The Dynamical Behaviour and Control of an Enterprise

Firm and Consumer as the System, Management as the Controller

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Master of Science Thesis

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Abstract

This thesis develops a method to forecast the cash flows that determine the value of an enterprise. From a control engineers perspective, three stages arise to develop such a method: the research, the system and the control part.

The research part presents the results of a control theoretic analysis of PricewaterhouseCoopers' (PwC) valuation methods. The subject of the analysis in this thesis considers an enterprise in the car leasing business. It addresses the fact that current cash flow forecast methods have limitations that can be overcome by using the tools of Systems and control. This part determines that three important agents influence the cash flow of an enterprise: the firm, its management and its consumer.

The system part develops an LTI model in such a way that its dynamical behaviour aligns with the behaviour of the firm and its consumer. This part uses bond graphs and the analogies of economic engineering to build a fifth-order mechanical system that corresponds to the firm and its consumer. The system includes the interaction between the firm and its customer in order to determine price, output, and resource allocation decisions.

The control part designs two controllers that align with the control actions of management in various market situations. Proportional-integral-derivative (PID) control as management deals with growth reference signals and unexpected disturbances in future market situations. Furthermore, management strives to maximize profit. This part designs an optimal controller that optimizes energy flows to maximize the profit of an enterprise by analogizing cash flow as energy.

The benefit of modeling an enterprise with a control theoretic approach is that the model incorporates high-order effects such as price changes and management decisions, which results in a more realistic representation of reality.

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Preface and Acknowledgments

This basis for this research originally stemmed from my passion for System and Control (S&C). During my master, I got interests in finance, especially on valuations. Through the field of economic engineering, I got the opportunity to develop knowledge on S&C and finance at the same time. During this thesis, I did not only learn about the theoretic difference between the two worlds, but did learn from the socially and way of thinking differences, which were enriching. I am very thankful for this opportunity.

Dr.Ir. M.B. Mendel, I am grateful for all your time and plenty of discussions on how economics and mechanics are linked. You are an incredibly enthusiastic person, which drives the people around you. You want to give your student not only lessons study material, but you also want to help the student as a person. Your group meetings create a very pleasant and therefore productive environment to work in, and your cooperative spirit has made the whole endeavour truly a pleasure.

Rob Jellema, thank you for all your time with helping me on the valuations department and coming to my presentation. I started with less knowledge as an intern that you are used to. But you and all valuation consultants at PwC brought me to a high level of finance knowledge. Manoel de Goeij, thank you for giving me the opportunity to do an internship at PwC. I hope the TU Delft and you have a long and good collaboration.

Also, I would like to thank the rest of my assessment committee, dr.ir. A.J.J. van den Boom and dr.ir. K. Hemmes, for taking the time to read through this whole report and reviewing it, for assessing my presentation and the work as a whole. It is very much appreciated.

Finally, my parents, friends and girlfriend thank you for your support and wise counsel.

I hope you enjoy your reading.

Justin Mulder

P.S. The important problems, solutions, findings and equations are emphasized by enclosing them in a cyan box.

Chapter 1

Introduction

1-1 Economic Engineering

Economic engineering is a new field of research that treats economic systems as causal dynamical systems. Through mechanical and electrical modeling, the internal dynamics of economic systems are revealed, and control formalisms can be applied. Analogies between economic phenomena and mechanical or electrical elements, are the foundation of the theory of economic engineering[2]. This thesis contributes to the field of economic engineering by applying the theory to corporate finance.

Chapter 3 presents engineering background material that is used throughout this thesis. This chapter explains the fundamentals of economic engineering and bond graph modeling.

1-2 Problem Statement of the Cash Flow Forecast

Business valuation is the process of determining the present value of an enterprise [3]. Typically, the valuation department of one of the Big Four accounting firms -PwC, Deloitte, EY, and KPMG- is hired to preform the valuation of an enterprise. Chapter 2 presents the research I conducted by doing an internship at PwC to investigate the procedures on valuing an enterprise.

There are two ways of determining the present value of an enterprise. The most common way is that valuation consultants predict the future cash flows of the enterprise and use the discounted cash flow model (DCF) to determine the present net value of an enterprise[3]. The other way to value an enterprise is the relative valuation method. Relative valuation compares the cash flow to that of its competitors. Both methods are highly dependent on the cash flow forecast of the enterprise [3].

The identified problem is that the current valuation methods do not incorporate high-order effects on the predicted cash flow and do not sufficiently take into account management decisions to various future market situations. Although the theoretical basis of the DCF is undisputed, it is limited in its use because of the practical issues surrounding the forecasting of the future cash flows.

1-3 The Firm and Consumer as a System

This thesis separates three important agents that influence the cash flow of an enterprise: the firm, its consumer and its management. The supply of goods, the inventory allocation, the demand and the satisfaction of the consumer are fundamental elements in the behavioural theory of a firm [4]. A firm wants to sell its goods for the best price to maximize its profits. The customer gets satisfaction from consuming a good or service. In order to consume a product, the consumer will first have to purchase or lease the product. The behaviour of the consumer influences the behaviour of the firm and vice versa.

Chapter 4 develops a system that has the behavioural dynamics of a firm in de car leasing business. Chapter 5 adds the consumer of the car leasing business to the system. The system is based on the analogs between masses, springs, and dampers on the one hand, and the demand and supply, storage, and expenses on the other hand. The chapter interpreters physical properties like force as the cost of holding an asset and energy as a cash flow of the firm.

The LTI system theoric approach contributes in two ways. The first contribution of this approach is that the forecast of the cash flow takes into account the high-order effects of e.g. a price change. The system gives one insight into what the reasons are of these price changes and what kind of impact they will have on other elements of the firm and consumer. The second contribution of this approach is that it enables us to apply control formalisms to the firm and its consumer.

Chapter 6 validates the system of the firm and its consumer by testing two scenario's and cross-checking the models outcome against the expected behaviour.

1-4 The Management as Controller

Good management is the backbone of any successful company. Employees are also very important, but it is management that ultimately makes the strategic decisions. Management seeks to make the correct decisions in the best interest of the firm. Examples are attaining a positive growth or maximizing the enterprise's profits.

The idea is to view management as a controller acting on the system to imitate the behaviour of management. Chapter 7 applies different types of control on the dynamical system of the firm and the consumer. The chapter applies two types of control to model the management of an enterprise. The first type of control is a PID controller that simulates management to retain an inventory level like an autopilot of an airplane retains a certain height. The second type is a optimal controller that strives to maximizes certain energy flows, like management strives to maximize profit.

The contribution of the application of control theory is that it adds management control to the model of an enterprise. The forecast of the enterprise's cash flow becomes more realistic by simulating management decisions in various market situations.

Chapter 2

Economic Background Material

2-1 Introduction

In order to find a method to value an enterprise as an control engineer, I first had to learn how valuation in practice works and identify its limitations that can be overcome by the tools of Systems and Control. My objectives for the PwC internship are stated as:

Learning objectives:

1. Learn how valuation works in practice
2. Find a valuation problem that seems solvable by an approach of a control engineer

The most common method used by consultants to determine the value of a firm is the discounted cash flow method, commonly referred to as DCF. Section 2-2 explains the DCF method, which involves the critical concepts of future payment surpluses and the weighted average cost of capital (WACC).

During my PwC internship, I worked on the valuation of a specific enterprise in the car leasing business. Section 2-3 shows what major value drivers of the revenues and the costs of an enterprise in the car leasing business. Section 2-4 explains how future revenues and costs are determined within the context of a car leasing enterprise. Section 2-5 concludes the main limitations of the forecast cash flow forecast that possibly can be overcome by a control engineer's approach.

Summarizing the important findings of this chapter:

Important findings:

- The valuation of a firm depends mainly on the predicted cash flows and the discount rate (WACC)
- The forecast of cash flows do not account for high-order effects
- The forecast of cash flows not sufficiently consider different management decisions in variant market situations

2-2 Discounted Cash Flow Method

The basic concept of the DCF model can be illustrated with a farm analog. A cow is worth as much as the milk it gives. Free cash flow (FCF) is represented by the milk produced by an enterprise. So an enterprise is worth as much as the FCF it generates.

Free cash flow (FCF) is the sum of the payment surpluses, which is generated through the operating business of an enterprise. The free cash flow can either be paid out to the financiers or kept within the firm for extra cash or investments. The financiers are the creditors or the shareholders of an enterprise.

An enterprise promises surplus payments for the future, defined as FCF_n with n being the period of the payment. The key concept used in the DCF method is the time value of money. A dollar today is worth more than a dollar tomorrow, because the dollar today gives a potential earning capacity. The consequence is that the future surplus payments have a lower value today (present value):

Valuation consultants calculate the present value (PV) of a future payment by discounting the payment with the cost of capital. The discount rate is what valuation consultants call the weighted average cost of capital (WACC), pronounced the "*Wack*." The WACC is the rate that an enterprise is expected to pay to all its financiers, which is the minimum rate of return of an enterprise. The present value of a future surplus payment is calculated as follows:

$$\begin{aligned} \text{Present value payment} &= \frac{\text{FCF}_n}{(1+r)^n} \\ n &= \text{years in future} \\ r &= \text{WACC} \end{aligned} \tag{2-1}$$

A cow is worth the sum of all the milk it gives. So the value of an enterprise is the sum of all discounted surplus payments from now to infinity.

$$\begin{aligned} \text{Value} &= \sum_{n=0}^{\infty} \frac{\text{FCF}_n}{(1+r)^n} \\ n &= \text{years in future} \\ r &= \text{WACC} \end{aligned} \tag{2-2}$$

However, given that one cannot predict the whole lifespan of an enterprise, a forecast period of T between 5 or 10 years is commonly used. After those 5 or 10 years, a cow is still worth a particular value: this value is called the terminal value. The terminal value is the estimated value of an enterprise beyond the forecast period.

$$
\begin{aligned}
\text{Terminal Value} &= \frac{(1+g)\text{FCF}_N}{r-g} \\
N &= \text{end-year forecast period} \\
r &= \text{WACC} \\
g &= \text{growth}
\end{aligned}
\tag{2-3}
$$

Combining the forecast period (Equation 2-2) and the terminal value (Equation 2-3), the equation for determining the value with the DCF method is as follows:

$$
\begin{aligned}
\text{Value} &= \sum_{n=0}^{N} \frac{\text{FCF}_n}{(1+r)^n} + \frac{\text{Terminal value}}{(1+r)^N} \\
n &= \text{year in future} \\
r &= \text{WACC} \\
N &= \text{end-year forecast period} \\
g &= \text{growth}
\end{aligned}
\tag{2-4}
$$

Figure 2-1 visualizes an example of the application of the DCF method.

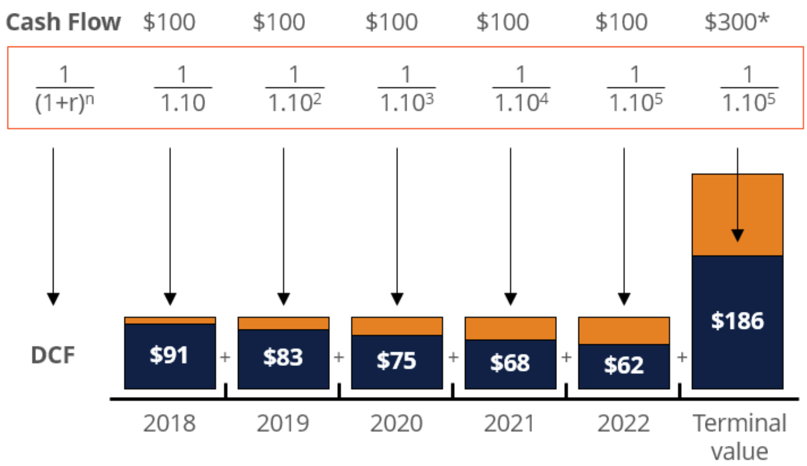


Figure 2-1: [1]The figure shows a visual example of Equation 2-4. The FCF payments are assumed to be $100 per year during the forecast period. The terminal value is assumed to be $300. The WACC is 10%. The present value of all payments is shown in the blue blocks. The total value is the sum of all the individual present values, which makes the value $565.

2-3 Value Drivers of a Car Leasing Enterprise

In order to build a system that represents a firm and its consumer, one should know how the enterprise generates their revenues and cost to determine the operating cash flow. Operating cash flow is a measure of the amount of cash generated by a enterprise's normal business operations[5]. The revenues and costs of an enterprise are called the value drives. An enterprise in the car leasing business has two major revenue drivers[3]:

- The leasing business
- The remarketing business

2-3-1 Revenues

The first value driver is the leasing business. The firm buys cars as assets, which are leased to consumers for a yearly fee [€/(#·yr)]. The firm's total revenue from leasing is:

$$\text{revenue leasing} = \text{avg. lease price} \cdot \text{leased cars} \tag{2-5}$$

The second major value driver is the remarketing business. After a five-year leasing period the car is sold on the second-hand market (remarketing).

Valuation consultants calculate the revenue from the remarketing business by multiplying the number of cars sold per year by the average price paid by the buyer:

$$\text{revenue remarketing} = \text{avg. remarketing price} \cdot \text{cars sold} \tag{2-6}$$

2-3-2 Costs

The costs of the leasing business concern the cost of possessing an asset. The total assets in possession are the leased cars plus the cars that are in inventory.

$$\text{total stock cars} = \text{leased cars} + \text{non-leased cars} \tag{2-7}$$

Storage, depreciation, insurance and maintenance of the cars are the costs. Valuation consultants make use of an average cost per car per year. [€/(#·yr)]:

$$\text{cost of leasing} = \text{avg. cost of leasing} \cdot \text{total stock of cars} \tag{2-8}$$

It can be concluded from Equation 2-5 and Equation 2-8 that the firm generates the highest possible earnings by reducing the number of cars standing still in the parking lot. These cars are making costs but no revenues.

Valuation consultants make no distinction between the costs of the cars Leased and that of the cars stored at the inventory. A car in the lease has a higher wear and tear rate than a car at the inventory. But a car at the inventory generates costs of storage. Car allocation is one limitation I want to solve:

> Valuation consultants make no distinction between the costs of the leased cars and the cars at the inventory.

Because a leasing firm sells the cars after a five-year leasing period, the residual value of a car is less than the initial purchase price of the car. After five years, the car is worth a certain amount (residual value). The profit the firm makes by remarketing a car is the demand price minus the residual value. The residual value of a car is considered the costs of the remarketing.

The yearly costs of the remarketing business are the average residual value of a car multiplied by the number of cars sold yearly.

$$\text{costs of remarketing} = \text{avg. residual value} \cdot \text{demand re-mrkt} \tag{2-9}$$

2-4 Cash Flow Forecast of a Car Leasing Enterprise

2-4-1 Revenues

The forecast of the future prices and future quantities is the hardest part of a valuation. The most common approach to forecast the future prices is to assume a yearly growth rate. The growth rate is based on analyst reports, expected inflation and current management plans:

$$\text{residual value}(t+1) = \text{residual value}(t) \cdot \text{residual value growth rate} \tag{2-10}$$

The forecast of the future demand quantities is done in the same manner as the prices. The demand quantity of the most recent book year is multiplied by an assumed growth rate.

$$\text{demand re-mrkt}(t+1) = \text{demand re-mrkt}(t) \cdot \text{demand remarkted growth rate} \tag{2-11}$$

Similarly the future lease prices and the demand for lease cars is predicted to increase with a growth rate:

$$\text{lease price}(t+1) = \text{lease price}(t) \cdot \text{lease price growth rate} \tag{2-12}$$

$$\text{leased cars}(t+1) = \text{leased cars}(t) \cdot \text{demand lease growth rate} \tag{2-13}$$

2-4-2 Costs

In the same manner as the lease price, the average leasing costs per car has a growth rate based on experience and analyst reports.

$$\text{cost leasing}(t+1) = \text{cost leasing}(t) \cdot \text{lease cost growth rate} \tag{2-14}$$

The total stock of cars is the number of cars in stock the year before plus the supply of cars minus remarketed cars.

$$\text{total stock cars}(t+1) = \text{total stock cars}(t) + (\text{supply} - \text{demand re-mrkt}) \cdot t \tag{2-15}$$

The residual value price and the demand of remarketing increases with a growth rate:

$$\text{residual value}(t+1) = \text{residual value}(t) \cdot \text{residual value growth rate} \tag{2-16}$$

$$\text{demand re-mrkt}(t+1) = \text{demand re-mrkt}(t) * \text{demand remarketing growth rate} \tag{2-17}$$

Example firm's cash flow

This example presents a simple cash flow prediction of the firm's remarketing business. Equations of Section 2-4 are used.
The revenue is predicted as:

Table 2-1: Revenue remarketing business

Year	**2019**	**2020**	**2021**	**2021**
Price per car [€/#]	1,000	1,010	1,020	1,030
Price growth rate		1%	1%	1%
Cars sold [#/yr]	200	204	209	213
Growth sales rate		2%	2%	2%
Revenue [€/yr]	**200,000**	**206,040**	**212,997**	**218,671**

The cost is predicted as:

Table 2-2: Costs remarketing business

Year	**2019**	**2020**	**2021**	**2021**
Residual value per car [€/#]	800	808	816	824
Price growth rate		1%	1%	1%
Cars sold [#/yr]	200	204	209	213
Growth sales rate		2%	2%	2%
Cost [€/yr]	**160,000**	**164,832**	**170.544**	**175.512**

2-5 Conclusions

This chapter concludes two cash flow forecast problems that potentially can be solved by using a control engineer's approach.

2-5-1 No High-Order Effects

Valuation consultants do not take into account the change in cost per car resulting from the change in quantity supplied. Economists refer to the supply cost as the marginal cost. Figure 2-2 shows the marginal cost curve, which demonstrates the relation between the quantity supplied and the cost per unit supplied. The change in supply price is not taken into account during the forecast.

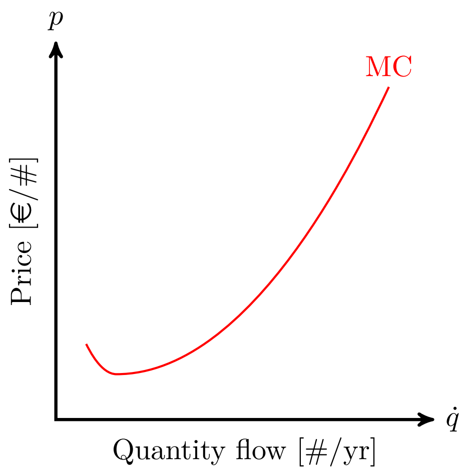


Figure 2-2: Marginal cost curve of an individual firm

Similarly, the forecast of future prices or quantities does not take into account the second-order effect of movements along the demand curve. Changes in the price will lead to changes in the demand quantity. The demand curve of Figure 2-3 is a graphical representation of the relation between the demand quantity and the asking price of a good.

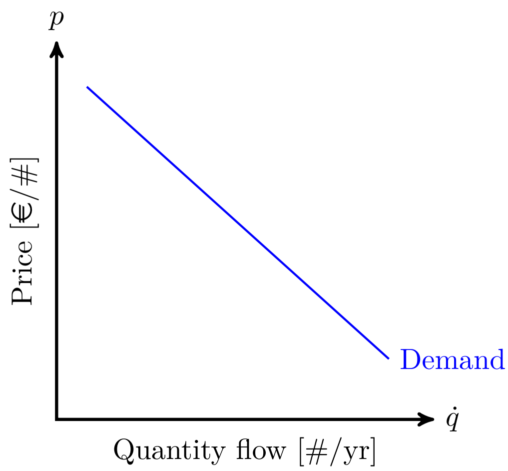


Figure 2-3: Demand curve

Eventually, every action of supply, demand, price change or extra holding costs influence each other. It is called the firm behavioural theory[4]. Resulting in the second limitation I want to solve:

> The prediction of prices or quantities do not influence each other. All the different revenues and costs are stand-alone calculations; no high-order effects are considered.

2-5-2 One Management Plan

Valuation consultants apply only one or two management plans. So management reacts the same, independently of the market situation. Management is not considered as a separate agent. Management plans are incorporated in the growth rates of the prices and quantities supplied or demanded. To make a forecast more accurate, I want to solve the following identified problem:

> Real-life management will react differently to various market or firm situations. Valuation consultants assume that management reacts the same way independently from the firm's situation.

Chapter 3

Engineering Background Material

3-1 Introduction

This chapter provides the background material that is used throughout the thesis.

Section 3-2 explains the fundamentals on the analogs of economic engineering.

Section 3-3 explains the junctions and storage elements in the bond graph theory. Bond graphs are a well-known tool for the graphical description of dynamic physical systems. By describing a system, it guides you to the derivation of system equations and matrices. Bond graphs are commonly used for expressing mechanical or electrical systems [1].

3-2 Economic Engineering

Economic engineering is a new field of research at DCSC TU Delft, which seeks to interpret economic systems as mechanical or electrical systems. It was Dr. M. Mendel, who introduced theories on economic engineering that are used in this thesis.

Mendel proposes an analog with economics similar to the analogs between mechanical systems and electrical systems. Momentum is a mechanical state with flux linkage as its electrical analog. Mendel's economic analogies start with the mechanical state-space coordinates displacement q and momentum p [2]:

$$q := \text{stock of assets} \qquad [\#] \tag{3-1}$$

$$p := \text{price per asset} \qquad [€/\#] \tag{3-2}$$

The displacement is defined as the number of assets held in stock. A positive unit stock is when a firm holds a quantity of stock higher than it aims to be. A negative unit stock is when a firm holds a quantity of stock lower than the firm aims to be. Every asset q_i has a corresponding price p_i. The momentum is defined as the price of one asset. By introducing the momentum as price and the displacement as the number of units in stock, the state of a firm can be described by the state vector $x = \left(q_1 \cdots q_N \; p_1 \cdots p_N\right)^T \in \mathbb{R}^{2N}$.

The time derivatives of the state-space coordinates give economic analogies with c.q. for velocity v and force F:

$$F := \text{benefit or cost} \qquad [€/(\# \cdot yr)] \tag{3-3}$$

$$v := \text{demand or supply flow of asset} \qquad [\#/yr] \tag{3-4}$$

The time derivative of the price is a benefit or a cost. An example of a cost or a benefit is the rent of an apartment or the wage of a worker. Bond graphs define a cost or benefit as an effort variable $e(t)$.

The time derivative of assets in stock is the change of assets in stock. Demand and supply are the most common asset flows used in economics. Demand and supply have the units of assets per unit of time (days, months, years). Bond graphs define a supply or demand as a flow variable $f(t)$.

The kinetic energy (KE) of a mass is calculated as:

$$\text{KE} = \int p \, dv \tag{3-5}$$

If one interprets the momentum and velocity as economic analogies, the price is integrated over its corresponding import or export, which is equivalent to cash flow. Thus, energy has cash flow as its economic analog. Cash flow is the money that moves (flows) in and out of ones business. The analog for energy is determined as cash flow, with euro per unit of time as its unit:

$$E := \text{cash flow} \qquad [€/yr] \tag{3-6}$$

The table of the analogs between mechanics and economics can be found in Section A-1

3-3 Bond Graphs

3-3-1 Junctions

Junctions split power between their connected elements. A 0-junction splits the flows and keeps the efforts constant, making the 0-junction a distribution point of assets. The following equations hold for the 0-junction of Figure 3-1a:

$$\begin{aligned} f_1 &= f_2 + f_3 \\ e_1 &= e_2 = e_3 \end{aligned} \tag{3-7}$$

In contrast, 1-junctions split the efforts and keep the flows constant, making the 1-junction an accumulation of benefits and costs. The following equations hold for the 1-junction of Figure 3-1b:

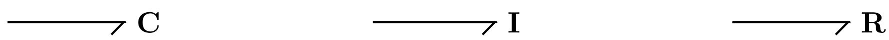
$$\begin{aligned} e_1 &= e_2 + e_3 \\ f_1 &= f_2 = f_3 \end{aligned} \tag{3-8}$$

(a) 0-junction **(b)** 1-junction

Figure 3-1: Multi-port elements

3-3-2 Storage Variables

Junctions and bonds represent the relation between elements in a system. Three single-port elements are used: springs (capacitors), masses (inductors) and dampers (resistors). Figure 3-2 illustrates these elements.



(a) Spring **(b)** Mass **(c)** Damper

Figure 3-2: Single-port elements used in bond graphs

Spring

A linear spring stores displacement according to Hooke's law[6]. Hooke's law states:

$$\begin{aligned} F &= kq \\ \text{with: } F &= \text{force} \\ k &= \text{a constant of stiffness} \\ q &= \text{deformation of the spring} \end{aligned} \tag{3-9}$$

The displacement is the integral over the stored flow of the spring. So Hook's law has the following relation:

$$\begin{aligned} F &= k \int \dot{q}\, dt \\ \text{with: } \dot{q} &= \text{velocity} \end{aligned} \tag{3-10}$$

Mass

A mass stores momentum p. Newton's second law of motion states that impulse occurs when a force acts over an interval of time[7]. Since the force is the time derivative of momentum, it follows that:

$$\begin{aligned} v &= \frac{p}{m} = \frac{1}{m}\int \dot{p}\,dt \\ \text{with: } m &= \text{a constant of object mass} \\ v &= \text{velocity of mass} \end{aligned} \tag{3-11}$$

Damper

A damper has a frictional effect on the effort or on the flow. A linear damper, with a constant of the linear friction force b, gives the following relation between the velocity and force:

$$b = \frac{F}{v} \tag{3-12}$$

Chapter 4

The Firm as Dynamical System

4-1 Introduction

This chapter develops a mechanical system where its dynamical behaviour aligns with the behaviour of a firm in the leasing business. The consumer generates revenue for the firm and the firm itself only generates costs. Costs obviously influence the behaviour of a firm. The two major cost drivers of a car leasing firm are the supply of cars and the holding costs. The supply is the purchase of goods to sell or lease to the consumer. Holding costs are those associated with storing inventory that remains unsold. For these reasons, this chapter considers the following problems:

Problems

1. Find a mechanical analog for inventory storage and model the change in holding costs.
2. Find a mechanical analog for the supply of cars and model the change in supply costs.

The generalized definition of inventory is stated as: "all work within the process of production-all work that is or has occurred prior to the completion of production" [8]. In the context of a firm in de car leasing business, the inventory is the total stock of cars stored. In this chapter I use the concept of market clearing to make the mechanical analog for the over-supply of cars that has to be stored.

Solution to problem 1

- Identify the market clearing with the mechanical analog as a 0-junction.
- Store the oversupplied cars in a spring.
- The mechanical analog of the total holding costs is the potential energy of the spring.

The supply is the amount per period a firm are willing to sell or lease to the consumer. In this chapter I use a costs-benefit analysis to model the willingness to pay for the supply of a good.

Solution to problem 2

- Identify the cost-benefit analysis with the mechanical analog as a 1-junction.
- Store the benefit that determines the willingness to pay in a mass.
- The mechanical analog of the total supply costs is the kinetic energy of the mass.

Section 4-2 explains how I model the inventory and Section 4-3 explains how I model the supply of a firm in the leasing business as a mechanical LTI system.

4-2 Inventory Clearing

4-2-1 Market Clearing

A firm shows similar behaviour to the process of the market clearing. The market clearing is that in a free market, the supply of whatever is traded is equated to the demand, so that there is no leftover supply or demand [9]. I identify that market clearing occurs within a firm. In Section 2-5, I put forwards the importance of tracking the holding costs, being the cost of the stock of cars that are not leased or remarketed. Too small stocks result in unfulfilled product requests. Too large stocks result in unnecessary costs. So a firm wants to have an optimal stock level at all times [10]. Thus, a firm strives to equate the supply to the demand. A market clearing within a firm is what I call the inventory clearing.

If the demand flow and the supply flow of cars are not equal, there will be a situation of over- or under-supply. In case the supply is higher than the demand, the result will be an over-supply. As seen in Section 3-2 supply or demand have a mechanical analog for velocity, or in generic terms a flow:

$$f_{\text{over-supply}} = f_{\text{supply}} - f_{\text{demand}} \tag{4-1}$$

In the end, a firm is interested in the total stock that causes holding costs. For this reason, the over-supply flow is integrated over time. Stock obviously has a mechanical analog as displacement:

$$q_{\text{firm stock}} = \int f_{\text{over-supply}}\, dt \tag{4-2}$$

4-2-2 Mechanical Analog

In this thesis, I assume that holding cost per car in stock grow linearly. The first storing place is assumed to be the cheapest, with the second one assumed to be more expensive due to, e.g., hired facilities being less close to the desired location.

By interpreting the volume of stock as displacement, and the holding costs per car as a force, the mechanical analog for the holding-cost curve is a spring. A spring tends to clear its displacement like a firm tends to clear their inventory.

Figure 4-1 shows a visualization of the analog between the curve and the spring. By having more stock, the holding costs per car will grow. One must imagine the spring attached to the curve to the F axis. As the stock rises, the displacement returns the force F. The returned force represents the holding costs resulting from the increased inventory.

If a firm costs per car per year increases, the firm wants to clear their inventory even more. A spring show the same behaviour with a higher displacement. A spring returns a higher force, so tries to loose its displacement even more.

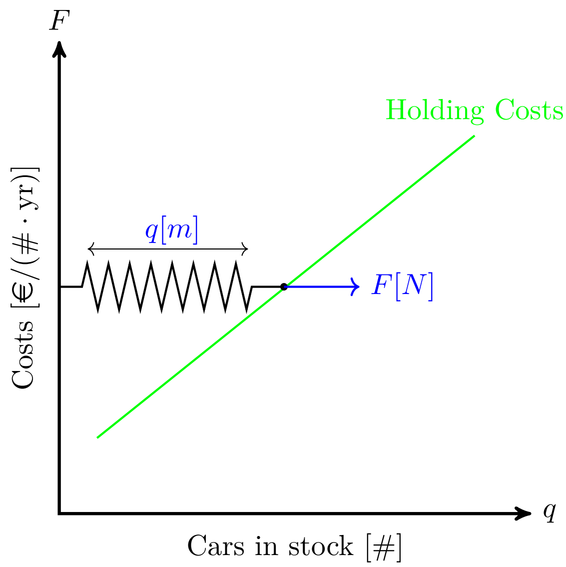


Figure 4-1: A visualisation of the analogy between holding cost curve and the behaviour of a spring.

The slope of the curve in Figure 4-1 is defined in mechanics as the stiffness of the spring. The stiffness k is expressed in the economic unit of [€/(#^{2}·yr)]. No exact economic analog is found for the stiffness k, but it represents the increase of costs per car as a result of having to store more inventory. So the stiffness k represents difficulty to store excess stock.

A spring is able to store the over-supply of cars and returns the holding costs of the oversupplied cars. A spring tends to clear its displacement like a firm clears its inventory. With Hooke's law[6] the following relation is defined:

$$F_{\text{holding cost}} = k_1 * q_{\text{stock}} \tag{4-3}$$

4-2-3 Bond Graph Model

The problem we identified earlier is that the element of over-supplied cars is currently not included in the forecast period of a firm. The 0-junction of Figure 3-1a is the accurate representation of the sum of flows of Equation 4-1.

A 0-junction sums all the flows of cars, that results in a over- or under-supply.

A spring stores the flow of over-supplied cars, which returns a force or effort that represents the holding cost per car. A C-element represents a spring in a bond graph. The visualization of the inventory bond graph is shown in Figure 4-2 with the corresponding flow and effort at every bond.

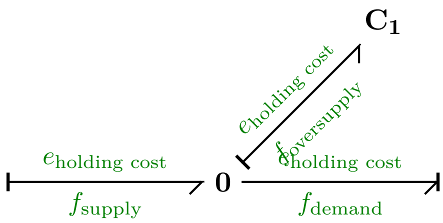


Figure 4-2: Bond graph of market clearing with 0 junction, three bonds and a spring

4-2-4 Inventory Shrinkage as Damping

A car in stock depreciates over time. Valuation consultants do not distinguish between the depreciation on a car in stock or on a car in use. Depreciation has two definitions [11]:

- The decrease in value of assets
- The allocation of the cost of assets to periods in which the assets are used

The decrease in value cannot be determined because the assets do not have a known value when stored at the inventory. Moreover, because the assets are not in use, the second type of deprecation does also not reflect the loss of inventory because the cars are not in use when they are stored at the firms facilities.

Inventory shrinkage is the term economists use to describe the loss of inventory. Inventory shrinkage is the displacement of the stock between the time of acquiring the goods and selling it to customers, due to inventory theft or damage. When your recorded and physical stock counts do not match, it is called inventory shrinkage [12]. "Displacement" is an ironic word choice because inventory shrinkage also results in displacement in the mechanical analog of the spring, due to friction.

Inventory shrinkage is a phenomenon that cannot be reversed without effort. One should hire a car mechanic to fix the car to its original state. This is why a damper already feels like a good mechanical analog for inventory shrinkage because friction is a energy loss that cannot be reversed without effort.

The economic calculation of inventory shrinkage is as follows [13]:

$$\text{Inventory Shrinkage} = \text{Recorded Inventory} * \text{Inventory Shrinkage Rate} \tag{4-4}$$

Inventory shrinkage is expressed in the unit of a flow [#/yr].

$$f_{\text{shrinkage}} := \text{Inventory Shrinkage} \qquad [\#/\text{yr}] \tag{4-5}$$

We know that the recorded inventory is modeled as q_{stock} of the firm. By substitution of $f_{\text{shrinkage}}$ and q_{stock} in Equation 4-4 the following relation comes out:

$$f_{\text{shrinkage}} = q_{\text{stock}} * \text{Inventory Shrinkage Rate} \tag{4-6}$$

Above relation is legitimate. The amount of cars that are lost due to shrinkage is dependent on the amount that is stored.

Due to the proportionality of a spring, the q_{stock} is substituted by the effort e_{cost} and stiffness k_1:

$$f_{\text{shrinkage}} = \frac{e_{\text{holding cost}}}{k_1} * \text{Inventory Shrinkage Rate} \tag{4-7}$$

Now I propose to model the inventory shrinkage as linear damping. A linear damper relates a flow and an effort with the damping constant b.

$$\frac{e(t)}{f(t)} = b \tag{4-8}$$

By substituting the flow of shrinkage and the effort of holding cost, a linear damper is capable of producing the inventory shrinkage through the following relation.

$$\boxed{\frac{e_{\text{holding cost}}}{f_{\text{shrinkage}}} = \frac{k_1}{\text{Inventory Shrinkage Rate}} = b_2 \qquad (4\text{-}9)}$$

Due to Equation 4-9, the damping constant b_2 depends on the stiffness of the spring k and the inventory shrinkage rate. Calculating the damping constant b_2 is not a problem because the stiffness k_1 and the inventory shrinkage rate are constants.

4-2-5 Bond Graph Model with Inventory Shrinkage

Taking into account the inventory shrinkage, the 0-junction of the market clearing consists of a new bond with the following flow distribution:

$$f_{\text{over-supply}} = f_{\text{supply}} - f_{\text{demand}} - f_{\text{shrinkage}} \tag{4-10}$$

An R-element visualizes a linear damper in a bond graph. The bond graph of the market clearing with inventory shrinkage is as follows:

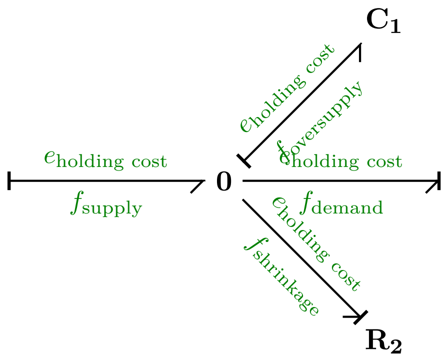


Figure 4-3: Bond graph of market clearing from Figure 4-2 with inventory shrinkage bond and damper

In above situation, the recorded car count stored in the spring through supply is not the same as the recorded car count going to the demand of the consumer. The cars do not physically disappear, but they get smelly, and damage occurs. due to the period of storage, the car is sold for less because it is not brand new anymore.

4-2-6 Holding Costs as Potential Energy

The holding costs are the costs associated with storing inventory that remains unsold or not leased. Unsold inventory is an important aspect of the total firm's cost especially in the leasing business because the total assets of the firm consist mainly of the cars.

The spring C_1 represent the inventory stock of the system. The potential energy of the spring is calculated as [14]:

$$E_{\text{pot}} = \int F dq = \frac{1}{2} k q^2 \tag{4-11}$$

By substituting the economic analogs of Section 3-2 for energy and displacement, we find that the potential energy of C_1 is the cash flow of the inventory stock. The integral of Equation 4-11 matches every car in stock with its corresponding holding cost. The sum of these costs is the total holding costs (THC). Consequently, the mechanical analog for the total holding costs is:

$$E_{\text{THC}} = \frac{1}{2} k_1 q_{\text{firm stock}}^2 \tag{4-12}$$

Graphically, E_{THC} is the area underneath the holding cost curve just like the potential energy of a spring would be:

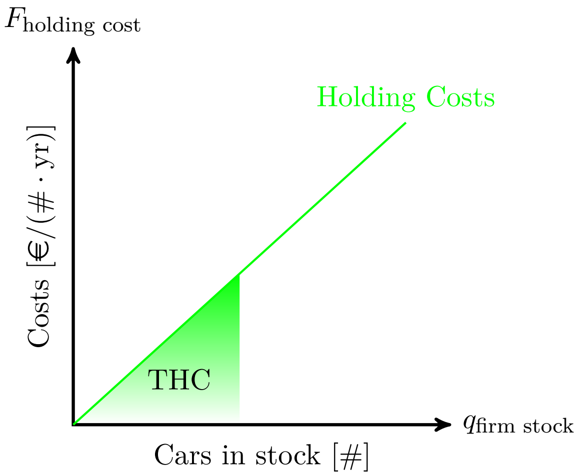


Figure 4-4: The area underneath the holding cost curve is the THC, which is equivalent to the potential energy of the spring C_1

4-3 Supply Cost-Benefit Analysis

4-3-1 Problem Statement

No matter what product one sells, the price that the firm is willing to pay for its supply has an impact on the supply of the firm. Equivalently, the price charged to the consumers affects the demand for the firm's product. Economist state that interaction between demand and supply of a product determines the price of a product (see Figure 4-5)

Economists usually put 'Quantity' on the x-axis. But the x-axis is a quantity flow because a quantity without time unit is not useful. For instance, a sales manager never states: "I sold three cars." Did the sales manager sell three cars in a year or in a day? The reference to a time unit is essential: "I sold three cars *today*."

A change in quantity supplied of a firm influences the future supply and demand of a firm. The influence of a quantity supply change is not taken into account in the valuation consults' model to forecast the cash flow of a firm.

4-3-2 Marginal Cost as Supply Curve

The supply curve of Figure 4-5 is the curve for the total market of a product. However, in valuation, one looks at one individual firm. The supply curve for a single firm is called the

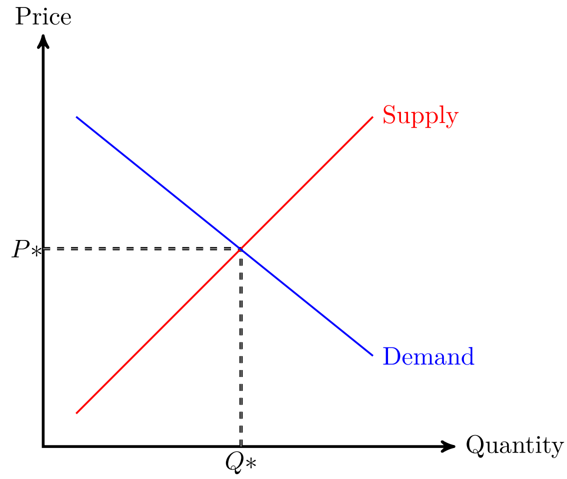


Figure 4-5: Example of Supply and Demand curves

marginal cost (MC) curve. The marginal cost is the change of costs at any given output [15]. This thesis only considers the linear part of the MC curve to model an LTI system. The MC curve is visualized in Figure 4-6.

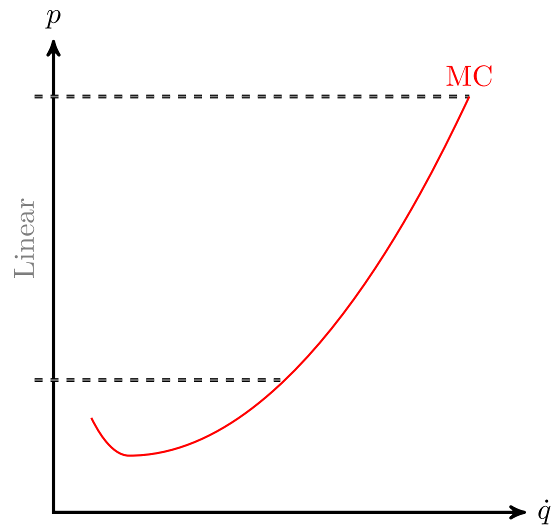


Figure 4-6: Marginal cost (MC) curve of an individual firm. The dashed lines indicate the linear part of the MC curve

4-3-3 Mechanical Analog

The price has momentum as its mechanical analogy, and the flow of supply has the analogy of velocity. The momentum of a particle is related to its velocity multiplied by the particle's mass m [14]:

$$p = mv \tag{4-13}$$

With the analogies of Section 3-2, the MC curve is modeled as a mass. A price change results in a change in supply flow, just like the velocity changes due to the momentum change of the mass. As a consequence, the slope of the curve represents the mass m. The economic analogy for a mass, as a property of a physical body and as a measure of its resistance to acceleration, is the inverse liquidity of the product. Liquidity describes the degree to which an asset or security can be quickly bought or sold in the market at a price reflecting its intrinsic value [2]. Money as example, has high liquidity, thus almost no mass, because it is quickly bought or sold at its intrinsic value. A change in supply does not change the price of money, which fits your expectations.

$$p_{\text{mc}} = m_1 v_{\text{supply}} \tag{4-14}$$

Figure 4-7 shows the visualization of the analogy between a mass and an MC curve.

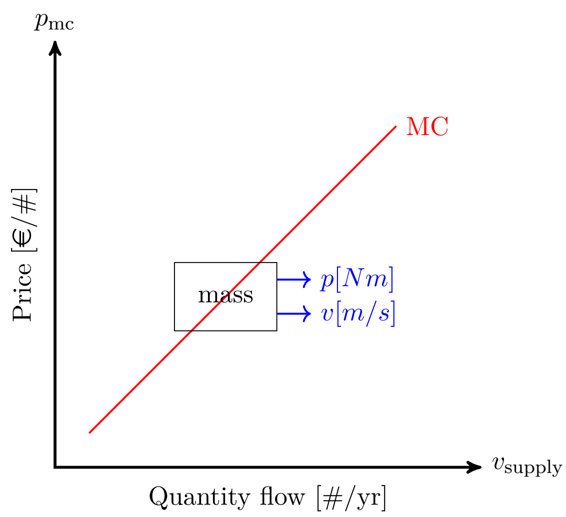


Figure 4-7: Visualisation of the analogy between a mass and an MC curve

4-3-4 Bond Graph Model

A firm performs a cost-benefit analysis to determine if a certain business decision is sound. Benefits should always outweigh the costs. How much the firm is willing to pay for a product is a particular case of a cost-benefit analysis. If the benefits outweigh the costs, the firm is willing to pay more for its supply - the inequality of costs and benefits results in net benefits. An example of costs is the holding costs, if the holding cost grow, the firm is willing to pay less for additional supply, because they already have more cost due oversupply. The benefit in this particular case is chosen by management, if management thinks more supply will result in more profit, they push more benefit $e_{\text{manag benefit}}$ to the supply.

$$e_{\text{net benefit}} = e_{\text{manag benefit}} - e_{\text{cost}} \tag{4-15}$$

A mass can store the effort of net benefit. The mass reacts with a momentum change, the integral of the effort, which represents the willingness to pay for a product, p_{mc}.

$$\int e_{\text{net benefit}} dt = p_{\text{mc}} \tag{4-16}$$

The price p_{mc} corresponds to a quantity flow of supply v_{supply}(see Figure 4-7).

The 1-junction of Figure 3-1b is the correct representation of the sums of efforts of Equation 4-15. An I-element denotes a mass element in a bond graph. The visualization of the cost-benefit analysis bond graph is shown in Figure 4-8 with the corresponding flow and effort at each bond.

A mass is able to store the net benefit and returns a flow of supply. A 1-junction constructs the cost-benefit analyses.

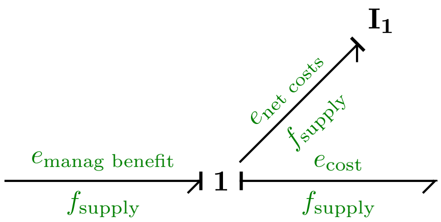


Figure 4-8: Bond graph of cost-benefit analysis with 1-junction, three bonds and a mass

4-3-5 Salary as Damping

A damper dissipates energy in a way that it cannot be retrieved, like heat. The main cost a leasing firm makes which it cannot retrieve is the salary paid to its employees.

A firm will need additional employees to increase the supply of cars. Additional efficient employees will be harder to come by for the same salary, which means that the salary per unit of car supplied will increase over time.

This thesis assumes that the salary will increase proportionally with the flow of supply.

Equation 4-8 defines the relation between an effort and a flow with constant b. Substituting f_{supply} and e_{salary} into Equation 4-8 gives the following relation:

$$\frac{e_{\text{salary}}}{f_{\text{supply}}} = b_1 \tag{4-17}$$

Salary of a leasing firm is given as a percentage per year of price per car supplied [3]. So by substitution of Equation 4-14 in above equation:

$$\frac{e_{\text{salary}} \cdot m_1}{p_{\text{mc}}} = \text{Salary percentage} \cdot m_1 = b_1 \tag{4-18}$$

4-3-6 Bond Graph Model with Salary

With salary as an additional damping effort, the 1-junction of the cost-benefit analysis consists of a new bond with the following effort distribution:

$$e_{\text{net benefit}} = e_{\text{manag benefit}} - e_{\text{cost}} - e_{\text{salary}} \tag{4-19}$$

An R-element visualizes a linear damper in a bond graph. The bond graph of the inventory clearing with salary is as follows:

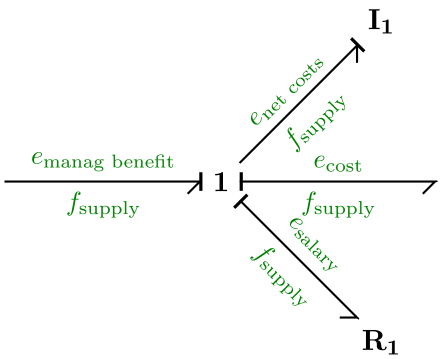


Figure 4-9: Bond graph of cost-benefit analysis with a salary damper

4-3-7 Supply Costs as Kinetic Energy

Supply costs are one of the biggest costs in almost every kind of firm. If a firm wants to sell a product, it first has to produce the product or buy the product from another party. One financial difference with the leasing business is that the costs of a car is the depreciation of a car. But to make this model more general, I consider the supply cost as the main costs of the product and not the depreciation of the product. This is just a matter of definition, not behavioral dynamics of the system.

In Section 3-2 the mechanical analog of a cash flow is energy. The kinetic energy of a mass, as we know as a mechanical engineer, is calculated as [14]:

$$E_{\text{kin}} = \int p dv = \frac{1}{2} m v^2 = \frac{p^2}{2m} \tag{4-20}$$

Even looking at the integral of Equation 4-20 the prices are multiplied with their corresponding supply, which leads to the total supply costs (TSC). This means by substituting the economic analog of momentum and energy in Equation 4-20 the formula for the TSC arises:

$$\boxed{E_{\text{TSC}} = \frac{p_{\text{mc}}^2}{2m_1}} \tag{4-21}$$

Graphically, the TSC is the area beneath the MC curve, see Figure 4-10. In mechanics the kinetic energy of a mass is this exact same area.

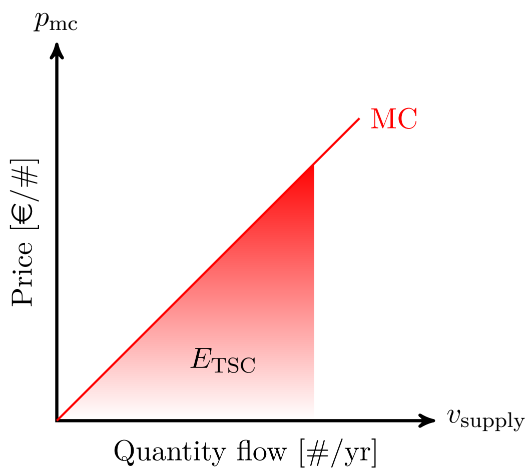


Figure 4-10: The area underneath the MC curve is the TSC, which is equivalent to the kinetic energy of the mass I_1

4-4 Connection of the Inventory Storage and the Supply

Assuming the supply of cars goes to the firm's storage, the inventory market-clearing and the MC cost-benefit analysis have a common bond. The common bond connects the bond graphs of Figure 4-3 and Figure 4-9:

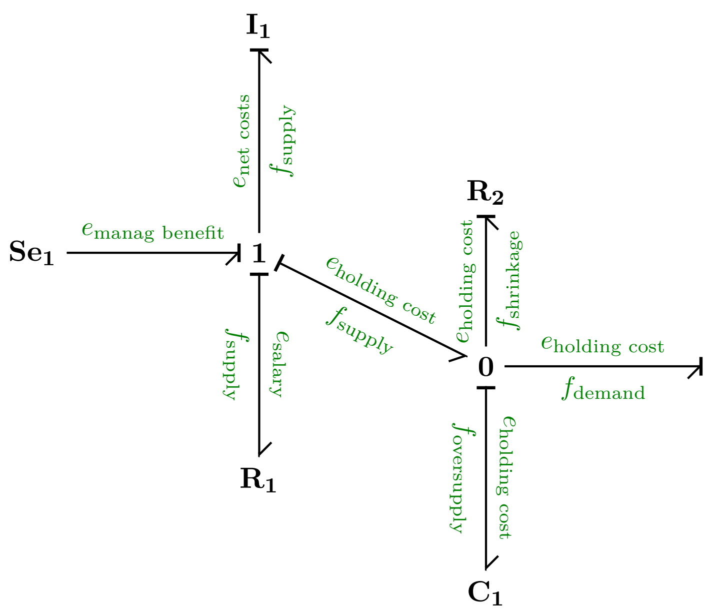


Figure 4-11: Bond graph of firm's supply and inventory

Physically speaking, if management pushes the mass with e_{benefit}, the mass I_1 will start moving. Economically, the firm starts to supply additional cars. The damper R_1 slows down the mass through friction, i.e., higher salary costs damp the supply. Because the management is considered an input, it is modeled as a source effort Se_1.

Because the mass has a velocity, it starts storing velocity into the spring C_1. Economically speaking, the supply of cars is stored. The spring returns a force (holding costs) and subsequently, the damper R_2 damps the spring, i.e., the stored cars undergo inventory shrinkage.

4-5 Conclusions

The inventory storage is modeled in a mechanical system as a spring. The spring counts the amount of cars that flow in and out of the inventory. The more the spring has to store, the more holding costs it will return. The firm wants to equate its supply and demand, which is modeled as a 0-junction. With the assumption that the holdings costs per car grow linearly per amount of cars stored this chapter concludes that:

> The mechanical analog of the inventory storage is a spring with a 0-junction.

The supply of a car leasing firm is modeled as a mass. It is important to see that the willingness to pay for your supply within a firm is done by a cost-benefit analysis. If the holdings cost grow per car, the firm does not want to supply even more cars than it already does. This chapter concludes the following solution:

> The mechanical analog of the supply is a mass with a 1-junction.

In order to determine the cash flow of a firm, the total costs need to be defined as a mechanical properties of the system. Because the spring stores its displacement and returns a force with their economic analogs of cars stored and costs per car per year respectively, this chapter concludes that:

> The mechanical analog of the holding costs is the potential energy of a spring.

Similarly, the mass stores its momentum and returns a velocity. Their mechanical analogs are price and supply flow of cars respectively, which gives us the following conclusion:

> The mechanical analog of the supply costs is the kinetic energy of a mass.

Chapter 5

The Consumer as Dynamical System

5-1 Introduction

The behaviour of a firm depends on the behaviour of the consumer. The dictionary gives a two clear definitions: "A consumer is an individual that buys goods for own consumption and not for the resale or for a commercial purpose. Or: A consumer is an individual who pays an amount of money required to consume goods and services produced." [16]

The dictionary already makes a distinction between the remarketing business and the leasing business of a firm in the car leasing business. If a consumer buys goods to consume, we are talking about the remarketing business. If a consumer pays money per car per unit of time to consume an asset, we are talking about the leasing business. This chapter handles the following problems:

Problems

- Find a mechanical analog for the remarketing consumer and its corresponding remarketing price.
- Find a mechanical analog for the leasing consumer and its corresponding lease price.

In this chapter I assume that the consumer has an imaginary stock and consume the cars when allocated at their stock (in the consumers possession). As in Chapter 4, I use the concepts of the market clearing and cost-benefit analyses to model the consumers.

Solutions

- The consumer has a imaginary stock with its mechanical analog as a spring.
- Use the market clearing and costs-benefit analyses of Chapter 4.
- The consumer pays a negative price and add a reference frame.

Section 5-2 models the demand of the remarketing consumer as a cost-benefit analysis. Section 5-3 shows that the remarketing consumer has an imaginary stock and model the remarketing stock as a market clearing. Section 5-4 shows the derivation of the leasing business as a spring.

5-2 Cost-Benefit Analysis in the Remarketing Business

5-2-1 Assumptions

The demand of the consumer is an essential aspect of the revenue of a firm. The demand curve is a well-known curve among economists. The curve represents the relation between the quantity demanded flow and the willingness to pay for a product.

The demand curve of an individual firm in a situation of pure competition is a straight line without a slope, see Figure 5-1. Pure competition occurs when there is a large number of (small) firms providing the same product. In case a firm increases its asking prices, the demand for its product will go down to zero: consumers could easily buy the product at another firm. Figure 5-1 shows how the demand for a firm's product under pure competition follows from the market-supply and -demand curve.

An example of pure competition is the farming business. A lot of individual farms each produce a small portion of the total supply. A farm will lose all of its demand in case it increases its asking price [15].

Under pure competition, a firm cannot influence the price. So I assume that the leasing firm of this thesis is not in the situation of pure competition.

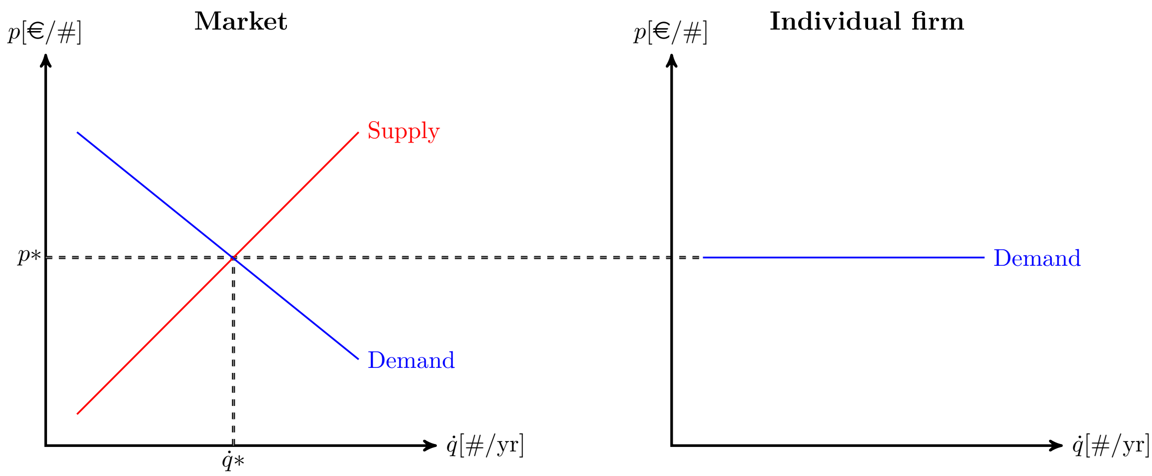


Figure 5-1: Demand curve of the market and of an individual firm under pure competition

I assume in this thesis a situation at the other extreme, namely that there is only one firm in the industry (a monopoly). Because the firm as a monopolist has a high influence on the total supply of the product, the firm will be a so-called price maker. A price maker is unlikely to take the equilibrium market price as its asking price: it will strive to maximize its profits.

A second assumption is that the demand curve is linear, because the goal of this thesis is to build an LTI system.

5-2-2 Mechanical Analog

From Section 4-3 we know that the mechanical analogy of the MC curve is a mass. The demand curve also depends on the states price p and quantity flow $\dot{q}$. It seems logical therefore to model the remarketing demand curve as a mass, just like the MC curve.

$$\dot{q}_{\text{demand}} := \text{demand flow for an a product} \qquad [\#/yr] \tag{5-1}$$
$$p_{\text{demand}} := \text{asking price for a product} \qquad [€/\#] \tag{5-2}$$

Parallel to Equation 4-14, the asking price and the flow of demand have the following relation:

$$p_{\text{demand}} = m\dot{q}_{\text{demand}} \tag{5-3}$$

One problem arises when we look at Equation 5-3. The mass m should be a negative number because the demand curve has a downward slope. A negative mass would have strange properties. For instance, if you push the mass away, it would accelerate towards you following Newton's second law of motion.

However, at a transaction the firm receives money, and the consumer gives money. Logically, from the firms perspective, the firms is paying a negative prices to sell the product. The firm is willing to pay a negative price from the firm's perspective.

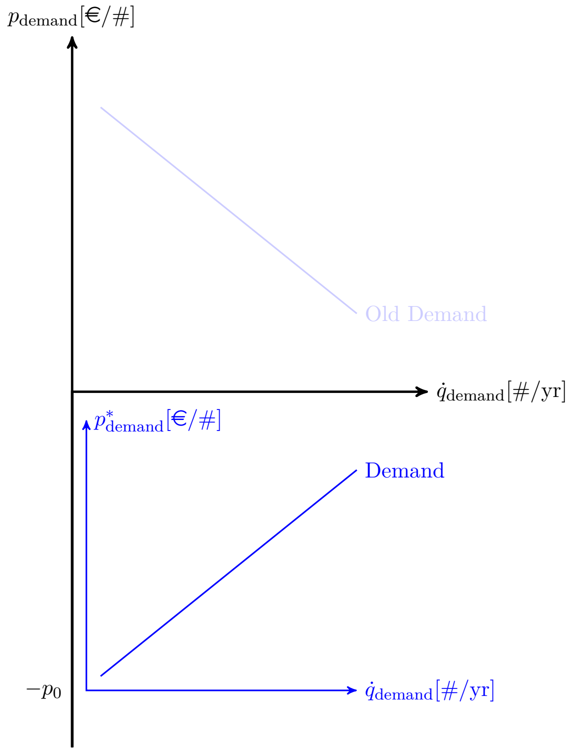


Figure 5-2: The demand- and supply-curve of an individual firm with the assumption that consumers are willing to pay a negative price

Following the assumption that firms willingness to pay a negative price, the remarketing demand curve changes to Figure 5-2.

Now, Equation 5-3 holds without the mass being negative and the demand curve has the same dynamics as the MC curve, only with a different reference frame and other constants of the mass m_i.

An mechanical example of the reference frame is the velocity of an object on earth. The velocity of an object is relative to the earth's velocity. But if we put the reference frame in space, the velocity of the object will be much higher, due to the velocity of the earth. However, a different reference frame does not influence the dynamics of the object.

The reference frame of the demand has changed so that the velocity (demand flow) is zero at momentum (demand price) $-p_0$:

$$p_{\text{demand}} = p^*_{\text{demand}} - p_0 \tag{5-4}$$

Consequently, the dynamics are described as:

$$p^*_{\text{demand}} = m_2 \cdot \dot{q}_{\text{demand}} \tag{5-5}$$

5-2-3 Bond Graph Model

The supply to the remarketing consumer is equal to the demand for the firm's remarketing business, because the firm supplies to the demanding consumer. Interpreting the consumer as a firm, the consumer also performs a cost-benefit analysis to determine its supply, just the like the firm determines its own supply in Section 4-3. So the bond graph of the cost-benefit analysis of the consumer has the same form as the cost-benefit analysis of the firm.

A cost-benefit analysis of the remarketing business is added to the firms bond graph, with two efforts determining the asking price of the car:

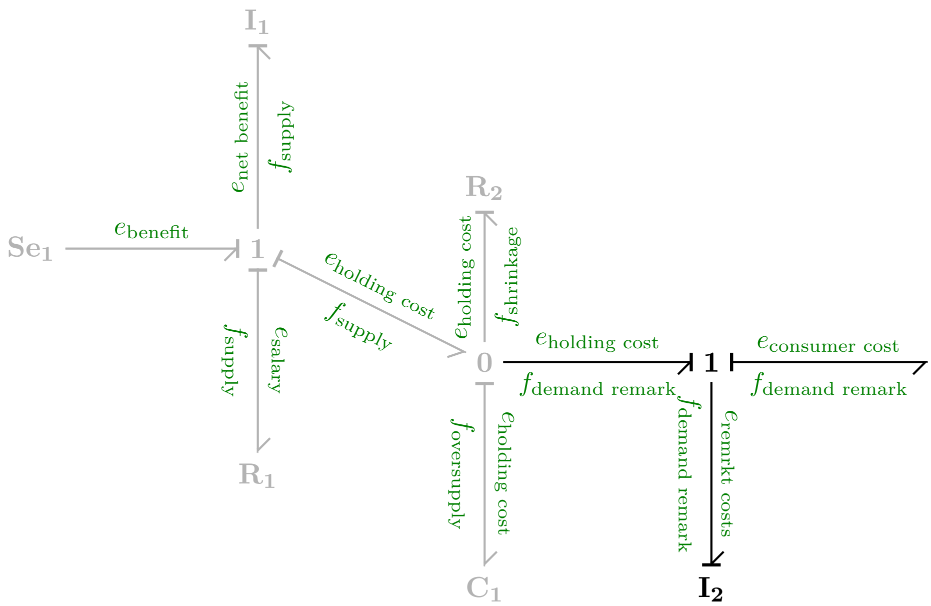


Figure 5-3: Bond graph of firm in car leasing business with their consumers cost-benefit analysis

The $e_{\text{holding cost}}$ in Figure 5-3 is a benefit within the cost-benefit analysis of the demand of the remarketing I_2 and that sounds contradictory. But if the firm has too many cars on the parking lot, the holding costs go up. The firm will ask a less negative price (see Figure 5-2) to reach a higher demand, which will clear the excess stock. Firms, for example, give a discount to get rid of their excess stock. The discount increases the demand for the product.

In the mechanical analogy, the spring C_1 pushes the mass I_2, the momentum of the mass I_2 increases, so that again the velocity of the mass increases.

An important issue is the costs which the consumer will have from owning an additional car. Section 5-3 explains the derivation of the effort $e_{\text{consumer cost}}$.

A 1-junction constructs a cost-benefit analyses for the mass I_2. Firstly, because the firms want to clear their excess stock. Secondly, because the consumers experience higher costs. The 1-junction combines holding- and consumer cost, resulting in a net cost:

$$e_{\text{remrkt cost}} = e_{\text{holding cost}} - e_{\text{consumer cost}} \tag{5-6}$$

The force $e_{\text{remrkt cost}}$ is integrated over time to determine the change in asking price.

$$\int e_{\text{remrkt cost}} dt = p^*_{\text{demand}} = m\dot{q}_{\text{demand}} \tag{5-7}$$

In case the pressure of selling the excess stock $e_{\text{holding cost}}$ outweighs the cost of the consumer $e_{\text{consumer cost}}$, the price p_{demand} becomes less negative and the demand $\dot{q}_{\text{demand remark}}$ increases.

5-2-4 Remarketing Revenue as Energy

In order to determine the cashflow of the firm, the reveneu of the remarketing business has te be defined. The mass I_2 generates the remarketing revenue of the firm. The difference with the costs is that every car is sold at the same price because as seller one cannot give every customer a different price at the exact same moment in time. The supply costs are different because every car is bought with its corresponding price. Figure 5-4 visualizes the total remarketing revenue (TRR):

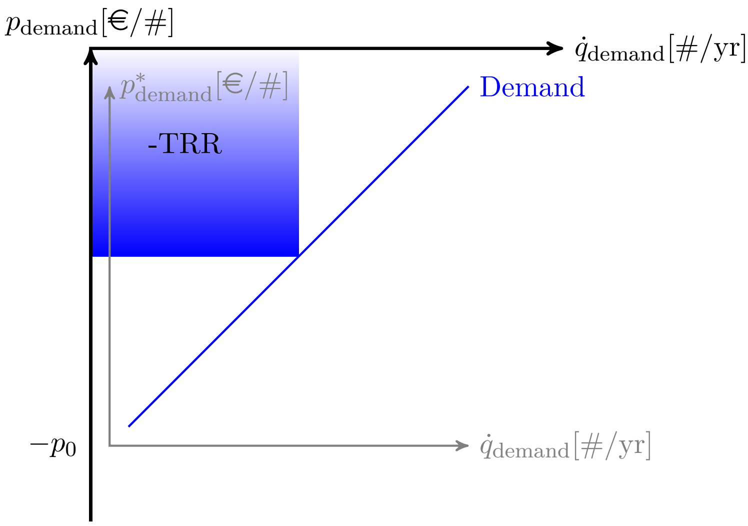


Figure 5-4: The demand- and supply-curve of an individual firm with the assumption that consumers are willing to pay a negative price

We know from Section 3-2 that the mechanical analog from a cashflow is energy. No exact mechanical analog for the revenue is found. But we know that it is a summation of elements with the units of energy. From Figure 5-4 we know that the TRR is defined as:

$$-E_{\text{TRR}} = (-p_0 + p^*_{\text{demand}}) \cdot q_{\text{demand}} \tag{5-8}$$

Substitute Equation 5-5 for q_{demand}:

$$-E_{\text{TRR}} = \frac{-p_0}{m_2} \cdot p^*_{\text{demand}} + \frac{p^{*2}_{\text{demand}}}{m_2} \tag{5-9}$$

5-3 Market Clearing in the Remarketing Business

5-3-1 Assumptions and Mechanical Analog

This thesis assumes that the consumer has an imaginary stock in order to let the consumer receive, possess and consume cars.

In case the consumers have more cars in stock, the benefit gained from acquiring an additional car declines. The declining benefit is known as the "law of diminishing marginal utility"[17]. Imagine having no cars at all in your possession; you will be pleased with your first car. But if you can buy two of the same car, you will be less pleased with the second car. The more cars you have in stock, the larger the discount has to be in order for you to buy an additional car, simply because an additional car gives you less benefit (or higher costs).

The mechanical analogy for the law of diminishing marginal utility is a spring. A spring stores the whole flow of cars supplied to the consumer $\dot{q}_{\text{demand remark}}$:

$$q_{\text{remark stock}} = \int \dot{q}_{\text{demand remark}} \, dt \tag{5-10}$$

The spring returns a force that represents the costs (negative benefit) to the consumer as a result of having more cars in stock. I assume that the cost to the consumer increases linearly with the stock that he holds. The visualization of the spring is shown in Figure 5-5.

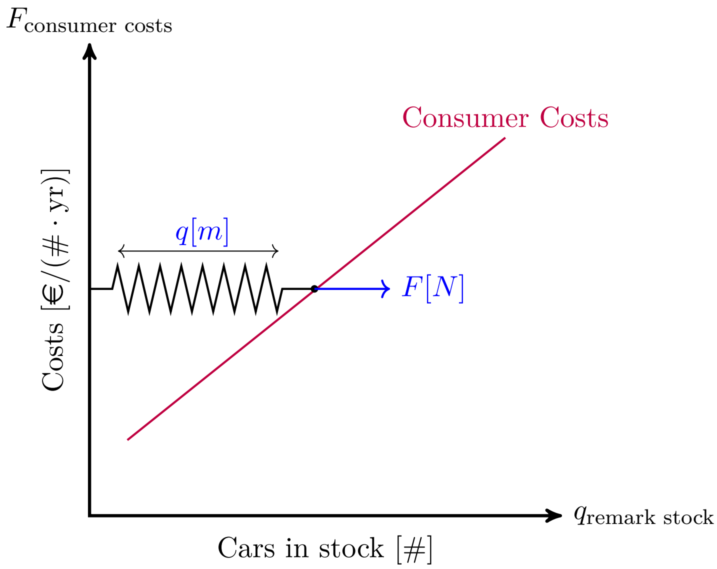


Figure 5-5: A visualization of the analogue between remarketing cost curve and the behavior of a spring.

The slope of the curve is defined as the stiffness of the spring k_3. No economic analogy for k_3 is found, since economists consider the consumer having a imaginary stock.

$$\boxed{F_{\text{consumer cost}} = k_3 * q_{\text{remark stock}} \tag{5-11}}$$

5-3-2 Consumer Consumption as Damping

To prevent market saturation, many companies have intentionally designed their products to "wear down" in such a way that they need replacement at some point. For example, a firm may start selling lamps that never burn out. These lamps will in the long term lead to zero revenue. Future sales depend on the rate of consumption, which is an important phenomenon to consider when modeling the consumer.

By connecting a damper to the market clearing, we add the element of consumption into the dynamics of a firm. Consumers use the car like a damper dissipates potential energy into heat. The car cannot retrieve its original state without external effort. The damper reduces the displacement of the spring. Economically speaking, the stock is consumed by the consumer.

Economist use the consumption rate to determine the consumption flow of cars [18]:

$$f_{\text{consumption}} = q_{\text{remark stock}} * \text{Consumption Rate} \tag{5-12}$$

Due to the proportionality of a spring, the $q_{\text{remark stock}}$ can be substituted by the effort $e_{\text{consumer cost}}$ and the stiffness k:

$$f_{\text{consumption}} = \frac{e_{\text{consumer cost}}}{k_3} * \text{Consumption Rate} \tag{5-13}$$

I propose to model consumption as a linear damping. A linear damper relates a flow and an effort to the damping constant b.

$$\boxed{\frac{e_{\text{consumer cost}}}{f_{\text{consumption}}} = \frac{k_3}{\text{Consumption Rate}} = b_4} \tag{5-14}$$

Due to Equation 4-9, the damping constant b_4 depends on the stiffness of the spring k_3 and on the consumption rate. This calculation is not a problem because the stiffness k_3 and the consumption rate are constants.

5-3-3 Bond Graph Model

The bond graph of the market clearing of the remarketing consumer is added to Figure 5-3 with a 0-junction. The 0-junction subtracts the flow of cars that is consumed from the demand of the firms cars, resulting in an excess demand flow:

$$f_{\text{excess demand}} = f_{\text{demand remark}} - f_{\text{consumption}} \tag{5-15}$$

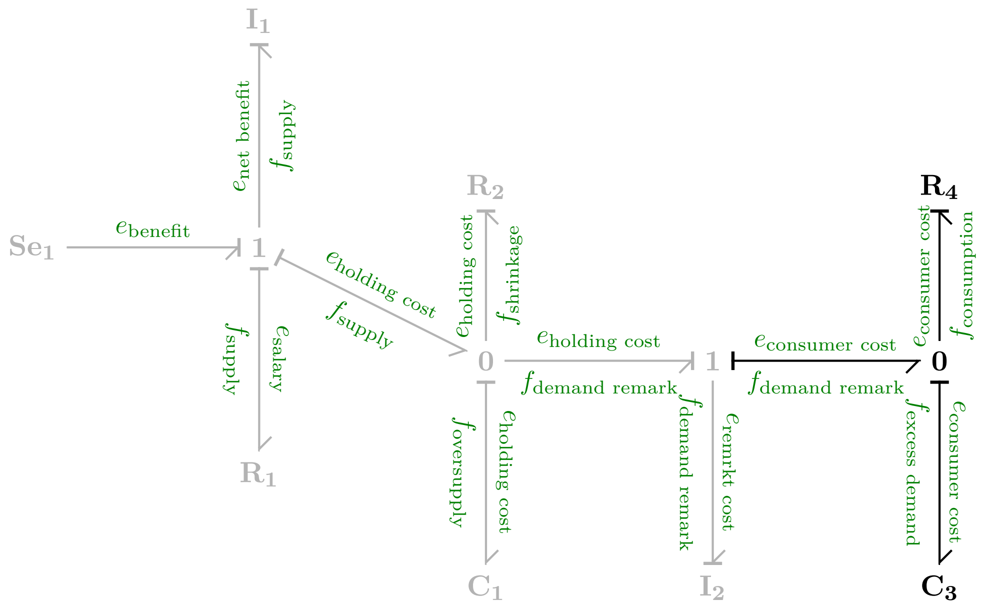


Figure 5-6: Bond graph of firm in car leasing business with their consumer's market clearing

5-4 Market Clearing in the Leasing Business

5-4-1 Assumptions

The second value driver of a car is the leasing business is the actual leasing of the car. For a monthly payment, customers lease a car for a 5-year period. The cars remain assets of the firm, but do not generate costs; they return a monthly benefit for the firm. I assume here that the demand line of the leasing revenues is linear, with a downwards slope.

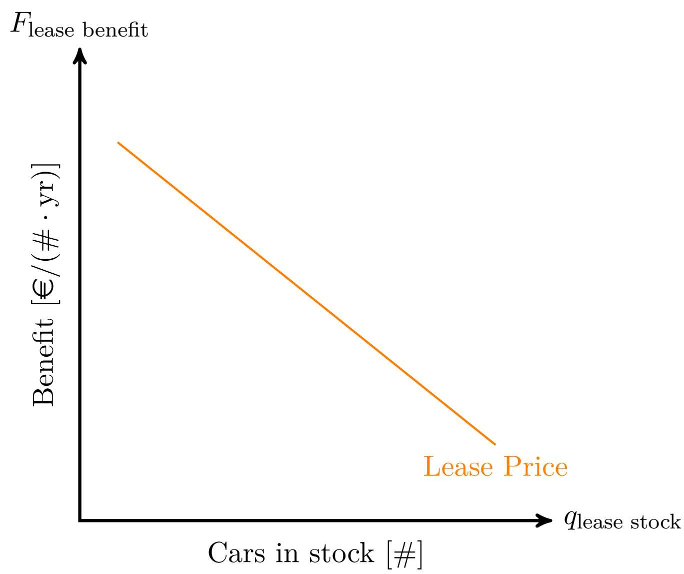


Figure 5-7: Leasing demand curve

5-4-2 Mechanical Analog

Following the theories of Max Mendel, the mechanical analogies for lease price and lease stock are:

$$F_{\text{lease benefit}} := \text{Lease price per car per unit of time} \qquad [€/(\# \cdot yr)] \tag{5-16}$$
$$q_{\text{lease stock}} := \text{Stock that is leased} \qquad [\#] \tag{5-17}$$

The mechanical analogy for imaginary stock of the consumer is a spring, because the spring is able to store the displacement $q_{\text{lease stock}}$ and to return a force $F_{\text{lease benefit}}$, following Hooke's law. However, we find the same problem here as we had with the demand curve of Section 5-2: a spring with a negative stiffness is not physically possible. As I did in Section 5-2, I assume to that the firm receives a negative price. By interpreting the lease price as a negative benefit, the curve does not have a downwards slope.

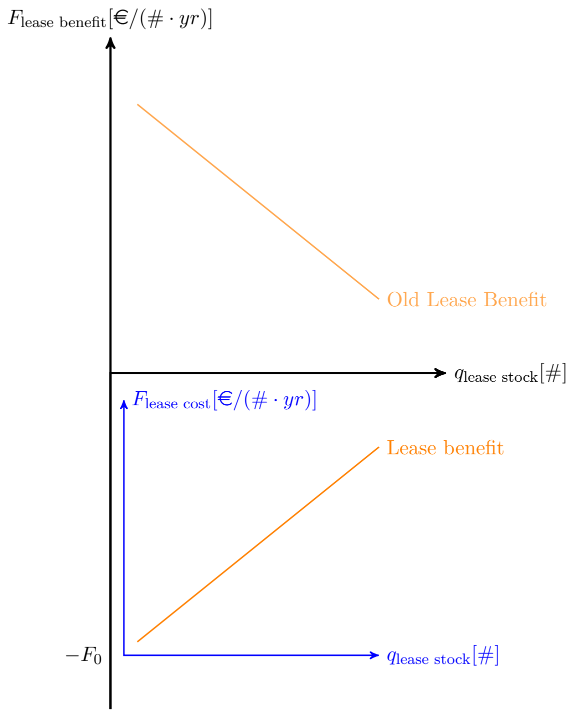


Figure 5-8: The demand- and supply-curve of an individual firm with the assumption that consumers have a negative willingness to pay

Add a reference frame with the states of $q_{\text{lease stock}}$ and returns a force $F_{\text{lease cost}}$ to have a starting lease price $-F_0$ to Figure 5-8

$$F_{\text{lease benefit}} = F_{\text{lease cost}} - F_0 \tag{5-18}$$

The relation between the lease cost dand lease stock being:

$$F_{\text{lease cost}} = k_2 q_{\text{lease stock}} \tag{5-19}$$

5-4-3 Depreciation as Damping

Depreciation is defined as: "the allocation of the cost of assets to periods in which the assets are used"[11]. The costs to the firm arise as the cars are consumed and in return, a lease payment is paid by the consumers. Consumption is equivalent to loss of energy. This means the stock is damped by the consumer. To calculate the depreciation, economist use the depreciation rate. They use the rate as a percentage. But looking at the units, the depreciation rate is expressed in a percentage per unit of time [18].

$$\text{Flow of depreciation}[\#/\text{yr}] = \text{Leased cars}[\#] \cdot \text{Depreciation rate}[\%/\text{yr}] \tag{5-20}$$

As already pointed out in Equation 5-20, the depreciation of cars is a flow of cars. The leased cars obviously represent a displacement of the stock held by the lease consumers.

$$f_{\text{depreciation}} = q_{\text{lease stock}} \cdot \text{Depreciation rate} \tag{5-21}$$

From Equation 5-19, it follows that:

$$f_{\text{depreciation}} = \frac{e_{\text{lease cost}}}{k} \cdot \text{Depreciation rate} \tag{5-22}$$

By interpreting the depreciation as linear damping, a linear damper has the following relation to de depreciation rate:

$$\frac{e_{\text{lease cost}}}{f_{\text{depreciation}}} = \frac{k_2}{\text{Depreciation rate}} = b_3 \tag{5-23}$$

5-4-4 Bond Graph Model

The supply of cars can flow into the firm stock or into the lease stock held by the leasing consumer. A C-element stores the stock held by the lease consumer. An R-element represents the depreciation of the cars used by the consumer.

The bond graph of Figure 5-6 is extended:

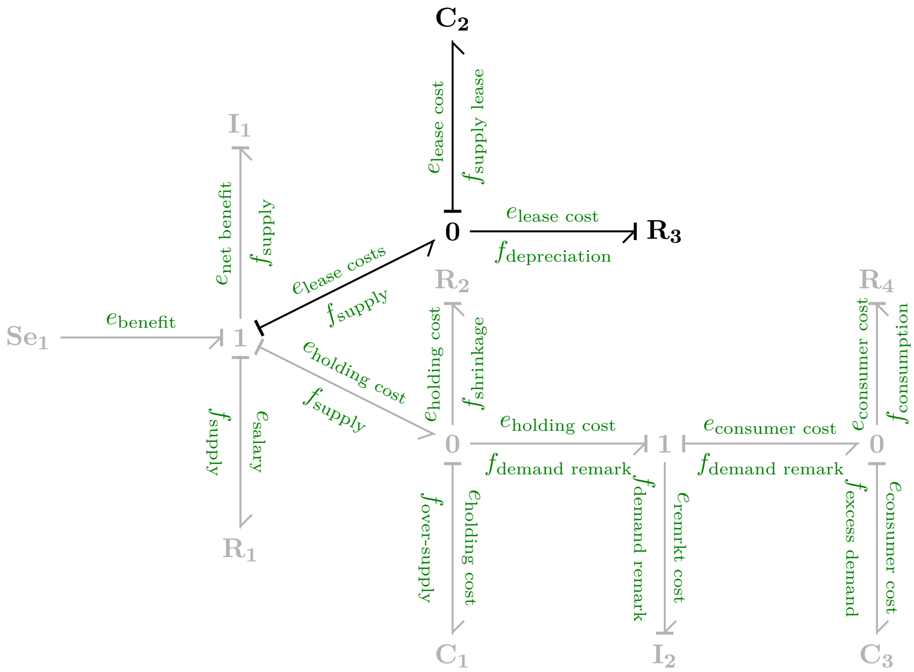


Figure 5-9: Bond graph of firm in car leasing business with their consumers market clearing

5-4-5 Leasing Revenue as Energy

The second revenue generator is the leasing of cars. Just like the remarketing price, C_2 leases the cars for one price as the same moment in time because the market is transparent. The leasing revenue is the lease price multiplied by the amount of cars that is leased. Figure 5-10 visualizes the total leasing revenue (TLR):

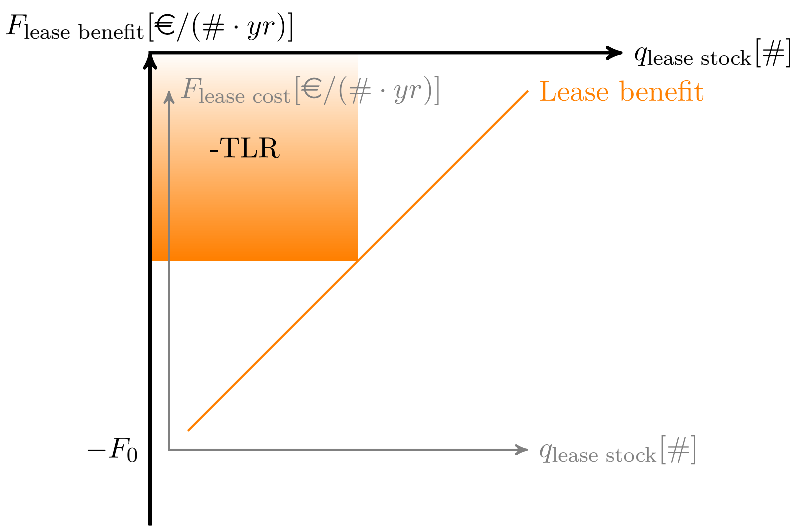


Figure 5-10: The demand- and supply-curve of an individual firm with the assumption that consumers have a negative willingness to pay

Keep in mind that real life contract are bonding for 5 years. This makes it harder to change the lease price. In order to model this 5 year contract, every contract has to be modelled as a seperate mechanical system. For this reason I assume is that the average lease price of all lease cars can be changed at every moment in time.

We know from Section 3-2 that the mechanical analog from a cash flow is energy. No exact mechanical analog for the leasing revenue is found. But we know that it is a summation of elements with the units of energy. From Figure 5-10 we know that the TLR is defined as:

$$-E_{\text{TLR}} = (-F_0 + F_{\text{lease cost}}) \cdot q_{\text{lease stock}} \tag{5-24}$$

Substitute Equation 5-19:

$$-E_{\text{TLR}} = -F_0 \cdot q_{\text{lease stock}} + k_2 \cdot q_{\text{lease stock}}^2 \tag{5-25}$$

5-5 Conclusions

A consumer receives, possess and consumer a car. By having an car in possession the consumer wants to have a larger discount on the second car. That means the consumer returns a negative benefit, a cost. Thus in order to model the consumer, this chapter concludes that the following:

The consumer has a imaginary stock with its mechanical analog of a spring and the consumer consumes by damping the spring.

The demand curve is a hard to model as a mass because the demand flow increase as the price decreases, this would lead to a negative constant of mass. The same problem arises with modeling the leasing demand curve as a spring. In order to solve this problem this chapter concludes:

The firm pays a negative price to the consumer and one should add a reference frame at the starting (lease) price to model demand.

Chapter 4 and Chapter 5 explain step-by-step how the leasing firm and its consumer is modeled as a mechanical system. Now we see that the high-order effects of changes in firm supply, firm stock and consumer behaviour influence each other, as shown in the modeled bond graph in Figure 5-11.

As stated in Chapter 3, the ordinary differential equations (ODE) that describe the system are not difficult to derive from a bond graph model. The states of the system are the storage variables of the springs and masses $x = \left(p_{\text{mc}}\ p^*_{\text{re-mrkt}}\ q_{\text{firm stock}}\ q_{\text{lease stock}}\ q_{\text{remrkt stock}}\right)^T \in \mathbb{R}^5$. Obviously, this is a fifth-order system. Only one input u from the firm managements effort is taken into account, which is controlling the firm's supply with source effort Se_1 .

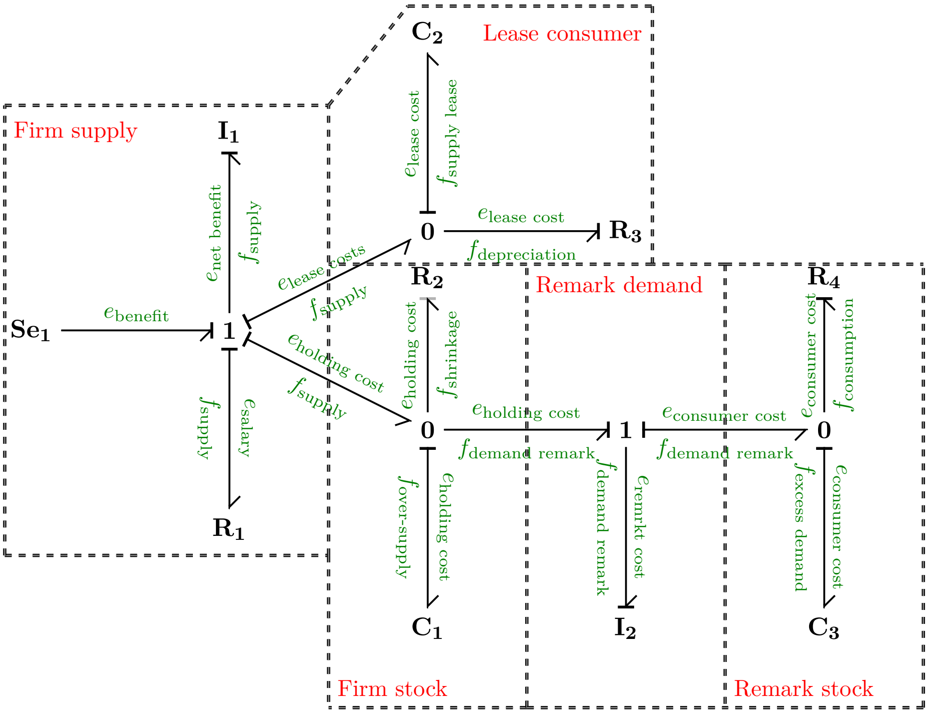


Figure 5-11: Bond graph of firm in car leasing business

The mass I_i has its corresponding constant m_i. The spring C_i has its corresponding stiffness k_i. The damper R_i has its corresponding damping constant b_i. This chapter concludes that the system of the firm and its consumer is described by the following state-space matrices:

State-space matrices

$$\dot{x} = Ax + Bu, \text{ with } x = \begin{bmatrix} p_{\text{mc}} \\ p_{\text{pre-mrkt}} \\ q_{\text{firm stock}} \\ q_{\text{lease stock}} \\ q_{\text{remrkt stock}} \end{bmatrix} \; A = \begin{bmatrix} -\frac{b_1}{m_1} & 0 & -k_1 & -k_2 & 0 \\ 0 & 0 & k_1 & 0 & -k_3 \\ \frac{1}{m_1} & \frac{-1}{m_2} & \frac{-k_1}{b_2} & 0 & 0 \\ \frac{1}{m_1} & 0 & 0 & \frac{-k_2}{b_3} & 0 \\ 0 & \frac{1}{m_2} & 0 & 0 & \frac{-k_3}{b_4} \end{bmatrix} \; B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{5-26}$$

Chapter 6

Scenario Analysis

6-1 Introduction

Simulation models are imitations of real-world systems. The goal of the system is to simulate a real-world firm and its consumer. This chapter validates the system of Equation 5-26 in order to determine if the model can be used to forecast the cash flows of an enterprise. Limitations of the system will potentially be found during the validation.

This chapter tests the system if behaves as one would expect. It puts forward two scenarios to validate the LTI system of the firm and its consumer:

Scenarios

1. Management decides to increase the supply because they believe it will increase the profits. (Section 6-3)
2. A sudden decrease of the firms stock q_{stock} occurs, e.g. a fire at the storage location. (Section 6-4)

To test the system against the stated scenarios, the mass, spring, and damper constants of the system need to be quantified. Section 6-2 substantiates the chosen constants. The constants are educated guesses of the real-world, but are not factual correct. This chapter only strives to give an idea on how this cash flow forecast method works.

6-2 Constants of the System

6-2-1 Mass Constants as Liquidity

The mass represents inverse of the liquidity of the product. A study done by Mahindra First Choice Wheels says that in 2018-19, the pre-owned car market has grown steadily and has crossed the 4 million unit mark making it bigger than the new car market. The new car market in 2018-19 stood at 3.4 million units, though a big growth from FY16-17, it's still far behind the used car market [19].

Because the used car market is bigger than the new car market, I assume that the used car market has a higher liquidity. The bigger the market, the easier it is to liquefy your asset, if the products are almost the same. Consequently, the slope of the supply curve m_1 is assumed to be steeper than the demand curve m_2.

The demand curve of Figure 5-2 has a starting price p_0. I assume the starting price is €10,000.

Table 6-1: Mass and liquidity constants with starting price of demand curve of the remarketing business

Mechanical	**Economic**	**Value**
Mass constant m_1	Liquidity^{-1} remarketing m_1	15 [kg] or $[\frac{€\cdot \mathrm{yr}}{\#^2}]$
Mass constant m_2	Liquidity^{-1} supply m_2	10 [kg] or $[\frac{€\cdot \mathrm{yr}}{\#^2}]$
Initial momentum p_0	Starting price remarketing p_0	10,000 [Nm] or $[\frac{€}{\#}]$

6-2-2 Spring Constants as Slope of Costs

A spring represents the stock of a consumer or a firm. The spring returns a force that represents the cost incurred by the agent from having that stock.

The starting point F_0 for the leasing price, when the lease consumer has a zero stock, is assumed to be €1,000 per car per year (see Figure 5-8) and is assumed to go down by €3.00 per car.

It is further assumed that the market for the remarketing consumer is bigger than the leasing market. Because of the bigger market, the cost for the remarketing consumer is expected to be less sensitive for each extra car in stock; the slope is assumed to be €2.50 per car.

The leasing firm is determined to keep its inventory low. The firm wants to push the cars out of its lot. With a higher spring stiffness, the firm will push harder. Thus, the holding cost has a relatively steeper slope than the consumer stocks. The holding cost is assumed to rise by €6 per car in stock.

Table 6-2: Spring constants with starting lease price of the demand curve of the leasing business

Mechanical	Economic	Value
Spring stiffness k_1	Slope holding cost k_1	$6\ [\frac{\text{N}}{\text{m}}]$ or $[\frac{€}{\#^2 \cdot yr}]$
Spring stiffness k_2	Slope lease cost k_2	$2.5\ [\frac{\text{N}}{\text{m}}]$ or $[\frac{€}{\#^2 \cdot yr}]$
Spring stiffness k_3	Slope remarketing cost k_3	$3\ [\frac{\text{N}}{\text{m}}]$ or $[\frac{€}{\#^2 \cdot yr}]$
Initial force F_0	Starting lease price F_0	2,000 [N] or $[\frac{€}{\# \cdot yr}]$

6-2-3 Damper Constants for Consumption

The dampers represent the loss of energy which the system will have as a result of salary or storing cars or using cars.

The depreciation rate of a car in use is assumed to be 17% per year. By substituting the stiffness k_2 and the depreciation rate of 17% in Equation 5-23, it follows that:

$$b_3 = \frac{k_2}{\text{Depreciation rate}} = \frac{2.5}{17\%} = 14.7 \tag{6-1}$$

The consumption rate of the consumer is assumed to be 12% per year. Because the car gets older, it will depreciate more slowly than a brand new lease car. By substituting the consumption rate and the the stiffness k_3 in Equation 5-14, it follows that:

$$b_4 = \frac{k_3}{\text{Consumption rate}} = \frac{2.5}{12\%} = 20.8 \tag{6-2}$$

The inventory shrinkage rate is much lower than the consumption rate and the depreciation rate, because the car is not in use. I assume that the inventory shrinkage rate is 5%.

$$b_2 = \frac{k_1}{\text{Inventory shrinkage rate}} = \frac{2}{5\%} = 40 \tag{6-3}$$

Valuation consultant assume that the salary costs are just a fraction of the total price paid per car supplied. I assume that the percentage per year spent on salaries is 1% of the total price.

$$b_1 = \frac{e_{\text{salary}}}{f_{\text{supply}}} = \frac{e_{\text{salary}}}{p_{\text{mc}}/m_1} = 1\% \cdot m_1 = 1\% \cdot 15 = 0.15 \tag{6-4}$$

Table 6-3: Damper constants of the loss of cash flow

Mechanical	Economic	Value
Damper b_1	Salary constant b_1	$0.15\ [\frac{\text{Ns}}{\text{m}}]$ or $[\frac{€}{\#^2}]$
Damper b_2	Inventory constant b_2	$40\ [\frac{\text{Ns}}{\text{m}}]$ or $[\frac{€}{\#^2}]$
Damper b_3	Depreciation constant b_3	$14.7\ [\frac{\text{Ns}}{\text{m}}]$ or $[\frac{€}{\#^2}]$
Damper b_4	Consumption constant b_4	$20.8\ [\frac{\text{Ns}}{\text{m}}]$ or $[\frac{€}{\#^2}]$

6-3 Scenario 1: Management's Higher Supply Decision as Step Input

In this scenario management decides to push the supply of the firm, because they expect that the enterprise will make more profit through this action. I assume that management can only influence the supply through the effort source Se_1, as seen in yellow in Figure 6-1. As a control engineer the push of the management can be seen as a step input and the results as the step input response.

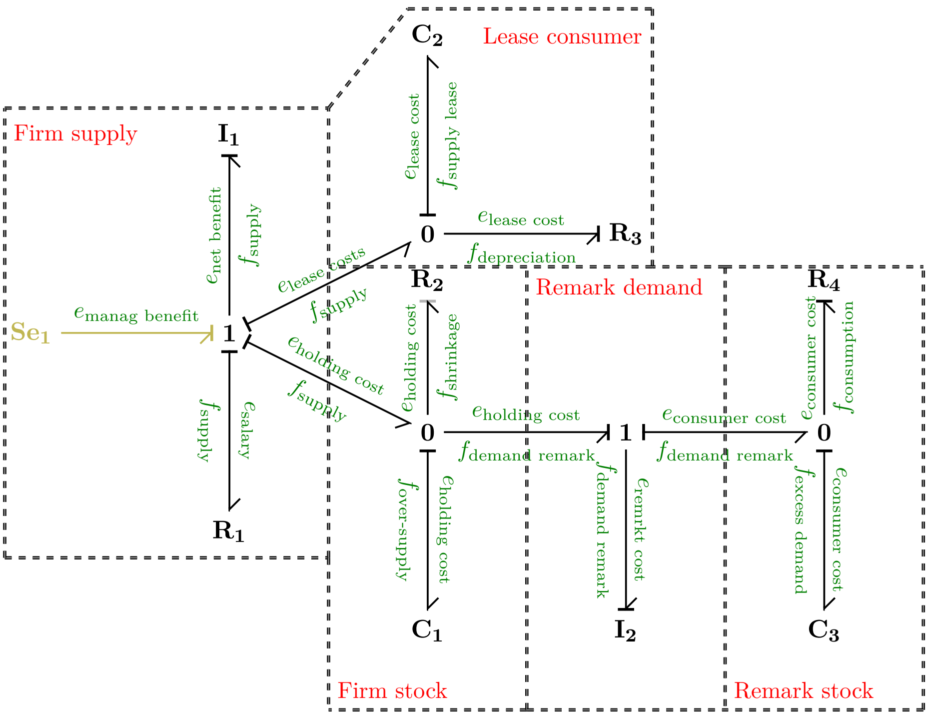


Figure 6-1: Bond graph of firm in car leasing business with highlighted management as source effort Se_1

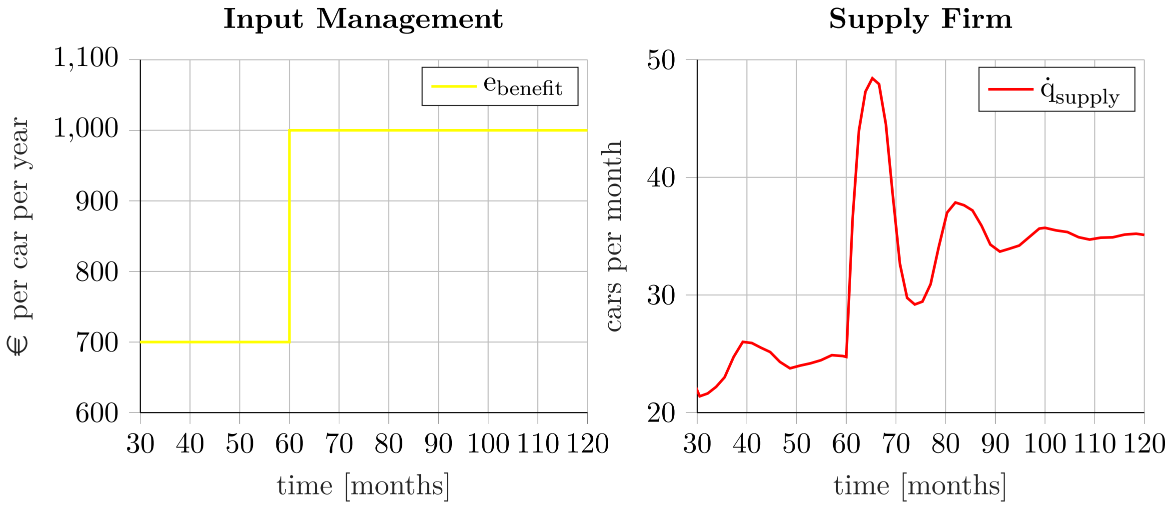
Before looking at the results, I reason on how the different elements of the firm are expected to behave:

- The willingness to pay for the supply increases, so the supply increases
- Because of the supply increase, the lease price will decrease. The firm wants to lease out more cars.
- The lease stock of the consumer will increase due to lower lease prices.
- The over-supply will cause an increase of stock at the inventory.
- The remarketing price will decrease because the firm incurs higher holding costs. The demand for inventory cars will increase due to the lower remarketing price.

6-3-1 Input and Supply

Valuation consultants use a maximum of 10 year or 120 months as forecast period: I will do the same for the sake of consistency. After 60 months, management increases e_{benefit} by €300 per car per year(Figure 6-2a), because expects increased profits with higher supply.

Due to managements higher effort, the momentum of mass I_1 increases. The momentum p_{mc} represents a willingness of the firm to pay a higher price for its supply. As a result, the supply of the firm will increase, as shown in Figure 6-2b.



(a) Management effort input

(b) The supply rises, and mechanically, the velocity of $m1$ increases

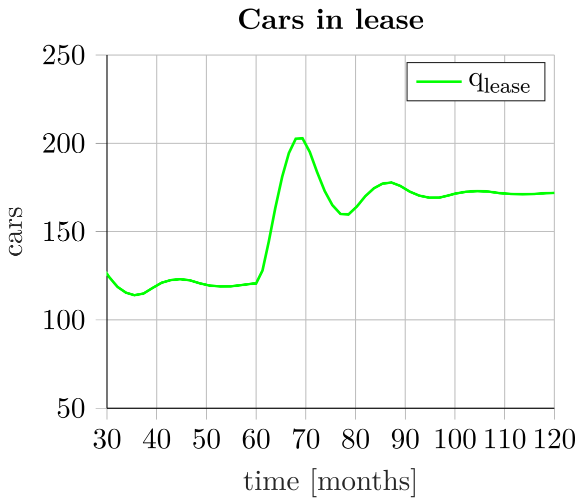
Figure 6-2: Management pushes the firm to supply more cars at month 60

The result of Figure 6-2b is what we expected the firm to do. If management pushes for more supply, the supply increases.

6-3-2 Leasing Business

Because the mass $I1$ accelerates, the spring C_2 starts to compress more. In economic terms, the leasing consumer starts to stock more cars (see Figure 6-3b). However, because the consumer has additional cars, they are willing to pay a lower lease price per car.

(a) Lease price development due to management change of supply



(b) Lease stock development due to management change of supply

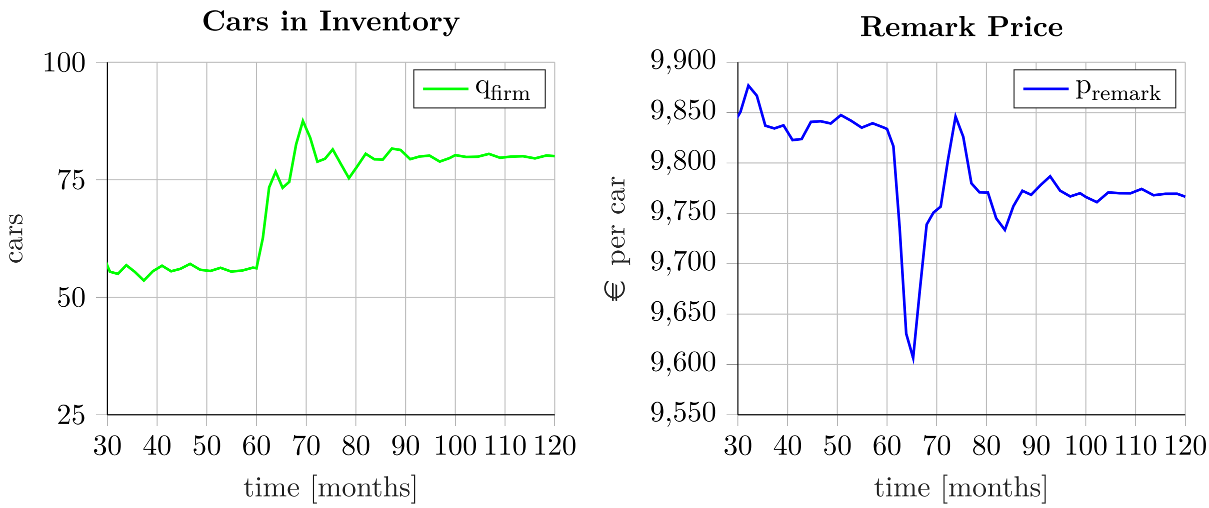
Figure 6-3: Management pushes the firm to supply more cars at month 60. With the aim of moving its surplus stock to the lease consumer.

The result of Figure 6-3 is in line with what we predicted. The firm leases out more cars because they have additional supply. To lease more cars out, the leasing enterprise decides to charge a lower leasing price.

6-3-3 Inventory and Remarketing Business

As the mass I_1 accelerates, the spring C_1 starts to compress more. In economic terms, the firm stores the additional supplied cars that are not being leased out, see Figure 6-4a.

As the inventory increases, the holding costs will also increase (see Figure 4-1). The firm will bring down its remarketing price in order to sell off the extra supply of cars, see Figure 6-4b. The lower remarketing prices result in a higher demand from the remarketing consumer.



(a) Lease price development due to management change of supply

(b) Lease stock development due to management change of supply

Figure 6-4: Management pushes the firm to supply more cars at month 60. Thereby the firm wants to get rid of its extra supply to the remarketing consumer.

Note that the firm does not sell all of its over-supplied cars, see Figure 6-4a. The firm makes a cost-benefit analysis of how much the holding costs grow and how much discount they have to give to the remarketing consumer. Conclusion is that having more cars in stock is the best option.

6-4 Scenario 2: Fire at Establishment as Impulse Response

In the second scenario, a fire at an inventory location occurs. I model the fire as a negative source flow of cars f_{fire} that removes cars in the spring C_1, as seen in Figure 6-5. As a control engineer, I interpreted this as an negative impulse to the stock of the firm .

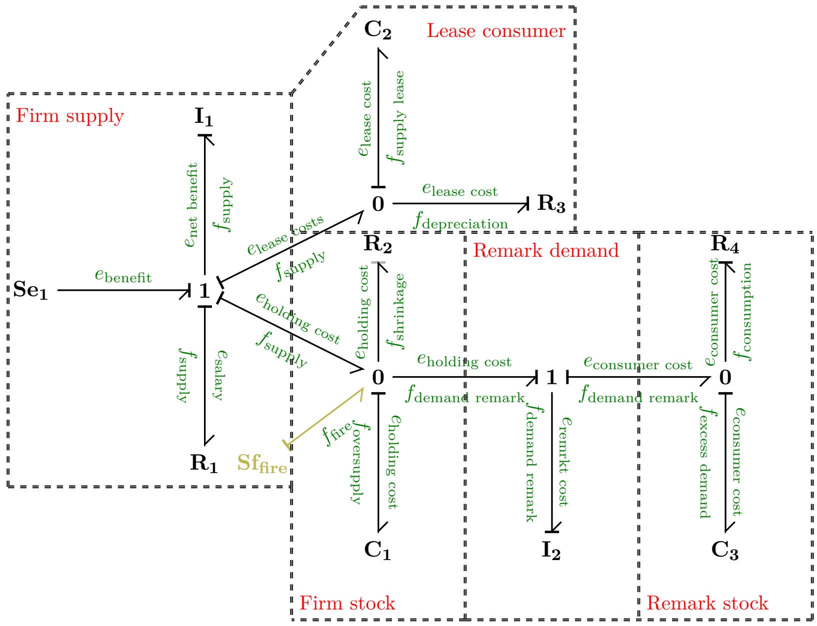


Figure 6-5: Bond graph of firm in car leasing business with fire as a source flow $\mathrm{Sf}_{\mathrm{fire}}$

.

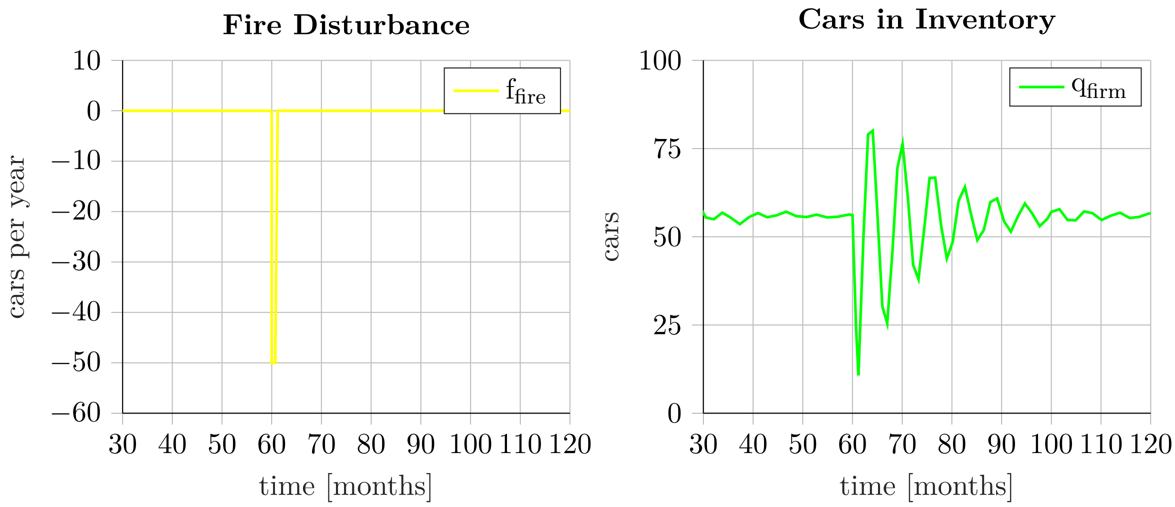
As we did with scenario 1, I first reason how the different elements of the firm can be expected to behave after a loss of inventory:

- Due to the fire, the inventory at that moment in time decreases.
- The supply of the firm should increase due to the lower stock of the inventory.
- The firm raises the asking price for the remarketing consumer because they have a shortage of cars.
- The leasing price is not affected because the inventory can only be sold to the remarketing consumer. Lease cars are always brand new.

6-4-1 Fire Disturbance and Firm's Inventory

The input f_{fire} is modeled as a high negative flow of cars in a very short period of time (Figure 6-6a).

The model returns stronger vibrations in the inventory level with a long settling time (Figure 6-6b). The strong vibrations and the long settling time occur because the system is not controlled by a management controller. I expect that, in my model, the firm's supply and remarketing price will show similar strong large vibrations.



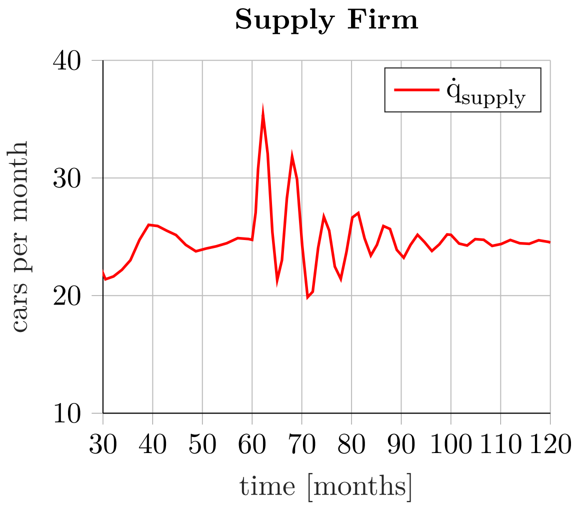
(a) The disturbance is modeled as a negative flow **(b)** The development of the inventory resulting from the fire

Figure 6-6: The fire removes cars from the inventory at the time of 60 months. The graph of the inventory shows strong vibration and long settling time

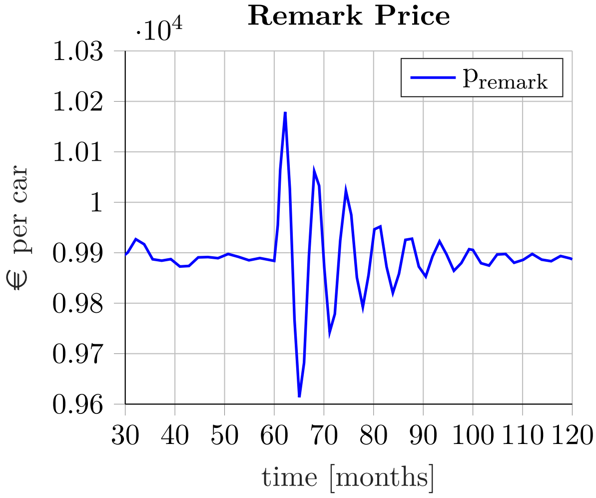
6-4-2 Supply and Remarketing Price

As seen in Figure 6-7a, the firm increases its supply of cars directly after the fire (at 60 months). However, the firm increases the supply too much, which results in over-supply. The over-supply causes a wave of vibrations in the system.

The remarketing price shows effects similar to what we see in the supply of firm. First, the remarketing price becomes more negative; which is good because there are fewer cars to sell. However, the price goes up too much, which results in an over-supply. Overall, the remarketing price also shows strong vibrations and a long settling time.



(a) The shortage of cars in stock pushes the firm to purchase more at the time of 60 months.

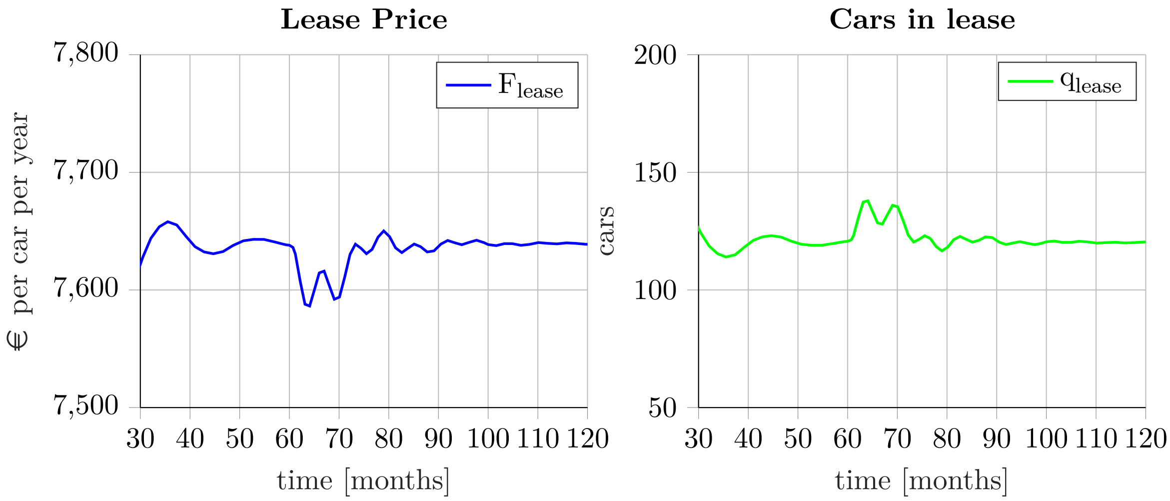


(b) The shortage of cars in stock pushes the remarketing price up at the time of 60 months

Figure 6-7: The supply and remarketing price show the expected behaviour at the time of 60 months. But as a result of the fire, the firm experiences strong vibrations with a long settling time.

6-4-3 Leasing Business

The expectation was that the leasing business was not going to be affected by the fire. However, Figure 6-8 shows that the fire does have a small impact on the leasing business too. Mechanically speaking, mass I_1 starts moving because of an additional supply of cars and the higher supply results in a displacement of spring C_2. Economically speaking, the firm lowers its leasing price, which leads to the leasing consumers having more cars in stock. The impact on the leasing business shows that there is a limitation in how the model works: the supply of leasing is not a separate mass like the supply of inventory C_1.



(a) Development of lease price due to the fire

(b) Development of lease stock due to the fire

Figure 6-8: The leasing business shows minor effects from the fire at the inventory location because the leasing consumer stock C_2 and the firms stock C_1 share the same mass I_1 that simulates the supply

6-5 Conclusions

The goal of this chapter is to prove that the a firm in the leasing business and its consumer can be modeled as the bond graph of Figure 5-11. With the help of two scenario's, the model is validated.

First I conclude, after reviewing all the results, that the model behaves as one would expect. This does not prove that the model simulates the behavior of the firm in a perfect manner, but it shows that the model gives a good representation of the essential elements of supply, demand, pricing, and inventory stock.

Secondly, what is missing in the model is the element of management control. A sudden change in the situation (for instance a fire), causes significant vibrations with a long settling time. A management that functions properly can foresee such situations and react to prevent the vibrations.

Chapter 7

The Management as Controller

7-1 Introduction

The advantage of modeling the firm and its consumer as an LTI system is that the tools of System and Control can be applied to the model. The LTI system opens new insights and possibilities to forecast the behaviour of an enterprise.

Control systems engineering deals with the control of continuously operating dynamical systems. This chapter applies control theory to the system of this thesis to simulate the management of an enterpise. The objective states:

> **Objective**
> Find a control formalism that simulates the management of an enterprise.

This chapter designs two control solutions to the problem:

> **Solutions**
>
> - PID control as management that controls inventory.
> - Optimal control as management that maximizes profit.

Section 7-2 checks the controllability and observability of the system with the given inputs and observably outputs and gives interpretations what this means as a management. Section 7-3 applies a proportional-integral-derivative controller (PID) to the system and interprets the different control actions as management decisions. Section 7-4 designs an optimal controller that strives to maximize profit under the given market situation of Chapter 6.

7-2 Controllability and Observability

The controllability checks if the system can move in its entire configuration space with the given inputs. If the controllability matrix $\begin{bmatrix} B & AB & A^2B & A^3B & A^4B \end{bmatrix}$ is full rank, all the states of the system are controllable[20]. For management it is important that they can influence the states of the system. For instance, it would be hard to control a firm and its cosumers as management if prices could not be controlled. As earlier mentioned, I assume that the leasing enterprise is a price maker. But a market under pure competition would be an example of an enterprise that could not influence the price willing to pay of the consumer. For this reason, the price maker assumption is essential in the controllabillity of the firm and its consumer.

The system matrices of Equation 5-26 are used. By means of this thesis paper width, I substitute the constants given in Section 6-2 to make the controllability matrix:

$$\begin{bmatrix} B & AB & A^2B & A^3B & A^4B \end{bmatrix} = \begin{bmatrix} 1.0000 & -0.0050 & -0.1000 & -1.1915 & 0.1714 \\ 0 & 0 & 0.2000 & -0.0160 & -0.1888 \\ 0 & 0.0333 & -0.0027 & -0.0231 & -0.0364 \\ 0 & 0.0333 & -0.0070 & -0.0019 & -0.0393 \\ 0 & 0 & 0 & 0.0200 & -0.0040 \end{bmatrix} \quad (7\text{-}1)$$

The matrix of Equation 7-1 is full rank. The management controller is capable to control the firm and its consumer.

As for observability, I assume that the output C-matrix of the system is an identity matrix. An identity matrix as output matrix is applicable if your are the only enterprise in this market. For example, if there were a competitor with the same product. One would never know how much the competitor is selling to the consumer, thus the stock states of the consumer are not outputs of the system.

$$y = Cx, \text{ with } C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (7\text{-}2)$$

7-3 PID as Management Control

Section 6-5 concludes the system without control is too sensitive for disturbances, which shows in high vibrations and long settling times. PID control as management is a potential solution to get rid of these vibrations. But does it make economic sense to make proportional, integral, and derivative actions as management?

In certain management models economist already make a distinction between the past, present and future. The "maturiteitsmodel" or maturity-model of M. Franssen distinguishes the control of management as past, present en future control.[21] This section uses this model to analogize PID-control to management control.

I consider the situation of Section 6-4 that there occurs a fire and we want inventory control. Specialists conclude that the ideal inventory is 70 cars in stock. Now we know what the reference signal is and the control variable[22]:

$$e_{\text{firm stock}} = r_{\text{firm stock}} - q_{\text{firm stock}} \tag{7-3}$$

The error term $e_{\text{firm stock}}$ is what a PID controller seeks to minimize.

7-3-1 Proportional as Present Control

Franssen states: "The design of the present control system is focused on the present."[21] Equivalently, the proportional term of the PID-control acts only on the error at the current time. [22]

$$u_p = K_p \cdot e_{\text{firm stock}} \tag{7-4}$$

If management were a P-controller for inventory control, they would act only on the shortage or overstock at that current moment. With a higher K_p, management will react more drastically on the offset of the desired stock.

Figure 7-1 shows three simulation result on different values of K_p. $K_p = 10$ shows a far too big steady-state error because the P-controller as management does not act enough on the shortage. The result of $K_p = 80$ has a smaller steady-state error, but worse settling time. $K_p = 400$ shows an even lower steady-state error but unrealistic and fast vibrations, which results in a long settling time.

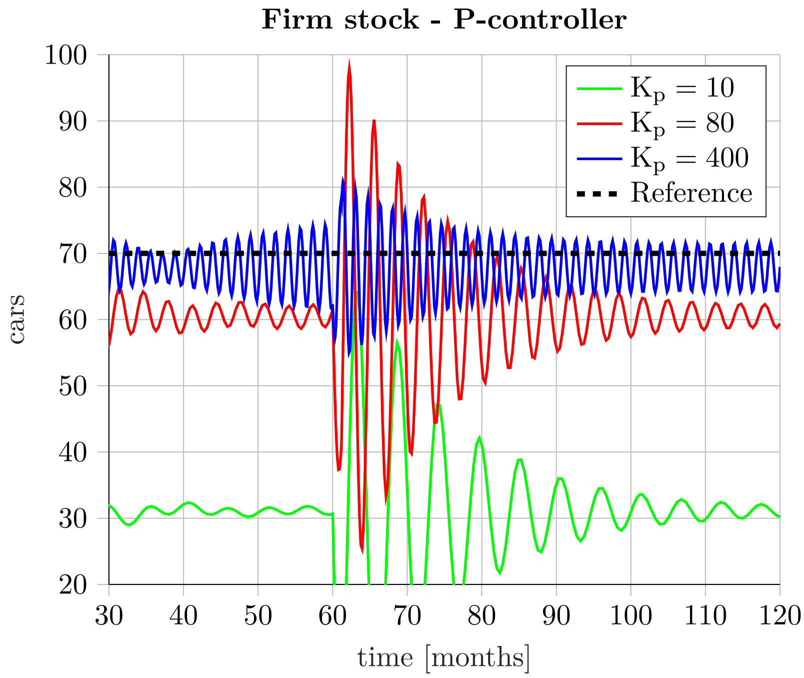


Figure 7-1: Impulse response of the system with a P-controller with different value of K_p. At time 60, the impulse fire of Section 6-4 occurs.

7-3-2 Integral as Past Control

Franssen states: "The design of the past control system is focused on the past."[21] Just like the integral term of a PID controller takes the past error into account. The I-term sums all the past errors by taking the integral over time. The I-term eliminates the steady-state error of the system.[22]

$$u_i = K_i \cdot \int_0^t e_{\text{firm stock}}(\tau) d\tau \tag{7-5}$$

Management looks at the past as a basis for the future. For instance, if there were a shortage for a couple of years, they would not be too worried if there is an overstock one year.

Similarly, if there is a steady-state error of shortage for a couple of years, they know they have to push the supply harder in comparison with the years before. Thus, the integral term makes sense to mimic the behaviour of management.

Figure 7-2 shows three simulation results on different values of K_i, with $K_p = 80$. As expected, the integral term eliminates the steady-state error. But as a consequence, it increases the frequency of the vibrations and the settling time. The steady-state error or the vibrations is the trade-off of the integral term.

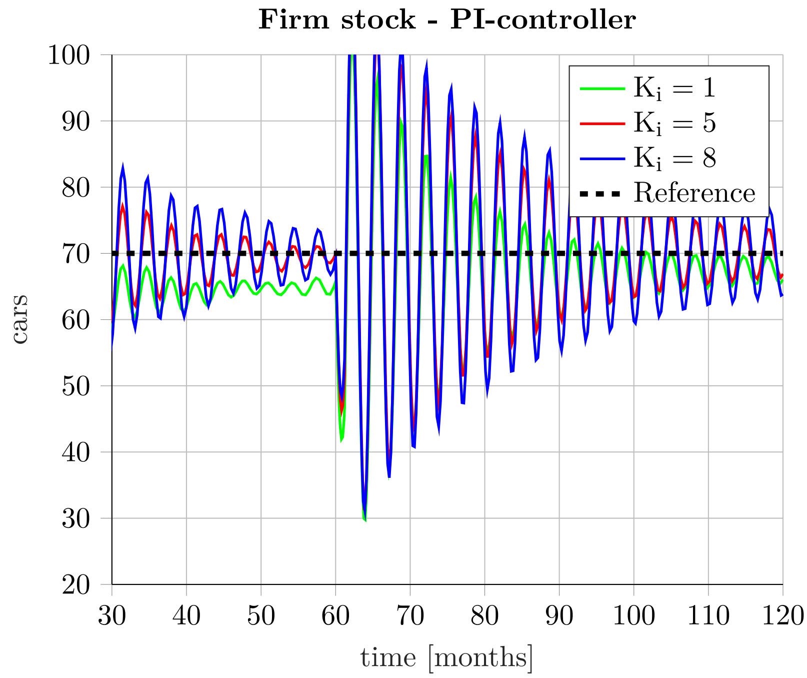


Figure 7-2: Impulse response of the system with a PI-controller with different value of K_i, with $K_p = 80$. At time 60, the impulse fire of Section 6-4 occurs.

7-3-3 Derivative as Future Control

Franssen states: "The future control system seeks to steer the enterprise into in the right direction of the future."

The derivative term of a PID controller looks the future by taking the derivative of the error over time. The D-term is not often used in the real-world controller because of the derivative is sensitive to noise. But, because my model's purpose is forecasting, the controller does not have to be used in real-world applications. Thus noise on the output is of no importance in this system. Consequently, the derivative term is applicable.[22]

$$u_d = K_d \cdot \frac{de_{\text{firm stock}}}{dt} \tag{7-6}$$

A prediction is an essential piece of information for management. This thesis strives to be a contribution to management's forecasts. The D-term predicts the error, so it plays a necessary part in mimicking management.

The downside is that the D-term only looks at the current derivative. A management takes into account a larger prediction horizon. I suggest for future research to design a MPC controller that can simulate a management with a larger prediction horizon.

Figure 7-3 shows three simulation results on different values of K_d, with $K_i = 5$ and $K_p = 80$. The D-term shows great results in decreasing the settling time and the amplitude of the vibrations. The higher K_d, the more management trusts their predictions.

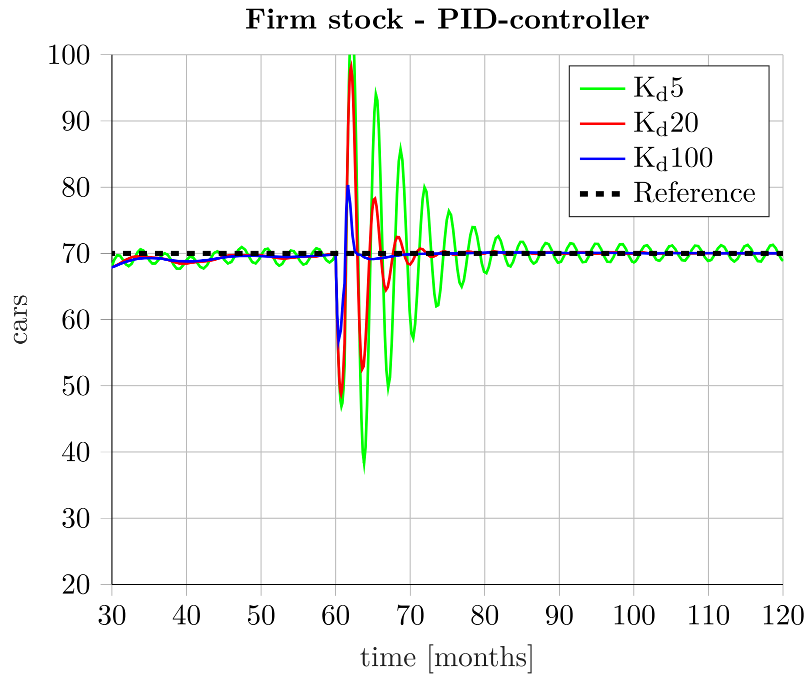


Figure 7-3: Impulse response of the system with a PID-controller with different value of K_d, with $K_i = 5$ and $K_p = 80$. At time 60, the impulse fire of Section 6-4 occurs.

7-3-4 Conclusions

A PID-controller takes into account the past, present and future, just like management does. The input of management to push or pull the supply is a summation of the proportional, integral and derivative input[22]:

$$u = u_p + u_i + u_d = K_p \cdot e_{\text{firm stock}} + K_i \cdot \int_0^t e_{\text{firm stock}} d\tau + K_d \cdot \frac{de_{\text{firm stock}}}{dt} \tag{7-7}$$

Therefore this section concludes the following:

A PID controller has the capabilities to simulate management.

Inventory control is just one example of an application of PID control as management. Management could have the objective to attain a certain growth or keep their products price constant.

A limitation of PID control is that the derivative term only considers the current derivative. Real-life management hires specialists to forecast further into the future and decides on that information. Other types of control could be applied to solve this problem.

7-4 Profit Maximization with Optimal Control

Every business or management has its own objective, the foremost one is to maximize profit. With excess profit, the other important objectives of management can be fulfilled like customer satisfaction, employee welfare, and social responsibilities.

In system and control, we know a control field that maximizes, or more often, minimizes. Optimal control deals with a control law for a dynamical system over a period with an objective function. The objective function would be, as being management, profit. With the help of an optimal controller a management can be simulated that makes the best choices to maximize the enterprises profit in variant market situations. It is hard for a valuation consultant to know how management decisions change when he changes one variable. Because optimal control is an advanced control method, the controller knows what the best input is for the system, as management knows what the best decision is to steer their firm. It can be argued if real management knows what the best decisions are at that particular time, but that is a different discussion.

7-4-1 Profit as Objective Function

I assume that the profit is defined as:

$$\text{Profit} = \text{Revenue} - \text{Costs} \tag{7-8}$$

Optimal control us a function $V(p_i, q_i)$ that minimizes the objective function of the controller. Because management want to maximize the profit, the controller should minimize the negative of the profit. The objective function is the summation of Equations 4-21, 4-12, 5-9 and 5-25:

$$\begin{aligned} V(p_i, q_i) &= \min \int_0^\infty - \text{profit } dt \\ &= \min \int_0^\infty - \text{Revenue} + \text{Costs } dt \\ &= \min \int_0^\infty - E_{\text{TRR}} - E_{\text{TLR}} + E_{\text{TSC}} + E_{\text{THC}} \, dt \\ &= \min \int_0^\infty \frac{-p_0}{m_2} \cdot p^*_{\text{demand}} + \frac{p^{*2}_{\text{demand}}}{m_2} + -F_0 \cdot q_{\text{lease stock}} + k_2 \cdot q^2_{\text{lease stock}} \\ &\qquad + \frac{p^2_{\text{mc}}}{2m_1} + \frac{1}{2} k_1 q^2_{\text{firm stock}} \, dt \end{aligned} \tag{7-9}$$

$V(p, q)$ also needs to be depended on the input. The weighing factor R given to the input represents the weight management is allowed to push the firm to supply in comparison with the profit is makes. This chapter assumes that management is allowed to push the supply as high as it wants; thus R of Equation 7-10 is small.

The integral of Equation 7-9 is essential in simulating a management that wants to maximize profit. Without the integral, the controller would optimizes profit in the present time. Consequently, the controller would never investment in order to make profits in the future.

With the states defined as Equation 5-26 and R as the weight of u, the the objective function in matrix form is:

$$V(x,u) = \min \int_0^\infty x^T Q x + Gx + u^T R u \, dt \text{ with } Q = \begin{bmatrix} \frac{1}{2m_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_2} & 0 & 0 & 0 \\ 0 & 0 & \frac{k_1}{2} & 0 & 0 \\ 0 & 0 & 0 & k_2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} G = \begin{bmatrix} 0 \\ -\frac{p_0}{m_2} \\ 0 \\ -F_0 \\ 0 \end{bmatrix} \tag{7-10}$$

With Q and G as the error weighted matrices. And R being the control weighted matrix.[23]

7-4-2 Optimal Regulator with Integral Feedback

LQR control is the most common type of optimal control which minimizes a quadratic objective function. A problem arises from using regular linearquadratic regulator LQR control because the objective function of Equation 7-10 is not quadratic. The affine term Gx gives trouble solving the differential equations. I introduce new states p_{n2} and q_{n2} to rewrite Equation 7-9 as an quadratic function.

$$p_{new} = p^*_{\text{demand}} - \frac{p_0}{2} \tag{7-11}$$

$$q_{new} = \sqrt{k_2} \cdot q_{\text{lease stock}} - \frac{F_0}{2\sqrt{k_2}} \tag{7-12}$$

Rewrite Equation 7-9:

$$\begin{aligned} V_n(p_1, p_{new}, q_1, q_{new}, u) = \int_0^\infty & \frac{p_{new}^2}{m_2} + q_{new}^2 + \frac{p_{\text{mc}}^2}{2m_1} + \frac{1}{2} k_1 q_{\text{firm stock}}^2 + u^T R u \\ & - \frac{1}{m_2}(\frac{p_0}{2})^2 - (\frac{F_0}{2\sqrt{k_2}})^2 \end{aligned} \tag{7-13}$$

Now the objective function is quadratic with two constant terms, which can be added back to the objective function later on.

With the new states p_{new} and q_{new}, the system matrices are rewritten as Equation 5-26. The constant on the states are interpreted as constant disturbances on the states of the system with a constant of $w = 1$.

$$\dot{x}_n = A x_n + G_n w + B u, \text{ with } x_n = \begin{bmatrix} p_1 \\ p_{new} \\ q_1 \\ q_{new} \\ q_3 \end{bmatrix} w = 1 \tag{7-14}$$

$$A_n = \begin{bmatrix} -\frac{b_1}{m_1} & 0 & -k_1 & -\sqrt{k_2} & 0 \\ 0 & 0 & k_1 & 0 & -k_3 \\ \frac{1}{m_1} & \frac{-1}{m_2} & \frac{-k_1}{b_2} & 0 & 0 \\ \frac{1}{m_1\sqrt{k_2}} & 0 & 0 & \frac{-1}{b_3} & 0 \\ 0 & \frac{1}{m_2} & 0 & 0 & \frac{-k_3}{b_4} \end{bmatrix} G_n = \begin{bmatrix} \frac{F_0}{2} \\ 0 \\ -\frac{p_0}{2m_2} \\ -\frac{F_0}{2b_3\sqrt{k_2}} \\ \frac{p_0}{2m_2} \end{bmatrix} B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

And the objective function of Equation 7-13 in matrix form:

$$V_n(x_n, u) = \min \int_0^\infty x_n^T Q_n x_n + u^T R_n u \text{ with } Q_n = \begin{bmatrix} \frac{1}{2m_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{m_2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2k_1} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} R_n = R \quad \text{(7-15)}$$

To solve this optimal control problem, a linear operator with integral feedback is an option.[24] A condition is that the disturbance matrix G_n is in the range space of the input matrix B. This condition satisfies if it is possible to choose a matrix M such that $G_n = BM$. This condition does not hold.

Because G_n is not in the range space of B, the linear operator with integral feedback of Anderson[24] cannot be applied.

7-4-3 Linear-Quadratic Regulator with Disturbance Rejection

Bingzhao Gao [25] proposes to use the Hamiltonian approach [26] to vehicle launch control . This paper rejects known time-varying disturbances. Our new system of Equation 7-4-2 has a constant disturbance, because $w = 1$ at all times. That should not give any problems.

The Hamiltonian function is chosen as:

$$H = x_n^T Q_n x_n + u^T R_n u + \lambda(A_n x_n + Bu + G_n w) \quad \text{(7-16)}$$

where λ is the multiplier function to be determined.

After derivation, the optimal control law u^* is designed as [25]:

$$u^* = K_x x_n + K_w w \quad \text{(7-17)}$$

with

$$\begin{aligned} K_x =& - R_n^{-1} B^T P \\ K_w =& - R_n^{-1} B^T h \\ h =& (PBR^{-1}B^T - A_n^T)^{-1} P G_n \end{aligned} \qquad (7\text{-}18)$$

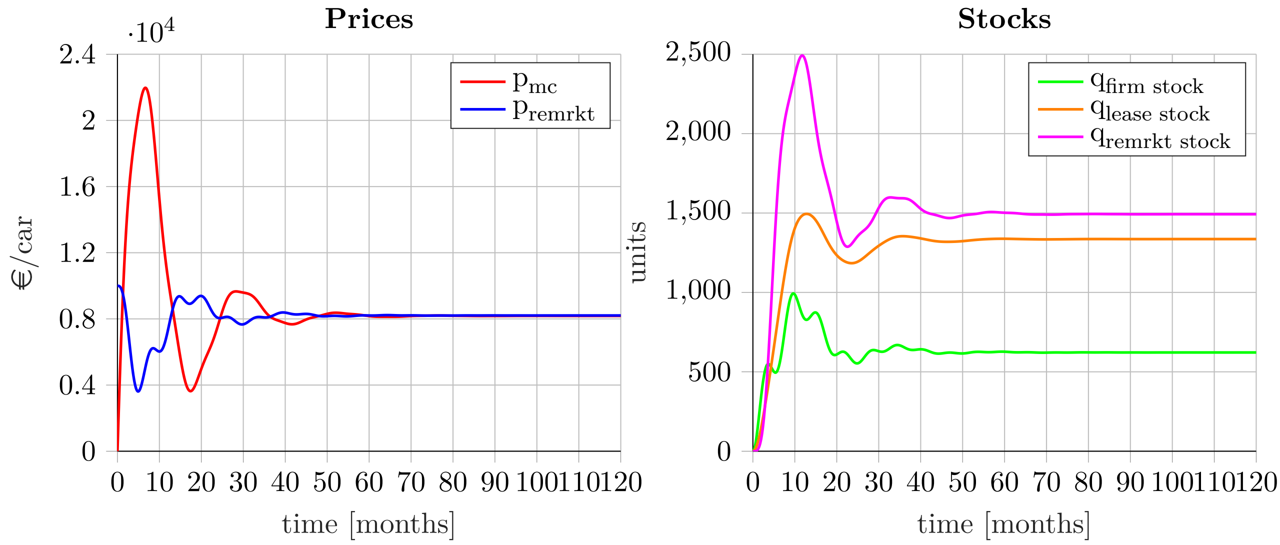
Appendix B shows the derivation of the control law of Equation 7-17.

7-4-4 Application Management as Optimal Controller

The system of Equation 7-4-2 is simulated with the optimal control law of Equation 7-17. Afterwards, the states are transformed back to the original system of Equation 5-26. Simulating the system with a management thats seeks the highest profit in current market situations.

Figure 7-4 presents the optimal prices and stock at every moment in time. The enterprise starts with no cars en no income. Logically, the enterprise starts buying a lot of cars to have inventory for their lease and remarketing consumer. Afterwards, the enterprise find a steady state. For example, the enterprise is willing to pay €8000 for its last supplied car at its steady state.

Keep in mind that the p_{remrkt} is the price for a second hand car with its complete value. The enterprise is selling depreciated cars because of the wear and tear. For example, if two cars are 50% depreciated, the model will see that as one car and sell this for its 100% value.



(a) Prices of the system with optimal controller as management

(b) Stocks of the system with optimal controller as management

Figure 7-4: The prices of the system with optimal controller as management

Because the enterprise starts investing in their stock. The enterprise making a lot of costs in the first year, as seen in Figure 7-5. In this situation, the leasing business has a higher contribution to te revenue than the remarketing. This is because the constants of the system are not identified. In practice these revenues should be divided proportionally,

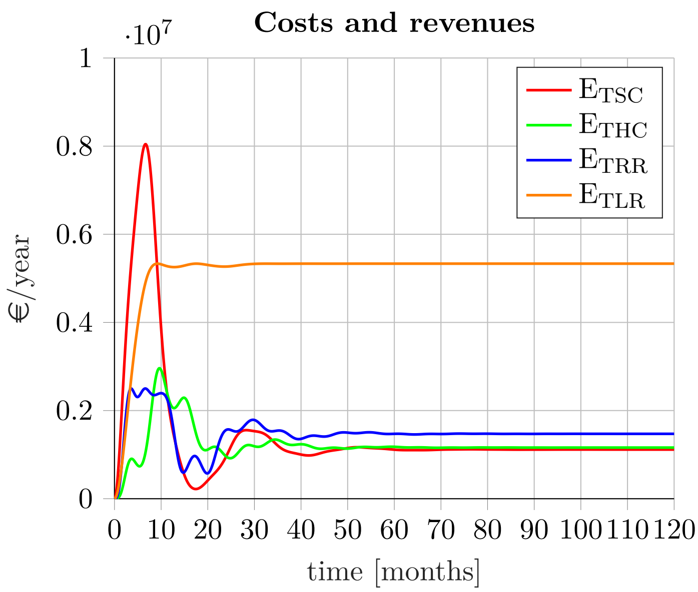


Figure 7-5: Revenues and costs of the system with optimal controller as management

Figure 7-6 presents the maximum profit the firm can make under these market situations. Logically, the first year the enterprise is not profitable because their first-year investment in their stock. After 50 months the enterprise has a steady-state and the management has found their maximum profit under these market situations.

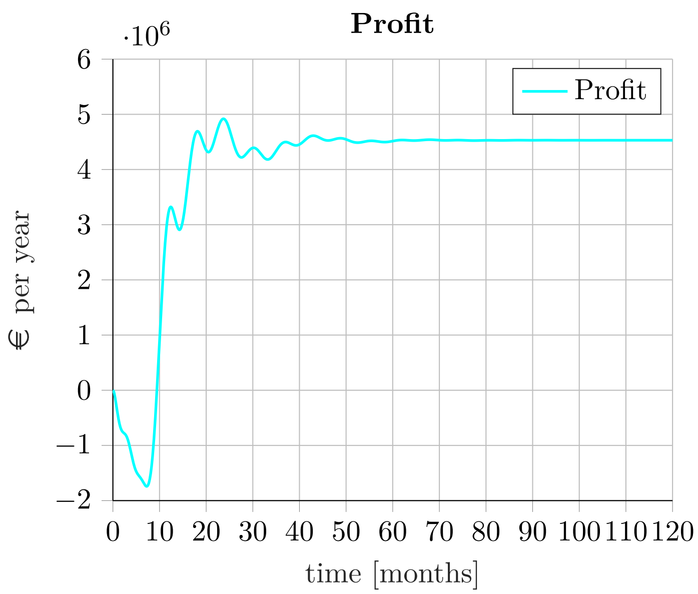


Figure 7-6: The profit with an optimal controller that simulates management by maximizing profit

7-4-5 Conclusions

This section designs an optimal controller to model a management that strives to maximize profit. The objective function is the hardest problem to solve. The objective function has non-quadratic terms excludes to use of regular LQR control. By introducing new states the optimal control is build with distrubance rejection. For this reason, this section concluges:

> A optimal controller with disturbance rejection has the capabilities to simulate a management that strives to maximize profit.

The optimal controller misses two important boundaries. The first boundary is that the actual enterprise is depended on how much money they have for their investments. This section assumes that the enterprise has unlimited access to money without any interest. Real-life management does commonly has a budget to make investments to their enterprise. The second boundary is that a real-life management is not capable to grow infinitely fast. Growing your enterprise costs time. The optimal controller of this section does not has a maximum growth boundary.

Chapter 8

Discussion

This chapter discusses the LTI model as a firm in the leasing business and its consumer in Section 8-1. And this chapter discusses the application of the controller simulating a management of an enterprise in Section 8-2.

8-1 The Firm and its Consumer as a System

The results of Chapter 6 indicate that the system modeled in this thesis gives a good representation of the phenomena within and between a firm and its consumer. However, the model of the firm assumes that the supply cost per asset increases by adding a quantity flow supplied. Valuation consultants often assume that the supply cost per quantity flow of a product is a constant, because smaller firms do not influence the price. Similarly, valuation consultant assume the inventory costs per product to be a constant. If I where to model the holding costs per quantity stored as a constant, the spring would not have the tendency to clear its inventory. Same argument could be put forward for the supply or demand and their price. This makes the market clearing and cost-benefit analysis only applicable on large companies or monopolies, which are price makers.

An advantage of the model is that the car leasing business and the remarketing business in my model are modeled as two separate businesses, with a common supply of cars. This is true in real-life. But a limitation is that the model is able to sell brand new cars to the consumer of the remarketing. In real life the supply of cars is leased first to the lease consumer and is sold after a 5-year contract. The supply of cars to the remarketing consumer should have a five year delay.

This thesis does not focus on the identification of the constants of the system (mass, spring, damper constants). To apply this method as a cash flow forecast method, these constants are essential to make the method work. This asks a whole different kind of due diligence of the accounting firms. For example, it seems hard to identify the cost curve of a consumer. To validate the system, one should predict the cash flow of an enterprise and wait a few years to check the results. This is a time consuming problem. Note that all cash flow forecast methods incur this same problem.

8-2 Management as Controller

This thesis applies PID and optimal control to model the management of an enterprise.

PID control makes a distinction between present, past and future control. Many would agree that management makes decisions based on the present, past and future. However, the derivative term of a PID-controller only considers the present derivative of the error. The current derivative does not include all future predictions a real life management has for their decisions. For the sake of the argument, the cash flow forecast of this thesis constructs a forecast of 10 years.

Optimal control simulates a management that seeks to maximize the profit of an enterprise. Maximizing profit is a common goal for a management. In this thesis the optimal controller makes the best decisions possible. It can be argued if a management always knows the best decision to maximize their profit. For instance, if valuation consultant know the management is controlled by non-interested shareholders, one should design an optimal controller with less performance results. Valuation consultants have a solution for this problem, bad management is often included in the discount rate (WACC) of an enterprise.

The optimal controller of Section 7-4 is able to invest an infinite amount of money and the enterprise can grow almost infinitely fast. This controller does not have any boundary conditions like maximum growth or minimum equity/debt ratio restrictions. A real-life management has these restrictions. Growing a company costs time and finding money to invest is not that easy. This thesis only proves that an optimal controller is able to simulate a management that wants to maximize profit.

Chapter 9

Conclusions and Recommendations

9-1 Conclusions

This thesis aims to construct a cash flow forecast method with the approach of economic engineering. Cash flow predictions are important to determine the value of an enterprise or the predictions are used to make business decisions. The identified problem is that the current valuation methods do not incorporate high-order effects on the predicted cash flow and do not sufficiently take into account management decisions to various future market situations. This thesis concludes that these problems can be solved with a control engineer's approach.

This thesis considers three agents that influence the cash flow of an enterprise: the firm, its management and its consumer.

The analogs of economic engineering and LTI system theory are applied to model a mechanical system that contains phenomena of a firm in the leasing business and its consumer. For instance, spring behaves like the inventory of a firm, a mass represent the supply of a firm and damping as the wear and tear of an asset. A market clearing is identified within a firm and the consumer. The firm wants to clear their inventory, and the consumer does not want more assets than it consumes. Both construct a cost-benefit analysis to buy or sell an asset at their willingness to pay. The analogs between economics and mechanics gives the opportunity to model high-order effects between the different elements of the firm and its consumer. Impulse and step responses - which are well known in the field of System and Control - are applied to see what disturbances and decisions influence the firm and its consumer. These responses are hard to model in the spreadsheets of current valuation methods. The results of the responses show the same behaviour as one would expect how a firm and its consumer would behave in certain scenarios. This leads me to conclude that a firm and its consumer can be modeled as a mechanical LTI system.

This thesis separates the management from the firm. This separation and by modeling the firm and its consumer as a mechanical system gives the opportunity to design a controller that simulates a well-functioning management. It has been proven that PID control and optimal control are able to simulate a management with objectives as inventory control or maximizing profit. A PID controller shows similar behaviour in making decisions as management does by considering the past, present and future of a state. Optimal control as maximizing profit is proven to be applicable on the system of this thesis. This leads me to conclude that different types of control are applicable in simulating a management.

Above contributions combined brings me to the conclusion that this thesis lays the foundation for developing a cash flow forecast method by using the tools of Systems and Control.

9-2 Recommendations

This thesis models a linear time-invariant system. Many phenomena, such as supply or demand, are not linear. I recommend to expand the system by using non-linear masses, springs and dampers to make the system a more realistic presentation of reality.

The system does not incorporate curve shifts due to non-price determinants of demand or supply. Examples of a demand curve shifts are when the consumer's income or tastes and preferences change[15]. Such as the popularity of the electric car increases [3]. De demand curve will shift to the right or to the left. Economists refer to curve shifts as important events when using supply or demand curves. I recommend to find a way to model a bond graph that makes it capable to add curve shifts.

An important aspect of valuation is the capitalization of an enterprise. For instance, if an enterprise has a low equity level (an enterprise in the car leasing business), it is common to use the dividend discount model (DDM)[5]. DDM bases the value of an enterprise on the dividend that is payed to the shareholders. When using the DDM, the equity debt ratio is important to determine the maximum growth of an enterprise, because the firm has to invest profit to remain at a certain equity level. I recommend to change the flow and effort of the system through gyrators, for example from asset flow to cash flow.

In order to determine the value of an enterprise other economic elements need to be modeled or identified. Taxes, discount rate, growth and inflation are examples of these elements. In recommend to add these elements to the model.

The discount rate (WACC) has great influence on the value of an enterprise as seen in Chapter 2. Although finding a mechanical interpretation for the discount rate is not in the scope of this thesis, the DCSC economic engineering group noticed that the Laplace domain shows high resemblance to the discount rate. The real part of the complex frequency variable emerges as the analog to the regular discount rate, while the imaginary part represents the volatility in the cash flow that serves as a measure of the risk. R. Grubbström (1967) already found applications of the Laplace transform on economic problems. [27]

Appendix A

A-1 Analogs between mechanics and economics

Overview of the economical analogs introduced by [2]:

Table A-1: Mechanics, electronics and valuations analogies with units[2]

Mechanics			**Electronics**			**Valuations**		
Force	F_x, F_y, F_z	[N]	Voltage	V	[V]	Cost	$\dot{p_1}, ..., \dot{p_n}$	[€/#·yr]
Velocity	v_x, v_y, v_z	[m/s]	Current	A	[A]	Flow of goods	$\dot{q_1}, ..., \dot{q_n}$	[#/yr]
Momentum	p_x, p_y, p_z	[kg·m/s]	Flux linkage	λ	[Wb]	Price	$p_1, ..., p_n$	[€/#]
Displacement	q_x, q_y, q_z	[m]	Charge	Q	[C]	Stock	$q_1, ..., q_n$	[#]
Mech. Energy	$E_{\text{kin}}, E_{\text{pot}}$	[J]	Elec. Energy	E	[J]	Cash flow	$E_1, ..., E_n$	[€/yr]

Appendix B

B-1 Derivation Linear-Quadratic Regulator with Disturbance Rejection

The Hamiltonian function is chosen as:

$$H = x_n^T Q_n x_n + u^T R_n u + \lambda(A_n x_n + Bu + G_n w) \tag{B-1}$$

where λ is the multiplier function to be determined. Now that the Hamiltonian is defined, we know that the control variation δu should be set to zero. The optimal control law u^* is found:

$$\frac{\partial H}{\partial u} = 0 \rightarrow R_n u^* + B^T \lambda^* = 0 \tag{B-2}$$

leading to

$$u^* = -R_n^{-1} B^T \lambda^* \tag{B-3}$$

With above equation, the state and the co-state equations are obtained as:

$$\dot{x}_n^* = \frac{\partial H}{\partial \lambda}_* \qquad \rightarrow \dot{x}^* = A_n x_n^* - B R_n^{-1} B^T \lambda + G_n w \tag{B-4}$$

$$\dot{\lambda}^* = -\frac{\partial H}{\partial x_n}_* \qquad \rightarrow \dot{\lambda}^* = -Q_n x_n^* - A_n^T \lambda^* \tag{B-5}$$

Now I introduce the solution of λ^*:

$$\lambda^* = P x_n^* + hw \tag{B-6}$$

The disturbance, h and P are time-invariant, thus the time derivative of Equation B-6 is:

$$\dot{\lambda}^* = P \dot{x}_n^* \tag{B-7}$$

Substituting Equation B-4,Equation B-5 and Equation B-6 into above equation leads to:

$$(Q_n + A_n^T P + P A_n - P B R_n^{-1} B^T P) x_n^* + (A_n^T h - P B R_n^{-1} B^T h + P G_n) w = 0 \tag{B-8}$$

P is determined by the Ricatti equation:

$$Q_n + A_n^T P + P A_n - P B R_n^{-1} B^T P = 0 \tag{B-9}$$

Afterwards, h is found by solving:

$$\begin{aligned} A_n^T h - P B R_n^{-1} B^T h + P G_n = 0 \\ h = (P B R_n^{-1} B^T - A_n^T)^{-1} P G_n \end{aligned} \tag{B-10}$$

The optimal control law u^* of Equation B-3 becomes[25]:

$$\boxed{u^* = K_x x_n + K_w w} \tag{B-11}$$

with

$$\begin{aligned} K_x =& - R_n^{-1} B^T P \\ K_w =& - R_n^{-1} B^T h \\ h =& (P B R^{-1} B^T - A_n^T)^{-1} P G_n \end{aligned} \tag{B-12}$$

P is found with the matlab function *lqr(sys,*Q_n*,*R_n*)*.

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Glossary

List of Acronyms

DCF	Discounted cash flow
WACC	Weighted average cost of capital
DDM	Dividend discount model
FCF	Free cash flow
MC	Marginal cost
PID	Proportional-integral-derivative
DCF	Discounted cash flow
THC	Total holding costs
TSC	Total supply costs
TRR	Total remarketing revenue
TLR	Total leasing revenue
DDM	Dividend discount model
LQR	Linear-quadratic regulator