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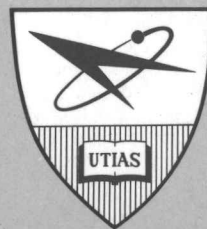
THE BEHAVIOUR OF FREE MOLECULE CYLINDRICAL LANGMUIR PROBES IN SUPERSONIC FLOWS,
AND THEIR APPLICATION TO THE STUDY OF THE BLUNT BODY STAGNATION LAYER

by

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TECHNISCHE HOOGESCHOOL DELFT
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TABLE OF CONTENTS

	<u>Page</u>
NOTATION	vi
1. INTRODUCTION	1
2. APPLICATION OF STATIC THEORY FOR COLLISIONLESS LANGMUIR PROBES	3
2.1 Review of Laframboise's Collisionless Theory for Probes at Rest	3
2.2 Determination of the Electron Temperature	5
2.2.1 The Semi-Log Plot Method	5
2.2.2 The Tangent Method	5
2.3 Number Density From Electron Current at Plasma Potential	6
2.4 Number Density From Ion Current	7
2.5 Some Criteria for the Validity of Collisionless Theory	10
3. EXPERIMENTAL APPARATUS AND INSTRUMENTATION	11
3.1 Description of Experimental Facility	11
3.2 Probes, Models and Instrumentation	14
3.2.1 Pressure Instrumentation and Impact Probes	14
3.2.2 Langmuir Probes and Associated Circuitry	15
3.2.3 Models	16
3.2.4 The Ion Mass Spectrometer	18
3.2.5 Measurement of Gas Temperature	18
4. EXPERIMENTAL FLOW FIELDS	19
4.1 The Free Jet	19
4.1.1 Theory	19
4.1.2 Experiment	20
4.2 The Mach 2.2 Nozzle	22
4.3 Identification of the Ion Species	23
4.4 Volume Recombination	24
5. EXPERIMENTAL STUDY OF FREE MOLECULE LANGMUIR PROBES IN PLASMA FLOWS	25
5.1 Introductory Comments and Experimental Method	25
5.2 On the Absence of a Length Effect	27
5.3 Experimental Curve of the Variation of Ion Current With the Ratio of Probe Radius to Debye Length	28
5.4 Variation of Ion Current with Probe Potential	30
5.5 Effect of Flow on Ion Current and Electron Temperature When Probe is Turned at an Angle to the Flow Velocity	31
5.6 Some Notes on the Electron Current	33
5.7 Non-Ideal Effects, Errors	35
5.7.1 Surface Effects	35
5.7.2 Mean Free Paths	37
5.7.3 Other Causes of Error	37
5.8 Discussion of Results	38

6.	STUDY OF THE STAGNATION REGION AHEAD OF A FLAT-NOSED CYLINDER IN LOW REYNOLDS NUMBER, SUPERSONIC FLOW	42
6.1	Introduction and Experiment	42
6.2	Interpretation of Ion Current Surveys in Terms of Ion Number Density	44
6.3	General Results	46
6.4	The Ion Current to the Wall at the Stagnation Point	49
6.4.1	Direct Measurement	49
6.4.2	Diffusion Flux Calculated from Measured Ion Number Density Gradient at Wall	50
6.4.3	Comparison with High Reynolds Number Boundary Layer Theory	51
6.5	The Ion Shock Wave	53
6.6	The Shock Detachment Distance	54
6.7	Some Experiments at Lower Reynolds Numbers	54
6.8	Discussion of Results	55
7.	SHOCK WAVE TRAVERSES	56
8.	CONCLUSIONS	59
	REFERENCES	60
	TABLES	
	FIGURES	
	APPENDIX A: Diffusion Coefficient, Viscosity Coefficient, Schmidt and Reynolds Numbers	
	APPENDIX B: The Mean Free Paths	
	APPENDIX C: Boundary Layer Calculation of the Ion Flux to the Stagnation Point Wall	

NOTATION

Roman letters

A_{eff}	effective cross section for loss of charged particles to probe, defined by Eq. (5.1)
A_p	area of cylindrical probe, πdl
Ar	symbol for argon
Ar^+	symbol for atomic argon ion
Ar_2^+	symbol for molecular argon ion
A^*	throat area of nozzle or free jet orifice
c	mass concentration of ions, $m_i n_i / \rho$
$\bar{c}$	average thermal speed
C_d	nozzle discharge coefficient: ratio of measured to ideal one-dimensional mass flow
d	diameter of probe, $2R_p$
D	diameter of cylindrical model; also, diameter of free jet orifice
D_i	ion diffusion coefficient
D_a	ambipolar diffusion coefficient, $(1 + T_e/T_i)D_i$
e	magnitude of electronic charge; also, base of natural logarithm
f	dimensionless stream function in boundary layer theory, defined by Eq. (C4)
g	temperature ratio in boundary layer theory, T/T_0
I_i, I_e	dimensionless ion and electron currents, defined by Eqs. (2.2) and (2.5), respectively
$I_i(\chi_f - 10)$	value of I_i at 10 dimensionless potential units below floating potential
I_p	dimensionless net current to probe, defined by Eq. (5.2)
j	current density indicated by probe, $J/\pi dl$
j_i, j_e	magnitudes of the ion and electron current densities indicated by probe

$j_e(0)$	value of j_e at plasma potential
$j_i(\chi_f-10)$	value of j_i at $10kT_e/e$ volts below floating potential
$j_{ }, j_{\perp}$	ion current densities measured with the probe axis parallel and perpendicular to flow velocity, respectively
J	total current drawn by probe
$J(\chi_f-10)$	total current drawn by probe at a potential $10kT_e/e$ volts below floating potential
k	Boltzmann's constant; also, denotes N/Sc in boundary layer theory
l	length of cylindrical probe
m_i, m_e	ion and electron masses, respectively
M	Mach number, $u/\sqrt{\gamma RT}$; also, atomic mass
n	number density, usually overall or neutral atom number density; also, denotes charged particle number density, where so specified.
n_i, n_e	ion and electron number densities, respectively; these two are assumed equal in bulk of fluid, and the symbols are used interchangeably for charged particle number density
n_i^*	normalized charged particle number density, $n_i/(n_i)_{\max}$
$(n_i)_{\max}$	maximum charged particle number density in shock layer
$(n_i)_{\delta}$	charged particle number density at outer edge of boundary layer
N	symbol for $\rho\mu/\rho_{\delta}\mu_{\delta}$
P	pressure; usually, local static pressure
P_i	measured impact pressure
P_o	static pressure in stagnation chamber
P_{tc}	static pressure in test chamber
P_w	pressure at the wall of the model, at the stagnation point
Pr	Prandtl number
Q_{ab}	cross-section for collisions of particle 'a' with particle 'b'
r	radial distance from axis

R	specific gas constant
R_p	radius of probe
Re_2	Reynolds number, based on body diameter and stagnation conditions behind normal shock
Re^*	throat Reynolds number, based on throat diameter and sonic conditions
s	dimensionless distance in boundary layer theory, defined by Eq. (C.21)
Sc	Schmidt number based on ambipolar diffusion coefficient, $\mu/\rho D_a$
Sc_0	value of Sc at outer edge of boundary layer
Sc_1	value of Sc upstream of normal shock
Sc_i	Schmidt number based on ion diffusion coefficient, $\mu/\rho D_i$
T	temperature; usually, local static temperature of gas
T_e	electron temperature
T_i	ion temperature; assumed to be equal to gas temperature
T_w	temperature of body wall at stagnation point
u	flow speed; also, velocity component defined in Fig. C.1
v	velocity component, defined in Fig. C.1
$\underline{v}$	velocity vector (barycentric velocity)
$\underline{V}$	diffusion velocity of ions, relative to gas as a whole
V	probe potential
V_f	floating potential
V_∞	plasma potential (see C.20.7)
x	coordinate, defined in Fig. C.1
y	distance along stagnation streamline, measured upstream from stagnation point
y_s	shock detachment distance; distance along stagnation streamline from nose of body to centre of bow shock
y^*	dimensionless y , y/D

z	axial distance downstream from exit plane of nozzle or free jet orifice.
Z	dimensionless ion concentration, defined by Eq. (C.7); also, charge of ion in units of electron charge
$Z'(0)$	value of $dZ/d\eta$ at $\eta = 0$.
<u>Greek letters</u>	
α	volume recombination coefficient
β	velocity gradient in boundary layer theory, defined by Eq. (C.4)
γ	ratio of specific heats, C_p/C_v
γ_i	electron yield for ion impact
Γ	particle flux
δ	total secondary electron yield; as subscript, it denotes conditions at outer edge of boundary layer
Δ	shock thickness, derived by maximum slope method from density profile
Δ_i	shock thickness, derived by maximum slope method from experimental ion current profile
ϵ_0	permittivity of free space
η	similarity variable in boundary layer theory; defined by Eq. (C.3)
θ	angle between probe axis and flow velocity
λ	mean free path
λ_e	thermal conductivity of electron gas
λ_{ab}	mean free path for collisions between a particle of species 'a' and one of species 'b'
λ_D	Debye length $(\epsilon_0 kT_e / e^2 n_e)^{1/2}$
Λ	quantity defined in Eq. (B.11), numerically in Eq. (B.14)
μ	viscosity coefficient
μ_i	ion mobility
μ_{io}	ion mobility at standard number density of 2.69×10^{25} atoms/m ³
ρ	gas density

χ_f dimensionless floating potential, relative to plasma,
 $e(V_f - V_\infty)/kT_e$

χ_p dimensionless potential difference between probe and
plasma, $e(V - V_\infty)/kT_e$

ω mass flow

Subscripts

a atoms

e electrons

i ions

w conditions at wall (usually at stagnation point wall)

r in radial direction

z in axial direction

δ conditions at outer edge of boundary layer

o refers to stagnation chamber

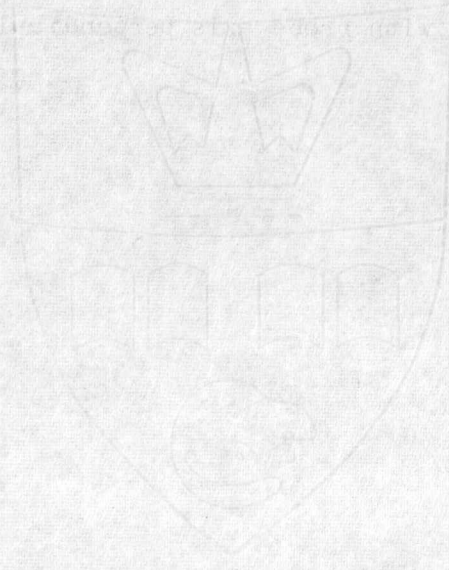
1 free stream conditions

2 conditions behind normal shock

|| measured with probe axis parallel to flow velocity

$\perp$ measured with probe axis perpendicular to flow velocity

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1. INTRODUCTION

The instrumentation requirements of an experimenter trying to obtain detailed information about the structure of rarefied flow fields are increased considerably when the electrical properties of the gas are of importance. There are then at least two additional quantities to contend with: the electron-ion number density and the electron temperature, both of which exert a direct influence on the transport properties of the charged species. It is precisely these quantities that can be inferred from electrostatic probe measurements.

The Langmuir probe consists, in its basic form, of a small conductive sensing element and some associated circuitry. In principle, it may be used for making local measurements of two quantities: the electron temperature and the electron number density, both of which may be obtained from an interpretation of the current drawn by the probe as a function of the voltage applied to it. However, to extract this information, the response of the probe must be known from theory. Several good reviews of Langmuir probe theory and application are available in the literature. Loeb (Ref. 1) gives a detailed account of earlier work in this area, and de Leeuw (Ref. 2) brings the survey closer to the present.

For an experimenter with flow studies in mind the salient fact which emerges from a literature survey is that, although in recent years the theory of stationary electrostatic probes of spherical and cylindrical geometry has reached an admirably high degree of sophistication (Refs. 3 - 7, for example) and for the most part gives results in good agreement with independent measurements (Ref. 8), our understanding of the response of moving probes is on a considerably more primitive footing, available theories being oversimplified and data from systematic experimentation scarce. Fortunately, there are situations where static theory is applicable even when the plasma is flowing at high speed, as for example in the case of an "infinitely long" cylindrical probe with its axis aligned with the flow velocity. This last consideration is a great incentive for the use of cylindrical probes in moving plasmas rather than for example spherical ones, at least when it is possible to employ them in the parallel orientation. Flat, disc-shaped probes can be used with confidence only under special circumstances, for available theories are applicable to this geometry only when the Debye length is very small compared to the disc diameter (i.e. the sheath is negligibly thin compared to the disc diameter). For finite Debye lengths the disc must be treated as a finite one, and this complicates the theoretical approach substantially.

Thus, the long, cylindrical probe appears to be the most attractive geometry to adopt for application in flowing plasmas. However, even here it is imperative to understand how the flow influences the probe's response when its axis is oriented at an angle to the velocity, for in many situations it is not possible to make all measurements with the probe in the parallel orientation. The orientation of a satellite-borne probe, for example, may be changing continually, while in a small scale laboratory experiment, it may be necessary to turn the probe transverse to the flow to obtain a sufficiently high resolution (this was the case in the experiments described in Sec. 6 of this paper). Alternatively, a knowledge of the influence of the velocity may permit the velocity itself to be determined from a comparison of measurements taken with the probe in different orientations.

This paper describes an experimental investigation which had two major objectives. The first of these was to make a systematic study of the behaviour of free molecule, cylindrical Langmuir probes in supersonic plasma flows, the idea being to first check the response of parallel probes against recent static theory, and then to attempt an experimental calibration of the probes for arbitrary orientations with respect to the flow velocity, using the parallel orientation as a reference.

Secondly, the purpose was to use such free molecule probes for making a detailed investigation of the charged particle number density and electron temperature distributions in the stagnation region ahead of a flat-nosed cylinder in supersonic, low Reynolds number flow. This particular model shape was selected largely because it is similar to the geometries commonly employed for sounding rockets and stagnation point Langmuir probes. Both the last-named devices are diagnostic tools which require a knowledge of the flow field in the neighbourhood of the sensing element for the interpretation of the data. A rocket-borne ion mass spectrometer, for example, is sensitive to the ion flux entering its sampling orifice rather than to the ion number density in the free stream, which is the quantity of interest, and an absolute calibration of it therefore requires a knowledge of the relationship between those two quantities. A stagnation point Langmuir probe, usually used under conditions where the gas density is so high that free molecule probes are impracticable, relies on essentially the same relationship to determine the free stream charged particle number density.

In the study described in this paper, particular emphasis was laid on the experimental determination of the relationship between the flux of ions to the wall at the stagnation point of the cylinder (measured independently) and the charged particle number density in the external flow (the latter measured with the free molecule Langmuir probes). The results were compared with high Reynolds number theory.

All the experiments discussed here were conducted in a highly non-equilibrium, cold argon plasma, in which the electron temperature was invariably large compared to that of the ions, and chemical reactions were to a good approximation frozen. The degree of ionization was of the order of 10^{-4} .

2. APPLICATION OF STATIC THEORY FOR COLLISIONLESS LANGMUIR PROBES

2.1 Review of Laframboise's Collisionless Theory for Probes at Rest

The most realistic collisionless probe theory published to date is that of Laframboise (Ref. 4), who has used the mathematical framework of Bernstein and Rabinowitz (Ref. 3) to calculate entire probe characteristics for both cylindrical and spherical probes at rest in a two-temperature plasma. Laframboise assumes that the plasma consists of two species of charged particles, one positive and one negative, each having a Maxwellian velocity distribution with its own characteristic temperature, and that whenever a particle of either species strikes the probe surface, it is annihilated and removed from the problem. His calculations are based on the Vlasov equations for the two species, coupled by Poisson's equation for the electric field, and the results are therefore not expected to be valid unless binary collisions can be ignored and the electric field consistent with Poisson's equation is the dominant influence on the motion of the charged particles.

In the discussion which follows, we shall assume that the positive particles are singly charged ions and the negative ones are electrons. We begin by defining the dimensionless potential difference between the probe and the plasma as follows:

$$\chi_p = \frac{e(V - V_\infty)}{kT_e} \quad (2.1)$$

in which V is the probe potential, V_∞ is the plasma potential, and kT_e/e is the electron temperature, in volts. Laframboise's results may be summarized as follows:

For $\chi_p < 0$,

$$j_i = en \sqrt{\frac{kT_e}{2\pi m_i}} \cdot I_i(\chi_p, R_p/\lambda_D, T_i/T_e) \quad (2.2)$$

$$j_e = en \sqrt{\frac{kT_e}{2\pi m_e}} e^{\chi_p} \quad (2.3)$$

For $\chi_p > 0$

$$j_i = en \sqrt{\frac{kT_i}{2\pi m_i}} e^{-\frac{T_e}{T_i} \chi_p} \quad (2.4)$$

$$j_e = en \sqrt{\frac{kT_e}{2\pi m_e}} \cdot I_e(\chi_p, R_p/\lambda_D, T_i/T_e) \quad (2.5)$$

where

j_i and j_e are the absolute magnitudes of the ion and electron current densities at the probe surface, respectively,

n is the charged particle number density in the undisturbed plasma,

T_i and T_e are the ion and electron temperatures, respectively,

m_i and m_e are the ion and electron masses

R_p/λ_D is the ratio of the probe radius to the Debye length,

k is the Boltzmann constant,

e is the magnitude of the electronic charge,

I_i and I_e are dimensionless ion and electron currents, calculated by Laframboise (both are of order unity at moderate probe potentials),

and

$$\lambda_D = \left(\frac{\epsilon_0 kT_e}{e^2 n} \right)^{\frac{1}{2}} \text{ is the Debye length.}$$

In other words, when the probe is at plasma potential, $\chi_p = 0$, either species arrives at the probe surface at a rate given by its random kinetic flux, $n\bar{c}/4$, and when a species is repelled, its flux to the surface is reduced from this value by the Boltzmann factor. When it is attracted, on the other hand, the situation becomes more complex. When the electrons are attracted, their flux increases over the random kinetic value by the factor I_e , calculated by Laframboise. But when the probe is made negative so as to attract ions, the ion flux soon reaches a magnitude of the order $en\sqrt{kT_e/2\pi m_i}$, which is $\sqrt{T_e/T_i}$ times the random ion flux and may be much larger than the latter if $T \gg T_i$. Again, I_i represents a dimensionless correction factor which emerges from theory.

Both I_i and I_e depend not only on the dimensionless potential difference between the probe and the plasma, χ_p , but also on the properties of the plasma through R_p/λ_D and T_i/T_e , as well as on the probe geometry. Both are generally of order unity at moderate probe potentials. The magnitudes of I_i and I_e for cylindrical and spherical probes are given in Ref. 4 for

$$|\chi_p| < 25$$

$$0 < T_i/T_e < 1$$

$$0 < R_p/\lambda_D < 10^2$$

So far we have discussed the problem from the viewpoint of probe theory, in which one assumes that the plasma properties are given and sets out to derive the functional relationship between the probe voltage and the electron and ion currents corresponding to these conditions. What we are really interested in, however, is the inverse problem: given the probe characteristic - that is, the plot of the net current drawn by the probe as a function of the voltage applied to it - what information regarding the ambient plasma can we infer from it, and how can we best extract this information from the characteristic? With this problem in mind, we can make some obvious but nevertheless important observations on the nature of a typical probe characteristic. What follows will be said with specific reference to a

cylindrical probe in a plasma consisting of Ar^+ ions and electrons, but the remarks have more general validity in a qualitative sense. The theoretical characteristic shown on Fig. 2.1 serves as an example.

(a) For $\chi_p < -10$, the electron current is negligible, and the current actually measured is the ion current, i.e. $|j| = j_i$.

(b) For $\chi_p > -10$, both j_i and j_e contribute to the total probe current, but they can be separated with reasonable accuracy by extrapolating the ion current curve from $\chi_p < -10$ to higher values of χ_p . The accuracy of the resulting approximation for j_i of course deteriorates progressively as less negative values of χ_p are reached, but the important point is that the electron current can be separated very conveniently in this way, for its magnitude increases rapidly as χ_p approaches zero and is generally very large compared to j_i in the region where the approximation for the latter becomes uncertain.

(c) For $\chi_p > -1$, $|j| = j_e$ to very good accuracy. The fact that this is true even for a range of electron repelling probe potentials is a consequence of the great disparity between the electron and ion masses.

The above observations split the characteristic into three distinctive regions according to voltage. These may be called the ion current region, the electron retarding region, and the electron current region, respectively. We shall see that the electron temperature may be inferred from the electron retarding region under a wide range of conditions, while the charged particle number density may in principle be estimated from either the electron or the ion current regions of the characteristic.

2.2 Determination of the Electron Temperature

2.2.1 The Semi-Log Plot Method

In the previous section it was shown that the electron current can be separated from the total probe current in the range $-10 < \chi_p < 0$. Now, Eq. (2.3) indicates that in this electron repelling range of probe bias,

$$\frac{d \ln j_e}{dV} = \frac{1}{\left(\frac{kT_e}{e} \right)} \quad (2.6)$$

In other words, a plot of $\ln j_e$ versus V should yield a straight line, the slope of which determines the electron temperature. Conversely, a linear plot of $\ln j_e$ against the probe voltage may be interpreted as an indication of a Maxwellian energy distribution. This method is therefore endowed with a built-in check for self-consistency.

2.2.2 The Tangent Method

A considerably more convenient and rapid - although perhaps somewhat less accurate - estimate for T_e may be obtained by a method which involves only the measurement of a voltage difference from the probe characteristic rather than a laborious semi-log plot of electron current. Recalling that j_i and j_e denote absolute magnitudes, we write

$$j = j_e - j_i$$

where j is the total current density, taken by convention to be positive when the electron current predominates. Substituting for j_e from Eq. (2.3), we find that

$$\frac{kT_e}{e} = \frac{j + j_i}{\frac{dj}{dV} + \frac{dj_i}{dV}} \quad (2.7)$$

Now if we evaluate the right-hand side of Eq.(2.7) at the floating potential, where by definition the total probe current is zero, and recall that j_i decreases as V increases, we obtain

$$\frac{kT_e}{e} = \frac{j_i(V_f)}{\left(\frac{dj}{dV}\right)_{V_f} - \left|\frac{dj_i}{dV}\right|_{V_f}} \quad (2.8)$$

If we approximate the ion current at the floating potential by a linear extrapolation from the region where the electron current is negligible, the construction shown in Fig.2.2 reveals that Eq. (2.8) becomes simply

$$\frac{kT_e}{e} = a \quad \text{volt} \quad (2.9)$$

where 'a' represents the voltage difference between the floating potential and the intersection point of the tangent to the total current curve at V_f and the extrapolated ion current.

This method is in principle the same as the "equivalent resistance" method described by Johnson and Malter (Ref. 9). Although it yields a very quick estimate for kT_e/e , it is more prone to error than the semi-log plot method because of the necessity for linear extrapolation of a not quite linear curve as well as the difficulty of drawing the tangent at the floating point accurately. It also lacks the automatic check on the nature of the electron energy distribution inherent in the latter.

2.3 Number Density From Electron Current at Plasma Potential

Since the ion current is negligible when the probe is at the local plasma potential, one should be able to measure the electron current at that point directly, and calculate the electron number density from (c.f. Eq. (2.3))

$$n = \frac{j_e(0)}{e \sqrt{\frac{kT_e}{2\pi m_e}}} \quad (2.10)$$

(the electron temperature being known independently from the electron retarding region of the characteristic) without reference to any probe theory other than the simple observation that when the probe is at plasma potential, the charged particle flux to it is given by the simple kinetic theory value.

Unfortunately, it is often difficult to determine the ideal electron current at plasma potential from an experimental probe characteristic. According to collisionless theory, the plasma potential may be equated to that value of probe potential at which the electron current breaks away from its exponential variation with voltage, i.e. the point at which $\ln j_e$ first deviates from its linear variation with probe potential. In many experiments it is found that even under ostensibly ideal conditions the experimental electron current characteristics are considerably more rounded near plasma potential than theory suggests and as a result the definition of the plasma potential is very uncertain. Clayden (Ref. 10), for example, found that the distortion of the electron current was particularly severe for probes of cylindrical geometry, and the same observation was made in the experiments described in this paper. Graf (Ref. 8), who also observed apparently excessive rounding with plane probes, although to a less pronounced degree, tried to overcome the consequent difficulties in determining the plasma potential by fitting an appropriate theoretical electron current curve, taken from Laframboise, over his experimental one so as to match the two at moderately negative and positive potentials, accepting the value of $j_e(0)$ indicated by the theoretical curve rather than by the experimental one. Earlier writers, who had no accurate theory for electron collection at positive potentials, often used the point of intersection of the linear extrapolations (on a semilogarithmic plot) of that part of the electron current where $\ln j_e$ varies linearly with V and the part at positive potentials where the curve flattens out (Refs. 10 and 11, for example), arguing in effect that all rounding is from spurious causes. Laframboise's theory indicates that this always leads to an overestimation of $j_e(0)$. Graf's method of matching appears more satisfactory, but it too is somewhat arbitrary and difficult to justify unless the experimental curve matches the theoretical one everywhere except in the immediate vicinity of the plasma potential.

Another difficulty with this method is often more restrictive than the possibility of distortion near the plasma potential, which is by no means always observed: the electron current is very large at positive potentials, generally several hundred times larger than the ion saturation current, and this may result in the probe acting as an influential part of the system as a whole rather than as a mere sensing element. This is a danger to which small laboratory systems for producing plasmas are particularly susceptible.

2.4 Number Density From Ion Current

In order to determine the charged particle number density from the ion current density, which is related to it by Eq. (2.2), an appropriate value of I_i must be available from theory. Now, I_i is a function of R_p/λ_D and is therefore itself dependent, through λ_D , on the number density we seek to establish. An iterative procedure can of course be used to find the value of I_i consistent with the experimental conditions, but we shall outline a method which circumvents this laborious procedure.

Let us first consider the function $I_i(\chi_p, R_p/\lambda_D, T_i/T_e)$, available in numerical form from Ref. 4. To simplify matters, we shall eliminate the probe potential as an independent variable by agreeing to make all ion current measurements at the dimensionless probe potential $\chi_p = \chi_f - 10$, i.e. at 10 dimensionless potential units ($10kT_e/e$ volts) below the floating potential. The floating potential is chosen as the reference point here because it can be located without ambiguity and, unlike the plasma potential, is available on an experimental characteristic even when the voltage applied to the probe is

restricted to electron repelling values. Ten dimensionless units below the floating potential is an appropriate voltage because it represents a probe potential where the electron current is entirely negligible, but at the same time is not too negative, in the sense that a characteristic showing both the floating potential and the point at $10kT_e/e$ volts below it will have a sufficiently extended voltage scale to allow an accurate determination of the electron temperature.

Note that the floating potential lies about 5 dimensionless units below plasma potential for Ar^+ ions. Equating the ion and electron currents given by Eqs. (2.2) and (2.3), respectively, we obtain the following implicit equation for χ_f :

$$e \chi_f = \left(\frac{m_e}{m_i} \right)^{\frac{1}{2}} I_i (\chi_f, R_p/\lambda_D, T_i/T_e)$$

On the basis of Laframboise's theoretical values for I_i , χ_f was found to have values between -4.5 and -5.5 when the ions are atomic argon ions and R_p/λ_D and T_i/T_e are anywhere in the ranges 0 to 100 and 0 to 1, respectively. Thus, the probe potential $\chi_p = \chi_f - 10$ corresponds to $\chi_p = -15 \pm 0.5$. Note also that this choice of dimensionless probe potential must be reconsidered when the actual voltage of the probe relative to the plasma which corresponds to it,

$$V - V_\infty \approx 15 \frac{kT_e}{e}$$

is large enough to induce secondary effects.

Figure 2.3 shows the theoretical dependence of $I_i (\chi_f - 10)$ on R_p/λ_D for values of $T_i/T_e = 0$ and 1. The curves are based on Laframboise's numerical results, with the ion mass number assumed to be 40 atomic units (although I_i is not directly dependent on ion mass, the dimensionless floating potential is). According to Laframboise, the curves for I_i for intermediate values of T_i/T_e can be approximated by linear interpolation between the limits 0 and 1.

A convenient procedure for determining the charged particle number density, n , can be developed as follows: first, we note that although n itself cannot be determined without prior knowledge of the value of I_i which is consistent with it, we can evaluate the product

$$n I_i = \frac{1}{e} \sqrt{\frac{2\pi m_i}{kT_e}} j_i$$

for kT_e/e can be determined as described in Sec. 2.2 and j_i can be measured from the current-voltage characteristic. Now, from the definition of λ_D ,

$$n = \frac{\epsilon_0 kT_e}{e^2 \lambda_D^2}$$

It follows that the quantity

$$\left(\frac{R_p}{\lambda_D}\right)^2 I_i = \frac{R_p^2}{\epsilon_0} \sqrt{\frac{2 m_i}{e}} \frac{j_i}{\left(\frac{k T_e}{e}\right)^{3/2}} \quad (2.11)$$

can also be evaluated, since all variables on the right hand side can be measured directly from the probe characteristic. This latter quantity is uniquely related to I_i , and we can convert the plot of I_i vs R_p/λ_D shown on Fig. 2.3 to a plot of I_i vs $(R_p/\lambda_D)^2 I_i$ (Fig. 2.4). This last figure provides a direct link between the directly measurable quantity $(R_p/\lambda_D)^2 I_i$ and I_i , and does away with the necessity for iterations.

To recapitulate, the following procedure will be used for determining the charged particle number density from the ion current region of a probe characteristic (the numerical data, in MKS units, are appropriate for cylindrical probes and atomic argon ions, as are Figs. 2.3 and 2.4):

- (1) Determine kT_e/e as described in Sec. 2.2.
- (2) Measure the probe current, J , at $10kT_e/e$ volts below floating potential and calculate the current density

$$J_i(\chi_f - 10) = J / 2\pi R_p \ell \quad \text{ampere/m}^2 \quad (2.12)$$

where R_p is the radius of the probe, and ℓ its length.

- (3) Calculate the quantity

$$\begin{aligned} \left(\frac{R_p}{\lambda_D}\right)^2 I_i(\chi_f - 10) &= 1.83 \times 10^8 R_p^2 \frac{J_i(\chi_f - 10)}{(kT_e/e)^{3/2}} \\ &= \frac{1.45 \times 10^7}{(\ell/d)} \frac{J(\chi_f - 10)}{(kT_e/e)^{3/2}} \end{aligned} \quad (2.13)$$

- (4) From Fig. 2.4, obtain the appropriate value of $I_i(\chi_f - 10)$.
- (5) Calculate the ion number density from Eq. (2.2),

$$n = \frac{1.01 \times 10^{16} j_i(\chi_f - 10)}{I_i(\chi_f - 10) \sqrt{\frac{kT_e}{e}}} \quad \text{m}^{-3} \quad (2.14)$$

2.5 Some Criteria for the Validity of Collisionless Theory

(1) Rostoker and Rosenbluth (Ref. 12) have demonstrated that the BBGKY hierarchy of equations, which represents the complete description of a plasma system, reduces to the Vlasov equation in the limit when the number of particles in a Debye cube becomes very large, i.e. $n \lambda_D^3 \rightarrow \infty$. The plasma is then "rarefied" in the sense that close encounters between particles are negligible in comparison with collective phenomena. In other words, one may assume that a particle feels only the macroscopic force fields, and neglect microscopic interactions.

For practical purposes this description may be considered appropriate when (in MKS units)

$$n \lambda_D^3 = 3.29 \times 10^5 \frac{T_e^3}{n} \gg 1 \quad (2.15)$$

(2) When $n \lambda_D^3$ is large but finite, a particle moving in the macroscopic force field will find its trajectory gradually altered by constant small-angle deflections resulting from encounters with other particles. In using Vlasov's equation it is assumed that the net effect of such deflections on the particle's trajectory is small over the distance characterizing the scale of the problem. This criterion takes the usual form,

$$\frac{\lambda}{d} \gg 1 \quad (2.16)$$

where λ represents the "mean free path", that is, the distance an average charged particle travels before its trajectory is significantly disturbed by interactions with other particles, and d the probe diameter.

The mean free path for encounters between charged particles when $n \lambda_D^3$ is large is evaluated in Appendix B.

(3) The ignorance of the neutral atoms which are generally present in a laboratory plasma is justified when the mean free paths for collisions between charged and neutral particles are large compared to the probe diameter,

$$\frac{\lambda_{\text{charge-neutral}}}{d} \gg 1 \quad (2.17)$$

(4) Since a probe acts as a sink for the charged particles it collects, it will always be a perturbation in the sense that supply currents will be set up in the body of the plasma. These currents can be ignored only if the number density and potential gradients which their presence implies are negligible, and if the currents themselves are negligible compared to the current which maintains the plasma.

A criterion for the latter requirement cannot be formulated without reference to a specific experimental system, but as regards the macroscopic gradients set up by the probe currents, it can be shown that for spherical probes these are small whenever the charged particle-neutral atom mean free paths and the mean free paths for collisions between electrons and ions are large compared to the probe diameter. Thus, this consideration is already implicit in criteria (2) and (3).

In the case of an ideal cylindrical probe, a similar proof cannot be formulated, for the well-known reason that an infinitely long line sink does not exist in cylindrical symmetry with finite density at $r \rightarrow \infty$. However, any practical cylindrical probe is of course finite, and from a distance will cause a perturbation similar to that of a point sink. Thus, the criterion for small macroscopic gradients would be similar to Eq. (2.17), except that the effective mean free path would be somewhat reduced from the actual one due to the length of the probe (by a factor of about $1/\ln(l/d)$).

3. EXPERIMENTAL APPARATUS AND INSTRUMENTATION

3.1 Description of Experimental Facility

The general features of the apparatus and instrumentation are shown schematically in Fig. 3.1. In principle, the arrangement is very simple. Gas (argon was used exclusively) is passed via a flow meter and control valve into the stagnation chamber, where it is ionized by RF induction. From there, it expands through either a nozzle or a free jet orifice to supersonic speed in the test section and then continues to the surge tank, from where it is removed by continuous pumping. Apart from the exploitation of the superior pumping capacity of the UTIAS low density wind tunnel rather than that of the (now defunct) interim plasma tunnel, the major difference between the basic layout in this and previous studies of steady state plasma flows at UTIAS (Refs. 8, 11, 13-14) is that the plasma was produced by RF induction instead of by an arc jet. This method was adopted because the arc jet used hitherto becomes unstable at mass flows as high as those which were necessary for the present experiments.

A comprehensive view of the apparatus is shown on Fig. 3.2. The UTIAS low density wind tunnel was first described in detail by Enkenhus (Ref. 15). Basically it is a cylindrical vacuum vessel, three feet in diameter and 18 feet long, from which are suspended nine booster-type oil diffusion pumps with a combined volumetric capacity of somewhat over 10,000 liters per second in the pressure range 0.001 to 0.070 torr. The boosters are backed by two large mechanical pumps which together have a pumping speed of about 460 liters per second. Originally there was one test section, located near one end of the tunnel, but in the past three years the system has undergone a considerable metamorphosis: a second test section, has been added in tandem with the original one, while a third test section couples via a short cylindrical section into the side of the surge tank and a molecular beam experiment taps into it from the other side.

The third test section was built specifically for the experiments which will be described in this paper. It has the shape of a rectangular box, 26 inches long, with a cross section of 22 by 22 inches, and is connected to the original low density tunnel by a short length of 20" diameter pipe. It can be detached from the tunnel at a flange in the connecting pipe and rolled out somewhat on rails, so that the main tunnel can be isolated with a blind flange and used with any of the other experiments requiring its pumping capacity while the test section described here is being prepared for the next experiment.

On one side of the test section is a large door with a window, giving access to the interior, while on the other side, on a plate which may be removed and replaced with a door or a window, is mounted a two-dimensional

traversing gear for Langmuir and pressure probes (Figs.3.3 and 3.4). Another, smaller, traversing gear is mounted on the roof of the test section, and further access to the interior may be gained through an aperture in the test section floor.

A notable feature of the larger traversing system is that the gear, and in fact the whole driving mechanism, are located outside the tunnel. The entire system is mounted on a finely finished panel which slides over an O-ring, held in place in a groove in a Teflon sheet, to provide a total of 15 cm motion parallel to the flow axis. For transverse motion, two tubes protruding into the test section through sliding O-ring seals in the panel may be moved in and out (over 25 cm) with an external driving mechanism. These tubes serve not only as probe supports, but also as leads from pressure probes to gauges outside the tunnel. Motion in the transverse direction is controlled by a remotely operated dual speed reversible motor, which moves the probes at a rate of either 0.6 or 0.2 cm per second, whereas in the direction parallel to the flow axis, the gear had to be moved manually in these tests. Helipot 10-turn potentiometers are coupled to the traversing gear to provide voltage signals proportional to distance in both directions of motion. This facilitated greatly the recording of quantities like ion current vs probe position, for example, since an x-y storage oscilloscope was available.

In addition to the two-dimensional motion afforded by the traversing gear as a whole, Langmuir probes mounted on it can be rotated about a vertical axis through more than 90° by external control.

The second traversing gear, mounted on the roof of the test section, allows for a total of 8 cm motion in the direction parallel to the flow, in which direction it is driven by a reversible 72 rpm synchronous motor at a constant linear speed of 0.635 mm per second. Whenever it was necessary to make an accurate Langmuir probe survey over a very small distance in the axial direction, as for example a survey of ion current through the shock layer ahead of a blunt body, the probe was mounted on this traversing gear, and the ion current could then be recorded against time on an oscilloscope as the probe advanced, yielding a plot of current vs distance. Whenever necessary, the position of the probe could also be read very accurately from a revolution counter. This second traversing gear also permits radial motion, but this feature was used only to swing the probe into or out of position in the stream.

A Vycor cylinder, six inches in diameter and 21 inches long, serves as the stagnation chamber. The working gas enters this chamber from a $\frac{1}{2}$ inch pipe, but is baffled by a circular plate and then by a fine copper screen before it passes into the stagnation chamber proper, where it is ionized by RF induction. This is shown in Fig. 3.5, which is drawn to scale, although details have been omitted. Under typical running conditions (Figs. 3.6 and 3.7) the stagnation pressure was about 1 torr and the stagnation temperature about 600°K . The Reynolds number based on chamber radius was about 10^2 , a value so low that problems with turbulence would in general not be expected to be severe. However, preliminary tests indicated that without the screen, the mass flow had to be restricted to relatively low values to avoid fluctuations in the flow. These fluctuations, definitely hydrodynamic in character, were visible in the stagnation chamber where the gas was luminous, and showed up as relatively low frequency noise on the characteristics of Langmuir probes in the test section. With the installation of the fine screen, however, the fluctuations appeared only at considerably higher mass flows than when the screen was absent. Nevertheless, it was this instability which put an upper

limit on the usable mass flow, and not the pumping capacity of the tunnel, although the situation seemed to be also influenced by other factors, for at a given mass flow the fluctuations were worse with the nozzle than with the free jet.

The RF power was supplied from a Philips PH 1412/00 RF oscillator, fed by high voltage DC power from the rectifier unit of a 12 KW Philips PH 1202/17 generator. The latter was modified slightly with the addition of a variable transformer at the input; this allowed the output power to be varied continuously from zero to maximum. The oscillator was used at a frequency of about 10 megacycles per second. The coil used in these experiments consisted of four 7.5" diameter turns of $\frac{1}{4}$ " copper tubing, connected in parallel, with 1.5" separating one turn from the next; it was water cooled. The oscillator unit was mounted on rails so that it could be rolled back and forth in the direction parallel to the stagnation chamber axis in order to provide convenient control over the position of the coil with respect to the nozzle throat. Under typical operating conditions, the downstream turn of the coil was about 15 to 35 cm upstream of the throat. The stagnation chamber itself was air cooled with a standard ten inch fan, mounted above it.

Early probe studies in this system revealed a noticeable oscillation in the plasma potential (not to be confused with the hydrodynamic instability mentioned earlier) at a well defined frequency of about 360 cycles per second. This was traced to a ripple in the high voltage output of the rectifier (such a ripple has a frequency of 360 cps in a full-wave, three-phase rectifier like the one discussed here), which apparently was transmitted through the oscillator into the plasma. The trouble was overcome with the addition of a simple R-C-L filter in the high voltage line between the rectifier and the oscillator.

The one nozzle used in these experiments is shown on Fig. 3.8. With an exit Mach number of 2.2, it had a simple conical diverging section with a half-angle of 10 degrees and an exit diameter of 6 cm. No attempt was made to design this nozzle with fastidious attention to contouring and boundary layer growth, mainly because at the time the nozzle was built no information was yet available on the temperatures to be expected in, nor in fact the general properties of, the RF heated gas in the stagnation chamber, and without this data the effort of careful nozzle design could hardly be justified. It was decided that a simple conical nozzle would be used for initial tests, and that improvements would be made later, if necessary, when data on operating conditions had been obtained. It turned out, however, that this nozzle produced a fairly satisfactory flow field when operated in an underexpanded condition, although as a result of significant boundary layer growth, the Mach number at the exit plane was about 2.2 instead of the 2.5 value one would have expected on the basis of one-dimensional isentropic theory. Provisions for water cooling the nozzle wall had been incorporated, but this feature was found to be unnecessary.

For tests at higher Mach numbers ($M > 4$, say) at low densities, free jets have decisive advantages over conventional nozzles. The application of these jets for wind tunnel testing has been discussed by Sherman (Ref. 16), and very useful semi-empirical equations defining the free-jet flow field have been given by Ashkenas and Sherman (Ref. 17). In our case, the free jet proved to yield a particularly versatile flow field for studies of Langmuir probe behaviour in high speed flows. Orifices of several sizes were tested, but the one which was used in the experiments reported here was a simple, square-edged aperture, with a diameter of 5.00 cm, in a 0.175 cm thick plate.

3.2 Probes, Models, and Instrumentation

3.2.1 Pressure Instrumentation and Impact Probes

The following four pressures were usually recorded in any particular run:

- (1) Stagnation pressure.
- (2) Impact pressure (this quantity could be measured at any point in the test section within the range of the larger traversing gear).
- (3) Nozzle exit pressure, when the nozzle was used.
- (4) Static pressure in the test section.

The first three were measured with a three-tube oil micromanometer, while a McLeod gauge (CVC Type GM 100A), vapour trapped with liquid nitrogen, was used for the static pressure in the test section. Commercial thermocouple and cold cathode gauges were also installed, the former for pressure monitoring during runs and the latter for measuring the base pressure of the tunnel as well as the pressure in the interior of the models and a mass spectrometer. The McLeod gauge served as a standard against which all other gauges were calibrated. According to the manufacturer's specifications, the accuracy of this instrument is better than 1% at pressures above 0.010 torr.

The bulk of the pressure measurements were made with the micromanometer. This instrument is similar to the one described by Enkenhus (Ref. 15) except for the fact that it is of all-glass construction. It consists of three separate manometer tubes arranged so that one end of each is connected to a common reservoir which is maintained at a pressure below 10^{-4} torr by a small diffusion pump, while glass stopcocks permit the other ends of the tubes to be connected either to the reference reservoir or to the points at which the pressure is to be measured. The difference in the two levels, which may be read accurately by means of an optical system travelling on a micrometer screw, then gives an indication of the pressure. In this case, coloured dibutyl phthalate was used as the manometer fluid, and by actual calibration against the McLeod gauge, one division on the micrometer drum (representing a 0.002 inch difference in elevation) was found to correspond to 0.00391 torr.

All impact pressure probes were of the same geometry and size. They were of the thin-lipped type with 10° external chamfer, an I.D. of 0.62 cm, a ratio of O.D. to I.D. of 1.25, and a ratio of length to outside diameter of 26. Except for the lower ratio of length to outside diameter used here, these probes were similar to those used by Ashkenas and Sherman (Ref. 17).

3.2.2 Langmuir Probes and Associated Circuitry

Cylindrical Langmuir probes were made by leaving a length of tungsten wire to protrude from a tubular Pyrex sheath. The sketch on Fig. 3.9 illustrates the construction of a typical probe, while Fig. 3.10 shows photographs of some of the probes used in the experiments. Tungsten wires with diameters of 0.025, 0.1, 0.2, and 0.3 mm were obtained in cleaned and straightened form from General Electric. In making a probe, a Pyrex tube about four millimeters in O.D. was first heated over the flame of a small propane torch, drawn out, and broken neatly at the narrow neck which resulted. The result was a tube, one end of which tapered gradually to a very fine diameter. By a process of repeated trials and examinations under a binocular microscope, one finally obtained a tube with its fine end just large enough to allow wire of the chosen diameter to be passed through it. This was done, and then, with the wire held in place so that it protruded from the glass sheath to the desired length, the region near the fine end of the Pyrex tube was placed over a very small flame until the glass constricted firmly over the wire and made a strong bond. This completed the working end of the probe. The construction of the rest of the probe is obvious from the sketch and the photographs.

The diameter and length of the collecting surface of each probe were measured carefully under a Leitz "Ortholux" microscope. This microscope was equipped with an eyepiece with graduations and a hairline movable with a micrometer screw. With the objective used, the diameter of the smallest probe (0.025 mm) corresponded to 100 units on the micrometer drum, and therefore - particularly since a number of diameter readings were always taken for a given probe - the error in this measurement should in no case be greater than about 1%. (The units of the micrometer drum were calibrated against a standard scale provided by Leitz.) The length of the collecting surface, on the other hand, could not be measured in the same way because the range of the micrometer eyepiece was not large enough. Instead, a vernier arrangement attached to the microscope was used for this purpose. This provided a measurement to within 0.05 mm, which represents 2% of the length of the shortest probe (2.5 mm). Thus, the area of even the smallest probe was measurable to within 3%.

Figure 3.11 shows the circuit for sweeping the probe voltage and recording the variation of probe current with probe voltage. The voltage was applied either in the form of a sine wave, the amplitude of which could be varied manually, or in the form of variable DC. Alternating voltages of various frequencies were originally tried, but a 60 cps signal was selected largely because it was conveniently available and slow enough not to produce appreciable "hysteresis" effects as a result of line capacitance. The probe current was measured as a voltage across the precision resistor R. The resistor of resistance 2R was inserted as shown to balance out from the reading that part of the voltage which results from leakage to ground (through the one megohm effective resistances across the oscilloscope inputs) rather than from the true probe current itself. Actually, any spurious indication of probe current was always eliminated anyway by using for the zero probe current line the plot of "current" vs probe voltage obtained when the probe was disconnected from the circuit at the point where the circuit entered the vacuum system; the balance resistor 2R merely served to keep the zero line as horizontal as possible.

The circuit used for plotting the ion current to a probe at constant potential against the probe's position was similar to that in Fig. 3.11, except that the 60 cps oscillator was removed, the point A grounded,

and the x-input taken from one of the potentiometer arrangements on the larger traversing gear rather than from the probe. As described before, the potentiometers provided voltage signals proportional to distance in both axial and transverse directions. This made it a simple matter to record a plot of current vs position on the x-y storage oscilloscope. When a probe was mounted on the smaller traversing gear, a time base provided a signal proportional to displacement, for the carriage was advanced at a constant rate by a synchronous motor.

At the beginning of each run the probes which were to be used were cleaned carefully. This was done, after steady running conditions had been reached, by moving the probe to be cleaned into the plasma flow and then increasing the amplitude of the voltage applied to it until the collecting surface glowed red. Measurements with an optical pyrometer indicated that the tungsten wire reached a temperature in the neighbourhood of 1300°K during this process. After cleaning, the probe remained in the argon atmosphere throughout the run and its characteristic was very reproducible. For example, if the characteristic was recorded on the storage oscilloscope directly after the probe had been cleaned, and then again half an hour or so later in the same run, the later characteristic would almost always be congruent with the earlier one, except possibly for a voltage shift of a fraction of a volt. This speaks also for the steadiness of the running conditions once initial drifts had been given time to die down.

The state of cleanliness of the probes is important not only because of the possibility that it might influence the quality of the characteristic (Ref. 18), but also because it can introduce an error into the determination of the true collecting area. It was generally found that the cleaning process mentioned was very effective in removing the black deposits of oxidation products left on the tungsten surface after the probe had been made, and that it led to very reproducible characteristics. However, examinations under a microscope after runs revealed that there was often a region of the wire adjacent to the Pyrex sheath which was still dark and not clean and shining like the rest of the probe, probably because the glass served as a heat sink and prevented the wire very close to it from reaching as high a temperature as the rest. This could often not be remedied by trying to heat the probe even more, for one was then faced with the danger of damaging it (several probes were accidentally vapourized as a result of over-zealous cleaning). As a result, as much as 10% of the collecting surfaces of the smallest probes (0.025 mm diameter) was often found after the runs to be "unclean", although the larger probes suffered less in this respect. However, on the strength of some experimental evidence to this effect (see Sec. 5.7.3), it was assumed in all calculations in this paper that the entire exposed length of the collecting surface of a probe, including any apparently imperfectly cleaned part, was effective in collecting charged particles.

3.2.3 Models

Two flat-nosed circular cylinders were used as models in the stagnation region studies. These were identical in every respect except that one had a small orifice in the thin wall at the stagnation point for the measurement of ion flux to the wall, whereas the other was instrumented with a thermocouple at the same point. The details are shown in Fig. 3.12.

It might be appropriate at this point to outline the reasoning behind the orifice method for measuring the ion flux to the wall of the electrically floating body. Since at the surface of such a body the electron and ion fluxes are equal in the steady state, the net current density there is zero and the magnitude of the electron-ion flux cannot be measured as current unless the two species of charged particles are separated. One way of accomplishing a separation without disturbing the external flow is to allow the charged particles to pass through a very small orifice into the interior of the body and then to repel the electrons and to measure the ion flux as a current. If the sampling orifice is smaller than the Debye length and the mean free path, it should appear to a charged particle in the flow exactly as any other part of the wall and, barring losses in the passage from the orifice to the collector, the measured ion flux should equal the flux of both electrons and ions to the wall in the absence of the orifice.

The necessity for avoiding losses of ions in the interior made it imperative to have the wall at the orifice as thin as possible and to maintain the interior of the model at low pressure by constant pumping with a diffusion pump. Although some difficulty was encountered in making a suitably small orifice, the following method was found to give the best results: a fine spray of abrasive powder* was directed at the thin stagnation point wall from the interior (the orifice was prepared before the nose cap was welded onto the cylindrical trunk of the model). This spray would gradually cut a crater-like hole into the metal, and by stopping the process when the tip of the crater broke through at the other side, one obtained an orifice with almost zero effective wall thickness. All this was done under a binocular microscope, and a measure of trial and error was of course necessary. The orifice which was finally accepted was carefully measured under the Leitz microscope mentioned earlier (Fig. 3.12); although slightly oblong in shape, it had an effective diameter, based on area, of 0.140 mm, the measurement of which was considered to be accurate to within a few percent.

The pumping system associated with the model is shown in Fig. 3.13. The whole arrangement of model, pump, etc., could be rotated about a vertical axis while the tunnel was under vacuum so as to make way for impact probe studies of the flow field. The spherical electrode (Fig. 3.12) in the interior served both to repel electrons and to collect ions when biased sufficiently negative with respect to the shell of the model. The ion current was measured with a Keithley 610B electrometer.

The other model was instrumented with a chromel-alumel thermocouple at its stagnation point. The 0.010 inch diameter thermocouple wire were forced through a small hole in the thin nose wall and attached with a tiny dab of silver solder. The protruding ends were ground flush with the nose. With the reference junction at room temperature, the thermocouple emf was recorded directly on a Tektronix 564 oscilloscope with a 3A3 differential amplifier. A measurement made by exposing the model to boiling water was in agreement with an accurate thermometer to within one percent, which was better than the expected measuring accuracy. The thermocouple calibration was taken from Ref. 19.

* An abrasive unit manufactured by S.S. White was used for this purpose.

3.2.4 The Ion Mass Spectrometer

A miniature ion mass spectrometer was utilized for local identification of the ion species present in the flow. This instrument, of the quadrupole type pioneered by Paul (Ref. 20), was of a compact design which rendered it particularly suitable for studying the relatively confined plasma flows available in the facility described here.

The design and operation principles of a quadrupole mass filter have been amply documented in Ref. 20. The particular spectrometer used here had a field radius of 0.164 cm and a field length of only 3.00 inches. It was enclosed in a vacuum envelope with a 1.50 inch diameter flat nose and a conical afterbody leading to the pumping system which kept the interior at low pressure (the same system was used for this as for the blunt-nosed models). This is shown schematically on Fig. 3.14. The sampling orifice, 0.015 inches in diameter, was located in the thin wall at the centre of the flat nose.

The principles of operation of a quadrupole mass filter may be outlined briefly as follows. Charged particles enter through the sampling orifice into the evacuated interior of the instrument, where they are accelerated by a DC electric field. The portion near the axis of the resulting beam then passes through a collimating orifice into the quadrupole field region. Now, the geometry of the quadrupole electrodes and the combination of DC and RF potentials applied to them (no magnetic fields are used) are such that a charged particle which has entered the field region can remain in stable motion only if it possesses a charge to mass ratio within limits chosen by the experimenter. The stable particles are able to traverse the field region and reach the collector, which records their arrival as a current, whereas the unstable ones are either ejected from the field region or lost by collision with the electrodes. As the range of stable masses can be made arbitrarily small, the system serves as a convenient mass filter.

3.2.5 Measurement of Gas Temperature

The stagnation temperature was inferred from a comparison of the stagnation pressure with the RF on and with the gas at room temperature, with the mass flow held at the same value for both measurements. With critical flow at the throat, the mass flow, ω , is given in terms of the stagnation pressure, P_0 , and the stagnation temperature, T_0 , by the equation

$$\omega = C_d \cdot \frac{P_0}{\sqrt{T_0}} \cdot \sqrt{\frac{\gamma}{R}} \left(\frac{2}{\gamma+1} \right)^{\frac{1}{2} \left(\frac{\gamma+1}{\gamma-1} \right)} A^* \quad (3.1)$$

where C_d is the discharge coefficient, defined as the ratio of the measured to the ideal one dimensional mass flow, and A^* is the throat area. It follows that if the mass flow remains constant and the discharge coefficient does not change as the stagnation temperature is decreased from its value under operating conditions to room temperature, then

$$(T_o)_{\text{hot}} = \left[\frac{(P_o)_{\text{hot}}}{(P_o)_{\text{r.t.}}} \right]^2 (T_o)_{\text{room temp.}} \quad (3.2)$$

This yields a stagnation temperature appropriate to some average temperature over the throat cross-section. The assumption that C_d did not change appreciably received some confirmation from measurements of Mach number at the nozzle exit plane. Since a discharge coefficient different from unit reflects a boundary layer displacement effect, a change in C_d should in the case of a nozzle manifest itself as a change in the exit plane Mach number. However, under typical operating conditions practically no systematic change in exit Mach number was observed when the stagnation temperature was decreased to room temperature. For a free jet orifice, boundary layer effects are generally substantially lower than for a nozzle at similar throat Reynolds numbers, particularly if the orifice is in a thin plane as in this case. According to Ref. 17, the effect of boundary layer displacement at the throat should be insignificant for throat Reynolds numbers (based on throat diameter) over 100 with the present orifice configuration.

The static temperature was calculated from T_o on the assumption that the flow from the stagnation chamber to the test region was isentropic, after this assumption had received ample support from pressure and Langmuir probe surveys (see Sec. 4).

4. EXPERIMENTAL FLOW FIELDS

4.1 The Free Jet

4.1.1 Theory

The free jet has in recent years been adopted for a variety of applications requiring high Mach number, low density flows, and the flow field itself is by now reasonably well understood. Reference 17, for example, gives a semi-empirical calibration of the free jet flow field for several orifice shapes and a wide range of throat Reynolds numbers.

The term "free jet" is applied to the unconstrained expansion of a gas from a sonic orifice into a low pressure chamber. If we examine such a flow some distance downstream of the orifice, we find that it consists of a barrel-shaped shock structure, inside which there is a region of isentropic flow exhibiting properties very similar to the adiabatic expansion from a "point source". For a gas with a given ratio of specific heats, the shape and size of the barrel shock and the axial location of the Mach disc, which terminates the shock structure on its downstream end, relative to the orifice are determined by the pressure ratio across the orifice, whereas the Mach number distribution in the isentropic flow field in the interior depends only on the orifice diameter (except for a modest dependence near the exit plane on the orifice's entry section and on viscous effects at sufficiently low throat Reynolds numbers). In this sense the behaviour of a free jet core is analogous to that of a conventional nozzle, where the Mach number distribution

is determined by the area ratio. The important difference between the two, and the one which makes the free jet the more attractive configuration for high Mach number, low density testing, is that the latter remains free of viscous effects to a much lower throat Reynolds number than a nozzle. On the

other hand, there are always axial gradients in a free jet which can in principle be eliminated in a nozzle by proper contouring.

In Ref. 17, Ashkenas and Sherman give the following equation for the centreline Mach number distribution in a free jet, valid for a gas with $\gamma = 5/3$ at distances more than one orifice diameter downstream of the exit plane:

$$M = A \left(\frac{z - z_o}{D} \right)^{\gamma-1} - \frac{\frac{1}{2} \left(\frac{\gamma+1}{\gamma-1} \right)}{A \left(\frac{z-z_o}{D} \right)^{\gamma-1}} + \frac{C}{\left(\frac{z-z_o}{D} \right)^{3(\gamma-1)}} \quad (4.1)$$

where D = orifice diameter
 z = distance downstream of orifice exit plane
 $A = 3.26$
 $C = 0.31$
 $z_o/D = 0.075$

This profile is shown on Fig. 4.1. The constants A and C in Eq. (4.1) were chosen so as to match the equation as closely as possible to an available computer solution based on the method of characteristics. Experimental evidence suggested, however, that a better result was obtained by replacing D with an effective diameter D_{eff} , the magnitude of which deviates somewhat from D , depending on the shape of the orifice and the throat Reynolds number when viscous effects become important. Details are given in Ref. 17. For a square-edged orifice similar to the one used in the tests described here, D_{eff}/D does not deviate appreciably from unity for throat Reynolds numbers (based on D) over about 80.

In the absence of dissipative effects in the core itself, Eq. (4.1) is valid for

$$1 < z/D < z_M/D$$

where z_M denotes the axial location of the Mach disc, given for P_o/P_{test} chamber greater than 15 by the empirical relation (Ref. 17)

$$\frac{z_M}{D} = 0.67 \sqrt{\frac{P_o}{P_{tc}}} \quad (4.2)$$

4.1.2 Experiment

The free jet was used mainly for studies of Langmuir probe behaviour (Sec. 5), where particularly low densities were required to maintain all probes under free molecule conditions. Stagnation temperatures and pressures are listed for some of those tests in Table 5.2. Argon was used as the working gas in all cases. The lowest value of Re^* , the Reynolds number based on orifice diameter and sonic conditions at the throat, was about 190. Viscous effects resulting from boundary layer growth at the orifice should therefore have been very small (Ref. 17). On the other hand, the shock waves forming the sides and bottom of the barrel structure are expected to be quite diffuse under such conditions.

The flow was surveyed with pitot and Langmuir probes, the idea being to establish whether a usable region of near-isentropic flow remained

in the core. A first approximation for the Mach number was determined from P_i/P_0 on the assumption that the flow was isentropic (viscous corrections, based on the data of Ref. 17, were applied to the pitot probe readings whenever necessary). Here, P_i represents the measured impact pressure and P_0 the pressure in the stagnation chamber. If it was then found that the resulting centre line Mach number profile agreed reasonably well with theory (Fig. 4.1), the assumption of isentropic flow was considered justified. The data in Fig. 4.1 were taken in a run in which the full pumping capacity of the tunnel was deliberately not used in order to keep the Mach disc which terminates the barrel shock sufficiently close to the orifice to be in range of the impact probe. The agreement between experiment and theory is reasonably good up to the "kink" in the profile; this latter was assumed in Ref. 17 to define the position of the Mach disc.

The Langmuir probe itself proved to be a very useful instrument for investigating the flow field. After initial point by point surveys had established that the electron temperature was practically constant over the region of interest (3000°K is a representative value) and that recombination was negligible, the following method was adopted for making quick estimates of the spatial variations of charged particle number density in the jet as well as for locating the boundaries of the isentropic core. With the probe biased at a fixed potential sufficiently negative with respect to the plasma to be well in the ion current region of the probe characteristic, axial and radial traverses of ion current through the jet were recorded. As explained in Sec. 3.1, this could be done rapidly and conveniently on the x-y oscilloscope, the y input being the current and x input the signal from one of the positions indicating potentiometers coupled to the traversing gear. An exact interpretation of such surveys in terms of number density is discussed in Sec. 6.2. At this point, it suffices to say that if the electron temperature is uniform and variations in plasma potential are not too severe over the region surveyed, then the ion current profile taken with a cylindrical probe parallel to the flow is qualitatively similar to the profile of charged particle number density.

Figure 4.2 shows a centre line impact pressure survey and a corresponding ion current profile covering the region of the Mach disc. Again, only a fraction of the tunnel's total pumping capacity was employed so as to have the Mach disc sufficiently far upstream. Radial profiles of ion current and impact pressure in the same run are shown in Fig. 4.3. The ion current surveys were taken with a 0.265 cm long cylindrical Langmuir probe with its axis aligned with that of the flow. The impact pressure was also measured with the probe parallel to the flow axis, and was not corrected for viscous and yaw effects.

The first thing we note is that substantial species concentration gradients must exist in the flow field, for the shock structure indicated by the ion current is considerably more diffuse than that shown by the P_i profiles. In fact, the concentration gradients apparently penetrate into the core of the flow even where the gas as a whole (the degree of ionization was only of the order 10^{-4}) should still be to a good approximation isentropic. The neutral atom mean free path just upstream of the Mach disc was about $\frac{1}{4}$ cm in the particular run pictured in Fig. 4.2. Assuming that the structure of

the Mach disc is similar to that of a normal shock, its thickness should have been no more than about 1.5 cm, or $0.3D$ (Ref. 21). The charged particle number density distribution, on the other hand, extended over a considerably greater distance. The reason for this appears to be severe upstream diffusion from both the Mach disc as well as the shock forming the sides of the "barrel". This phenomenon is demonstrated under different circumstances in Sec. 6, and is discussed with reference to theory in Sec. 6.5. It is shown there that, irrespective of pressure level, marked separation of the species may be expected whenever the electron temperature is very large compared to that of the ions.

It is evident from this example that if the gas is ionized the Langmuir probe is an indispensable tool when it comes to locating the optimum position for testing in a free jet. In Ref. 17, a position just upstream of the Mach disc was recommended as the most desirable from the point of view of minimum gradients. In a situation like the present, on the other hand, special consideration must be given to the charged particle distribution. For example, a location at $z/D = 2.7$, say, would be considered acceptable on the basis of the P_i surveys of Figs. 4.2 and 4.3, but is in fact unacceptable for tests where charged particle properties are important, for the core has already been entirely "filled" by diffusion at that station.

Figure 4.4 shows radial profiles of ion current under conditions more representative of the experiments described in Sec. 5. Traverses with the probe both parallel and perpendicular to the flow axis are shown, and we observe how the orientation relative to the flow velocity becomes significant as soon as the probe moves into the region of high speed flow.

We note lastly that the plasma potential in the test section was usually only a few volts above ground, in contradistinction with the large potential differences reported in Ref. 22 for an RF system in an insulated vessel. Although the initial discharge did take place in an insulated chamber, the potential of the decaying plasma adjusted itself to a diffusion controlled level somewhat positive to the grounded metal parts (i.e. either the nozzle or the plate with the free jet orifice) with which it established contact prior to expanding into the test section.

4.2 The Mach 2.2 Nozzle

The nozzle was employed primarily for the experiments reported in Sec. 6, where a flow of higher density and greater uniformity was required than that available in the free jet. The nozzle itself was described in Sec. 3.1. As a result of its simple conical diverging section, it had to be operated in an underexpanded condition to avoid compression zones near the exit plane. For example, in balanced operation Langmuir probe surveys indicated that a diffuse but nevertheless prominent compression zone was present only three centimeters downstream of the exit plane. On the other hand, when the test chamber pressure was decreased to half the value of the exit pressure, this compression region moved far downstream and was entirely outside the region employed for testing. This is illustrated in Fig. 4.5(a), which shows the centre line ion current profile in a run where $P_e \approx 2P_{tc}$. It is evident that after emerging from the conical nozzle the gas expands for about 7 cm, but thereafter enters a compression zone, the beginning of which is indicated clearly by an increase in the ion current.

The center line Mach number (Fig. 4.5(b)) was calculated from P_i/P_0 on the assumption that the core of the flow upstream of the recompression zone was isentropic. This assumption received support from the fact that the ratio P_e/P_0 , where P_e is the static pressure measured at the nozzle exit plane, yielded a Mach number compatible with the value of the center line Mach number obtained from P_i/P_0 . The determination of the radial Mach number distribution (Fig. 4.6) was based on the ratio P_i/P , where P was taken as a constant across the flow at the value of the static pressure appropriate to the centre line Mach number and the stagnation pressure.

An interesting feature of the flow was that although the Mach number profile near the exit plane had a sizable flat core, the radial charged particle number density profile had already taken on a characteristic diffusion profile over the entire cross section (Fig. 4.6). This is not unexpected, for the electron-ion boundary layer is thicker than that of the gas as a whole whenever the electron temperature is substantially larger than that of the ions, as was the case here. This same phenomenon is evident in a different context in the experiments described in Sec. 6, as well as the theoretical analysis in Appendix C.

4.3 Identification of the Ion Species

The identity of the ion species is of direct importance in both the interpretation of Langmuir probe measurements and the calculation of diffusion fluxes. It is well known that either atomic or molecular ions, or both, may be present in a decaying noble gas plasma. On the other hand, the conditions under which a given species predominates are not sufficiently well understood to allow a quantitative prediction of the ion composition to be made in the present experiments. For this reason it was of importance to determine the relative abundance of the ion species by direct measurement.

Ion mass spectra were obtained in the nozzle flow with the small quadrupole mass filter described in Sec. 3.2.4. The instrument was mounted with its sampling orifice on the centre line 7.5 cm downstream of the nozzle exit plane, as shown in Fig. 3.14. At this station the flow had lost its "flat" core, but was still reasonably supersonic over the 1.5 inch diameter of the spectrometer nose; this was attested to by the observation of a distinct luminous shock layer ahead of the nose under operating conditions. The situation was therefore very similar to that studied in Sec. 6, with the charged particles arriving at the sampling orifice by ambipolar diffusion through the boundary layer at the stagnation point.

A series of runs covering typical operating conditions revealed only one species of ion in the flow: the atomic argon ion, Ar^+ . There was no evidence of the molecular ion, Ar_2^+ , or the doubly ionized atomic ion. If these were present their concentrations were less than a few percent of that of Ar^+ . The measurements covered the mass range $0 < M/Z < 90$, where M is the ion mass in atomic units and Z is the number of electronic charges it carries. A resolving power $M/\Delta M$ of about 18 was used, ΔM being the half-width of the mass peak. The stagnation pressure P_0 ranged from about 0.8 to 1.3 torr, T_0 from 450 to 750°K, and the downstream side of the region of RF excitation in the stagnation chamber was about 15 to 35 cm upstream of the nozzle throat.

4.4 Volume Recombination

The magnitude of the recombination coefficient was estimated in these tests by measuring the rate at which the ion concentration, $c = m_i n_i / \rho \approx n_i / n$, decreased along the stream centre line. The value of n_i was calculated from ion current data according to the method which will be outlined in Sec. 6.2.1 (a parallel probe was used so as to simulate static conditions) and the overall gas density $\rho \approx m_i n$ was determined from the Mach number and the stagnation conditions. A typical profile is shown on Fig. 4.7.

In the absence of both recombination and diffusion, the ion concentration would remain constant. In general, however, the distribution of c in a steady state flow field is governed by the conservation equation

$$\rho \underline{v} \cdot \nabla c = -\alpha n_i^2 - \nabla \cdot (n_i \underline{V})$$

where $\underline{v}$ is the barycentric velocity of the gas, $\underline{V}$ the diffusion velocity of the ions relative to the gas as a whole, and α the recombination coefficient. We have assumed here that the rate at which ions are lost from unit volume through recombination is given by αn_i^2 . When the barycentric velocity is in the axial direction and cylindrical symmetry obtains, the above equation reduces to

$$\rho u \frac{\partial c}{\partial z} = -\alpha \rho^2 c^2 - \frac{1}{r} \frac{\partial}{\partial r} (r n_i V_r) - \frac{\partial}{\partial z} (n_i V_z)$$

In the nozzle flow field to which Fig. 4.7 applies the diffusion terms represent a net loss of ions from the region near the axis. An upper limit for the value of α can therefore be obtained by neglecting the last two terms on the right hand side. The solution is then

$$\frac{c}{c_o} = \frac{1}{1 + \alpha n_{io} u z}$$

where the subscript (o) denotes conditions at $z = 0$. Fig. 4.7,

$$n_{io} = 2.0 \times 10^{12} \text{ cm}^{-3}$$

$$u \approx 7.2 \times 10^4 \text{ cm sec}^{-1}$$

and

$$c/c_o \approx 0.9 \text{ for } z = 3.0 \text{ cm}$$

This implies that

$$\alpha < 3 \times 10^{-9} \text{ cm}^3 \text{ sec}^{-1}$$

The actual value of α was probably much smaller than the upper limit given above, for an estimate of the magnitudes of the diffusion terms showed that the entire observed decrease in the ion concentration could have been accounted for by diffusion alone. Such a low recombination coefficient indicates that dissociative recombination (Refs. 23 and 24, for example) could not have been the predominant process. This is not unexpected, in view of the fact that no molecular argon ions were found. Our magnitude of α is more in line with that for collisional-radiative recombination and is consistent with

earlier measurements under similar conditions of electron temperature and charged particle number density (Ref. 25).

At any rate, it is clear that the flow was to a very good approximation frozen in these experiments as far as recombination was concerned. The electron number density was in fact higher, and the recombination therefore should have been more severe, in the run used for illustration in Fig. 4.7 than in most other experiments.

5. EXPERIMENTAL STUDY OF FREE MOLECULE LANGMUIR PROBES IN PLASMA FLOWS

5.1 Introductory Comments and Experimental Method

The purpose of the experiments described in this section was to make a careful study of the behaviour of collisionless Langmuir probes in flowing plasmas, in preparation for their use as diagnostic tools in supersonic flows.

All experiments were conducted in the free jet flow field described in Sec. 4.1, with as many as three Langmuir probes mounted simultaneously in the test region for the purposes of comparison. When three probes were used, for example, they were arranged as shown in Figs. 3.4 and 3.10(b), with one probe on the stream centre line and the other two about 1 cm above and below it, respectively. All three could be rotated in unison about a common vertical axis passing through their geometric centers. At the axial position chosen for most runs ($z/D = 2.6$) the flow was reasonably uniform over a region within 2.7 cm of the axis (c.f. Fig. 4.4) and all probes were therefore always exposed to flow conditions which were identical within a few percent. However, the gradients in the axial direction were not entirely negligible over a distance of one centimeter, which represents the length of the longest probe used; the electron number density decreased by almost 15% in this distance. Nevertheless, the average effect of this gradient should have been small because the geometric centres of all three probes were located at the same axial station.

Argon was used exclusively as the working gas, and the charged species in the test region were identified as electrons and singly charged atomic argon ions (see Sec. 4.3). Table 5.1 shows the results from a series of experiments on the collection of ions by probes of various diameters under a range of flow conditions. Readings were taken with the probe axes parallel and perpendicular to the flow velocity in this series of tests, one probe being used at a time while the others floated electrically. The parameters defining the flow conditions for the various runs are listed in Table 5.2, and Table 5.3 identifies the dimensions of the probes used. Section 2 outlines the manner in which all probe characteristics were interpreted. The results are discussed fully in the following separate subsections.

An important consideration in any experiment is the problem of charged particle depletion by the probe. In a steady state frozen flow of the sort used here, the charged particles arrive at the test region by a process of convection from the stagnation chamber where they are created. When a probe is placed in the flow, it acts as a sink for the ions and/or electrons and disturbs the flow, not only because of the resulting loss of charged particles, but also because, except in the special case when no current

is drawn, it tends to cause a charge imbalance which results in a current being set up in the plasma between the probe and the other parts of the system remote from it. Unless care is taken, these effects may be so strong as to make the probe an important part of the system as a whole, rather than a mere sensing element. One way of ensuring that a probe does not disturb the flow appreciably is to make it so small in comparison to the region being probed, that the rate at which it consumes ions and/or electrons is negligible compared to the rate at which the charged particles cross a significant area of that region. This may best be expressed by defining an effective "cross-section" for the probe, A_{eff} say, which represents the area of the free stream being effectively depleted of the species of charged particles collected by the probe. For example, if the probe draws a total current J_p , its cross section A_{eff} would be defined by

$$J_p = e n_e u A_{eff} \quad (5.1)$$

where n_e is the electron number density in the free stream and u is the free stream flow speed. Let us write

$$J_p = A_p n_e \sqrt{kT_e / 2\pi m_i} \cdot I_p \quad (5.2)$$

where A_p represents the probe area and I_p is a dimensionless current which is of order unity when ions are collected and of order $\sqrt{m_i/m_e}$ when electrons are collected. Also, since in the central core of the free jet expansion the flow is approximately isentropic down to the test region upstream of the Mach disc, we can express u in terms of the Mach number and the stagnation temperature. For $\gamma = 5/3$,

$$u = \frac{\sqrt{5RT_0}}{\sqrt{1 + 3/M^2}} \quad (5.3)$$

Combining the above expressions, we find that

$$A_{eff} = I_p \sqrt{1 + 3/M^2} \sqrt{\frac{1}{10\pi} \left(\frac{T_e}{T_0} \right)} A_p \quad (5.4)$$

It is worthwhile to note that the severity of the depletion caused by the probe is independent of the electron number density level. Typical running conditions for these tests may be represented by $M^2 \gg 1$ and $T_e/T_0 \approx 4$, say. When a probe draws ions, $I_p \approx 5$, whereas the electron current part of the probe characteristic involves values of I_p about 100 times larger. With these values, a comparison of the effective "cross-sections" for the smallest and largest probes used (e.g. probes 11 and 15, respectively, in Table 5.3) with the area of the 5.4 cm diameter uniform flow region yields the following results:

Table of $A_{\text{eff}}/A_{\text{flow}}$

	ion current	electron current
Smallest probe	2×10^{-4}	2×10^{-2}
Largest probe	0.8×10^{-2}	0.8

It is evident that for any probe except the smallest, the loss of particles from the flow is prohibitive when electrons are collected. The two percent depletion for the smallest probe size may still be considered acceptable, but the next largest probe (e.g. probe 16) would already cause a depletion of about 25%, while the largest probe would consume practically all electrons in the uniform part of the incident flow.

For this reason, attention was confined to the ion current region of the probe characteristics. The voltage sweep applied to the probes was limited so that the electron current never reached values much larger than the ion current, nevertheless including a sufficient portion of the electron retarding region of the probe characteristic to allow the electron temperature to be determined by the usual semi-log plot method.

We note lastly that although the original purpose of these tests was to study the behaviour of probes under free molecule conditions, it was found (see Sec. 5.7.2) that the criteria for such conditions were not satisfied in all respects: the ion-ion mean free path, as defined by Spitzer's self-collision time, was small compared to the probe diameter. The term "free molecule" is nevertheless applied since all other mean free paths were generally large compared to the probe diameter, including the ion-atom and electron-atom mean free paths which are usually assumed to determine whether particle collection is collision free or not. The peculiarities which may result from the small ion-ion mean free path are discussed in Sec. 5.8.

5.2 On the Absence of a Length Effect

As probe theory is a complex enough subject even for the simplest geometries, it is understandable that all available analyses begin with the assumption that a cylindrical probe is infinitely long. In practice one simulates infinite length by making the length to diameter ratio of the probe sufficiently large for end effects to be negligible. However, the question of how low an l/d ratio can be tolerated before end effects begin to exert a significant influence must be answered by experiment. This point is of considerable interest to the experimenter, who, being usually concerned with small systems, finds it desirable to keep the probes as small, and therefore as short, as possible, both to avoid excessive particle depletion and to have good spatial resolution.

Most probes in the experiments described in this paper had length to diameter ratios near 100. In order to ascertain whether they were subject to end effects, some tests were run in which three probes having equal 0.025 mm diameters but length to diameter ratios of 99, 218 and 430, respectively, were compared with each other. Altogether three runs were made,

with plasma conditions defined by $(R_p/\lambda_D)^2 I_i(\chi_f - 10)$ values* of 5.3, 7.0 and 16.8, respectively, based on measurements with the probe axes parallel to the flow velocity. These values lie near the lower end of the R_p/λ_D spectrum covered in the experiments reported here and should therefore represent the most likely conditions for end effects to occur.

However, in no case was a length effect detected, either with the probe oriented parallel to the flow velocity or at an angle to it. This is illustrated in Fig. 5.1, which shows the values of $j_i(\chi_f - 10)$ measured with the different probes, plotted against the length to diameter ratio, each current value normalized by the average value for that run and that probe orientation. The points in the figure represent the average values, and the error bars the absolute scatter in the measurements from all three runs. No systematic deviation from unity can be observed. The measurement of electron temperature was likewise independent of probe length.

We can conclude, therefore, that even for the smallest value of probe radius to Debye length encountered in these experiments, a probe with a l/d ratio of about 100 behaves as if it were infinitely long. The author undertook no systematic experimentation to actually determine the minimum acceptable length to diameter ratio, but reference may be made to some measurements by Clayden (Ref. 26), which indicated that there is no change in the ion current per unit length measured by a cylindrical probe as the l/d ratio was decreased to as low a value as 30. In his measurements, the lowest value of $(R_p/\lambda_D)^2 I_i(\chi_f - 10)$ investigated appears to have been about 150. This result is of importance here because in some tests probes with length to diameter ratios of 50 and 34 were used in addition to those with the usual value of 100, and Clayden's results confirm that these probes, too, should not have been influenced by end effects.

5.3 Experimental Curve of the Variation of Ion Current With the Ratio of Probe Radius to Debye Length

An important consequence of the foregoing result that the cylindrical probes respond as if they were infinitely long is that a probe with its axis aligned with the flow should in no way be sensitive to the flow speed, but should behave as if it were in a stationary plasma. This is the great advantage of using cylindrical probes for studying flowing plasmas, for the theories available for static probes are far more advanced and reliable than the analyses for moving probes. This section describes how parallel probes were used in a bootstrap technique to obtain an experimental curve of $I_i(\chi_f - 10)$ versus $(R_p/\lambda_D)^2 I_i(\chi_f - 10)$ for the case $T_i/T_e \ll 1$ **. As discussed in Sec. 2.4, such a curve forms the backbone of the method of inferring the electron number density from ion current measurements.

A series of runs was made in which two or three probes of different diameters were placed simultaneously in the free jet flow field. From the current-voltage characteristic of each probe, the electron temperature was determined by the semi-log plot method and the ion current density measured at $\chi_f - 10$. These two quantities were used in Eqs. (2.13) and (2.14) to evaluate the products $(R_p/\lambda_D)^2 I_i(\chi_f - 10)$ and $n_i I_i(\chi_f - 10)$ for each probe. In this way three values of $n_i I_i$ were found for three different values of

* See Sec. 2.4 for definition of terminology.

** $0.01 < T_i/T_e < 0.1$. See Table 5.2.

$(R_p/\lambda_D)^2 I_i$. Since the plasma conditions for three probes were the same, i.e. n_i was the same, it was possible to determine the magnitude of I_i for each of the probes relative to an arbitrary reference, which we shall denote by I_{ref} . The three ratios so obtained defined, as it were, part of an experimental curve of I_i/I_{ref} versus $(R_p/\lambda_D)^2 I_i$.

Next, the experiment was repeated at a different plasma density by adjusting the r.f. source in such a way that a new set of $(R_p/\lambda_D)^2 I_i$ values were obtained, shifted slightly from the previous values but still overlapping with them. As before, the relative magnitudes of the corresponding I_i values could be established. However, the overlap with the previous triplet of points allowed the magnitudes of the second triplet of I_i values to be fixed also with respect to I_{ref} , giving three additional points on the curve. Continuing in this manner, an experimental curve of I_i/I_{ref} versus $(R_p/\lambda_D)^2 I_i$ was obtained over the range $7 < (R_p/\lambda_D)^2 I_i < 4000$, which was as large a variation of the latter quantity as experimental conditions permitted.

Of course, a "bootstrap" technique like this cannot yield an absolute comparison with theory unless the arbitrarily chosen reference value can be evaluated by means of an independent measurement of electron number density. Although no method for doing this was available in these experiments, the magnitude of I_{ref} could still be fixed indirectly by referring to the measurements of Graf (Ref. 8), who showed that for $(R_p/\lambda_D)^2 I_i$ values in the range 230 to 3500,* a cylindrical Langmuir probe indicated an electron number density in close agreement with microwave measurements if Laframboise's theory for ion collection was used as the basis of interpretation. The conditions under which Graf carried out his comparison differed from those discussed here in two major respects: first, he studied a low speed subsonic plasma flow rather than a hypersonic flow; and secondly, T_i/T_e was in the neighbourhood of $1/3$ rather than close to zero as in these tests. However, if we presume that a long probe aligned with the flow velocity simulates static conditions, then the first difference is in effect removed. Furthermore, since Graf's results for $T_i/T_e \approx 1/3$ agreed with Laframboise's theory for that value, it is reasonable to assume that the presents results should agree with the theory for $T_i/T_e \rightarrow 0$ (the theory is in fact rather insensitive to the temperature ratio). It should be noted that in Graf's experiments also, all mean free paths were large compared to the probe diameter except the ion-ion mean free path. In this respect, his conditions were similar to the ones in the present tests. However, the actual value of the ion mean free path relative to the probe diameter was about $\frac{1}{4}$ in his experiments, whereas it was usually considerably smaller than this in the present ones. Since Graf's results nevertheless agree with Laframboise's theory, it was assumed as a working hypothesis that the same was true in the present experiments in the range of $(R_p/\lambda_D)^2 I_i$ covered by Graf, and therefore the value of I_{ref} was adjusted so as to bring the experimental curve into agreement with theory in that range.

Figure 5.2 shows the results of manipulating the experimental data in the manner described (the results are also listed in Table 5.1 (a)). This plot may be interpreted as an empirical curve for $I_i(\chi_f - 10)$ in terms of $(R_p/\lambda_D)^2 I_i(\chi_f - 10)$.

* $10 < R_p/\lambda_D < 50$

The empirical curve matches the theoretical one closely for the higher values of $(R_p/\lambda_D)^2 I_i$, but deviates from it sharply when R_p/λ_D decreases below the limit at which the ion current should become orbital motion limited. Instead of reaching a constant value as theory predicts, $I_i(\chi_f - 10)$ apparently continues to increase as R_p/λ_D is decreased below this point. As this behaviour is qualitatively similar to the theoretical result for a spherical probe in the case $T_i/T_e = 0$ (Ref. 4), one might suspect that it may be caused by the Debye length being so large that the field penetrates sufficiently far into the plasma to make the probe take on some of the character of a spherical probe rather than behaving as an infinitely long cylindrical one. However, this conjecture appears not to be justified, for the experiments which indicated that no length effect was present were conducted under precisely those conditions which represent the region where the experimental curve deviates from the theoretical one. We shall see in Sec. 5.8 that a likely cause for this deviation was the short mean free path for ion-ion collisions.

In concluding the discussion of the effects associated with probe diameter, it may also be remarked that the electron temperature determined by the usual semi-log plot method showed no systematic dependence on probe diameter. This is evident from the data in Table 5.1(b).

5.4 Variation of Ion Current With Probe Potential

To complete the comparison of the experimental data from parallel probes with Laframboise's static theory for ion collection, it remains to investigate the shape of the ion current versus probe potential curve for various values of R_p/λ_D . Experimental characteristics were compared with theory in the following manner. First, the quantity $(R_p/\lambda_D)^2 I_i(\chi_f - 10)$ was calculated from the data of a particular probe characteristic, and then $I_i(\chi_f - 10)$ was determined from Fig. 5.2. The empirical curve of that figure was used where the experimental data diverges from theory. This led to a value of R_p/λ_D . The experimental voltage scale was converted to dimensionless units by division with kT_e/e . Now, the dimensionless potential which appears in the theory, χ_p , refers to the potential difference between the probe and the undisturbed plasma. In the experiment, the floating potential was easily located (the point where the net current is zero), but the plasma potential was not. In order to evaluate χ_p , it was therefore necessary to calculate the difference between the plasma and floating potentials on the basis of the theory, as outlined in Sec. 2.4. Finally, the ion current was reduced to the theoretical scale according to

$$I_i(\chi_p) = \frac{j(\chi_p)}{j(\chi_f - 10)} I_i(\chi_f - 10) \quad (5.5)$$

This procedure matched the dimensionless current at $\chi_f - 10$ to the value indicated by the experimental data in Fig. 5.2.

The results from some randomly chosen characteristics are shown in Fig. 5.3. Only measurements at dimensionless potentials below $\chi_f - 5$ are shown, for this represents the limit of the region where electron current is negligible. We see that there is good agreement with theory where such agreement is expected. The fact that the curves for $R_p/\lambda_D < 2.8$ (i.e. $(R_p/\lambda_D)^2 I_i(\chi_f - 10) < 33$) fall above the corresponding theoretical curves is of course a direct consequence of our acceptance of the empirical data of Fig. 5.2 as the basis of matching, rather than the theoretical curve of Laframboise.

5.5 Effect of Flow on Ion Current and Electron Temperature When Probe is Turned at an Angle to the Flow Velocity

Initial tests indicated that when a probe was turned perpendicular to the flow velocity, the ion current decreased. Furthermore, the variation of the ion current with the orientation of the probe appeared to be a complicated one which depended on the flow speed, on the ratio of probe radius to Debye length and also on the potential difference between the probe and the plasma. This is illustrated in Fig. 5.4, which shows the ion current, measured at a fixed probe potential, versus angle of attack for two probes of different diameters under the same flow conditions, and Fig. 5.5, which shows similar plots for a single probe at two different potentials. In both figures the currents are normalized with respect to the values read with the probe aligned with the velocity. Subsequent experiments demonstrated that when a probe was turned perpendicular to the flow the ion current could either increase or decrease with respect to the value measured with the probe aligned with the flow, the behaviour being dictated by flow conditions.

Before we advance the discussion of the dependence of the ion current on the probe's orientation angle, it is important to point out that the electron temperature indication was found to be independent of that angle. Table 5.1(b) lists the values of T_e obtained with the various probes in parallel and perpendicular attitudes. On the average, the transverse probe orientation yielded an electron temperature about one percent higher than the parallel orientation. This is so small compared with typical scatter that it cannot be interpreted as an indication of a true difference between the results for these two orientations. The scatter is - at least for the most part - due to the basic inaccuracy of the graphical interpretation. Figure 5.6 provides a more direct demonstration of the total insensitivity of the electron temperature indication to the orientation of the probe. There, the plots of $\ln j_e$ versus probe voltage, from which the electron temperature was determined, are shown for $\theta = 0$ and 90° , where θ denotes the angle between the probe axis and the velocity vector. The data were taken from the oscillogram shown on Fig. 5.5. The reader is also referred to Sec. 7 for further illustration of this point (see Fig. 7.4).

In the series of runs which provided the data for Table 5.1, the ion current density j_i ($\chi_f - 10$) was measured with the probes perpendicular as well as parallel to the flow velocity. The ratio $j_\perp / j_\parallel$ was found to vary from about 0.5 to 1.6, depending on running conditions. Now, the fact that the Mach number had the same value, 5.8, in practically all runs rules this parameter out as a cause for the variation in the current ratio; nor could the dominant influence have come from the ratio of probe radius to Debye length, for this quantity was found to have approximately the same value in runs in which $j_\perp / j_\parallel$ was markedly different. The dimensionless quantity which was found to change consistently with $j_\perp / j_\parallel$, however, was $u / \sqrt{kT_e / m_i}$.

A reasonable argument for the use of this parameter to correlate the flow velocity effect* can be derived from the consideration of the limiting case of very large flow velocity. Since in the parallel orientation a cylindrical probe collects ions as if it were in a stationary plasma, we have, from Eq. (2.2),

* Similar arguments have been used previously, as for example by Clayden (Ref. 26).

$$J = en_i \sqrt{\frac{kT_e}{2\pi m_i}} I_i \cdot 2\pi R_p \ell$$

In the perpendicular attitude, on the other hand, the probe current will be given in the limit $u \rightarrow \infty$ by the rate at which the probe sweeps out ions,

$$J \rightarrow en_i u \cdot 2R_p \ell$$

Hence in this limit,

$$\frac{j_{\perp}}{j_{\parallel}} \rightarrow \sqrt{\frac{2}{\pi}} \cdot \frac{1}{I_i} \cdot \frac{u}{\sqrt{\frac{kT_e}{m_i}}} \quad (5.6)$$

It is immediately clear that - at least in this limit - the ion current ratio depends on the quantity $u/\sqrt{kT_e/m_i}$, rather than on the conventional speed ratio (or Mach number) and is, furthermore, influenced through I_i by the dimensionless probe potential χ_p as well as by R_p/λ_D and T_i/T_e .

In Fig. 5.7(a) the data from Tables 5.1(a) and 5.2 are plotted to show $j_{\perp}/j_{\parallel}$ vs $u/\sqrt{kT_e/m_i}$. As the assumption of isentropic flow appeared to describe the situation reasonably well in the central core of the free jet expansion, the dimensionless speed, which is given for the various runs in Table 5.2, was calculated from (c.f. Eq. (5.3))

$$\frac{u}{\sqrt{\frac{kT_e}{m_i}}} = \frac{1}{\sqrt{1 + \frac{3}{M^2}}} \sqrt{\frac{5 T_o}{T_e}} \quad (5.7)$$

For convenient identification the points are coded according to run, using the same symbols as in the table. Note that they represent the ratios of currents, measured from probe characteristics, at the particular dimensionless probe potential $\chi_p = \chi_f - 10$, i.e. $10kT_e/e$ volts below floating potential. Referring to Table 5.1(a) it is clear that, despite some scatter, $j_{\perp}/j_{\parallel}$ for a particular dimensionless speed ratio (i.e. particular run) is generally higher for larger R_p/λ_D . This is particularly evident for low values of R_p/λ_D in the region $u/\sqrt{kT_e/m_i} < 1$. From Fig. 5.7(a), we see that although there appears to be an initial decrease in $j_{\perp}/j_{\parallel}$ as the flow velocity increases from zero, this ratio seems to increase when $u/\sqrt{kT_e/m_i}$ increases further.

The variation of the ion collection with flow velocity and R_p/λ_D - or $(R_p/\lambda_D)^2 I_i$, which is uniquely related to it - can be more explicitly displayed by using the data from tests in which the ion current was measured, at some fixed potential of the probe with respect to ground, as a function of the orientation angle, θ . Since for an "infinite" probe the effect of flow velocity should depend only on the component of the velocity normal to the probe axis, i.e. on $u \sin \theta$, turning the probe from $\theta = 0$ to $\theta = 90^\circ$ should yield data simulating conditions from $u = 0$ to the actual flow speed used. Such plots of j_i vs $u \sin \theta$, normalized with respect to the value of j_i at $\theta = 0$, are shown on Fig. 5.7(b). The curves correlate reasonably well with the data from Fig. 5.7(a), although some allowance must be made here for the fact that these measurements were generally not taken at exactly $\chi_p = \chi_f - 10$, as in the case of the points on Fig. 5.7(a); the actual fixed probe potentials were found to correspond to dimensionless potentials between $\chi_f - 10$ and $\chi_f - 17$ after T_e had been determined.

The curves of $j/j_{\parallel}$ versus $u/\sqrt{kT_e/m_i}$ have the same general shape for all values of $(R_p/\lambda_D)^2 I_i$. As the dimensionless speed increases from zero, the current ratio first drops below unity, reaches a minimum, and then begins what appears a monotonic climb. The dip, which invariably takes place in the region $u/\sqrt{kT_e/m_i} < 1$, is particularly accentuated for small values of R_p/λ_D and becomes systematically shallower as R_p/λ_D increases.

Since practically all these experiments were done at the same high Mach number, there remains the question of whether the conventional Mach number also enters the correlation of the dependence of $j/j_{\parallel}$ on flow speed. Some evidence that this is not the case, at least when $T_i/T_e \ll 1$, is given in Sec. 7. Also, although no systematic investigation of the orientation dependence of j_i was carried out in the Mach 2.2 nozzle flow field, the readings of $j_{\perp}/j_{\parallel}$ which were taken agreed with the correlation of Fig. 5.7(a).

It is instructive to contrast the experimental ratio $j_{\perp}/j_{\parallel}$ to the corresponding ratio of neutral particle fluxes, which can be calculated theoretically. In the free molecule limit the velocity distribution function of the neutral atoms striking the probe is represented by a drifting Maxwellian (Ref. 27), and the flux of atoms per unit probe length, Γ , may be calculated in a straightforward manner (e.g. Ref. 10). The ratio $\Gamma_{\perp}/\Gamma_{\parallel}$, it turns out, depends only on the Mach number, and increases monotonically from unity M increases. The calculation is simple in the limit $u \rightarrow \infty$, where the particle flux to a perpendicularly oriented cylinder is given by

$$\Gamma_{\perp} = nu \cdot 2R_p$$

whereas when the cylinder is aligned with the flow velocity, Γ is determined by the thermal flux,

$$\Gamma_{\parallel} = \frac{nc}{4} \cdot 2\pi R_p = n \sqrt{\frac{kT}{2\pi m}} \cdot 2R_p$$

Thus, in this limit,

$$\begin{aligned} \frac{\Gamma_{\perp}}{\Gamma_{\parallel}} &= \sqrt{\frac{2\gamma}{\pi}} M \\ &= 1.05 M \text{ for } \gamma = 5/3 \end{aligned} \tag{5.8}$$

Although this expression is exact only in the limit $M \gg 1$, it does in fact give a good approximation to the exact value down to Mach numbers as low as 2 (where the difference is about 5%). In most of the experiments being discussed, $M = 5.8$, and the neutral atom flux to the collecting surface therefore increased by a factor of 6 when the probe was turned from a parallel to a perpendicular orientation.

5.6 Some Notes on the Electron Current

If conditions permit a probe to be operated at positive as well as negative potentials without excessive charge depletion, the electron-ion number density can be determined from either the ion or electron current. From Eqs. (2.2) and (2.3) we find that for a stationary or parallel probe the ratio of the electron current at plasma potential to the ion current at $10T_e/e$ volts below floating potential should be

$$\frac{j_e(0)}{j_i(\chi_f - 10)} = \frac{1}{I_i(\chi_f - 10)} \sqrt{\frac{m_i}{m_e}} \quad (5.9)$$

$$= \frac{271}{I_i(\chi_f - 10)} \quad \text{for Ar}^+ \text{ ions.}$$

The measurement of this ratio would therefore afford a convenient check on the magnitude of I_i .

Unfortunately, as has already been pointed out in the introduction of this section, even the smallest probes used collected, when operated at positive potentials with respect to the plasma, about 2% of the entire convected electron flux in the core of the free jet flow field, and with the next larger probe size this figure increased to 25%. For this reason electron currents were generally avoided in this study. However, since a fair amount of electron current data was nevertheless taken with the smallest probes (0.025 mm diameter, l/d ratio of 100), it might be of interest to review the results.

Most of the measurements were taken in the free jet at Mach numbers from about 4 to 6. Because only the smallest probes could be used, the Debye length was relatively large compared to the probe radius: $(R_p/\lambda_D)^2 I_i$ ranged from about 7 to 30, i.e. R_p/λ_D was in the range 1 to 2.5. The results may be discussed with reference to the typical example shown on Figs. 5.8 and 5.9.

The first point to note is that when the probe was turned from a parallel to a perpendicular orientation relative to the flow velocity, the electron current characteristic changed only in the region of the "knee" on the semilogarithmic plot, i.e. near plasma potential. Invariably, the deviation from the exponential variation occurred at a lower value of electron current when the probe was perpendicular to the flow. Secondly, the experimental electron current characteristics were only qualitatively similar to the corresponding theoretical ones. For example, in Fig. 5.8 we have tried to fit a theoretical characteristic to the experimental one by first bringing the two into congruence in the electron retarding region and then attempting to match them also at positive probe potentials. It is obvious from Fig. 5.9, which shows the result on an linear plot, that the curves cannot in fact be matched, for they do not acquire the same slope at higher positive potentials, at least not in the voltage range recorded. As a result of this basic disagreement it was not possible to interpret the experimental electron current characteristic on the basis of Laframboise's theory, at any rate not without serious reservations, and a check of I_i through Eq. (5.9) was thus precluded. It might be said, however, that the electron current was on the whole considerably smaller than expected. For example, if the value of $j_e(0)$ indicated by the "matching" theoretical curve in Figs. 5.8 and 5.9 had been accepted, the resulting ratio of $j_e(0)/j_i(\chi_f - 10)$, with the denominator taken from a parallel probe measurement, would only have been about one half that predicted by Eq. (5.9) with I_i taken from the empirical curve of Fig. 5.2. Similar results were obtained with the probe in a subsonic flow (in the stagnation region of the blunt-nosed model used in the tests described in Sec. 6).

It is difficult to draw any quantitative conclusions from this result, for the "matching" procedure used to "estimate" $j_e(0)$ was of course entirely arbitrary and unsatisfactory. We can therefore conclude only that the electron collection could not have taken place under the idealized circumstances to which the theory was tailored.

5.7 Non-Ideal Effects, Errors

5.7.1 Surface Effects

The theory upon which our interpretation of the probe characteristic rests assumes that the surface of the probe acts as an ideal sink for the charged particles which strike it. In point of fact, a real metal surface may reflect some charged particles and act as a source of others.

Let us consider the case of a probe biased sufficiently negative with respect to the plasma to repel all electrons, and confine our attention to a tungsten probe in an argon plasma. There are then two effects which may decrease the ion current indication below the ideal value: first, some of the incident ions may be reflected, and secondly, secondary ions may be ejected from the surface. Now according to Ref. 28 (p. 656), the fraction of incident Ar^+ ions reflected from a tungsten surface is smaller than 10^{-3} at low energies, while Ref. 29 indicates that secondary ion ejection is also entirely negligible, for the threshold energy of sputtering due to the impact of Ar^+ ions on W is 33 volts.

A more likely cause for error is the emission of electrons from the surface, which would lead to an overestimation of the ion current. Thermionic emission was negligible because of the low temperature of the probe. An argument can also be presented against significant photoemission. Since any source of radiation giving rise to the latter would have been concentrated in the stagnation chamber where the plasma was generated, it follows that when the probe was turned from a parallel to a transverse orientation, thereby increasing by several orders of magnitude the area of it exposed to the stagnation chamber, the effects of photoemission must have increased substantially. If photoemission had been important, this would have meant a proportional increase in ion current, rather than the decrease which was generally observed.

Careful consideration must be given, however, to the effects of ion and metastable atom bombardment. The ejection of electrons from metals by impinging ions is expressed in terms of an electron yield, γ_i , which represents the average number of electrons emitted per incident ion. The value 0.10 has been accepted (Ref. 30) for low energy argon ions striking atomically clean tungsten, but when foreign atoms are adsorbed on the surface, the electron yield is generally much lower. Hagstrum's (Ref. 30) work on this problem is of interest in this respect. He observed how γ_i changed with time as an originally atomically clean tungsten target was bombarded with argon ions, and found that in the presence of small amounts of residual gases and impurities γ_i decreased from the original value of 0.10 to below 0.05, the drop being particularly rapid when N_2 was present in the system. The reason for the decrease was apparently that the process of electron ejection by ions was influenced strongly by the presence of an adsorbed layer of foreign atoms on the metal surface, whether these were atoms of argon, nitrogen or other gases. In the tests we are concerned with here, the condition of the probe

surface had invariably reached a steady state before experimentation commenced. This was evident from the reproducibility of the probe characteristics over periods as long as an hour. Since the steady state was one in which the tungsten surface was covered by adsorbed gases (the 1300°K cleaning temperature was not sufficient to remove the last monolayer, and in any case even the 0.002% N₂ present as an impurity in the commercial argon used was sufficient to build up a monolayer of nitrogen in less than one minute), it is reasonable to assume that the electron yield was less than 0.05. In other words, this effect could have caused the ion current to be overestimated by no more than 5%. No correction for this possibility was made in the interpretation of the probe readings.

Atoms in metastable states cause secondary emission of electrons in much the same manner as ions. In fact, the mechanisms involved are so similar that the yield for electron emission is practically the same in both cases (Ref. 31). It follows that the probe current which results from the emission of electrons by metastable atom bombardment can be comparable in magnitude to the total ion current drawn by the probe only if the metastable atom concentration is very much larger than the charged particle concentration. If metastables contributed at all to the ion current measurements in these tests, their influence was probably very small, for when a cylindrical probe was turned from a parallel to a perpendicular orientation, the ion current indication generally decreased, whereas the metastable flux increased by a factor of 6, as was argued in Sec. 5.5. We can say with certainty that when the probe was aligned with the flow, the contribution of metastables to its ion current measurement could not have amounted to more than 1/6 of the current measured in the transverse orientation, and was almost certainly very much smaller than this amount since it is highly unlikely that the true ion current was zero in the transverse orientation.

In conclusion, then it is unlikely that secondary causes made a predominant contribution to the ion current measurements.

When electron currents are measured, there are two effects to consider: reflection and secondary ejection, both of which cause a decrease of the electron current relative to the ideal value. The magnitude of these effects is governed by the magnitude of the total secondary electron yield, δ , which is defined as the average number of external electrons produced per incident electron, the former including both reflected and emitted electrons. The author was unable to find suitable data on δ for very low energy primary electrons. However, for kT_e of the order of one eV, secondary emission should be entirely negligible at negative probe potentials. As for reflection, Langmuir and Mott-Smith (Ref. 32) have shown that for electron repelling probe potentials the shape of the electron current characteristic is unchanged when the reflection coefficient depends only on energy and the electron velocity distribution is Maxwellian. It follows that the electron temperature determination should be unaffected by reflection, but the entire curve would be shifted toward the plasma potential. The concomitant shift in floating potential would lead to an underestimation of $J_1(\chi_f - 10)$, but this should not be serious unless the reflection is very severe.

At sufficiently positive potentials, any reflected or ejected electrons will eventually be recaptured by the probe, but their presence will alter the potential distribution in the vicinity of the probe and thereby exert a secondary influence. It is likely, however, that the predominant

cause for the distortion of the electron current characteristic which was observed in the present tests at positive potentials was the excessive depletion of electrons from the stream rather than some effect associated with surface phenomena.

5.7.2 Mean Free Paths

Appendix B outlines the methods and supplies the data for evaluating the mean free paths for collisions between the various species of particles. In the series of runs referred to in this Section (see Table 5.2) the magnitudes of the mean free paths relative to the diameters of the probes used may be summarized as follows:

	min.	av.	max.
λ_{aa}/d	8	32	112
λ_{ee}/d	3	180	10 ³
λ_{ia}/d	3	11	45
λ_{ii}/d	0.03	0.08	0.5

The electron-ion mean free path is approximately equal to λ_{ee} , and λ_{ie} is much larger than λ_{ii} . The electron-argon atom mean free path was invariably very large in these experiments, for the electron energies were near those for the Ramsauer-Townsend minimum in the cross section curve. From Ref. 33, it is clear that the average cross section for elastic scattering of electrons by argon atoms was less than about 10^{-16} cm². This means that

$$\lambda_{ea} \approx \frac{1}{n Q_{ea}} > \frac{T(^{\circ}\text{K})}{P(\mu\text{Hg})} \text{ cm}$$

where P is expressed in micron Hg. The value of λ_{ea} was thus never less than about 20 cm in all these runs.

We conclude that the probes were generally free molecular with respect to all species except the ions. As regards the latter, the situation was rather peculiar in that collisions between ions and particles of all other species were negligible (thus, no diffusion effects), but the ion fluid itself was almost in continuum flow relative to the probes. We shall discuss some possible consequences of this in Sec. 5.8.

5.7.3 Other Causes of Error

Ignorance of the effective collecting area of the probe is an ever present possibility of error in number density measurements. Contamination of the metal with nonconducting material will decrease, and coating of the glass insulation with a conducting layer increase, the true collecting area compared to that of the exposed wire itself. Great care was taken in these experiments to avoid both these possibilities. The probes were always examined carefully under a microscope after use. No deposits on the glass sheath were detected. On the other hand, it often happened that part of the wire adjacent to the glass was found to retain a black colour despite vigorous cleaning at the beginning of the experiment (see Sec. 3.2.2). Com-

parison of readings taken in the plasma flow, under invariant test conditions, before and after such a deposit was removed by heating, indicated that such a blacker region was fully conductive.

Wehner and Medicus (Ref. 18) have demonstrated a number of more insidious sources of error, but many of these are negated by the discovery that the probe characteristic was time invariant after proper cleaning. Slow changes in the work function of the probe's surface were of no concern in these experiments where absolute values of potential were not measured. Nor was it likely that the distortion of the electron current characteristic was caused predominantly by a nonuniform work function over the collecting surface (Refs. 18 and 34), for similar results were obtained with a number of different probes. Oscillations in the plasma may also cause distortion of the probe characteristic (Ref. 35). Oscillations at the RF generating frequency of about 10 Mc/sec were in fact picked up on the ion current signals when an oscilloscope with sufficiently fast risetime was employed (the usual amplifier-oscilloscope combination was limited to frequencies below 500 Kc/s). Since their amplitude did not change with probe orientation, these oscillations could not have been a cause for the changes in both the ion and electron currents which we have attributed to the flow velocity.

Another cause for concern is the possibility of attracted particles being "trapped" in orbits which correspond to negative total energy (the potential energy is taken to be zero at infinity). The existence of such orbits has been explicitly recognized in the literature (e.g. Refs. 3, 4 and 5). However, since a free particle approaching the probe from infinity cannot, by virtue of its positive energy, enter a trapped orbit, it is appropriate in a collisionless theory to assume that such orbits are unpopulated. This has been done in all theoretical works published to date. On the other hand, collisions, no matter how infrequent, may knock particles into and out of trapped orbits. In a practical situation, the residence time of a particle in a trapped orbit would be governed by collisions and also influenced by the non-ideal shapes of laboratory probes. A cylindrical probe, for example, is always of finite length in practice, and trapped particles can therefore leak out of and enter through the ends of the configuration. It is difficult to comment quantitatively on this matter. However, it is likely that a significant population of ions in trapped orbits would result in a decrease of the ion current from the ideal case where the possibility of trapping is neglected, for the presence of the orbiting ions around the probe would serve to shield the probe's field more effectively.

5.8 Discussion of Results

The work in this section constitutes an experimental calibration of the ion current collected by a cylindrical Langmuir probe under conditions such that

$$T_i/T_e \ll 1 \quad (T_e < 5000^\circ\text{K})$$

$$1 < R_p/\lambda_D < 54^*$$

and $0 < u/\sqrt{kT_e/m_i} < 2$, approximately.

Most of the data were taken with probes having length to diameter ratios

* $7 < (R_p/\lambda_D)^2 I_i (\chi_F - 10) < 4000$

of about 100, although ℓ/d ratios of 50 and 34 were also employed to study the higher R_p/λ_D values. All mean free paths were large compared to the probe diameter except the ion-ion mean free path, which was generally small.

The basis for interpreting this data as a calibration is that a cylindrical probe is uninfluenced by flow when its axis is parallel to the flow velocity and it is sufficiently long. This property was invoked in two instances: first, to justify our matching of the I_i data to theory at higher values of R_p/λ_D (Sec. 5.3), and secondly, in the assumption that only the normal component of the flow velocity influences the ion collection (Sec. 5.5). Evidence of sufficient probe length rests on the finding that a four-fold increase in the probe's length had no effect on the results (Sec. 5.2).

The following were the major results:

(1) The value of electron temperature indicated by the electron retarding region of the probe characteristic was independent of R_p/λ_D , flow speed and the orientation of the probe relative to the flow velocity.

The same conclusion has been reached previously by French (Ref. 11) and Clayden (Ref. 36).^{*} In fact, the evidence which has by now accrued is sufficient to demonstrate that the electron temperature can be determined uniquely with a probe of any geometry, irrespective of its orientation, as long as the probe is small compared to the electron mean free path and the flow speed is small compared to the electron thermal speed. We refer to the present results and those of French and Clayden mentioned above, all of which illustrate the independence of orientation in supersonic flows, and to those of Clayden (Ref. 36) with spherical and cylindrical probes and of Graf (Ref. 8) with flat-faced and cylindrical probes, which demonstrate the independence of probe shape.

(2) For a probe in a stationary plasma, or a probe parallel to the velocity in a flow, the experimental variation of the dimensionless ion current I_i with R_p/λ_D (Fig. 5.2) agreed with Laframboise's theory for $R_p/\lambda_D > 2.8$, but deviated from it at lower values of R_p/λ_D .

The result that the dimensionless ion current rises above the orbital motion limited value ** at low R_p/λ_D is rather unexpected at first

* Brundin (Ref. 44) cites French's data as an indication that a transverse probe reads a higher electron temperature than a parallel one. This is not the conclusion French himself drew from his results; French argued (p. 32, Ref. 11) that the occasional large differences which he observed in the two temperature indications were spurious and entirely unconnected with flow effects.

** The "orbital motion limit" refers here to the region at low R_p/λ_D where the dimensionless ion current levels off to a constant value for a cylinder. The term is somewhat inappropriate in the context of the Bernstein-Rabinowitz formulation, but appears to have been accepted in the literature.

sight, for this is the current that an entirely "unshielded" probe would draw in a plasma of Maxwellian ions. It is possible that this apparently anomalous behaviour may be attributed to the fact that the ion-ion mean free path was small compared to the probe diameter. This contention is based on a comparison of the theory by Laframboise (Ref. 4) with that of Allen, Boyd and Reynolds (Ref. 37) who treated the collisionless case for zero energy ions. In the formulation by Laframboise the current is calculated by a detailed analysis of the orbits of the particles being attracted from infinity, where their velocity distribution is specified, while the earlier theory of A-B-R assumes a priori that the attracted particles move along straight trajectories directed radially inward toward the probe. Both analyses account fully for the space-charge field resulting from charge separation, and therefore predict a dependence on R_p/λ_D . Now, it appears reasonable to assume that the two formulations should give identical results in the limit where the ion energy is very small compared to the electron energy and the dimensionless probe potential is fairly large compared to unity. It is well known, however, that although this is indeed true for spherical probes, the two theories give different results for probes of cylindrical geometry. This disagreement was in fact anticipated by A-B-R (in a cryptic appendix), who therefore confined their attention to the sphere. The A-B-R curve of $(R_p/\lambda_D)^2 I_1$ on Fig. 5.2 was obtained from the data of Chen (Ref. 6), who recently published extensive computations based on their formulation for both cylinders and spheres.* This curve applies to zero energy ions and a dimensionless probe potential $\chi_p = -15$, which is sufficiently close to $\chi_p = \chi_f - 10$ for purposes of comparison. We see that the A-B-R and Laframboise curves are similar in shape at higher values of R_p/λ_D , but differ in magnitude. As R_p/λ_D decreases, however, Laframboise's curve reaches the orbital motion limit and levels off to a constant value, but the A-B-R curve continues to increase indefinitely, with a shape very similar to that indicated by the experimental data. The dramatic difference between the results of the two theories is apparently brought about by a rather subtle point (Refs. 5 and 6). The A-B-R theory contains the explicit assumption that the angular momentum of the particles is zero, whereas in the Bernstein-Rabinowitz formulation used by Laframboise, the angular momentum at $r = \infty$ becomes indeterminate. Thus the assumption that all values of angular momentum are possible at infinity remains implicit in the theory of Laframboise even in the limit of zero ion energy.

It is clear that the theory of Laframboise obtains when the mean free paths of the attracted particles are very large compared to the probe diameter. When the ion-ion mean free path is small, on the other hand, the A-B-R description is favoured. For example, in the limit when the ion fluid behaves as a continuum but does not interact with the other species, the equations governing its motion reduce formally to the collisionless equations for the motion of individual particles if the external field predominates over the conversion of thermal to kinetic energy, as would be the case when $T_i \ll T_e$ and $eV \gg kT_e$. In this case the velocity refers of course to the average velocity of the ions, and is directed radially inward. The formulation thus reduces precisely to that of A-B-R.

* Chen expresses his results in terms of $\xi_p = R_p/\lambda_D$ and a quantity J . For cylindrical probes, Chen's J is equivalent to $(I_1/2\sqrt{\pi})R_p/\lambda_D$ in the notation of this paper.

We recall that the absolute magnitude of the experimental I_i curve was determined (Sec. 5.3) by matching the curve to that of Laframboise at high R_p/λ_D . It might be thought, in view of the argument given above, that matching to the Allen, Boyd and Reynolds curve would have been more appropriate. There are, however, some indications which favour the choice of Laframboise's curve. For one thing, at high R_p/λ_D Graf obtained good agreement between microwave and Langmuir probe measurements, using Laframboise's theory, under conditions where the magnitude of λ_{ii}/d was about $\frac{1}{4}$, which is somewhat larger than the magnitude of this ratio in the present tests, but still small compared to unity. Some evidence that the experimental curve of Fig. 5.2 may in fact be applicable under conditions where λ_{ii}/d is somewhat larger than unity was obtained in the tests described in Sec. 6 of this paper, where it was found that the use of the experimental I_i curve yielded electron number densities in very good agreement with independent measurements of ion flux to the wall of a model. However, the situation there was again somewhat different in that the ion-atom mean free path was only slightly larger than the probe diameter rather than being very large compared to it, as was generally the case in the experiments described above.

In any case, the experimental curve of I_i shown in Fig. 5.2 was accepted as a working hypothesis throughout this work. It is obviously desirable to investigate this problem further under conditions where the various effects which are possible can be isolated and studied systematically.

(3) The effect of a flow velocity component transverse to the probe axis is demonstrated in Fig. 5.7. As the flow velocity increased, the ion current first decreased, reached a minimum somewhere at $u/\sqrt{kT_e/m_i} < 1$, and then began what appears to be a monotonic increase. The magnitude of the initial decrease depended on R_p/λ_D , being more severe for smaller values of this quantity.

Theories for ion collection by moving cylindrical probes have been published by Smetana (Ref. 38) and Kanai (Ref. 39). Both these works resort to some severe simplifying assumptions to make the problem tractable. Neither shows an initial decrease of current as the normal component of the velocity increases. It is interesting to note, however, that a similar analysis for spherical probes (Ref. 40) does show such a dip, qualitatively very similar to the results of the present experiments.

The author is not aware of earlier experimental work showing an initial decrease of ion current with flow velocity. Although measurements of $j_{\perp}/j_{\parallel}$ have been published (e.g. Refs. 11 and 41), they were taken under conditions where the current ratio was greater than unity, and orientation angles intermediate to 0° and 90° were not investigated. Both Refs. 11 and 41 reported experimental values of $j_{\perp}/j_{\parallel}$ that agreed reasonably well with free molecule calculations of particle flux neglecting the effect of the probe's field. In the light of the results of the present investigation, this agreement must be regarded as fortuitous. In both references mentioned, R_p/λ_D was large and the electron and ion temperatures were about equal. The present data indicates that under such conditions the conventional neutral particle flux calculation should give a fair approximation for $j_{\perp}/j_{\parallel}$ in a supersonic flow.

6. STUDY OF THE STAGNATION REGION AHEAD OF A FLAT-NOSED CYLINDER IN LOW REYNOLDS NUMBER, SUPERSONIC FLOW

6.1 Introduction and Experiment

This section describes a series of experiments in which free molecule Langmuir probes were used to study the shock layer and shock wave ahead of a flat-nosed, electrically floating, cylinder in a supersonic plasma flow. This particular model shape was chosen because it has been the object of much analysis, being the most common geometry employed for the "stagnation point Langmuir probes" which have found application in higher density flows where free molecule probes are impractical (Refs. 42 - 45, for example), and also because it provides a flow field with large variations in both electron number density and flow speed and therefore sets a severe test for the usefulness of the collision-free Langmuir probe as a diagnostic tool. The probe data were supplemented by an independent measurement of the ion flux to the wall at the stagnation point of the electrically floating model. The relationship between this last quantity and the free stream charged particle number density is of basic importance not only in laboratory experiments involving stagnation point Langmuir probes, but also in atmospheric measurements with rocket borne ion mass spectrometers sampling at the stagnation point.

Most of these experiments were carried out in the Mach 2.2 nozzle flow field described in Sec. 4.2, with the model and probe arranged as shown in Fig. 6.1. The models have been described fully in Sec. 3.2.3, in which the method of measuring the ion flux to the wall was also outlined. Only probes of the smallest size (0.025 mm diameter, 2.5 mm length) were employed in these tests, and they were always mounted with their axes perpendicular to the flow velocity. This orientation was dictated by resolution requirements. In the perpendicular attitude the probe was to a good approximation exposed to uniform conditions over its entire length, all measurements being taken along the stagnation streamline, and its diameter was much smaller than any distance over which substantial gradients in the flow direction were present.

The free stream can be described briefly as follows:

Mach number ≈ 2.2

degree of ionization $\sim 10^{-4}$

volume recombination negligible (see Sec. 4.4)

$T_e/T_i > 10$

$\lambda_D \ll$ boundary layer thickness

Typical values for the stagnation pressure and temperature, respectively, were 1 torr and 500°K. Thus, the free stream was a weakly ionized gas, in a highly nonequilibrium state, but nevertheless a true plasma in the sense that the electron and ion number densities were equal over the entire flow field except for the very thin sheath near the wall. Not also that, although the probes were so small as to be reasonably collision-free, the flow over the model was in the continuum regime, with a Reynolds number based on body diameter and stagnation conditions behind the bow shock in the range

$$70 < Re_2 < 125$$

The flow relative to the probes is defined by the conditions in the shock layer, where $T_0 = 500^\circ\text{K}$, $T_e = 3000^\circ\text{K}$, $P = 0.7$ torr, and $(n_i)_{\text{max}} = 10^{18} \text{ m}^{-3}$ are representative figures. From Appendix B, we find that under these conditions $\lambda_{aa} = 0.15 \text{ mm}$, $\lambda_{ia} = 0.04 \text{ mm}$, $\lambda_{ii} = 0.09 \text{ mm}$, and $\lambda_{ee} = 2.2 \text{ mm}$. The value of λ_{ea} is very large, as is shown in Sec. 5.7.2. Comparing these to the probe diameter of $d = 0.025 \text{ mm}$, we see that the probes were near the free molecule flow limit, the critical mean free path being that for ion-atom collisions, which is only slightly larger than the probe diameter. Passing through the bow shock into the free stream, λ_{ia} increases, whereas λ_{ii} decreases (to about 0.05 mm).

The procedure in a typical run may be outlined as follows: first, sufficient time was allowed after operating conditions had been set for the steady state to be reached in all respects. Almost one hour was necessary to ensure that the wall temperature at the nose had come to its final value. Then, Langmuir probe characteristics were taken at a series of locations along the stagnation streamline, covering the region from the model's nose to the free stream. From these the floating potential of the probe was determined, and the probe was then biased at a fixed potential with respect to ground such that it would operate in the region of the characteristic where the ion current had become "saturated" and the electron current was negligible. The idea was to have the probe at approximately $10kT_e/e$ volts below floating potential, so that the data of Figs. 5.2 and 5.7(b) could subsequently be utilized to interpret the readings. It was usually possible to set the bias fairly close to this value simply by making use of the observation that the difference between the floating potential and the potential at which the total current levelled off to the ion current value is about $5kT_e/e$. A survey of ion current versus probe position was then recorded, starting from the body wall and moving out through the shock layer and the bow shock to the free stream. Finally, the ion flux to the wall was measured as a current to the spherical collector in the interior of the model, and after this the model was swung out of the stream to make way for impact pressure surveys. It is worthwhile to stress again that the flow conditions were found to be very steady throughout the runs. A probe characteristic was always recorded just prior to swinging the model out of the flow in order to check for changes against the data taken at the beginning of the run and excellent agreement was obtained. Flow properties were also checked before and after the model was swung out of the stream. Both stagnation chamber and nozzle exit pressures remained invariant, indicating that the presence of the model did not disturb the ordinary gasdynamic properties of the free stream flow. The electron number density also remained constant, but the plasma potential increased noticeably (by several times kT_e/e) when the model was removed from the flow. This should not necessarily be interpreted as a penetration of the potential downstream of the atom shock wave to positions upstream of it, as described theoretically by Jaffrin (Ref. 46), for the areas of the model and its support being bathed by the plasma were not negligible compared to the surface area of the nozzle, which was the only (grounded) metal part with which the plasma established contact before emerging into the test chamber. Lastly, the stagnation chamber pressure was recorded, the R.F. power turned off, and, without altering the mass flow P_0 , was again read, this time with the gas in the stagnation chamber at room temperature. A comparison of the "hot" and "cold" stagnation pressures yielded a value for stagnation temperature, as discussed in Sec. 3.2.5.

The temperature of the wall at the stagnation point, T_w , was determined by repeating the experiment with the sister model which was instrumented with a thermocouple. Flow conditions could be reproduced very well in every respect in successive runs. In all tests in which this model was used, T_w/T_0 was found to be $0.85 \pm 2\%$ once steady state conditions had been reached (the two percent here refers to scatter, not error).

6.2 Interpretation of Ion Current Surveys in Terms of Ion Number Density

The electron temperature and number density distributions over a particular region may be obtained from current-voltage characteristics, provided that measurements at a sufficiently fine network of points are available. In these experiments, however, a simpler and more graphic method was used, largely because, as will be seen in the following section, the electron temperature turned out to be uniform over the entire region surveyed. The shape of the ion number density distribution was inferred from plots of ion current versus position, taken at a fixed probe potential. Complete probe characteristics were taken only at a few representative points to determine the electron temperature and check the floating potential and the absolute magnitude of the number density.

The profiles of ion current along the stagnation streamline were qualitatively similar to the corresponding ion number density profiles. Moving in toward the nose of the model from the upstream direction, the ion current was constant in the free stream, then increased as the probe passed through the bow shock into the shock layer, levelled off again briefly, and finally dropped toward zero as the probe entered the boundary layer and approached the wall of the model. As explained previously, care was taken in these surveys to set the probe potential as close as possible to $10kT_e/e$ volts below the average floating potential, so that the data of Sec. 5 could be used for interpretation. Any influence arising from the variation of the plasma potential over the region traversed was found to be small, for the profiles of $j/j_{\max}$, where $j_{\max}$ represents the maximum value of the ion current in the shock layer, measured with the probe at constant potential with respect to ground were closely matched by similar profiles measured at exactly $10kT_e/e$ volts below the local floating potential.

The ion current drawn by a probe oriented transverse to the flow direction may be expressed in the form

$$j_{\perp} = e \sqrt{\frac{kT_e}{2\pi m_i}} n_i I_i \cdot \left(\frac{j_{\perp}}{j_{\parallel}} \right) \quad (6.1)$$

where I_i retains the significance it had in the static case, and is a function of χ_p , R_p/λ_D and T_i/T_e . For $\chi_p = \chi_f - 10$ and $T_i/T_e \ll 1$, it is given in Fig. 5.2. The quantity $j_{\perp}/j_{\parallel}$ is available, in terms of $u/\sqrt{kT_e/m_i}$ and $(R_p/\lambda_D)^2 I_i(\chi_f - 10)$ - or R_p/λ_D if Fig. 5.2 is consulted - from the experimental curves of Fig. 5.7(b). The latter are also applicable to the case $T_i/T_e \ll 1$ and $\chi_p \approx \chi_f - 10$. Thus, we have sufficient information to calculate the relationship between the ion number density and the ion current measured at a probe potential corresponding to $\chi_p = \chi_f - 10$, provided that $T_i/T_e \ll 1$ and the value of $u/\sqrt{kT_e/m_i}$ is known.

In these experiments the greatest uncertainty in applying this method to account for the flow velocity was caused by the difficulty of esti-

mating u . Although the flow speed was accurately known in the free stream, it could not be calculated with any confidence for positions in the low Reynolds number shock layer. In the boundary layer region, the problem was further complicated by the fact that the flow speed of the ions, which is presumably the speed on which the probe response depends, obviously differed, because of diffusion, from the very low speed of the neutral gas.

Because of these difficulties flow effects were ignored entirely in the subsonic region behind the bow shock, and the ion current profiles converted to number density profiles by the steps outlined in Sec. 2.4 for static probes, except that the empirical curve of Fig. 5.2 was used for I_i instead of the theoretical one. Actually, the profiles of n_i obtained in this manner were in the end accepted as regards shape only, for to avoid as far as possible any errors which might have been incurred as a result of the fact that the fixed probe potential in the surveys was in general not exactly 10 dimensionless units below floating potential, the absolute value of the ion number density profile was afterwards determined by matching its height to the n_i values obtained from the complete current-voltage characteristics which were always taken to supplement the ion current surveys. For those, too, the empirical curve of Fig. 5.2 was used whenever necessary. The maximum value of n_i in the shock layer, obtained in this way, is denoted by $(n_i)_{\max}$. The influence of the flow speed on the readings in the shock layer should in general not have been serious. For example, in Run 18 - 1, which was typical, the data of Fig. 5.7(b) implies the perpendicular probe should have underestimated the static ion current by 8% at a speed u_2 corresponding to the speed on the downstream side of a normal shock with the experimental upstream conditions. In the shock layer, the component of the gas velocity directed toward the wall decreases as the wall is approached, but the charged particle diffusion speed increases. An estimate shows that for the case at hand the diffusion speed at the point near the wall where the ion number density was one tenth of $(n_i)_{\max}$ was such as to cause the ion current to be underestimated by about 9%, according to the data of Fig. 5.7(b). In other words, although the flow speed of the gas decreased toward the wall, the ion diffusion speed increased, and the net effect was such that the total ion speed tended to remain constant. Thus, flow effects should not have exerted a serious influence on the shape of the n_i profile in the subsonic region, although they may have caused the absolute magnitude of n_i to be slightly underestimated.

We note that the conditions in these tests were not in all respects similar to those under which the data of Fig. 5.2 were taken. For one thing, in the shock layer, where the gas was at approximately the stagnation temperature, the ratio T_i/T_e had the relatively high value of about 1/6. However, judging by the apparent insensitivity of the theoretical I_i curve of Laframboise (Fig. 2.4), it is reasonable to assume that this temperature ratio was sufficiently small to justify the use of data for $T_i/T_e \ll 1$ without appreciable error. A more serious cause for concern was the fact that the ion-ion mean free path was not small compared to the probe diameter, as in the experiments described in Sec. 5 from which the probe calibration was taken; rather, it was usually somewhat larger than the probe diameter. There was also a danger of diffusion effects influencing the probe response, for the ion-atom mean free path was only moderately larger than the probe diameter. Despite these differences, however, the experimental data of Fig. 5.2 was used as a working hypothesis throughout this section to interpret the probe readings.

Flow effects were accounted for in the measurements through the bow shock and in the free stream. There, it was possible to obtain a reasonable approximation for the speed by assuming that the flow in the shock was one dimensional. The ion number density and (ion) flow speed were related by

$$n u = n_{\text{ref}} u_{\text{ref}} \quad (6.2)$$

where n_{ref} was equated to the maximum ion number density in the shock layer, $(n_i)_{\text{max}}$, obtained as described before, and u_{ref} was assumed, as a first approximation, to be equal to u_2 , the speed on the downstream side of a normal shock with the experimental upstream conditions. A calibration of $n_i/(n_i)_{\text{max}}$ versus $j_{\perp}/j_{\text{max}}$ was then calculated according to Eq. (6.1), with I_i taken as before from Fig. 5.2 and $j_{\perp}/j_{\parallel}$ from Fig. 5.7(b). This calibration was then used to convert the experimental ion current profile to a number density distribution. As we shall see in Sec. 6.3, the normal shock density jump was not quite realized across the bow shock in the experiments, and the use of u_2 for the reference speed might therefore not have been quite appropriate. However, the assumption of one dimensionality was under those conditions not free of suspicion, and further refinements based on this assumption did not seem to be warranted.

Note that precisely the same conversion procedure would have been necessary if a series of probe characteristics had been taken rather than a single plot of ion current against position. The only advantage in the former case would have been the more accurate knowledge of the dimensionless potential at which the ion current was measured.

6.3 General Results

Figure 6.2 shows a photograph of the model in position in the flow*. Under operating conditions, the shock layer was clearly visible as a more intensely luminous region ahead of the nose, with a fairly distinct outer boundary delineating the location of the bow shock.

The results from a series of runs where stagnation streamline surveys were taken are summarized in Table 6.1. We note first that in all these runs the electron temperature was found to be constant throughout the entire region surveyed, from the face of the model out to positions in the free stream (Fig. 6.3). This result is not unexpected. For one thing, a reasonable boundary condition for the electron temperature at the body wall in this case is $dT_e/dy = 0$, (see for instance Ref. 60), since at the floating wall, which in order to reduce the electron flux to equality with the ion flux assumes a bias heavily negative with respect to the plasma adjacent to it, all but a few percent of the incident electrons are reflected elastically in the sheath** without making contact with the metal. In other words, one

* The stagnation and static conditions for all runs referred to here are listed in Table 6.1.

** We are dealing with a situation in which the electron mean free path is much larger than the Debye length, i.e. an electron does not suffer collisions in its passage through the sheath.

mean free path away from the wall only the very highest energy tail of the electron velocity distribution as a whole is distorted as a result of the proximity of the surface, and thermal accommodation should therefore be practically nonexistent even in this otherwise continuum situation. Secondly, thermal contact-poor between electrons and other species - is extremely good within the electron gas itself. For example, an order of magnitude analysis of the electron energy equation used by Grewal and Talbot (Ref. 47) in their study of the electron temperature distribution in a normal shock wave shows that the length scale for appreciable percentage change of T_e in an electron gas is given by

$$L \approx \frac{\lambda_e}{n_e k u} \quad (6.3)$$

where λ_e is the electron thermal conductivity, k the Boltzmann constant, and u the flow speed. From Eq. (A.7) of the same reference, in c.g.s. units,

$$L \approx \frac{2 \times 10^{11} T_e^{5/2}}{n_e u \ln \Lambda} \quad (\text{cm}) \quad (6.4)$$

where Λ has the same meaning as in Appendix B of this paper. In order to obtain a conservative estimate for L , we represent the present experiments by $T_e = 3000^\circ\text{K}$, $n_e = 10^{12} \text{ cm}^{-3}$, and $u = 10^5 \text{ cm/sec}$, and find that $L > 10^2 \text{ cm}$. This is to be compared with the shock standoff distance, which was less than one centimeter.

Figure 6.4 displays surveys of ion current along the stagnation streamline for three runs at different Reynolds numbers, while the "boundary layer" regions - if this term can still be applied - of the same runs are shown on a more extended scale in Fig. 6.5 (the "noise" on the curves is caused mainly by sticking of the traversing gear carriage; the middle oscillogram in Fig. 6.5 shows a particularly bad example of this). We recall that when the electron temperature is uniform, a survey of ion current is at least qualitatively similar to a profile of electron number density, and can in principle be converted to the latter. In these figures, the wall of the model is at the right, where the ion current approaches zero, and the free stream at the left. We see clearly the region of constant ion current (or number density) in the free stream, the rise through the bow shock, followed by a fairly uniform region which in higher Reynolds number flows becomes the "inviscid flow" region, and then a sharp drop toward zero near the wall as the electrons and ions diffuse through the neutral gas to the surface, where they are lost by surface recombination.

The three runs selected for illustration show the thickening of the boundary layer and the bow shock as Re_2 decreases from 120 to 107, and then the merging of these two regions at the lowest Reynolds number of 84. It should be stressed, however, that the shock and boundary layer we are here concerned with are those defined by the number density distribution of the charged particles, and not those of the neutral atoms or the gas as a whole. We shall see in subsequent sections that these two may differ, and that the degree of difference between them is governed largely by the magnitude of the Schmidt number Sc , based on the ambipolar diffusion coefficient. When this effective Schmidt number is considerably less than unity, as is generally the case when $T_e \gg T_i$, the electron-ion and neutral atom distributions may differ substantially.

Figures 6.6 - 6.10 show some electron-ion number density profiles obtained from the ion current surveys by the method outlined in Sec. 6.2. All profiles are normalized with respect to the maximum electron-ion number density in the shock layer, $(n_i)_{\max}$. In the "boundary layer" region, where flow effects on the probe data were neglected, the difference between the number density and ion current profiles is brought about entirely by the variation of the dimensionless ion current I_i with R_p/λ_D . In the bow shock, on the other hand, this correction competes with that for flow speed, which tends to override it, particularly at the lowest number densities encountered (e.g. Run 16-3), and to bring the number density curve above the ion current curve.

We observe first that the jump in electron-ion number density across the bow shock is not as great as one would expect on the basis of the density ratio based on the usual normal shock analysis. Rather than the value 2.5 which is appropriate for a Mach number of 2.2, it is about 1.8 for the two higher Reynolds number cases, where it is still possible to separate the bow shock and the boundary layer, and decreases to about 1.5 for the lowest Reynolds number case shown, where the boundary layer and shock have merged entirely. The question arises whether these low electron-ion density jumps were real, or may be attributed, wholly or partly, to the falsification of the Langmuir probe response by some non-ideal influence. As regards the last-named possibility, one may make the hypothesis that ion-atom collisions were beginning to affect the probe's response. As was noted in Sec. 6.1, the ion-atom mean free path was usually only moderately larger than the probe diameter behind the shock wave, and diffusion effects could therefore have caused a reduction of the ion current measured by the probe. Such an effect would have been more serious on the downstream side of the shock than on its upstream side, where the mean free path was larger. However, this hypothesis appears to be contradicted by the finding that the collisionless interpretation of the ion current was consistent with the independent measurements of ion flux to the wall of the model (see Sec. 6.4.2).

A true reduction of the electron-ion number density downstream of the shock is not unreasonable. First, it is evident from the fact that the charged particle number density must approach zero at the wall that it will be prevented from reaching its full value downstream of the bow shock whenever the region of influence of the wall - i.e. the boundary layer, loosely speaking - extends sufficiently far upstream to interact with the shock. Such an interaction will occur when the boundary layer and shock thicknesses are comparable to the shock detachment distance (as was the case in these tests). Furthermore, since the shock detachment was about equal to the nose radius of the model, it follows that under these conditions the centre line electron-ion number density distribution will also be influenced by diffusion losses in the radial direction. Radial diffusion losses may in fact have been abnormally high in these experiments because the radial ion concentration distribution was already non-uniform in the free stream (Fig. 4.6). Note that the short, reasonably flat region of n_i between the bow shock and the boundary layer in the higher Reynolds number runs does not necessarily imply that the shock and boundary layer did not merge. In the absence of dissipative phenomena, n_i should actually increase in the shock layer as the flow velocity decreases toward zero and the gas density increases from its value behind the shock to the corresponding stagnation value (this increase is about 15% for the present free stream Mach number).

It is important to stress again that because of the low values of the ambipolar Schmidt number, Sc , the scale of the electron-ion diffusion phenomena was substantially larger in these tests than the scale of the dissipative regions in the neutral atom flow field. Under such conditions it is to be expected that deviations from high Reynolds number theory will occur earlier - i.e. at higher Reynolds numbers - for phenomena associated with charged particle diffusion than for phenomena which depend only on the neutral atom flow field (such as the heat transfer, when the gas is weakly ionized).

Returning lastly to the "boundary layer" region of the shock layer, we observe on Figs. 6.9 and 6.10 that the charged particle number density variation becomes linear as the wall is approached. This is of course to be expected, for very near the flat wall, where $y/D \ll 1$, the diffusion problem becomes practically one dimensional. We stress again that the conversion of j_i to n_i was based on the experimental curve shown on Fig. 5.2 whenever the operating point was in the region where that curve diverged from the theoretical one. For comparison, Figs. 6.9 and 6.10 also show the n_i profiles which would have resulted had the theoretical plot of I_i vs $(R_p/\lambda_D)^2 I_i$ been employed instead of the empirical one. In Fig. 6.10, for example, the two curves diverge dramatically near the wall. Note that the curve based on the experimental calibration of I_i is more satisfying physically, for it does take on the expected linear shape near the wall, whereas the other one does not. We shall also see that the ion flux to the wall calculated on the basis of the empirical probe calibration agreed well with the value measured with the internal ion collector.

6.4 The Ion Current to the Wall at the Stagnation Point

The flux of ions at the wall of a body in continuum flow is of basic importance in many experiments, as for example those involving measurements with a stagnation point Langmuir probe or a rocket borne ion mass spectrometer sampling at the stagnation point. In the tests being described here, this quantity was measured directly at the stagnation point and compared with the ambipolar diffusion flux appropriate to the charged particle number density gradient there established by the traverses with free molecule Langmuir probes. A theoretical description of the problem is not at present available for the flow regime studied, but the measurements are compared with the high Reynolds number boundary layer theory which forms the framework of the majority of the stagnation point theories published to date.

6.4.1 Direct Measurement

The method of measuring the ion flux to the wall of the electrically floating body has already been outlined in Sec. 3.2.3. To recapitulate, both electrons and ions were allowed to pass through an 0.140 mm diameter orifice in the thin wall at the stagnation point into the evacuated interior of the model, where the two species of charged particles were separated by biasing the collector sufficiently negative to repel all electrons, and the ion flux was measured as a current to the collector.

Figure 6.11 shows a typical plot of collector current versus bias voltage, the latter measured with respect to the floating body. At sufficiently negative potentials, the current levelled off, as expected, to what we identify as the ion current, but still showed a slight increase with further bias. This latter was interpreted to indicate a penetration of the

interior electric field into the region exterior to the model, an effect which was reasonable since λ_D was not large compared to the orifice radius. It was compensated for by accepting for $(j)_w = e(n_i v_i)_w$ that current which resulted from a linear extrapolation of the collector current from highly negative potentials to the zero current potential.

There remains the question of what effect the pressure in the interior of the model had on the current measurement. Although at a point just upstream of the baffle valve (see Fig. 3.13) the pressure was about 10^{-4} torr under representative operating conditions, a free molecule calculation taking into account the conductance of the path to the pump (Ref. 48) indicated that the pressure near the nose of the model should have been of the order of 10^{-3} torr. Experiments showed that a higher pressure in the interior had the effect of shifting the levelling off of the current to higher values of negative bias. For example, in a test where the pressure in the interior was allowed to reach 0.06 torr, the ion current at -15 volts bias was reduced from the low pressure value by about 10% and approached the latter only at considerably higher bias values. However, since this represented a rather extreme example, it was concluded that any errors from this cause could not have been serious.

Table 6.2 lists the wall current densities measured in this manner in four runs (the retention of only two figures is an acknowledgement of the small degree of ambiguity introduced by the necessity for the extrapolation). Also shown for comparison are the ambipolar diffusion fluxes for the respective runs, calculated from the ion number density profiles at the wall. These will be discussed in Sec. 6.4.2.

6.4.2 Diffusion Flux Calculated From Measured Ion Number Density Gradients At Wall

When a body is floating electrically and the Debye length is very small, the rate at which ions and electrons arrive at its wall is governed entirely by the ambipolar diffusion flux. At the wall, where the charged particle number density approaches zero, this flux is given by the expression

$$n_i V_i = -D_i \left(1 + \frac{T_e}{T_i} \right) \frac{\partial n_i}{\partial y} \quad (6.5)$$

where D_i is the binary diffusion coefficient of the ions in the neutral atoms. In general there are additional terms present on the right hand side of Eq. (6.5) because the driving force for diffusion is actually governed by concentration and pressure gradients, but these terms disappear when n_i approaches zero. It follows that if the diffusion coefficient is known, a measurement of the ion number density gradient near the wall will suffice to specify the flux to the wall. This allows the Langmuir probe measurements in the shock layer to be checked against the independent measurement of ion flux to the wall at the stagnation point.

Writing

$$\begin{aligned} n_i^* &= \frac{n_i}{(n_i)_{\max}} \\ y^* &= \frac{y}{D} \end{aligned} \quad (6.6)$$

where $(n_i)_{\max}$ is the maximum ion number density in the shock layer and D is the body diameter, and expressing the flux as an equivalent current, we obtain

$$j_w = e(n_i V_i)_w = \frac{e(n_i)_{\max} D_i(P_w T_w)}{D} \left(1 + \frac{T_e}{T_w}\right) \left(\frac{dn_i^*}{dy^*}\right)_w \quad (6.7)$$

If we specialize to the case at hand by substituting for D_i from Eq. (A.7), this becomes

$$j_w = 1.84 \times 10^{-25} \frac{(n_i)_{\max}}{D \cdot P_w (\text{torr})} \left(\frac{T_w^2}{11.5 + \sqrt{T_w}}\right) \left(1 + \frac{T_e}{T_w}\right) \left(\frac{dn_i^*}{dy^*}\right)_w \quad (6.8)$$

Ampere /m²

Here, all quantities are in MKS units except P_w , which is expressed in torr. In the experiments, P_w was equated to the measured impact pressure*, T_w was measured directly, T_e and $(n_i)_{\max}$ were obtained from probe characteristics, and $(dn_i^*/dy^*)_w$ from the number density profile inferred from the ion current survey.

Table 6.2 lists $(dn_i^*/dy^*)_w$ and the resulting calculations of j_w for four runs in which the number density profile was measured very carefully in the vicinity of the wall. We see that the calculated wall flux agrees remarkably well with the directly measured value. Since the expression for the diffusion coefficient for atomic argon ions in their parent gas appears to be well confirmed by experiment near the temperatures of interest here (Ref. 49), the agreement may be interpreted as a favourable reflection on the accuracy of the probe measurements. We stress once more that the empirical curve of I_i vs $(R_p/\lambda_D)^2 I_i$ shown on Fig. 5.2 was employed to interpret the probe readings. The use of the theoretical curve would in some instances have resulted in substantially different values of both $(dn_i^*/dy^*)_w$ and $(n_i)_{\max}$. This has already been commented on in reference to Fig. 6.10, which shows a case in which $(n_i)_{\max}$ would have been unaltered had the theoretical curve been accepted, but the slope at the wall would have been considerably increased. In Run 16-3 the number density was particularly low and the difference therefore even more dramatic: the use of the theoretical curve would in that case have led to a diffusion flux at the wall some 70% higher than the directly measured value, whereas very good agreement was obtained with the flux calculation based on the empirical curve.

6.4.3 Comparison With High Reynolds Number Boundary Layer Theory

The conventional boundary layer approach to the present problem is outlined in Appendix C. It is shown there that the application of high Reynolds number theory yields a relationship between j_w , the current equivalent of the ion flux to the stagnation point wall, and $(n_i)_\delta$, the ion number density at the boundary layer edge. For the case at hand (flat-nosed cylinder, $M_1 = 2.2$, Ar⁺ in Ar, frozen flow, $\lambda_D \ll D$), this relationship is expressed by Eq. (C.25). In terms of the inviscid flow in the shock layer as a whole, $(n_i)_\delta$ represents the stagnation value of n_i , and in frozen flow is related to the free stream ion number density, $(n_i)_1$, by

* Reynolds number corrections were negligible for both impact probe and model.

$$(n_i)_\delta = \frac{\rho_{02}}{\rho_1} \cdot (n_i)_1 \quad (6.9)$$

where ρ_{02} represents the stagnation density of the gas behind the shock, and ρ_1 the free stream density. For $M_1 = 2.2$ and $\gamma = 1.67$, $\rho_{02}/\rho_1 = 2.91$. Using this in Eq. (C.25), we obtain the following expression for the free stream ion number density in terms of the ion current to the wall:

$$(n_i)_1 = 6.25 \times 10^{20} \frac{\sqrt{D} \sqrt{P_w(\text{torr})} \sqrt{T_w + 11.5}}{\left(1 + \frac{T_e}{T_w}\right) \sqrt{T_o + 142}} \frac{1}{T_o} \frac{j_w}{Z'(0)} \quad (6.10)$$

where all quantities are in MKS units except P_w , which is expressed in torr. The quantity $Z'(0)$ is given for $T_w/T_o = 0.8$ (the actual value in the experiments was 0.85) in Fig. C.2, as a function of Sc_δ , the value of the ambipolar Schmidt number at the outer edge of the boundary layer, i.e. at stagnation conditions. Referring to Appendix A, we obtain

$$Sc_\delta = \frac{2.51}{\left(1 + \frac{T_e}{T_o}\right)} \left(\frac{T_o + 11.5 \sqrt{T_o}}{T_o + 142} \right) \quad (6.11)$$

Equation (6.10), or its voltage dependent generalization, forms the basis of the "stagnation point Langmuir" probe technique, in which one relies on continuum theory to provide a relation between the ion current measured at the stagnation point and the free stream ion number density.

Table 6.3 shows a comparison of the free stream ion number density calculated from Eq. (6.10) and the directly measured value of j_w with the value of $(n_i)_1$ measured in the free stream with the free molecule Langmuir probes. We see that the boundary layer theory underestimates $(n_i)_1$ by about 50%. Now, it was established in the previous section that the shape and absolute magnitude of the experimentally determined n_i distribution near the wall was in good agreement with the measured j_w . Since the present disagreement is more than can be accounted for by the difference in the experimental and theoretical density ratios across the bow shock, the profiles must also be dissimilar in shape near the wall. This is shown in Fig. 6.12, where theoretical profiles of $n_i/(n_i)_\delta$ are compared to experimental profiles of $n_i/(n_i)_{\max}$ in the "boundary layer" region. The theoretical profiles were obtained from the data shown in Fig. C.3, and converted to the physical scales by means of Eq. (C.21).

The difference between the results of the two methods is presumably caused by a failure of the high Reynolds number boundary layer theory at these low Reynolds numbers. This point will be discussed further in Sec. 6.8.

6.5 The Ion Shock Wave

It was apparent early in these experiments that the maximum slope shock thickness obtained from the ion number density profiles was about twice as large as the accepted value for the neutral atom shock thickness. This was not surprising, for the weakly ionized gas in which the tests were conducted behaved as a mixture in which the minority species had a very high effective mobility, and variations in the species concentrations were therefore to be expected in any region of gradients.

The separation of the constituents of a neutral gaseous mixture in a shock wave has been considered by Sherman (Ref. 50), whose analysis of the shock structure in a binary mixture of inert gases predicts pronounced separation of the species if the constituents are of disparate mass.

The analogous problem for the weakly ionized inert gas has been solved by Sinclair (Ref. 51). Sinclair considered a frozen, partially ionized gas, consisting of electrons, ions and atoms of an inert gas, with a degree of ionization so low that the charged particles make no significant contribution to the overall transfer of momentum and energy. This allowed him to first obtain the neutral atom shock structure and then calculate the charged particle distribution consistent with it. The electron temperature was assumed a priori to remain constant, at an arbitrarily assigned value, over the region of the electron-ion and neutral atom number density variations, and charged particle diffusion was assumed to be ambipolar (in other words, the Debye length was assumed to be much smaller than the shock thickness). Sinclair's calculations for argon at $M_1 = 2.2$ and $T_0 = 450^\circ\text{K}$ are shown in Fig. 6.13, where the charged particle number density profiles through the shock are plotted with T_e as parameter.

Experiment and theory are compared in Fig. 6.14. The points represent ion shock thicknesses, Δ_i , measured directly from the ion current profiles by the maximum slope method. They are plotted against the free stream value of the ambipolar diffusion coefficient, which governs (in this case through T_e) the degree of the ion-atom separation. The ambipolar Schmidt number is used here, rather than T_e itself, because it is the magnitude of this dimensionless parameter which is of basic importance in the separation process.

It is evident that the experimental plot appears to follow the theoretical one in shape, but falls about 50% higher than the latter. However, much of this difference can immediately be attributed to both theoretical and experimental shortcomings. For one thing, it is well known that the Navier-Stokes theory, which is basically equivalent to the 13 moment method which Sinclair used, underestimates the shock thickness at higher Mach numbers. For argon at $M_1 = 2.2$, the measured atom shock thickness is 25% greater than that predicted by the Navier-Stokes theory (Ref. 21), and we may therefore expect the experimental ion shock thickness to fall above the theoretical one by about the same amount. An additional 10% of the difference may be attributed to our measuring Δ_i directly from the ion current profiles rather than from the n_i profiles derived from them (the conversion of j_i to n_i in the shock wave was a very time-consuming process). It is clear from Figs. 6.6 and 6.7, for example, that this procedure in general caused an overestimation of Δ_i by about 10%. (The low density runs, where the shock had merged entirely with the boundary layer, were disregarded for these measurements.) In addition to these factors, the experimental

simulation of one-dimensional normal shock conditions was obviously poor. As was discussed in Sec. 6.3, the shock structure was in all likelihood influenced by three-dimensional effects and by downstream gradients.

6.6 The Shock Detachment Distance

The shock detachment distance, y_s , we refer to here is the separation, measured along the stagnation streamline, between the nose of the model and the maximum slope centre of the bow shock. The values of y_s/D listed in Table 6.1 were measured directly from the ion current profiles, as indicated in Fig. 6.15. These values scatter about an average of 0.45 and, as shown on Fig. 6.15, do not reveal a systematic variation with Re_2 in the narrow Reynolds number range covered in the experiments. In fact, our value of 0.45 for $Re_2 \approx 10^2$ is in excellent agreement with the experimental data for very high Reynolds numbers. This is evident from Fig. 8 of Ref. 52, where such data for flat-nosed cylinders are summarized.

Bailey et al. (Ref. 52) also found that for flat-nosed cylinders the shock detachment distance remained very close to the high Reynolds number value down to Re_2 values as low as 80. In their measurements, however, the shock detachment distance was measured to the outer edge of the luminous region ahead of the model. Also, their experiments were done at higher Mach numbers and lower ratios of wall temperature to stagnation temperature. In the present tests the centre of the ion shock was used to define y_s . Note that according to the theoretical n_i profiles shown on Fig. 6.13, the ion shock lies somewhat upstream of the neutral shock, but for the case at hand the centre of the ion shock should coincide approximately with the upstream edge of the neutral shock.

6.7 Some Experiments at Lower Reynolds Numbers

A few runs were also made with the model in the free jet flow field, where higher Mach numbers and therefore lower Reynolds numbers could be obtained. It was soon apparent, however, that the severe axial gradients in that flow, together with the interaction of the model's shock layer with the barrel shock of the free jet, distorted the flow about the model significantly from the shape it would have had in the absence of free stream gradients. We shall nevertheless show some of the results from these tests, for they are of qualitative interest.

Figure 6.16 shows an ion current profile along the stagnation streamline of the model for a run in which the Mach number just upstream of the bow shock was about 4.5, and the corresponding Re_2 value was 18. The value of the Schmidt number, based on stagnation conditions, was 0.39. The ion current profile is entirely merged and bears little similarity to a high Reynolds number n_i profile. The axial gradients in the free stream are evident from the ion current profile, and also from the free stream Mach number distribution shown in Fig. 4.1 (the nose of the model was at $z/D = 2.3$ in this run, while the upstream edge of the bow shock was at about $z/D = 1.9$). As in the higher Reynolds number experiments, the electron temperature was found to be constant from the wall of the model to points in the free stream. Qualitatively similar results have been reported by Clayden (Ref. 36).

The entirely three dimensional nature of the n_i distribution in the stagnation region may be inferred from the ion current surveys in Fig. 6.17. These represent traverses through the shock layer in the direction

transverse to the free stream flow velocity. The interaction with the barrel shock was obviously serious in this run, but the results should nevertheless be qualitatively similar to what they would have been had the free stream been uniform.

6.8 Discussion of Results

We have seen that with careful interpretation the free molecule Langmuir probe can be used with considerable success for the detailed exploration of continuum flow fields. This study of the stagnation region of a flat-nosed cylinder in a Mach 2.2 flow has yielded the following information.

(1) The electron temperature was found to be constant in the bow shock and the entire shock layer, from points in the free stream to the face of the cylinder.

Similar results have been obtained previously by Sonin (Ref. 14). A discussion has been given in Sec. 6.3.

(2) The merging of the charged particle number density distributions in the shock and boundary layer was observed at a value of Re_2 of about 100, with the ambipolar Schmidt number based on stagnation conditions in the range 0.3 to 0.5.

Since the scale of the electron-ion diffusion phenomena relative to the scale of the dissipative regions in the neutral flow field is governed by the magnitude of the ambipolar Schmidt number, the merging of the n_i profile, for example, or the behaviour of any measurable which is related to it, cannot be correlated with the Reynolds and Mach numbers only. The ambipolar Schmidt number (at stagnation conditions, say) must also be included as a significant dimensionless parameter.

(3) The n_i profiles established with free molecule Langmuir probes near the wall of the model implied diffusion fluxes which were in good agreement with the directly measured values of the ion flux to the stagnation point wall.

On the other hand, the interpretation of the measured j_w on the basis of high Reynolds number theory gave a free stream ion number density about 50% lower than the value measured with free molecule probes. This disagreement was traced to the dissimilarity of the experimental and theoretical n_i profiles, both in the region near the wall, where the experimental curve was more extended than the theoretical one, as well as through the bow shock, across which the experimental n_i jump was less than the value expected at high Reynolds numbers.

Brundin (Ref. 44) reports measurements with stagnation point Langmuir probes at Re_2 values between 2.5 and 47. In two of his three runs (covering Re_2 values from 2.5 to 20) he also made free stream measurements with free molecule, cylindrical Langmuir probes with their axes aligned with the flow velocity. Brundin did not accept the available free molecule probe theories, but argued that since he found little systematic difference between the readings of the two different diameter stagnation point probes which he employed in each run, the conventional high Reynolds number boundary layer interpretation of the stagnation point probe must have remained valid down

to the lowest Reynolds numbers in his tests, $Re_2 \approx 2.5$. He accepted his stagnation point probe readings, and arrived at the conclusion that the dimensionless ion current I_i , defined in Sec. 5, must have had a value of 2.5 for his cylindrical probes. Now, the static free stream conditions in Brundin's experiments were similar, as regards T_i/T_e and the mean free paths, to those in the experiments described in Sec. 5. If we re-interpret Brundin's free molecule probe readings in terms of the data in Fig. 5.2, we find that I_i should have had a value of 1.20 in both of the runs for which he reports free molecule probe measurements. This means that his stagnation point probes gave a free stream n_i about 50% lower than the free molecule cylindrical probes.

The fact that Brundin's stagnation point probe measurements at the lower Reynolds numbers apparently underestimated $(n_i)_1$ by about the same factor as the equivalent measurements in the present experiments at $Re_2 \approx 100$ must be regarded as fortuitous. For one thing, a direct comparison between the two experiments cannot be made in terms of Reynolds number alone. Brundin's measurements were made at hypersonic Mach numbers and at quoted stagnation temperatures of about 10,000°K, with considerably lower electron temperatures. The high T_0 and low T_e/T_0 combine to give the ambipolar Schmidt number based on stagnation temperature a high value in Brundin's case, and as a result the electron-ion diffusion effects which were so pronounced in the present experiments may have been less prominent under his conditions. Brundin made no detailed investigation of the structure of the shock layer, and therefore no definite conclusion can be drawn in this respect. We have seen semiquantitatively in Sec. 6.7, however, that at the low Schmidt numbers characteristic of the present experiments, the shock layer was entirely merged at Re_2 values comparable to, and in fact even considerably higher than, those in Brundin's tests.

(4) The diffusive separation of the ions and atoms in the bow shock was demonstrated and compared with theory. The results bore out the general features of the theory, but a conclusive quantitative comparison was precluded because of the poor experimental simulation of the normal shock conditions, at least as regard the charged particles.

7. SHOCK WAVE TRAVERSES

The application of the Langmuir probe to traversing a normal shock wave in a shock holder seemed a natural extension of the work described in the previous section. In particular, it was thought that traverses with both parallel and transverse probes would not only yield data on shock structure, but also provide a convenient check on the probe calibration, the density jump across the shock being known. Let it be said at the outset that the experiments were failures in both these respects, for it proved to be impossible to create a suitable one dimensional shock wave in the facility available. However, the tests did furnish some relevant information, and will therefore be discussed.

The simple shock holder used for these experiments is sketched in Fig. 7.1. It consisted of an open-ended tube, the front end chamfered externally at 10° to a thin lip, with slots in the sides and bottom to allow a probe to be passed into the interior. A movable perforated disc downstream of the tube served to position the shock at the mouth of the holder. Again, only the smallest, 0.25 cm long, probes were used. They were mounted on the larger traversing gear and could be rotated about the geometric centres of their collecting surfaces to permit surveys to be taken in both the parallel

and transverse orientations without having to interrupt the operation of the tunnel. All surveys were taken along the axis of the flow. The potentiometers coupled to the traversing system allowed the ion current profiles to be plotted directly on the x-y storage oscilloscope.

Three runs were made, at flow conditions given in Table 7.1. Ion current surveys through the shock are shown on Fig. 7.2; the top curve in each oscillogram represents data taken with the probe axis parallel, and the bottom one with it perpendicular, to the flow velocity. The oscillogram for Run 29-3 also shows the corresponding ion current surveys obtained after the shock holder had been removed from the flow. In all three runs, the lip of the shock holder was at $z = 11.8$ cm, and an attempt was made to centre the ion shock at that station.

It is clear that the normal shock conditions were simulated very poorly. Not only was the ion shock thick compared to the shock holder diameter (about 2 cm versus 3.5), but there were considerable gradients in the flow properties in both the axial and radial directions. The former are evident from the ion current surveys as well as from the axial Mach number distribution in Fig. 4.1 compared to the shock thickness. The radial gradients are illustrated in Fig. 7.3. Since the flow about the axis of a free jet is very similar to the spherical expansion from a source, uniform conditions can be expected only over an area which subtends a very small solid angle at the orifice. In this case a compromise had to be reached between this requirement and the necessity for making the holder diameter as large as possible compared to the shock thickness. Obviously, the result left a lot to be desired in both respects. Figure 7.3 indicates that the spherical nature of the flow was quite prominent over the dimensions of the shock holder, the Mach number being lower and the electron density higher at the centre of the shock relative to its edge. As a result, the shock wave was somewhat concave, as it should be in a spherical flow field.

These shortcomings precluded quantitative comparisons with one dimensional shock theory. Nevertheless, some interesting results should be pointed out. We refer first to the ion current survey in Fig. 7.2 (Run 29-1) which indicates a local increase on the upstream side of the shock. Such a bump was observed only when the stagnation pressure was low, i.e. when the flow was diffuse. The result was quite reproducible and independent of probe potential. Now, this bump could have been either a real one in n_e or T_e , or else a peculiarity of the probe's response. If the effect is a peculiarity of the probe's response, it implies that a parallel probe - the bump is quite pronounced in the $j_{||}$ profile, and hardly visible in the $j_{\perp}$ profile - is influenced by flow speed. This, we have argued in Sec. 5, was unlikely, and we have in fact explicitly denied this possibility in interpreting the results of Sec. 5.3 as a calibration of the static probe. Point by point surveys showed that T_e was constant throughout the compression region (Fig. 7.4). A localized increase in n_e is difficult to explain in this particular case. On the other hand, the same effect was observed in traverses through the Mach disc of the free jet shock structure, and in that case an explanation was evident.

When axial $j_{||}$ traverses were made through the free jet Mach disc at successively lower and lower stagnation pressures, it was found that eventually a similar bump appeared in the $j_{||}$ profiles, and grew more pronounced as the stagnation pressure was decreased further. We see it in very mild form in Fig. 4.2(b), for example, although considerably more prominent

records were also obtained. At the lower Reynolds numbers the sides of barrel shock associated with the flow of the neutral gas had become quite thick, but as a result of the low magnitude of the ambipolar Schmidt number in these tests, the corresponding electron-ion number density distributions were even more diffuse. Consequently, the electron-ion number density distributions associated with the sides of the barrel shock merged at the centre line, and indicated an increase in j_i , before the Mach disc itself caused a sharper increase in n_i . These details could be inferred from radial ion current surveys. A similar explanation cannot be given in the case of the surveys shown on Fig. 7.2, where, as we see from Fig. 7.3, the barrel shock of the free jet was not close to the shock holder. However, it is likely that in this case, too, the bump in j_i represents a real increase in n_i .

Figure 7.5 shows plots of $j_{\perp}/j_{\parallel}$ obtained from the data on Fig. 7.2. Note that in the free stream upstream of the shock, the Mach number was increasing, and the gas temperature and electron-ion number density decreasing, with the axial distance z . The only flow property which remained practically invariant was $u/\sqrt{kT_e/m_i}$, for the gas had almost reached its escape speed and T_e was constant. The fact that $j_{\perp}/j_{\parallel}$ remained constant in the free stream in all cases indicates again that the dependence of the ion current on flow speed correlates with $u/\sqrt{kT_e/m_i}$ rather than with the conventional Mach number. Another relevant point here is that the increase of $j_{\perp}/j_{\parallel}$ through the shock wave was consistent with a normal-shock decrease of flow speed and the flow speed correlation of Fig. 5.7(b). In Run 29-1, for example, where $u/\sqrt{kT_e/m_i} = 0.79$ upstream of the shock, the ion current ratio was about 0.83 downstream of the shock, where $(R_p/\lambda_D)^2 I_i (\chi_f - 10) \approx 11$. From Fig. 5.7(b), this implies that $u_2/\sqrt{kT_e/m_i} \approx 0.19$, i.e. $u_1/u_2 \approx 4.2$, compared with the ratio of about 3.6 predicted by the one dimensional shock relations. For Runs 29-2 and 29-3 the speed ratio was 3.2 and 4.0, respectively.

A reason for finding fair agreement in the speed ratio but not in the electron-ion number density ratio may be advanced as follows. Since the Reynolds number Re_2 based on shock holder diameter was in the range 17 to 24, which is sufficiently high for impact pressure corrections not to be serious (Ref. 17), the jump in the overall gas properties across the shock wave was probably reasonably well represented by the one dimensional relations. The electron-ion shock, on the other hand, was substantially more diffuse than the neutral shock, for the ambipolar Schmidt number was very small. Its thickness, we have seen, was almost as great as the shock holder diameter. It was therefore to be expected that the combined effects of three dimensional upstream diffusion and losses by diffusion to the shock holder walls would distort the centre line electron-ion number density distribution from the one dimensional shape, but there was no reason to expect the axial velocity component of the charged species to have been significantly different from that of the gas as a whole upstream and downstream of the shock.

8. CONCLUSIONS

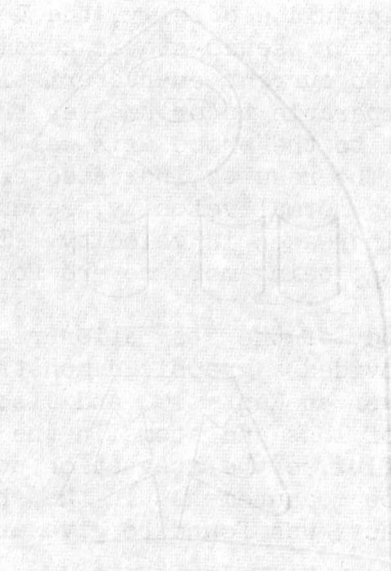
This investigation has covered two areas: the response of free molecule cylindrical Langmuir probes in flowing plasmas, and the application of such probes to the study of the flow in the stagnation region of a blunt body. The detailed conclusions from these two studies are listed in Sections 5.8 and 6.7, respectively.

To recapitulate briefly, the Langmuir probe experiments indicated that under static or pseudo-static conditions, the theory of Laframboise represents the ion current characteristic adequately at large R_p/λ_D , but not when R_p is comparable to or smaller than λ_D . The influence of the plasma velocity normal to the probe axis was found to depend on the quantities $u/\sqrt{kT_e/m_i}$ and R_p/λ_D . The results indicated clearly that the ion current decreases with increasing normal velocity, reaches a minimum, and then increases again for subsequent increases in velocity. The magnitude of the initial decrease depends on R_p/λ_D , being more severe for smaller values of the latter.

The study of the stagnation region of the flat-nosed cylinder in supersonic flow provided a graphic demonstration of the merging of the bow shock and boundary layer at $Re_2 \approx 100$ and also furnished evidence of the diffusive separation of ions and atoms in the shock wave. Independent measurements of the ion flux to the stagnation point wall were in good agreement with the Langmuir probe measurements in the shock layer. High Reynolds number boundary layer theory was found to give an inadequate description of the flow at $Re_2 \approx 100$.

The Langmuir probe was also used to great advantage as an aid in qualitative flow description (Sec. 4). Ion current surveys, which could be simply and rapidly recorded, yielded immediate qualitative information on the structure of the flow field, revealing the approximate locations of shocks and compression zones.

Thus, the free molecule Langmuir probe is unquestionably an extremely useful tool in studies of complicated plasma flow fields. On the other hand, it is also clear that the quantitative interpretation of its readings must be done with great care, for there are still many aspects of probe behaviour which are imperfectly understood. For example, the effects of small ion-ion mean free paths on an otherwise free molecule probe certainly merit further systematic investigation. A more quantitative definition of the limits of free molecule theory - there are here a number of limits to consider because of the many types of particle interactions which are possible - would be of great interest, particularly for cylindrical probes. At the present state of the art, it is the author's opinion that microwave methods, for example, although not themselves suitable for local measurements on a very small scale, should be used whenever possible to supplement probe data and check probe response under typical operating conditions. Much of the uncertainty which often arises in probe work could be avoided with such methods of cross checking.



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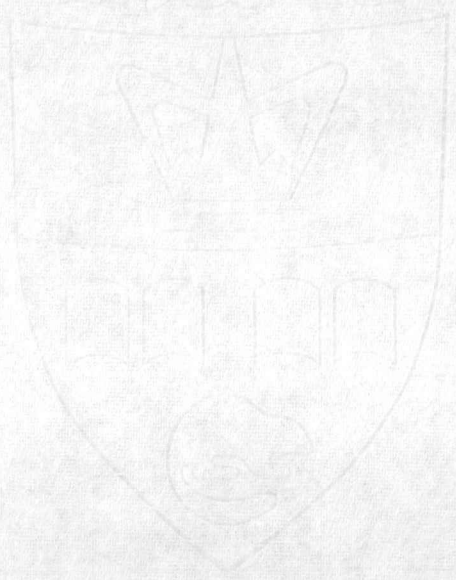


TABLE 5.1

Results from Tests With Free Molecule Langmuir Probes*(a) Ion Current Measurements

Run	Symbol	Probe No.	$(R_p/\Lambda_D)^2 I_i(\chi_f - 10)^{**}$	$I_i(\chi_f - 10)^{**}$ experimental	$j_{\perp}/j_{\parallel}$ at $\chi_p = \chi_f - 10$
26-2	●	9	7.18	7.32	0.50
		10	60.2	3.43	0.63
26-3	△	9	22.9	5.23	0.62
		10	193	2.46	0.80
32-4	□	11	7.40	6.82	0.47
		10	55.0	3.57	0.59
34-1	+	16	40.8	4.25	0.58
		14	108	2.88	0.76
		15	196	2.44	0.81
34-2	○	16	384, 400	1.96, 2.06	1.21, 1.18
		14	1160	1.50	1.26
		15	2330	1.42	1.26
35-2	▽	16	202	2.58	0.95
		14	562	1.82	1.01
		15	1090	1.63	0.99
35-3	▲	16	102	3.12	0.79
		14	282	2.20	0.88
		15	530	1.90	0.88
35-6	●	16	601	1.91	1.51
		14	1790	1.47	1.65
		15	3640	1.38	1.60

* Running conditions are listed in Table 5.2 and probes are described in Table 5.3

** Based on measurements with probe parallel to flow

TABLE 5.1 (continued)

(b) Electron Temperature Measurements

Run	Probe No.	kT _e /e (volt)		Percent Difference
		Probe Parallel	Probe Perpendicular	
26-2	9	0.352	0.345	-2
	10	0.372	0.330	-12
26-3	9	0.369	0.369	0
	10	0.369	0.369	0
32-4	11	0.315	0.312	-1
	10	0.315	0.298	-6
34-1	16	0.369	0.344	-7
	14	0.356	0.356	0
	15	0.356	0.361	+1
34-2	16	0.124	0.123	+1
	14	0.122	0.121	-1
	15	0.122	0.122	0
35-2	16	0.178	0.191	+7
	14	0.185	0.185	0
	15	0.183	0.181	-1
35-3	16	0.270	0.261	-3
	14	0.265	0.282	+6
	15	0.275	0.282	+3
35-6	16	0.122	0.134	+10
	14	0.122	0.151	+21
	15	0.120	0.130	+8

Average difference between electron temperature readings in parallel and perpendicular orientations: 1.1%

Root mean square difference: 6.7%

TABLE 5.2

Running Conditions in Tests With Free Molecule Langmuir Probes

Run	Stagnation Conditions		Conditions at Probes						
	P_o (torr)	T_o (°K)	M	$P \times 10^3$ (torr)	T (°K)	T_e (°K)	T_i/T_e	$\frac{u}{\sqrt{kT_e/m_i}}$	n_e (m^{-3})
26-2	0.742	544	4.8	3.3	63	4060	0.016	0.77	1.25×10^{17}
26-3	1.06	970	4.8	4.7	112	4280	0.026	1.00	5.82×10^{17}
32-4	0.837	547	5.8	1.7	45	3600	0.013	0.83	0.98×10^{17}
34-1	0.747	558	5.8	1.5	46	4150	0.011	0.78	0.70×10^{17}
34-2	1.06	993	5.8	2.1	82	1430	0.057	1.79	4.85×10^{17}
35-2	0.942	855	5.8	1.9	71	2140	0.033	1.35	2.94×10^{17}
35-3	0.900	772	5.8	1.8	64	3160	0.020	1.06	1.80×10^{17}
35-6	1.26	1480	5.8	2.5	123	1510	0.082	2.13	7.42×10^{17}

Note: The given T_e refers to the average value

TABLE 5.3

Langmuir Probes

Probe No.	$d = 2R_p$ (mm)	l (mm)
9	0.0248	2.65
10	0.105	10.2
11	0.0252	2.50
14	0.208	10.3
15	0.305	10.5
16	0.105	10.0

TABLE 6.1

Results from Stagnation Streamline Surveys

Run	T _o °K	P _o torr	M ₁	Re ₂	T _e °K	$\frac{\Delta i}{\lambda_1}$	Sc ₁	y _s /D	(n _i) _{max} m ⁻³
9-2	545	1.08	2.23	93	3830	7.24	0.135	0.43	
9-3	718	1.26	2.20	78	3270	7.11	0.217	0.43	
9-4	614	1.41	2.28	105	4580	7.27	0.128	0.43	
10-1	531	1.09	2.19	97	3630	8.03	0.142	0.45	
10-2	422	0.961	2.19	114	3930	7.87	0.100	0.45	
15-1	478	1.23	2.23	124	3000	7.84	0.147	0.44	
15-2	445	0.977	2.20	108	3300	8.17	0.126	0.47	
16-1	478	1.22	2.24	120	2400	7.30	0.180	0.450	1.42 x 10 ¹⁸
16-2	453	0.981	2.23	104	3010	8.30	0.137	0.468	0.827 x 10 ¹⁸
16-3	444	0.721	2.13	84	5320	9.12	0.084	0.503	0.368 x 10 ¹⁸
17-1	453	0.985	2.23	107	2860	7.14	0.145	0.462	1.05 x 10 ¹⁸

TABLE 6.2

Ion Current at the Stagnation Point

Run	From Langmuir Probe Measurements			Direct Measurement	Difference
	$(n_i)_{\max}$ m^{-3}	$\left(\frac{dn_i^*}{dy^*}\right)_w$	j_w A/m^2	j_w A/m^2	
16-1	1.42×10^{18}	8.47	5.18	5.0	+4%
16-2	0.827×10^{18}	7.19	3.28	3.9	-16%
16-3	0.368×10^{18}	5.75	2.75	2.9	-5%
17-1	1.05×10^{18}	6.40	4.10	3.9	+5%

TABLE 6.3

Comparison With High Reynolds Number Boundary Layer Theory

Run No.	j_w measured directly A/m^2	Sc_δ	Free Stream Electron Number Density (m^{-3})		Ratio (a)/(b)
			(a) calculated from j_w using b.l. theory	(b) measured with free molecule probe in free stream	
16-1	5.0	0.491	3.2×10^{17}	7.82×10^{17}	0.41
16-2	3.9	0.384	2.1×10^{17}	4.55×10^{17}	0.46
16-3	2.9	0.225	1.0×10^{17}	2.50×10^{17}	0.40
17-1	3.9	0.400	2.6×10^{17}	5.78×10^{17}	0.45

TABLE 7.1

Operating Conditions in Tests With Shock Holder

Run	29-1	29-2	29-3
P_o (torr)	0.827	1.14	1.17
T_o ($^{\circ}K$)	537	792	983
T_e ($^{\circ}K$)	3810	3500	2710
$u_1 / \sqrt{\frac{kT_e}{m_i}} *$	0.79	1.00	1.27
average (R_p/λ_D) ² I_i in ion current traverse (approximately)	10	20	50

* Free stream conditions (1) were taken to be
at $z \approx 10.5$ cm, where $M \approx 5$

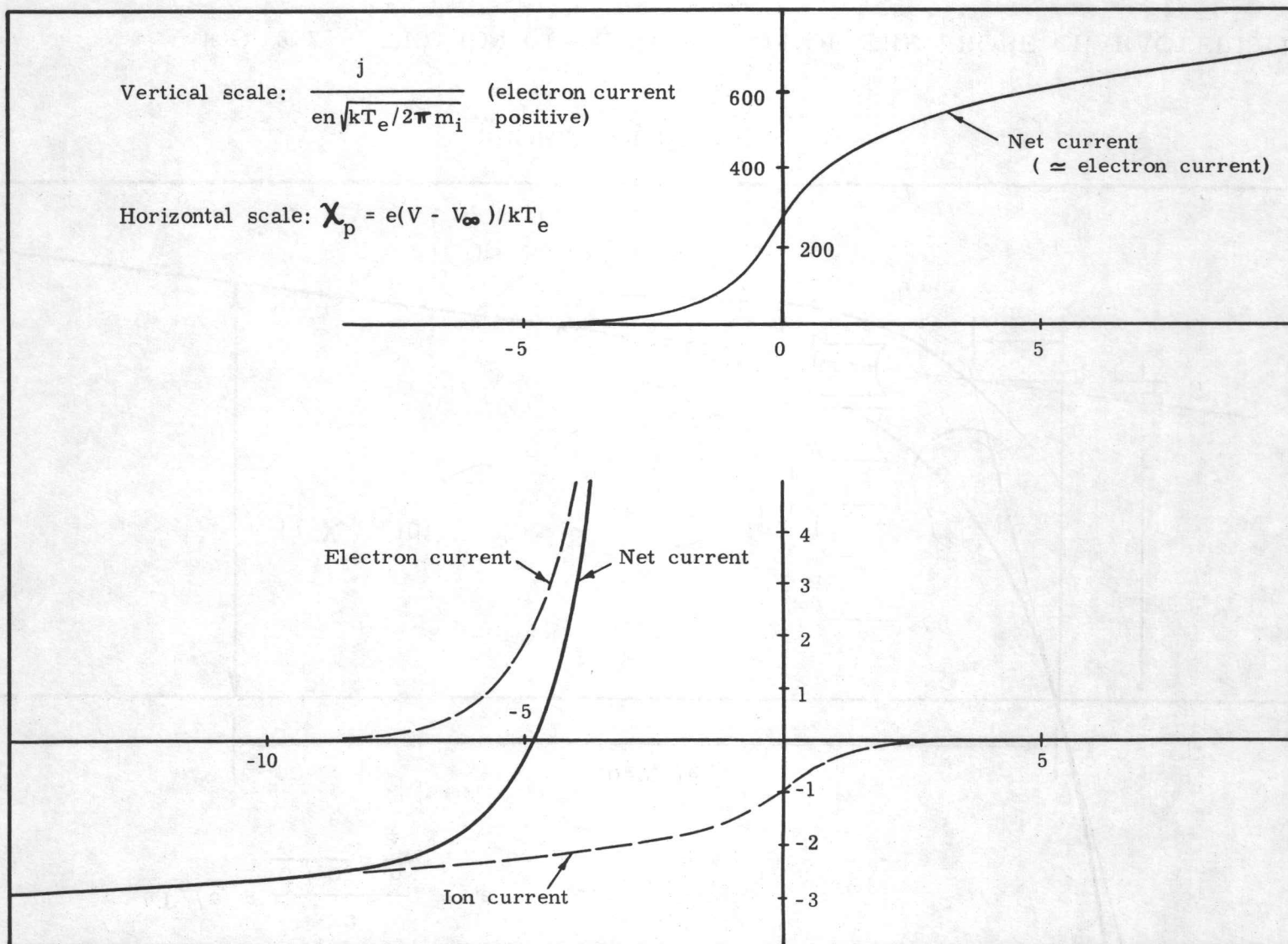


FIG. 2.1 THEORETICAL CURRENT-VOLTAGE CHARACTERISTIC FOR CYLINDRICAL PROBE. $R_p/\lambda_D = 10$, $T_i/T_e = 1$, ATOMIC ARGON IONS (FROM LAFRAMBOISE).

PROBE CURRENT (ELECTRON CURRENT POSITIVE)

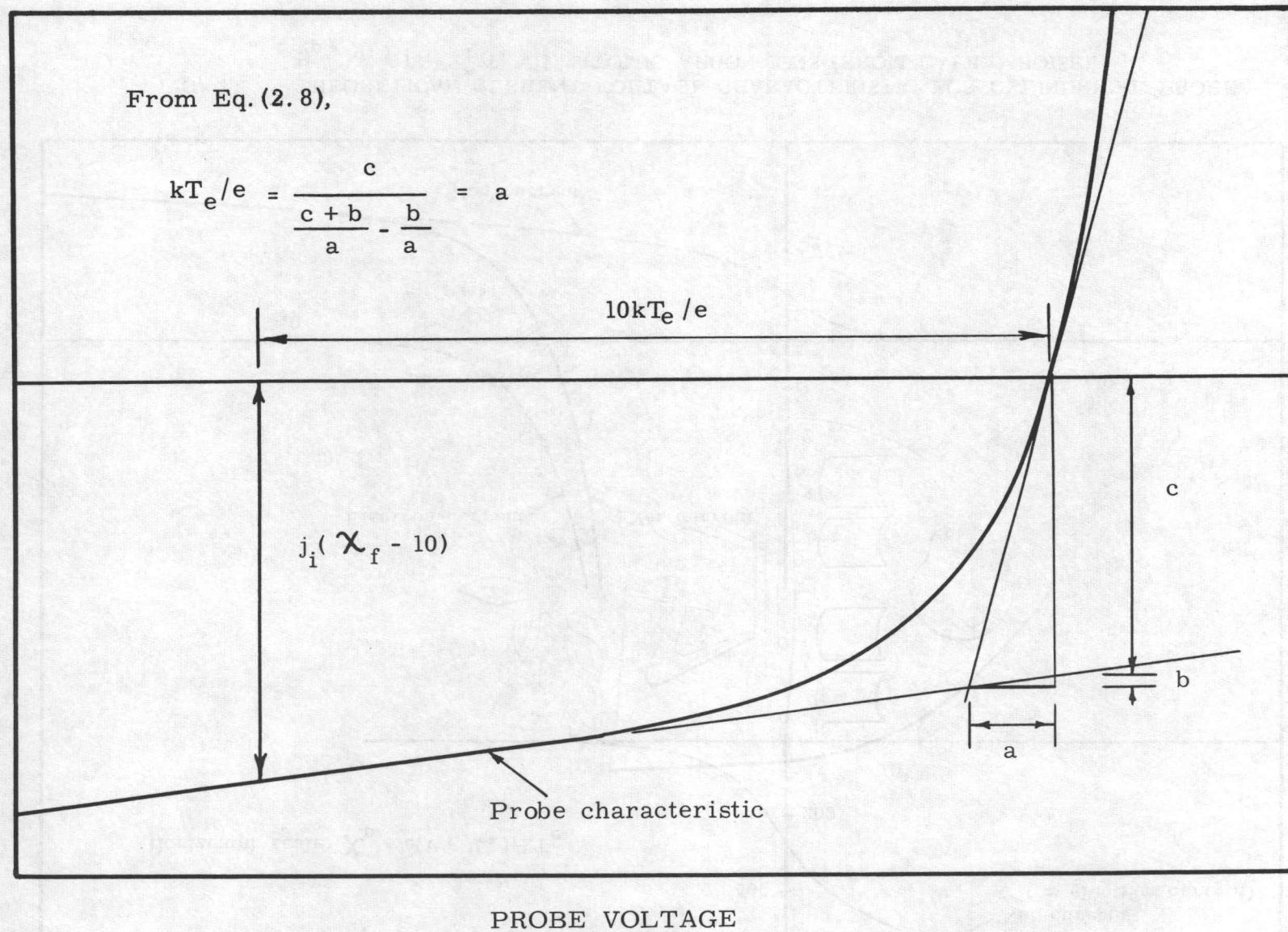


FIG. 2.2 THE ION CURRENT REGION OF THE PROBE CHARACTERISTIC

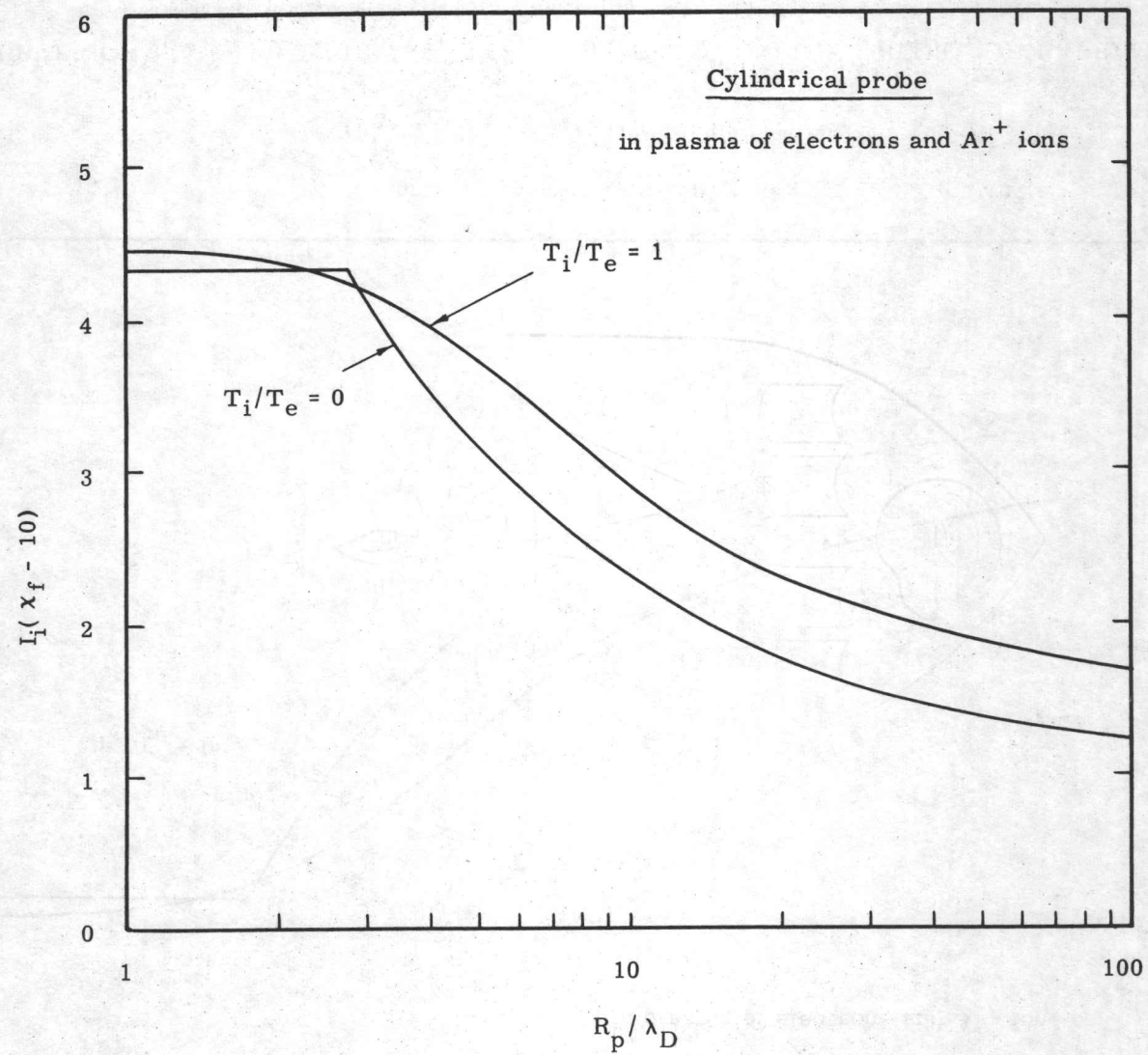


FIG. 2.3 CURVES OF $I_1(x_f - 10)$ VS R_p/λ_D , BASED ON THE THEORY OF LAFRAMBOISE

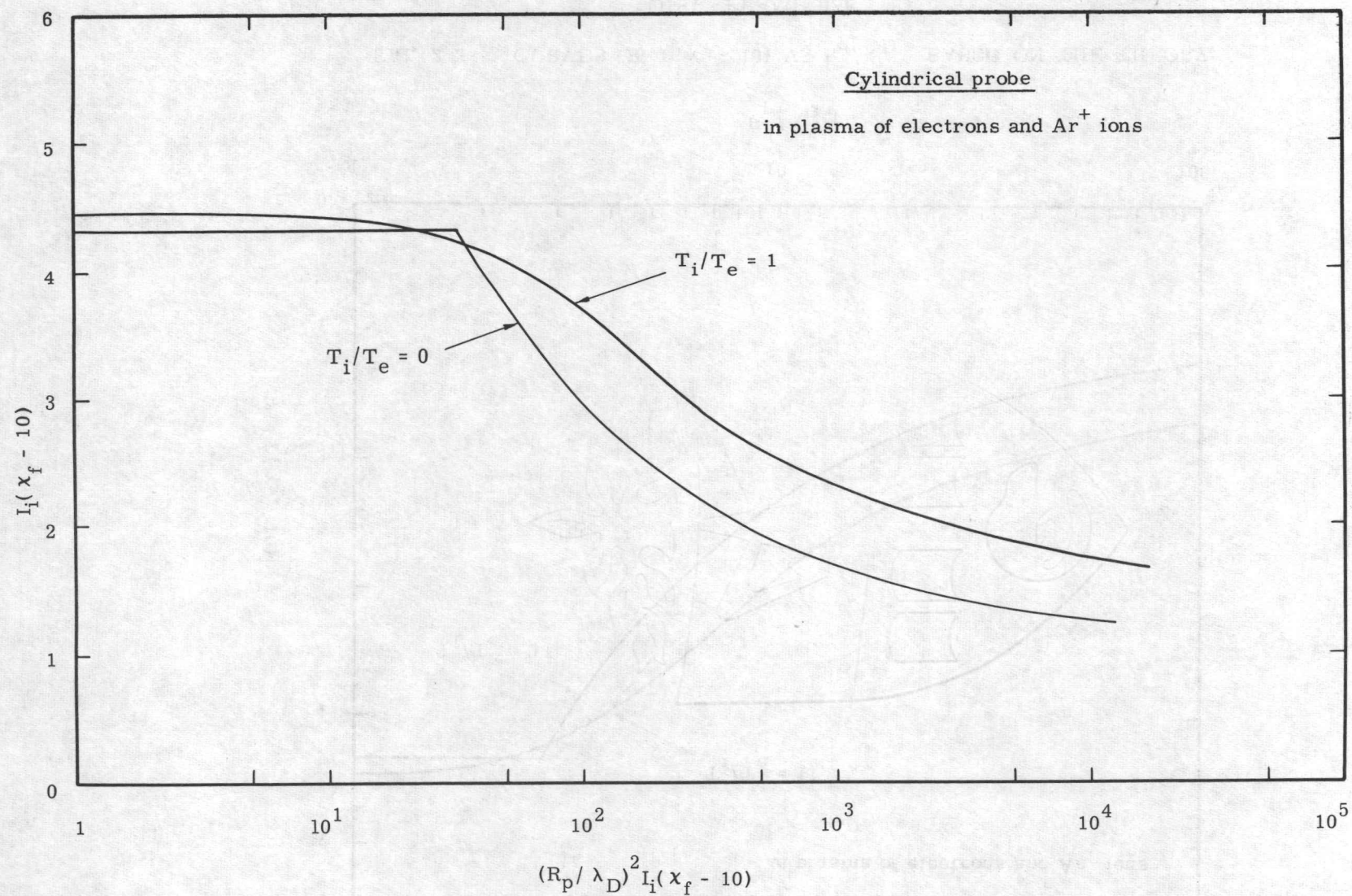


FIG. 2.4 CURVES OF $I_i(x_f - 10)$ VS $(R_p / \lambda_D)^2 I_i(x_f - 10)$, BASED ON THE THEORY OF LAFRAMBOISE

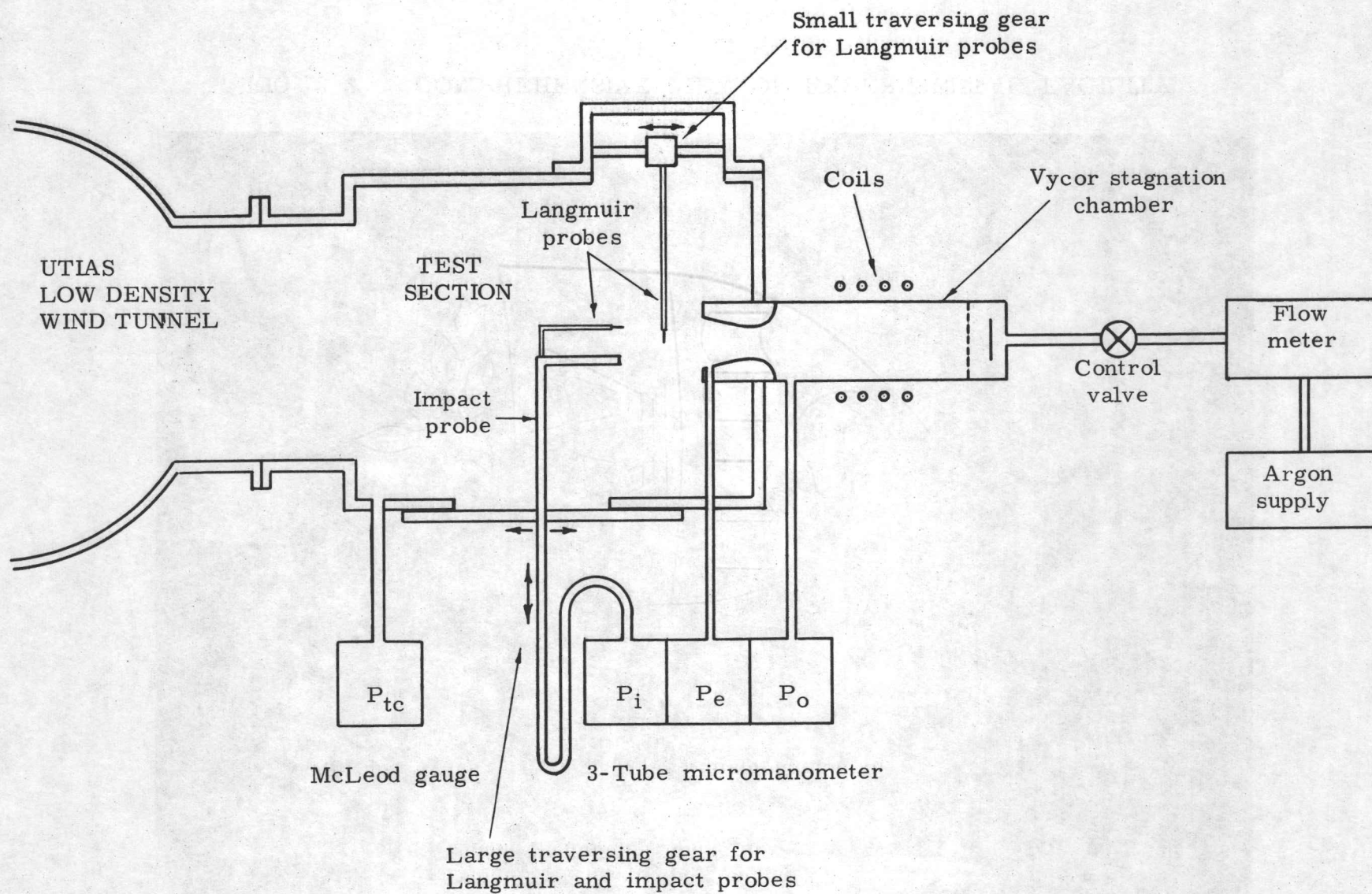


FIG. 3.1 SCHEMATIC DIAGRAM SHOWING GENERAL FEATURES OF APPARATUS

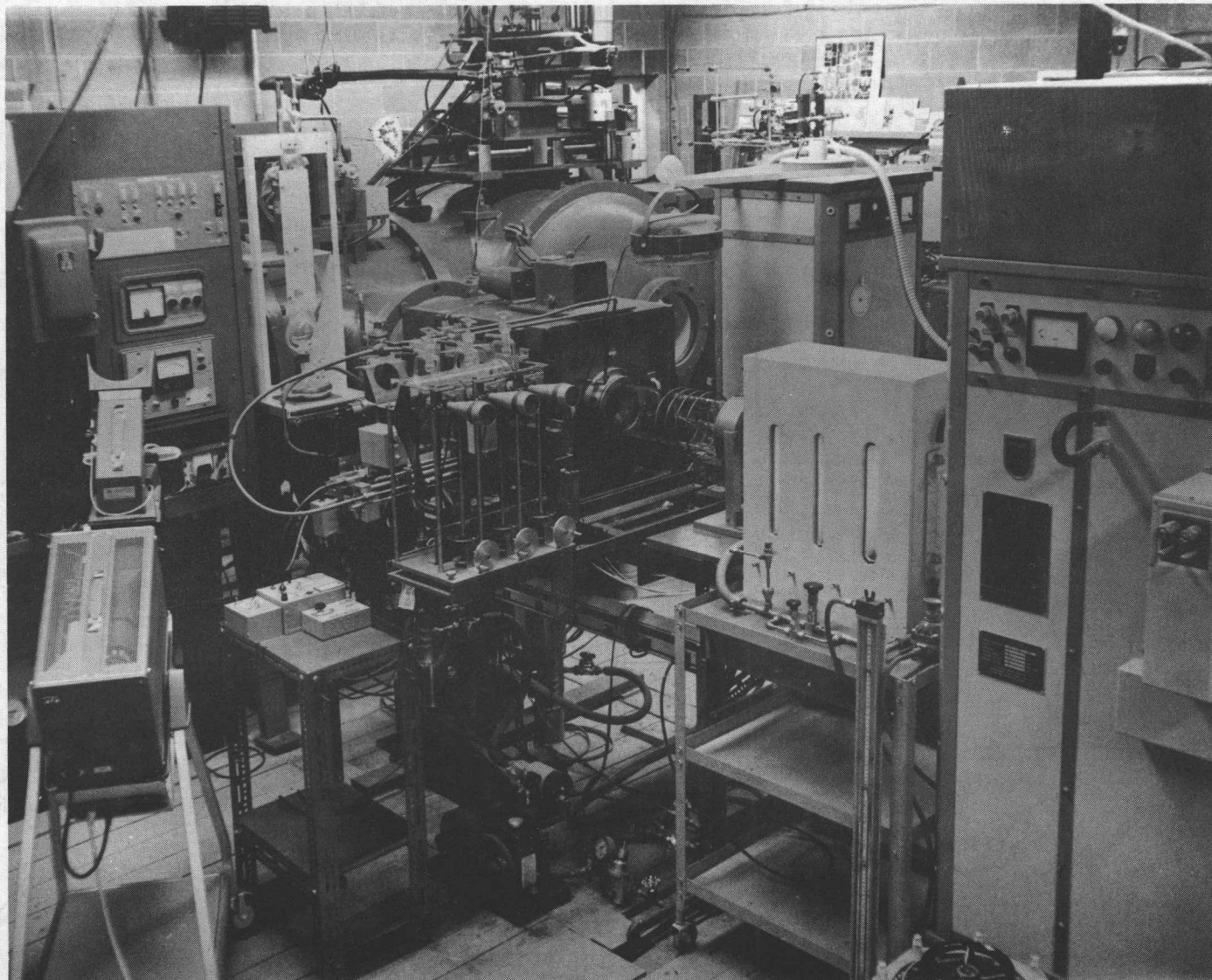


FIG. 3.2 COMPREHENSIVE VIEW OF EXPERIMENTAL FACILITY

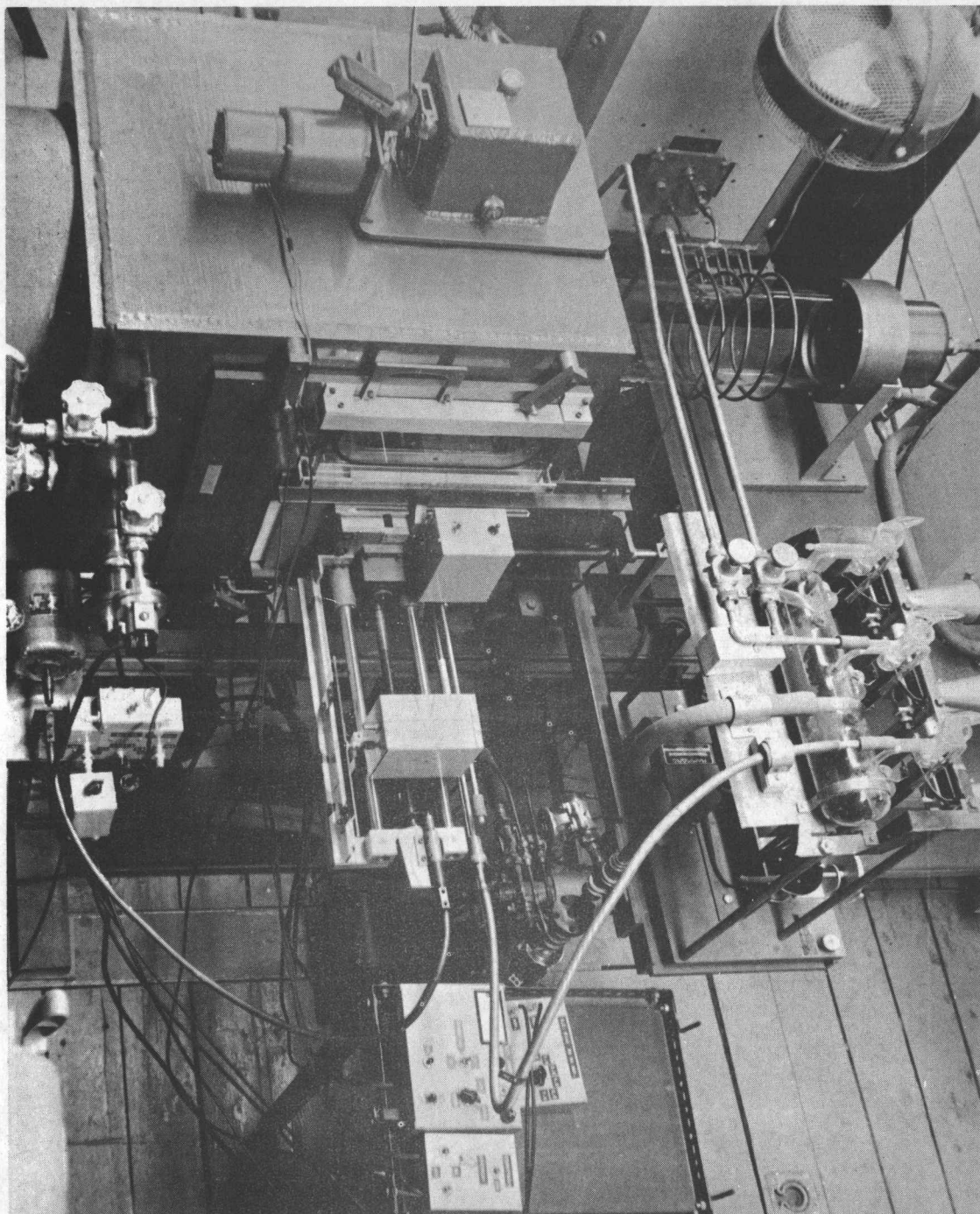


FIG. 3.3 EXTERIOR VIEW OF TEST SECTION AND TRAVERSING GEAR

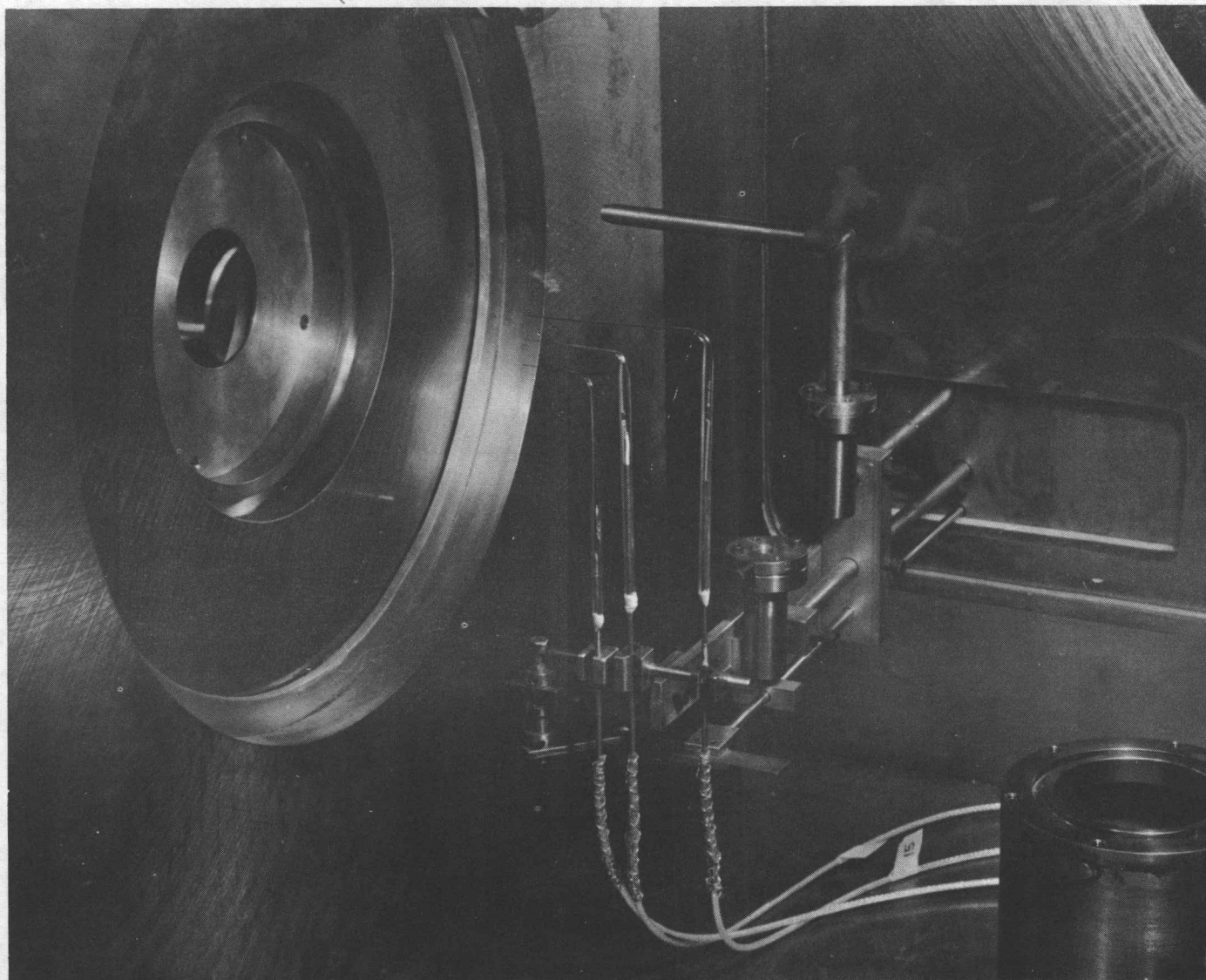
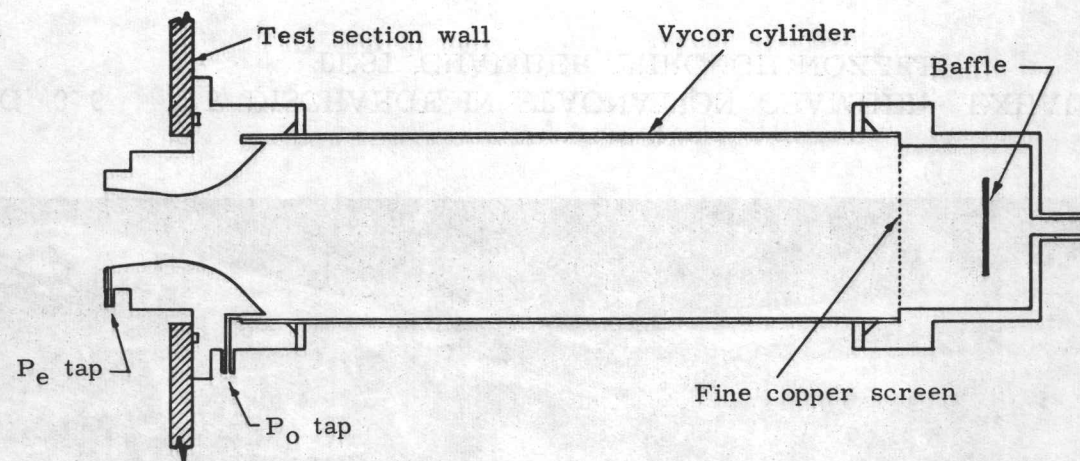
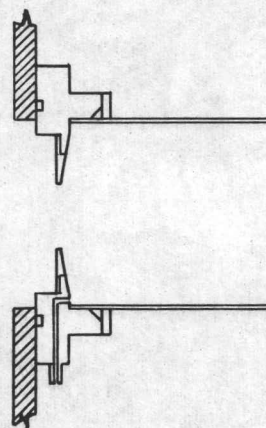


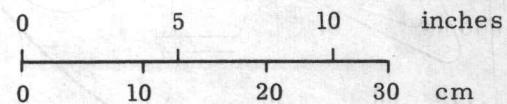
FIG. 3.4 INTERIOR OF TEST SECTION, SHOWING PROBES ON TRAVERSING
 GEAR AND FREE JET ORIFICE



(a) With nozzle



Scale



(b) With free jet orifice

FIG. 3.5 DETAILS OF STAGNATION CHAMBER

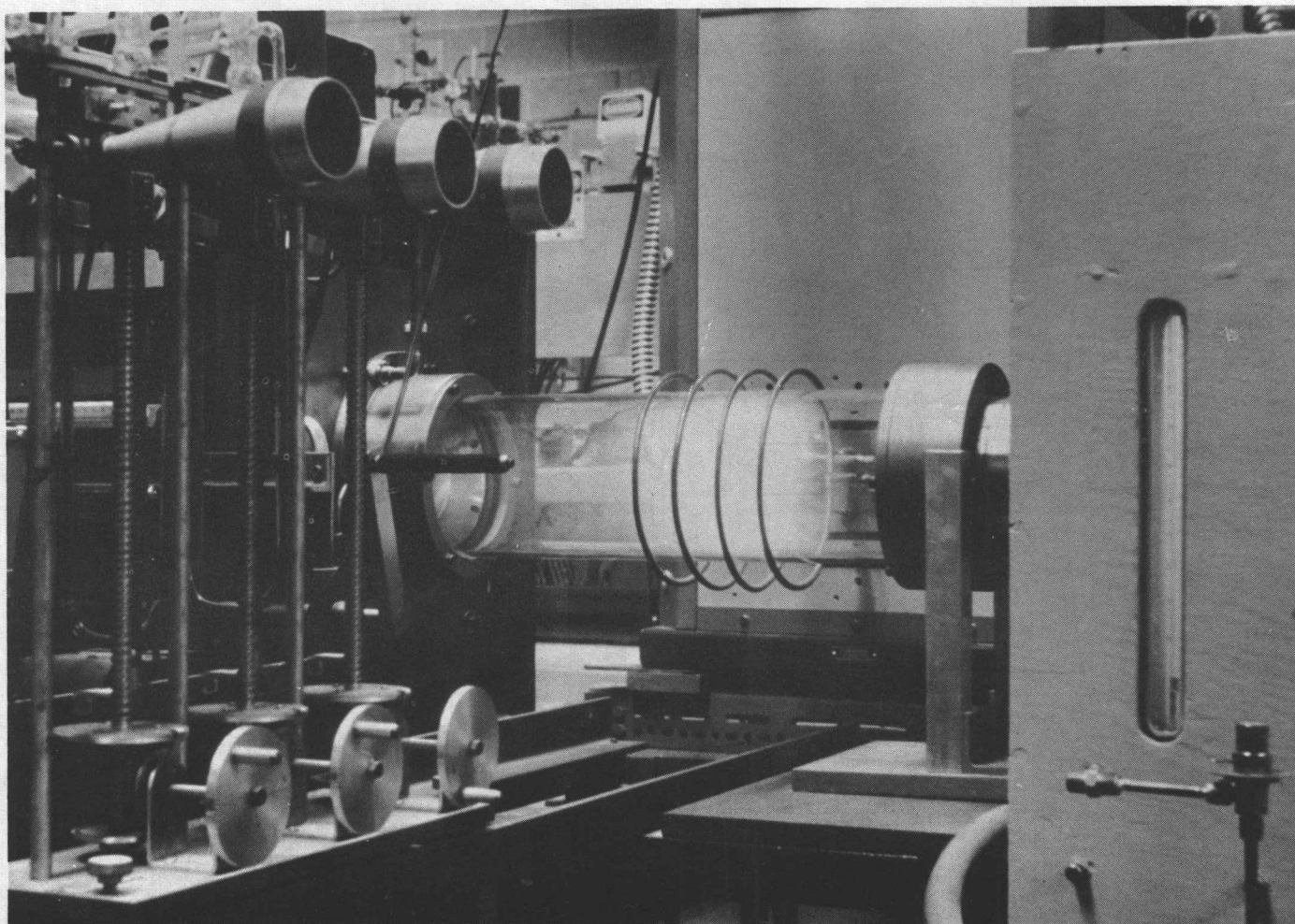


FIG. 3.6 RF DISCHARGE IN STAGNATION CHAMBER, EXHAUSTING INTO TEST CHAMBER THROUGH NOZZLE

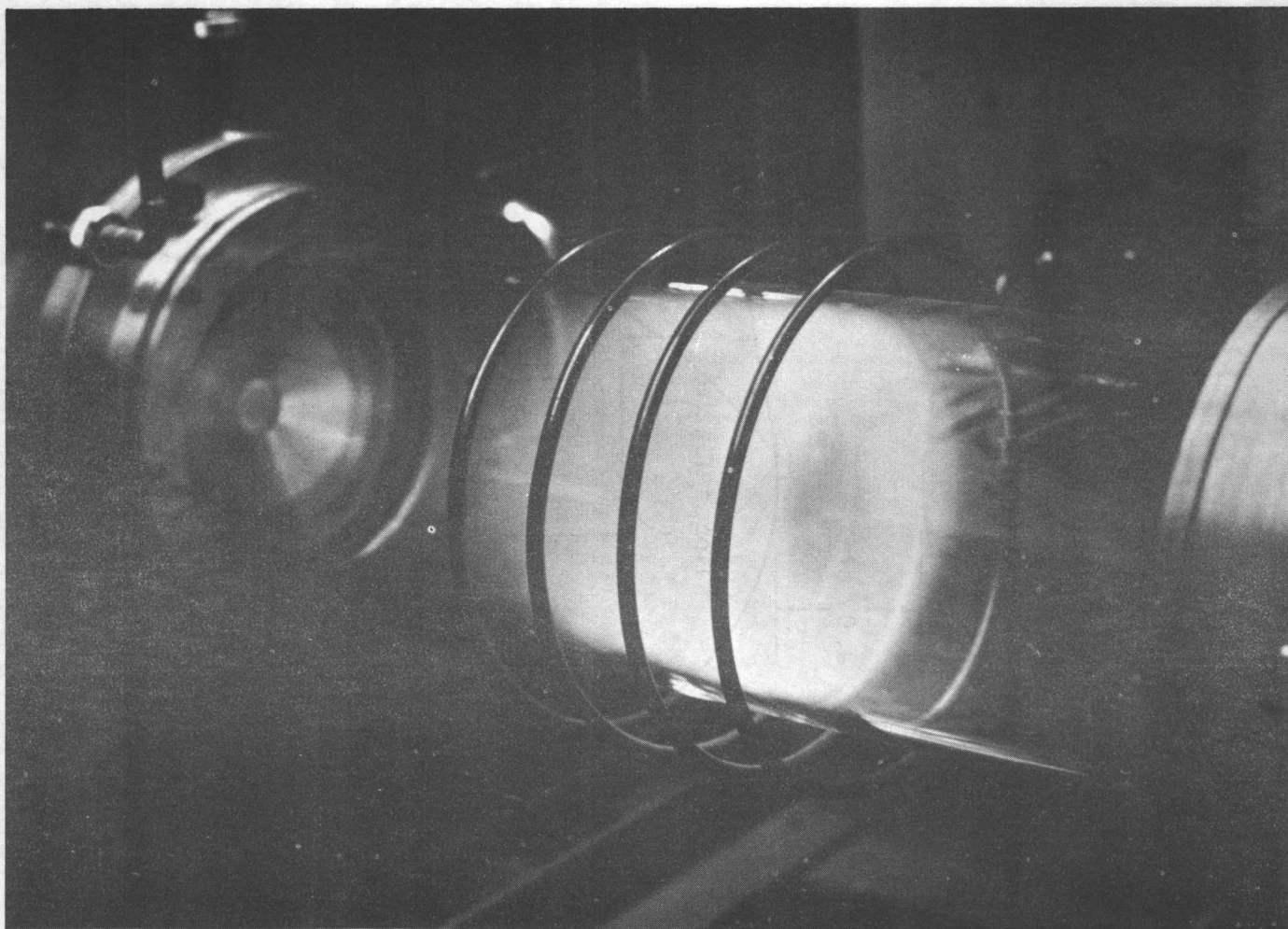


FIG. 3.7 RF DISCHARGE IN STAGNATION CHAMBER, EXHAUSTING INTO TEST CHAMBER THROUGH FREE JET ORIFICE

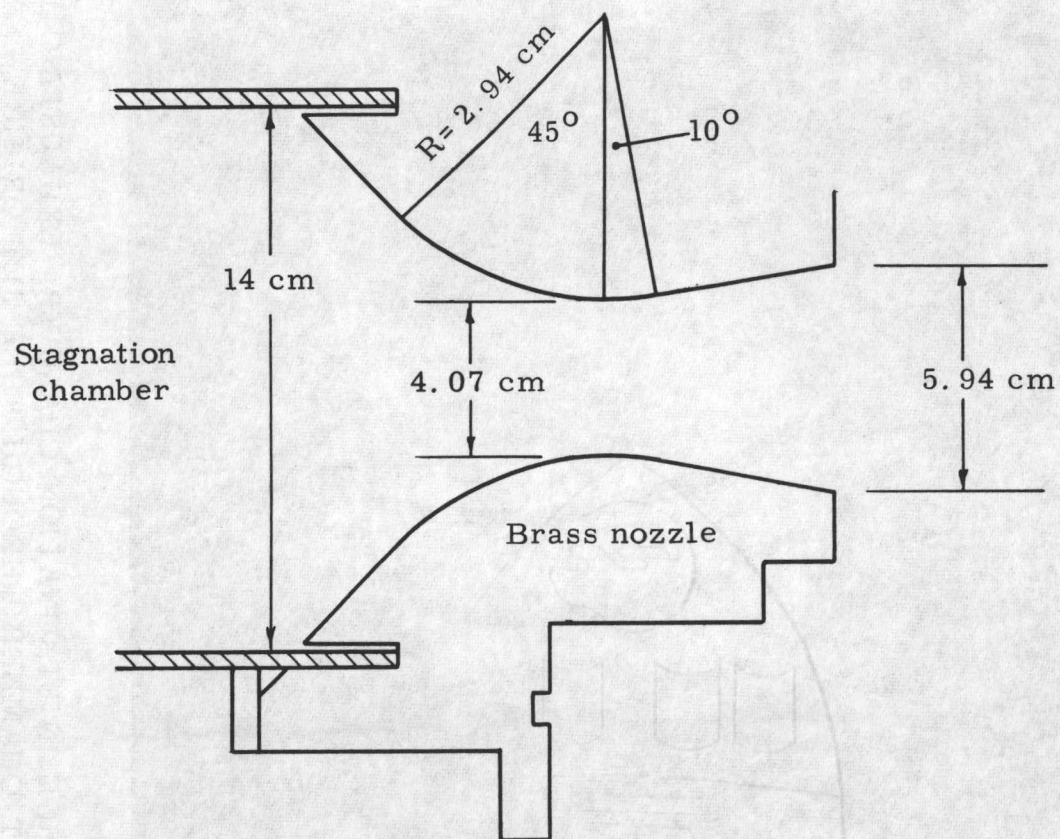


FIG. 3.8 MACH 2.2 NOZZLE

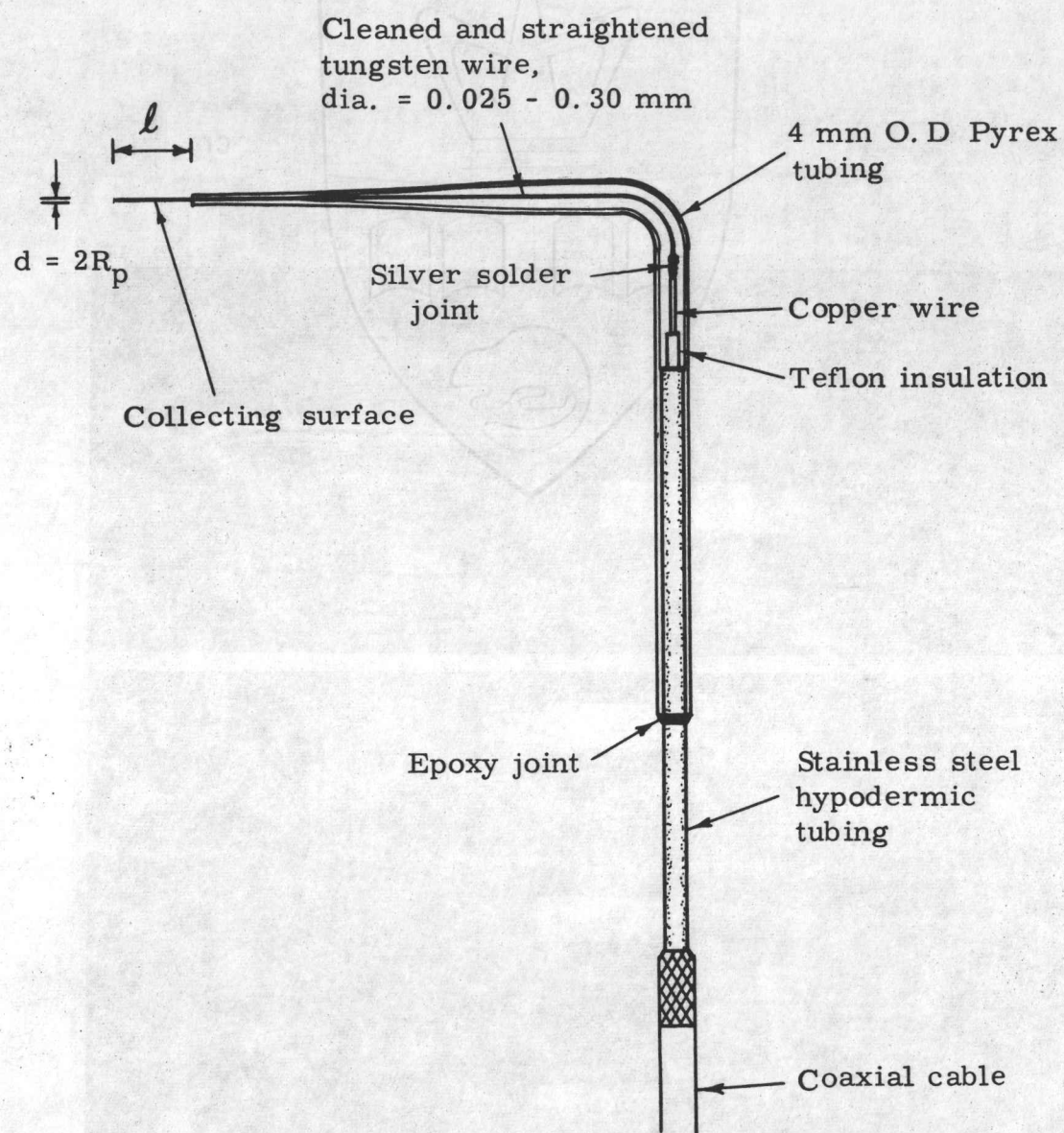


FIG. 3.9 LANGMUIR PROBE CONSTRUCTION

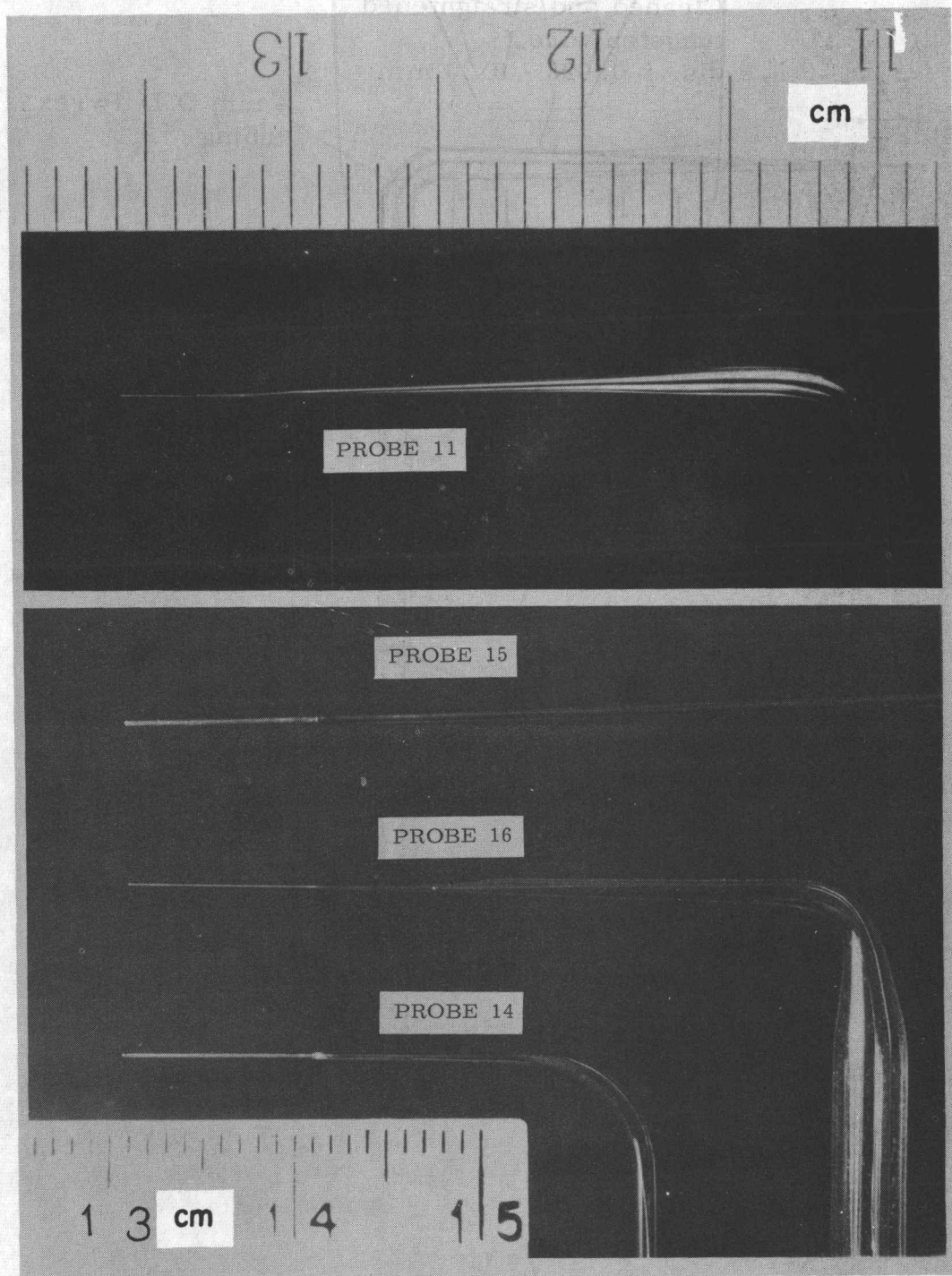


FIG. 3.10 LANGMUIR PROBES

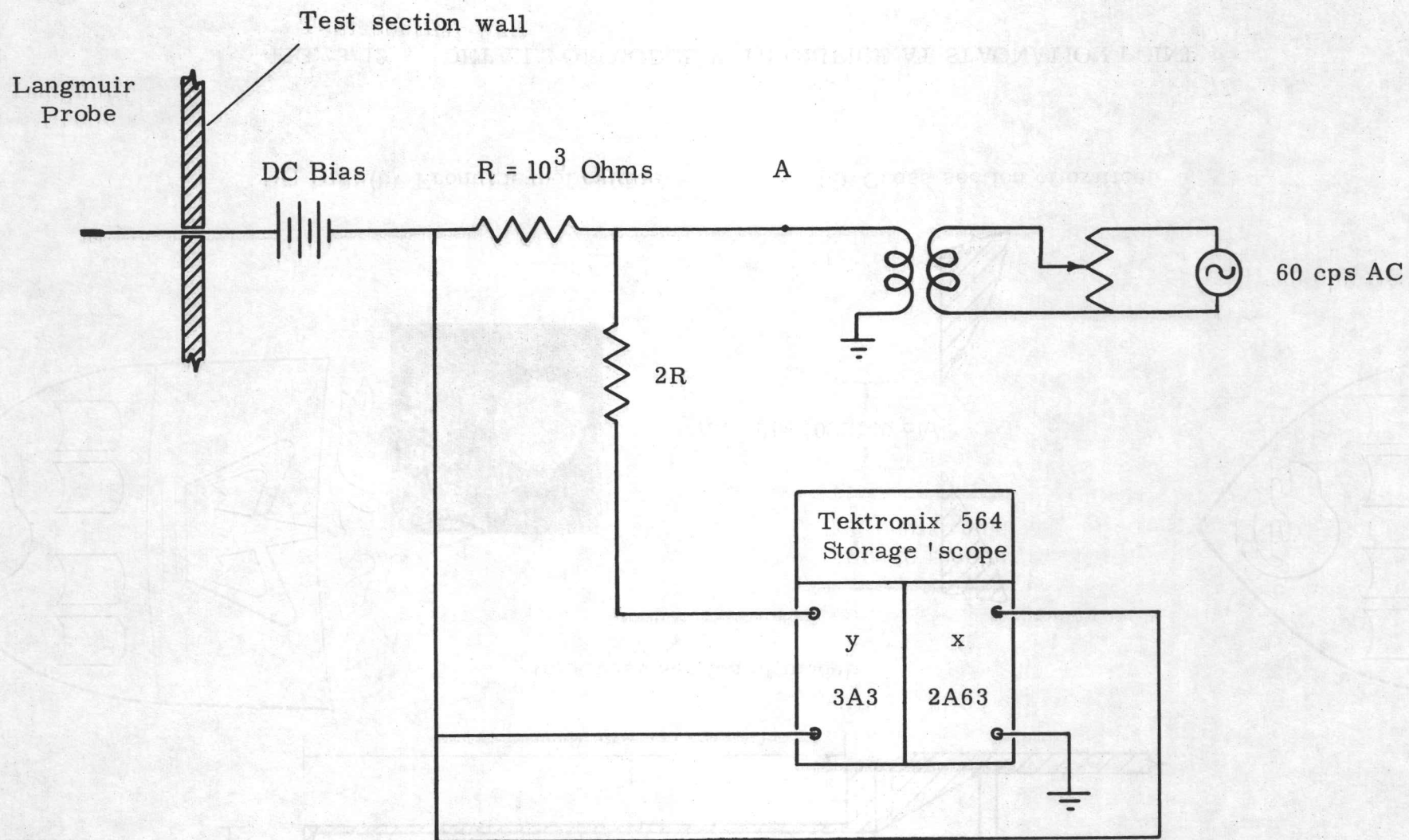
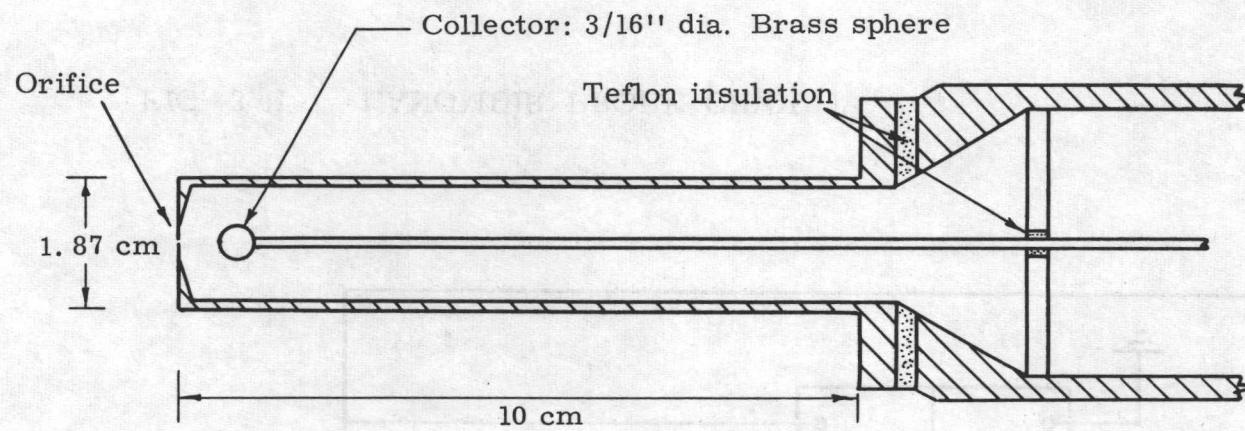
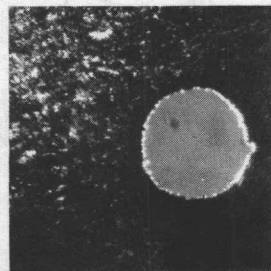


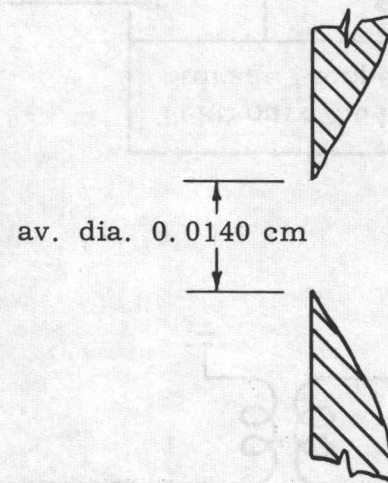
FIG. 3.11 LANGMUIR PROBE CIRCUIT



(a) Cross section of model



(b) Front view of orifice



(c) Cross section of orifice

FIG. 3.12 DETAILS OF MODEL WITH ORIFICE AT STAGNATION POINT

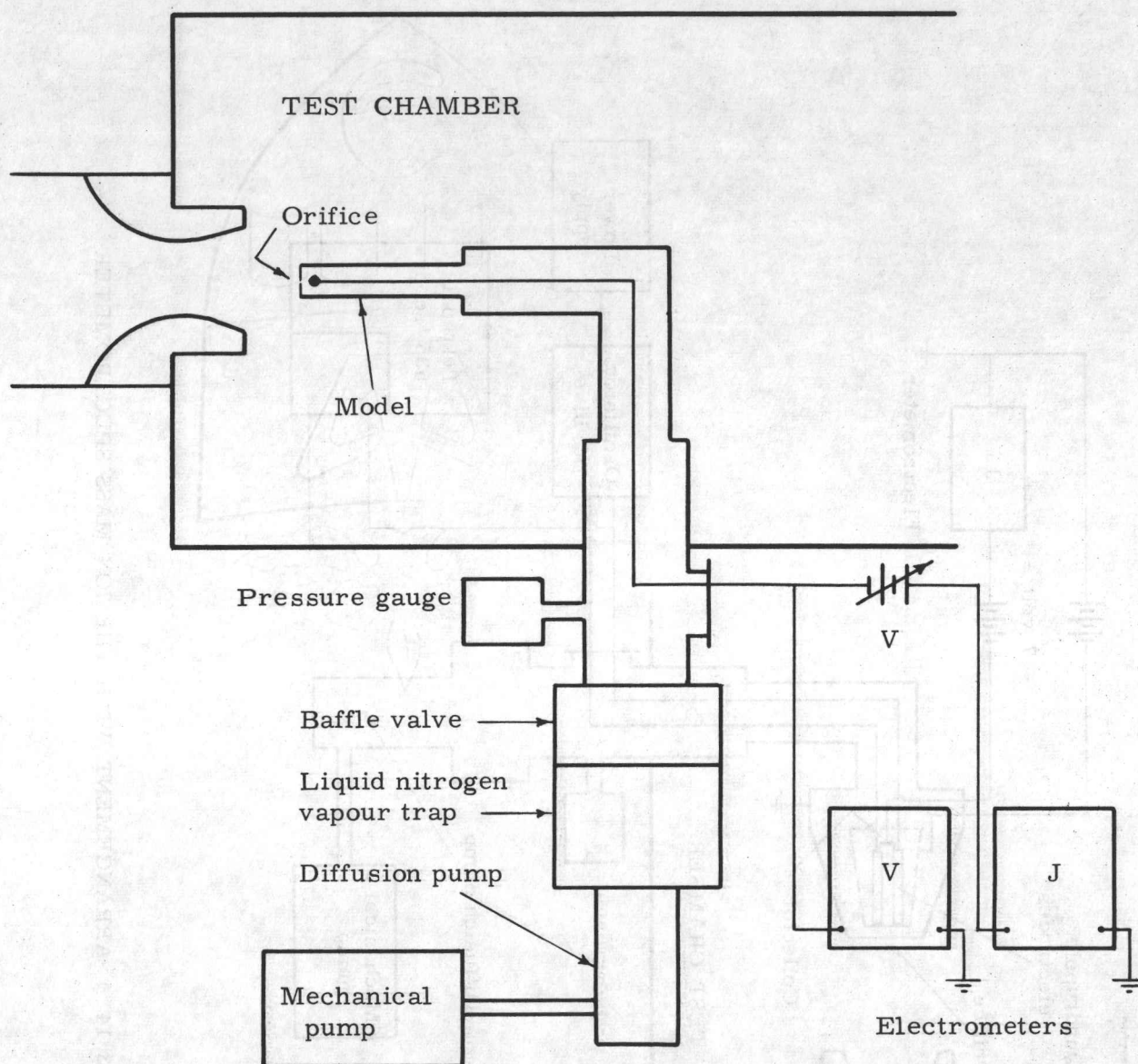


FIG. 3.13 EXPERIMENTAL ARRANGEMENT FOR THE MEASUREMENT OF THE ION FLUX TO THE WALL AT THE STAGNATION POINT.

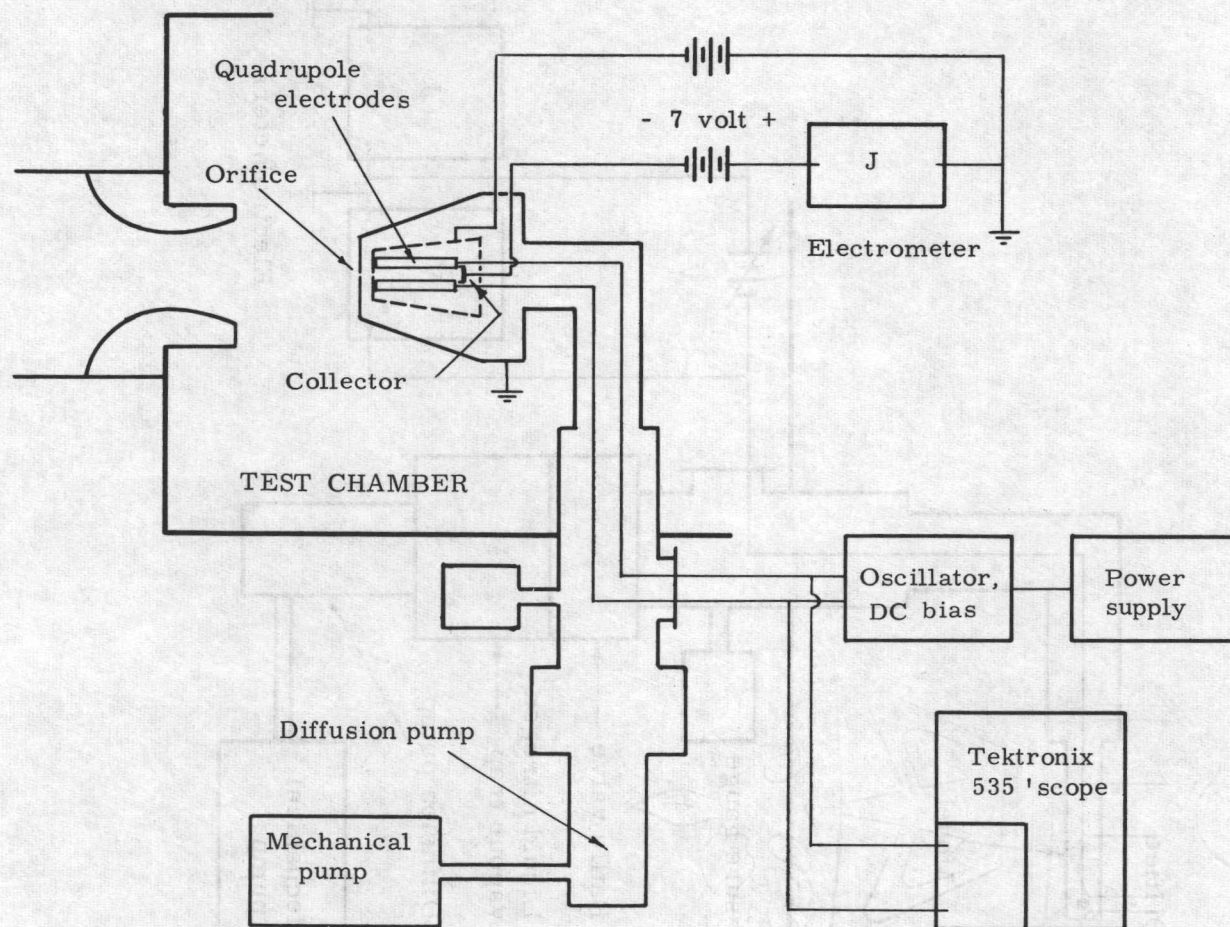


FIG. 3.14 ARRANGEMENT WITH THE ION MASS SPECTROMETER

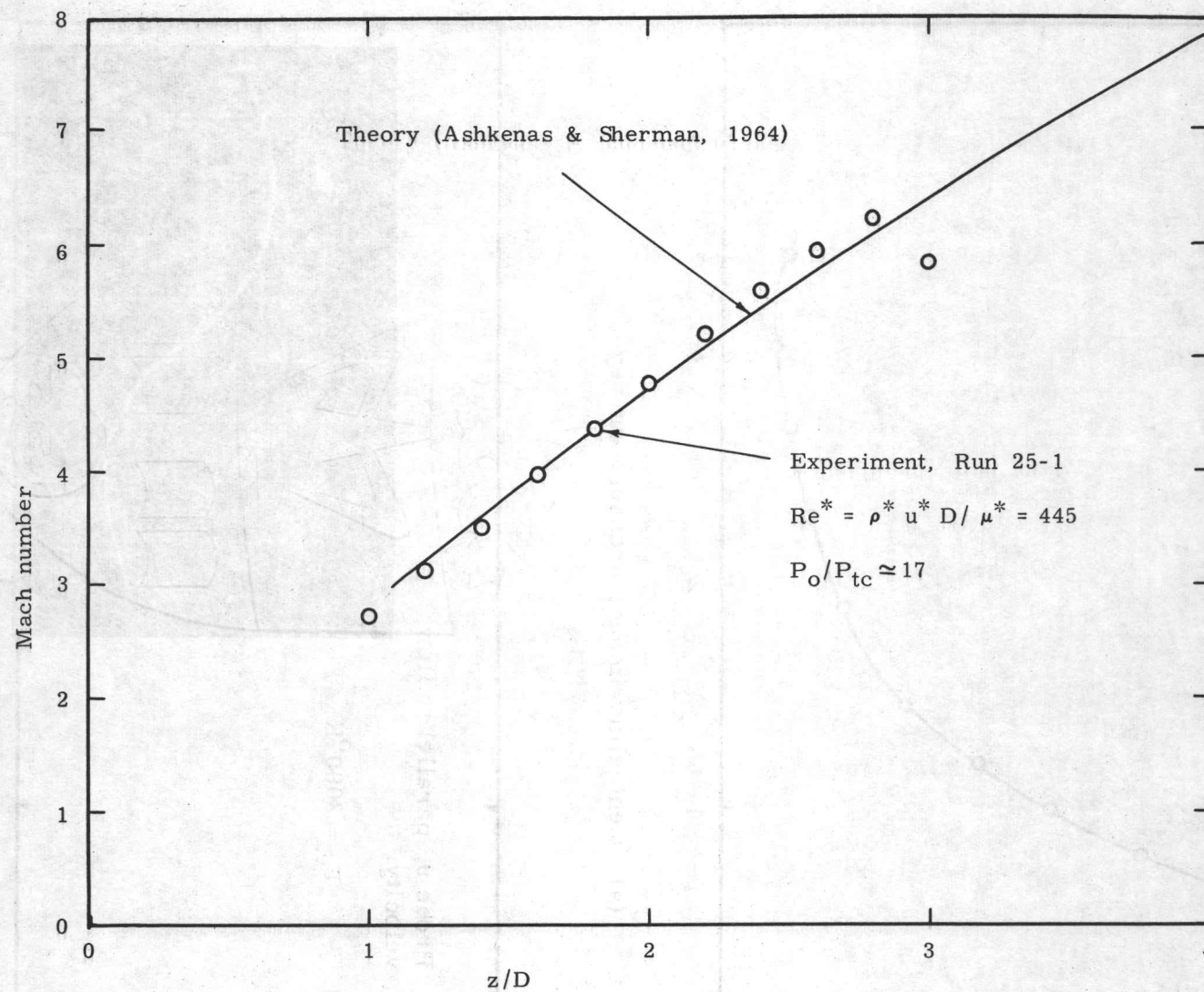
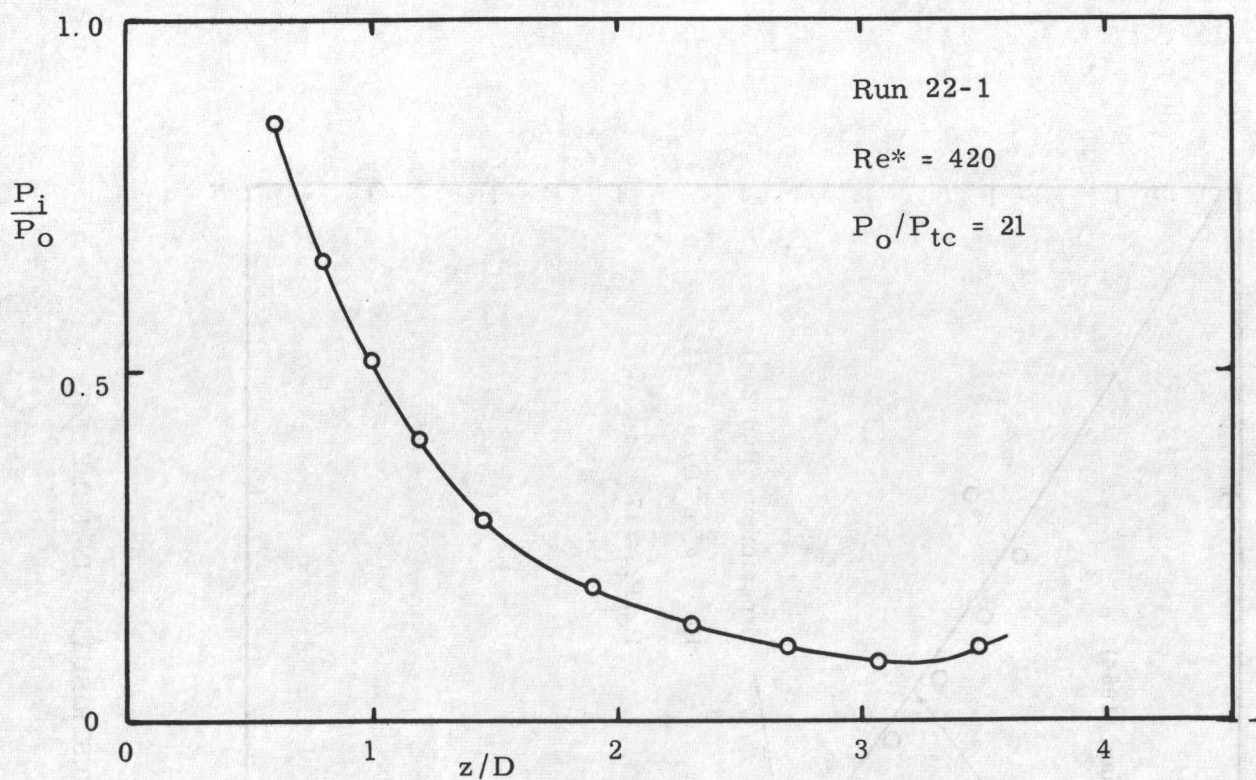
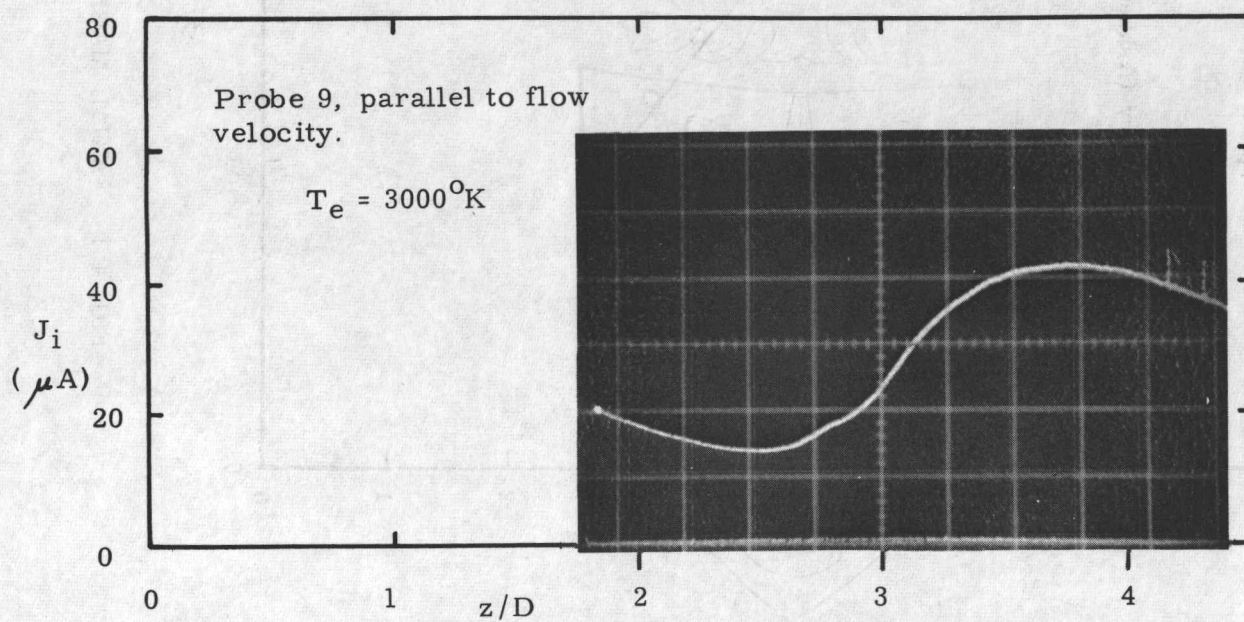


FIG. 4.1 AXIAL MACH NUMBER DISTRIBUTION IN FREE JET

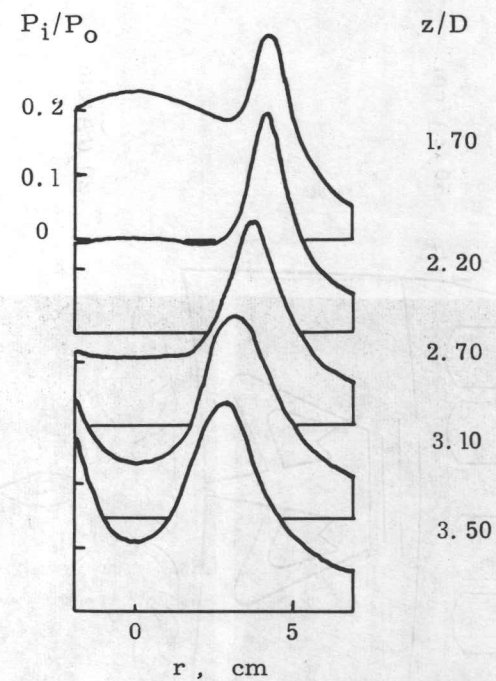
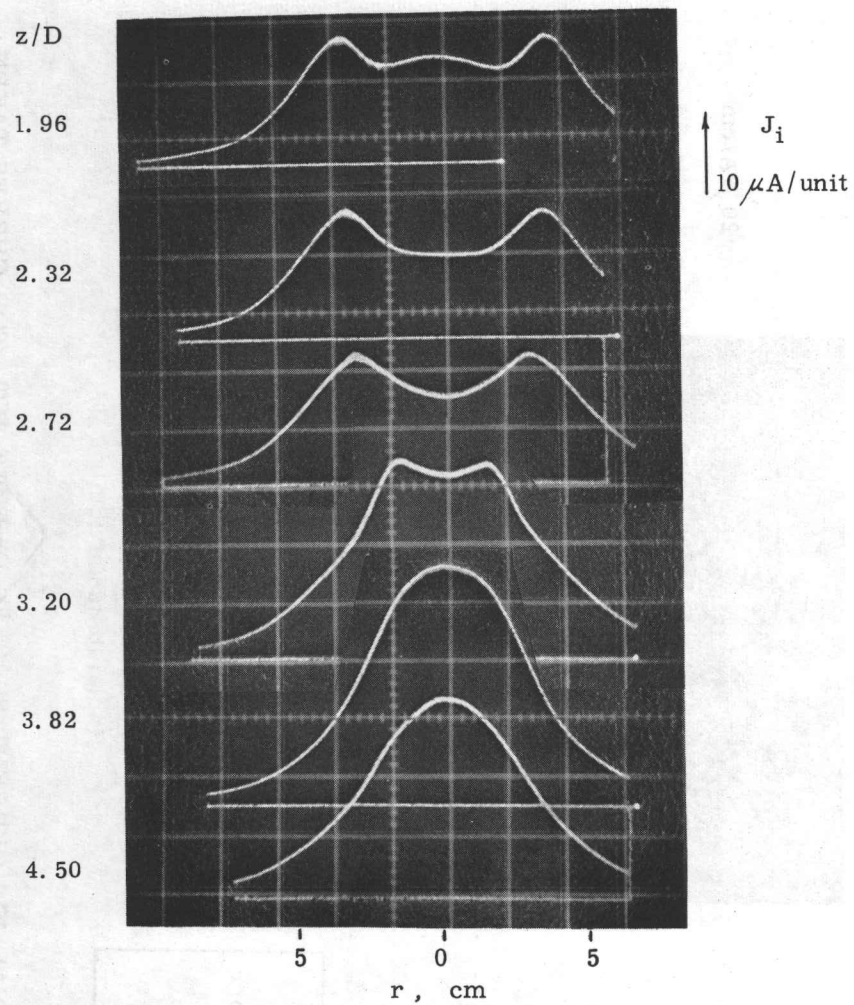


(a) Centreline impact pressure survey



(b) Centreline ion current survey

FIG. 4.2 AXIAL SURVEYS IN FREE JET



Run 22-1

$Re^* = 420$

$P_0/P_{tc} \approx 21$

FIG. 4.3 RADIAL SURVEYS IN FREE JET

Axial station

Current scale

$z/D = 1.60$

J_i $50 \mu A/cm$

$z/D = 2.20$

$20 \mu A/cm$

$z/D = 2.60$

$20 \mu A/cm$

$z/D = 3.40$

$20 \mu A/cm$

Run 34-1

$P_O = 0.747$ Torr

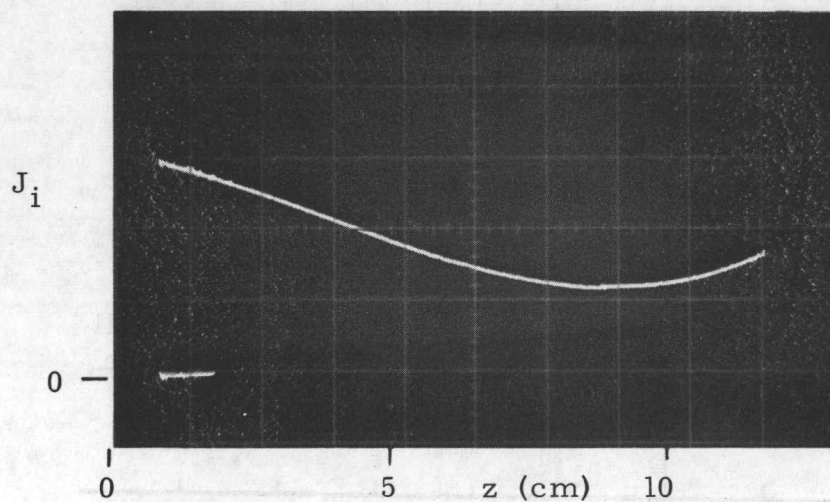
$T_O = 558$ °K

Probe 16

10 5 0 5

Radial distance (cm)

FIG. 4.4 RADIAL ION CURRENT SURVEYS IN FREE JET. TOP CURVES TAKEN WITH PROBE PARALLEL, AND BOTTOM ONES WITH IT PERPENDICULAR, TO THE FLOW AXIS.



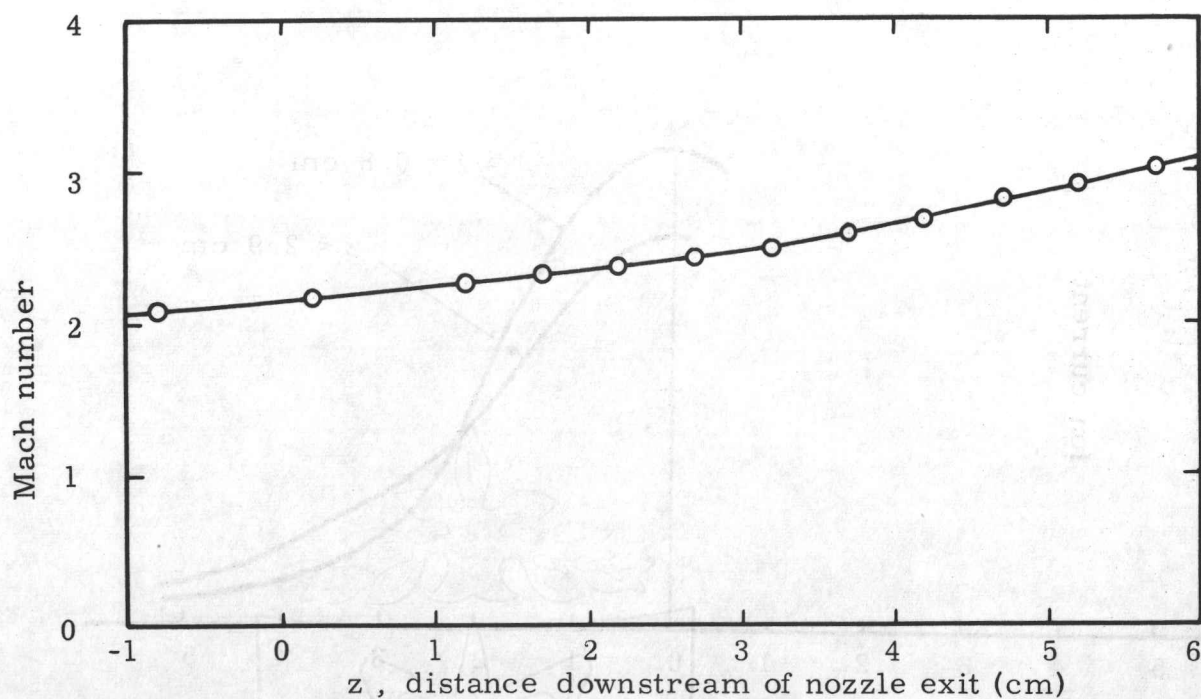
Run 4-1

$P_o = 1.28$ Torr

$T_o = 765^\circ\text{K}$

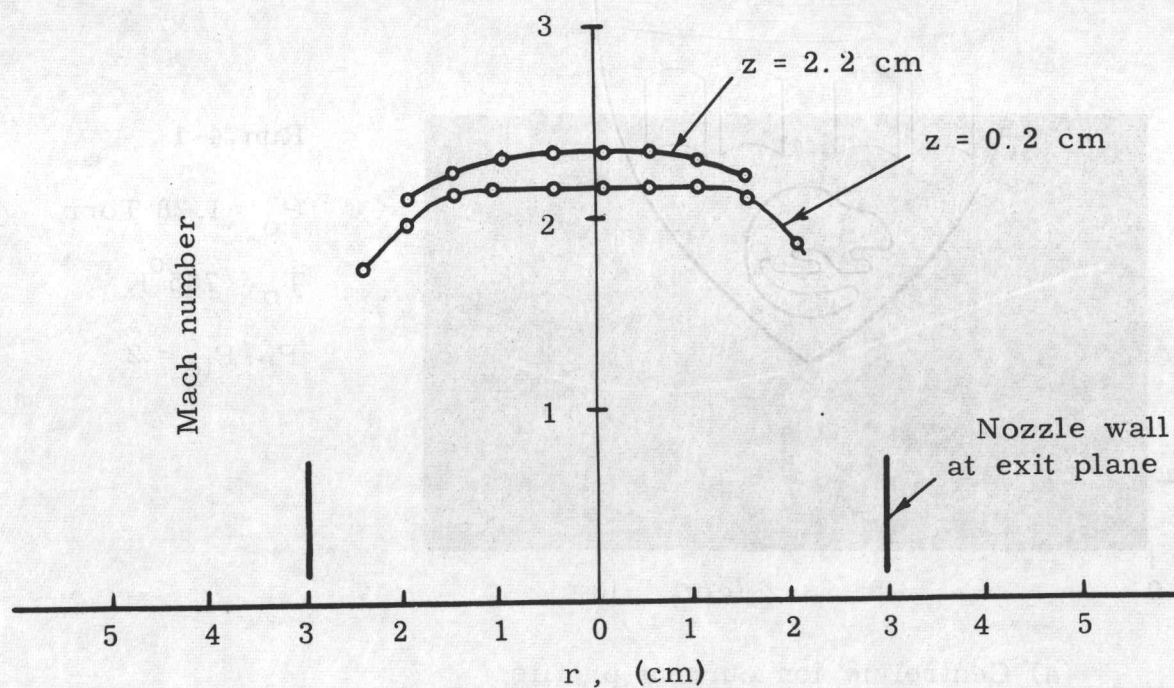
$P_e/P_{tc} = 2$

(a) Centreline ion current profile
(with parallel probe)

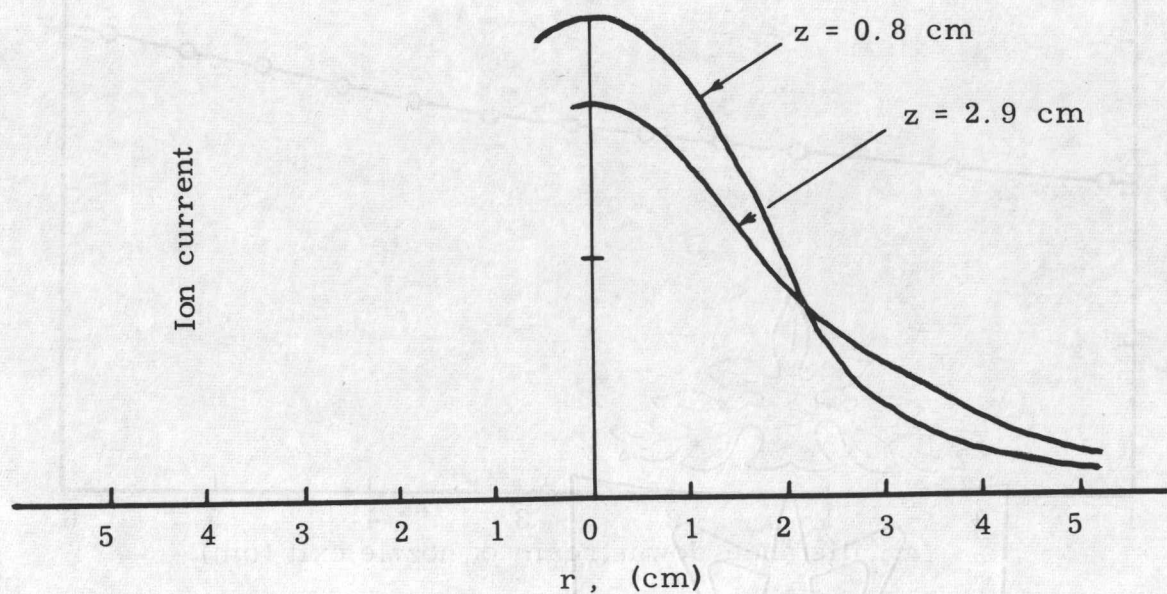


(b) Centreline Mach number profile

FIG. 4.5 AXIAL SURVEYS IN NOZZLE FLOW FIELD

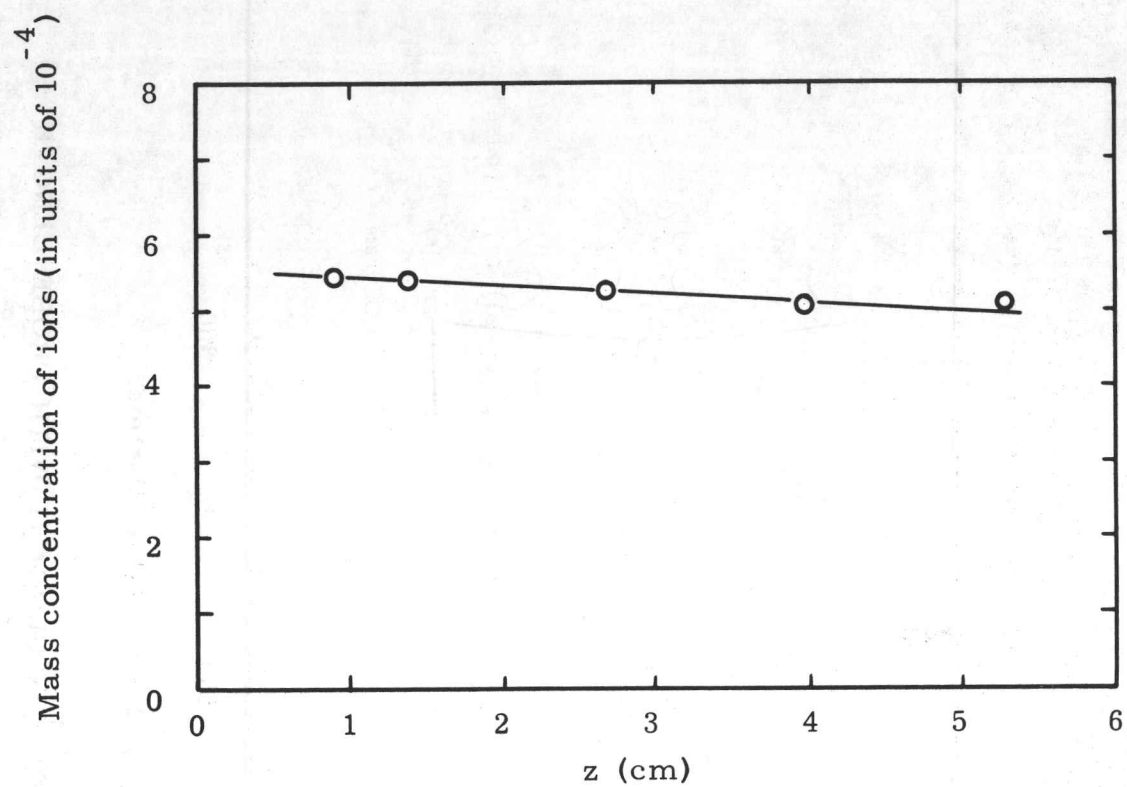


(a) Radial Mach number profiles



(b) Radial ion current profiles
(with parallel probe)

FIG. 4.6 RADIAL SURVEYS OF NOZZLE FLOW FIELD
 $P_0 = 1.28 \text{ Torr}$, $T_0 = 765^\circ\text{K}$



Run 4-1

FIG. 4.7 PROFILE OF ION CONCENTRATION ALONG CENTRELINE OF NOZZLE FLOW

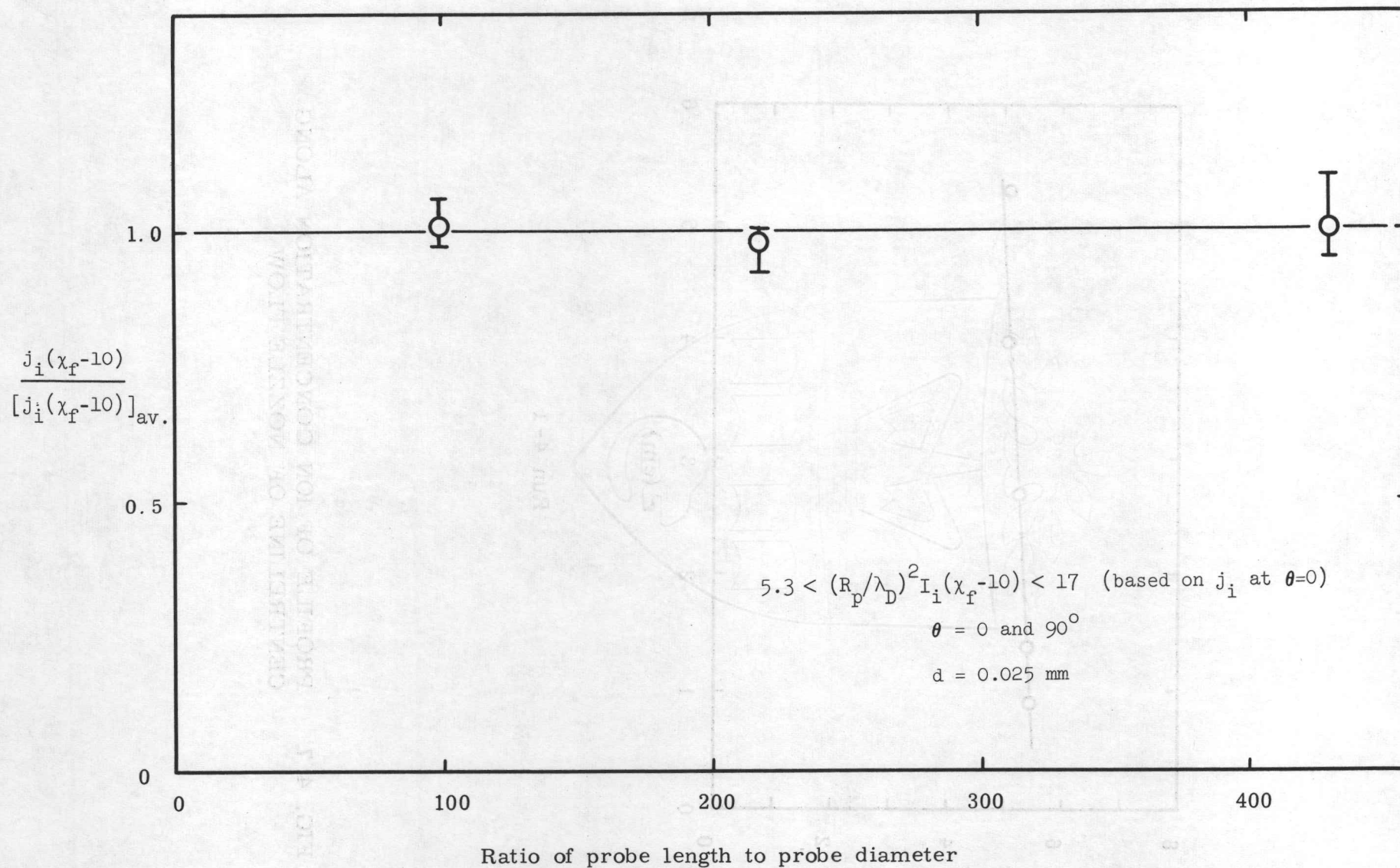


FIG. 5.1 VARIATION OF ION CURRENT WITH PROBE LENGTH

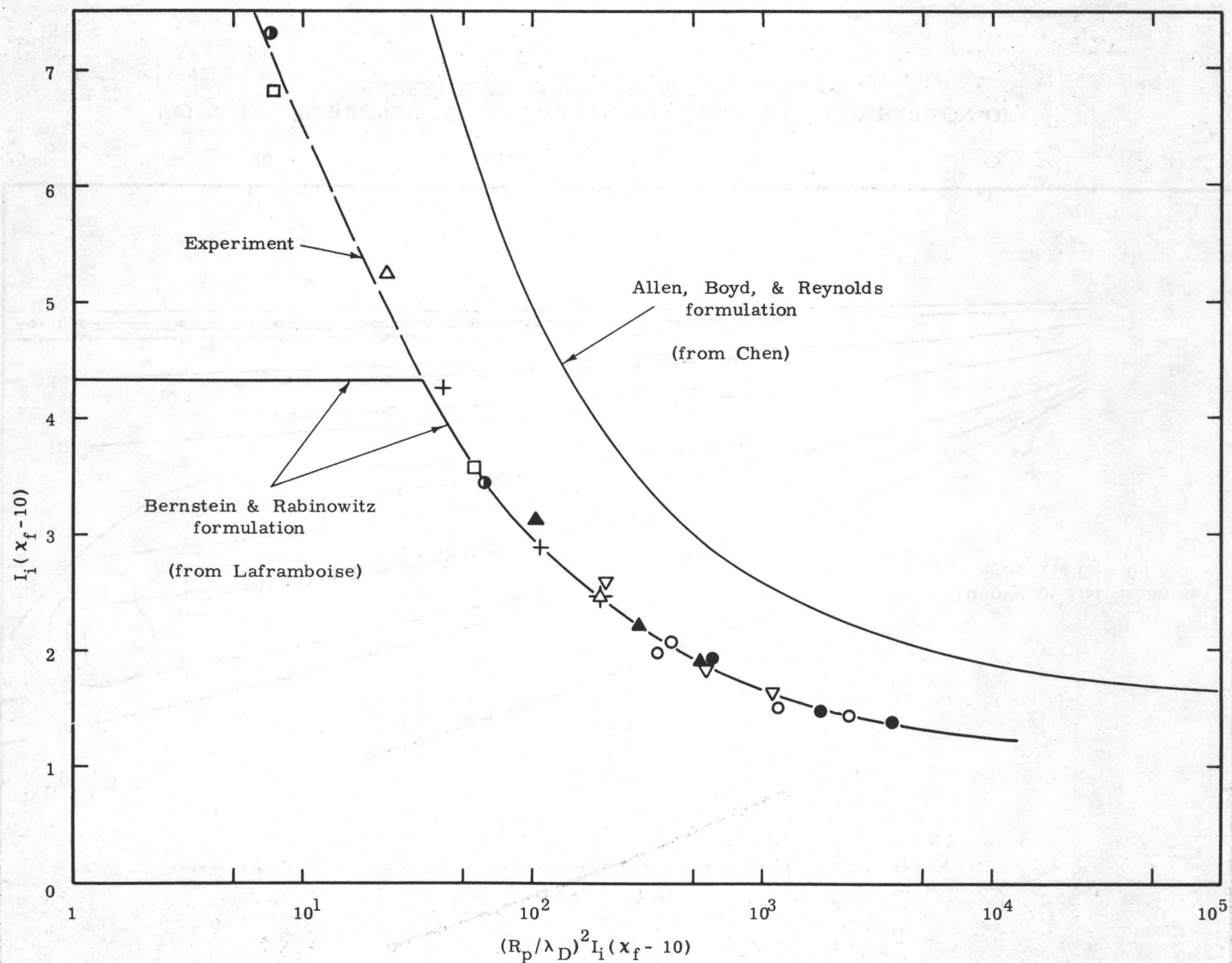


FIG. 5.2 VARIATION OF $I_i(x_f - 10)$ WITH $(R_p/\lambda_D)^2 I_i(x_f - 10)$: COMPARISON OF EXPERIMENT AND THEORY FOR $T_i/T_e \ll 1$

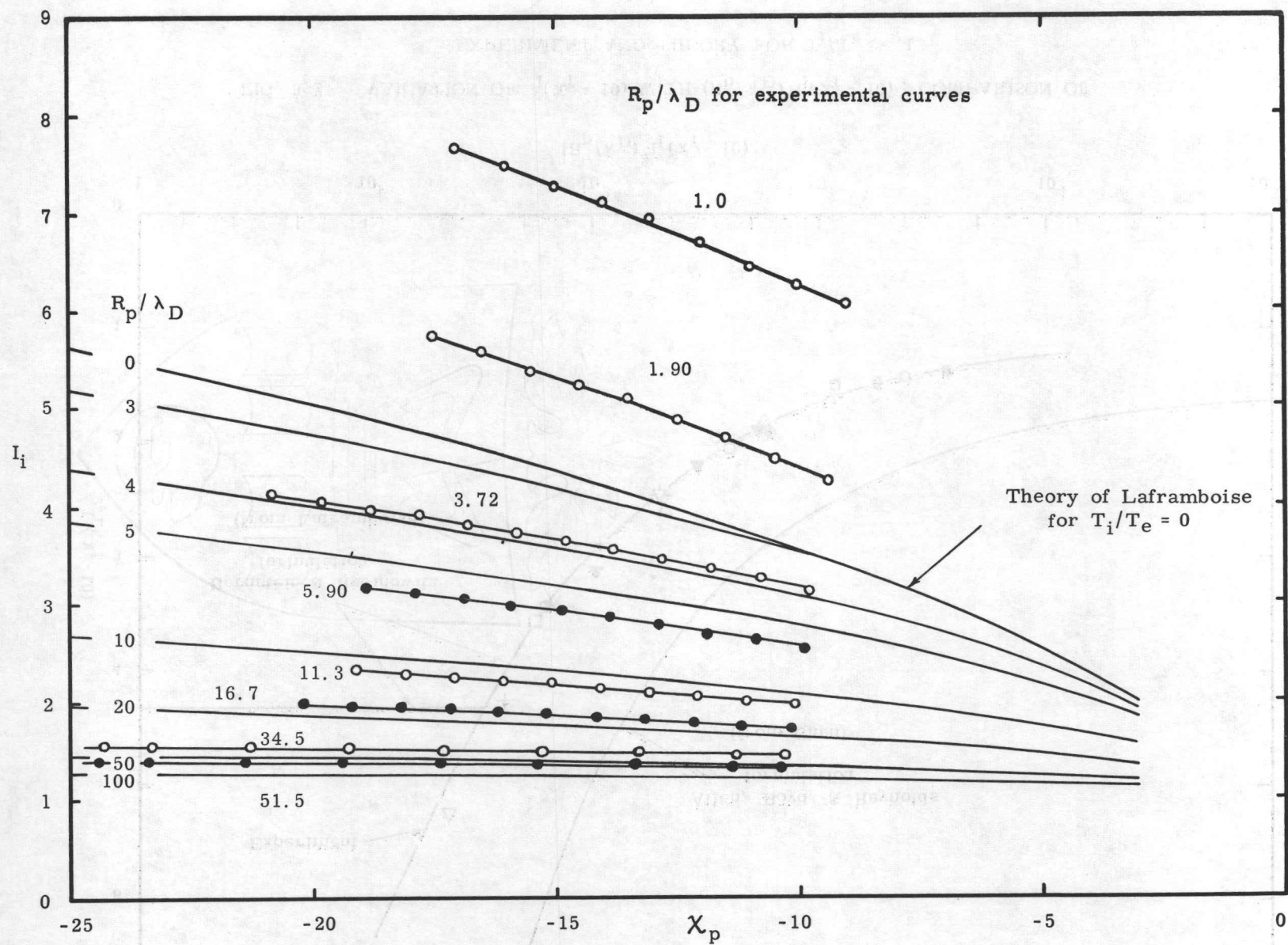


FIG. 5.3 VARIATION OF I_i WITH PROBE POTENTIAL : COMPARISON OF EXPERIMENT AND THEORY FOR $T_i/T_e \ll 1$

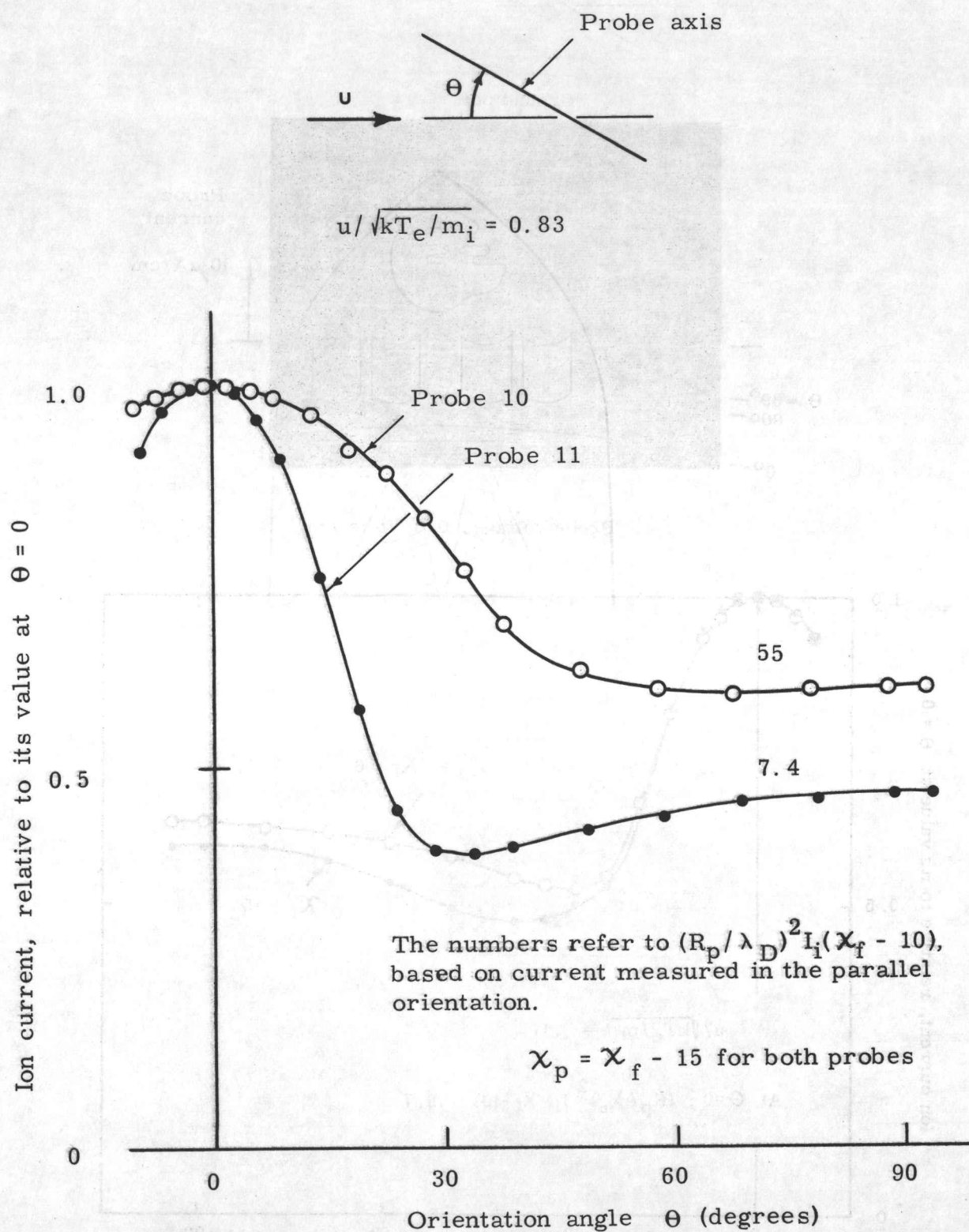


FIG. 5.4 VARIATION OF ION CURRENT WITH ORIENTATION FOR TWO PROBES WITH DIFFERENT DIAMETERS UNDER THE SAME FLOW CONDITIONS

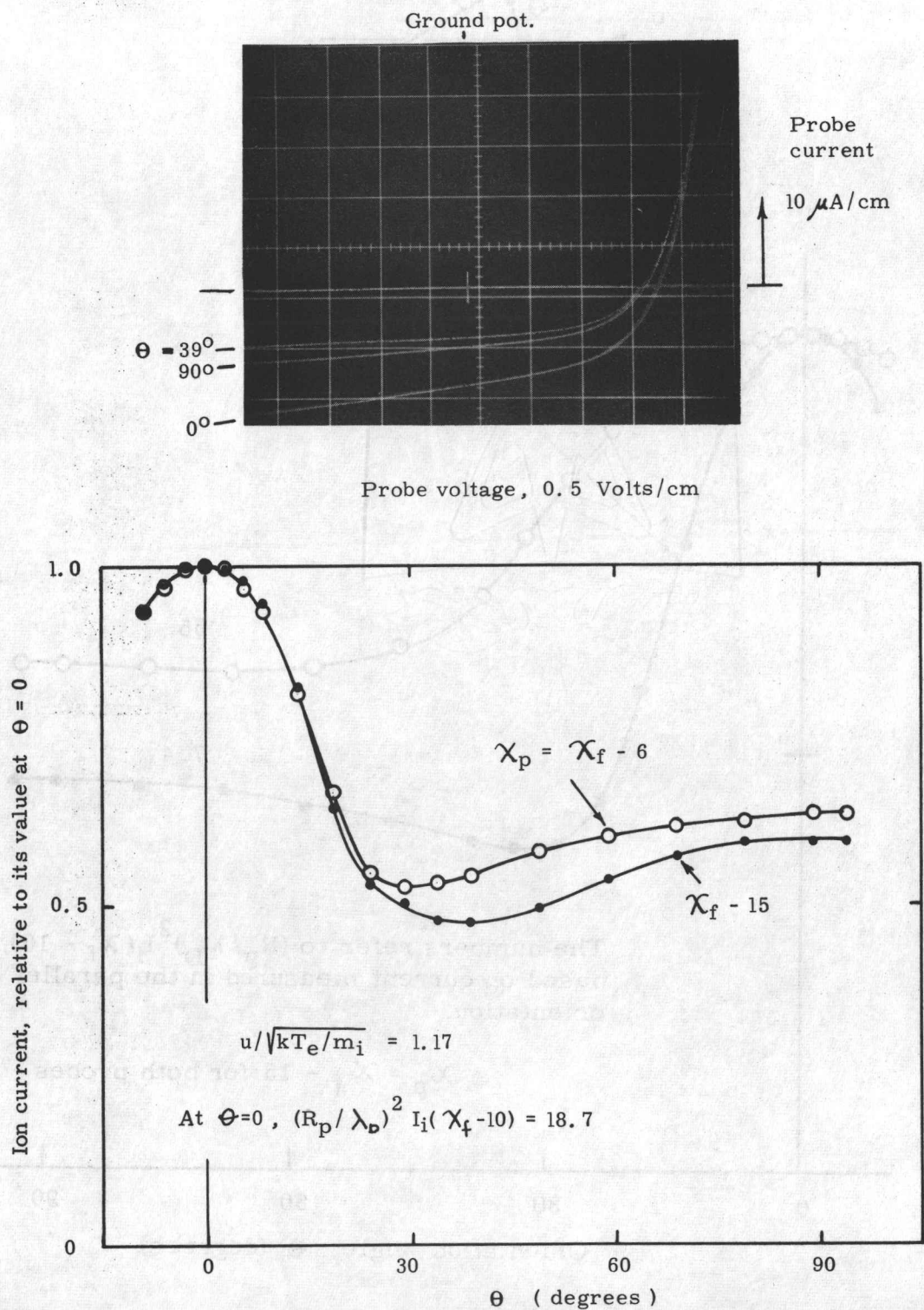


FIG. 5.5 VARIATION OF ION CURRENT WITH ORIENTATION AT TWO DIFFERENT PROBE POTENTIALS

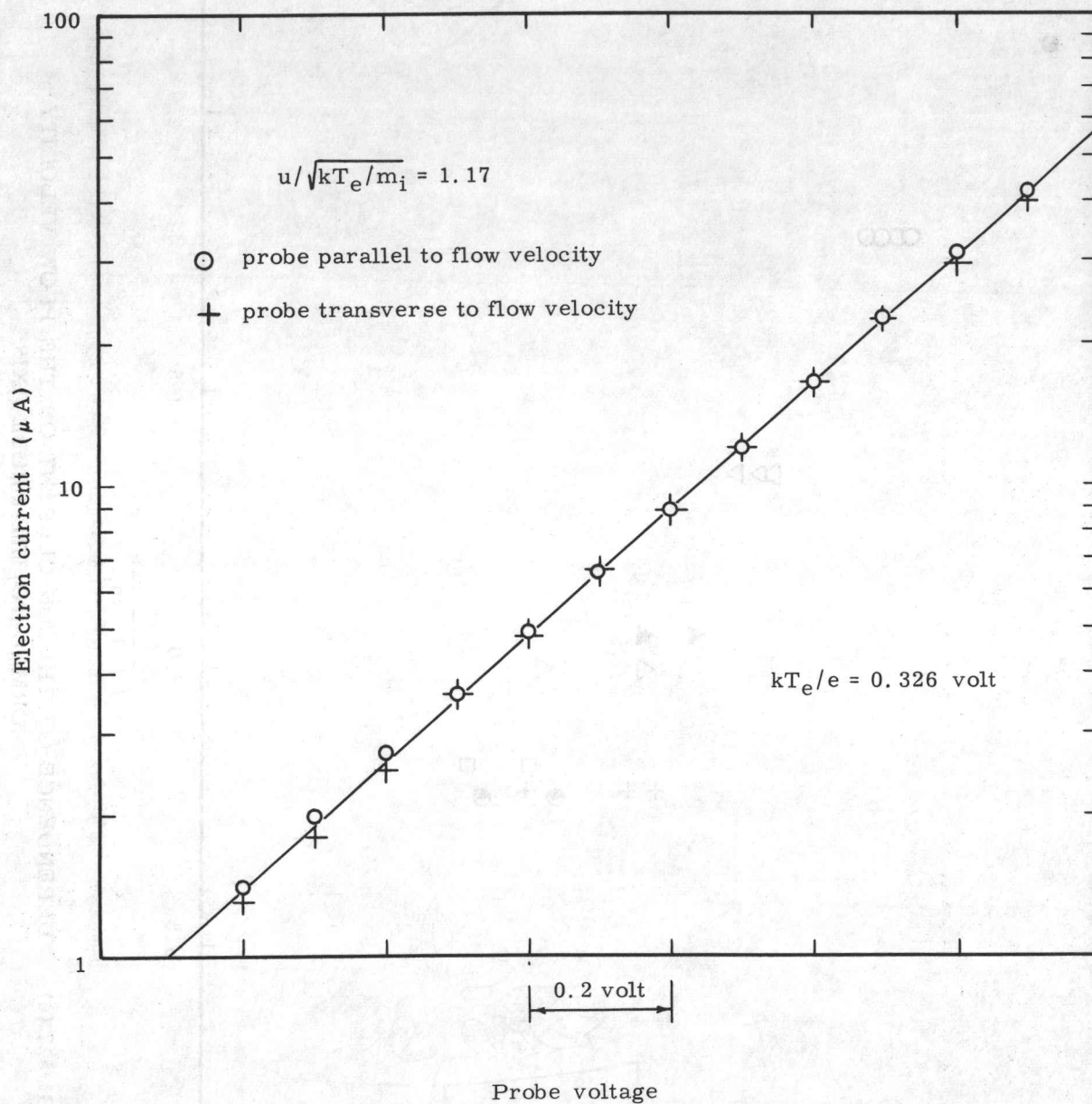


FIG. 5.6 COMPARISON OF ELECTRON CURRENT MEASURED WITH PROBE ORIENTED PARALLEL AND PERPENDICULAR TO FLOW VELOCITY (DATA TAKEN FROM OSCILLOGRAM ON FIG. 5.5)

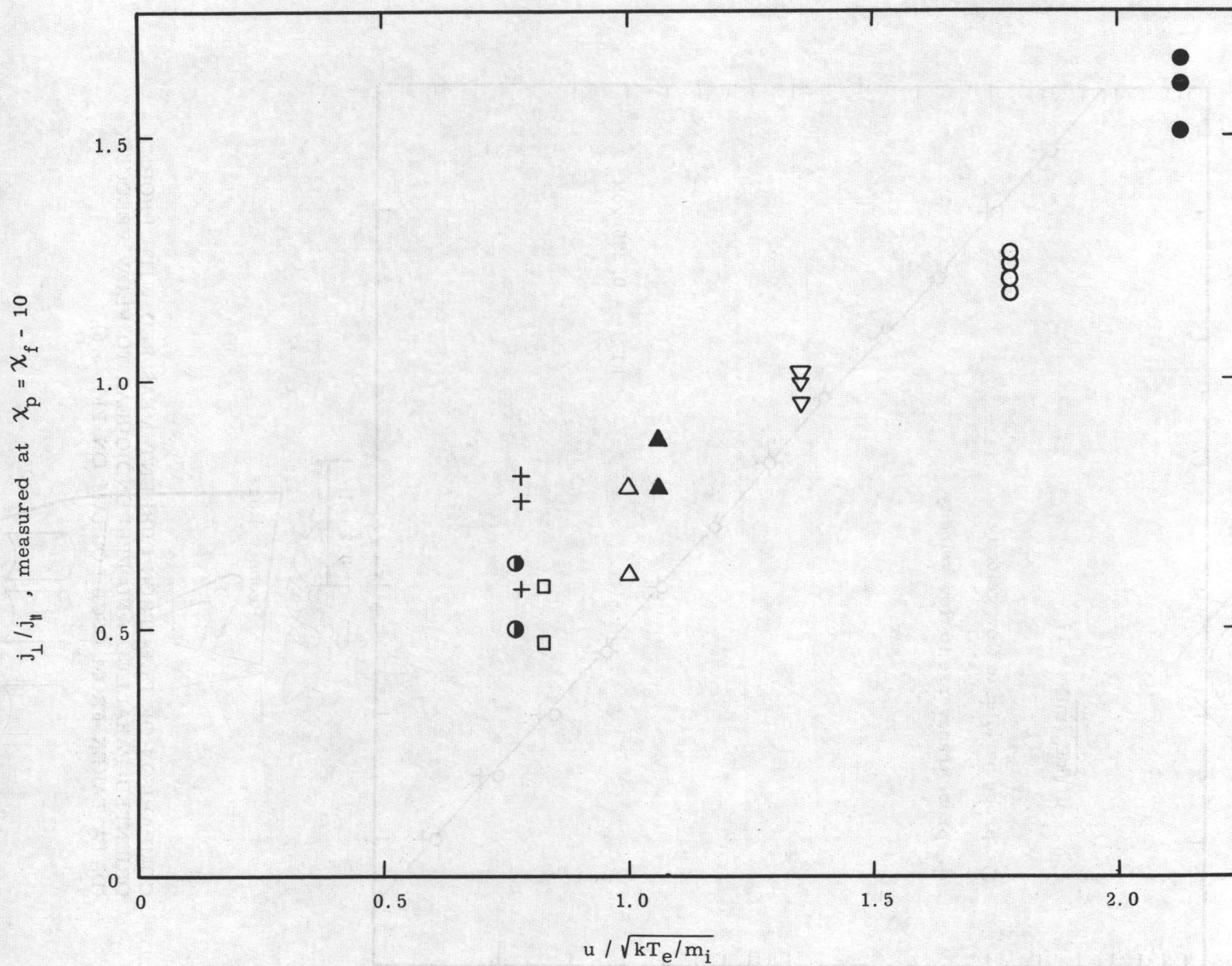


FIG. 5.7 (a) DEPENDENCE OF THE ION CURRENT ON THE FLOW VELOCITY NORMAL TO THE PROBE AXIS

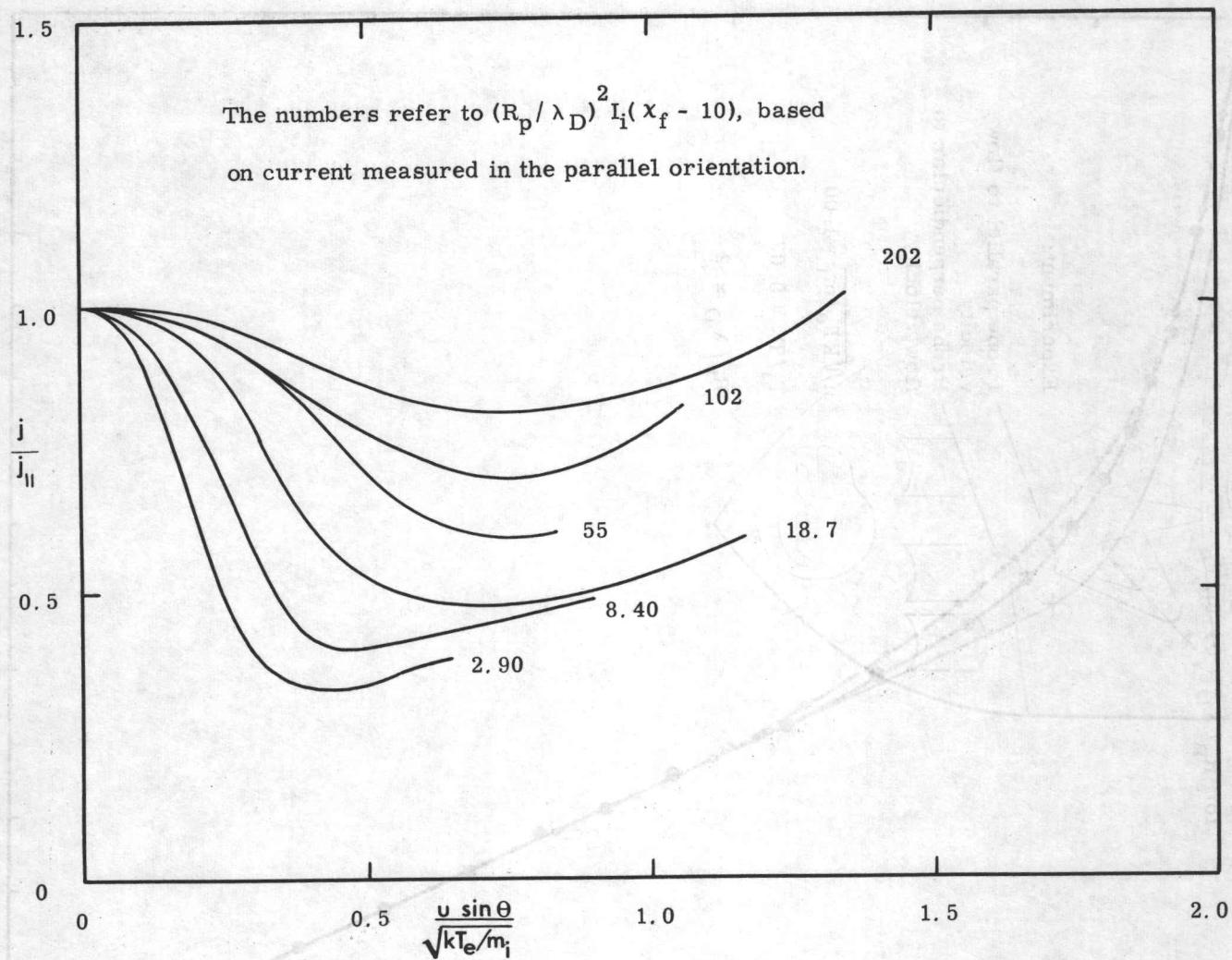


FIG. 5.7 (b) DEPENDENCE OF THE ION CURRENT ON THE FLOW VELOCITY NORMAL TO THE PROBE AXIS

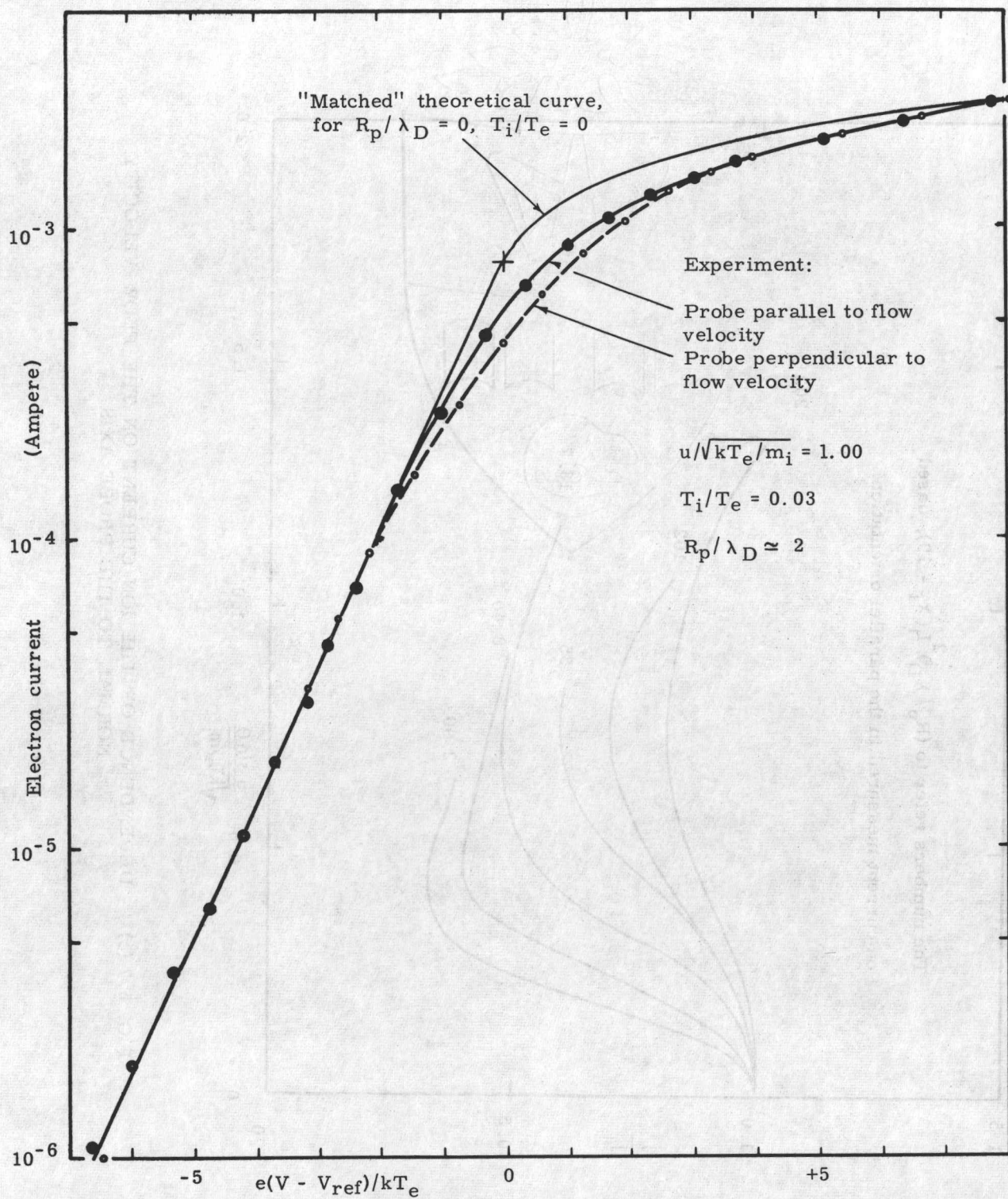


FIG. 5.8 SEMI-LOG PLOT OF ELECTRON CURRENT VS PROBE POTENTIAL

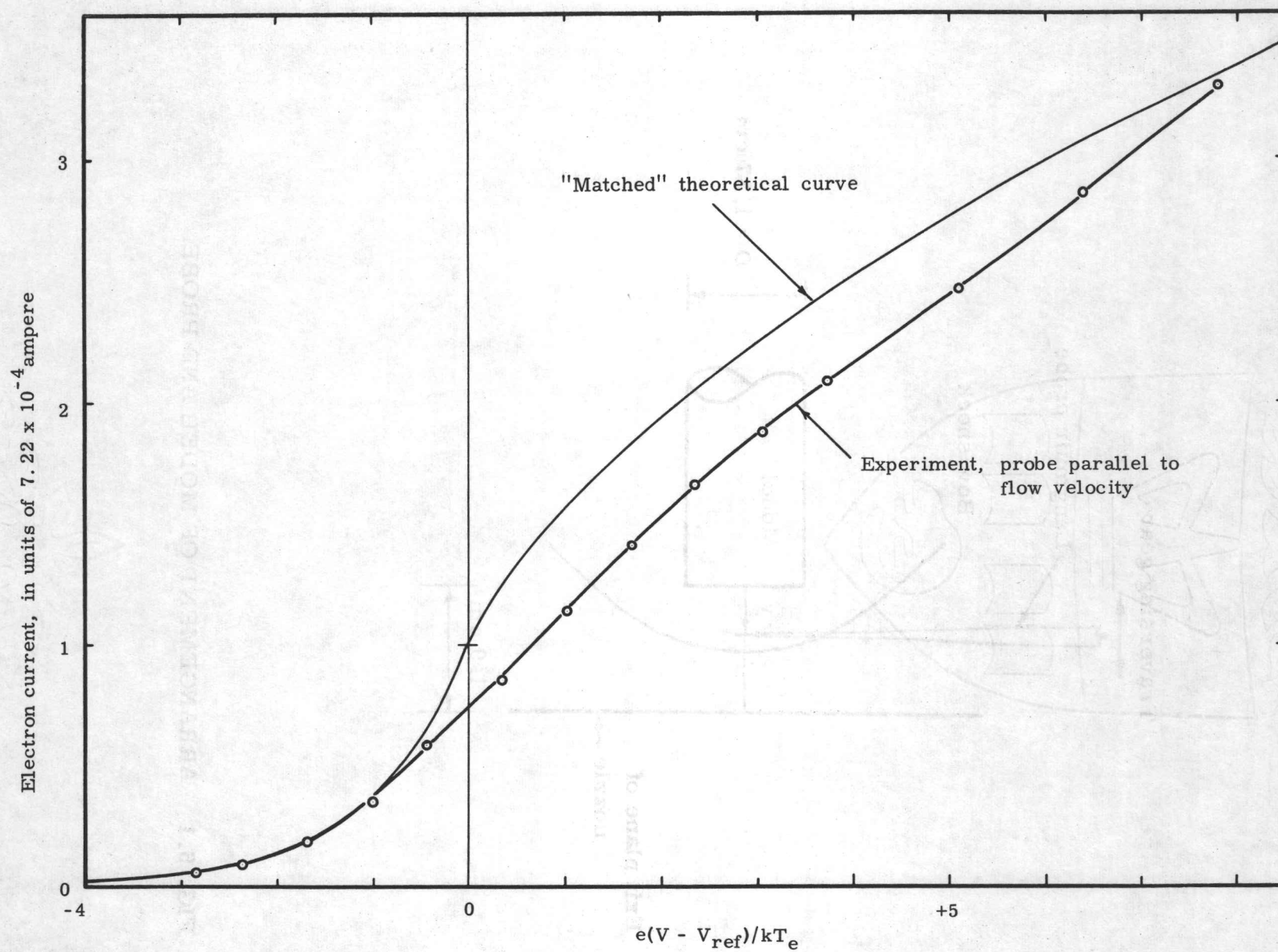


FIG. 5.9 LINEAR PLOT OF ELECTRON CURRENT VS PROBE POTENTIAL
(SAME DATA AS IN FIG. 5.8)

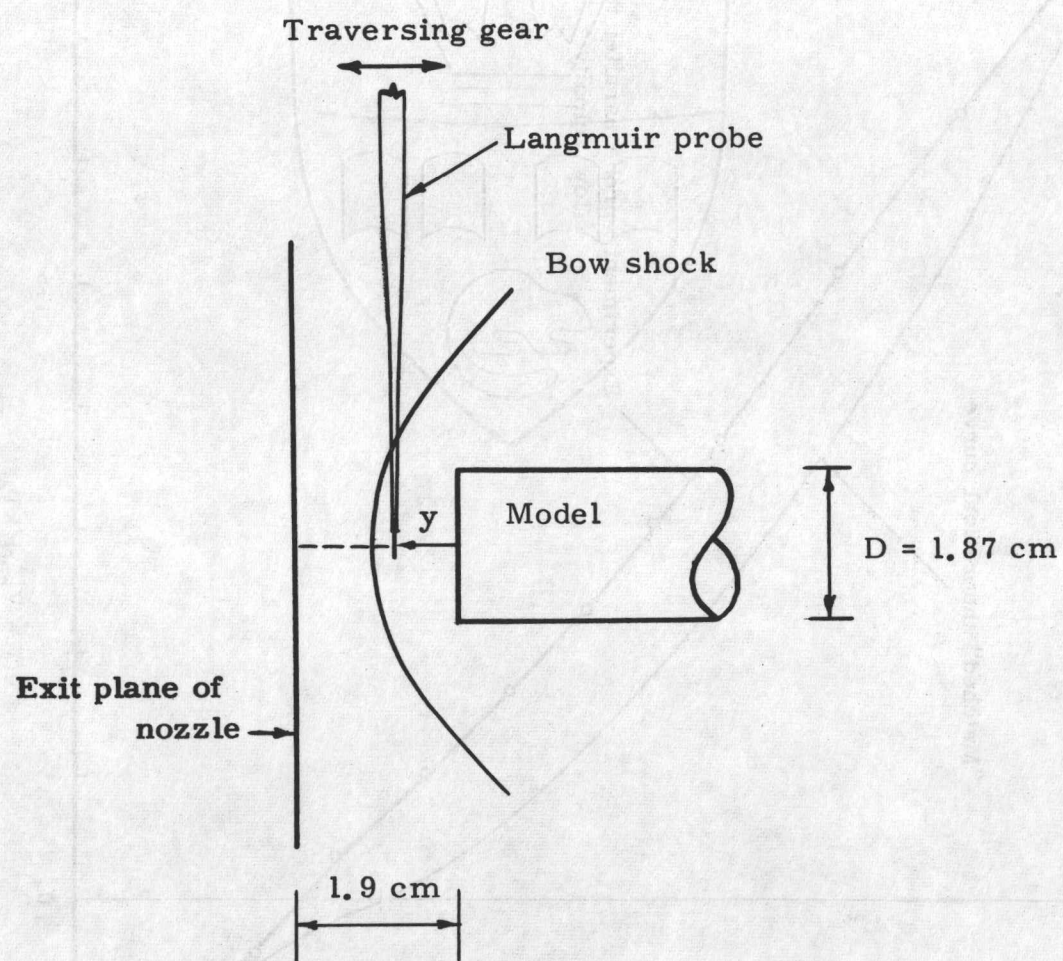


FIG. 6.1 ARRANGEMENT OF MODEL AND PROBE

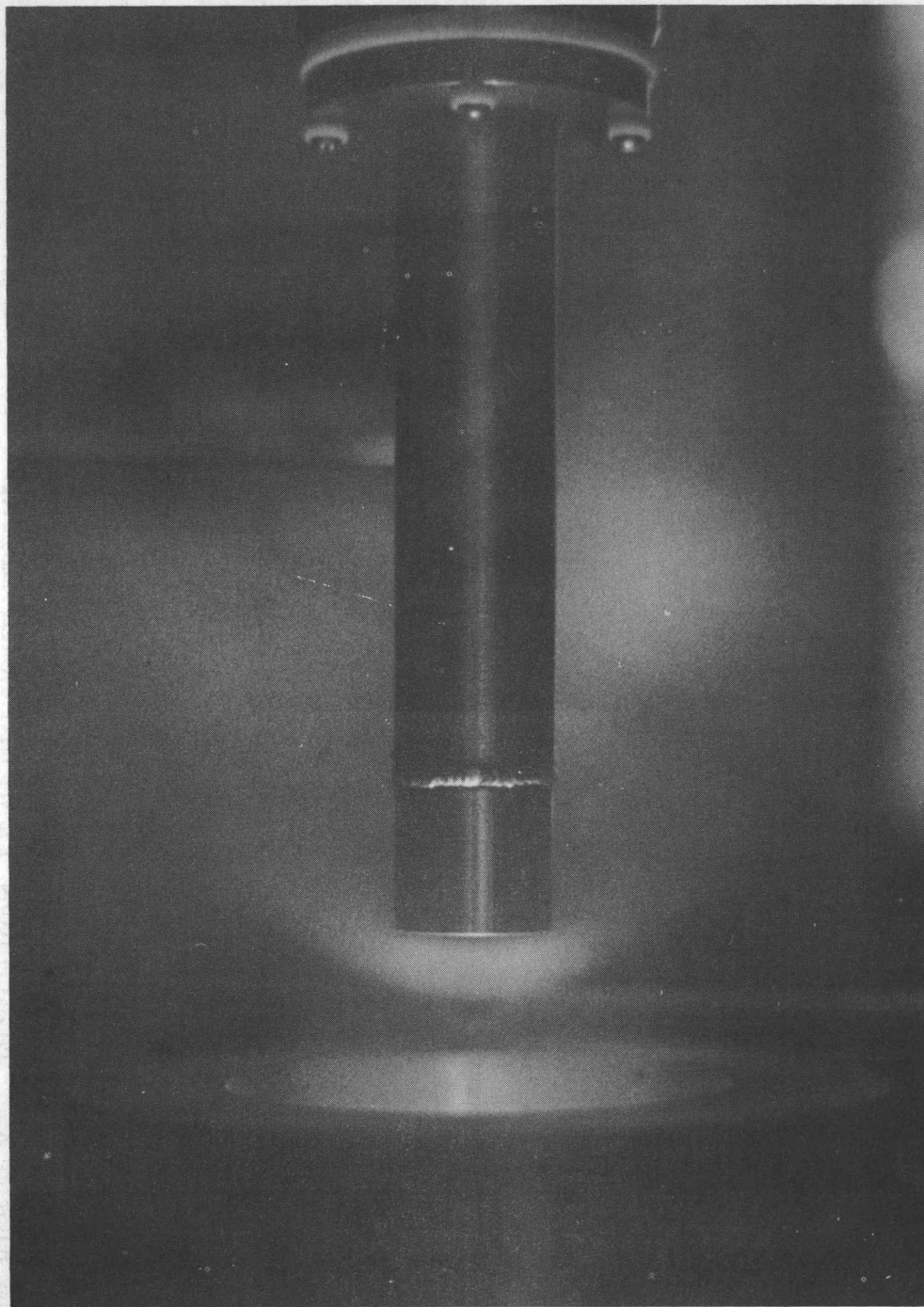


FIG. 6.2 PHOTOGRAPH OF MODEL IN MACH 2.2 FLOW (RUN 15-1)

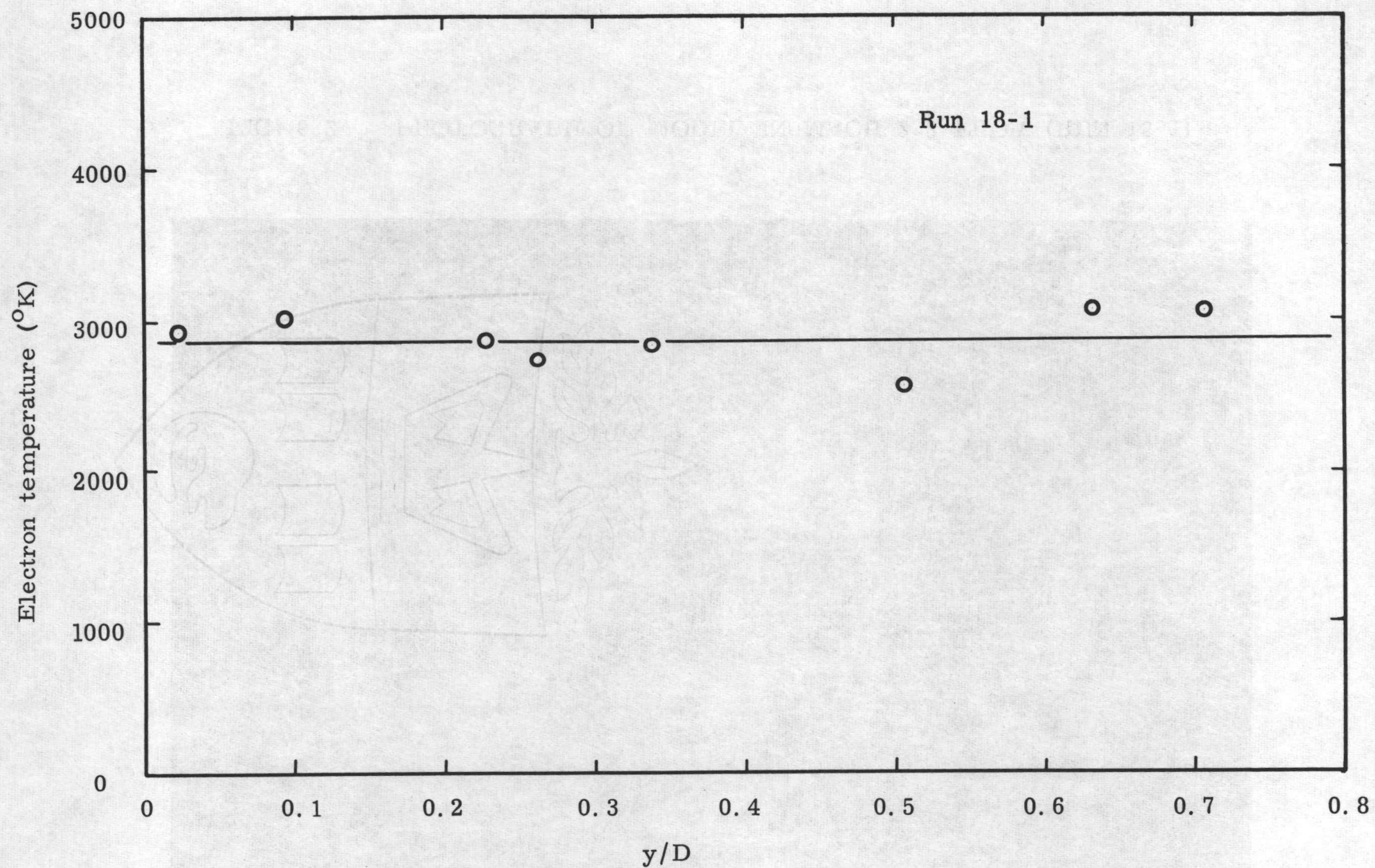
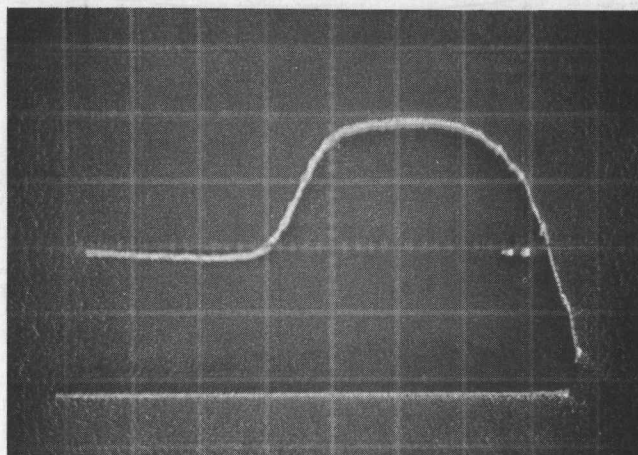


FIG. 6.3 ELECTRON TEMPERATURE DISTRIBUTION ALONG STAGNATION STREAMLINE (SEE ALSO FIG. 6.7)

Run 16-1

$Re_2 = 120$

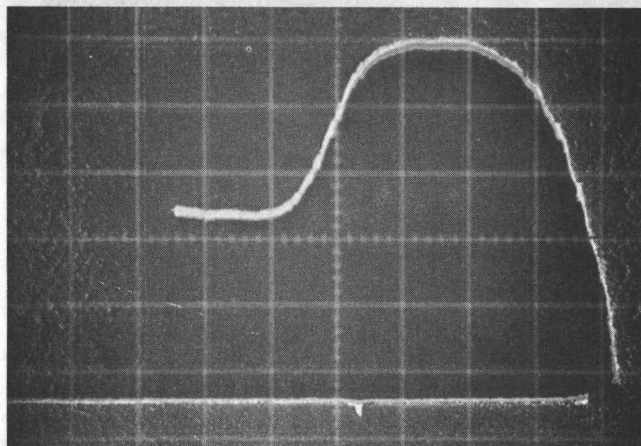


J_i

$10 \mu A/unit$

Run 18-1

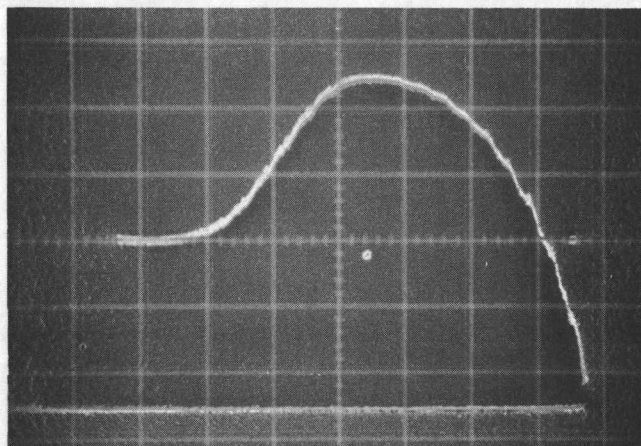
$Re_2 = 107$



$6.67 \mu A/unit$

Run 16-3

$Re_2 = 84$



$5 \mu A/unit$

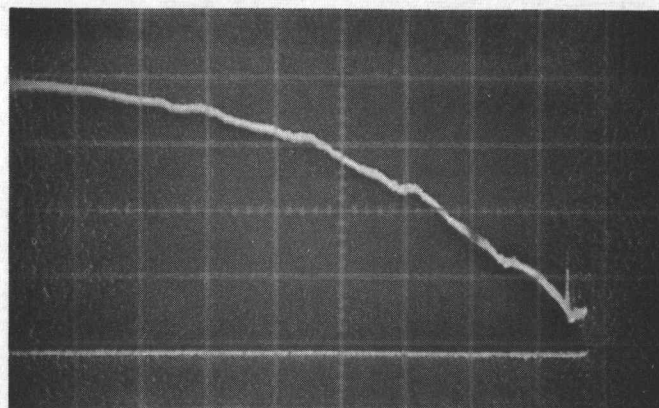
$y, 1.96 mm/unit$

FIG. 6.4

ION CURRENT PROFILES ALONG STAGNATION STREAMLINE

Run 16-1

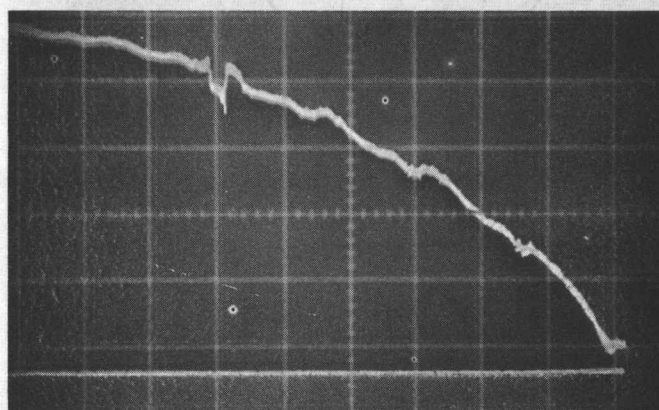
$Re_2 = 120$



J_i
↑
 $10 \mu A/\text{unit}$

Run 18-1

$Re_2 = 107$

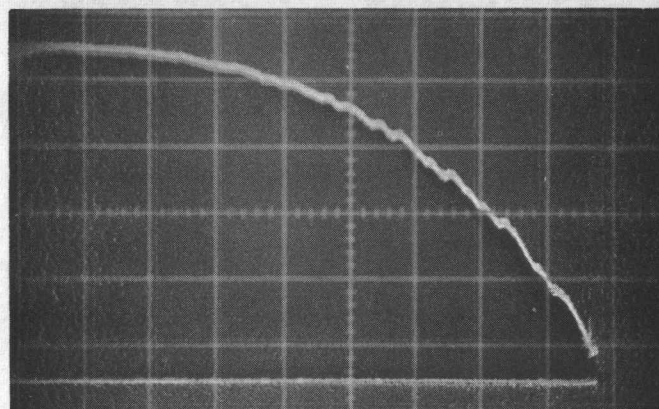


↑
 $6.67 \mu A/\text{unit}$

←
 $y, 0.392 \text{ mm/unit}$

Run 16-3

$Re_2 = 84$



↑
 $5 \mu A/\text{unit}$

←
 $y, 0.784 \text{ mm/unit}$

FIG. 6.5 ION CURRENT PROFILES NEAR WALL

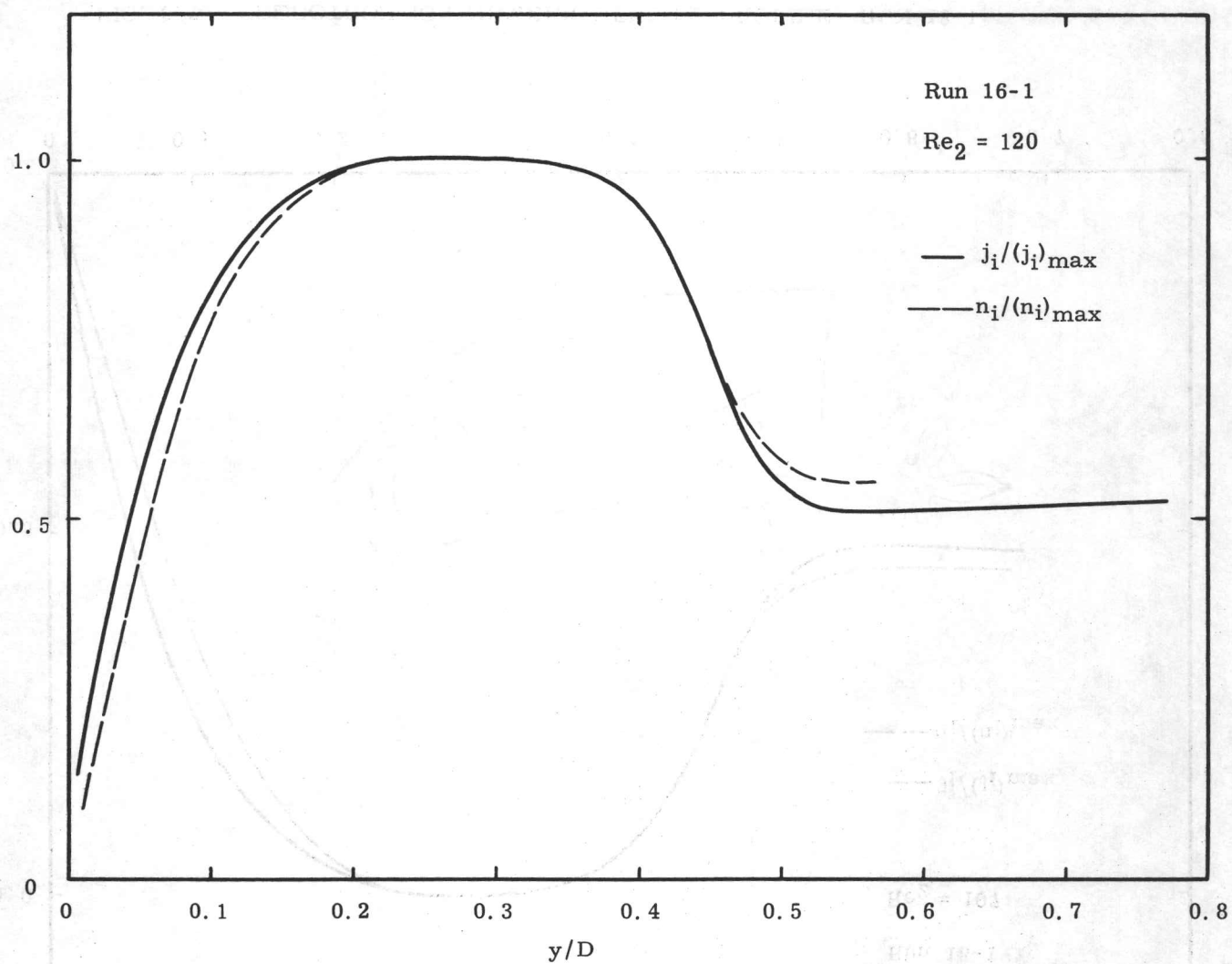


FIG. 6.6 ELECTRON-ION NUMBER DENSITY PROFILE, RUN 16-1

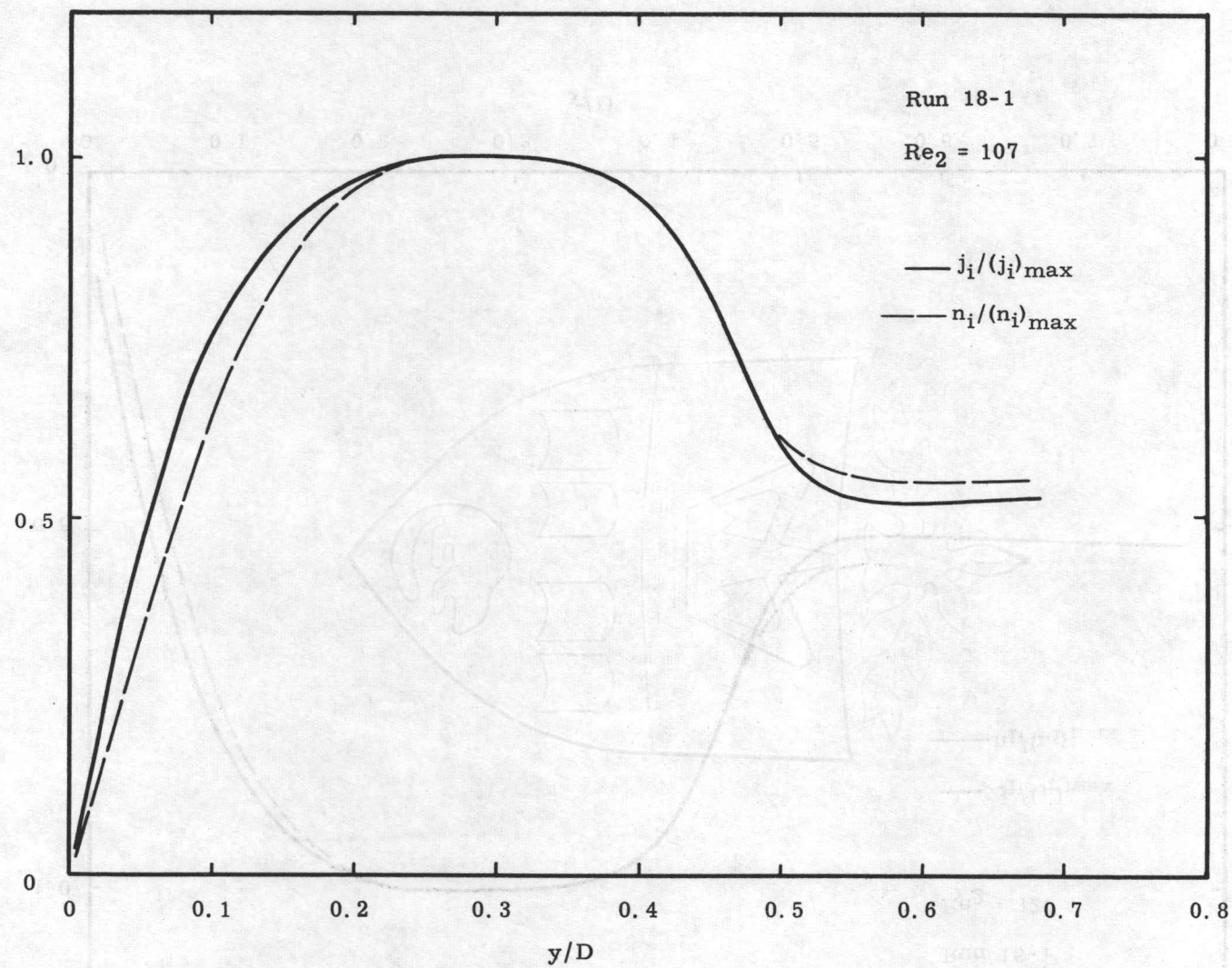


FIG. 6.7 ELECTRON-ION NUMBER DENSITY PROFILE, RUN 18-1

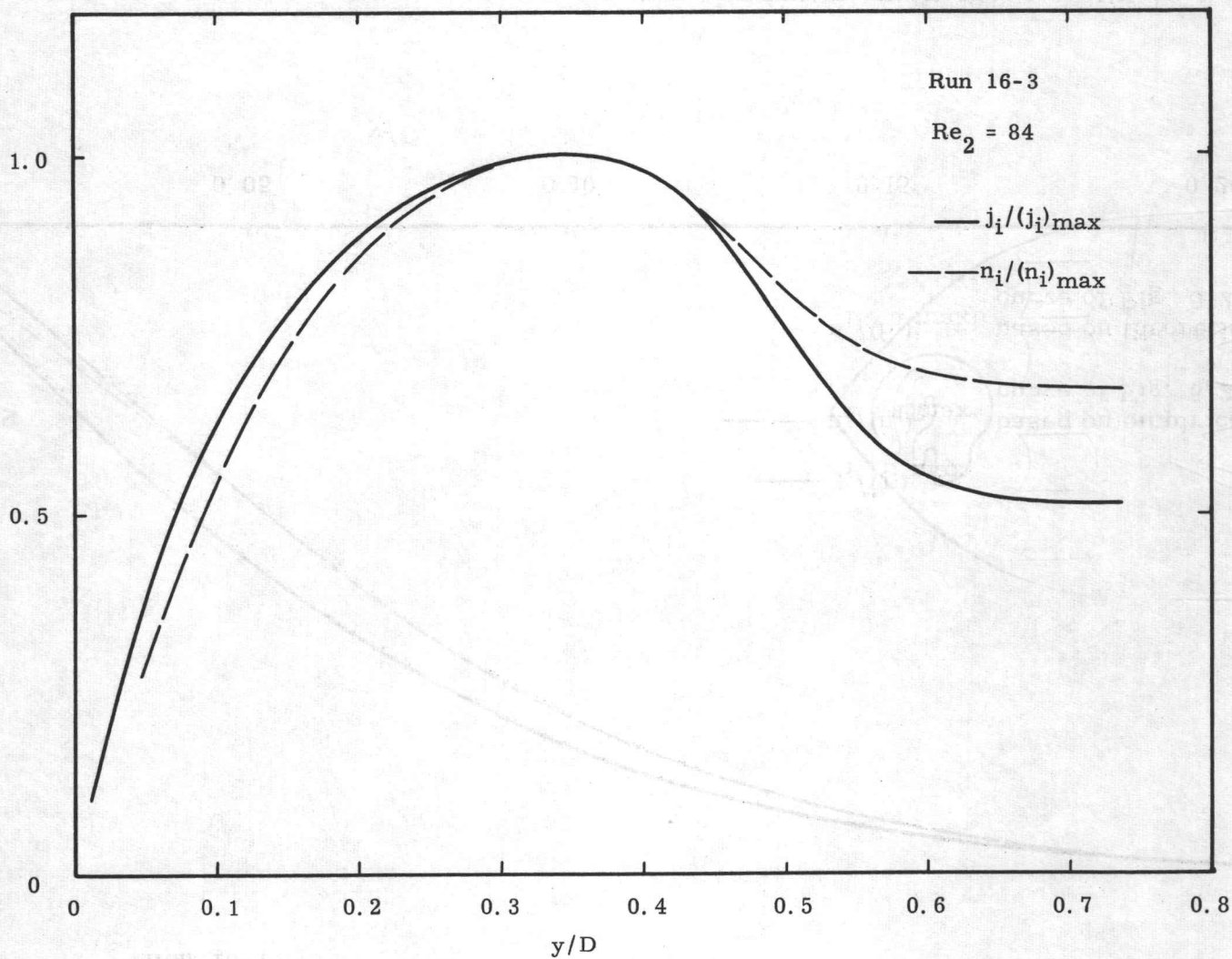


FIG. 6.8 ELECTRON-ION NUMBER DENSITY PROFILE, RUN 16-3

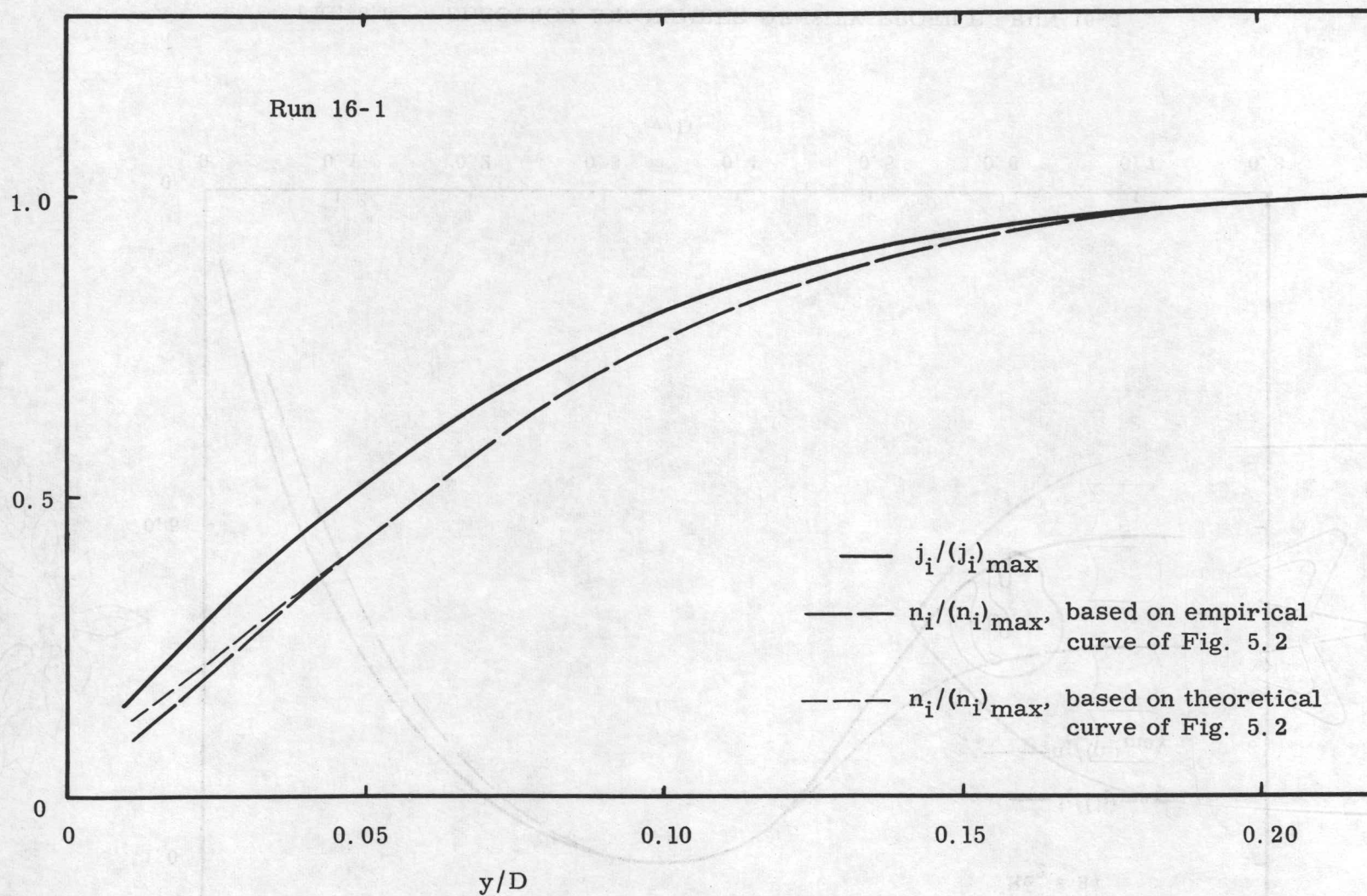


FIG. 6.9 ELECTRON-ION NUMBER DENSITY PROFILE NEAR WALL, RUN 16-1

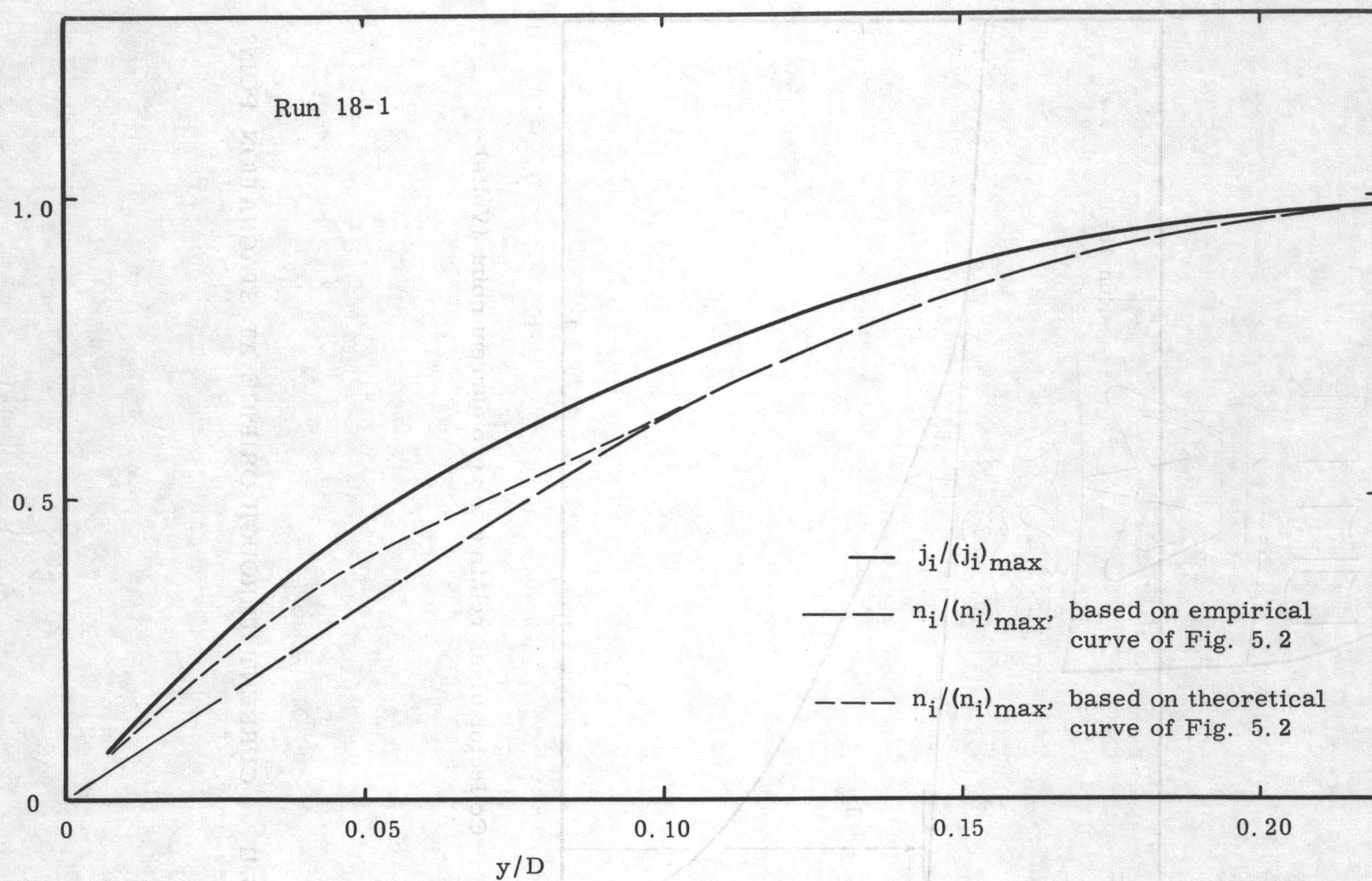


FIG. 6.10 ELECTRON-ION NUMBER DENSITY PROFILE NEAR WALL, RUN 18-1

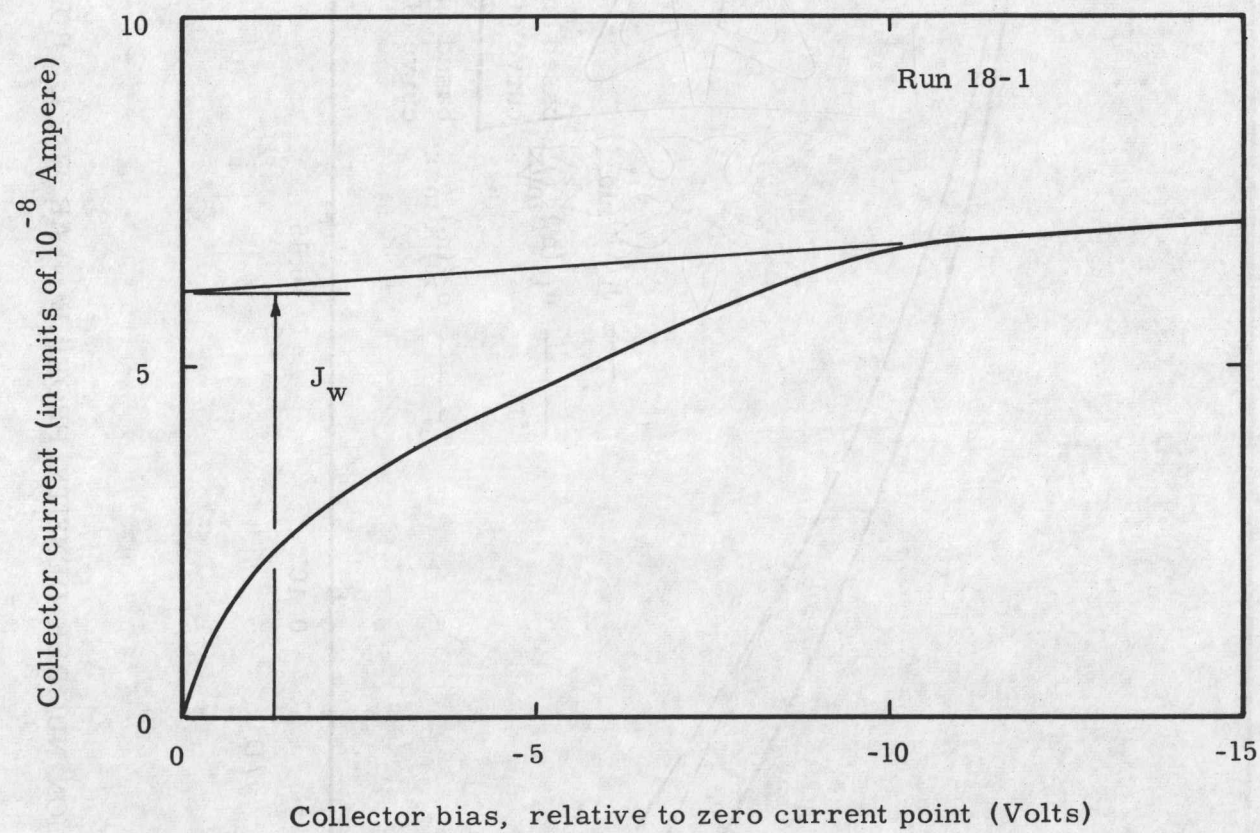


FIG. 6.11 CURRENT THROUGH ORIFICE AT STAGNATION POINT

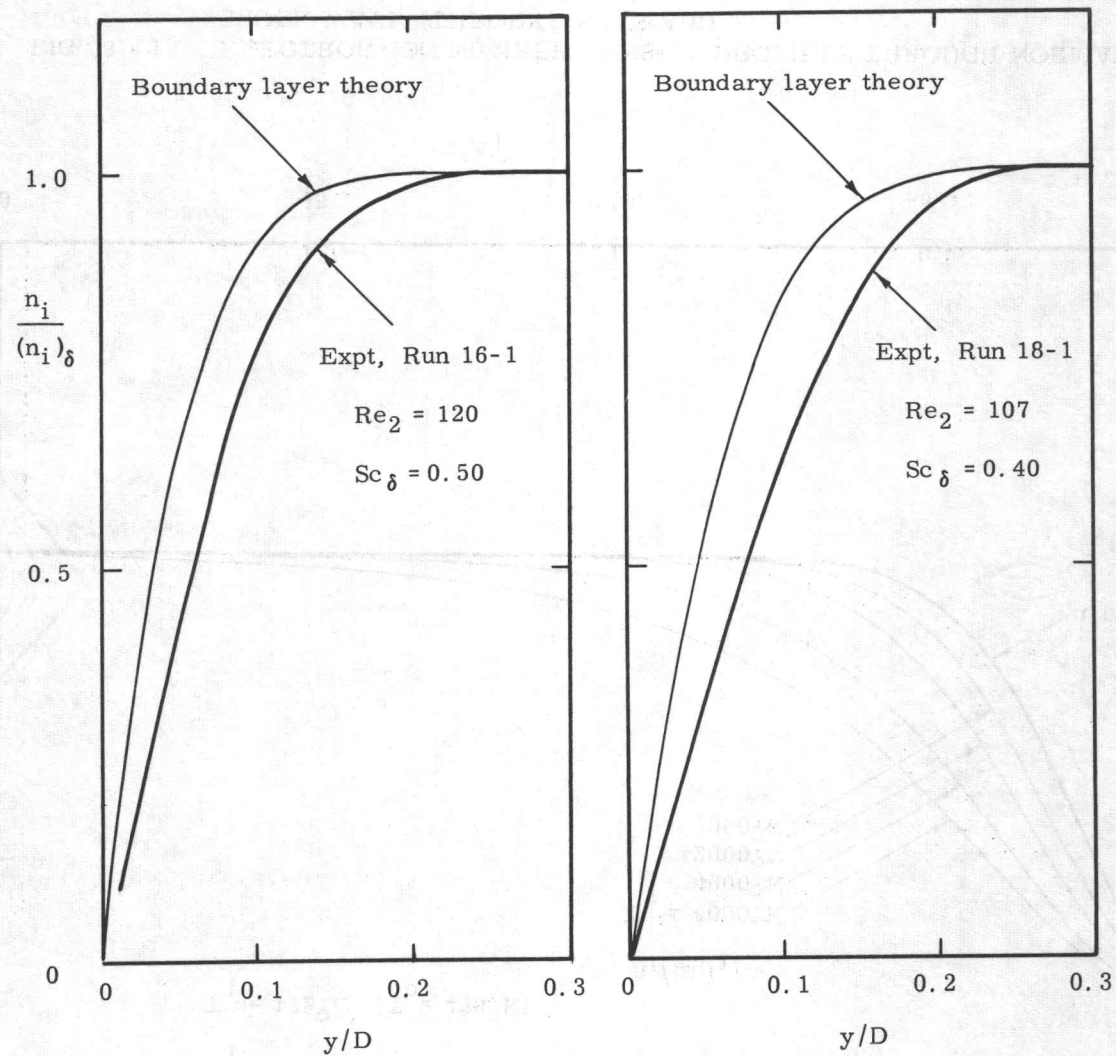


FIG. 6.12 COMPARISON OF SHAPES OF n_i PROFILES WITH HIGH REYNOLDS NUMBER BOUNDARY LAYER THEORY

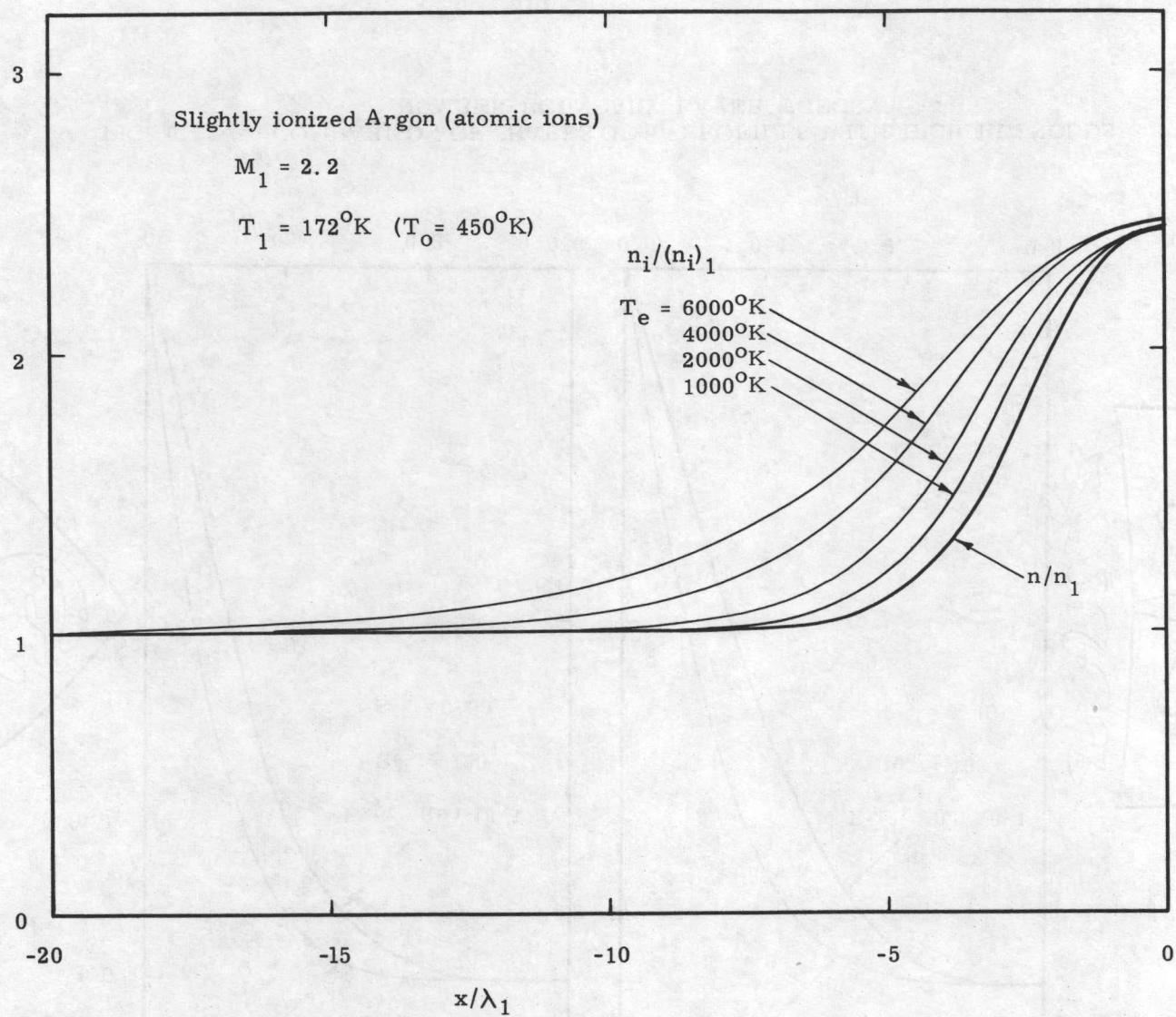


FIG. 6.13 ELECTRON-ION NUMBER DENSITY PROFILES THROUGH NORMAL SHOCK WAVE (THEORY, SINCLAIR)

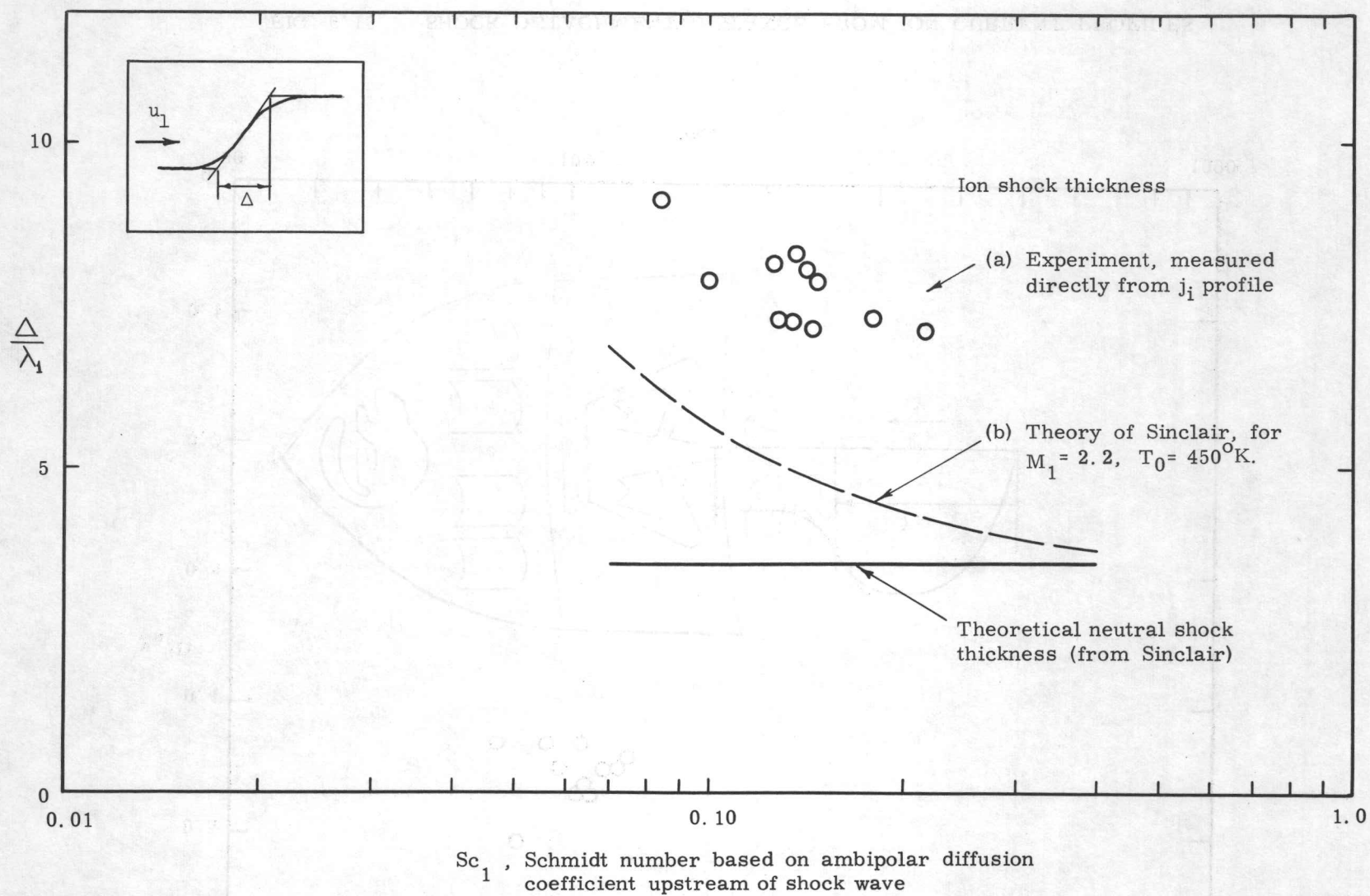


FIG. 6.14 ION SHOCK THICKNESS : EXPERIMENT AND THEORY

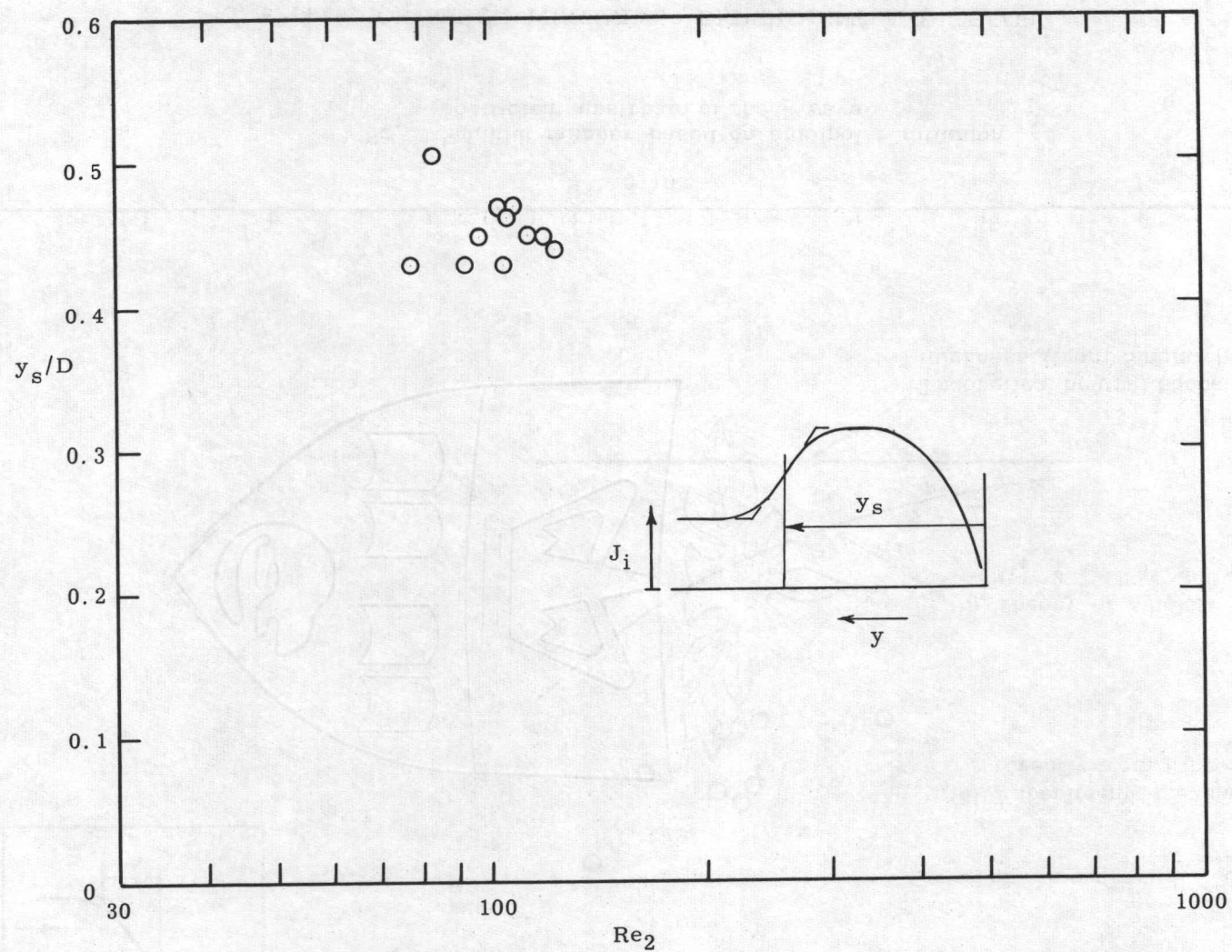


FIG. 6.15 SHOCK DETACHMENT DISTANCE FROM ION CURRENT PROFILES

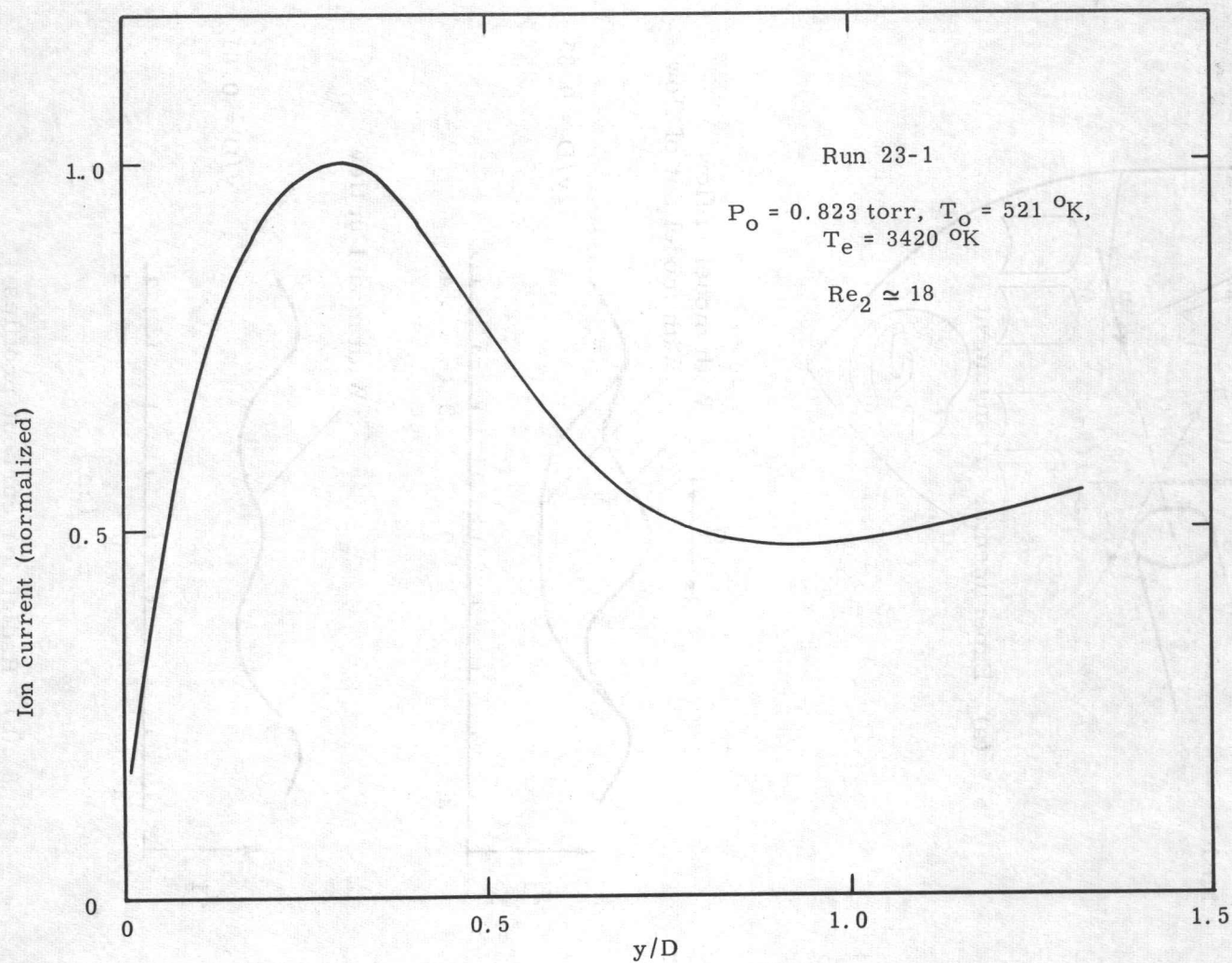
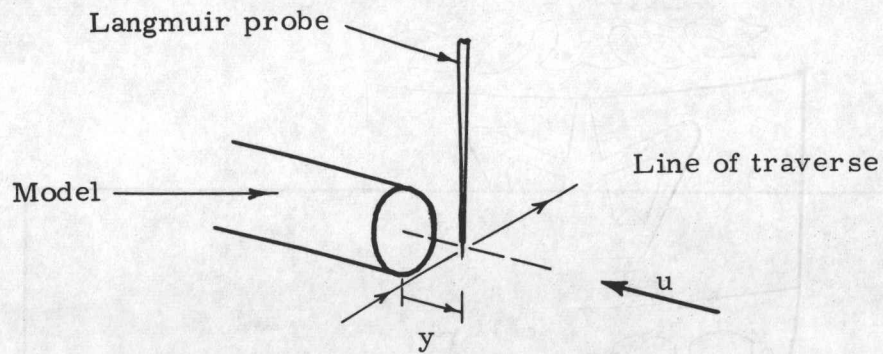
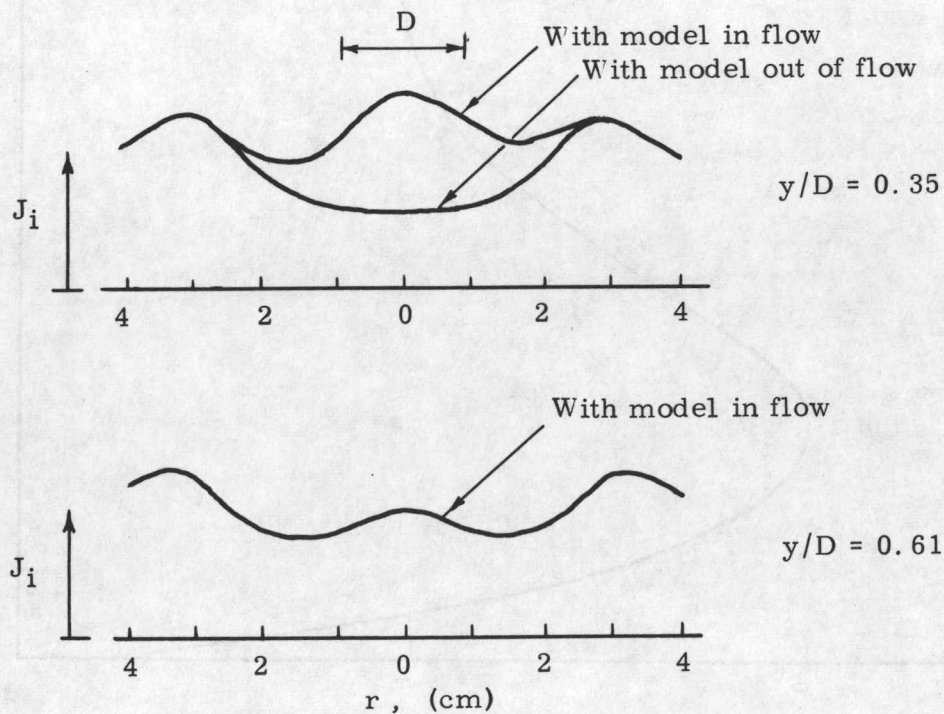


FIG. 6.16 EXPERIMENT WITH MODEL IN FREE JET FLOW FIELD:
ION CURRENT PROFILE ALONG STAGNATION STREAMLINE



(a) Experimental arrangement



(b) Radial ion current profiles

FIG. 6.17 EXPERIMENT WITH MODEL IN FREE JET FLOW FIELD: RADIAL ION CURRENT PROFILES THROUGH STAGNATION REGION

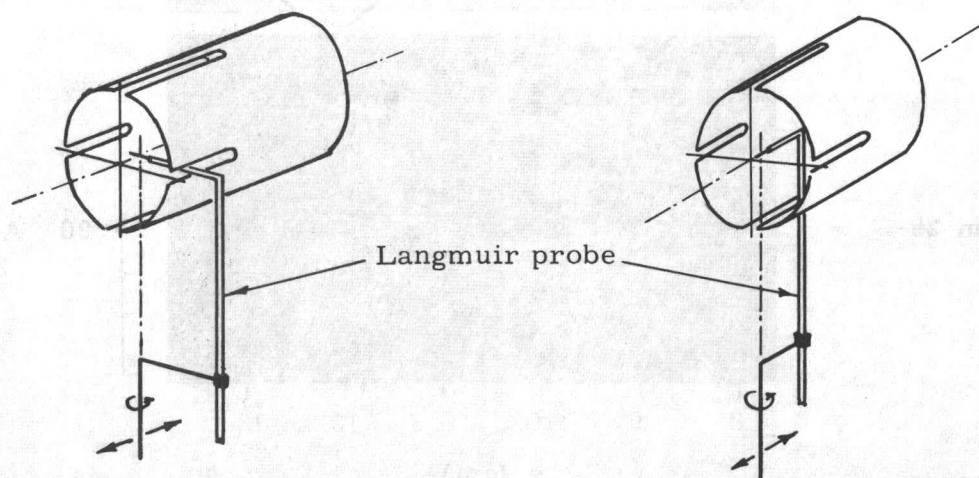
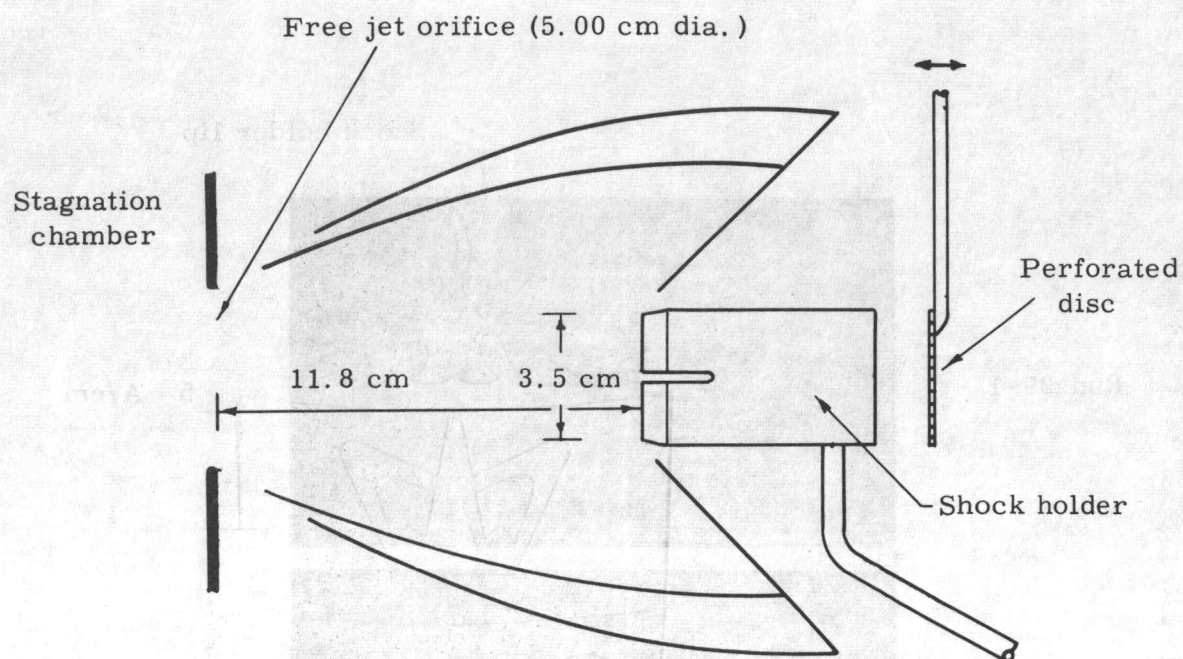


FIG. 7.1 ARRANGEMENT IN TESTS WITH SHOCK HOLDER

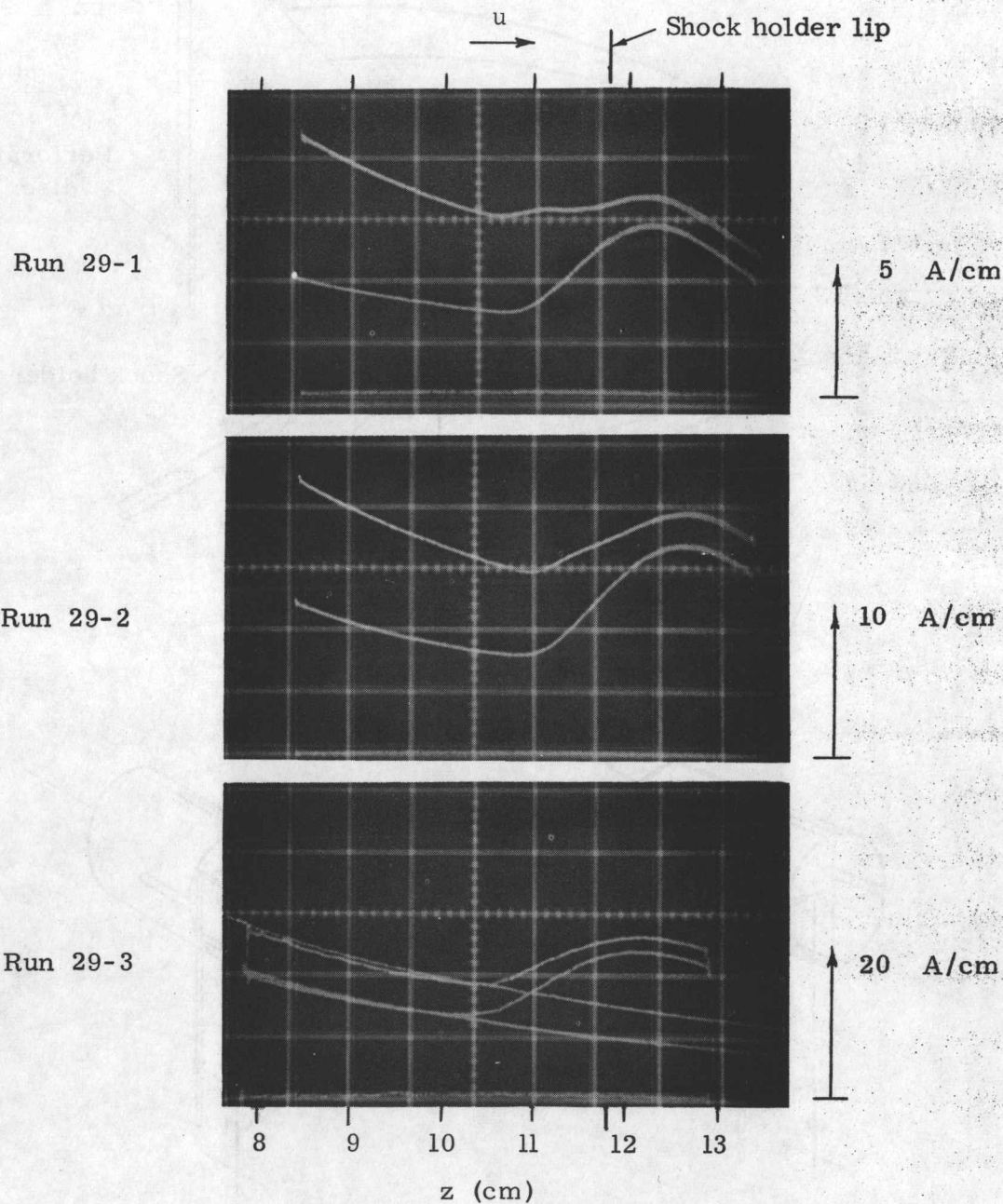


FIG. 7.2. ION CURRENT PROFILES ALONG SHOCK HOLDER AXIS. TOP CURVES TAKEN WITH PROBE PARALLEL, BOTTOM ONES WITH IT PERPENDICULAR, TO FLOW VELOCITY.

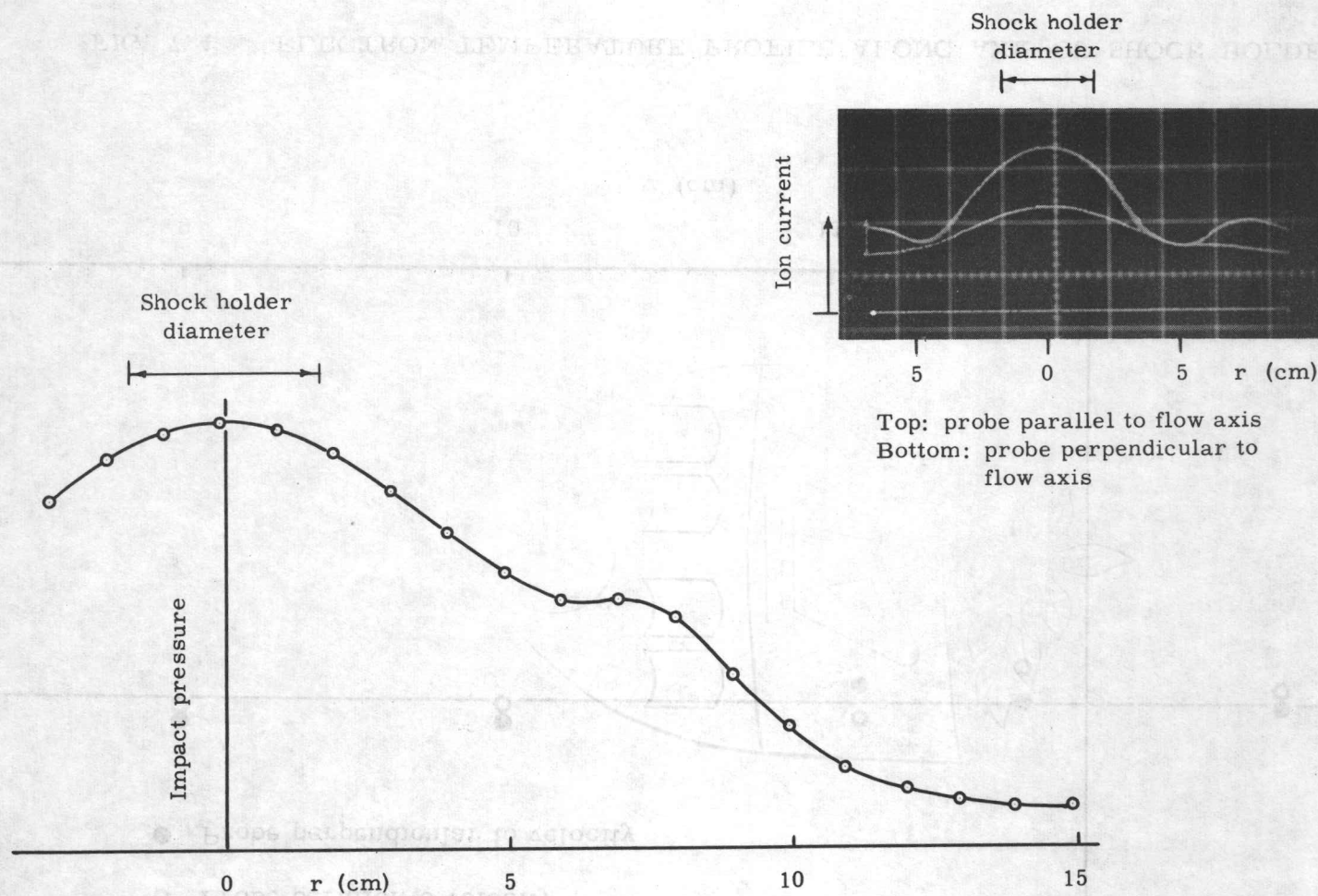


FIG. 7.3 RADIAL PROFILES OF IMPACT PRESSURE AND ION CURRENT IN FREE JET, AT $Z = 11.8$ CM. RUN 29-3.

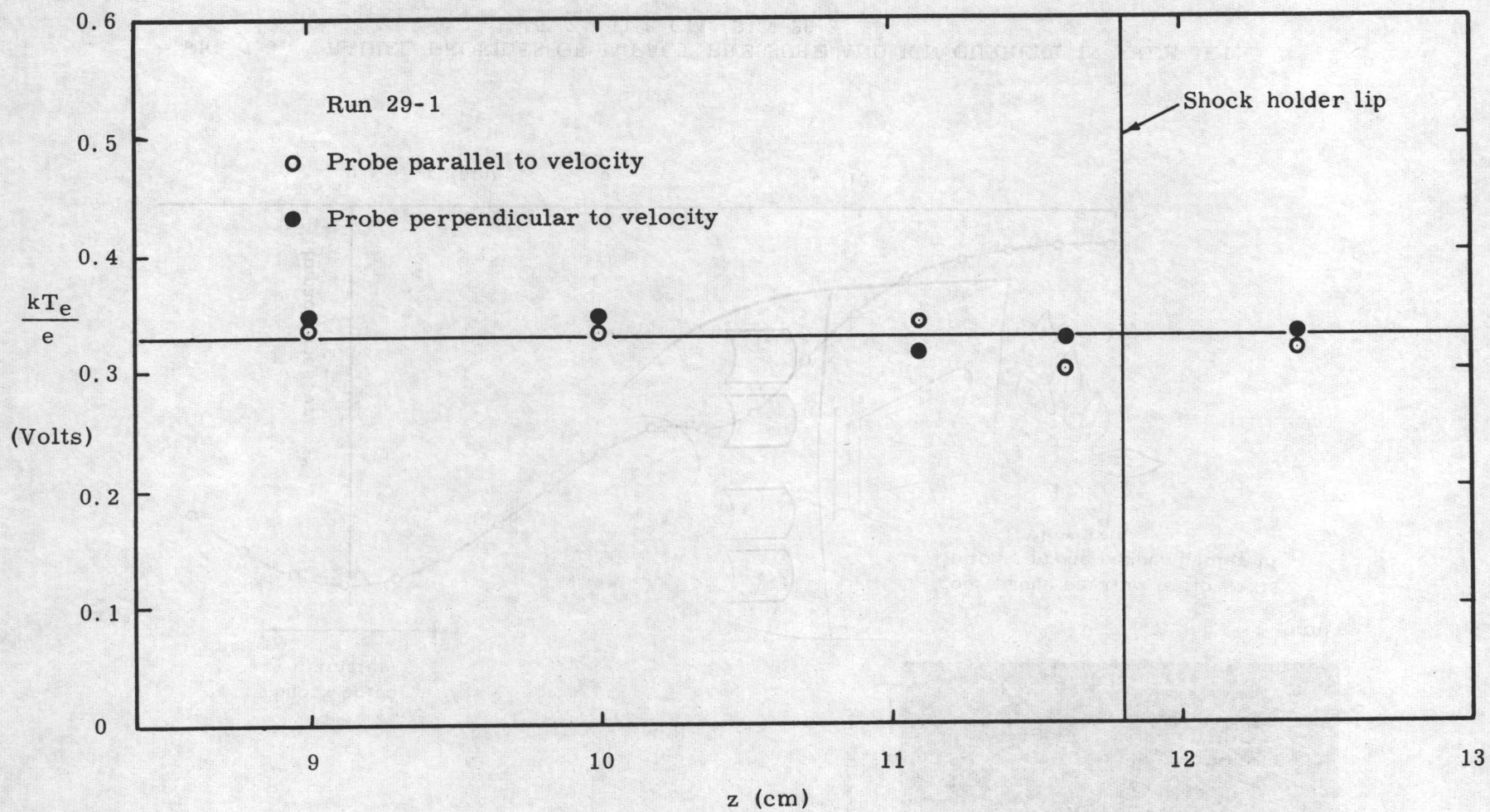


FIG. 7.4 ELECTRON TEMPERATURE PROFILE ALONG AXIS OF SHOCK HOLDER

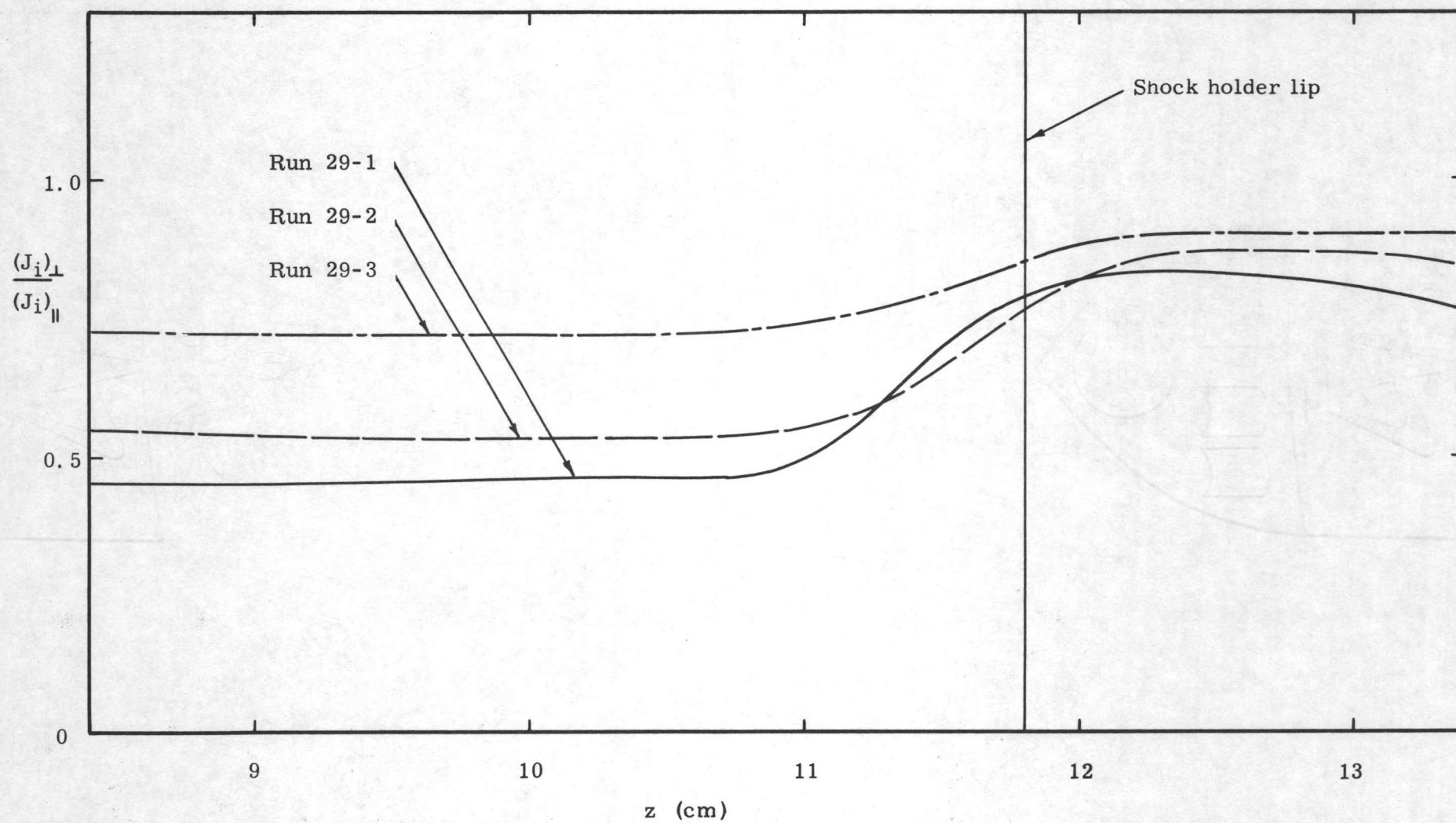
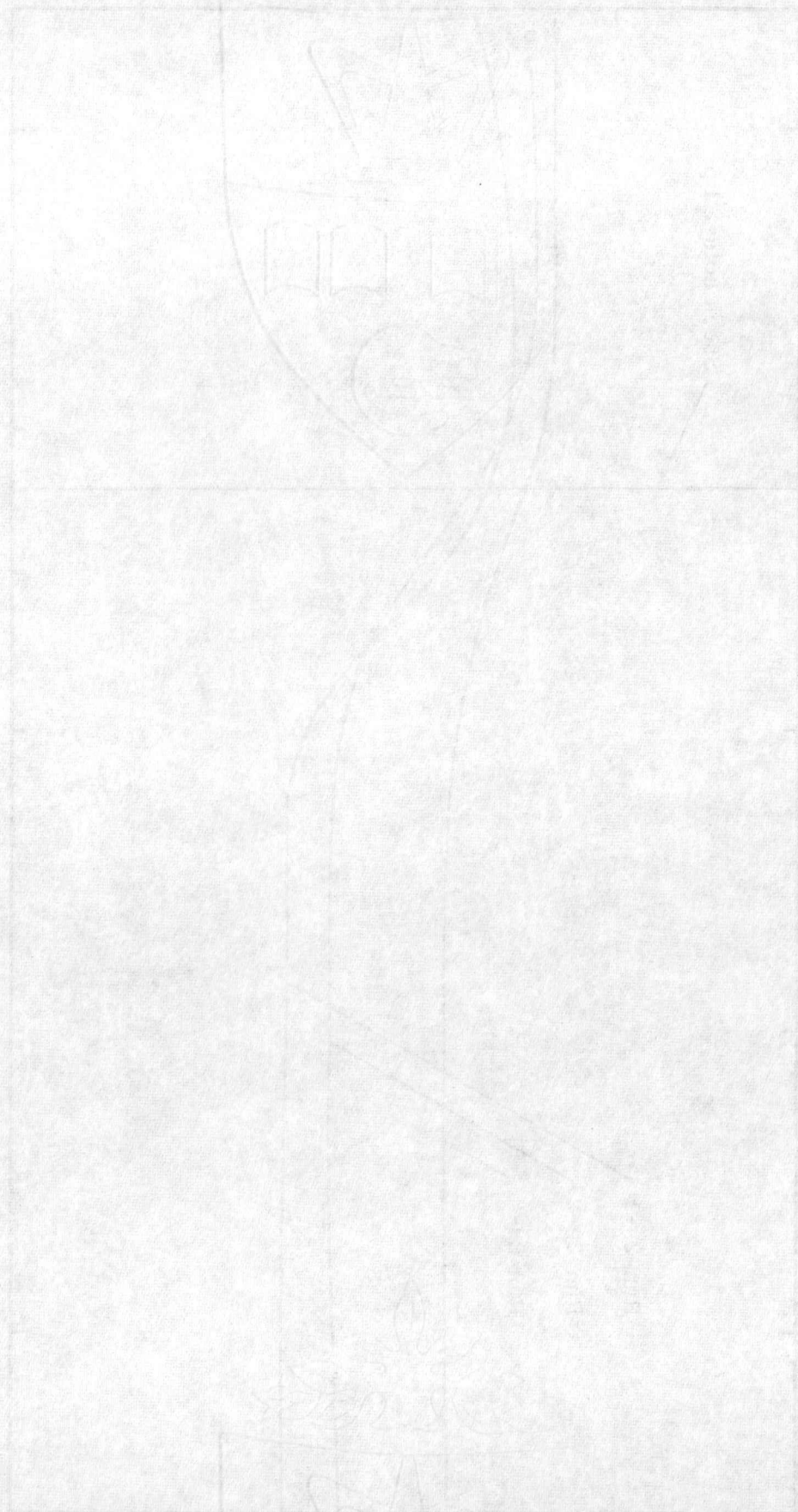


FIG. 7.5 DISTRIBUTION OF $(J_i)_\perp / (J_i)_\parallel$ ALONG AXIS OF SHOCK HOLDER



APPENDIX A

Diffusion Coefficient, Viscosity Coefficient, Schmidt and Reynolds Numbers

A.1 Diffusion Coefficient for Atomic Argon Ions in Argon

Most data on the motion of ions in neutral gases is given in terms of the mobility, μ_i , which is related to the diffusion coefficient, D_i , by Einstein's relation,

$$D_i = \frac{kT}{e} \mu_i \quad (A.1)$$

The mobility μ_{i0} which is usually given in the literature represents the value of μ_i at the standard gas number density of $n_0 = 2.69 \times 10^{25}$ atoms/m³, which corresponds to STP conditions of 760 torr and 273°K. The mobility at other densities is related to μ_{i0} by the equation

$$\mu_i(n, T) = \frac{n_0}{n} \mu_{i0}(T) \quad (A.2)$$

Thus,

$$D_i = \frac{k}{e} \frac{P_{STP}}{T_{STP}} \cdot \frac{T^2}{P} \mu_{i0}(T) \quad (A.3)$$

Chanin and Biondi (Ref. 49) give the following expression for $\mu_{i0}(T)$ for Ar⁺ ions in Ar:

$$\frac{1}{\mu_{i0}} = 2.08 \times 10^{-2} T^{\frac{1}{2}} + 0.24 + \frac{1.33}{T} + \frac{1.43 \times 10^2}{T^2} \quad \text{volt-sec/cm}^2 \quad (A.4)$$

which is valid for 50°K < T < 1200°K. Their experiments (in the temperature range 77°K to 300°K) were in good agreement with this relation. At higher temperatures it suffices to retain only the first two terms on the right hand side of Eq. (A.4), and one then has

$$\mu_{i0} = \frac{4.81 \times 10^1}{\sqrt{T} + 11.5} \quad \text{cm}^2/\text{volt sec.} \quad (A.5)$$

Equation (A.5) differs from Eq. (A.4) by less than 2% at 200°K and by about 6% at 100°K. Substituting Eq. (A.5) into Eq. (A.3), we obtain

$$D_i P = 1.60 \times 10^{-4} \frac{T^2}{\sqrt{T} + 11.5} \quad \text{m}^2/\text{sec} - \text{N/m}^2 \quad (A.6)$$

or

$$D_i P(\text{torr}) = 1.15 \times 10^{-6} \frac{T^2}{\sqrt{T} + 11.5} \quad \text{m}^2/\text{sec-torr} \quad (A.7)$$

where P is expressed in N/m² in Eq. (A.6) and in torr in Eq. (A.7).

A.2 Viscosity Coefficient for Argon

For $T > 100^\circ\text{K}$, the viscosity coefficient was calculated from the Sutherland Formula (Ref. 53),

$$\mu = \frac{K T^{3/2}}{T + C} \quad \text{kg/m sec} \quad (\text{A.8})$$

where $K = 1.93 \times 10^{-6}$

$$C = 142$$

At temperatures below 100°K , the Lennard-Jones potential calculation of the viscosity was used (Ref. 54, p. 528), with $\epsilon/k = 124^\circ\text{K}$ and $\sigma = 3.418 \text{ \AA}$. Above 100°K Eq. (A.8) agrees with the Lennard-Jones calculation to within less than 2%.

A.3 The Schmidt Number

Based on the diffusion coefficient of atomic argon ions in their parent gas, D_i (not the ambipolar diffusion coefficient), the Schmidt number is given by

$$Sc_i = \frac{\mu}{\rho D_i} \quad (\text{A.9})$$

Taking the viscosity, μ , from Eq. (A.8) and D_i from Eq. (A.6), we obtain

$$Sc_i = 2.51 \left(\frac{T + 11.5 \sqrt{T}}{T + 142} \right) \quad (\text{A.10})$$

This should be accurate to within a few percent from over 1000°K down to 200°K , and may be stretched to lower temperatures for less stringent requirements.

The ambipolar Schmidt number, based on the ambipolar diffusion coefficient, is given by

$$Sc = \frac{Sc_i}{\left(1 + \frac{T_e}{T}\right)} \quad (\text{A.11})$$

where T_e is the electron temperature.

A.4 Re_2 , the Reynolds Number Based on Free Stream Mass Flux and Stagnation Temperature

Re_2 is defined by

$$Re_2 = \frac{\rho_1 u_1 D}{\mu_0} \quad (\text{A.12})$$

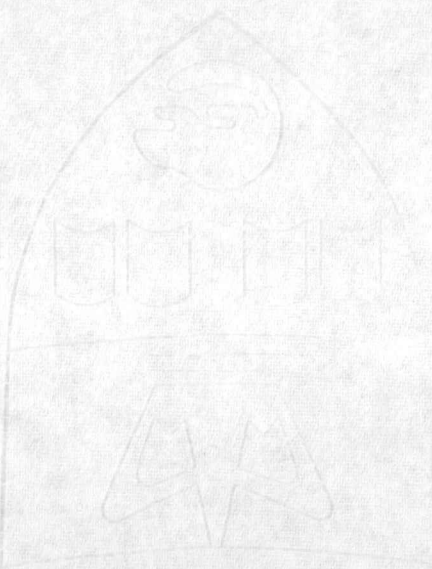
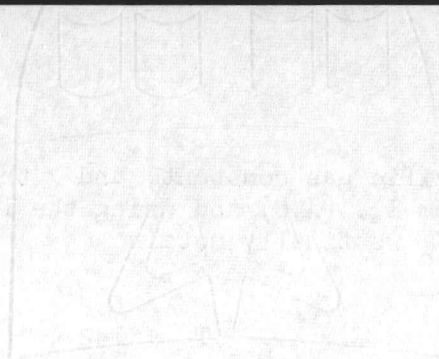
where the subscript 1 denotes free stream conditions and 0 stagnation conditions. In terms of Mach number and stagnation conditions, this equation takes the form

$$\frac{Re_2}{D} = \sqrt{\frac{\gamma}{R}} \cdot \frac{1}{\mu(T_0)} \cdot \frac{P_0}{\sqrt{T_0}} \cdot M_1 \left(\frac{P_1}{P_0} \right) \sqrt{\frac{T_0}{T_1}} \quad (\text{A.13})$$

where R is the specific gas constant, and γ the specific heats ratio. Substituting for μ from Eq. (A.8) and using the isentropic relations (Ref. 55) for P_1/P_0 , and T_1/T_0 , we finally obtain

$$\frac{Re_2}{D} = 6.19 \times 10^6 \left(\frac{T_0 + 142}{T_0^2} \right) P_0 (\text{Torr}) \left(\frac{M_1}{1 + \frac{M_1^2}{3}} \right)^2 \quad (\text{A.14})$$

where all quantities are in MKS units except P_0 , which is expressed in torr.



APPENDIX B

The Mean Free Paths

B.1 Mean Free Path for Collisions Between Neutral Argon Atoms

The concept of a mean free path emerges unambiguously only when a gas of rigid sphere molecules is considered. For more realistic force fields it is customary to define an equivalent mean free path in terms of the viscosity coefficient of the gas (Ref. 56, p. 147),

$$\lambda = \frac{16}{5} \left(\frac{m}{2\pi kT} \right)^{\frac{1}{2}} \frac{\mu}{\rho} \quad (\text{B.1})$$

Equation (B.1) defines the neutral atom mean free path in this paper. For argon,

$$\lambda_{aa} = 1.38 \times 10^{-1} \frac{T^{\frac{1}{2}}}{P(\text{torr})} \mu \quad \text{meter} \quad (\text{B.2})$$

where μ is in MKS units and P in torr. For $T > 100^\circ\text{K}$, we may use Eq. (A.8) for μ and obtain

$$\lambda_{aa} = \frac{2.66 \times 10^{-7}}{P(\text{torr})} \left(\frac{T^2}{T + 142} \right) \text{m} \quad (\text{B.3})$$

For $T < 100^\circ\text{K}$, the Lennard-Jones result for μ was used, as described in Sec. A.2.

B.2 Mean Free Path for Collisions Between Ar^+ Ions and Ar Atoms

The ion-atom mean free path is defined by

$$\lambda_{ia} = \frac{1}{\sqrt{2} n Q_{ia}} \quad (\text{B.4})$$

where n is the number density of atoms, and Q_{ia} is the cross section for collisions between ions and atoms. In this case the appropriate collision cross section is the one associated with the binary diffusion coefficient D_i . From Ref. 56 (p. 195), the mutual diffusion coefficient of species 1 and 2 is related to the appropriate collision cross section by

$$D_{12} = \frac{3}{8} \sqrt{\frac{\pi}{2}} \frac{1}{(n_1 + n_2) Q_{12}} \left(\frac{m_1 + m_2}{m_1 m_2} k T \right)^{\frac{1}{2}} \quad (\text{B.5})$$

In this particular case $(n_1 + n_2)$ is to a very good approximation equal to the number density of the neutral atoms and both m_1 and m_2 may be equated to the atom mass. Substituting into Eq. (B.4) and (B.5), we obtain

$$\lambda_{ia} = \frac{8}{3\sqrt{2}\pi} \sqrt{\frac{m}{kT}} D_i \quad (\text{B.6})$$

With D_i taken from Eq. (A.7), this becomes

$$\lambda_{ia} = \frac{0.852 \times 10^{-7}}{P(\text{torr})} \frac{T^{3/2}}{11.5 + \sqrt{T}} \quad (\text{B.7})$$

A comparison of Eqs. (B.7) and (B.3) yields the result

$$\frac{\lambda_{ia}}{\lambda_{aa}} = 0.320 \frac{T + 142}{T + 11.5 \sqrt{T}} \quad (\text{B.8})$$

In other words, $\lambda_{ia} \approx 0.3 \lambda_{aa}$ over most of the temperature range of interest in the present work.

B.3 The Mean Free Path for Electron-Atom Collisions

The electron-atom mean free path may be obtained from the relation,

$$\lambda_{ea} = \frac{1}{n Q_{ea}} \quad (\text{B.9})$$

where the collision cross section Q_{ea} must be taken from empirical data or from quantum mechanical calculations (e.g. Ref. 33). The factor $\sqrt{2}$ which appears in the denominator of Eq. (B.4) is not appropriate in this case, where the electron moves in an essentially stationary target gas.

B.4 Mean Free Paths for Collisions Between Charged Particles

In a plasma where there are a great many particles in a Debye cube (cube with side equal to the Debye length), the conventional definition of a "mean free path" in terms of binary encounters becomes meaningless. However, Spitzer (Ref. 57) defines a "deflection time", t_D , which represents the time in which a particle of mass m , charge Ze and speed v is gradually deflected through 90° by a number of small-angle deflections:

$$t_D = \frac{1}{8\pi n_1 P_o^2 v \ln \Lambda \Psi(v/c_{m1})} \quad (\text{B.10})$$

Here, the unsubscripted quantities refer to the "test particle", which we are following, and the subscript 1 refers to the "field particles" with which the test particle interacts. The latter are assumed to have a Maxwellian distribution of velocities. The notation in Eq. (B.10) is defined as follows:

$$\begin{aligned} n_1 &= \text{number density of field particles} \\ P_o &= ZZ_1 e^2 / 4\pi \epsilon_0 m v^2 \quad (\text{MKS units}) \\ \Lambda &= \lambda_D / P_o \\ c_m &= (2kT/m)^{1/2} \\ \Psi(x) &= \text{erf}(x) - (1/2x^2)[\text{erf}(x) - x \text{erf}^1(x)] \end{aligned} \quad (\text{B.11})$$

The distance travelled by the test particle in the time t_D may be used to represent its mean free path, i.e.

$$\lambda = v t_D = \frac{1}{8\pi n_1 P_o^2 \ln \Lambda \Psi(v/c_{m1})} \quad (\text{B.12})$$

In the case of a group of thermal particles interacting with each other, Spitzer defines a "self-collision time" by giving v the root mean square value for the group, $(3kT/m)^{1/2}$. The quantity v/c_m then becomes $(3/2)^{1/2}$ and Ψ takes the value 0.714. The corresponding mean free path, as defined by Eq. (B.12), is

$$\lambda_{\alpha\alpha} = \frac{18\pi}{0.714} \left(\frac{\epsilon_0 k}{Z^2 e^2} \right)^2 \frac{T^2}{n \ln \Lambda} \quad (\text{B.13})$$

where α represents either electrons or ions. In MKS units, with $Z = 1$,

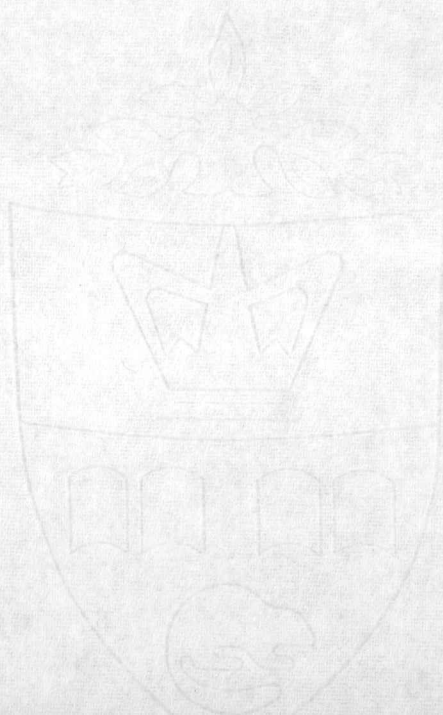
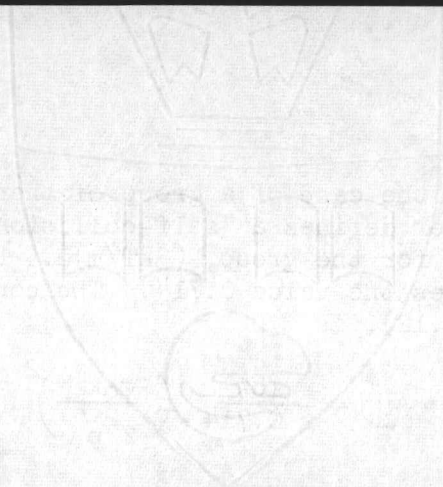
$$\lambda_{\alpha\alpha} = 1.80 \times 10^9 \frac{T^2}{n \ln \Lambda} \quad (\text{B.13b})$$

where

$$\Lambda = 1.24 \times 10^7 \sqrt{\frac{T^3}{n}} \quad (\text{B.14})$$

In Fig. B.1, the self-interaction mean free path is plotted in terms of n and T .

By making use of the fact that $m_e \ll m_i$, one can also show from Eq. (B.12) that $\lambda_{ei} \sim \lambda_{ee}$ and $\lambda_{ie} \gg \lambda_{ii}$.



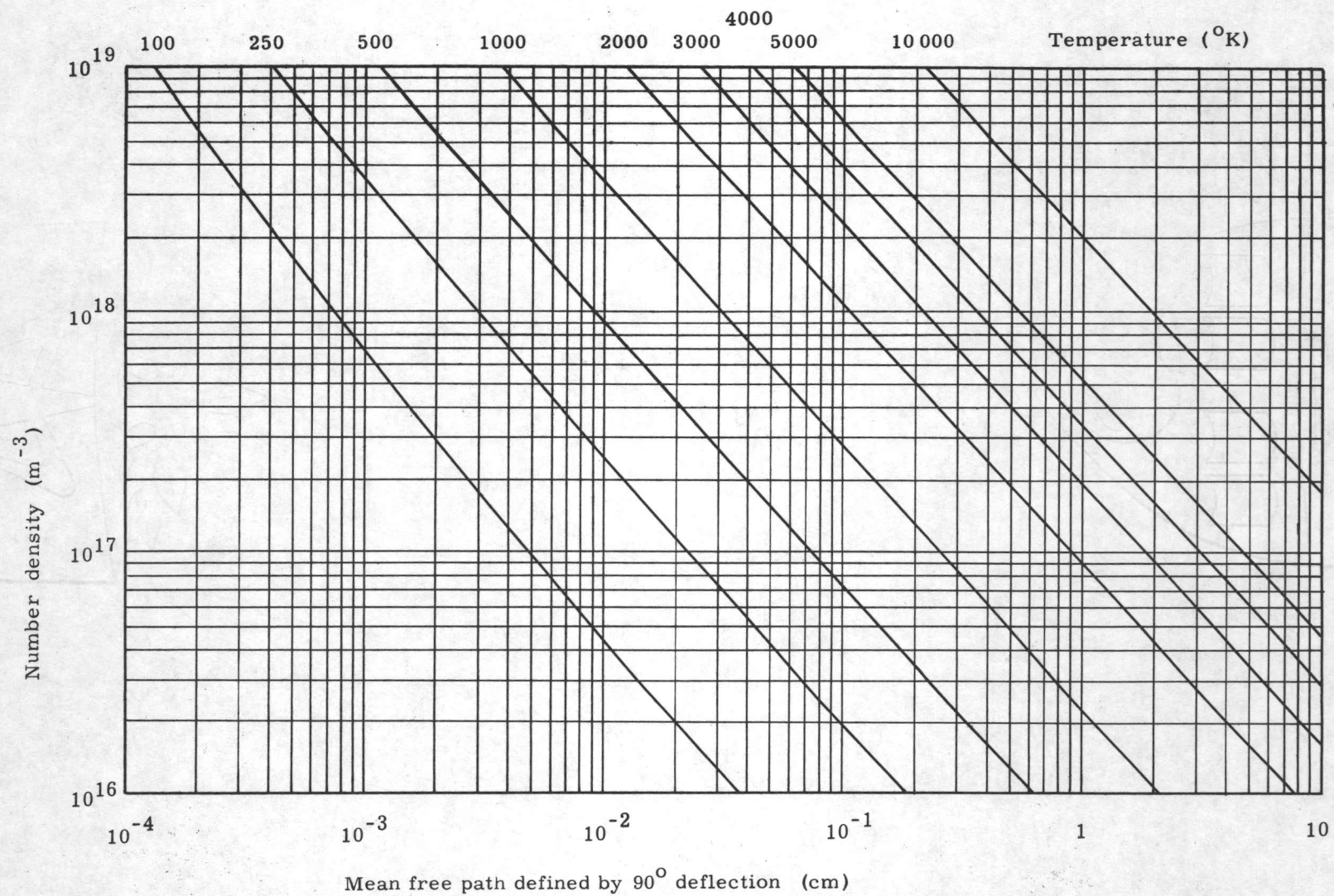
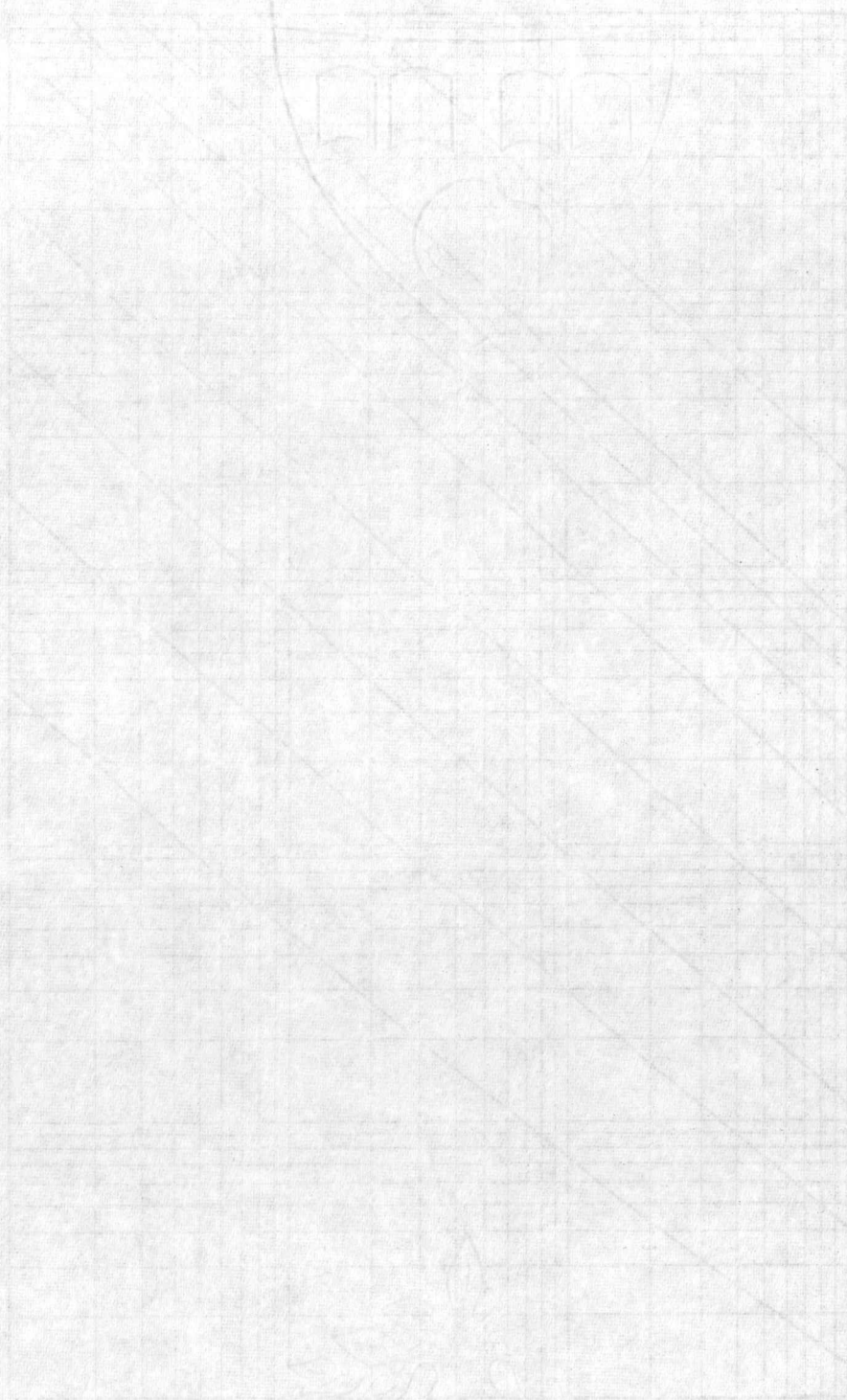


FIG. B.1 THE ELECTRON-ELECTRON OR ION-ION MEAN FREE PATH



1000 2000 3000 4000 5000 6000 7000 8000 9000 10000

APPENDIX C

Boundary Layer Calculation of the Ion Flux to the Stagnation Point Wall

The steady, supersonic flow over the stagnation point region of a blunt, axisymmetric body has in the past decade been the subject of numerous investigations, both theoretical and experimental. When the gas is ionized, the general boundary layer problem is complicated by the necessity for solving the diffusion equations of the individual species simultaneously with the overall equations of motion. We shall here restrict our attention to cases where the degree of ionization is very small. The charged species then make no significant contribution to the overall transfer of momentum and energy, and the problem separates into two parts: first, the determination of the flow field of the gas as a whole, and then the determination of the charged particle distribution which is consistent with that flow field. This problem has by now been quite thoroughly discussed in the literature (e.g. Refs. 42-45, 58-61), for on it depends the interpretation of the stagnation point Langmuir probe. We shall therefore merely outline briefly its formulation for the case at hand and obtain a numerical solution for the specific conditions of the experiments described in Sec. 6.

The first aspect of the problem reduces to the solution of the compressible boundary layer equations of a one-component gas. At a very blunt stagnation point, these may be written in the form (Refs. 62, 63):

$$\text{Momentum:} \quad (Nf'')' + ff'' + \frac{1}{2} [g - (f')^2] = 0 \quad (\text{C.1})$$

$$\text{Energy:} \quad \left(\frac{N}{Pr} g' \right)' + f g' = 0 \quad (\text{C.2})$$

where the derivatives are with respect to a similarity variable, η , defined by

$$\eta = \sqrt{\frac{2 \beta \rho_\delta}{\mu_\delta}} \int_0^y \frac{\rho}{\rho_\delta} dy \quad (\text{C.3})$$

and $f' = u/u_\delta$

$$g = T/T_\delta$$

$$N = \rho\mu/\rho_\delta \mu_\delta \quad (\text{C.4})$$

Pr = Prandtl number

$$\beta = (du_\delta/dx)_{x=0}$$

The coordinates x and y and the velocity components u and v are shown on Fig. C.1. The symbol δ used as a subscript denotes the outer edge of the boundary layer. Equations (C.1) and (C.2), with their boundary conditions

$$\begin{aligned} f(0) = f'(0) = 0 & \quad ; \quad f'(\infty) = 1 \\ g(0) = T_w/T_\delta & \quad ; \quad g(\infty) = 1 \end{aligned} \quad (\text{C.5})$$

define, together with the condition $\partial P / \partial y = 0$, the properties of the neutral gas in the boundary layer. Numerical solutions for inert gases are given by Bade (Ref. 63).

Once the overall flow field has been defined, the distributions of the charged minority species follow from the appropriate diffusion equations. Here we shall specialize to the case where electrons and only one species of ions are present, and where the flow is frozen in the sense that no recombination takes place in the gas itself (although the surface of the body is assumed, as usual, to have infinite catalytic efficiency for charged particle recombination). We assume further that the Debye length is smaller than the mean free path, and therefore much smaller than the boundary layer thickness. In other words, the thickness of the sheath adjacent to the surface is very small, and charge separation in the boundary layer is negligible. The distribution of electron-ion number density, $n = n_i \approx n_e$, in the boundary layer is then governed by the equation (Refs. 44 and 61, for example):

$$\left(\frac{N}{Sc} Z' \right)' + f Z' = 0 \quad (C.6)$$

where

$$Z = \frac{n/\rho}{(n/\rho)_\delta} \quad (C.7)$$

and $Sc = \mu/\rho D_a$ is the Schmidt number based on the ambipolar diffusion coefficient. The boundary conditions for Eq. (C.6) are

$$Z(0) = 0 \quad ; \quad Z(\infty) = 1 \quad (C.8)$$

Equation (C.6) can be solved formally as follows: first, we let $N/Sc = k(\eta)$, say and throw the equation into the form

$$Z'' + \left(\frac{k' + f}{k} \right) Z' = 0 \quad (C.9)$$

Integrating once, we have

$$Z' = Z'(0) \frac{k(0)}{k(\eta)} \exp. \left[- \int_0^\eta \frac{f}{k} d\eta \right] \quad (C.10)$$

Integrating once more and applying the boundary condition $Z(\infty) = 1$, we obtain the formal solution

$$Z(\eta) = Z'(0) k(0) \int_0^\eta \frac{1}{k} \exp. \left[- \int_0^{\eta'} \frac{f}{k} d\eta'' \right] d\eta' \quad (C.11)$$

where

$$Z'(0) = \frac{1}{k(0) \int_0^\infty \frac{1}{k} \exp \left[- \int_0^\eta \frac{f}{k} d\eta' \right] d\eta} \quad (C.12)$$

Now the quantity we are interested in, the ion flux to the wall, is determined entirely by the ion concentration gradient when the body is floating electrically (Ref. 61). The electron-ion diffusion flux at the wall is then given by

$$\Gamma_w = (n V)_w = - \rho D_a \frac{\partial}{\partial y} \left(\frac{n}{\rho} \right) \Big|_w \quad (C.13)$$

where V represents the diffusion velocity common to both charged species. Expression this flux as an equivalent current, $j_w = e \Gamma_w$, and transforming to the variables of the similarity solution (c.f. Eqs. (C.3) and (C.7)), we obtain (dropping the sign)

$$j_w = e n_\delta D_{aw} \frac{\rho_w^2}{\rho_\delta^2} \sqrt{\frac{2 \beta \rho_\delta}{\mu_\delta}} Z'(o) \quad (C.14)$$

The value of $Z'(o)$ must be obtained from a solution of Eq. (C.12) with f and g taken from data appropriate to the experimental conditions. Now the quantity k , being defined by

$$k = \frac{N}{Sc} = \frac{1}{Sc_\delta} \frac{\rho^2}{\rho_\delta^2} \frac{D_a}{D_{a\delta}} \quad (C.15)$$

is in general a complicated function of g . For our present purposes, we shall sacrifice some rigour in favour of substantial simplification by making the following two assumptions:

$$D_i \propto T^2/P \quad (C.16)$$

$$T_e \gg T \quad (C.17)$$

Equation (C.16) applies rigorously when the ion is not of the same species as the neutral atoms, for example, or when it is the molecular ion in its atomic parent gas (Ref. 64). It is also a reasonable approximation for Ar^+ ions in Ar when the temperature variation is modest, as was the case in the present experiments. Nor is Eq. (C.17) a bad approximation for the experiments of Sec. 6.

With these two assumptions, k reduces to

$$k = \frac{1}{Sc_\delta} \cdot \frac{1}{g} \quad (C.18)$$

and equations (C.11) and (C.12) assume the forms

$$Z = \frac{Z'(o)}{g(o)} \int_0^\eta g \exp \left[- Sc_\delta \int_0^{\eta'} g f d\eta'' \right] d\eta' \quad (C.19)$$

$$Z'(o) = \frac{g(o)}{\int_0^\infty g \exp \left[- Sc_\delta \int_0^{\eta'} g f d\eta'' \right] d\eta'} \quad (C.20)$$

Dr. W.L. Bade of the Research and Advanced Development Division of the Avco Corporation supplied the author with numerical values of f and g , in intervals of $\Delta\eta = 0.1$ from $\eta = 0$ to $\eta = 9$, for $g(o) = 0.8$ (a good approximation for the experiments, where $g(o)$ was 0.85) and a viscosity coefficient varying $T^{3/4}$ (a good approximation for argon). Using these data, Eqs. (C.19) and (C.20) were evaluated for series of values of Sc_δ between 0.1 and 3. The calcula-

tions were performed by D.P. Jackson of UTIAS on an IBM 7090 computer, using Simpson's rule. The results for $Z'(0)$ are shown in Fig. C.2. Figure C.3 shows profiles of Z , plotted against the dimensionless distance (c.f. Eq. (C.3))

$$S = \int_0^\eta g \, d\eta = \sqrt{\frac{2 \beta \rho_\delta}{\mu_\delta}} \cdot y \quad (C.21)$$

rather than the variable η itself, which is nonlinearly related to the physical distance y . Here we see clearly how the thickness of the electron-ion diffusion layer varies with the ambipolar Schmidt number, Sc_δ . When Sc_δ is about unity, this layer has approximately the same thickness as the overall boundary layer. When $Sc_\delta \ll 1$, on the other hand - as is the case when $T_e \gg T$ - the diffusion layer penetrates considerably further than the ordinary boundary layer. The maximum value of Sc_i (see Appendix B) is below 3 for Ar^+ ions in Ar, at least for gas temperatures below 1200°K, and hence Sc would have a value of only 1.5 even if the electron and ion temperatures were equal.

In order to complete the evaluation of Eq. (C.14), we also require a value for β . It is through this quantity that the nose shape of the body influences the solution (see Ref. 65). Following Brundin (Ref. 44), we use Shugayev's (Ref. 66) expression for β for a flat-nosed cylinder:

$$\beta = \frac{v_\infty}{y_s} \cdot \frac{1}{4} \left[\frac{1}{A} + \frac{2}{\sqrt{A(\gamma+1)}} \frac{a}{1-a} \right] \quad (C.22)$$

where

$$a = \frac{\gamma-1}{\gamma+1} \left(1 + \frac{2}{\gamma-1} \cdot \frac{1}{M_\infty^2} \right)$$

$$A = \frac{\left[\frac{(\gamma+1)^2}{4\gamma} \right]^{\frac{1}{\gamma-1}}}{a \left(1 - \frac{\gamma-1}{2} \frac{1}{M_\infty^2} \right)^{\frac{1}{\gamma-1}}}$$

Here, y_s denotes the shock detachment distance at the stagnation point, M the Mach number, γ the specific heat ratio, and v_∞ the velocity, directed toward the stagnation point, of the undisturbed flow upstream of the bow shock.

Remaining consistent to the high Reynolds number limit implicit in the boundary layer theory, we use Serbin's (Ref. 67) empirical expression for the shock detachment distance

$$\frac{y_s}{D} = \frac{0.515}{\left(\frac{\rho_\delta}{\rho_\infty} - 1 \right)^{\frac{1}{2}}} \quad (C.23)$$

rather than our experimentally measured values. For $M_\infty = 2.2$, a combination of Eqs. (C.22) and (C.23) gives, for $\gamma = 5/3$,

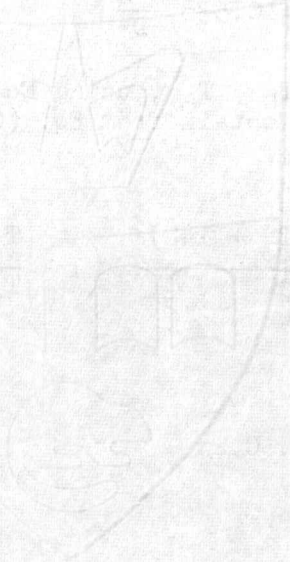
$$\beta = 0.536 \frac{v_\infty}{D} \quad (C.24)$$

Furthermore, assuming that $T_\delta = T_o$, using Eq. (5.3) for v_∞ , Eq. (A.8) for μ , Eq. (A.7) for D_i , we obtain from Eq. (C.14), for $M = 2.2$,

$$\frac{j_w}{n} = \frac{5.50 \times 10^{-22}}{D^{\frac{1}{2}}} \cdot \frac{(T_o + 142)^{\frac{1}{2}}}{T_w^{\frac{1}{2}} + 11.5} \left(1 + \frac{T_e}{T_w}\right) \frac{T_o}{[P_w(\text{torr})]^{\frac{1}{2}}} Z'(o) \quad (C.25)$$

ampere-metre

where $Z'(o)$ is given in Fig. C.2.



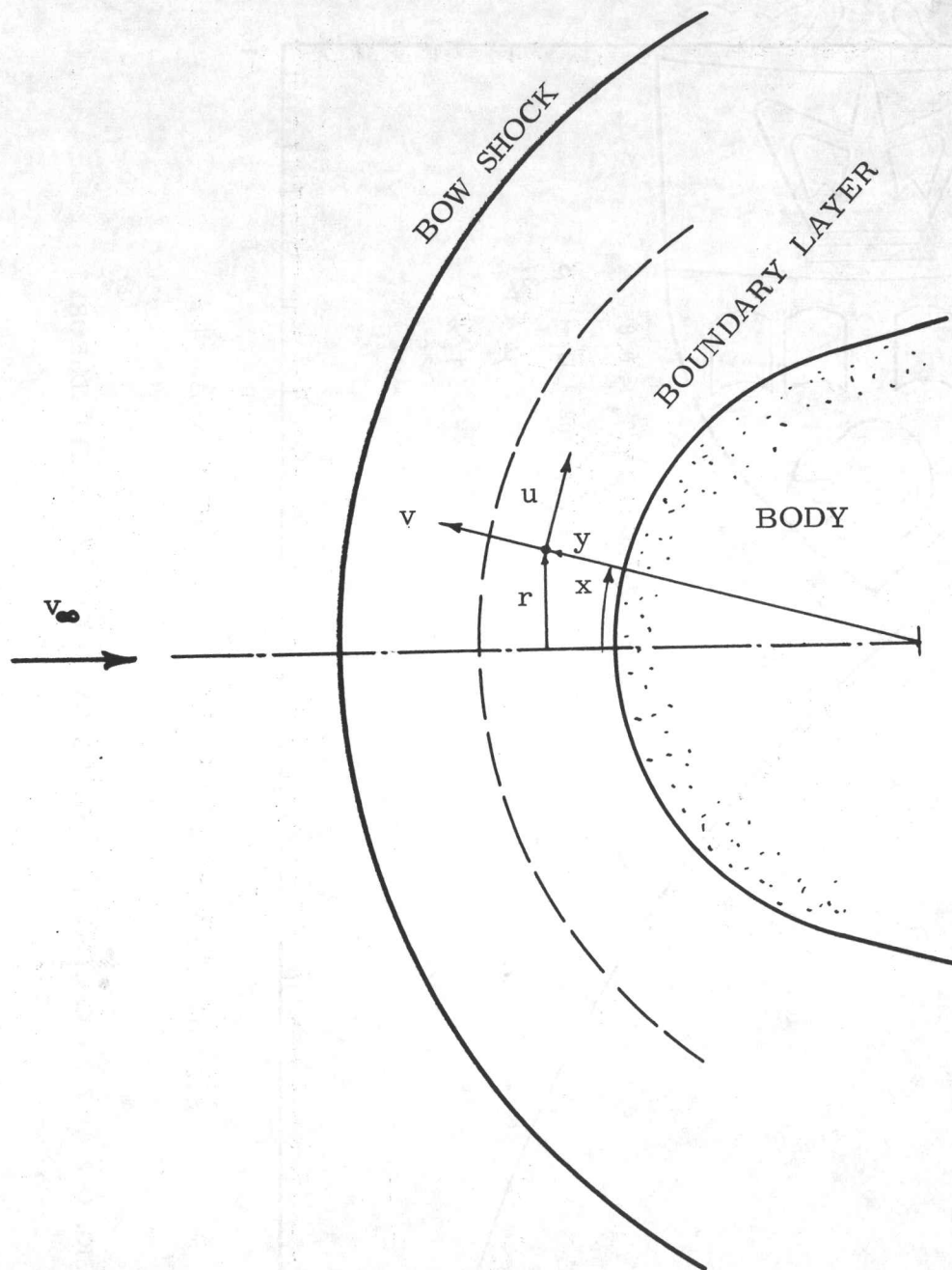


FIG. C. 1 COORDINATE SYSTEM IN BOUNDARY LAYER THEORY

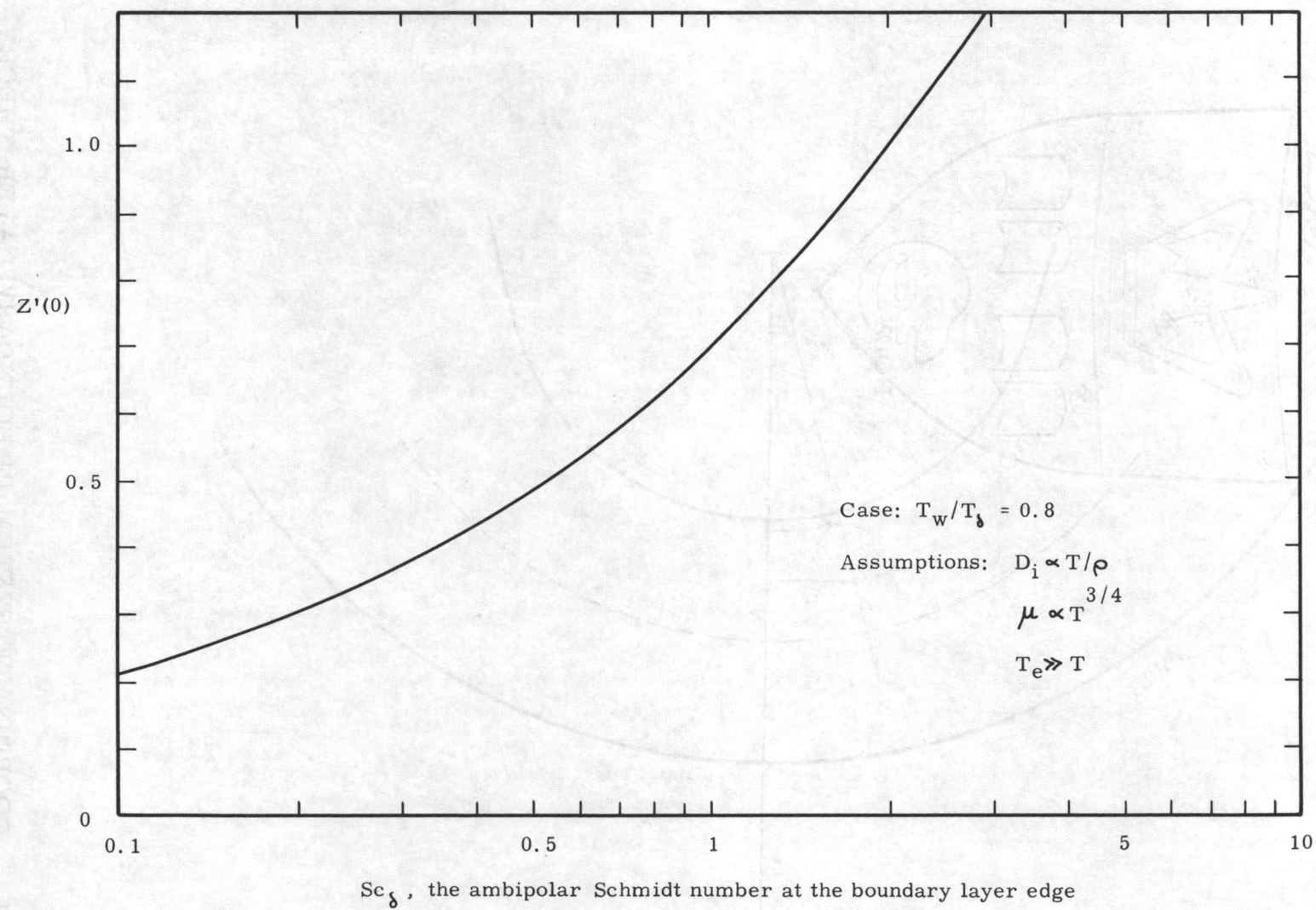


FIG. C.2 $Z'(0)$ V.S. Sc_δ , AS GIVEN BY BOUNDARY LAYER THEORY

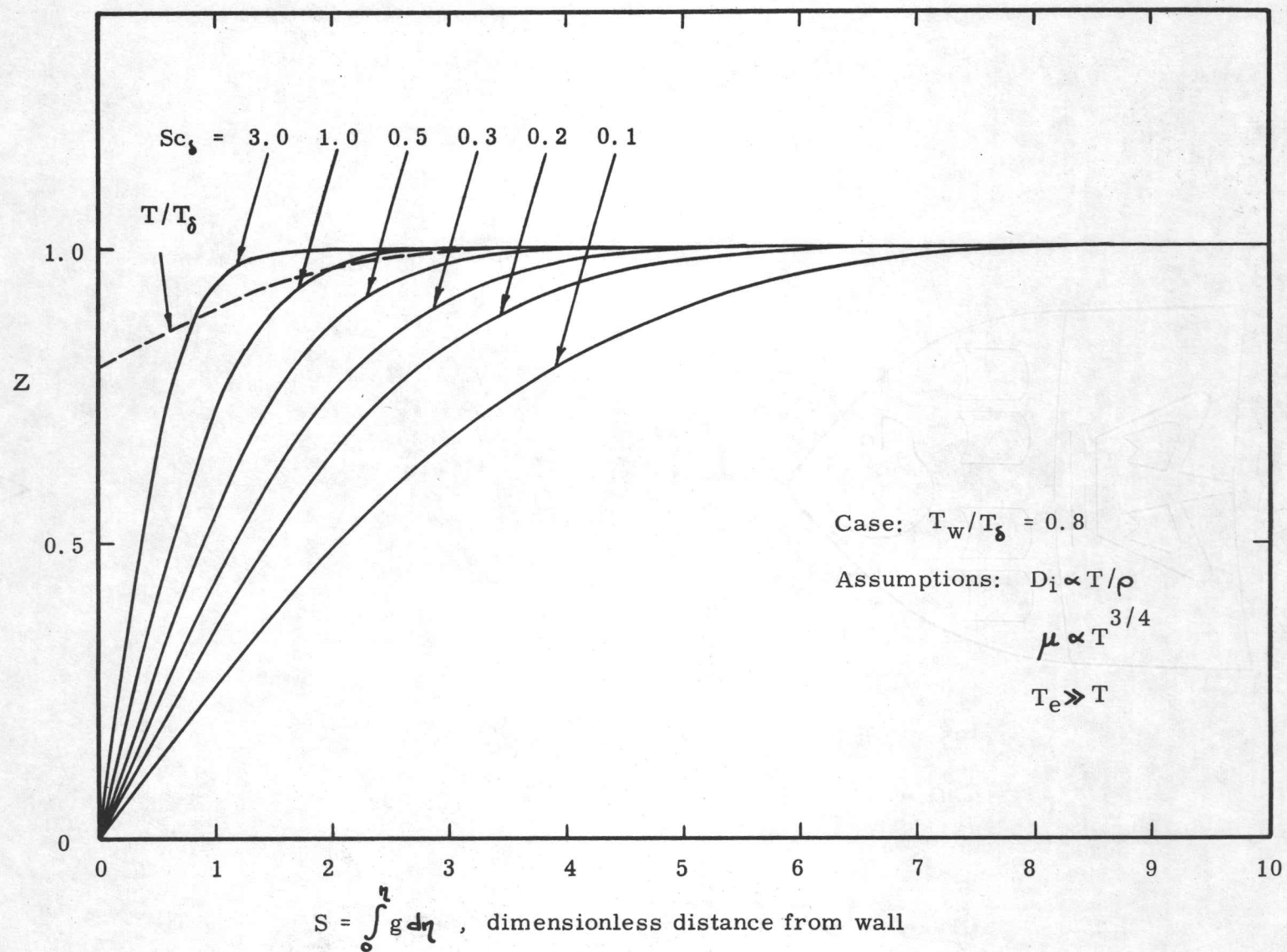


FIG. C.3 DISTRIBUTION OF ELECTRON-ION CONCENTRATION IN STAGNATION POINT BOUNDARY LAYER, ACCORDING TO BOUNDARY LAYER THEORY