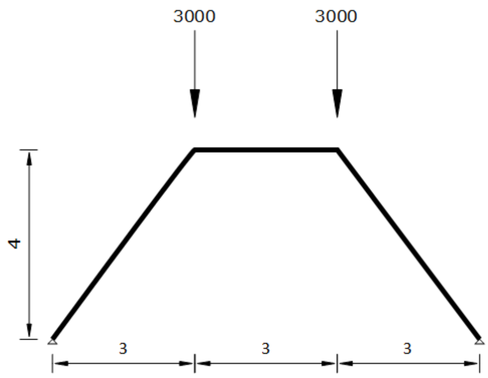


Figure 17. Example 1 - equal loads, thickness = 0,10m (Ros, 2016).

Grasshopper old definition	2250 N
Grasshopper renewed definition	2250 N
Oasys GSA Suite	2250 N

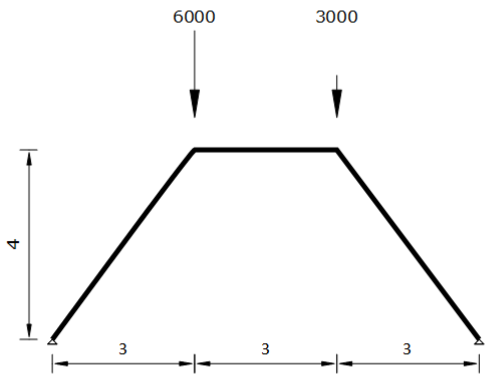


Figure 18. Example 2 - unequal loads, thickness = 0,10m (Ros, 2016).

Grasshopper old definition	3374 N
Grasshopper renewed definition (see Appendix 1)	3375 N
Oasys GSA Suite	3375 N

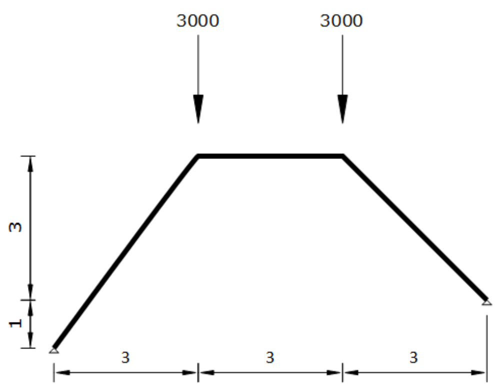


Figure 19. Example 3 - unequal supports, thickness = 0,10m (own work).

Grasshopper old definition	-
Grasshopper renewed definition (see Appendix 2)	2562 N
Oasys GSA Suite	2564 N

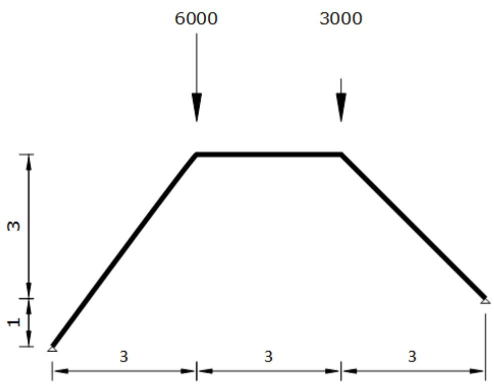


Figure 20. Example 4 - unequal supports and unequal loads, thickness = 0,10m (own work).

Grasshopper old definition	-
Grasshopper renewed definition (see Appendix 3)	3874 N
Oasys GSA Suite	3875 N

Discussing the results

One thing that was not taken into account as much as it should be in the P2 report (Ros, 2016) was the effect of the thickness of an arc on its outcome. This might be important as the bending moment is causing an inverted parabola as a function of the horizontal support reaction, having less of a slope as the thickness decreases, while the normal force is linear decreasing as a function of the horizontal support reaction.

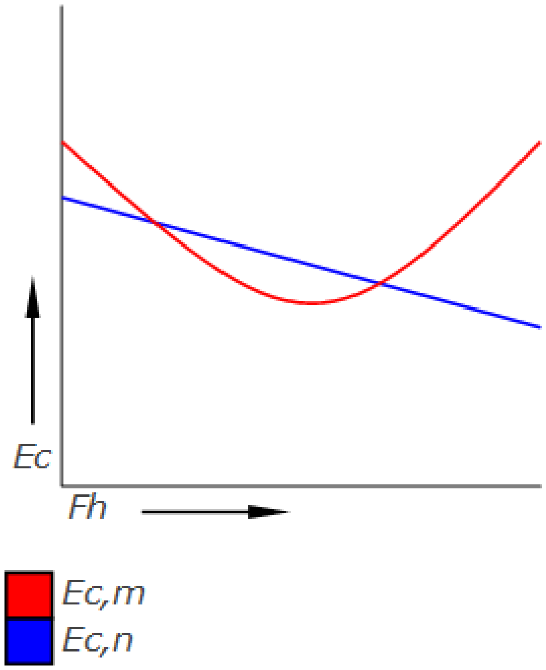


Figure 21. Share of the Ec,n and Ec,m for the total Ec (own work).

This means that whenever the thickness of the arc increases the share of the bending moment will be smaller, thus the total complementary energy will be more determined by the normal forces acting on the arc (hence the 12M/t part in the formula presented by van Dijk and Borgart (2014).

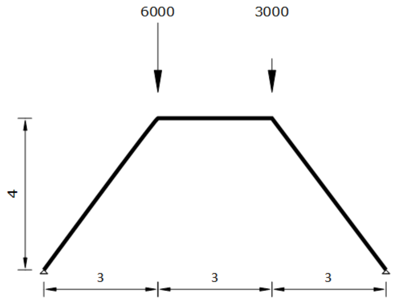


Figure 22. Example 2 - unequal loads (Ros, 2016).

Thickness	GSE (N)	Fh Ec,m (N)	Ec,m (kNm)	Fh Ec,tot (N)	Ec,tot (kNm)	Dev.
0,10m	**3375**	3375	11725	**3375**	12003	0,0%
0,25m	**3373**	3375	1876	**3377**	2154	0,1%
0,50m	**3366**	3375	469	**3385**	747	0,6%
1,00m	**3340**	3375	117	**3413**	394	2,2%
2,00m	**3242**	3375	29	**3515**	301	8,4%

It can be seen that for every thickness the Fh related to the minimal Ec,m is the same, respectively 3375N. This is because this is the 'perfect shape' for the line of thrust, as it cuts the top segment of the arc in exactly two pieces:

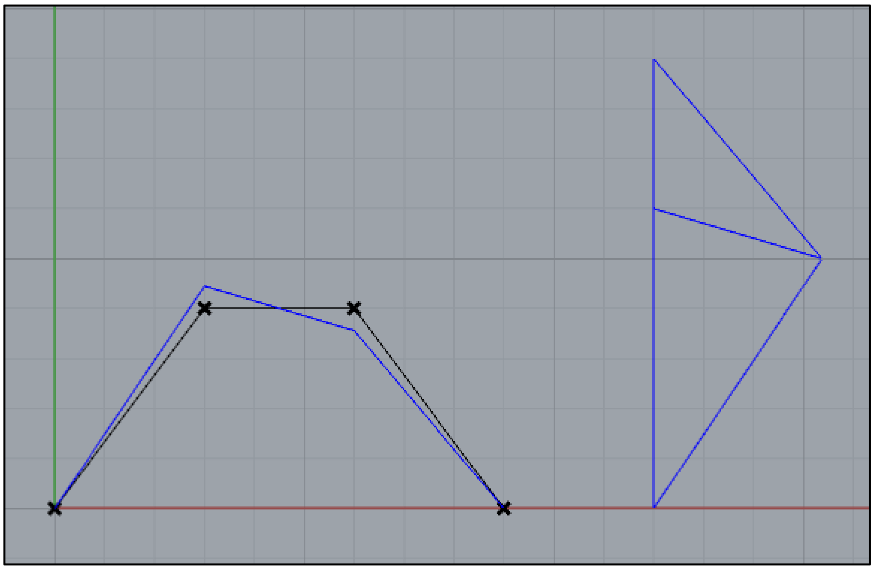


Figure 23. Perfect line of thrust for example 2 (own work).

This is because this shape always gives the least amount of eccentricity in this example, and therefore creating the least amount of bending moments, thus giving the least amount of surfaces underneath the squared bending moment as well:

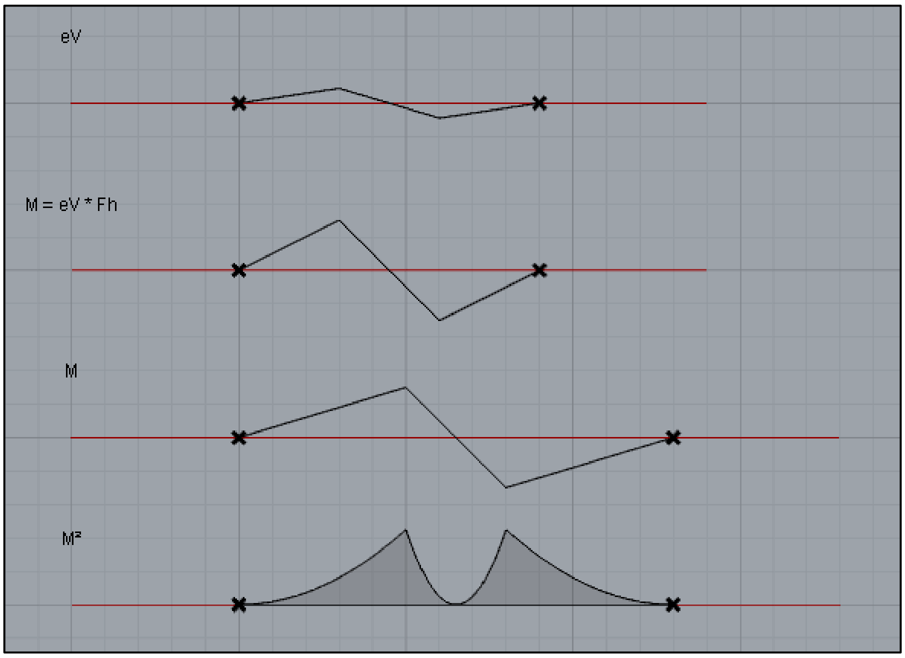


Figure 24. Eccentricity and bending moments for example 2 (own work).

As the thickness of the arc increases the complementary energy caused by this bending moment decreases (hence the 12M/t –part in the formula presented by van Dijk and Borgart (2014)). However, the shape of the course will be the same, but lower. Therefore the Fh related to the minimal Ec,m will be the same in each situation.

The reason that makes the Ec,tot differ 'so much' from the result of GSE is because as the plot for the Ec,m becomes smaller (having the same shape as for any other thickness) the slope changes as well. Therefore the slope of the linear Ec,n becomes of greater part of the Ec,tot, thus changing the result.

However, as both the results from GSE and the Ec,tot result from the definition float way from the Ec,m in opposite directions, it might give a better solution to discard the Ec,n in future results. This should not give any trouble for the design, as a well-designed arc should cope with as much normal forces as possible and the least amount of bending moments. This also means that the arc can be designed much slimmer, which is something good as it decreases the quantity of material needed for a structure. Besides, a very tall arc will look rather silly:

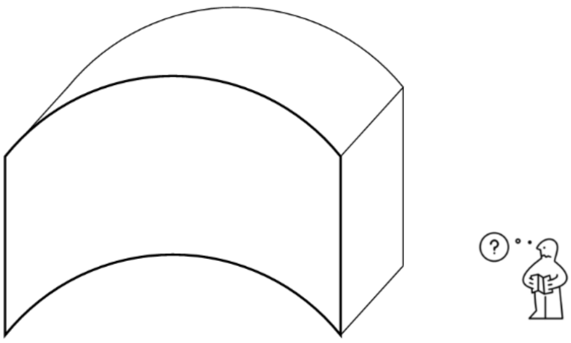


Figure 25. A rather sily arc (own work).

Calculating a shell – 3 supports

For understanding how a shell works it is important to first understand how an arc works. Trying to calculate an arc is a statically indeterminate problem, as there are multiple valuables unknown which are both needed to find the correct solution:

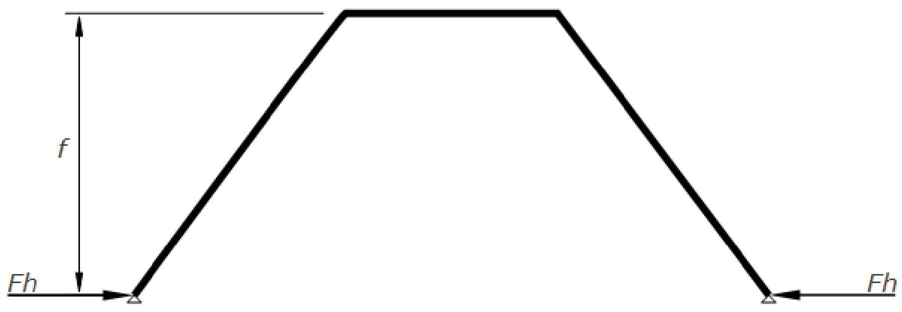


Figure 26. An arc being a statically indeterminate problem (own work).

Both the height of the line of thrust (giving the slopes of the line of thrust and therefore giving the correct horizontal support reaction as the vertical support reactions can be calculated using the energy equilibrium) and the optimal horizontal support reaction (giving the correct slopes for the force polygon and therefore giving the correct line of thrust as well) are unknown. However, they are related to each other and can both be solved by adding a third value, in the previously mentioned situation as being the total complementary energy. However, this method used cannot be used for any three dimensional shape as there are even more values unknown (the height of the plane of thrust and three (!) horizontal support reactions):

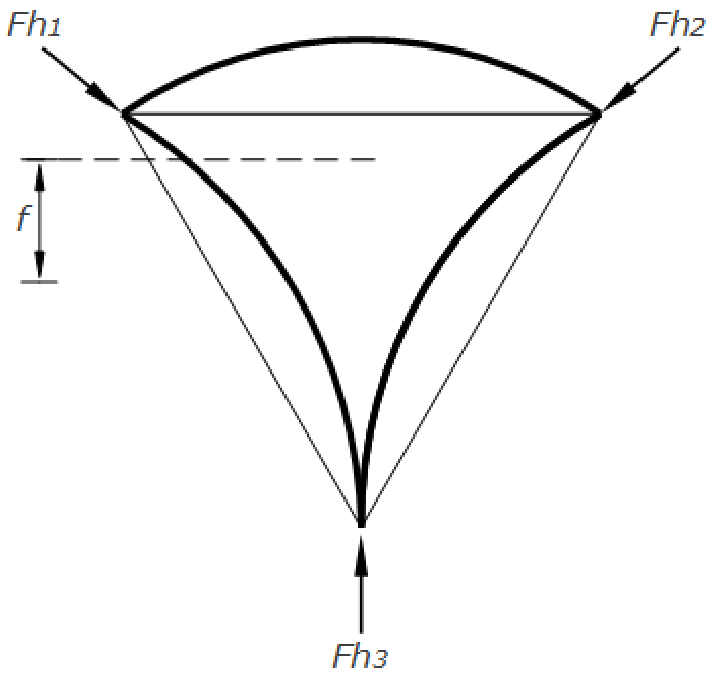


Figure 27. Four unknown values for a shell supported on 3 points (own work).

The first thing for understanding how a shell is acting when it is loaded with a (equally distributed) force is to know how a cable is acting when loaded by a force. As a cable can only cope with tensile stresses, it will change its shape to be able to withstand these forces:

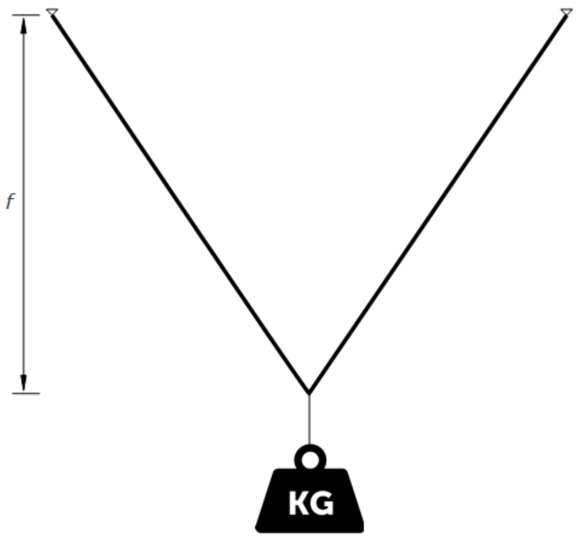


Figure 28. A cable with a weight attached will change its shape accordingly (own work).

When a weight is attached to a cable will change its shape by a vertical length f. When two weights are attached to the cable, a new shape will appear:

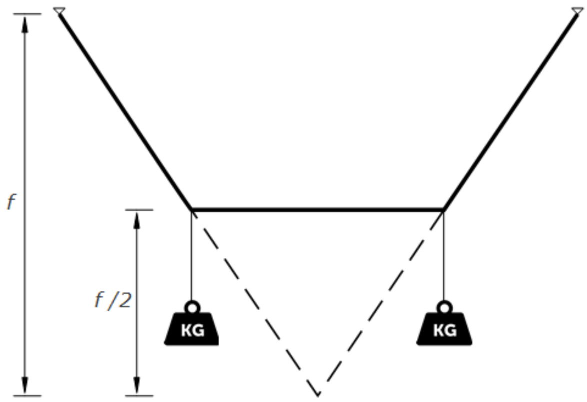


Figure 29. Two weights attached to a cable (own work).

It should be noted that when the weights are attached (being half of the weight in the previous image) the cable will reach a maximum lowest point of ½f. The process of dividing the weights in half and placing them halfway the cable parts can be done an infinite amount of times, getting closer each time to an arc.

As a regular arc is best to cope with compressive stresses and a cable with tensile stresses, the previous examples can be inverted and be used for an arc in the same way, dividing the length and the weights to create a more arc-like shape:

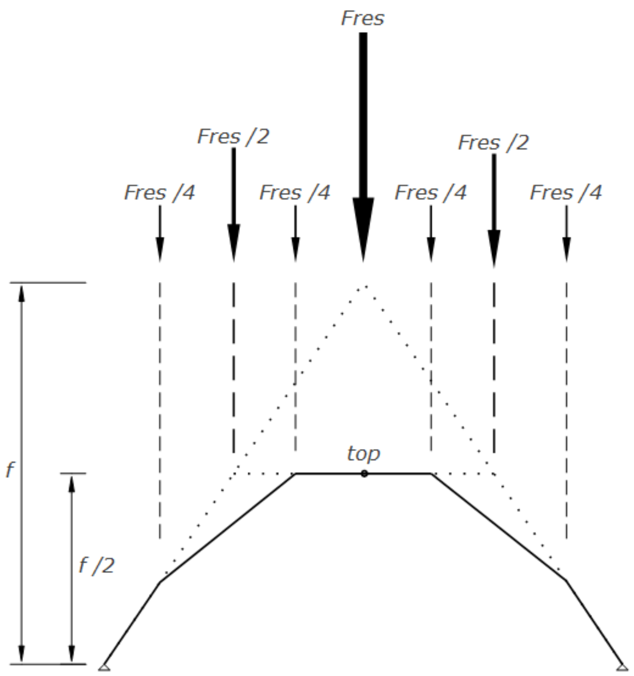


Figure 30. Dividing lengths and forces to create an arc-like shape (own work).

It should be noted that the top of the arc never gets any higher than the height after the first dividing, respectively half of the first force (the resultant of the equally distributed load). This indicates that the top of the arc is in line with the resultant force, being halfway of the distance that the force would be changing the shape if it was a cable.

The theory

Usually a shell structure is calculated under an equally distributed load. According to the theory of a weight attached to a cable it is important that this load is transformed to a resultant force at a specific location on the shell.

To apply this two dimensional theory on a three dimensional shape it is important to create a projection of this shape on a plane perpendicular to the loads. In the most common situation gravity and own load, both in the Z-direction, will need to be projected on the XY-plane, which can be seen as the surface the structural shell will be placed upon.

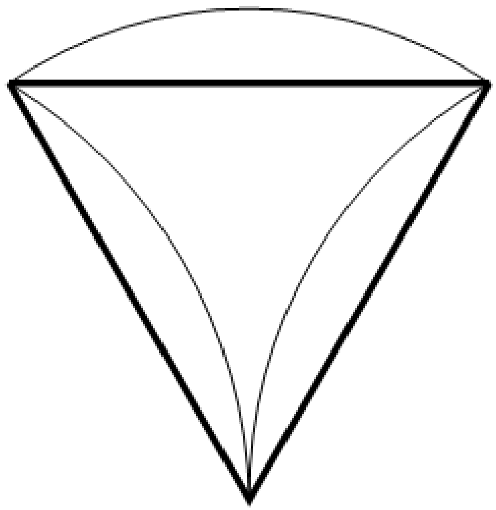


Figure 31. Projecting the three-dimensional shell (own work).

This triangular projection will have a certain surface:

$$a = \frac{w * h}{2}$$

Knowing this surface and the equally distributed load the size of the resultant force can be calculated:

$$F_{res} = a * q$$

The exact location where this resultant force will affect the shell can also be found with this projection. As it was originally an equally distributed load (a force acting on a two dimensional plane), it is safe to say that it will only appear on the projection. Whatever slope the original shell may have, it does not affect the outcome.

To find this point of engagement (also known as the centroid) it is a common method to let gravity help. Attach the projection to a point where it is free to rotate within the plane and draw a line going down from the rotational point. When this method is used for multiple spots within the plane a point should appear where these vertical lines meet, which is the centroid of the plane:

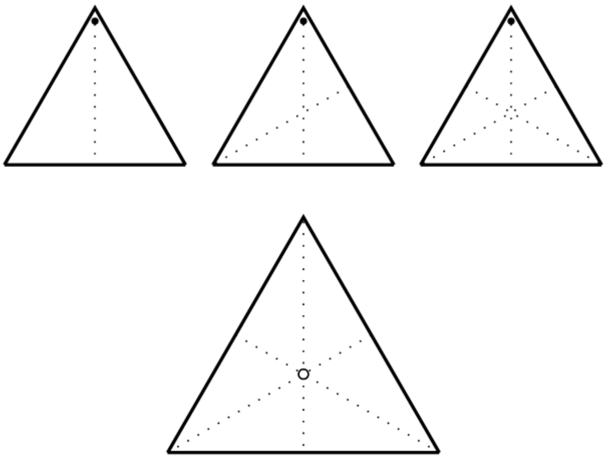


Figure 32. Finding the centroid of a plane (own work).

This method can be done much more simple by using a computer. It should also be noted that in case of finding the centroid of a triangular shape, the centroid is very likely to have a certain ratio within this line (Csonka, 1987):

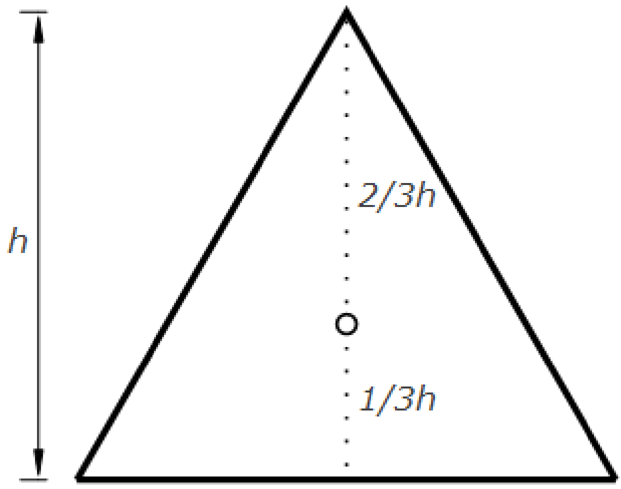


Figure 33. A ratio for the centroid within a triangular plane (own work).

With the size and point of engagement of the resultant force known, it is (just like with the two dimensional arc) possible to calculate the vertical support reactions by using the equilibrium of energy:

$$\sum F = 0$$

For a three dimensional shell this would be:

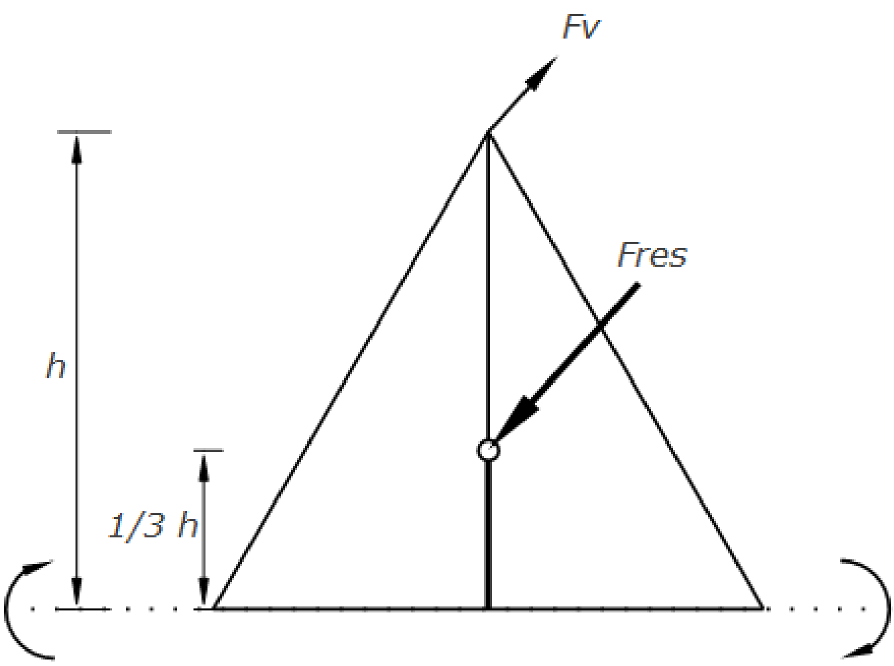


Figure 34. Calculating the vertical support reaction for a three-dimensional shell (own work).

In which the rotation (bending moment) will be calculated over a line connecting two different support points.

$$M = (F_{res} * \frac{1}{3}h) - (F_v * h)$$

Which leads to:

$$F_v = -\left(\frac{1}{3} * F_{res}\right)$$

Which matches the rule of thirds by Csonka (1987).

With the resultant force and the vertical support reactions known all which is needed to make the step towards the horizontal support reactions is the angle of the plane of thrust in the supports. As the forces will be transferred to the supports through this plane of thrust, a simple geometric equation will be the last formula needed towards the wanted values.

To find the plane of thrust the theory of the cable must be used. The height (the value for 'f' in the cable theory) will be used as the main variable in this situation, as the other three unknown values (the horizontal support reactions) will be directly dependant on this height. With the vertical component already calculable, the correct plane of thrust is the one thing that makes everything else calculable.

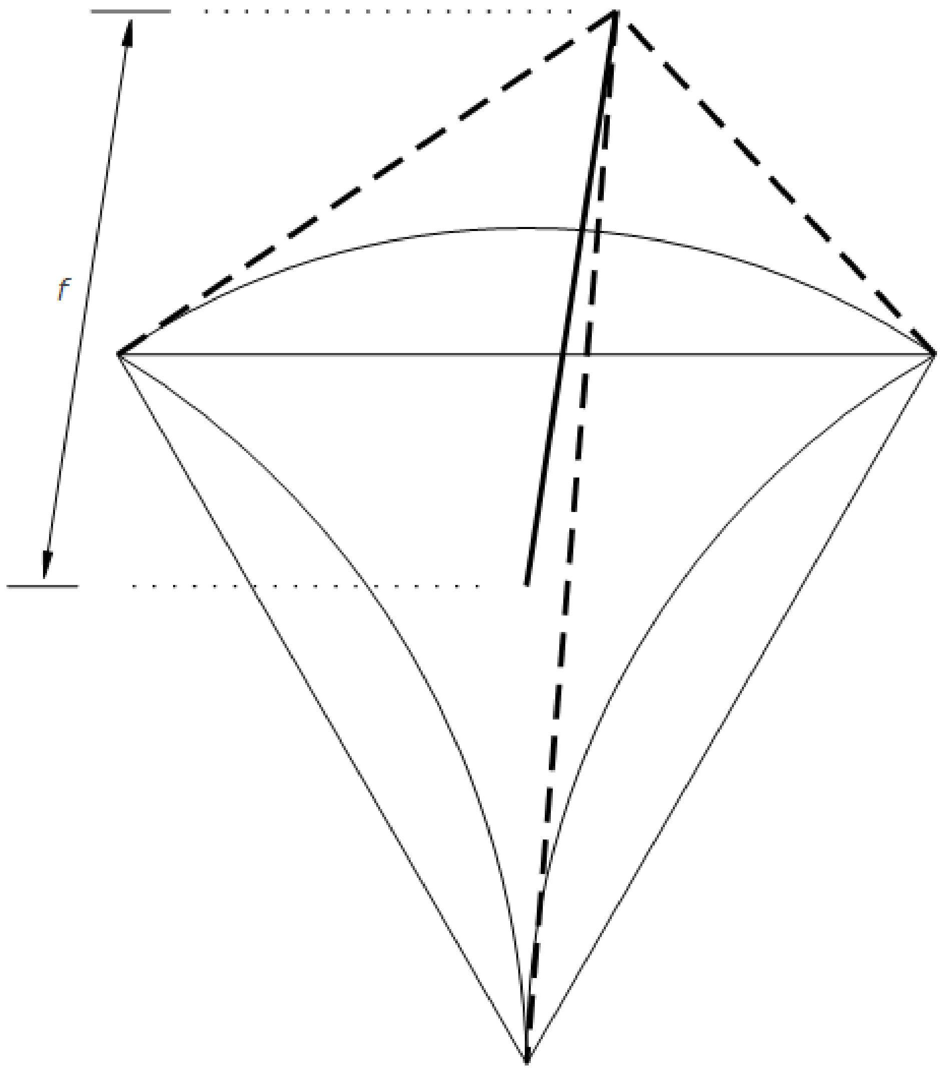


Figure 35. The weighted cable theory implied upon a triangular shell (own work).

Just like with the cable theory this three-dimensional triangle should be refined to smooth the pyramid to become a perfect shell. This shell will be the plane of thrust in which the correct angle can be retrieved for transforming the vertical support component into the horizontal component. In the book written by Csonka (1987) lies one of the main clues for coping with this refinement:

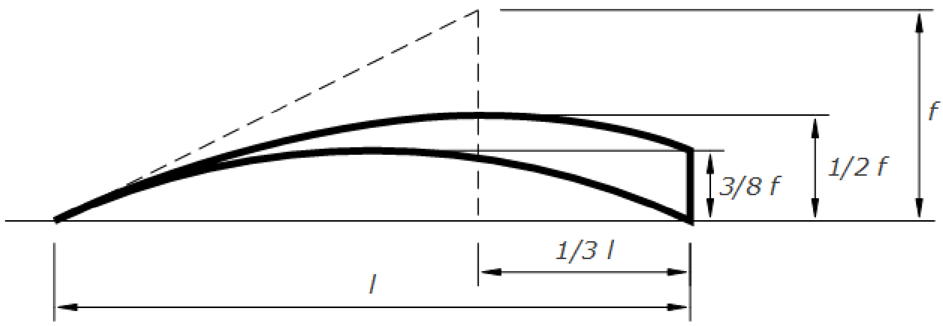


Figure 36. Ratios within a perfect shell on three supports (own work, based on Csonka (1987)).

It can be seen that the top of the shell will have a height of half of the f-value. The tops of the arcs determining the shape of the shell will be 3/4th of the top of the shell, therefore being 3/8th of the f-value. It should also be noted that the support of the shell has the same slope as the line connecting the support to the f-point, floating above the centroid of the projection.

From the top view of the shell some other major clues can be found:

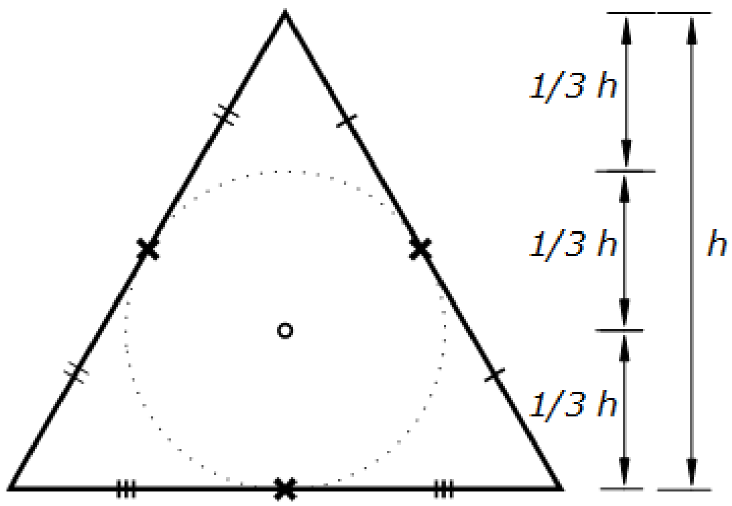


Figure 37. Top view: several important findings (own work, based on Csonka (1987)).

The centre of the edges of the shell are the highest points of these arcs, being 3/4th the height of the top of the plane of thrust. Given that the plane is a perfect shell for transporting loads to the supports, these centre points on the edges can be connected with a spherical shape (in this case a perfect circle, as the example is an equilateral triangle) as each point with a specified planar distance from the top should have the same vertical difference. It can be found that this circle also cuts the 2/3rd-part from the support towards the top of the shell (Csonka, 1987) is cut in half by this circle.

This finding seems to conclude that a new ratio appears. As a planar distance is cut in half, the vertical component seems to decrease to 3/4th of the height of the top point.

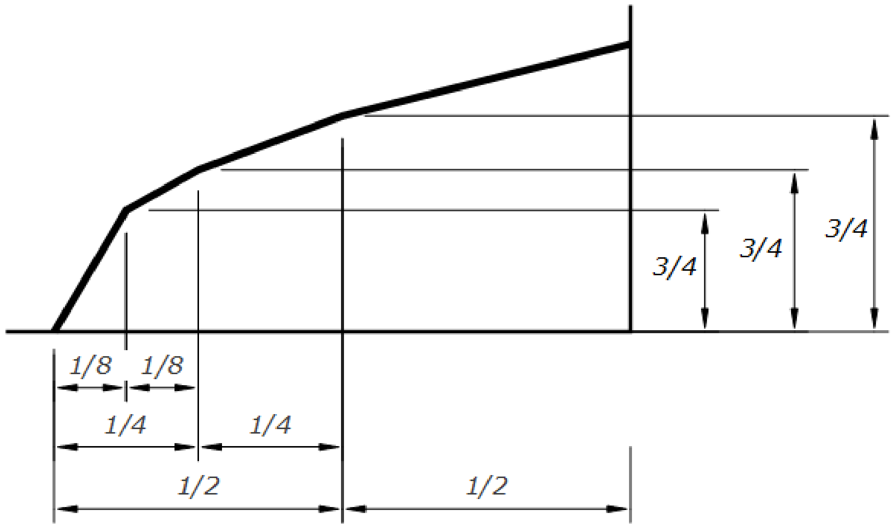


Figure 38. Creating a perfect shell by using a fixed ratio (own work).

By repeating this process an infinite amount of times, a perfect shell should appear. This is a process that can very simply be inserted into a computer, changing the slopes and the shape of the plane of thrust for each f-value that the user likes to enter.

The last thing that needs to be done is to find the optimal plane of thrust for a given shell. Earlier in this report it has been concluded that most likely it is safe to discard the results from the normal forces. Therefore we can apply the reduced sum of the moments (Beranek, 1974):

$$\bar{M} = \frac{m_{xx} + m_{yy}}{1 + \nu}$$

Giving

$$q_n = \frac{\partial}{\partial_n} * \left(\frac{m_{xx} + m_{yy}}{1 + \nu}\right)$$

When this is interpreted in a graphical manner, the correct plane of thrust will be the plane (dependant on the f-value) with the volume closest to the volume of the original shell (Beranek, 1974).

The Grasshopper definition

To start the definition for Grasshopper, a shell-structure is needed. For now, this should be inserted as a three dimensional surface. For this example a not-so-smooth shell has been used, as it will later in this explanation become clearer what the role of the plane of thrust is.

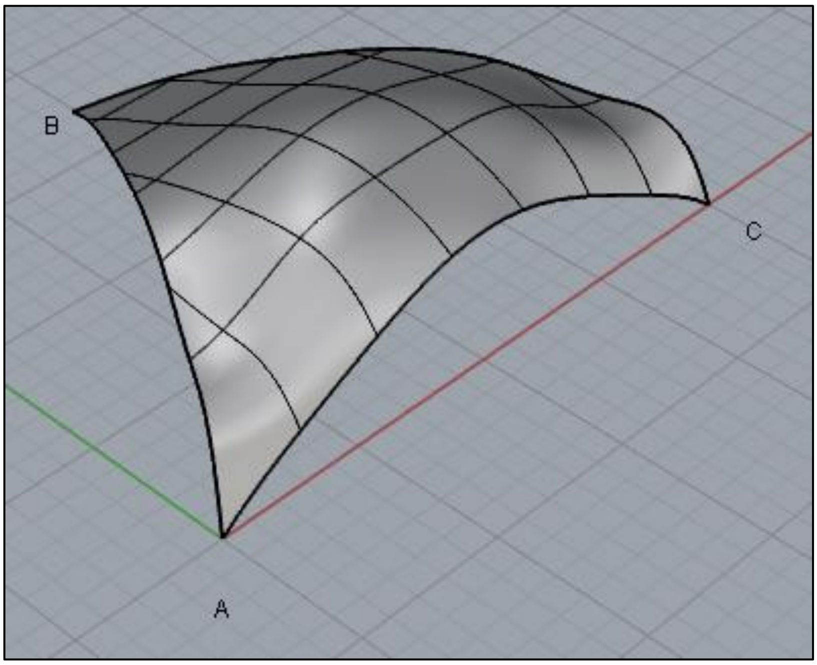


Figure 39. Insert a shell into the definition (own work).

The definition is able to extract the edges and supports from this ‘shell’. The edges are needed for the projection and the supports are needed for the energy equilibrium and the plane of thrust.

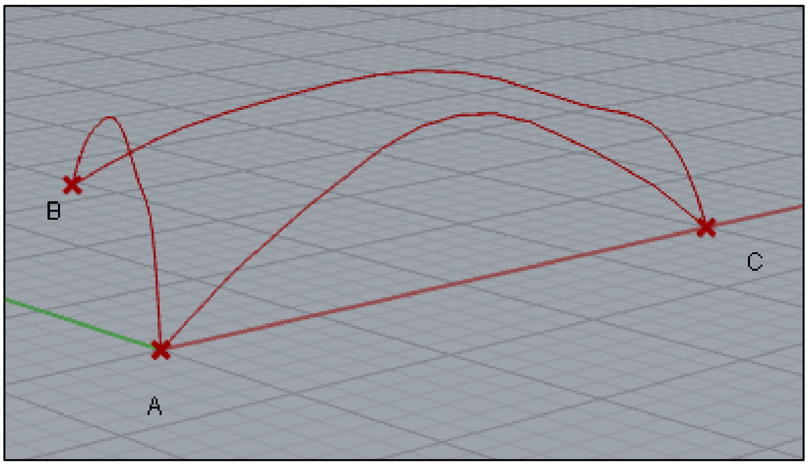


Figure 40. Extracted edges and supports (own work).

These edges determine the projection, of which the definition will calculate the centroid. From this centroid the variable f-value will rise, perpendicular to the projection plane.

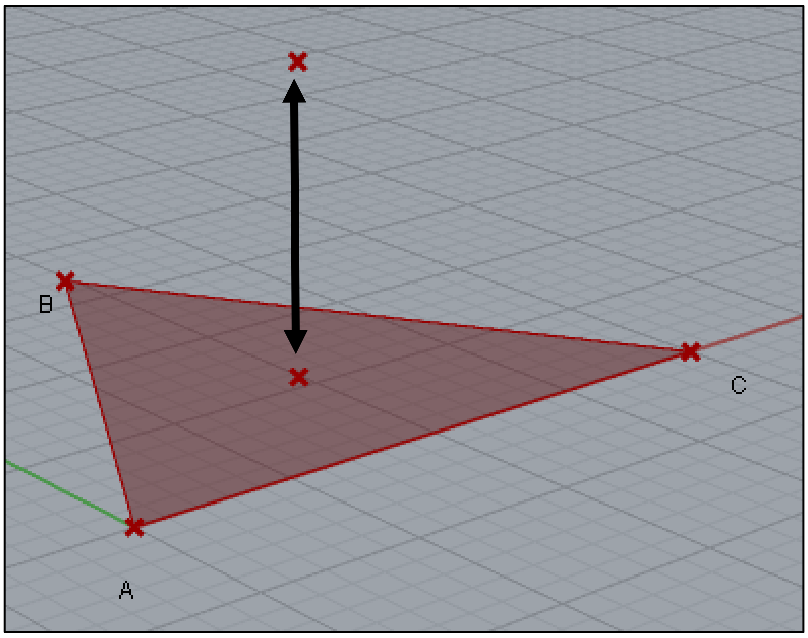


Figure 41. The projection with its centroid giving two coordinates for the f-value (own work).

The chosen f-value will be cut in half, giving the top of the plane of thrust. With the distance of the top of the shell now determined, the definition is able to create circles of $3/4^{th}$ the height and half the diameter (according to the theory presented following the findings of Csonka (1987)), creating an 'igloo-like' appearance.

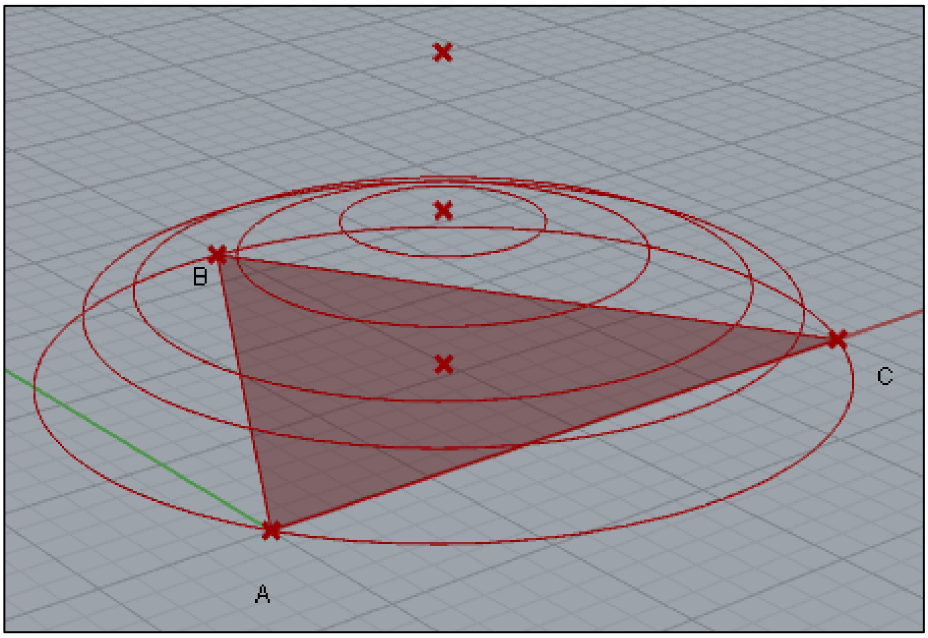


Figure 42. Circles based on ratios create an igloo (own work).

The circles forming this igloo can be used as the base for a surface, thus creating the plane of thrust for a specific value of f.

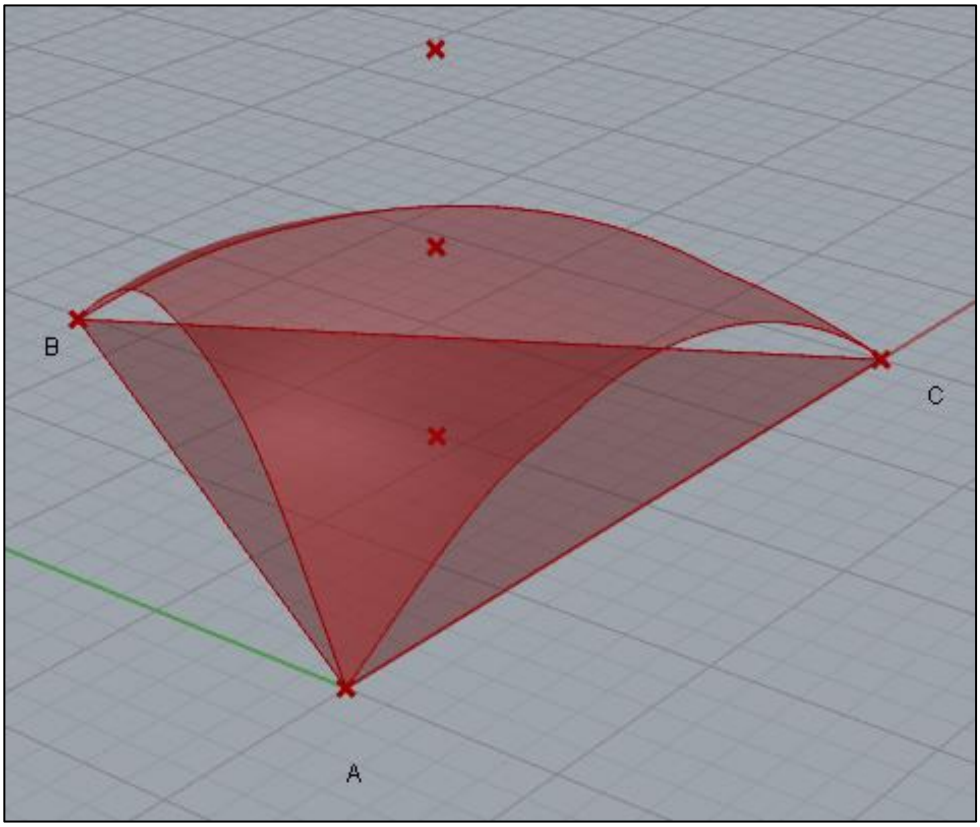


Figure 43. A shell is constructed from the igloo-circles (own work).

As a final control it needs to be checked if the slopes at the supports of the plane of thrust match with the point related to the chosen f-value.

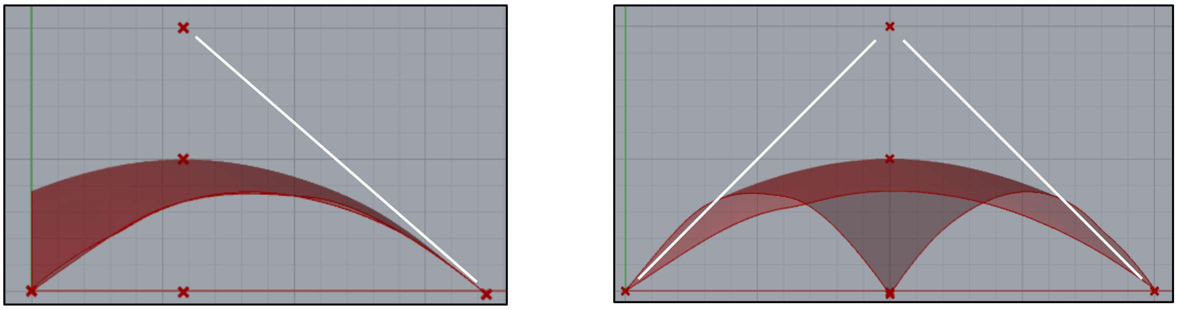


Figure 44. Side view and front view showing that the slopes match the f-value (own work).

Now that the correct definition for creating the plane of thrust has been written, it is important to be able to compare the plane of thrust to the original shell.

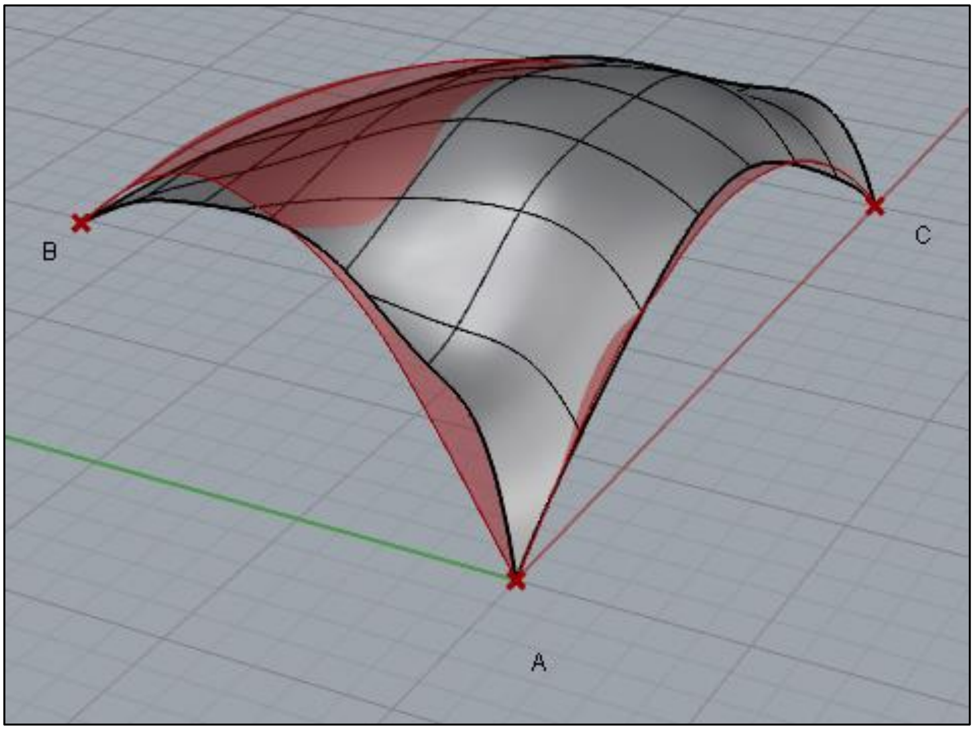


Figure 45. Comparing the shell to the plane of thrust (own work).

By sliding with the value for f, the closest value should be found for the volumes of the plane of thrust and the original shell.

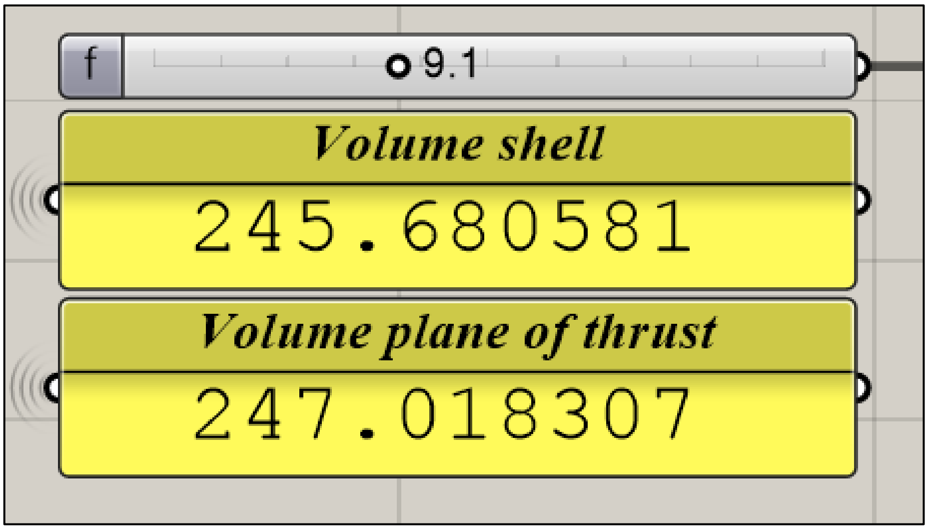


Figure 46. Finding the smallest difference in volume by changing the f-value (own work).

When the most optimal plane of thrust is found, the slopes at the supports (leading all the way up to the point determined by the f-value) can be retrieved.

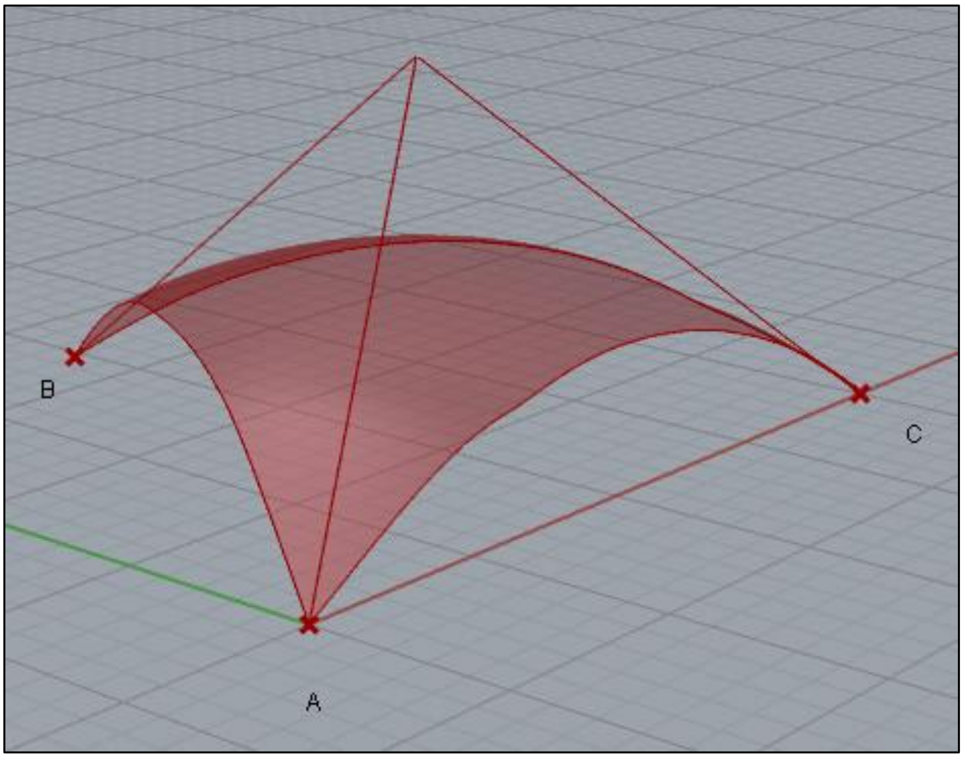


Figure 47. Finding the slopes at the support of the plane of thrust (own work).

With the vertical components and the angles of the supports known the definition is capable of calculating the resultant- and horizontal forces in the supports by using a simple geometric formula.

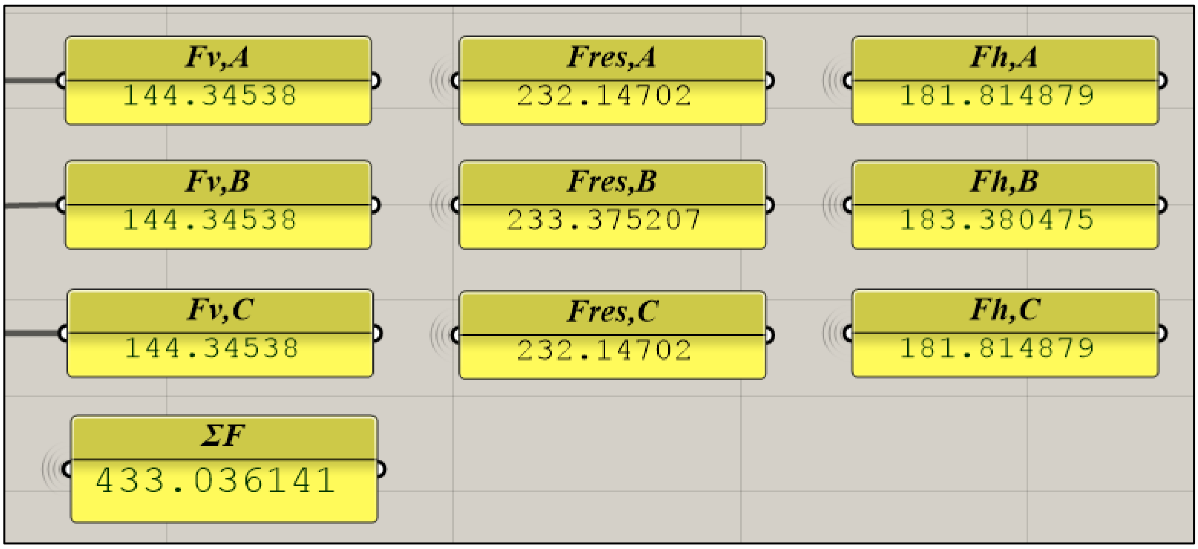


Figure 48. All the support reactions are calculated (own work).

Validating the results

The results that the definition returns need to be validated to be able to see whether these are accurate enough to validate this new method. However, when trying to get results from DIANA several errors occur:

```
GEOMETRY: NR= 1
SEVERITY : ABORT
ERROR CODE: /DIANA/LB/DS30/2236
ERRORMSG.A: Element is not of planar shape.
CURRENT element 3
MATERIAL: 1
GEOMETRY: 1
DATA : 1
TYPE : CQ24P
DIANA-JOB ABORTED
```

Figure 49. DIANA error (own work).

Apparently the shell that has been made in Rhinoceros 5.0 for the explanation of the definition is not 'meshable' into planes. DIANA also gives a notification of which element this would be:

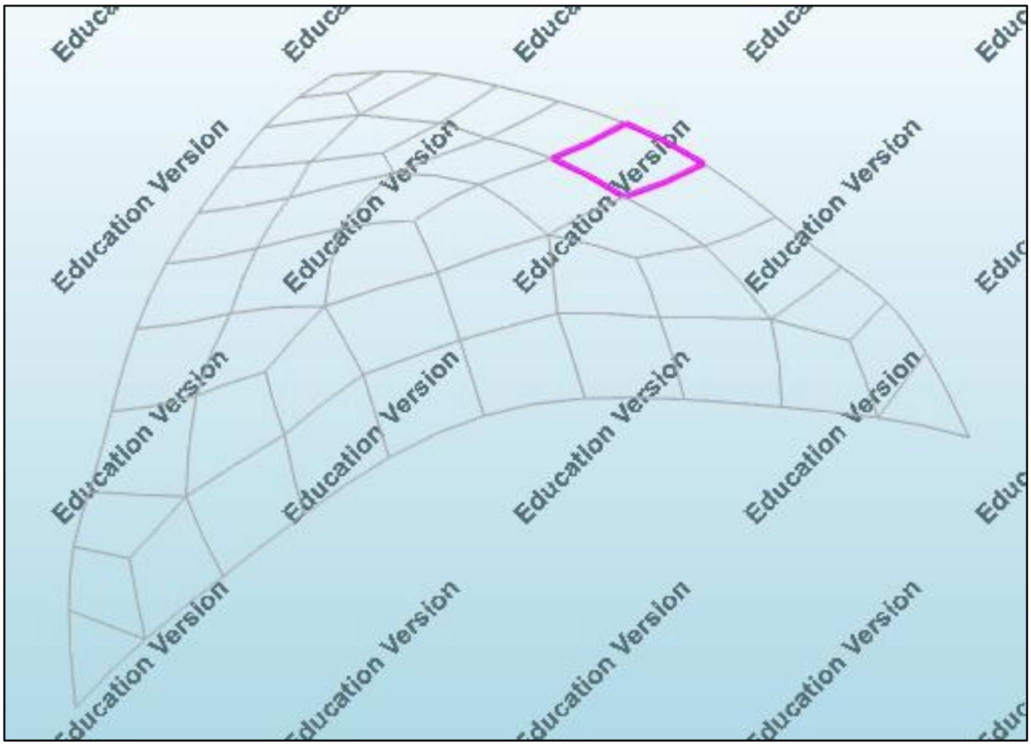


Figure 50. Element 3 being not planar (own work).

In an attempt to get some valid results from the definition, a new shell has been made. In an attempt to avoid getting planar issues yet again, the shell has been smoothed with care. However, the same error message appears:

```
GEOMETRY: NR= 1
SEVERITY : ABORT
ERROR CODE: /DIANA/LB/DS30/2236
ERRORMSG.A: Element is not of planar shape.
CURRENT element 383
MATERIAL: 1
GEOMETRY: 1
DATA : 1
TYPE : CQ24P
DIANA-JOB ABORTED
```

Figure 51. DIANA error (own work).

Yet again one of the planes is not actually a plane.

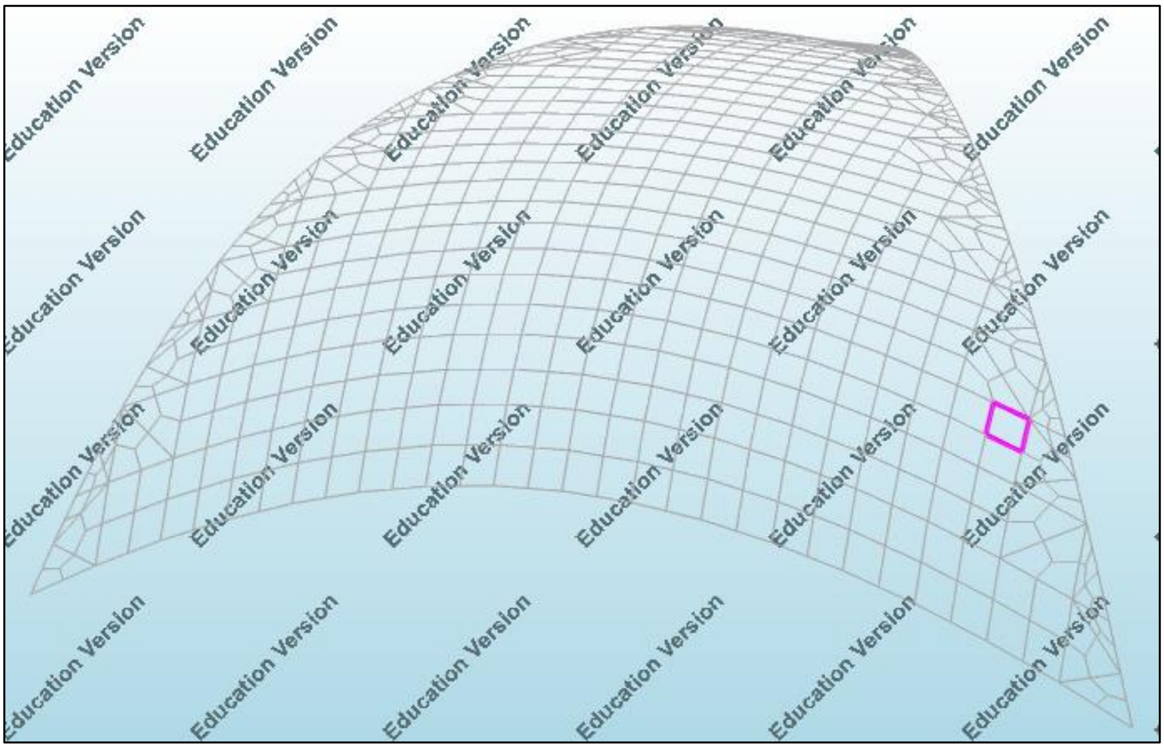


Figure 52. Element 383 not being planar (own work).

These errors are a huge disappointment for the validation of the definition, as this report is not able to prove whether the new method is accurate enough.

The only thing that can be done at this point is to validate the shell with a formula similar to the one presented by Beranek in 1974:

$$z(x,y) = h - \left(\frac{h_o}{4a_0^2}(x^2 + y^2) \right)$$

In which:

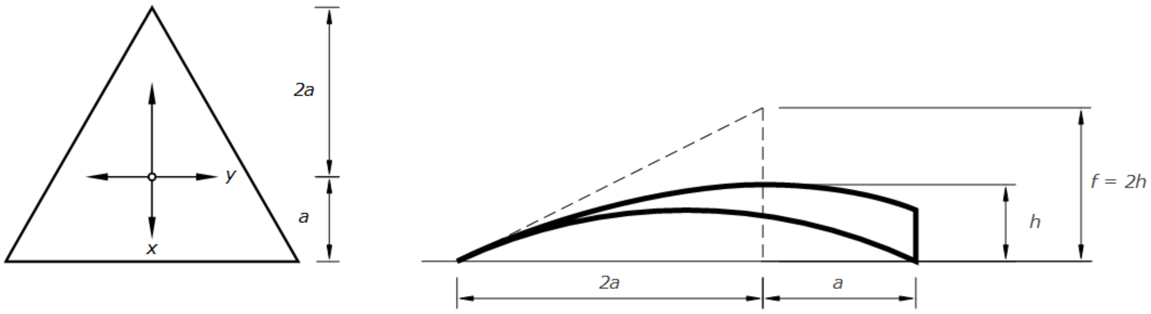


Figure 53. Visual explanation of Beranek's formula (1974) (own work).

With

h = 5,50 m
a ≈ 5,77 m

Which returns:

x

y

Formula	**0**	**1**	**2**	**3**	**4**	**5**
0	5,5	5,46	5,33	5,13	4,84	4,47
1	5,46	5,42	5,29	5,09	4,80	4,43
2	5,33	5,29	5,17	4,96	4,67	4,30
3	5,13	5,09	4,96	4,76	4,47	4,10
4	4,84	4,80	4,67	4,47	4,18	3,81
5	4,47	4,43	4,30	4,10	3,81	3,43

Definition	**0**	**1**	**2**	**3**	**4**	**5**
0	5,5	5,46	5,33	5,12	4,84	4,49
1	5,46	5,41	5,29	5,08	4,80	4,44
2	5,33	5,29	5,15	4,94	4,65	4,28
3	5,12	5,08	4,94	4,72	4,42	4,10
4	4,84	4,80	4,65	4,42	4,12	3,84
5	4,49	4,44	4,28	4,03	3,84	3,51

It can be seen that the results are very similar to each other, having a maximum deviation of 2,3% at the point with x=5, y=5 .

Also, the results of the formula presented by Beranek (1974) seems to validate the theory of the ellipses connecting points at different heights as well, as for example point x=4,y=1 returns the same value as x=1,y=4 for both the formula as well as the definition.

Room for improvement

When reviewing the points of improvement from the previous report (Ros, 2016), several have been implemented into a new definition. It is now possible to calculate an arc with different heights of the two support points. Also the definition has become appreciable faster compared to the definition presented in the previous report (Ros, 2016) by simply removing the possibility of adjusting the amount of segments. Also, all results have been validated with Oasys GSA Suite.

However, it is still not possible to cope with an equally distributed load, even though this would be very simple to add, as the method to cope with a resultant force is already implemented.

For the shell on 3 supports the results however are a bit disappointing. Unfortunately the results from the new method (that however seems very feasible compared to Beranek's findings (1974)) could not be validated by Finite Elements Method as DIANA gave errors that could not be solved without some expert help.

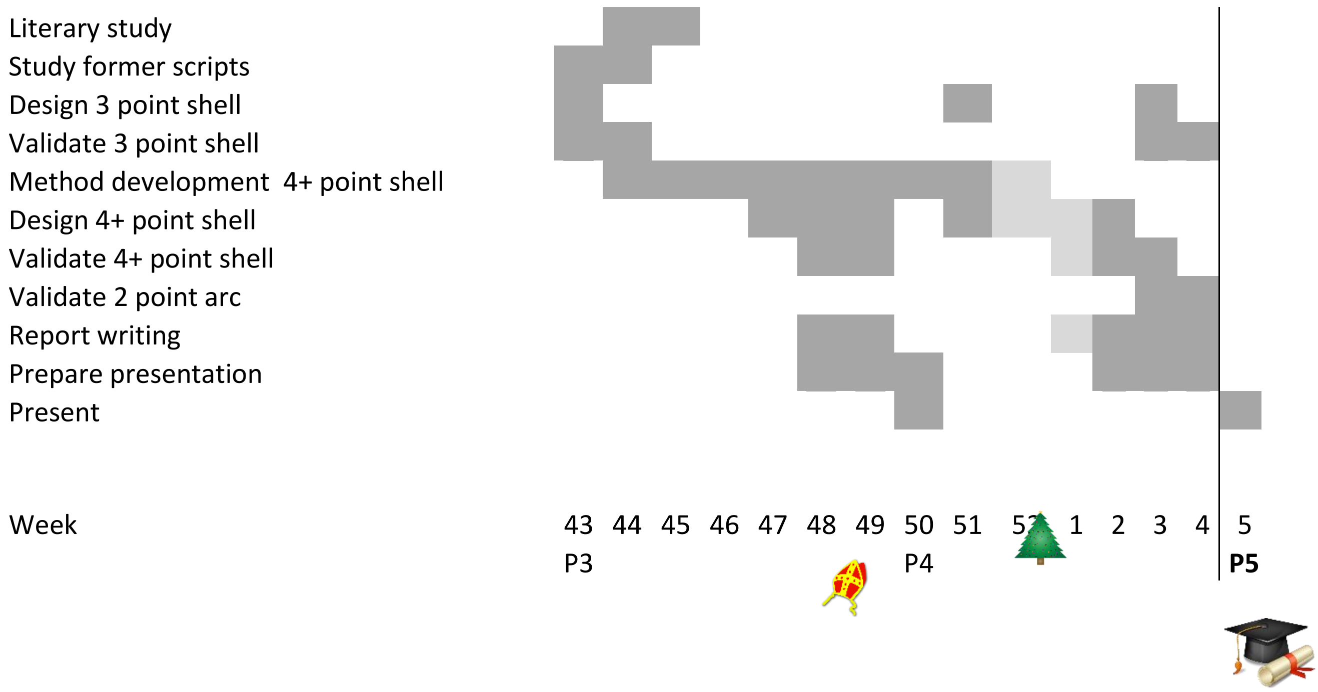
When receiving some useful tips for using the DIANA software (Eigenraam, 2016) a shell was used that was created out of two meshes. The explanation that was received worked for this specific input, solving an earlier error within the geometry.

It was later found that the volume Grasshopper finds for a mesh consisting of two separate meshes is different compared to the same shape being just one mesh. Therefore a new shell was created, this time being constructed out of a surface, adjusted by moving several control points. However, this surface gave trouble as well with the DIANA-software, as some of the panels that were created by DIANA were not planar, the software aborted all actions from that point on, on that particular shell.

For the P4 report and presentation it is of great importance that understanding of DIANA needs to be gathered, as validating a new method to an existing one is one of the key elements for presenting a new theory. Also, more irregular shells on three supports will be examined as well.

Also, for the P4 report it would be the following step to also present a method for finding the correct support reactions of a shell with more than three support points.

Plan for the remaining weeks



References

Beranek, W. (1974). *Vlakke Constructiedelen Elasticiteitstheorie - Basisvergelijkingen, Schijven, Platen en Wapenen.* Technische Hogeschool Delft, Delft.

Csonka, P. (1987). *Theory and Practice of Membrane Shells*. Budapest: Akademiai Kiado.

Eigenraam, P. (2016). [DIANA explanation].

Ros, J. (2016). *Graphically calculating arcs and shells by using the lowest complementary energy - P2 Report.* TU Delft, Delft.

van Dijk, N., & Borgart, A. (2014). *Graphic Statics in Arches and Curved Beams - Finding Force Equilibrium through Total Minimum Complementary Energy.* TU Delft, Delft.

Figure references

Appendix 1

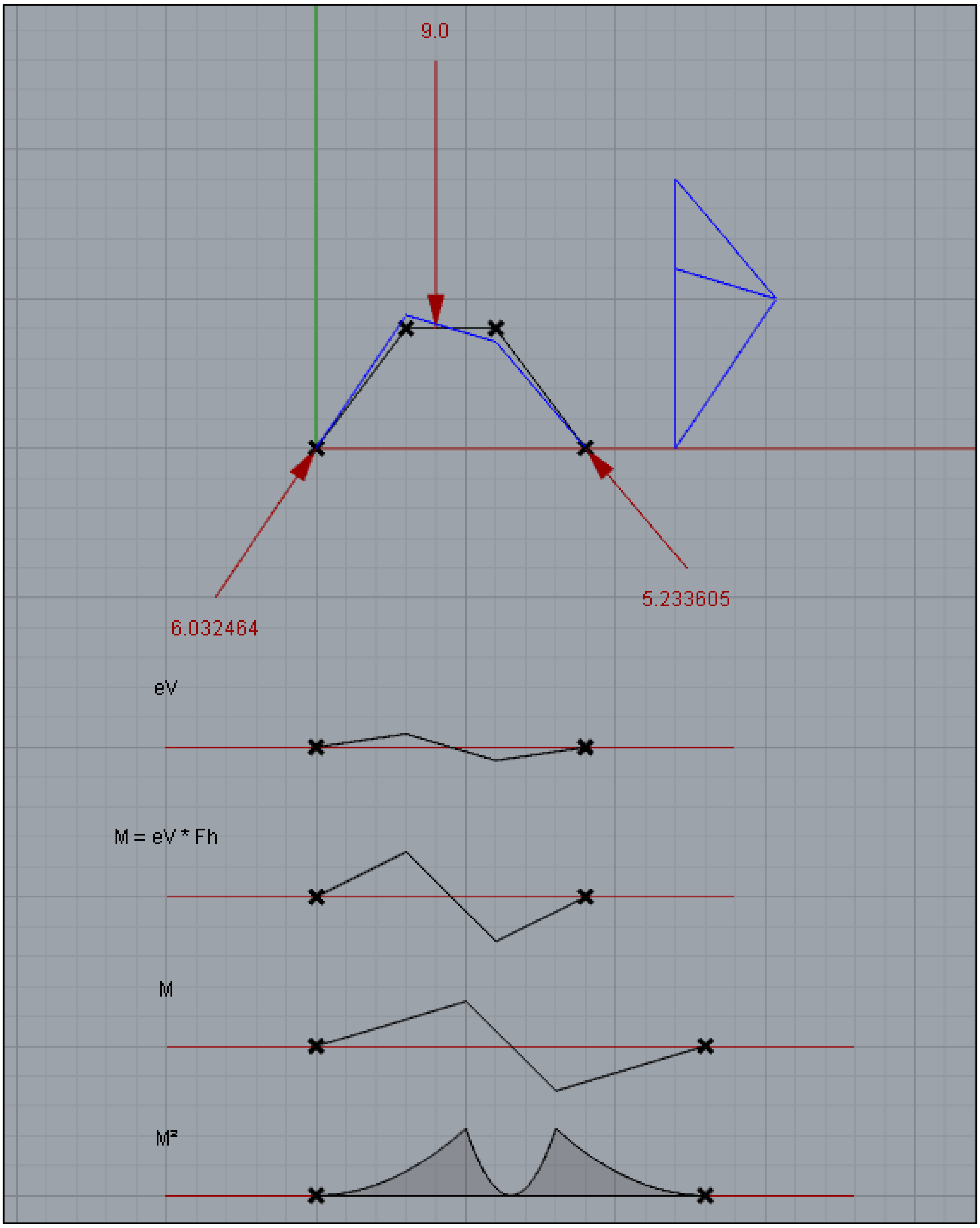


Figure 54. Example 2 - Grasshopper model (own work).

Appendix 2

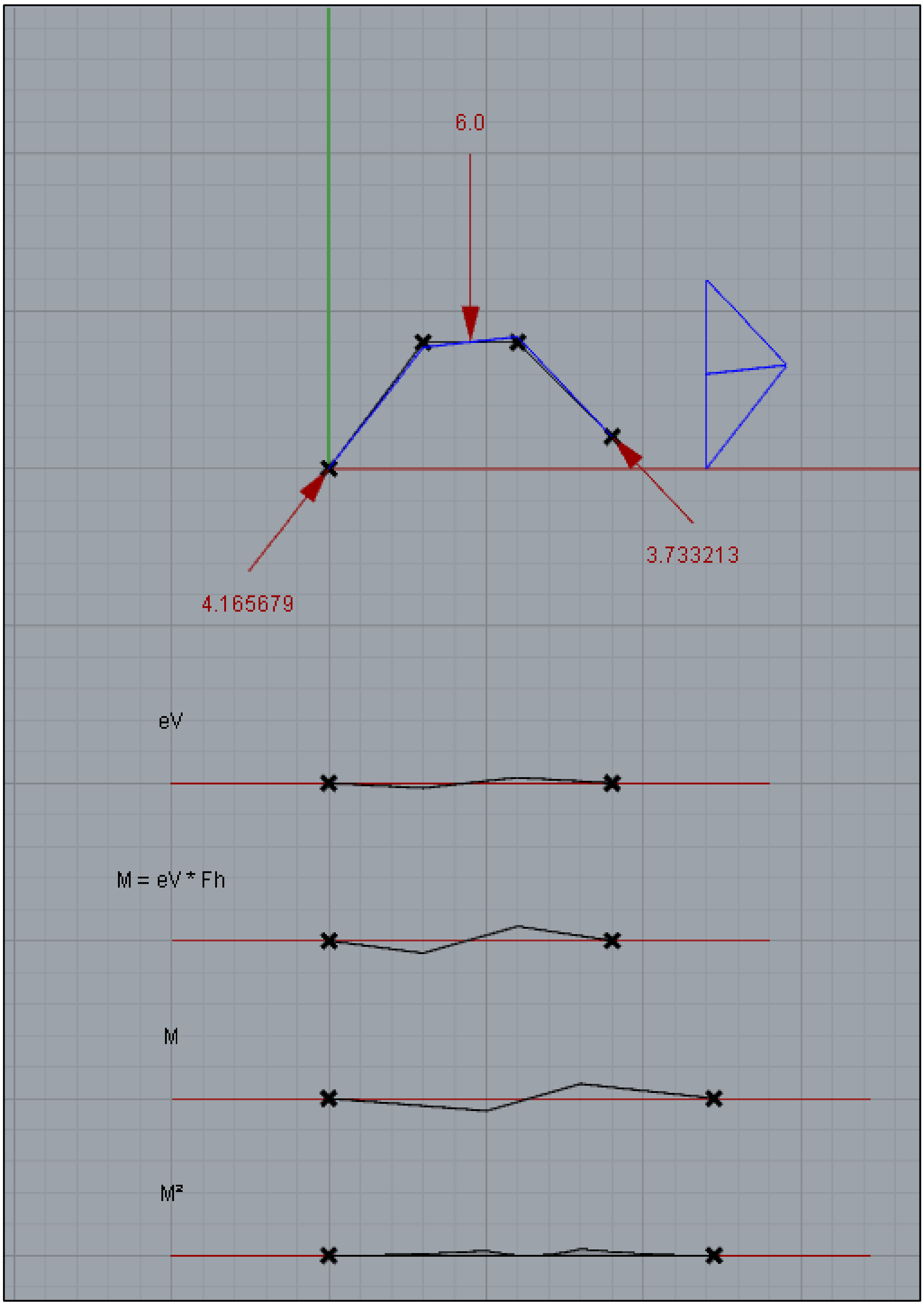


Figure 55. Example 3 - Grasshopper model (own work).

Appendix 3

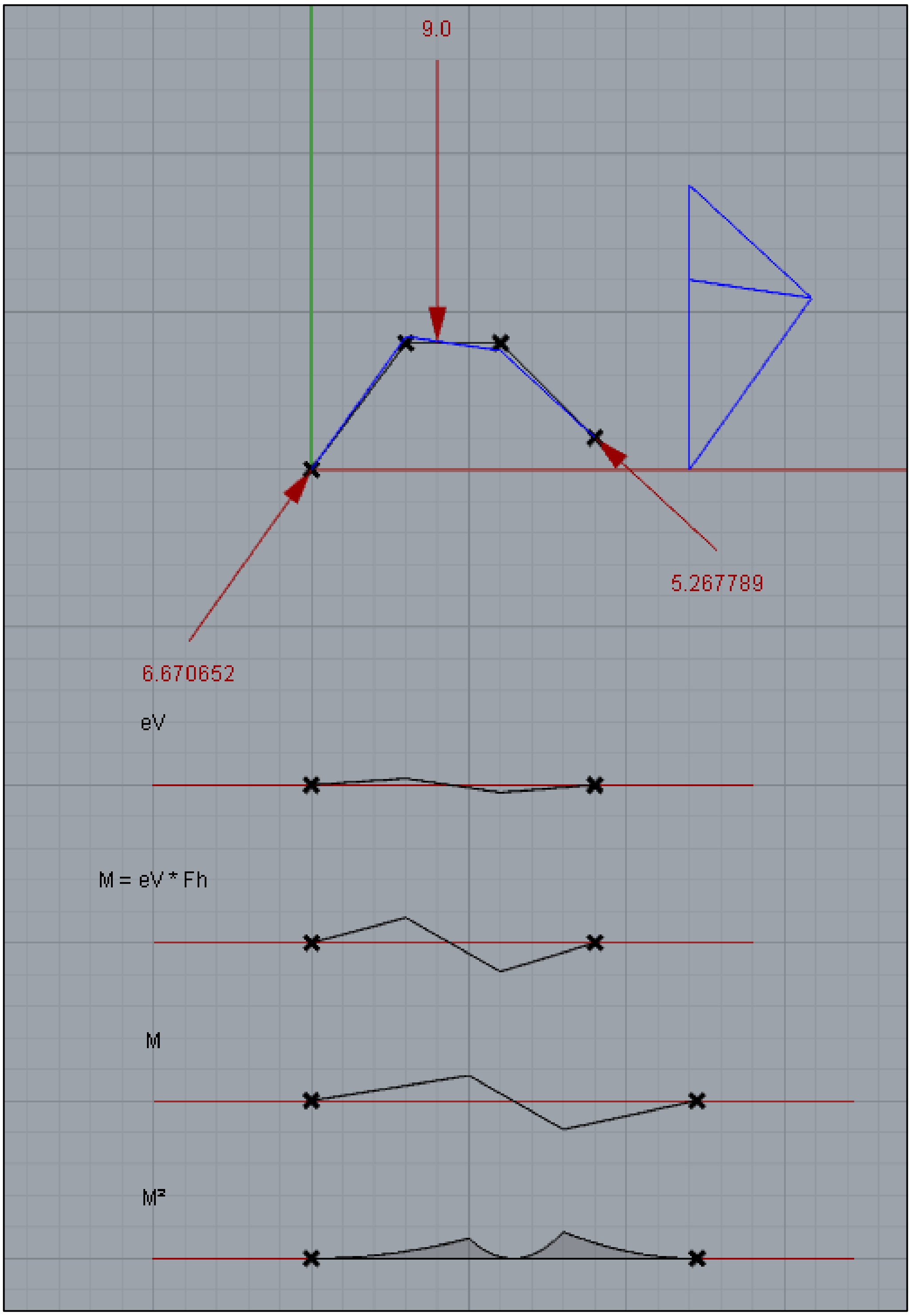


Figure 56. Example 4 - Grasshopper model (own work).