

Boussinesq modelling of wave-induced particle velocities

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Abstract

In this thesis, the modelling of the horizontal velocity field under waves using Boussinesq-type models is studied. Special attention is paid to the prediction of bottom velocities in view of sediment-transport formulations.

Boussinesq-type models describe the propagation of weakly non-linear shallow-water waves. The depth-dependence of the velocity field has been removed in deriving the equations, which are therefore stated in the surface elevation and one single horizontal velocity variable, the so-called computational velocity. After integration of the Boussinesq-type equations, the horizontal velocity field must be reproduced in some kind of postprocessing. Many different forms of Boussinesq-type equations exist. They differ in frequency-dispersion and shoaling characteristics as well as in the used computational velocity variable.

Various different forms of Boussinesq-type equations are compared in view of their capability in reproducing the horizontal velocity field. A set of equations with good frequency-dispersion and shoaling behaviour and stated in terms of the depth-averaged horizontal velocity is selected for numerical treatment. The method to reconstruct the horizontal velocity field from the depth-averaged velocity is described.

The selected equations are implemented in an existing time domain Boussinesq model (Beji and Battjes, 1994) supplemented with a formulation to obtain the horizontal velocity field. The model is validated against surface elevation data from three different wave flume experiments (see Dingemans, 1994a and Luth et al., 1994).

The time domain model is used to predict the horizontal velocity field, especially bottom velocities, obtained from wave flume experiments of non-breaking regular waves propagating over a submerged bar (Luth et al., 1994). The results show encouraging agreement with the measurements. The comparison is limited inshore by the wave breaking process.

In order to predict horizontal velocities in breaking waves, a spectral Boussinesq model in which a spectral breaking term is incorporated (Eldeberky and Battjes, 1995), is used. This model is based on the assumption of uni-directional wave propagation and therefore only yields surface elevations. The spectral model is extended to compute the depth-averaged horizontal velocity and, subsequently, the complete horizontal velocity field.

The extended spectral model is verified against wave flume experiments of irregular (partially) breaking waves propagating over a nearly monotonic sandy beach (Arcilla et al., 1994 and Roelvink and Reniers, 1995). In the comparison of computations with measurements, the focus is on bottom velocity time series and resulting third odd velocity moments. Good results are obtained for long waves. For shorter waves a significant underestimation of the crest values and velocity moments is found.

It is concluded that the accuracy in predicting horizontal velocities with Boussinesq models is comparable to the accuracy in predicting surface elevations. The latter is limited by the inaccurate frequency dispersion of Boussinesq models in deeper water.

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1 Introduction

1.1 General

The prediction of sediment transport in the coastal zone is a main interest in morphological problems. Sediment transport by waves may be due to net flow, long wave flow and short wave orbital motion, and to interactions between these.

A net transport occurs in the case of net velocities such as the return flow under breaking waves, net drift velocities associated with standing long waves and the net velocity effect introduced through short wave asymmetry.

The short wave motion increases the time-averaged amount of suspended sediment and therefore the amount of sediment transported by the net flow. The interaction between long and short waves can also contribute to a net transport. Roelvink and Stive (1989) demonstrate the importance of this interaction. The net transport effect was shown to be the result of a strong correlation between the amount of suspended sediment and the long wave velocity; the short wave amplitude varies on the time scale of the wave groups which causes the amount of suspended sediment to vary on the same time scale. The strong correlation between suspended load and the long wave velocity occurs since the long waves act on the time scale of the wave groups as well.

Because of the complexity of the mechanisms contributing to sediment transport, no accurate enough transport formulation is available at the moment. Reasonably good results are obtained using the Bailard formulation (see Bailard, 1981) which is based on a concept first introduced by Bagnold (1966). The sediment transport is assumed to be related to the energy dissipation near the bed. The energy dissipation is described as a function of the instantaneous velocity near the bed. In the formulation a distinction is made between bed load and suspended load. For the time-averaged transport rate $\langle i(t) \rangle$ is found:

$$\begin{aligned}\langle i(t) \rangle &= \langle i_b(t) \rangle + \langle i_s(t) \rangle \\ \langle i_b(t) \rangle &= s_{b1} \langle |u(t)|^2 u(t) \rangle - s_{b2} \tan \beta \langle |u(t)|^3 \rangle \\ \langle i_s(t) \rangle &= s_{s1} \langle |u(t)|^3 u(t) \rangle - s_{s2} \tan \beta \langle |u(t)|^5 \rangle\end{aligned}\tag{1.1}$$

where $\langle i_b(t) \rangle$ and $\langle i_s(t) \rangle$ are the bed load and suspended load transport rate, respectively, averaged over the short wave and wave group scale, $\tan \beta$ is the bottom slope and $u(t)$ is the instantaneous near-bed velocity which consists of a wave group averaged component $\bar{u}$ and an oscillatory component $\tilde{u}$. The parameters s_{b1} , s_{b2} , s_{s1} and s_{s2} containing sediment parameters, can be found in Bailard (1981). The terms containing the even velocity moments $\langle |u|^3 \rangle$ and $\langle |u|^5 \rangle$ are directed downslope and are usually an order of magnitude smaller than the terms containing the odd velocity moments $\langle |u|^2 u \rangle$ and $\langle |u|^3 u \rangle$.

It is obvious that the correct prediction of the instantaneous near-bed velocity and consequently the velocity moments, is essential for the prediction of sediment transport. For this purpose, wave propagation models are needed which are able to describe horizontal as well as vertical wave asymmetry. Boussinesq models include both of these shallow-water effects.

1.2 Scope and aim of the present study

As was mentioned in Section 1.1, there is a need of wave propagation models suited for the prediction of near-bed velocities in shallow water. Wave propagation in shallow water involves non-linear shallow-water effects such as the generation of bound sub- and super-harmonics and near-resonant triad interactions. The process of triad interactions involves energy exchange between three interacting wave modes and leads to substantial cross spectral energy transfer in relatively short distances. The strength of the interactions is governed by the phase mismatch between the free and bound wave numbers, which implies that the process will be enhanced for decreasing water depths where eventually the waves become non-dispersive. During shoaling more and more energy will be transferred into bound sub- and super-harmonics which will travel phase-locked to the primary wave train. Simultaneously, part of the energy can be released as free harmonics, for example during the passage over a submerged bar. These non-linear interaction processes lead to significant changes in the spectral shape and the wave profile and can be satisfactorily modelled using Boussinesq-type equations. The linear dispersion and shoaling properties of the used equations are important for a correct representation of the non-linear energy transfer.

Boussinesq-type models, which describe the propagation of weakly non-linear shallow-water waves, were found to yield good predictions for the free surface elevation (see e.g. Beji and Battjes, 1994 and Eldeberky and Battjes, 1995). Further, since Boussinesq-type equations are integrated over the water depth, they require relatively little computational effort and are therefore attractive from an economical point of view. Many different forms of Boussinesq-type equations exist, which differ in their frequency-dispersion and shoaling characteristics and therefore in their range of practical applicability.

The vertical dependence has been removed in Boussinesq-type equations, which means amongst others that the horizontal velocity field is represented by a single velocity variable, the so-called computational velocity. Therefore, time domain Boussinesq-type models provide, besides the free surface elevation, only the computational velocity variable. This computational velocity can be chosen in different ways, e.g. the near-bed velocity, the depth-integrated velocity and the surface velocity. The choice of the computational velocity influences the frequency-dispersion and shoaling characteristics. Except for Boussinesq-type equations stated in terms of the near-bed velocity, the determination of the near-bed velocity requires the reconstruction of the horizontal velocity field in some kind of postprocessing.

Besides the direct time domain approach, a spectral approach can be used by rewriting the time domain equations into the frequency domain (see Madsen and Sørensen, 1993). In this procedure uni-directional wave propagation is assumed, resulting in spectral Boussinesq equations stated in terms of the surface elevation only. When interested in the prediction of near-bed velocities, an expression must be found to determine the horizontal velocity field after solving the equations for the surface elevation.

In view of the above, the principal research questions are whether and under which restrictions near-bed velocities can be modelled using Boussinesq-type equations. Those questions are addressed in this study. As a starting point both a time domain Boussinesq model and a spectral Boussinesq model supplemented with a spectral breaking term, which were both available at Delft University of Technology, were used.

The objectives of the present study can now be specified as follows:

- 1 Determination and implementation of the time domain Boussinesq-type equations which are most suitable for the prediction of near-bed velocities. The determination of the near-bed velocities can be done either directly, using equations stated in the near-bed velocity, or indirectly by reconstruction of the velocity field after integration of the equations. In the latter case, a method to reconstruct the velocity field has to be found.
- 2 Extension of the spectral model with a formulation for the determination of the horizontal velocity field, especially the near-bed velocities.
- 3 Verifying the computation of the near-bed velocity with both the time domain model and the spectral model against laboratory measurements of non-breaking and both non-breaking and breaking waves, respectively.

1.3 Outline of report

The general lay-out of the report is as follows. Chapter 2 gives an overview of equations for water waves and the approximations behind them. In Chapter 3, different forms of the time domain Boussinesq equations are compared in view of their capability in determining the near-bed velocities, and the equations selected for numerical treatment are presented.

In Chapter 4, the numerical method used to determine the near-bed velocities on the basis of the selected time domain equations is presented. Chapter 5 is about the validation of the time domain model with respect to surface elevations. The verification of time-domain velocity computations against laboratory measurements of both regular and irregular non-breaking waves is presented in Chapter 6.

The extension of the spectral model to compute near-bed velocities is described in Chapter 7. The extended model was tested against laboratory measurements of both non-breaking and breaking irregular waves, results of which are presented in Chapter 8.

Finally, conclusions and recommendations are given in Chapter 9.

The figures and tables in the report are numbered in a systematic way. The first character indicates the number of the chapter. The figure or table can be found in the text of this chapter if the second character is a digit and in the Appendices if the second character is a letter.

2 Equations for water waves

2.1 Introduction

In principle, the motion of water for an incompressible Newtonian fluid is well described by the continuity equation and the Navier-Stokes equations of motion. Problems arise however in solving the equations. This is especially the case with water waves where the difficulty is compounded by the unknown position of the free surface and the relatively complicated (non-linear) boundary conditions to be applied there.

To simplify the problem the fluid motion is often assumed to be irrotational. As long as breaking does not occur, this is a good approximation for waves invading still water of not too small a depth. Then, there is no initial vorticity and the boundary layers at the bottom and the free surface, from where vorticity can enter the fluid by the effects of viscosity, are relatively thin. For an incompressible fluid with constant viscosity, the effects of viscosity in the Navier-Stokes equations vanish under the assumption of irrotationality. In that case the Navier-Stokes equations reduce to the Euler equations.

The equations describing the irrotational motion of an incompressible fluid with constant density ρ and constant viscosity read

$$\nabla \cdot \bar{u} = 0 \quad (2.1)$$

$$\bar{u}_t + (\bar{u} \cdot \nabla) \bar{u} + \nabla \left(\frac{p}{\rho} + gz \right) = 0 \quad (2.2)$$

$$\nabla \times \bar{u} = 0 \quad (2.3)$$

in which p denotes the pressure. The velocity in the directions (x,y,z) is denoted by

$$\bar{u} = (u, v, w)$$

The direction Oz is chosen vertically upwards and the still-water level is taken at $z = 0$. The free surface is $z = \zeta(x,y,t)$, the bottom is $z = h(x,y)$. Suffices are representing partial differentiation. Equation (2.1) is the continuity equation, representing conservation of mass. The Euler equations for conservation of momentum in two horizontal directions and in vertical direction are given by Equation (2.2). Equation (2.3) is the irrotationality condition.

Those equations have to be completed with boundary conditions at the bottom and free surface. From the fact that fluid particles move with the free surface, the kinematic boundary condition at $z = \zeta(x,y,t)$ is found to be

$$\zeta_t + u\zeta_x + v\zeta_y = w \quad (2.4)$$

The dynamic boundary condition is that the pressure at the surface is zero:

$$p = 0 \quad (2.5)$$

The velocity in a direction perpendicular to the bottom is assumed to be zero, which yields on $z = -h(x, y)$

$$uh_x + vh_y + w = 0 \quad (2.6)$$

For an irrotational motion the velocity field is derivable from a scalar potential. A velocity potential $\phi(x, y, z, t)$ may be introduced such that

$$\vec{u} = \nabla \phi$$

Equation (2.1) then gives the Laplace equation:

$$\nabla^2 \phi = 0$$

The water motion is now completely described by Equations (2.1-6). Unfortunately, analytical solutions for these equations are for special cases, whereas numerical solutions can only be obtained with a considerable amount of computation. In order to find wave propagation formulations which can be solved either analytically or numerically, further approximations are necessary. The appropriate model depends on the spatial and temporal scales involved, such as: 1) the ratio of depth to wave length h/L or equivalently kh where the wave number $k = 2\pi/L$; 2) the ratio of amplitude to depth a/h ; and 3) the ratio of amplitude to wave length a/L or equivalently ak .

In Section 2.2 the Stokes expansion, which at first order gives the linear theory, is considered. The Stokes expansion is valid if the Stokes parameter $(ak)/(kh)^3$ is a small quantity and is therefore not converging in shallow water ($kh \ll 1$). The Stokes solution is formulated in both the horizontal space (the propagation space) and the vertical direction (the cross space). Therefore the vertical structure of the waves has to be obtained as part of the solution.

A different approach to find approximate models is based on separating the propagation space and cross space; by making assumptions about the vertical structure the vertical coordinate can be eliminated. This results in wave propagation equations which are formulated in the propagation space only and therefore can be solved relatively fast. This is a very suitable approach for long waves ($kh \ll 1$) because of the weak depth-dependence of the velocity field. Within this shallow-water restriction the finite-amplitude theory and Boussinesq-type approximations, which are described in Section 2.3 and 2.4 respectively, can be obtained. The drawback of this type of wave propagation models is that they are only valid for (fairly) long waves. In Section 2.3 it is shown that the fact that the wave length L should be very much longer than a characteristic depth h also limits the possible bottom slope h_x .

There are various ways to derive the finite-amplitude theory and Boussinesq-type equations. As stated before the main point in all derivations is the integration of the equations with respect to the vertical coordinate z . At some point in the derivations assumptions about the vertical structure have to be made. It is preferable that the implications of these assumptions are made explicit by using non-dimensional and scaled variables in deriving the equations. The two important scaling parameters are the shallowness parameter h/L and the amplitude parameter a/h . It is convenient to represent these parameters by μ and ε respectively:

$$\mu = \left[\frac{h}{L} \right]^2$$

$$\varepsilon = \frac{a}{h}$$

The assumptions made about the vertical structure depend on the relative magnitude of ε with respect to μ . The finite-amplitude theory is obtained for $\varepsilon = O(1)$. If ε and μ are of the same order the Boussinesq-type equations are obtained. It will be demonstrated that the shallowness parameter μ can be associated with the effects of frequency dispersion, while ε is representing the terms which give rise to amplitude dispersion.

In Peregrine (1967, 1971) and Dingemans (1973, 1994b), who both used a perturbation approach to derive the equations, the scaling of the variables and the implications for the resulting equations is dealt with in great detail. In the sections on the finite-amplitude theory and the Boussinesq approximation basically their approach is followed, although the derivation is only broadly outlined.

The finite-amplitude and Boussinesq-type equations will be given in one horizontal dimension denoted by x . The y -coordinate and components are easily included if it is necessary to generalise the 1D equations to the 2DH situation.

2.2 Linearized equations

If the wave slope parameter ak is small, a good way to find approximate solutions to Equations (2.1-6) is to linearize the free-surface boundary conditions and apply them to the undisturbed position of the free surface ($z = 0$). The linearized equations then become in terms of the velocity potential

$$\nabla^2 \phi = 0 \quad (2.7)$$

$$\zeta_t = \phi_z \quad \text{and} \quad \phi_t + g\zeta = 0 \quad \text{on} \quad z = 0 \quad (2.8)$$

$$\phi_x h_x + \phi_y h_y + \phi_z = 0 \quad \text{on} \quad z = -h(x, y) \quad (2.9)$$

Equation (2.7) is the Laplace equation. The boundary conditions at the surface are given by Equation (2.8), whereas Equation (2.9) gives the boundary condition at the bottom. From this set of equations $\phi(x, y, z, t)$ and $\zeta(x, y, t)$ can be solved.

Since the problem is now linear with fixed boundaries, many techniques are available for its solution. In case of a horizontal bottom a simple solution of these linearized equations is given by a wave consisting of a single Fourier component. The dispersion relation, the relation between the cyclic frequency ω and the wave number k , corresponding to this solution is given by Equation (2.10). Note that the phase celerity $c = \omega/k$.

$$\frac{c^2}{gh} = \frac{\tanh kh}{kh} \quad (2.10)$$

For $kh \rightarrow 0$ the dispersion relation reduces to $c^2 = gh$. Hence, if the wave is long enough ($kh \ll 1$), the phase celerity is independent of the wave number and the wave is non-dispersive. Short waves ($kh \gg 1$) on the other hand, are highly dispersive.

It is important to consider the approximations implied by the linearization. Ratios between the neglected terms and terms which have been retained in the free-surface boundary condition can be worked out using the solution for a horizontal bottom. The condition that those ratios be small is found to be satisfied by

$$ak \ll 1 \quad (2.11)$$

if kh is not small and by

$$a \ll h \quad (2.12)$$

if kh is small.

Stokes extended this solution of the linearized equations to non-linear wave motion by invoking some perturbation method. If the solution for the constant depth situation is considered to be the first-order approximation in a perturbation expansion in terms of ak , then further terms can be found as corrections to the first approximation. The ratio of the second-order correction term to the linear solution is found to be small within the conditions (2.11) and (2.12) unless $kh \ll 1$. In that case a further requirement is that the Stokes parameter is a small quantity:

$$\frac{(a/h)^3}{(ak)^2} = \frac{(ak)}{(kh)^3} \ll 1 \quad (2.13)$$

This condition is very stringent since it implies that

$$ak \ll (kh)^3$$

where kh is already less than 1. Therefore it can be concluded that the Stokes expansion, which at first order gives the linear theory, is not converging in shallow water unless the wave slope parameter ak is extremely small. Hence, for fairly long waves a different approach is needed.

2.3 Classical finite-amplitude shallow-water equations

The finite-amplitude equations are non-linear long-wave equations without frequency dispersion. The equations can be derived by assuming that $\varepsilon = O(1)$ and $\mu \ll 1$. The latter means that the wave properties vary little in a distance of the same order of the depth. Integration of Equations (2.1-3) with respect to z using the boundary conditions at the bottom and free surface (2.4-6), eliminates the vertical coordinate. In this integration procedure terms of $O(\mu)$ are neglected. The condition of irrotationality (2.3) gives on integration that the velocity is independent of the depth to $O(\mu)$. The vertical acceleration is $O(\mu)$ and is neglected. Hence, the integration of the vertical momentum equation yields that the pressure is hydrostatic to $O(\mu)$.

Omitting the dependence of the second horizontal coordinate, Equations (2.1-6) yield upon integration the finite-amplitude equations for the 1DH case:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} [(h + \zeta)u] = 0 \quad (2.14a)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + g \frac{\partial \zeta}{\partial x} = O(\mu) \quad (2.14b)$$

The terms in the left hand sides of the equations are $O(1)$.

Note that the finite-amplitude equations give on linearization the linear theory for $kh \rightarrow 0$.

The above equations can also be derived by making one of the equivalent assumptions: that the pressure is hydrostatic or that the horizontal velocity is uniform with the depth.

The non-dimensionalisation and scaling also shows that the bottom slope dh/dx is at most

$$\left| \frac{dh}{dx} \right| = O(\delta) \quad (2.15)$$

in which

$$\delta = \mu^{1/2}$$

This means that at least one primary wave length should fit in the inhomogeneity region.

The phase velocity, defined as the velocity of a point of constant elevation, for these finite-amplitude waves is given as

$$c^2 = g(h + \zeta) \quad (2.16)$$

Two conclusions can be drawn from this expression. At first, since the phase velocity does not depend on the wave number k there is no frequency dispersion. And secondly, the higher part of the wave travels faster than the lower part which implies that the wave front continually steepens. No permanent solutions are possible. This process is called amplitude dispersion.

The steepening in fact means that a description in Fourier modes needs more harmonics. To counteract the steepening of the wave front, frequency dispersion should be included; the effect of frequency dispersion is that the higher harmonics i.e. shorter waves travel much slower than the basic wave.

2.4 Boussinesq-type equations

The Boussinesq-type equations describe non-linear long waves with frequency dispersion and amplitude dispersion of comparable magnitude. They are derived taking ε and μ to be of the same order. Basically, the frequency dispersion is introduced in the equations by taking into account the effect of the curvature of the streamlines (or the vertical acceleration) on the pressure distribution. As was mentioned in Section 2.3, the assumption that $\mu \ll 1$ implies that the bottom slope $|dh/dx|$ is $O(\mu^{1/2})$ at most.

While integrating the equations, terms of $O(\mu)$ are now retained and terms of $O(\varepsilon\mu, \mu^2)$ are neglected. As opposed to the finite-amplitude shallow-water equations, in the Boussinesq-type equations the vertical acceleration is retained to some extent; the local vertical acceleration is accounted for. This results in a pressure distribution which is no longer hydrostatic and a horizontal velocity field which is now parabolic instead of uniform over the depth. Integration of the irrotationality condition now yields integration constants of the horizontal velocity. Those constants have to be recombined into a velocity variable at a certain level. So, there is a choice of the velocity variable to be used in stating the equations. For example, the velocity at the still-water line $z = 0$, the velocity at the bottom $z = -h$ or the depth-averaged velocity can be used.

With the vertically averaged velocity $\bar{u}$ as computational velocity, the equations read in one horizontal dimension and for variable depth

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} [(h + \zeta) \bar{u}] = 0 \quad (2.17a)$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + g \frac{\partial \zeta}{\partial x} = \frac{1}{2} h \frac{\partial}{\partial t} [(h \bar{u})_{xx} - \frac{1}{3} h \bar{u}_{xx}] + O(\varepsilon\mu, \mu^2) \quad (2.17b)$$

The Boussinesq-type equations only differ from the finite-amplitude shallow-water equations by the third-order derivative terms in the right-hand side of the equations. In the set of equations (2.17) the third-order derivatives only appear in the momentum equation (2.17b); the vertically averaged continuity equation (2.17a) is exact when written in terms of $\bar{u}$. If the equations are written in non-dimensional variables the relative magnitude of the various terms in the equations is clear directly from the appearance of the parameters ε and μ (see for example Peregrine, 1971 and Dingemans, 1994b). It is found that the third-order terms contain the small parameter μ . Because μ is a measure for the wave length at a given still-water depth those terms are a measure for the frequency dispersion. Further, it is obvious that the non-linear terms in both the continuity equation and the momentum equation contain the parameter ε , which was introduced as an amplitude measure. As was demonstrated in Section 2.3, the non-linear terms give rise to amplitude dispersion. Therefore it can be concluded that the Boussinesq-type equations describe wave propagation for the case that frequency dispersion and amplitude dispersion are balanced. The limiting cases, however, are obtained for $\varepsilon \ll \mu$ (linear long waves) and $\varepsilon \gg \mu$ (finite-amplitude long waves). In contrast with the finite-amplitude shallow-water equations, the Boussinesq-type equations can have permanent solutions. An example of a permanent wave form is a cnoidal wave in which all higher harmonics are phase-locked.

Solution of a specific set of Boussinesq-type equations yields values for the surface elevation and the computational horizontal velocity. Since the vertical dependence has been removed, expressions for the pressure and the horizontal and vertical velocity as a function of the depth have to be determined in some kind of postprocessing. The pressure distribution and the velocity field can be reconstructed using the expansions which were used to obtain the specific set of equations. The horizontal velocity field, for example, reads in terms of $\bar{u}$ (Dingemans, 1994b)

$$u(x, z, t) = \bar{u} - \left[\frac{1}{2} h + z \right] (h \bar{u})_{xx} + \left[\frac{1}{6} h^2 - \frac{1}{2} z^2 \right] \bar{u}_{xx} + O(\mu \epsilon^2, \mu^2 \epsilon) \quad (2.18)$$

In this expression terms of $O(\mu \epsilon^2, \mu^2 \epsilon)$ are neglected. The parabolic correction terms are $O(\mu)$. Note that the latter terms are neglected in the finite-amplitude shallow-water theory where the horizontal velocity is assumed to be uniform over the depth.

Instead of a formulation in the vertically averaged velocity $\bar{u}$, a formulation in terms of the volume fluxes q is often used. The vertically averaged velocity $\bar{u}$ can be determined from the flux as follows:

$$\bar{u}(x, t) = \frac{q(x, t)}{h(x) + \zeta(x, t)} \quad (2.19)$$

Rewriting the equations in the averaged velocity (2.17) using Equation (2.19) yields the corresponding equations written in terms of q :

$$\frac{\partial \zeta}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (2.20a)$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left[\frac{q^2}{h + \zeta} \right] + g(h + \zeta) \frac{\partial \zeta}{\partial x} = \frac{1}{2} \frac{\partial}{\partial t} \left[h^2 \frac{\partial^2 q}{\partial x^2} - \frac{1}{3} h^3 \frac{\partial^2}{\partial x^2} \left[\frac{q}{h} \right] \right] + O(\epsilon \mu, \mu^2) \quad (2.20b)$$

The characteristics of the equations are not changed by rephrasing the equations in $\bar{u}$ in terms of the flux q .

3 Comparison of various Boussinesq-type models

3.1 Introduction

As indicated in Chapter 2, Boussinesq-type equations provide a good and highly efficient way to reproduce the effects of wave phenomena for (fairly) long waves. Besides the sets of equations presented in Chapter 2, many other forms of Boussinesq-type equations have been derived, some of which will be considered in this chapter. The purpose is to find the most suitable model for the numerical modelling of the horizontal velocity field and especially the near-bottom velocities. This model should meet the following requirements:

- 1 The equations should have excellent frequency-dispersion and shoaling behaviour since it can be expected that accurate prediction of velocity variables is only possible if the free surface elevation is described well.
- 2 The equations should be formulated in such a way that after solution of the set of equations the velocity field can be reconstructed.

Different sets of Boussinesq-type equations, which differ only in the third-order frequency-dispersion terms, can be obtained by choosing a different computational (horizontal) velocity variable. A closer study of the derivation by for example Dingemans (1994b) shows that the choice of the computational velocity slightly influences the way the vertical acceleration is accounted for. However, within the order of approximation used in deriving the Boussinesq-type equations the various formulations for the vertical acceleration are equivalent.

Although the different models are asymptotically equivalent, the (linearized) sets of equations have different dispersion relations. Those dispersion relations follow for a Fourier component for the constant depth situation. In Section 3.2.1, the most common forms are derived using the equations written in $\bar{u}$ as a starting point. Their linear frequency-dispersion characteristics are compared in Section 3.2.2.

The applicability of the models described in Section 3.2 is limited because of the inaccurate representation of the frequency dispersion. Especially for shorter waves large discrepancies exist between the phase velocity obtained by those models and measurements. The result of inaccurate frequency dispersion therefore is that the computed wave form is wrong. Recently, a lot of effort has been spent on improving the frequency-dispersion characteristics of Boussinesq-type models. Improved frequency dispersion typically leads to another form of the Boussinesq-type equations. An overview of the recent achievements is given in Section 3.3. Only Boussinesq models are considered in which the frequency dispersion is improved *without* introducing still higher derivatives than the third; higher-order frequency dispersion leading to fifth-order derivatives might be a next step in improving the applicability of Boussinesq-type models. It should be stressed that, although the frequency dispersion can be improved, the incorrect representation of the phase velocity for shorter waves is a direct result of the basic assumption made in deriving the Boussinesq equations that we are dealing with (fairly) long waves with respect to the local depth.

For horizontal bottom the differences of behaviour of the various Boussinesq-type models are wholly attributable to different frequency-dispersion characteristics. The models described in Section 3.3 all have the same optimum frequency-dispersion behaviour for the corresponding cases of horizontal bottom. The depth-dependent terms, however, may differ between the various models, which gives rise to different shoaling characteristics. It is important to note that good frequency dispersion is more important than good shoaling behaviour since inaccurate shoaling behaviour does not necessarily mean that the computed wave form is totally wrong. The shoaling characteristics of the sets of equations with improved frequency dispersion are discussed in Section 3.4. In Section 3.5 a final comparison is made between the various models and a choice is made for the model to be used in this study.

3.2 Standard forms of the equations

3.2.1 The equations

In Section 2.4, a set of Boussinesq-type equations (2.17) was presented in which the vertically averaged velocity $\bar{u}$ was used as computational velocity. An expression for the horizontal velocity field (2.18) was given as well. It was mentioned that in the Boussinesq-type equations terms of $O(\epsilon\mu, \mu^2)$ and in the expression for the horizontal velocity terms of $O(\mu\epsilon^2, \mu^2\epsilon)$ are neglected. This will not be stated explicitly in the equations any more. For the sake of convenience, the continuity and momentum equation in terms of $\bar{u}$ are repeated here. Without stating the order of the neglected terms they read

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} [(h + \zeta)\bar{u}] = 0 \quad (3.1a)$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + g \frac{\partial \zeta}{\partial x} = \frac{1}{2} h \frac{\partial}{\partial t} [(h\bar{u})_{xx} - \frac{1}{3} h \bar{u}_{xx}] \quad (3.1b)$$

The corresponding expression for the velocity field in terms of $\bar{u}$ is

$$u(x, z, t) = \bar{u} - \left[\frac{1}{2} h + z \right] (h\bar{u})_{xx} + \left[\frac{1}{6} h^2 - \frac{1}{2} z^2 \right] \bar{u}_{xx} \quad (3.2)$$

By choosing a different computational velocity in the derivation, equations stated in that specific velocity can be obtained. Here a different approach is followed which demonstrates that the various models are asymptotically equivalent. The set formulated in terms of $\bar{u}$ (3.1) is chosen as a starting point. Using the parabolic relation between $u(x, z, t)$ and $\bar{u}$ (3.2) this set can be rewritten *within the same order of approximation* to the sets of equations formulated in the bottom velocity $u_b = u(x, z = -h, t)$ and the MSL velocity $u_m = u(x, z = 0, t)$.

By using Eq. (3.2) the following expressions for the bottom velocity u_b and the MSL velocity u_m can be derived:

$$u_b = u(x, z = -h, t) = \bar{u} + \frac{1}{2} h (h\bar{u})_{xx} - \frac{1}{3} h^2 \bar{u}_{xx} \quad (3.3a)$$

$$u_m = u(x, z = 0, t) = \bar{u} - \frac{1}{2} h (h\bar{u})_{xx} + \frac{1}{6} h^2 \bar{u}_{xx} \quad (3.3b)$$

Within the order of approximation used in deriving Boussinesq-type equations the second-order derivatives of the various computational velocity variables are equal: $\bar{u}_{xx} = (u_b)_{xx} = (u_m)_{xx}$ and $(h\bar{u})_{xx} = (hu_b)_{xx} = (hu_m)_{xx}$. This can be used to solve Eqs. (3.3) with respect to $\bar{u}$:

$$\bar{u}(x, t) = u_b - \frac{1}{2}h (hu_b)_{xx} + \frac{1}{3}h^2 u_{b,xx} \quad (3.4a)$$

$$\bar{u}(x, t) = u_m + \frac{1}{2}h (hu_m)_{xx} - \frac{1}{6}h^2 u_{m,xx} \quad (3.4b)$$

Substitution of Eq. (3.4a) in the Eqs. (3.1) leads to a set of Boussinesq-type equations written in terms of the bottom velocity u_b :

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} [(h + \zeta) u_b] = \frac{\partial}{\partial x} \left[\frac{1}{2}h^2 (hu_b)_{xx} - \frac{1}{3}h^3 u_{b,xx} \right] \quad (3.5a)$$

$$\frac{\partial u_b}{\partial t} + u_b \frac{\partial u_b}{\partial x} + g \frac{\partial \zeta}{\partial x} = \frac{\partial}{\partial t} \left[h(hu_b)_{xx} - \frac{1}{2}h^2 u_{b,xx} \right] \quad (3.5b)$$

It can be seen that now also in the right-hand side of the continuity equation third-order frequency-dispersion terms occur.

The expression for the velocity profile in terms of u_b is obtained by substitution of Eq. (3.4a) in Eq. (3.2):

$$u(x, z, t) = u_b - (z + h)(hu_b)_{xx} + \left(\frac{1}{2}h^2 - \frac{1}{2}z^2 \right) u_{b,xx} \quad (3.6)$$

Note that this expression can also be obtained by applying a Taylor expansion about the bottom and truncating the series after the second derivative terms.

In the same way Eq. (3.4b) can be substituted in Eqs. (3.1) to obtain the continuity and momentum equations in terms of the MSL velocity u_m :

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} [(h + \zeta) u_m] = - \frac{\partial}{\partial x} \left[\frac{1}{2}h^2 (hu_m)_{xx} - \frac{1}{6}h^3 u_{m,xx} \right] \quad (3.7a)$$

$$\frac{\partial u_m}{\partial t} + u_m \frac{\partial u_m}{\partial x} + g \frac{\partial \zeta}{\partial x} = 0 \quad (3.7b)$$

No third-order derivatives occur in the right-hand side of the momentum equation. The right-hand side of this equation however, is $O(\varepsilon\mu, \mu^2)$ and the momentum equation is therefore not exact. It can be argued that a momentum equation of the form (3.7b) is exact if the velocity at the free surface $u_s = u(x, z = \zeta, t)$ is chosen; since the dynamic boundary condition at the free surface prescribes that the pressure is zero, the Boussinesq correction of the pressure distribution is zero at the free surface.

The expression for the velocity field written in terms of u_m follows from Eq. (3.4b) and Eq. (3.2):

$$u(x, z, t) = u_m - z(hu_m)_{xx} - \frac{1}{2}z^2 u_{m,xx} \quad (3.8)$$

Upon comparison it can be seen that the various models differ only in the third-order terms in the right-hand members, which are $O(\mu)$. Although the various models are asymptotically equivalent they differ in practical applicability. For engineering applications large differences in frequency-dispersion characteristics occur.

3.2.2 Frequency-dispersion behaviour

The model with the linear dispersion relation which, also for shorter waves, remains closest to the exact one for linear waves can be expected to have the widest applicability. For the sake of convenience, the dispersion relation corresponding to Stokes first-order theory (2.10) is repeated here:

$$\frac{c^2}{gh} = \frac{\tanh kh}{kh} \quad (3.9)$$

The linear dispersion relations corresponding to each of the forms of the Boussinesq-type equations follow for a Fourier component for the constant depth situation. It turns out that the different dispersion relations can be written in the following form (Madsen et al., 1991):

$$\frac{c^2}{gh} = \frac{1 + bk^2 h^2}{1 + (b + \frac{1}{3})k^2 h^2} \quad (3.10)$$

where the value of b is different for the different models: $b = 0$ for set (3.1) in $\bar{u}$; $b = 1/6$ for set (3.5) in u_b ; and $b = -1/3$ for set (3.7) in u_m .

A comparison of these phase velocities with the exact linear one is shown in Fig. 3.1. It is seen that the dispersion relation with $b = -1/3$ deviates very much from the exact relation, and for $kh > 1.7$ solutions to the dispersion relation cannot be found. It is also seen that the set written in $\bar{u}$ has the best frequency-dispersion properties of these three Boussinesq-type models. However, also for the latter model the difference between the model phase velocity and the exact linear one is considerable for shorter waves. Note that this not only concerns the basic wave but also the higher harmonics generated by non-linearity. It is clear that the representation of the frequency dispersion limits the applicability of Boussinesq-type models in deeper water.

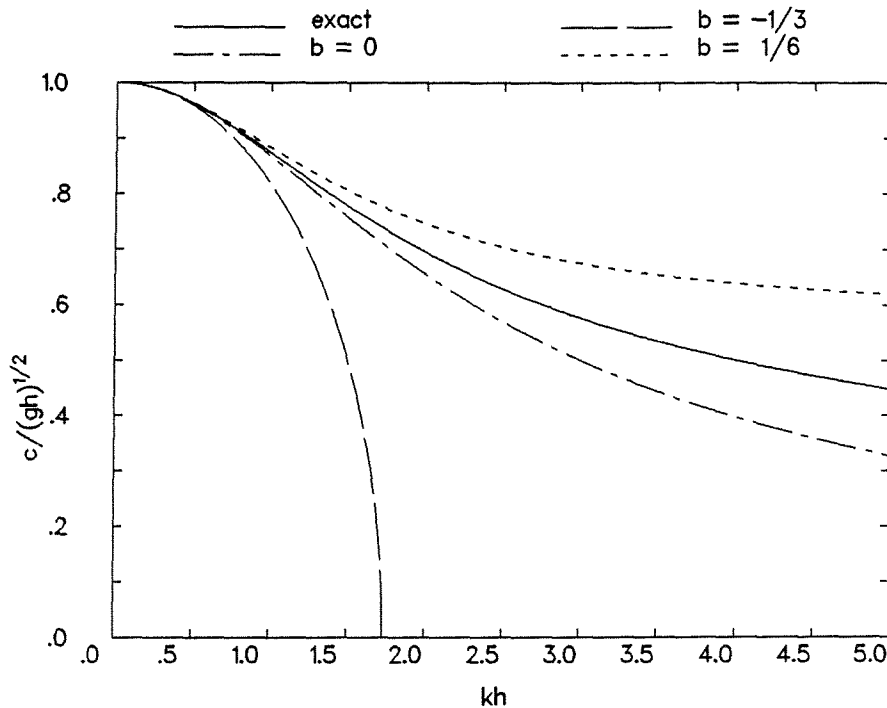


Figure 3.1 Comparison of phase velocities c for $b = 0$, $b = 1/6$ and $b = -1/3$ with the exact linear one, normalised with $(gh)^{1/2}$

3.3 Equations with improved frequency dispersion

3.3.1 Overview of the methods to improve the frequency dispersion

Although the basic assumption in the derivation of Boussinesq models is that the waves are (fairly) long with respect to the depth, it is desirable to be able to treat as short waves as possible. Therefore, Boussinesq-type models with a dispersion relation as close as possible to the exact linear dispersion relation have to be found. Reformulated, improving the frequency-dispersion properties means finding the Boussinesq-type equations for which the linear dispersion relation follows (3.10) in which b can be considered as a curve fitting parameter in obtaining the best agreement with Stokes first-order theory.

The optimum value of b can be obtained by comparing Eq. (3.10) with the Padé approximant of the exact linear phase velocity. This method has been suggested by Witting (1984) and taken up by Madsen et al. (1991). A Padé expansion of order $[1/1]$ gives the best possible results without introducing higher derivatives than the third in the Boussinesq-type equations. The $[1/1]$ Padé expansion of c^2 in the expansion parameter $(kh)^2$ is given as

$$[1/1] \frac{c^2}{gh} = \frac{1 + pk^2h^2}{1 + (q + \frac{1}{3})k^2h^2} \quad (3.11)$$

The parameters p and q of the Padé approximant can be obtained by comparing the Taylor expansions of Eq. (3.9) and Eq. (3.11). It is found that $p = 1/15$ and $q = 2/5$. Comparing

Eq. (3.10) with Eq. (3.11) after substitution of the values for p and q shows that for the Padé approximant the b -value is $b = 1/15$.

The dispersion relation for $b = 0$ (corresponding to Eqs. (3.1) in $\bar{u}$) and $b = 1/15$ (the Padé approximant) is shown in Fig. 3.2, which shows that $b = 1/15$ gives much better agreement than $b = 0$. Madsen et al. (1991) showed that this conclusion holds for the group velocity as well.

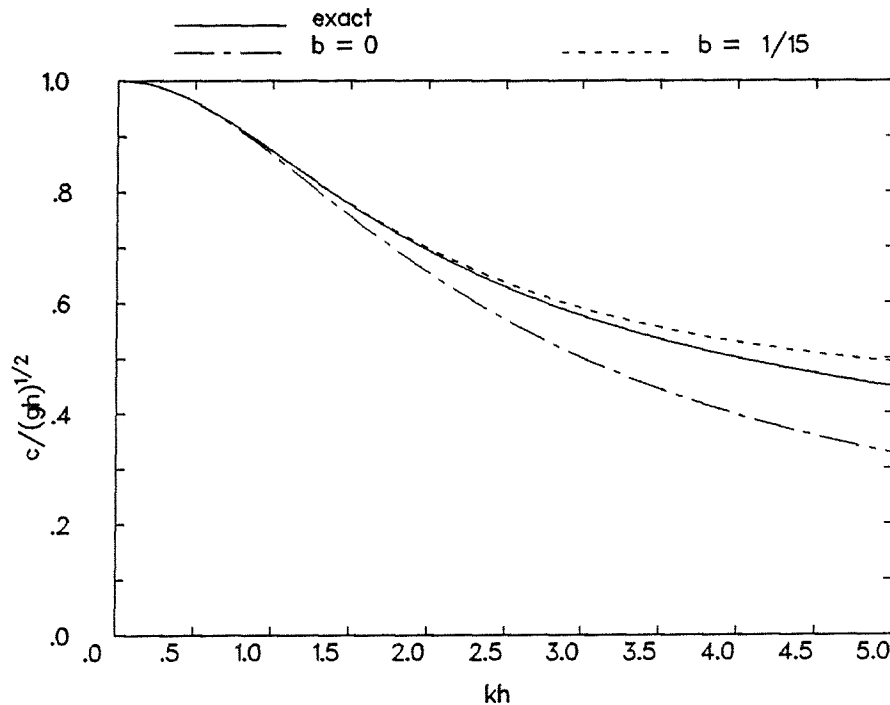


Figure 3.2 Comparison of phase velocities c for $b = 0$ and $b = 1/15$ with the exact linear one, normalised with $(gh)^{1/2}$

Some different dispersion relations have been proposed by various researchers. Mooiman (1991) for example suggested a Padé expansion of c , but this was found to yield a slightly less good agreement than the approximation of c^2 . Nwogu (1993) minimised the sum of deviations of the phase velocity according to Eq. (3.10) and the exact linear one in the range $0 < kh < \pi$. In this way an optimum value of $b = 1/17.5$ is found. The difference in the resulting frequency-dispersion behaviour with $b = 1/15$ is not large. The defect of the choice of $b = 1/17.5$ is that for very long waves a less accurate velocity is found. The Padé expansion resulting in $b = 1/15$ is usually better over the whole range. The latter value will therefore be used in the computations performed in this study. Dingemans (1994b) worked out higher-order improvement of the frequency dispersion resulting in higher-order derivatives than the third. This is considered to be outside the scope of this study.

It should be noted that the procedure to obtain differential equations corresponding to the linear dispersion relation is not unique. Moreover, the problem of finding the corresponding equations can be dealt with in a couple of ways. Nwogu (1993) introduced a very elegant way to solve this problem. Bearing in mind that the level at which the computational velocity is evaluated determines the dispersion relation, it was assumed that with a horizontal velocity evaluated at some distance $z = -\alpha h$ below the still water level a behaviour close to the exact

linear one could be found. Nwogu worked this out for the choice of $b = 1/17.5$. Dingemans (1994b) took up this matter and determined the parameter α in such a way that, for horizontal bottom, again the same linear dispersion relation is obtained as follows from a Padé expansion of c^2 . The latter corresponds to the choice of $b = 1/15$ in the dispersion relation (Eq. 3.10). Then as optimum value for α is found $\alpha \approx 0.553$. This means that the horizontal velocity is evaluated approximately at mid-depth, a level which is often taken as the level for which the velocity closely approximates the averaged velocity. Unfortunately, this model was found to have very bad shoaling behaviour (Dingemans, 1994b) and is therefore not very promising for numerical evaluation.

Another method to find the corresponding Boussinesq-type equations was presented by Madsen et al. (1991, 1992) and is based on adding higher-order terms consisting of third derivatives in x en t in the horizontal momentum equation by invoking the long-wave equation. Dingemans (1994b) derived various possible sets of Boussinesq-type equations obeying the optimum linear dispersion relation using operator correspondence. The problem with this method to improve the frequency dispersion is that the original meaning of the computational velocity is lost so that it is not possible to reconstruct the velocity field after the computation. Within the framework of this study, it is important that the interpretation of the velocity variable used in the improved equations is unambiguous. The fact that the interpretation of the velocity variable is unknown holds for equations obtained by the procedure of Madsen et al. as well since it makes use of order relations to rewrite and add terms in the equations. Nevertheless, in special cases both the method of Madsen et al. and the method used by Dingemans yield Boussinesq-type equations for which the interpretation of the computational velocity is unambiguous. This is the case if no higher-order derivatives are present in the continuity equation, which means that the vertically averaged velocity or the flux is the computational velocity variable. The equations which meet this requirement are explained in Section 3.3.2. This is done on the basis of the procedure by Madsen et al. which is easier to illustrate than the method of operator correspondence.

3.3.2 Procedure by Madsen et al.

Madsen et al. (1991, 1992) gave a procedure to improve the frequency dispersion based on adding higher-order terms to the right-hand members of the equations. They applied the procedure to two-dimensional Boussinesq-type equations written in terms of the fluxes and valid for small bottom slopes. In one horizontal dimension their starting equations thus were a reduction for a mildly-sloping bottom of Eqs. (2.20). The assumption of small bottom slopes can be quantified by introducing the parameter $\beta = h_x$. From Eq. (2.15) follows that the Boussinesq-type equations are valid for $\beta^2 \leq \mu$. Small bottom slopes are now characterized by $\beta^2 \ll \mu$. This implies that terms of order β^2 , i.e. bottom-slope squared and bottom-curvature terms, can be neglected in the Boussinesq-type equations. Restricting the terms in the right-hand side of the momentum equation (2.20b) to terms linear in h and h_x and dropping the contributions with $(h_x)^2$ and h_{xx} yields

$$\frac{\partial \zeta}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (3.12a)$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left[\frac{q^2}{h + \zeta} \right] + g(h + \zeta) \frac{\partial \zeta}{\partial x} = \frac{1}{3} h^2 \frac{\partial^3 q}{\partial t \partial x^2} + \frac{1}{3} h h_x \frac{\partial^2 q}{\partial t \partial x} \quad (3.12b)$$

Since the non-linear and frequency-dispersion terms in the momentum equation are of higher order than the remaining terms, the momentum equation (3.12b) provides the following order relation:

$$\frac{\partial q}{\partial t} + gh \frac{\partial \zeta}{\partial x} \approx 0 \quad (3.13)$$

By spatial differentiation this order relation leads to the approximation

$$\frac{\partial^3 q}{\partial t \partial x^2} + 2gh_x \frac{\partial^2 \zeta}{\partial x^2} + gh \frac{\partial^3 \zeta}{\partial x^3} \approx 0 \quad (3.14)$$

Addition of bh^2 times the left-hand side of Eq. (3.14) to the right-hand side of Eq. (3.12b) leads to the equations with improved frequency behaviour as derived by Madsen et al. for the one-dimensional situation:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (3.15a)$$

$$\begin{aligned} \frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left[\frac{q^2}{h + \zeta} \right] + g(h + \zeta) \frac{\partial \zeta}{\partial x} = & \left(\frac{1}{3} + b \right) h^2 \frac{\partial^3 q}{\partial t \partial x^2} + bgh^3 \frac{\partial^3 \zeta}{\partial x^3} + \\ & + hh_x \left[\frac{1}{3} \frac{\partial^2 q}{\partial t \partial x} + 2bgh \frac{\partial^2 \zeta}{\partial x^2} \right] \end{aligned} \quad (3.15b)$$

The choice of $b = 1/15$ leads to equations which have optimum frequency-dispersion properties. The set of equations without improved frequency dispersion behaviour (3.12) is recovered from the new set of equations for the choice of $b = 0$.

Dingemans (1994b) applied the above described procedure to Eqs. (2.20), which are written in terms of the flux q and are valid for larger bottom slopes ($\beta^2 = O(\mu)$). In this way the following equations valid for larger bottom slopes are found, of which Eqs. (3.15) are a reduction for small bottom slopes:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial q}{\partial x} = 0 \quad (3.16a)$$

$$\begin{aligned} \frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left[\frac{q^2}{h + \zeta} \right] + g(h + \zeta) \frac{\partial \zeta}{\partial x} = & h^2 \frac{\partial}{\partial t} \left[\left(\frac{1}{2} + b \right) \frac{\partial^2 q}{\partial x^2} - \frac{1}{6} h \frac{\partial^2}{\partial x^2} \left[\frac{q}{h} \right] \right] + \\ & + bgh^2 \frac{\partial^2}{\partial x^2} \left[h \frac{\partial \zeta}{\partial x} \right] \end{aligned} \quad (3.16b)$$

The corresponding set stated in the vertically averaged velocity $\bar{u}$ is given by:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} [(h + \zeta) \bar{u}] = 0 \quad (3.17a)$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + g \frac{\partial \zeta}{\partial x} = h \frac{\partial}{\partial t} \left[\left(\frac{1}{2} + b \right) \frac{\partial^2(h\bar{u})}{\partial x^2} - \frac{1}{6} h \frac{\partial^2 \bar{u}}{\partial x^2} \right] + b g h \frac{\partial^2}{\partial x^2} \left[h \frac{\partial \zeta}{\partial x} \right] \quad (3.17b)$$

For the choice of $b = 0$ the set (3.1) in $\bar{u}$ without improved frequency behaviour is obtained. The sets in u_b and in u_m cannot be recovered by the choice of $b = 1/6$ and $b = -1/3$ respectively.

For later use the reduction of Eqs. (3.17) for small bottom slopes is given here as well:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} [(h + \zeta) \bar{u}] = 0 \quad (3.18a)$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + g \frac{\partial \zeta}{\partial x} = \left(\frac{1}{3} + b \right) h^2 \frac{\partial^3 \bar{u}}{\partial x^2 \partial t} + b g h^2 \frac{\partial^3 \zeta}{\partial x^3} + h h_x \left[(1 + 3b) \frac{\partial^2 \bar{u}}{\partial x \partial t} + 3 b g h \frac{\partial^2 \zeta}{\partial x^2} \right] \quad (3.18b)$$

For the choice of $b = 1/15$ the sets (3.15) and (3.18) valid for small bottom slopes and the two sets valid for larger bottom slopes (3.16) and (3.17) all have the same optimum frequency-dispersion properties. Since the sets only differ in the bottom-curvature terms, also no differences with respect to the linear shoaling behaviour occur between the four sets; linear shoaling behaviour is determined by the relation between a_x/a and h_x/h , in which a is the wave amplitude and h is the undisturbed water depth.

Dingemans showed that equations which differ in the depth-dependent terms and therefore in shoaling behaviour from the three above described sets can be derived by applying the procedure to the equations written in terms of $\bar{u}$ (3.1) instead of q . Only the resulting equations in terms of $\bar{u}$ and without the restriction of small bottom slopes are given here:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} [(h + \zeta) \bar{u}] = 0 \quad (3.19a)$$

$$\begin{aligned} \frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + g \frac{\partial \zeta}{\partial x} = & \frac{1}{2} (1 + 3b) h \frac{\partial}{\partial t} \left[\frac{\partial^2(h\bar{u})}{\partial x^2} - \frac{1}{3} h \frac{\partial^2 \bar{u}}{\partial x^2} \right] + \\ & + \frac{3}{2} b g h \left[\frac{\partial^2}{\partial x^2} \left(h \frac{\partial \zeta}{\partial x} \right) - \frac{1}{3} h \frac{\partial^3 \zeta}{\partial x^3} \right] \end{aligned} \quad (3.19b)$$

Although the dispersion relation of this alternative set of equations also obeys Eq. (3.10), it can be seen that the third-order terms in the momentum equation (3.19b) are clearly different from the terms in Eq. (3.17b). It is therefore expected that the linear shoaling characteristics of Eqs. (3.17) and Eqs. (3.19) differ.

As was seen in Section 3.2, the use of a different computational velocity variable leads to different third-order terms in the right-hand members of the equations and therefore to a different dispersion relation. Conversely, the procedure of adding third-order terms to the equations changes the interpretation of the velocity variable. Nevertheless, since no higher-order terms were added to the continuity equation of Eqs. (3.15-19), it is assumed that the computational velocity variable in these equations has not been changed and is still the flux

q or the vertically averaged velocity $\bar{u}$. Note that this also means that the horizontal velocity field can be reconstructed after computation of q or $\bar{u}$ and ζ by means of the parabolic expression for the horizontal velocity profile.

Since we are primarily interested in the prediction of near-bottom velocities, it would be desirable to obtain Boussinesq-type equations with improved frequency dispersion which are written in the bottom velocity u_b instead of in the averaged velocity $\bar{u}$. The procedure of Madsen et al. then should be worked out starting with the equations (3.5) written in the bottom velocity u_b . In this case both in the momentum equation and the continuity equation dispersive terms are present. Following the procedure by Madsen et al. therefore means changing the third-order terms in the momentum equation as well as in the continuity equation (see Dingemans, 1994b) which means in fact changing the meaning of the computational velocity. It can be concluded that for the prediction of velocity variables only the equations in $\bar{u}$ or the corresponding equations in q are suitable.

Beji and Battjes (1994) used a set of equations which they derived from Madsen et al. (1991, 1992), but formulated in terms of $\bar{u}$ rather than in the flux q . The equations read

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} [(h + \zeta) \bar{u}] = 0 \quad (3.20a)$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + g \frac{\partial \zeta}{\partial x} = \left(\frac{1}{3} + b \right) h^2 \frac{\partial^3 \bar{u}}{\partial x^2 \partial t} + b g h^2 \frac{\partial^3 \zeta}{\partial x^3} + h h_x \frac{\partial^2 \bar{u}}{\partial x \partial t} \quad (3.20b)$$

The momentum equation of this set however, differs from Eq. (3.18b), which is the equivalent of the momentum equation as derived by Madsen and Sørensen (1992), but formulated in terms of the vertically averaged velocity $\bar{u}$ instead of in the flux q ; the terms with $b h_x$ in Eq. (3.18b) are omitted in Eq. (3.20b). This discrepancy between the bottom-slope terms does not affect the frequency dispersion but may give rise to different linear shoaling behaviour. This will be studied in the next section.

3.4 Linear shoaling behaviour

All models described in Section 3.3.2 have the same frequency-dispersion behaviour for the corresponding cases of horizontal bottom, at least for the optimum value of the parameter b . However, the differences are found in the depth-dependent terms. The differences in shoaling behaviour can be studied for slowly-varying bottom which means that terms with $(h_x)^2$ and h_{xx} are disregarded.

Madsen and Sørensen (1992) investigated the shoaling behaviour of their equations (3.15). Dingemans (1994b) studied the shoaling behaviour for both the alternative set (3.19) described in Section 3.3.2 and the corresponding set in q . The fact that those two models resulted in the same shoaling behaviour demonstrates that indeed the rephrasing of a specific set of Boussinesq-type equations in $\bar{u}$ to equations in q and vice versa does not affect the shoaling properties. Dingemans also studied the linear shoaling of Eqs. (3.17) which naturally gave the same results as found by Madsen et al. since the linear shoaling properties of a specific set of Boussinesq-type equations for large bottom slopes is unaffected if the equations are reduced to small bottom slopes.

In the following, a linear shoaling analysis will be made of the equations (3.20) presented by Beji and Battjes. Afterwards the shoaling behaviour of these equations will be compared with the shoaling behaviour of the equations after Madsen et al. (3.15-18) and the alternative to the latter equations (3.20) as derived by Dingemans.

We consider the linearized version of Eqs. (3.20):

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} (h \bar{u}) = 0 \quad (3.21a)$$

$$\frac{\partial \bar{u}}{\partial t} + g \frac{\partial \zeta}{\partial x} = \left(\frac{1}{3} + b \right) h^2 \frac{\partial^3 \bar{u}}{\partial x^2 \partial t} + b g h^2 \frac{\partial^3 \zeta}{\partial x^3} + h h_x \frac{\partial^2 \bar{u}}{\partial x \partial t} \quad (3.21b)$$

The corresponding wave equation is derived by eliminating ζ or $\bar{u}$ from these equations. Neither ζ nor $\bar{u}$ can exactly be eliminated from the linearized model. Therefore an approach is followed as used by Dingemans (1994b). After differentiation of the linearized continuity equation with respect to t , and multiplication with h and subsequent differentiation of the linearized momentum equation with respect to x , $\bar{u}$ can be eliminated from the left-hand side of the momentum equation. This yields

$$\frac{\partial}{\partial x} \left(g h \frac{\partial \zeta}{\partial x} \right) - \frac{\partial^2 \zeta}{\partial t^2} = \left(\frac{1}{3} + b \right) \frac{\partial^2}{\partial x \partial t} \left[h^3 \frac{\partial^2 \bar{u}}{\partial x^2} \right] + b g \frac{\partial}{\partial x} \left[h^3 \frac{\partial^3 \zeta}{\partial x^3} \right] + \frac{\partial^2}{\partial x \partial t} \left[h^2 h_x \frac{\partial \bar{u}}{\partial x} \right] \quad (3.22)$$

The terms with $\bar{u}$ in the right-hand side of this equation are to be rewritten in terms of ζ . If we neglect higher-order derivatives of h , from the linearized continuity equation can be found

$$h \frac{\partial \bar{u}}{\partial x} = - \left[\frac{\partial \zeta}{\partial t} + \bar{u} \frac{\partial h}{\partial x} \right] \quad (3.23a)$$

$$h^2 \frac{\partial^2 \bar{u}}{\partial x^2} = -h \frac{\partial^2 \zeta}{\partial t \partial x} + 2 \frac{\partial \zeta}{\partial t} \frac{\partial h}{\partial x} \quad (3.23b)$$

Using these approximations and again restricting the analysis to terms linear in h_x , Eq. (3.22) becomes

$$\frac{\partial}{\partial x} \left(g h \frac{\partial \zeta}{\partial x} \right) - \frac{\partial^2 \zeta}{\partial t^2} = - \frac{\partial^2}{\partial x \partial t} \left[\left(\frac{1}{3} + b \right) h^2 \frac{\partial^2 \zeta}{\partial x \partial t} + \left(\frac{1}{3} - 2b \right) h h_x \frac{\partial \zeta}{\partial t} \right] + b g \frac{\partial}{\partial x} \left[h^3 \frac{\partial^3 \zeta}{\partial x^3} \right] \quad (3.24)$$

We now consider a progressive-wave solution of the form

$$\zeta(x, t) = a(x) e^{i(\omega t - \phi(x))} \quad (3.25)$$

where ω is the cyclic frequency, a is the local wave amplitude and ϕ is the phase function, which is related to the local wave number by $\phi_x = k(x)$. Besides the water depth also the wave number and the wave amplitude are considered to be slowly varying functions of x , and consequently products of derivatives and higher derivatives of these quantities will be neglected in the following. The next step is to substitute Eq. (3.25) in Eq. (3.24). To the lowest order all x -derivatives of h , a and k are neglected and, as expected, with $c = \omega/k$ the

linear dispersion relation (3.10) is obtained. Collecting the terms to the next order (the terms with h_x , a_x and k_x) yields

$$\begin{aligned} & a_x(2ghk + 4bgh^3k^3 - 2(b + \frac{1}{3})h^2\omega^2k) + \\ & k_x(gha + 6bgh^3k^2a - (b + \frac{1}{3})h^2\omega^2a) + \\ & h_x(gka + 3bgh^2k^3a) \end{aligned} \quad (3.26)$$

After elimination of the frequency ω from this expression by using the linear dispersion relation we get

$$\alpha_1 \frac{a_x}{a} + \alpha_2 \frac{k_x}{k} + \alpha_3 \frac{h_x}{h} = 0 \quad (3.27)$$

where

$$\begin{aligned} \alpha_1 & \equiv 2(1 + 2bk^2h^2 + b(b + \frac{1}{3})k^4h^4) \\ \alpha_2 & \equiv 1 + 6bk^2h^2 + 5b(b + \frac{1}{3})k^4h^4 \\ \alpha_3 & \equiv 1 + (4b - \frac{2}{3})k^2h^2 + 3b^2k^4h^4 \end{aligned} \quad (3.28)$$

By differentiation of the linear dispersion relation (3.10) a relation between k_x/k and h_x/h can be found. After elimination of ω the following solution is obtained:

$$\frac{k_x}{k} = -\alpha_4 \frac{h_x}{h} \quad (3.29)$$

where

$$\alpha_4 \equiv \frac{1}{2} \left[\frac{1 + (2b - \frac{1}{3})k^2h^2 + b(b + \frac{1}{3})k^4h^4}{1 + 2bk^2h^2 + b(b + \frac{1}{3})k^4h^4} \right] \quad (3.30)$$

Substitution of Eq. (3.29) in Eq. (3.27) gives the resulting linear shoaling equation:

$$\frac{a_x}{a} = -\alpha_5 \frac{h_x}{h} \quad (3.31)$$

where the shoaling coefficient α_5 is given by

$$\alpha_5 \equiv \frac{\alpha_3 - \alpha_2\alpha_4}{\alpha_1} \quad (3.32)$$

As could be expected, the only difference with the shoaling behaviour derived by Madsen et al. for their equations (3.15) is the expression for α_3 since this coefficient is an indication for the effect of the bottom-slope terms. The coefficient α_3 for Eq. (3.15) is denoted by α_{3m} and given by:

$$\alpha_{3m} \equiv 1 + (4b - \frac{2}{3})k^2h^2 + b(3b + \frac{2}{3})k^4h^4 \quad (3.33)$$

Thus, the shoaling coefficient corresponding to Eqs. (3.15) is given by Eq. (3.32) with α_3 replaced by α_{3m} .

The coefficients α_{1d} , α_{2d} and α_{3d} for Eqs. (3.19) derived by Dingemans (1994b) are given by:

$$\begin{aligned} \alpha_{1d} &\equiv 2 + 4bk^2h^2 + 2b(b + \frac{1}{3})k^4h^4 \\ \alpha_{2d} &\equiv 1 + 6bk^2h^2 + 5b(b + \frac{1}{3})k^4h^4 \\ \alpha_{3d} &\equiv 1 + 2(2b - \frac{1}{3})k^2h^2 + 3b(b + \frac{1}{3})k^4h^4 \end{aligned} \quad (3.34)$$

The shoaling coefficient for Eqs. (3.19) follows from Eq. (3.32) if α_{1d} , α_{2d} and α_{3d} are used instead of α_1 , α_2 and α_3 .

To be able to compare the shoaling behaviour of the three considered sets of equations to the exact linear shoaling behaviour, we need the shoaling coefficient α_s for Stokes first-order theory. By the use of the concept of energy flux conservation, α_s can be found to be (see for example Dingemans, 1994b):

$$\alpha_s \equiv khT \frac{(1 - khT)(1 - T^2)}{[T + kh(1 - T^2)]^2} \quad (3.35)$$

where $T = \tanh kh$

The shoaling coefficients of the three sets of equations are compared with this exact linear one in Figure 3.3.

From Fig. 3.3 it can be concluded that both the equations by Beji and Battjes (3.20) and the Eqs. (3.19) have bad shoaling behaviour for $kh > 1$. For the equations after Madsen et al. (3.15-18) we see a good correspondence for the whole considered range $0 < kh < 5$. It is remarkable that the small discrepancy between the bottom-slope terms between the equations of Madsen et al. and the equations of Beji and Battjes gives rise to large differences in linear shoaling characteristics.

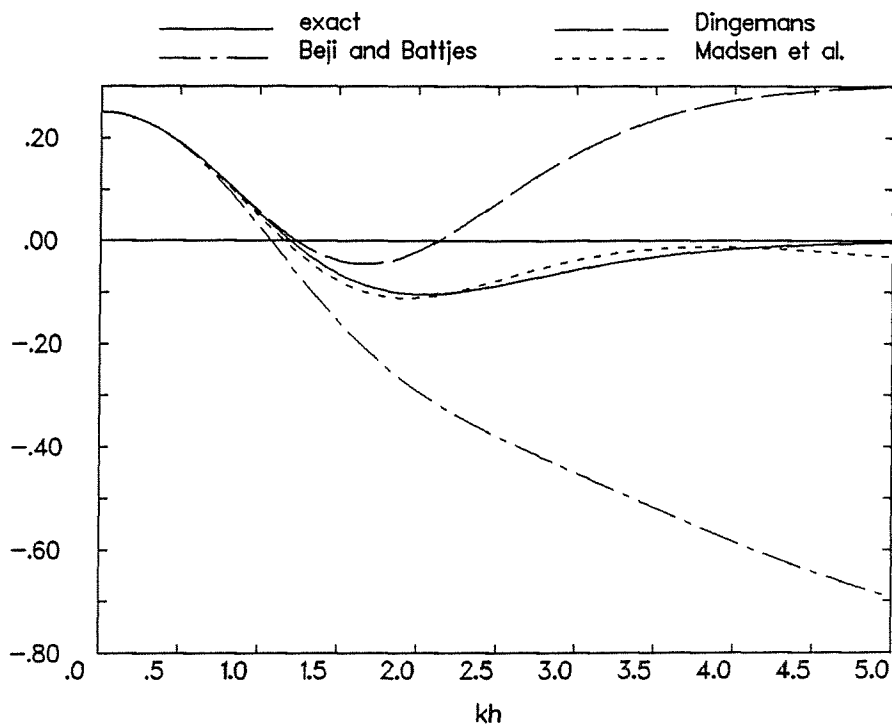


Figure 3.3 Graph of linear shoaling coefficients of the sets after Madsen et al. (3.15-18), the set by Dingemans (3.19) and by Beji and Battjes (3.20) and the exact linear one

3.5 Choice of equations for numerical treatment

In Section 3.3 it was demonstrated that Boussinesq-type equations with improved frequency dispersion have to be stated in $\bar{u}$ or in q in order to retain the unambiguous interpretation of the velocity variable. The equations meeting this requirement were given in Section 3.3.2. The comparison of those equations with respect to shoaling behaviour indicates preference for the sets (3.15-18) after Madsen et al.

The time domain model developed at Delft University of Technology (Beji and Battjes, 1994) is presently based on Eqs. (3.20) which are stated in $\bar{u}$. Those equations are to be replaced by Eqs. (3.17). In this way a model is obtained which is valid for larger bottom slopes than the present model and in which good frequency behaviour is combined with good shoaling behaviour. The fact that both the new equations (3.17) and the old equations (3.20) are written in terms of $\bar{u}$ limits the necessary changes to the numerical scheme and the program code. Further, the time domain model is extended to reconstruct the horizontal velocity field after solving ζ and $\bar{u}$ from the Boussinesq equations. The time domain model is dealt with in Chapters 4 through 6.

In the DUT spectral model (Eldeberky and Battjes, 1995) spectral Boussinesq-equations which were based on Eqs. (3.15) are used. This model is extended to obtain the horizontal velocity field. Chapters 7 and 8 are about the spectral Boussinesq model.

4 Numerical solution using time domain equations

4.1 Introduction

As was mentioned in Chapter 3, a time domain Boussinesq model based on Eqs. (3.20) was developed at Delft University of Technology. In the following this model will be referred to as the DUT1 model. This model was extended to reconstruct the horizontal velocity field after integration of the Boussinesq equations. Also the governing equations were changed; Eqs. (3.20) were replaced by Eqs. (3.17). The new equations have better shoaling behaviour and are valid for larger bottom slopes than the old equations. The new model will be denoted by the DUT2 model.

In discretizing the new equations, which contain different third-order derivatives in the right-hand side of the momentum equation compared to the old equations, the numerical method as used in the DUT1 model is followed. The resulting finite difference approximations and the treatment of the boundaries are given in Section 4.2.1 and 4.2.2 respectively. The numerical scheme used in the DUT1 model is essentially based on the technique used by Peregrine (1966, 1967) but it differs in the conservative formulation of the continuity equation and the treatment of the advective term.

In the method of Peregrine a time-centered implicit scheme is used to discretize the differential equations. Those equations are easily solved by using a predictor-corrector method as follows. An explicit formulation of the continuity equation is used to obtain an estimation of the surface elevation at the new unknown time level, which is used in determining the non-linear gravity term and the Boussinesq correction terms in ζ in the implicit discretization of the momentum equation. By doing so, the discretized momentum equation is reduced to a three-diagonal system, which is efficiently solved by the Double Sweep or Thomas algorithm. This gives the velocity at the new time level, which is used in the implicit formulation of the continuity equation to obtain a better approximation of the surface elevation at the new time level. Peregrine argued that this technique gives a better convergence compared to straightforward approximations.

The explicit determination of the terms in ζ in the momentum equation however, requires a relatively small time step, especially in deep water where the total Boussinesq effect becomes a balance between relatively large terms (Madsen et al., 1991, 1992). Too large a time step will result in artificial damping of the wave energy. The time step restriction however, is preferable above the five grid points wide momentum operator resulting from an implicit discretization of the terms in ζ . Madsen et al. also considered the possibility of iteration, which allows for a larger time step to be used but turned out to make the computational effort comparable to or larger than the explicit determination of the terms in ζ , because of the number of iterations necessary to obtain an accurate solution.

Section 4.2.3 deals with the numerical method used to reconstruct the horizontal velocity field. The numerical solution as described in Section 4.2 was implemented using the programme code of the DUT1 model which is written in Fortran 77. The structure and the

documentation of the programme code was improved as well. The Fortran programme is briefly discussed in Section 4.3.

4.2 Discretization procedure

4.2.1 Finite difference approximations

For the sake of convenience, the Boussinesq-type equations as used in the DUT1 en DUT2 model, Eqs. (3.20) and (3.17) respectively, are repeated here. In the DUT1 model the following equations are used:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} [(h + \zeta) \bar{u}] = 0 \quad (4.1a)$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + g \frac{\partial \zeta}{\partial x} = \left(\frac{1}{3} + b \right) h^2 \frac{\partial^3 \bar{u}}{\partial x^2 \partial t} + b g h^2 \frac{\partial^3 \zeta}{\partial x^3} + h h_x \frac{\partial^2 \bar{u}}{\partial x \partial t} \quad (4.1b)$$

The DUT2 model on the other hand is based on

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} [(h + \zeta) \bar{u}] = 0 \quad (4.2a)$$

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + g \frac{\partial \zeta}{\partial x} = h \frac{\partial}{\partial t} \left[\left(\frac{1}{2} + b \right) \frac{\partial^2 (h \bar{u})}{\partial x^2} - \frac{1}{6} h \frac{\partial^2 \bar{u}}{\partial x^2} \right] + b g h \frac{\partial^2}{\partial x^2} \left[h \frac{\partial \zeta}{\partial x} \right] \quad (4.2b)$$

The equations are formulated in terms of the vertically averaged velocity $\bar{u}$ since the momentum equation written in q as applied by Madsen et al. is more difficult to discretize and solve. For completeness, it should be mentioned that the equations and the resulting finite difference approximations can easily be extended to two horizontal dimensions.

The finite difference equations as used in the DUT1 model (Beji and Battjes, 1994) read, with the terms in the right-hand side of the momentum equation denoted by RHS1,

$$\frac{\zeta_i^{j+1} - \zeta_i^j}{\Delta t} + \frac{[(h + \zeta) \bar{u}]_{i+1}^{j+1} - [(h + \zeta) \bar{u}]_{i-1}^{j+1} + [(h + \zeta) \bar{u}]_{i+1}^j - [(h + \zeta) \bar{u}]_{i-1}^j}{4 \Delta x} \quad (4.3a)$$

$$\frac{\bar{u}_i^{j+1} - \bar{u}_i^j}{\Delta t} + \bar{u}_i^{j+1/2} \frac{\bar{u}_{i+1}^{j+1} - \bar{u}_{i-1}^{j+1} + \bar{u}_{i+1}^j - \bar{u}_{i-1}^j}{4 \Delta x} + g \frac{\zeta_{i+1}^{j+1*} - \zeta_{i-1}^{j+1*} + \zeta_{i+1}^j - \zeta_{i-1}^j}{4 \Delta x} = \text{RHS1} \quad (4.3b)$$

where the indices i and j stand respectively for the integers multiplying the increments in space and time, and $\bar{u}^{j+1}$ and ζ^{j+1} are the unknown variables. Note that central differences formulations are used both for time and space derivatives which are computed at $(i, j + 1/2)$.

The * used in the approximation of the momentum equation indicate values of the unknown surface elevation at time level $j + 1$ estimated by an explicit use of the continuity equation:

$$\frac{\zeta_i^{j+1} - \zeta_i^j}{\Delta t} + \frac{[(h + \zeta)\bar{u}]_{i+1}^j - [(h + \zeta)\bar{u}]_{i-1}^j}{2\Delta x} \quad (4.4)$$

So far exactly the same discretization is used in the DUT2 model. Comparison of Eqs. (4.1) and (4.2) however shows that different third-order derivatives occur in the right-hand side of the momentum equations. The right-hand side of Eq. (4.2b), denoted by RHS2, was discretized as follows:

$$\begin{aligned} \text{RHS2} = & \left(\frac{1}{2} + b \right) h_i \frac{(h\bar{u})_{i+1}^{j+1} - 2(h\bar{u})_i^{j+1} + (h\bar{u})_{i-1}^{j+1} - (h\bar{u})_{i+1}^j + 2(h\bar{u})_i^j - (h\bar{u})_{i-1}^j}{\Delta x^2 \Delta t} \\ & - \frac{1}{6} (h_i)^2 \frac{\bar{u}_{i+1}^{j+1} - 2\bar{u}_i^{j+1} + \bar{u}_{i-1}^{j+1} - \bar{u}_{i+1}^j + 2\bar{u}_i^j - \bar{u}_{i-1}^j}{\Delta x^2 \Delta t} \\ & + b g h_i \left[\frac{(h_{i+2} + h_{i+1})(\zeta_{i+2}^{j+1} + \zeta_{i+2}^j) - (h_{i+2} + 2h_{i+1} + 2h_i)(\zeta_{i+1}^{j+1} + \zeta_{i+1}^j)}{8\Delta x^3} + \right. \\ & + \frac{(h_{i+1} - h_{i-1})(\zeta_i^{j+1} + \zeta_i^j) + (h_i + 2h_{i-1} + 2h_{i-2})(\zeta_{i-1}^{j+1} + \zeta_{i-1}^j)}{8\Delta x^3} + \\ & \left. - \frac{(h_{i-1} - h_{i-2})(\zeta_{i-2}^{j+1} + \zeta_{i-2}^j)}{8\Delta x^3} \right] \quad (4.5) \end{aligned}$$

Note that also in the determination of the Boussinesq terms use is made of the explicit determination of the surface elevation at the new time level. The procedure is summarized as follows: with the provisional surface elevation values at the new time level solved from Eq. (4.4), the new time level velocity field can be obtained from Eq. (4.3b) with RHS2 instead of RHS1. Subsequently, the surface elevations at the new time level are improved by applying Eq. (4.3a). Iteration between the momentum equation and continuity equation could have been carried out but was found to be unnecessary to obtain accurate estimates for the surface elevation (Beji and Battjes, 1994). To find out whether this conclusion also holds for the accuracy of the velocity values, tests with monochromatic incoming waves were run in which extra iteration was carried out; the updated surface elevation was used to obtain new estimates for the velocities using Eq. (4.3b) again. In this way not only the surface elevation values, but also the velocity values were corrected. It was found that in case of small enough spatial and time steps to reproduce accurate surface elevations, the accuracy of the velocities was satisfactory as well without this extra iteration.

Since all terms in the finite difference equations are evaluated at the time level $j + 1/2$, the averaged velocity $\bar{u}_{i,j+1/2}$ arises in the advective term. In the original formulation Peregrine (1966, 1967) uses the old time level value $\bar{u}_{i,j}$ as an approximation. Beji and Battjes (1994) however showed that, especially for steep waves, this causes a gradual but consistent decay in wave height as time progresses. In order to estimate $\bar{u}_{i,j+1/2}$ they used a Taylor series expansion. Truncation at the first order yields

$$\bar{u}_i^j = \bar{u}_i^{j+1/2} - \frac{\Delta t}{2} \left[\frac{\partial \bar{u}}{\partial t} \right]_{i,j+1/2} + \dots \quad (4.6)$$

Not all terms in this expansion are known and the unknown term $\bar{u}_t$ has to be replaced. The time derivative $\bar{u}_t$ can be approximated by employing the momentum equation for the shallow water approximation, $\bar{u}_t \approx -\bar{u}\bar{u}_x - g\zeta_x$, in which $\bar{u}_x$ can be replaced by using the linearized continuity equation $\bar{u}_x = -\zeta_t/h$. This results in

$$\bar{u}_i^{j+1/2} = \bar{u}_i^j + \frac{\Delta t}{2} \left[\frac{1}{h} \frac{\partial \zeta}{\partial t} \bar{u} - g \frac{\partial \zeta}{\partial x} \right]_{i,j+1/2} \quad (4.7)$$

The factor $\bar{u}_{i,j+1/2}$ at the right can be handled in different ways. Beji and Battjes used a Taylor series expansion as in Eq. (4.6), in which now $\bar{u}_t$ is approximated as $\bar{u}_t \approx -g\zeta_x$.

In computations, however, they also used expansions at higher order. In this work in the first instance the expansion at first order was used. However, since the effect of this first-order expansion compared with the old time level formulation by Peregrine was found to be insignificant, the latter formulation was used. In situations in which a more accurate estimation of $\bar{u}_{i,j+1/2}$ is needed, an explicit formulation of the momentum equation in which half a time step is used could be applied. This however was not considered.

If we disregard the effects of the Boussinesq terms in the right-hand side of the momentum equation and the effects of the explicit determination of the terms in ζ in the momentum equation, the numerical method reduces to the θ -method with $\theta = 0.5$. An idea of the characteristics of the numerical method can therefore be obtained by considering the θ -method. Petit (1994) considered the dissipation and dispersion for a hyperbolic equation of the form $u_t + cu_x = 0$ for a number of integration schemes, including the θ -method, as a function of the number of points per wave length $L/\Delta x$. Figure 4.1 shows the errors in the numerical phase celerity c_n for $\theta = 0.5$. Those errors depend on $L/\Delta x$ and the CFL number $\mu = c\Delta t/\Delta x$. Note that the CFL number should be less than 2 for stability. Further, no dissipation is present in the scheme.

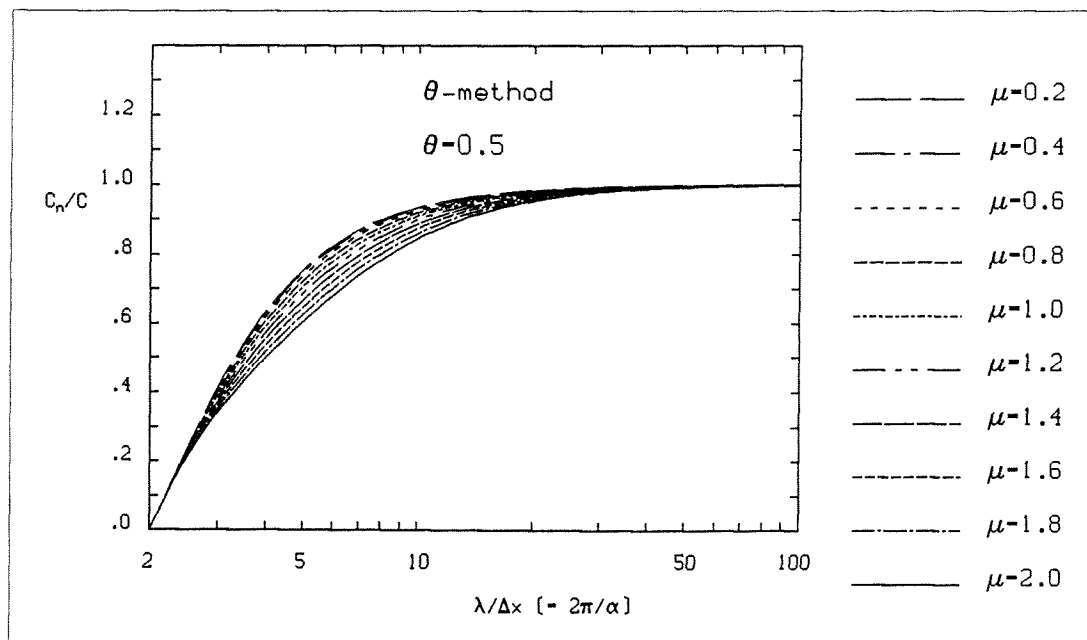


Figure 4.1 Ratio of numerical phase celerity c_n to c versus $L/\Delta x$ for the θ -method with $\theta=0.5$, from Petit (1994)

4.2.2 Initial and boundary conditions

The most straightforward initial condition is the unperturbed state. Arbitrary surface elevations and velocities at time $t = 0$ can be dealt with however. Further, both at the ingoing boundary and the outgoing boundary, surface elevation as well as velocity values are prescribed. Note that for the limiting case of $\varepsilon \gg \mu$ the Boussinesq equations reduce to the shallow-water equations. The mathematical nature of the equations then requires at most one boundary condition since only one characteristic is entering the computational domain. The higher-order Boussinesq terms in the right-hand side however change the nature of the equations and it becomes possible to prescribe two boundary conditions at each boundary as long as $\varepsilon = O(\mu)$. The boundary conditions at the first node $i = 1$ are handled as follows. The surface elevation $\zeta_1(t)$ is specified either as input from measurements or as some other appropriate function specified at each time step:

$$\zeta_1(t) = f(t) \quad (4.8)$$

Beji and Battjes used two different formulations for the depth-averaged velocity $\bar{u}_1(t)$ at the ingoing boundary, i.e. computed from the linear theory and computed from the continuity equation for a progressive wave.

In all computations with the DUT2 model, the latter formulation was chosen because it is mass-conserving:

$$\bar{u}_1(t) = \frac{c_1 \zeta_1(t)}{h_1 + \zeta_1(t)} \quad (4.9)$$

The computation of the second-order derivatives of $\bar{u}$ and $h\bar{u}$ with respect to x , which are needed for the reconstruction of the horizontal velocity field, is very sensitive to small disturbances. It was found that this problem could partially be solved by applying a slow start of the following form:

$$\begin{aligned} \zeta_1(t) &= f(t) \cdot \frac{1}{2} \left[1 - \cos \left(\frac{4\pi t}{T} \right) \right] & t < \frac{T}{4} \\ \zeta_1(t) &= f(t) & t \geq \frac{T}{4} \end{aligned} \quad (4.10)$$

Equation (4.10) was therefore used in the computations as described in Chapter 6. At the outgoing boundary, $i = n$, the radiation condition was used both for the surface displacement, when solving the continuity, and for the velocity, when solving the momentum equation. This condition was not completely satisfactory (Beji and Battjes, 1994); waves with phase velocities different from $c = (gh)^{1/2}$ gave rise to large reflection. Because no appreciable improvement was found when using a sponge layer condition, they performed a series of cold start computations. In the computations with the DUT2 model a different approach is followed; the computational domain is chosen long enough, so that reflected waves cannot manifest themselves in the domain of interest before the end of the computation.

The third-order term in ζ which is the third term in Eq. (4.5) requires some additional effort at the boundary nodes. Since this term is a third-order spatial derivative, it is necessary to know ζ_0 , the surface displacement just outside the upwave boundary node $i = 1$, for the node $i = 2$ at which the computation starts, and ζ_{n+1} for the node $i = n-1$ at which the computation ends. These values are obtained by parabolic extrapolation as suggested by Beji and Battjes:

$$\begin{aligned}\zeta_0^j &\approx 3\zeta_1^j - 3\zeta_2^j + \zeta_3^j \\ \zeta_{n+1}^j &\approx 3\zeta_n^j - 3\zeta_{n-1}^j + \zeta_{n-2}^j\end{aligned}\quad (4.11)$$

Likewise, values for h_0 en h_{n+1} are needed. These are estimated by means of linear extrapolation, which is consistent with the linear interpolation carried out between the input values of the water depth h to determine the values of h_i . Linear extrapolation yields for h_0 en h_{n+1} :

$$\begin{aligned}h_0^j &\approx 2h_1^j - h_2^j \\ h_{n+1}^j &\approx 2h_n^j - h_{n-1}^j\end{aligned}\quad (4.12)$$

4.2.3 Reconstruction of the horizontal velocity field

Solving the equations as indicated in Section 4.2.2 provides the surface elevation and the vertically averaged velocity $\bar{u}$. To obtain velocity values at various levels, for example the bottom velocity $\bar{u}_b$, the horizontal velocity field has to be reconstructed by means of the parabolic expression for the velocity field as a function of the depth as assumed in deriving the Boussinesq equations. This expression stated in terms of $\bar{u}$ was given as Eq. 3.2 in Chapter 3 and is repeated here:

$$u(x, z, t) = \bar{u} - \left[\frac{1}{2}h + z\right](h\bar{u})_{xx} + \left[\frac{1}{6}h^2 - \frac{1}{2}z^2\right]\bar{u}_{xx}\quad (4.13)$$

Equation (4.13) shows that the second-order derivatives $(h\bar{u})_{xx}$ and $\bar{u}_{xx}$ have to be determined after computation of the vertically averaged velocity $\bar{u}$ at each position at the new time level $j+1$. This was at first dealt with as follows:

$$\bar{u}_{xx}^{j+1} = \frac{\bar{u}_{i+1}^{j+1} - 2\bar{u}_i^{j+1} + \bar{u}_{i-1}^{j+1}}{\Delta x^2}\quad (4.14a)$$

and

$$[(h\bar{u})_{xx}]_i^{j+1} = \frac{h_{i+1}\bar{u}_{i+1}^{j+1} - 2h_i\bar{u}_i^{j+1} + h_{i-1}\bar{u}_{i-1}^{j+1}}{\Delta x^2}\quad (4.14b)$$

It appeared however that these second-order derivative estimates were very sensitive. Small disturbances in the computed velocity values $\bar{u}$ gave rise to vastly erroneous results for the

second-order derivatives. Those disturbances in $\bar{u}$ with a wave length of the order of the spatial mesh size, are due to numerical errors. A similar problem was encountered by Quinn et al. (1994) in calculating second-order derivatives from PIV velocity data. They pointed out that the standard deviation in calculating the mean velocity $\bar{u}$ can be greater than the difference between means at adjacent positions, giving large errors in $\bar{u}_x$ and consequently $\bar{u}_{xx}$. In order to smooth out this statistical variation they applied a least squares polynomial fit to the $\bar{u}$ -values for the calculation of $\bar{u}_{xx}$.

Various techniques were considered to improve the calculation of $(h\bar{u})_{xx}$ and $\bar{u}_{xx}$. At first a higher-order discretization was used. Instead of the three grid points wide central differences formulation (4.14), formulations based on five and seven points respectively were used. Calculation of $(h\bar{u})_{xx}$ and $\bar{u}_{xx}$ in this way did not yield better results. Hereafter another approach was used in which the discretization (4.14) was applied with a spatial mesh size $2\Delta x$ instead of the spatial step Δx as used in solving ζ and $\bar{u}$. This method was unsuccessful as well.

The use of a procedure to smooth out the calculated mean velocity $\bar{u}$ and applying Equation (4.14) to the smoothed values was found to give good results. Several low-pass digital smoothing filters which eliminate the high frequencies and cause the low frequencies to pass through, were considered. Designing a filter in fact means finding the transfer function $H(\omega)$ which is 1 for the frequencies which should pass through and 0 for the frequencies which should be eliminated. We require that the smoothing procedure eliminates all harmonics higher than the fifth. A Fourier transformation of the transfer function gives the impulse response function $g(x)$. Multiplying the original mean velocity $\bar{u}(x)$ signal at each time step with this impulse response function gives the smoothed signal $\bar{u}^s(x)$.

The used filters are given by

$$\begin{aligned}\bar{u}_i^s &= g_0 \bar{u}_i + \sum_{k=1}^n g_k (\bar{u}_{i-k} + \bar{u}_{i+k}) \\ g_i &= \frac{(w_i h_i)}{\sum (w_i h_i)} \\ w_i &= 1 + \cos \left[\frac{\pi i}{n} \right] \quad |i| < n \\ w_i &= 0 \quad |i| \geq n \\ h_i &= \frac{1}{\pi i} \sin \left[\frac{2 \pi i}{m_p} \right]\end{aligned}\tag{4.15}$$

in which i is again the integer multiplying the increments in space. After trying various filters it was found that the computed signal was best filtered with $n = 12$, $m_p = 8$, leading to g_k as given in Table 4.1.

The corresponding transfer function H as a function of $2/nx$ where nx is the number of points per wave length $L/\Delta x$, is shown in Figure 4.2. In the computations performed with the DUT2 model the spatial mesh size was such that $80 < nx < 400$. For the fifth harmonic then follows $16 < nx < 80$ and thus $0.03 < 2/nx < 0.13$. Figure 4.2 shows that waves for which $2/nx > 0.4$ and thus $nx < 5$ are filtered from the signal.

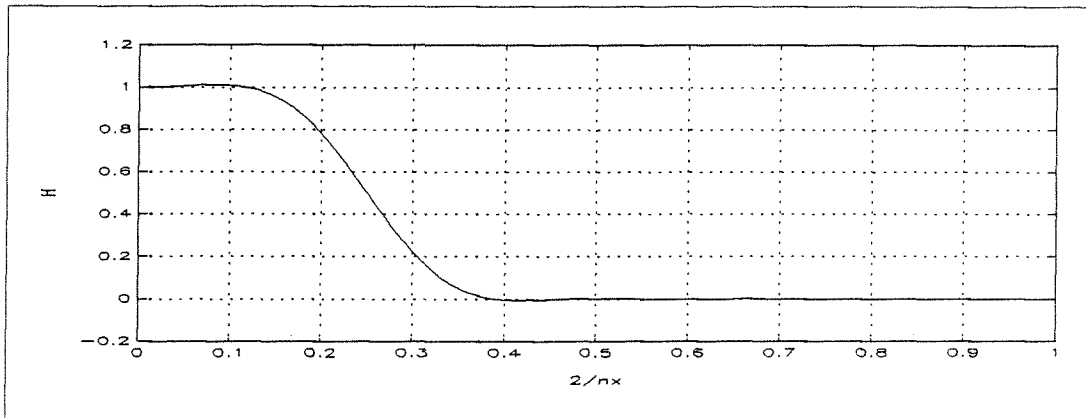


Figure 4.2 Transfer function H as a function of $2/nx$

k	0	1	2	3	4	5	6
g_k	0.2509	0.2221	0.1490	0.0643	0	-0.0284	-0.0266
k	7	8	9	10	11	12	
g_k	-0.0120	0	0.0037	0.0021	0.0003	0	

Table 4.1 Coefficients g_k

After computation of the computational velocity $\bar{u}$ at the new time level $j+1$ and subsequent calculation of the values of $(h\bar{u})_{xx}$ and $\bar{u}_{xx}$ in the above described way the velocity profiles at the new time level $j+1$ can be obtained from Eq. (4.13) as follows:

$$u(i, z, j+1) = \bar{u}_i^{j+1} - \left[\frac{1}{2} h_i + z \right] [(h\bar{u})_{xx}]_i^{j+1} + \left[\frac{1}{6} (h_i)^2 - \frac{1}{2} z^2 \right] \bar{u}_{xxi}^{j+1} \quad (4.16)$$

The values of z for which the velocity values as a function of t have to be calculated can be defined in the model input. In the computations with the DUT2 model the bottom velocities corresponding with $z = -h$ and the velocities at approximately mid-depth were calculated at various locations as a function of t . Since the values of $(h\bar{u})_{xx}$ and $\bar{u}_{xx}$ are part of the model output, it is possible to reconstruct the complete horizontal velocity field if necessary.

4.3 Programme implementation

The DUT1 programme which is written in Fortran 77 was used as a basis for the DUT2 model. Before implementation of the new equations and the reconstruction of the velocity field, far-reaching changes were made to the programme code to enhance its readability and generality. The code was subdivided in various subroutines and the input data and computational parameters which were first part of the code were placed in an input file. In this way only this input file needs to be changed to perform computations with different wave conditions, geometry or computational parameters such as the spatial and time steps. Further, comment lines were inserted in the code.

After these improvements the right-hand side of the momentum equation RHS1 was replaced by RHS2 (4.5) with the use of Eqs. (4.11) and (4.12) at the boundaries. Further, the seaward boundary condition was replaced by (4.10) and the computation of the horizontal velocity field was included as described in Section 4.2.3. The implementation of the new equations and the velocity field computation were verified by performing computations with monochromatic waves with $\varepsilon \ll \mu$. Since those waves behave to close to linear wave theory, i.e. small non-linear contributions, the computations could be compared with the linear wave theory. Computations with a horizontal as well as a slowly-varying bottom were performed.

5 Validation of the time domain model with respect to surface elevations

5.1 Introduction

The generation of higher harmonics by non-linearity produces both free and locked harmonics. Freely moving wave components, as opposed to phase-locked components, are components moving according to their dispersion characteristics. Phase-locked components are moving with the velocity of the basic wave. The inaccurate frequency dispersion for freely moving short wave components is the most important reason that often a discrepancy between measurements and computations with Boussinesq-type models is found.

As part of the MAST G8M Coastal Morphodynamics research programme a comparison was made between flume measurements based on a bar-type geometry and computations with Boussinesq-type models of various institutes, including the DHI model (Dingemans, 1994a). As argued by Dingemans such a bar-type geometry provides a discriminating test between the various models; on the upward slope the effects of non-linearity generate higher harmonics and on the downward slope the difference between the phase velocities of the locked and free harmonics increases rapidly because of the rapidly increasing water depth. On the downward slope the wave form therefore changes very fast. The shallow horizontal part of the bar is so short that there the phase difference between the free and locked harmonics is still small. Especially on the downward slope, a good representation of the phase velocity by the computational models is needed. In the intercomparison study it was found that the differences between the various models indeed are most prominent on the downward slope.

The measurements used in the MAST comparison were performed by Klopman of DELFT HYDRAULICS in 1993. Three different measurement conditions with regular waves were considered. The surface elevations as measured in those tests were used for comparison with computed free surface elevations with various models. Beji performed a similar flume experiment at the Delft University of Technology and compared those measurements with computations with the DUT1 model (Beji and Battjes, 1994).

In order to determine its usability the computations as described in the above mentioned intercomparison study were also performed with the new DUT2 model, which is an extension of the model of Beji (see Chapter 4). Computations with the DUT1 model were performed to serve as a reference. In this way the influence of the extension to larger bottom slopes and the improved shoaling behaviour of the DUT2 model can be determined. Further the performance of the DUT2 model can be compared with the performance of the DHI model (Madsen and Sørensen, 1992). The equations used in the DHI model are a reduction for small bottom slopes of the equations used in the DUT2 model.

The DELFT HYDRAULICS measurements are described in Section 5.2. The computations performed with both the DUT2 model and the DUT1 model are dealt with in Section 5.3. Section 5.4 is about the comparison of the computations with the measurements. Finally, Section 5.5 summarizes the conclusions.

5.2 Flume experiments

The experiments to be used are three regular wave tests performed by Klopman in January 1993 as part of the EU-sponsored Large Installations Programme (LIP). Those tests have a linear scale of 2 compared to the Beji measurements. In accordance with the computations in the MAST study, the computations performed in this study are based on the geometry of Beji and Battjes. Therefore the measurements of Klopman have to be rescaled to this geometry to compare the computations and measurements; the surface elevations have to be divided by 2 and the time by $\sqrt{2}$.

In the flume experiments by Klopman a trapezoidal bar is used (see Luth et al., 1994). The offshore slope is 1:20 and the shoreward slope is 1:10. The (unscaled) depth is 80 cm seaward and shoreward of the bar and 20 cm on the horizontal part of the bar. The lay-out of the experiments is given in Figure 5.1. The experiments have very accurate active wave absorption so that the reflection from the wave absorber can be neglected.

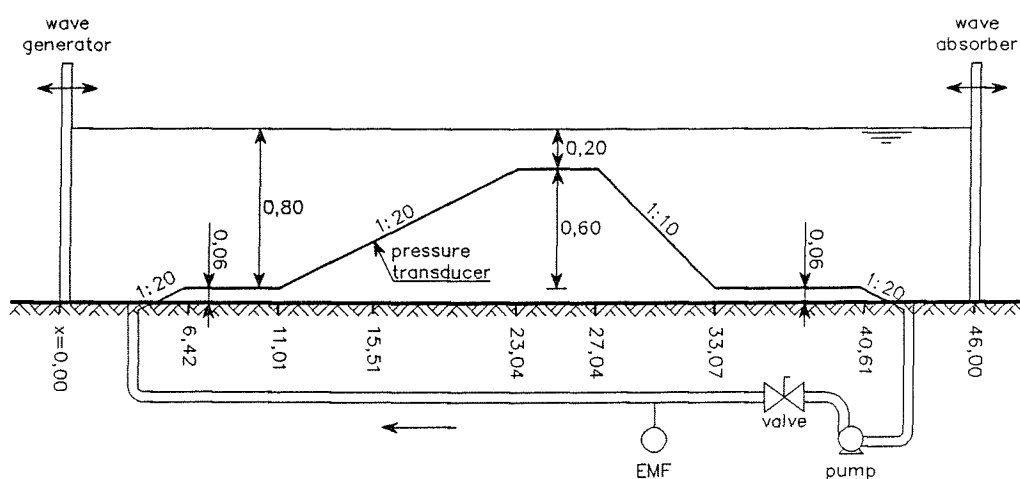


Figure 5.1 Experimental set-up of the Scheldt flume experiments (all dimensions in m), from Luth et al. (1994)

For the computations this geometry is rescaled to the geometry of Beji which yields an undisturbed water depth of 40 cm and 10 cm depth above the bar. The geometry as used in the computations is shown in Figure 5.2. This rescaled geometry is not completely equivalent to the geometry in the measurements but the differences are assumed to be insignificant.

Table 5.1 gives the locations of the stations at which the surface elevations were followed in the measurements and computations. More wave gauges were used behind and in front of the bar than in the experiments performed by Beji. The location of the wave gauges and the measurements are always given in rescaled quantities. When unscaled quantities are used this is mentioned explicitly.

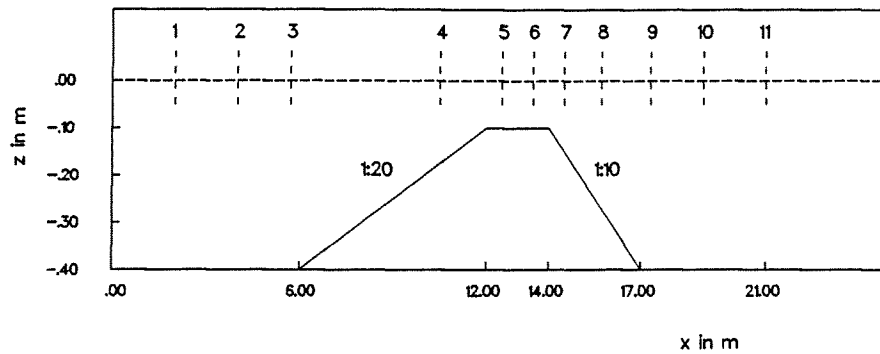


Figure 5.2 The rescaled bottom geometry and the location of the wave gauges as used in the computations

Station	x (m)	Series	Remarks
1	2.00	01	in front of bar
2	4.00	02	in front of bar
3	5.70	both	in front of bar, no computations
4	10.50	01	on upward slope
5	12.50	02	on the bar
6	13.50	01	on the bar
7	14.50	02	on downward slope
8	15.70	01	on downward slope
9	17.30	02	just behind the bar
10	19.00	01	further behind the bar
11	21.00	02	far behind the bar

Table 5.1 Distances of the wave gauges from the wave maker

For each measurement condition the measurements were carried out in two different series; the experiment had to be repeated with another position of the free surface elevation measuring probes. One wave gauge was kept fixed at 5.70 m and therefore measurements at that location are provided in both sets. Figure 5a shows time-windows of 60 s unscaled time of the measurements for the three tests A, B and C. Notice that "be" comes from Beji, "a", "b" and "c" denote the measurement condition, and "01" and "02" denote the fact that the measurements were performed twice with different locations of the wave probes.

The three measuring conditions which are considered are:

- A $T = 2.02$ s and $H = 2.0$ cm
- B $T = 2.525$ s and $H = 2.9$ cm
- C $T = 1.01$ s and $H = 4.1$ cm

In test A and C the waves are non-breaking. In test B spilling breakers occur between 13.3 and 15.3 m. The shallowness parameter $T(g/h)^{1/2}$ yields for the three conditions, with $h = 0.4$ m in the deep part:

- A $T(g/h)^{1/2} = 10.0$
- B $T(g/h)^{1/2} = 12.5$
- C $T(g/h)^{1/2} = 5.0$

The long wave region can be defined as $T(g/h)^{1/2} > 7$ which is equivalent with $kh < 1$. So in test A en B we are dealing with a long basic wave, while test C is characterised by a rather short basic wave.

5.3 Numerical experiments

All computations were performed with both the DUT1 and the DUT2 model. For the b -value in the models both $b = 0$ and $b = 1/15$ were chosen, the latter corresponding with improved frequency dispersion.

The duration of the tests should be enough to obtain permanent wave profiles at the farthest station at 21 m. Over 21 m we have an average depth of 31 cm and thus with $c = (gh)^{1/2}$ a mean celerity of $c \approx 2$ m/s. So it takes about 10 s for the main wave to travel the distance of 21 m. The measurements have a duration of about 60 s unscaled time which is rescaled a good 40 s. Therefore the duration of the numerical experiments was chosen to be 40 s as well. This provides enough reserve for the higher harmonics to reach the farthest station before the end of the computation.

The outgoing boundary, where the radiation condition is used with $c = (gh)^{1/2}$, appeared to cause reflection. Waves with phase velocities different from $c = (gh)^{1/2}$ cannot leave the domain without reflection. For this reason, the position of the outgoing boundary was chosen such that the reflected waves cannot manifest themselves in the domain of interest before the computation ends at $t = 40$ s. The horizontal part behind the bar with a depth $h = 0.4$ m was taken so long that the basic wave with a celerity of 2.0 m/s reaches the outgoing boundary at the end of the computation. This yields for the position of the outgoing boundary $x = 75$ m.

The surface elevation boundary condition at $x = 0$ m was a sinusoidal wave for all runs. In test A the signal $\zeta(t) = a \cos 2\pi t/T$ was taken as a boundary condition. In test B and C this same signal was found to give rise to disturbances close to the boundary, which grew progressively in time. This might be the result of the larger amplitude of the signal in test B and C compared to test A. The instabilities could be avoided by using the signal $\zeta(t) = a \sin 2\pi t/T$ which provides a slower start.

The discretization of the advective acceleration term as suggested by Beji was simplified. The factor $u_{i,j+1/2}$ was approximated by the old time level value $u_{i,j}$. The differences between the results with both formulations were found to be insignificant, at least in this situation. Further, all cases were calculated without extra iteration between the momentum and continuity equation.

Beji and Battjes (1994) performed their computations with Δx equal to 1/70 of the initial wave length and Δt equal to 1/50 of the initial wave period. For the fourth free harmonic these resolutions are then approximately 1/8 of the wavelength and 1/12 of the wave period, respectively. In the computations performed with the DUT1 model for the MAST comparison

study Δt equal to 0.02525 s was used for all three measurement conditions. The spatial resolution then is restricted by the fact that the CFL number has to be less than 2. The latter resolutions are more stringent than the resolutions used by Beji. In this work it was found that the time step of 0.02525 s was sufficient for obtaining good results before the downslope region. Behind the downslope region, where the higher harmonics become particularly manifest, even smaller resolutions are needed for convergence of the solution. The time and spatial steps necessary for convergence are very small, which is a result of the numerical method used. As can be seen in Figure 4.1, the numerical error in the phase velocity is increasing rapidly for a smaller number of spatial steps per wave length. Therefore, a spatial step which is small enough to have an insignificant error in the numerical phase celerity for the basic wave can cause large errors for higher harmonics. Figure 4.1 also shows that the influence of the CFL number on the numerical celerity is of less importance. Further, the explicit determination of the terms in ζ in the momentum equation requires a relatively small time step.

Note that besides numerical inaccuracy, incorrect representation of the frequency dispersion by Boussinesq models can also cause errors in the phase celerity. For $b = 1/15$ the Boussinesq equations overestimate the phase celerities (see Figure 3.2), while numerical inaccuracy can lead to underestimation of the phase celerity. Therefore it is possible that those two errors cancel and that the best numerical solution does not give the best correspondence with measurements. For $b = 0$ the Boussinesq model leads to an underestimation of the phase celerity.

In test A we have a wave period of $T = 2.02$ s and with $c = (gh)^{1/2}$ a length of the basic wave of $L = 4$ m. Using the spatial and time resolutions as suggested by Beji gives $\Delta x = 1/70 L \approx 5$ cm and $\Delta t = 1/50 T = 0.0404$ s. Then we have the CFL number

$$CFL = \sqrt{gh} \frac{\Delta t}{\Delta x} = 1.6$$

Comparison of computations with these Δx and Δt and with halved Δx and Δt , which approximately correspond with the steps used in the MAST study, showed significant differences in the results for most of the stations. A further halving of the steps learned that for convergence of the solution the halved steps are sufficient for the stations seaward of the downward slope, but that the farthest stations require still smaller spatial and time steps. It appeared that for test A Δt had to be chosen equal to 0.0101 s which corresponds to 1/200 of the initial wave period. For the spatial step, $\Delta x = 1.25$ cm was sufficient which corresponds to approximately 1/320 of the initial wave length. For the CFL number we still have $CFL = 1.6$ at the seaward boundary. Reduction of either the time and spatial steps or the CFL number did not change the solution any more. For the fourth free harmonic for which the wave length is 0.4 m, the spatial resolution is then approximately 1/30 of the wave length. This can be seen to give only small numerical errors in the phase celerity (Figure 4.1). The same spatial and time steps as used in test A appeared to be satisfactory for test B. In test C, Δt was chosen as 1/200 of the initial wave period which yields $\Delta t = 0.00505$ s. Since we have a rather short basic wave in test C, the requirement that the resolution of the third or fourth harmonic is about 1/30 of the wave length is very stringent. Therefore Δx is determined so that the CFL number is about 1. This leads to $\Delta x = 1$ cm.

5.4 Comparison of computations and measurements

5.4.1 Introduction

A comparison between computations and measurements can be carried out in a number of ways. Dingemans compared the computed and measured wave profiles at the various locations without matching the time elapsed since the start of the computations and measurements. In this way information on the correspondence of the wave shape is provided but time information is lost. In this study a different approach is followed. The time shifting between computations and measurements is determined at the first wave probe at $x = 2$ m. Comparisons between measurements and computations at the various locations are given with the time origin of the measurements shifted with respect to the time origin of the computations over this time difference at the first wave probe. It is obvious that this provides a more severe test than the comparison in the MAST study. Note that this procedure was carried out after rescaling of the measurements by dividing the surface elevations by 2 and the time by $\sqrt{2}$.

Since we have two measurement series for each wave condition, first the time shifting between these series 1 and series 2, denoted by τ_{12} , had to be determined. This was performed at the station at $x = 5.7$ m, since that is the only station which was kept fixed while repeating the experiment. Afterwards the time shifting τ_1 between the first measurement series at $x = 2$ m and the computations at $x = 2$ m was determined. The necessary time shifting for all stations provided by the first measurement series, is then given by τ_1 . The measurements for the stations provided by the second measurement series, have to be shifted over $\tau_1 + \tau_{12}$. For condition B and C it was found that the same results were found when matching computations and measurements at 4 m: $\tau_2 \approx \tau_1 + \tau_{12}$ and $\tau_2 + \tau_{21} \approx \tau_1$. This knowledge was used for condition A for which at the start of the measurements the waves had already reached the station at $x = 2$ m. The various time shiftings were computed as the time shifting for which the covariance between the series was maximal by using a DELFT HYDRAULICS covariance programme. For all four computations (i.e. with the DUT1 and DUT2 model for both $b = 0$ and $b = 1/15$) the same results were found. The necessary shiftings with respect to the time origin of the computations were found to be:

bea01	τ	=	-5.13 s
bea02	τ	=	-5.90 s
beb01	τ	=	-14.71 s
beb02	τ	=	-5.55 s
bec01	τ	=	-5.98 s
bec02	τ	=	-5.66 s

The comparison between measurements and computations at the various stations is presented graphically using both a time window of 0 to 40 s so that the evolution of the wave field can be followed from the beginning, and a time-window of four times the wave period to provide more detail. This latter time-window was chosen at the end of the computations so that most of the higher-harmonic components have reached the farthest stations and the waves are approximately stable.

The computations with $b = 0$ with both the DUT1 and the DUT2 model were used to verify the new model. For $b = 0$ the two models have the same linear frequency and linear shoaling behaviour. The difference between the models then is compounded by the order

of approximation of the bottom-curvature terms; in DUT1 model the bottom-curvature terms in the right-hand side of the momentum equation are neglected while the terms linear in h_x are contained. The DUT1 model is therefore only valid for small bottom slopes. In the DUT2 model on the other hand, the momentum equation contains contributions with h_{xx} and is valid for larger bottom slopes. For all three conditions the results of the DUT1 and the DUT2 model with $b = 0$ were practically equal. This indicates that apparently the experiments are such that the reduction to small bottom slopes is justified in this situation and that the adjustments made to the existing programme code were done correctly.

5.4.2 Measurement condition A

A comparison between measurements and computations with the DUT2 model is shown for 0 to 40 s in Figure 5b. It can be seen that the model simulates the observed behaviour fairly well. The results at the stations behind the bar, from 17.3 m onwards, show increasing differences between the computations and measurements. More detail is provided in Figure 5c, where the results of the DUT2 model are plotted together with the measurements using a time-window of four times the wave period at the end of the computation. The correspondence between the measurements and computations is good up to 17.3 m. On the downward slope, the difference in celerity between the higher harmonic bound and free waves increases fast and a good representation of the phase celerity of the higher-harmonic components becomes more important. Behind the bar the errors in the phase celerity of these shorter wave components manifest themselves clearly and cause the correspondence to be less good. For this experiment computations with $b = 1/17.5$, the b -value introduced by Nwogu, might give better results for the stations from 17.3 m onwards. As mentioned in Chapter 3, for shorter waves $b = 1/17.5$ gives a better agreement between the computed and the (exact) linear phase velocities, but $b = 1/15$ is usually better for the whole range of kh . The importance of good frequency behaviour is stressed in Figure 5d, where the results with the DUT2 model for $b = 1/15$ and $b = 0$ are plotted together. The results of both computations are very much the same for all stations before the downslope region. From comparing Figures 5c and 5d it can be seen that from 14.5 m onwards the performance of the model is less good when $b = 0$ is chosen instead of $b = 1/15$. Figure 3.2 indicates an underestimation of the computational velocities for $b = 0$ and a small overestimation for $b = 1/15$. This can be seen clearly in Figures 5c and 5d. It is remarkable that for the stations up to and including 13.5 m the results with $b = 0$ are slightly closer to the measurements than the results with $b = 1/15$.

The computations with the DUT1 model are given in Figure 5e. Comparison with Figure 5c and 5d shows that from 17.3 m onwards differences between the two models occur. Compared to the DUT2 model the DUT1 model gives too high higher-harmonic peaks. This could possibly be attributed to the less good linear shoaling behaviour of the latter model for increasing values of kh (see Figure 3.3). The two models have the same linear frequency-dispersion behaviour. This frequency-dispersion behaviour can be seen to be more important than the shoaling behaviour; for larger values of kh , for which the DUT2 model behaves better than the DUT1 model, the frequency dispersion is not accurate enough to obtain a good correspondence between measurements and computations.

Because of the other way of representation of the results in the MAST study (Dingemans, 1994a) and this study it is hard to directly compare the DHI results with the results obtained by both the DUT1 model and the DUT2 model. However, by inspection of the DHI results in the MAST study it can be concluded that the DHI results are similar to those obtained by both the DUT1 and the DUT2 model but not exactly the same.

5.4.3 Measurement condition B

The results for measurement condition B are presented in the same way as the results for condition A in Figures 5f, 5g and 5h. However, no comparison of the results for $b = 0$ and $b = 1/15$ is given for test B and C. Test B is characterised by a longer basic wave than test A, which means that the generation of higher harmonics in test B is stronger and that still higher harmonics are generated. This causes discrepancies between the measurements and computations to occur at an earlier stage than in test A; already for the stations on top of the bar significant differences between measurements and computations can be seen. The overall behaviour of the model however is fairly good in spite of the fact that some wave breaking occurs in the measurements.

The results with the DUT1 and the DUT2 model are similar for the stations up to and including 15.7 m. However, at the horizontal part behind the bar the latter model performs significantly better than the DUT1 model. The phase velocities can be seen to be represented in the same way, but the amplitudes predicted by the DUT1 model are consequently too high.

The overall behaviour is comparable with the DHI results for condition B.

5.4.4 Measurement condition C

The results for condition C are plotted in Figures 5i, 5j and 5k. Although we have a rather short basic wave for condition C, the correspondence of the measurements and computations is good. Only for the stations at 17.3 m and 19 m large discrepancies between measurements and computations occur. The correspondence at 21 m is again very good, which might be the result of a different phase relation between the various harmonic components. Especially at 15.7 m, 19 m and 21 m the DUT2 model performs better than the DUT1 model.

The agreement of the DHI results with the measurements seems to be less good than the results obtained with the DUT2 model. Also the results of the DUT1 model as presented in the MAST study seem to be less good than the results of the same model presented here. The latter could be the result of numerical errors caused by too large time and spatial steps applied in the computations for the MAST comparison.

5.5 Conclusions

The performance of the DUT2 model is quite satisfactory. Discrepancies between measurements and computations arise for shorter wave components. These discrepancies are ascribed to inaccurate frequency dispersion. Better results with Boussinesq-type models could possibly be obtained by further improving the frequency dispersion by taking into account higher-order frequency dispersion. Although correct shoaling behaviour was found to be less important than correct frequency dispersion, the improved linear shoaling of the DUT2 model as

compared to the DUT1 model resulted in a better correspondence with the measurements. Besides improved shoaling, another difference between the two models is the assumption about the bottom slope; the DUT1 model is only valid for small bottom slope, while the DUT2 model is valid for larger bottom slopes. In these experiments the assumption of small bottom slopes was found to be justified, but in other situations this can possibly give more pronounced differences between the two models.

6 Verification of time-domain velocity computations

6.1 Introduction

The research on Boussinesq-type models has been primarily focussed on the prediction of surface elevations. The knowledge about the capability in reproducing the velocity field, which is of particular interest to sand-transport predictions is limited. Recently, efforts have been spent on testing Boussinesq-type models against velocity data. Both Brocchini et al. (1992) and Quinn et al. (1994) considered velocity data for waves breaking on a gently sloping beach. The velocity data were acquired with the technique of Particle Image Velocimetry (PIV) and compared with results obtained with amongst others a Boussinesq model. The description of the breaking process was based on the concept of the surface roller developed by Deigaard (Schäffer et al., 1993).

The experiments were carried out on a 1:30 slope with regular waves with a period of 1 s and an offshore wave height of 0.032 m. With a water depth of about 15 cm, the shallowness parameter yields for this condition $T(g/h)^{1/2} = 8$, which means that a rather long wave is dealt with. For the basic wave $\varepsilon = a/h \approx 0.2$ and $\mu = (h/L)^2 \approx 0.02$. Brocchini et al. (1992) worked out comparisons on velocity data both on mean velocity spatial series and on vertical profiles. It was found that the Boussinesq model gave an overestimation of the velocity of about 10%. A more extensive comparison was carried out by Quinn et al. (1994). Quinn et al. discerned three stages in the comparison. First the computed depth-averaged velocity components were compared with the experimental data. It appeared that the computed depth-averaged velocities slightly exceeded the measured values. Secondly, the validity of the reconstruction of the velocity field by imposing a parabolic profile was considered. The horizontal velocity field was calculated based on the *measured* depth-averaged velocities and compared with the measured profile. It was found that the agreement in general was quite good, especially in the near-bed zone. The parabolic profile however, was seen to exceed the measured values near the surface, particularly in the crest locations. Finally, the *calculated* depth-averaged horizontal velocity values were used to calculate the velocity profiles and a comparison with the measured values was made. It was concluded that the approach to obtain a velocity profile from mean velocities predicted by a Boussinesq model is quite valid.

In this study a further comparison of velocity data and computations was made using the DUT2 model. Both regular and irregular wave tests were considered. Since wave breaking is not incorporated in the model equations, the comparison of measurements and computations is mainly made for locations outside the surfzone. The regular wave tests were obtained in a test programme performed with waves breaking partially over a bar in a wave-current flume at DELFT HYDRAULICS. These flume experiments and simulations are dealt with in Section 6.2. In Section 6.3, a comparison is made between computations and measurements performed at DELFT HYDRAULICS for irregular waves on a monotonic beach. Section 6.4 summarizes the conclusions.

6.2 Regular waves

6.2.1 Flume experiments

The experiments were performed by DELFT HYDRAULICS using regular waves breaking on a bar, as part of the EU-sponsored Large Installations Plan (LIP). The set-up of the experiments is reported in Luth et al. (1994) and is the same as in the experiments used in Chapter 5. In contrast with Chapter 5 however, the unscaled geometry is used here (Figure 5.1). The flume was 45 m long, 1 m wide and had a still water depth of 80 cm on each side of the bar.

From these experiments two regular wave conditions S and R were selected for comparison with computations. In the first series of tests, a small wave height was used to produce waves which behave close to linear wave theory. Those series of tests are denoted with S01, in which "0" indicates that no current is used and "1" denotes the position of the measurement equipment along the flume. The wave conditions used in the S01 series are $T = 1.50$ s and $H = 0.06$ m. The shallowness parameter yields for this condition, with $h = 0.8$ m in the deep part, $T(g/h)^{1/2} = 5$ which implies that we are dealing with a rather short basic wave. The waves are non-breaking. Horizontal and vertical velocity measurements were carried out at $x = 15.5$ m and 21.72 m from the wave maker using two 3-beam, 2-component Laser Doppler Anemometers. Both locations are on the 1:20 upward slope of the bar. Wave elevation records were made at 6 locations along the flume using 6 wave height gauges, the last of which was located on the top of the bar at $x = 25.0$ m. The last wave gauge is not considered in the comparison with computations. The tests were carried out 11 times, allowing for velocity measurements at 11 different points along a vertical line near the middle of the flume at each location. The horizontal distances of the wave height meters and the LDV equipment measured from the wave maker are given in Tables 6a and 6b respectively. The vertical distances of the LDV equipment to the bottom for each of the 11 runs are given in Table 6c, in which the last two characters of the name of each run indicate the run number. Note that the waterdepth $h = 57.6$ cm at $x = 15.5$ m, and 26.6 cm at $x = 21.72$ m.

In the second series of tests a higher wave height is used such that the waves are breaking over the bar. The conditions are $T = 1.80$ s and $H = 0.3$ m and therefore $T(g/h)^{1/2} = 6$ for the basic wave at the deepest part. Since no breaking is incorporated in the time domain equations only those tests should be used for which the measurement equipment is located outside the wave breaking region. The selected tests are named R01 and R02 with "1" and "2" again indicating the cross-section number of the measurement equipment along the flume. Tables 6d and 6e show the locations of the wave gauges and LDV equipment which could be used in the comparison with computations. Note that the two locations at $x = 15.5$ m and 19.5 m at which the horizontal velocity measurements took place are on the upward slope of the bar. At the location at 15.5 m the waves are shoaling but not yet breaking. The location at 19.5 m occurs in the transition zone after wave breaking but before the increase in set-up and therefore outside the inner surf zone. Despite the wave breaking the latter location was also used in the comparison with measurements. The tests were repeated several times to obtain velocity measurements at various heights. In the comparison with computations only the runs as indicated in Table 6f were used. Note that the waterdepth $h = 57.6$ cm at $x = 15.5$ m, and 37.6 cm at $x = 19.5$ m.

There are two small errors in the LDV signals (see Luth et al., 1994) with a maximum of 5% in the horizontal velocity components. Because of the rather complicated procedure which should be followed to remove the errors, the LDV signals are used without correction.

The wave conditions can be compared with those used by Quinn et al. (1994) in terms of the parameters $\varepsilon = a/h$ and $\mu = (h/L)^2$. With $L = cT \approx (gh)^{1/2}T$, $\mu \approx h/(gT^2)$. For wave condition S this yields $\varepsilon \approx 0.04$ and $\mu \approx 0.04$ at the deepest part and $\varepsilon \approx 0.15$ and $\mu \approx 0.01$ at the shallowest part. For wave condition R, $\varepsilon \approx 0.2$ and $\mu \approx 0.03$ at the deepest part and $\varepsilon \approx 0.8$ and $\mu \approx 0.006$ at the shallowest part. It can be concluded that in series R we are dealing with a longer and more non-linear wave than in series S and that the wave conditions used by Quinn et al. with $\varepsilon \approx 0.2$ and $\mu \approx 0.03$ can be classified between the series S and R.

6.2.2 Numerical experiments

Since the considered wave gauges and LDV equipment were located before the horizontal part of the bar, the minimum duration of the tests was determined in such a way that permanent wave profiles could be obtained at $x = 20$ m. With an average depth of 70 cm over 20 m we have with $c = (gh)^{1/2}$ a mean celerity of $c \approx 3$ m/s. So it takes less than 10 s for the main wave to travel the distance of 20 m. The celerity of the third harmonic is $c = gT/2\pi$ which yields, with $T = 0.5$ s (a third of the smallest wave period $T = 1.50$ s in the tests), $c = 0.8$ m/s. A duration of 25 s should therefore provide enough reserve for the higher harmonics to reach the farthest stations. The duration of the tests was chosen to be 70 s.

The position of the outgoing boundary was chosen to be $x = 100$ m to ensure that reflection does not affect the computational results. A wave with a celerity of $c = 2.6$ m/s can then travel the distance from ingoing to outgoing boundary and back to $x = 20$ m in 70 s.

In contrast with the computations in Chapter 5, the surface elevation boundary at $x = 0$ had to be bolstered with the slow start (4.10) to decrease the disturbances in the computed velocity signal at the various vertical locations. In the experiments, waves are generated by sending a control signal according to a second-order Stokes wave theory to the wave board (Klopman and Van Leeuwen, 1990). This second-order wave board control includes corrections for the suppression of spurious free sub-harmonic and super-harmonic components. In test S, the non-linear contributions are small and the waves generated by the wave board are approximately sinusoidal. Therefore, in test S a sinusoidal wave $\zeta(t) = a \sin 2\pi t/T$ was taken as a boundary condition and the depth-averaged velocity was calculated from Eq. (4.9). In test R however, a different boundary condition had to be chosen because of the highly non-linear conditions; a harmonic analysis of the wave paddle signal showed that second and third harmonics were significantly present in the signal. At first the wave paddle drive signal was used as the velocity boundary condition. The surface elevation boundary was calculated using Eq. (4.9). This however resulted in a signal for $\zeta(t)$ for which the time-averaged value did not equal zero resulting in a net flux over the boundary. In order to avoid the generation of a net flux, a surface elevation instead of a velocity had to be imposed. The surface elevation signal corresponding to the wave paddle signal was determined by using a DELFT HYDRAULICS programme based on higher-order Stokes theory. This gave the cosine Fourier components corresponding to the wave conditions used in test R. To provide a slower start the boundary condition was formulated in sine components as follows:

$$\zeta(t) = 0.1418 \sin(\omega t) + 0.0298 \sin(2\omega t - \frac{\pi}{2}) + 0.0074 \sin(3\omega t - \pi) \quad (6.1)$$

where $\zeta(t)$ is expressed in meters.

This expression (6.1) in combination with Eqs. (4.9) and (4.10) was found to provide an adequate boundary treatment for test series R.

In Chapter 5, it was already shown that small time and spatial steps were necessary for convergence of the solution for ζ and $\bar{u}$. To find the necessary resolutions, Δt and Δx were varied choosing Δt and determining Δx such that the CFL number for the basic wave in the deepest part was equal to 1. For test S it turned out that $\Delta t = 0.01$ s and $\Delta x = 2.5$ cm were sufficient. In test R, it was necessary to halve these steps to obtain an accurate solution. For this test the smoothing procedure proved to be good enough with $\Delta t = 0.01$ s and $\Delta x = 2.5$ cm, but unfortunately could not filter the disturbances from the signal of $(h\bar{u})_{xx}$ and $\bar{u}_{xx}$ if the halved steps were used. Therefore, computations were made with $\frac{1}{2}\Delta t$ and $\frac{1}{2}\Delta x$ but with determination of $(h\bar{u})_{xx}$ and $\bar{u}_{xx}$ from Eq. (4.14) using Δx . This could not solve the problem, which indicated that the wave length of the numerical disturbances depends on the step size. Also no appreciable improvement was obtained using a different filter which smoothed out still longer waves. Eventually the computation for test R was performed with $\frac{1}{2}\Delta t$ and Δx and therefore a small CFL number. In this way an accurate enough solution was found with moderately smooth results for $(h\bar{u})_{xx}$ and $\bar{u}_{xx}$. Apparently, Δx is the more important parameter where the generation of the disturbances in $(h\bar{u})_{xx}$ and $\bar{u}_{xx}$ is concerned and Δt where the accuracy of the solution for ζ and $\bar{u}$ is concerned.

The computations from 0 - 70 s show an increase of the mean water level as time progresses and a long-wave disturbance in the mean velocity. These effects are mainly noticeable in test R. For this condition, Figure 6e shows the computed surface elevations and depth-averaged velocity at some locations along the flume using a time-window from 0-70 s. It can be seen that the phenomenon occurs from 30 s onwards for the station at $x = 8.00$ m and still later on for stations further from the wave maker. Since 1) the scheme is internally mass-conserving; 2) the prescribed time-averaged surface elevation at the seaward boundary is zero; and 3) reflected waves cannot manifest itself at $t = 30$ s, it is likely that the treatment of the seaward boundary where both the velocity and the surface elevation are prescribed generates sub-harmonic components. This problem could not be solved. Finally, it should be remarked that all computations were performed with $b = 1/15$.

6.2.3 Comparison of computations and measurements

From Tables 6a and 6b it can be seen that for each run wave elevation records at $x = 8.00$ m are available. Those wave elevation records were used to determine the time origin of each run with respect to the computations. In this way the measurements and computations could be compared in such a way that the phase difference between them was zero for the station at $x = 8.00$ m. Since the start of the experiments was not provided in the measurements, the necessary time shifting τ of the computations with respect to the measurements was determined omitting the start of the computations and requiring that the maximum value of τ is $\frac{1}{2}T$. The results are summarized in Table 6g for all used runs for both condition S and R.

Wave condition S

A comparison between measured and computed surface elevations is shown in Figure 6a for a time-window of four times the initial wave period. The time-window is chosen in such a way that permanent wave profiles are obtained in the computations. It can be seen that the overall agreement between measurements and computations is good for all stations. At the crest locations the computed values systematically overestimate the amplitude; the agreement is good at the trough locations.

The comparison with velocity data is carried out as follows. At two vertical positions time records for the computed and measured horizontal velocities are given: 1) at the bottom just outside the boundary layer; and 2) at the level where the computational (depth-averaged) velocity may be found. From the measurements it could be seen that run number s0106 provides velocity values just outside the boundary layer. The depth-averaged value of a parabolic profile is found at $z \approx -0.4 h$. This depth is best approximated in run number s0112 for $x = 15.5$ m and in run s0115 for $x = 21.72$ m. At the corresponding vertical distances from the still water level, $z = -22.6$ cm and -11.6 cm respectively, the horizontal velocity records were computed from the computational velocity by imposing the parabolic profile as explained in Chapter 4. Note that at these depths the deviation from the computational velocity is still small. In Figures 6b and 6c the comparison of measured and computed horizontal velocities is given at both $x = 15.5$ m and 21.72 m; Figure 6b shows the velocity at a depth close to the depth where the computational velocity is expected while Figure 6c shows the bottom velocity. Figure 6b shows an overestimation of the crest amplitude, as was already seen for the surface elevations in Figure 6a. Apparently, the computational velocity values exhibit the same behaviour as the surface elevation. The trough amplitude of the bottom velocities on the contrary is slightly underestimated by the Boussinesq model.

Figure 6d shows the measured and computed velocity profiles at $x = 15.5$ m and 21.72 m for both a crest location and a trough location. This comparison was obtained as follows. In the time records for the computational velocity a crest and a trough location were selected. Using the values for $(h\bar{u})_{xx}$ and $\bar{u}_{xx}$ at these moments, the parabolic velocity profiles were obtained. Taking into account the time shiftings as given in Table 6g, the corresponding measurements were provided by run numbers s0105 to s0115. Tables 6h and 6i sum up the percent error $(u_{meas} - u_{comp})/u_{meas}$ between the measured and computed velocity profile for both the crest and trough situation for the locations at $x = 15.5$ m and 21.72 m respectively. Note that for $x = 21.72$ m no measurements are available in the upper part of the profile and that the measured velocities at the second location from the bottom were used in the comparison with the computed bottom velocities in Figure 6c. The first data point from the bottom is shown in Figure 6d but is not listed in Tables 6h and 6i.

Figure 6d shows a quite good agreement between the flume and numerical experiment at the trough locations. At the crest locations the agreement is good in the near-bed zone. The Boussinesq model can be seen to perform better at $x = 21.72$ m than at $x = 15.50$ m. Further it can be seen that, except for the lower part of the crest profile at 21.72 m, the model gives a systematic overestimation of the velocity. Brocchini et al. reported an almost constant overestimation of the velocity of about 10%. Here it is found however, that the overestimation of the velocity values at the trough locations decreases if the vertical distance from the bottom increases. In the crest locations on the contrary, the deviations between measurements and computations increase with increasing distance from the bottom.

The bottom velocity is quite well predicted by the Boussinesq model; in the crest locations errors of 6% for both locations and in the trough locations errors of 17% and 8% are found.

Because of the small non-linear contributions for condition S, a relatively symmetrical velocity profile is found. The more non-linear wave in test series R is more interesting from the point of view of sand transport predictions.

Wave condition R

As was mentioned in Section 6.2.3, no permanent wave profiles are obtained in the computations due to a sub-harmonic variation in the computational results (see Figure 6e). Comparison with measurements shows that this phenomenon does not occur in the flume experiment and that the agreement between measurements and computations is optimal when considering a time-window as soon as possible after the start of the computations. Therefore the comparison with measurements is made after 25 s, which was estimated to be the time necessary for the higher harmonics to reach the farthest stations.

The comparison of surface elevations is presented in Figure 6f. For the stations up to $x = 15.70$ m the agreement is slightly less good than obtained with the lower wave height in test series S. The wave breaking is clearly visible for the last two stations. It can be seen that although the peaks in the computed signal are too high the overall agreement is still quite good. This is in accordance with the observation that the primary effect of wave breaking is the reduction of the total wave energy without affecting the evolution of the spectral shape significantly (Beji and Battjes, 1993).

In Figure 6g the measured and computed velocities are shown at an elevation close to that where the computational velocity can be found. Therefore the influence of the parabolic terms as superimposed on the computational velocity is still small there. At $x = 15.50$ m, a systematic overestimation of the velocity values by the Boussinesq model is seen, as was the case for condition S. At $x = 19.50$ m the peaks are far too high. This however is a result of the absence of a dissipation term in the time domain model. The spectral model might solve this problem. The bottom velocities are presented in Figure 6h. The computed signals at $x = 15.50$ m and 19.50 m show a higher harmonic disturbance which is introduced by the fact that the smoothing procedure used in computing $(h\bar{u})_{xx}$ and $\bar{u}_{xx}$ was not completely effective for condition R. Despite this disturbance it can be concluded that at $x = 15.50$ m the trough amplitude of the bottom velocity is overestimated. To a smaller extent this was already found for condition S.

6.3 Irregular waves

6.3.1 Flume experiments

Within the framework of the Large Installations Programme (LIP), measurements of hydrodynamics and sediment transport for irregular waves breaking on a beach were carried out in DELFT HYDRAULICS' Delta flume (Roelvink and Reniers, 1995). Two different geometries were considered in the measurement programme; in this study only the first geometry consisting of a Dean-type beach was used. For this geometry three different incident wave conditions were considered; narrow-banded, random waves were chosen such that the wave condi-

tions resulted in a stable (sub-test 1a), erosive (sub-test 1b) and accretive beach (sub-test 1c), respectively. Each sub-test was subdivided in a number of wave hours. EMF current meters attached to a movable carriage and three movable wave gauges remained in a fixed position during each wave hour. The EMF current meters were attached to the carriage at various distances from the bed allowing for velocity measurements at various heights. The sampling interval for all instruments was 0.1 s. The bed profile was measured in between the wave hours.

The DUT2 model was tested against sub-test 1a. For this test, the incident significant wave height $H_{m0} = 0.9$ m and the incident peak period $T_p = 4.9$ s. The bed profile for test 1a is shown in Figure 6.1. Here $x = 0$ corresponds to the position of the wave board and $z = 0$ to the flume bottom. The water depth above the flume bottom $h = 4.1$ m. From 65 m up to about 140 m, some wave breaking can be observed. The intense wave breaking takes place from 140 m onwards. Since no breaking formulation is incorporated in the DUT2 model, comparison of measurements and computations was only performed at $x = 100$ m, the measurement location closest to the wave board. The movable carriage was positioned at $x = 100$ m during the second wave hour. The water depth $h = 1.8$ m at $x = 100$ m. During the second wave hour surface elevations were measured at $x = 20$ m and 90 m.

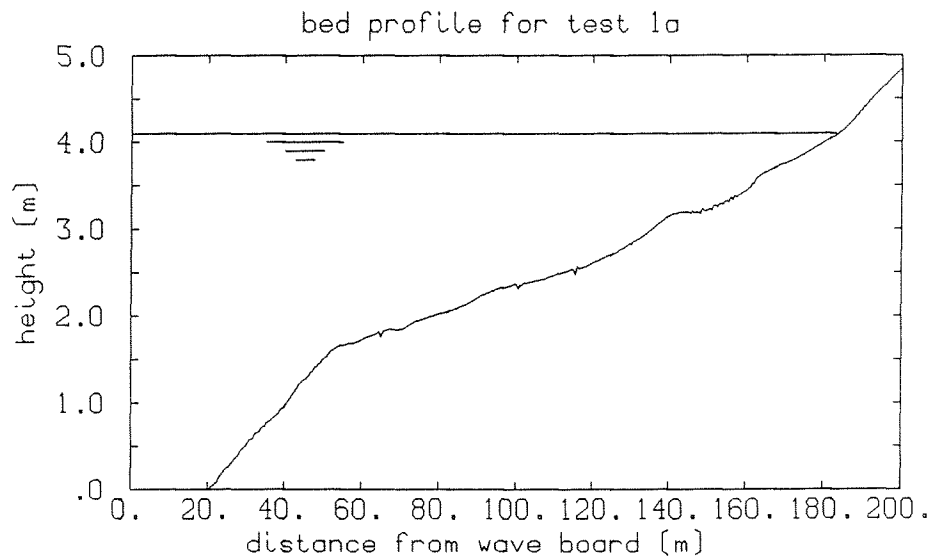


Figure 6.1 Bed profile for sub-test 1a of the LIP Delta Flume experiments

At the deepest part the shallowness parameter is $T_p(g/h)^{1/2} = 8$. We are therefore dealing with rather long waves. This can also be observed when comparing the measured horizontal velocity records at $x = 100$ m; the differences between the bottom velocities and the velocities at approximately mid-depth are small. Therefore, only bottom velocities were considered in the comparison with computations. The wave conditions can be quantified in terms of the parameters $\varepsilon = H_{m0}/2h$ and $\mu = (h/L)^2$ with $L = cT \approx (gh)^{1/2}T$. At the seaward boundary we find $\varepsilon \approx 0.1$ and $\mu \approx 0.02$.

6.3.2 Numerical experiments

The DUT2 model was used to compute time series of the surface elevation and the bottom velocity at $x = 90$ m and 100 m, respectively. The surface elevation boundary condition was provided by the measured surface elevation time records at $x = 20$ m during the second wave hour. The boundary condition was bolstered with the slow start (4.10).

The requirement that permanent wave profiles be obtained at $x = 100$ m, determines again the minimum test duration. The celerity of the third super-harmonic is $c = gT/2\pi$ which yields, with $T = 1/3 T_p = 1.6$ s, $c = 2.5$ m/s. So it takes 32 s for the third super-harmonic to travel from the ingoing boundary at $x = 20$ m to $x = 100$ m. The comparison of measurements and computations was performed from 50 s onwards to ensure that enough reserve was provided for the higher harmonics to reach the farthest stations.

Unfortunately, the computations demonstrated unstable behaviour, comparable with the phenomenon observed in the regular wave test with the higher wave height (condition R). In Section 6.2.2 it was already noted that this might be the result of the treatment of the seaward boundary condition. This erroneous behaviour seems to be enhanced for higher values of the non-linearity parameter ε . Attempts made at solving the instability problem, among which the use of an upwind discretization at the boundaries, were unsuccessful. Since the unstable behaviour became noticeable after more than 100 s, it was decided to compare the model output and the experimental data from 50 s up to 100 s only.

The bed profile used in the computations was the profile measured after the first wave hour. From $x = 150$ m onwards the depth was kept constant at $h = 0.8$ m. The outgoing boundary was positioned at $x = 255$ m to ensure that the reflected waves could not reach the output stations before the end of the computation.

The spatial and time steps were determined in such a way that the CFL number at the seaward boundary was 1. A convergent solution was found for $\Delta t = 0.025$ s and $\Delta x = 0.13$ m. Since Δt was smaller than the sampling interval in the measurements, interpolation had to be carried out between the values of the ingoing surface elevation signal.

At first, the bottom velocity was computed from the computational velocity by means of the parabolic distribution of the horizontal velocity, Eq. (2.18). The procedure (4.15) was successfully applied to smooth the signal of $\bar{u}$. Small irregularities in the bed profile however, gave rise to vastly erroneous results for $(h\bar{u})_{xx}$ and consequently the bottom velocity. This problem could have been solved by smoothing the bed profile before the computation or the signal of $h\bar{u}$. However, in view of the small contributions of the parabolic correction terms because of the almost uniform velocity profile, it was decided to compute the bottom velocity for the approximation of horizontal bottom. Neglecting the bottom-slope and bottom-curvature terms in Eq. (2.18) yields:

$$u(x, z, t) = \bar{u} - \left[\frac{1}{3} h^2 + zh + \frac{1}{2} z^2 \right] \bar{u}_{xx} \quad (6.1)$$

The bottom velocity then follows from Eq. (6.1) for $z = -h$.

Finally, it should be remarked that the computation was performed with $b = 1/15$.

6.3.3 Comparison of computations and measurements

Figure 6i shows the comparison of measurements and computations for the surface elevation at $x = 90$ m and the bottom velocity at $x = 100$ m, respectively. The discrepancies between the predictions and the computations are larger for the bottom velocity than for the surface elevation, which is ascribed to the fact that the comparison for the bottom velocity was made further from the wave board.

It can be seen that the model overestimates the amplitudes, especially the bottom velocity amplitudes. This is possibly the result of the absence of a dissipation formulation in the model to describe wave breaking; the LIP experiments for test 1a show a small decrease of the significant wave height from 65 m up to 140 m, after which the wave breaking becomes stronger. Besides the overestimation of the amplitudes, phase errors can be observed in the predicted time series, due to the inaccuracy of the linearized dispersion relation of the model. This inaccuracy becomes significant for the higher harmonics.

6.4 Conclusions

The numerical method used in the DUT1 and DUT2 models for the integration of the Boussinesq equations, gives rise to instabilities which become more significant for increasing values of the non-linearity parameter ε . Development of a model based on a different numerical scheme is recommended.

The calculation of the velocity profile from the computational velocities is very sensitive since second-order derivatives $(h\bar{u})_{xx}$ and $\bar{u}_{xx}$ must be determined. The used smoothing procedure was able to filter high-frequency disturbances from the computational velocity signal in most situations. Irregularities in the bed profile can give rise to erroneous results for the parabolic correction terms as well.

The regular wave tests S and R show that the discrepancies between predictions and measurements for the computational (mean) velocities are comparable with the discrepancies observable in the surface elevation; both for the surface elevation and for the mean velocity, the model slightly overestimates the crest amplitudes and underestimates the trough amplitudes. The velocity profile obtained from the computational velocity is in good agreement with the measured profile for the lower half of the profile. The trough amplitude of the bottom velocity is slightly overestimated. It can be concluded that the accuracy in predicting the computational velocity is directly related to the accuracy of the prediction of the free-surface elevation. Further, the determination of the bottom velocity from the computational velocity by the parabolic extrapolation yields good results.

Computations for the irregular wave test 1a show an overestimation of the bottom velocities which can be (partly) ascribed to the absence of a dissipation formulation in the model. Therefore, and because of the rather short predicted time series, this simulation was not suited for the comparison of predicted and measured velocity moments. The DUT spectral amplitude Boussinesq model, which includes a spectral breaking term, will be used to simulate amongst others this Delta flume test 1a (see Chapter 7 and 8).

7 Determination of the velocity field using a frequency domain formulation

7.1 Introduction

Besides the direct time domain approach as used in the preceding chapters, a frequency domain approach based on evolution equations and Fourier series approximations can be used. Madsen and Sørensen (1993) developed a set of coupled evolution equations for complex Fourier amplitudes based on Boussinesq equations with improved dispersion characteristics and valid for a mildly sloping bottom. They found that the agreement between the time domain equations and the evolution equations is most satisfactory. The only discrepancy is found in the highest waves where the evolution equations underestimate the peak values.

Boussinesq-type models are basically conservative (non-dissipative). In order to extend the applicability of Boussinesq models to the surf zone, the influence of wave breaking should be modelled and incorporated in either the time domain or spectral equations. The effect of wave breaking is the dissipation of energy on the main frequencies, while low-frequency energy might be released. Additional low-frequency energy can be generated due to the horizontal oscillation of the breaker line, which occurs during the breaking of waves in a group or of irregular waves. The low-frequency waves will reach the shoreline and be almost totally reflected, forming a standing wave pattern. Schäffer et al. (1993) modelled the process of wave breaking using the concept of surface rollers and incorporated this in a time domain Boussinesq model. The wave breaking can also be modelled in a spectral form by using a spectral dissipation function. Such a dissipation function was incorporated in a spectral amplitude model based on the above mentioned evolution equations by Eldeberky and Battjes (1995). The resulting DUT spectral model was used to predict surface elevations and surface elevation spectra. The results showed encouraging agreement with observed surface elevations as well as spectra (Eldeberky and Battjes, 1995).

In this study the DUT spectral model was used in order to predict horizontal velocities, velocity spectra and the third odd velocity moment, which is an important parameter in describing the time-averaged sediment transport rate. Section 7.2 globally describes the equations on which the DUT model is based. In Section 7.3, the extension of the model in order to compute horizontal velocity values is discussed. The testing of the model against flume measurements is described in Chapter 8.

7.2 The DUT spectral model

7.2.1 The governing equations

As mentioned in the introduction, the spectral model consists of a set of coupled evolution equations for complex Fourier amplitudes based on Boussinesq-type equations supplemented with a quasi-linear dissipation term to account for wave breaking. The evolution equations for the complex amplitudes were derived by Madsen and Sørensen assuming uni-directional wave propagation. Starting point of the derivation were the Boussinesq-type equations (3.12) in terms of the flux with improved frequency dispersion and valid for small bottom slopes.

Solutions for the surface elevation are formulated in terms of Fourier series with spatially varying coefficients:

$$\zeta(x,t) = \sum_{p=-\infty}^{\infty} A_p(x) \exp[i(\omega_p t - \psi_p(x))] \quad (7.1)$$

where p indicates the rank of the harmonic, A_p is the complex Fourier amplitude, ω_p is the cyclic frequency and $d\psi_p/dx = k_p(x)$ is the wave number in the linear approximation. Note that $\omega_{-p} = -\omega_p$, $\psi_{-p} = -\psi_p$, $A_{-p} = A_p^*$ with $*$ denoting the complex conjugate. The cyclic frequencies are determined by $\omega_p = p\Delta\omega$ where $\Delta\omega$ is the lowest frequency of interest. The wave number k_p can be determined from Eq. (3.10), the linear dispersion relation of Eqs. (3.12). Although all wave numbers in this formulation (7.1) correspond to free waves, the spatial variation of the Fourier amplitudes actually allows for combinations of bound and free waves for all frequencies.

A lowest-order estimate of the flux q is obtained by combining the expansion for ζ (7.1) with the continuity equation (3.12a) while neglecting the first derivatives of A_p and k_p :

$$q(x,t) \approx \sum_{p=-\infty}^{\infty} \frac{\omega_p}{k_p} A_p(x) \exp[i(\omega_p t - \psi_p(x))] \quad (7.2)$$

Inserting Eq. (7.1) and Eq. (7.2) in the Boussinesq equations (3.12) yields after some algebraic manipulations first-order evolution equations for the complex Fourier amplitudes. In this procedure, first derivatives of A_p , k_p and h are assumed to be small and products of derivatives and higher derivatives of these quantities are neglected in the formulation.

The evolution equations were supplemented with a semi-empirical dissipation formulation for the total energy loss due to breaking which reduces the spectral amplitudes in the same proportion without affecting the spectral shape (Eldeberky and Battjes, 1995). With p covering the interval from $p = 1$ to ∞ , the evolution equations then read

$$\frac{dA_p}{dx} = -\beta_5 \frac{h_x}{h} A_p - i2g (F_p^+ + F_p^-) + d_{bp} \quad (7.3)$$

where the dissipation term d_{bp} represents the contribution to the spatial rate of change of the complex amplitude due to wave breaking. The dissipation term is proportional to the complex amplitude A_p with a frequency-independent factor equal to the ratio of the total local rate of random-wave energy dissipation per unit area due to breaking over the total local rate of energy flux per unit width. The complete expression for the dissipation term can be found in Eldeberky and Battjes (1995). The first term in the right-hand side of Eqs. (7.3) represents linear shoaling. The second and the third term represent the triad sum and difference interactions respectively. The terms F_p^+ and F_p^- are defined by

$$F_p^+ = \sum_{m=1}^{p-1} \frac{\alpha^+}{2\beta_1} A_m A_{p-m} \exp[-i(\psi_m + \psi_{p-m} - \psi_p)]$$

$$F_p^- = \sum_{m=1}^{\infty} \frac{\alpha^-}{\beta_1} A_m^* A_{p+m} \exp[-i(\psi_{m+p} - \psi_m - \psi_p)] \quad (7.4)$$

Expressions for the shoaling coefficient β_5 and the interaction coefficients β_1 , α^+ and α^- can be found in Eldeberky and Battjes (1995).

7.2.2 Numerical solution

A solution to the evolution equations (7.3) is found by numerically integrating these first-order ordinary differential equations using the fourth-order Runge-Kutta method. The model input comprises:

- 1 the bottom geometry and output locations
- 2 the b -value to control the frequency-dispersion behaviour
- 3 the breaking coefficient $\gamma = H_m/h$ in which H_m is the maximum wave height
- 4 the bandwidth Δf used in the simulations
- 5 the number of frequency components P
- 6 the spatial step Δx
- 7 an upwave boundary condition for the integration

The upwave boundary condition is a set of complex amplitudes A_p^+ ($p = 1, 2, 3 \dots P$). Those amplitudes can be obtained from measured time records of the surface elevation by the use of a standard fast Fourier transformation algorithm.

The result of the numerical integration is a set of complex amplitudes A_p ($p = 1, 2, 3 \dots P$) at each location. The phases of the complex amplitudes A_p predicted by the evolution equations only contain the non-linear contribution. The complex amplitudes A_p^+ are found by adding the linear part of the phase function as follows:

$$A_p^+ = A_p \exp[-i\psi_p] \quad (7.5)$$

The linear phases ψ_p in Eq. (7.5) are obtained by integrating k_p spatially.

The output of the model consists of the complex amplitudes A_p^+ . The surface elevations at each location are then defined by

$$\zeta(x, t) = 2 \sum_{p=1}^P A_p^+(x) \exp[i(\omega_p t)] \quad (7.6)$$

and are obtained by transformation of the complex amplitudes A_p^+ into surface elevations by using inverse fast Fourier transformation.

7.3 Computation of the horizontal velocity field

Since we are interested in horizontal velocity values at different heights an expression has to be found for the horizontal velocity as a function of the depth in terms of the complex Fourier amplitudes.

The lowest-order approximation of the flux q is given by Eq. (7.2). In principle the depth-averaged velocity $\bar{u}$ can be obtained from

$$\bar{u} = \frac{q}{h + \zeta} \quad (7.7)$$

in which ζ is determined by Eq. (7.1). In order to simplify the expression for $\bar{u}$ and its second-order derivatives Eq. (7.7) is linearized:

$$\bar{u}(x,t) \approx \frac{q}{h} \approx \sum_{p=-\infty}^{\infty} \frac{\omega_p}{k_p h} A_p(x) \exp[i(\omega_p t - \psi_p(x))] \quad (7.8)$$

The depth-averaged velocity amplitude is thus obtained from the surface elevation amplitude by multiplication of each Fourier component by $\omega_p/(k_p h)$. Note that in this way only the purely oscillating part of the horizontal velocity is predicted by the model; the time-averaged component of the velocity is eliminated upon linearization.

In order to compute the horizontal velocity profile as a function of the depth a parabolic profile is imposed. As a first approximation we use the reduction of Eq. (2.18) for the constant depth situation:

$$u(x,z,t) = \bar{u} - \left[\frac{1}{3} h^2 + zh + \frac{1}{2} z^2 \right] \bar{u}_{xx} \quad (7.9)$$

The depth-averaged velocity $\bar{u}$ in this expression is given by Eq. (7.8). If we neglect all derivatives of A_p , k_p and h , the second derivative $\bar{u}_{xx}$ can be computed by multiplying each Fourier component of $\bar{u}$ by $-k_p^2$ or equivalently by multiplying each component of ζ by $-(k_p \omega_p)/h$. The complete expression for the velocity field in terms of Fourier series then becomes

$$u(x,z,t) = \sum_{p=-\infty}^{\infty} U_p(x,z) \exp[i(\omega_p t - \psi_p(x))] \quad (7.10a)$$

with

$$\frac{U_p(x,z)}{A_p(x)} = \frac{\omega_p}{k_p h} \left[1 + k_p^2 \left[\frac{1}{3} h^2 + zh + \frac{1}{2} z^2 \right] \right] \quad (7.10b)$$

The numerical model was extended to compute the complex amplitudes $U_p^+ = U_p e^{-i\psi}$ which contain both the non-linear and linear contribution to the phase function. This is done at each location and at various vertical positions by multiplication of A_p^+ by the expression in the right-hand side of (7.10b). The values for z can be specified in the input for the model. Note that the bottom velocity is obtained for $z = -h$. Besides the complex amplitudes U_p^+ , raw spectral density values $2U_p^2/\Delta f$ were computed. After the simulations the velocity time records can be obtained from the complex amplitudes by means of an inverse fast Fourier transformation.

The testing of the model against velocity measurements is discussed in Chapter 8.

8 Verification of frequency-domain velocity computations

8.1 Introduction

The DUT spectral model was used to predict surface elevations from given complex Fourier amplitudes obtained from measured time records in laboratory experiments at the upwave boundary (Eldeberky and Battjes, 1995). The model was also used together with the assumption of random, independent initial phases to calculate the evolution of the energy spectrum of random waves. The model yielded realistic predictions of the spectral evolution as well as the surface elevation. It was found however, that the model tends to underestimate the crest elevation of the highest waves. This might be due to the transformation of the time domain equations into the frequency domain; Madsen and Sørensen (1993) already noted that the evolution equations underestimate the peak values of the highest waves compared to the corresponding time domain equations.

In this study, the extension of the DUT spectral model to predict horizontal velocities (see Chapter 7) was verified against test 1a and 1c of the LIP Delta flume experiments (Roelvink and Reniers, 1995). The extended model was used to predict bottom velocity time series, bottom velocity spectra and bottom velocity moments. Eldeberky and Battjes (1995) used the same tests, which they refer to as tests C and D respectively, for the comparison of measured and computed surface elevation time series and spectra. In this work, all computations were performed with an upwave boundary condition consisting of a set of complex amplitudes obtained from measured time records. Note that Eldeberky and Battjes (1995) computed energy spectra with the assumption of random, independent initial phases.

In Chapter 1, it was already mentioned that the odd velocity moments are important in describing sediment transport, while the contribution of the even velocity moments is usually an order of magnitude smaller. The focus of this study is on the third odd velocity moment, which determines the bed load transport rate. Roelvink and Stive (1989) showed that the most important contributions to the third odd velocity moment are given by:

$$\langle |u|^2 u \rangle = 3 \langle |u_{hi}|^2 \bar{u} \rangle + \langle |u_{hi}|^2 u_{hi} \rangle + 3 \langle |u_{hi}|^2 u_{lo} \rangle + \dots \quad (8.1)$$

where the instantaneous near-bed velocity u is assumed to consist of a wave group averaged component $\bar{u}$, a short wave averaged oscillatory component u_{lo} and a short wave component u_{hi} . The $\langle \rangle$ indicate time-averaging over the short wave and wave group scale. The first term in Eq. (8.1) is related to the return flow under breaking waves. The second and the third term represent the contribution to the velocity moment of the oscillatory part of the instantaneous velocity; the second term is related to the short wave asymmetry, whereas the third term is associated with the interaction between the long wave velocity and the slowly varying short wave velocity variance. The return flow effect was disregarded in the comparison with measurements since the DUT spectral model only predicts the oscillatory velocity component (see Chapter 7).

In Section 8.2. the Delta flume experiments are dealt with. The numerical experiments are described in Section 8.3. Section 8.4 is about the comparison of the computations with the measurements. Finally, Section 8.5 summarizes the results.

8.2 Flume experiments

The LIP Delta flume experiments (Roelvink and Reniers, 1995) have already been described in Section 6.3.1. It was mentioned that both in sub-test 1a and 1c a Dean-type beach was used. The combination of wave conditions and water level resulted in a stable beach in the case of sub-test 1a and in an accretive beach for sub-test 1c. In test 1a, the incident wave conditions are such that wave breaking is strong. The monotonic sandy beach profile (Figure 8.1) allows for wave breaking to take place over a large distance; the experiments showed a gradual decrease of the significant wave height from 100 m onwards up to about 140 m, beyond which the wave breaking gets strong. In test 1c on the contrary, a barred beach profile is used (see Figure 8.2). The wave breaking is mild and is concentrated behind the bar, the crest of which is located around 138 m.

Each sub-test was subdivided in a number of wave hours, during each of which the water level and wave conditions were kept constant. EMF current meters attached to a movable carriage and three movable wave gauges remained in a fixed position during each wave hour. Therefore, each wave hour provided measurements at different locations. An overview of the locations of the EMF current meters during each of the wave hours for sub-test 1a and sub-test 1c is given in Table 8.1. Only those locations are given for which a comparison between measurements and computations was made. The sampling interval for all instruments was 0.1 s. The bed profile was measured in between the wave hours.

In the computations the sequence of wave hours was simulated in one single run, using a typical bed profile. Because of the small changes in the bed profile in sub-test 1a during the sequence of stages, the typical bed profile was arbitrarily chosen as the profile measured after the 4th wave hour (see Figure 8.1).

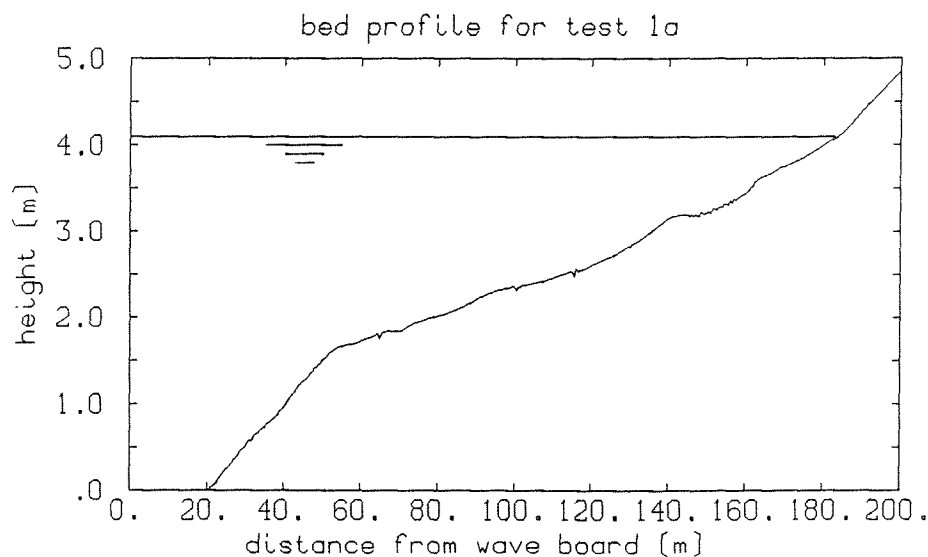


Figure 8.1 Bed profile for sub-test 1a of the LIP Delta Flume experiments

The profile development in sub-test 1c is strongest just beyond the bar. In order to properly simulate the wave propagation for the stations beyond the bar, which are provided by the 7th to 9th wave hours, the typical bed profile for sub-test 1c was chosen as the bed profile measured after the 6th wave hour (see Figure 8.2). The model assumption of slowly varying bottom ($\beta = h_x \ll \mu^{1/2}$) can be seen to be in contrast with the relatively steep slope beyond the bar crest in sub-test 1c (see Eldeberky and Battjes, 1995).

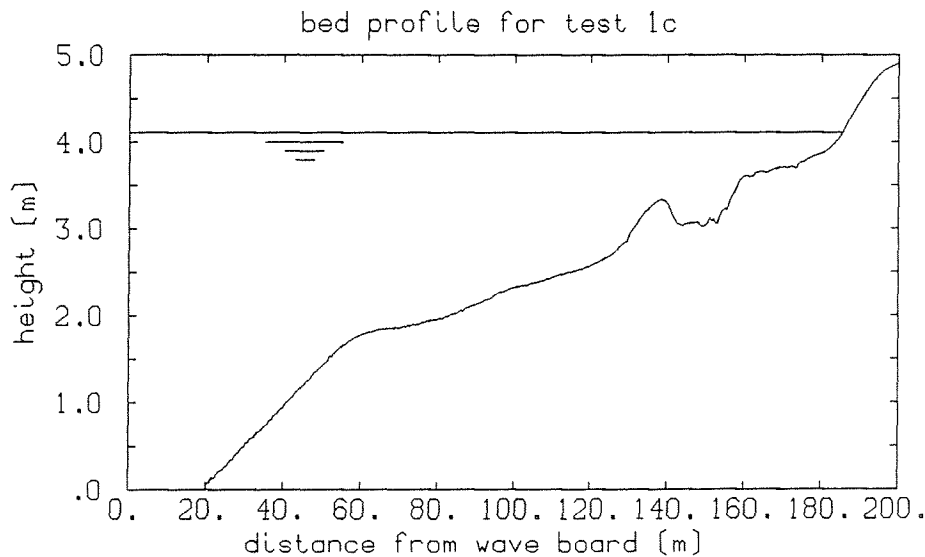


Figure 8.2 Typical bed profile for sub-test 1c of the LIP Delta Flume experiments

Sub-test 1a		Sub-test 1c	
wave hour	EMF location (m)	wave hour	EMF location (m)
2	100	1	65
3	115	3	102
4	130	2	115
5	138	7	138
6	145	8	145
7	152	9	152

Table 8.1 Selected wave hours and corresponding positions of the EMF current meters measured from the wave board, for sub-test 1a and sub-test 1c

During each wave hour free surface elevations were measured at $x = 20$ m. The measured time record during the 5th (sub-test 1a) and the 7th wave hour (sub-test 1c) were used as the upwave boundary condition in the simulations. The other records at 20 m were used to determine the time difference between the measurements of the various wave hours; the time at which sampling was started was slightly different for each wave hour. This was done using a DELFT HYDRAULICS covariance programme. For sub-test 1a besides bottom velocities time records also surface elevation records at $x = 109$ m, 132 m and 154 m were considered to validate the computations against the computations by Eldeberky and Battjes (1995). Those surface elevation records were measured during the 3rd, 5th and 8th wave hour.

The EMF current meters were attached to the carriage at various distances from the bed allowing for velocity measurements at various heights in the lower half of the profile. In both sub-tests however, the differences between the bottom velocities, measured at 10 cm above the bed, and the velocities at approximately mid-depth were small. Therefore, only bottom velocities are considered in the following.

In both tests narrow-banded random waves were generated at the wave board. In sub-test 1c, a second-order wave board control signal was applied so that the generation of spurious free sub- and super-harmonics is suppressed. In sub-test 1a on the contrary, spurious free waves were excited at the wave board as a result of the use of first-order wave board control. For both tests, the wave board control signal included compensation for waves reflected from the beach to prevent re-reflection at the wave board.

For both tests, the incident wave conditions are listed in Table 8.2, in which T_p is the peak period and H_{m0} the significant wave height. The non-linearity parameter $\epsilon = a/h = H_{m0}/2h$ and the dispersion parameter $\mu = (h/L)^2$ with $L = cT \approx (gh)^{1/2}T$ are calculated at the upwave boundary. The shallowness parameter $T_p(g/h)^{1/2}$ at the upwave boundary is computed as well. The water depth at this boundary is $h = 4.1$ m for both tests.

Test	T_p (s)	H_{m0} (m)	$T_p(g/h)^{1/2}$	ϵ	μ
1a	4.9	0.9	7.6	0.11	0.017
1c	8.0	0.6	12.4	0.07	0.007

Table 8.2 Wave parameters for sub-test 1a and sub-test 1c

Dingemans (1994b) mentioned that in practical computations good results are obtained for ϵ as large as 0.25 while μ should be typically smaller than 0.02 for the shortest wave component. It can be seen that for both tests, the primary wave meets the requirement that ϵ be small. This does not hold however, for the higher harmonics in sub-test 1a. This is further considered in Section 8.4.3. The shallowness parameter for sub-test 1c is large compared to sub-test 1a and the experiments used in Chapters 5 and 6.

Because of the larger shallowness parameter for sub-test 1c, the Boussinesq model can be expected to yield more accurate phase celerities for sub-test 1c than for sub-test 1a. It should be remarked however that, since the strength of the triad interactions is governed by the phase mismatch between the bound and free wave numbers, the generation of higher harmonics is stronger in sub-test 1c than in sub-test 1a. Further, Table 8.2 shows that the non-linearity is higher in sub-test 1a than in sub-test 1c, which also implies that the wave breaking is stronger.

8.3 Numerical experiments

As was mentioned in Section 8.2 one single run was made to obtain output at each station indicated in Table 8.1. The upwave boundary condition at $x = 20$ m was obtained from the selected measured time records of the surface elevation (see Section 8.2) by the use of a standard fast Fourier transformation (FFT) algorithm. Besides the boundary condition, the bottom geometry and the output locations, the model input comprises:

- 1 the b -value to control the frequency-dispersion behaviour, which was chosen as $b = 1/15$
- 2 the breaking coefficient γ for which $\gamma = 0.85$ was chosen in accordance with the γ -value used by Eldeberky and Battjes (1995)
- 3 the bandwidth Δf
- 4 the number of frequency components P
- 5 the spatial step Δx

The bandwidth Δf and the number of frequency components P are determined so that all harmonics are represented adequately. The sampling interval of the measurements which is $\Delta t = 0.1$ s, determines the maximum cut-off frequency f_{Nyq} :

$$f_{Nyq} = \frac{1}{2\Delta t} = 5 \text{ Hz} \quad (8.2)$$

This frequency was chosen as the maximum frequency in the computations by Eldeberky and Battjes (1995). Choosing a maximum frequency of 5 Hz in the simulations however, resulted in an extremely noisy signal for the bottom velocities. This appeared to be due to the amplification of the higher harmonics in the computation of the second-order derivative of the depth-averaged velocity with respect to x . Inspection of surface elevation spectra obtained from measurements showed that no appreciable energy was present above a frequency of about 1 Hz for both sub-test 1a and 1c. Choosing a maximum frequency of 1 Hz gave smooth results for the bottom velocity time series. Besides, an enormous reduction of the computational effort was achieved; the computational time appeared to be proportional to the square of the number of frequency components P . A maximum frequency of 1 Hz implies a time interval Δt of 0.5 s. Therefore, the time series of the surface elevation with a sampling rate of 1 Hz were thinned out before using a FFT programme to obtain the upwave boundary condition.

The number of frequency components P is directly related to the choice of the bandwidth Δf or equivalently, the choice of the length of the simulated time period $T = 1/\Delta f$. The value for $T = 2P\Delta t$ was chosen such that P was an integer power of 2, since FFT procedures are most efficient in that case. It is obvious that the choice of the simulated time T implies a compromise between the resolution and the computational effort. For sub-test 1a, at first a sequence of 4 runs of 512 s was made, with upwave boundary conditions obtained from successive parts of the measured time series. The raw bottom velocity spectra which appeared to vary between the realizations were averaged over the realizations as well as over a number of frequency bands in order to decrease the statistical error. It appeared however, that a more accurate representation of the longest waves was obtained when using longer time records. Despite the appreciable increase of the computational effort, the comparison of measurements and computations was eventually performed using one simulation with a length $T = 2048$ s for both tests. In that way, the accurate representation of the sub-harmonic components was ensured and the postprocessing of the output facilitated; the averaging of the bottom velocity spectra and moments over various realizations was unnecessary.

Thus, for the length of the upwave boundary time series $T = 2048$ s was chosen, with a time interval $\Delta t = 0.5$ s; the record contained 4096 data points. The first part of the measurements which was sampled just after the wave paddle was started was disregarded. Applying the FFT yielded a set of complex amplitudes A_p^+ ($p = 1, 2, 3, \dots, P$) with $P = 2048$. The bandwidth was $\Delta f = 1/T = 4.883 \cdot 10^{-2}$ Hz.

For the spatial step at first $\Delta x = 0.25$ m was chosen for both tests, in accordance with the spatial step used by Eldeberky and Battjes (1995). It appeared however, that without loss of accuracy $\Delta x = 0.5$ m could be used, which reduced the computational time. A further reduction of the computational effort might be obtained by using a still larger spatial step.

An overview of the computational parameters used in the numerical simulations is given in Table 8.3.

Test	b	γ	P	Δf (Hz)	Δx (m)
1a and 1c	1/15	0.85	2048	$4.883 \cdot 10^{-2}$	0.5

Table 8.3 Computational parameters for sub-test 1a and sub-test 1c

8.4 Comparison of computations and measurements

8.4.1 Introduction

The output of the model contained at the specified locations sets of complex amplitudes A_p^+ and U_p^+ ($p = 1, 2, 3 \dots P$) as well as raw spectral density values $2A_p^2/\Delta f$ and $2U_p^2/\Delta f$ ($p = 1, 2, 3 \dots P$). Note that the complex amplitudes U_p^+ only represent the oscillating part of the bottom velocity. Therefore, the measured bottom velocity time records were demeaned before comparison with computations. Further, in the comparison the time shifting of the various wave hours with respect to the typical wave hour (see Section 8.2) was taken into account. The computed complex amplitudes were transformed into time series using inverse FFT. Further, the comparison was carried out on bottom velocity spectra, covariance functions between the short wave velocity variance and the long wave velocity and bottom velocity moments.

Spectra

In order to obtain raw spectral density values $2U_p^2/\Delta f$ from the measurements, FFT was used. Subsequently, both for the measurements and the computations a suitable averaging in the frequency domain was applied. The number of neighbouring frequency bands k over which the raw spectral data were averaged was chosen so that the statistical error was sufficiently reduced without resulting in too large a loss of spectral resolution. The averaging was done by merging $k = 32$ consecutive spectral density estimates, yielding a number of degrees of freedom in the spectral estimates of 64. The spectral resolution thus was $k\Delta f = 0.0156$ Hz.

The statistical error amounted to

$$\frac{1}{k^{\frac{1}{2}}} = 18 \% \quad (8.3)$$

Covariance functions

The bottom velocity spectra computed from the model output appeared to largely underestimate the sub-harmonic spectral peak. This could be due to the assumption of uni-directional wave propagation in the model so that waves reflected from the beach are not represented in the simulations. To verify this, for both measurements and computations the covariance between the slowly varying short wave velocity variance and the long wave velocity was determined for different time lags τ . The covariance function $C(\tau)$ at each station is defined as

$$C(\tau) = \text{Cov} (u_{hi}^2(t), u_{lo}(t-\tau)) \quad (8.4)$$

where u_{hi} is the short wave component and u_{lo} is the short wave averaged oscillatory component or long wave component. The covariance functions so determined represent the phase coupling between long waves and the short wave groups at a certain station. Waves reflected on the beach are indicated by a correlation at a negative time lag corresponding to the time it takes for a short wave group to travel from the station to the beach and for a long wave to travel back. A negative correlation at zero time lag can be explained by the presence of bound long waves, long waves which are forced by the short wave groups and travel at the same speed as these wave groups. In the covariance functions free long waves coming in before the wave groups are represented by a positive peak at positive time lag, next to the negative peak at zero time lag due to the bound long waves.

To determine the covariance functions the bottom velocity time series were divided in a high-frequency part u_{hi} and a low-frequency part u_{lo} . The bottom velocity time series $u(x,t)$ are represented by the following truncated Fourier series (compare Eq. (7.6)):

$$u(x,t) = 2 \sum_{p=1}^P U_p^+(x) \exp[i(2\pi p \Delta f t)] \quad (8.5)$$

The short wave bottom velocity $u_{hi}(t)$ and the long wave bottom velocity $u_{lo}(t)$ are then given by

$$\begin{aligned} u_{hi}(x,t) &= 2 \sum_{p=m}^P U_p^+(x) \exp[i(2\pi p \Delta f t)] \\ u_{lo}(x,t) &= 2 \sum_{p=1}^{m-1} U_p^+(x) \exp[i(2\pi p \Delta f t)] \end{aligned} \quad (8.6)$$

For the lowest short wave frequency $m\Delta f$, half the peak frequency was used yielding $m\Delta f = 0.102$ Hz and $m = 209$ for sub-test 1a, and $m\Delta f = 0.0625$ Hz and $m = 128$ for sub-test 1c.

Filtering according to Eq. (8.6) was applied to the sets of complex amplitudes U_p^+ ($p = 1, 2, 3 \dots P$) for both measurements and computations. The low-pass and high-pass filtered sets of complex amplitudes were transformed into time series of $u_{lo}(t)$ and $u_{hi}(t)$, respectively. The latter time series were then squared. The covariance functions (8.4) at each location were obtained by the use of a covariance package.

Third odd velocity moments

Since we only consider the oscillatory velocity components, Eq. (8.1) is reduced to

$$\langle |u|^2 u \rangle = \langle |u_{hi}|^2 u_{hi} \rangle + 3 \langle |u_{hi}|^2 u_{lo} \rangle + \dots \quad (8.7)$$

All three terms in Eq. (8.7) were determined for both measurements and computations. It appeared that indeed the sum of the short wave and long wave contribution to the third odd velocity moment approximately equalled the moment computed on the basis of the time series for u . The moments were determined as the covariance at zero time lag:

$$\langle |u|^2 u \rangle = \text{Cov} (u^2(t), u(t)) \quad (8.8)$$

Note that the long wave contribution to the third odd velocity moment is equal to three times the covariance between the long wave velocity and the slowly varying short wave velocity variance at zero time lag.

8.4.2 Results

Sub-test 1a

In Figure 8a, the computed surface elevation time series are compared with measurements using a time-window of about 10 times the wave period. The results correspond with the results presented in Eldeberky and Battjes (1995). Inspection of the results shows that the peak values of the highest waves are largely underestimated by the model and that the form of the predicted wave profile somewhat differs from the measured one with the differences coming more significant closer to the beach. These discrepancies can be partly attributed to phase errors resulting from the inaccuracy of the dispersion relation of the Boussinesq equations, which become significant for larger propagation distances. The underestimation of the crest values was already noted by Madsen and Sørensen (1993) in a comparison of the results of their time domain Boussinesq model and the corresponding spectral Boussinesq model. They could not give an explanation for this discrepancy which seems to be the result of the transformation of the equations from the time domain into the frequency domain.

The computed and measured bottom velocity time series are shown in Figures 8b.1 and 8b.2. It can be seen that the accuracy in predicting the bottom velocity is comparable to the accuracy in predicting the surface elevation. The major discrepancy between computations and measurements is again found in the crest values, resulting in a too symmetric computed signal. In Figure 8c the horizontal velocities at approximately mid-depth are presented, showing that the horizontal velocity field is indeed approximately uniform.

Spectral density functions of the measured and computed bottom velocities are depicted in Figure 8d. For the stations up to 130 m the agreement in the spectral evolution is good. For the stations from 138 m onwards, where the wave breaking is strong, a discrepancy in the magnitude of the sub-harmonic spectral peak occurs. This effect was not noticeable in the surface elevation spectra (see Eldeberky and Battjes (1995)) which can be attributed to the fact that the ratio of velocity amplitude and surface elevation amplitude increases for decreasing frequencies. The underestimation of the magnitude of the sub-harmonic peak might be partly due to a too strong reduction of the long wave amplitudes as a result of the

wave breaking formulation in the model. Another plausible explanation could be the assumption of uni-directional wave propagation in the model. As a result the computations do not describe the long waves which are reflected from the beach forming a standing wave pattern. Note that since most short wave energy is dissipated on the beach, the reflection of short waves is negligible in the flume experiments.

To verify the latter explanation, the covariance functions of the short wave bottom velocity variance and the long wave bottom velocity were determined. These covariance functions are presented in Figure 8e. For the stations at $x = 100$ m and 115 m the predictions are quite reasonable. The model reproduces the minimum at zero time lag due to the bound waves and the maximum at positive time lag due to the (spurious) incoming free waves. This maximum is rather large due to the first-order wave generation at the paddle. The measurements for these two stations show a small covariance at negative time lag which is not reproduced by the model. This could not only be due to reflected waves but also to statistical variability in the covariance functions. For the stations from 130 m onwards however, for which the model underestimated the sub-harmonic spectral peak, the discrepancies are not only found at negative time lag but also at the time lags corresponding to the bound waves and free waves coming in before the wave groups. It can be seen that the effect found in the flume experiments that the correlation at zero time lag becomes positive close to the shore is not fully reproduced by the model. Therefore, no definite conclusions about the discrepancies in the spectra for the low frequencies can be drawn on the basis of the covariance functions.

The measured and computed third odd velocity moments are shown in Figure 8f. A distinction is made between the total velocity moment and the long wave and short wave contribution to the velocity moment. It can be seen that the long wave contribution to the total velocity moment is small compared to the short wave contribution. Therefore, the discrepancies found in the covariance functions at zero time lag hardly influence the total velocity moment. The short wave contribution and consequently the total velocity moment are largely underestimated by the model, except for the last two stations where strong wave breaking takes place. The underestimation of the short wave contribution is a direct result of the underestimation of the crest values of the highest waves as noted in the bottom velocity time series.

Sub-test 1c

In Figures 8g.1 and 8g.2 the computed bottom velocity time series are compared with measurements using a time-window of about 10 times the wave period. The agreement is quite reasonable. The phenomenon that the crest elevations are underestimated is not found for this sub-test. The form of the computed signals is in better agreement with the measurements than was found for sub-test 1a. The phase errors are smaller, which is a direct result of the more accurate frequency dispersion for smaller values of the dispersion parameter μ (see Table 8.2). The phase errors become noticeable for larger propagation distances.

The velocity spectra, depicted in Figure 8h, show a good agreement between measurements and computations for the first three stations. The predicted spectrum at $x = 138$ m, corresponding with the top of the bar, not only shows an underestimation of sub-harmonic spectral peak but of the primary peak as well. The latter could be due to the fact that the model

responds instantaneously to the rapidly decreasing water depth whereas the breaking in the flume experiments does not start until behind the bar. The spectra at the last two stations show that in the computations the waves decompose without a distinct primary peak, which is in contrast with the flume experiments. Eldeberky and Battjes (1995) noted this phenomenon in the surface elevation spectra as well and ascribed this to the relatively steep bottom beyond the bar crest which is in contrast with the assumption of slowly varying bottom.

The covariance functions are shown in Figure 8i. The feature in the measurements at decreasing negative time lag due to reflected long waves is not reproduced by the model. A very large covariance at negative time lag is found at the bar crest at $x = 138$ m. This might indicate that a standing wave pattern be found in the flume experiments with a node in the surface elevation and hence large velocity amplitudes at the bar crest. This would also explain the large sub-harmonic primary peak in the measured velocity spectra at $x = 138$ m which is not reproduced by the model.

From Figure 8j we see an encouraging agreement between the third odd velocity moments in the flume and numerical experiments, especially up to the bar crest. The contribution of the interaction between the long and short waves to the total velocity moment is small, which was also seen in sub-test 1a. The short wave contribution is predicted extremely well indicating that the short wave asymmetry is predicted accurately. This was already seen in the time series which, in contrast with sub-test 1a, do not demonstrate the underestimation of the peak values.

8.4.3 Discussion of the results

Both sub-test 1a and 1c show that the third odd velocity moments are dominated by the contribution of the short wave asymmetry. This contribution is very well predicted in sub-test 1c. In sub-test 1a however, the model largely underestimates the short wave asymmetry, which is a direct result of the underestimation of the crest values of the highest waves.

The underestimation of the crest values might be partly due to the inaccurate frequency dispersion of the Boussinesq equations for shorter wave components. As can be seen in Figure 3.2 the percent error in the phase celerity increases for increasing values of kh . Both for sub-test 1a and for sub-test 1c, the error in phase celerity of the primary wave, which is rather long, is negligible. Discrepancies however, can be found in the phase celerity of the freely moving higher harmonics. These errors can be computed as

$$\frac{c_{Bous} - c}{c} * 100 \% \quad (8.9)$$

where c is the exact linear phase velocity which can be obtained from Eq. (3.9) and c_{Bous} is the phase velocity corresponding to the Boussinesq equations. The latter can be computed from Eq. (3.10) using $b = 1/15$. For both tests this was done for the second and third super-harmonic for values of kh corresponding to a water depth of 4.1 m (at the upwave boundary) as well as a water depth of 1.8 m (at $x = 100$ m). The results are listed in Table 8.4.

	$h = 4.1 \text{ m}$		$h = 1.8 \text{ m}$	
	$T = 1/2 T_p$	$T = 1/3 T_p$	$T = 1/2 T_p$	$T = 1/3 T_p$
test 1a	2.0	16	0.1	1.7
test 1c	0.04	1.1	0.01	0.1

Table 8.4 Percent errors in the phase velocity for sub-test 1a and 1c for the second and third harmonic at $x = 20$ and $x = 100 \text{ m}$

From Table 8.4, it can be seen that as a result of the smaller peak period T_p , the errors in sub-test 1a are considerably larger than in sub-test 1c, especially for the third harmonic. Note however, that the third harmonic is stronger represented in sub-test 1c than in sub-test 1a. The larger part of the phase errors in the computations originate in the deepest part close to the wave board and become more noticeable for larger propagation distances. These phase errors could result in the underestimation of the crest values in sub-test 1a.

It is also likely that the underestimation of the crest values originates from the transformation of the time domain equations into the frequency domain. Madsen and Sørensen (1993) remarked that the spectral model yielded lower crest values than the time domain counterpart of the spectral equations but could not give an explanation for this phenomenon.

In the procedure to obtain the spectral equations it is assumed that the complex amplitudes $A_p(x)$ vary slowly. To verify this assumption the recurrence length can be computed. Due to the interplay between the free and bound second harmonic, the second harmonic amplitude is periodic with the recurrence length L_r . Dingemans (1994b) demonstrated that the recurrence length L_r can be expressed as

$$L_r = \frac{2\pi}{k_2 - 2k_1} \quad (8.10)$$

where k_1 and k_2 are the wave numbers pertaining to the free first and second harmonics respectively. The ratio of recurrence length L_r to the wave length of the first harmonic L_1 is a measure for the rate at which the complex amplitudes $A_p(x)$ vary. This ratio L_r/L_1 is computed at both the upwave boundary and at $x = 100 \text{ m}$. The wave length of the second harmonic L_2 is computed from linear theory. The wave length of the first harmonic is computed using $c^2 = g(h + \zeta)$. At both $x = 20$ and 100 m , ζ was chosen as $\zeta = H_{m0}/2 = 0.45 \text{ m}$ for sub-test 1a and $\zeta = H_{m0}/2 = 0.3 \text{ m}$ for sub-test 1c (see Roelvink and Reniers, 1995). The results are listed in Table 8.5.

	$h = 4.1 \text{ m}$	$h = 1.8 \text{ m}$
test 1a	0.7	1.2
test 1c	2.0	2.9

Table 8.5 Ratio L_r/L_1 of recurrence length to first harmonic wave length for sub-test 1a and 1c at $x = 20$ and $x = 100 \text{ m}$

The amplitudes $A_p(x)$ can be said to be slowly varying if the ratio L_r/L_I is at least 1 or 2. Table 8.5 shows that in sub-test 1a the assumption of slowly varying complex amplitudes underlying the transformation of the equations into the frequency domain, is violated. Sub-test 1c on the contrary, meets the requirement having a considerably larger peak period T_p . This difference between sub-test 1a and 1c might explain the underestimation of the crest values only found in sub-test 1a.

8.5 Conclusions

The computation of the bottom velocities from the surface elevation yields results with an accuracy comparable to the accuracy of the predicted surface elevations. The accuracy in predicting the velocity moments is governed by the accuracy in predicting the bottom velocity time series. The third odd velocity moments are very well predicted in sub-test 1c, whereas sub-test 1a shows a large underestimation of the velocity moments due to an underestimation of the crest values of the highest waves. The latter might be explained by the smaller peak period in sub-test 1a compared to sub-test 1c resulting in: 1) phase errors due to inaccurate frequency dispersion of the Boussinesq equations; and 2) the violation of the assumption of slowly varying complex amplitudes $A_p(x)$ underlying the derivation of the spectral equations. To provide definite conclusions about this, additional simulations should be performed. Recommendations about this and other matters, can be found in Chapter 9.

9 Conclusions and recommendations

9.1 Introduction

In this thesis, the modelling of the horizontal velocity field under waves using Boussinesq-type equations was studied. In view of sediment transport predictions the focus was on the prediction of near-bed velocity time series and resulting velocity moments. The conclusions of this study are formulated in Section 9.2. A distinction is made between the form of the Boussinesq-type equations, the time domain modelling and the spectral modelling. The recommendations are presented in Section 9.3.

9.2 Conclusions

Form of the Boussinesq-type equations

Many different forms of Boussinesq-type equations exist, which differ in linear frequency-dispersion and shoaling characteristics. The equations with optimum linear frequency dispersion can be found by matching the expansion of the exact linear dispersion relation in the expansion parameter $(kh)^2$ with the dispersion relation corresponding to the Boussinesq-type equations. In this study only those equations were considered in which the frequency dispersion is optimized using a first-order expansion of the exact linear dispersion relation. This leads to equations in which at most third-order (mixed) derivatives to x and t appear.

Improving the frequency dispersion basically means changing the third-order derivatives in the equations, which also changes the interpretation of the velocity variable in which the equations are stated. Since we are interested in the prediction of the horizontal velocity field, an unambiguous interpretation of the computational velocity variable is required. Only the sets of equations stated in terms of the depth-averaged velocity $\bar{u}$ or equivalently the flux q were found to meet this requirement. Those sets of equations with improved frequency dispersion and stated in terms of $\bar{u}$ or q , differ in the slope-dependent terms which give rise to large differences in shoaling behaviour. The equations with the best shoaling behaviour (Eqs. 3.17) were selected for numerical treatment and implemented in an existing time domain numerical model. Also a spectral model based on the reduction of Eqs. (3.17) for mildly sloping bottom and supplemented with a wave breaking formulation was used. After the integration of the Boussinesq equations the horizontal velocity field has to be reconstructed by using a parabolic velocity profile (Eq. 3.2) in which second-order derivatives in h and $\bar{u}$ appear.

Time domain modelling

The new time domain model was successfully validated against measured surface elevation time series of regular waves propagating over a submerged bar. The validation demonstrated the importance of good frequency dispersion. Discrepancies between measurements and computations arose for shorter wave components due to the still not accurate enough frequency dispersion. Note that, although further improvement of the frequency-dispersion characteristics is possible, the inaccuracy of the frequency dispersion is a direct result of the assumption of (fairly) long waves underlying the Boussinesq-type equations. The better linear

shoaling behaviour and the applicability for larger bottom slopes of the new time domain model as compared to the old model resulted in slightly better predictions. The accuracy of the frequency dispersion behaviour however, which is the same for both the old and the new model, was found to be more important than good shoaling behaviour.

The new time domain model was tested against velocity measurements. Note that the application was limited inshore by the wave breaking process. The numerical computation of the horizontal velocity profile from the computational velocity $\bar{u}$ appeared to be very sensitive since the second-order derivatives $(h\bar{u})_{xx}$ and $\bar{u}_{xx}$ must be determined. Small disturbances in the computational velocity signal gave rise to vastly erroneous results for the second-order derivatives and consequently for the horizontal velocity field. A low-pass filter was applied to the computational velocity signal before calculation of the second-order derivatives. This could adequately smooth out the high-frequency disturbances in most situations. Further, in applying the time domain model it was found that the numerical method gave rise to instabilities which become more significant for increasing values of the non-linearity parameter ε .

Simulations of regular waves propagating over a submerged bar showed that the accuracy in predicting the computational (mean) velocity is comparable to the accuracy in predicting the surface elevation; both for the surface elevation and for the mean velocity the model slightly overestimated the crest amplitudes and underestimated the trough amplitudes. The velocity profile obtained from the computational velocity variable by using the parabolic extrapolation was found to yield good results, especially for the lower half of the profile. The bottom velocity was very well predicted, with a slight overestimation of the trough amplitude.

Spectral modelling

The spectral model, which yields complex amplitudes for the surface elevation, was extended to compute the depth-averaged velocity $\bar{u}$ and subsequently the complete horizontal velocity field using a parabolic expression in terms of $\bar{u}$. The depth-averaged velocity is obtained from the surface elevation by linearization of the continuity equation. Therefore only the purely oscillating part of the horizontal velocity is predicted by the model.

Two irregular wave tests, in which wave breaking occurs on a mainly monotonic beach, were used to verify the computation of the bottom velocity. In both tests the horizontal velocity was approximately uniform. The tests mainly differ in the peak period. The tests showed that the discrepancies between the predicted and measured velocity time series are comparable to the discrepancies found in predicting the surface elevation. This was already found in the simulations with the time domain model. The longer wave test yielded very good results for the bottom velocity time series and consequently the third odd velocity moment. For the shorter wave test however, a significant underestimation of the crest values of the higher waves was found, resulting in too symmetric signals. This yielded a large underestimation of the third odd velocity moments. Two possible explanations were given for the underestimation of the crest values in the shorter wave test:

- 1 the inaccuracy of the linearized dispersion relation corresponding to the Boussinesq equations, which becomes significant for shorter wave components
- 2 the assumption of slowly varying complex amplitudes in horizontal direction, which underlies the transformation of the time domain equations into the frequency domain

The latter assumption was shown to be violated in the shorter wave test.

9.3 Recommendations

The applicability of the Boussinesq models is limited by the inaccurate frequency dispersion for shorter wave components. The range of applicability may be extended by higher-order improvement of the frequency dispersion, leading to fifth-order derivatives (see Dingemans, 1973, 1994b).

For further applications, it is important to improve the numerical scheme of the time domain model. Besides, attention should be paid to the determination of the second-order derivatives necessary for the reconstruction of the horizontal velocity field.

An important extension to the spectral model could be the description of the time-averaged velocity component, in addition to the oscillatory part of the velocity. This must be investigated further.

The spectral model is very time-consuming in the case of irregular wave trains. It should be investigated whether a reduction of the computational time by increasing the spatial step can be established without loss of accuracy.

The underestimation of the crest values of the highest waves by the spectral model in comparison with the time domain counterpart (see Madsen and Sørensen, 1993) should be investigated. Therefore, the transformation of the time domain equations into the frequency domain must be studied. This might yield an explanation for the underestimation of the crest values as compared to measurements in the shorter irregular wave test.

Additional verification of velocity predictions obtained from both the spectral model and an improved version of the time domain model should be carried out. Comparison on velocity time series and moments for non-breaking irregular wave tests with *both* models may yield insight in the accuracy of the prediction of velocity moments with the time domain model and the origin of the underestimation of the crest values by the spectral model.

Further, the regular wave tests, against which the time domain model was verified, could be simulated using the spectral model. In those regular wave tests, a less uniform velocity field was found than in the irregular wave tests simulated with the spectral model in this study. It would also provide a comparison between the performance of both models.

10 Acknowledgements

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Appendix A

Figures to Chapter 5

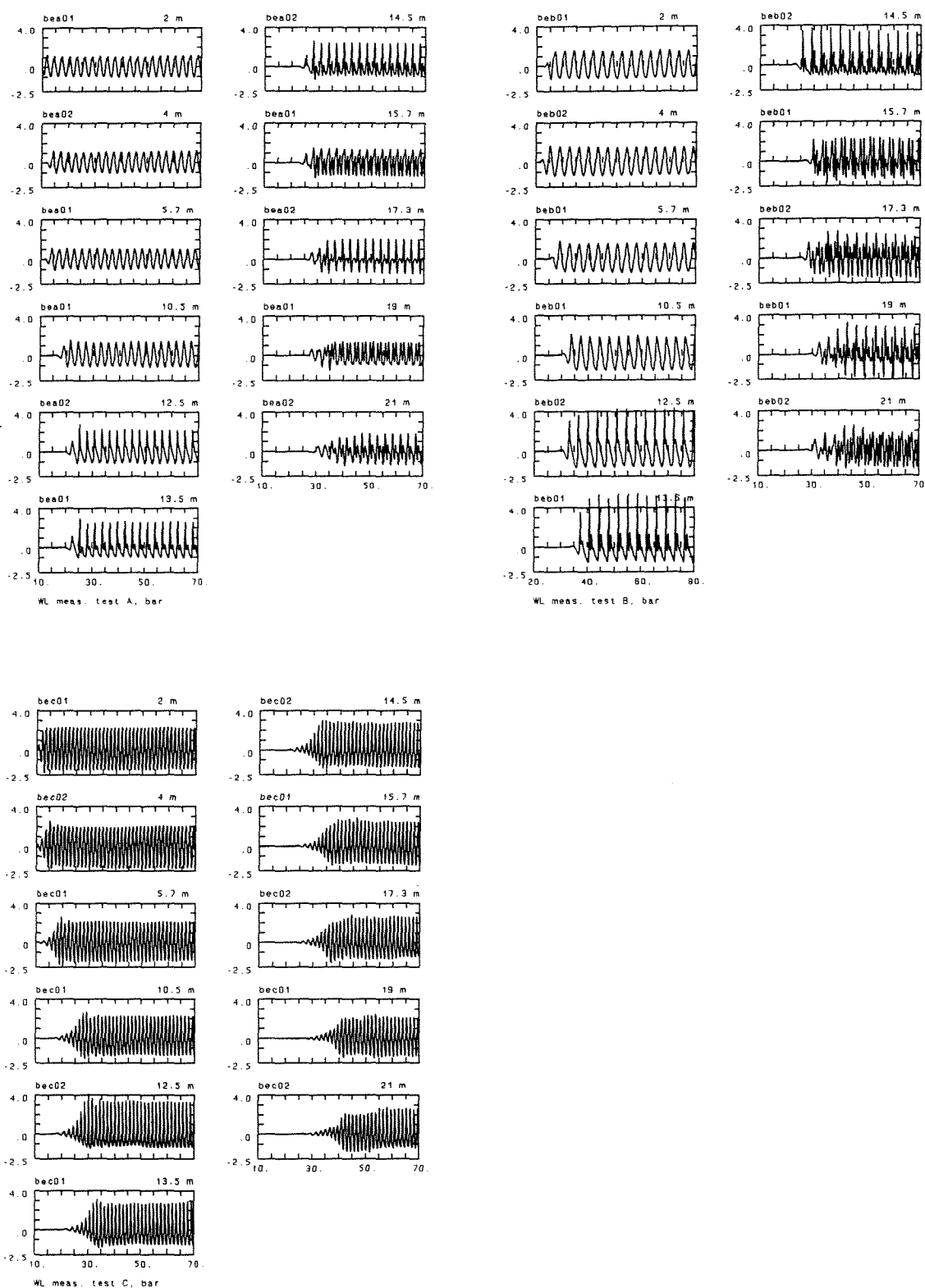


Figure 5a *WL measurements in the Scheldt flume for conditions A, B and C. Time-window of 60 s with unscaled time and scaled height (from 20 to 80 s for beb01, from 10 to 70 s for all other series). From Dingemans (1994a)*

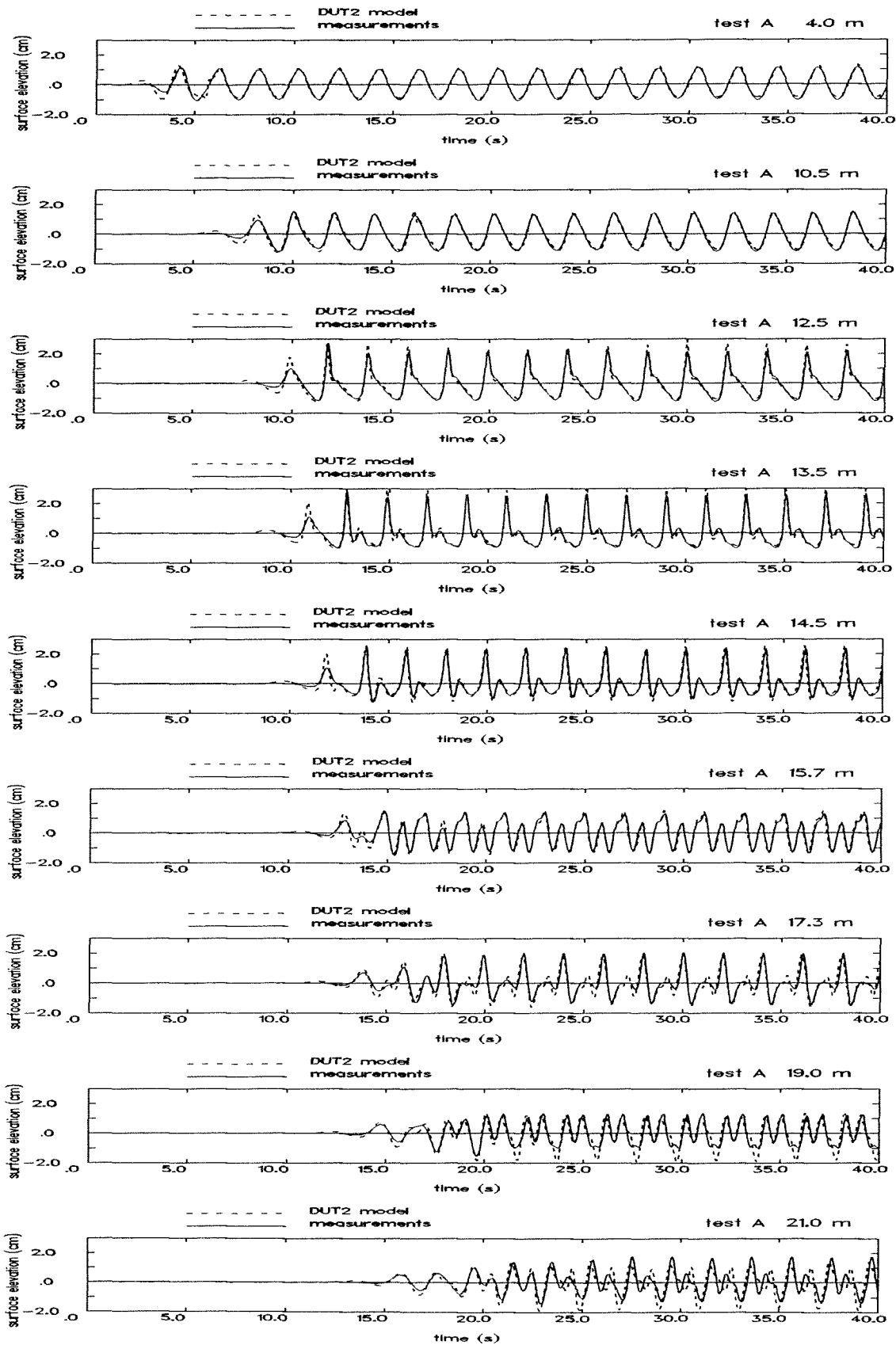


Figure 5b Comparison of measurements with results of DUT2 model with $b = 1/15$, test A, time-window from 0 to 40 s

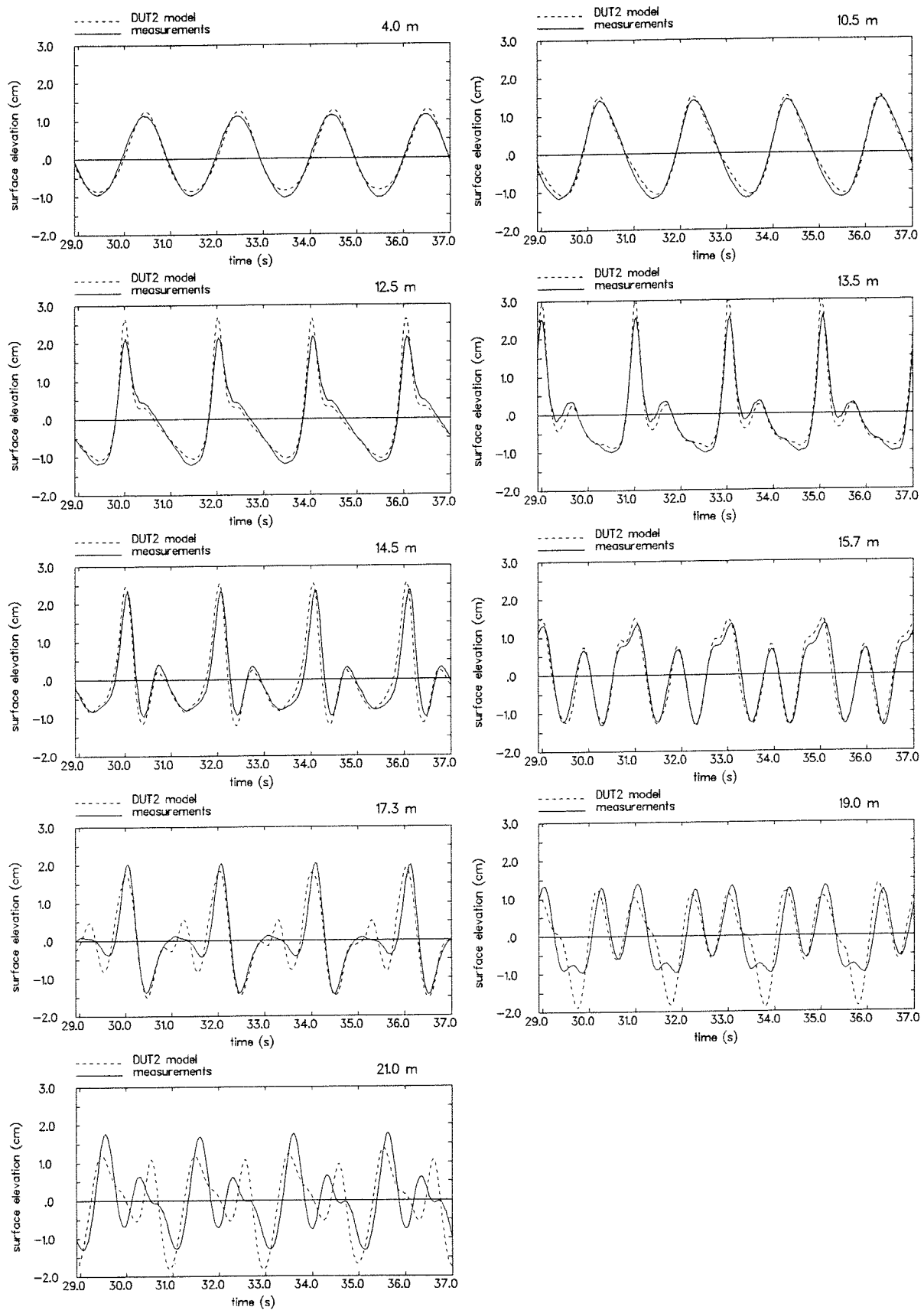


Figure 5c *Comparison of measurements with results of DUT2 model with $b = 1/15$, test A, time-window is four times the initial wave period*

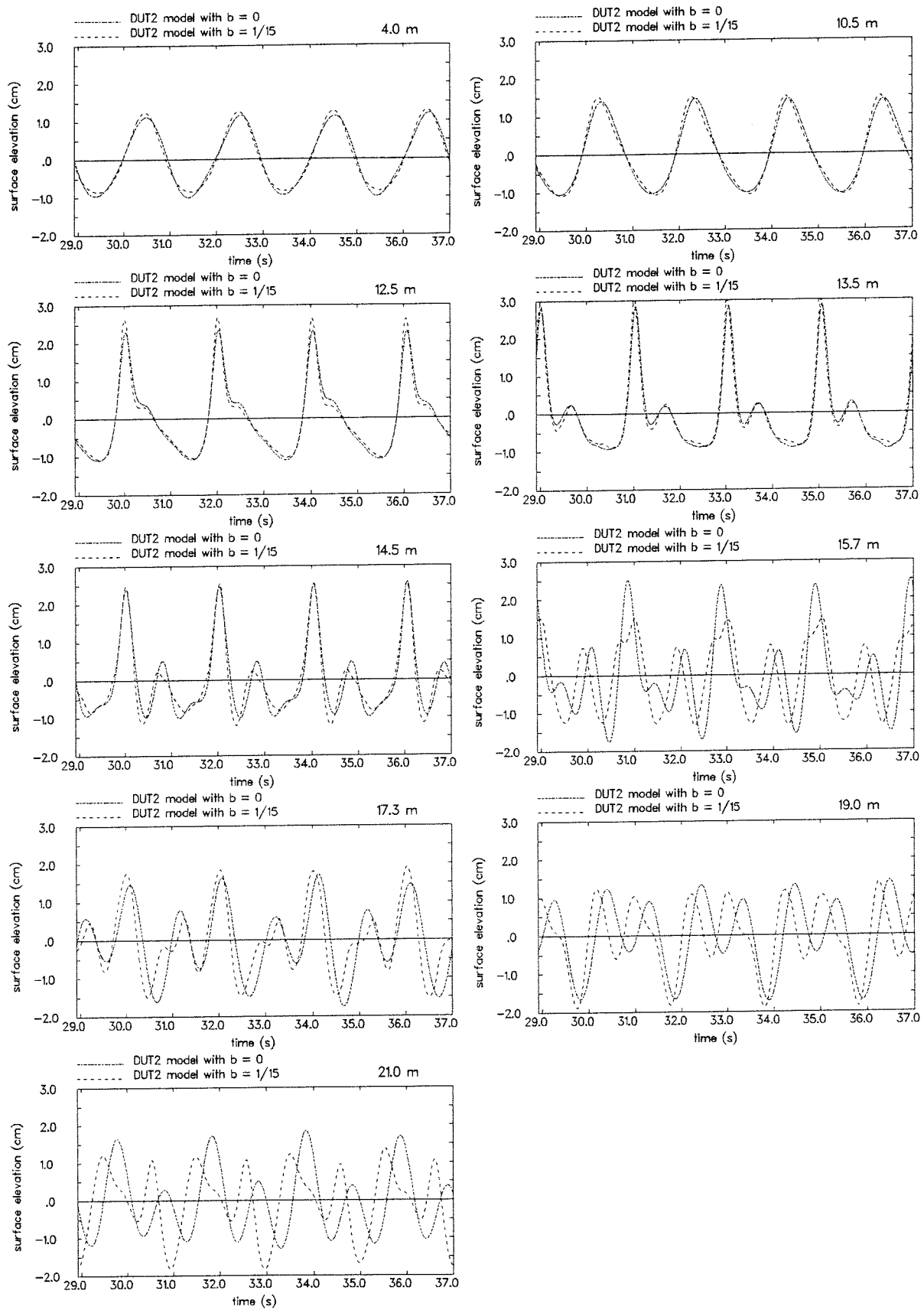


Figure 5d Comparison of the results of DUT2 model with $b = 1/15$ and $b = 0$ respectively, test A, time-window is four times the initial wave period

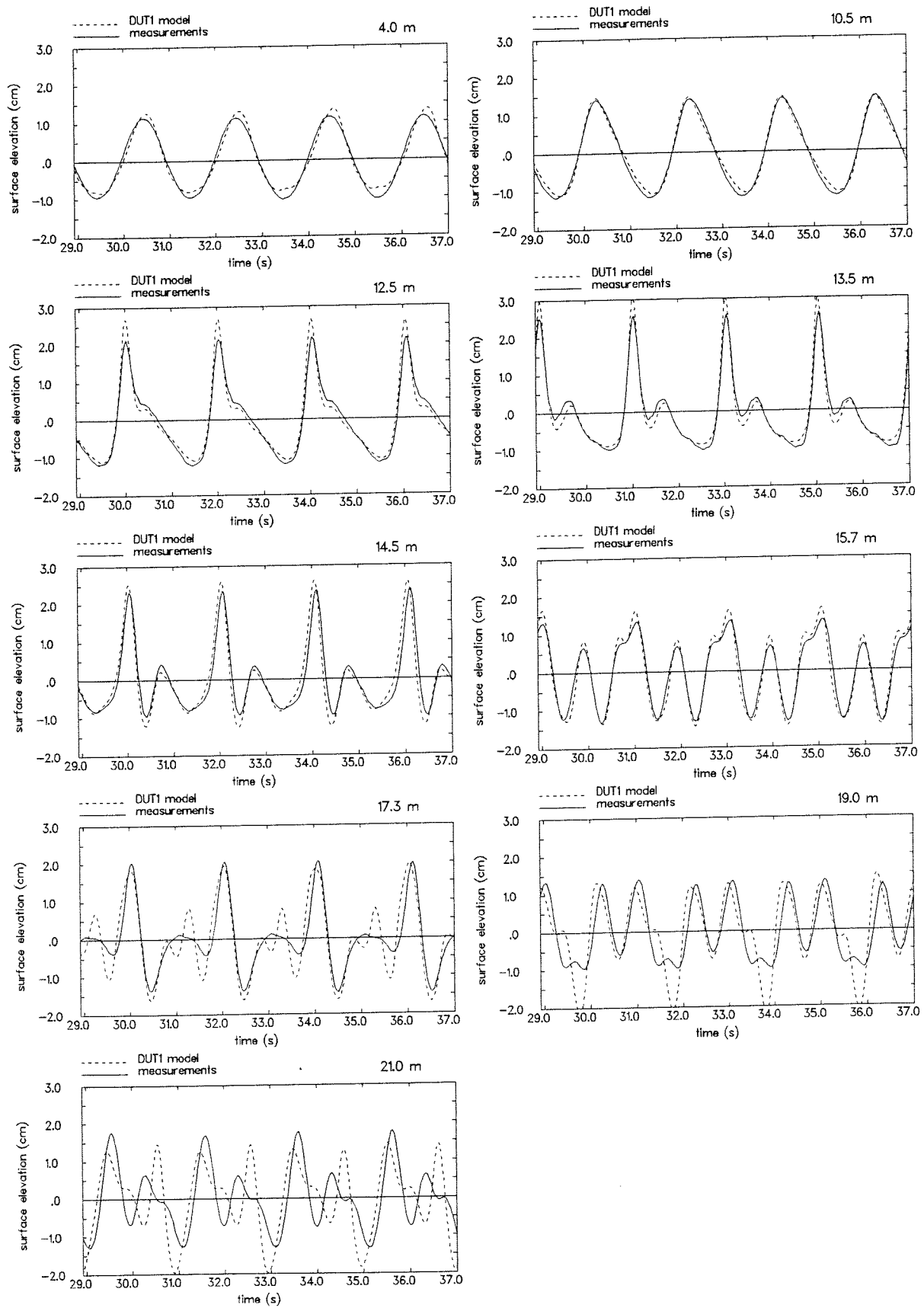


Figure 5e *Comparison of measurements with results of DUT1 model with $b = 1/15$, test A, time-window is four times the initial wave period*

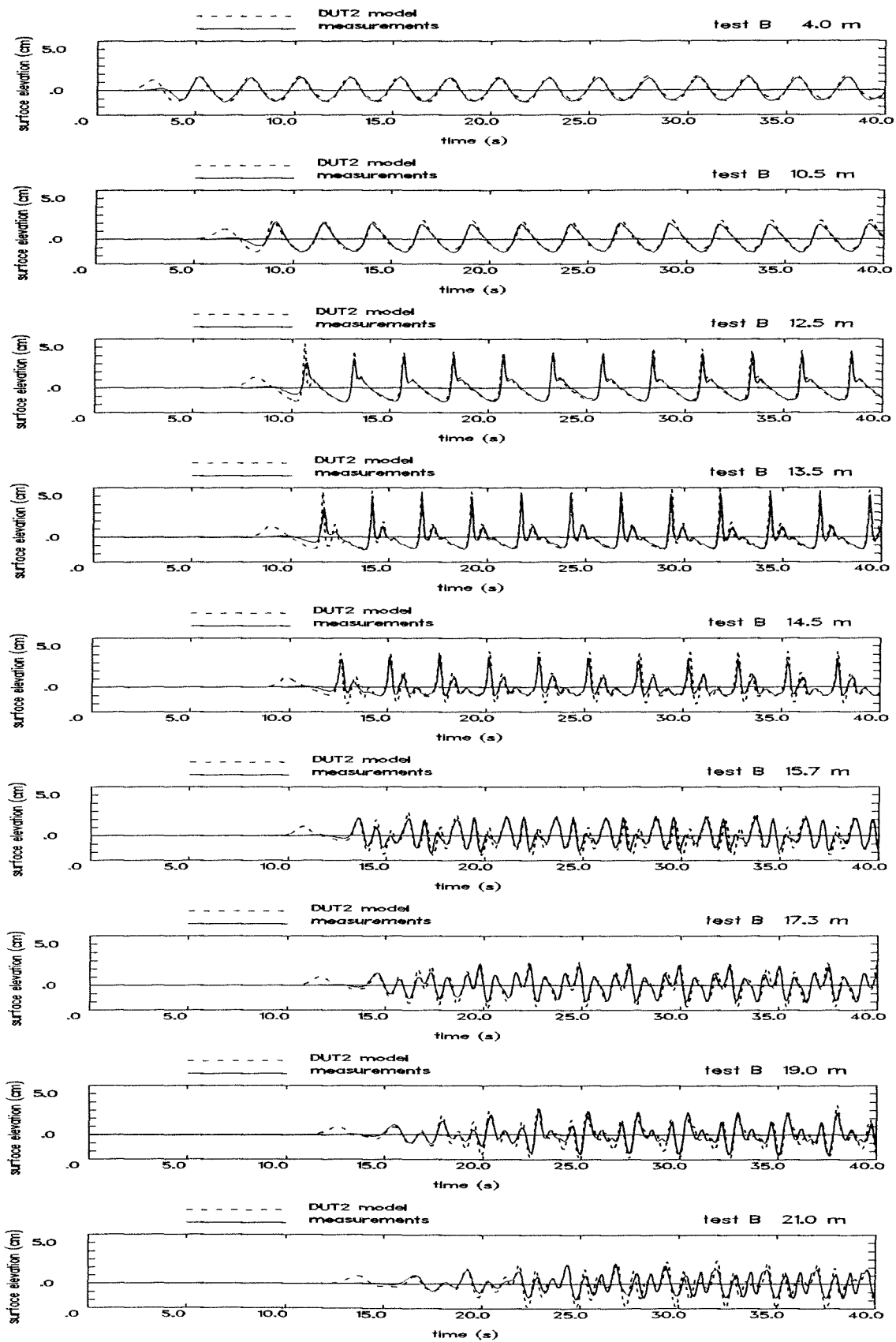


Figure 5f Comparison of measurements with results of DUT2 model with $b = 1/15$, test B, time-window from 0 to 40 s

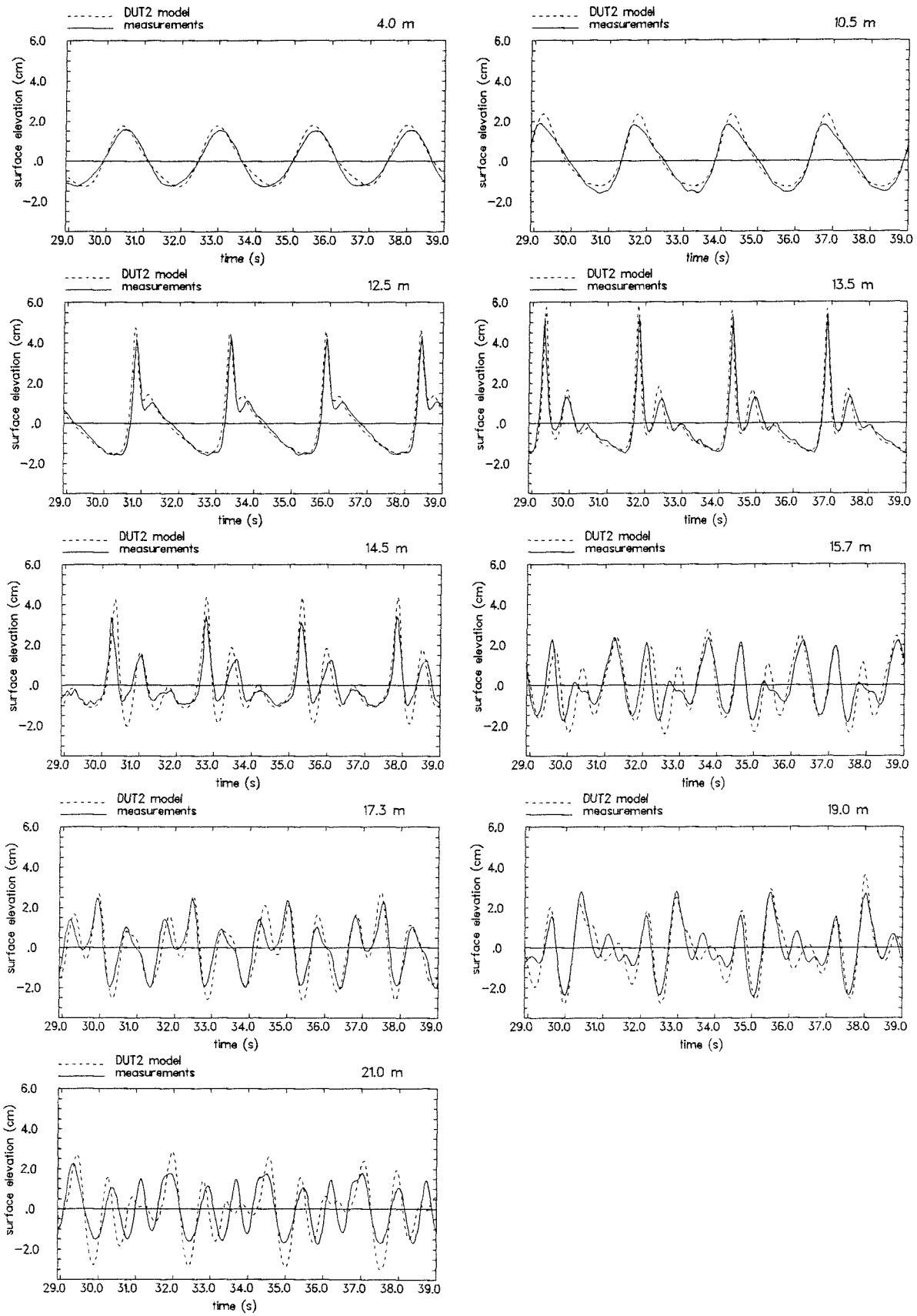


Figure 5g *Comparison of measurements with results of DUT2 model with $b = 1/15$, test B, time-window is four times the initial wave period*

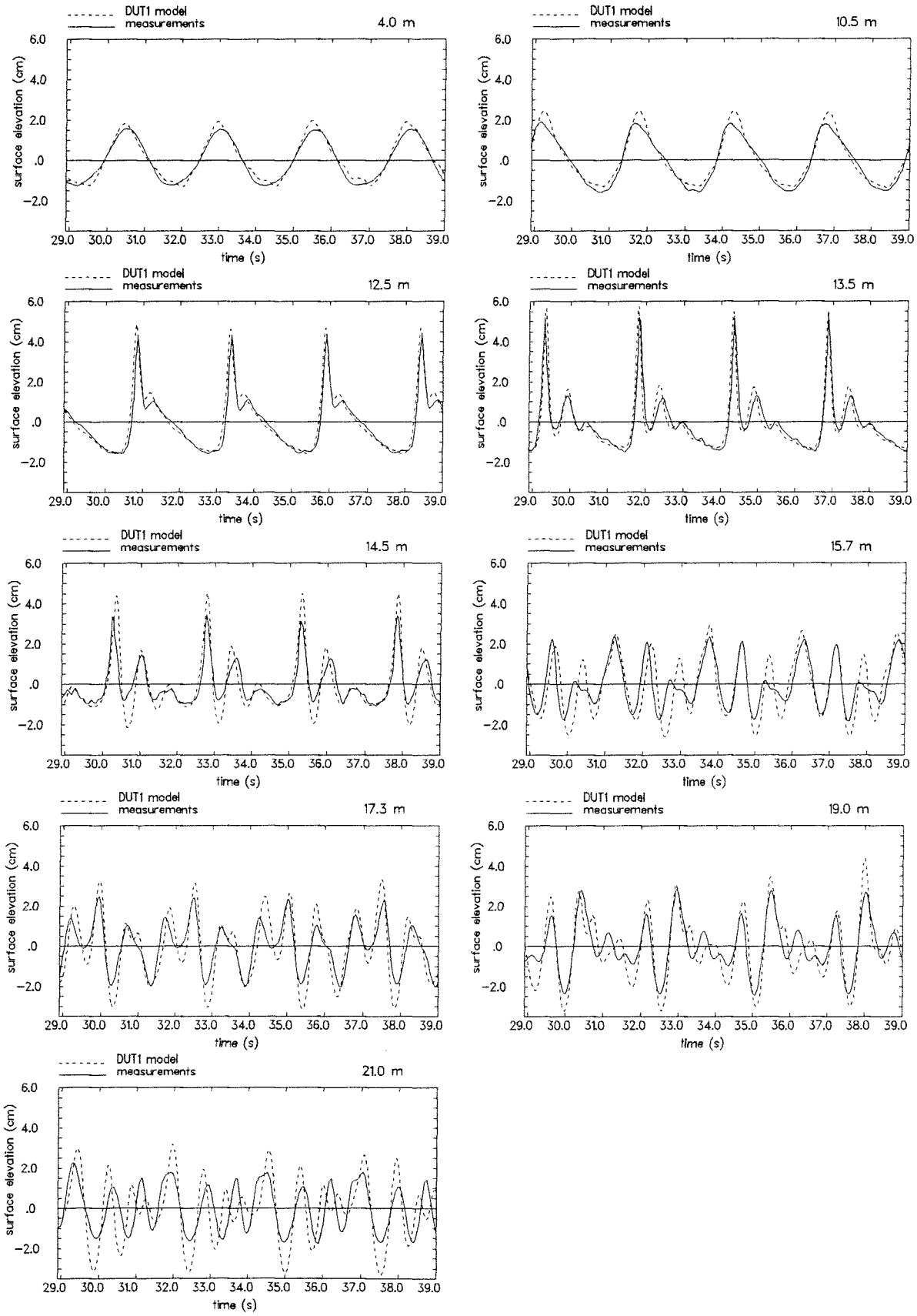


Figure 5h *Comparison of measurements with results of DUT1 model with $b = 1/15$, test B, time-window is four times the initial wave period*

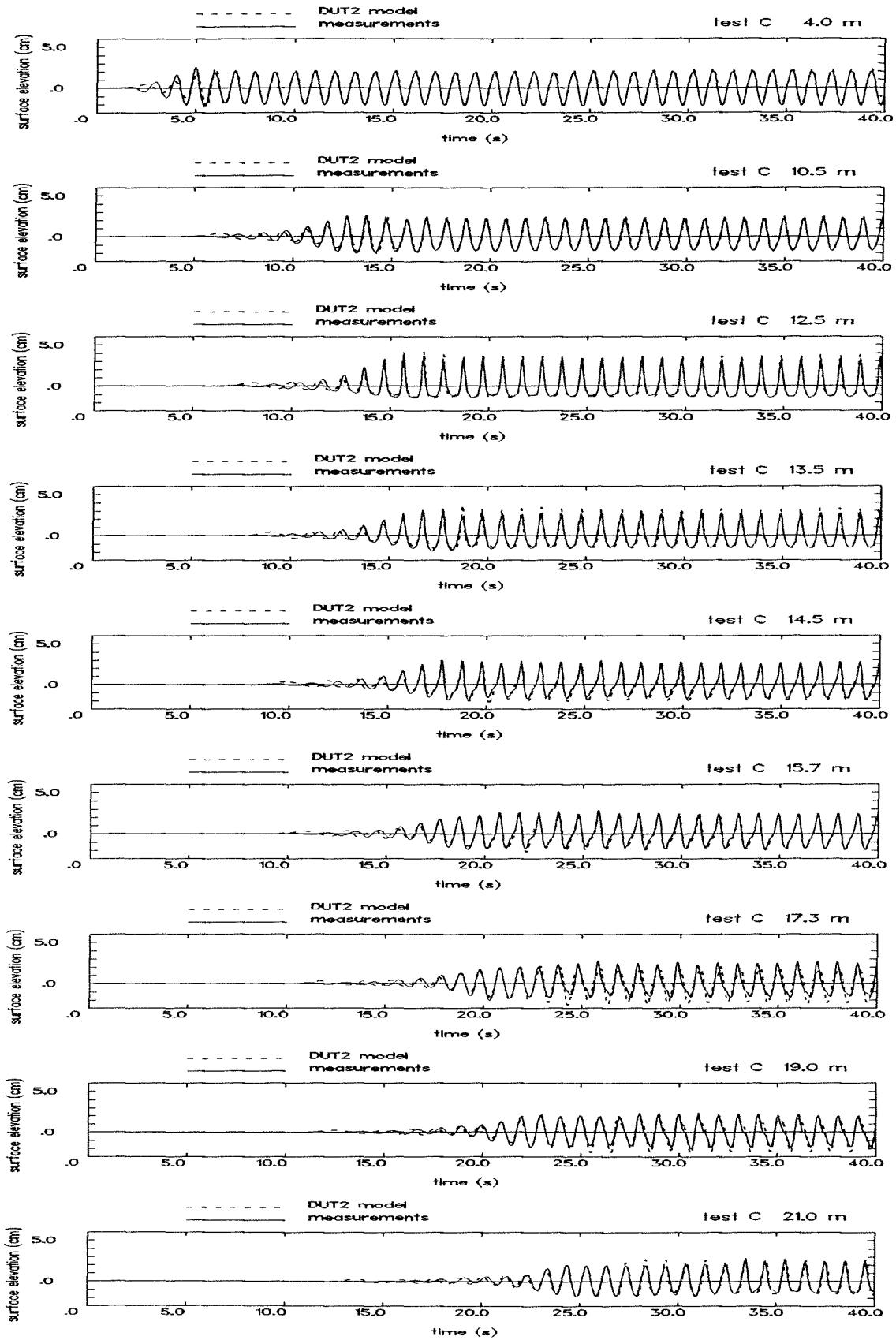


Figure 5i Comparison of measurements with results of DUT2 model with $b = 1/15$, test C, time-window from 0 to 40 s

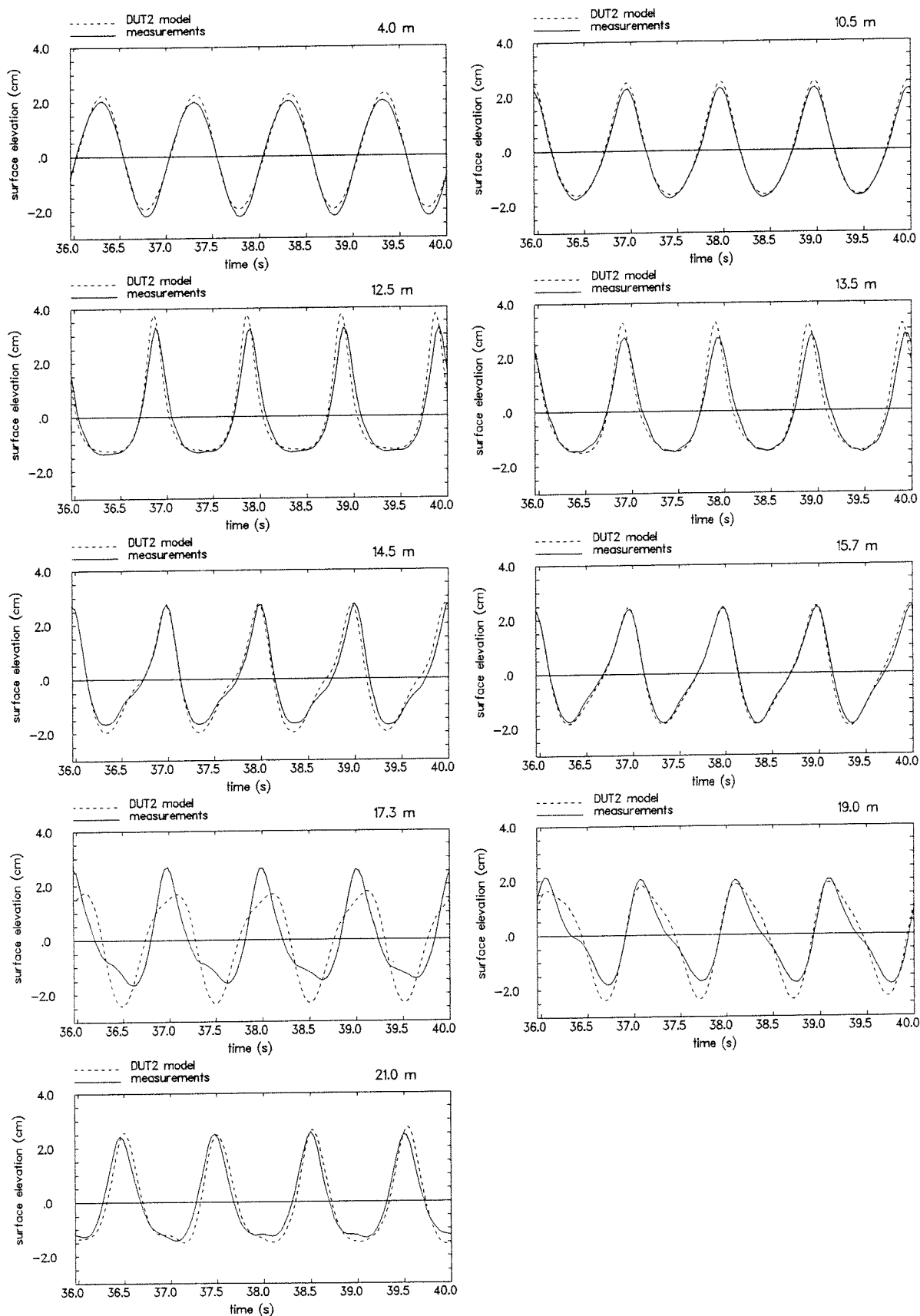


Figure 5j *Comparison of measurements with results of DUT2 model with $b = 1/15$, test C, time-window is four times the initial wave period*

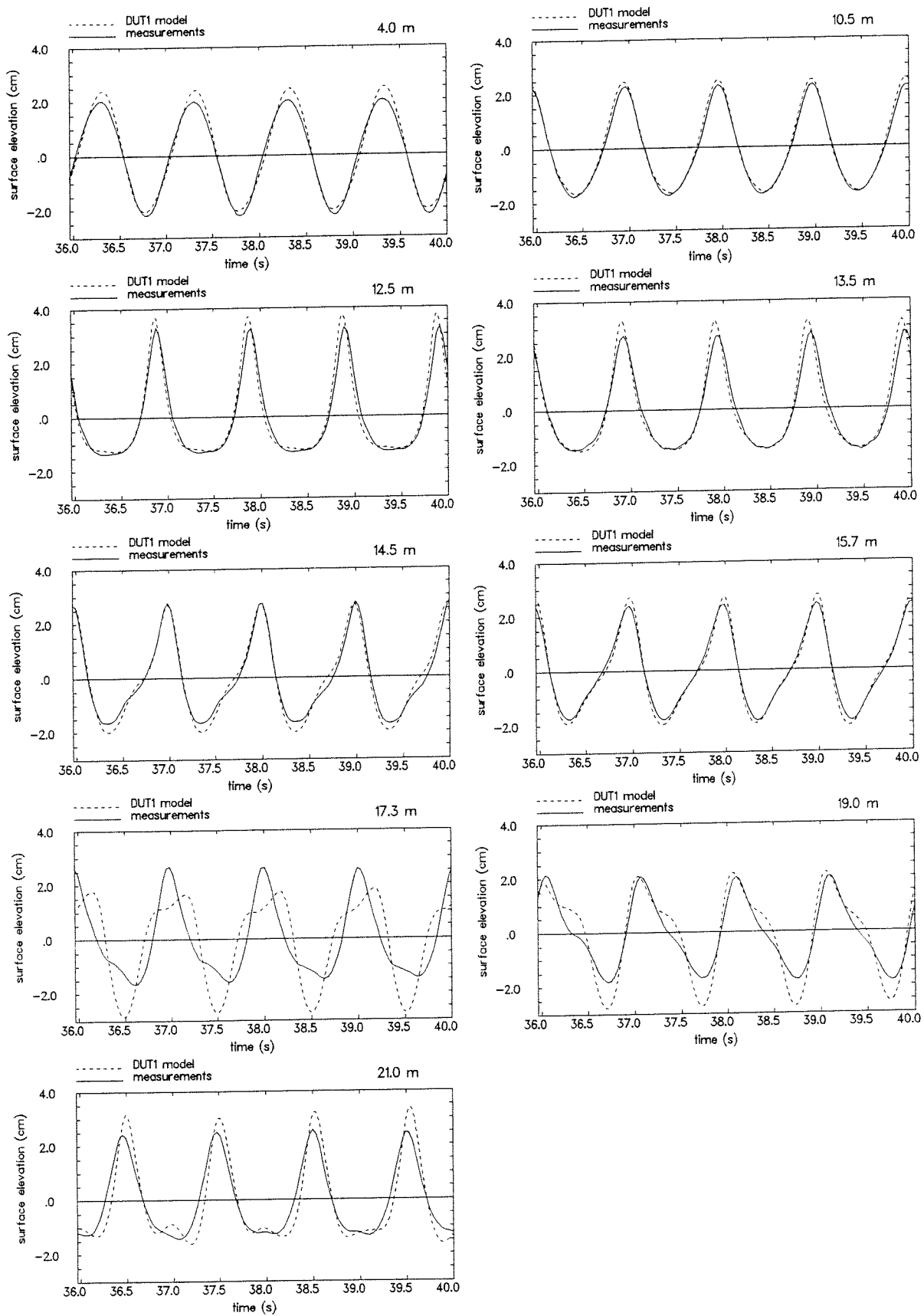


Figure 5k *Comparison of measurements with results of DUT1 model with $b = 1/15$, test C, time-window is four times the initial wave period*

Appendix B

Tables to Chapter 6

wave height meter	1	2	3	4	5
x location (m)	8.00	14.95	15.70	21.52	22.02

Table 6a *Horizontal distances of the wave height meters to the wave maker in tests S01*

LDV system	1	2
x location (m)	15.50	21.72

Table 6b *Horizontal distances of the LDV equipment to the wave maker in tests S01*

run number	vertical position (cm)	
	LDV 1	LDV 2
s0105	1.0	0.8
s0106	2.0	1.2
s0107	5.0	1.6
s0108	10.0	2.0
s0109	15.0	3.0
s0110	20.0	4.5
s0111	27.5	6.0
s0112	35.0	8.0
s0113	42.5	10.0
s0114	50.0	12.0
s0115	57.0	15.0

Table 6c *Vertical distances of the LDV equipment to the bottom in tests S01*

wave height meter	1	2	3
<i>x</i> location (m) in tests R01	8.00	14.95	15.70
<i>x</i> location (m) in tests R02	8.00	18.95	19.70

Table 6d *Horizontal distances of the wave height meters to the wave maker in tests R01 and R02*

LDV system	1
<i>x</i> location (m) in tests R01	15.50
<i>x</i> location (m) in tests R02	19.50

Table 6e *Horizontal distances of the LDV equipment to the wave maker in tests R01 and R02*

run number	vertical position of LDV 1 (cm)
r0106	2.0
r0112	35.0
r0203	2.5
r0208	20.0

Table 6f *Vertical distances of the LDV equipment to the bottom in tests R01 and R02*

run number	τ (s)
s0105	0.41
s0106	-0.03
s0107	0.51
s0108	0.10
s0109	0.28
s0110	-0.04
s0111	-0.27
s0112	-0.44
s0113	0.32
s0114	-0.48
s0115	-0.65
r0106	-0.07
r0112	0.30
r0203	-0.02
r0208	-0.81

Table 6g *Time shiftings with respect to the time origin of the computations for tests S01, R01 and R02.*

	through at $x = 15.50$ m			crest at $x = 15.50$ m		
z (m)	u_{meas} (cm/s)	u_{comp} (cm/s)	percent error (%)	u_{meas} (cm/s)	u_{comp} (cm/s)	percent error (%)
-55.6	-7.1	-8.3	-17	7.2	7.6	-6
-52.6	-7.3	-8.4	-15	7.0	7.7	-10
-47.6	-7.7	-8.5	-10	6.9	7.9	-15
-42.6	-7.7	-8.8	-14	7.7	8.3	-8
-37.6	-8.2	-9.2	-12	7.8	8.8	-14
-30.1	-8.8	-9.8	-11	8.3	9.7	-17
-22.6	-10.1	-10.8	-6	8.4	11.1	-32
-15.1	-11.6	-11.9	-3	9.6	12.8	-33
-7.6	-13.5	-13.3	1	10.4	14.7	-41

Table 6h *Percent error on velocity profiles at $x = 15.50$ m in tests S01*

	through at $x = 21.72$ m			crest at $x = 21.72$ m		
z (m)	u_{meas} (cm/s)	u_{comp} (cm/s)	percent error (%)	u_{meas} (cm/s)	u_{comp} (cm/s)	percent error (%)
-25.4	-13.1	-14.1	-8	16.2	15.3	6
-25.0	-13.5	-14.1	-4	16.2	15.3	6
-24.6	-13.4	-14.1	-5	16.2	15.4	5
-23.6	-13.1	-14.1	-8	15.5	15.4	1
-22.6	-14.1	-14.1	0	15.6	15.6	0
-20.6	-13.5	-14.2	-5	15.3	15.8	-3
-18.6	-14.2	-14.2	0	15.6	16.3	-5
-16.6	-14.3	-14.4	-1	16.3	16.8	-3
-14.6	-14.2	-14.5	-2	17.0	17.5	-3
-11.6	-14.2	-14.7	-3	17.1	18.7	-9

Table 6i *Percent error on velocity profiles at $x = 21.72$ m in tests S01*

Appendix C

Figures to Chapter 6

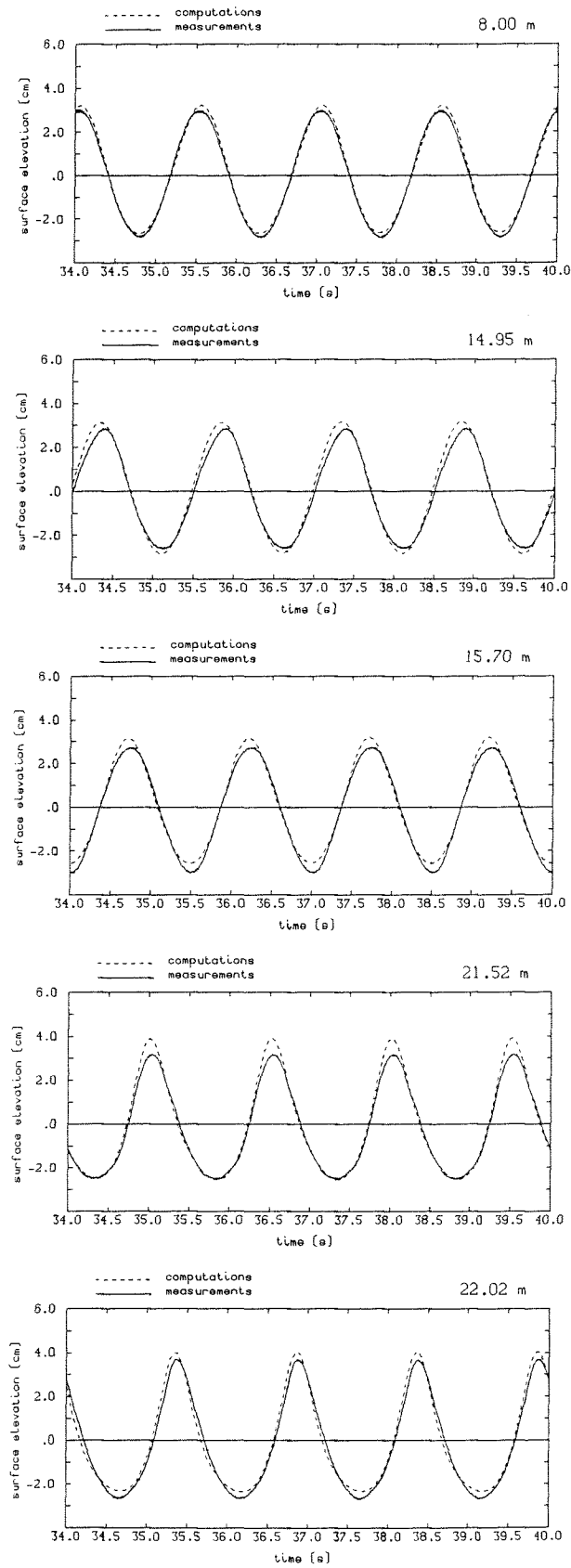


Figure 6a *Comparison of measured and computed surface elevations, condition S*

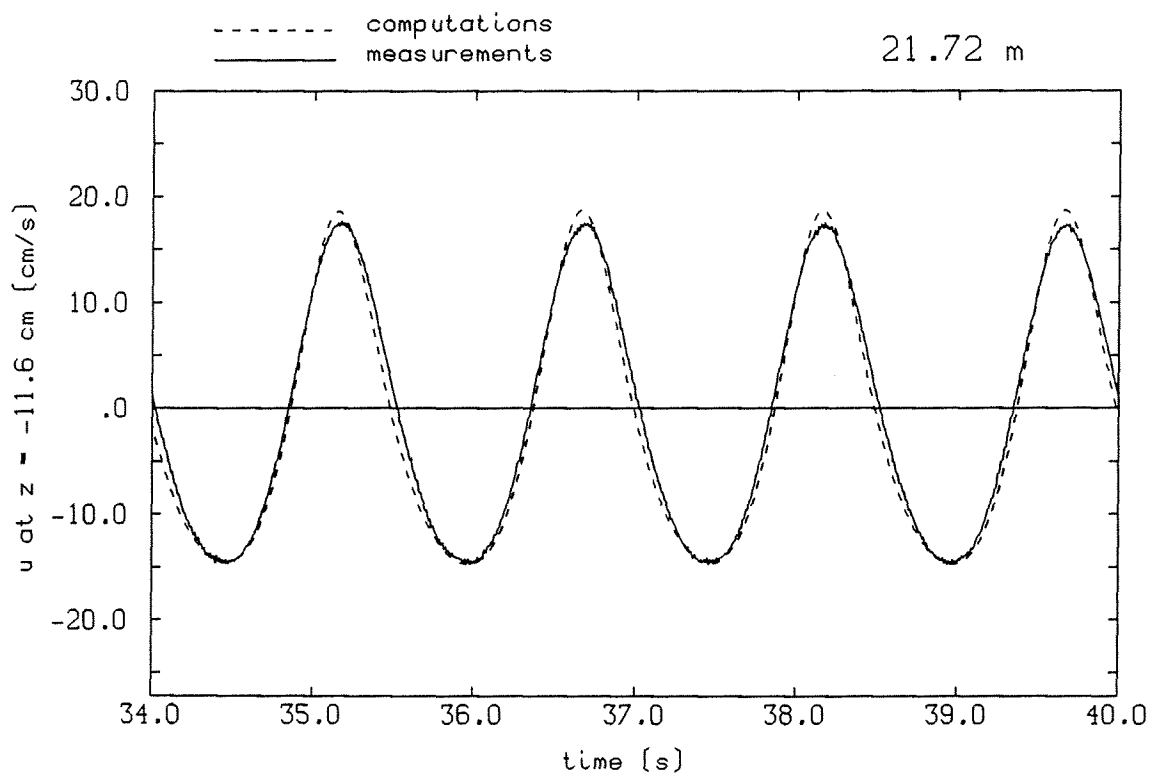
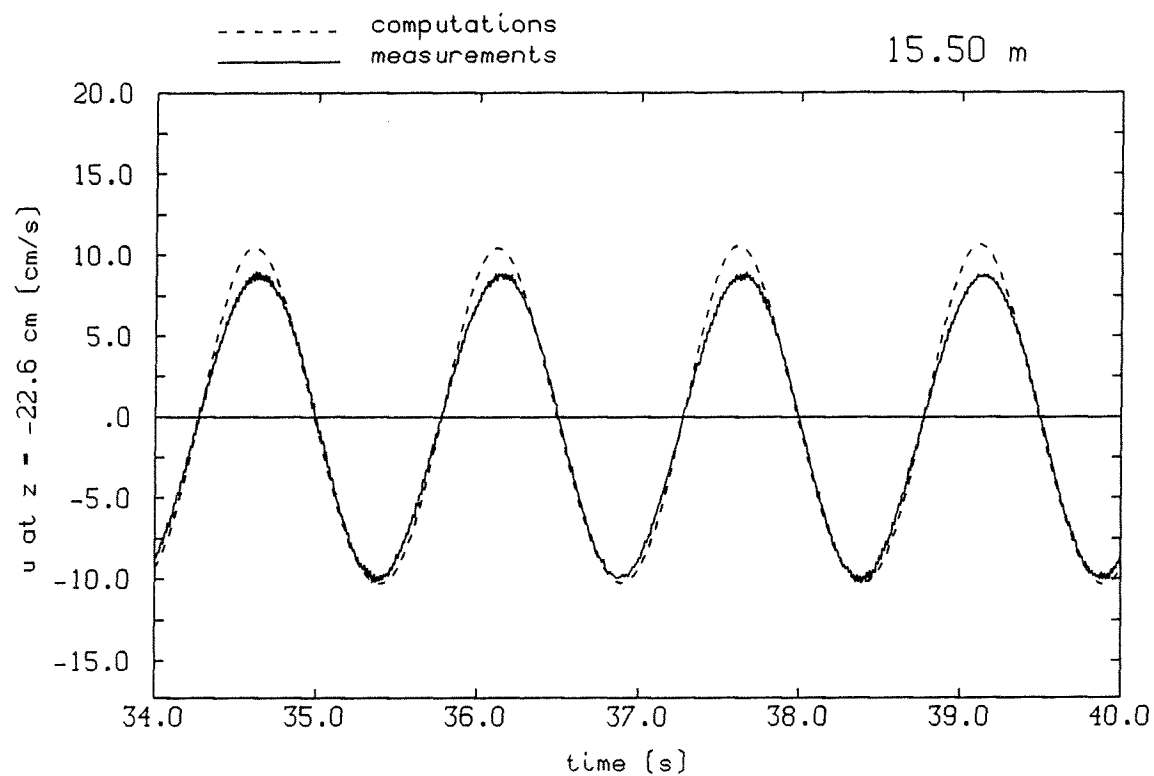


Figure 6b Comparison of measured and computed horizontal velocity at $x = 15.50$ m and 21.72 m respectively, with z such that a velocity close to the depth-averaged velocity is found, condition S

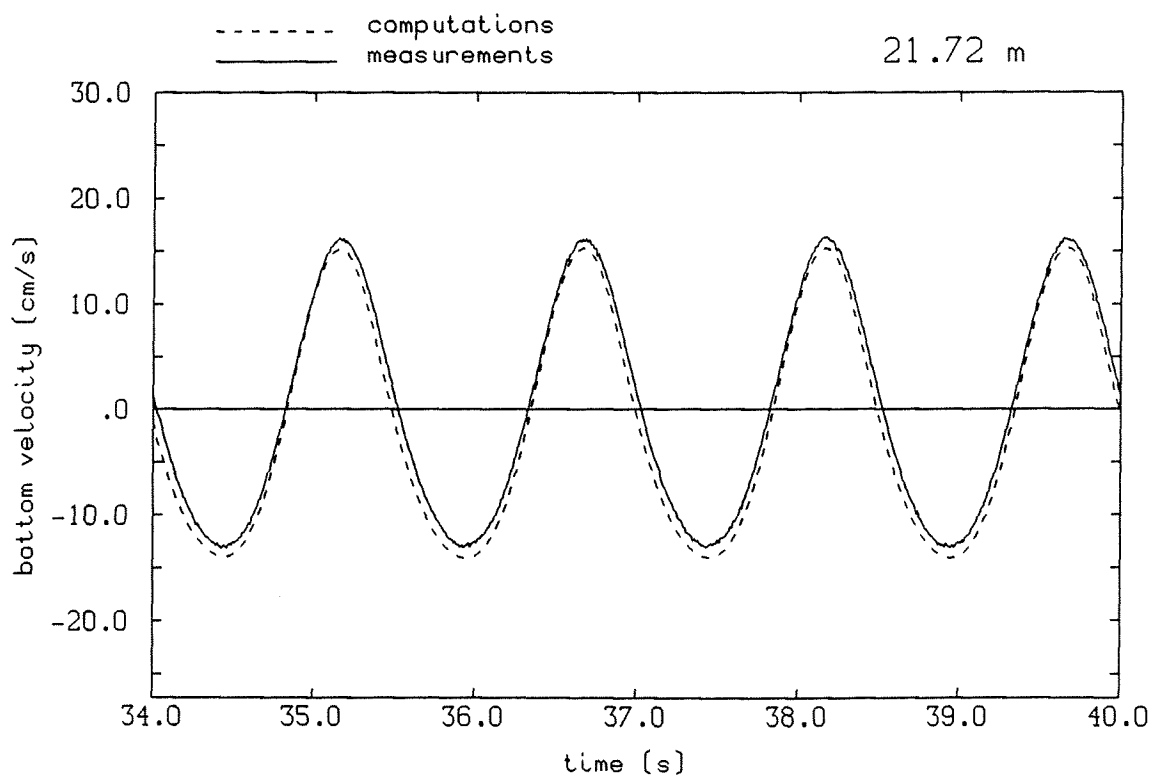
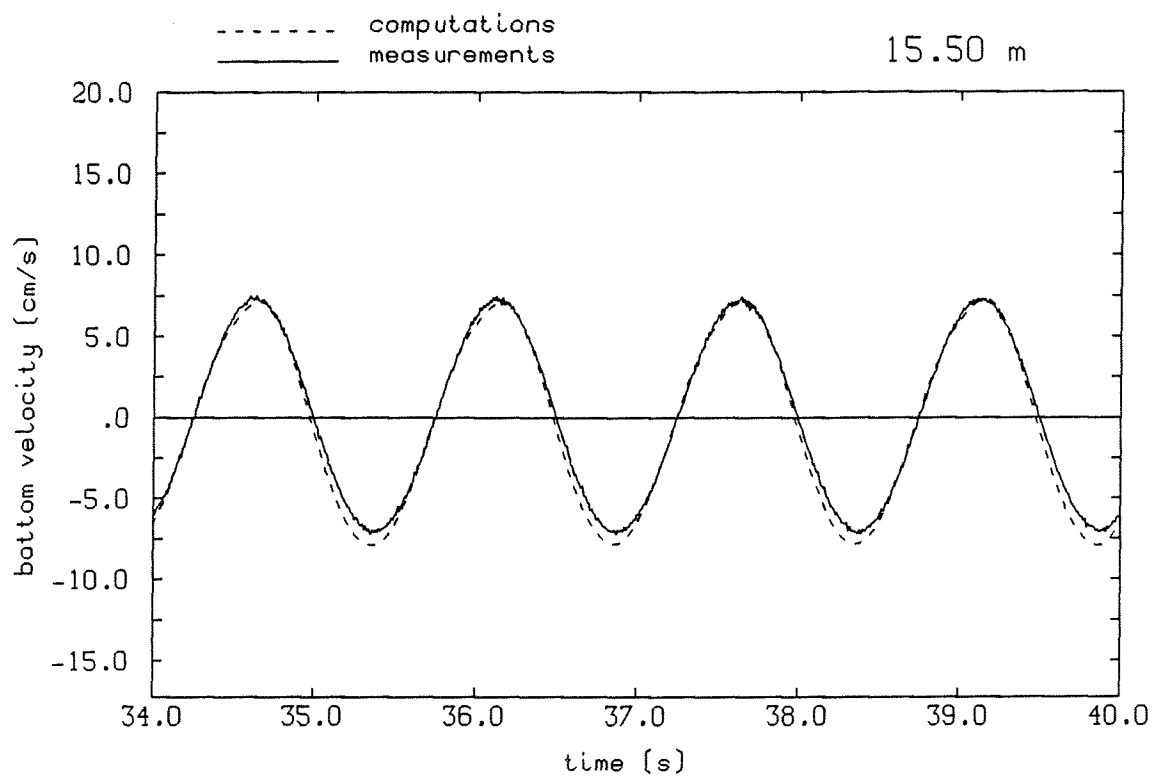


Figure 6c *Comparison of measured and computed bottom velocity at $x = 15.50$ m and 21.72 m respectively, condition S*

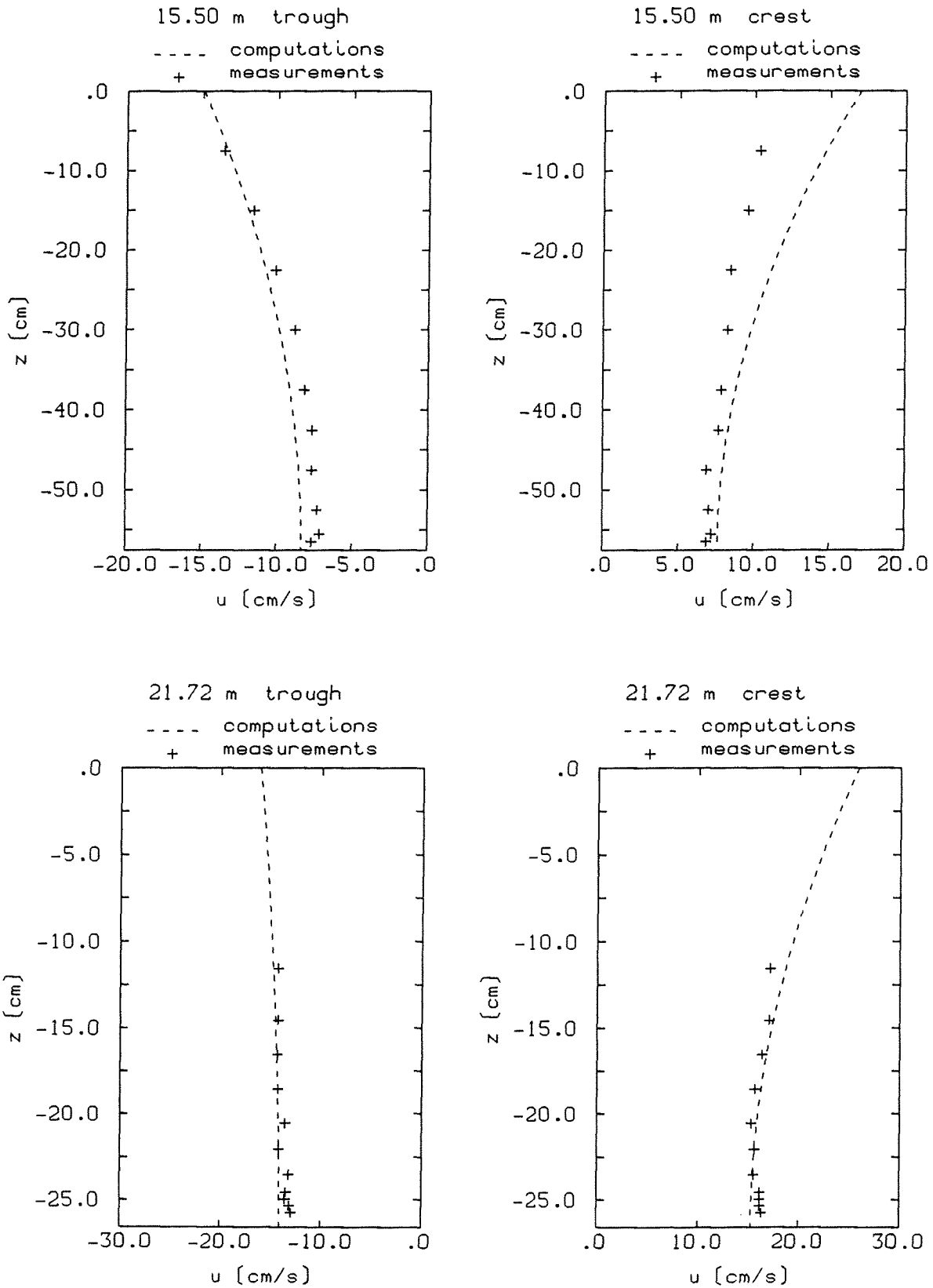


Figure 6d Comparison of measured and computed horizontal velocity profile at $x = 15.50$ m and 21.72 m respectively, for two different wave phases (corresponding to a crest and a trough in the computations at the both locations), condition S

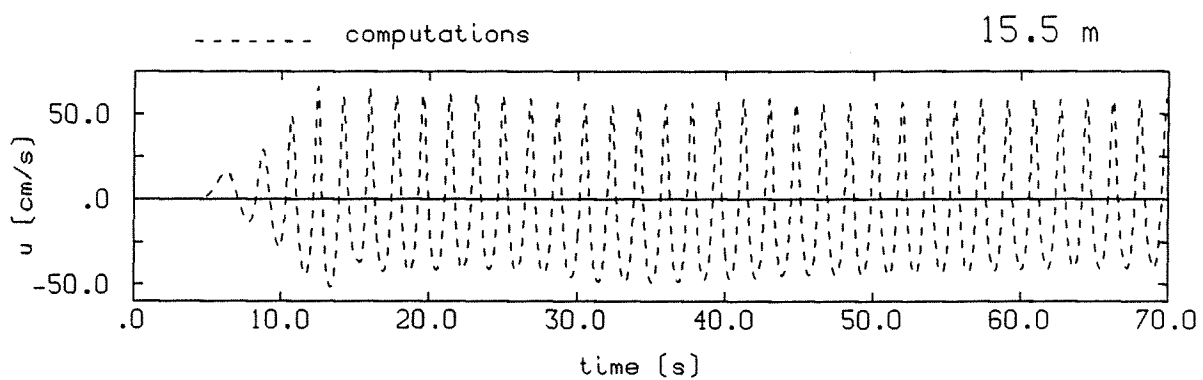
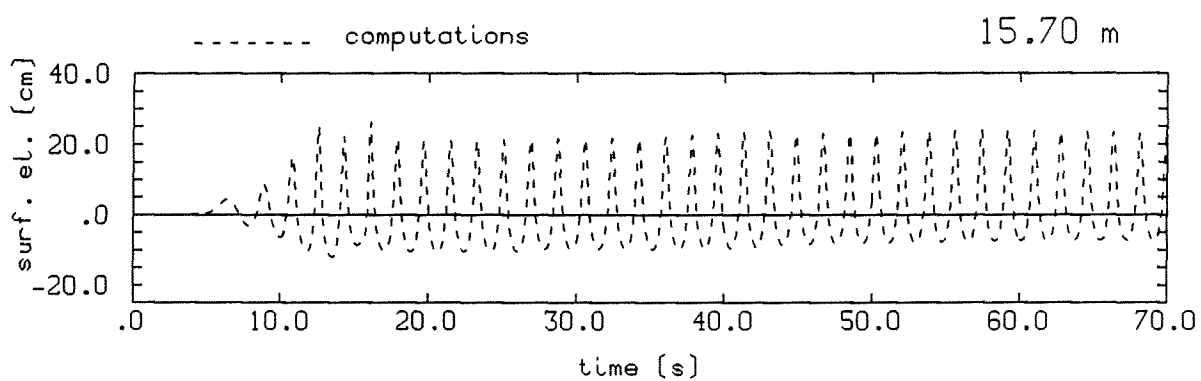
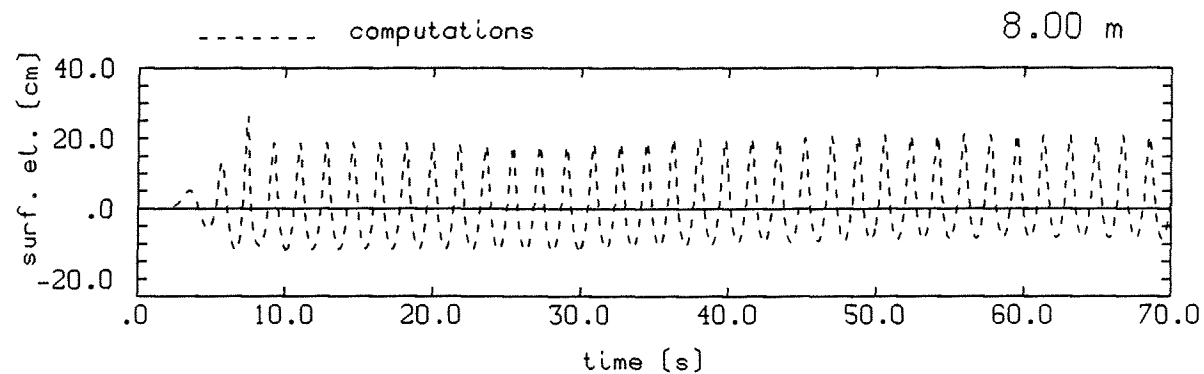


Figure 6e *Computed surface elevations at $x = 8.00$ m and 15.70 m and computational velocities at $x = 15.50$ m, time-window from 0 to 70 s, condition R*

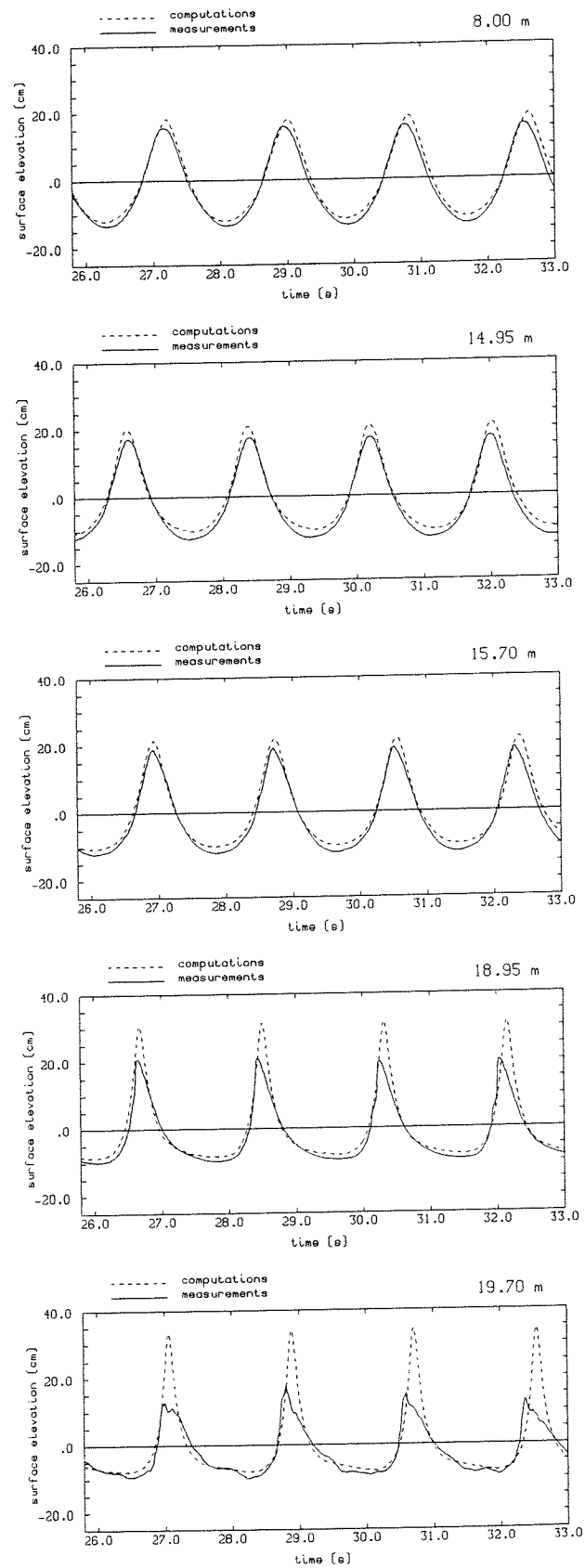


Figure 6f *Comparison of measured and computed surface elevations, condition R*

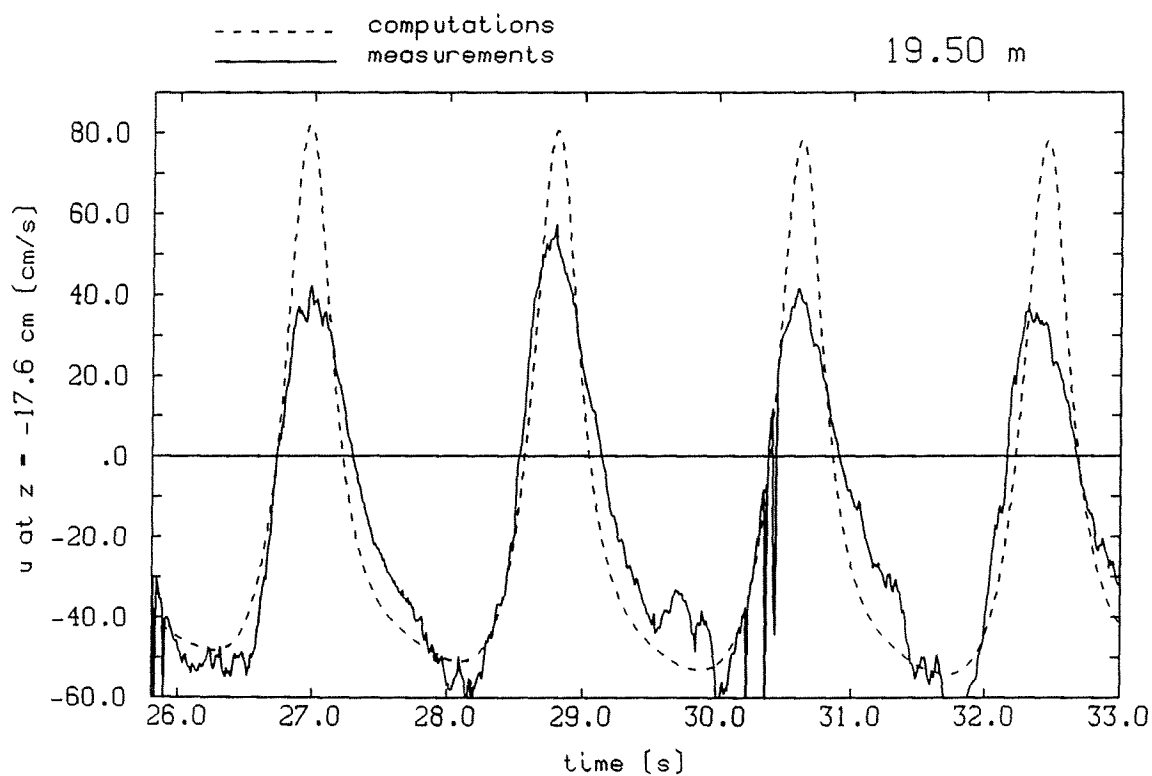
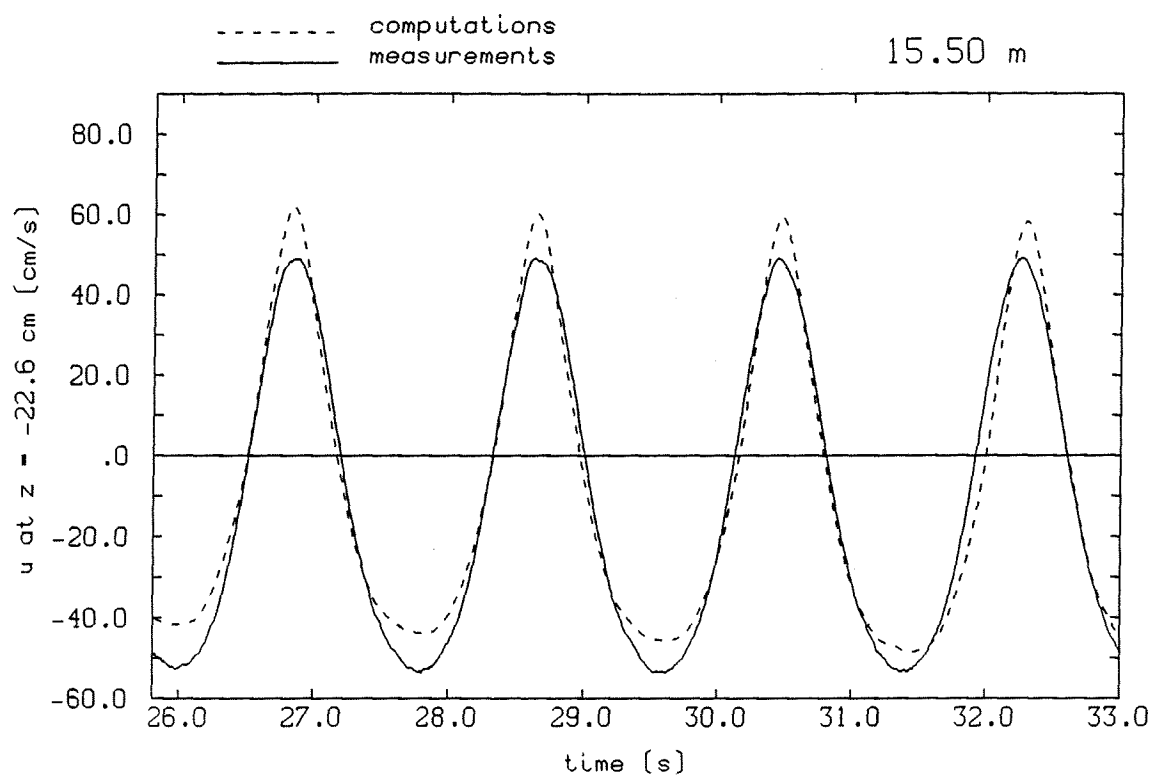


Figure 6g

Comparison of measured and computed horizontal velocity at $x = 15.50$ m and 21.72 m respectively, with z such that a velocity close to the depth-averaged velocity is found, condition R

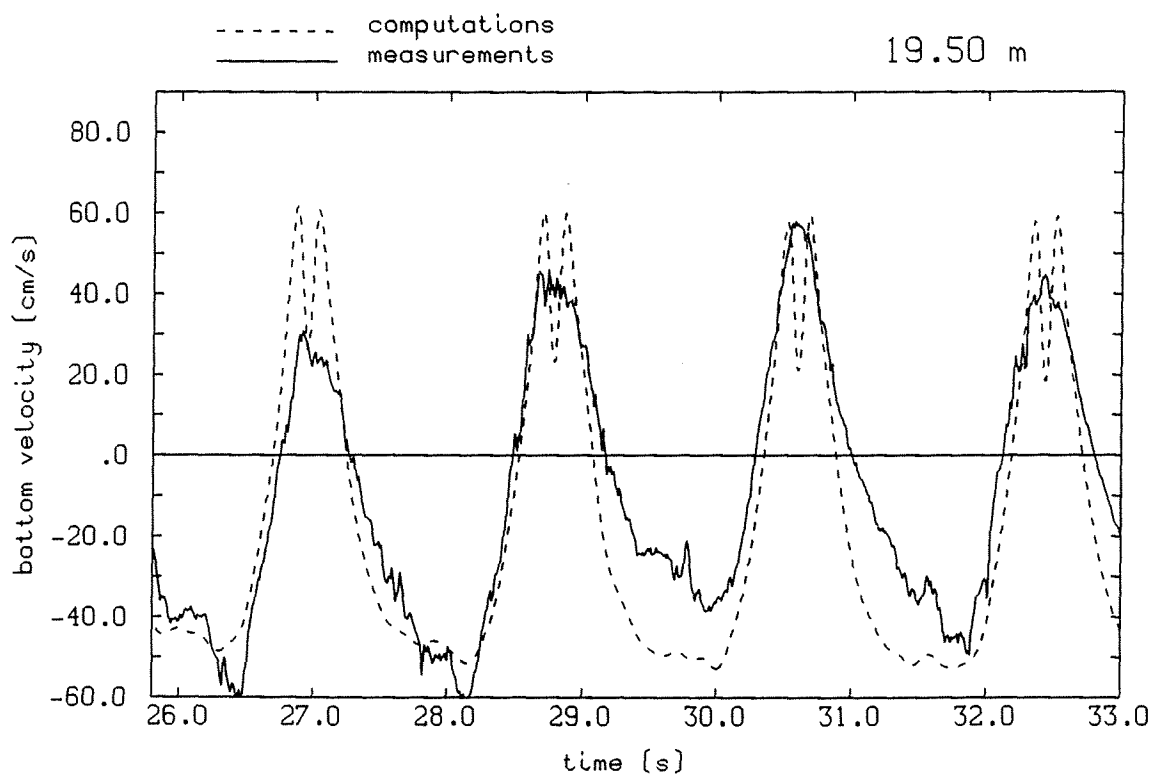
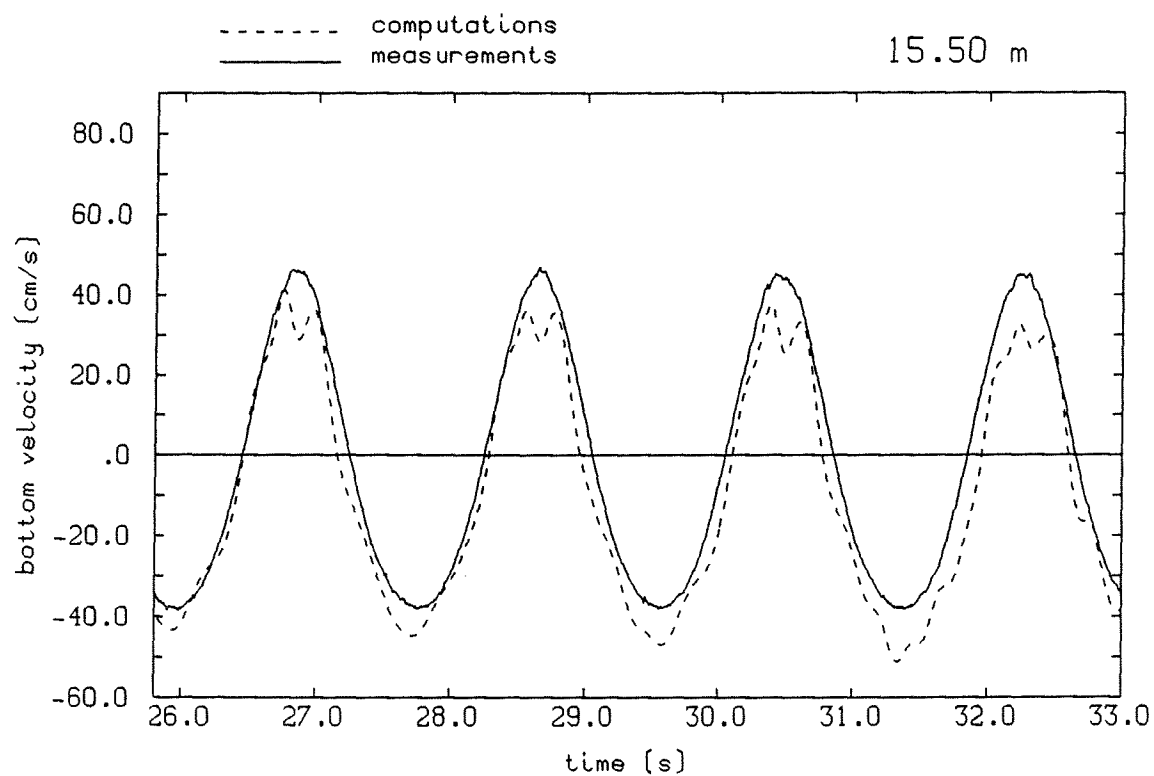


Figure 6h *Comparison of measured and computed bottom velocity at $x = 15.50$ m and 21.72 m respectively, condition R*

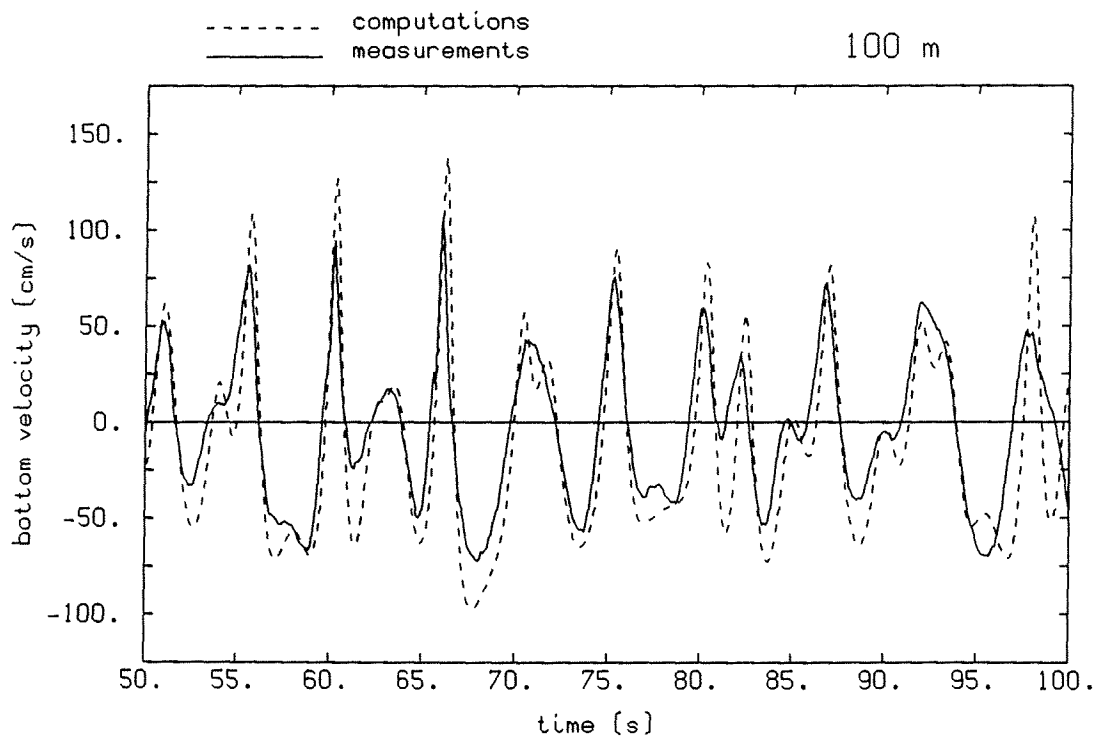
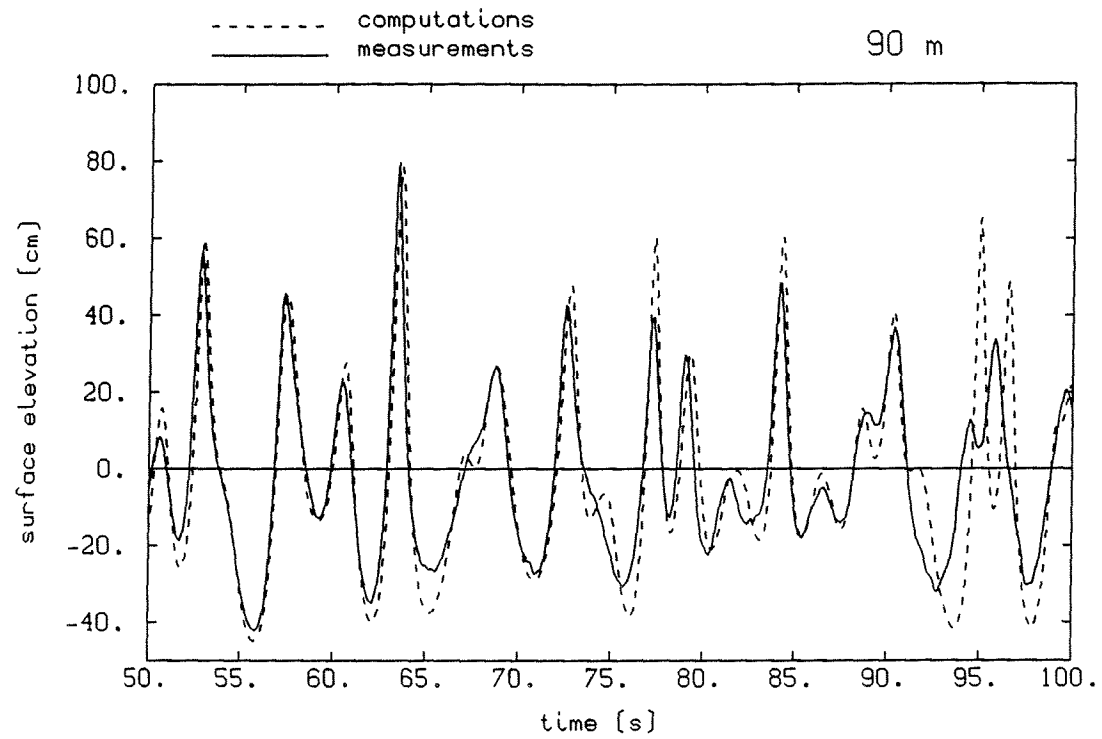


Figure 6i *Comparison of measured and computed time series of surface elevation and bottom velocity respectively, for test 1a, computations with DUT2 model*

Appendix D

Figures to Chapter 8

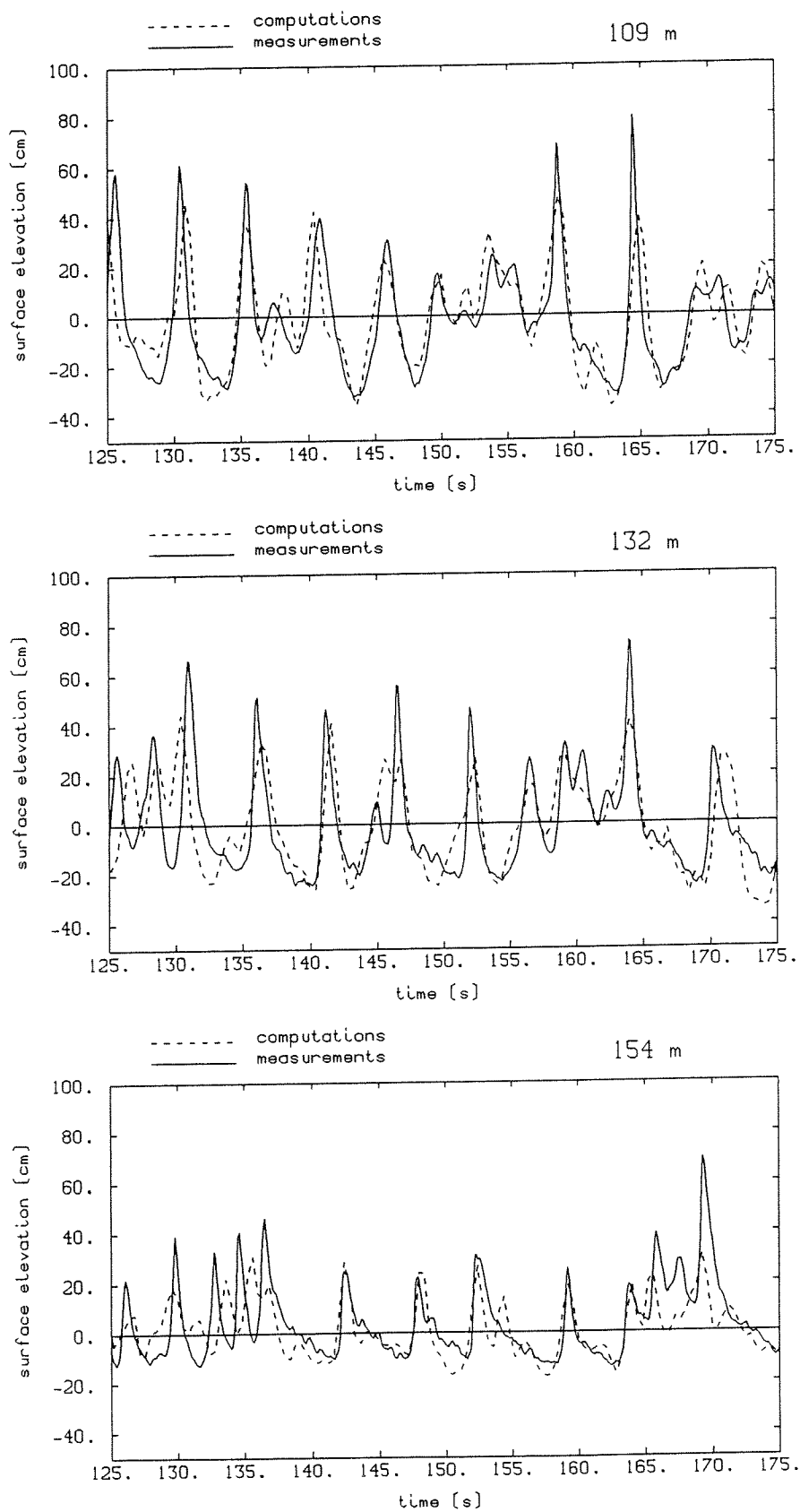


Figure 8a

Comparison of time series of measured and computed surface elevations for test 1a

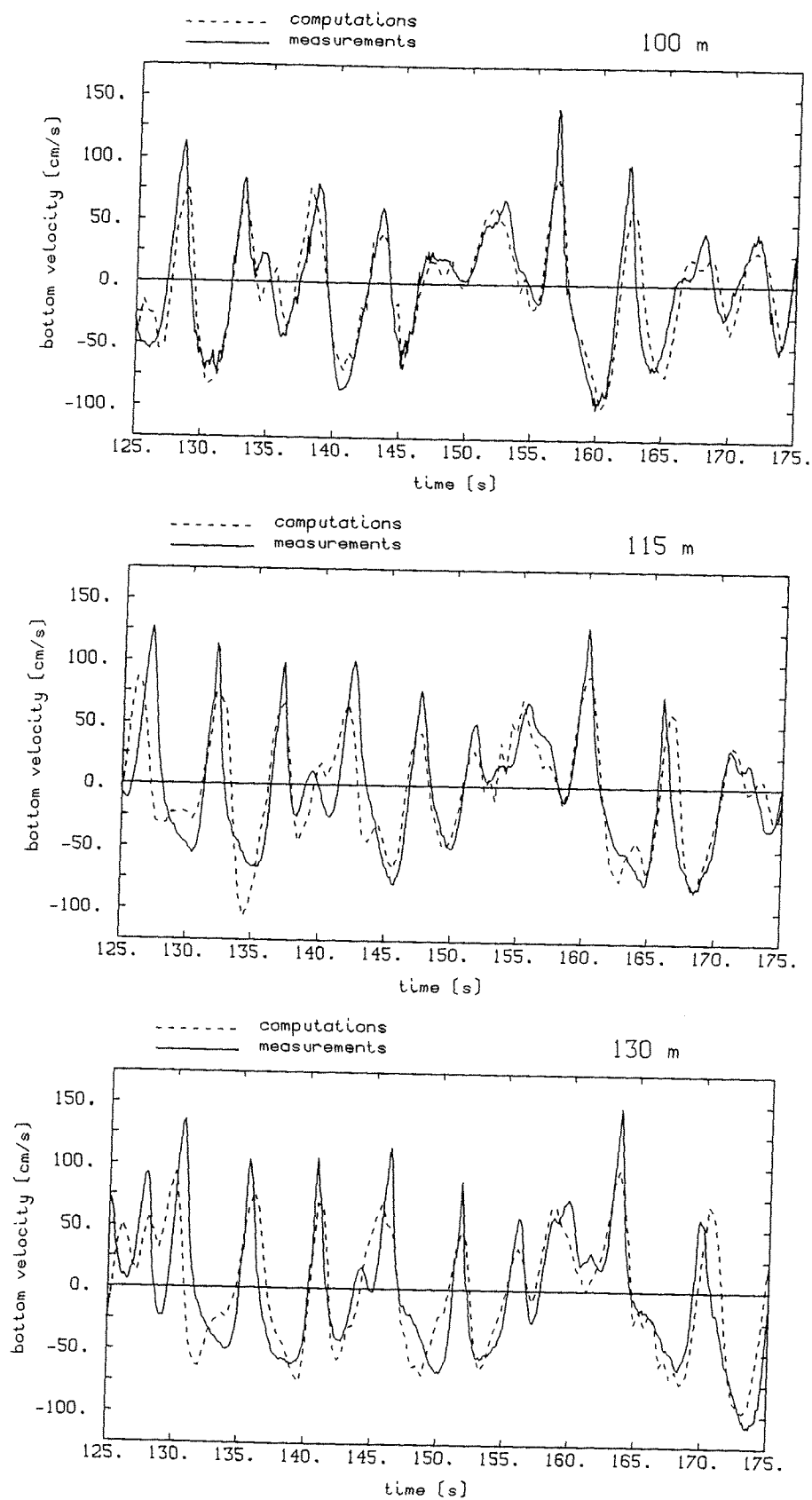


Figure 8b.1 *Comparison of time series of the measured and computed bottom velocity for test 1a; see also Figure 8b.2*

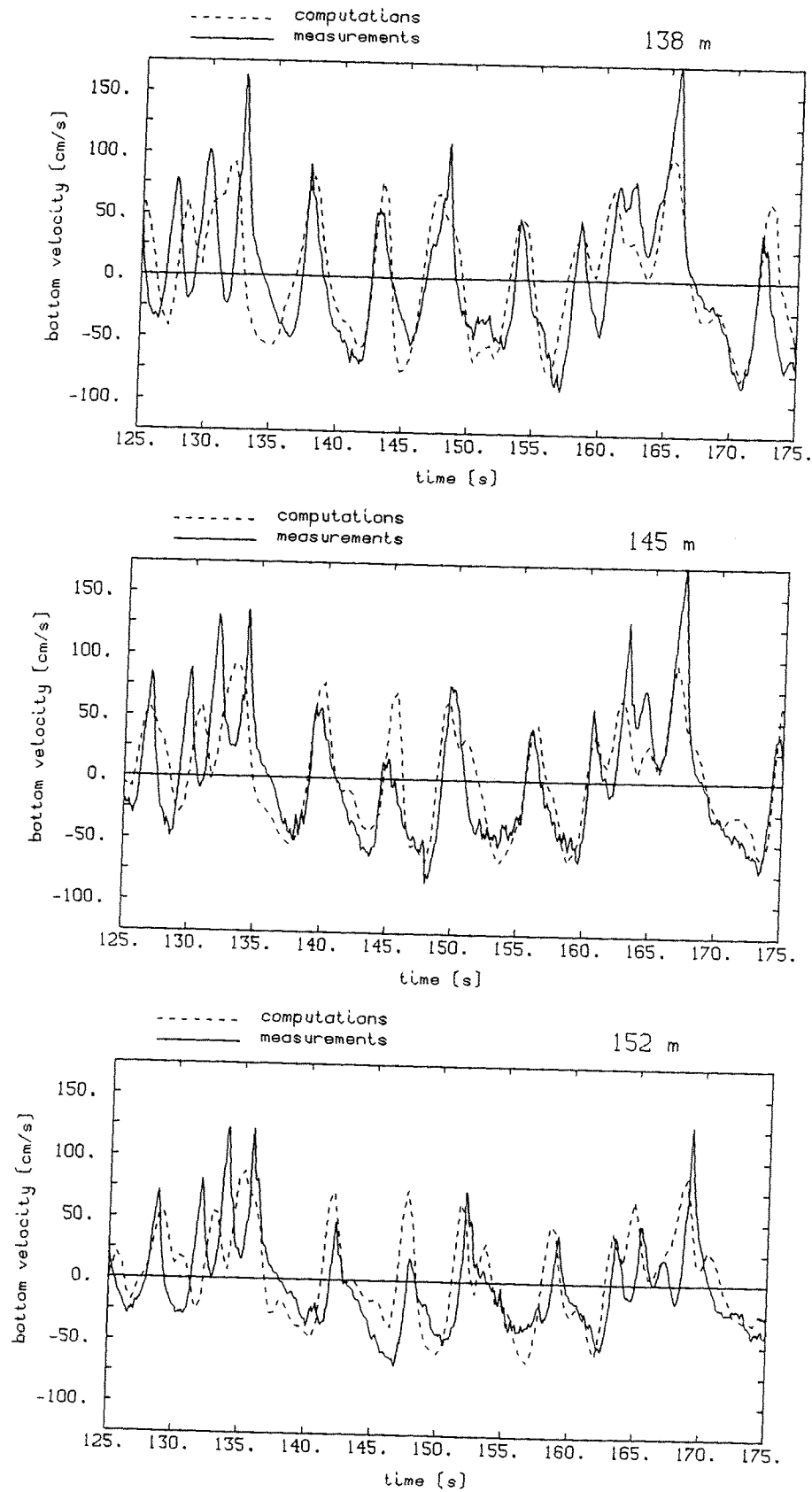


Figure 8b.2 Comparison of time series of the measured and computed bottom velocity for test 1a; see also Figure 8b.1

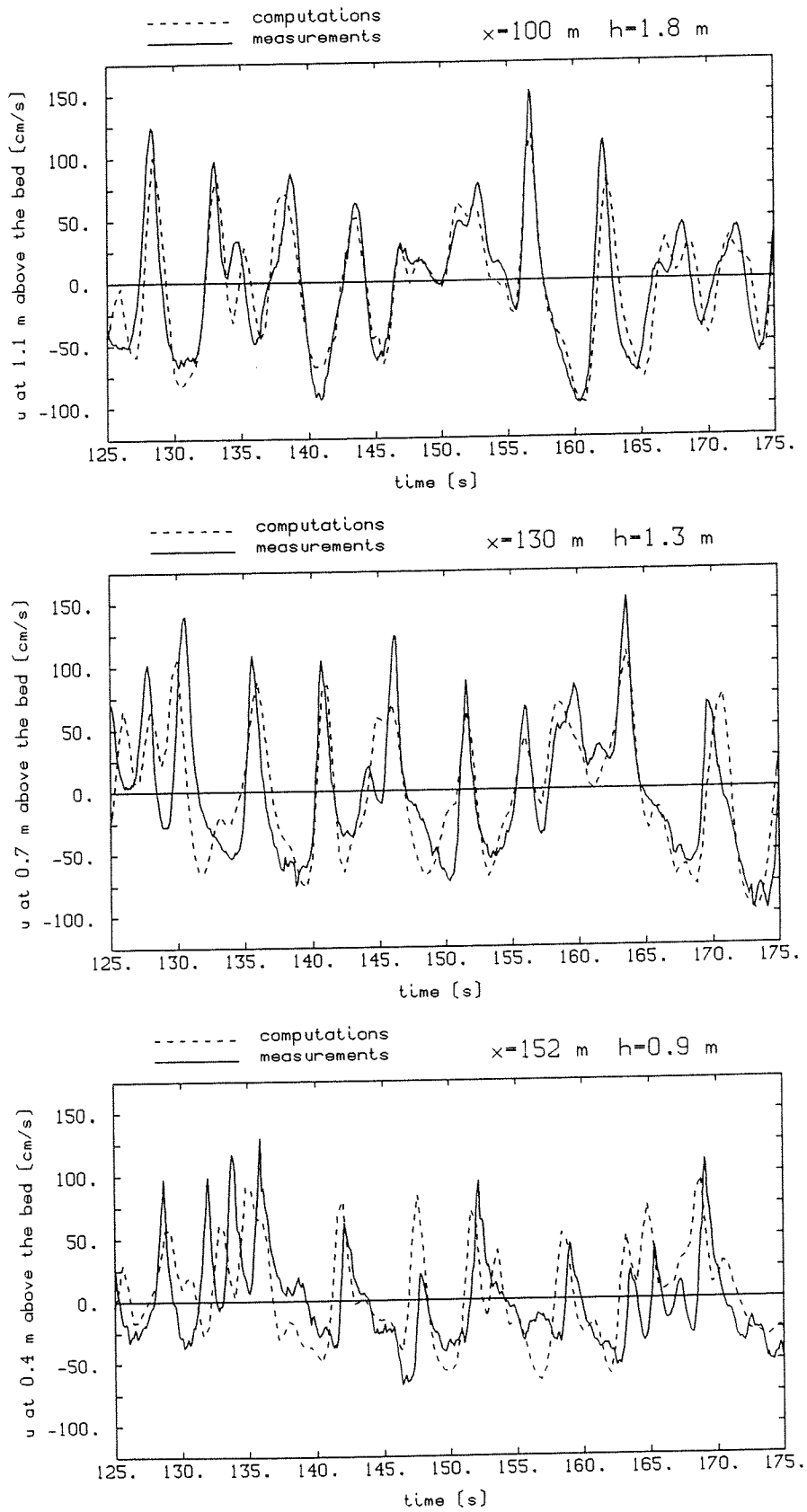


Figure 8c

Comparison of time series of the measured and computed velocity at some distance from the bed for test 1a

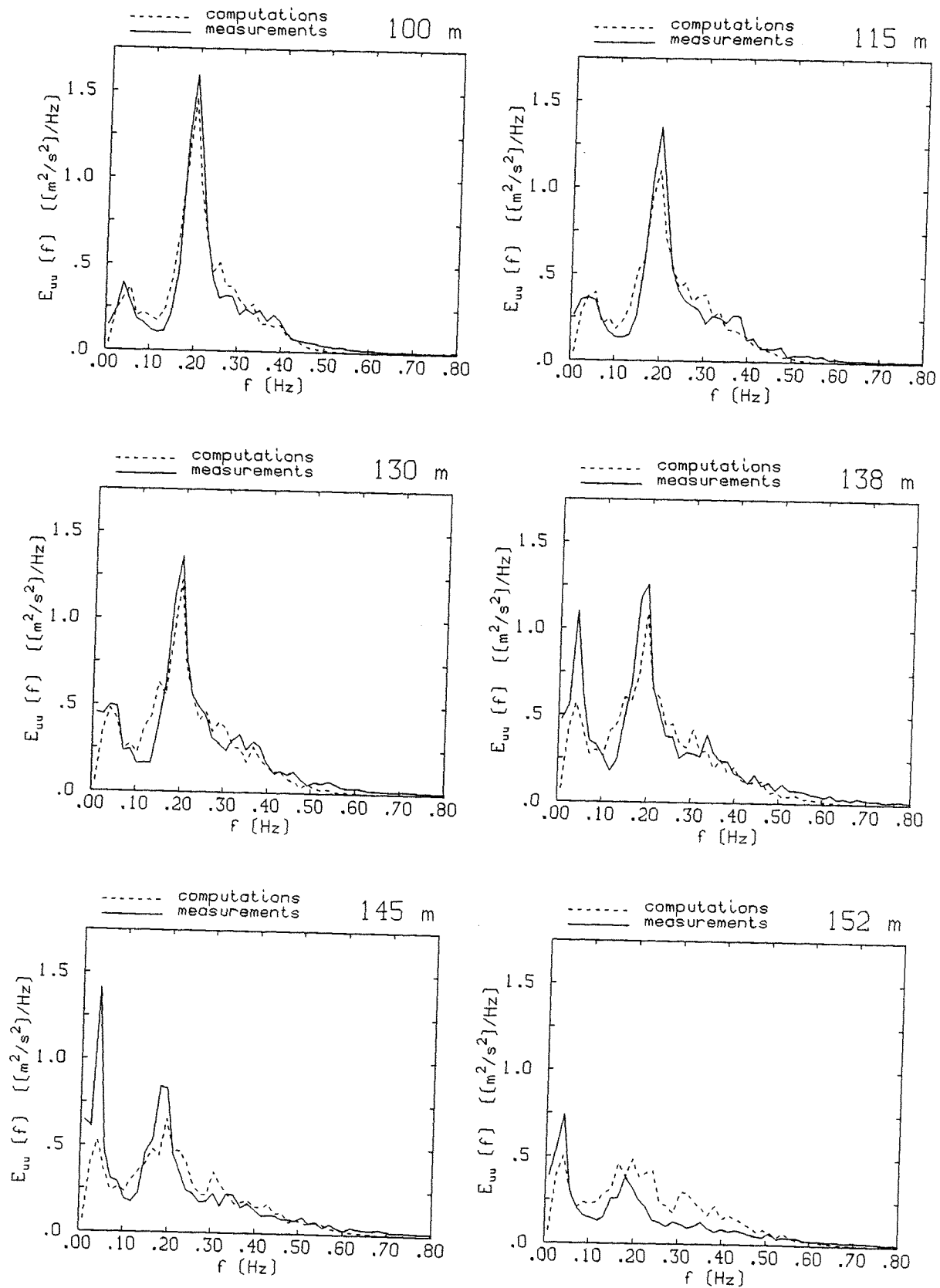


Figure 8d *Spectral density functions of the bottom velocity for test 1a*

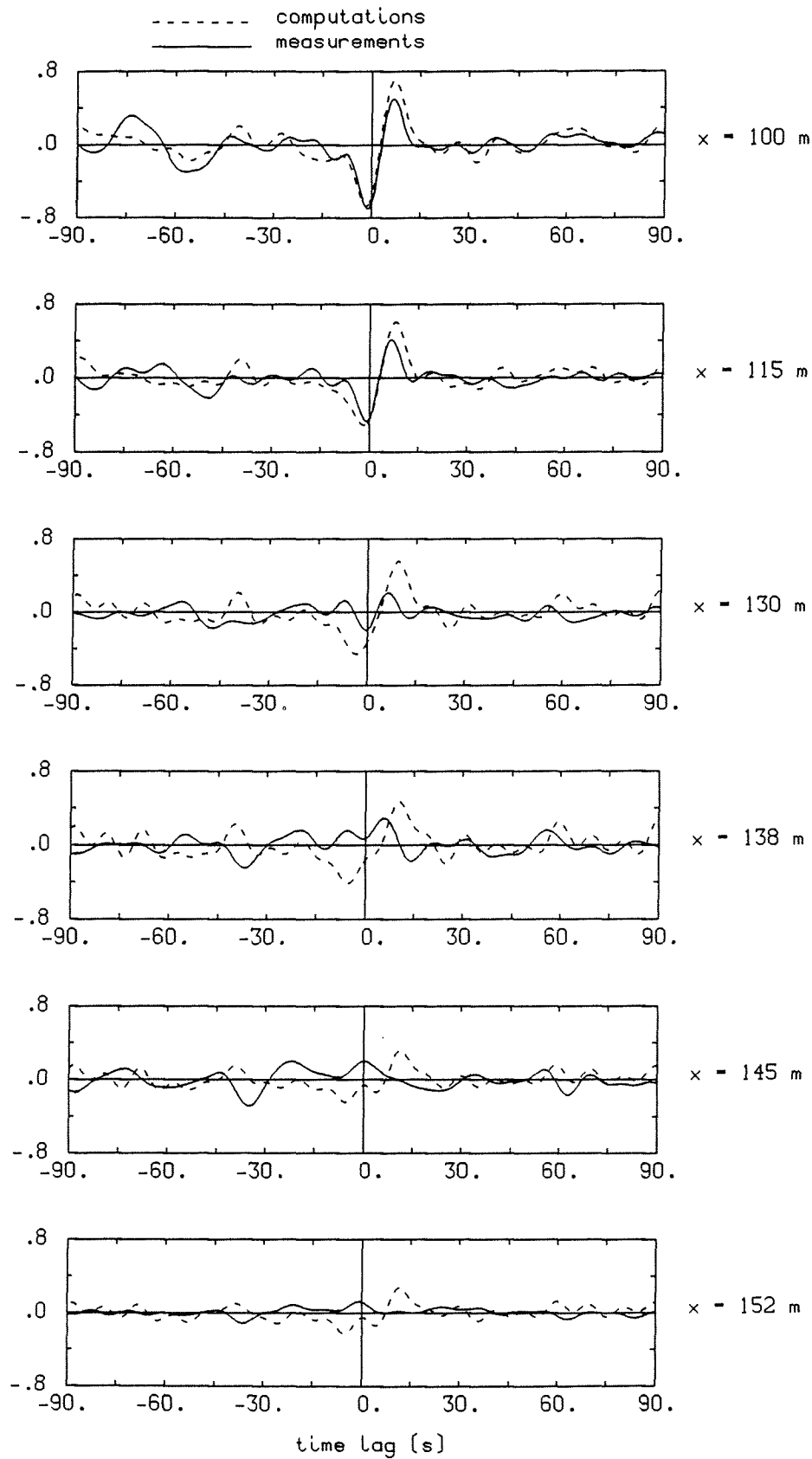


Figure 8e Covariance function ($10^{-2} \text{ m}^3/\text{s}^3$) of the squared short wave bottom velocity and the long wave bottom velocity component for test 1a

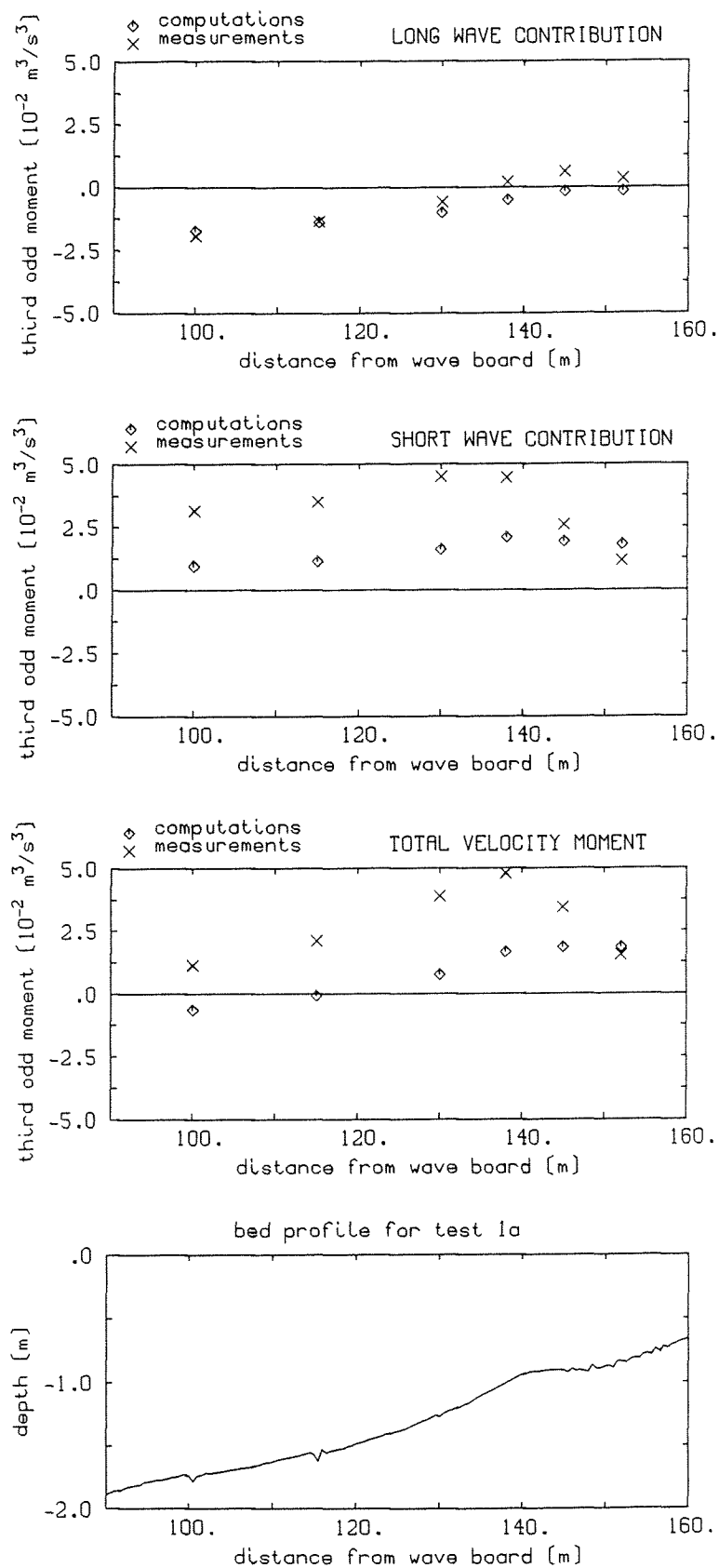


Figure 8f

Comparison of measured and computed third order odd bottom velocity moments: long wave contribution, short wave contribution and total moment respectively; test 1a

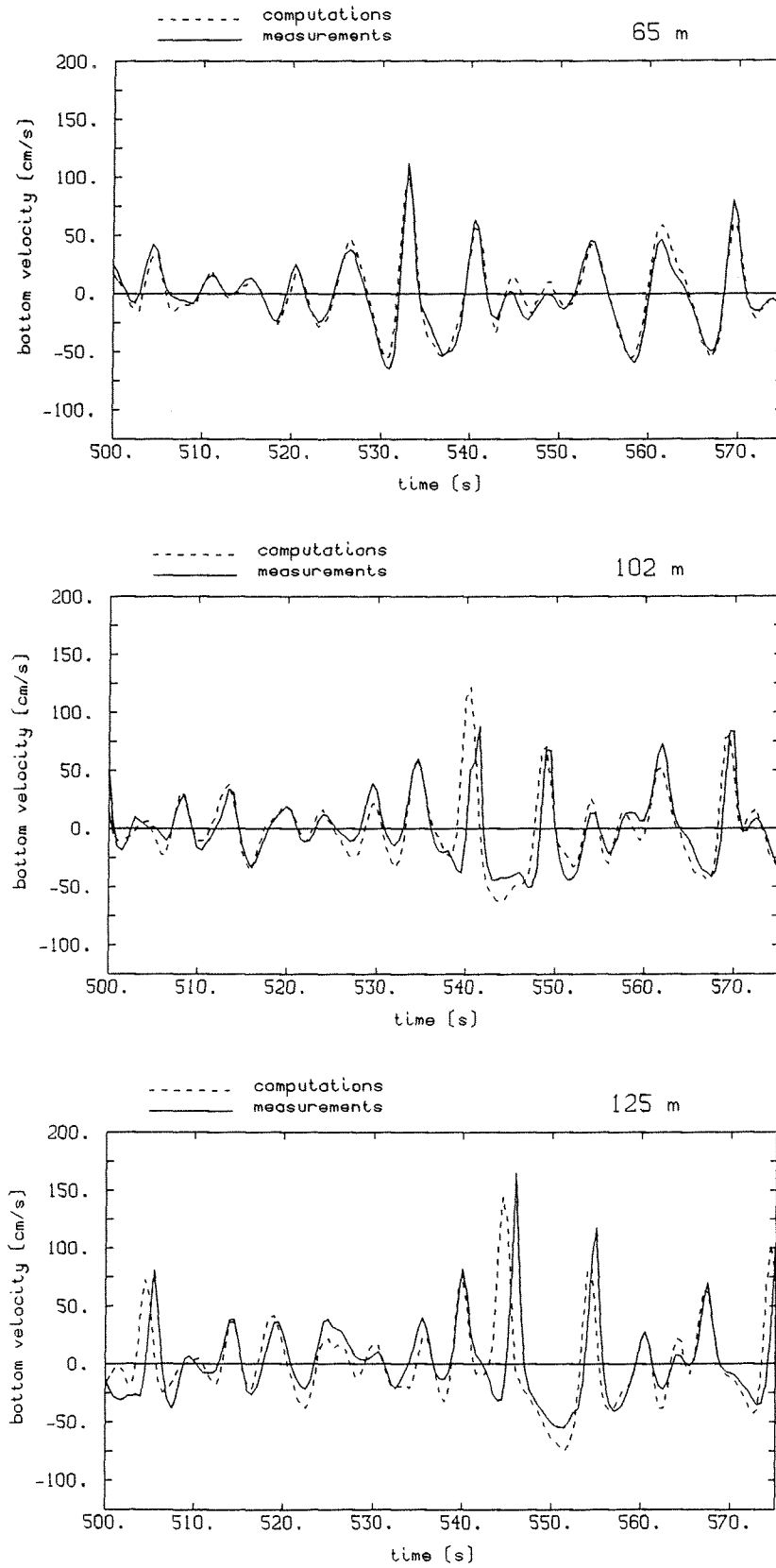


Figure 8g.1 *Comparison of time series of the measured and computed bottom velocity for test 1c; see also Figure 8g.2*

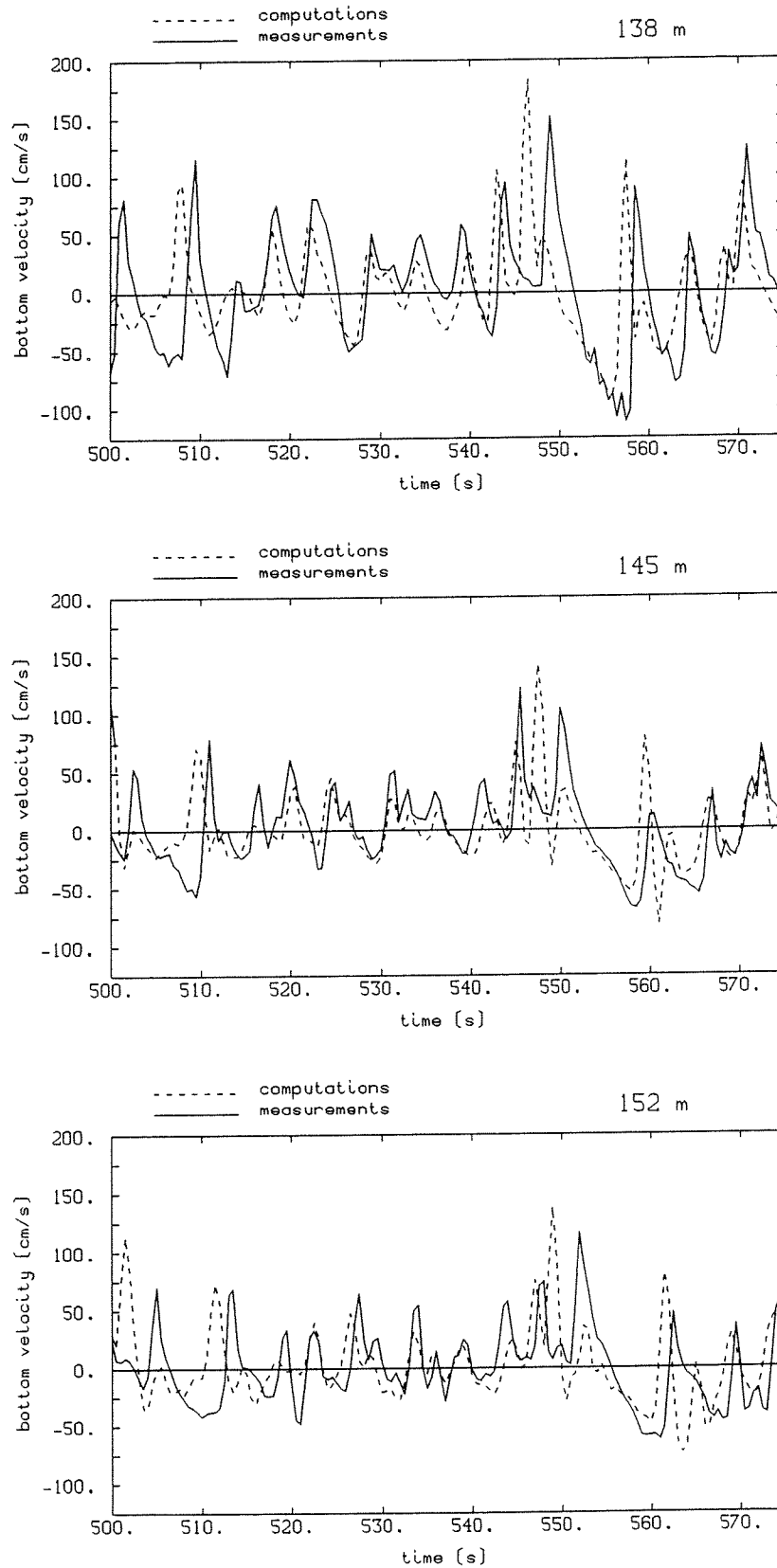


Figure 8g.2 Comparison of time series of the measured and computed bottom velocity for test 1c; see also Figure 8g.1

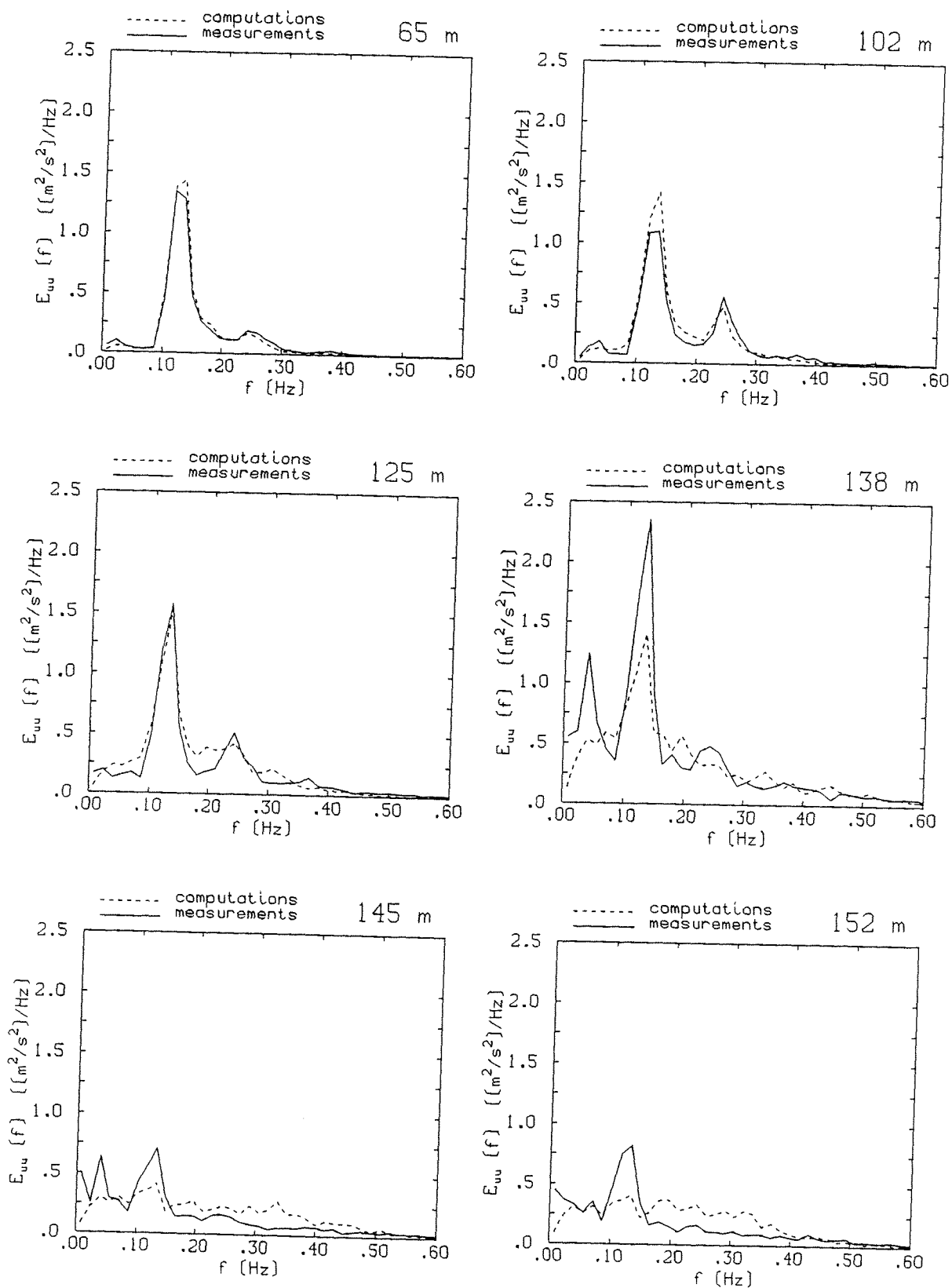


Figure 8h *Spectral density functions of the bottom velocity for test 1c*

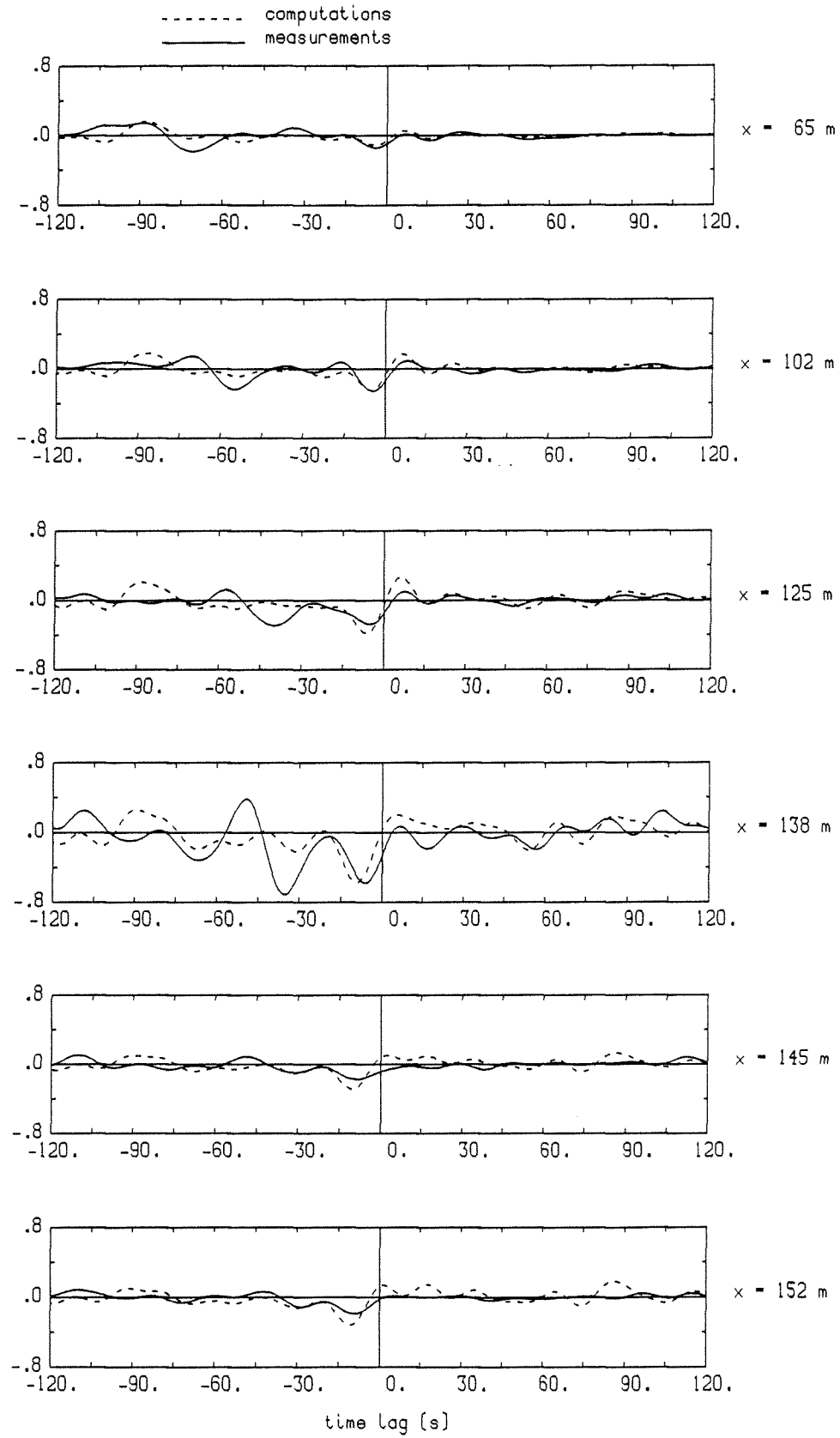


Figure 8i

Covariance function ($10^{-2} \text{ m}^3/\text{s}^2$) of the squared short wave bottom velocity and the long wave bottom velocity component for test 1c

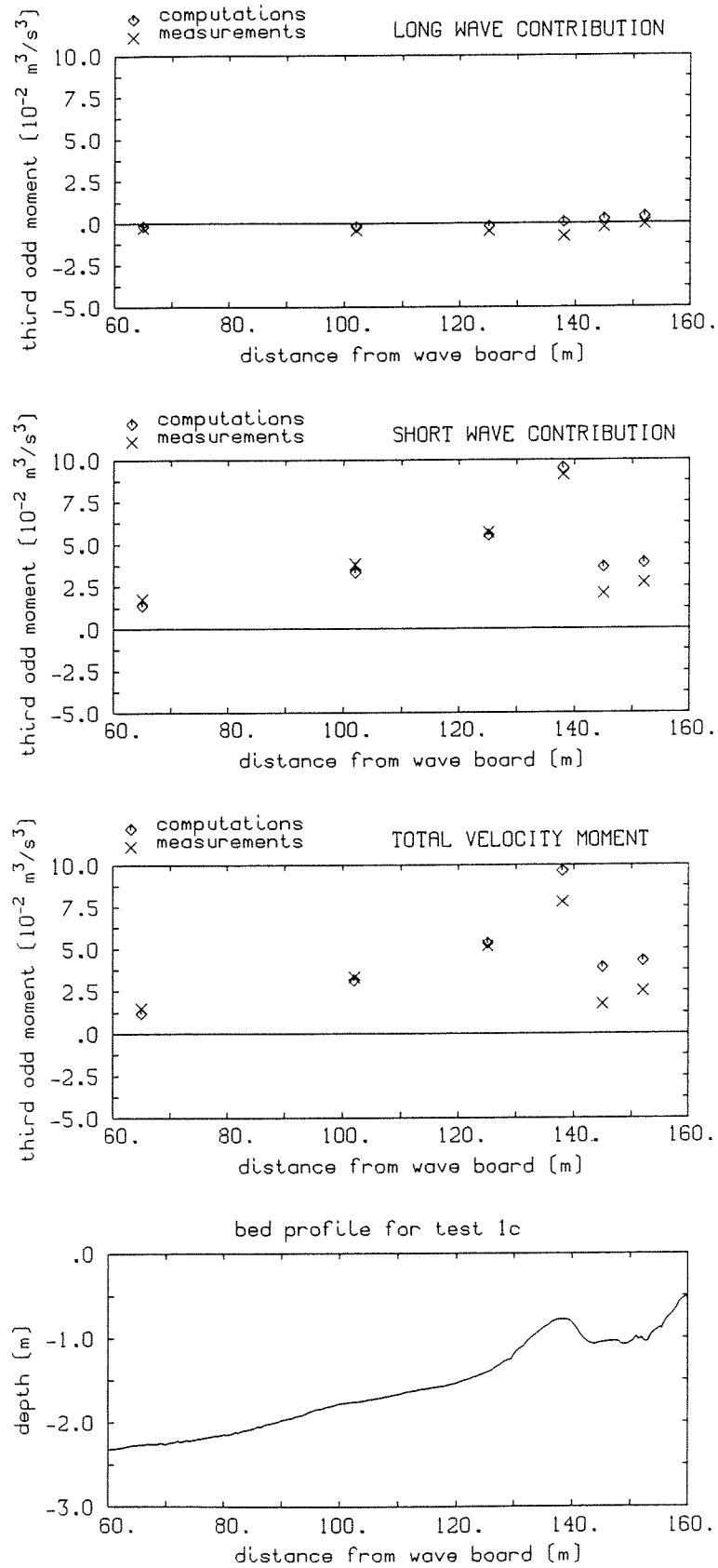


Figure 8j

Comparison of measured and computed third order odd bottom velocity moments: long wave and short wave contribution and total moment respectively; test 1c