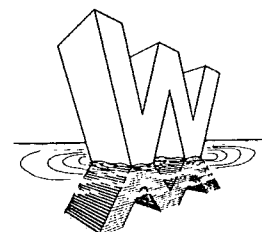


Discussion of the one-line calculations
for the nourishments on Sylt (1972, 1978).

By S.E.A. Jansen.



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Introduction and justification

In August 1987 a "Manual on Artificial Beach Nourishment" has been published by the Dutch Department of Public Works, Rijkswaterstaat, and Delft Hydraulics [RWS DH87]. (For shortness in the rest of this report this publication will be referred to as just "the Manual".)

The theoretical and experimental background of the recommendations given in this manual is elaborated in a number of annexes. Of these annexes the numbers V and VI are of importance to this study. Annex V discusses the "Hindcast computations of some projects" and Annex VI looks at "Coastal morphology theories" (line modelling).

The Department of Civil Engineering of the Delft University of Technology, section Coastal Engineering, has considered it useful to have a critical look at the computations of annex V.

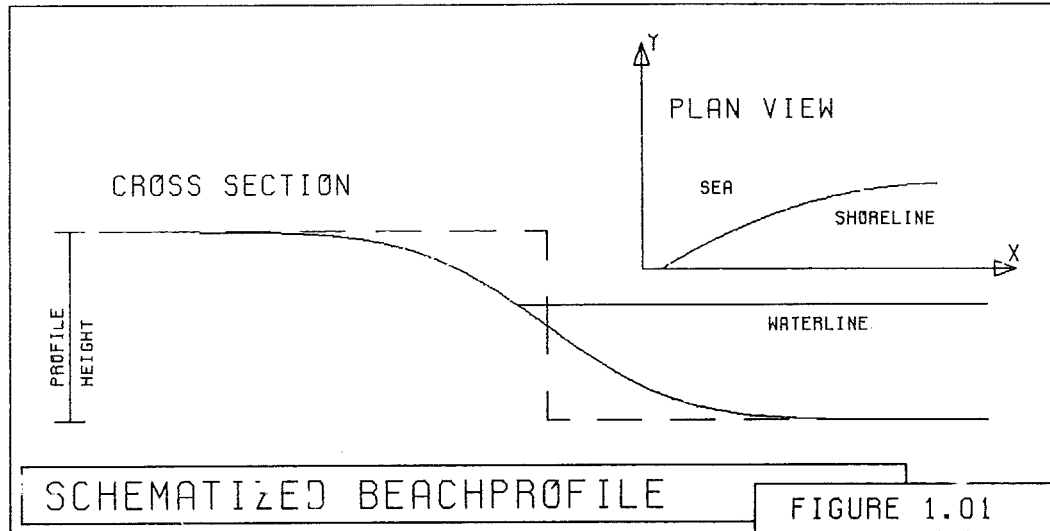
Therefore the objectives of this study have been to:

Reexamine the computations of the case study of Sylt and evaluate especially the assumptions used there.

Since line models have been the most important tools in the calculations of Annex V, the first chapter of this report will discuss one-line theory. In particular the attention will be focused on two factors that can limit the application of the one-line model: the profile height and the angle of wave incidence. In the second chapter the computations of Annex V as far as they involve one-line schematization for the case of Sylt are reviewed and commented. In the concluding chapter (Chapter 3) the results of the analysis will be summarized.

1 One-line theory

The most simple form of line modelling can be achieved by the so-called one-line theory. The coastal profile is schematized by one line according to the figure below.



Detailed descriptions of this kind of modelling will not be given here but can be found in Annex VI [RWSDH87] or in the literature, for example [DUT80], [DH82] or [Pel56]. In this report there only will be given a recapitulation and discussion of the aspects of the theory as far as necessary for this study.

1.1 GeneralEquations

Using the one-line theory has the pleasant consequence, that it leads to the well known diffusion equation for describing the position of the shoreline as a function of the time. This equation yields,

$$\frac{\partial y}{\partial t} = - \frac{s}{h} * \frac{\partial^2 y}{\partial x^2} \quad (1.01)$$

where h : height of the schematized beach profile
 s : coastal constant for the longshore transport
 t : time
 x : x-coordinate along the shoreline
 y : coordinate perpendicular to the x-axis

In words: the progress or the retreat of the shoreline is proportional to the curvation of the shoreline in x-direction.

This diffusion equation is derived from combining the equation of continuity for the transported sand,

$$\frac{\partial S}{\partial x} + h * \frac{\partial y}{\partial t} = 0 \quad (1.02)$$

with the equation of motion for the transported sand,

$$S(B) = S_0 - s * \frac{\partial y}{\partial x} \quad (1.03)$$

where B : coastline direction with respect to the x-axis
 S : longshore sand transport capacity (a function of the coastline direction)
 S_0 : longshore sand transport capacity along a straight coastline parallel to x-axis

Assumptions

Equation (1.02) is a straightforward equation with which there is not any fault to find. The only assumption used here is that there is no transport of sand from the considered area transversal to the coast.

Equation (1.03) however consists of quite a number of assumptions brought together in one equation.

The first important simplification is that the longshore transport is a function of the coastline direction only,

$$S = S(\beta) \tag{1.04}$$

Some (from coastal engineering point of view) less important mathematical assumptions about differentiability and neglecting second (and higher) order terms lead to a Taylor series,

$$S(\beta) = S \Big|_{\beta=0} + \beta * \frac{dS}{d\beta} \Big|_{\beta=0} + O(\beta^2) \tag{1.05}$$

The second simplification of importance is that the angle β is considered small. So it can be posed,

$$\beta = \tan[\beta] = \frac{\delta y}{\delta x} \tag{1.06}$$

As a direct effect of the approximation of the transport capacity by a Taylor series (1.05) the following relation can be posed,

$$\frac{\delta S}{\delta \beta} = -s \tag{1.07}$$

When as (1.05) suggests the coastal constant should be defined only for $\beta=0$, this would be not as much as a simplification, but its value to the model should be dependent upon the acceptability of the approximation of (1.05). However the definition is a little different. By (1.07) a more general relation has been posed. Formula (1.07) expresses a linear relationship between changes in coastline direction and changes in sand transport capacity without any restrictions to the coastline direction.

When the relations (1.06) and (1.07) are substituted in (1.05) the equation of motion for the transported sand (1.03) is obtained.

Analytical solution

Of course these simplifications have their disadvantages. In fact it is one of the objectives of this study to evaluate the effects of these on the usefulness of the model. But a substantial advantage of this approach is the possibility to find an analytical solution of equation (1.01).

Detailed description of analytical solutions in the case of coastal engineering line modelling can be found in Annex VI [RWSDH87] or in the literature [CERC87], [DH82]. A more general review of analytical solutions of a diffusion equation can be obtained from the mathematical handbooks on partial differential equations, for instance by Smirnow [Smi64].

Here the description will be confined to the solution, which represents the case of a given initial shape of a stockpile-type beachfill, which mathematically can be represented by a Dirac function (see Appendix A for more details). This solution which has the shape of a Gaussian curve is given by,

$$y = CA * \exp[-CB * x^2] \quad (1.08)$$

$$\text{with } CA = \frac{V_f}{\text{sqrt}[4 * \text{Pi} * h * s * (t + T_i)]}$$

$$\text{and } CB = \frac{h}{4 * s * (t + T_i)}$$

where CA : time dependent coefficient A
 CB : time dependent coefficient B
 h : schematized profile height
 Pi : well known constant with value 3.14159...
 s : coastal constant according to (1.07)
 t : time
 Ti : initial period between t=0 of the Dirac-function (fictive) and the time of placement of the fill
 Vf : total volume of the fill
 x : x-coordinate of a shoreline point
 y : shoreline position

More information about this particular solution can be obtained from Appendix A, where the mathematical terms are explained in more detail.

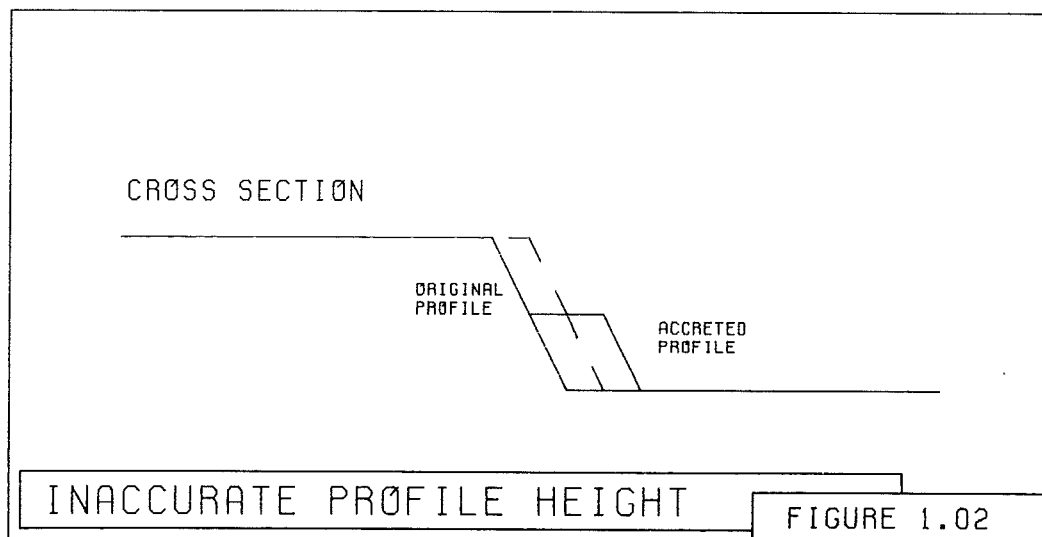
1.2 Difficulties in one-line schematization

Height of the active profile

First it will be specified what is meant by the active coastal profile height. It is the height over which there is erosion or accretion. Because the profile is schematized to one-line also the expression height of the schematized profile will be used. To determine a proper value for the schematized coastal profile height "h" is a problem, which has its origin in the so solid seeming equation of continuity. For a correct use of this equation it is necessary to find a fairly accurate value of the active profile height.

Because the equation of continuity is a budget equation it is important to know how much sand is transported through the boundaries of the considered area. In this model the transport of sand through the upper and the lower boundary is assumed to be zero. Thus accretion or erosion results directly from gradients in transport through the side boundaries (i.e. gradients in longshore transport capacity).

Seeing this one could suggest to take a safe (exaggerated) height. Although in this way it is assured that no sand passes through the upper and lower boundary of the budget area, the calculated retreat or progress of the shoreline will not be accurate. This is shown below in Figure 1.02.



The ratio of the accreted area of sand [m^2] and the chosen profile height [m] gives the progress of the shoreline [m]. Since the height is assumed too large the progress will be calculated too small.

Through its coefficients "CA" and "CB" the chosen value of "h" also affects the shoreline position computed by means of the analytical solution (1.08). This influence will be analysed further in Section 2.3 (Par. profile height). For now the treatment of this effect will be confined to the remark that

the accuracy of the profile height has an important influence on the accuracy of the computation as a whole.

From the above mentioned it is clear that the determination of the active profile height has to be done with care. Trying to express it in a formula the following has been suggested,

$$h = d + e \quad (1.09)$$

Where d : depth from the undisturbed water-level to the point where the coastal profile becomes about horizontal
 h : height of the active coastal profile
 e : maximum elevation of the beach above the undisturbed water level

For the undisturbed waterlevel the mean sea-level can be used. Further elaboration of (1.09) still leaves two problems.

Firstly the somewhat subjective judgement on what slope is "about horizontal". Especially in the case of flat slopes this judgement can not be very critical. A few conditions however have to be satisfied,

- * The depth "d" has to be taken at a point wide outside the breaker zone to enclose all wave induced longshore currents within the budget area.
- * But as mentioned before one can not shift this point too far seaward because that would decrease the value of the calculations.

Secondly the determination of the maximum elevation of the waterline. Hereby a distinction has to be made between an eroding and an accreting shoreline.

For an eroding beach the value of "e" can be obtained rather simple. Since the process of erosion affects the whole frontal part of the sand dune, the elevation of the beach up to the dune height has to be used.

In the case of accretion three components contribute to "e", the tidal range, the wave set-up and the wave run-up. Besides the influence of transport of sand in landward direction by wind should be accounted for. This factor is rather difficult to quantify.

For all three components a reasonable estimation has to be given. A limited accuracy is not critical.

The tidal range as a component is not very difficult to determine if measurements are available. The mean high tide value can be used.

The set-up and the wave-run up can both be considered a function of wave characteristics and slope characteristics. Important factors are the significant wave height, the wave length and the angle of incidence as far as it concerns wave characteristics and the angle of the beach slope and the roughness (grain size) of the beach material for the slope characteristics.

For the wave run-up Hunt has given a formula which can be found in the Manual (Section 4.2.3) as formula (4.16). So the wave run-up can simply be calculated when is known what kind of wave condition match the average longshore transport proces.

For the wave set-up a similar "simple" approach can be carried out, using formula (4.15) of the same section of the Manual.

If several measurements (in time) of the coastal profile are available one could forget about everything posed above and instead try to estimate the height "h" by comparison of measured profiles of the same place for different points of time. But then one has to be prepared to venture upon the hazardous field of measurement errors and measurement inaccuracy. In the Manual [RWSDH87], Section 4.5.3 such a method is described. In fact the described method needs yet another assumption, viz. that the coastal profile is assumed to move horizontally over its whole active height as a result of accretion and erosion. This is an assumption that is usually considered essential for line models in general. In the next paragraph (horizontal translation of the active profile) this assumption will be discussed further.

If there are only a limited number of measurements available there is also the possibility of doing an "expert" estimation at the height "h" by studying the coastal profile. Hereby it is useful to keep in mind the considerations mentioned at the beginning of this paragraph.

Things become very complicated if one tries to combine these longshore transport calculations with transversal transport calculations. Using the theory of Swart for determining the value of "h" brings on more problems than it solves.

As a conclusion it can be posed that a working solution can be found most effectively in a rule of the thumb. A detailed calculation does not seem to be more reliable.

The following procedure is necessary,

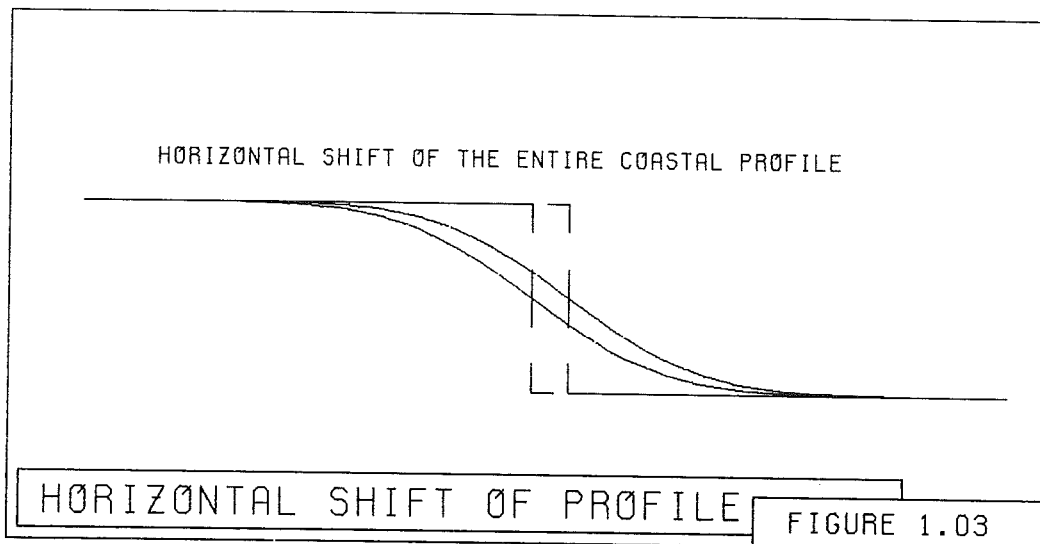
- ⇒ Choose a wave height, which occurs once or twice a year.
- ⇒ Calculate the depth at which this chosen wave breaks.
- ⇒ Estimate the value of "d" from formula (1.09) by taking three times this depth.
- ⇒ Determine whether the beach is likely to erode or to accrete.
 - Estimate the value of "e" in case of an eroding beach by taking the dune height above the undisturbed water-level.
 - Estimate the value of "e" in case of an accreting beach by calculating the wave set-up and the wave run-up for the chosen wave height and the tidal amplitude. If important add an extra height to account for the sand transport by wind.
- ⇒ The sum of these two factors gives the value of the height of the schematized coastal profile.

The method is simple and not very intensive. The calculations can be carried out with only a few formulas. The crucial point however is to determine a representative value for the wave height. It seems best to choose a height that occurs not too often in a year (for instance twice a year). The angle of incidence can be an average angle. One could also consider to apply a value of zero (wave incidence perpendicular to the coastline).

It should be realized however that this procedure is acceptable when insufficient data is available on the actual beach profile. When the geometry of the beach profile is known a good approximation of "h" should be achieved by judging the profiles critically, while the suggestions of this paragraph can be helpful.

Horizontal translation of the active profile

As mentioned earlier the assumption that the active coastal profile as a whole shifts only horizontally (see figure below) is usually considered an essential one.

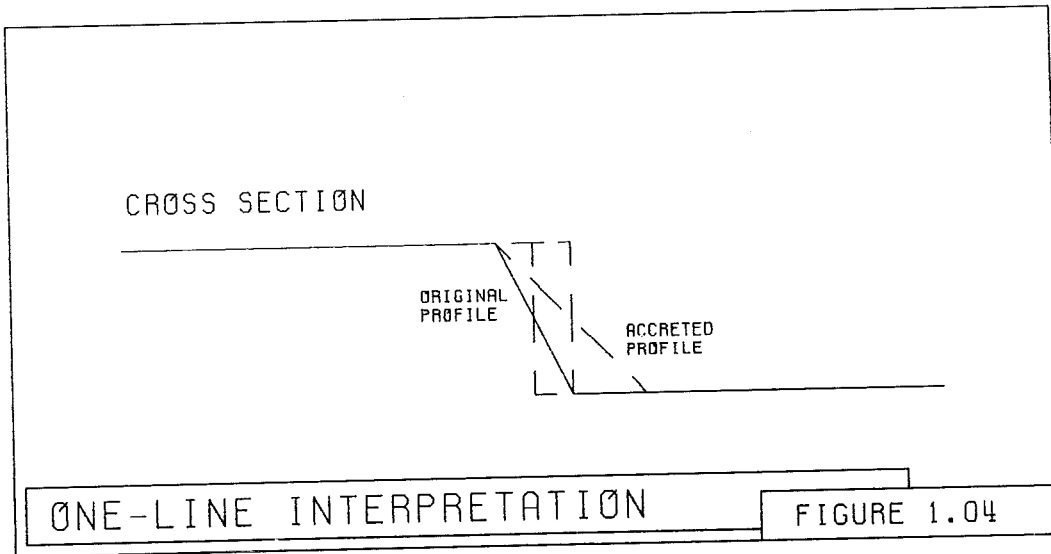


Basically this is a very meaningful consideration, because the idea behind it is the existence of a characteristic profile fitting the average wave conditions. One can not call it an equilibrium profile in case of a gradually accreting (please notice that further on in this paragraph the term accretion will be used, but everything is also valid for erosion) profile, which by definition is not in equilibrium.

Problems arise when a measured profile has to be judged. It is difficult to tell what relation exists between the measured profile and the so-called "characteristic profile". This last profile can be regarded as the outcome of long-term average wave conditions, while a measurement can only be seen as a snapshot of the coastal profile. The process of accretion is likely to be influenced by erratic disturbances. These may only involve a part of the active profile and cause a redistribution of sand within the profile on a more delayed time scale.

The assumption of horizontal translation is only true from a very global point of view. So a lot of reasoning carried out with this assumption in mind can only be regarded as essential in a kind of mental model. However in Figure 1.04 there is shown another way to look at a line model without considering this assumption as an essential one.

The first interpretation of the line model leads to the insight that the place of the schematized coastline is unimportant. The progress of the coastline has the same value everywhere in the active profile. But the second interpretation attaches an important meaning to the position of the schematized coastline. The progress of this line, which is not by definition equal to that of the active profile as a whole, can be considered as a characteristic value to describe the progress of the active coastal profile.



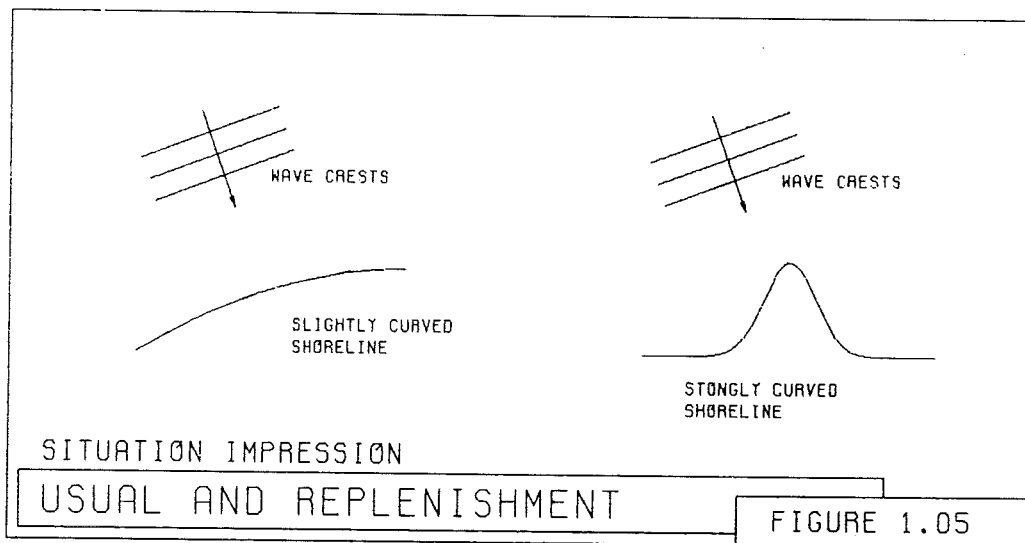
In Figure 1.04 the original profile schematized by the leftmost vertical dashed line is accreted to the dashed profile, which is represented through the rightmost vertical dashed line. The progress of the schematized profile can be regarded as an average progress of the whole coastal profile.

As a conclusion it can be posed that the second view is perhaps less elegant, but certainly more applicable to the data as it is gathered.

Equation of motion

As explained before the equation of motion for the transported sand (1.03) has been derived from the set of equations (1.04) through (1.07). From this set all four equations can be seen as assumptions and the validity of at least three of these is dependent upon the magnitude of the gradient of the coastline in longshore direction.

To illustrate this a little more two different situations of a coastline are given in the figure below.



The picture on the left-hand side can be considered as a "usual" situation of a coastline with a small gradient in longshore direction. For the coastline pictured on the right-hand side the opposite is valid. Such a situation can result from a stockpile type replenishment as mentioned in Section 1.1 (Par. analytical solution).

In this paragraph both situations will be referred to as the "usual" and the "replenishment" situation. This is only a matter of referencing these situations, it is not meant to generalize.

It will be explained below that the simplification (1.04) through (1.07) are only valid for the "usual" situation with additional restriction that only wave influence is regarded.

As it is expressed by equation (1.04) the longshore transport is considered a function of the coastline direction only. This is an important simplification since it limits the application of the line models to the case of sand transport induced by waves only. It will be clear that the sand transport is not a function of the coastline direction in case of tide induced sand transport. One only has to look at the sketch of the "replenishment" situation to reason that out. Since the transport of sand is highly dependent on the shear stress (and thus of the velocity) at the bottom, the contraction of the streamlines at the top of the fill (angle is zero) will definitely lead to another quantity of longshore transport than beside the fill where also a zero angle can be found.

But even if the application of the line model is restricted to areas where tidal influences are negligible, equation (1.04) gives some problems when one tries to use it for the "replenishment" situation.

Due to refraction and diffraction the distribution of the wave height along the breakerline will show gradients. A more or less constant wave height along the breakerline as in the "usual" situation can not be expected. It is obvious that a larger wave height working over the same limited depth will result in a larger velocity (and shear stress) component at the sea bottom and therefore will induce a larger amount of sand transport. In addition the gradients in wave height will cause gradients in wave set-up and thus local currents which might not be negligible.

Thus serious attention has to be given to the assumptions used when applying a line theory for the "replenishment" situation. The simplifications (1.05) through (1.07) are a further elaboration of equation (1.04) and they have to be evaluated thoroughly too.

Seeing all the problems with the basic concept according to (1.04) a valid linear approach to this problem should be a good achievement. It seems good policy to cut off second and higher order terms from the Taylor-series. Also the longshore transport can be considered a smooth (about sinusoidal) function of the coastline direction (see Figure 1.06). Therefore differentiability is not in question.

So if equation (1.04) is accepted equation (1.05) follows from it when is realized that such an approximation is valid only for small changes in the coastline direction " β " with respect to $\beta=0$. In the "usual" situation this condition can be satisfied by choosing a convenient system of axis.

The validity of simplification (1.06) is dependent on this same condition, that the angle of the coastline with the x-axis has to be small. In this context "small" has to be understood as less than about 20 degrees (0.35 rad).

In that respect (1.06) will not be a surprise, it is a well known approximation in mathematics.

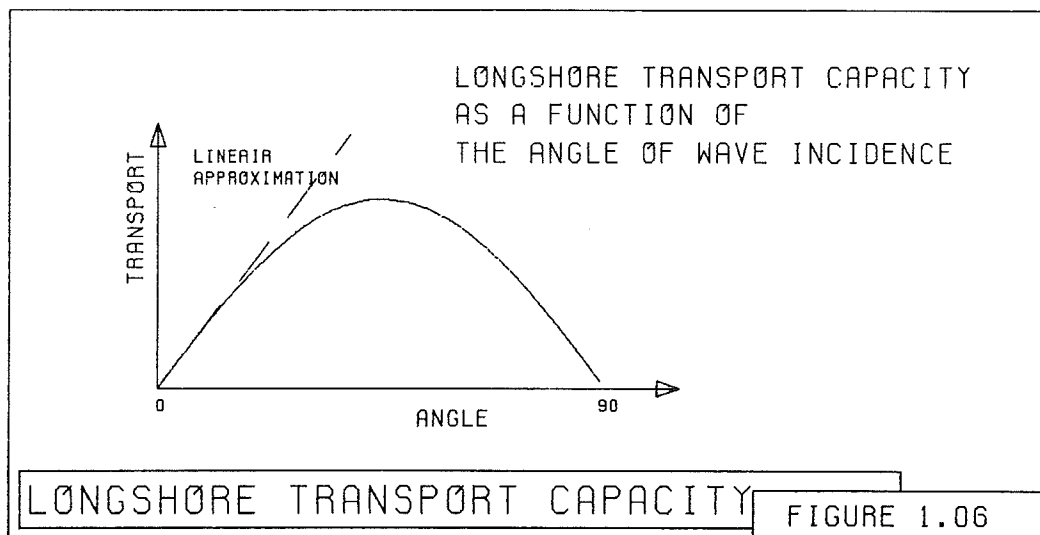
The justification of (1.07) can be found in Figure 1.06, where the longshore transport capacity is plotted against the angle of wave incidence. This relation can be derived from the CERC formula,

$$S = 0.020 * H_0^2 * cbr * \sin[\phi_0 r] * \cos[\phi_0 r] \quad (1.10)$$

which, using the expression for the double angle can also be written as,

$$S = 0.010 * H_0^2 * cbr * \sin[2*\phi_0 r] \quad (1.11)$$

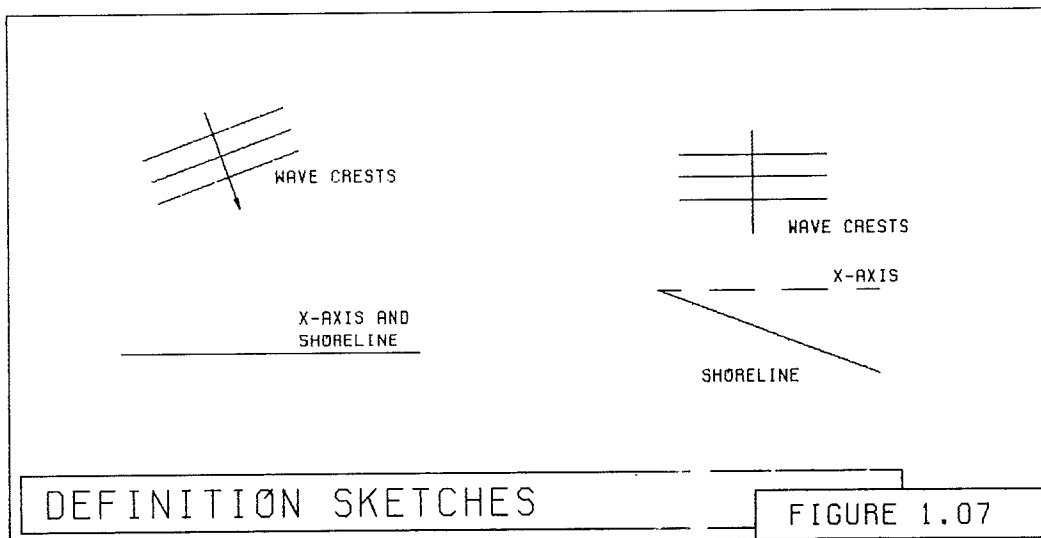
where S : longshore sand transport capacity
 H_0 : deep water significant wave height
 cbr : wave speed at the breakerline
 $\phi_0 r$: angle of wave incidence at deep water relative to the coastline direction



Although strictly formula (1.07) is defined by substitution of $B=0$, it can be noticed from Figure 1.06 that a somewhat wider range is also permitted. But from the sinusoidal shape of the curve as a whole it can also be seen that, when applied on the full range of angles the definition of (1.07) has to give problems. Within its full range the gradient of the curve is not a constant.

Basically (1.06) and (1.07) are two different simplifications. The first one approximates the coastline angle with respect to the x-axis, the second one gives an approximation of the change of longshore transport under varying coastline direction, assuming this variation is small. Their similarity is that they are both valid only when small changes are considered.

To evaluate the influence of these simplifications the following situation is analysed. A replenishment with the shape of a Gaussian curve is placed on a horizontal shoal. That the toe of the top of the fill does not extend outside the shoal is considered essential for this model. Otherwise the schematized profile height "h" can not be defined properly. To start the analysis the basic concept behind the equation of motion has to be reviewed. Figure 1.07 provides two definition sketches.



The leftmost picture of Figure 1.07 shows the basic situation of a straight coastline parallel to the x-axis, which is attacked by waves incidencing obliquely. In this case an equilibrium longshore transport can be defined as,

$$S_0 = S \Big|_{\beta=0} \tag{1.12}$$

- Where S : longshore transport capacity as a function of the coastal direction
- S₀ : transport for β=0 under a given (constant) angle of wave incidence
- β : coastline direction with respect to the x-axis

When a small rotation (positive counter clockwise) is applied on the coastline the following approximation can be made using a Taylor series,

$$S(\beta) = S \Big|_{\beta=0} + \beta * \frac{dS}{d\beta} \Big|_{\beta=0} \tag{1.13}$$

This in fact is the same approach as in (1.05). Here the terms of second (and higher) order have been neglected also.

Formula (1.13) is valid for a fixed angle of wave incidence and gives a good approximation of the longshore transport when the changes in coastline direction are small.

The influence of the sinusoidal shape of Figure 1.06 on the behaviour of a beachfill modelled by (1.08) will be analysed in further detail in a separate report (Computer programme GaussLine). There a numerical method is used to analyse the differences that are introduced by taking in account the sinusoidal relation of Figure (1.06).

Consider also the rightmost picture of Figure 1.07. In this case the angle of wave incidence ϕ equals zero, but the coastline direction has a fixed value of " $-\beta_0$ ".

As far as it concerns the longshore transport the two situations are identical. This yields that rotating of the coastline at given angle of incidence has the same effect on the transport as rotating the angle of wave incidence at a fixed coastline direction. So only the difference between these two angles is important.

Defining " β_r " as,

$$\beta_r = \beta - \phi \tag{1.14}$$

the angle of wave incidence relative to the coastline direction, the longshore transport can be regarded a function of " β_r ": $S = S(\beta_r)$. Thus instead of (1.13) it is convenient to approximate as follows,

$$S(\beta_r) = S \Big|_{\beta_r=0} + \beta_r * \frac{dS}{d\beta_r} \Big|_{\beta_r=0} \tag{1.15}$$

Since $\beta_r = \phi = 0$, the angle of wave incidence relative to the coastline direction, now the first term of (1.15) equals zero by definition and (1.15) therefore reduces to,

$$S(\beta_r) = \beta_r * \frac{dS}{d\beta_r} \Big|_{\beta_r=0} \tag{1.16}$$

To analyse this a little further at first a case of wave incidence perpendicular to the unreplenished coastline is considered. For this situation the Taylor-series of (1.16) reduces to,

$$S(\beta) = \beta * \left. \frac{dS}{d\beta} \right|_{\beta=0} \quad (1.17)$$

Applying as an approximation for small values of "β" the leftmost part of approximation (1.06), instead of the rightmost part, and substituting relation (1.07) as used in Section 1.1 yields,

$$S(\beta) = -s * \tan[\beta] \quad (1.18)$$

From the CERC formula (1.11) the next relation follows,

$$\frac{dS}{d\beta} = \frac{d}{d\beta} [0.010 * H_0^2 * cbr * \sin[2*\phi_{0r}]] \quad (1.19)$$

Since the waves incidence perpendicular to the (original) coastline " ϕ_{0r} " may be substituted by β,

$$\frac{dS}{d\beta} = \frac{d}{d\beta} [0.010 * H_0^2 * cbr * \sin[2*\beta]] \quad (1.20)$$

Disregarding the influence of "cbr" by considering its value constant under varying "β" an expression for the coastal constant "s" can be found through combining its definition of (1.07) with (1.20),

$$s = - \left[0.010 * H_0^2 * cbr * 2 * \cos[2*\beta] \right]_{\beta=0} \quad (1.21)$$

and substitution of $\beta=0$ yields,

$$s = -0.010 * H_0^2 * cbr * 2 \quad (1.22)$$

The CERC formula therefore can be written as,

$$S(\beta) = -s * 0.5 * \sin[2*\beta] \quad (1.23)$$

Looking at (1.23) and (1.18) an evaluation can be made by comparing the value of the expressions " $0.5*\sin[2\beta]$ " and " $\tan[\beta]$ " for different values of "β".

The comparison is elaborated in the following table,

β [DEG]	β [RAD]	One-line	CERC
0.0	0.00000	0.00000	0.00000
5.0	0.08727	0.08749	0.08682
10.0	0.17453	0.17633	0.17101
15.0	0.26180	0.26795	0.25000
20.0	0.34907	0.36397	0.32139
25.0	0.43633	0.46631	0.38302
30.0	0.52360	0.57735	0.43301
35.0	0.61086	0.70021	0.46985
40.0	0.69813	0.83910	0.49240
45.0	0.78540	1.00000	0.50000
50.0	0.87266	1.19175	0.49240
55.0	0.95993	1.42814	0.46985
60.0	1.04720	1.73205	0.43301
65.0	1.13446	2.14450	0.38302
70.0	1.22173	2.74747	0.32139
75.0	1.30900	3.73203	0.25000
80.0	1.39626	5.67124	0.17101
85.0	1.48353	11.42982	0.08683

Tabel 1.1 - Influence of coastline direction

The table shows the value of the one-line approximation, " $\tan[\beta]$ ", and that of the CERC schematization, " $0.5 \cdot \sin[2\beta]$ " for increasing " β ". The angle " β " in this comparison has to be interpreted as the coastline direction.

It can be noticed that for angles less than 20 degrees the one-line and the CERC approximation yield almost the same result. The difference increases above 20 degrees. The accepted accuracy therefore is an important factor, when questions are asked about the suitability of a line model.

Now the case of waves incidencing obliquely will be regarded. The difference with the situation just analysed is not essential, but the situation is a little more complicated. Unlike the first case the relative angle " β_r " is not equal anymore to the coastline direction " β " and (1.14) has to be used. Again starting from (1.11) and (1.16) a set of formulas can be derived equivalent to (1.17) through (1.23) only with " β_r " instead of " β ". In fact this new set can be seen as the general case, while the first situation (wave perpendicular to the original shoreline) has to be considered a special case. Thus it can be concluded that the previous analysis resulting in Table 1.1 is valid here also, when " β " is replaced by " β_r ".

The interpretation however of the numeric values is a little more difficult, because the value of " ϕ_0 " influences the scope of tolerable coastline angles,

$$-\beta_a < \beta_r < +\beta_a \quad \longleftrightarrow \quad -\beta_a + \phi_0 < \beta < +\beta_a + \phi_0 \quad (1.24)$$

Where β : coastline direction with respect to the x-axis
 β_a : angle up to which the inaccuracy of the one-line schematization is accepted
 β_r : angle of wave incidence relative to the coastline direction
 ϕ_0 : deep water angle of wave incidence

The rightmost expression of (1.24) can mean a serious restriction on the applicability the line model.

Whether it can be used or not is not only dependent on the accepted inaccuracy " β_a ", but also on the actual angle of wave incidence " ϕ_0 ".

2 One-line calculations

2.1 General

On Sylt, an island in the northern part of West-Germany, two replenishments have been undertaken. The first one in 1972, the second one in 1978, both to be considered as stockpile-type beach fills.

Of each replenishment three series of measured data have been gathered from Führböter [Füh74] for the 1972 nourishment and from Pätzold [Pät80] for the 1978 nourishment. Table 2.1 shows the three measurement dates for the 1972 - (left) and the 1978 replenishment (right).

Measurement No	Measurement date	Lapse day (yr)	Measurement date	Lapse day (yr)
1	Oct 11, 1972	- -	Jul 30, 1978	- -
2	May 17, 1973	217 (0.59)	Sep 9, 1978	38 (0.10)
3	Feb 28, 1974	506 (1.38)	Oct 1, 1979	427 (1.17)

Table 2.1 - Measurement dates

In Annex V of the Manual [RWSDH87] these data have been elaborated further. The objectives of that analysis have been to:

- * Evaluate the usefulness of the one-line model for predictive purposes (in Annex V called "relevance").
- * Evaluate the accuracy of the one-line model when applied on the available data (in Annex V called "accuracy" too).

The calculations as they appear in Annex V are reviewed in Section 2.2 and they will be discussed in Section 2.3.

2.2 Calculations from Annex V

In this section it will be attempted to streamline the calculations described in Annex V in order to obtain a clear view not only on the results they produce but also on the problems that remain.

Presented in their original form they can be found in Annex V, pages 2-35.

Conventions

First a few conventions used in this- and the next section will be explained.

The three series of measured data will be referred to as t1, t2 and t3, where the digits 1 through 3 denote chronology. If the replenishment year is of importance it will be referred to as for instance 72.t1 or 78.t3.

Elaboration of the measured data

The in situ measured data has been gathered by a division of the German Department of Public Works located in Husum. A further elaboration has been carried out by means of a computer programme of the Braunschweig University of Technology able to deal with cubic calculations.

Hereby the volume of sand above several chosen planes of reference has been computed per 100m coastline. The calculated sand volumes have been related to the last available measured data before nourishment (thus not necessarily the same for each part of the coastline).

So the volumetric changes per 100m coastline have been obtained. A more detailed description of this procedure can be found in the literature [Füh76].

In Annex V these computed changes have been the starting point of the analysis. Thus both the above mentioned references (horizontal plane as well as initial situation) have been accepted. For the horizontal plane of reference though each time the deepest available level has been selected.

The volumetric changes (m^3 per 100m) have been converted to a coastline position through deviding them by 100m and then again by the profile height "h". The choice of "h" will be point of further discussion, later on in this section (Par. calculation method A) and also in Section 2.3 (Par. profile height).

In this way a schematized coastline has been obtained according to the second interpretation mentioned in Section 1.2 (Par. horizontal translation of the active profile).

Survey

The elaborated computations according to the one-line schematization can roughly be divided in four types:

- A: Using the deformation of the top of the beach fill between $t=t_1$ and $t=t_2$ as reference data it is attempted to calculate a value of the coastal constant "s", which could be used to predict the regression of the coastline at time $t=t_3$.
- B: The same attempt with respect to the coastal constant "s", but now by using the displacement in time of the contraflexure points of the Gaussian curve as essential data.
- C: From available wave data it has been tried to calculate the coastal constant "s" according to its definition, equation (1.07).
- D: In order to include known behaviour of the coastline in the model the original coastline has been described as a retreating parabola. The Gaussian curve as a result of a stockpile-type replenishment has been superposed on this form. The accuracy of the representation of the initial shape at $t=t_1$ and the accuracy of the computational results at $t=t_3$ has been evaluated through comparison with the measured data.

For the 1972 nourishment the methods typed A,B and C have been applied. The wave data used for method C have been gathered by Dette [Det74].

For the 1978 nourishment calculations have been made according to the methods A,B and D.

Calculation method A

Starting point has been the idea to schematize the stockpile-type beach fill as a Dirac-function. As a solution of the diffusion equation (1.01) formula (1.08) becomes valid.

This formula and the expressions for its coefficients are repeated here below as formula (2.01) through (2.03),

$$y = CA * \exp[-CB * x^2] \quad (2.01)$$

with coefficients "CA" and "CB" according to,

$$CA = \frac{Vf}{\text{sqrt}[4 * Pi * h * s * (t + Ti)]} \quad (2.02)$$

$$CB = \frac{h}{4 * s * (t + Ti)} \quad (2.03)$$

where CA : time dependent coefficient A
 CB : time dependent coefficient B
 h : schematized profile height
 Pi : well known constant with value 3.14159...
 s : coastal constant according to (1.07)
 t : time
 Ti : initial period between t=0 of the Dirac-function (fictive) and the time of placement of the fill
 Vf : total volume of the fill
 x : x-coordinate of a shoreline point
 y : position of the schematized coastline

The regression in time of the top of the the fill is obtained by substituting x=0 in formula (2.01), which leaves,

$$y = CA = \frac{Vf}{\text{sqrt}[4 * Pi * h * s * (t + Ti)]} \quad (2.04)$$

In this method it is assumed that the volume of the fill "Vf" is known and that the value of the schematized profile height "h" can be estimated. So if at two different points in time the position of the top of the fill is known, the values of the (assumed) constants "s" and "Ti" can be calculated.

In Annex V the measured data has been used to estimate through curve fitting the coefficients "CA" and "CB" of the Gaussian curve. This curve fitting procedure has been carried out for both the data of t=t1 and t=t2 separately.

The curve fitting procedure apparently has involved a least-squares method on the available shoreline points. As mentioned before in this section (Par. elaboration of the measured data) these points have been obtained each 100m by elaboration of volumetric changes. The applied method does not involve a check on the budget of sand volume. It purely minimizes the squares of the deviation in shoreline position.

As a result the values at $t=t_1$ and $t=t_2$ of the coefficients "CA" and "CB" have been obtained. The estimated values of "CA" are of importance for this calculation method, the values of "CB" will be used in the next paragraph (Calculation method B).

Since the volume of the fill has not been an influencing factor in the curve fitting procedure, the representation of the fill can be used as a standard for acceptance of the estimation. In Annex V an inaccuracy of less than 10% has been accepted. As a consequence the volume of the beachfill is not exactly a constant. This can be noticed from Table 2.2. The differences found in Table 2.3 have besides this aspect of inaccuracy also another cause, as will be explained later on in this paragraph.

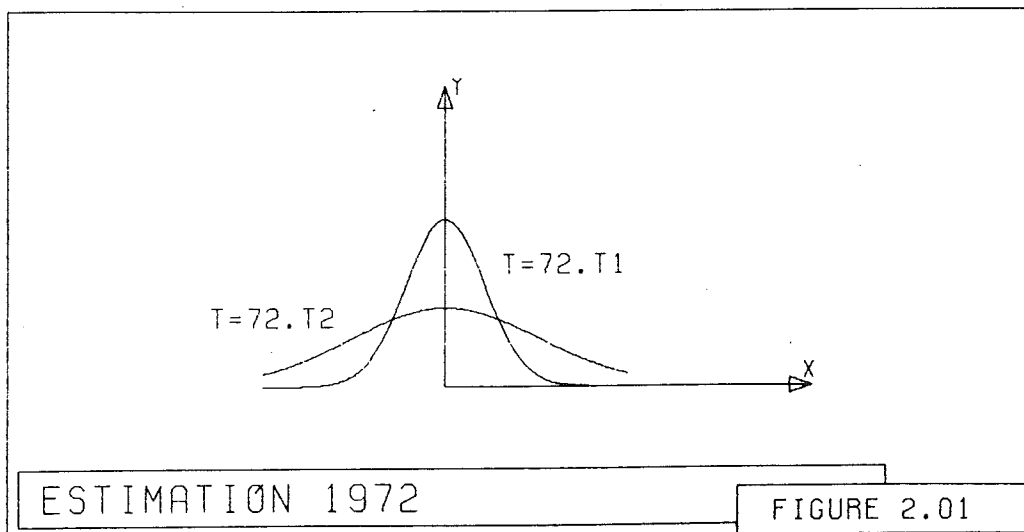
For the 1972 nourishment a profile height "h" of $h=6.0m$ has been posed. The volume of the beachfill "Vf" as follows from the observed volumetric changes has the value of $Vf=770,000m^3$.

The results of the computations for the 1972 nourishment are shown in Table 2.2. The values of "CA" and "CB" resulting from the fitting procedure are given as well as the resulting values of "Vf", which indeed show a less than 10% difference.

1972:		$h=6.0m, Vf=770,000m^3$		
	Time after t_1 [days] [yr]	CA [m]	CB [1/m ²]	Vf [m ³]
t_1	- -	222	$1.07E-5$	721,000
t_2	217 0.59	104	$1.85E-6$	813,000

Table 2.2 - Curve fitting 1972

These results are also graphically presented in Figure 2.01.

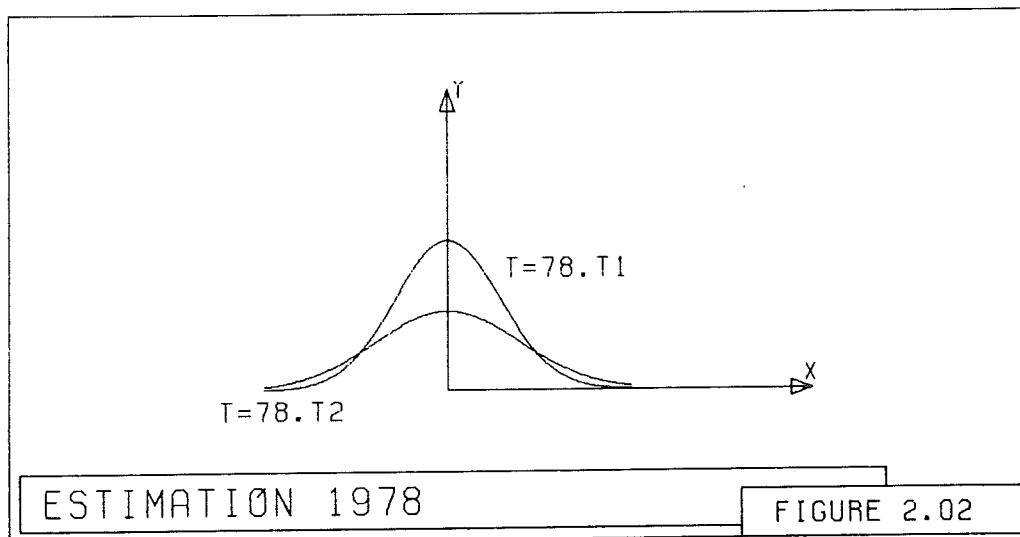


For the 1978 nourishment the same procedure has been applied. In this case a profile height of $h=8.5\text{m}$ has been used and for the replenished volume a value of $V_f=1,160,000\text{m}^3$ applies. The results can be found in Table 2.3,

1978:		$h=8.5\text{m}, V_f=1,160,000\text{m}^3$		
	Time after t1 [days] [yr]	CA [m]	CB [1/m ²]	Vf [m ³]
t1	- -	159	0.60E-5	978,000
t2	38 0.10	83.5	0.32E-5	683,000

Table 2.3 - Curve fitting 1978

A plot of both estimations can be found in Figure 2.02.



Comparing Table 2.2 and 2.3 the difference in the values of the profile height "h" is remarkable. In Section 1.2 (Par. height of the active profile) the meaning of the schematized profile height has been discussed and later on in Section 2.3 (Par. profile height) the influence of such a variation on the results will be analysed.

An other substantial (but in Annex V recognized) problem has been the occurrence of losses. In Annex V they are considered seaward losses, since along the shoreline a sufficient interval (up to 8000m) has been included in the volumetric computations.

This type of loss vanishes beyond the boundaries of the model and no compensation can be found within the model. In fact it creates the suspicion that the value of the profile height "h" has been chosen too small. This will be discussed further in Section 2.3 (Par. losses).

As a consequence one has to go to extra efforts to include those losses in the computation. It even could be posed that their occurrence is in contradiction with the model so that the usefulness of a further analysis of this data becomes questionable. However in Annex V it is decided to continue the analysis. There it is also attempted to account for the losses as much as possible.

In the case of Sylt the result of the losses has been that the three series of measured data have become insufficient for predictive purposes as meant in the beginning of this section. The losses were observed for both nourishments, between 72.t2 and 72.t3 for the 1972 fill. For the 1978 fill losses occurred in the initial phase, between 78.t1 and 78.t2 and also between 78.t2 and 78.t3.

This is the main reason for the differences in the values of "Vf" for the 1978 replenishment (see Table 2.3). In Section 2.3 (Par. losses) these losses will be discussed further.

For the 1972 nourishment the method to account for these losses has been the following:

- * The curve fitted estimation on $t=72.t2$ (see Table 2.2) has been used as a starting point.
- * However the volume of the fill has been set to $V_f=846,000m^3$, according to a unexplained increase of the fill volume reported by Dette [Det77]. In the literature [Füh76] this increase is considered to be caused by the interception of sand from the natural littoral transport.
- * An amount of loss (observed by Dette also) of about $259,000m^3$ over a beach length of about 3000m and a profile height of 6m corresponds with a retreat of the schematized coastline of 14.3m.
- * The measured serie 72.t3 has been shifted seawards over a distance of 14.3m to compensate for the loss.
- * The shifted coastline 72.t3 has been curve fitted.
- * The found curve has been shifted back landwards over the same distance of 14.3m to fix its real position.
- * With both the estimations of 72.t2 and 72.t3, and using (2.04), it is possible to compute the value of the coastal constant "s" based on this (second) observation period.

For the 1978 nourishment the conclusion was drawn of omitting the first serie of measured data "78.t1" from the calculations, because the interval of thirty-eight days with considerable loss was regarded too unreliable for a accurate estimation of the coastal constant "s".

The period between 78.t2 and 78.t3 has been analysed according to the same method applied to the 1972 fill:

- * The curve fitted estimation on $t=78.t2$ (see Table 2.3) has been taken as the starting point.
The fill volume has been $V_f=683,000m^3$.
- * The amount of observed loss of about $204,000 m^3$ over a beach length of 1600m and a profile height of 8.5m corresponds with a retreat of 15m of the schematized coastline.
- * The measured serie 78.t3 has been shifted seawards over a distance of 15m to compensate for the loss.
- * The shifted coastline 78.t3 is curve fitted.
- * The found curve is shifted back landwards over the same distance of 15m to fix its real position.
- * With both the estimations of 78.t2 and 78.t3, and using (2.04), it is possible to compute the value of the coastal constant "s" based on this observation period.

From this procedure it becomes clear that the procedure to compensate for the losses brings quite some trouble, while the value of the results remains uncertain. Especially the predictive value of the model disappears. It can be noticed also that in Annex V this purpose is not mentioned any further. So the calculation method A has provided three valuations of the coastal constant "s", which are summarized in the next table,

Data of	Applied method	s [m ³ /yr/rad]
72.t1 - 72.t2	eq.(2.04), original	0.96E6
72.t2 - 72.t3	eq.(2.04), with shift	1.20E6
78.t1 - 78.t2	data not considered	--
78.t2 - 78.t3	eq.(2.04), with shift	0.58E6

Table 2.4 - Results calculation method A

Calculation method B

Another way to use the available data is to analyse the displacement in time of the points of contraflexure of the Gaussian curve.

By curve fitting the measured data can be represented by a Gaussian curve. The previous paragraph has already shown some examples of that. From the formula of the Gaussian curve (2.01) the position of the points of contraflexure can be derived by differentiation to the place "x" twice and equalizing the result to zero. This yields,

$$y'' = (2*CB*x^2 - 1) * 2*CA*CB * \exp[-CB*x^2] = 0 \quad (2.05)$$

After using formula (2.03) to eliminate coefficient "CB" it can be concluded that the the position of the points of contraflexure is independent of the volume of the fill and can be described by,

$$xcf = \sqrt{\frac{1}{2 * CB}} = \sqrt{\frac{2 * (t+Ti) * s}{h}} \quad (2.06)$$

where CB : time dependent coefficient according to (2.03)
 h : schematized profile height
 s : coastal constant according to (1.07)
 t : time
 Ti : initial period between t=0 of the Dirac-function
 (fictive) and the time of placement of the fill
 xcf : x-coordinate of the contraflexure point

For the schematized profile height "h" an assumption is made and the curve fitted coastlines provide pairs of ("t","xcf") data for each nourishment.

In Annex V the policy of elaborating a hindcast computation with the first two measured series (t=t1 and t=t2) and after that a forecast computation to predict the shoreline position at t=t3 is not continued anymore. Instead for this calculation method the available data has been regarded all in one, where the following procedure has been chosen.

According to formula (2.06) the displacement in time of the contraflexure point in x-direction is proportional to the square root of the time. Therefore by plotting

1st: "xcf" against "sqrt[t]"
 or 2nd: "xcf²" against "t"

and by linear regression on this data a value for "s" (and "Ti") can be found.

As the rightmost expression of equation (2.06) can be written as follows,

$$x_{cf}^2 = \frac{2*s}{h} * t + \frac{2*s}{h} * T_i \tag{2.07}$$

the slope of the regression line represents the quotient of "2*s" and "h". Since a value of "h" is posed the value of the coastal constant "s" can be computed. The intersection of the regression line and the time-axis provides a value for "-Ti".

In Annex V the second of the previous two alternatives has been elaborated, "x_{cf}²" has been plotted against "t". The proposition that the computation is independent of the fill volume will be discussed in further detail in Section 2.3, but in Annex V the method of regarding the displacement of the points of contraflexure is considered there a very pleasant one because then the losses do not have to be accounted for.

Nevertheless there is a possibility of elaborating the measured data in two different ways:

- * The position of the contraflexure points is derived from direct curve fitting of the measured coastline. This option can be considered as "not taking in account the losses".
- * The position of the contraflexure points can also be derived from a curve fitted coastline, which is shifted seawards and landwards to account for the losses. This procedure of shifting the coastline has been described in the previous paragraph (Calculation method A). Strictly considered this is in contradiction to the pleasant property (no influence of losses) mentioned above.

The computations have been made in both ways for both nourishments.

The curve fitted estimations of the coastline for the data measured at t=72.t1, 72.t2, 78.t1 and 78.t2 have been treated before. They can be found in Table 2.2 and Table 2.3 (see Calculation method A).

In addition to this data the curve fitted estimations of t=72.t3 and 78.t3 are listed below,

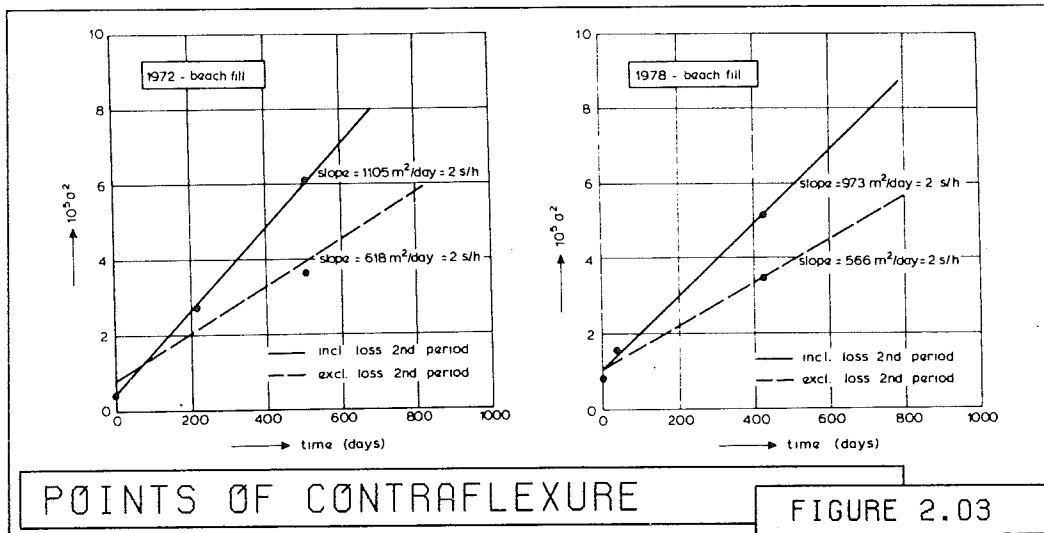
time	method	CA	CB	CS
72.t3	actual	60.1	0.14E-5	-
72.t3	shifted	72.3	0.83E-6	14.3
78.t3	actual	44.5	0.14E-5	-
78.t3	shifted	59.2	0.97E-6	15.0

Table 2.5 - Curve fitting for t=t3

In Table 2.5 both ways of analysis have been listed. With "actual" is meant a direct fit of the measured data, while "shifted" indicates a fit of a shifted coastline. The coefficient "CS" (in [m]) represents the shifted distance and the formula of the Gaussian curve is supplemented with this extra term,

$$y = CA * \exp[-CB * x^2] - CS \tag{2.08}$$

Using the coefficients "CB" from Table 2.2, 2.3 and 2.5 the leftmost part of equation (2.06) provides four values of the position of the contraflexure points for both replenishments. The square of these values can be plotted against the time, which has been done in Figure 2.03. This figure is a reproduction of originally two figures in Annex V (fig.2.3, page 8 and fig.2.5, page 14).



POINTS OF CONTRAFLEXURE FIGURE 2.03

In Annex V for each replenishment two regression lines are calculated, each based on three of the four available data points. The dashed lines represent the data not shifted, while the full lines correspond with the case of shifted coastlines.

The results of the calculations are summarized in the following table,

Fill	Method	s [m ³ /yr/rad]
1972	original	0.68E6
1972	shifted	1.20E6
1978	original	0.88E6
1978	shifted	1.50E6

Table 2.6 - Results calculation method B

Calculation method C

Wave data can also be used to compute a value for the coastal constant "s". The procedure to estimate its value involves using the wave data as a starting point. Through a refraction (and if necessary a diffraction) computation the wave characteristics at the edge of the breaker zone can be calculated. Henceforward the longshore transport capacity can be derived as a function of the place. Now using the definition of (1.07) it is possible to compute the coastal constant by taking the ratio of the change in sand transport capacity and the change in direction (rotation) of the coastline.

In Annex V a computer programme (KC programme) has been used to elaborate numerically the calculations just mentioned.

The programme is based on the one-line theory and uses a CERC formula, (1.15), alike approach of computing the transport capacity. It does only account for parallel depth-contours.

But as a whole the KC programme offers a rather wide choice of possibilities (see [Cas75] for documentation) such as for instance the option of taking in account the influence of diffraction around a breakwater. Between the wave data can be differentiated with respect to their direction, wave periods and wave heights. Tidal influence and wave set-up can also be included in the calculation, which finally produces a value of "s" based on all requested calculation options.

However since the treatment in Annex V of this type of computations is not very extensive it is not possible to point out exactly, which options of the KC programme have been used. But from Section 2.2.3 of Annex V it becomes clear that the wave data have been gathered from wave height meters, placed on different distances from the shore. Six direction sectors of 22.5 degrees have been taken in account, from South-South-West through West until North-West. For several periods the significant wave height has been calculated, but an overall average has been used as input for the programme. No remarks are made about the bottom schematization, which has to be done in accordance with the assumption of parallel depth-contours. Therefore it is not clear how exactly the refraction calculation has been executed.

Calculations have been made for the northern part of Sylt and for the southern part separately. The results are listed in the following table,

Location	s [m ³ /yr/rad]
northern part of Sylt	0.37E6
southern part of Sylt	0.82E6

Table 2.7 - Results calculation method C

Calculation method D

In Annex V also an attempt has been made to develop a more refined method of analysis for the case of Sylt.

This involves a description of the regression in time of the original coastline by a retreating parabola. When besides the replenished volume of sand is modelled by the shape of a flattening Gaussian curve, both effects can be analysed together using the method of superposition.

However seeing all problems with the previous three calculation methods such an analysis has to be regarded as too ambitious. The methods that are used in Annex V seem artificial and remarkable results are not obtained also.

Therefore this method will not be treated in further detail in this report. The computations in their original form can be found in Annex V, pages 16-35.

2.3 Comment on the one-line calculations

In this section the computations of Annex V will be discussed further in order to evaluate the usefulness of the applied methods. First a few general remarks will be made, later comment on the calculation methods will follow.

Losses

First striking problem has been that of the substantial sand losses from the considered area. Not only their occurrence itself but also the relation of the losses to the number of measurement series has appeared to be unfortunate.

As explained in Section 2.2 the calculation of a value for the coastal constant "s" requires measured data at two different points in time. So theoretically a number of three measurement series is sufficient for a hindcast computation of "s" using the first two and with this calculated result a forecast computation, which can be compared with the third series. However the occurred losses confuse the whole procedure.

As mentioned in Section 2.2 (Par. calculation method A) serious losses have been observed for both nourishments. Between 72.t2 and 72.t3 for the 1972 fill, in the initial phase, between 78.t1 and 78.t2 and also between 78.t2 and 78.t3 for the 1978 fill.

The 1972 losses seem to be caused by heavy storm conditions, while those of 1978 (first period) have to be regarded as initial losses in combination with a more than average amount of wave attack. For the second observation period of 1978 nourishment no explanation of the losses is given.

This kind of reasoning which can be found in Annex V and in the literature [Füh76] is not very satisfying. As already explained in Section 1.2 (Par. height of the active profile) the application of the one-line model demands a proper treatment of the equation of continuity (1.02). In this equation the transport of sand through the upper and the lower boundary of the budget area is assumed to be zero. Disregarding special situations (e.g. dredging from the budget area) losses other than through the (far) side boundaries of the budget area might not occur.

Thus the described seaward losses are in conflict with the propositions of the used model. No valid reasons seem available to identify a "special" situation in this case.

The observed losses can be explained by posing that the value of the profile height "h" has been chosen too small. Then sand passing through for instance the lower boundary of the budget area disappears from the model causing a loss.

It is exactly this essential choice of the profile height that plays an important role throughout the whole calculation. Its influence on the computations therefore will be investigated in further detail in the next paragraph.

Profile height

In Section 2.2 (Par. calculation method A) the differences in the value of the schematized profile height "h" have been noticed already.

In Annex V of the Manual three different values have been used for what should be regarded as three similar calculations: a computation of "s" according to the schematization of the one-line model.

To be more specific:

- * For the 1972 nourishment a profile height of $h=6\text{m}$ has been posed without any explanation. Probably this choice has been forced upon by the nature of the measured data. Only data has been available of volumetric changes above $\text{MSL}-4.0\text{m}$.
- * For the 1978 nourishment (calculation method A and B) a value of $h=8.5\text{m}$ ($\text{MSL}+4.5\text{m} - \text{MSL}-4.0\text{m}$) has been chosen because the loss percentage for a profile height of $h=11\text{m}$ ($\text{MSL}+4.5\text{m} - \text{MSL}-7.0\text{m}$) appeared to be only 3% higher.
- * For the 1978 nourishment (calculation method D) $h=12\text{m}$ ($\text{MSL}+4.0\text{m} - \text{MSL}-8.0\text{m}$) has been used in order to take all available data in account.

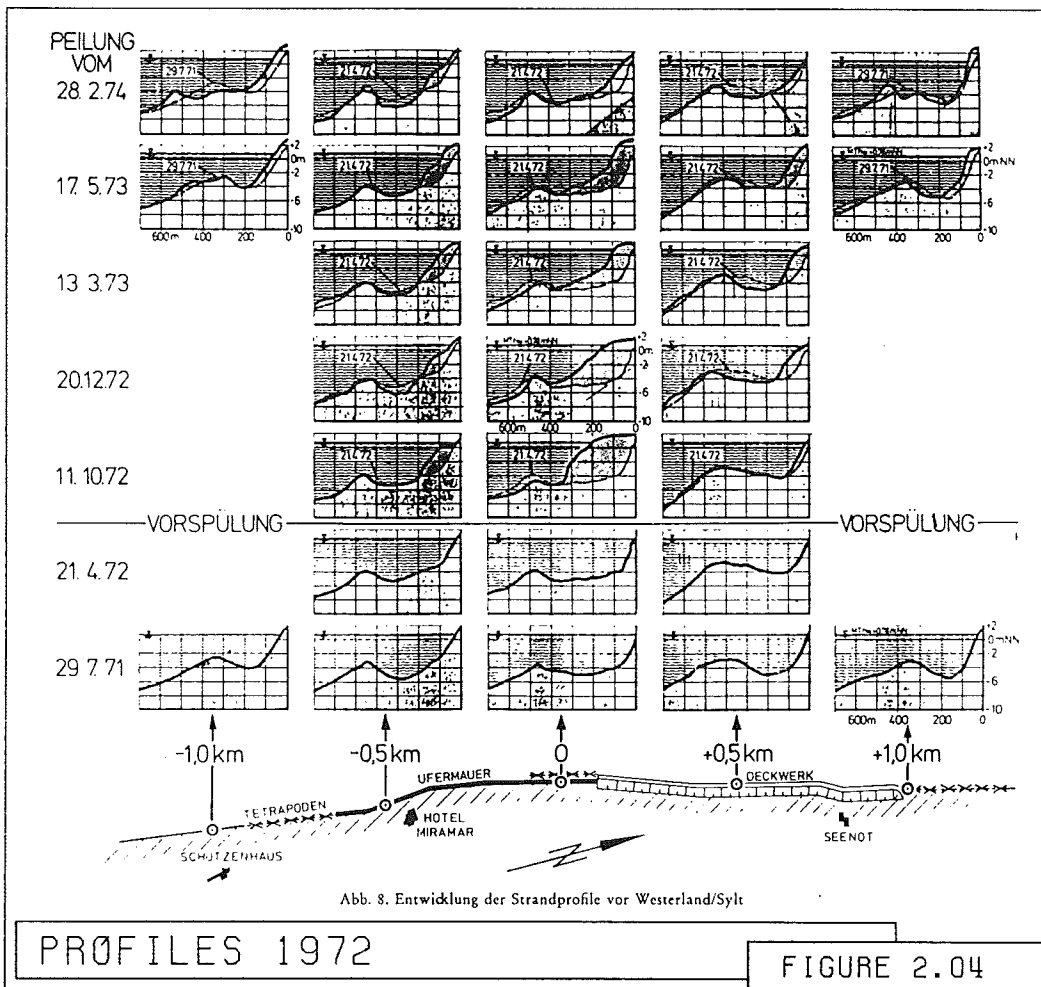
It thus becomes clear that the problem of finding an accurate profile height has been considered primarily as a difficulty within each separate calculation. No attention has been given to the fact that the differences between those values mount up a factor two.

Apparently when a lower boundary of the budget area had to be chosen, the standard of acceptation only has been, whether the representation of the fill volume just after nourishment is accurate enough. The possibility of future redistribution of sand along the cross-sectional profile has not been taken in consideration. In this way an important possibility for the occurrence of seaward losses has been created.

In Section 1.2 (Par. height of the active profile) the importance of using a fairly accurate value of "h" has been explained. It is necessary for a proper application of the equation of continuity, which is a budget equation.

From the evaluation of the 1972 replenishment by Führböter [Füh76] it can be seen that the a profile height of $h=6\text{m}$ is too small. This can be examined from volumetric computations, which show a decrease in time, and also from the cross-sectional profiles.

To illustrate this the following figure has been copied from [Füh76], page 41.



From Figure 2.04 it can be noticed that a profile height of $h=6\text{m}$ (MSL+2.0m - MSL-4.0m) is not sufficient. Obviously the part of the profile under MSL-4.0m is affected too by the nourishment.

Also in the "Plan for coastal protection of Sylt" [ALWH85] it is shown that there is a substantial transport of sand to the part of the coastal profile under MSL-5m. This applies to investigations carried out in the period 1978-1982.

As a result in this period after the 1978 nourishment volumetric changes are observed in the area between MSL-6.0m and MSL-8.0m. Therefore in this plan further investigations with a boundary of MSL-10.0m are suggested. This would give a profile height of about 14m.

It can be posed that the availability of the proper measured data is a necessity for the correct application of the one-line model. In Annex V this apparently has not fully been realized. Knowledge of behaviour of the volumetric changes above a level of MSL-4.0m only, yielding a profile height of $h=6\text{m}$, has to be considered as insufficient.

Looking at the profiles of Figure 2.04 and regarding also the available literature [Füh76], [ALWH85] it becomes clear that the lower boundary of the budget area should be taken at MSL-8.0m at least. The upper boundary of the budget area is dependent on the height up to which the nourishment has been constructed (MSL+4.0m). A realistic profile height therefore should be h=12m rather than h=6m.

Since the applied values of the profile height show a considerable divergence it is useful to analyse the influence of these differences on the results of the calculations. At first this will be carried out by means of analysing the sensitivity of the analytical solution (2.01) for the value of the profile height "h".

Therefore the formulas (2.01) through (2.03) have to be regarded. The profile height contributes to the coefficients "CA" (2.02) and "CB" (2.03). Using these formulas a shoreline position can be calculated, when for all influencing factors a value is chosen. Since this analysis is meant to investigate the influence of "h" on the shoreline position, only "h" will be varied during the process, while all other factors will have to remain constant.

Here the variation of the profile height has been achieved by choosing a constant reference value (h=12m) and defining a proportional disturbance "dh" (%) with respect to this reference value. With all other factors constant the coastline position "y" becomes a function of the profile height only, so $y=y(h)$. The coastline position for the reference value h=12m can be written now as "y(12m)" and for an introduced disturbance of dh% as "y(12m- 0.01 * dh * 12m)".

For "h" the value of h=12m has been chosen as a reference, because the investigations mentioned above lead to the conclusion that out of the three used values this is probably the most realistic one. The minus sign in above expression denotes a decrease of "h" with respect to its reference value. This because also smaller values of "h" have been used in Annex V.

To express in a dimensionless form the differences in coastline position that are introduced by a disturbance of dh% the following formula can be used,

$$\text{devy}(h) = \frac{y(12m * (1-0.01*dh)) - y(12m)}{y(12m)} * 100\% \quad (1.01)$$

Formula (2.09) gives the deviation in the shoreline position "y" as a result of a decrease of "h" of dh%. Hereby "devy(h)" is expressed as a percentage of deviation with respect to the undisturbed (dh=0) situation.

Formula (2.09) can be evaluated for different values of "dh", when for all other factors a (constant) reference value is chosen. The following values have been used in this analysis:

- h = 12.0 [m]
- s = 1.00E+6 [m³/yr/rad]
- t+Ti = 1.0 [yr]
- Vf = 1.00E+6 [m³]
- x = 0 (400) [m]

Combining (2.01) - (2.03) and (2.09) it can be noticed that the value of the volume of the beachfill "Vf" is unimportant. But to achieve realistic coastline positions this factor is not omitted.

For the coastal constant "s", the time factor "t+Ti" and the fill volume "Vf" reference values have been chosen, which are rather "neat" and which lie within the range of their own occurrence. Hereby it has been noticed that in this way the value of "Ti" will be a constant in this analysis. When the value of "h" is changed this perhaps is not a realistic starting point. However the combined influence of "Ti" and "h" will be the next item of investigation (see next page).

As mentioned before a reference value of h=12m has been chosen for the profile height.

The analysis has been carried out for the top of the beachfill (x=0) and for x=400m, which is about the x-position of a point of contraflexure (the exact x-position: xcf=408.24m). The results can be found in table 2.8.

	x=0m		x=400m	
dh [%]	devy [%]	y [m]	devy [%]	y [m]
0	0	81.4	0	50.4
1	0.5	81.8	1.0	50.9
5	2.6	83.5	5.1	53.0
10	5.4	85.8	10.6	55.7
25	15.5	94.0	30.2	65.6
40	29.1	105.1	56.4	78.8
50	41.4	115.2	79.8	90.6
75	100.0	162.9	186.7	144.5
90	216.2	257.5	387.1	245.4

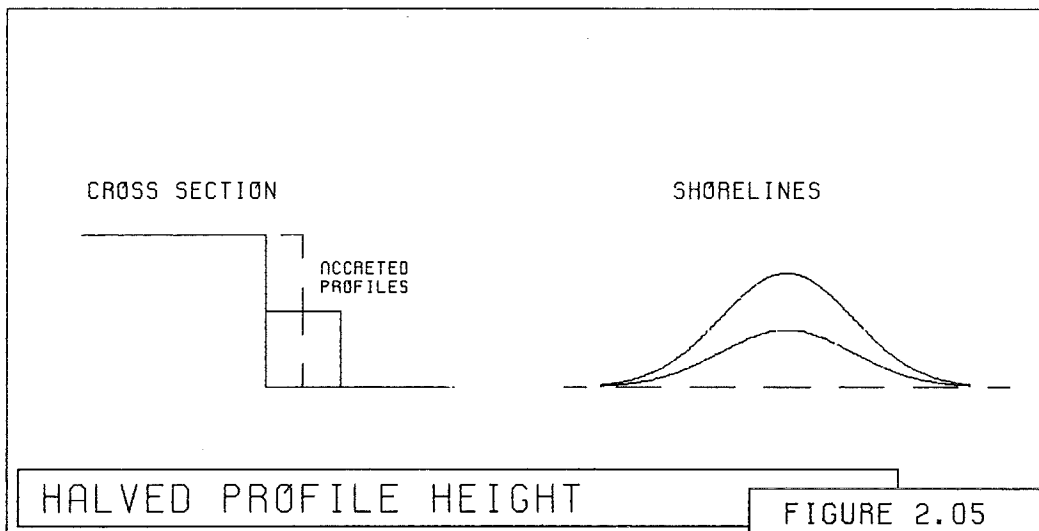
Table 2.8 -- Influence of dh

For the case of x=0m can be noticed that a 50% lower value of dh (thus dh=6m) yields a 41% higher value of "y". For dh=25% (thus h=9m) the deviation amounts 16%. These are rather substantial differences.

For increasing "x" the absolute deviation caused by a given percentage of "dh" is about (not exactly) constant. However because the value of "y" decreases the relative deviation only increases.

Returning to the case of $x=0m$ it can be concluded that when a deviation of about 5% is accepted for this particular situation a profile height has to be known with an accuracy of about 10%.

The results given in Table 2.8 are rather remarkable. A non-linear relation between "dh" and "devy" is found. This is a little unexpected because the method in which the coastline position "y" is usually determined suggests a linear relation. This is illustrated in Figure 2.05.



A replenished volume of sand distributed over a profile height twice as small yields a coastline with y -coordinates twice as large.

Therefore the same problem is looked at in somewhat different way. Instead of starting at a variation in "h", while all other factors remain constant, a condition of proportionality will be set on the initial situation at $t=0$. Thus it is posed that on $t=0$ the y -position of the shoreline has to be proportional to the profile height, just as Figure 2.05 suggests. As can be seen by considering formula (2.01) this condition can be satisfied by posing that regardless the value of "h" the coefficient "CB" (2.03) at $t=0$ has to have a constant value. In fact this describes a situation denoted as "situation 2)", which will be treated in full detail later on in this section (Par. about calculation method B).

For now the following example will illustrate the plausibility of this approach. Given are two chosen values for the profile height, $h_1=h$ and $h_2=2*h$.

Under the condition that "CB" is a constant, formula (2.03) yields, when $t=0$ and "s" is a constant, a twice as large value of "Ti2" with respect to "Ti1" ($Ti_2=2*Ti_1$, "Ti1" and "Ti2" correspond to "h1" and "h2"). With this result and under the important assumption that the fill volume "Vf" is a constant it can be seen from formula (2.04) that at $t=0$ the value of

the coefficient "CA2" (and thus of the shoreline position) has a value $\sqrt{4}=2$ times smaller, which corresponds exactly to the condition of proportionality.

It is clear now that a chosen value of "h" has a considerable influence on the value of "Ti" and thus also affects the analytical solution (2.01).

In order to analyse the influence of the combined changes in "h" and "Ti" on the calculated shoreline position the behaviour of the analytical solution for three different values of "h" has been computed. The values used are given in Table 2.9.

h [m]	Ti [yr]	other constants
6.0	0.15	s = 1.00E6 m ³ /yr/rad
8.0	0.20	Vf = 1.00E6 m ³
12.0	0.30	

Table 2.9 -- Used values of h

So in this analysis three different values of "h" have been used, which correspond closely to those applied in Annex V. Hereby the values of "Ti" have been appropriately adjusted in order to satisfy the condition of proportionality.

In the initial situation at $t=0$ all three analytical solutions are equivalent. They can easily be compared when multiplied by a correction factor. The solution of $h=6\text{m}$ has been taken as a starting point. Thus the values of the solutions of $h=8\text{m}$ and $h=12\text{m}$ have been multiplied by the factors "4/3" and "2" respectively in order to obtain quantitatively comparable results.

The behaviour in time of the three different solutions has been computed. The results can be found in the Tables 2.10 and 2.11. They are graphically illustrated in the Figures 2.06 through 2.09.

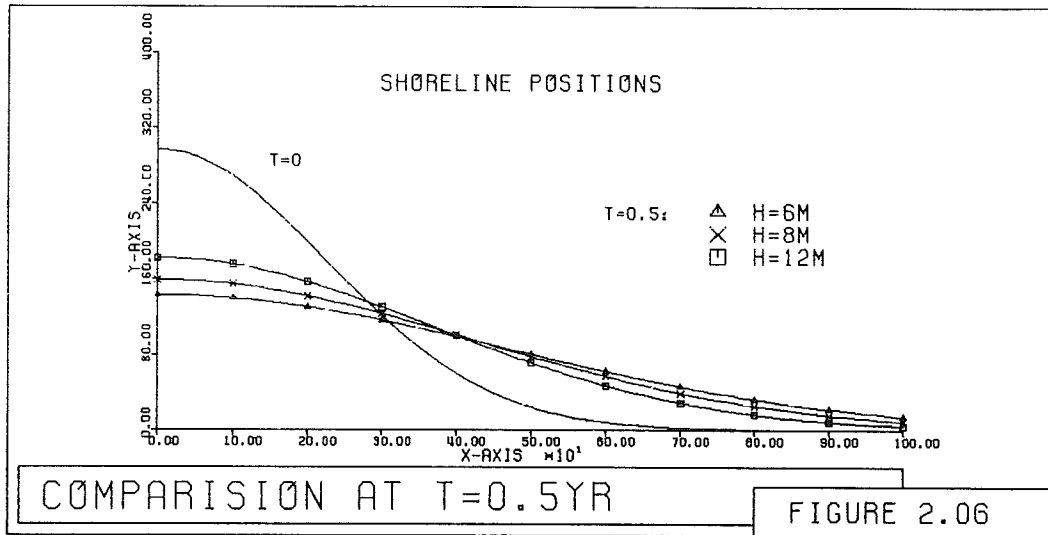
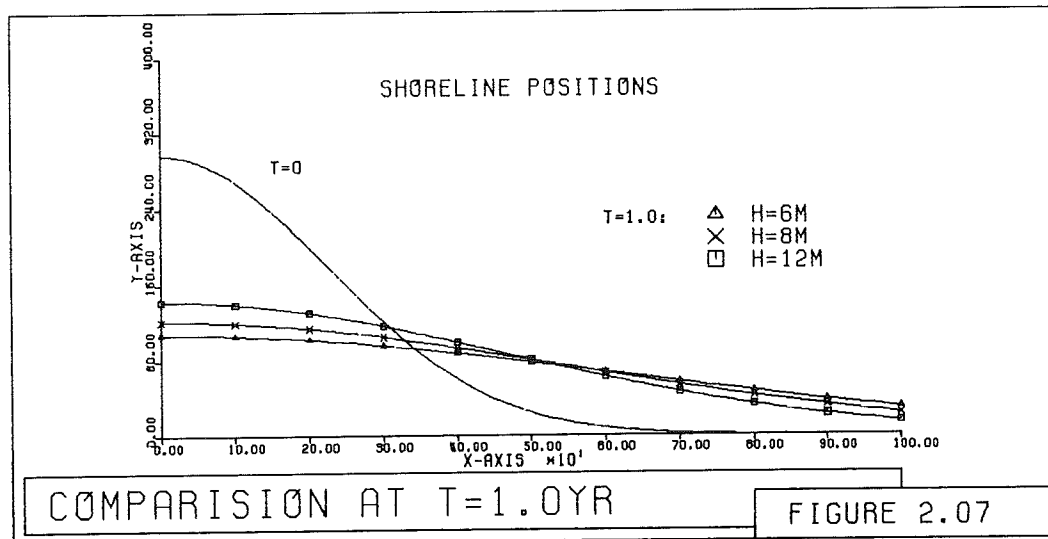


Figure 2.06 shows the situation at t=0.5yr and also for t=1.0yr a similar plot can be made (Figure 2.07). As mentioned before the shoreline y-coordinates corresponding to h=8m have been multiplied by a correction factor of "4/3" in order to obtain quantitatively comparable results. For the case of h=6m a correction factor "2" has been used.



And also for $t=1.5\text{yr}$ (Figure 2.08) these three values of the profile height show the same tendencies. In each figure the shoreline at $t=0$ has also been plotted in order to give a more clear impression of the absolute differences.

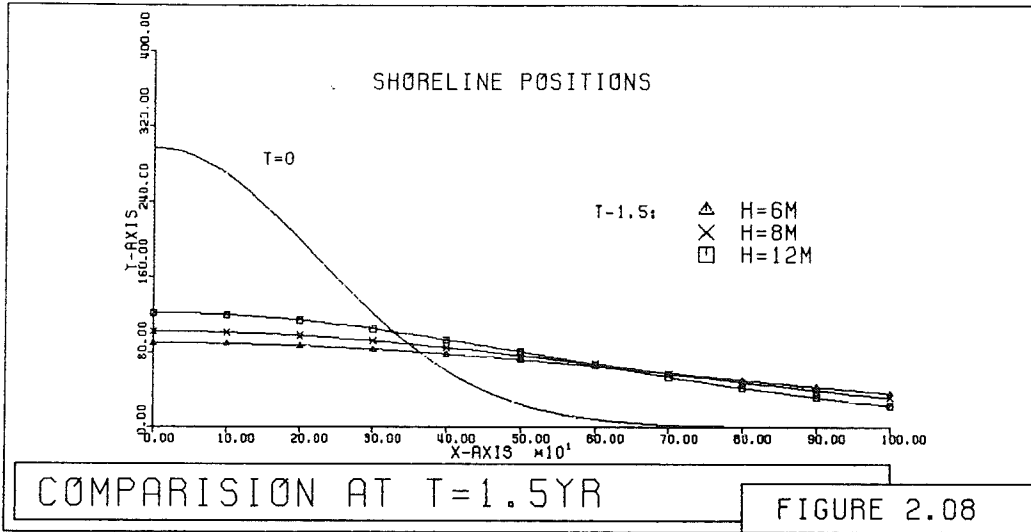
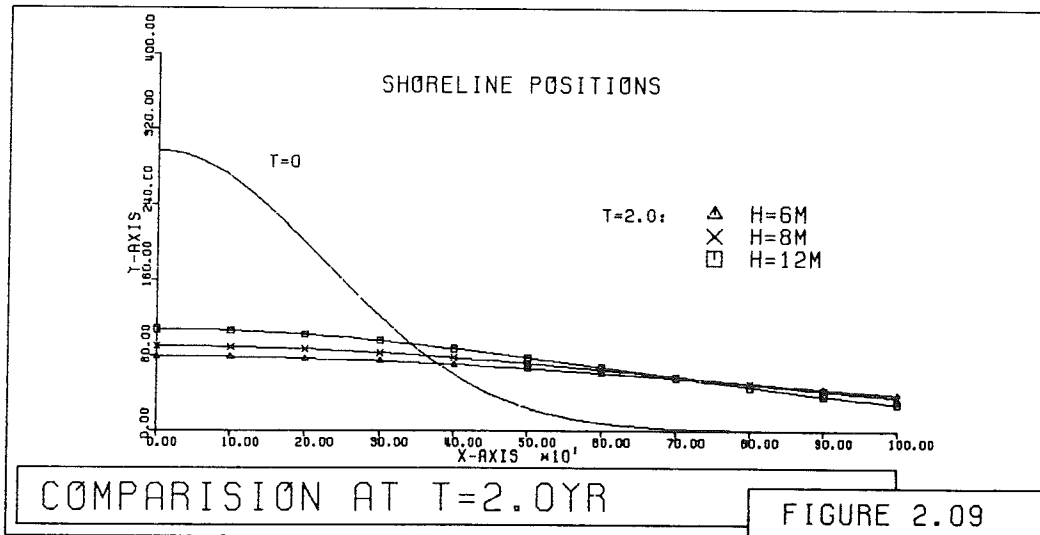


Figure 2.09 shows the situation at $t=2.0\text{yr}$.



The previous figures give a good impression of the influence of the profile height. At $x=0\text{m}$ a larger value of "h" causes the shoreline to be positioned more seawards. For the far sides of the fill (for instance $x=1000\text{m}$) a reversed effect can be noticed. This behaviour can be understood when it is realized that the shoreline modelled with the smallest value of "h" shows the largest gradients in x-direction. Thus a larger regression of the top of this shoreline can be expected. The differences in coastline position can be evaluated quantitatively from Table 2.10 for $x=0$.

time [yr]	shoreline position (y) at $x=0$		
	h=6m [m]	h=8m [m]	h=12m [m]
0.0	297.36	297.36	297.36
0.5	142.48	158.94	182.09
1.0	107.39	121.39	142.84
1.5	89.66	101.99	121.39
2.0	78.54	89.66	107.39

Table 2.10 -- Numeric values for $x=0$

In Table 2.11 the results for $x=1000\text{m}$ are given numerically.

time [yr]	y at $x=1000\text{m}$		
	h=6m [m]	h=8m [m]	h=12m [m]
0.0	0.01	0.01	0.01
0.5	14.21	9.13	4.28
1.0	29.14	22.93	14.21
1.5	36.12	31.45	22.93
2.0	39.03	36.12	29.14

Table 2.11 -- Numeric values for $x=1000\text{m}$

Table 2.10 and 2.11 show once again the equivalence of the solutions at $t=0$. However the differences between them that develop in time are rather substantial.

At $x=0$ they come to about 30 meters (=10%). Moving sideways along the x-axis the influence of the profile height decreases at first, but increases again as soon as the points of contraflexure have been passed. At $x=1000\text{m}$ differences are found of about 15 meters, which yields a considerable relative error.

It thus can be concluded that the influence of value of the profile height on the behaviour of the analytical solution is rather substantial. The computed shoreline position is strongly dependent on the value of "h".

A difference in this value of a factor two introduces a computed difference in shoreline position of about 10%. Since this is of the same order as the found inaccuracy of the calculation methods, it is very important to give good attention to the choice of "h".

From this result it also becomes obvious that a calculation method based on the displacement in time of the points of contraflexure has to be preferred to a method that uses the behaviour of the top of the beachfill as essential data. The first one will not be influenced by the profile height.

The previous analysis shows the error in the shoreline position when an inaccurate value of the profile height is used and all other values are known (exactly). Especially this applies to the value of the fill volume "Vf", which is assumed to have a constant value in time.

Thus the previous analysis is valid for the following case.

- * A correct profile height has been chosen and the budget area encloses the nourished volume of sand during the whole observation period.
- * An accurate shoreline position can be found using the analytical solution of (2.01).
- * The analysis shows the effect of an enlarged profile height with respect to this correct value.

However when as has been the case in Annex V a too small value of "h" has been used the situation is a little different. Because the budget area is taken too small now, transport of sand through the upper and (or) lower boundary can occur. When this happens, as has been apparently in the case of Sylt in Annex V, the volume of the fill can not be considered a constant anymore. The previous analysis then loses a part of its value.

That the profile height is a factor, which has to be determined with care, remains valid, but the influence of this too small profile height on the accuracy of the computations becomes unpredictable. The moment at which the loss occurs, the place from where it is eroded and the volume of the eroded sand become important. The one-line model can not be properly applied in such a situation.

There is also another effect that confuses a proper analysis of such a situation. In Annex V two different shoreline positions have been compared using the same inaccurate value for both of the situations. So the question could be asked how bad it will be to do it wrong consistently. Unfortunately also this effect is not easy to quantify.

Small angle of wave incidence

As explained in Section 1.2 (Par. equation of motion) the condition that the angle of wave incidence relative to the coastline direction can be regarded small is an important one when employing a line-type kind of modelling. The justification of the simplifications (1.06) and (1.07) is dependent on the validity of this assumption.

The accepted inaccuracy of the computation limits the applicability of the theory. Considering the generally rather global nature of the coastline calculations a value of about 30 degrees (0.52 rad) is often accepted, but a more strict limitation can also be used. In Table 1.1 (Section 1.2 also) an indication has been given of the introduced error through comparison with the CERC formula.

In the same section some remarks have been made about a "replenishment situation" (Figure 1.05).

Indeed when looking at Figure 1.05 the strong suggestion is raised of substantial angles of wave incidence. So serious doubts arise about the correctness of the used calculation methods.

When in the case of Sylt the occurring relative angles have been too large indeed, the application of the one-line theory is not allowed or usable only under the acceptance of considerable inaccuracy.

To investigate how bad the situation really has been it is most convenient to return to the curve fitted approximations of the coastline of calculation method A.

Table 2.2, 2.3 and 2.5 provide the curve fitted estimations of the coastline assuming a Gaussian curve shape.

As is known from mathematics the extreme values of the slope of a function occur at the points of contraflexure. Since a Gaussian curve can be expressed in formula (2.01), the necessary values can be derived easy.

The leftmost part of equation (2.06) gives an expression for the x-position of the points of contraflexure,

$$x_{cf} = \text{sqrt} \left[\frac{1}{2 * CB} \right] \quad (2.10)$$

This formula is valid for the rightmost part of the Gaussian curve (positive values of "x"). For the leftmost part of the beachfill the difference is just a minus sign. Because that produces positive angles, here the leftmost part of the fill will be regarded. Differentiation of (2.01) to the place once and substituting "-x_{cf}" for "x" yields with (2.10),

$$y'_{cf} = CA * \text{sqrt} [2 * CB] * \text{exp} [-0.5] \quad (2.11)$$

By taking the inverse-tangent function this slope can be reduced to an angle,

$$\beta_{cf} = \text{arctan} [y'_{cf}] \quad (2.12)$$

With the coefficients "CA" and "CB" from the Tables 2.2, 2.3 and 2.5 and using the formulas (2.11) and (2.12), the maximum occurring angle of the coastline with respect to the x-axis can be computed. Of course this relates to an angle according to the modelled coastline.

time	dt [yr]	CA [m]	CB [1/m ²]	y' of	Bcf [deg]
72.t1	-	222	1.07E-5	0.623	31.9
72.t2	0.59	104	1.85E-5	0.384	21.0
72.t3	1.38	60.1	0.14E-5	0.061	3.5
78.t1	-	159	0.60E-5	0.334	18.5
78.t2	0.10	83.5	0.32E-5	0.128	7.3
78.t3	1.17	44.5	0.14E-5	0.045	2.6

Table 2.12 - Maximum angle of coastline

In Table 2.12 an indication can be obtained of the occurring angles. It gives a rather logical picture of gradually flattening Gaussian curves. The value of the coefficients "CA" and "CB" decreases in time and so does the maximum coastline angle.

As a first obvious result it can be noticed that the (modelled) coastline angles (with respect to the x-axis) are tolerably small. This is certainly valid for the 1978 nourishment, where the angles are smaller than twenty degrees, but also for the 1972 nourishment although this situation is a little less favourable.

Of course the angle of wave incidence with respect to the actual coastline is also dependent on the angle of wave incidence itself (on deep water), the refraction pattern and the real shape of the coastline (there is a deviation from the model).

To evaluate the applicability of the one-line model the angle of wave incidence relative to the coastline direction (1.14) has to be considered. As a standard (1.24) can be used.

In Appendix B the wave data as presented in Section 2.2.3 of Annex V has been used to compute an average angle of wave incidence " ϕ_0 ". From this appendix it can be seen that this average angle yields about $\phi_0=39$ degrees.

Accepting only a range of $\beta_a=20$ degrees the one-line model is valid only for that part of the Gaussian curve within the interval of 19 degrees to 59 degrees. This restricts the application of the one line model to the starting situation of the 1972 nourishment and yet only for a limited part of that starting curve. A more generous acceptance, $\beta_a=30$ degrees for instance, does not improve the possibility to use this schematization.

Using (1.24) as a standard it thus can be concluded that the application of one-line model for this particular situation of Sylt is not allowed.

About calculation method A

Starting point of method A is the behaviour in time of formula (2.04). It is a rather obvious method, which in the case of Sylt is obstructed by the occurring losses. These change not only the volume of the fill but may influence also the value of the initial period "Ti" (see next paragraph for a further discussion on this last point).

An acceptable way of compensating for the decreasing volume of the fill "VF" is the method of shifting the coastline. In fact such an approach consists of considering the original coastline as gradually retreating, while the fill volume stays constant.

In formula this can be expressed as a modification of (2.08),

$$y = CA * \exp[-CB*x^2] - \frac{P}{h} * t \quad (2.13)$$

Where P : volume of loss per meter coastline per year

The leftmost part of the formula represents the Gaussian curve and the rightmost part expresses a uniform erosion of the coastline in time.

As long as the erosion is distributed uniformly along the entire fill this is a valid schematization. If however the losses occur locally, concentrated at a certain part (for instance the top) of the fill, such a schematization gives problems.

As mentioned before in this section (Par. profile height) the application of this method demands an accurate value of the profile height "h". This has been a serious problem in the computations of Annex V. When the value of "h" is not known too well or when the necessary data are not available calculation method B should be preferred.

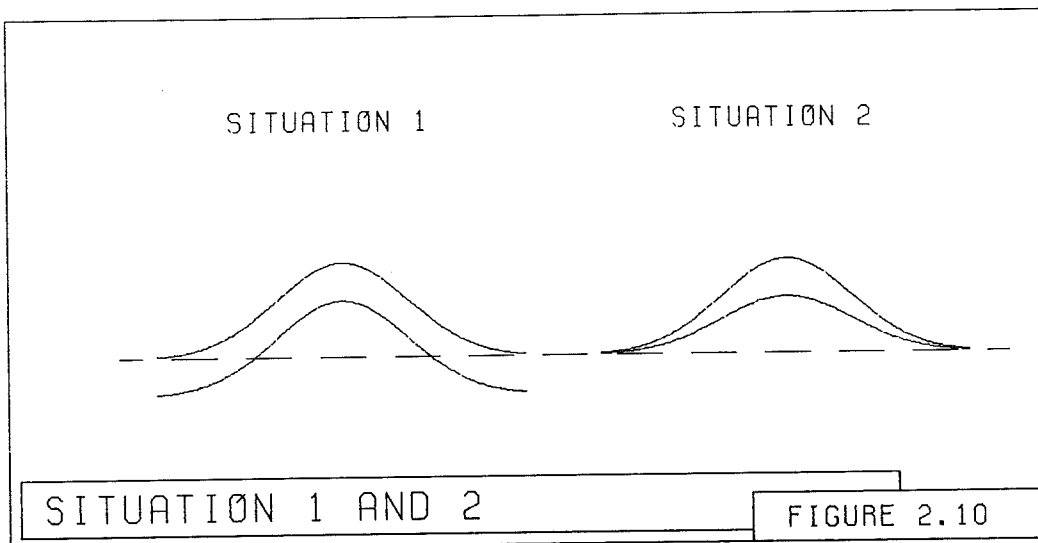
About calculation method B

This method looks at the displacement in time of the points of contraflexure of the Gaussian curve. As already stated in Section 2.2 formula (2.06) suggests that the x-position of these points "xcf" is independent of the volume of the beachfill. Therefore in Annex V this method is considered to be uninfluenced by the losses. Obviously this should be a very convenient method.

Unfortunately this point of view is not fully correct. Because analysing further the value of "xcf" should not change in the case of occurring loss. There are two situations in which such a schematization is justified.

- 1) The first one is already discussed in the previous paragraph. The loss is assumed to be uniformly distributed along the entire fill. Now it can be posed that the original coastline erodes while the beachfill keeps it supplemented volume. The development of the fill in time then follows the behaviour of the analytical (Gaussian curved) solution. An other way to look at it is that the coastline can be shifted as a correction for the losses.
- 2) The second one can be derived from the formulas (2.01) through (2.03) and (2.06). If it is proposed that the value of "xcf" remains unchanged when losses occur at a given time "t", formula (2.06) implies that "CB" has to be a constant. Returning to (2.03) the value of "Ti" has to be constant also, because "s" and "h" are constants. Now evaluating (2.02) and (2.01) a change in fill volume "Vf" has a direct effect on the value of "y", since all other variables are constant. This means that the y-position of the shoreline (for all x) is proportional to the volume of the fill. Therefore when losses cause a decrease of "Vf" (say with z%) the y-position of the entire Gaussian curve changes proportionally (z% with respect to its previous value). So this is a way to compensate for the loss, where the beachfill is posed to be eroded while the original shoreline is regarded immovable.

Both situations are illustrated in Figure 2.10, left for the first schematization, an eroding original coastline and a fixated fill volume, right for the second schematization, an eroding beachfill and a fixated original coastline.



Again it can be concluded that the nature of the erosion prescribes whether calculation method B can be used or not. If a schematization according one of the two situations above is not valid the factor "Ti" becomes important.

From (2.02) it can be noticed that "Ti" is a parameter of the model, which fits the relation between the used mathematical solution and the measured data at a given point of time, when "Vf", "h", "s" and the coastline position "y" are known. Unless one of the two previous ways of compensating the loss can be applied a change in "Vf" will affect "Ti" also. Its value then has to be recalculated and method B can not be used.

About calculation method C

Deriving a value of the coastal constant "s" from the wave data through the KC programme is a known method, which is valid when appropriate schematization is possible.

As far as it concerns the wave data there seems little problem. The wave data cover an extensive period of time and are (partly) measured close to the area of nourishment.

About the bottom schematization, as already mentioned in Section 2.2, no explanation is given in Annex V. However the computations have been carried out not for the exact nourishment situation, but just for two different coastal directions (one representative for the northern part of Sylt, the other for the southern part). So problems with respect to the geometry of the applied model are not to be expected.

A notable result can be achieved when the values for the transport capacities necessary to derive a coastal constant (according to (1.07)) are considered. In Annex V these are given (see also Appendix B):

$$S = 580,000 \text{ m}^3/\text{yr} \text{ for the northern part}$$

$$S = 410,000 \text{ m}^3/\text{yr} \text{ for the southern part}$$

Also given is a difference of 17 degrees (0.30 rad) in coastal direction. Since the replenishment area is about in the middle part of the island the coastal constant can be approximated by applying its definition (1.07) on this data. This yields,

$$s = \frac{\text{difference in "S"}}{\text{difference in "B"}} = \frac{170,000}{0.30} = 0.57\text{E}6 \text{ m}^3/\text{yr}/\text{rad}$$

This is a result that agrees rather well with the values of calculation method D (1978) and also with some of the other results (see Table 2.13 and 2.14).

Both the results from Table 2.5 and also this last estimation reflect the influence of the sinusoidal shape of Figure 1.06 on the definition of the coastal constant "s" according to (1.07).

Only when its value is defined more strictly (as mentioned in Section 1.1, Par. assumptions and as suggested in the conclusions, Chapter 3) the coastal constant "s" can really be a "constant".

Comparison

In the following tables the results of the calculation methods will be compared. For the 1972 nourishment,

Calculation method	Coastal constants [m ³ /yr/rad]	Remarks
A	0.96E6	unshifted
A	1.20E6	shifted
B	0.68E6	unshifted
B	1.20E6	shifted
C	0.37E6	northern
C	0.82E6	southern

Table 2.13 -- Comparison of 1972 nourishment

And for the 1978 nourishment,

Calculation method	Coastal constants [m ³ /yr/rad]	Remarks
A	--	unshifted
A	0.58E6	shifted
B	0.88E6	unshifted
B	1.50E6	shifted
D	0.66E6	total

Table 2.14 -- Comparison of 1978 nourishment

First all computed values can be regarded in order to obtain an overall view of the results.

It becomes clear that the deviation is rather wide, viz. from "0.37E6" for the 1972.C option to "1.50E6" for the 1978.B option. This is about a factor four. Through leaving this last two (in fact extreme) values out, a range from "0.58E6" to "1.20E6" results, which is about a factor two.

Secondly the comparison can be made between the 1972 and the 1978 nourishment. Hereabout there is little specific information to extract.

The impression that the values of 1972 are relatively higher is frustrated by the low value of "0.37E6" and for the 1978 nourishment the opposite is valid. The values seem relatively lower but there also occurs a calculated value of "1.50E6". Thus a conclusion about whether the coastal constant behaves really as a "constant" in time or not can not be drawn.

Thirdly the results with respect to one replenishment can be considered.

For both nourishments it can be noticed that shifting of the coastline results in higher values of "s". With method A this is a relatively small increase, with method B the increase amounts about a factor two.

As has been previously explained the nature of the losses determines the suitability of the method and of the submethod (shifting or not shifting).

From the data gathered in Table 2.13 and Table 2.14 it is difficult to draw further specific conclusions about which method should be preferred.

Sensitivity analysis

The first comparison has shown a substantial deviation in computed values for the coastal constant "s".

To analyse the influence of the inaccuracy of "s" the same method can be used as has been applied for "h" earlier in this section (Par. profile height). The effect of a disturbance in "s" can be evaluated through regarding the combined result of the formulas (2.01) through (2.03). The constant "s" appears in "CA" and in "CB". Similar to (2.09) the effect of a disturbance of "ds%" of "s" on the shoreline position "y" can therefore be derived from (2.01) as,

$$\text{devy}(s) = \frac{y(s*(1+0.01*ds)) - y(s)}{y(s)} * 100\% \quad (2.14)$$

For different values of "ds" a value of the expression (2.14) can be computed, when for all other factors a constant reference value is chosen. Hereby the effect of positive (denoting an increase of "s") and negative values of "ds" (denoting a decrease) has been investigated. In this analysis the following values have been used:

h = 12.0 [m]
 s = 1.00E6 [m³/yr/rad]
 t+Ti = 1.0 [yr]
 Vf = 1.00E6 [m³]
 x = 0 [m]

The results can be found in Table 2.15.

ds [%]	devy [%]	y [m]
-50	41.4	115.2
-25	15.5	94.0
-10	5.4	85.8
-5	2.6	83.5
-1	0.5	81.8
0	0	81.4
1	-0.5	81.0
5	-2.4	79.5
10	-4.7	77.6
25	-10.6	72.8
50	-18.4	66.5

Table 2.15 -- Influence of ds

From Table 2.15 it can be seen that a 50% higher value of "s" causes a 18% smaller value of "y", which in the case of Sylt corresponds with a difference in coastline position of about 10m to 20m. This also is of the same order as the found inaccuracy of the used calculation methods.

3 Conclusions and recommendations

Now the results of the analyses of chapter 1 and 2 will be summarized. A short motivation of each conclusion will be given, but further justification can be found in the indicated sections.

Conclusions

- 1) It is most convenient to consider the longshore transport capacity "S" a function of "Br" only. Hereby "Br" is the angle of wave incidence relative to the coastline direction (1.14).

explanation: It is not impossible to look at "S" as a function of "β" but that complicates the analysis, because the angle of wave incidence "φ0" has to be regarded also.

<Sect. 1.2, Par. equation of motion>

- 2) By (1.07) the value of the coastal constant "s" is not defined uniquely. A better definition should be,

$$\left. \begin{array}{l} dS \\ \hline dBr \end{array} \right|_{Br=0} = -s \quad (3.01)$$

explanation: By this definition the value of the coastal constant is related to the tangent in Br=0 of the transport capacity function (Figure 1.06).

<Sect. 1.1, Par. assumptions>

<Sect. 1.2, Par. equation of motion>

- 3) For the analysis of the 1972 nourishment at Sylt in Annex V an inaccurate profile height has been used. The results therefore may not be considered as reliable, although they do not have to be inaccurate.

explanation: A profile height of h=6.0m has been used in that analysis. In the literature [Füh76] and [ALWH85] it has been shown that this value is too small. The computed values however are within the range of the 1978 results.

<Sect. 2.3, Par. profile height>

- 4) In order to achieve an accuracy of 5% in the position of the shoreline the profile height has to be known with an accuracy of about 10%.

explanation: This result is valid for the top of the fill. For increasing or decreasing "x" the situation is worse.

<Sect. 2.3, Par. profile height>

- 5) The shoreline position calculated by means of the analytical solution (2.01) is strongly dependent on the chosen value of the profile height "h". Its value therefore has to be determined with care. When the value of "h" is in doubt calculation method B should be applied rather than calculation method A.

explanation: The influence is especially significant at the top of the beachfill (x=0). The shoreline position at the points of contraflexure is uninfluenced by "h".

<Sect. 2.3, Par. profile height>

- 6) The application of the one-line model in the case of Sylt is not allowed when (1.24) is accepted as a standard.

explanation: The representative angle of wave incidence relative to the direction of the unreplenished shoreline yields 39 degrees. So the occurring relative angles are too wide.

<Sect. 2.3, Par. small angle of wave incidence>

- 7) If the occurring losses can be modelled according to situation 1), an eroding original coastline and a fixated fill volume, or 2), an eroding beachfill and a fixated original coastline, of Figure 2.10, calculation method B can be used.

explanation: This method is a very convenient one since its results are not influenced by the exact amount of loss. However the nature of the losses determines the suitability of this method.

<Sect. 2.3, Par. about calculation method B>

- 8) The sensitivity of the calculated coastline positions at Sylt to the computed variation in the value of the coastal constant "s" appears to be of the same order as the inaccuracy found.

explanation: Therefore it is not clear if the inaccuracy of the model causes the deviation in "s" or that other factors are responsible.

<Sect. 2.3, Par. sensitivity analysis>

Recommendation

Further analysis with respect to the definition of the one-line model is necessary.

explanation: Then it will be easier to determine the standards, which have to be used when the applicability of a one line model is in question.

References

- [ALWH85] Amt für Land- und Wasserwirtschaft Husum.
Fachplan Küstenschutz Sylt (Plan for coastal protection of Sylt).
Husum, January, 1985
- [Cas75] Casteleyn, J.A.
Numerieke berekeningen van de verandering van kustvormen onder invloed van golfaanval (Numerical computations of shoreline changes under wave attack).
Delft University of Technology, Department of Civil Engineering, Section Coastal Engineering, September, 1975
- [CERC87] Coastal Engineering Research Center.
Analytical solutions of the one-line model of shoreline change.
Vicksburg (MS), October, 1987
- [Det74] Dette, H.H.
Wellenmessungen und Brandungsuntersuchungen vor Westerland/Sylt (Wave measurements and surf investigations at Sylt/North Sea).
Mitteilungen Leichtweiss-Institut für Wasserbau, Heft 40, 1974
- [Det77] Dette, H.H.
Effectiveness of beach deposit nourishment.
ASCE, Coastal Sediments '77,
Charleston, November, 1977
- [DH82] Delft Hydraulics.
Morphological aspects of Gioia Tauro Port, Theoretical analysis of shoreline computations.
Report M1857 (Vol.II), December, 1982
- [DUT80] Delft University of Technology, Department of Civil Engineering, Section Coastal Engineering.
Coastal Engineering (Vol.II), Harbour and beach problems.
Delft, 1980
- [Füh74] Führböter, A.
A refraction groyne build by sand.
Proc. 14th International Conference on Coastal Engineering, Copenhagen, 1974

- [Füh76] Führböter, A. et al.
Beurteilung der Sandvorspülung 1972 und
Empfehlungen für die künftige Strand erhaltung
am Weststrand der Insel Sylt (Evaluation of the
1972 nourishment and recommendations for the
maintenance of the beach of Sylt/North Sea).
Die Küste, Heft 29, 1976
- [Pät80] Pätzold, U.
- [Pel56] Pelnard-Considère, R.
Essai de theorie de l'evolution des formes de
rivage en plages de sable et de galets (Essay on
the theory of evolution of the form of beaches
and bars).
Societe Hydrotechnique de France, IVes Journees
de l'Hydraulique, Paris, 1956
- [RWSDH87] Rijkswaterstaat (Dutch Department of Public Works)
& Delft Hydraulics.
Manual on Artificial Beach Nourishment (with
Annexes).
Centre for Civil Engineering, Research Codes
and Specifications, Gouda, August, 1987
- [Smi64] Smirnow, W.I.
Lehrgang der höheren Mathematik (Course in
higher mathematics).
Verb. Deutscher Verlag der Wissenschaften,
Berlin, 1964

Notation

The symbols used in the main report (chapters 1-3) have the following meaning:

Capitals

CA	Top of the beachfill, time dependent coefficient according to (2.02)
CB	Curvature of the Gaussian curve, time dependent coefficient according to (2.03)
CS	Shifted distance of the coastline
H0	Deep water significant wave height
P	Volume of loss per meter coastline per year
Pi	Well known constant with value 3.14159...
S	Longshore sand transport capacity
S0	Longshore sand transport capacity along a straight coastline parallel to x-axis
Ti	Initial period between $t=0$ of the Dirac-function (fictive) and the time of placement of the fill
Vf	Total volume of the fill

Lowercase

cbr	Wave speed at the breakerline
d	Depth from the undisturbed water-level to the point, where the coastal profile becomes about horizontal
devy	Deviation of the shoreline position caused by a disturbance
dh	Disturbance in "h", profile height
ds	Disturbance in "s", coastal constant
e	Maximum elevation of the beach above the undisturbed water-level
h	Schematized profile height
s	Coastal constant according to (1.07)
t	Time
x	x-position of a shoreline point
xcf	x-coordinate of the contraflexure point
y	y-position of a shoreline point
y'cf	Shoreline slope with respect to the x-axis at the point of contraflexure

Greek

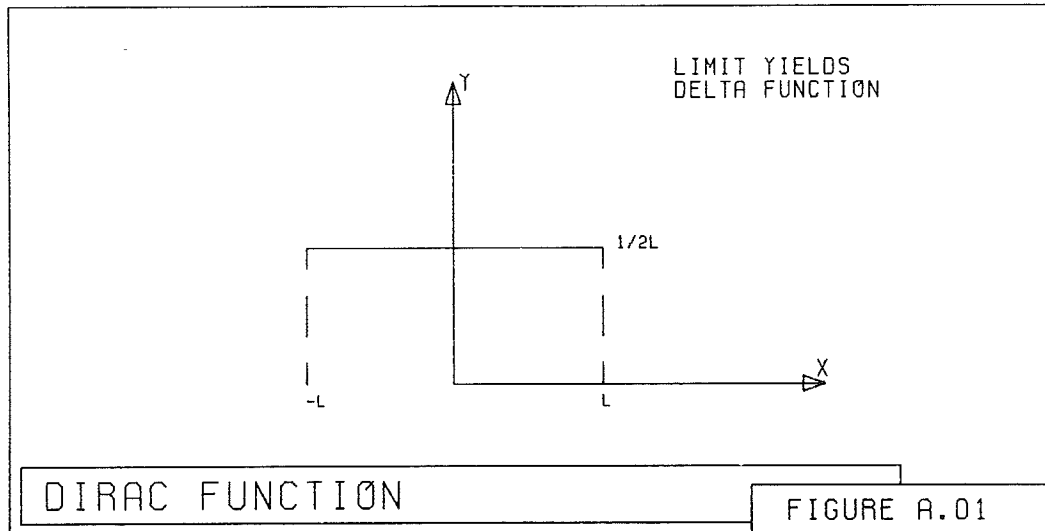
β	Coastline direction with respect to the x-axis
β_a	Angle up to which the inaccuracy of the one-line schematization is accepted
β_{cf}	Coastline direction at the point of contraflexure
β_r	Angle of wave incidence relative to the coastline direction
ϕ_0	Deep water angle of wave incidence
ϕ_{0r}	Angle of wave incidence at deep water relative to the coastline direction

Appendix A

In this appendix the analytical solution of section 1.1 (par. analytical solution) will be explained further.

Dirac function

To get an impression of the nature of the Dirac function (also called delta function) Figure A.01 has to be considered.



In Figure A.01 a function has been plotted which is given by,

$$g(x) = \frac{1}{2L} \quad \text{for } -L < x < +L \quad (\text{A.01})$$

$$\text{and } g(x) = 0 \quad \text{for } x < -L \quad \text{or } x > +L \quad (\text{A.02})$$

It can be noticed that the area under the curve equals one. In mathematical terms this can be expressed by formulating that the integral from minus infinite to plus infinite is equal to one,

$$\int_{-\infty}^{+\infty} g(x) \, dx = 1 \quad (\text{A.03})$$

This property is valid for each value of "L" as long as "L" does not equal zero. But one could take (A.03) as a demand and then force the function "g(x)" to act on a shorter and shorter interval.

As a result of this limiting operation an idealized situation can be obtained, which can be expressed as,

$$\delta[x] = +\infty \quad \text{for } x=0 \quad (\text{A.04})$$

$$\text{and } \delta[x] = 0 \quad \text{for } x<0 \text{ or } x>0 \quad (\text{A.05})$$

where (A.03) is still valid. In this idealized situation a sort of spike remains, but with an area equal to one.

Although one could consider this as nonsense or at least as inconsistent this concept can be useful. In fact such idealized situations are used quite often in mechanics, for instance the concept of a concentrated load on a beam, which applies on an area equal to zero.

In this case the Dirac function can be used to advantage, because at first it is expected to describe the geometry of the particular beachfill of Sylt (stockpile type) rather well. On the other hand it is possible to find an analytical solution for this initial shape so that modelled behaviour can be compared with measured data too.

One of the properties of the Dirac function, which is a consequence of its special nature, is given by,

$$\int_{-\infty}^{+\infty} \delta[x] * g(x) dx = g(0) \quad (\text{A.06})$$

where "g(x)" is a given function. Expression (A.06) can be understood when it is realized that the Dirac function has been defined in such a way that only the point $x=0$ contributes to the integral. In addition the magnitude of this contribution equals one, since the area under the Dirac function has been defined as equal to one in (A.03). Formula (A.06) will be used in the following paragraph.

Derivation of the solution

To obtain the analytical solution of the diffusion equation (1.01) in the case of a Dirac function as an initial shape the Fourier transform method can be used.

This technique involves converting a given partial differential equation into an ordinary differential equation, using a Fourier transformation. Hereby also the initial and boundary conditions have to be taken in account.

Such an ordinary differential equation is often much easier to solve. When a solution has been found this function has to be inverse transformed to obtain the solution of the original problem. This step is usually the most bothersome one, because in its general form it requires a rather extensive knowledge of mathematical complex function theory.

Luckily this actual case is a known problem which is treated in the handbooks. The solution of the diffusion equation (1.01) with the Dirac function of (A.03) through (A.05) as an initial shape is given by a function called the "normal distribution" function known from mathematical statistics. It has the shape of a "hat" as can be seen from Figure A.02 and has originally been discovered by De Moivre. Then it has been neglected for about a hundred years but was rediscovered by Gauss. Therefore especially his name is connected to this function. In this report also the name "Gaussian curve" is used to reference it.

The derivation of this analytical solution will not be explained in full detail, only the main lines will be treated. More specific information on the Fourier transform method can be obtained from the mathematical handbooks, for instance by Smirnow [Smi64].

The Fourier transform of a given function "y(x,t)" to the place is denoted by " $\hat{y}(u,t)$ " and can be expressed as,

$$\hat{y}(u,t) = \frac{1}{\sqrt{2\pi}} * \int_{-\infty}^{+\infty} y(x,t) * \exp[-iux] dx \quad (\text{A.07})$$

Where i : imaginary number defined by $i^2 = -1$
 t : time
 u : parameter
 x : place

As can be noticed the dependency of "x" and "t" has been transformed to a dependency of "t" only, while a parameter "u" has been introduced.

The diffusion equation (1.01) can be written as,

$$\frac{\partial y}{\partial t} = D * \frac{\partial^2 y}{\partial x^2} \quad (\text{A.08})$$

with as initial condition: $y(x,0) = \delta[x]$
 and as boundary conditions: $y(+\infty,t) = 0$
 $y(-\infty,t) = 0$

For shortness the diffusion coefficient is denoted by "D". The equation can be Fourier transformed using (A.07) and partial integration. The result yields,

$$\frac{d\hat{y}}{dt} = -D * u^2 \hat{y} \quad (\text{A.09})$$

Equation (A.09) is a ordinary differential equation in "t", which can be solved rather easily,

$$\hat{y}(u,t) = C(u) * \exp[-D * u^2 * t] \quad (A.10)$$

Where C(u) is a integration constant dependent on "u". Hereby the boundary conditions have been used.

At first a generalized initial condition will be used.

$$y(x,0) = g(x) \quad (A.11)$$

Fourier transformation of this initial condition gives,

$$\hat{y}(u,0) = \hat{g}(u) \quad (A.12)$$

The function "g(x)" is not yet specified, later on the Dirac function will be substituted. With this initial condition the integration constant "C(u)" can be found,

$$C(u) = \hat{g}(u) \quad (A.13)$$

So the solution of the transformed diffusion equation (A.09) yields,

$$\hat{y}(u,t) = \hat{g}(u) * \exp[-D * u^2 * t] \quad (A.14)$$

Now the inverse transformation has to be carried out. Tables found in the mathematical handbooks on Fourier transformation supply the global form of the solution, while theorems can be used to fit the coefficients. The elaboration will not be given in detail, but with a little effort the following solution of the original diffusion equation can be derived,

$$y(x,t) = \frac{1}{\sqrt{4*\pi*D*t}} * \int_{-\infty}^{+\infty} g(k) * \exp\left[-\frac{(x-k)^2}{4*D*t}\right] dk \quad (A.15)$$

This solution contains an integral with "k" as an integration parameter.

Here the generalized initial condition can be replaced by the actual initial condition of a Dirac function,

$$g(k) = \delta[k] \quad (A.16)$$

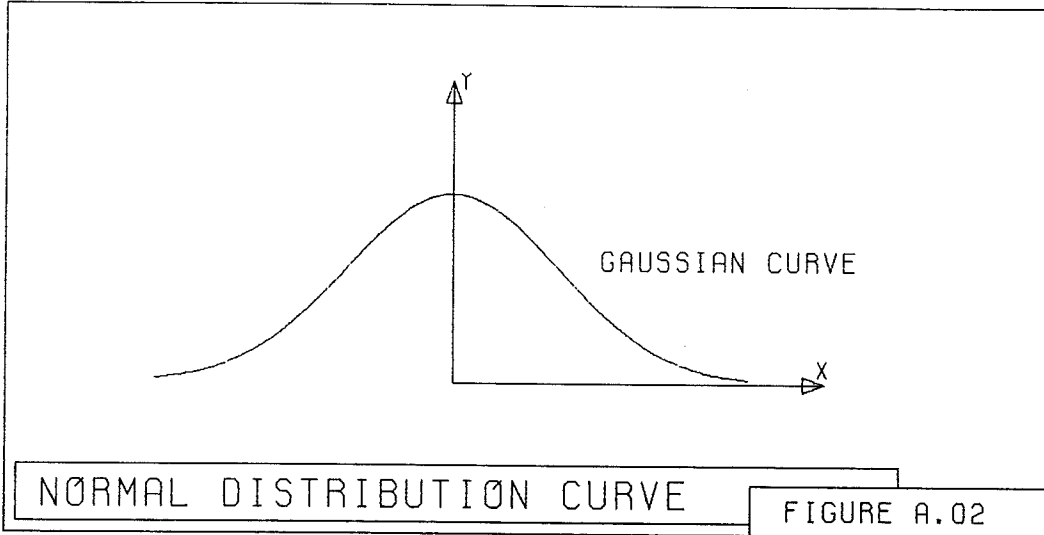
At this point formula (A.06) can be used and the integration of (A.15) yields,

$$y(x,t) = \frac{1}{\sqrt{4*\pi*D*t}} * \exp\left[-\frac{x^2}{4*D*t}\right] \quad (A.17)$$

This equation represents the normal distribution function known from mathematical statistics.

Gaussian curve

The Gaussian curve has been plotted in Figure A.02.



As treated before the Dirac function is a sort of spike with an area equal to one (A.03). It can be noticed that because of the Gaussian curve is a normal distribution function the area under the curve is equal to one by definition.

The normal distribution function is characterised by two parameters, its mean and its standard deviation.

The function is symmetrical with respect to its mean. In Figure A.02 the mean of the distribution is found for $x=0$ and the y -axis is the axis of symmetry. The value of the function at $x=0$ is about 0.4 (0.39894) provided the standard deviation equals one. The standard deviation is an indication of the width of the distribution. It represents the distance from the axis of symmetry to the points of contraflexure. Approximately two-thirds (68% to be precise) of the area under the function falls within one standard deviation distance of the mean. For two and three times the standard deviation these percentages yield about 95% and 99% respectively.

Since this basic solution represents the case of a unit area, formula (A.16) should be corrected for the actual area of the fill. Returning to the derivation of the analytical solution it can be seen that a multiplication factor "Af" should be added to the Dirac function (area equal to one) in (A.16). This yields instead of (A.17),

$$y(x,t) = \frac{Af}{\sqrt{4\pi Dt}} * \exp\left[\frac{-x^2}{4Dt} \right] \tag{A.18}$$

To introduce the fill volume in this last result it should be realized that in the model a schematized profile height "h" is used. Thus the volume of the beachfill "Vf" can be defined as,

$$V_f = A_f * h \quad (A.19)$$

Also the diffusion coefficient "D" from (A.08) has to be substituted by,

$$D = \frac{s}{h} \quad (A.20)$$

Using (A.19) and (A.20) the analytical solution for an actual case of a stockpile-type beachfill can be expressed as,

$$y = CA * \exp[-CB * x^2] \quad (A.21)$$

with coefficients "CA" and "CB" according to,

$$CA = \frac{V_f}{\sqrt{4 * \pi * h * s * t}} \quad (A.22)$$

$$CB = \frac{h}{4 * s * t} \quad (A.23)$$

Now a good solution is obtained, but unfortunately for a rather unreal problem. The idealized situation of a Dirac function will never occur, but it is possible to model an actual stockpile-type beachfill by this function.

Because the initial situation of the fill will not be a Dirac function, but may resemble a Gaussian curve, a new axis of time can be introduced starting from the moment of placement of the fill. Then the actual initial situation can be fitted into the model by posing that fictively a Dirac function was placed a period of time "Ti" previous to the actual placement. Introducing "Ti" and the new time axis yields the following formulas,

$$y = CA * \exp[-CB * x^2] \quad (A.24)$$

with coefficients "CA" and "CB" according to,

$$CA = \frac{V_f}{\sqrt{4 * \pi * h * s * (t + T_i)}} \quad (A.25)$$

$$CB = \frac{h}{4 * s * (t + T_i)} \quad (A.26)$$

where CA : time dependent coefficient A
CB : time dependent coefficient B
h : schematized profile height
Pi : well known constant with value 3.14159...
s : coastal constant according to (1.07)
t : time
Ti : initial period between t=0 of the Dirac-function
(fictive) and the time of placement of the fill
Vf : total volume of the fill
y : position of the schematized coastline

This set of expressions (A.24) through (A.26) can be found in section 1.1 (par. analytical solution) and has been used in this report.

Appendix B

In this appendix the wave data as presented in section 2.2.3 of Annex V will be used to obtain a representative angle of wave incidence for the project area at Sylt.

Wave data

In Annex V the distribution of wave energy over six direction sectors is given. These sections of 22.5 degrees each have their range (counterclockwise) from North-West to South-South West. The distribution of wave energy over these sections can be found in Table B.1.

direction	wave energy [%]
NW	5
WNW	15
W	15
WSW	26
SW	13.5
SSW	13.5

Table B.1 -- Wave energy

As can be verified the total percentage of wave energy over this six direction amounts 88%.

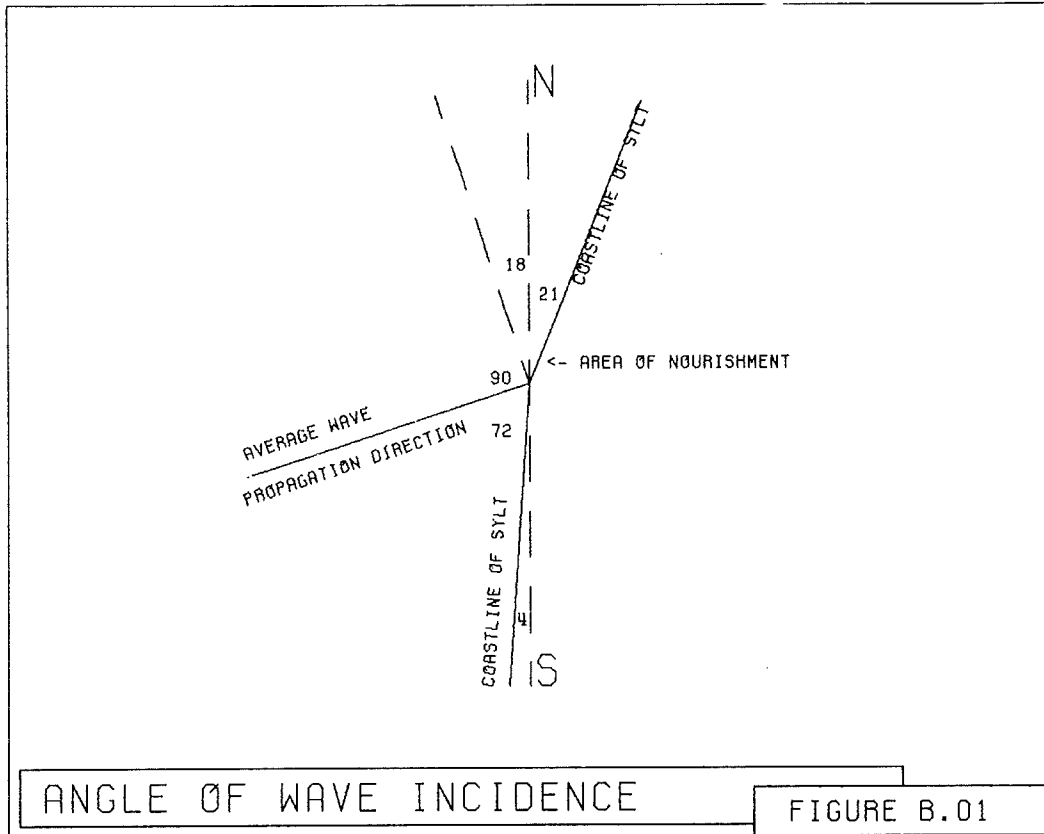
Representative wave direction.

As already noticed in section 2.2 (par. Calculation method C) one overall average significant wave height has been used as an input for the computations of Annex V. Holding to this approach only an average direction of wave incidence has to be determined to find a representative wave direction. This involves the computation of the median of the wave energy distribution with respect to its directions.

As a result an average direction of wave propagation can be found of 72 degrees with respect to the north (so about WSW). Since the angle of wave incidence is defined perpendicular to this direction a value of $\phi_0 = 72 - 90 = -18$ degrees (with respect to the north) is found.

Relative angle

The situation at Sylt is graphically illustrated in figure B.01.



ANGLE OF WAVE INCIDENCE

FIGURE B.01

In this sketch also the coastline of Sylt is plotted. As can be seen from Figure B.02 (,which is copied from [ALWH85], page 4,) the coastline direction of Sylt with respect to the north amounts 4 degrees for the southern half of the island and 21 degrees for the northern half of the island.

Because the area of replenishment is located in the "northern half", its coastline direction with respect to the north is 21 degrees.

Thus for the angle of wave incidence relative to the coastline a value is found of $\beta_r = 39$ degrees. This result is used in the analysis of section 2.3 (par. small angle of wave incidence).

Direction of the sand transport

From the average angle of wave incidence it can also be concluded that the transport direction of the sand for both the the northern and the southern part of Sylt will be from the south to the north. This is implicitly used in section 2.3 (par. About calculation method C), where a similar transport direction has been assumed for both parts of the island.

