Gravity Field Modeling on the Basis of GRACE Range-Rate Combinations

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Abstract. A new functional model is proposed for gravity field modeling on the basis of KBR data from the GRACE satellite mission. This functional model explicitly connects a linear combination of gravitational potential gradients with a linear combination of range-rate measurements at several successive epochs. The system of observation equations is solved in the least-squares sense by means of the pre-conditioned conjugate gradient method. Noise in range-rate combinations is strongly dependent on frequency, so that a proper frequency-dependent data weighting is a must. The new approach allows a high numerical efficiency to be reached. Both simulated and real GRACE data have been considered. In particular, we found that the resulting gravity field model is rather sensitive to errors in the satellite orbits. A preliminary gravity field model we obtained from a 101 day set of GRACE data has a comparable accuracy with the GGM01S model derived by CSR.

Keywords. GRACE, range-rate combinations, frequency-dependent weighting, Earth's gravity field

1 Introduction

The GRACE (Gravity Recovery And Climate Experiment) satellite mission was launched in March 2002 mainly for the purpose of high-precision mapping of the Earth's gravity field (Tapley et al., 2004). The mission consists of two satellites co-orbiting at about 480 km altitide with a 220 ± 50 km along-track seperation. The satellites are equipped with a K-band ranging (KBR) measurement system, thanks to which the inter-satellite range-rates can be continuously determined with an accuracy of better than 0.5 μm/s (Biancale et al., 2005). Other important on-board sensors are: (i) GPS receivers needed to determine the satellite orbits and to synchronize time tags of KBR measurements of the two satellites, (ii) accelerometers, which measure non-gravitational satellite accelerations, and (iii) star cameras needed to determine the satellite attitudes. A number of functional models for processing GRACE KBR data have been already proposed and applied, e.g. variational equation approach (Tapley et al., 2004; Reigber et al., 2005), energy balance approach (Jekeli, 1999; Han et al., 2005b), acceleration approach (Rummel, 1979), the approach based on integration of short arcs (Ilk et al., 2003) and the gradiometry approach (Keller and Sharifi, 2005). In the paper, we propose a new approach for gravity field modeling, which is based on so-called range-rate combination. The structure of the paper is as follows. In Sect. 2, we present the theoretical basis of the proposed approach. In Sect. 3, we tackle some implementation issues. To verify the approach, one-month GRACE data set is simulated and processed (Sect. 4). Next, we process 101-day set of real GRACE data (Sect. 5). Finally, conclusions are given and the future outlook is discussed.

2 Functional Model

The functional model makes use of a local frame at each particular epoch (see Figure 1(a)). In the frame, the X-axis is defined as the Line-Of-Sight; the Z-axis is orthogonal to the X-axis in the plane formed by two satellites and the center of the Earth (i.e. this axis is approximately radial) and the Y-axis is orthogonal to the X- and Z-axes forming a right-hand frame (i.e. the Y-axis is cross-track). In order to build up one observation equation, three succesive epochs are considered (say, $i-1$, i and $i+1$). Let us introduce inter-satellite average accelerations between the epochs $i-1$ and i (i.e. $\bar{\mathbf{g}}_{i-}$) and between the epochs i and $i+1$ (i.e. $\bar{\mathbf{g}}_{i+}$) as:

$$\bar{\mathbf{g}}_{i-} := \frac{\int_{-\Delta t}^{0} \mathbf{g}(t_i + s)\,\mathrm{d}s}{\Delta t}, \bar{\mathbf{g}}_{i+} := \frac{\int_{0}^{\Delta t} \mathbf{g}(t_i + s)\,\mathrm{d}s}{\Delta t},$$

where $\mathbf{g}(t)$ is point-wise inter-satellite acceleration as a function of time, and Δt is sampling rate. Obviously, the following equalities hold:

$$\Delta t \cdot \bar{\mathbf{g}}_{i-} = \mathbf{v}_i - \mathbf{v}_{i-1}, \quad (1)$$

$$\Delta t \cdot \bar{\mathbf{g}}_{i+} = \mathbf{v}_{i+1} - \mathbf{v}_i, \quad (2)$$

where $\mathbf{v}_{i-1}$, $\mathbf{v}_i$ and $\mathbf{v}_{i+1}$ are inter-satellite velocities at three successive epochs. The accelerations in the left-hand side of Equations (1) and (2) can be related to the gravitational potential gradients, while the inter-satellite velocities in the right-hand side can be related to the range-rates. As a result, a linear combination of three successive range-rates d_{i-1}, d_i, d_{i+1} can be directly related to the average inter-satellite accelerations $\bar{\mathbf{g}}_{i-}$ and $\bar{\mathbf{g}}_{i+}$ (Ditmar and Liu, 2006):

$$\begin{aligned} &\nu_i(\bar{g}^{\mathbf{z}}_{i-} + \bar{g}^{\mathbf{z}}_{i+}) - (\tau_{i-})\bar{g}^{\mathbf{x}}_{i-} - (\tau_{i+})\bar{g}^{\mathbf{x}}_{i+} \\ &= (\epsilon_{i-})d_{i-1} + \epsilon_i d_i + (\epsilon_{i+})d_{i+1}, \end{aligned} \quad (3)$$

where $\bar{g}^{\mathbf{x}}_{i\pm}$ and $\bar{g}^{\mathbf{z}}_{i\pm}$ are the X- and Z-component of the vector $\bar{\mathbf{g}}_{i\pm}$ at the epoch of i, respectively; ν_i, τ_{i-}, τ_{i+}, ε_{i-}, ε_i, and ε_{i+}, the so-called navigation parameters, are functions of the unit vectors $\mathbf{e}_{i-1}$, $\mathbf{e}_i$, and $\mathbf{e}_{i+1}$ that define the line-of-sight directions at the three successive epochs (see Figure 1(b)).

Strictly speaking, equation (3) is only valid in the 2-D case, i.e. if all 3 line-of-sight unit vectors coincide with the orbital planes of the satellites. However, real data can be reduced to the (locally) 2-D case by applying small corrections to d_{i-1}, d_i and d_{i+1}, respectively. The corrections are calculated from the Y-components of velocity differences at epochs $i-1$ and $i+1$ projected onto the X- and Z-axes of the epoch i.

As the orbit radius approaches infinity, equation (3) turns into a double-differentiation formula:

$$\frac{g^{\mathbf{x}}_{i+} - g^{\mathbf{x}}_{i-}}{\Delta t} = \frac{d_{i-1} - 2d_i + d_{i+1}}{(\Delta t)^2}. \quad (4)$$

A series of average inter-satellite accelerations $g^{\mathbf{x}}_{i\pm}$ and $g^{\mathbf{z}}_{i\pm}$ can be easily related to a set of Stokes coefficients (or to other parameters if the gravity field representation is not the spherical harmonic expansion). In particular, the algorithm to compute $g^{\mathbf{x}}_{i\pm}$ and $g^{\mathbf{z}}_{i\pm}$ from stockes coefficients can be as follows: (1) Compute the gravitational accelerations at the satellite locations; (2) Compute the inter-satellite differences ("point-wise inter-satellite accelerations"); (3) Apply the averaging filter (Ditmar and Van Eck van der Sluijs, 2004); (4) At each epoch, compute the line-of-sight component $g^{\mathbf{x}}_{i\pm}$ and the orthogonal component $g^{\mathbf{z}}_{i\pm}$.

3 Implementation

Equation (3) can be written as a matrix-to-vector multiplication: $\mathbf{Ax} = \mathbf{d}$, where $\mathbf{x}$ is the set of gravity field parameters, $\mathbf{d}$ is the set of Range-Rate Combinations (RRCs), and $\mathbf{A}$ is a design matrix. These equations have to be solved in the least squares sense. The number of unknown parameters (stokes coefficients) in the least-squares adjustment can be up to tens of thousands, and the number of data can reach tens of millions. Therefore, a tailored, numerically efficient adjustment algorithm is advisable. A reasonable choice is an algorithm based on the pre-conditioned conjugate gradient method. Such an algorithm can be split into a number of basic operations, each of which can be efficiently implemented: (1–2) Multiplication of the matrices $\mathbf{A}$ and $\mathbf{A}^T$ to a vector. These steps can be implemented as a fast synthesis and co-synthesis (Ditmar et al., 2003) combined with the application of an averaging filter (Ditmar and Van Eck van der Sluijs, 2004) (3) Multiplication of the inverse data covariance matrix $\mathbf{C}_{\mathbf{d}}^{-1}$ to a vector. This step can be implemented as a low-level conjugate-gradient algorithm (Ditmar and Van Eck van der Sluijs, 2004; Klees and Ditmar, 2004). (4) Pre-conditioning, i.e. solving the system of linear equations where the original normal matrix is replaced by its approximation ("a pre-conditioner"). In the proposed functional

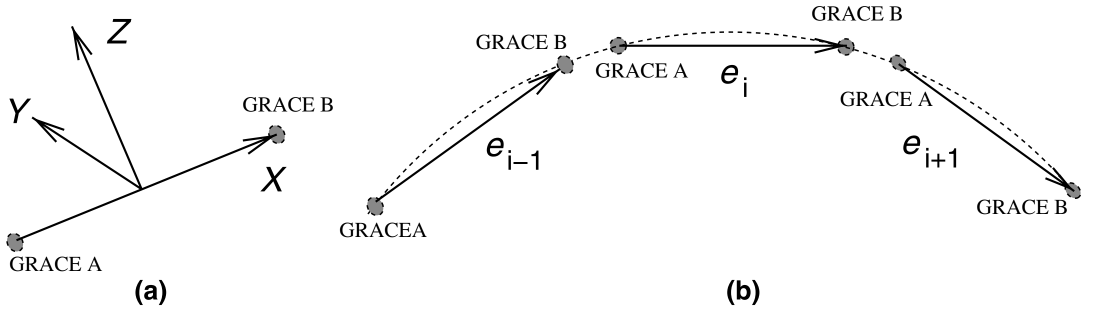

Fig. 1. (**a**): Definition of the working frame; (**b**): Unit vectors of line-of-sight directions at three successive epochs.

model, a block-diagonal approximation of the normal matrix can be obtained by making a number of not very unrealistic assumptions (e.g. that the orbit is perfectly circular; the gravity field between the satellites changes linearly; temporal change of the gravity field at a given point in an inertial frame caused by the Earth rotation can be neglected within a sampling interval, etc.)

4 Simulation

To test the proposed functional model, a numerical experiment was performed. One-month satellite orbits of two satellites with 5-sec sampling were simulated in compliance with parameters of the GRACE mission. The force model was defined as the gravity field model EIGEN-CG03C truncated at degree 150. The simulated orbits were used to compute both range-rates and the navigation parameters. Next, "observed" RRCs were derived. Furthermore, reference RRCs were computed according to the left-hand side of Equation (3) on the basis of the EGM96 model truncated at degree 150. The residual ("observed" minus "reference") RRCs were used to compute the corrections to the EGM96 Stokes coefficients by the least-squares adjustment. The obtained gravity field model (reference model + corrections) was compared with the "true" one (see the noise-free case in Figure 2). A perfect agreement between the derived and "true" model can be considered as a proof of validity of the proposed functional model. The remaining small differences between the models presumably stem from a limited accuracy of the orbit integration. The least-squares adjustment took only 31 min (the SGI Altix 3700 super-computing system with eight processing elements was used).

The data sets presented above were further used to estimate how noise from different sources propagates into the gravity field model in the proposed approach. Errors of following origins are simulated and added to corresponding quantities:

1. **Case I – Noise in range-rates**: white noise with a RMS (root-mean-square) of 0.5 μm/s.
2. **Case II – Noise in satellite orbits**: noise with a RMS of 10 mm and an auto-correlation factor of 0.995 (Reubelt et al., 2003) is added to the 'true' orbit of the satellite A. Furthermore, noise with a RMS of 10, 1, or 0.1 mm and with the same auto-correlation factor of 0.995 is resepctively added to the baselines of two satellites (3-D differences of the satellite positions). Then, the orbits of satellite B used is computed by adding the noisy baselines to the noisy orbit of the satellite A.

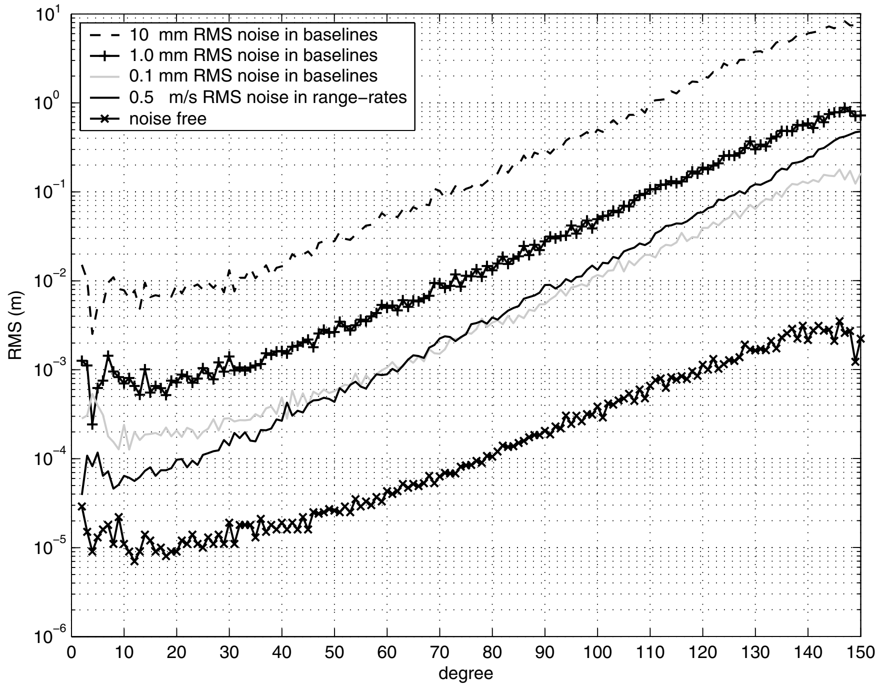

Fig. 2. Difference between the obtained models and the "ture" model in terms of geoid height errors per degree (noise-free data, Case I and Case II noisy data are used, respectively).

Figure 3(a) shows the square root of power spectral density of noise in RRCs for the Case I. The figure clearly displays that computation of RRCs heavily amplifies noise at high-frequencies, therefore, a frequency-dependent data weighting is a must. So that relatively high weights are assigned to low frequencies and low weights to high frequencies. To achieve that, we made use of the noise PSD approximated by an analytic function (Ditmar and Van Eck van der Sluijs, 2004; Ditmar et al., 2007). Figure 3(b) compares the results obtained with and without frequency-dependent data weighting. It can be seen that the frequency-dependent weighting improves the model accuracy both at low and high degrees.

Propagation of orbit noise (Case II) into the gravity field model is shown in Figure 2. As seen, the resulting gravity field model is rather sensitive to errors in the baselines of two satellites. This means the orbit noise may become the dominant factor in the error budget. The accuracy of baselines between two satelites is of particular importance. A similar conclusion was aslo made earlier by Jekeli (1999) and Han et al. (2005a) in the context of the energy balance approach.

5 Real Data Processing

A GRACE gravity filed model up to degree or order 150 was derived from 101 days of GRACE science data spanning the interval from July 9, 2003 to October 17, 2003. The following main data sets were used: (1) Reduced-dynamic orbit of satellite A (30-sec sampling); (2) Relative baseline vectors between satellite A and B (10-sec sampling); (3) Non-gravitational accelerations (1-sec sampling); (4) Attitude data (5-sec sampling); (5) K-band inter-satellite range-rates (5-sec sampling). The items (1) and (2) are kindly provided by Kroes et al. (2005), and the items (3)–(5) are the L1B products which are distributed by JPL PODAAC User Services Office (Case et al., 2004). The principal procedure of real data precessing is similiar to that of simulated data. Additionally, we subtracted temporal variations caused by tides, as well as by atmospheric and ocean mass changes. The data sets used for this purpose, are DE405 numerical ephemerides (Standish, 1998), GOT00.2 ocean tide model (Ray, 1999) and AOD1B atmospheric and ocean de-aliasing product (Case et al., 2004). Daily bias and scale factor in terms of non-gravitational RRCs are estimated altogether with the parameters of gravity field model.

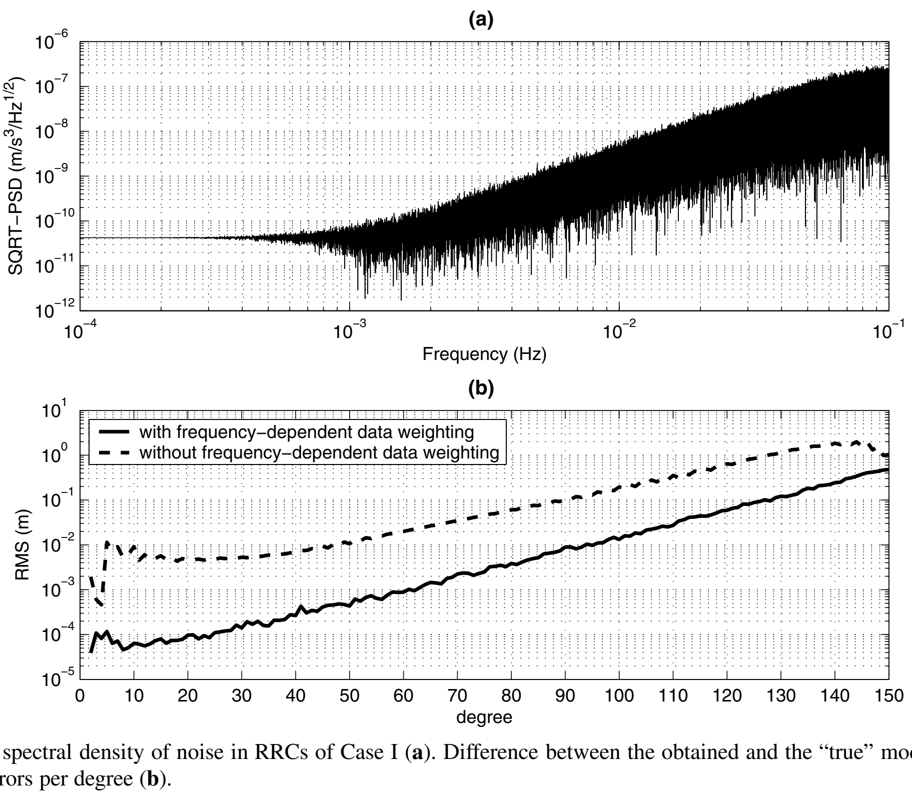

Fig. 3. Power spectral density of noise in RRCs of Case I (**a**). Difference between the obtained and the "true" model in terms of geiod height errors per degree (**b**).

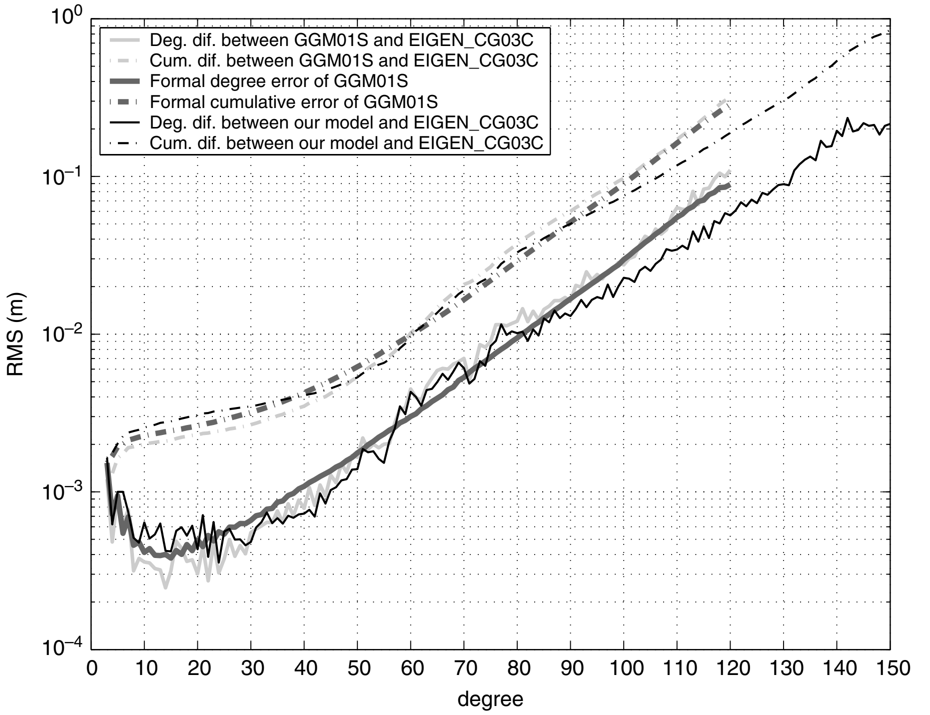

Fig. 4. Geoid height difference between our model and EIGEN-CG03C (*thin black line*), between GGM01S and EIGEN-CG03C (*thick grey line*): cumulative geiod height difference (*dash line*) and geiod height difference per degree (*solid line*). Formal degree error of GGM01S (*thick black solid line*) and formal cumulative error of GGM01S (*thick black dash line*). Degree 2 is excluded.

The difference between our model and the state-of-the-art EIGEN-CG03C model (Förste et al., 2005) is shown in terms of geoid heights in Figure 4. For comparison, the difference between GGM01S (Tapley et al., 2004) and EIGEN-CG03C models, as well as the formal error of GGM01S model are also shown. The GGM01S model was produced from 111 days of GRACE data. As can be seen, our model has less than 2-cm geoid height difference up to degree and order 70, and 20-cm difference up to degree and order 120 with respect to the EIGEN-CG03C model. This is comparable with or even better than the GGM01S model. Unfortunately, our model shows a somewhat lower accuracy at low degrees (below 35). There are at least three possible reasons for that. The first reason is temporal gravity filed variations. The data span of the GGM01S model is from April to November, though only 111 days data were selected. Thus, temporal gravity field variations in this model are largely averaged out. The second reason is satellite orbit error. Unlike the developers of the GGM01S model, we have not minimized these errors by adding corresponding nuisance parameters to the list of unknowns at the stage of least-squares adjustement. The third reason to explain low accuracy at low degrees could be the noise in the navigation parameters. The navigation parameters are computed from the baselines of two satelites, which are determined by GPS data. The navigation parameters certainly contain noise, therefore it could propagate into the RRCs. To what an extent this noise influences of the noise on the final gravity field is not known yet.

6 Conclusion and Future Outlook

A new approach has been proposed for gravity field modeling from GRACE KBR data. It is based on usage of so-called range-rate combinations. A numerical study and real data processing prove the validity of the approach. Although the research is by far not complete, it is already clear that the new approach can produce models with an accuracy comparable to that provided by other techniques. In the future, more nuisance parameters have to be estimated in the course of least-squares adjustment. In particular, nuisance parameters related to orbit errors will be incorporated. Furthermore, more modern ocean tides models will be used. It also goes without saying that more GRACE data should be used for computation of a more accurate mean gravity field model.

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