Report no.130-M

Discussion of the papers:

"A NEW APPRAISAL OF STRIP THEORY",

By: Vassilopoulos and Mandel,

and

"THE DISTRIBUTION OF THE HYDRODYNAMIC FORCES ON A HEAVING AND PITCHING SHIPMODEL IN STILL WATER",

By: Gerritsma and Beukelman.

Both presented at the Fifth Symposium on Naval Hydrodynamics, Bergen 1964.

By: T.R. Dyer.

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1. Introduction.

The paper by Vassilopoulos and Mandel [1] rigorously examined much of present seakeeping theory, and is especially valuable for emphasis on developing a basis for practical ship design application. The paper by Gerritsma and Beukelman [2] contains significant experimental results, and a clear concise presentation of strip theory. It is a meaningful bridge between theory and physical phenomena. However, these two papers have discrepancies between them, and the paper by Vassilopoulos and Mandel [1] disagrees with the results of Korvin-Kroukovsky [3]. The differences have been examined below, with respect to: first, the strip theory; second, the choice of axes; and third, the experimental results in [2].

The two papers, [1] and [2] disagree in the evaluation of some motion derivatives. Let it be emphasized that no disagreement exists as to the form of the coefficients of the equations of motion. The distinction between the coefficients of the equations of motion, and the motion derivatives, is important. The coefficients, a, b, c, . . A, B, C . . ., contain motion derivatives. Which motion derivatives appear in each coefficient is independent of the method of evaluation of the hydrodynamic forces. Since both papers present final results for fixed axes, it is gratifying, that agreement exists on the motion derivatives contained in each coefficient. Motion derivatives are expressed as $Z_{\dot w}$, Z_w, $M_{\dot g}$, M_g etc., in the notation of [1]. The disagreement in evaluation of motion derivatives is due to differences in the application of strip theory to the determination of hydrodynamic forces.

These differences in application are due solely to one differing assumption. This involves what Gerritsma and Beukelman [2] refer to as "the effect of forward speed", however, "the effect of forward speed on strip theory" is more precise. [2] clearly shows the necessity of a forward speed consideration, but two types of considerations are included: those arising from strip theory evaluation of the motion derivatives, and those arising only from the fixed axis mechanics of a rigid body.

2. Strip theory.

Differences in strip theory application explain all discrepancies between [1] and [2]. Vassilopoulos and Mandel [1] state that: "Each of the strips is assumed to belong to a specific infinite cylinder oscillating at zero forward speed and its behaviour is assumed independent and isolated from the neighbouring strip".

Since a system moving with forward speed is considered, the rate of change of added mass comes into the formula for the hydrodynamic force, as is put forward in 2 , while the flow in each strip is independent of the flow in the neighbouring strips. This is not considered in [1], which states: "The introduction of terms dependent on the rate of change of added mass over the ship length is inconsistent with the use of two-dimensional theory". Therefore, it seems that [1] assumes "the effect of forward speed on strip theory" to be negligible.

Table I compares (columns 3 and 4) the final results of [1] and [2], each expression of a motion derivative is enclosed in brackets . Disagreement exists in coefficients B, C and E, and the apparent agreement in some other coefficients is due only to cancellation of speed effects. The speed effects will be untangled from the mathematics, showing their exact roles, and uncovering no errors in either [1] or [2] .

The "effect of forward speed on strip theory" may be analysed by subdivision into a "three-dimensional correction" and a "speed correction". Consider first the "three-dimensional correction", which expresses change of added mass along the ships length. It can be seen in Fig. 7 and 8 of [2], that forward speed has little effect on a' and b', for the two-dimensional cylindrical form of the midship section (section No. 4). Conversely, the three-dimensional forward and after sections show marked speed dependence in b', the sectional damping coefficient. Korvin-Kroukovsky, [3] page 123, takes sectional area to be a function of time. This area, of the specific section instantaneously in contact with the hypothetical sheet of water, must obviously change as the ship progresses through the sheet. m' and N' are functions of sectional area, and so must also be functions of time. The "three-dimensional correction" thus is expressed by considering m' a function of time, and $\frac{dm'}{dt} \neq 0$.

The "speed correction" is considered separately from the "three-dimensional" correction, in order to clearly show its relation to other velocity terms. The "speed correction" is found by considering X, the distance from the body-axis origin to the sheet of water, to be a function of time, as the ship progresses through the sheet.

TABL I. Coefficients of equations of m ion.

1 Coeffi-cients	2 Motion derivatives	3 Results of [2], with strip theory <u>corrected</u> for forward speed.	4 Results of [1]	5 Results of [2], with strip theory <u>not corrected for</u> forward speed
a	$-Z_{\dot{w}}$	$\left[\int_L m'\,dx\right]$	$\left[\int \mu\,(x)\,dx\right]$	$\left[\int_L m'\,dx\right]$
b	$-Z_w$	$\left[\int_L N'\,dx\right]$	$\left[\int N\,(x)\,dx\right]$	$\left[\int_L N'\,dx\right]$
c	$-Z_{z_o}$	$\left[\rho\, g\, A_w\right]$	$\left[\rho\, g \int B(x)\,dx\right]$	$\left[\rho\, g\, A_w\right]$
d	$-Z_{\dot{q}}$	$-\left[\int_L m'\,x\,dx\right]$	$-\left[\int \mu\,(x)\,x\,dx\right]$	$-\left[\int_L m'\,x\,dx\right]$
e	$-(Z_q + u_o Z_{\dot{w}})$	$-\left[\int_L N'\,x\,dx\right] + V\,[m]$	$-\left[\int N(x)\,x\,dx\right] + U_o\left[\int \mu\,(x)\,dx\right]$	$-\left[\int_L N'\,x\,dx\right] + V\,[m]$
g	$-(Z_\theta + u_o Z_w)$	$-\left[\rho\, g\, S_w\right] + V\left[\int_L N'\,dx\right]$	$-\left[\rho\, g \int B(x)\,x\,dx\right] + U_o\left[\int N(x)\,dx\right]$	$-\left[\rho\, g\, S_w\right] + V\left[\int_L N'\,dx\right]$
A	$-M_{\dot{q}}$	$\left[\int_L m'\,x^2\,dx\right]$	$\left[\int \mu\,(x)\,x^2\,dx\right]$	$\left[\int_L m'\,x^2\,dx\right]$
B	$-(M_q + u_o M_{\dot{w}})$	$\left[\int_L N'\,x^2\,dx + V\int_L m'x\,dx\right] - V\int_L m'x\,dx]$	$\left[\int N(x)\,x^2\,dx\right] - U_o\left[\int \mu\,(x)\,x\,dx\right]$	$\left[\int_L N'\,x^2\,dx\right] - V\left[\int_L m'x\,dx\right]$
C	$-(M_\theta + u_o M_w)$	$\left[\rho\, g\, I_w\right] - V\left[\int_L N'\,x\,dx + V\,m\right]$	$\left[\rho\, g \int B(x)\,x^2\,dx\right] - U_o\left[\int N\,(x)\,x\,dx\right]$	$\left[\rho\, g\, I_w\right] - V\left[\int_L N'x\,dx\right]$
D	$-M_{\dot{w}}$	$-\left[\int_L m'\,x\,dx\right]$	$-\left[\int \mu\,(x)\,dx\right]$	$-\left[\int_L m'\,x\,dx\right]$
E	$-M_w$	$-\left[\int_L N'\,x\,dx + V\,m\right]$	$-\left[N\,(x)\,x\,dx\right]$	$-\left[\int_L N'\,x\,dx\right]$
G	$-M_{z_o}$	$-\left[\rho\, g\, S_w\right]$	$-\left[\rho\, g \int B(x)\,x\,dx\right]$	$-\left[\rho\, g\, S_w\right]$

Notes: 1) For convenience, the notation of [1] and [2] is mixed, in all cases intent should be clear.

2) For consistency, coefficients in column 3 have been rearranged relative to their form in Table VI of 2.

Such a correction is independent of the third dimension and would also appear in the consideration of two-dimensional cylinders. This correction is, however, confused by similarity to terms arising from the mechanics of movable axis systems. Care must be taken to distinguish between these similar terms.

The role of "the effect of forward speed on strip theory" is most easily seen by carrying out the theoretical derivation of [2] , but eliminating all "effect of forward speed" corrections. All such corrections will be identified by braces $\{ \}$. Since all assumptions should now agree, the resulting motion derivatives should agree with those of [1].

Refering now to article 4.1, Strip Theory, of [2], for pure heave we have, with the notation of [2]:

$$F_H' = -\frac{d}{dt}(m'\dot{Z}_o) - N'\dot{Z}_o - 2\rho g y Z_o.$$

Differentiating, we obtain:

$$F_H' = -(m'\ddot{Z} + \left\{\frac{dm'}{dt}\right\}\dot{Z}_o) - N'\dot{Z}_o - 2\rho g y Z_o.$$

Noting that the "three-dimensional correction":

$$\left\{\frac{dm'}{dt}\right\} = \frac{dm'}{dx}\frac{dx}{dt} + \ldots = \frac{dm'}{dx}\frac{dx}{dt} = -\left\{V\frac{dm'}{dx}\right\},$$

we obtain, as in [2]:

$$F_H' = -m'\ddot{Z}_o - (N' - \left\{V\frac{dm'}{dx}\right\})\dot{Z}_o - 2\rho g y Z_o \ldots \qquad (5)$$

But neglecting the "the effect of forward speed" we have:

$$(F_H')_{2D} = -m'\ddot{Z}_o - N'\dot{Z}_o - 2\rho g y Z_o, \ldots \qquad (5)_{2D}$$

where 2D indicates neglect of the "three-dimensional" and "speed" corrections. Integrating we obtain:

$$(F_H)_{2D} = -(\int_L m'dx)\ddot{Z}_o - (\int_L N'dx)\dot{Z}_o - \rho g A_w Z_o \ldots \qquad (6)_{2D}$$

Notice that coefficient $b = \int_L N'dx$ is the same with and without corrections, because:

$$\left\{V\int_L \frac{dm'}{dx}dx\right\} = 0$$

for the case of $m' = 0$ at $x = -L/2$ and $x = +L/2$, see [2] and [5].

Now considering the moment we see:

$$M'_H = (x\,m')\ddot{Z}_o + (N'_x - \left\{V x \frac{dm'}{dx}\right\})\dot{Z}_o + 2\rho g x y Z \ldots \quad (7)$$

and:

$$(M'_H)_{2D} = (X m')\ddot{Z}_o + (N' x)\dot{Z}_o + 2\rho g y Z_o \ldots \quad (7)_{2D}$$

Integrating we obtain:

$$(M_H)_{2D} = (\int_L m' x\,dx)\ddot{Z}_o + (\int_L N' x\,dx)\dot{Z}_o + \rho g S_w Z_o \ldots \quad (8)_{2D}$$

Notice that, in [2], the coefficient E =

$$\int_L N' x\,dx + V m,$$

because:

$$-\left\{V \int_L x \frac{dm'}{dx}\,dx\right\} = + V m$$

when integrated by parts, for $m' = 0$ at the ends, see [2] and [5].

Next we must consider the ship in pure pitch:

$$F'_p = -\frac{d}{dt}(m' \dot{Z}_o) - N'(\dot{Z}_o) - 2\rho g y Z_o.$$

An expression for Z_o must be found. If we consider the point where the movable x-axis pierces the hypothetical sheet of water, letting the displacement of that point be Z_o, we have $Z_o = (-x\,\theta)$, and differentiating we have:

$$\dot{Z}_o = (-x\dot{\theta} - \dot{x}\,\theta) = (-x\dot{\theta} + V\theta),$$

which agrees with [2]. Bear in mind that this is an expression of the velocity of a point on the ship, relative to <u>fixed</u> <u>axes</u>, thus x must be taken as a function of time (see Fay [6]). However, if x is not a function of time, then $\dot{Z}_o = -x\dot{\theta}$. This will lead to results disagreeing with those of [1]. Thus it appears that Vassilopoulos and Mandel [1] do, in this case, consider x a function of time. This is not done explicitly, but is a consequence of the conversion from moving to fixed-axis systems. [1] develops the form of the coefficients independently of the motion derivatives, and is thus able to consider the effect of forward speed on the rigid body mechanics, while not considering its effect on the strip theory. This is because [1] considers only u_o, the constant velocity of the ship and not $\dot{x}$.

The parameter x appears only as a result of strip theory; and is thus considered, in [1], only in the final evaluation of the coefficients. Now continuing, but <u>not</u> neglecting $V\theta$, we have:

$$F_p' = -\frac{d}{dt} m'(-x\dot{\theta} + V\theta) - N'(-x\dot{\theta} + V\theta) + 2\rho g y x \theta.$$

Noting that in [2] the sign of $2\rho g y x \theta$ must be positive. We then have:

$$F_p' = -m'(-x\ddot{\theta} - \left\{\dot{x}\dot{\theta}\right\} + V\dot{\theta}) - \left\{\frac{dm'}{dt}\right\}(-x\dot{\theta} + V\theta) -$$
$$-N'(-x\dot{\theta} + V\theta) + 2\rho g y x \theta,$$

and:

$$F_p' = m' x\ddot{\theta} + (N' x + \left\{\dot{x} m'\right\} - V m' - \left\{x V \frac{dm}{dx}\right\})\dot{\theta} +$$
$$+ (2\rho g y x + \left\{V^2 \frac{dm}{dx}\right\} - N' V)\theta \ldots \qquad (9)$$

At this stage the "speed correction" is neglected by [1], and $\left\{\dot{x} m'\right\}$ disappears, yielding:

$$(F_p')_{2D} = m' x\ddot{\theta} + (N' x - V m')\dot{\theta} + (2\rho g y x - N' V)\theta \ldots \qquad (9)_{2D}$$

Integration gives us:

$$(F_p)_{2D} = (\int_L m' x\,dx)\ddot{\theta} + (\int_L N' x\,dx - Vm)\dot{\theta} + (\rho g S_w - V\int_L N'\,dx)\theta \ldots (10)_{2D}$$

Note that the coefficient e, in [2], is also:

$$\int_L N' x\,dx - Vm, \text{ because } \int_L \dot{x} m'\,dx \text{ and } V\int_L x \frac{dm'}{dx}\,dx$$

cancel, and that g, in [2], is:

$$\rho g S_w - V\int_L N'\,dx, \text{ because } \int_L \frac{dm}{dx}\,dx = 0.$$

Now considering moments:

$$M_p' = -m' x^2\ddot{\theta} - (N' x^2 + \left\{\dot{x} m' x\right\} - V m' x - \left\{x^2 V \frac{dm'}{dx}\right\})\dot{\theta} -$$
$$-(2\rho g y x^2 + \left\{V^2 x \frac{dm'}{dx}\right\} - N' V x)\theta \ldots \qquad (\sim 11)$$

and:

$$(M_p')_{2D} = -m' x^2\ddot{\theta} - (N' x^2 - V m' x)\dot{\theta} + (2\rho g y x^2 - N' V x)\theta \ldots \qquad (11)_{2D}$$

and integrating:

$$(M_P)_{2D} = -(\int_L m' x^2 \, dx)\ddot{\Theta} - (\int_L N' x^2 \, dx - V\int_L m' x \, dx)\dot{\Theta} -$$

$$-(\rho g I_W - V\int_L N' x \, dx)\dot{\Theta} \ldots \qquad (12)_{2D}$$

Comparing this with [2] we see that in coefficient B:

$$\dot{x}\int_L m' x - V\int_L m' x - \int x^2 V \frac{dm'}{dm} \, dx = 0,$$

when the last integral is evaluated by parts for $m' = 0$ at the ends. This leads to a coefficient with seemingly no $u_o M_{\dot{w}}$ term. However, we now see that $u_o M_{\dot{w}} = + V\int_L m' x \, dx$, which is canceled by a $-V\int_L m' x \, dx$ term in M_Θ.

A comparison of these results (column 5) with those of [1] (column 4) in Table I shows the results to be identical. This shows that the differences between [1] and [2] do, indeed, result only from a differing assumption regarding the effect of forward speed on strip theory evaluation of the hydrodynamic terms. If the integrals in [2] and [5] , mentioned above, are applied to Korvin-Kroukovsky's 3 coefficients e, B, C and E, they are seen to agree with those of Gerritsma and Beukelman [2] , with "the effect of forward speed on strip theory" corrections included. The coefficients in [3] are more general, not requiring $m' = 0$ at the ends. Thus, Korvin-Kroukovsky's disagreement with [1] does not result from "erroneous time-differentiation", but only from a differing assumption regarding the application of strip theory.

The key to the theory, in [2] , is in the $\frac{d}{dt}(m' \dot{Z}_o)$ terms, which give rise to the "three-dimensional correction" and lead to use of the "speed correction". Those speed terms found in all papers, due to rigid body mechanics, arise from the expression:

$$\dot{Z}_o = -x\dot{\Theta} + V\Theta,$$

which appears in [1] , as:

$$w = \dot{Z}_o + u_o \Theta.$$

3. Choice of axes.

The work in [1] is based on the work of Abkowitz [4]. Abkowitz' derivation of the equations of motion is performed first on a system of movable axes, then converted to fixed axes. Gerritsma and Beukelman [2], and Korvin-Kroukovsky [3] work with fixed axes. It is sometimes suggested that differences arise from these two approaches. Obviously the physical phenomena, and thus the motion derivatives, do not change as man changes imaginary axis systems. However, since the coefficients are associated with different parameters in the different systems (eg. $\dot{Z}$ and $\dot{Z}_o$), their forms must change. Coefficients e, g, C and D all contain a term of the product of V and a motion derivative, when written for fixed axes. But, in a movable axis system these velocity dependent terms disappear. These velocity terms are quite similar to some of the "effect of forward speed on strip theory" corrections. In order to demonstrate the effect of axis systems, and to show that the speed corrections for strip theory are independent of axis systems, the fixed-axis results of [2] will be converted to movable axes. These results can then be compared with the movable-axis results of [1].

To clearly see what happens to the "speed effect on strip theory" terms, they will remain inside braces $\{\ \}$, and the equations will remain in expanded form.

Considering first the equations presented for fixed axes, in the form of [1], the notation of [2]:

$$F = \rho \nabla (\ddot{Z}_o), \quad \text{and:} \quad M = k_{yy}^2 \rho \nabla \ddot{\theta},$$

and:

$$F = (\int m' \, dx)\ddot{Z}_o + (\int_L N' \, dx)\dot{Z}_o + (\rho g A_w)Z_o - (\int_L m' x \, dx)\ddot{\theta} -$$

$$- (\int_L N' x \, dx + \left\{\int_L \dot{x} m \, dx\right\} - V m + \left\{V \int_L x \frac{dm'}{dx} dx\right\})\dot{\theta} -$$

$$- (\rho g S_w + \left\{V^2 \int_L \frac{dm}{dx} dx\right\} - N' V)\theta ,$$

and:

$$M = (\int_L m' x^2 \, dx)\ddot{\theta} + (\int_L N' x^2 \, dx + \left\{\int_L \dot{x} m' x \, dx\right\} - V\int_L m' x \, dx - \left\{V\int_L x^2 \frac{dm'}{dx} dx\right\}\dot{\theta} +$$

$$+ (\rho g I_w + \left\{V^2 \int_L x \frac{dm'}{dx} dx\right\} - V\int_L N' x \, dx)\theta - (\int_L m' x \, dx)\ddot{Z}_o -$$

$$- (\int_L N' x \, dx - \left\{V\int_L x \frac{dm'}{dx} dx\right\})\dot{Z}_o - \rho g S_w Z_o$$

Now these equations may be transformed to those for movable axes by expressing Z_o of the centre of gravity in terms of Z, as follows:

$$Z_o = Z \cos \Theta \approx Z$$

$$\dot{Z}_o = \dot{Z} - V \Theta$$

$$\ddot{Z}_o = \ddot{Z} - V \dot{\Theta}$$

Notice that these expressions are identical with those in [1]. In this case x is <u>not</u> a function of time, for these equations express the relation of two motions of the same point, moving with the ship. Now substituting these relations for Z_o terms, and canceling equal terms of opposite sign, we have:

$$F = \rho \nabla (\ddot{Z} - V \dot{\Theta}), \text{ and: } M = k_{yy}^2 \rho \nabla \ddot{\Theta},$$

and:

$$F = (\int m' dx)\ddot{Z} + (\int N' dx)\dot{Z} + (\rho g A_w)Z - (\int_L m' x\, dx)\ddot{\Theta} -$$

$$-(\int_L N' x\, dx + \left\{ \int_L \dot{x} m\, dx \right\} + \left\{ V \int_L x \frac{dm'}{dx} dx \right\})\dot{\Theta} -$$

$$-(\rho g S_w + \left\{ V^2 \int_L \frac{dm}{dx} dx \right\})\Theta,$$

and:

$$M = (\int_L m' x^2 dx)\ddot{\Theta} + (\int N' x^2 dx + \left\{ \int_L \dot{x} m' x\, dx \right\} - \left\{ V \int x^2 \frac{dm'}{dx} dx \right\} \dot{\Theta} +$$

$$+ (\rho g I_w + \left\{ V^2 \int_L x \frac{dm'}{dx} dx \right\})\Theta - (\int_L m' x\, dx)\ddot{Z} -$$

$$- (\int_L N' x\, dx - \left\{ V \int_L x \frac{dm'}{dx} dx \right\}) \dot{Z} - \rho g S_w Z.$$

The "effect of speed on strip theory" terms remain; and if motion derivatives are substituted for the various terms, the results are identical to those of [1], for movable axis systems. Evidently then, the arbitrary choice of axis-system has no effect on strip theory, and no errors have been made in [1] or [2] . All velocity dependent terms now present are the result only of strip theory.

4. Comparison with experimental results.

As seen in Table I, three coefficients show discrepancies between [1] and [2] , these are B, C and E. The theoretical values of these parameters, due to both [1] and [2] should be compared with the experimental results of [2] . In Fig. 13, [2] indicates that strip theory should be corrected, for the determination of E. In Fig. 15, [2] gives values of A and B. Coefficient A includes the speed terms of C, which were moved by division by ω^2.

These speed terms are equal to V E, in both [1] and [2] . Since E seems to require speed terms in the strip theory, so also does coefficient A, or C. The experimental agreement with the theoretical A is quite good in [2] . Thus only B remains to be considered. Theory and experiment do not agree well in [2] . However, as seen in Table II, the theoretical values from [1] show even worse agreement, for ω = 6 rad/sec.

TABLE II. Values of coefficient B

ω = 6 rad/sec.

F_n	0.15	0.20	0.25	0.30
Experiment	6.3	6.1	6.0	5.7
Theory in 2	7.7.	7.7	7.7	7.7
Theory in 1	4.0	2.8	1.6	0.4

It must be concluded that the strip theory does require a correction for forward speed, and that with such a correction satisfactory evaluation is obtained for all coefficients, except B.

5. Summary and conclusions.

The derivation of the equations of motion appearing in [1] , and due to Abkowits [4] , seems a more rigorous and satisfying method. However, it does not attempt to evaluate the motion derivatives. When such evaluation was made, in [1] , it was assumed that forward speed did not affect the strip theory. The experimental results of [2] do not appear to justify this assumption. The method in [2] , due in part to Korvin-Kroukovsky [3] , derives the equations of motion and evaluates the motion derivatives in one process. Experimental results agree with the results of this method. However, the method does not seem as elegant or flexible as that due to Abkowitz.

The assumption regarding the effect of forward speed on strip theory is the <u>only</u> difference between these papers. No errors have been made by either authors, or by Korvin-Kroukovsky. It seems, to this discusser, most practical to use the derivation of Abkowitz, and Vassilopoulos and Mandel, to study the equations of motion; and to use the method of Korvin-Kroukovsky, Watanabe, and Gerritsma and Beukelman, to evaluate the motion derivatives. It is, therefore hoped that this discussion will contribute to the understanding of these two approaches.

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