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The collaborative berth allocation problem with row-generation algorithms for stable cost allocations

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ABSTRACT

Recent supply chain disruptions and crisis response policies (e.g., the COVID-19 pandemic and the Red Sea crisis) have highlighted the role of container terminals as crucial and scarce resources in the global economy. To tackle these challenges, the industry increasingly aims for advanced operational collaboration among multiple stakeholders, as demonstrated by the ambitions of the recently founded Gemini alliance. Nonetheless, collaborative planning models often disregard the requirements and incentives of stakeholders or simply solve idealized small instances. Motivated by the above, we design novel and effective collaboration mechanisms among terminal operators that share the resources (berths and quay cranes). We first define the collaborative berth allocation problem and propose a mixed integer linear programming (MILP) model to minimize the total cost of all terminals, referred to as the coalitional costs. We adopt the core and the nucleolus concepts from cooperative game theory to allocate the coalitional costs such that stakeholders have stable incentives to collaborate. To obtain solutions for realistic instance sizes, we propose two exact row-generation-based core and nucleolus algorithms that are versatile and can be used for various combinatorial optimization problems. To the best of our knowledge, the proposed row-generation approach for the nucleolus is the first of its kind for combinatorial optimization problems. Extensive experiments demonstrate that the collaborative berth allocation approach achieves up to 28.44% of cost savings, increasing the solution space in disruptive situations, while the proposed core and nucleolus solutions guarantee the collaboration incentives for individual terminals.

1. Introduction

Disruptions in global supply chain networks and crisis response policies such as the Covid-19 pandemic and the Red Sea crisis have recently highlighted the importance of container terminals as scarce resources in the networked global economy. The container crisis, in particular, has demonstrated the need for enhanced resilient container terminal operations. On the other hand, advances in digital technology have stimulated collaborative planning through convenient information sharing in recent years. As a result, collaborative planning strategies, in which multiple stakeholders can provide service cooperatively based on resource sharing, are increasingly targeted by the industry to enhance performance in peak-demand situations and to compensate for temporarily limited capacity at certain nodes in the network. Through these strategies, economic, environmental, and intangible benefits can be obtained (Cleophas, Cottrill, Ehmke, & Tierney, 2019). Accounting for over 90% of global trade, the maritime shipping industry has great opportunities and challenges to deal with when adopting this new trend of collaboration. Over the past two decades, compound annual growth in maritime trade has been 2.9% (UNCTAD, 2021). The increasing rate urges terminals to expand their capacity to ensure the efficiency of port service and enhance port resilience when facing enormous disruptions, such as the breakdowns of the COVID-19 pandemic. However, constructing the port and its supporting facilities requires substantial investments and would incur long-term influences on the environment. Consequently, Maersk and Hapag-Lloyd AG, as prominent companies with ownership of multiple terminals, have announced the new Gemini alliance aiming at extensive operational collaboration and an interconnected ocean network with industry-leading reliability (Hapag-Lloyd, 2024).

Container terminals act as an essential intermediary hub for sailing voyages, and the efficiency of the port-of-call operations significantly impacts the smooth transport of cargo. Berth planning involves determining the berthing time and position for incoming vessels and is therefore one of the most critical decisions for terminal operators. Effective and efficient berth planning ensures the optimal utilization

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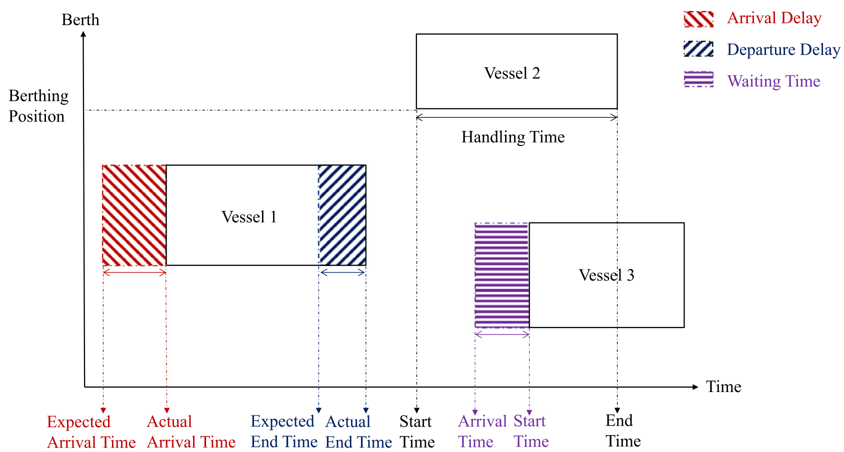

Fig. 1. An illustration of the two-dimensional berthing plan.

of available resources, minimizing vessel waiting times and increasing overall productivity. The classic Berth Allocation Problem (BAP) formalizes the decisions of when and where to discharge (or load) the incoming vessels. Fig. 1 illustrates the BAP in a two-dimensional diagram. The two dimensions are the berthing line (the position that vessels can berth) and the timeline (the planning horizon), respectively. Each rectangle represents the berthing time and position allocated to each calling vessel. Bierwirth and Meisel (2010, 2015) classify the models and algorithms developed in BAP according to different features, while the general goal is to make the two-dimensional space occupied by as many rectangles as possible without overlapping and within the limits of capacity.

The Collaborative Berth Allocation Problem (CBAP) is more complex than that shown in Fig. 1, as it deals with coordinating multiple parties; thus, new models are required to support decision-makers, especially from the operational level. Typically, two types of collaboration, vertical and horizontal, are recognized in the literature (as reviewed in Section 2). Partnerships in vertical collaboration are between different levels of the supply chain, while those in horizontal collaboration happen at the same level. Concerning vertical CBAP (VCBAP), Golias, Saharidis, Boile, Theofanis, and Ierapetritou (2009) and Venturini, Iris, Kontovas, and Larsen (2017) propose collaborative berth allocation models considering the cooperative relationship between shipping lines and terminals, that is, the terminal managers can take part in deciding the arrival time of vessels. The horizontal CBAP (HCBAP) has appeared in practice as an effective form of collaboration (Kavirathna, Kawasaki, & Hanaoka, 2019; Peng, Zhou, & Li, 2015), while the relevant studies are relatively limited. Only Imai, Nishimura, and Papadimitriou (2008) and Lyu, Negenborn, Shi, and Schulte (2022) formulate berth allocation models that allow one vessel to transfer to another terminal within the port based on sharing berths among terminals.

All the studies mentioned above assume that the collaboration has already formed. However, individual interests remain a primary concern for all stakeholders, and they may be reluctant to share their resources if they cannot obtain clear benefits. In this context, successful collaborative planning requires efficient shared-resource allocation methods to improve overall performance and appropriate incentives to convince individual participation, such as to steer effective collaborations (Schulte, Lalla-Ruiz, Schwarze, González-Ramırez, & Voß, 2019). Cooperative game theory provides theoretical approaches, such as the core and the nucleolus, to allocating the coalitional costs to individuals appropriately (Guajardo & Rönnqvist, 2016). Specifically, the core ensures that collaborative members do not incur costs exceeding those associated with working independently, and the nucleolus aims to maximize the number of members within the collaboration. Notably, both methods require prior knowledge of costs associated with all potential coalitions. However, these methods tend to be computationally challenging in implementation because enumerating costs for all potential coalitions quickly becomes impossible when the coalitional costs are intertwined with (np-hard) combinatorial optimization problems, even with a limited number of participants. Thus, to overcome this limitation, we introduce two row-generation-based methods to calculate the core (subsequently referred to as the RG-based core) and the nucleolus mechanism to efficiently generate attractive cost allocations to individual members involved in the collaboration.

To sum up, this work proposes a collaborative berth allocation model in which multiple terminals within one port serve the calling vessels cooperatively. Besides, to facilitate successful collaborative berth allocation, we propose new optimization approaches building on two major concepts in cooperative game theory (i.e., the core and the nucleolus) to find attractive cost allocations, overcoming computational difficulties than simple enumeration-based methods, and thereby incentivizing individual terminals to form a coalition that they do not leave. The computational experiments show that the proposed model can result in significant cost savings for the entire coalition, while crucially maintaining stable collaboration incentives. Moreover, considering the realistic instance sizes, for instance, Hong Kong Port with five terminal operators (Ma, Wong, Leung, & Chung, 2020) and Busan New Port with five terminal operators (Kim, Lee, & Kim, 2022), the proposed cost allocation algorithms for calculating the core and the nucleolus can provide satisfying solutions for six collaborative terminals to ensure joining the collaboration is always attractive for individuals. In summary, we make the following main contributions:

(1) We present a new mathematical model for collaborative bert allocation to minimize the entire cost of all terminals within one port by serving the calling vessels cooperatively, in which we consider the trade-off between the duration time of the vessel and the extra transhipment cost caused by the transfer of vessels.
(2) To ensure stable collaboration incentives, we develop a row generation algorithm to obtain the core solution of cost allocation based on cooperative game theory. The idea is to make it clear to individuals how much they stand to gain to avoid some players benefiting greatly while some even not, thereby maintaining a stable collaboration.
(3) We further strengthen collaboration stability based on the core solutions by suggesting a novel mechanism to find the nucleolus solution for cost allocations of the collaborative berth allocation problem.
(4) With the proposed row-generation-based core algorithm and nucleolus mechanism, we provide general-purpose approaches to achieve attractive and stable cost (or profits) allocations for collaborative combinatorial optimization problems. To the best of

our knowledge, the proposed row-generation approach for the nucleolus is the first of its kind for combinatorial optimization problems.

The subsequent section reviews the studies on collaborative berth allocation and articulates the research gap. Section 3 presents the MILP formulation of the HCBAP model and the related cooperative game. Section 4 describes the RG-based core algorithm and the nucleolus calculation mechanism for allocating the coalitional costs. Section 5 showcases the experimental results, and Section 6 offers a discussion and key insights. Finally, Section 7 provides conclusions and discusses future work.

2. Literature review

In this section, we present an extensive review of collaborative berth allocation. First, we describe two typical types of collaboration in BAP: vertical collaboration between the shipping line and the terminal in Section 2.1 and horizontal collaboration among multiple terminals in Section 2.2. Our focus is the mathematical model that can support terminal operators in making berth allocation decisions; therefore, we also categorize integrated problems with berth allocation as BAP. In Section 2.3, we cover the application of cooperative game theory to collaborative planning in maritime shipping and identify the research gap explicitly in Section 2.4.

2.1. BAP with vertical collaboration

For vertical collaboration, the berthing plan is often organized based on the interaction between the shipping company and the terminal manager. For example, the shipping company can slow down their sailing speed according to the busyness level of the terminal, thereby alleviating terminal congestion and reducing unnecessary fuel costs. The terminal operators can participate in adjusting the arrival time of vessels, which distinguishes VCBAP from the traditional BAP significantly, and terminal resources restrict the duration time of the vessel. Therefore, coordinating the sailing voyage and terminal operation is critical for VCBAP.

Although the concept of collaboration is not explicitly proposed, vessels' arrival time is firstly regarded as a decision variable in the BAP model proposed by Golias et al. (2009). They aim to reduce fuel costs by minimizing the waiting time of vessels at the terminal. However, it is only partially reasonable since the fuel consumption during the sailing voyage is more prominent than that of mooring periods at the quayside (Schrooten, De Vlieger, Panis, Styns, & Torfs, 2008). Therefore, Du, Chen, Quan, Long, and Fung (2011) propose a more elaborate BAP model by considering the fuel consumption in both sailing and mooring periods. Furthermore, they transform the nonlinear model into a mixed integer second-order cone programming model to overcome the problem-solving complexity. Lang and Veenstra (2010) compare the cost of different berthing plans by simulating different scenarios of vessel arrival time. Their experiment results show that the flexible arrival time suggested by the terminal performs better than treating it as a previously-known parameter in terms of terminals' operational efficiency and shipping lines' fuel consumption. Alvarez, Longva, and Engebrethsen (2010) establish a discrete event simulation model by integrating speed optimization with BAP, demonstrating significant benefits of reductions in fuel consumption and dwell time. Besides, there are also some innovative considerations in the literature. Wang, Liu, and Qu (2015) incorporates the utility conception of shipping lines in the model, where a higher bunker and inventory cost decreases the utility. Yu, Tang, and Song (2022) emphasizes vessel service differentiation and develops a bi-objective model for the integrated collaborative berth allocation and quay crane assignment problem.

It is worth noting that all the above studies focused on a single terminal. Only two papers consider the multi-port setting. Venturini et al. (2017) propose a multi-port berth allocation problem, in which shipping lines and multiple ports decide the berthing position and berthing time for each vessel at each port coordinately. They aim to minimize the total fuel consumption of the shipping line and the operation cost of terminals along the entire shipping route. The proposed MIP model performs well for small-scale instances but needs to improve when the size increases. Then, Martin-Iradi, Pacino, and Ropke (2022) propose an exact algorithm based on branch-and-cut-and-price procedures to solve the instances reflecting the real-world scenarios.

2.2. BAP with horizontal collaboration

In horizontal collaboration, different terminals can work together to provide the discharging (or loading) service, potentially sharing the information of the calling vessels and facilities to fully use terminal resources (Kavirathna, Kawasaki, Hanaoka, & Bandara, 2020). However, some of these terminals are competitors and may not be willing to collaborate. In this regard, on top of an efficient berth allocation plan, it is essential to convince terminals about collaborating as more benefits can be obtained.

There have been studies demonstrating the benefits of consolidating container terminals. Budipriyanto, Wirjodirdjo, Pujawan, and Gurning (2015) propose a conceptual framework of a collaborative operational system among terminals. The results show that terminal collaboration can reduce vessels' waiting time, balance resource utilization, and increase overall profits. Saeed and Larsen (2010) investigate the coalitions forming by different combinations among three terminals at Karachi Port, which models a Bertrand game with one outside competitor, the coalition, and the terminal in Karachi Port (if any) that has not joined the coalition. Wong, Ma, and Leung (2018) conduct empirical research on facilitating terminal coalition at the Hong Kong Port, consisting of five terminal operators. Kim et al. (2022) simulate the effects of sharing berth resources among terminals within one port using scenario analysis. Pujats, Konur, and Golias (2021) classify forming terminal coalition as intra-port collaboration, and they develop quantitative tools to analyze the dynamics of individual profit of terminal operators and their willingness to cooperate. The authors pre-assume two cooperation schemes and seven transfer fee policies and then investigate the changes in profits before and after collaboration.

However, limited studies contribute to the HCBAP model in forming terminal coalitions. Imai et al. (2008), and Cho, Park, and Lee (2021) address a variation of BAP at multi-user terminals, which assign vessels that would usually be served at the terminal to an external terminal due to waiting time limitations. Nevertheless, these models tacitly assume that the cooperative alliance is already formed and, thus, ignore the rational decisions of individual terminals as a requisite to form such alliances.

2.3. Cooperative game theory models

In real cases, terminal operators pursue enhancing their own interests (Kavirathna et al., 2020). Thus, convincing individual terminals to collaborate and abide by the coalitional decisions is a significant concern. Cooperative game theory has been increasingly applied in collaborative maritime shipping in recent decades: shipping alliances (Agarwal & Ergun, 2010), network design (Buer & Haass, 2018) and hinterland transport (Giudici, Lu, Thielen, & Zuidwijk, 2021). Their focus is generally on allocating the coalitional benefits appropriately to incentivize individual players to stay in the collaboration (Özener & Ergun, 2008), thereby maintaining collaboration stability. The core (Tinoco, Creemers, & Boute, 2017) and ϵ-core (Lai, Cai, & Hall, 2022) for collaboration stability and the nucleolus for enhancing stability (Guajardo & Jörnsten, 2015) are applied in collaborative transportation problems. However, their computational time grows exponentially with the increased number of players; thus, they fail to deal with realistic problems.

For the CBAP, most researchers only consider minimizing the overall cost; however, studies covering rational individual considerations are limited. The collaborative mechanism for berth allocation proposed by Wang et al. (2015) implies the cooperative and competitive relationship between the terminal and the shipping line to ensure that the berthing plan is mutually beneficial to both parties. Sabar, Chong, and Kendall (2015) embed the game theory in their heuristics for solving the BAP, and Saeed and Larsen (2010) design a two-stage Bertrand non-cooperative game for terminals within one port. Martin-Iradi et al. (2022) design a cooperative game consisting of the shipping line and the terminal. They apply the Shapley Value Method (SVM) and Equal Profit Method (EPM) to allocate the joint cost among the individual member fairly. Similarly, Guo, Zheng, Liang, and Wang (2023) apply the SVM and the core to allocate the total cost of each group and select the stable groups. Based on the previous results, they propose a new integer programming model to determine the collaborative groups with the maximum revenue. However, in dealing with the cooperative part, the study depends on the enumeration method, which greatly limits the number of partners joining the game.

2.4. Overview and research gap

According to the classification scheme in Bierwirth and Meisel (2010), we use three attributes to describe the problem properties of the CBAP model. The spatial attribute reflects the quay layout in discrete (DS) or continuous (CN) berths. The handling time attribute concerns the way of dealing with vessel handling time in the model: fixed (FX), quay crane dependent (QD), and position dependent (PD). The performance measure attribute lists six different evaluation criteria: the waiting time of vessels (wait), handling time of vessels (hand), departure tardiness of vessels (tard), extra container transshipment cost (extra), fuel consumption (fuel), and utility calculation of terminals (utility).

The relevant CBAP literature is exhibited in Table 1, sorted by collaboration types, problem properties, solution methods, and whether considering collaboration stability. The last row highlights our research. From it, we can observe that although the concept of collaborative berth planning has been recognized over the last decade, the matching planning models are still in their infancy. Especially for horizontal collaboration among multiple terminals, most studies only analyze the potential advantages, while few can support making decisions on berth allocation from the operational level. Furthermore, effective cost allocation models to allocate the coalitional benefits for maintaining collaboration stability are still lacking. Thus, other than developing an instructive CBAP model from the operational planning perspective, we further present supportive cost allocation methods that are vital to enable a stable collaboration in practice. This paper aims to provide decision-support tools for practitioners in maritime shipping to facilitate collaborative berth allocation effectively.

3. The collaborative berth allocation problem as a cooperative game

The proposed HCBAP model described in Sections 3.1 and 3.2 is based on the discrete and dynamic BAP, in which multiple terminal operators serve the calling vessels cooperatively by sharing the berths and quay cranes. In this paper, we assume that a port has multiple terminal operators, where each operator manages either one terminal or several terminals. Thus, the optimization of berthing plans for terminals under the same operator's government is assumed to be decided internally, while the focus of our model lies in its emphasis on inter-operator collaboration, where berthing plan is approached through the sharing of resources and information among different terminal operators. This collaborative approach aims to enhance overall efficiency by encouraging different terminal operators to work together, as discussed by Budipriyanto et al. (2015), Kavirathna et al. (2019), Lyu et al. (2022). This model strongly depends on the collaboration among terminal operators; therefore, the benefits of such collaboration need to be explicit to convince individual terminal operators to join and stay in the coalition. To incentivize such collaboration in practical scenarios, we develop cost allocation schemes to maximize the individual benefits of each terminal operator. Thus, we further define a cooperative game using the objective value of the HCBAP model as the characteristic function. The proposed collaborative berth allocation game is intended to make collaboration more appealing by ensuring that each terminal operator gains from the shared efforts.

For ports with a single operator, centralized decision-making naturally optimizes port efficiency without the need for collaboration, which is out of the scope of the proposed collaborative berthing model in this paper.

3.1. Problem description of HCBAP

A calling vessel can be regarded as the terminal's customer. The terminal operator earns money by providing customers with discharging (or loading) services. Delays in this service directly damage customer satisfaction, which will influence the development of the terminal and the benefits of terminal operators in the long term. Without collaboration, the vessels can only wait at terminals governed by their contracted terminal operator until the berths and quay cranes are available. With collaboration among terminal operators, the vessels can be berthed at terminals governed by other operators providing better time slots while incurring extra container transshipment costs. Thus, terminal operators need to balance the service quality and service cost during the planning. Based on the BAP model in Kim and Moon (2003), our HCBAP model incorporates a trade-off between extra container transshipment costs caused by vessel transfer between terminals and the cost of vessel tardiness. We assume a coalition of terminal operators is willing to share the resources (e.g., berths and relevant information about the incoming vessels) with each other. If the contracted terminal operator cannot serve the calling vessel because of disruptive events or if providing the service is economically unreasonable, these vessels are allowed to transfer to another terminal operator in the coalition for loading or discharging, and the saving costs are shared.

Figs. 2 and 3 illustrate the berthing plan without and with the collaboration based on an example port with four terminals governed by three terminal operators. In Fig. 2, terminal operators independently make their own planning on berth allocation for their respective vessels. That is, each terminal operator's berth planning is limited by its internally accessible berth and quay crane resources. While in the collaborative case shown in Fig. 3, the calling vessel can berth at any terminal governed by the operators of the coalition given that the necessary berth resources are available. In practice, the yard location of containers loaded to (or discharged from) the calling vessel has been decided before. Therefore, extra container transshipment fees due to vessel transfer between terminals have to be considered in the cost calculation. In this way, the proposed HCBAP model achieves a simultaneous berthing plan for multiple terminals governed by multiple terminal operators within a port.

Above all, we conclude our problem as a collaborative berth allocation problem based on horizontal collaboration among terminal operators within one port. We frame the problem as concentrating all the vessel's calling information and making the berthing plan optimally, considering the available resources of all collaborative terminals and providing decision support for both cases with and without disruptions. Specifically, we only consider the quayside operation, and the inland schedule of container transportation is out of our scope. Thus, we assume all the related containers have already arrived at the terminal, waiting for the loading or unloading operations.

Table 1
Overview of the CBAP model in the literature.

Type	Reference	Problem properties			Solution method	Stability consideration
		Spatial attribute	Handling attribute	Performance measure		
Vertical Collaboration	Golias et al. (2009)	DS	FX	$\Sigma(wait + tard)$	H	
	Lang and Veenstra (2010)	DS	QD	$\Sigma(fuel + tard + extra)$	SM	
	Du et al. (2011)	CN	FX	$\Sigma(tard + wait + fuel)$	MIP	
	Wang et al. (2015)	DS	FX	$\Sigma(utility - extra)$	MIP	✓
	Venturini et al. (2017)	DS	PD	$\Sigma(wait + hand + tard + fuel)$	MIP	
	Yu et al. (2022)	CN	QD	$\Sigma(tard + wait + fuel)$	MIP	
	Martin-Iradi et al. (2022)	DS	PD	$\Sigma(wait + hand + tard + fuel)$	B&P, CG	✓
Horizontal Collaboration	Imai et al. (2008)	DS	PD	$\Sigma(wait + hand + tard + fuel)$	H	
	Hendriks, Armbruster, Laumanns, Lefeber, and Udding (2012)	CN	QD	$\Sigma(hand + extra)$	MIP	
	Dulebenets, Golias, and Mishra (2018)	DS	FX	$\Sigma(hand + tard + extra)$	H	
	Ma et al. (2020)	DS	FX	$\Sigma(hand + tard + extra)$	SM	
	Cho et al. (2021)	CN	QD	$\Sigma(hand + tard + extra)$	H	
	Pujats et al. (2021)	–	QD	$\Sigma(utility)$	SM	✓
	Xu, Du, Li, and Zhang (2021)	CN	PD	$\Sigma(tard+extra)$	LR	
	This paper	DS	QD	$\Sigma(hand + wait + tard + extra)$	MIP, CG	✓

Note: LR, lagrange relaxation; B&P, branch and price; SM, simulation; H, heuristics; CG, cooperative game theory.

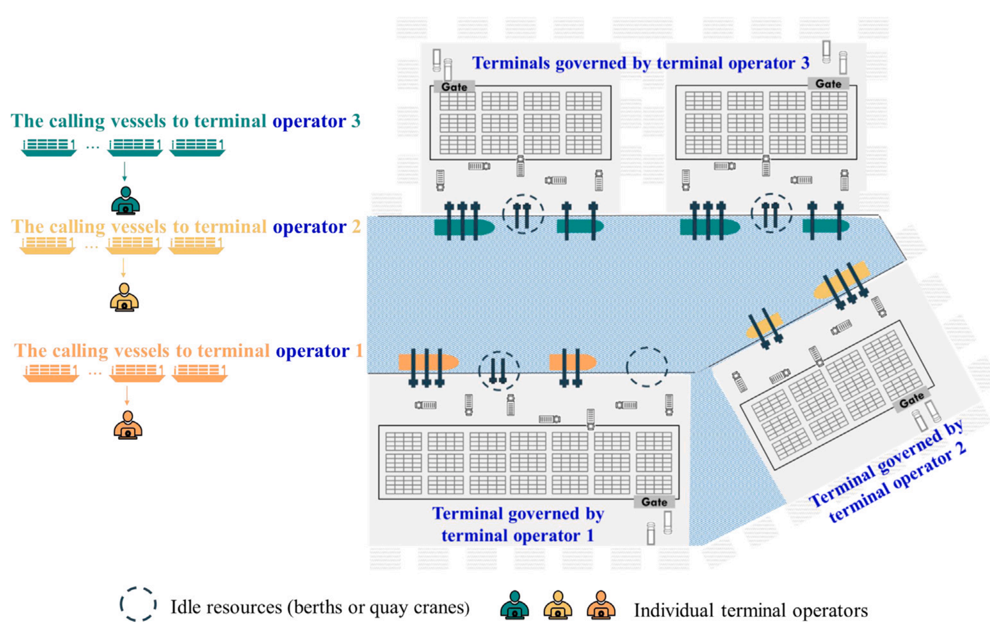

Fig. 2. Internal berthing plan by each terminal operator.

3.2. The HCBAP model formulation

This section first introduces assumptions of our HCBAP model as follows:

(1) Berth positions are discrete and one vessel can only occupy one berth position.
(2) The loading/discharging operation by quay cranes for each vessel is assumed to be conducted continuously without interruption.

Next, we develop a mixed-integer linear programming (MILP) model for the proposed collaborative berth allocation problem with horizontal collaboration among terminal operators. The objective is to minimize the total service costs of all terminals. The model outputs (i) which

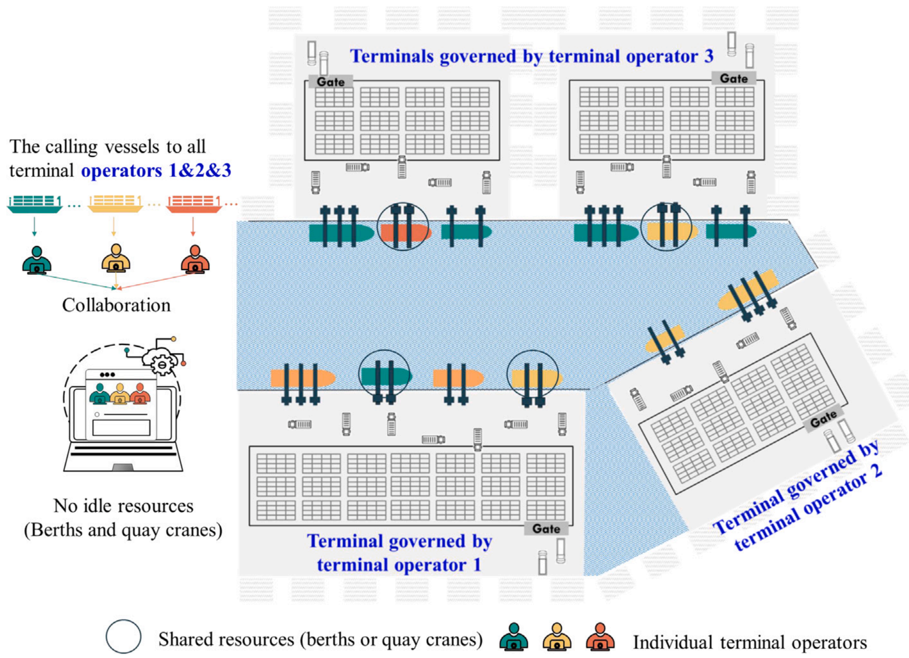

Fig. 3. Berthing plan with collaboration.

Table 2
Notation of sets, parameters, and decision variables.

Notation	Explanation
Sets	
V	set of all vessels.
T	set of one-hour time periods.
M	set of all terminals within the port.
B_m	set of berths at terminal $m \in M$.
Parameters	
c^{ι}_{im}	unit cost of inter-terminal container transshipment for vessel $i \in V$ from its contracted terminal to terminal $m \in M$, given in units of USD per TEU.
c^{ω}_{i}	penalty rate of vessel $i \in V$ for waiting service given in units of USD per hour.
c^{δ}_{i}	penalty rate of vessel $i \in V$ for departure tardiness given in units of USD per hour.
c^{κ}	operation cost rate given in units of USD per Quay Crane(QC)-hour.
a_i	expected arrival time of vessel $i \in V$.
d_i	expected departure time of vessel $i \in V$.
w_i	requirement for QC-hour of vessel $i \in V$.
g_i	the number of container units (TEU) required to be loaded or discharged on vessel $i \in v$.
Q_m	the total number of available quay cranes at terminal $m \in M$.
Decision variables	
z	the total cost for serving all calling vessels.
s_i	starting time of vessel $i \in V$.
e_i	ending time of vessel $i \in V$.
x_{im}	binary variables equal to 1 if vessel $i \in V$ is berthed at terminal $m \in M$, and 0 otherwise.
$\bar{x}_{imk}$	binary variables equal to 1 if vessel $i \in V$ is berthed at berth $k \in B_m$ of terminal $m \in M$, and 0 otherwise.
y_{ijmk}	binary variables equal to 1 if vessel $i \in V$ and vessel $j \in V$ are both assigned to berth $k \in B_m$ of terminal $m \in M$ and vessel $i \in V$ is processed before vessel $j \in V$, and 0 otherwise.
q_{imt}	the number of quay cranes assigned to vessel $i \in V$ when time $t \in T$ at terminal $m \in M$.
r_{it}	binary variables equal to 1 if vessel $i \in V$ is operated when time $t \in T$, and 0 otherwise.
γ_{imt}	binary variables equal to 1 if vessel $i \in V$ is operated when time $t \in T$ at terminal $m \in M$, and 0 otherwise.

berth at which terminal the vessel is served, (ii) the vessels' berthing time, (iii) the number of quay cranes operating within the service time, and (iv) the departure time. We list the used notations in Table 2. Based on the notations, the proposed HCBAP model is formulated as follows:

$$\min \quad z = \sum_{i\in V}\sum_{m\in M} c^{\iota}_{im} g_i x_{im} + \sum_{i\in V} c^{\omega}_{i}(s_i - a_i) + \sum_{i\in V} c^{\delta}_{i}(e_i - d_i)^{+} + \sum_{i\in V}\sum_{m\in M}\sum_{t\in T} c^{\kappa} q_{imt} \tag{1}$$

Subject to:

$$\sum_{m\in M} x_{im} = 1 \quad \forall i \in V \tag{2}$$

$$\sum_{m\in M}\sum_{k\in B_m} \bar{x}_{imk} = 1 \quad \forall i \in V \tag{3}$$

$$s_i \le tr_{it} + M(1-r_{it}) \quad \forall i \in V, t \in T \tag{4}$$

$$e_i \ge (t+1)r_{it} \quad \forall i \in V, t \in T \tag{5}$$

$$\sum_{t\in T} r_{it} = e_i - s_i \quad \forall i \in V \tag{6}$$

$$\sum_{t\in T} q_{imt} \ge w_i x_{im} \quad \forall i \in V, m \in M \tag{7}$$

$$\sum_{i\in V} q_{imt} \le Q_m \quad \forall t \in T, m \in M \tag{8}$$

$$M(\gamma_{imt}-1) - q_{imt} \le 0 \quad \forall i \in V, m \in M, t \in T, \tag{9}$$

$$q_{imt} \le M\gamma_{imt} \quad \forall i \in V, m \in M, t \in T \tag{10}$$

$$\sum_{m\in M} \gamma_{imt} = r_{it} \quad \forall i \in V, t \in T \tag{11}$$

$$\sum_{k\in B_m} \bar{x}_{imk} = x_{im} \quad \forall i \in V, m \in M \tag{12}$$

$$s_j \ge e_i - M(1-y_{ijmk}) \quad \forall i \in V, j \in V, i \neq j, m \in M, k \in B_m \tag{13}$$

$$y_{ijmk} + y_{jimk} \le 0.5(\bar{x}_{imk} + \bar{x}_{jmk}) \quad \forall i \in V, j \in V, i \neq j, m \in M, k \in B_m \tag{14}$$

$$y_{ijmk} + y_{jimk} \ge \bar{x}_{imk} + \bar{x}_{jmk} - 1 \quad \forall i \in V, j \in V, i \neq j, m \in M, k \in B_m \tag{15}$$

$$s_i, e_i \in \{a_i, \dots, |T|\} \quad \forall i \in V \tag{16}$$

$$x_{im}, \bar{x}_{imk}, r_{it}, \gamma_{imt} \in \{0,1\} \quad \forall i \in V, m \in M, t \in T, k \in B_m \tag{17}$$

$$y_{ijmk} \in \{0,1\} \quad \forall i \in V, j \in V, i \neq j, m \in M, k \in B_m \tag{18}$$

The objective function (1) minimizes the total service costs z of all terminals, consisting of four parts. The first part is the cost caused by extra container transshipment between terminals, defined as inter-terminal transportation (ITT) by Tierney, Voß, and Stahlbock (2014). The parameter c^{ι}_{im} represents the unit cost associated with transporting a single container from the designated terminal of vessel i to an alternative terminal m. This cost is derived based on the fuel expenses incurred during the container transfer process between terminals (Gharehgozli, de Koster, & Jansen, 2017), and it is computed following the equation outlined in Eq. (19). Specifically, the term l_{im} denotes the spatial distance between two terminals. The parameter α corresponds to the fuel expenditure per unit distance, reflecting the variable costs associated with fuel consumption during the container transhipment process. Furthermore, the parameter β indicates the capacity of the container transhipment medium, defined as the maximum number of containers that can be conveyed simultaneously in a single trip. Consequently, the unit transportation cost c^{ι}_{im} is intrinsically linked to these factors, considering both the physical distance of transport and the efficiency of container movement in relation to fuel utilization and capacity constraints. The second part is the penalty cost for waiting after the vessel arrives at the port. The third part is the penalty cost for the tardiness of vessels' departure time. The fourth part is the operation cost of quay cranes.

$$c^{\iota}_{im} = \frac{\alpha l_{im}}{\beta} \tag{19}$$

Constraint (2) ensures that each vessel is operated at only one terminal. Constraint (3) ensures that each vessel is operated at only one berth of one terminal. Constraints (4)–(6) determine the service starting time and ending time for vessels. Constraint (7) enforces that the total number of assigned quay cranes must satisfy the vessel's requirement. Constraint (8) ensures that the number of quay cranes operating simultaneously cannot exceed the maximum number available at the terminal. Constraints (9)–(10) denote the relationship of variables r_{imt} and q_{imt}, implicating that no quay crane is assigned when the vessel is not berthed. The consistent setting of the corresponding variables related to berthing position ($\bar{x}_{imk}$ and x_{im}) and handling time (γ_{imt} and r_{it}) is incorporated in Constraints (11)–(12). Constraints (13)–(15) ensure no overlapping exists for vessels that are served at the same berth of the same terminal. Constraint (16) restricts the service starting time to be after the vessel's arrival and the service ending time to be within the planning horizon. Constraints (17)–(18) define the remaining decision variables.

The third part of the objective function is nonlinear, and we linearize the objective function by defining an auxiliary variable $\mu_i = (e_i - d_i)^+$. Additional constraints (20) and (21) are added for restricting μ_i.

$$\mu_i \ge 0 \quad \forall i \in V \tag{20}$$

$$\mu_i \ge e_i - d_i \quad \forall i \in V \tag{21}$$

Therefore, we reformulate the model as follows:

$$\min \quad z = \sum_{i\in V}\sum_{m\in M} c^{\iota}_{im} g_i x_{im} + \sum_{i\in V} c^{\omega}_i (s_i - a_i) + \sum_{i\in V} c^{\delta}_i \mu_i + \sum_{i\in V}\sum_{m\in M}\sum_{t\in T} c^{\kappa} q_{imt} \tag{22}$$

Subject to Constraints (2)–(21).

4. Cost allocation algorithms for stable and attractive collaboration

The HCBAP model proposed in Section 3.2 supports the centralized system in which multiple terminal operators are involved to cooperate on berth allocation planning for the calling vessels. From the perspective of global optimization, combining all terminal operators into the grand coalition means expanding the decision-making space for the berth allocation problem, which is cost-saving for the overall coalition. Essentially, it aims to minimize the overall cost based on the assumption that the collaboration has already formed. However, a genuine concern for individual terminals is whether staying in the coalition is in their best interest; if not, they may choose to form a sub-coalition or work independently without collaboration. Thus, a reasonable cost allocation strategy to convince individual terminals is crucial for a stable collaboration in practice. In this regard, we first define a cooperative game in Section 4.1 using the objective value of the HCBAP model as the characteristic function. Next, we adopt the core and the nucleolus concepts from cooperative game theory to allocate the coalitional costs among collaborative terminals, which can guarantee that no subgroups can yield better benefits than the grand coalition. In detail, the first part of Eq. (23) states that all the costs need to be allocated to the coalition members, and its second part means that the cost allocated to players in the grand coalition is less than any subgroup. Therefore, the grand coalition is always more beneficial for terminal operators under the core. For the nucleolus, one solution found in the core, so the nucleolus solution also observes Eq. (23). Hence, under the definition of the core and the nucleolus, no subgroups of terminal operators will perform better than the grand coalition formed by all terminal operators.

$$\text{Core}(M, f) = \left\{ f \in \mathbb{R} | \sum_{m\in M} f_m = C(M), \text{and} \sum_{m\in S} f_m \le C(S) \quad \forall S \subset M, S \neq \emptyset \right\} \tag{23}$$

Where M represents the grand coalition, f_m represents the cost allocated to player (terminal operator) m, S represents a sub-group of the grand coalition, and $C(X)$ represents the coalitional costs of coalition X.

In general, the core can guarantee that no collaborative terminals cost more than working alone, and the nucleolus aims to allow as many terminals to save costs within the collaboration as possible, while both methods require the costs of all potential coalitions to be pre-known. That is to say, the calculation of the core and the nucleolus needs multiple iterations of the MILP model to obtain the costs for all possible coalitions as inputs. Due to the NP-hard nature of the optimization problem which is already difficult to solve in a one-player setting. If we want to obtain Core and Nucleolus solutions, we need to solve this problem for every subset of players, that is, up to $2^6 - 1$ (null player coalition) or 63 times. To address this challenge, we develop two efficient algorithms based on the two essential concepts in cooperative game theory: the core in Section 4.2 and the nucleolus in Section 4.3, combing with the combinatorial optimization model of HCBAP, to allocate the coalitional costs while keeping the stability of collaboration. Finally, we give an example as an illustration of the proposed RG-based core and the nucleolus mechanism in Section 4.4.

4.1. The cooperative berth allocation game

Our cooperative game is based on the HCBAP model, in which a set of terminal operators $M = \{1, 2, \dots\}$ participate as players, and $\mathcal{N}$ is the set of all non-empty subsets of M. Each element in $\mathcal{N}$ represents one possible terminal coalition S. In other words, each S is actually a set of some terminal operators, and there are $2^{|M|} - 1$ different S, that is, $\mathcal{N} = \{S_1, S_2, \dots, S_{2^{|M|-1}}\}$. For simplicity, we just use S in the following sections. As stated above, $S \subseteq M$ and $S \in \mathcal{N}$. Specifically, the coalition formed by all players is called the *grand coalition*. The *characteristic function* $C(S)$ represents the impact of a coalition S in the cooperative game theory, which in this case is defined as the objective value of the HCBAP model. For the cost of players who make decisions independently without collaboration, we call it *stand-alone* cost.

The characteristic function satisfies the following two conditions required in cooperative game theory:

$$C(\emptyset) = 0 \tag{24}$$

$$C(S) + C(T) \geq C(S \cup T) \quad \forall S, T \subseteq M, S \cap T = \emptyset \tag{25}$$

Eq. (24) states that there is no cost in an empty coalition. Eq. (25), referred to as *subaddivity*, ensures that forming a coalition can always generate no more cost than when operating independently. Furthermore, a cost allocation vector $\boldsymbol{f} = \{f_1, f_2, \dots, f_n\}$ denotes the cost allocated to each player. Subsequently, two properties *efficiency* and *individual rationality* are defined in Eqs. (26) and (27), respectively. Efficiency means all costs in the coalition are distributed to individuals, and individual rationality states that the cost allocated to each individual cannot exceed its stand-alone cost.

$$\sum_{m \in M} f_m = C(M) \tag{26}$$

$$f_m \leq C(\{m\}) \quad \forall m \in M \tag{27}$$

4.2. Cost allocation in the core

The core is one of the most widespread concepts for stable collaboration in cooperative game theory. Let us denote the cost allocation vector $\boldsymbol{f}(S) = \sum_{j \in S} f_j$. Given the total cost $C(M)$, a solution consisting of the cost distribution over all players and satisfying the condition of the core is referred to as *imputation*. In addition to efficiency, it requires no sub-coalition to incentivize individual players to leave the grand coalition. Therefore, we formulate the core solution of our cost allocation problem in Eq. (23). According to Driessen (2013), the characteristics function of our cooperative berth allocation game satisfies submodularity defined by Eq. (28). It follows the setting of the HCBAP model proposed in Section 3, where more terminal operators participating in the alliance means expanding the decision space of the HCBAP model from a global optimization perspective. Thus, it opens the possibility of finding better (less overall costs) berth allocation results. Shapley (1971) states that (i) a game is a convex game if its characteristic function is submodular, and (ii) the core is nonempty in convex games. Thus, we conclude that our cooperative game is a convex game, and the core solution exists.

$$C(S \cup \{i\}) - C(S) \geq C(T \cup \{i\}) - C(T) \quad \forall S \subset T \subset N \setminus \{i\} \tag{28}$$

However, the problem is how to calculate the core solution. As a result of Eq. (23), the number of constraints denoted in the core grows exponentially to $2^{|M|}-1$, making the model difficult to solve. To address this challenge, Drechsel and Kimms (2010) introduced a row generation approach that mitigates this computational burden by reducing the need to calculate each subgroup. Based on this idea, we develop a Row-Generation-based (RG-based) core algorithm consisting of a linear programming model in the master problem and a MILP model in the sub-problem. The linear programming model can be solved quickly, and its output provides fixed values that simplify the MILP model in the sub-problem. This significantly reduces the total solution time, making our RG-based algorithm a more efficient method for calculating the core. In doing so, we first formulate the master problem (**MP**) in Section 4.2.1 and the subproblem (**SP**) in Section 4.2.2, then we introduce the procedure of the RG-based core algorithm in Section 4.2.3.

4.2.1. MP model

The difficulty of calculating the core directly by Eq. (23) is that each possible terminal coalition S will add one more constraint and the core solution needs to satisfy all the constraints. Thus, the master problem of the row generation approach is to calculate the cost allocation with some limited constraints. The current result of the master problem then assists in finding more constraints in the subproblem.

As stated already, in order to calculate the cost allocation based on a limited number of constraints, we define the following parameter Θ and decision variables in the master problem:

Parameter:

- Θ: the set of potential coalition S, initialized as $\Theta = \{\{1\}, \{2\}, \dots, \{n\}\}$

Decision variables:

- δ: the minimum cost savings considering all the coalition S in current Θ.
- f_m: the cost allocated to player m.

Then, we formulate the **MP** model of our RG-based core algorithm as follows:

$$\min \quad \delta \tag{29}$$

Subject to:

$$\sum_{m \in S} f_m - \delta \leq C(S) \quad \forall S \in \Theta \tag{30}$$

$$\sum_{m \in M} f_m = C(M) \tag{31}$$

$$f_m \in \mathbb{R}, \forall m \in M \tag{32}$$

$$\delta \geq 0 \tag{33}$$

The value of the objective function (29) indicates whether the core is empty. If $\delta = 0$, then the core of the problem is not empty, and the value of f_m is the cost allocation in the core. In contrast, if $\delta > 0$, the problem has an empty core, which means the grand coalition is unstable regardless of the cost allocation solution. Constraints (30)–(31) embody the core definition. Constraints (32)–(33) define the decision variables.

4.2.2. SP model

To search for the sub-coalition S' that violates the core definition the most, the following decision variable ξ_m and parameter ζ_{im} need to be further defined in the subproblem.

Parameters:

- ζ_{im}: binary parameters equal to 1 if vessel $i \in V$ is the contracted customer of terminal m, and 0 otherwise.
- $\hat{f}_m$: stand-alone costs of terminal m without any collaboration with other terminals.

Decision variables:

- ξ_m: binary variables equal to 1 if terminal m belongs to the coalition S', and 0 otherwise.

Continuing the notion used in Section 3.2, the **SP** model is given as:

$$\max \sum_{m \in M} f_m \xi_m - \left[Obj - \sum_{m \in M} \hat{f}_m (1 - \xi_m) \right] \tag{34}$$

$$Obj = z \tag{35}$$

$$x_{im} \geq \zeta_{im}(1 - \xi_m) \quad \forall i \in V, m \in M \tag{36}$$

$$x_{im} \leq \zeta_{im} + \xi_m \quad \forall i \in V, m \in M \tag{37}$$

and constraints (2)–(21).

The objective function (34) outputs the sub-coalition $S' \notin \Theta$ that violates the core condition most. Constraints (36) and (37) state the relationship of the definition of ζ_{im} and ξ_m, restricting terminal $m \notin S'$ can only serve its own customer vessels.

4.2.3. RG-based core algorithm

Algorithm 1 describes procedures of the proposed RG-based core algorithm.

Algorithm 1: Algorithm for Calculating the Core based on Row Generation Method

```
1  Initialization: Θ = {{1}, {2}, ..., {n}}
2  Run model MP (4.2.1), obtain the value δ and f_m for m ∈ M
3  if δ > 0 then
4  |   Stop, and the problem has an empty core
5  else
6  |   Run model SP(4.2.2) to find a sub-coalition
   |     S' = arg max_{S' ∉ Θ} ( Σ_{m∈S'} f_m − C(S') )
7  |   if S' exists then
8  |   |   Θ = Θ ∪ {S'}
9  |   |   Return to line 2
10 |   else
11 |   |   Current f_m for m ∈ M are in the core
12 |   end
13 end
   Output: One core solution f
```

The set of all possible coalitions, Θ, is initialized $\Theta = \{\{1\}, \{2\}, \ldots, \{n\}\}$ in line 1. Starting from Θ, we run the **MP** model to obtain the value δ and f_m for $m \in M$. If $\delta > 0$, the problem has an empty core, and the algorithm stops. Otherwise, if $\delta = 0$, the **SP** model aims to find a sub-coalition $S' \notin \Theta$ that maximizes $\sum_{m \in S'} f_m - C(S')$. It aims to search for the coalition S' that violates the core definition most. If S' exists, update $\Theta = \Theta \cup \{S'\}$, and return to line 2; otherwise, the current vector $\boldsymbol{f}$ is cost allocation in the core.

4.3. Cost allocation in the nucleolus

The core solution can provide stable outcomes for cost allocation. However, it is not necessarily unique, and different cost allocations can be in the core (Guajardo & Jörnsten, 2015; Xiao & Fang, 2023). In such cases, decision-makers require additional support to select one among them. Therefore, the problem of choosing a cost allocation in the core is raised. In this regard, the nucleolus, introduced by Schmeidler (1969), is another well-known allocation rule in cooperative game theory. Most importantly, the nucleolus is unique and lies in the core (non-emptiness of core demonstrated in Section 4.2), making it an attractive and preferred choice over other shared cost allocation methods for decision makers in the collaboration (Nguyen & Thomas, 2016).

However, calculating the nucleolus can be challenging when the number of players increases, especially when it intertwines with solving combinatorial optimization problems. Despite these challenges, the nucleolus provides more substantial support for decision-makers on cost allocation than the core solution due to its superior stability properties and unique nature. Therefore, we propose an effective mechanism to compute the nucleolus for the collaborative berth allocation game. In this section, we first provide a mathematical definition of the nucleolus in Section 4.3.1. Next, we present the general framework for finding the nucleolus in Section 4.3.2. We detail the designed verifying and updating algorithms in Section 4.3.3 and Section 4.3.4, respectively.

4.3.1. Definition

Since there might be multiple solutions of the core, we are interested in defining the best one among these outcomes. The nucleolus is one of cooperative game theory's most widely known solution concepts. It is considered the "most stable" cost allocation in the sense that it lexicographically minimizes dissatisfaction among all possible coalitions (Perea & Puerto, 2019). In this part, we first define nucleolus from a mathematical view and then describe the tight sets and balancedness proposed by Kohlberg (1971).

- **Definition of the Nucleolus:** We denote the excess of a coalition S as $e(S, \boldsymbol{f}) := C(S) - \sum_{m \in S} f_m = C(S) - \boldsymbol{f}(S)$, where the cost allocation vector is denoted by $\boldsymbol{f}$. It reflects how satisfied the players in coalition S are with the corresponding cost allocation in vector $\boldsymbol{f}$. For any $\boldsymbol{f}$, let $Y(\boldsymbol{f}) = \left(e(\boldsymbol{f}, S_1), \ldots, e(\boldsymbol{f}, S_{2^n-2})\right)$ be excess values of $2^n - 2$ coalitions with respect to cost allocation $\boldsymbol{f}$ that are stored in a none-decreasing order, n is the number of players in the coalition. The vector $Y(\boldsymbol{f})$ is said to be lexicographically greater than another vector $Y(\bar{\boldsymbol{f}})$ if there exists $h \leq 2^n - 2$ such that $Y_i(\boldsymbol{f}) = Y_i(\bar{\boldsymbol{f}}), \forall 1 \leq i < h$ and $Y_h(\boldsymbol{f}) > Y_h(\bar{\boldsymbol{f}})$. We annotate $Y(\boldsymbol{f}) \succeq Y(\bar{\boldsymbol{f}})$. The nucleolus is defined as $\boldsymbol{f}$ that makes $Y(\boldsymbol{f}) \succeq Y(\bar{\boldsymbol{f}})$ for any $\bar{\boldsymbol{f}}$.
- **Tight Sets:** For the cost allocation $\boldsymbol{f}$, the following sets are defined: $\Psi_0(\boldsymbol{f}) = \{\{m\}, m = 1, \ldots, n : f_m = C(\{m\})\}$, $H_0(\boldsymbol{f}) = \{M\}$ and $H_k(\boldsymbol{f}) = H_{k-1}(\boldsymbol{f}) \cup \Psi_k(\boldsymbol{f})$. For $\forall k \geq 1$,

  $$\epsilon_k(\boldsymbol{f}) = \min_{S \notin H_{k-1}(\boldsymbol{f})} e(S, \boldsymbol{f}),$$

  $$\Psi_k(\boldsymbol{f}) = \{S \notin H_{k-1}(\boldsymbol{f}) : e(S, \boldsymbol{f}) = \epsilon_k(\boldsymbol{f})\}.$$

  We regard $\Psi_k(\boldsymbol{f})$ as tight sets in the sense that all possible coalitions that can obtain the same excess $\epsilon_k(\boldsymbol{f})$ are included. In particular, $\Psi_0(\boldsymbol{f})$ is the set of players that cannot gain cost savings from collaboration under the cost allocation $\boldsymbol{f}$; in other words, those players are on the boundary of violating their individual rationality.
- **Balancedness:** Given a set $K_0 \subseteq 2^M$, a set of coalitions $A \subseteq 2^M$ is called K_0-balanced if there exist vector $\tau \in \mathbb{R}_{\geq 0}^{|K_0|}$ and vector $\sigma \in \mathbb{R}_{>0}^{|A|}$ such that

$\mathbf{u}(M) = \sum_{S\in K_0} \tau^{\top}\mathbf{u}(S) + \sum_{S\in A} \sigma^{\top}\mathbf{u}(S).$

Here, $\boldsymbol{f}(S) = \Sigma_{m\in S} f_m = \boldsymbol{f}^{\top}\mathbf{u}(S), \forall S \subseteq M$. More specifically, if player m joins coalition S, its corresponding mth element in vector $\mathbf{u}(S)$ is 1, otherwise 0.

Example 1. Given a 3-player game with costs $C(\{1\}) = -1$, $C(\{2\}) = -2$, $C(\{3\}) = 5$, $C(\{1,2\}) = -6$, $C(\{1,3\}) = -7$, $C(\{2,3\}) = -8$, and $C(\{1,2,3\}) = -12$. If starting from imputation $\boldsymbol{f}^0 = [-1,-4,-7]$, then $\epsilon_1(\boldsymbol{f}^0) = -1$, $\Psi_0(\boldsymbol{f}^0) = \{\{1\}\}$, and $\Psi_1(\boldsymbol{f}^0) = \{\{1,2\}\}$. Hence, the current tight set $\Psi_1(\boldsymbol{f}^0)$ is not $\Psi_0(\boldsymbol{f}^0)$-balanced. If we improve $\boldsymbol{f}^1 = [-2.5,-4,-5.5]$, then $\epsilon_1(\boldsymbol{f}^1) = 0.5$, $\Psi_0(\boldsymbol{f}^1) = \emptyset$, and $\Psi_1(\boldsymbol{f}^1) = \{\{1,2\},\{3\}\}$. After this step, we can see $\Psi_1(\boldsymbol{f}^1)$ is $\Psi_0(\boldsymbol{f}^1)$-balanced.

4.3.2. The proposed framework

We propose an efficient mechanism for computing the nucleolus, which addresses the challenge of calculating each possible coalitional cost for finding nucleolus when the cost is the output of a combinatorial optimization problem. The proposed framework of the mechanism is based on the Kohlberg criterion by Kohlberg (1971) and the improvement on reducing the number of algorithmic iterations by Benedek, Fliege, and Nguyen (2021) and Xiao and Fang (2023). However, their methods to calculate the nucleolus require that the cost of each possible coalition is known. This is quite challenging when each required cost is actually the output of a combinatorial optimization problem. To tackle this issue, in this work, we develop an efficient mechanism consisting of the verifying algorithm and the updating algorithm, which combines our HCBAP optimization model into the iterative search process of the nucleolus. This mechanism effectively avoids the complexity of calculating each possible coalitional cost but incorporates the characteristics of nucleolus to approach the nucleolus solution iteratively with the idea of gradient descent. As described in Algorithm 2, starting from a core solution obtained in Section 4.2, we verify if the current cost allocation is the nucleolus via the verifying algorithm. If the current cost allocation $\boldsymbol{f}$ passes the verifying algorithm, the nucleolus is found; otherwise, our updating algorithm determines which terminals' costs to increase or decrease and by how much, generating a new cost allocation vector that is then verified until the nucleolus is found.

Algorithm 2: The Framework for Calculating the Nucleolus

Input : grand coalition M, core solution $\boldsymbol{f}$
Output: The nucleolus solution $\boldsymbol{f}$

1 *Initialization:* $H_0 = \{M\}$ *and* $k = 1$, $\Psi_0 = \{\{m\} : f_m = C(\{m\}), m \in M\}$
2 Run verifying algorithm:
3 **while** $Rank(H_{k-1}) < n$ **do**
4 Calculate $\epsilon_k = \min_{S\notin H_{k-1}} \{C(S) - \boldsymbol{f}(S)\}$;
5 Form $\Psi_k = \{S \notin H_{k-1} : e(S,\boldsymbol{f}) = \epsilon_k\}$
6 **if** $\bigcup_{j=1}^{k} \Psi_j$ *is* Ψ_0*-balanced* **then**
7 $H_k = H_{k-1} \cup \Psi_k$, and $k = k+1$
8 **else**
9 Output the current largest Ψ_0-balanced set $U \subset \Psi_k$;
10 $\boldsymbol{f}$ is not the nucleolus, go to line 16
11 **end**
12 **end**
13 **if** $\boldsymbol{f}$ *is the nucleolus* **then**
14 stop
15 **else**
16 Run updating algorithm:
17 Generating direction λ and step size ρ;
18 Update $\boldsymbol{f} = \boldsymbol{f} + \rho\lambda$;
19 Return to line 1
20 **end**

4.3.3. Verifying algorithm

The verifying algorithm is to verify if a cost allocation $\boldsymbol{f} = \{f_1, \ldots, f_m\}$ is the nucleolus solution. As shown in Algorithm 2, the tight sets $\Psi_j (j = 1,2,\ldots,k)$ is formulated iteratively, observing $\epsilon_1(\boldsymbol{f}) > \epsilon_2(\boldsymbol{f}) > \cdots > \epsilon_k(\boldsymbol{f})$. The algorithm stops either at line 10 of Algorithm 2, where the union of tight sets is found not Ψ_0-balanced, or at line 12 of Algorithm 2, where the rank of H_{k-1} reaches n. The output U_k is the union that satisfies Ψ_0-balancedness and contains the largest number of possible coalitions. $U = \Psi_k$ means the checkness is passed in this iteration k, while $U \subset \Psi_k$ means the Ψ_0-balancedness check fails. The cost allocation $\boldsymbol{f}$ that can pass the Ψ_0-balancedness check in each iteration $k < n$ is the nucleolus we are finding.

4.3.4. Updating algorithm

The adjustment is to keep the excess of coalitions that already pass the balancedness check but increase that of the most unsatisfied coalitions in the unbalanced set. The procedure to compute a direction vector λ and step size ρ is described in Algorithm 3, which is supported by Proposition 1 and Corollary 1.

Algorithm 3: Updating Algorithm for Generating λ and ρ

Input : Ψ_0, Ψ_k, U
Output: Direction vector λ and step size ρ

Initialization: $\Pi = \emptyset$
Obtain an adjusting direction λ via **UDP**
for $\forall S \in \mathcal{N}$ **do**
if $1 - \lambda(S) > 0$, *and* $1 - \lambda(S) \notin \Pi$ **then**
$\Pi = \Pi \cup \{1 - \lambda(S)\}$
end
end
$\rho = -\infty$
for $\forall \chi \in \Pi$ **do**
$obj = \max\big(\epsilon_k(\boldsymbol{f}) - e(S,\boldsymbol{f})\big)$
s.t. $1 - \lambda(S) = \chi$
if $\frac{obj}{\chi} > \rho$ **then**
$\rho = \frac{obj}{\chi}$
end
end
$\rho = \max\left(\{\frac{C(\{j\})-f_j}{\lambda_j} : \lambda_j < 0\} \cup \rho\right)$

Proposition 1. *If there exists a coalition $\bar{S}$ (possibly more than one) such that $\epsilon_k(\boldsymbol{f}+\rho\lambda) = e(\bar{S},\boldsymbol{f}+\rho\lambda)$ at iteration k, then when $\lambda(S) < 1$, $\lambda(\bar{S}) \geq 1$ and $\rho < 0$, the distance between the excess of each coalition and the current minimal satisfaction decreases after adjustment.*

Proof. $\forall S \in M$, at iteration k, the change of excess after adjustment is $e(S,\boldsymbol{f}+\rho\lambda) - e(S,\boldsymbol{f}) = C(S) - \big(\boldsymbol{f}(S)+\rho\lambda(S)\big) - \big(C(S)-\boldsymbol{f}(S)\big) = -\rho\lambda(S)$. The distance between the excess of each coalition and the current minimal satisfaction for $\boldsymbol{f}$ is $\ell(\boldsymbol{f}) = e(S,\boldsymbol{f}) - \epsilon_k(\boldsymbol{f})$, and for $\boldsymbol{f}+\rho\lambda$, it is $\ell(\boldsymbol{f}+\rho\lambda) = e(S,\boldsymbol{f}+\rho\lambda) - \epsilon_k(\bar{S},\boldsymbol{f}+\rho\lambda)$. The gap in the distance after adjustment is $\Delta = \ell(\boldsymbol{f}+\rho\lambda) - \ell(\boldsymbol{f}) = \rho\big(\lambda(\bar{S})-\lambda(S)\big) \leq \rho\big(1-\lambda(S)\big) < 0$. □

Based on Proposition 1, we formulate the following model, denoted as **UDP**, to find an adjusting direction λ.

$$\min \sum_{\Omega\in\Psi_k\setminus U} \sum_{j\in\Omega} \lambda_j \tag{38}$$

Subject to:

$$\sum_{j\in\Omega} \lambda_j \geq 1 \quad \forall\Omega \in \Psi_k \setminus U \tag{39}$$

$$\sum_{j\in\Omega} \lambda_j \geq 0 \quad \forall\Omega \in \Psi_0 \tag{40}$$

$$\sum_{j\in\Omega} \lambda_j = 0 \quad \forall\Omega \in U \setminus \Psi_k \tag{41}$$

Table 3
Results of an illustrative example.

Terminal	Stand-alone	RG-based core		The nucleolus	
		Cost	Improvement	Cost	Improvement
1	939.00	939.00	0.00%	939.00	0.00%
2	1482.00	1086.7	26.67%	1038.60	29.91%
3	1693.00	1217.3	28.09%	1265.40	25.26%
Total	4114.00	3243.00	21.17%	3243.00	21.17%

Recall that three sets are formed when we check the Ψ_0-balancedness: Ψ_0, Ψ_k, and U. Ψ_0 contains all players whose distributed costs cannot be increased, that is, they are on the boundary of violating individual rationality. Ψ_k is the tight set, and U is the largest Ψ_0-balanced set. We aim to find one coalition $\bar{S}$ satisfying Proposition 1 restricted in set $\Psi_k \setminus U$, which can guarantee the balanced set unchanged. The optimal solution is to find a direction vector satisfying $\lambda(\bar{S}) = 1$, $\lambda(S) < 1$, and $\Delta < 0$.

Corollary 1. *At iteration k, for* $\forall \lambda(S) < 1$ *and* $\rho < 0$*,*

$$\rho \geq \frac{\epsilon_k(\boldsymbol{f}) - e(S, \boldsymbol{f})}{1 - \lambda(\boldsymbol{S})}.$$

Proof. For any S at iteration k, $e(S, \boldsymbol{f}) \geq \epsilon_k(\boldsymbol{f})$. Thus, $e(S, \boldsymbol{f} + \rho\lambda) = e(S, \boldsymbol{f}) - \rho\lambda(S) \geq \epsilon_k(\boldsymbol{f} + \rho\lambda) = \epsilon_k(\boldsymbol{f}) - \rho\lambda(\bar{S})$. After rearranging, we get $e(S, \boldsymbol{f}) + \rho\big(\lambda(\bar{S}) - \lambda(S)\big) \geq \epsilon_k(\boldsymbol{f})$. Given $\lambda(\bar{S}) = 1$ and $\lambda(S) < 1$, $\rho \geq \frac{\epsilon_k(\boldsymbol{f}) - e(S,\boldsymbol{f})}{1-\lambda(S)}$. □

As we decrease ρ from 0, the smallest moving step should be the largest ρ. Therefore, for cost allocation $\boldsymbol{f}$ at iteration k, the following model is proposed to calculate the adjusting step size ρ.

$$\max \quad \frac{\epsilon_k(\boldsymbol{f}) - e(S, \boldsymbol{f})}{1 - \lambda(S)} \tag{42}$$

Subject to:

$$\lambda(S) < 1 \tag{43}$$

Specifically, the objective function (42) is non-linear. Thus, we decompose the problem into several subproblems. The general procedure is to group S with the same value of $1 - \lambda(S)$, and then find the maximum value of $\epsilon_k(\boldsymbol{f}) - e(S, \boldsymbol{f})$ within each group. Finally, we choose the maximum one among all groups. The details are described from line 3 to line 15 in Algorithm 3. Besides, individual rationality should also be considered. Therefore, at iteration k, the step size is:

$$\rho = \max\left(\left\{\frac{\epsilon_k(\boldsymbol{f}) - e(S, \boldsymbol{f})}{1 - \lambda(S)} : \lambda(S) < 1\right\} \cup \left\{\frac{C(\{j\}) - f_j}{\lambda_j} : \lambda_j < 0\right\}\right).$$

4.4. An illustrative example for comparing the core and the nucleolus

We have shown the overall cost savings for all terminals participating in the collaboration. In this section, we focus on the cost allocated to the individual terminal so that they will stay in the coalition, in other words, maintaining collaboration stability. First, we illustrate a small instance's core and nucleolus relationship with $|N| = 10$, $|M| = 3$, and $|B_m| = 3$. The results are shown in Table 3. In this case, the cost of grand coalition is $C(\{1, 2, 3\}) = 3243$, and the stand-alone cost of each terminal is $C(\{1\}) = 939$, $C(\{2\}) = 1482$, and $C(\{3\}) = 1693$.

Fig. 4 presents the individual cost of each terminal by two different cost allocation methods. Compared with the stand-alone method, individual costs of Terminal operator 2 (T2) and Terminal operator 3 (T3) are reduced by joining the collaboration, and Terminal operator 1 (T1) remains the same.

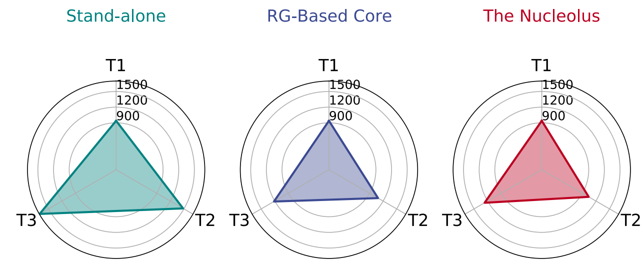

Fig. 4. Different individual costs of an illustrative example with three Terminals.

We first analyze the individual cost of T1 with the core and the nucleolus. The coalition is stable even though T1 has no extra benefits. First, it respects the definition of the core, which requires that no sub-coalition (including working alone) can bring more profits than forming the coalition. Thus, T1 can also not get more if they work alone. However, there are many other benefits for T1 to join the coalition. For example, by joining the coalition, they can improve the service level by providing candidate space for customers. It may not bring extra profits in some instances, but it may save a lot in other cases. No matter in which cases, T1 will not perform worse than working alone. Therefore, even for some members with no extra benefits, the core and the nucleolus can still maintain collaboration stable according to cooperative game theory.

Next, we compare the cost savings between T2 and T3. The overall cost savings achieved by forming the grand coalition are 871. Fig. 5 illustrates the allocation of this 871 under two cost allocation methods: the RG-based core and the nucleolus. The figure demonstrates that the difference in cost savings between T2 and T3 is smaller under the nucleolus than the RG-based core. This indicates that the nucleolus method distributes the total cost savings from collaboration more equitably among the terminals, thereby providing a stronger incentive for stable collaboration. In other words, the nucleolus method enhances stability more effectively than the core method.

We use Barycentric coordinates to illustrate the cost allocation in Fig. 6, where the vertex is defined as the maximum cost (stand-alone cost) each terminal can accept, and each point inside the triangle represents a cost allocation. The definition of the core maps a stable area in which there is no incentive for terminals to leave the grand coalition. As can be seen, the RG-based core falls into the stable zone, and the nucleolus is also in the core.

5. Computational experiments

In this section, numerical experiments are carried out to evaluate the performance of the proposed HCBAP model and the developed cost allocation algorithms for the stability of collaboration. The MILP model is solved using CPLEX 12.7 with a time limit of 7200 s. We code the presented algorithms in C++, and the experiments are conducted on the computer using one node with 12 cores, 2x Intel XEON E5-6248R 24C 3.0 GHz, and 192 GB of RAM.

5.1. Instances

We created instances containing three vessel classes with the corresponding cost rates (Meisel & Bierwirth, 2009) and loaded or discharged container quantity (Lyu, Jin, & Hu, 2020), shown in Table 4. The notation U represents a uniform distribution of the given range. The vessel set of each instance consists of 60% Feeder, 30% Medium, and 10% Jumbo. The expected arrival time is randomly generated within the planning horizon (T = 168 h), and the expected departure time is also obtained successively. The number of QCs equipped at each terminal is set between 2 to 10. Besides, for each terminal, the cost rate

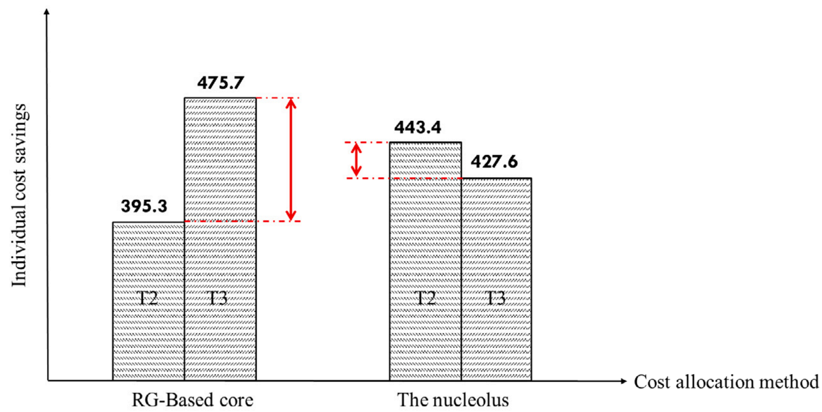

Fig. 5. Allocations of the overall cost savings.

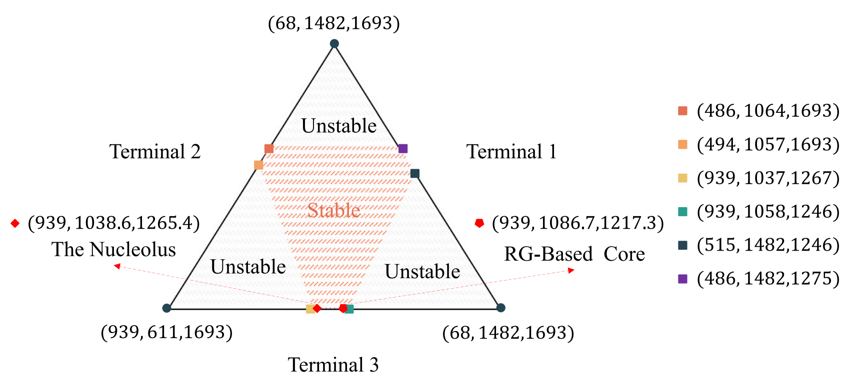

Fig. 6. Illustration of the relationship between RG-based core and the nucleolus.

Table 4
Vessel-related parameters setting (unit is given in 10 dollars).

Class	Unit cost of vessel $i \in V$			Container quantity(g_i)
	QC-hour demand (w_i)	Waiting (c_i^{ω})	Tardness (c_i^{δ})	
Feeder	U[5,15]	U[100,199]	U[100,199]	U[500,3500]
Medium	U[15,50]	U[200,299]	U[200,299]	U[3500,5000]
Jumbo	U[50,65]	300	300	U[5000,7500]

of QC service is set as $c^{\kappa} = 10$ (Meisel & Bierwirth, 2009). We pre-set the terminal where each vessel is contracted to visit, and the distance between terminals is generated randomly.

In the disruption scenarios, vessel delays were introduced at varying levels of intensity. Specifically, delays were applied to randomly selected proportions of the total vessel fleet, with three distinct levels: 20%, 40%, and 60% of vessels experiencing delays. The duration of these delays was generated using a uniform distribution, denoted as U[5,15], meaning that the delay time for each affected vessel was randomly determined within a range of 5 to 15 units of time.

5.2. Improvement with the HCBAP model

To evaluate the effectiveness of the proposed collaborative strategy for berth allocation, we compare the results of our HCBAP model with the stand-alone planning method. The stand-alone method reflects an independent decision-making process without collaboration among terminals: the vessel can only wait at the contracted terminal until there are available berths and QCs, while our HCBAP model allows the vessels to transfer to other terminals for a better berthing plan. Therefore, in the experiments, we compare the total costs of all terminals by stand-alone method and by our HCBAP model respectively to show the benefits of the proposed HCBAP model.

When one vessel's arrival or departure is delayed compared to the initial plan, it will impact the following vessels' berthing plan. The decision-makers must adjust the initial plan to avoid further mess in the port. This is one typical disruption scenario that happens frequently in practice. Considering the setting of our HCBAP model is based on consolidating berthing resources of multiple terminal operators, which broadens the choices that can be made by terminal operators for the berth allocation process, thus, its benefits are expected to perform better under disruption scenarios. To compare the HCBAP model's performance in dealing with disruption caused by vessel delay, we define the total berth allocation costs indicated by the objective function as recovery costs when testing the model in cases of disruptions.

In Table 5, the first column states the instance properties, including the number of vessels $|V|$, the number of terminals $|M|$, and the number of berths at each terminal $|B_m|$. The columns denoted by "Z" show the total cost defined by function (1) in Section 3.2. Besides, "C_{delay}" reports the cost of tardiness caused by vessel delay, and "C_{trans}" displays the container transshipment cost because of vessel transfer between terminals. The column "T_{opt}" is the time for solving the HCBAP model. The increase in transshipment cost (+), the decrease in tardiness cost (-), and the total cost savings (-) by the HCBAP model are indicated in column"IC_{trans}", "DC_{delay}", and "Z_{save}" respectively.

Table 5
Comparison between HCBAP model and stand-alone method.

Instance	Stand-alone method			HCBAP model				Improvement		
$\|V\|-\|M\|-\|B_m\|$ (1)	Z (2)	C_{trans} (3)	C_{delay} (4)	Z (5)	T_{opt} (6)	C_{trans} (7)	C_{delay} (8)	IC_{trans} (9)	DC_{delay} (10)	Z_{save} (11)
10-3-3	4010.00	0.00	2306.67	2419.80	16.85	156.46	560.00	+3.90%	−43.56%	−39.66%
10-5-3	3592.67	0.00	1619.33	2826.68	4.04	130.34	723.00	+3.63%	−24.95%	−21.32%
12-3-4	3989.33	0.00	1872.67	3318.33	9.13	150.33	1051.33	+3.77%	−20.59%	−16.82%
12-5-4	5942.33	0.00	2599.00	4670.84	44.06	213.51	1114.00	+3.59%	−24.99%	−21.40%
18-3-4	6905.67	0.00	3715.67	4586.02	14.03	161.68	1234.33	+2.34%	−35.93%	−33.59%
18-4-5	5699.67	0.00	2749.67	4567.96	36.86	234.96	1416.33	+4.12%	−23.39%	−19.86%
20-4-5	7647.33	0.00	4094.00	5164.93	89.04	362.93	1248.67	+4.75%	−37.21%	−32.46%
20-5-5	5952.33	0.00	2682.33	4636.83	67.97	247.14	1119.67	+4.15%	−26.25%	−22.10%
25-5-5	8314.00	0.00	4150.67	6496.58	428.29	355.91	1977.33	+4.28%	−26.14%	−21.86%
28-5-6	13 015.00	0.00	6838.33	8728.07	216.58	526.73	2024.67	+4.05%	−36.99%	−32.94%
30-6-6	9147.00	0.00	4677.00	7052.57	132.57	419.24	1833.33	+4.58%	−31.09%	−22.90%
35-6-7	15 835.00	0.00	7491.67	11 085.76	453.51	738.09	2004.33	+4.66%	−34.65%	−29.99%
40-6-8	21 452.80	0.00	7405.00	12 678.57	2479.23	902	3027.00	+4.20%	−59.12%	−40.90%
45-7-8	22 079.00	0.00	12 159.00	12 735.38	5507.40	945.38	1870.00	+4.28%	−46.60%	−42.32%
Average	–	–	–	–	–	–	–	+4.02%	−33.68%	−28.44%
10-3-3-dr	5262.67	0.00	3559.33	3304.77	25.23	165.77	1065.67	+3.15%	−70.06%	−37.20%
10-5-3-dr	4876.67	0.00	2903.33	3605.09	1.58	132.75	1499.00	+2.72%	−48.37%	−26.07%
12-3-4-dr	5417.33	0.00	3300.67	4099.99	1.13	186.66	1796.67	+3.45%	−45.57%	−24.32%
12-5-4-dr	7841.33	0.00	4498.00	5946.87	24.47	237.54	2366.00	+3.03%	−47.40%	−24.16%
18-3-4-dr	10 065.67	0.00	6849.00	6288.21	7.92	205.21	2893.00	+2.04%	−57.76%	−37.53%
18-4-5-dr	7518.33	0.00	4601.67	5877.75	22.45	247.42	2713.67	+3.29%	−41.03%	−21.82%
20-4-5-dr	8867.67	0.00	5281.00	5862.39	118.76	409.73	1899.33	+4.62%	−64.03%	−33.89%
20-5-5-dr	9112.67	0.00	5842.67	6198.61	50.01	337.28	2591.33	+3.70%	−55.65%	−31.98%
25-5-5-dr	13 959.00	0.00	7563.67	8251.23	53.81	406.90	3681.00	+2.91%	−51.33%	−40.89%
28-5-6-dr	17 516.13	0.00	10 894.67	11 701.61	224.35	582.61	4942.33	+3.33%	−54.64%	−33.20%
30-6-6-dr	15 089.67	0.00	10 286.33	8591.81	408.07	442.81	3349.00	+2.93%	−67.44%	−43.06%
35-6-7-dr	23 001.33	0.00	11 749.00	14 146.30	563.46	827.63	4975.33	+3.60%	−57.65%	−38.50%
40-6-8-dr	22 898.80	0.00	8798.00	13 758.43	473.03	761.43	4247.00	+3.33%	−51.73%	−39.92%
45-7-8-dr	37 576.60	0.00	14 937.00	17 315.72	1453.04	976.72	6419.00	+2.60%	−57.03%	−53.92%
Average	–	–	–	–	–	–	–	+3.19%	−54.98%	−34.75%

We test 28 instance scales with up to 45 vessels and seven terminals. Instances with "−dr" represent the disruption caused by arrival delays of calling vessels in this paper. The average results of three different instances for each scale are displayed in Table 5. As seen in the table, while the collaborative strategy can incur extra container movements between terminals (denoted by the parameter c^t_{im}), the HCBAP model exhibits significant potential in alleviating the overall costs incurred by all terminals through the reduction of time inefficiencies resulting from vessels awaiting service at their designated terminal. Regarding the instances without disruption, although collaborative berth planning incurs extra transshipment costs C_{trans}, it dramatically reduces the tardiness cost C_{delay}. On average, with a 4.02% increase in transshipment cost, the tardiness cost can be decreased by around 33.68%; consequently, our model can result in around 28.44% savings for the total cost. Our HCBAP model also performs well when dealing with disruption, with around 34.75% savings compared with the stand-alone method. Furthermore, the reduction of vessel tardiness also shows excellent potential for releasing port congestion. Thus, the HCBAP model proposed in this paper significantly improved over the stand-alone method without collaboration.

As demonstrated in Table 5, the proposed HCBAP model exhibits superior performance in mitigating the negative effects of disruptions, achieving an approximate 34.75% cost savings compared to the stand-alone method. To further illustrate the effectiveness of the HCBAP model in handling disruptions, we compare the additional costs incurred post-disruption, both with and without collaborative strategies, as depicted in Fig. 7. The results indicate that the HCBAP model significantly reduces recovery costs associated with disruptions, particularly those caused by arrival delays of calling vessels. Moreover, Fig. 7 reveals that as the instance size increases, the cost-saving benefits of the HCBAP model become even more pronounced compared to stand-alone methods. This highlights the model's scalability and its potential for greater economic efficiency in larger, more complex scenarios.

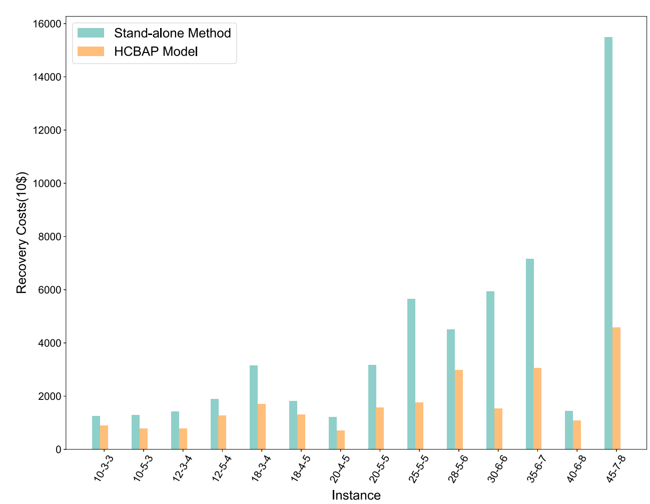

Fig. 7. Comparison of recovery costs in disruption scenarios.

Consequently, the HCBAP model not only improves cost-effectiveness but also enhances the overall resilience in disruption scenarios.

5.3. Results of cost allocations by RG-based core and the nucleolus

In this section, we have calculated the core and the nucleolus based on cooperative game theory to allocate the coalitional costs among collaborative terminals. As described in Section 4, the calculation of the core and the nucleolus requires multiple iterations of the MILP model, and thus it is computationally quite challenging even with a limited number of participants. To better illustrate the computational advantages of our RG-based approach, we compare the time required for solving without it. Solving each subset individually, or "one by one", for the smallest instance (10-3-3-I1) takes 2227.34 s in a single-player setting. To obtain solutions for the core and nucleolus with three players, however, this berth allocation problem (BAP) must be solved for every subset of players, leading to a computational time of around $2227.34 \times 7 = 15591.38$ seconds, or approximately 4.33 h. By contrast, with our RG-based method, the total computation time required is 112 s for the core and 782 s for the nucleolus. For the largest instance (30-6-6-I2), solving it in a one-player setting takes 7208.84 s. Extending this to calculate the core and nucleolus solutions across all player subsets would require an estimated $7208.84 \times (2^6 - 1) \simeq 126$ h, that is, over 5 days. With our RG-based approach, we can achieve a solution in just 2 h. The accepted computational time for operational BAP is within the maximum 3 h (Buhrkal, Zuglian, Ropke, Larsen, & Lusby, 2011; Venturini et al., 2017). This highlights that solving the MILP for each subset individually is impractical. In contrast, as shown in this section, the proposed RG-based algorithm effectively meets the required time constraints.

5.3.1. Comparison with benchmark

For evaluating the performance of the proposed algorithms for calculating the core and the nucleolus, in this section, we compare the cost allocation results for individuals obtained by our algorithms with those obtained by the Proportional to Stand-alone Costs (PSC) method (An & Chen, 2022). The PSC method is based on the rules concluded by Flisberg, Frisk, Rönnqvist, and Guajardo (2015), which distributes costs among all players according to their stand-alone costs, and the formulation of PSC is as follows:

$$f_i = \frac{C(\{i\})}{\sum_{j \in M} C(\{j\})} C(M),$$

where f_i is the cost allocated to player i, $C(\{j\})$ is the stand-alone cost of player j, and $C(M)$ is the coalitional cost of the grand coalition M.

There are two main allocation concepts in cooperative that grant stability in cooperative game theory: the core and the nucleolus (Moulin et al., 1989). We provide exact solutions for both methods. That is, we can claim with certainty that the provided allocations are stable in terms the relevant theoretical foundations. We present the numerical results of our RG-based Core and the nucleolus in Table 6, comparing them with the PSC method. In addition to the same notations as Table 5, for each cost allocation method, we report the total cost of grand coalition "$\mathrm{Z_{HCBAP}}$", the running time in seconds "Time", the number of terminals that can obtain cost savings "$\mathrm{NO_m}$", and the minimum percentage of individual cost savings "Min", the maximum value"Max" as well as the average value "Ave". Individual rationality has been considered a necessary condition in the core and the nucleolus; thus, we only check that for the PSC method, represented by " IR".

As we can see from column (3) in Table 6, using the PSC cost allocation method, only 16 of 33 instances can satisfy the requirement of individual rationality, which means the individual terminal will cost more within collaboration than without in most cases. Compared with PSC, the definition of our RG-based core and the nucleolus have considered individual rationality as strict constraints, guaranteeing no terminals perform worse than working alone. Notably, in columns (10) and (15), 0.00% cost savings means there are some terminals whose costs in the collaboration are the same as working alone. As long as the costs are not increased, for those terminals, there are many other benefits to form the coalition. For example, by joining the coalition, they can improve their service level by providing more candidate space for their customers. It may not bring extra profits in some instances, but it may save a lot in other cases. No matter in which cases, they will not perform worse than working alone. Therefore, we can say the coalition is stable as long as there is no worse performance in profits in the coalition than working alone. However, by the PSC cost allocation method, many negative figures are shown in columns (5), implying that some players whose costs have even increased after joining the collaboration; for those players, they may choose to leave the coalition, and thus, the collaboration is unstable. Comparing the last row of columns (5), (10), and (15), the average of the minimum cost savings for all 33 instances is −15.76% by the PSC method, while it is 0.00% by our RG-based Core and 4.48% by the nucleolus, showing a considerable improvement on collaboration stability by the proposed RG-based Core and the nucleolus than the PSC method.

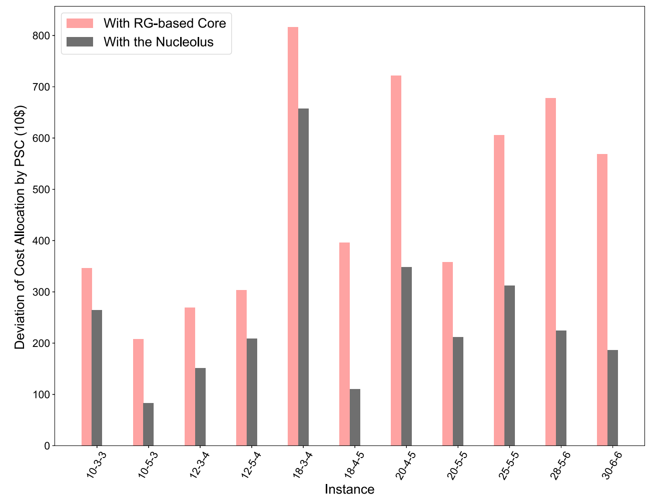

Fig. 8. Deviation of PSC compared with RG-based core and the nucleolus.

To highlight the results by the RG-based core and the nucleolus, we define the term of the great deviation of the cost allocation as the maximum difference of individual cost compared with the PSC method. For example, the great deviation of the cost allocation with RG-based core is calculated by $max_{i\in M}|Ind^i_{RG} - Ind^i_{PSC}|$, where Ind^i_{RG} and Ind^i_{PSC} represent the individual cost of terminal operator i by RG-based core and the PSC method, respectively, and M represents the coalition formed by all terminal operators. In Fig. 8, we illustrate the great deviation of the cost allocation obtained by the PSC compared with the proposed RG-based core and the nucleolus for individual terminals in the coalition. It further highlights the necessity of applying the proposed cost allocation methods rather than the naive PSC method.

5.3.2. The convergence analysis

To analyze the convergence of the proposed RG-based core algorithm and the nucleolus algorithm, we provide a detailed depiction of their iterative processes in Table 7 and Fig. 9. The example instance considered involves $|V| = 10$, $|M| = 5$, and $|B_m| = 3$. f_i represents the cost allocated to player i. The right vertical axis Max violation represents the output of the SP model in Algorithm 1, which approaching 0 indicates that no solution exists in the sub-problem that violates the main problem, signifying that the RG-based core algorithm has converged. The left vertical axis Step size corresponds to the adjustment parameter ρ in Algorithm 2. When it reaches 0, the algorithm has successfully found the nucleolus solution. Fig. 9 clearly demonstrates the iterative improvement of the RG-based core and the nucleolus solutions, thus validating the convergence of the proposed algorithms.

5.3.3. The advantages of the nucleolus over the RG-based core

By analyzing the number of terminals that obtain individual cost savings shown in columns (9) and (14) of Table 6, we observe that the nucleolus solution can always make more terminals gain benefits than the RG-based core solution. This effect is gradually remarkable in larger instances with more vessels and terminals. That means more

Table 6
Numerical results of RG-based core and the nucleolus.

Instance	Z_{HCBAP}	PSC method (An & Chen, 2022)					RG-based core					The nucleolus				
$\|V\|-\|M\|-\|B_m\|-ID$ (1)	(2)	IR (3)	No_m (4)	Min (5)	Max (6)	Ave (7)	Time (8)	No_m (9)	Min (10)	Max (11)	Ave (12)	Time (13)	No_m (14)	Min (15)	Max (16)	Ave (17)
10-3-3-I1	2340.00	✓	3	51.05%	89.31%	65.87%	112	2	0.00%	100.00%	54.07%	782	2	0.00%	74.90%	38.94%
10-3-3-I2	2700.90	✓	3	21.03%	41.27%	34.11%	94	1	0.00%	57.25%	19.08%	201	2	0.00%	29.93%	19.28%
10-3-3-I3	2218.19	–	2	−2.90%	31.43%	19.27%	43	1	0.00%	77.21%	25.74%	56	2	0.00%	54.71%	31.82%
10-5-3-I1	2655.48	–	4	−88.20%	37.40%	7.39%	170	2	0.00%	100.00%	26.08%	8128	5	4.35%	38.34%	25.74%
10-5-3-I2	2689.29	✓	5	19.84%	61.93%	47.42%	67	1	0.00%	80.50%	16.10%	1712	4	0.00%	37.02%	19.48%
10-5-3-I3	2501.59	–	3	−87.37%	24.10%	−9.43%	310	2	0.00%	100.00%	31.81%	2318	4	0.00%	45.21%	23.56%
12-3-4-I1	3046.88	✓	3	15.60%	53.42%	36.67%	63	1	0.00%	36.52%	12.17%	24	2	0.00%	23.45%	13.82%
12-3-4-I2	3152.78	–	1	−25.77%	22.40%	−4.08%	28	1	0.00%	73.23%	24.41%	76	2	0.00%	41.69%	26.28%
12-3-4-I3	3755.33	✓	3	12.57%	62.19%	31.47%	73	1	0.00%	23.91%	7.97%	119	2	0.00%	27.38%	12.80%
12-5-4-I1	3592.20	✓	5	6.57%	68.75%	34.30%	47	1	0.00%	24.01%	4.80%	263	2	0.00%	11.25%	4.16%
12-5-4-I2	4054.55	✓	5	11.34%	83.09%	44.34%	195	3	0.00%	100.00%	42.07%	1118	3	0.00%	100.00%	27.26%
12-5-4-I3	4808.75	✓	5	16.46%	58.93%	34.76%	25	1	0.00%	54.96%	10.99%	3721	3	0.00%	48.48%	18.01%
18-3-4-I1	4752.79	–	2	−99.82%	12.15%	−40.28%	30	1	0.00%	38.66%	12.89%	119	2	0.00%	27.90%	14.68%
18-3-4-I2	4604.88	✓	3	20.80%	60.30%	40.45%	47	1	0.00%	87.19%	29.06%	522	2	0.00%	70.97%	34.44%
18-3-4-I3	4400.38	–	1	−12.46%	42.09%	7.90%	49	2	0.00%	100.00%	54.12%	506	2	0.00%	85.51%	54.57%
18-4-5-I1	4575.86	–	3	−3.49%	49.50%	21.95%	58	1	0.00%	52.58%	13.14%	6808	4	7.86%	28.86%	21.12%
18-4-5-I2	4004.69	✓	4	22.85%	52.35%	41.91%	241	1	0.00%	91.18%	22.80%	3627	3	0.00%	34.30%	21.21%
18-4-5-I3	4122.58	–	3	−56.83%	34.27%	1.41%	43	1	0.00%	67.34%	16.83%	3374	4	13.51%	32.80%	24.45%
20-4-5-I1	5032.56	–	3	−19.80%	39.56%	17.59%	59	1	0.00%	69.70%	17.42%	5510	4	11.38%	33.56%	19.38%
20-4-5-I2	4744.90	✓	4	36.50%	81.20%	59.43%	92	1	0.00%	81.89%	20.47%	4718	3	0.00%	62.18%	26.12%
20-4-5-I3	5717.34	✓	4	37.97%	71.69%	53.23%	79	2	0.00%	100.00%	42.25%	9867	4	7.14%	66.12%	31.48%
20-5-5-I1	5032.56	✓	5	8.68%	38.29%	24.07%	63	2	0.00%	100.00%	24.14%	9215	5	10.52%	38.08%	25.98%
20-5-5-I2	5717.34	–	4	−10.33%	29.37%	14.13%	66	2	0.00%	100.00%	21.29%	4636	5	12.75%	38.14%	24.89%
20-5-5-I3	6427.92	–	1	−92.77%	6.05%	−41.13%	195	2	0.00%	100.00%	20.79%	11 147	5	1.12%	32.90%	21.24%
25-5-5-I1	4914.55	–	3	−64.10%	36.63%	2.93%	83	2	0.00%	100.00%	24.92%	7784	5	17.09%	35.44%	25.88%
25-5-5-I2	4637.01	✓	5	27.80%	41.69%	35.13%	629	2	0.00%	100.00%	20.68%	23 006	5	3.72%	42.61%	21.87%
25-5-5-I3	4718.05	✓	5	8.84%	40.44%	24.46%	67	1	0.00%	54.27%	10.85%	1746	2	0.00%	36.64%	10.56%
28-5-6-I1	8242.96	–	3	−22.75%	41.38%	5.80%	181	1	0.00%	63.45%	12.87%	54 632	5	5.53%	35.72%	22.49%
28-5-6-I2	6958.65	–	4	−0.03%	40.71%	22.92%	238	2	0.00%	100.00%	24.05%	38 183	5	24.73%	40.07%	29.66%
28-5-6-I3	8433.45	–	1	−178.15%	23.44%	−79.64%	101	2	0.00%	100.00%	20.48%	48 793	5	6.76%	37.09%	20.58%
30-6-6-I1	7282.64	–	5	−12.82%	31.73%	14.09%	151	2	0.00%	100.00%	25.83%	43 352	4	0.00%	48.70%	27.73%
30-6-6-I2	6796.29	✓	6	14.03%	54.12%	29.33%	174	2	0.00%	100.00%	17.01%	34 083	6	7.67%	29.43%	19.97%
30-6-6-I3	7078.78	–	4	−74.37%	37.08%	1.79%	355	2	0.00%	100.00%	30.95%	86 400	6	13.87%	40.82%	29.94%
Average				−15.76%	45.40%	18.17%			**0.00%**	**79.81%**	**22.97%**			**4.48%**	**43.34%**	**23.92%**

Table 7
Iterative RG-based core and the nucleolus allocations for an example instance.

Iteration	RG-based core						The nucleolus					
	Max violation	f_1	f_2	f_3	f_4	f_5	Step size	f_1	f_2	f_3	f_4	f_5
1	345.70	12 117.00	10 540.00	7517.00	12 726.00	10 897.15	76.61	12 078.69	10 501.69	7291.71	12 432.35	11 492.69
2	186.98	12 117.00	10 540.00	7517.00	12 092.15	11 531.00	21.76	12 046.05	10 469.05	7302.60	12 475.88	11 503.58
3	293.65	12 117.00	9906.15	7517.00	12 726.00	11 531.00	50.58	12 046.05	10 317.30	7403.76	12 577.04	11 452.99
4	123.00	12 117.00	10 540.00	7330.02	12 279.13	11 531.00	8.46	12 020.68	10 291.93	7420.68	12 593.96	11 469.91
5	0.00	12 117.00	10 540.00	7330.02	12 279.13	11 531.00	0.00	12 020.68	10 291.93	7420.68	12 593.96	11 469.91
6	0.00	12 117.00	10 540.00	7330.02	12 279.13	11 531.00	0.00	12 020.68	10 291.93	7420.68	12 593.96	11 469.91
7	0.00	12 117.00	10 540.00	7330.02	12 279.13	11 531.00	0.00	12 020.68	10 291.93	7420.68	12 593.96	11 469.91

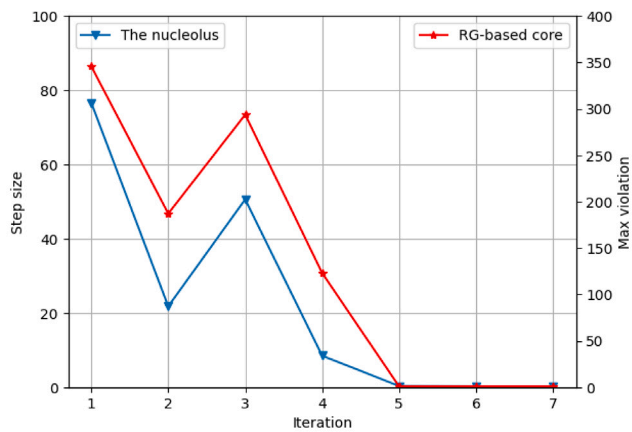

Fig. 9. Illustration of iterative processes for the proposed algorithms.

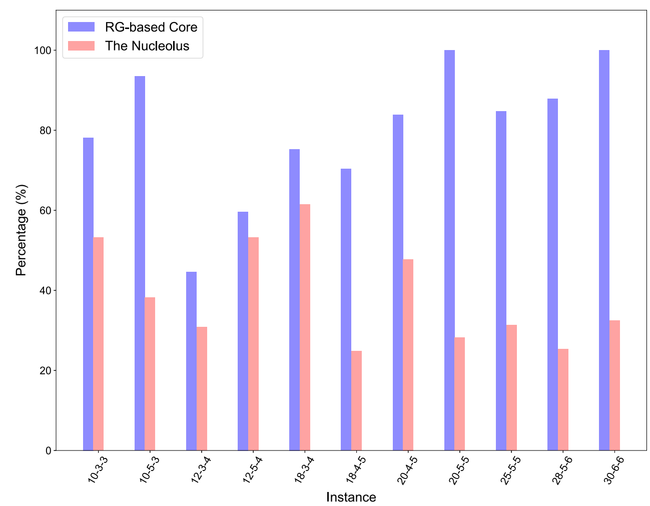

Fig. 11. Difference between Max and Min of RG-based core and the nucleolus.

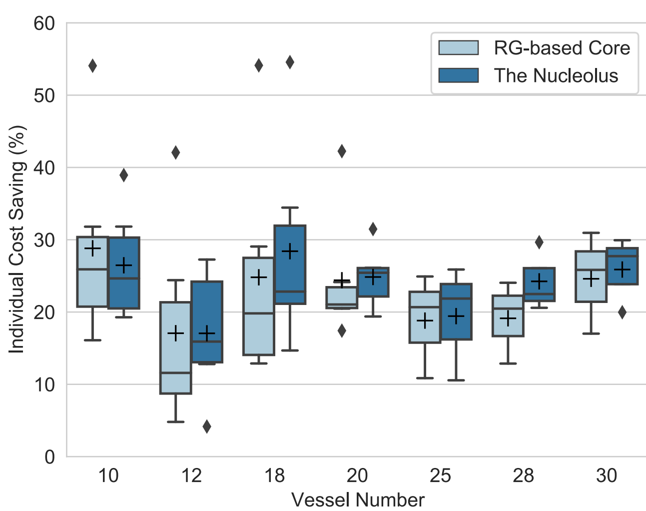

Fig. 10. Analysis of individual cost savings for instances with different vessel size.

individual terminals can be firmly convinced not to leave the coalition; thus, collaboration stability is improved. The minimum and maximum cost savings among terminals further confirm our observation. As seen from Fig. 10, the statistical analysis reveals that the nucleolus method can produce superior individual cost savings compared to the RG-based core. The average minimum cost savings of the nucleolus solution is 4.48%, larger than 0.00% of the RG-based core solution, emphasizing the critical role of the nucleolus in strengthening collaboration stability. In contrast, the average maximum savings is 79.81% in the core, while it turns smaller to 43.34% in the nucleolus solution. Consequently, compared to the RG-based Core, the nucleolus significantly decreases the variance of individual cost savings by 49.67% (on average of all considered instances), indicating that more terminals gain from the cost savings generated by collaboration, and consistently establishing stronger collaboration incentives for terminals.

As shown in Fig. 11, the difference between Min and Max of the RG-based core is considerably larger than that of the nucleolus. In other words, the nucleolus solution allocates the total cost savings brought by collaboration more "evenly" to each terminal. Thus, more terminals benefit from collaboration so that they have a clear incentive to collaborate, and thereby, the stability is further enhanced.

From the computational time shown in Table 6, we can also see that finding a core solution is much faster than finding a nucleolus solution. Thus, the decision-makers can choose the cost allocation strategy according to practical situations. When the core solution is not satisfying enough to convince individual terminals, the nucleolus solution becomes more significant even though its calculation is time-consuming.

6. Discussion and key insights

The following discussion presents key findings from our study on horizontal collaborative berth planning. While the collaborative strategy can incur extra container movements between terminals, the proposed berth allocation based on inter-terminal collaboration exhibits significant potential in alleviating the overall costs by reducing time inefficiencies resulting from vessels awaiting service at their designated terminal. These findings highlight the potential of collaboration in cost savings and facilitate its practical implementation.

(1) *The proposed collaborative berth allocation approach demonstrates a significant cost reduction potential for terminal operators, in both conventional and disruptive scenarios:* Our experimental results showcase significant (28.44%) savings in overall costs in conventional scenarios. Specifically, despite a 4.02% increase in transhipment

costs, the tardiness cost decreased by around 33.68%. In disruptive scenarios, the total cost savings reach 34.75% with a notable reduction of 54.98% in tardiness costs, indicating the positive impact of terminal collaboration on alleviating port congestion after disruptions.

(2) *Collaboration among terminals is an effective means to reduce recovery costs after disruptions:* In managing disruptions, terminal operators have to face additional costs associated with berthing plan adjustment, referred to as recovery costs. However, our results indicate that collaboration among terminals provides an economically viable opportunity for vessels to transfer to another terminal for earlier service. On average, the proposed collaborative berth allocation approach has been shown to reduce recovery costs by 54%, with a range of 30% to 70%, thus significantly enhancing the resilience of terminal operations.

(3) *Stable and attractive collaboration incentives (with the core and nucleolus) can be achieved at a moderate cost:* Terminal operators may decline to join a coalition if they do not see clear benefits by comparing their individual costs/gains in a collaborative setting vs. a non-cooperative setting (also referred to as a stand-alone setting). The results of our numerical experiments calculating the core and the nucleolus ensure that the cost allocated to individual terminals does not exceed their stand-alone costs, on average, achieving savings of 22.97% and 23.92% for individual costs, respectively, thereby maintaining stable collaboration incentives. That is, ensuring stable and attractive collaboration incentives (based on the core and nucleolus) results in average savings that are only about 5 percentage points lower than the hypothetical collaboration optimum (28.44%).

(4) *Simple allocation methods bear the risk of unstable collaboration incentives:* Applying the PSC cost allocation method (An & Chen, 2022), which distributes costs based on stand-alone costs, the experimental results reveal an undesirable trend. In over half of the instances (17 out of 33) examined, individual costs for terminal operators remained increasing despite a decrease in overall costs. The PSC cost allocations have largely deviated from our RG-based core and the nucleolus solution, where the maximum deviation of individual costs with RG-based core and the nucleolus can occupy 36.27% and 35.26% of the total costs, respectively. These findings highlight the inherent instability of collaboration among terminal operators when cost allocations provided by the core and nucleolus are not adequately considered.

(5) *The nucleolus increases the number of terminals benefiting from collaboration for all instances:* In our experimental instances, the nucleolus solution outperforms the RG-based core in all instances examined regarding the number of terminals achieving cost reductions. That is, the nucleolus allocations yield more terminals with actual improvements, while the core solutions have more terminals that do not improve (i.e., remain with the standalone costs) by collaboration. Moreover, compared to the RG-based core solution, the nucleolus significantly decreases the variance of individual cost savings in a coalition by 49.67% (on average over all considered instances), indicating that more terminals gain from the cost savings generated by collaboration. Thus, the nucleolus solutions consistently establish stronger collaboration incentives for terminals.

7. Conclusions and future work

Collaboration has become vitally important as a strategy in the maritime sector to respond to disruptions in global supply chain networks, for instance, imposed by the COVID-19 pandemic or the Red Sea crisis. The container crisis further highlighted that container terminals have a crucial role as scarce resources in these global networks. As a result, new alliances and digital platforms are introduced in an attempt to facilitate collaborative planning of maritime transport operations. Related research, nevertheless, often either entirely disregards the collaboration incentives of the involved parties or assumes unrealistically small problem sizes, which significantly limits the application potential for real-world problems. Consequently, collaboration becomes unstable. That is, actors may be hesitant to engage in collaboration or leave a collaboration because they do not perceive a clear benefit in comparison to a non-collaborative scenario.

In this work, we suggest a collaborative berth allocation approach and propose new row-generation-based algorithms that allow us to obtain exact solutions for stable collaborative berth allocation, based on the game theoretic concepts of the core and the nucleolus. The core ensures stable cost allocation in which a non-collaborative strategy or splitting up will never be more attractive than collaboration. The nucleolus maximizes the number of participants with a clear gain from the collaboration, thus providing the most stable collaboration incentives.

We find the proposed collaborative berth allocation approach leads to significant average cost savings (28.44%) in comparison to the non-collaborative strategy, even after deducting costs related to additional container movements. We further observe that ensuring stable and attractive collaboration incentives (based on the core and nucleolus) results in average savings of 22.97% and 23.92%, which are only slightly below the hypothetical optimum of unconstrained collaboration. Comparing these results to those of a simple cost allocation method from the related literature, we find that the simple method violates the stability criteria of the core (i.e., collaboration leads to increased costs for players) in almost 17 of 33 considered instances. We also see that delays caused in disruption scenarios are reduced when applying the proposed collaborative optimization approaches, and related recovery costs are reduced by 54.98% on average. Finally, our results demonstrate that the nucleolus increases the number of actors with clear collaboration benefits, showcasing an average 49.67% decrease in the variability of individual payoffs in comparison to the core solutions.

These findings extend earlier research on the multiport berth allocation (Martin-Iradi et al., 2022; Venturini et al., 2017) by investigating a new form of collaboration in berth allocation and proposing exact and stable collaboration mechanisms based on the core and the nucleolus. To the best of our knowledge, the proposed row-generation approach is the first of its kind to obtain exact nucleolus solutions for combinatorial optimization problems. Both row-generation algorithms provide general-purpose solution approaches for a large set of related (np-hard) collaborative assignment, routing, or scheduling problems. On the other hand, in terms of practical implications, this study confirms the potential to explore advanced collaboration for efficient and resilient maritime transport and highlights the importance of using stable allocation methods to create strong and lasting collaboration incentives. In some ports, all terminals are governed by a single operator. This would be attractive if the core was empty. However, across the considered problem instances, we can consistently find solutions that are in the core. That is, considering our (operational) results, the formation of such sub-coalitions is rather unattractive, and one might suggest the involved terminal operators to rather explore the formation of coalitions that obtain solutions inside the core, for instance, by adopting platform business models as already established in urban mobility.

Given the practical relevance of the problem, it is important to note that the number of deep-sea terminal operators within a port is typically not very large. For instance, in the Port of Hong Kong, there are nine terminals, which are managed by fewer than five distinct operators (Hong Kong Maritime and Port Board, 2023). In our work, the maximum size of the experimental examples involves up to six terminal operators, reflecting a realistic scenario that aligns with current industry practices. While the proposed two cost allocation algorithms obtain optimal results for a realistic number of players in the required time, there are computational limitations that may become relevant for larger related problems. However, heuristic algorithms for MILP model may sometimes converge to a local optimum, providing only an approximation that cannot always satisfy the prerequisite of the cooperative game theory. Consequently, the accuracy of core and nucleolus calculations, which rely on the precise optimal values for possible coalitions, may be compromised. Such approximations could result in some players identifying better cost allocations on their own, reducing their incentive to participate in the coalition.

In the future, developing highly efficient exact algorithms, could broaden the scope of applications. Recently, some papers on integrated column- and row-generation approaches have been published with promising results. However, these works do not consider a master problem that ensures the game theoretic core conditions. Thus, integrating column and row-generation would require a novel type of algorithm, which certainly would make a promising ambition for future work. Moreover, future research could also set out for an integrated collaborative berth allocation model combining the horizontal collaboration among terminals and the vertical collaboration between shipping companies and terminal operators for higher performance on cost savings (or profit improvement).

CRediT authorship contribution statement

Xiaohuan Lyu: Writing – review & editing, Writing – original draft, Visualization, Methodology, Implementation, Conceptualization. **Eduardo Lalla-Ruiz:** Writing – review & editing, Methodology, Supervision, Conceptualization. **Frederik Schulte:** Writing – review & editing, Writing – original draft, Methodology, Supervision, Conceptualization.

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