Delft University of Technology
Faculty of Architecture and the Built Environment

A Topology Optimization Process for Discrete Modular Design based on Discrete Element Modelling for generating reconfigurable funicular structures

by
Qinglu Chen
(5287413)

MSc Architecture, Urbanism and Building sciences
Building Technology

Thesis Supervisor

dr. Pirouz Nourian | Architectural Engineering + Technology, Design Informatics
dr. Simona Bianchi | Architectural Engineering + Technology, Structural Design & Mechanics

Advisor

dr. Anjali Mehrotra | Department of Materials, Mechanics, Management and Design (3MD)

June 30, 2022

Abstract

The paper presents a generative design process based on topology optimization methodology for configuring masonry structures. In this approach, structures consisting of stackable interlocking blocks are modelled as discrete elements using the Discrete Element Method, approximating their mechanical behaviours. A process is devised to result in funicular structures that can be built using a limited set of modular masonry blocks with the aim to lower the environmental costs in terms of embodied carbon, monetary costs, and construction labour. Additionally, this process aims to increase the reuse and reconfigurability potential of the stackable blocks by seeking utmost modularity in the topological design of the underlying 3D tiling/tessellation.

Topology optimization is widely known as a methodology for generating geometrically elaborate structures, which typically minimize the use of the material. These approaches typically use the Finite Element Method to formulate and solve the governing differential equations for computing their objective functions, assuming that the structure to be designed is a virtually continuous distribution of material that is refinable within a continuum. However, at a more general level, the idea of topology optimization can also be applied to inherently discrete problems by creating algorithms based on the Discrete Element Modelling approach (O'Sullivan, 2011), or a particle system at a quasi molecular level. The proposed approach is applicable in the design of funicular structures, with the potential for form-finding of waste-free and reconfigurable, structural geometries that are constructible using a limited stock of modular blocks.

The paper introduces a discrete topology optimization process in three steps: 1) defining a space-filling 3D tiling/tessellation consisting of interlocking blocks as a graph colouring of a voxel grid; 2) defining the objective function of a topology optimization problem based on a Discrete Element Modeling approach; 3) assembling a topology optimization algorithm using the gradient-based Optimality Criteria method adapted to work with the DEM-based governing equations, deriving an objective function and related gradient equations(O'Shaughnessy et al., 2021).

The proposed methodology allows designers to find static equilibrium configurations for funicular structures defined by their desired space by minimizing potential energy between blocks. The method is validated for simply designing space discretized as interacting particles, whose optimum solutions compare to those from a typical continuum-based algorithm(Bendsøe & Sigmund, 2004).

Acknowledgments

Topology optimization is widely known as a methodology for generating geometrically elaborate structures, which typically minimize the use of the material. These approaches typically assume that the structure to be designed is virtually a continuum. To apply TO in inherently discrete problems, such as compression-only structures, is state of the art. This thesis mainly focuses on developing the mathematical theory and shows the possibility of bridging Topology optimization and discrete problems.

As an architect, I witness how we design and construct, leading to further material extraction, extensive energy consumption, waste production and environmental degradation, and realize the building industry is transitioning to reduce destruction. I believe that the strategy of optimizing structure is one of those.

I want to thank my first mentor, Pirouz Nourian, for guiding my research and ongoing assistance in exploring and developing the mathematical process. It is my honour to be one of his students, and I am looking forward to more collaboration in the future.

I want to thank my second mentor, Simona Bianchi, for guiding the structural part of this research and explaining the knowledge of masonry structure. I want to thank my advisor, Anjali Mehrotra, for taking the time to explain the field of Discrete Element Modelling. Hopefully could have more opportunities to cooperate henceforth.

I would also like to thank a few people who helped with the important parts of developing this method. I want to thank Nan Bai for inspiring me on how to deal with the last step of the mathematical process. Shervin Azadi kindly gave me an idea of how to implement the tessellation pattern. I am extremely indebted to Baolian Liu's help with the output visualization. I am grateful to receive endless kindness during this period.

This research is surely a starting point, and I would like to continue with this topic if possible. Thank you all!

CONTENTS

1. Research framework

1.1 Introduction

This chapter gives an overview of the research by illustrating its framework, motivation, and the methods used concerning the formulated problem and research questions derived from it. It also delineates the boundaries of the research focusing primarily on the topics of computational design, structural mechanics and architectural design.

1.2 Background and necessity

According to International Energy Agency and the United Nations Environment Programme (2021), global material extraction, waste production and greenhouse gas emissions from the building sector (full life cycle) are more than 35% (UNEP, 2021). In such a state of affairs, the construction industry is irresponsibly using natural resources, and the way of construction is depleting those. Also, when considering where building materials come from, all these materials are initially taken from the earth but had to go through complex processes to become building products which further increase the pollutions to air, soil and water.

The world population will increase by at least 2.1 million in the next thirty years. With the population growth, the demand for resources by the building industry is predictable growing. The way we are designing and constructing will lead to further material extraction, extensive energy consumption, waste production and environmental degradation. After realizing the impacts, the building industry is transitioning to reduce destruction.

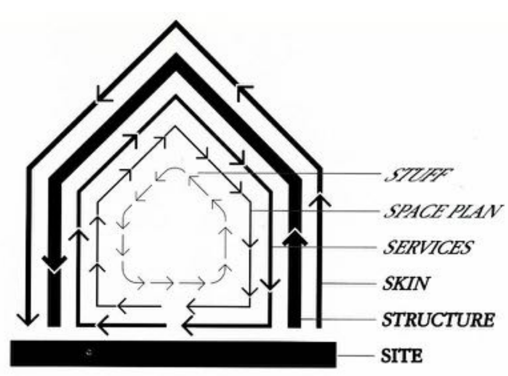


Figure 1

Stewart Brand's six layers of building which age at different rates.(Brand, 1994)

In "how building learns", StewartBrand (1994) introduced the six layers of the building, which age at different rates. The strategies to every layer have long-term economic, social, and environmental implications. As part of the building elements, the structure's life is typically most extended and is statics, compared to other sectors. Moreover, when looking into weight distribution in a building, 3/4 of the weight of the building is in its structure. Therefore, designing and constructing structures more sustainably becomes an urgent topic. To achieve this, Philippe Block (2020) proposed three strategies:

- reducing structural volume by having a better structural geometry to achieve carbon emissions-reducing.
- addressing resources depletion through the funicular form, which means compression-only form.
- finding waste-free efficient, economically viable strategies by digital tools toward structural geometries to decrease building material waste.

This research will explore the possibility toward these three objectives.

1.3 Problem Statement

The previous section discussed that current approaches to design and construct structure lack the ability to model and fabricate waste-free efficient prototypes. This section will further define the problems.

When focusing on reducing structural volume, topology optimization is an efficient and common means to generate geometrically precise structures which highly optimize stiffness and minimize the use of the material in relatively short periods of time. However, this technique is typically used on small objects such as connections, hinges and beams. When it comes to building scale, different methods might be needed to implement topology optimization.

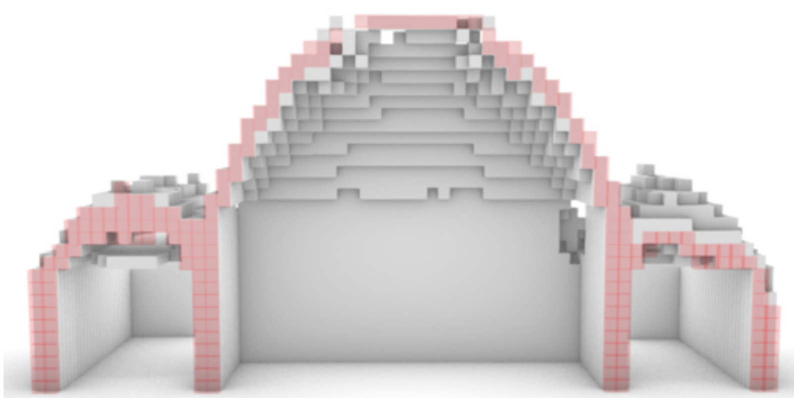


Figure 2

Topology optimization applied in building scale (example) (van Dijk, 2020)

The strategy behind topology optimization in generating architectural structures is to assign force-dependent loads to voxels (or pixels). However, these approaches assume that such structures are successively defined, neglecting their compositions. When these apply to the funicular structure, which means the compression-only structure, the created algorithm can only be used in the early stage of form-finding instead of configuring and fabricating buildings.

In addition, the objective function is computed multiple times when solving a discretized topology optimization problem, typically using the Finite Element Method (FEM). The FEM is highly efficient for linear elastic analyses but becomes problematic when applied to inherently discrete problems, such as masonry structure analyses, fragmentation and structural collapse.

Overall, current topology optimization approaches are not applicable in funicular structures as part of the modelling and fabricating, resulting in material waste and structurally non-efficient geometries.

1.4 Scope and limits

The research involves in three scientific fields, including architectural design, mechanics engineering and computer science. The intersection of three disciplines shown in figure 2. The following statements describe the self-limitations of the methods and tools used to achieve the above research objectives within these three disciplines.

- Structural Topology Optimization
- Numerical analysis
- Compression-only structure
- Statics equilibrium
- 3D tessellation

Specifically, the three-dimension tessellation is a combination between the space-filling system and structural mechanics, which requires computer science to be integrated.

Limits:

- The structural loads and safety factors will be based on Eurocode standards.
- Mortar or other connections between two blocks will not be taken into account for formulating and structural analysis.
- Stress tensor will not be discussed in this research.
- The structural analysis will be a linear static analysis so that the stiffness matrix will be constant, and the solving process is relatively short compared to a nonlinear analysis on the same model.
- The material of masonry blocks is considered as idealized isotropic material.

1.5 Research objective

The main objective of this research is to propose a methodology to implement discrete building blocks into the topology optimization process to generate funicular structures for architecture that is applicable for construction. The proposed method aims to enhance element optimization (shaping, configuring) of

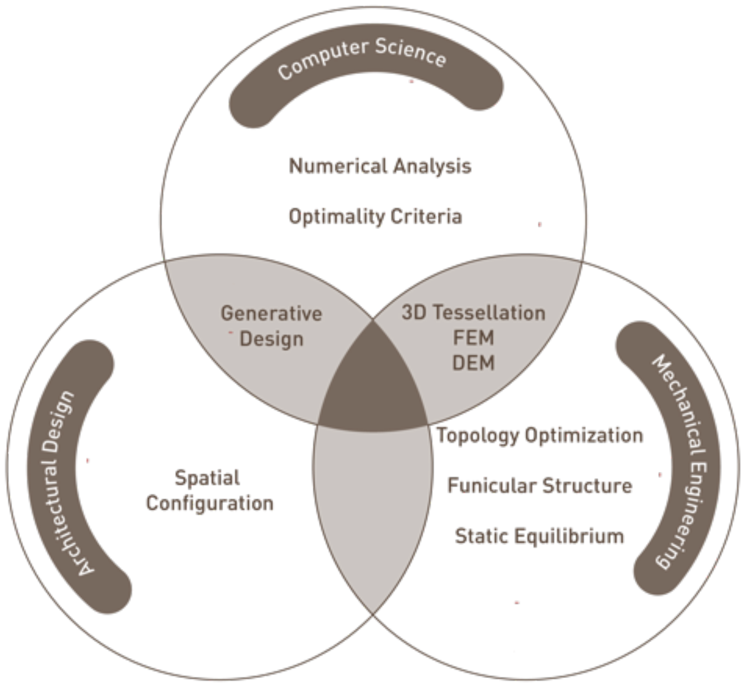


Figure 3

Scope of the research

material properties relative to their structural performance. In addition, to improve the relation between modelling, analysis, and construction processes by providing a more efficient and integrated workflow.

When applying topology optimization to masonry architecture, the first insight is how to configure the blocks. In other words, a space-filling grid that differs from the typical voxel grid is needed to provide a sound structural performance. As the focus is on masonry architecture, this three-dimension grid has to well perform under compression-only structures.

The second sub-objective is implementing this 3D grid to the topology optimization process. As mentioned in the previous section, the objective function is typically the Finite Element Method, which will become problematic when solving inherently discrete problems. Therefore how to formulate the objective function would be explored in this research.

This research will be performed mainly in Python. Abaqus is used as a structural simulation tool, which can cooperate with Python to do the structural verification.

1.6 Research questions

The main research question is:

How to implement discrete building blocks into the topology optimization to design funicular structures for architecture applicable for later construction processes?

For this, several sub-questions arise:
- How does topology optimization work, including mathematical methodology and optimization algorithms?
- What kinds of topology optimization methods have been applied to solve inherently discrete problems?
- How can we shift the current voxel system in topology optimization to a grid system with a more efficient load path for compression-only structures? This is related to a three-dimension tessellation in space.

1.7 Methodology

The research and design consist of five steps based on the literature study achievements. A visual framework of the research is shown in Figure 3.

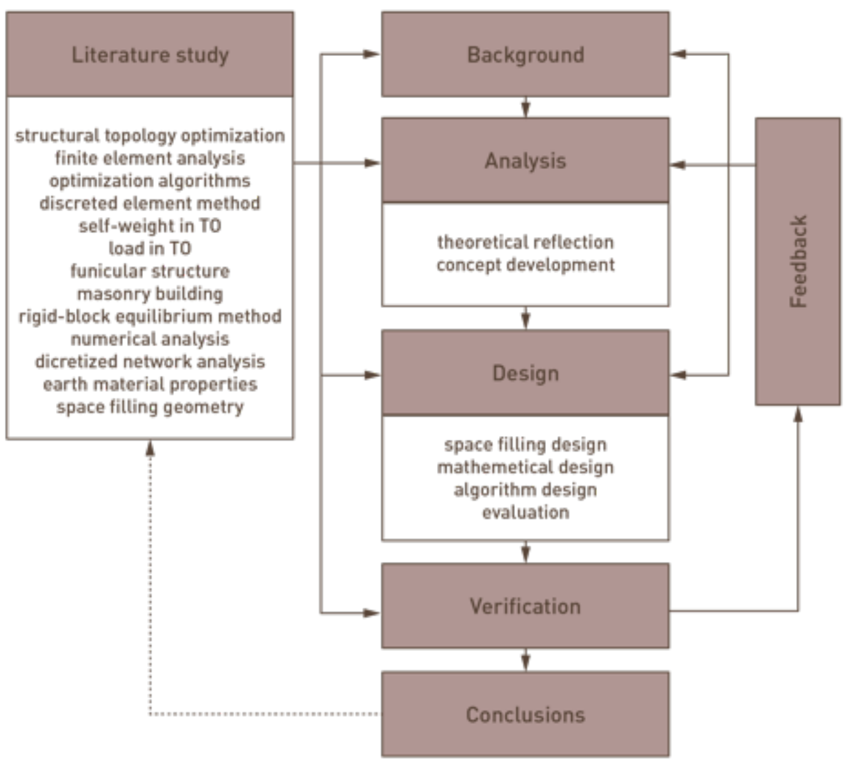


Figure 4

Research and design framework

Background

This research is based on the literature study of mainly scientific background, such as Scopus, Google Scholar, Web of Science, Researchgate and the repository of TU Delft. The queries were formulated with keywords derived from three major concepts (discrete method, structural topology optimization, masonry architecture) subdivided into research terms (synonyms for each concept). The results were evaluated based on their relevance and reliability. Also, the essential background knowledge required for the thesis is being learned through online open courses, including Numerical modelling, Matrix method in data analysis and Dynamic mechanics.

Analysis
This step includes theoretical reflection and concept development. The broad study of literature within the scope is done in the previous step, and the topic can break down into several subproblems.

Design
At this stage, a feasible space-filling grid will be developed. Also, the concept of mathematical model and algorithm design about how to implement this grid and the load case to topology optimization process will be proposed.

Verification
Verification will be done by using existing simulation software to analyze the test case's output. Moreover, make the comparison with typical generated results.

Conclusion
This step includes a summary and reflection of the design and the developed methodology.

1.8 Planning and organization

The planning and organization (Figure 4) of this research was divided into 5 phases, corresponding to the project presentation dates.

-The P1 phase entails literature study and acquisition of relevant resources to be used for the research, after which the objective and main research question are established.

-The P2 phase is reserved for literature study, analyzing relevant cases and gaining mathematical knowledge, which might be necessary for this project. Also, the research framework and design methodology concept are established at this stage.

-The way to formulate topology optimization problem need to be finalize at this stage and start with python coding for both 2D and 3D TO algorithm.

-The P4 phase of this research focuses on performing Structural Topology Optimization simulations on various 2D and 3D TOY problems, before progressing to the application on a building.

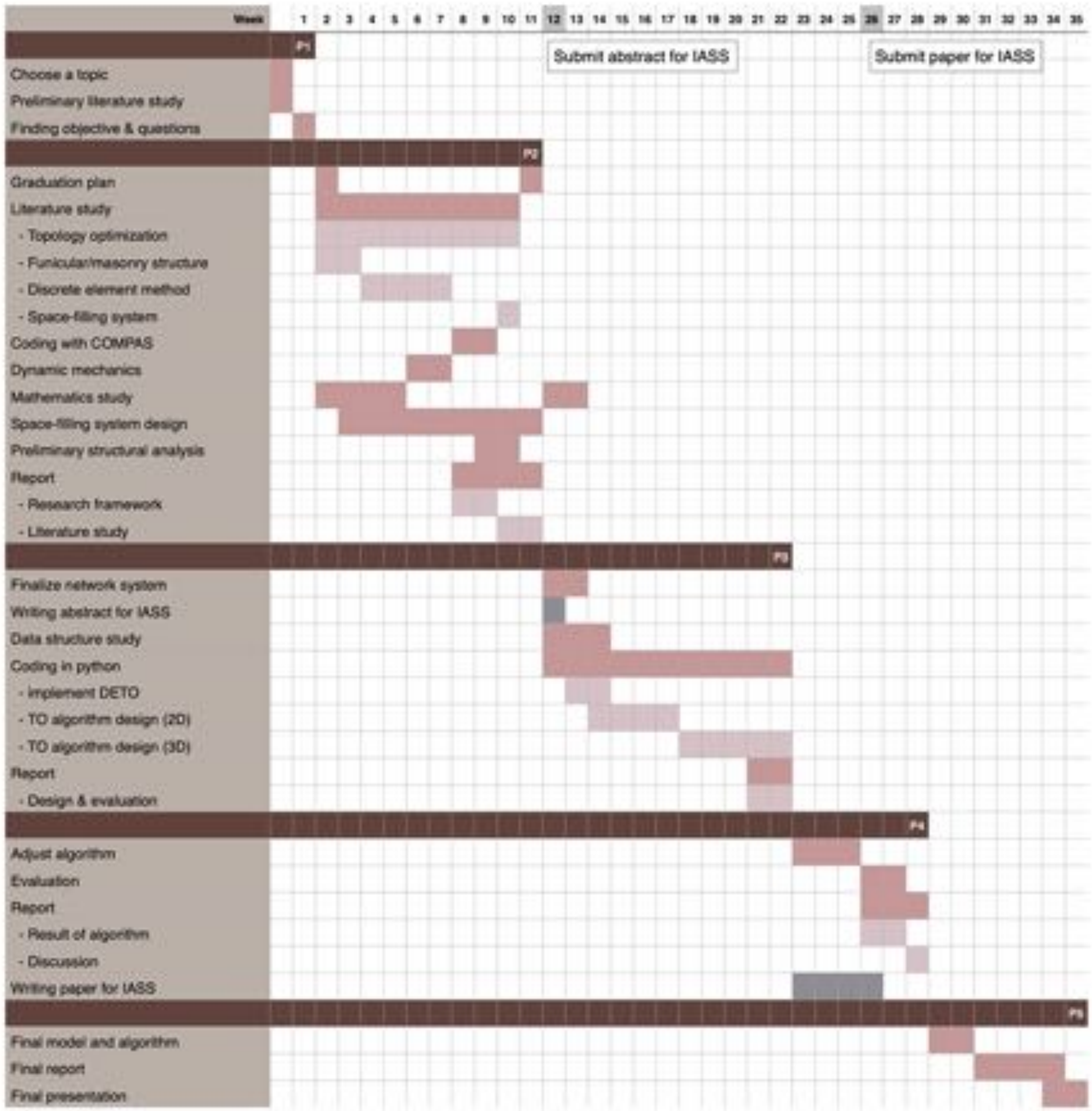


Figure 5

Graduation planning

- The P5 phase is reserved for finalizing the script, its code, and the report.

2. Literature study

2.1 Introduction

As mentioned in design methodology, the design consists of three steps: space-filling grid system design, mathematical design, and optimization algorithm design. This chapter will review literature based on the research question, clarify some theoretical concepts and introduce desktop design results.

2.2 Discrete element modelling

This section aims to give an overview of the Discrete Element Method. Also, as mentioned in the design methodology, the first step before the topology optimization process is to redefine the grid system. It is essential to understand the general tessellation in three-dimensional space and different approaches to discretize space.

2.2.1 Discretizing space

Tessellation
The term tessellation describes the division of the space with no overlaps and no gaps. These tessellations can also generalize in three-dimensional space. Certain polyhedra can fill three-dimensional space, including the cube, the rhombic dodecahedron, the truncated octahedron, and triangular, quadrilateral, and hexagonal prisms, among others (Branko, 1977).

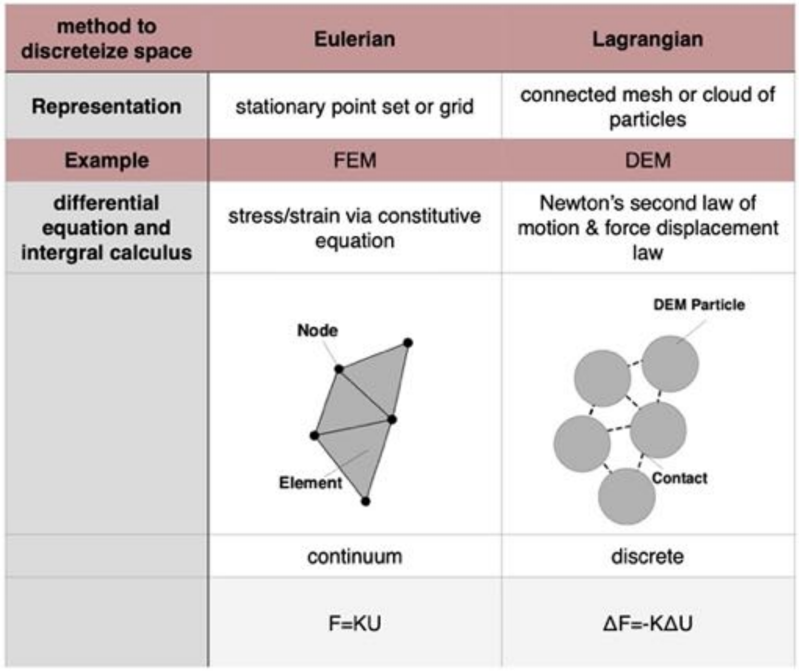
Approaches to discretize space
There are two main ways to discretize space, which are the Lagrangian method and the Eulerian method. In Eulerian models, the differential equation and integral calculus are related to stress and strain via constitutive equation. Node and element need to be defined within a fixed grid. In contrast, the representation of the Lagrangian model is a connected mesh or cloud of particles, where Newton's classical mechanics dominate each particle. The distinct element method is within this group and is widely used to model the granular materials discretely.

2.2.2 Discrete Element Method

P. A. Cundall (1979) first formulates the Discrete Element Method. In the absence of the damping, the DEM algorithms are concerned with solving the equilibrium equations for the system of particles (O'Sullivan Catherine, 2004).

$$Ma + K\Delta x = \Delta f$$

where M is the mass matrix, K is the stiffness matrix, Δf is the incremental force vector, and Δx is the incremental displacement vector. This equation is similar to the global stiffness matrix as in the finite



method to discreteize space	Eulerian	Lagrangian
Representation	stationary point set or grid	connected mesh or cloud of particles
Example	FEM	DEM
differential equation and intergral calculus	stress/strain via constitutive equation	Newton's second law of motion & force displacement law
	Node Element	DEM Particle Contact
	continuum	discrete
	$F=KU$	$\Delta F=-K\Delta U$

Figure 6

comparison of the Eulerian method and the Lagrangian method

element analysis.

DE modelling is typically applied in arch structures simulation to understand the mechanical behaviour of masonry structures. To formulate masonry blocks within DEM, four features need to be understood, including the element, the contacts, the displacement and loads.

Element

The elements in DEM can be rigid or deformable. In masonry block analysis, the element can behave rigidly or deformable, although rigid and FEM-subdivided elements are more common (V. Sarhosis, 2019). According to J. V. Lemos (2007), experimental evidence has confirmed that assuming blocks act as rigid bodies is justified. A reference point is chosen typically at the centre of gravity of the body to describe the behaviour. The degrees of freedom of a rigid element cooperate with the translation vector and the rotation vector.

Contacts

Duran (2000) divided the numerical technique used in DEM into soft-sphere models and hard-sphere models. The significant difference between the two approaches is whether the particle can be considered deformation or penetration or not.

Both types of methods are time-dependent or transient. After identifying neighbours, two blocks are

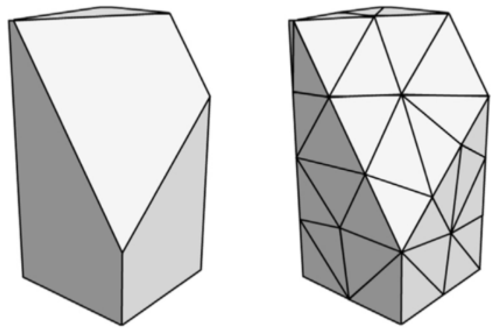


Figure 7

Rigid and deformable polyhedral block (J. Lemos, 1998)

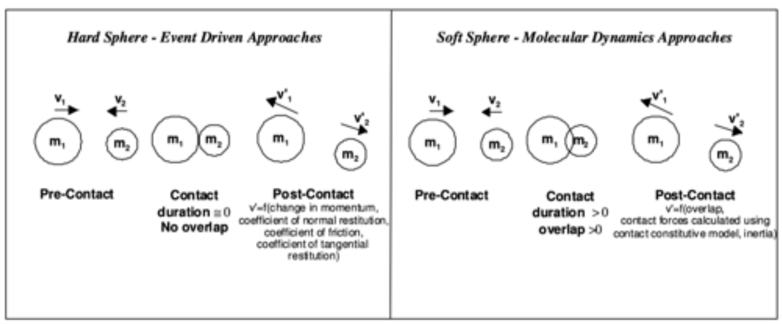


Figure 8

“Hard Sphere” and “Soft Sphere” approaches to DEM (O’Sullivan, 2011)

tested for contact by the contact detection algorithms. When a common plan between two blocks occurs, subcontacts are created with the help of the nodes being located on the block face. Based on the contact logic described above, two sets of subcontacts are formed in parallel when two blocks come together, each carrying subcontact forces.

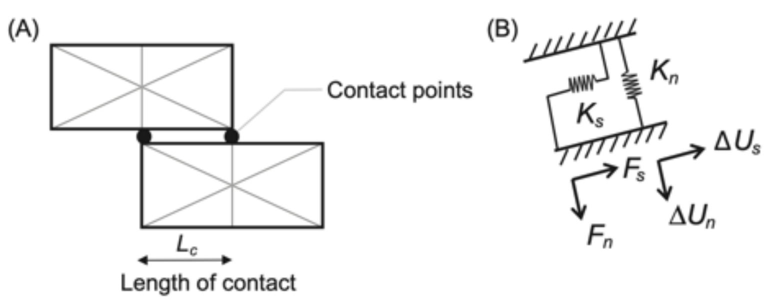


Figure 9

(A) Block-to-block contact; (B) interactions at contact. (V. Sarhosis, 2019)

The mechanical behaviour of contacts is modeled by considering the contact stiffness defined in both the normal and shear directions. The behaviour is governed by the joint normal and shear stiffness within the elastic range:

$$\begin{cases} \Delta F^n = -k_n \cdot \Delta U^n \cdot A_c \\ \Delta F^s = -k_s \cdot \Delta U^s \cdot A_c \end{cases}$$

where ΔF^n; ΔF^s is the normal and the shear force increment (resultant for the subcontact); k_n; k_s is the joint normal and the joint shear stiffness; ΔU^n; ΔU^s is the normal and the shear displacement increments belonging to the subcontact; and A_c is the subcontact area.

The maximum shear force is related to the normal force and the angle of friction.

$$F^s_{max} = F^n \cdot tan(\varphi)$$

Displacement
The DEM techniques based on the stiffness matrix of the system, which is mentioned at the beginning of this section, is not applied in masonry modelling (V. Sarhosis, 2019). The current time-stepping method starts from an initial state and through a series of small, finite time intervals. The equation can be written as:

$$M(t) \cdot a(t) = f(t, u(t), v(t))$$

where M is the inertia matrix, and f is the generalized force vector with time points, displacements and velocities.

Loads
Loads in DEM can be several types of loads, including external and internal loads.

2.2.3 Static equilibrium as constraints

Whiting et al. (2012) present a method to compute the gradient for the stability of a structure composed of rigid blocks and optimize masonry structures. Although the proposed methodology is for analyzing and optimizing existing structures, the way they formulate feasible constraints can be a reference.

To be structurally feasible, the force in a structure must satisfy static equilibrium, friction constraints, and additional constraints dependent on the material.

Static equilibrium
Static equilibrium conditions require that net force and net torque for each block equal zero, accounting

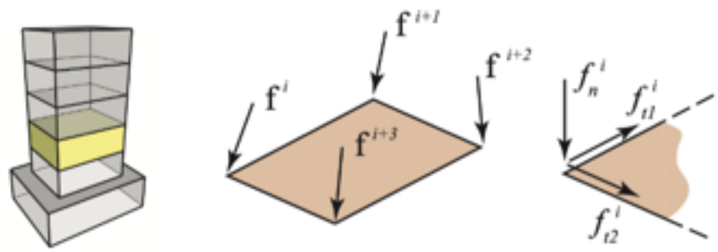


Figure 10

Model of contact forces at interfaces between blocks. (Wei, 2012)

for self-weight of the structure and external applied loads. The equation that combines equilibrium constraints for each block within a linear system is (Livesley, 1978),

$$A_{eq} \cdot f + w = 0$$

where w is a vector containing the self weights of each block, external forces can also be added using w; vectors f is the vector of interface forces, and A_{eq} is the matrix of coefficients for the equilibrium equations. The explanation of static equilibrium matrix can refer to the Appendix D.

Compression Constraint

According to limit analysis of masonry, the material can be assumed to have zero tensile strength. The condition can be expressed as:

$$f_n^i \geq 0, \forall i \in interface\, vertices$$

Friction constraint

A friction constraint is applied at all vertices of the block interfaces. For each forces $f_n^i, f_{t_1}^i, f_{t_2}^i$ the two in-plane forces are constrained within the friction coefficient times the normal force.

$$\left|f_{t_1}^i\right|, \left|f_{t_2}^i\right| \leq \alpha f_n^i, \forall i \in interface\, vertices$$

where α is the coefficient of static friction, and typically equal to 0.7.

Combining friction constraints over the entire assemblage of blocks in the structure gives a sparse linear system of inequalities:

$$A_{fr} \cdot f \leq 0$$

To summarize, the constraints for static equilibrium can be described as above.

2.3 Topology optimization

This section of the report will follow the mathematical methodology in topology optimization.

2.3.1 What is structural optimization

To understand topology design methods for structural optimization, the first thing to do is have an idea about structural optimization. Structural optimization aims to maximize the utility of a fixed quantity of resources to fulfil a given objective. There are three different types of optimization approaches under the head of structural optimization: size optimization, shape optimization, and topology optimization (Sigmund, 2001).

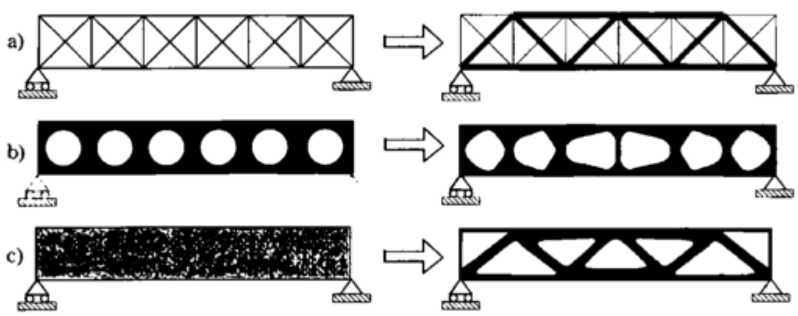


Figure 11

Three categories of structural optimization. a) Sizing optimization, b) shape optimization, c) topology optimization (Bendsøe & Sigmund, 2004)

In Size optimization, the final shape of structure is prescribed, and the method is to optimize the size of the components for that structure, while in Shape optimization, the form is unknown. The shape or boundary is represented as either an equation or control points that can move (Querin, Victoria, Alonso, Ansola, & Martíh, 2017).In both cases, the number of elements stays still.

On the other hand, in Topology optimization, which is also considered the most general form of structural optimization (G. I. Rozvany, 1995), part of the structure will be deleted and generate a new shape. The inputs of this method are the design space, support conditions and the applied load.. It achieves the optimal by minimizing the design variants. This method generally uses the finite element method (FEM) as the meshing ease to find the members to be removed.

2.3.2 Topology optimization methods

As an optimization algorithm, topology optimization methods can be grouped into two categories, including the gradient-based Optimality Criteria method and the non-gradient-based Heuristic method.

Optimality Criteria are more mathematically rigorous. They satisfy a set of criteria concerning the structure behaviours. These methods are suitable for problems with a large number of design variables and a few constraints (Querin, Victoria, Alonso, Ansola, & Martíh, 2017). The Heuristic methods are often inspired by the process of natural selection, which are easier to understand and provide various solutions. However, these also result in challenges to guarantee global optimality. The table below lists the methods within these two groups. In this research, the main focus will be lied in Optimality Criteria methods.

Optimality Criteria	Heuristic methods
Homogenization	Fully Stressed Design
Solid Isotropic Material with Penalization (SIMP)	Computer-Aided Optimization (CAO)
Level Set Method	Soft Kill Option
Growth Method for Truss Structures	Evolutionary Structural Optimization (ESO)
	Bidirectional ESO (BESO)
	Sequential Element Rejection and Admission (SERA)
	Isolines/Isosurfaces Topology Design (ITD)

Figure 12

Topology optimization methods according to Querin, Victoria, Alonso, Ansola, and Martíh (2017)

2.3.3 Topology optimization methods using discrete elements

Another common classification of methodologies suggested by Kentli (2020) is if its discrete elements are used or not. As the design will consider masonry blocks as discrete elements, the research regarding mainly used methods in topology optimization will focus on those using discrete elements.

The main methods that use discrete elements include:

-Ground structure method(GSM),
-Solid isotropic material with penalization method (SIMP),
-Homogenization method (HM),
-Evolutionary structural optimization (ESO),
-Level-set method (LSM),
-Meshless methods.

After a broad study of related papers, several conclusions can be drawn, and details can be found in Appendix A.

The most common application for using GSM is to solve discrete truss topology problems. The Homogenization method solves shape optimization problems where the topology is made from microscale voids that produce a porous structure(Querin, Victoria, Alonso, Ansola, & Martí, 2017). The applications related to masonry blocks using HM mainly optimize thermomechanical (Matteo Bruggi, 2013) or structural performance within a single block, resulting in hollow masonry blocks(Gabriele Milani, 2017).

The concept of the homogenization approach has developed in several different directions, and the direct consequence was the development of the SIMP method (Bendsøe & Sigmund, 2004), which is also known as the density approach. Gabriele Milani (2017) analyzed multistory masonry walls by using this approach; Whiting et al. (2012) developed an approach to formulate masonry blocks and modify their geometry; O'Shaughnessy et al. (2021) combined the Discrete element method with SIMP for topology optimization of discrete elements(disks) by minimizing interaction energy.

Instead of local density variables, the level set method and the meshless method operate with the boundaries, which also be coined the Lagrangian (boundary follow mesh) approach. As the focus of this research, these two methods will not be discussed.

Based on the above study, this report will first elaborate on the Solid isotropic material with penalization method (SIMP) and compare it with the Discrete Element Topology Optimization method, which is developed established on SIMP.

2.3.4 SIMP-based Topology optimization

SIMP-based Topology Optimization is currently the most commonly used technique and is simplified by Sigmund (2001). Multiple design variables, constraints, and objectives can be included in the methodology.

The first step of a numerical TO problem is to define the design space, external loads and supports. The void area also needs to be determined within the design boundary. The design space is discretized into individual elements (pixel in 2D and voxel in 3D), and each member is associated with design variable $x_e \in [0; 1]$, which represent material density at that point. 1 represent full solid at the member, while 0 means the material is deleted.

The general set-up for the topology optimization problem is actually a material distribution problem, object to minimize compliance. The objective function can be written as (Sigmund, 2001):

$$\min_x C(x) = U^T K U = \sum_{e=1}^{N} {u_e}^T k_e u_e$$

where **K** is the global stiffness matrix, **U** is the global displacement vectors, and **F** is the global force vectors. **N** is the number of elements, which equal to *(nelx × nely)*. k_e is the element stiffness matrix, and it depends on x_e as a power law (typically $p = 3$).

$$k_e = (x_e)^p k_0$$

Eq 1. can only be solved when the following constraints are satisfied.

$$subject\,to : \begin{cases} \frac{V(x)}{V_0} = f_{volfrac} \\ KU = F \\ 0 < x_{min} \leq x \leq 1 \end{cases}$$

The first constraint determines the solid target volume, where $V(x)$ is the target material volume; V_0 is the whole design domain with $x = 1$ for each element; $f_{volfrac}$ is the preset volume fraction and $f \in (0, 1)$. The second constraint is a basic formula to calculate the displacement in each node, using stiffness and the forces work on this node, which is also known as Finite element analysis. The third constraint set a range for x_e between x_{min} and 1. To avoid singularity, x_{min} can be very small but not equal to zero (normally $x_{min} = 10^{-3}$).

Numerous methods can solve the topology optimization problem, and this report focuses on the Optimality Criteria(OC) method. In this method, an updating scheme for design variables is used to find the new values of x_{new} (Bendsøe, 1995).

$$x_e^{new} = \begin{cases} if\ x_e B_e^\eta \leq max(x_{min}, x_e - m), & max(x_{min}, x_e - m) \\ if\ max(x_{min}, x_e - m) < x_e B_e^\eta < min(1, x_e + m), & x_e B_e^\eta \\ if\ min(1, x_e + m) \leq x_e B_e^\eta, & min(1, x_e + m) \end{cases}$$

In here, m is a positive move=limit, $\eta (= 1/2)$ is a numerical damping coefficient to improve convergence and B_e can be found from the following optimality condition.

$$B_e = -\frac{\partial C(x)}{\partial x_e} (\lambda \frac{\partial v(x)}{\partial x_e})^{-1}$$

where λ is a Lagrangian multiplier, that is related to constraint $v(x)$ and changes at every step of the optimization process.

The expression of the sensitivity analysis is partial differential that obtained by the combination of cost function and penalization scheme.

$$\frac{\partial c}{\partial x_e} = -p(x_e)^{p-1} u_e^T k_0 u_e$$

The integrated effect of equations 4, 5 and 6 is to eliminate material from less dense areas and move the design towards a solid-void only solution (Masoero et al., 2021). The procedure flowchart can refer to Figure 13.

2.3.5 Discrete Element Topology optimization method

Commonly, TO tools do not comprise nonlinear material behavior by default (Nadine Stoiber, 2021). However, to solve the discrete problem in TO, a Discrete Element Topology optimization method is developed by O'Shaughnessy et al. (2021). The proposed approach is based on the established framework of SIMP Topology optimization to achieve a system with Discrete Element Modelling (DEM), and the method only address performance at static equilibrium.

The Discrete element method is a numerical approach describing a discrete, interacting particles system, first formulated by P. A. Cundall (1979). More details about this technique can refer to section 2.2. In this section, only the essential functions related to this method are shown. Functions of the interaction force between particles i and j emerging from a harmonic potential are:

$$U_{ij} = \frac{1}{2}k_{ij}(r_{ij} - r_0)^2$$
$$F_{ij} = -k_{ij}(r_{ij} - r_0)$$

where k_{ij} is the stiffness of the spring, r_{ij} is the inter-particle distance, and r_0 is the equilibrium distance.

Instead of minimizing the displacement, the objective of the DEM-based method is to minimize the interaction energy. Also, the basic grid differs from the FEM-based method, which is the hexagonal lattice. Comparing the procedure of DETO with SIMP method, some steps are changed to apply it to DEM system. First of all, the design variable and the cost function is changed. To minimize the total interaction energy, where $U_{tot} = \sum_{i,j} U_{i,j}$, the optimization problem becomes:

$$\min_x C(x) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j>i}^{N} k_{ij}(r_{ij} - r_0)^2$$

$$subject\,to: \begin{cases} \frac{V(x)}{V_0} = f_{volfrac} \\ 0 < x_{min} \leq x_i \leq 1 \end{cases}$$

Instead of individual elements, the cost function features the pairs of particles. The penalization scheme is also changed since k_{ij} is associated with pairs of particles under the DEM context. The sensitivity expression is still written by combining the definition of cost function with the penalized k_{ij}.

$$k_{ij} = x_i^p x_j^p k_0$$

$$\frac{\partial c}{\partial x_i} = \sum_{j \neq i} -p(x_i)^{p-1} x_j^p k_0 (r_{ij} - r_0)^2$$

Above equations together complete the basic formulation of DEM-based Topology optimization. The

Figure 13

Implemented elements, goal and procedure of SIMP method (left) and Discrete Element Topology Optimization method (right).

proposed methodology of Discrete Element Topology Optimization provides an idea of how to formulate to address the optimization on discrete problems.

References

Abdulla, K. F., Cunningham, L. S., & Gillie, M. (2017). Simulating masonry wall behaviour using a simplified micro-model approach. *Engineering Structures*, *151*, 349–365. https://doi.org/https://doi.org/10.1016/j.engstruct.2017.08.021

Avdić, I. (2019). Bio-inspired approach to early stage structural form finding.

Bendsøe, M. P., & Sigmund, O. (2004). *Topology optimization: Theory, methods and applications*. Springer.

Bendsøe, M. P. (1995). *Optimization of structural topology, shape, and material*. Springer.

Block, P. (2020). *Philippe block: Strength through geometry[video]*. Melbourne School of Design. https://www.youtube.com/watch?v=a4NJCuJ1geg

Brand, S. (1994). *How buildings learn: What happens after they are built*. Viking Press.

Branko, G. (1977). Regular polyhedra—old and new. *Aequationes Mathematicae*, *15*(1), 118–120. https://doi.org/https://doi.org/10.1007/bf01837888

Duran, J. (2000). *Sands, powders, and grains : An introduction to the physics of granular materials*. New York: Springer.

Dyskin, A. V., Estrin, Y., Pasternak, E., Khor, H. C., & Kanel-Belov, A. J. (2005). The principle of topological interlocking in extraterrestrial construction. *Acta Astronautica*, *57*(1), 10–21.

G. I. Rozvany, U., M. P. Bendso[U+0338]e. (1995). Layout optimization of structures. *Applied Mechanics Reviews*, *48*(2), 41–119. https://doi.org/https://doi.org/10.1115/1.3005097

Gabriele Milani, M. B. (2017). Simple homogenization-topology optimization approach for the pushover analysis of masonry walls. *International Journal of Architectural Heritage*, *12*(3), 395–408. https://doi.org/https://doi.org/10.1080/15583058.2017.1323248

Kentli, A. (2020). Topology optimization applications on engineering structures. *Truss and Frames - Recent Advances and New Perspectives*. https://doi.org/https://doi.org/10.5772/intechopen.90474

Lemos, J. V. (2007). Discrete element modeling of masonry structures. *International Journal of Architectural Heritage*, *1*(2), 190–213. https://doi.org/https://doi.org/10.1080/15583050601176868

Lemos, J. (1998). Discrete element modelling of the seismic behaviour of stone masonry arches. *Computer Methods in Structural Masonry*, *2*(4), 220–227.

Livesley, R. K. (1978). Limit analysis of structures formed from rigid blocks. *International Journal for Numerical Methods in Engineering*, *12*, 853–871.

Masoero, E., O'Shaughnessy, C., Gosling, P. D., & Chiaia, B. M. (2021). Topology optimization using the discrete element method. part 2: Material nonlinearity. https://doi.org/https://doi.org/10.31224/osf.io/ubwaq

Matteo Bruggi, A. T. (2013). Design of masonry blocks with enhanced thermomechanical performances by topology optimization. *Construction and Building Materials*, *48*, 424–433. https://doi.org/http://dx.doi.org/10.1016/j.conbuildmat.2013.07.023

Mele, T. V., & many others. (2017-2021). COMPAS: A framework for computational research in architecture and structures. [http://compas.dev]. https://doi.org/10.5281/zenodo.2594510

Nadine Stoiber, B. K. (2021). Topology optimization in concrete construction: A systematic review on numerical and experimental investigations. *Structural and Multidisciplinary Optimization*, *64*(4), 1725–1749. https://doi.org/https://doi.org/10.1007/s00158-021-03019-6

O'Shaughnessy, C., Masoero, E., & Gosling, P. D. (2021). Topology optimization using the discrete element method. part 1: Methodology, validation, and geometric nonlinearity. https://doi.org/https://doi.org/10.31224/osf.io/c6ymn

O'Sullivan, C. (2011). *Particulate discrete element modelling: A geomechanics perspective*. Spon Press.

O'Sullivan Catherine, B. o. (2004). Selecting a suitable time step for discrete element simulations that use the central difference time integration scheme. *Engineering Computations*, *21*(2/3/4), 278–303. https://doi.org/https://doi.org/10.1108/02644400410519794

P. A. Cundall, O. D. S. (1979). A discrete numerical model for granular assemblies. *Geotechnique*, *29*(1), 47–65. https://doi.org/https://doi.org/10.1680/geot.1979.29.1.47

Querin, O. M., Victoria, M., Alonso, C., Ansola, R., & Martí, P. (2017). Discrete method of structural optimization * *additional author for this chapter is prof. dr.-ing. dr.-habil. george i. n. rozvany. *Topology Design Methods for Structural Optimization*, 27–46. https://doi.org/https://doi.org/10.1016/b978-0-08-100916-1.00003-9

Querin, O. M., Victoria, M., Alonso, C., Ansola, R., & Martíh, P. (2017). Introduction. *Topology Design Methods for Structural Optimization*, 1–13. https://doi.org/https://doi.org/10.1016/b978-0-08-100916-1.00001-5

Rita Esposito, J. G. R., Francesco Messali. (2016). Tests for the characterization of replicated masonry and wall ties.

Schek, H. (1974). The force density method for form finding and computation of general networks. *Computer Methods in Applied Mechanics and Engineering*, *3*, 115–134.

Sigmund, O. (2001). A 99 line topology optimization code written in matlab. *Structural and Multidisciplinary Optimization*, *21*(2), 120–127. https://doi.org/https:/doi.org/10.1007/s001580050176

Softmax function. wikipedia. (n.d.). https://en.wikipedia.org/wiki/Softmax_function

UNEP. (2021). 2021 global status report for buildings and construction: Towards a zero emission, efficient and resilient buildings and construction sector, 15.

V. Sarhosis, K. B., José V. Lemos. (2019). Discrete element modeling. *Numerical Modeling of Masonry and Historical Structures*, *48*(6), 469–501. https://doi.org/https://doi.org/10.1016/b978-0-08-102439-3.00013-0

van Dijk, F. (2020). Structurally informed design of interlocking block assemblages using limit analysis.

Wang, Z., Song, P., Isvoranu, F., & Pauly, M. (2019). Design and structural optimization of topological interlocking assemblies. *ACM Transactions on Graphics*, *38*(6), 193. https://doi.org/https://doi.org/10.1145/3355089.3356489

Wei, W. E. J. (2012). *Design of structurally-sound masonry buildings using 3d static analysis*. Massachusetts Institute of Technology.

Whiting, E., Shin, H., Wang, R., Ochsendorf, J., & Durand, F. (2012). Structural optimization of 3d masonry buildings. *ACM Transactions on Graphics*, *31*(6), 1–11. https://doi.org/https://doi.org/10.1145/2366145.2366178

Appendix

Appendix A: Topology optimization methods using discrete elements

The classification is based on the mainly used methods using discrete elements

Main method	Application	Methods	Authors
Ground structure method (GSM) Dorn et al. (1964)	multi-load truss systems	with linear programming	Sokol and Rozvany (2013)
	truss systems	with simulated annealing	Zhang et al. (2017)
	tensegrity structures	with mixed integer linear programming	Xu et al. (2017)
	large-scale pin-jointed frames		Sokol (2011)
	materials' nonlinear behavior		Ramos and Paulino (2014)
	skyscraper and arch bridge		Zhang et al. (2016)
	truss systems	with particle swarm optimization (PSO) algorithm	Shakya et al. (2017)
	multi-material	using discrete filtering scheme	Zhang et al. (2017)
Solid isotropic material with penalization method (SIMP) Bendsøe(1989)	3D printing applications	with BESO	Shao (2019)
	a continuum-type problem		Lógó (2012)
	eliminate gray areas	with simulated annealing	Garcia-Lopez et al. (2011)
	3D stress-constrained topology optimization problems		Gebremedhen et al. (2019)
	a multiobjective conductivity problem	with finite volume method (FVM) to solve the energy equation	Marck et al. (2012)
	structure problem of mold (short-fiber-reinforced polymer material)	with projected gradient method	Ospald and Herzog (2017)
	a MBB beam & a cantilever beam (lightweight cellular material)	with BESO	Qiao et al. (2018)
	flywheel rotor		Tsai and Cheng (2012)
	an electric vehicle body		Yang et al. (2011)
	control the length scale of structural members		Zhang et al. (2014)
	cellular structures with multiple types of microstructures		Zhang et al. (2018)
	multiple materials	introduce power functions with scaling and translation coefficients and the cost properties	Zuo and Saitou (2016)
	enhance thermal insulation of masonry blocks	Rational Approximation of Material Properties (RAMP)	Bruggi and Taliercio (2013)

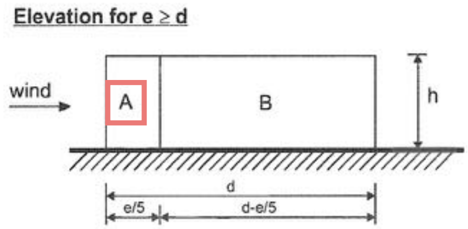
Main method	Application	Methods	Authors
	discrete problems	with DEM	Connor et al. (2021)
	multistories masonry wall		Milani and Bruggi (2018)
	variations of the geometry (3D masonry buildings)		Whiting et al. (2012)
	structure (periodically perforated material)		Allaire et al. (2019)
	metamaterials	Nonlinear homogenization at finite strains	Zhang and Khandelwal (2019)
	magnetic composite materials	asymptotic HM (nonlinear)	Lee et al. (2019)
	hyperbolic acoustic metamaterials	with a level-set-based method	Noguchi et al. (2019)
Homogenization method (HM) Bendsøe and Kikuchi (1988)	optimal truss and frame (discrete structure)		Larsen et al. (2019)
	masonry wall	with an adaptive meshing algorithm	Milani and Bruggi (2018)
	reinforced masonry wall		Milani and Bruggi (2015)
	microstructures with auxetic behavior	with evolutionary algorithms	Kaminakis et al. (2015)
	optimal masonry blocks (structural and thermal performance)		Vantyghem et al. (2016)
	fluid dynamics analogy	BESO	Daróczy and Jármai (2014)
	trusses	with simultaneous topology and size optimization	Tomšič and Duhovnik (2014)
	smoothing truss	with XFEM, isoline design approach	Abdi et al. (2014)
	die components	with Abaqus FEM software	Azamirad and Arezoo (2014)
	water distribution network	multiobjective real-code population-based incremental learning (RPBIL) and a hybrid algorithm of RPBIL with differential evolution (DE)	Bureerat and Sriworamas (2013)
	the rotary lobe of root vacuum pumps		Chen et al. (2013)
	plate structure under harmonic loading		Chen (2014)
Evolutionary structural optimization (ESO) Xie and Steven (1993)	cable-truss structure	combination of ground structure approach, nonlinear finite element analysis, and quantum-inspired evolutionary algorithms	Finotto et al. (2012)
	PBC made of two-phase composites	BESO	Huang et al. (2012)

Main method	Application	Methods	Authors
	hinge-free compliant mechanisms	BESO	Li et al. (2014)
	vibration problems of acoustic-structure systems	BESO	Picelli et al. (2015)
	a cantilever composite laminate under uniform in-plane pressure	BESO	Sun et al. (2011)
	constrained layer damping plates		Wang et al. (2014)
	automatic hole generation	bidirectional evolutionary level-set method	Zhu et al. (2015)
	multiple constraints of displacement and frequency	BESO	Zuo et al.(2011)
Level-set method (LSM)	local mesh modifications		Allaire et al. (2013)
	geometric uncertainty and related problems	shape and topology optimization	Chen and Chen (2011)
	a third dimension for 2D problems to adjust new hole positions	shape and topology optimization, hole insertion method	Dunning and Alicia Kim (2019)
	stress-related topology optimization problems		Guo et al. (2011)
	minimized mass under stress constraints	Lagrangian approach	Emmendoerfer and Fancello (2014)
	a light-scattering layer for solar cell applications		Otomori et al. (2014)
	irregular shape problems	with a body-fitted, nonuniform finite element mesh	James and Martins (2012)
	structural shape and topology optimization problems	with XFEM, Lagrangian approaches and combined shape and topology optimization	Wei et al. (2010)
	reduction in the vibration of structure		Shu et al. (2011)
	structural-acoustic system		Shu et al. (2014)
	elliptic boundary value problems	with projection Lagrangian method, shape and topology optimization	Zhu et al. (2011)
	3D isotropic microstructures	discrete approach	Challis et al (2008)
	mimic leaf venation	with element-free Galerkin method	Lin et al. (2019)
	meshless shape	meshless Galerkin level-set method	Wang and Luo (2011)
	multi-material optimization problems	with SIMP and using EFG method	Cui et al. (2017)

Main method	Application	Methods	Authors
Meshless methods	geometrically nonlinear structures	density variable approach with EFG	He et al. (2014)
	continuum structures	with EFG	Yang et al. (2017)
	free vibrating continuum structures	with EFG and SIMP	Zheng et al. (2012)
	same mesh can be used for both finite element calculations and shape representation	Lagrangian approaches and combined shape and topology optimization	Christiansen et al. (2014)
	nonlinear hyperelastic structures	with EFG	Zhang et al. (2018)
	geometrically nonlinear continuum structures	with EFG	Zheng et al. (2015)

Appendix B: Wind load calculation and related Eurocode

Wind load calculation	
Location	**Paris**
Terrain category	IV
z_0 [m]	1
z_{min} [m]	10
$c_e(z)$	1.2
Zone	A
c_{pe1}	-1.4



$q_p(z_e)=c_e(z) \times q_b$ [kN/m^2]	0.42
$w_e=q_p(z_e) \times c_{pe1}$ [kN/m^2]	0.588

Table 4.1 — Terrain categories and terrain parameters

	Terrain category	z_0 m	z_{min} m
0	Sea or coastal area exposed to the open sea	0,003	1
I	Lakes or flat and horizontal area with negligible vegetation and without obstacles	0,01	1
II	Area with low vegetation such as grass and isolated obstacles (trees, buildings) with separations of at least 20 obstacle heights	0,05	2
III	Area with regular cover of vegetation or buildings or with isolated obstacles with separations of maximum 20 obstacle heights (such as villages, suburban terrain, permanent forest)	0,3	5
IV	Area in which at least 15 % of the surface is covered with buildings and their average height exceeds 15 m	1,0	10
NOTE: The terrain categories are illustrated in A.1.			

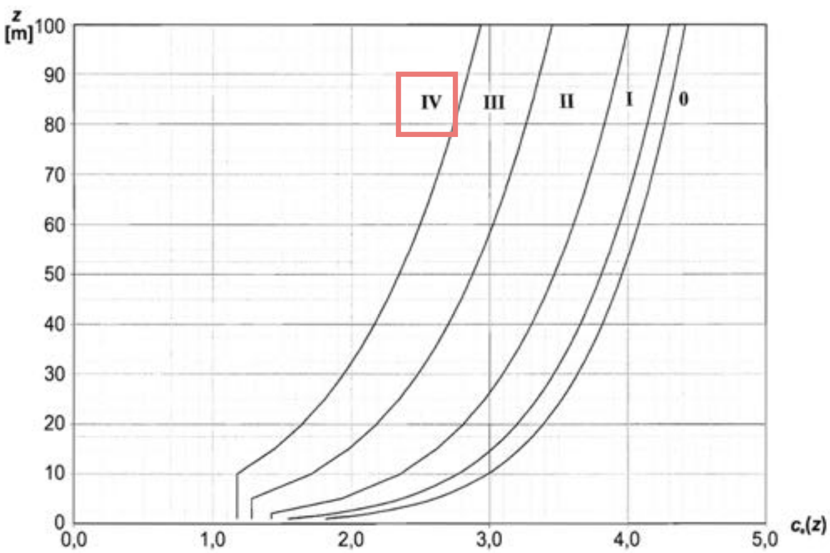
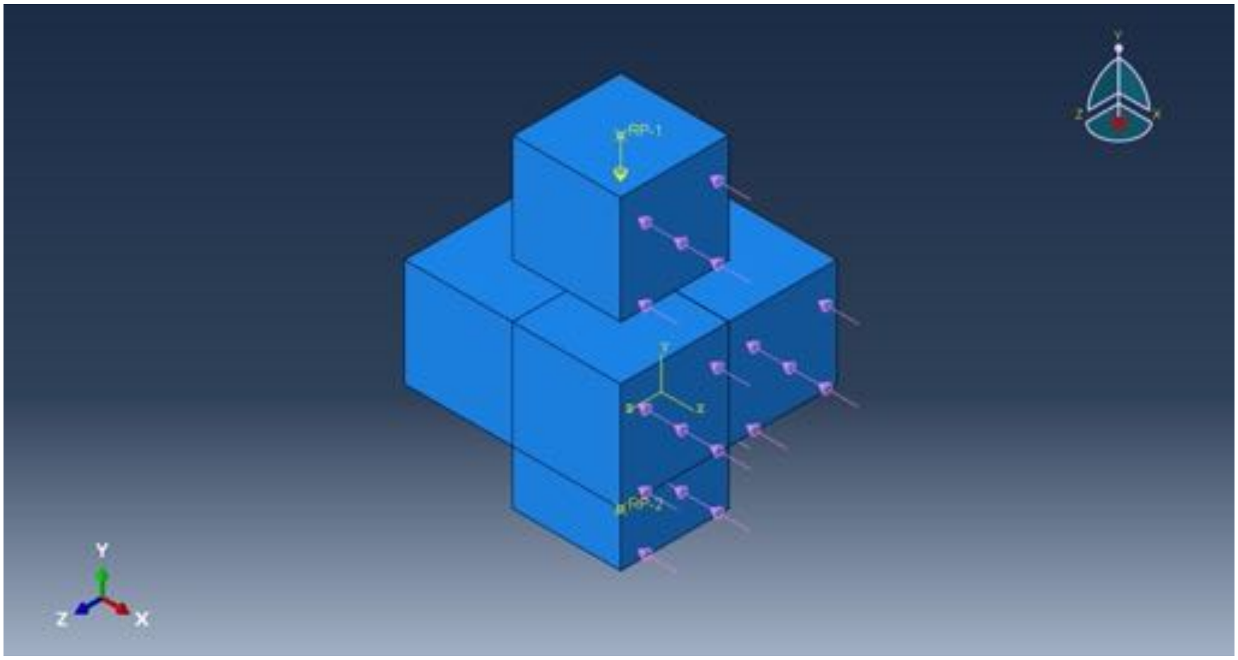


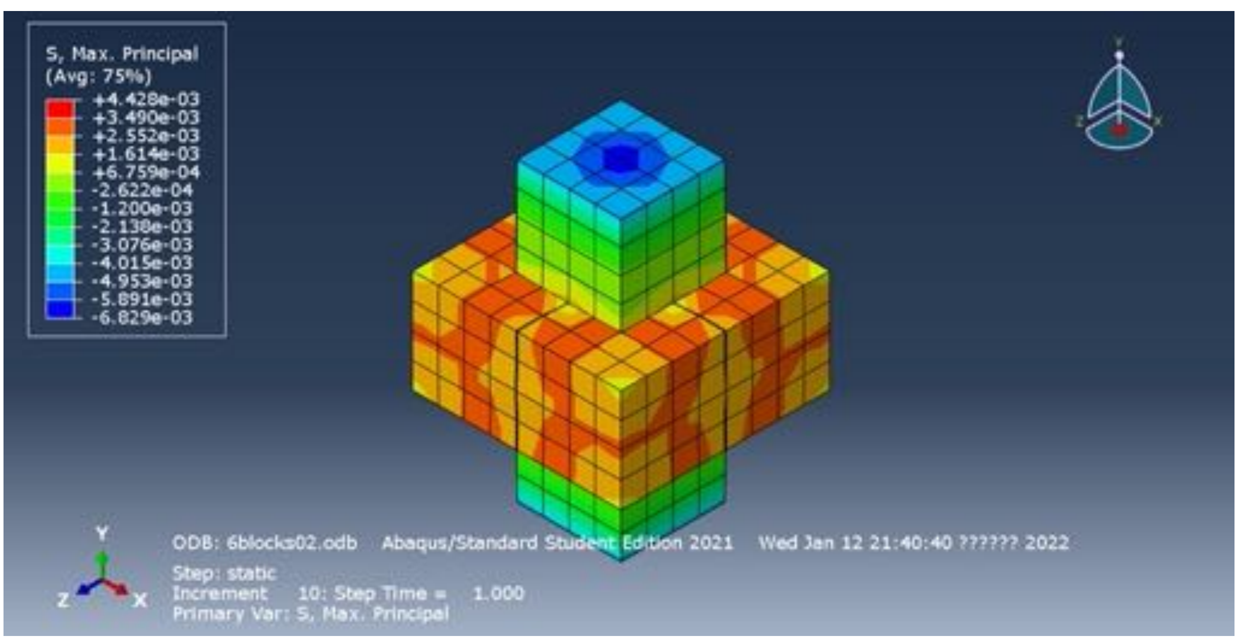
Figure 4.2 — Illustrations of the exposure factor $c_e(z)$ for c_O=1,0, k_I=1,0

Table 7.1 — Recommended values of external pressure coefficients for vertical walls of rectangular plan buildings

Zone	A		B		C		D		E	
h/d	$c_{pe,10}$	$c_{pe,1}$	$c_{pe,10}$	$c_{pe,1}$	$c_{pe,10}$	$c_{pe,1}$	$c_{pe,10}$	$c_{pe,1}$	$c_{pe,10}$	$c_{pe,1}$
5	-1,2	-1,4	-0,8	-1,1	-0,5		+0,8	+1,0	-0,7	
1	-1,2	-1,4	-0,8	-1,1	-0,5		+0,8	+1,0	-0,5	
≤ 0,25	-1,2	-1,4	-0,8	-1,1	-0,5		+0,7	+1,0	-0,3	

Appendix C: Abaqus results of two modular blocks


RP-1
RP-2
Y
Z
X


S, Max. Principal
(Avg: 75%)
+4.428e-03
+3.490e-03
+2.552e-03
+1.614e-03
+6.759e-04
-2.622e-04
-1.200e-03
-2.138e-03
-3.076e-03
-4.015e-03
-4.953e-03
-5.891e-03
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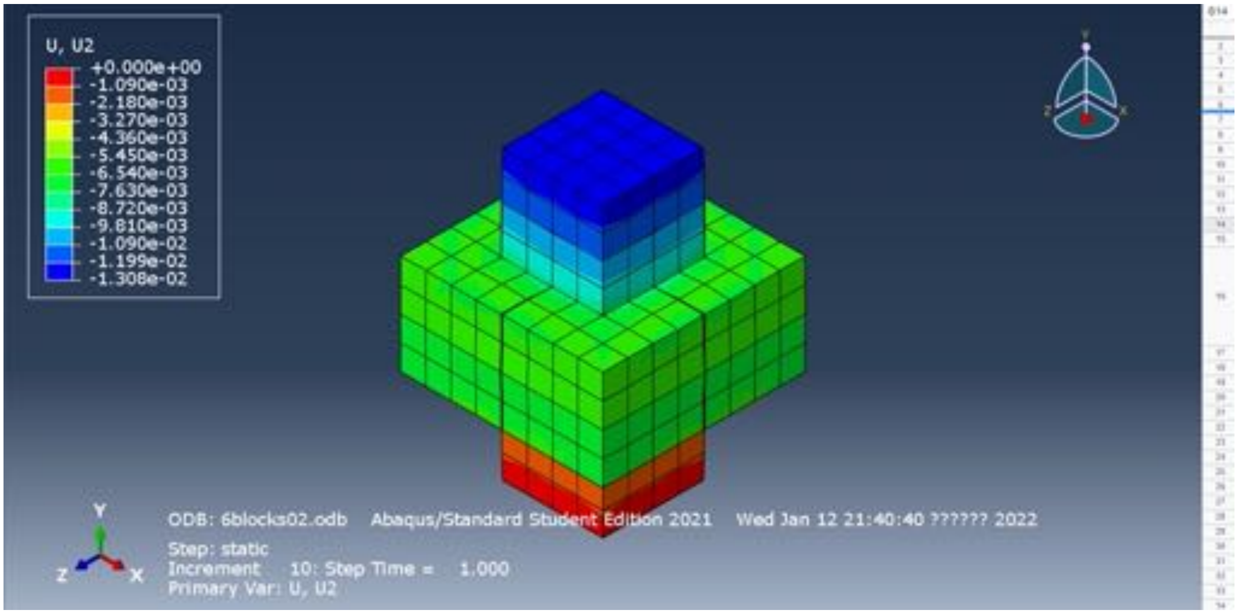
Primary Var: S, Max. Principal
Y
Z
X


U, U2
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-1.090e-03
-2.180e-03
-3.270e-03
-4.360e-03
-5.450e-03
-6.540e-03
-7.630e-03
-8.720e-03
-9.810e-03
-1.090e-02
-1.199e-02
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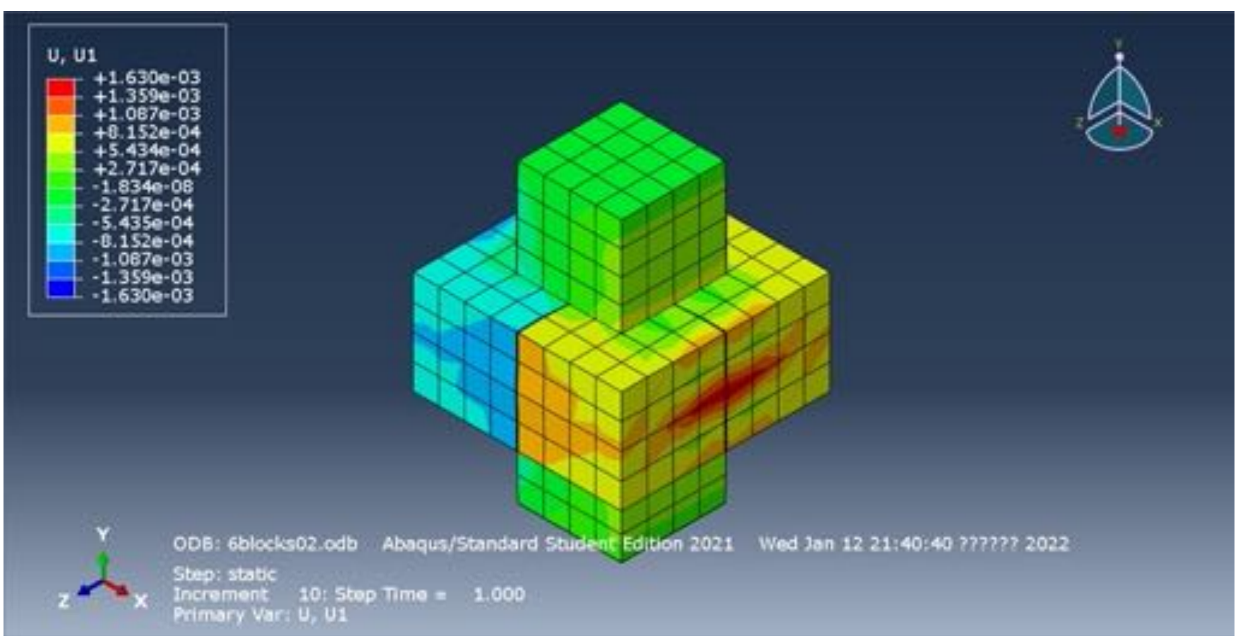
Increment 10: Step Time = 1.000
Primary Var: U, U2


U, U1
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+1.359e-03
+1.087e-03
+8.152e-04
+5.434e-04
+2.717e-04
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-2.717e-04
-5.435e-04
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-1.087e-03
-1.359e-03
-1.630e-03
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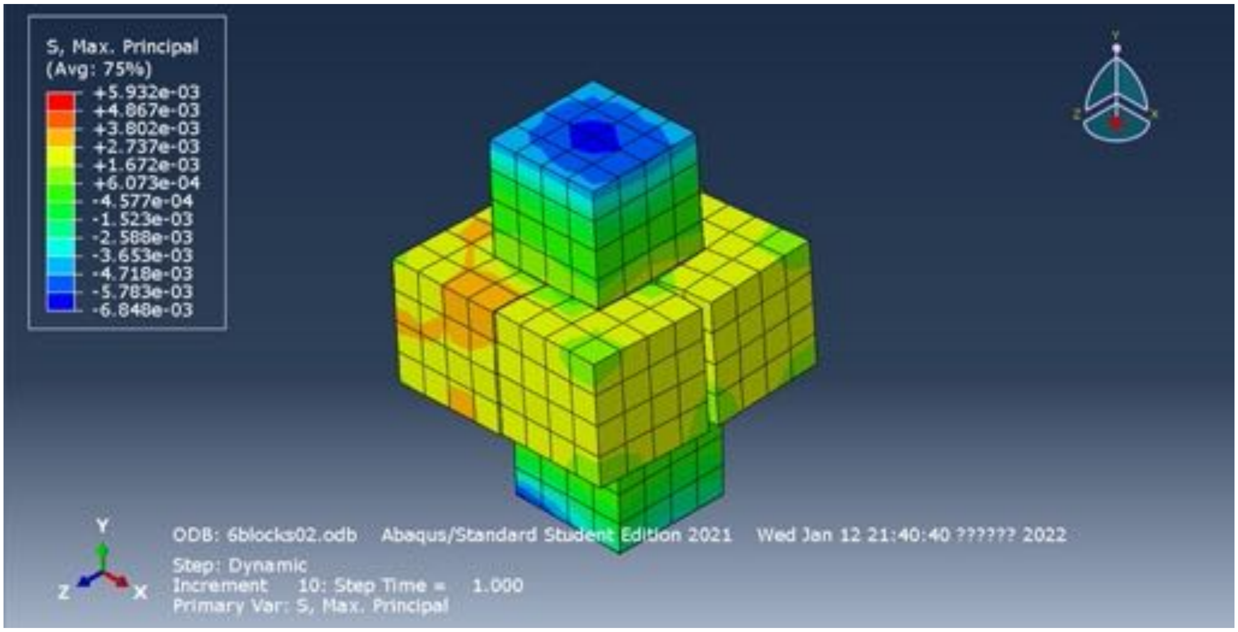
Step: static
Increment 10: Step Time = 1.000
Primary Var: U, U1


S, Max. Principal
(Avg: 75%)
+5.932e-03
+4.867e-03
+3.802e-03
+2.737e-03
+1.672e-03
+6.073e-04
-4.577e-04
-1.523e-03
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-3.653e-03
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-6.848e-03
Y
Z
X
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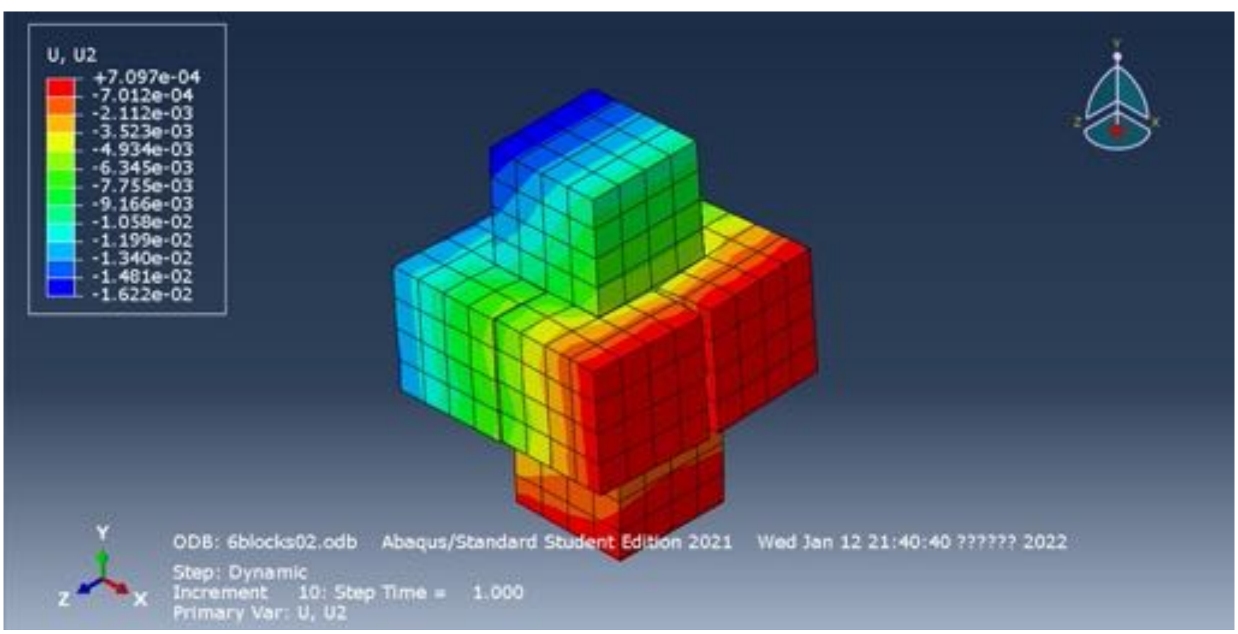
Increment 10: Step Time = 1.000
Primary Var: S, Max. Principal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-7.012e-04
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-3.523e-03
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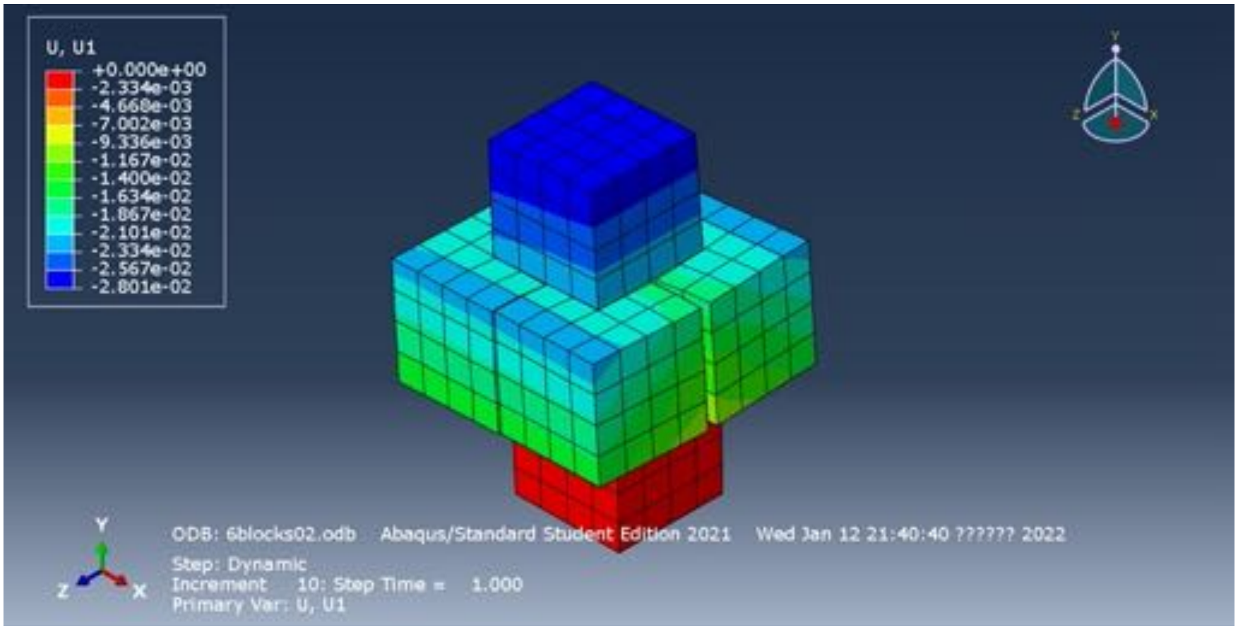
-1.199e-02
-1.340e-02
-1.481e-02
-1.622e-02
Y
Z
X
ODB: 6blocks02.odb Abaqus/Standard Student Edition 2021 Wed Jan 12 21:40:40 ?????? 2022
Step: Dynamic
Increment 10: Step Time = 1.000
Primary Var: U, U2


U, U1
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-9.336e-03
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-1.867e-02
-2.101e-02
-2.334e-02
-2.567e-02
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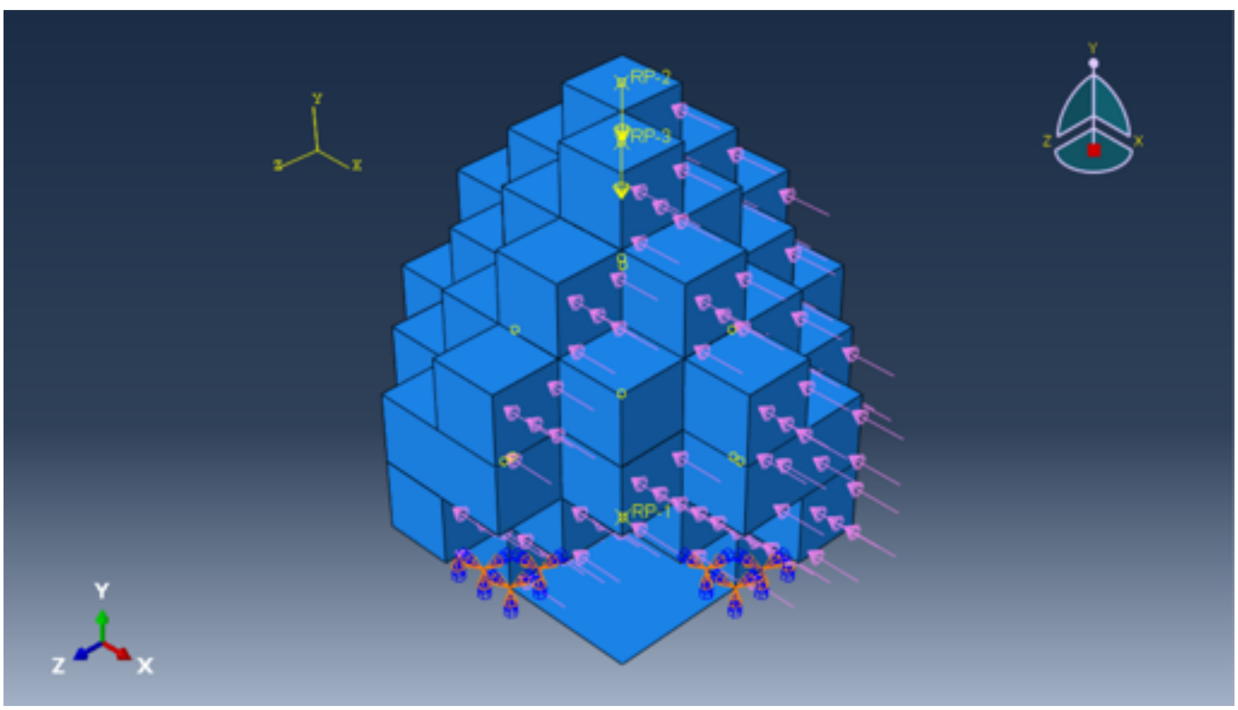
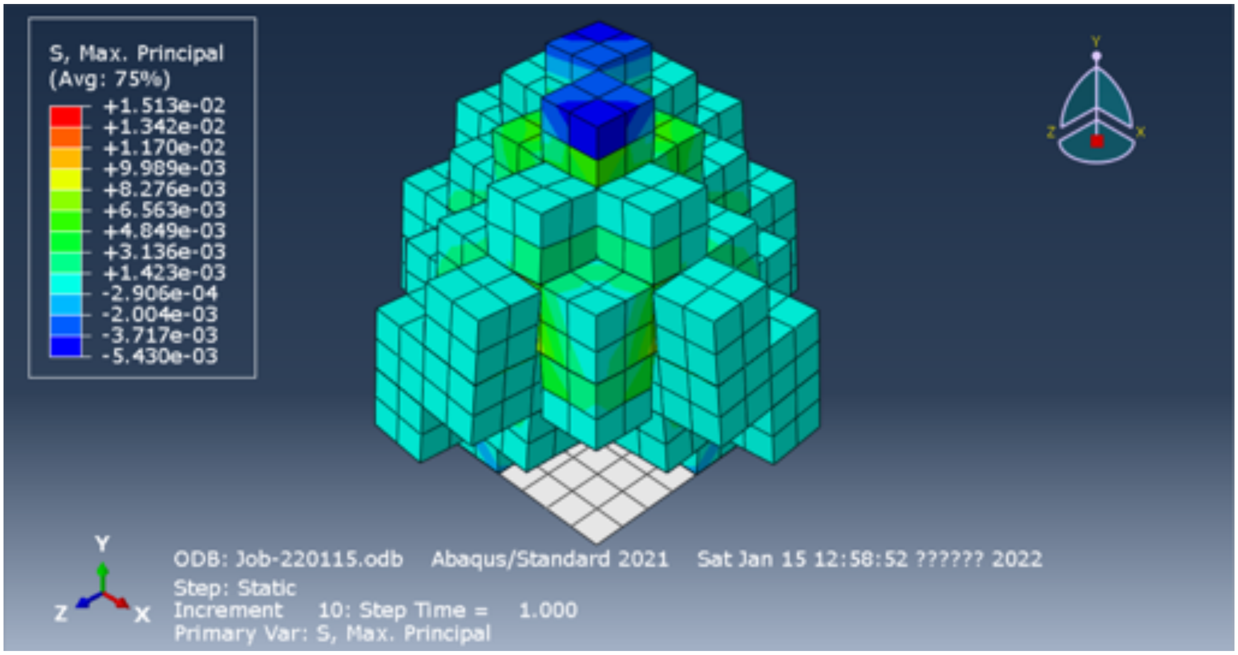
ODB: 6blocks02.odb Abaqus/Standard Student Edition 2021 Wed Jan 12 21:40:40 ?????? 2022
Step: Dynamic
Increment 10: Step Time = 1.000
Primary Var: U, U1
Y
Z
X


RP-2
RP-3
RP-1
Y
Z
X


S, Max. Principal
(Avg: 75%)
+1.513e-02
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+1.170e-02
+9.989e-03
+8.276e-03
+6.563e-03
+4.849e-03
+3.136e-03
+1.423e-03
-2.906e-04
-2.004e-03
-3.717e-03
-5.430e-03
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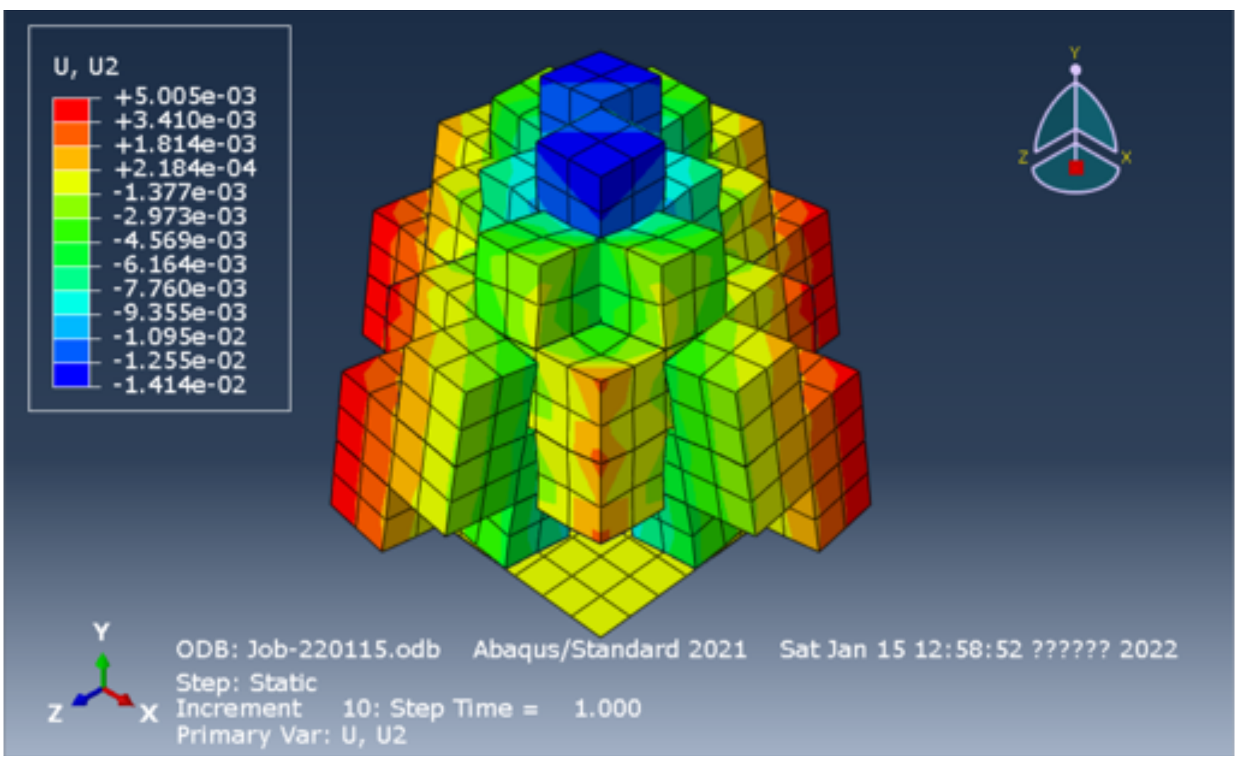
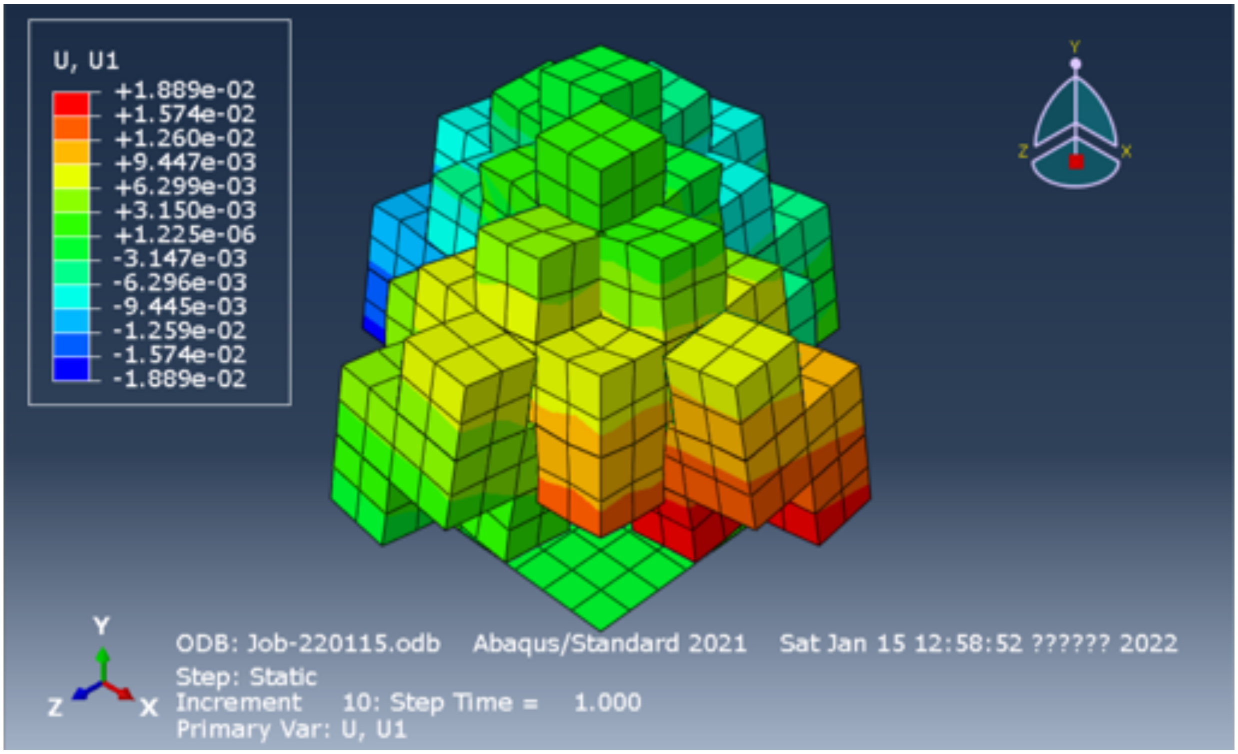
Increment   10: Step Time =   1.000
Primary Var: S, Max. Principal


U, U2
+5.005e-03
+3.410e-03
+1.814e-03
+2.184e-04
-1.377e-03
-2.973e-03
-4.569e-03
-6.164e-03
-7.760e-03
-9.355e-03
-1.095e-02
-1.255e-02
-1.414e-02
ODB: Job-220115.odb   Abaqus/Standard 2021   Sat Jan 15 12:58:52 ?????? 2022
Step: Static
Increment   10: Step Time =   1.000
Primary Var: U, U2


U, U1
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+1.574e-02
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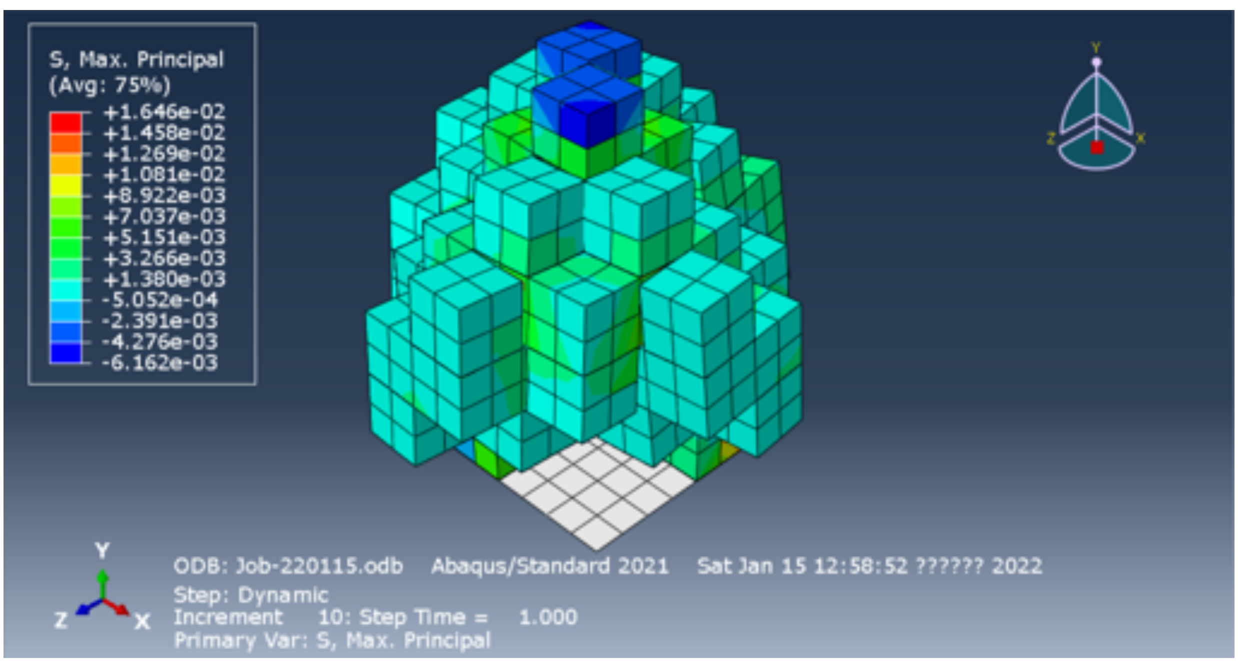
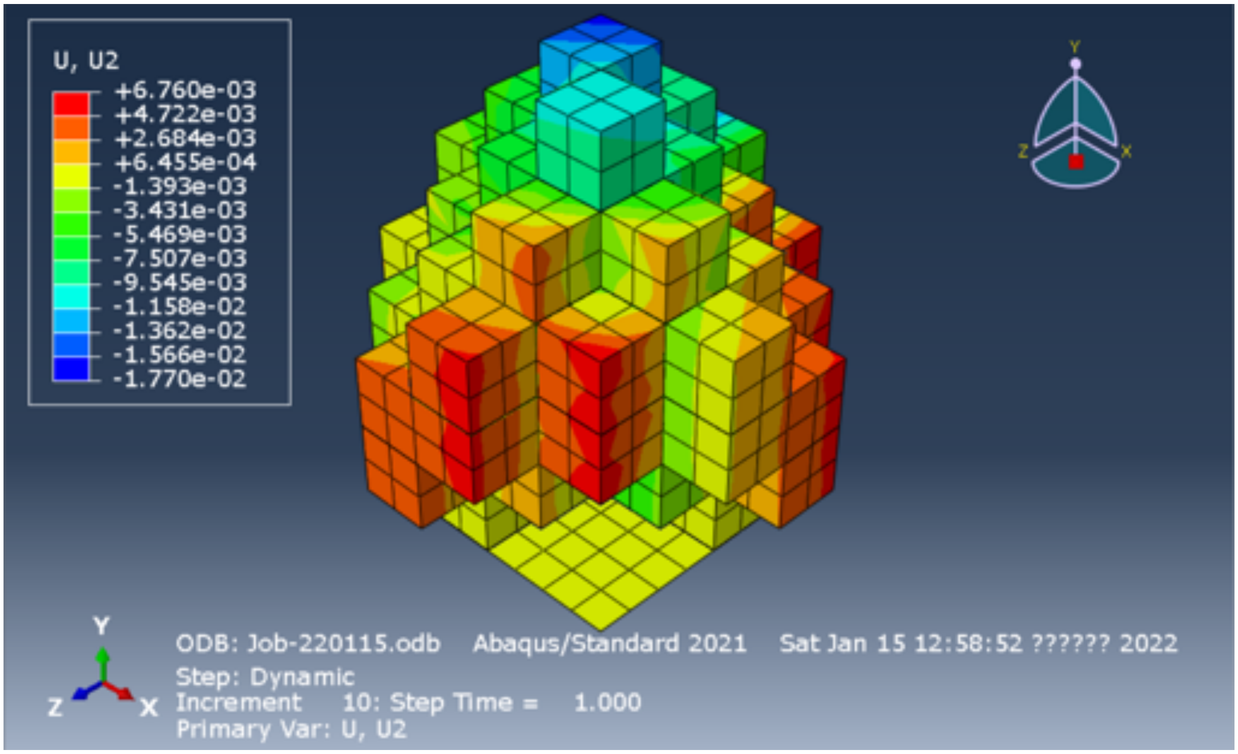
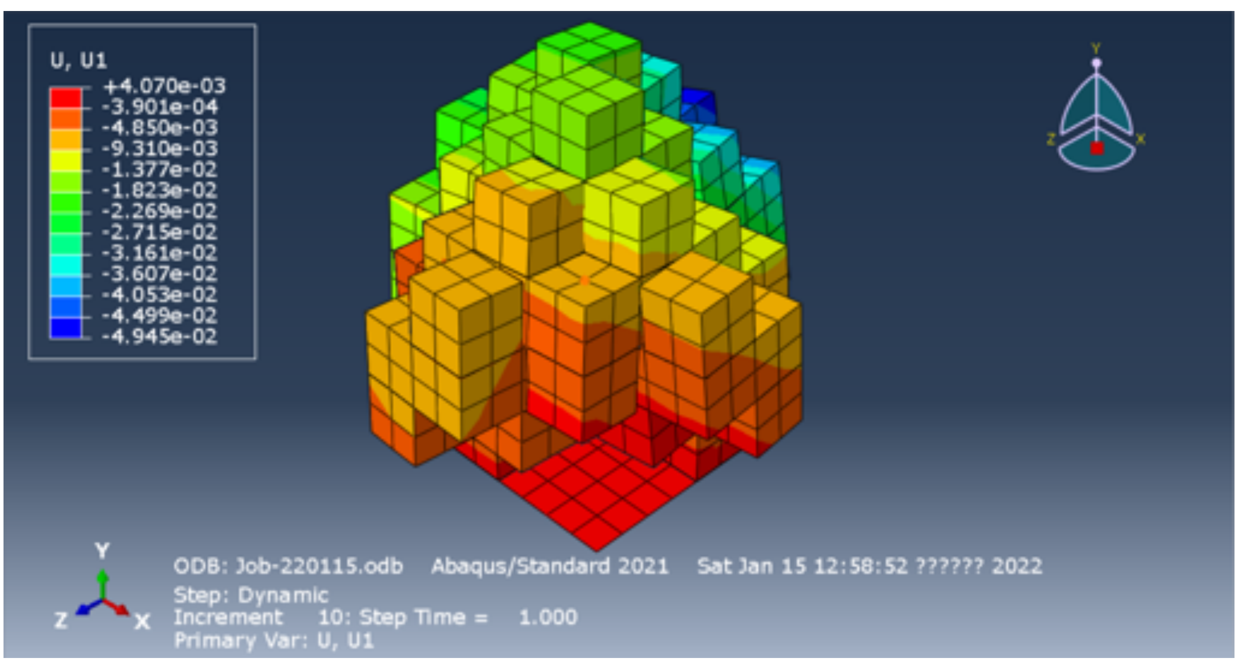
+9.447e-03
+6.299e-03
+3.150e-03
+1.225e-06
-3.147e-03
-6.296e-03
-9.445e-03
-1.259e-02
-1.574e-02
-1.889e-02
ODB: Job-220115.odb Abaqus/Standard 2021 Sat Jan 15 12:58:52 ?????? 2022
Step: Static
Increment 10: Step Time = 1.000
Primary Var: U, U1


S, Max. Principal
(Avg: 75%)
+1.646e-02
+1.458e-02
+1.269e-02
+1.081e-02
+8.922e-03
+7.037e-03
+5.151e-03
+3.266e-03
+1.380e-03
-5.052e-04
-2.391e-03
-4.276e-03
-6.162e-03
ODB: Job-220115.odb Abaqus/Standard 2021 Sat Jan 15 12:58:52 ?????? 2022
Step: Dynamic
Increment 10: Step Time = 1.000
Primary Var: S, Max. Principal


U, U2
+6.760e-03
+4.722e-03
+2.684e-03
+6.455e-04
-1.393e-03
-3.431e-03
-5.469e-03
-7.507e-03
-9.545e-03
-1.158e-02
-1.362e-02
-1.566e-02
-1.770e-02
ODB: Job-220115.odb Abaqus/Standard 2021 Sat Jan 15 12:58:52 ?????? 2022
Step: Dynamic
Increment 10: Step Time = 1.000
Primary Var: U, U2


U, U1
+4.070e-03
-3.901e-04
-4.850e-03
-9.310e-03
-1.377e-02
-1.823e-02
-2.269e-02
-2.715e-02
-3.161e-02
-3.607e-02
-4.053e-02
-4.499e-02
-4.945e-02
ODB: Job-220115.odb Abaqus/Standard 2021 Sat Jan 15 12:58:52 ?????? 2022
Step: Dynamic
Increment 10: Step Time = 1.000
Primary Var: U, U1