THE MAXIMUM BENDING MOMENT RESISTANCE OF PLATE GIRDERS

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INTRODUCTION

In many steel structures like buildings, industrial halls and bridges, standard hot-rolled sections like IPE, HEA, HEB, HEM, HED and UNP in Europe and similar profiles in other regions of the world are used. The range of hot-rolled sections is limited and therefore fabricated plate girders are used when the requirements for stiffness, strength, stability and economy are not fulfilled.

Such a plate girder is built up with steel plates for the top and bottom flange and for the web, welded together to an I-shape cross section, single or double symmetric. Using this type of plate girders, a high degree of optimization of material use is possible by using different plate thicknesses and widths for the flanges and thickness and height for the web over the span of the girder adapted to the distribution of bending moments and the shear forces.

Optimizations can be carried out for many aspects, but in this paper the bending moment resistance of a plate girder, given a certain weight per unit length, is the main topic for optimization. For a long time, this was not or hardly of interest in Western countries at all, especially because the costs of structures were mainly determined by labour cost and hardly by material cost. For nowadays structures life cycle costs and the environmental impact of structures become of more influence on the design, next to the increasing cost of steel by expanding demand by booming economies like China, Brazil, India and other upcoming economies. So, optimization for minimal use of materials has become highly important.
Using higher steel grades, applying most material in the flanges and increasing the lever arm between both flanges are the main possibilities to maximize the bending moment resistance of a plate girder under pure bending of a certain amount of steel. In many cases when using hot-rolled sections the deflection is the decisive design criterion and therefore using high steel grades seems not useful. For plate girders with a very slender web the stiffness is not decisive and the strengths of the material can be better exploited, so using higher steel grades can be very useful.

1 BENDING MOMENT RESISTANCE

1.1 General

In case of a fixed cross-sectional area more material is placed in the web by increasing the lever arm, reducing the left-over material for the flanges. The lever arm can also be increased by increasing the web height and decreasing the web thickness. This process is restricted by ending up with a practical web thickness, to make welding of the section possible and also the handling of the plate girder.
The slenderness of the flanges, expressed in the width to thickness ratio b/t_f of the flange plate, is restricted such that at least yielding of the outer fibre is possible to ensure that the plate girder exhibits at least a “not brittle like” post critical behaviour. In terms of cross-sectional classification, as given in EN1993-1-5 [1] based on local instability of the elements, the compressed flange has to be at least not slender (not class 4).
The slenderness of the web, expressed in the height to thickness ratio $\beta_w = h_w/t_w$, is not restricted expressed in this classification. Reaching the elastic critical plate buckling stress in the web is hard-

ly of influence on the bending behaviour of the plate girder. The post-critical plate buckling behaviour of the web plate can easily be exploited using the effective width theory.
However, EN1993-1-5 restricts the web slenderness $\beta_w \leq \beta_{w.\max}$ by a specific phenomenon, called "flange induced buckling". Basler and Thürlimann [2] described this phenomenon as "vertical buckling of the compressed flange into the web", based on the result of only one laboratory test. Several equations are derived, assuming different minimum values for the ratio of area between the web area and the flange area, $\rho = A_w / A_f$ and for the residual stress σ_r in this compressed flange. Theses equations are presented in Chapter 1.2.
The bending moment resistance of a plate girder with a slender web is studied by several researchers. Some of these researchers determine the bending moment resistance M_u of a plate girder with a slender web by multiplying the elastic bending moment resistance M_{el} by a reduction factor ξ. This reduction factor ξ according to Basler and Thürlimann [2] is described in Chapter 1.3 and according to Veljkovic and Johansson [3] in Chapter 1.5, based on the effective width theory.
Herzog [4] determined the bending moment resistance M_u based on the reduction of the stress in the compressed part of the web. This is presented in Chapter 1.4.
Abspoel [5] determines the bending moment resistance M_u based on an effective web as well for the compressive part as for tensile part of the web, see Chapter 1.6.

1.2 Vertical buckling of the compressive flange into the web

The model to determine the maximum web slenderness is based on column buckling of the web by compressive stresses due to curvature of the plate girder. The general equation is shown in *Eq. (1).*

$$\beta_{w.\max} = \sqrt{\frac{\pi^2 E}{24 \cdot \left(1-\upsilon^2\right)} \cdot \frac{A_w}{A_{tf}} \cdot \frac{1}{f_{y.tf} \cdot \varepsilon_{tf}}} \tag{1}$$

Herein $f_{y.tf}$ is the yield stress of the top flange (compressed) and ε_{tf} is the strain in this flange. The strain of the flange is assumed to be larger than the yield strain, $\varepsilon_{tf} \geq \varepsilon_{y.tf} = \frac{f_{y.tf}}{E}$, to guarantee that the flange fully yields to obtain some deformation capacity. To be sure $\varepsilon_{tf} \geq \varepsilon_{y.tf}$, the strain ε_{tf} of the top flange is taken equal to $\varepsilon_{tf} = \frac{f_{y.tf} + \sigma_r}{E}$, wherein σ_r is the residual stress in the compressed flange. Substitution of this strain into *Eq.(1)* results into *Eq.(2).*

$$\beta_{w.\max.I} = \sqrt{\frac{\pi^2 E}{24 \cdot \left(1-\upsilon^2\right)} \cdot \frac{A_w}{A_{tf}} \cdot \frac{1}{f_{y.tf} \cdot \frac{f_{y.tf} + \sigma_r}{E}}} = 0.67 \cdot \sqrt{\frac{A_w}{A_{tf}}} \sqrt{\frac{E^2}{f_{y.tf} \cdot \left(f_{y.tf} + \sigma_r\right)}} \tag{2}$$

Basler and Thürlimann [2] mentioned that the ratio of area ρ will not be taken smaller than 0.5. Substitution of the ratio of area $\rho = 0.5$ in *Eq.(2)*, gives for the maximum web slenderness $\beta_{w.\max}$:

$$\beta_{w.\max.II} = \frac{0.48E}{\sqrt{f_{y.tf} \cdot \left(f_{y.tf} + \sigma_r\right)}} \tag{3}$$

A second simplification of *Eq.(1)* is found by assuming a residual stress level of $\sigma_r = \frac{f_{y.tf}}{2}$ for mild steel and so the maximum web slenderness $\beta_{w.\max}$ becomes:

$$\beta_{w.\max.III} = \sqrt{\frac{\pi^2}{36 \cdot \left(1-\upsilon^2\right)}} \cdot \frac{E}{f_{y.tf}} \cdot \sqrt{\frac{A_w}{A_f}} = 0.55 \cdot \frac{E}{f_{y.tf}} \cdot \sqrt{\frac{A_w}{A_f}} \tag{4}$$

In EN1993-1-5 [1] this *Eq.(4)* is adopted. A third simplification of the maximum web slenderness is given by taken into account a ratio of area $\rho = 0.5$ as well as a residual stress level of $\sigma_r = \frac{f_{y.tf}}{2}$:

$$\beta_{w.\max.IV} = \frac{0.40E}{f_{y.tf}} \tag{5}$$

For example: the maximum web slenderness is $\beta_{w.\max} = 360$ for mild steel S235.

1.3 Bending moment resistance according to Basler and Thürlimann [2]

Basler and Thürlimann assumed an effective width of $30t_w$ for the part of the web under compression as shown in *Fig. 1a)* for a web with a web slenderness $\beta_w = h_w/t_w = 360$ for steel grade S235.

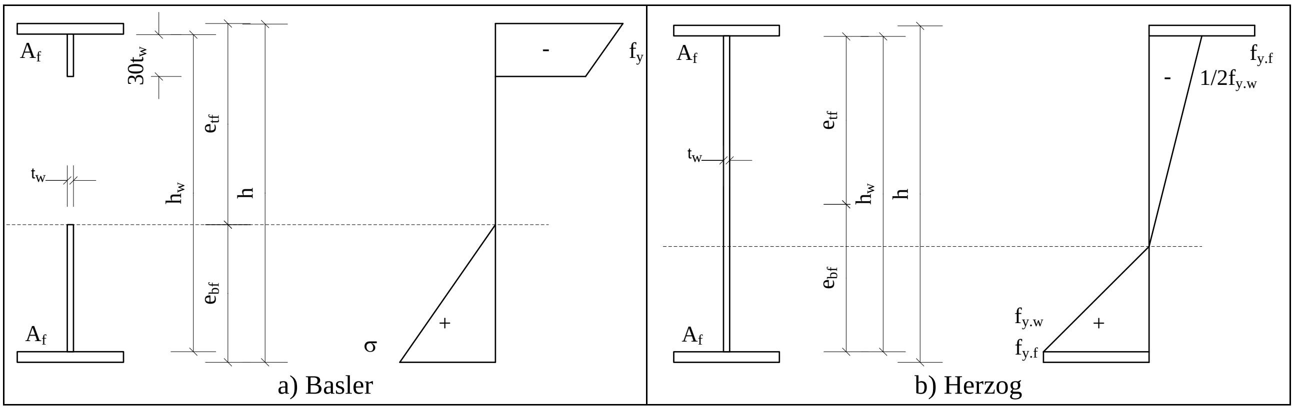

Fig. 1. Cross sections and stress distributions

The position of the neutral axis is found by equilibrium of the parts in tension and compression, assuming a linear stress distribution as shown in *Fig. 1a)*. Basler and Thürlimann gave several equations to determine the bending moment resistance based on such reduction factor ξ. One of them is adopted in AISI [6] for web slenderness's higher than the web slenderness $\beta_0 = 5.7\sqrt{\frac{E}{f_y}}$:

$$\xi = \frac{M_u}{M_{el}} = 1 - 0.0005 \cdot \frac{A_w}{A_f}\left(\frac{h}{t} - 5.7\sqrt{\frac{E}{f_y}}\right) \tag{6}$$

1.4 Bending moment resistance according to Herzog [4]

Herzog did not use an effective cross section, but a reduced yield stress in the compressed part of the web to take into account the influence of plate buckling of a slender web on the bending moment resistance, see *Fig. 1b)*. From equilibrium of the parts in compression and tension it is found $e_{tf} = \frac{2}{3} \cdot h_w$ and based on this, a so called unmodified bending moment resistance is determined:

$$M^o_{Uo} = A_f \cdot f_{y.f} \cdot \left(h_w + t_f\right) + \frac{1}{9} \cdot A_w \cdot f_{y.w} \cdot h_w \tag{7}$$

This unmodified bending moment resistance has to be multiplied with reduction factors K_1, K_2 and K_3 taken into account the influence of torsional buckling, horizontal buckling and vertical buckling of the compressed flange into the web respectively. In case the lateral buckling is prevented by lateral support of the compressed flange, the bending moment resistance is given by *Eq. (8)*.

$$M_u = K_1 \cdot K_3 \cdot M^o_{Uo} = \sqrt{\frac{16t_f}{b}} \cdot \left(1.17 - \frac{\beta_w}{2000}\right) \cdot M^o_{Uo} \tag{8}$$

Both reduction factors are smaller than or equal to 1.0.

1.5 Bending moment resistance according to Veljkovic and Johansson [3]

Veljkovic and Johansson published the bending moment resistance for plate girders with a very slender web, based on the effective width as presented in EN1993-1-5 [1]. For the effective cross-sectional area and the stress distribution, see *Fig. 2a)*.

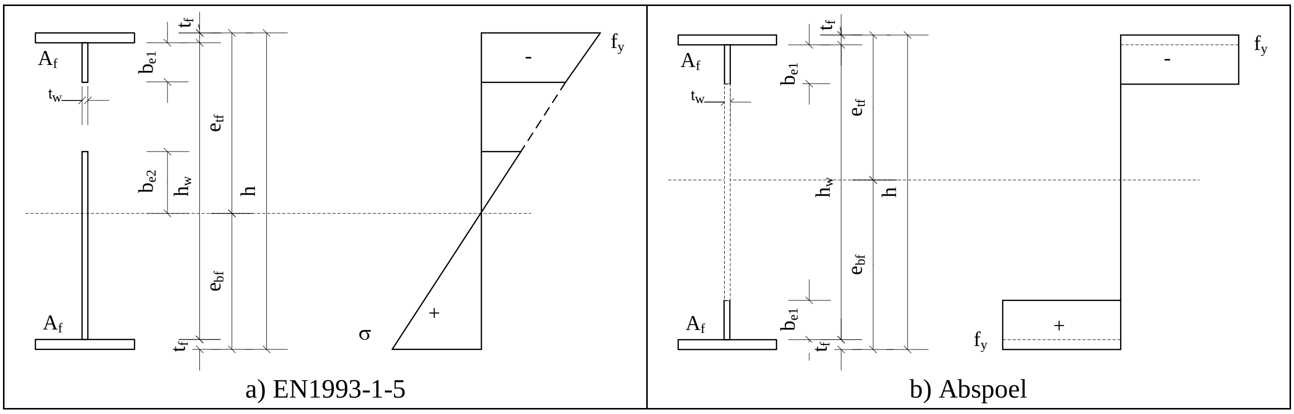

Fig. 2. Cross sections and stress distributions

They presented an equation for the reduction factor ξ for web slenderness's higher than the limitation for cross section class 3, so for $\beta_w \geq 124\varepsilon = 124\sqrt{\frac{235}{f_y}}$, see *Eq. (9)*:

$$\xi = \frac{M_u}{M_{el}} = \frac{M_{Rk}}{W \cdot f_{y.f}} = \left[1 - 0.1\frac{A_w}{A_f}\left(1 - 124\varepsilon \cdot \frac{t_w}{h_w}\right)\right] \tag{9}$$

1.6 Bending moment resistance according to Abspoel [5]

Based on laboratory tests on 10 specimens conducted at the Stevin II laboratory of the Delft University of Technology with web slenderness's of 400, 600 and 800 and based on results of FEM-calculations, Abspoel concluded that the effective width of the web influences not only the compressive part but also the tensile part of the web and the plate girder behaves like a truss. The effective width of the cross section and the stress distribution is given in *Fig.2b)*. The bending moment resistance is determined according to *Eq. (10)*.

$$M_u = M_{u.Abspoel} = A_{tf} \cdot \left(h_w + t_f\right) \cdot f_y + b_{e1} \cdot t_w \cdot \left(h_w - b_{e1}\right) \cdot f_y \tag{10}$$

The effective width b_{e1} is determined according to the EN1993-1-5 [1], see also *Fig.2a)*.

2 ELABORATIONS ON THE BENDING MOMENT RESISTANCE

2.1 Comparison bending moment resistances

It is of interest to determine the dimensions of the plate girder, the web height h_w, the web thickness t_w, the flange width b and the flange thickness t_f, in such a way that the maximum bending moment resistance $M_{u.\max}$ is found. It is assumed that the compressed flange is not susceptible to plate buckling by using $\frac{b}{t_f} \leq 24\varepsilon = 24\sqrt{\frac{235}{f_y}}$. For a certain cross-sectional area A_{tot} and web slenderness β_w, the bending moment resistance M_u is maximized by varying the ratio of area ρ. All dimensions can be expressed in the total cross-sectional area A_{tot}, the web slenderness β_w and the ratio of area ρ, see *Eq. (11)* to *Eq. (14)*:

$$h_w = \sqrt{\frac{A_{tot} \cdot \rho \cdot \beta_w}{(2+\rho)}} \tag{11}$$

$$t_w = \sqrt{\frac{A_{tot} \cdot \rho}{(2+\rho) \cdot \beta_w}} \tag{12}$$

$$b = \sqrt{\frac{24\varepsilon \cdot A_{tot}}{(2+\rho)}} \tag{13}$$

$$t_f = \sqrt{\frac{A_{tot}}{24\varepsilon \cdot (2+\rho)}} \tag{14}$$

For steel grade S460 and a cross-sectional area $A_{tot} = 1200\ \text{mm}^2$ the bending moment resistances according to Basler, Herzog, Veljkovic plus Johansson and Abspoel are presented in *Fig. 3*. The results of FEM calculations are also presented in *Fig. 3)*. From *Fig. 3)* it can be seen that the maxi-

mum bending moment resistance $M_{u.\max}$ is continuously increasing with increasing web slenderness β_w, except the one according to Herzog. This is caused by the additional reduction factor K_3.
The maximum web slenderness according to the FEM-calculations is found for the web slenderness $\beta_{w.max}$ = 1000 for steel grade S460.

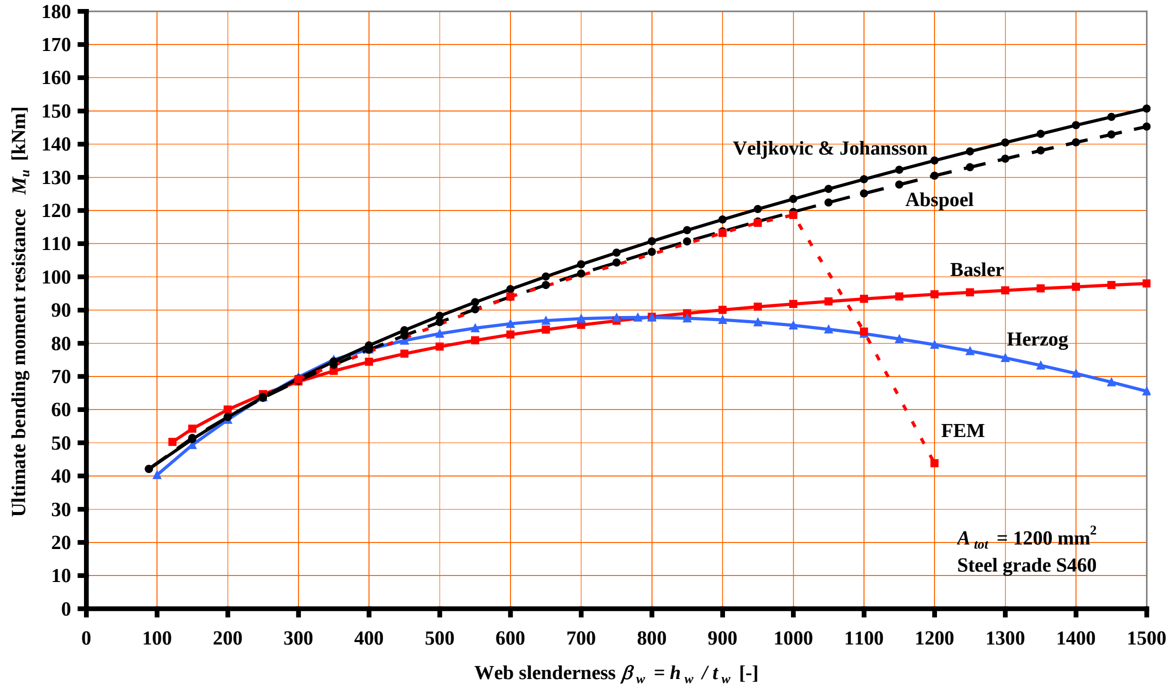

Fig. 3. Maximum bending moment resistance $M_{u.max}$ depending on the web slenderness β_w

To prove that the curves represent the maximum bending moment resistance $M_{u.\max}$, additional calculations are made by using different ratios of area ρ for several constant web slenderness's β_w.

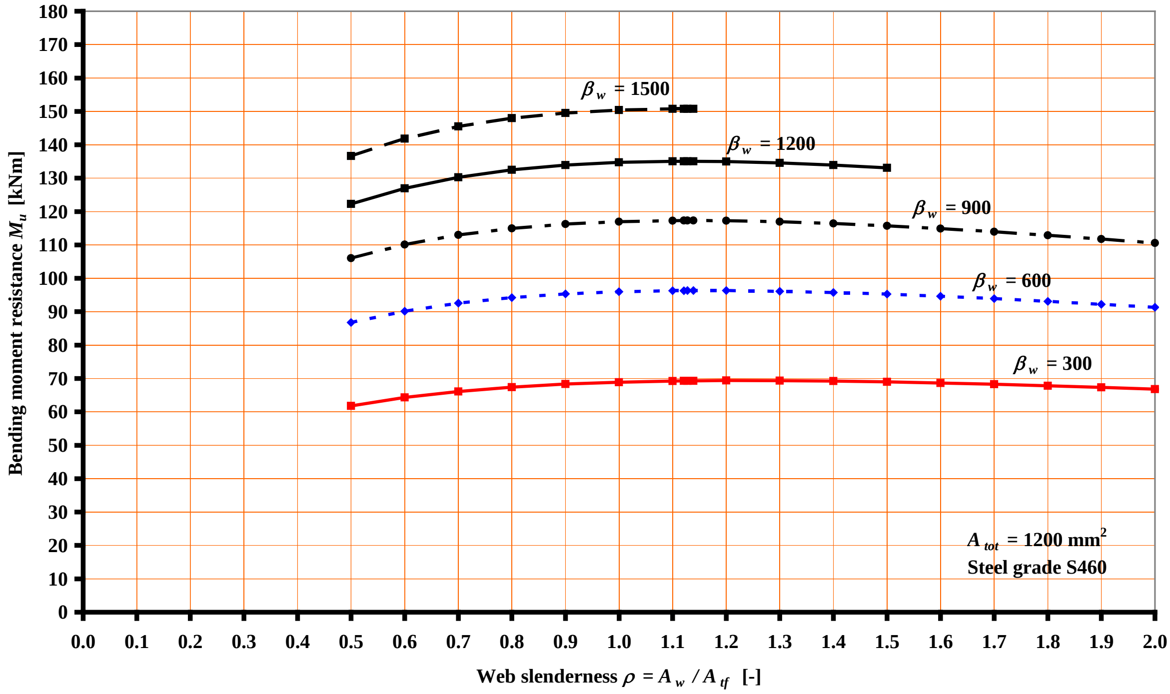

Fig. 4. Maximum bending moment resistance M_u according to Veljkovic & Johansson depending on the ratio of area ρ

The dimensions change and so does the reduction factor ξ, but also the accompanying elastic bending moment resistance M_{el}. *Fig. 4* shows that the maximum bending moment resistance $M_{u.\max}$ is found for specific ratios of area ρ. For smaller and higher values for the ratios of area ρ a smaller bending moment resistance M_u is determined. The maximum bending moment resistances $M_{u.\max}$ at the top of the curves of *Fig. 4* correspond with the bending moment resistances represented in *Fig. 3*.

3 OPTIMIZING THE BENDING MOMENT RESISTANCE OF PLATE GIRDERS

The maximum bending moment resistance $M_{u.\max}$ of plate girders made of a certain amount of steel depends on the reduction factor ξ, the elastic bending moment resistance M_{el}, the maximum web slenderness $\beta_{w.\max}$ and the yield stress f_y:

$$M_{u.\max} = \xi \cdot M_{el} = \xi \cdot W_{el.\min} \cdot f_y \qquad (15)$$

From above maximizations to realise the graph as shown in *Fig. 3),* follows that the product of $\xi \cdot W_{el.\min}$ has to be maximized to find the maximum bending moment resistance $M_{u.\max}$.

4 CONCLUSIONS

The following can be concluded:

1. The maximum bending moment resistance $M_{u.\max}$ for a certain amount of steel depends on the maximum of the product $\xi \cdot W_{el.\min}$ and not on the maximum of the reduction factor ξ alone;
2. The maximum web slenderness $\beta_{w.\max}$ is not determined by vertical buckling of the compressed flange into the web and is much higher than based on EN1993-1-5;
3. The model to determine the bending moment resistance $M_{u.\max}$ according to Abspoel gives good results compared with the results of the experiments and the FEM.

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