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Abstract: Railway ballasted track stiffness is an important indicator to identify supporting condition that ensures the facility is well designed and functioned. Although many studies have been performed on track stiffness based on experimental tests and finite element methods, the factors influencing the track stiffness have not been completely confirmed yet, especially the influences from ballast and subgrade layers at a mesoscopic level. To address this research gap, a coupled the discrete element method (DEM) and the finite difference method (FDM) model is utilised to study the factors influencing on the track stiffness from the particle level. Factors (related to ballast layer properties) are bulk density, thickness and stiffness, and other factor (related to subgrade properties) is elastic modulus. Additionally, the relationship between the track stiffness and the mechanical behaviour of ballast is analysed. This study quantified the influences of track components on the track stiffness and accordingly proposed how to improve it from the ballast and subgrade layers at the mesoscopic level, which can provide the guidance for railway ballasted track design and maintenance.

Keywords: Discrete element method, Finite difference method, Hybrid simulation, Track

stiffness, Railway ballasted track

INTRODUCTION

Railway ballasted tracks are widely used all over the world, and the main advantages of ballasted tracks (compared to slab track) are low construction cost and easy maintenance work. The performance of the ballasted track in terms of loading strongly depends on the track stiffness, which is expressed by the ratio of the static load to the corresponding track deflection. Until now, plenty of studies have demonstrated that the track stiffness has significant influences on the vehicle ride quality (Lundqvist and Dahlberg 2005; Xu et al. 2020), the track dynamic behaviour (Frohling et al. 1996; Li and Berggren 2010) and track long-term degradation (Milosavljević et al. 2012; Grossoni et al. 2016). More importantly, some studies pointed out that track stiffness is a key indicator for the demand of maintenance work (Sussman et al. 2001; Pita et al. 2004). Therefore, understanding track stiffness more deeply can provide clearer guidance for assessing and improving track performance.

To understand the track stiffness, many studies have been performed to confirm how various track components influence track modulus and stiffness. Some researchers concluded that improving the track substructure materials (ballast, subballast and subgrade layers) can enhance the track stiffness performance (Selig and Li 1994; Khordehbinan 2010; Mosayebi et al. 2016; Sussman and Selig 1999). Their theoretical models assumed the ballast layer with springs and dampers but ignored the discontinuity, inhomogeneity and the randomness of ballast assembly. Particularly, the effects of ballast layer characteristics (e.g. rearrangement) on the track stiffness cannot be revealed by the model from ballast particle level (Qian et al. 2018).

Some other researchers performed experimental tests to study the track stiffness, and obtained the macromechanical load-deflection characteristic of the whole track structure (Oscarsson and Dahlberg 1998; Priest and Powrie 2009). However, the meso-mechancical characteristic of ballast layer under the static load hardly can be investigated from experimental tests. In addition, the track stiffnesses that are measured in the field are of great randomness (due to the existence of uncertain factors), and experimental tests are not feasible to perform parametric study (due to difficulties in variable control). Thus, the relationship between the track stiffness and the meso-mechanical behaviour of ballast is rarely analysed, and the factors influencing the track stiffness have not been completely investigated yet.

To address the limitation of earlier studies, the hybrid discrete-continuum approach is applied in this study for the meso-analysis of track stiffness. The DEM is an effective and reliable approach to present the granular material properties of ballast assembly, e.g. density, degradation, particle size and particle shape (Guo et al. 2020a), and has been successfully applied in many ballast-related studies, such as, under sleeper pads (Li and McDowell 2018), ballast particle acceleration (Liu et al. 2019) and friction sleeper (Guo et al. 2020b). The hybrid discrete-continuum approach has been proved to be an effective solution for the ballasted track studies involving the subgrade (Shao et al. 2017; Ngo et al. 2017; Li et al. 2019; Shi et al. 2020a).

In this study, the DEM is utilised to build the ballast layer, sleeper and rail to study the track stiffness. Ballast particles are modelled with irregular geometry shapes, and the compacted ballast assembly under different sleepers had non-uniformly distributions (for different supporting conditions). To analyse the influence of subgrade on the improving of track stiffness, the subgrade layer is also considered. Considering the impossibility of numerical calculation of the subgrade with huge amounts of soil particles in DEM, the subgrade is simulated with the FDM by considering it as a continuous medium. The coupled DEM-FDM model of railway ballasted track and subgrade is realized by exchanging the force and displacement data. Subsequently, the coupled model is verified by comparing the numerical results of track stiffness to those in references, and then the verified model is used to study the factors influencing track stiffness, as well as the relationship between track stiffness and ballast behaviour.

MODEL DESCRIPTION AND VERIFICATION

Model description

Figure 1 shows the two-dimensional coupled DEM-FDM model of ballasted track and subgrade. The coupled model has 13 sleepers with the length at 8.4 m, and the ballast layer thickness (under the sleeper) is 0.35 m. Each longitudinal spacing between two adjacent concrete sleepers is 0.6 m. Besides, the height of the sleeper is 0.19 m that is the size of the Chinese Type III mono-block sleeper, and this type of sleeper can be found in (Guo et al. 2020b). The subgrade consists of three parts as shown in Figure 1: the surface layer of subgrade (0.6 m), the bottom layer of subgrade (1.9 m) and the subgrade body (3.1 m). The FDM model of the subgrade is built according to the Chinese standard for the heavy haul railway (National Railway Administration of P.R. China 2017). In the coupled model, the x-axis represents the longitudinal direction of the ballasted track, and the y-axis represents the vertical direction of the ballasted track. For the subgrade boundary conditions, in the plane of the model, at y = -5.6 m, the displacement of bottom boundary nodal was fixed (u_x, u_y = 0); in the planes at x = 0 and x = 8.4 m, the displacement was constrained u_x (u_x = 0).

As shown in Figure 1, the ballasted track (rail, sleepers and ballast layer) is built with the DEM software, Particle Flow Code (PFC), in which the ballast particles can be built in irregular shapes. More than 100 different shapes of ballast particles are applied in the ballasted track model, and the modeling of irregular shapes ballast can be found in the reference (Zhang et al. 2016). The ballast layer is built by compacting a certain number of particles with irregular shapes to an adequately-compacted state. The particle size distribution of the ballast layer is the same as that in the reference (Shi et al. 2020b), as shown in Table 1. The sleepers are built by combining 548 discs as a Clump (rigid block), and the rail is built by bonding discs together as a beam with linear parallel bonds. The linear parallel bonds present a physical performance similar to the cement, which can glue together the two contacting discs (Guo et al. 2020a).

The subgrade is built by plane-stress solid elements in the FDM software, Fast Lagrangian Analysis of Continua (FLAC), and the linear-elastic constitutive model is used to simulate the subgrade. Table 2 and Table 3 summarize the main parameters used in the ballasted track and subgrade models, respectively. Finally, a series of interface elements (walls) are created between the FLAC and PFC to implement the coupling process of force and displacement exchanges. These interface walls correspond to the nodal of the FDM subgrade surface and the wall positions update at the beginning of each calculation cycle.

Specifically, the hybrid simulation is achieved by the exchange of contact forces and velocities between the two kinds of software. Since both the PFC and FLAC are developed by the Itasca company, they have a parallel configuration (I/O socket) that can transfer data between each other. The data exchange between the two software packages is managed by the I/O socket using the FISH function (computer language developed by Itasca). The boundary nodal velocities in the FLAC (server) are outputted along with the updated coordinates, and then these data are inputted into the PFC (client) through the I/O socket connections. The coordinates and velocities are used to update the boundary wall coordinates, afterwards, the contact forces of wall-particle at the boundary wall are calculated using the force-displacement law. Eventually, the contact forces are converted to the nodal forces and applied to the boundary nodal in the FLAC. More detailed descriptions about the discrete-continuum ballasted track and subgrade model can be found in (Shi et al. 2020b).

Support stiffness to sleeper

In general, track stiffness is measured by the rail deflection under a static load, by which global track stiffness can be measured. The global track stiffness can be further classified as two parts: 1) above the sleepers (principally from the rail and rail pad) and 2) under the sleepers (from the ballast and subgrade). Due to the rail and rail pad stiffnesses are easy to control, and the support conditions of the sleepers have not been adequately studied. Therefore, the sleeper support stiffness (the relationship between load and deflection of the sleeper) from the perspective of the ballast and subgrade is focused in this study.

The secant stiffness (defined in Equation 1) is applied to calculate the sleeper support stiffness, as this method can minimise the influences of poor ballast-sleeper contacts (Ebersöhn and Selig 1994). It is calculated based on the load-deflection test for a chosen load range (From F_a to F_b). It is a common phenomenon that small gaps exist between sleeper and ballast, in other words, the sleeper in most cases partially or completely lost contacts with ballast, causing the hanging sleeper (Olsson and Zackrisson 2002; Augustin et al. 2003).

$$k = \frac{F_b - F_a}{z_b - z_a} \tag{1}$$

where Z_b is the final sleeper elevation; Z_a is the initial sleeper elevation.

The range of loading for analysis is dependent on transportation and vehicle types (e.g. heavy haul or high-speed railways). Because the stiffness of track components is non-linear (especially the ballast), and the different static load ranges applied to the sleeper lead to different stiffness results. In this study, the load range of heavy haul railway (freight vehicle) with an axis load of 22 t is used, According to the field tests performed by Zhang et al. (2018), the maximum rail pad forces induced by the locomotive with the axle-load of 22 t is between 58.2~79.7 kN. Thus, the secant stiffness is calculated to be in the range of 10 - 80 kN to eliminate the effect of hanging sleeper. Note that, the load value of 40 kN is used to apply on the sleeper in this half-track numerical model, which is equivalent to the effect of applying a force of 80 kN to a three-dimensional track. The preloading is carried out by applying a static force of 40 kN at the sleeper before the measurement to eliminate the voids between the sleeper and ballast.

Figure 2 shows the schematic diagram of the sleeper positions to where the loads (*F*) were applied. As shown in Figure 2, $F(t)$ are simultaneously applied on Numbers 1, 4, 7, 10 and 13 unfastened sleepers (i.e. fasteners were removed), the corresponding sleepers' displacements are recorded at the same time. The loads F are applied by the increment rate of 2 N/s until 40 kN, and the load F is obtained:

$$F(t) = 2000 + 2 \times t \tag{2}$$

Afterwards, using the same initial model, the Number 2, 5, 8 and 11 sleepers are performed the same process, as well as on the Number 3, 6, 9 and 12 sleepers. Finally, the sleeper support stiffnesses of all 13 sleepers are obtained.

Sleeper support stiffness verification

As described above, the DEM and FDM are coupled by data exchange at the interface walls, and the walls update according to the nodal of subgrade surface. Figure 3 shows typical displacement-force curves of the sleeper, interface walls and the corresponding node of the subgrade surface. From Figure 3, the displacements of the interface walls in PFC and corresponding FLAC nodes show a high correlation, which implies the data are reliably transmitted between the DEM model and the FDM model.

From Figure 3, it can be seen that the relationship between applied force and sleeper displacement is not linear, which is consistent with the experimental tests performed by others (Frohling et al. 1996; Oscarsson and Dahlberg 1998; Sussman and Ebersöhn 2001). The initial stiffness (From 0 kN to 10 kN) is affected by the insufficient contacts between the sleeper and ballast, which is also known as the seating stiffness. To further validate the coupled model in calculating sleeper support stiffness, the calculated values of sleeper support stiffness and ballast layer stiffness are compared with the previous measurement results, as shown in Table 4. The ballast layer stiffness is defined as a vertical load divided by the ballast layer deflection (the sleeper displacement subtracts subgrade surface displacement). The comparison shows that the simulation results are in consonance with the measurements. Summarily, the coupled model for the sleeper support stiffness analysis is validated.

EFFECT OF TRACK COMPONENT PARAMETERS ON SLEEPER SUPPORT STIFFNESS

In this section, a parametric study with variable track component parameters is carried out to confirm how much the factors influence on sleeper support stiffness. The parameters of track components include the density, thickness and stiffness of the ballast layer and the elastic modulus of different subgrade layers.

Effect of bulk density on sleeper support stiffness

Figure 4 shows the sleeper support stiffness and the bulk density of the ballast layer under each sleeper, and the bulk density is measured at different areas (Area 1, 2 and 3). The "Area 1" and "Area 2" mean the rectangles below each sleeper with a width of 0.15 m and 0.3 m, respectively. The "Area 3" means an isosceles trapezoid with the sleeper bottom as its upper base and two bottom angles at 45 degrees. In the following analysis, if no further description is made, the bulk density value and other index values are measured from "Area 2". From Figure 4, the sleeper support stiffness is found to scatter between 50 MN/m and 63 MN/m. The bulk densities under different sleepers are in the range of 1890 kg/m^3 to 1950 kg/m^3, which is consistent with the field measurement results that the bulk density of fully-compacted ballast layer is about 1900 kg/m^3 (Tutumluer et al. 2013). From Figure 4, it can be seen that the sleeper support stiffnesses significantly varies from one sleeper to its adjacent sleepers, and the bulk densities under different sleepers are considerably different. The conclusion can be drawn that the relationship between the sleeper support stiffness and the bulk density under this sleeper is not obvious.

To further explore the influence of bulk density on the sleeper support stiffness, the ballast layer with different compact states is analysed. Compaction states of "Tamp 1" to "Tamp 4" means the compaction time, which is that more load cycles were applied on the ballasted track. The bulk density of the ballast layer increases with the compaction time, as shown in Figure 5(a). From Figure 5(b), the sleeper support stiffness also increases with the increase of the compaction time. Summarily, improving the bulk density can increase the sleeper support stiffness to a certain degree, which is also helpful to improve the carrying capacity of ballasted tracks.

Furthermore, Figure 6 shows the relationship between the sleeper support stiffness and the bulk densities of different compaction states, where each point represents the average value of all 13 sleepers under different compaction states. From Figure 6, there is a good linear relationship between the increment of bulk density and the increment of sleeper support stiffness. Thus, the bulk density has significant influences on the sleeper support stiffness.

Effect of ballast layer thickness on sleeper support stiffness

The ballast layer supports the imposed wheel load and transmits the forces from the rail and sleeper to the subgrade at an acceptable level. The design approaches of ballast layers from different countries that are used to decide the thickness of the ballast layer were discussed and compared in the reference (Burrow et al. 2007). In this study, ballast layers with a thickness of 0.4 m, 0.5 m and 0.6 m are chosen to analyse how the thickness of ballast layers influences the sleeper support stiffness.

Figure 7(a) shows the initial bulk density of these ballast layers with different thicknesses. The bulk densities of the ballast layers with the thicknesses of 0.4 m, 0.5 m and 0.6 m are about 1922 kg/m^3, 1934 kg/m^3 and 1930 kg/m^3, respectively, which means their bulk densities were approximately the same. As shown in Figure 7(b), increasing the thickness of the ballast layer is also beneficial to improving the sleeper support stiffness, which is consistent with the studies performed in the references (Gallego et al. 2011; Kim et al. 2019).

The mean values and the standard deviations of the sleeper support stiffnesses under different ballast layer thicknesses are presented in Table 5. As the thickness of ballast bed increases from 0.4 m to 0.6 m, the sleeper support stiffness increases marginally by 17%, while the standard deviation does not show a clear increasing trend. Besides, the conclusion can be drawn that the effect of bulk density on the sleeper support stiffness is greater than the thickness of the ballast layer

Effect of ballast layer stiffness on sleeper support stiffness

According to the references (Ngo et al. 2016; Chen and McDowell 2016; Indraratna et al. 2016;

Zhang et al. 2016), when applying the DEM to simulate the ballast particles, the contact stiffness between ballast particles varies from 1×10^8 N/m to 5×10^8 N/m. Hence, three different contact stiffnesses (1×10^8, 3×10^8 and, 5×10^8 N/m) are chosen for comparison to confirm the influences of contact stiffness on the sleeper support stiffness.

At the beginning of the numerical simulation, three modeled ballast layers, each of which is made by ballast particles with one of the three contact stiffnesses (1×10^8, 3×10^8 and 5×10^8 N/m), are stabilized, by performing cyclic loadings until the models reach a certain condition that the ratio of average unbalanced force to average contact force reached 0.01. Subsequently, the numerical simulations are carried out on how different contact stiffnesses influence on the sleeper support stiffness.

Figure 8 shows that the sleeper support stiffness increases with the increase of the contact stiffness, and the mean values of the sleeper support stiffness are 35.07 MN/m, 56.88 MN/m and 65.98 MN/m, respectively. Besides, Figure 8 shows the deviation of the sleeper support stiffness reduces as the decrease of the contact stiffness, and the standard deviations are 2.49 MN/m, 3.69 MN/m and 4.3 MN/m, respectively. Therefore, the conclusion can be made that the increase of contact stiffness makes the sleeper support stiffness and the deviation of sleeper support stiffnesses increasing.

Effect of subgrade elastic modulus on sleeper support stiffness

To confirm the effect of different subgrade layer elastic modulus on the track performance, a practical range of elastic modulus values for each subgrade layer is chosen. Table 6 presents the elastic modulus of the variable subgrade used for parametric study.

Figure 9 shows the effects of subgrade elastic modulus on the sleeper support stiffness. From

Figure 9(a), it can be seen that the elastic modulus of the subgrade surface has insignificant influences on the sleeper support stiffness. In this regard, the statistical analysis of the sleeper support stiffness under different elastic modulus of the subgrade surface was carried out. The mean values of the sleeper support stiffness are 55.66 MN/m, 56.88 MN/m and 56.92 MN/m, respectively. As shown in Figure 9(b) and Figure 9(c), the sleeper support stiffnesses increase with the increase of the elastic modulus of different subgrade layers. In general, the increase of subgrade stiffness causes the sleeper support stiffness increasing. Furthermore, it can be seen from Figure 9 that the part of subgrade influencing sleeper support stiffness most is the elastic modulus of subgrade body.

RELATIONSHIP BETWEEN SLEEPER SUPPORT STIFFNESS AND BALLAST BEHAVIOUR

The relationship between the sleeper support stiffness and the meso-mechanical behaviour of ballast under vertical loading is presented below.

Ballast particle behaviour

Figure 10 shows that the sleeper support stiffness under the conditions that some degrees of freedom of the ballasts were constrained. The “Fix spin” means the rotation of ballast particles is constrained, and “Fix x-component displacement” means the movement of ballast particles in the x-direction is restricted. As shown in Figure 10, the “Fix spin” has a greater influence on the sleeper support stiffness than the “Fix x-component displacement”. Furthermore, the value of sleeper support stiffness in the condition of fixing both ballast spin and x-component displacement is almost the same as the condition of fixing ballast particles spin, which indicates that the x-component displacement of ballast particles is mainly caused by the rotation of the ballast particles.

To further prove the influences of ballast particles rotation on x-component displacement, the average rotation angle and x-component displacement of ballast particles are presented. In addition, the average azimuthal angle before and after loading is also presented, which indicates the rotation direction of ballasts. The azimuthal angle is the angle between the long axis of ballast particle and the vertical axis, and the long axis of ballast particle is the longest dimension of one ballast particle among three dimensions (length, width and height), explained in Guo et al. 2019.

Figure 11(a) shows the average rotation angle and x-component displacement of ballast particles. From Figure 11 (a), it can be observed that the change of ballast x-component displacement is about 0.04 mm and the ballast particle rotation is about 0.9° after the force applied on the sleeper. The average azimuthal angles of ballast particles before and after loading are shown in Figure 11(b). The results show that the directions of ballast rotation after applied forces increase the average azimuthal angles, which indicates that the increase of ballast average azimuthal angles will allow the ballast layer to withstand greater loads.

Contact forces and stress

Figure 12 shows the relationship between sleeper support stiffness and the average contact force in the ballast layer. It indicates that there is a good negative correlation between sleeper support stiffness and ballast contact forces. The main reason is that the overlaps between ballast particles increase as the contact forces increase, due to the contacts applied in the DEM models between ballast particles are the linear contact with spring and dashpot. Therefore, a larger sleeper deformation is formed by accumulating the overlap between ballast, and then bringing up small sleeper support stiffness. As well knows, the greater the contact force between the ballast, the more likely the ballast is to wear and break. Consequently, the results can be drawn that the ballast in the areas with larger sleeper support stiffness is more prone to deteriorate.

To further investigate the mesoscopic contact force chain of ballast particles, Figure 13 shows the distribution of the contact force chains in the DEM ballasted track and the vertical stress contour σ_{yy} in the FDM subgrade. Each contact force is represented at the contact points by a red line oriented in the direction of the force and with the thickness proportional to its intensity. As shown in Figure 13, the force chain structure in the ballast layer and the stress concentration phenomenon in the surface layer of the subgrade are obvious at the force-applied sleepers. For example, the red force chains are wider under sleeper Number 1, 4, 7…, to which the forces are applied. The force chains transmitting in the ballast layer approximately coincides with the cone distribution, which is consistent with the assumption that the force is pyramid distribution in the ballast layer (Zhai et al. 2004). Besides, the force chains (the contacts between ballast particles and the sleeper-ballast contact) in the ballast layer are obviously different under the different sleepers, which can be the reason of sleeper support stiffnesses significantly vary from one sleeper to its adjacent sleepers.

CONCLUSIONS

In this paper, the hybrid discrete-continuum approach is applied for the macroscopic and mesoscopic analysis of sleeper support stiffness. After validating the coupled model, the factors influencing the sleeper support stiffness are analysed, including the bulk density and thickness of the ballast layer, the contact stiffness of ballast particles and the elastic modulus of subgrade. Finally, the influences of ballast restriction on sleeper support stiffness and the mesoscopic analysis of the contact force chains in the ballast layer are presented. The following conclusions can be drawn for this study:

(1) There is a good linear relationship between the increment of ballast density and the increment of sleeper support stiffness, and the best remedy technical of increasing the sleeper support stiffness is increasing the density of the ballast layer.

(2) With the thickness of ballast bed increases from 0.4 m to 0.6 m, the sleeper support stiffness increases from 57.43 MN/m to 67.21 MN/m, in general, the increase of ballast layer thickness causes the sleeper support stiffness increasing slightly.

(3) The sleeper support stiffness and the deviation of sleeper support stiffnesses increase with an increase of the contact stiffness, and the elastic modulus of subgrade body influence on the sleeper support stiffness most among subgrade layers.

(4) Under the vertical force applied on the sleeper, the x-component displacement of ballast particles mainly caused by the rotation of the ballast particles.

(5) The sleeper support stiffness is considerably related to the contact forces between ballast particles, and the ballast in the areas with larger sleeper support stiffness is more prone to deteriorate.

DATA AVAILABILITY STATEMENT

Some or all data, models, or code that support the findings of this study are available from the corresponding author upon reasonable request (All data).

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Table 1 Particle size distribution of the ballast layer

Mesh size (mm)	22.5	31.5	40	50	63
Percentage passing by mass in Chinese design standards (%)	0～3	1～25	30～65	70～99	100
Percentage passing by mass of the ballasted track model (%)	0	13	45	88	100

Table 2 Parameters in the DEM model of ballasted track

Parameters	Value	Unit
Disk thickness	1.3	m
Rail particle density	490	kg/m^3
Rail particle radius	75	mm
Fastener particle density	3184	kg/m^3
Fastener particle radius	20	mm
Sleeper particle density	3129	kg/m^3
Sleeper particle radius	5	mm
Ballast particle density	2600	kg/m^3
Ballast particle radius	4	mm
Rail particle parallel bond radius	37.27	mm
Rail particle normal parallel bond contact stiffness	1.427×10^{12}	N/m^3
Rail particle shear parallel bond contact stiffness	5.5297×10^{11}	N/m^3
Rail particle normal/shear parallel bond strength	1×10^{10}	N
Rail particle normal/shear contact stiffness	2.765×10^{11}	N/m
Fastener particle normal/shear bond stiffness	1×10^{10}	N/m
Fastener particle normal/shear contact stiffness	1.2×10^{8}	N/m
Ballast/Sleeper particle and vertical wall stiffness	3×10^{8}	N/m
Ballast particle friction coefficient	0.7	-

Table 3 Parameters in the FDM model of subgrade

Components	Poisson's ratio	Young modulus (MPa)	Density (kg/m^3)	Thickness (m)
Surface layer of subgrade	0.25	180	1950	0.6
Bottom layer of subgrade	0.25	110	1900	1.9
Subgrade body	0.3	80	1800	3.1

Table 4 Comparison of the simulation results and measured results

Parameters	Numerical simulation results (MN/m)	Measurement results (MN/m)	References
Sleeper support stiffness	50-63	25-85	Brough, et al. 2006
		46.48-51.29	Cano et al. 2016
Balast layer stiffness	105-163	71.98 -193.52	Ma, et al. 2016

Table 5 Mean values and standard deviations of sleeper support stiffness

Ballast layer thickness (m)	Mean values (MN/m)	Standard deviations (MN/m)
0.4	57.43	3.45
0.5	60.93	4.57
0.6	67.21	4.04

Table 6 Variable subgrade elastic modulus used for parametric study

Parameters	Nominal value	Values used to keep all other parameters at nominal value
	Modulus of elasticity (MPa)	
Surface layer of subgrade	180	150(soft),210(stiff)
Bottom layer of subgrade	110	80(soft),140(stiff)
Subgrade body	80	50(soft),110(stiff)

Figure 1 Coupled DEM-FDM model of ballasted track and subgrade

Figure 2 Schematic diagram of force exertion

Figure 3 Applied force versus measured displacements of interval walls, nodals and sleeper

Figure 4 Sleeper support stiffness and bulk density of the ballast layer

Figure 5 Sleeper support stiffness and ballast layer density under different compaction states: (a) The density of ballast layer; (b) Sleeper support stiffness

Figure 6 Relationship between sleeper support stiffness and bulk density

Figure 7 Bulk densities and sleeper support stiffnesses of ballast layers under different ballast layer thicknesses: (a)bulk density; (b)sleeper support stiffness

Figure 8 Sleeper support stiffnesses of different ballast particle stiffness

Figure 9 Effects of subgrade elastic modulus on sleeper support stiffness: (a) surface layers of subgrade; (b) bottom layers of subgrade; (c) subgrade body

Figure 10 Sleeper support stiffness of constrained ballast particles

Figure 11 Behaviour of ballast particles before and after loading: (a) x-component displacement and rotation; (b)azimuthal angle

Figure 12 Sleeper support stiffness and ballast contact force

Figure 13 Discrete-finite coupled model under the sleeper loads